APPENDIX B LITERATURE REVIEWS

Table B.1 - Review of Literature on Steady State Strength Concepts

Year	Author(s)	Description of Study	Summary and Conclusions
1936	Casagrande	• Discussion of design methods and stability of earth slopes and fills Discussed definition of stability, shear resistance of cohesionless soils, and volume change during deformation Defined the term critical void ratio	• Stability is not a soil property since it depends on load environment Critical void ratio is void ratio at which "a cohesionless soil can undergo any amount of deformation or actual flow without volume change." Critical void ratio is defined as the void ratio at which continuous deformation at constant shearing stress produces no further volume change Soils above the critical void ratio reduce volume under continuous deformation, and can be unstable if saturated Soils below the critical void ratio are stable and are not susceptible to flow failures For coarse, well-graded sands and gravels, the critical void ratio corresponds to a loose state For medium or fine sands, the critical void ratio occurs in between the loosest and densest state The more uniformly graded a soil, the higher the critical void ratio
1969	Castro	• "Liquefaction of sands." Ph.D. Thesis	• Undrained steady state strength (S_{us}) is a function of void ratio only, and independent of initial stress state, type of loading (cyclic or monotonic), and initial fabric
1975	Castro	• Compared true liquefaction and cyclic mobility	• Liquefaction occurs when shear strength is significantly reduced and soil flows like a liquid until the stresses are reduced to the reduced value of strength Cyclic mobility occurs when soil undergoes excessive deformation due to softening during cyclic loading Undrained steady state strength (S_{us}) is a function of void ratio only, and independent of type of loading (cyclic or monotonic)

Year	Author(s)	Description of Study	Summary and Conclusions
1977	Castro and Poulos	• Described liquefaction and cyclic mobility concepts Discussed factors affecting both types of behavior Reviewed liquefaction failures documented in literature	• Liquefaction: "saturated sand looses a large percentage of its shear resistance (due to monotonic or to cyclic loading) and flows in a manner resembling a liquid until the shear stresses acting on the mass are as low as its reduced shear resistance." Cyclic mobility: "progressive softening of a saturated sand specimen when subjected to cyclic loading at a constant water content Liquefaction can only occur in contractive soils with initial states above the steady state line; can be triggered by either static or cyclic loads Cyclic mobility can occur in soils with any initial state (at least in the laboratory), but requires stress reversal Either type of behavior results in increased pore pressures and large strains Clean, fine, loose uniform sands are most susceptible to liquefaction Large strains during cyclic mobility are due to redistribution of void ratio in laboratory specimens; may not occur *in situ* Void ratio at steady state is identical to Casagrande's critical void ratio
1981	Poulos	• Defined steady state of deformation Added constant velocity requirement to definition of steady state Described differences in steady state and critical state concepts	• Steady State: "The state in which the mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress, and constant velocity." Steady state requires large strains, "well beyond those that normally can be reached in triaxial tests." Differences in steady state (SS) and critical state (CS): SS requires constant velocity during shear deformation SS requires flow structure (particle orientation) to destroy initial fabric; CS ignores fabric and particle orientation

Year	Author(s)	Description of Study	Summary and Conclusions
1982	Castro et al.	• Study of liquefaction susceptibility to cyclic loading Performed triaxial tests using both conventional and lubricated end platens Triaxial tests were load-controlled	• Undrained steady state strength (S_{us}) is a function of void ratio only, and independent of initial stress state, type of loading (cyclic or monotonic), and initial fabric Both grain size distribution and angularity have significant influence on SSL; slight changes in gradation (same shape) cause significant changes in position but not slope of SSL Liquefaction susceptibility is not independent of consolidation history; must consider state path during consolidation Triaxial test is limited to strains of 20% to 30%, strains at steady state may be much larger Lubricated ends reduced scatter in SSL and average line was slightly higher; results too similar to be conclusive Strain control appears adequate, needs to be verified Initial specimen nonuniformities and their propagation during test are important

Year	Author(s)	Description of Study	Summary and Conclusions
1985	Been, and Jefferies	• Presents state parameter Ψ, which is suggested as a single parameter to describe the behavior of sand Shows that Ψ can be used as a normalizing parameter for empirical data and for constitutive modeling For testing, used lubricated end platens for all: CD strain controlled ICUstress controlled ACU ICUcyclic ACU DSS Sample (75mm x 150mm) preparation was achieved by underwater pluviation or moist compaction: six equal lifts @ 5% w.c. compacted to predetermined height Asserts that sand behavior is dependent upon volume change tendency instead of void ratio or stress state	• Pluviation always resulted in dilative samples, so moist tamping was used to get contractive samples SSL is defined as locus of points in void ratio-stress space at which a soil deforms under constant effective stress, void ratio, and velocity slope of SSL increase with % fines (increasing compressibility) The state at point just before the stress path reaches the M-C failure envelope (the phase transformation point) will determine whether the sample will be contractive or dilative: If $\Psi < 0$ initially, then $\Psi < 0$ at phase transformation If $\Psi >> 0$ initially, then $\Psi > 0$ at phase transformation If Ψ is only slightly greater than 0 initially, then Ψ can be negative at phase transformation At phase transformation, response of sample to undrained loading changes; it either liquefies or becomes very dilatant Ψ can be used to normalize: peak drained friction angle, peak undrained shear strength, $\overline{A}_f$, angle of phase transformation Casagrande's critical void ratio concept implies that the dilation rate is zero, not that ε_V is zero (often mis-interpretated); this corresponds to the state $\Psi = 0.02$

Year	Author(s)	Description of Study	Summary and Conclusions
1985	Poulos, Castro, and France	• Outlined a 5 step procedure for liquefaction susceptibility analysis based on steady state concepts	• 5 step liquefaction evaluation procedure: Based on in situ tests (SPT), identify saturated zones that might be contractive Based on a stability analysis assuming fully mobilized steady state strengths in other zones, determine the driving shear stresses in the potentially liquefiable zone Determine S_{us} for the potentially liquefiable zone based on undisturbed samples and CU triaxial tests, correcting for void ratio changes Compare in situ S_{us} to the driving shear stress Determine if the earthquake can cause enough strain to reduce the strength to the steady state value Factor of safety is ratio of undrained steady state shear strength to driving shear stress S_{us} is: the minimum strength of a saturated contractive soil during undrained shear a function of *in situ* void ratio only; independent of structure or method or rate of loading very sensitive to minor void ratio changes Magnitude of disturbance controls occurrence of liquefaction in susceptible soils Slope of SSL ranges from 0.05 for round grains to 0.3 for very angular grains
1985	Castro, Poulos, and Leathers	• Evaluated the Lower San Fernando Dam slide in context of steady state shear strength	• Analysis based on steady state is consistent with observed failure Steady state occurred in most tests by 20% axial strain In situ strength was estimated as 5% of strength determined in laboratory; attributed increase to disturbance during sampling and reconsolidation

Year	Author(s)	Description of Study	Summary and Conclusions
1985	Poulos, Robinsky, and Keller	• Discussion of liquefaction resistance of tailings deposits	• Two condition required for liquefaction to occur: driving shear stresses are greater than the steady state shear strength earthquake stresses (or strains) are sufficient to trigger the reduction in strength to the steady state strength (i.e. overcome the peak strength)
1987	Been et al.	• Discussion of back-analysis of Nerlerk berm liquefaction slides Nerlerk berm was constructed underwater using undensified hydraulic sand fill Interpreted CPT data to get state parameter, stress state, and density, and compared to laboratory SSL to evaluate liquefaction potential	• Failure was not caused by liquefaction, as original paper suggests Based on the state parameter concept and CPT data, the Nerlerk sands were dilative (for 90% of tests, Ψ = -0.03, whereas $\Psi \geq$ 0.02 for liquefaction to occur) Accuracy of Ψ based on CPT data is $\pm$0.025 Peak ϕ' and dilation rate $(d\varepsilon_V/d\varepsilon_1)$ are unique functions of Ψ, with scatter due to fabric differences: $\phi'_{TRX} = 32(1-5\Psi/3)$ $d\varepsilon_V/d\varepsilon_1 = -3.5\Psi$

Year	Author(s)	Description of Study	Summary and Conclusions
1987	Castro	• Reviewed literature on earthquake induced liquefaction and suggests framework for comparing different behavior Used original definition of liquefaction, which requires a loss in strength, not just excessive pore pressure or strain, or the presence of sand boils Liquefaction: "the phenomena wherein the shear resistance of a mass of soil decreases when subjected to monotonic or cyclic loading at constant volume (undrained loading conditions) so that the mass undergoes very large unidirectional shear strains – it appears to flow – until the shear stresses are as low or lower than the reduced shear resistance." This requires the presence of driving static shear stresses that exceed the reduced (steady state) shear strength. Discussed various methods of deformation analysis, when liquefaction does not occur (stability is not lost), but excessive strain might cause damage	• Suggested framework: zero driving shear stress - contraction or increase in pore pressure resulting in settlement and/or sand boils driving shear stress - liquefaction and loss of stability resulting in flow slides, sinking of heavy structures, and floating of light structures driving shear stress - limited shear distortion, but no loss of stability, resulting in slumping of slopes, building settlement, and lateral spreading For condition with no driving shear stresses, volume change occurs (either instantaneously or as developed pore pressures dissipate). Real concern is the prediction of settlements (almost never uniform, and primarily a function of shear strain), not prediction of sand boils Occurrence of sand boils: is more likely in soils with low SPT blow counts is more likely for stronger shaking is less likely with increasing % fines At steady state, all particle re-orientation and breakage are complete, and any memory of the initial structure is erased Slope of SSL is a function of grain shape, vertical position is very sensitive to gradation SSL is actually a line in 3D space of void ratio, effective normal stress on the failure plane, and shear stress on the failure plane

Year	Author(s)	Description of Study	Summary and Conclusions
1987	Castro (cont.)		• Hazen (1920) was first to use liquefaction to describe the flow failure of Calaveras Dam in CA Liquefaction failures occur in soil at the pre-earthquake density (no drainage occurs) due to the short period of time that failure takes Dense (dilative zones) may drain (loosen), so drained strengths should be used Two condition required for liquefaction to occur: driving shear stresses are greater than the steady state shear strength earthquake stresses (or strains) are sufficient to trigger the reduction in strength to the steady state strength (i.e. overcome the peak strength) Deformation analysis can be performed by: Seed method based on stress analysis and cyclic tests Newmark method based on threshold stress (or acceleration) level and accumulation of deformation analytical procedures based on laboratory constitutive relationships Sand boils involve no driving shear stress and depend upon the shear modulus (controls strain and pore pressure development), permeability, and compressibility Liquefaction (flow slides) and limited deformation (lateral spreading) both require driving shear stresses and are dependent upon the steady state undrained shear strength
1987	Jefferies and Been	• Discussed limitations of critical state for sand behavior	• Stress dilatancy is an implicit part of critical state description of sand behavior Dilatancy causes bearing capacity factors to be underestimated

Year	Author(s)	Description of Study	Summary and Conclusions
1988	Alarcon-Guzman et al.	• Discussion of various observed behaviors during undrained loading of sands Presented concept of transition zone in which observed behavior depends range of strain Defined phase transformation as onset of strain hardening (dilation) after limited softening Performed torsional shear tests on Ottawa20-30 sand	• Steady state during drained shear (critical void ratio line) is not necessarily the same as that during undrained shear (steady state line) Transition zone lies above the steady state line (F line) and below the critical void ratio line (S line) Defined three regions of state diagram: Loose: above strain softening; peak occurs at small strains Transition: strength decreases to a residual value over a limited shear strain range; beyond this range, strain hardening occurs (pore pressure decreases) Dense: below S line and F line; strain hardening; no peak occurs; dilative tendency develops at very small strains Sudden pore pressure increase at onset of strain softening is due to collapse of soil structure Difference in F line and S line is due to structural collapse and particle rearrangement at onset of softening; increases with uniformity and decreases with angularity Phase transformation line is defined by friction angle at which dilation begins for specimens with initial states in the transition zone
1988	Jefferies	• Examined the validity of relationships derived from calibration chamber tests for determining the state parameter Ψ, and consequently density, based on CPT data	• The state parameter concept assumes that some key parameters of sand behavior are a function of the difference in the in situ void ratio and the void ratio at steady state for a particular stress state A unique relationship between Ψ and ϕ' exists If the position of the SSL line is known and the in situ mean stress is estimated, the in situ void ratio may be determined directly from CPT data

Year	Author(s)	Description of Study	Summary and Conclusions
1988	Kramer	• Examined historical liquefaction flow slides with particular attention to the initial equilibrium shear stress level	• Slides often occurred shortly after an adjacent drawdown in water level Very small changes in stress state may initiate liquefaction if the initial shear stress level is high Flow slides will follow liquefaction if driving shear stresses exceed the steady state strength
1988	Kramer and Seed	• Examined the stress conditions required to initiate liquefaction and the parameters that influence these stress conditions	• Static liquefaction resistance (the shear stress increase required to cause liquefaction) increases with relative density and confining pressure, and decreases greatly with increasing initial shear stress Factor of safety defined by Poulos et al. (1985) is only valid if steady state deformation is achieved. This ignores the requirement of a stress state necessary to reach steady state (peak strength must be exceeded) Liquefaction evaluation should include both the potential for initiation of liquefaction as well as the effects of liquefaction if it occurs (shear strength reduction)

Year	Author(s)	Description of Study	Summary and Conclusions
1989	Kramer	• "Uncertainty in steady-state liquefaction evaluation procedures." Discussed uncertainty in the steady state procedure Evaluated the effect of uncertainties on the results of steady state analyses Presented charts of a reduction factor (based on slope of SSL and COV of slope) that S_{us} as determined must be divided by to decrease the probability (to various values) that S_{us} is overestimated For a probability of 1% (R is primarily a function of slope, but increases somewhat with COV): if C_{SS} = 0.1 and COV = 0.2, R = 2 if C_{SS} = 0.02 and COV = 0.2, R = 30 Average reduction factor for 1% probability for 25 sands with published data was 15 (very high)	• Uncertainties include void ratio changes during sampling and the slope of the steady state line (due to scatter in the data from compacted specimens) Significant uncertainty exists in strengths determined by the steady state method Conservative interpretation of the steady state strengths is required to minimize the error associated with uncertainty in the method The slope of the SSL is usually very flat (0.01 to 0.10, COV typically 20%), so that small changes in void ratio result in large errors in effective stress The steady state procedure is extremely sensitive to in situ and laboratory void ratio measurement Uncertainty could be reduced if scatter in steady state data could be reduced COV is independent of number of tests, so more tests will not decrease uncertainty Since effective stress measurement in triaxial tests is very accurate, most scatter is a result of inaccurate void ratio determination (both gross measurement or nonuniformity)

Year	Author(s)	Description of Study	Summary and Conclusions
1991	Castro	• Discussion of "Uncertainty in steady-state liquefaction evaluation procedures" by Kramer (1989) Highlighted problems with Kramer's analysis	• Most of the published data were not from tests performed specifically for determination of the slope of the SSL SSL is not a straight line in e vs. σ_3', but is usually concave downward (esp. at higher stresses) Data from Dobry et al. (1985) were erroneous (Vasquez-Herrera 1988), so should have been discarded Slopes of SSL under 0.04 are "highly unusual"; more typical is 0.08 to 0.15 with COV<10%. For these values, R(1%) = 1.4 to 2 and R(10%) = 1.3 to 1.4 Presents two reasons steady state strength determination is difficult: strength is very sensitive to void ratio in natural soil deposits, void ratio is highly variable spatially Steady state strength method does not apply only to procedure outlined in Poulos et al. (1985), but to broader concepts that S_{us} is: the minimum strength for contractive soils not equal to zero only a function of void ratio the correct strength for evaluating liquefaction flow potential "The undrained steady state strength (S_{us}) is a critical parameter for understanding the behavior of soil masses during earthquakes. Thus the profession should continue to strive for better ways of determining S_{us}, and for an objective assessment of current methods."

Year	Author(s)	Description of Study	Summary and Conclusions
1991	Kramer	• Closure of "Uncertainty in steady-state liquefaction evaluation procedures" by Kramer (1989) Agreed with Castro that better methods of determination of S_{us} are needed Rebutted Castro's objections	• Data were selected objectively and were all determined in accordance with Poulos et al. (1985) 29 data sets were evaluated using statistical procedures, and only four showed a statistically significant curvature in the SSL. These four were removed from the analysis Some limited liquefaction (phase transformation) points were included, but it has been shown that the liquefaction SSL encompasses the limited liquefaction response Data from Dobry et al. (1985) were indeed erroneous

Year	Author(s)	Description of Study	Summary and Conclusions
1992	Castro, Seed, Keller, and Seed	• Analysis of Lower San Fernando Dam failure of 1971 using steady state concepts Steady state strengths determined at four different labs from "undisturbed" samples from boreholes Void ratios were corrected for: changes due to sampling changes due to test setup and consolidation changes over time based on settlement data changes due to the earthquake shaking differences in upstream and downstream shell Back-analysis was based on a variety of means for accounting for dynamic effects (acceleration) and the strength of the slide mass extending into the reservoir In situ void ratios and samples were based on the downstream shell, while the slide occurred in the upstream shell (assumed to be symmetrical) Testing program used: drained and undrained tests isotropic and anisotropic consolidation preparation by moist compaction, pluviation, and slurry consolidation lubricated and rough end platens Steady state strengths determined (and corrected) were compared to strengths back-calculated from slide movements All samples were very loose (contractive)	• The mobilized strengths (back-calculation) were less than those determined in the laboratory by one-half to one standard deviation Slide was a result of a loss of strength triggered by the earthquake, and not the increased load due to acceleration since the slide occurred about 40 seconds after shaking stopped Steady state analysis indicated that the dam was unstable following the earthquake, but still overestimated the strength as back-calculated from the slide movements At steady state, σ_3', σ_{fp}', and S_{us} are related only by ϕ_{SS}' Results from the 4 labs were in good agreement Slope of the SSL was independent of: sample preparation method (initial fabric) initial state (consolidation state) stress path (test type) Most "undisturbed" specimens were consolidated to very high effective stresses, and "approximately" reached the steady state at 10% to 20% axial strain Sources of void ratio change (in order of decreasing magnitude were consolidation (and test setup), earthquake, difference between upstream and downstream, sampling No correlation of S_{us} with % fines Considerable scatter in the corrected values of S_{us} for 22 undisturbed samples. As a result, a large number of tests is required, as well as a very conservative approach, in order for the corrected laboratory strengths to agree with the back-calculated strengths
1994	Poulos	• Defined steady state deformation and its relationship to stability Discussed and compared peak strength and steady state strength Presented uses of the steady state concept	• Steady state strength is a fundamental strength, and should be used for stability analyses Even stable soil masses, with driving stresses below the steady state strength, are susceptible to intolerable deformations

Table B.2 - Review of Literature on the Effects of the Stresses in the Membrane

Year	Reference	Description of Study	Summary and Conclusions
1952	Henkel and Gilbert	• Experimental study of membrane and filter paper effects on undrained strength • Four series of tests were performed on 1.5"x 3" specimens of remolded soft clay: one unconfined and three with different membrane thicknesses • Strain rate = 2%/min • Membranes were characterized by extension modulus • Theory based on RCC deformation • Membrane and soil were assumed to be incompressible • Membranes were assumed to be isotropic, linearly elastic, and to have equal moduli in compression and extension • Does not account for initial stresses in membrane	• Membrane strength contribution is: 1.) independent of soil strength 2.) proportional to membrane stiffness 3.) independent of cell pressure • "Rubber only" test (using membrane, but with zero cell pressure) showed greater increase in strength (based on unconfined strength) than triaxial test with cell pressure • The clay tested failed at 15% axial strain; the correction is a function of the strain at failure, but not linearly proportional • If no buckling occurs, there is no circumferential tension in the membrane; compression shell theory accounts for axial force • If buckling occurs, hoop stress theory is used to account for circumferential tension Based on theory only, both corrections are proportional to strain

Year	Reference	Description of Study	Summary and Conclusions
1965	Duncan and Seed	• Examined influence of filter paper drains, membrane stresses, and piston friction on measured strength properties • Applied corrections to tests on Bay Mud and examined the influence of the corrections on the resulting strength parameters • Extended compression shell theory of Henkel and Gilbert to account for volume change in the specimen and changing membrane thickness (assumed RCC deformation, isotropic linear elastic membrane with E_{tens} = E_{comp} and ν=0.5, no buckling)	• For undrained tests on Bay Mud, ignoring the sum of the corrections resulted in an overestimate of the strength by 10% to 20%. The consolidation stress was overestimated by 5% to 10% • Neglecting the corrections resulted in: 1.) A_f too low 2.) drained strength envelope too high; (c' too high, ϕ' too low) 3.) PSR at failure too high 4.) stress-strain curve too steep • Membrane corrections were about 5% less than the corrections measured experimentally by Henkel and Gilbert, whereas the corrections proposed by Henkel and Gilbert were 15% less than the measured correction • The membrane correction to the axial stress was about 3% of the deviator stress at failure (or 1.5% of the consolidation stress) for the tests on Bay Mud • The membrane correction to the lateral stress was about 0.5% of the consolidation stress for the tests on Bay Mud

Year	Reference	Description of Study	Summary and Conclusions
1967	Duncan and Seed	• Analysis of errors in plane strain and triaxial tests due to: 1.) membrane 1.) filter paper drains 1.) piston friction 1.) end plate friction (plane strain only) • Triaxial tests on 1.4" x 3.5 " specimens of Bay Mud (a soft NC clay) were performed to investigate influence of the corrections on the peak strength	• Filter paper strength was reached at 2% to 3% axial strain, so filter paper correction is constant for values of strain beyond 2 to 3% • Membrane corrections are the same as those presented in Duncan and Seed (1965); graphical form of these corrections was presented • Piston friction was dependent upon lateral forces on bushing and seal, and should be determined for each seal separately • For triaxial tests on Bay Mud, the magnitude of the combined corrections was 10 to 20% of the uncorrected strength • The filter paper correction accounted for about half of the combined total correction, and the membrane and piston friction accounted for about one fourth each • The membrane correction to the lateral stress was less than 1% of the confining pressure for the tests on Bay Mud • Since corrections are not isotropic, stresses at the end of consolidation are not isotropic if corrections are applied to consolidation phase of the test • Ramifications of neglecting corrections: 1.) Measured strength values are too high 1.) Consolidation pressure is too high 1.) Pore pressure parameter A_f is too low 1.) Drained strength envelope is too high; c' is too high, ϕ' is too low 1.) PSR at failure is too high 1.) Stress-strain curve is too steep
1988	Germaine and Ladd	• Examined factors influencing testing of saturated clays	• Can use axial force correction for various area corrections, although the force correction still assumes RCC deformation • Bulging causes greater error in radial stress than in axial stress • Initial membrane diameter should be within 10% of the specimen diameter

Year	Reference	Description of Study	Summary and Conclusions
1988	La Rochelle et al.	• Present observational method of making area and membrane corrections • Suggested corrections are based in part on theory and in part on observed behavior and specimen measurements at the end of the test • Performed tests on rubber dummies to simulate bulging deformation and on plexi-glass dummies pre-cut along 45° and 60° slip planes to simulate slip plane deformation • Used modulus defined by Henkel and Gilbert	• Modulus increased with thickness and decreased with strain. Suggest using secant modulus at 10% strain • Membrane applies an initial confining stress to the specimen since it is stretched slightly • Data presented by Henkel and Gilbert are better matched by their correction if correction for initial stress is also applied • Suggest applying correction for axial stress if no buckling occurs or applying the correction for the lateral stress if buckling does occur (only use one or the other) • No buckling occurred for confining stresses greater than 1 ATM • Tests on rubber dummy indicated that all corrections (Henkel and Gilbert, Duncan and Seed, and La Rochelle et al) underestimated the effect of the membrane on the axial stress • For shear plane failure, membrane effect depends upon: 1.) membrane stiffness 1.) specimen stiffness 1.) specimen-membrane friction (which is a function of the normal stress on this interface) 1.) angle of inclination 1.) overall shear plane geometry • The bulging correction should be applied until failure occurs, then a correction for shear plane failure based on specimen measurements at the end of the test should be used • The means of applying the membrane correction (i.e. decrease σ_1 increase σ_3) affects the measured stress path (and measured c′). Vertical drop in stress path (half of each) is probably a good compromise

Table B.3 - Review of Literature on the Effects of Membrane Penetration

Year	Reference	Description of Study	Summary and Conclusions
1957	Newland and Allely	• Examined difference in peak and residual friction angle as a result of volume change during shear • Attempted to explain why energy methods cannot explain this difference • Tested lead shot in drained compression • Examined membrane misfit	• Volume changes in drained tests are erroneously large since the membrane initially does not fit exactly the surface of the sample • Dilation rate is related to orientation of contact surfaces at granular interlocks • If dilation rate is zero, the residual shear strength should be attained but the shear strength is actually greater than the residual since all interlocks do not act independently
1959	Newland and Allely	• Identified the effect of membrane penetration on volume change in simulated undrained tests • Performed undrained tests and simulated undrained tests, where the pore water volume was held constant by adjusting the cell pressure, on saturated lead shot	• Even if no cavitation occurs during an undrained test, expansion may occur as a result of the membrane penetration as the effective stress increases • For isotropic loading, the membrane penetration effect may be measured by subtracting three times the measured axial strain from the measured volumetric strain • Effect on measured strength is negligible for materials finer than coarse sand • The membrane penetration effect decreases as membrane thickness and sample size increase
1963	Roscoe et. al	• Examined two methods to measure membrane penetration effect: 1.) same as method of Newland and Allely 1.) measuring volume change for tests on 1.5 inch specimens with brass rods of varying diameters embedded along the axis, and extrapolating the measured volume for a rod diameter of 1.5 inches	• The effect of the rod on the packing of the sand in the specimen resulted in questionable results for method 2 • The method of Newland and Allely should be used to measure membrane penetration
1964	El-Sohby	• Tested sand specimens with an aluminum internal core cylinder	• The internal cylinder method (Roscoe et. al 1963) is preferred method of determining membrane penetration • Anisotropy effects hinder the method based on isotropic volume change (Newland and Allely 1959)
1965	Thurairajah and Roscoe	• Tested glass beads to study membrane penetration effects	• Relationship between penetration volume and the logarithm of effective stress is linear

Year	Reference	Description of Study	Summary and Conclusions
1966	Lee, I. K.	• Studied membrane penetration effects using the method of replacing the core of the sample with a stiff cylinder	• Core replacement method is a viable way to accurately measure volumetric stress-strain relationships
1967	Steinbach	• Tested 18 samples with varying grain size distributions and mean grain sizes • Proposed relationship for membrane penetration based on isotropic behavior and measured volumetric and axial strains	• Membrane effect is independent of gradation, but depends primarily on mean grain size
1973	Frydman et. al	• Performed isotropic compression tests on monosized glass spheres to determine factors affecting membrane penetration • Formulated relationship between membrane penetration and particle size	• Major factor affecting penetration is grain size. • Particle shape, grain size distribution, density, and type of material have minor effects. • Gradation will not affect membrane penetration for specimens with the same mean grain size • Membrane penetration (measured as volume change per unit surface area) is linearly related to the logarithm of cell pressure, with a slope that is linearly related to the logarithm of the mean grain size: slope = $0.014\log(d_{50})-.001$
1974	Raju and Sadasivan	• Examined membrane penetration determination by the core method • Modified top platen to allow for more uniform stress on both the core and the sand • Modified method of determining membrane penetration	• Factors affecting membrane penetration are: 1.) effective confining pressure 1.) pore characteristics at sample surface: grain size, shape, gradation, and porosity 1.) membrane characteristics: thickness, extension modulus 1.) surface area • Rigid platens should not be used • Volume change should be plotted against total sand volume, not against rod diameter, in order to get linear relationship

Year	Reference	Description of Study	Summary and Conclusions
1977	Lade and Hernandez	• Method for determination of penetration was based on isotropic behavior (three times the axial strain was employed) • Derived expressions for Skempton's pore pressure parameters to account for membrane penetration • Performed tests on uniform sands and glass beads and compared measured pore pressure parameters to the predicted ones • Performed ICU tests on sands with square shims as an inner lining to the membrane to reduce flexibility and compared with conventional ICU tests	• In undrained triaxial tests, membrane penetration results in a decrease in the rate of measured pore pressure change, which affects the effective stress path (moves to the right of actual) • Effective strength envelope is independent of stress path, and hence is independent of membrane penetration • Membrane flexibility is the volume change due to membrane penetration per unit change in effective confining pressure • Membrane penetration affects measured values of B, but the effect decreases as confining pressure increases
1977	Kiekbush and Schuppener	• Performed tests using liquid rubber as a lining in normal membranes in an effort to reduce membrane penetration • Determined the slope of the membrane penetration curve in same manner as Frydman et. al for various grain sizes	• Membrane penetration affects measured volume change in drained tests where cell pressure is changing, and affects measured pore pressure changes in undrained tests • In contractive soils, membrane penetration results in measured pore pressures that are too low, and as a result, measured undrained strength is unconservative • Using liquid rubber on inside of membrane reduced membrane penetration and increased measured pore pressures by as much as 100% • Membrane penetration effect decreases as sample size increases • Cannot simply correct for membrane penetration in undrained tests, but must try to eliminate it
1980	Raju and Venkataramana	• Performed tests with polyethylene strips as a lining in membrane, and with the membrane coated by polyurethane • Performed "compensated" tests by increasing or decreasing the pore water volume to correct for membrane penetration • Suggested analytical correction for penetration effect	• If effective stress changes, volume changes occur due to membrane penetration • In contractive samples, pore pressures are underestimated; in dilative samples, pore pressures are overestimated • Membrane penetration results in unconservative results in cyclic undrained tests • Polyurethane coating on membrane reduce penetration by 85%, and resulted in increased pore pressure development in loose saturated sand samples

Year	Reference	Description of Study	Summary and Conclusions
1981	Ramana and Raju	• Proposed a method of compensation for membrane penetration during triaxial testing • Discussed pore pressure response and stress path behavior resulting from the new constant-volume method	• Membrane penetration significantly influences pore pressure development and stress path behavior in conventional CU tests but does not alter the effective strength envelope • For loose sands, membrane penetration must be accounted for to avoid errors in pore pressure measurement • For cyclic triaxial tests, the resistance to liquefaction is overestimated compared to tests where penetration is completely eliminated • Constant volume tests can overcome membrane penetration effects in static undrained tests • Problems with the method include non-continuous compensation, collapse due to unstable soil structure, considerable scatter, complicated application
1982	Ramana and Raju	• Proposed three analytical expressions to estimate errors due to membrane penetration and apply corrections to measured data	• Volume changes due to membrane penetration depend on grain size, effective confining pressure, density index, and sample geometry • Volume change, unit membrane penetration, and volumetric strain can be calculated with reasonable accuracy according to the proposed formulas • Membrane penetration can be minimized by adopting large diameter test samples
1984	Baldi and Nova	• Performed qualitative analysis to determine which factors influence membrane penetration and to what approximate extent	• Apparent volumetric strain due to membrane penetration decreases linearly as diameter increases • Grain size has major influence on membrane penetration • Confining pressure, rigidity, and thickness of membrane have a non-negligible role in membrane penetration • Proposed method to correct measured values of pore pressures results in doubling the measured values • Correction factor depends on the volumetric stiffness of the sample, which in turn depends on the state of stress • Effect of membrane penetration is more crucial for high values of stress ratio than for isotropic compression

Year	Reference	Description of Study	Summary and Conclusions
1984	Vaid and Negussey	• Reviewed previous studies of techniques to measure and correct for membrane penetration effects • Proposed two new alternative methods, both based on hydrostatic compression in the triaxial cell, to measure membrane penetration • Performed tests on Ottawa sand to demonstrate new methods for measuring membrane penetration • The first new method (1) involves plotting the measured volumetric strain vs. diameter for a given change in effective stress for specimens of different diameters, and thus requires several tests • The second method (2) involves measuring the axial strain during hydrostatic unloading and assuming isotropic response, and requires only one test	• Previous methods for determining membrane penetration, including the technique of the internal coaxial core and the assumption of isotropic response to hydrostatic loading, are invalid because basic assumptions are violated • For most sands, radial strains are greater than axial strains in a triaxial specimen loaded hydrostatically • In the core method, the volume of soil is not linearly proportional to rod diameter, so the linear plot is invalid • Even if nonlinear relationship is used in the core method, the assumption that the volumetric strain is the same even if the rod diameter (and hence soil volume) varies is violated • The assumption that, for a given change in effective stress, the diameter of the core does not influence the volumetric strain is usually not satisfied due to the mixed boundary conditions • The assumption that membrane penetration is a function of the surface area of the specimen/membrane interface is satisfied for a given density and effective stress • The hollow cylinder method of Frydman et. al (1973) is valid, but the test equipment is not common • The multiple diameter method (1) results in equal penetration volume change in loading and unloading • Both methods predicted essentially the same membrane penetration, with the multiple diameter method (1) predicting slightly more penetration for large increases in effective stress • Using method 1 to correct for membrane penetration, the sand response during hydrostatic loading was not isotropic, with greater lateral strains than axial strain, but the response to unloading was isotropic • The relationship between unit penetration and effective stress appears to be log-linear

Year	Reference	Description of Study	Summary and Conclusions
1987	Lin and Selig	• Described a procedure to fit a hyperbolic curve to the membrane penetration relationship • Procedure is a modification of the procedure suggested by Vaid and Negussey (1984) • Method assumes isotropic behavior in both loading and unloading	• A linear relationship between unit membrane penetration and the logarithm of effective stress is not appropriate at low effective confining pressures • The relationship between unit membrane penetration and effective stress is more closely approximated by a hyperbolic function than by a logarithmic function
1992	Evans et. al	• Examined reduction of membrane penetration effects on gravel by washing sand into the specimen (sluicing) prior to testing • Compared results of cyclic undrained triaxial tests for sluiced and unsluiced gravel specimens	• Membrane penetration increases liquefaction resistance of gravel in cyclic undrained triaxial tests, so test results are unconservative • Membrane penetration effects artificially inhibit pore pressure generation in cyclic undrained tests • Membrane penetration is a function of: effective stress level, membrane thickness and modulus, and peripheral void size • In drained tests, membrane penetration affects only volume change measurements • In undrained tests, contractive specimens develop less pore pressure, and have higher strength due to membrane penetration • In undrained tests, dilative specimens develop less negative pore pressures, and have lower strength due to membrane penetration • Sluicing gravel specimens with sands reduces membrane penetration, so that the measured liquefaction resistance is only about 60% of that for unsluiced specimens

Year	Reference	Description of Study	Summary and Conclusions
1992	Evans	• Studied effect of membrane compliance on final specimen density for unsluiced and sluiced gravel specimens. • Used both hydrostatic drained tests and cyclic undrained tests on 2.8 inch specimens • Used isotropic unloading response to measure membrane penetration	• Membrane penetration causes excessive pore water redistribution during undrained loading, so that the density of the specimen changes during loading • Undrained tests are not constant volume in terms of the soil specimen, only in terms of the entire system • Void ratio in the interior of the specimen changes as pore water moves to or from the perimeter voids • Using multiple membranes reduces the membrane penetration for two reasons 1.) the thicker membrane resists penetration 1.) the adhesion between membranes resists rebound of the membrane penetration when the effective stress decreases • For gravel, only 80% of the membrane penetration volume change during loading is recovered during rebound or unloading • Relative density can double for loose gravel specimens in undrained cyclic tests due to membrane compliance • Sluicing gravel specimens with sand reduces membrane penetration: to values expected for specimens of the sand alone, and to about 15% of that for unsluiced gravel • Reducing membrane penetration during consolidation phase increases accuracy for all but dilative specimens • Gravel responds isotropically to hydrostatic unloading • Cyclic undrained loading resistance increases due to membrane compliance

Table B.4 - Review of Literature on the Effects of Specimen End Friction

Year	Author(s)	Description of Study	Summary and Conclusions
1960	Bishop et al.	• Closure of 'Factors controlling the strength of partially saturated cohesive soils.' *Proceedings, Research Conference on the Shear Strength of Cohesive Soils,* Boulder CO Discussed the effects of rates of testing and pore pressure non-uniformities and equalization on measured strengths Presented data showing increase in drained cohesion intercept and drained friction angle with an increase in rate of shear for CU tests on compacted clays Used ceramic probes to measure pore pressure differences between the ends and the middle of the specimen	• In undrained conditions, nonuniform pore pressures develop in triaxial tests (greater at the ends) Strength results in terms of pore pressures at the base of the sample can be misleading Variation of pore pressure development with strain rate can be due to: lag in pore pressure measurement device equalization of nonuniform pore pressures with time viscous effects on soil behavior Variation of pore pressure with height in the specimen decreases as strain rate decreases At conventional strain rates, nonuniform pore pressures develop, so measurement should be made at the center of the specimen or the strain rate should be decreased When stress paths are to be measured, the strain rate should be selected based on the time to each desired point on the stress path, rather than based on the time to failure

Year	Author(s)	Description of Study	Summary and Conclusions
1960	Shockley and Ahlvin	• "Nonuniform conditions in triaxial test specimens." Performed CD tests and vacuum triaxial tests on fine sand of varying density Measured stresses in large specimens with embedded pressure cells Measured volumetric strain for increments of specimen height Froze specimens at 10% axial strain to examine void ratio variation with both height and radius Estimated volumetric strain in vacuum triaxial tests using grid drawn on membrane prior to tests	• End restraint results in nonuniform total stresses and strains in sands "Even for constant volume tests on saturated sands, there are nonuniform density or volume changes throughout the specimens as they are subjected to axial compression" Average volumetric strain does not accurately depict volumetric strain in the failure zone Density can vary by 8 pcf to 10 pcf within the specimen at only 10% axial strain Dilation occurs in failure zone and contraction occurs at ends for all specimens, regardless of density Vertical stress on midplane is greatest at the axis of the cylinder Vertical stress on ends is greatest at the edges Variations in strains match variations in stresses Strain gauges indicated that the axial strain near the ends was nearly zero, and the maximum axial strain occurred just below the midplane Nonuniformities are important where bulging failure occurs To measure critical void ratio, initial uniformity and density change measurements in different regions of the specimen must be accomplished Initial uniformity within 1 pcf was achieved for a clean, fine, subrounded sand Contribution to deviator stress by the membrane was less than 1 psi for axial strains less than 20%
1960	Larew	• Discussion of 'Nonuniform conditions in triaxial test specimens' by Shockley and Ahlvin (1960)	• Three ways to cope with end friction: lubrication segmented ends conical ends Paraboloidal ends produce more nearly uniform conditions

Year	Author(s)	Description of Study	Summary and Conclusions
1960	Phalen	• Discussion of 'Nonuniform conditions in triaxial test specimens' by Shockley and Ahlvin (1960) Looked at static equilibrium based on axial load only (ignores confining pressures) and shear plane Tried to explain failure due to additional driving stress of soil weight and end friction Used inconsistent reasoning and incorrect assumptions	• Test results support Shockley and Ahlvin (1960) Shear stress on failure plane is minimum (based on larger area at mid-height) Specimen weight should be considered Shear stress is function of unit weight and ϕ
1960	Whitman et al.	• Discussion of 'Nonuniform conditions in triaxial test specimens' by Shockley and Ahlvin (1960) CU tests on batch-consolidated Buckshot clay (1.4-inch diameter specimens using filter strips as side drains) Water content measured on three pucks (top, middle, bottom) for each specimen	• No trend with $\sigma_3'_{con}$ or principal stress ratio Water content of middle third was an average of 0.6% lower (middle third is denser) for NC clay For OCR > 2.5, density of middle is lower than ends (middle dilates), variation increases with OCR Variation is due primarily to non-uniform shear stresses during shear. Since variation does not increase with consolidation stress, variation is not due to non-uniform consolidation (presumably) Water migration during "undrained" shear is considered a drained process Variation of water content is consistent with pore pressure generation during shear Shrinkage limit was lower for middle third due to dispersed structure as a result of shear Using correct e in the failure region will affect plots of water content vs. strength and Hvorslev's parameters, but not strengths; especially for undisturbed samples with inherent nonuniformities to begin with

Year	Author(s)	Description of Study	Summary and Conclusions
1960	Shockley and Ahlvin	• Closure of 'Nonuniform conditions in triaxial test specimens.'	• Whitman, Ladd, and Cruz added data that substantiate the results in the paper Larew is too ambitious. Correct geometry for conical ends would be a function of the actual stress pattern in the specimen during loading. If this was known, conical ends would not be needed. Also, the stress conditions vary from specimen to specimen, and within the specimen during one test. Phalen looked at simple conditions and neglected the confining effect of shear at ends until the end of his discussion. He discussed inward movement of particles during axial loading. This seems wrong. He neglected confining pressure (unconfined test) and discussed tensile stresses. His comments on soil weights are good.

Year	Author(s)	Description of Study	Summary and Conclusions
1964	Rowe and Barden	• "Importance of free ends in triaxial testing." Examined the importance of free (lubricated) ends to reduce errors associated with nonuniform deformation of the sample Tried the following means to reduce end restraint: radially segmented platens on rollers lead shot as an interface a water bag interface Performed direct shear tests to examine the effects of varying the platen type, lubrication type, normal stress, and rate of shear on the friction characteristics Discussed methods of pore water pressure measurement and leakage past the membrane in tests of long duration	• End restraint affects measured stress-strain response of sands Enlarged lubricated end platens can remove "dead zones" near the ends and reduce piston friction Stresses and deformations approach the homogeneous case as end restraint is reduced Lubricated ends reduce the concentration of dilation in local (failure) zones and delay formation of a slip surface Lubricated ends allow the use of shorter samples, which minimize density variations and the possibility of buckling Test time can be reduced due to shorter sample length and reduced internal pore pressure gradients Even with lubricated ends, local density and/or pore pressure variations can occur, especially at large strains Lubricated ends result in slightly lower drained strengths than conventional ends do In undrained tests, pore pressure is more affected by end restraint than strain or deviator stress at failure are Segmented end platens, and lead shot or water bag interfaces are not satisfactory solutions A layer of grease between the platen and a latex membrane was the recommended lubricated end scheme Apparent friction angles of 1° or less are possible with latex-grease lubricated ends Latex-grease system is rate dependent (viscous), but basically independent of normal stress Platen material is not important as long as it is smooth Soap was the most frictionless lubricant, but dissolved in the pore water Electrical pressure transducers provide a means of automatically recording pore pressure response

Year	Author(s)	Description of Study	Summary and Conclusions
1964	Lee and Seed	• Discussion of 'Importance of free ends in triaxial testing' by Rowe and Barden (1964) Performed CD tests on sand Performed direct shear tests on: latex and MoS_2 powder latex and molykote silicone grease Used 0.125 inch diameter metal dowel protruding into the sample 0.375 inches to pin sample to the end platens	• Agreed with the findings of Rowe and Barden Latex and silicone grease provides the best lubricated end Thickness of the latex is important, and 0.015 inches seems to be a good maximum thickness Apparent friction angle for silt was 1°, but for Ottawa sand was 8° (compared to 15° for sand on steel) Sands can penetrate the latex and develop stress concentrations at the grains Apparent friction angle is independent of rate of shear Platen material has little effect (same conslusion as Rowe and Barden)
1964	Olson and Campbell	• Discussion of 'Importance of free ends in triaxial testing' by Rowe and Barden (1964) Performed CU tests on sedimented samples of sodium kaolinite to investigate end restraint effects Used various L/D ratios between 1.12 and 3.5 with conventional ends For a constant L/D ratio, examined: plexiglass platens with drainage stones unpolished Teflon and filter paper drains polished Teflon and powdered Teflon polished Teflon, latex rubber, and silicone oil	• Lubricated ends did not reduce bulging Lubricated ends reduced measured effective stress friction angle by 3°, but did not affect total stress envelope Lubricated ends reduced pore pressures induced during shear Samples with L/D greater than 3 buckled Increasing L/D from 1.2 to 3.6 reduced the deviator stress at failure by 3 psi (10%) Increasing L/D from 2 to 3 reduced the effective stress friction angle by 0.5 ° Pore pressures at failure were unaffected by varying L/D ratio Rowe and Barden were correct in recommending the use of lubricated ends and short samples Tests on clay support the results of Rowe and Barden for sands

Year	Author(s)	Description of Study	Summary and Conclusions
1964	Poulos	• Discussion of 'Importance of free ends in triaxial testing' by Rowe and Barden (1964) Outlined recommendations for preventing membrane leakage	• Remarks on membrane leakage: Use measuring system that is as incompressible as possible Minimize temperature fluctuations Use burettes of sufficiently small diameter and prevent evaporation from them Do not use materials that absorb water Prevent air diffusion from high to low pressure side of the measuring system Typical o-ring leakage should only be one order of magnitude greater than diffusion through the membrane, which might be on the order of 0.00001 cc/hr O-rings should be stretched to 40% strain when in place to provide an adequate seal
1964	Turnbull	• Discussion of 'Importance of free ends in triaxial testing' by Rowe and Barden (1964) Compared triaxial tests to tests on concrete cylinders	• "Elimination of the friction between the specimen ends and the platens is not considered to be a useful solution, because it results in horizontal deformations of the specimen that do not occur under natural conditions." Strain level should not be used to describe deformation conditions, since it does not apply to any specimen size Slower strain rates are desirable
1965	Barden and McDermott	• "Use of free ends in triaxial testing of clays." Extends work of Rowe and Barden (1964) to extension tests, cyclic tests, and plane strain tests	• End restraint results in nonuniform pore pressures and migration of water in clay specimens End restrain affects observed stress-strain response in clay specimens Free ends can improve uniformity of stress and deformation Calculated σ_3' is in error due to pore pressure gradient Reduction of strain rate will reduce pore pressure gradient, but will increase time allowed for water migration, which increases void ratio variations, and affects shear strength

Year	Author(s)	Description of Study	Summary and Conclusions
1965	Bishop and Green	• Performed drained tests on sands Used lubricated and conventional ends Used 3 different L/D ratios Lubrication - latex and silicone grease 4-inch diameter specimens were tamped under water	• End restraint affects observed stress-strain response, extent of effect not clear for sands If L/D is large enough, effects are minimal except on ε_V for sands at large strains End restraint increases apparent strength, but increase is a function of L/D. Apparent strength gain is negligible for L/D=2 Lubrication reduces scatter in ϕ' by ~0.3 degrees for a given density Lubrication had no significant effect on strength for L/D=2, but decreased "apparent" strength for L/D<2. Ends did not expand, whether lubricated or not (for L/D=2) End restraint does not appear to affect volumetric strain at failure
1965	Blight	• Examined measurement of $\overline{A}$ for CU tests on clays Isolated middle section of specimen using disc of membrane	• Restraint results in non-uniform pore pressures and water migration in clay specimens Segmented specimens result in better measurement of pore pressures Conventional tests overestimate $\overline{A}$ for normally consolidated clays and underestimate it for over-consolidated clays Appears to be no significant difference in strength for conventional and free end tests $\overline{A}$ for heavily over-consolidated clay decreases with strain rate (also a slight increase in strength parameters in terms of effective stresses)

Year	Author(s)	Description of Study	Summary and Conclusions
1965	Januskevicius and Vey	• Measured internal stresses in specimens for non-lubricated and lubricated ends Stresses in dry sand specimens measured using piezo-electric transducers (1-inch diameter x 0.1-inch thick)	• Quality of results is very poor Strains vary with length in conventional tests, but in free end tests, gage-measured strains and stresses agreed with average stresses and calculated overall strain Strains are greater in mid-section (~ 1.2 to 1.5 avg.) and smaller at ends (~ ¼ to ½ avg) Strains increase from axis toward cylindrical surface For free ends, RCC deformation occurred until 3% axial strain Accuracy of strain measurement decreases with strain rate
1968	Duncan and Dunlop	• Performed series of tests with and without restrained ends to determine how end restraint influences soil strength, strength parameters, pore pressure characteristics, and stress-strain behavior Addressed the question of cost effectiveness with respect to lubricated end tests	• Cap and base restraint results in non-uniform volumetric strains in drained tests and moisture migration within the sample in undrained tests. These consequences are unimportant unless they cause changes in strength, pore-water pressure, or stress-strain behavior End restraint causes a slight increase in the slope of the stress-strain curve, a slight decrease in strain at failure, and a small increase in compressive strength in undrained tests on clay, all of which are negligible For undrained tests, the influence of restraint on pore-pressure depends on the loading rate Quick tests on clays using lubricated ends produce non-uniform pore pressures If L/D is sufficiently large, lubricated end tests are only cost effective for drained testing of sands when unusually accurate volumetric strain measurement at large strains is required
1968	Kirkpatrick and Belshaw	• "On the interpretation of the triaxial test."	• End restraint results in non-uniform strains for sands Deformation pattern approaches homogeneous conditions as restraint is reduced for sands Conical, quasi-rigid zones occur with rough platens: at axis: axial strain at middle >>axial strain at ends (~0) at perimeter: axial strain at middle ≈ axial strain at ends

Year	Author(s)	Description of Study	Summary and Conclusions
1970	Kirkpatrick and Younger	• Expanded work of Kirkpatrick and Belshaw (1968) radial and circumferential strain to include axial strain Performed vacuum tests on 250-mm diameter samples with L/D =1 and 2 End conditions: Double layer of greased latex on glass, top and bottom for L/D =1 Lubrication on one end, none on other for L/D=1 Both ends rough for L/D=1 to 2 Estimated internal displacement by x-raying grid of lead shot; error ~ +/- 0.2 mm Used platens that prevented tilting	• Axial strains in lubricated specimens with L/D=1 are uniform to 10% axial strain If one end is rough, axial strain concentrates at lubed end If both ends are rough, axial strain is concentrated in center at strains as low as ε_a = 4%. Variation gets worse with strain increase Variation increases from perimeter toward the axis In some lubricated end tests, non-uniformity developed at ε_a=5 to 6%, but strains remained uniform until non-uniform lateral deformations were observed In non-lubricated end tests, the ratio of axial strain at the middle to the average axial strain was as high as 2 to 3, and the ratio of axial strain at the ends to the average axial strain was as low as 0.1 to 0.2
1972	Raju et al.	• "Use of lubricated and conventional end platens in triaxial tests on sands." Reviewed theoretical and experimental studies of stress and strain in triaxial tests Performed drained tests on dry sand specimens with mean grain sizes of 0.3 mm to 0.6 mm and diameters up to 3 inches Compared conventional ends (L/D=2) and lubricated ends (L/D=2, L/D=1) based on: strength stress-strain volume change Tried using elastic ends with properties similar to soil	• Lubricated ends (LE) result in uniform stresses and deformations up to the peak stress No failure surface develops with LE No peak in stress-strain for dense sands with LE Slightly lower drained strength with LE (~1 degree for loose, ~ 3 degrees dense) Axial strain level at peak deviator stress is greater for LE (10 to 12% compared to 6 to 8%) Could only continue conventional end tests to 7 to 10% axial strain; with LE could carry up to 18% (in terms of uniform deformation) Less dilation occurs with LE Conventional ends result in bulging for loose specimens and distinct failure plane for dense specimens Distinct failure plane and peak in stress-strain response is not a property, but is a function of test conditions

Year	Author(s)	Description of Study	Summary and Conclusions
1973	Dickin	• Discussion of 'Use of lubricated and conventional end platens in triaxial tests on sands.' by Raju et al. (1972) Drained tests on saturated sand specimens, 102-mm diameter, L/D=1, consolidated to 1.4 kg/cm^2 Vibrated specimens to achieve D_r~100% Used double layer of greased membranes on end platens	• Disagrees with conclusion that peak in stress strain is function of test and not a true soil response Proposes that a "less-pronounced peak" may result from lubricated ends
1974	Kirkpatrick et al.	• Investigated influence of lubrication on stress distributions Tested 8″ diameter x 8″ high samples of dry 20-30 Ottawa sand Platens: heavy aluminum, 9.5″ diameter x 1.5″ thick Performed vacuum tests using lubricated and non-lubricated ends Used proving ring to measure load; diaphragm pressure gages inset in platens to measure normal stress on base platen Total of 13 gages: 4 different radii on 2 diameters @ 90 degrees from each other Gages only good to 24psi Lubrication: double layer of latex and silicon grease Tests: 3 lubricated and 3 non-lubricated on both loose and dense	• For sands, "severely non-uniform conditions exist in samples tested with rough (non-lubricated) platens (Kirkpatrick & Belshaw 1968, Kirkpatrick & Younger 1970) With effective lubrication, strains are fairly homogeneous Three regions of stress-strain curve: Elastic Transition Ultimate Plots of ratio of measured axial stress to average axial stress showed variation from 1.5 at the axis to 1.75 at the perimeter for non-lubricated, with variation increasing with strain For lubricated tests, the ratio was proportional to r, with values ranging from ~ 0.75 to 1.25; scatter decreased as strain increased Scatter in measured data was worse for loose samples, decreased as axial strain increased, and increased from the axis to the perimeter Elastic range agrees with published theoretical solutions, but measured stress at the axis is lower than theoretical

Year	Author(s)	Description of Study	Summary and Conclusions
1978	Lee	• First study of end restraint specifically for undrained tests on saturated sands Cited Castro's recommended test procedure Used Rowe's grease/rubber system Conducted direct shear tests on Monterey 20 sand (mean grain size of 0.6 mm) in 3-inch square square shear box at a displacement rate of 0.001in/sec Rowe and Barden: if ϕ<1 degree, assume frictionless Barden and McDermott: benefits of LE for drained triaxial tests on sands are worth the trouble Duncan and Dunlop: benefits of LE for undrained triaxial tests on clays are not worth it, except for special research Duncan and Dunlop: sliding friction is less than static Tested 1.4-inch diameter samples with L/D $\approx$ 2.4 Lubrication scheme: 2 layers of 0.015-inch thick latex & high vacuum silicone grease Used strain rate of 10%/hr (based on t_{100}<2 hrs Sand: G_s=2.68, e_{max}=1.03, e_{min}=0.61, d_{50}=0.2 mm, C_u=1.6 Drainage tube was used to keep specimen concentric with platen was 1/8-inch diameter x ½-inch long Looked at shear-strain distribution in triaxial specimen	• Effects of using free ends instead of L/D=2 in drained tests: flatter initial slope of stress-strain curve (initial tangent modulus ~60% less) slightly lower strength (0% to 10%) greater dilation or less contraction (less tendency for volume change) Effects of free ends instead of L/D=2 in undrained tests on clays: slightly lower undrained strength (~5%), about same as scatter similar pore pressure and effective stress parameters unless rate is 50 times normal rate Can run test faster due to uniform strains without pore pressure errors Possible problems: stress concentration under coarse particles grease squeezes out (limits practical time for consolidation phase) effectiveness may decrease with load rate (viscosity) grease may "set up" with time (sliding will break) Direct shear results are independent of rate Measured friction angle higher than 1 degree in direct shear, especially for thin rubber (~0.05mm), was closer to 1 degree for thick rubber (0.38mm) Two layers of lubrication was better than one Increasing grain size increases maximum shear stress; effect is most significant in range of d_{50}=1 mm In direct shear, no peak response for no lubrication; with lubrication, peak and residual response occurs Drained friction angle changes with dilatancy and crushing, which change with shear strain End friction has more effect on the shear-strain distribution for undrained than drained tests

1978	Lee (cont.)		• More grain crushing occurs in mid-section for conventional ends (more shear-strain) It appears that LE increase the volume change tendency Both contractive and dilative sands have higher strength when tested with free ends Drained tests: More dilation in dense sands with LE Less contraction in loose sands with LE Undrained tests: dense sands have greater tendency for negative pore pressure development, higher effective stress, and higher deviator stress with LE contractive sands have less tendency for pore pressure development, higher effective stress, higher deviator stress with LE No differences in tests with lubricated or conventional ends up to 4 to 5% axial strain In CU tests with LE: ϕ' is ~1 degree lower criticial confining pressure, is higher In CD tests with LE: criticial confining pressure is higher LE results in tendency for more dilation, regardless of confining pressure LE increase measured strength based on effective stress in terms of deviator stress by ~20% (for sand tested) Conclusions: ϕ' only slightly influenced by ends σ'_{3crit} significantly influenced by ends Ends affect volume change tendency, and thus strength Effect is a function of σ'_{3crit} : for clays $\sigma'_{3crit} \approx 0$ so S_u is affected very little

Year	Author(s)	Description of Study	Summary and Conclusions
1979	Arthur and Dalili	• Examined different greases and fillers for rubber on rubber lubrication schemes Used modified direct shear test where latex sheet was pulled from between the two stationary halves of the shear box	• Adding PTFE filler to silicone grease appears to increase the effectiveness
1981	Norris	• Studied the effect of the thickness of lubricated end caps on the shear strength of sand Discussed Poisson effect in thick latex ends Tested Monterey 0 sand (d_{50} = 0.42 mm)	• Thick latex layers can cause artificial reduction in shear strength Optimal end membrane thickness is a function of sample height, consolidation pressure, and mean particle size Ratio of end membrane thickness to mean particle size should be increased if consolidation pressure, height, or particle size uniformity increases Optimal end membrane thickness should minimize both the end restraint and the Poisson effect Initial end membrane thickness estimate should be 150% of the mean particle size of the material Conventional end platen tests are preferred when feasible (e.g. when only strength is needed) due complexity of test preparation associated with lubricated ends
1982	Sarsby et al.	• Studied the extent to which free ends compress during triaxial testing (bedding error) Performed isotropically and anisotropically consolidated drained tests by applying four load cycles	• Bedding errors are only significant in elastic range (small strains) or for cyclic tests Axial deformation due to distortion of lubricated ends can be up to 80% of recorded deformation in the elastic range Bedding error is linearly proportional to log σ_1' for a constant effective stress ratio path (e.g. k_0) Linearity with effective stress is independent of porosity, grain shape, and grain size Magnitude of bedding error increases non-linearly with particle size (for sandy gravel ≈ 30 times that for fine silt) Bedding error during virgin loading is greater than that during reloading Magnitude of bedding error increases with sample porosity

Year	Author(s)	Description of Study	Summary and Conclusions
1984	Tatsuoka et al.	• Performed four types of tests on fine sand (d_{50} = 0.16 mm, e = 0.73) to gain insight into the methods of lubricating end platens: Compression tests to determine the rate at which lubrication (grease, oil) is squeezed out Direct shear tests on lubrication layers between sand and a smooth platen Direct shear tests on lubrication layers between two parallel smooth platens to evaluate lubrication quality of lubrication Triaxial compression tests with different height to width ratios at low confining pressures to evaluate the lubrication effectiveness Lubrication methods consisted of latex disks, various greases, and mixtures of grease and powder (MoS_2 and PTFE) Configurations: rubber/grease/steel rubber/grease/rubber/grease/steel rubber/grease/rubber/ steel rubber/rubber/grease/steel Grease thickness was about 0.05 mm Latex thickness was about 0.3 mm Displacement rate for direct shear tests was 0.2mm/min	• Apparent friction angle decreases as normal stress increases Powder filler increases viscosity, but also increases shear resistance For fine sand (d_{50} = 0.16 mm), the apparent friction angle can be less than 1° for normal stresses in the range of 150 KN/m^2 to 670 KN/m^2 One layer of grease between 2 latex discs provides the best lubrication for normal stresses between 150 KN/m^2 and 670 KN/m^2 In cyclic loading tests of long duration, the quality and viscosity of oil is not sufficient, but grease is acceptable Multiple-layer lubrication methods provide slightly better lubrication during first loading than single-layer methods, but significant bedding errors are introduced Apparent friction angle of silicone grease is independent of shear rate for rates between 0.1 mm/min and 50 mm/min Pre-loading to about 650 KN/m^2 prior to testing at normal stresses below 50 KN/m^2 causes severe deterioration of the lubrication

Year	Author(s)	Description of Study	Summary and Conclusions
1986	Head	• Chapter 22: Special Triaxial Test Procedures Described two special triaxial test procedures: Use of lubricated end caps for applying the load to test samples that exhibit "plastic" deformation Use of a special type of lubricated end cap for fissured soils Outlined advantages, limitations, and recommendations for using generally adopted lubricated end cap system Outlined reasons for using lubricated end caps and discussed several previous attempts to correct end restraint problem Outlined recommendations on lubricated end platen components (platens, membrane thickness), friction angle, sample length, mode of failure, time to failure, extension tests, pore pressure coefficients, and stress-strain curves	• Sample length can be decreased to the sample diameter (L/D=1) Enlarged end platens should be used for tests with axial strain levels greater than 10% End membrane thickness should be 150% of the mean particle size Time to failure in CU tests can be decreased by a factor of 5 (minimum of 3 hours) because pore pressure development is more uniform Lubricated ends decrease necking in extension tests UU tests (without lubrication) should be run slower to prevent excessive pore pressure gradients
1993	Goto et al.	• "Quality of the lubrication layer used in element tests on granular materials." Performed cyclic direct shear tests on granular soils (fine sands to gravels) with a lubrication layer (silicone grease and latex disk and variations) to evaluate the quality of lubrication Varied the following in study: void ratio grease type grease thickness presence of glass beads type of powder (fly ash, slag, glass, MoS_2, PTFE) quantity of powder layer configuration Configurations: rubber/grease/steel rubber/grease/rubber/grease/steel rubber/grease/rubber/ steel rubber/rubber/grease/steel Grease thickness was about 0.05 mm Latex thickness was about 0.3 mm	• The friction characteristics of the lubrication scheme are independent of void ratio Lubrication quality decreases as stress concentration at the grains increases (as grain size increases) Apparent friction angle decreases as normal stress on the lubrication layers increases For fine sand (d_{50} = 0.16 mm), the apparent friction angle can be as low as 0.15 degrees for normal stresses in the range of 196 KN/m^2 to 588 KN/m^2 For medium sand (d_{50} = 0.62 mm), the apparent friction angle can be as low as 0.6 degrees For gravel (d_{50} = 1.85 mm), the apparent friction angle can be as low as 0.8 degrees for normal stresses in the range of 150 KN/m^2 to 700 KN/m^2 For medium sand and gravel (d_{50} = 0.62 mm and d_{50} = 1.85 mm), adding powder to the grease helps prevent the grease from squeezing out at the granular contacts If too much powder is added to the grease, the shear resistance can increase significantly Fly ash is a good powder additive

Table B.5 - Review of Literature on Analytical Solutions for End Effects

Year	Author(s)	Description of Study	Summary and Conclusions
1902	Filon	• Attempted to find an analytical solution to the problem of an elastic cylinder subjected to the following boundary conditions: average vertical stress is equal to the axial force divided by the cross-sectional area vertical displacement is constant on the ends radial displacement is equal to zero at the intersection of the ends and the radial surface shear stress is equal to zero on the radial surface radial stress is equal to zero on the radial surface	• Solution is not exact; higher order terms would help convergence: "The analytical complexity of such a complete solution would, however, be very great, and would render it quite beyond the reach of arithmetical expression, and consequently valueless for the purposes of the engineer and the physicist." Failure occurs first at the intersection of the ends and the radial surface Based on a numerical example, yield begins when the average vertical stress is approximately one half to one third of that required for yield of a cylinder with free ends Numerical results contradict test results, for which the constrained cylinders are stronger
1944	Pickett	• Attempted to find an analytical solution to the compressed cylinder problem using the Fourier Method Boundary conditions: shear stress is equal to zero on the radial surface radial stress is equal to zero on the radial surface vertical displacement is constant on the ends radial displacement is equal to zero on the ends	• Fourier Method can be used to solve any problem where the boundary conditions can be expressed by a Fourier series Solutions are exact in the limit (infinite series) Solution suggests that the maximum vertical stress ($r=a$ on the ends) might be equal to the average vertical stress divided by (1-2n), but no proof is offered Filon's solution satisfied all of the boundary conditions except for zero radial displacement on the ends, which was not satisfied due to an incorrect distribution of the shear stress on the ends Stresses are discontinuous at the intersection of the ends and the radial surface
1951	D'Apolonia and Newmark	• Attempted to find an analytical solution to the compressed cylinder problem using lattice, or framework, analogy techniques	• Solution shows trends and variations in stresses that are similar to those for the solution of Pickett, but the magnitudes of the stresses are slightly less

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1957	Balla	• Attempted to find an analytical solution to the compressed cylinder problem based on a stress function in cylindrical coordinates Boundary conditions: shear stress is equal to zero on the radial surface radial stress is equal to zero on the radial surface vertical displacement is constant on the ends Includes a coefficient for the roughness of the end plate	• Solution uses products of polynomials, sine and cosine functions, and Bessel functions Results compare well with the results of Filon's solution
1960	Balla	• Attempted to find an analytical solution to the compressed cylinder problem based on a stress function in cylindrical coordinates Boundary conditions: shear stress is equal to zero on the radial surface radial stress an circumferential stress are equal and constant on the radial surface vertical displacement is constant on the ends radial displacement is equal to zero at the intersection of the ends and the radial surface Includes a coefficient for the roughness of the end plate Allows for axial and radial loads	• Stress function used for solution was similar to that of Balla (1957) Expression for vertical stress did not converge near the ends or near the radial surface (all boundaries of the cylinder) Balla suggests that when the ratio of vertical stress to radial stress is equal to the ratio of length to diameter that the shear stress is zero Solution did not recognize the significance of Poisson's ratio

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1969	Perloff and Pombo	• Solved restrained cylinder problem using an elastic-plastic constitutive law and the finite element method Boundary conditions: vertical displacement is constant on the ends radial stress is constant on the radial surface radial displacement is equal to zero on the axis radial displacement is equal to zero on the ends Constitutive law was bilinear and could model strain softening or hardening Load was applied in increments of 5% axial strain	• Using L/D ratio of 2 does not correct for influence of end restraint in strain softening materials Yielding begins halfway between the ends and the mid-plane, then progresses first toward the mid-plane and finally toward the ends Axial strains are uniform with height before and after yield, except near the ends Radial strains are uniform with radius except near the ends Observed stress-strain variation is most significant after yield, and is minimal for perfect plasticity and strain hardening, and severe for strain softening Yield stress can be smaller or larger than for free end case, since confinement and stress concentration have opposite effects For strain hardening, material appears slightly stiffer than actual after yield For strain softening, material appears much less stiff than actual after yield, and variation decreases as L/D increases L/D = 3 appears to erase effect of end restraint, even for strain softening For most of the specimen, circumferential stress is the smallest normal stress Effect of end restraint on observed stress-strain is dependent upon the constitutive law, and appears to be very important for strain softening Using a large L/D will produce better test results if end restraint cannot be eliminated

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1970	Girijavallabhan	• Solved axially loaded cylinder problem with complete end restraint using a linearly elastic constitutive law and the finite element method Solved for L/D=1, ν=0.25, and E=1 using 240 elements Presented correction factors to apply to the observed Young's Modulus to get the true Young's Modulus	• Filon's solution was incomplete Pickett's solution used Fourier-Bessel functions, experienced slow convergence, and did not provide solution near outer edges D'Appolonia and Newmark applied lattice analogy technique to the problem Balla combined polynomial, Fourier, and Bessel functions into closed form solution Pickett's solution agreed fairly well with the FEM solution Resulting stress distributions are primarily a function of Poisson's ratio and L/D ratio End restraint effect increases as Poisson's ratio increases Ratio of vertical stress to average vertical stress increases infinitely at the intersection of the ends and the radial surface