Amplitude Balancing in $\tau$-$p$ Domain

by

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(ABSTRACT)

An approach to balance the amplitude of seismic data in the $\tau$-$p$ domain is introduced in this study. The idea of amplitude balancing technique is based on the following observation: In the $\tau$-$p$ domain, direct wave, ground-roll, primary reflection, multiple and refraction arrivals are located at different regions. These regions can be viewed as signal region and noise region. By increasing the amplitudes in the signal region and suppressing the amplitudes in the noise region, so called amplitude balancing in $\tau$-$p$ domain, the signal-to-noise ratio of seismic data can be improved.

The $\tau$-$p$ domain amplitude balancing scheme is tested and calibrated on synthetic seismic data using AIMS® package. The modeled data is also used to illustrate transformation (slant stacking) to and from $\tau$-$p$ domain. The signal-to-noise ratio enhancement using amplitude balancing in $\tau$-$p$ domain is illustrated. This general discussion also includes aliasing effect of slant stack and deconvolution in $\tau$-$p$ domain.

After the calibration with synthetic data, the amplitude balancing in $\tau$-$p$ domain is applied to real seismic data recorded on the Atlantic Coastal Plain near Richmond, Virginia and Aiken, South Carolina to explore the possibilities of enhancing the quality of seismic data. Processing of synthetic and real data is carried out on VAX 11/785 and Sun Sparc10 workstation at the Regional Geophysics Laboratory at Virginia Polytechnic Institute and State University using DISCO® seismic data processing package.

The results suggest that $\tau$-$p$ domain amplitude balancing can be combined into conventional seismic data processing sequence to improve the signal-to-noise ratio and thus give a better imaged

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1 AIMS® is trademark of GeoQuest International, Inc.

2 DISCO® is trademark of CogniSeis Development, Inc.
seismic section. Extensive tests carried out indicate that choice of ray parameter range, the degree of amplitude change, are important aspects of the processing in \(\tau-p\) domain.

In this study, a complete data processing was carried out to generate a stack section of NRC line 2 in Virginia while the amplitude balancing in \(\tau-p\) domain was incorporated into a conventional processing scheme. The \(\tau-p\) domain processing of NRC line 2 improved the data quality. The signal-to-noise ratio enhancement obtained by the amplitude balancing in \(\tau-p\) domain led to test the method to improve weak reflections from within the Dunbarton Triassic basin on SRP line 2EXP in South Carolina. After the application of amplitude balancing in \(\tau-p\) domain, CMP gathers showed enhanced signal-to-noise ratio, although the improvement became almost indiscernible after stack.
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Chapter 1. Introduction

Conventionally seismic data acquisition, processing and interpretation are all done in a distance-time (x-t) domain. Radon transform (Radon, 1917) can be used in a discrete form to transform 2-D seismic data from conventional distance-time domain onto delay time-ray parameter (τ-p) domain. The transform process is also called slant stack (Claerbout, 1975). Over the past few decades, slant stack processing has been applied to seismic data for different purposes. Velocity estimation from multifold seismic reflection data (Schultz and Claerbout, 1978; Gray and Golden, 1983), plane wave decomposition by slant stack (Phinney et al., 1981), migration of seismic wavefield (Hubral, 1980; Ottolini and Claerbout, 1984; Ruter, 1987), inversion of seismic reflection data (Clayton and McMechan, 1981; Diebold and Stoffa, 1981; Thorson and Claerbout, 1985) and general quality enhancement of seismic data (Tatham, 1984; Hampson, 1986; Yilmaz, 1987; Mitchell and Kelamis, 1990) are a few examples for use of τ-p domain processing. Most of the processes involving the application of slant stack follow a general procedure which begins with a forward slant stack, then applies special processing steps in τ-p domain and ends with an inverse slant stack. The approach introduced here enhances the quality of seismic data by amplitude balancing in τ-p domain.

In τ-p domain, primary reflections and multiple events form elliptical arrival patterns, direct wave, ground-roll, refraction become isolated points. The region in τ-p domain that is limited to the ellipses of primaries is defined as signal region while the region that includes other arrivals such
as direct wave, ground-roll, refraction and multiples is defined as noise region. Since the primary reflections are separated from the other events in $\tau$-$p$ domain, it is feasible to attenuate the amplitudes of the unwanted arrivals in the noise region. Attenuating the amplitudes in the noise region is called amplitude balancing in $\tau$-$p$ domain. The process enhances the signal-to-noise ratio when the data is transformed back to $x$-$t$ domain.

The organization of this study is that a review of $\tau$-$p$ transform and a discussion on related aliasing problem are given in Chapter 2. In Chapter 3, the scheme of amplitude balancing in $\tau$-$p$ domain is introduced and discussed. Synthetic data is used for testing the amplitude balancing in $\tau$-$p$ domain. Examples of slant stack processing of synthetic data are presented to interpret results from the real data sets. In chapter 4, two seismic lines recorded on the Atlantic Coastal Plain near Richmond, Virginia and Aiken, South Carolina are used to document the effect of the amplitude balancing in $\tau$-$p$ domain on seismic reflection data. Conclusions are given in chapter 5.
Chapter 2. Forward and Inverse $\tau$-$p$ Transformation

Most of conventional seismic reflection data processing is carried out in $x$-$t$ domain. Slant stack is a method that decomposes 2-D seismic data in $x$-$t$ domain and maps onto $\tau$-$p$ domain where $\tau$ and $p$ represent time and slowness (or, ray parameter), respectively. Transforming two-dimensional seismic data to $\tau$-$p$ domain using a slant-stack implementation has many advantages (Tatham, 1984). In $\tau$-$p$ domain, wavefields that represent direct wave, ground-roll, primary reflections, multiples and refractions are located at different regions. Therefore, it is possible to improve the signal-to-noise ratio of the reflected arrivals by suppressing the amplitudes of the other arrivals in $\tau$-$p$ domain.

**Review of $\tau$-$p$ transform**

Let’s assume a plane wave in an isotropic, homogeneous medium propagating upward to a flat surface with a wave speed of $V$ (Figure 1 on page 5). The plane wavefront at AB position will reach the point O on the surface after a time interval of $dt$. The angle of incidence $(i)$ is defined
as the angle between the wavefront AB and the horizontal flat surface AO. From the right triangle ACO in Figure 1 on page 5, it is obvious that the incident angle \( i \) is given by

\[
\sin i = \frac{V dt}{dx}
\]

(2.1)

where the \( dx \) is a small distance on the surface and \( V \) is the velocity. This relation is usually expressed as:

\[
p = \frac{dt}{dx} = \sin \frac{i}{V}
\]

(2.2)

where \( p \) is called ray parameter. For a horizontal multilayered earth model, the above relation extends to:

\[
\frac{\sin i_1}{V_1} = \frac{\sin i_2}{V_2} = \ldots = \frac{\sin i_n}{V_n} = p
\]

(2.3)

where \( i_n \) is the incidence angle in layer \( n \) of \( V_n \) velocity. The above relation indicates that any ray path can be represented by a \( p \) value or a related emergence angle \( (i) \).

The mathematical expression for Radon transform is developed by the German mathematician J. Radon early in 1917. If the variation of time \( t \) is given as a function of ray parameter \( p \), from equation (2.2):

\[
t = \tau + px
\]

(2.4)

Using (2.4), a wave field of \( F(x,t) \) in \( x-t \) domain can be represented by \( F(x, \tau + px) \) and summed over distance \( x \) to give \( \hat{F}(\tau, p) \) as wave field in \( \tau-p \) domain:

\[
\hat{F}(\tau, p) = \int_{-\infty}^{\infty} F(x, \tau + px) dx
\]

(2.5)

where \( x \) is offset, \( t \) is recorded time, \( \tau \) is the intercept time at zero offset, and \( p \) is apparent slowness.

The discrete form of this transform (Turner, 1990) is:
Figure 1. Plane wavefront of a seismic wave intersecting the surface of the earth: Angle $i$ is (1) the angle between the vertical and the ray associated with the wavefront, and (2) the angle between the horizontal surface and the wavefront itself. $V$ is the propagation velocity of the seismic wavefront, and $dx/dt$ is the apparent horizontal phase velocity of the wave as it sweeps across the horizontal surface (after Tatham, 1984).
\[ \hat{F}(\tau, p) = \sum_{l=1}^{n} F(x_l \tau + p x_l) \]  

(2.6)

where \( n \) is the number of traces at different offsets available. This discrete summation (slant stack) transforms a 2-D seismic wavefield from \( x-t \) domain to \( \tau-p \) domain. The following is a brief description of such a process. For a given zero-offset intercept time \( \tau \), all amplitudes along a straight line of slope \( p \) are summed to form the output at that \( \tau-p \) coordinate. This process is repeated for all \( p \) values at all zero offset intercept times (\( \tau \)) to produce the wavefield in \( \tau-p \) domain for all \( \tau-p \) coordinates. In \( x-t \) domain, the amplitude as a function of time for a given offset is called a trace. Similarly, in \( \tau-p \) domain, the resulting amplitudes as a function of time for a given \( p \) form a \( p \) trace.

The inverse Radon transform (Phinney et al., 1981) is given by

\[ F(x, t) = \frac{1}{2\pi} \frac{d}{dt} H_i \left[ \int_{-\infty}^{\infty} \hat{F}(p, t - px) dp \right] \]  

(2.7)

where \( H_i \) is Hilbert transform. Slopes in \( \tau-p \) domain \( \frac{\Delta \sigma}{\Delta p} \) correspond to offset information. Summing amplitudes along straight lines with slopes of \( \sigma \) brings data back to \( x-t \) domain.

Since the summation has only taken place along the straight lines representing different \( p \) values during forward \( \tau-p \) transform, the arrivals that form a straight line in the \( x-t \) domain are mapped into a point in \( \tau-p \) domain. For the same reason, an arrival as a single point in the \( x-t \) domain is mapped onto a straight line in the \( \tau-p \) domain. It follows that hyperbolic trajectories in the \( x-t \) domain are mapped onto elliptical trajectories in the \( \tau-p \) domain. Let's examine this transformation graphically using Figure 2 on page 7.

In Figure 2 on page 7, point \( A \) in the \( x-t \) domain maps to point \( A' \) in the \( \tau-p \) domain. Point \( B \) maps to point \( B' \) at a smaller \( \tau \) and a larger \( p \) representing the straight line tangential to point \( B \). Point \( C \) maps to point \( C' \) at \( \tau = 0 \) and marks a specific \( p \) that represents a straight line asymptotic to the hyperbola in \( x-t \) domain.
Figure 2. Slant stack transforms 2-D data from x-t domain onto τ-p domain. A hyperbola on a CDP gather maps onto an ellipse in τ-p domain (from Yilmaz, 1987). Compare corresponding points labeled A, A', B, B', C, and C'.

Chapter 2. Forward and Inverse τ-p Transformation
Figure 3 on page 9 shows the relations of patterns of different arrivals in \(x-t\) domain (a) and in \(\tau-p\) domain (b) for an earth model of four horizontal layers. There are three hyperbolic reflections A, B, and C; one direct arrival D to represent ground-roll; one direct arrival of body wave \(H_1\); and one linear arrival of refracted waves \(H_2\). In \(\tau-p\) domain, these events map onto different places and are designated with the same letters. Hyperbolic arrivals of reflections become ellipses \(A, B,\) and \(C\); direct arrival of body wave goes into point \(H_1\); linear arrival of refracted waves goes into point \(H_2\); another linear event, the ground-roll arrival D, was not included in the figure because the choice for the maximum value of \(p\) \((p_{\text{max}})\) is not large enough to include the slow apparent velocity of this arrival. It is obvious that the direct arrival D would have been mapped to a point in \(\tau-p\) domain at \(\tau = 0\) and \(p = \frac{1}{V_D}\) \((V_D\) is the apparent horizontal phase velocity for direct arrival D) if a larger \(p_{\text{max}}\) were used.

**Aliasing in the \(\tau-p\) transform**

Because seismic data usually is not sampled with a sufficiently small spatial interval, spatial aliasing limits the application of \(\tau-p\) processing to seismic reflection data. Spatial aliasing arises when the spatial sample interval is too large with respect to the wave propagation velocity and frequency. To avoid spatial aliasing, the spatial sample interval \(\Delta x\) must be selected to satisfy the following relation (Stoffa *et al.* 1980):

\[
\Delta x \leq \frac{V}{2f_{\text{max}}}
\]  

(2.8)

or

\[
\Delta x \leq \frac{\lambda_{\text{min}}}{2}
\]
Figure 3. Construction of the slant stack (t-p transform): Left panel (A) schematically represents events on a seismic field record. Hyperbolae A, B, and C represent reflections. Linear events D, H₁, and H₂ represent a direct arriving ground roll (D), a direct body wave, and a refraction. The right panel (B), represents the equivalent t-p domain. Events that interfere in the left panel are separated in the right panel. Reflection hyperbolae A, B, and C become ellipses A, B, and C. The linear event D has a slope of 980 μs/m and is thus beyond the range of p values in this transform. Direct arrival of body wave H₁ and refraction H₂ become points H₁ and H₂ in t-p domain. (modified from Tatham et al., 1983b).
where $V$ is the lowest horizontal phase velocity and $f_{\text{max}}$ is the maximum available frequency in the recorded data and $\lambda_{\text{min}}$ is the corresponding wavelength.

The aliasing in the $\tau$-$p$ transform is discussed in detail by Turner (1990). To avoid the aliasing in inverse $\tau$-$p$ transform, the increment $\Delta p$ can be selected to satisfy:

$$\Delta p < \frac{1}{x_{\text{r}}f_{\text{max}}}$$  \hspace{1cm} (2.9)

where $x_{\text{r}}$ is the range of $x$ values. Some selected examples of the synthetic seismic data regarding aliasing in $\tau$-$p$ transform are discussed in the next chapter.
Chapter 3. Amplitude Balancing in $\tau$-$p$ Domain

Generally, the seismic root-mean-square velocity ($V_{RMS}$) increases by depth and the reflection arrivals from deeper reflectors (i.e. B in Figure 3) have a smaller amount of normal moveout than the arrivals from shallower reflectors (i.e. A in Figure 3). The moveout forms hyperbolic patterns in $x$-$t$ domain and the hyperbolic patterns exhibit less curvature by depth. A hyperbola transforms to an ellipse by slant stack where the curvature of ellipse is also determined by the RMS velocity at a depth in question (Cutler et al., 1980). The higher the RMS velocity, the smaller the ellipse is. In $\tau$-$p$ domain, this moveout pattern change results in elliptical patterns that occupy smaller areas by depth (Figure 3). The envelope of the ellipses, each representing a different reflector, forms a boundary that separates the reflected energy from others. It is interesting to note that the region on the left hand side of this envelope has a general shape of an inverted half-cone (Figure 3) regardless of the velocity function and dips of the reflectors (Diebold et al. 1981). This envelope divides the $\tau$-$p$ plane into two regions. The one on the left of this envelope (Figure 3) is called the signal region and the area left outside the signal region is called the noise region in this study.
Amplitude balancing in $\tau$-$p$ domain

In $\tau$-$p$ domain, the information (arrival time, amplitude, and phase) carried out by reflections are limited to the signal region. The direct arrivals share a common point at zero time with the ellipse of the first reflector if the velocity of the first layer is constant. The arrivals of multiple of primary reflections lie in both signal region and noise region. For small offsets, multiples and reflections that coincide at the same time have very close trajectories in $x$-$t$ domain, therefore transformations of these data into $\tau$-$p$ domain result in the ellipses that are quite close to each other at smaller $p$ values. A multiple represents a low velocity for its time and therefore has larger moveout than the reflection that coincides at the same two-way travel time. This moveout difference increases by offset. In $\tau$-$p$ domain, this translates into the ellipses that become separated at higher values of the ray parameter ($p$). If there is a velocity increase within the first layer, then the low velocity direct arrivals are also in the noise region. The arrivals of ground-roll are limited to the noise region at smaller times and at high $p$ values.

Transformation of the seismic reflection data into separable signal and noise regions of $\tau$-$p$ domain leads to an application that enhances the signal-to-noise ratio. Since the signal and noise fields are separated in $\tau$-$p$ domain, one can attenuate the amplitudes in the noise region in $\tau$-$p$ domain to enhance the signal-to-noise ratio when the data is transformed back to $x$-$t$ domain. This process as a new application introduced in this study is called the amplitude balancing in $\tau$-$p$ domain.

Ideally, it is desirable to define a boundary between signal and noise regions to attenuate the amplitudes in the noise region. In practice, the boundary separating the signal and noise regions is usually not clearly defined and therefore a complete cancellation of the amplitudes of the noise region is not always possible. Furthermore, the boundary changes from one shot or CDP to another due to variations in geology, surface velocity, coupling conditions, and other random factors. Even if this boundary is distinct, it is impractical to examine and define the boundary in $\tau$-$p$ domain for each gather to cancel the amplitudes of the noise region of each gather. In this study, as a
compromised solution, a 2-D gain function is defined in \( \tau-p \) domain and used to attenuate the amplitudes of the noise region. A smoothly varying 2-D gain function is used to multiply each amplitude in \( \tau-p \) domain by a different scalar in a way that boosts the signal-to-noise ratio.

The steps of the amplitude balancing in \( \tau-p \) domain to attenuate noise are:

- Transform a typical CMP gather into \( \tau-p \) domain using a wide range of ray parameters;
- Identify the ellipses for reflections in \( \tau-p \) domain;
- Determine the envelope of these ellipses of reflections from different reflectors;
- Define a 2-D gain function that shows no or minimum change along the envelope while it reflects the desired amount of attenuation for noise region in the direction perpendicular to the envelope;
- Apply this initial 2-D gain function to the gather in \( \tau-p \) domain;
- Inverse transform the noise attenuated \( \tau-p \) data back to \( x-t \) domain;
- Examine the effects of processing by comparing the input and output data;
- Repeat the above steps while modifying the gain function in \( \tau-p \) domain until satisfactory results are obtained;

A successful application of the amplitude balancing in \( \tau-p \) domain is expected to suppress the unwanted noise such as air wave, ground-roll and thus enhances the reflected events. In addition to the amplitude balancing in \( \tau-p \) domain, a narrower range for the ray parameter can be defined in \( \tau-p \) transform to leave out some unwanted arrivals of low velocity events, such as air waves, ground-roll, etc.

**Synthetic data**

The application of the amplitude balancing in \( \tau-p \) domain given above is tested using synthetic data. Two dimensional synthetic seismic data is generated using the Advanced Interpretive Mod-
eling System (AIMS) software package on the Regional Geophysics Laboratory VAX computer to test the proposed amplitude balancing in \( \tau-p \) domain. A 2-D subsurface model used to generate a synthetic seismic record for testing the amplitude balancing in \( \tau-p \) domain is shown in Figure 4. The model consists of seven horizontal layers above a half space. Each layer is considered to have a constant velocity \( (V_{\text{in}}) \). Model parameters such as depth, thickness, interval velocity \( (V_{\text{int}}) \), root-mean-square velocity \( (V_{\text{RMS}}) \), average velocity \( (V_{\text{ave}}) \), and depth in two-way travel time \( (t_0) \) are tabulated in Table 1.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Depth (m)</th>
<th>Thickness (m)</th>
<th>( V_{\text{int}} ) (m/s)</th>
<th>( V_{\text{RMS}} ) (m/s)</th>
<th>( V_{\text{ave}} ) (m/s)</th>
<th>( t_0 ) (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Layer 1</td>
<td>200</td>
<td>200</td>
<td>1800</td>
<td>1800</td>
<td>1800</td>
<td>222</td>
</tr>
<tr>
<td>Layer 2</td>
<td>500</td>
<td>300</td>
<td>2200</td>
<td>2030</td>
<td>2020</td>
<td>495</td>
</tr>
<tr>
<td>Layer 3</td>
<td>600</td>
<td>100</td>
<td>3200</td>
<td>2193</td>
<td>2153</td>
<td>557</td>
</tr>
<tr>
<td>Layer 4</td>
<td>750</td>
<td>150</td>
<td>4000</td>
<td>2477</td>
<td>2372</td>
<td>632</td>
</tr>
<tr>
<td>Layer 5</td>
<td>1000</td>
<td>250</td>
<td>4700</td>
<td>2904</td>
<td>2707</td>
<td>739</td>
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<tr>
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<td>3066</td>
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</tr>
<tr>
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<td>200</td>
<td>6300</td>
<td>3636</td>
<td>3292</td>
<td>911</td>
</tr>
</tbody>
</table>

The synthetic shot record generated from the subsurface model given in Table 1 using the ray tracing option in the AIMS modeller is shown in Figure 5. The record consists of 96 traces with a trace length of 1.2 seconds and sample rate of 4 ms. The near and far offsets of 0 m and 1900 m, respectively, were assumed with a group interval represented by an offset increment of 20 m. A Ricker wavelet with peak frequency of 37 Hz \( (f_{\text{max}} = 80 \text{ Hz}) \) is used in modeling.

The shot gather obtained from the AIMS modeller is transformed onto \( \tau-p \) domain using the ray parameters from \( p_{\text{min}} = 0 \text{ m/s} \) to \( p_{\text{max}} = 600 \text{ m/s} \) with an increment of \( \Delta p = 6.32 \text{ m/s} \) when the total number of \( p \) traces is \( n_p = \frac{p_{\text{max}} - p_{\text{min}}}{\Delta p} + 1 = 96 \). The \( p_{\text{max}} \) chosen here is determined by \( \frac{1}{V_{\tau}} \), and the \( \Delta p \) chosen here is determined by equation (2.8) where \( f_{\text{max}} \) is about 80 Hz and \( x \) is 1900 m. The result of slant-stack is shown in Figure 6.

In Figure 6, it is clear that the hyperbolic reflection trajectories in \( x-t \) domain become elliptical trajectories in \( \tau-p \) domain. The crossed hyperbolic reflection trajectories in \( x-t \) domain in Figure 5 become separated elliptical trajectories in \( \tau-p \) domain in Figure 6.
Figure 4. Subsurface model for synthetic seismic data: The subsurface model used to generate a general synthetic seismic record consists of seven horizontal layers over a half space. Each layer is considered to have a constant velocity ($V_m$). Model parameters such as depth, thickness, interval velocity ($V_i$), root-mean-square velocity ($V_{RMS}$), average velocity ($V_{RMS}$), and depth in two-way travel time ($t_0$) are tabulated in Table 1.
Figure 5. Synthetic shot record: The shot gather is generated from the subsurface model shown in Figure 4. The record consists of 96 traces with a trace length of 1.2 seconds and sample rate of 4 ms. The near and far offsets are 0 m and 1900 m, respectively. The group interval is represented by the offset increment of 20 m. A Ricker wavelet with peak frequency of 37 Hz ($f_{max} = 80$ Hz) is used in modeling.
Figure 6. Forward $\tau$-$p$ transform of synthetic data: This data is the $\tau$-$p$ transform of the synthetic data shown in Figure 5. The range of the ray parameters is defined by $p_{\text{min}} = 0$ $\mu$s/m and $p_{\text{max}} = 600$ $\mu$s/m. The number of $p$ traces is $n_p = 96$. The crossed hyperbolic reflection trajectories in $x$-$t$ domain in Figure 5 become separated elliptical trajectories in $\tau$-$p$ domain in the figure. There are two kinds of linear streaks presented in $\tau$-$p$ transformed data. One of them is parallel to the timing lines at the two way reflection times (N) due to near-offset truncation. The second kind of linear streaks is tangent to the ellipses at higher $p$ values (F) due to far-offset truncation. Also, there are some high frequency noise build up in slant stacked data. These high frequency noises are mainly located in areas of high $p$. 

Chapter 3. Amplitude Balancing in $\tau$-$p$ Domain
There are two kinds of linear streaks presented in \( \tau-p \) transformed data (Figure 6). One of them is parallel to the timing lines at the two way reflection times (N in Figure 6). This type of linear streaks is caused by the nonzero contributions to the summation due to near-offset truncation of \( x-t \) data (Schultz et al., 1976). The second kind of linear streaks is tangent to the ellipses at higher \( p \) values (F in Figure 6). This type of linear streak is caused by the nonzero contributions to the summation due to far-offset truncation. Also, there are some high frequency noise build up in slant stacked data. This high frequency noise, which is also observed by Yilmaz (1987), is mainly located in areas of high \( p \).

To test the software, the data shown in Figure 6 is transformed back to \( x-t \) domain using the inverse \( \tau-p \) transformation. The result of this inverse transform is shown in Figure 7 which is a reconstruction of the data in Figure 5. The reconstruction was carried out without any additional processing in \( \tau-p \) domain. From the comparison of the data in Figure 5 and in Figure 7, it is obvious that the original data are recovered in spite of some noise introduced by \( \tau-p \) processing steps.

The same shot gather shown in Figure 5 was transformed onto \( \tau-p \) domain using \( p_{\text{min}} = 0 \) \( \mu s/m \), \( p_{\text{max}} = 1200 \) \( \mu s/m \), and \( n_t = 96 \). These parameters correspond to a \( \Delta p = 12.6 \) \( \mu s/m \). The result of this \( \tau-p \) transformation is shown in Figure 8 where there is much more high frequency noise, especially in high \( p \) region. In the discussion of aliasing problem of slant stack, it was given that the ray parameter increment \( \Delta p \) should satisfy equation (2.8). Using \( x_c = 1900 \) m and \( f_{\text{max}} = 80 \) Hz, \( \Delta p \) is 6.58 \( \mu s/m \). By doubling the value of \( p_{\text{max}} \) while keeping the same number of \( p \) traces, \( \Delta p \) was doubled. The high frequency noise in Figure 8 is the result of a larger \( \Delta p \). The inverse \( \tau-p \) transform of the data in Figure 8 yields the data shown in Figure 9 where the noise is much higher than in the data shown in Figure 7 even though the same number of \( p \) traces was used. This illustrates that insufficient sampling in the \( p \) direction leads to aliasing in the inverse \( \tau-p \) transform.

The effects of ray parameter range in \( \tau-p \) transform was also tested. The synthetic shot gather shown in Figure 5 is transformed onto \( \tau-p \) domain using \( p_{\text{min}} = 0 \) \( \mu s/m \), \( p_{\text{max}} = 1200 \) \( \mu s/m \), and \( n_t = 192 \). These ray parameters ensure a \( \Delta p \) which satisfies the sampling criteria of \( \tau-p \) transform while \( p_{\text{max}} \) is greater than what is needed to cover the velocities of the data. The \( \tau-p \) transformed synthetic shot gather using the above parameters is shown in Figure 10 on page 23.

Chapter 3. Amplitude Balancing in \( \tau-p \) Domain
Figure 7. Inverse $\tau$-$p$ transform of synthetic data: This $x$-$t$ domain data is recovered from the $\tau$-$p$ data shown in Figure 6 using inverse $\tau$-$p$ transform.
Figure 8. Forward $\tau$-$p$ transform of synthetic data: This data is the $\tau$-$p$ transform of the synthetic data shown in Figure 5. The range of the ray parameters is defined by $p_{\text{min}} = 0$ $\mu$s/m, $p_{\text{max}} = 1200$ $\mu$s/m. The number of $p$ traces is $n_p = 96$. 

Chapter 3. Amplitude Balancing in $\tau$-$p$ Domain
Figure 9. Inverse $\tau$-$p$ transform of synthetic data: This data is the inverse $\tau$-$p$ transform of the $\tau$-$p$ data shown in Figure 8 on page 20. Because of a larger $\Delta p = 12.6 \, \mu s/m$, the aliasing effect is evident. Compare with the data in Figure 7 on page 19.
Figure 10, it is evident that high frequency noise is introduced by high $p_{\text{max}}$, especially in high $p$ region. The inverse $\tau$-$p$ transform of the data (Figure 10) is shown in Figure 11 on page 24.

There is some noise in the data shown in Figure 11. Compared with Figure 9, the data in Figure 11 exhibit less noise and better reconstruction, especially for high $p$ region. However Figure 11 is more noisy when it is compared with Figure 7 since the choice of a too big $p_{\text{max}}$ produces some noise, especially at small offset. This example illustrates that a satisfactory reconstruction of an $x$-$t$ data requires a good selection of $p_{\text{min}}$, $p_{\text{max}}$, and $\Delta p$. It appears that the choice of $\Delta p$ is more critical than $p_{\text{max}}$.

The effects of random noise in $\tau$-$p$ transformation were also tested. The synthetic shot gather shown in Figure 5 was regenerated with additive pseudo random noise and the result is shown in Figure 12. The level of the pseudo random noise was defined by the intensity reduction relative to the amplitude of the strongest event in the section. A 36 dB noise level was added to the synthetic data, Figure 5, and the result is shown in Figure 12.

The synthetic data with 36 dB pseudo random noise (Figure 12) was transformed onto $\tau$-$p$ domain using $p_{\text{min}} = 0 \mu s/m$, $p_{\text{max}} = 600 \mu s/m$, and $n_x = 96$ and the result is shown in Figure 13. The linear streaks observed in Figure 6 are buried in pseudo random noise and become less visible in Figure 13. The long tails of ellipses are shortened due to cancellation of pseudo random noise. Since the velocity structure of the model is known, the inverse of these RMS velocities gives the $p$ values where these elliptical tails should reach if an infinitely long spread length were used. The data needed to reconstruct these hyperbolic trajectories are within the signal region defined by the envelope of seven elliptical trajectories.

The amplitude balancing in $\tau$-$p$ domain is utilized to improve the signal-to-noise ratio by increasing the trace amplitudes within the signal region while suppressing the trace amplitudes within the noise region. The straight line defined by points ($\tau = 1100$ ms, $p = 0 \mu s/m$) and ($\tau = 0$ ms, $p = 600 \mu s/m$) is the approximated boundary separating the signal and noise region in $\tau$-$p$ domain as shown in Figure 14. A 2-dimensional amplitude balancing scheme (Table 2) was designed using 24 dB at ($\tau = 0$ ms, $p = 0 \mu s/m$), 12 dB at ($\tau = 1100$ ms, $p = 0 \mu s/m$), and at ($\tau = 0$ ms, $p = 600 \mu s/m$), 0 dB at ($\tau = 1200$ ms, $p = 0 \mu s/m$), ($\tau = 1100$ ms, $p = 600 \mu s/m$), and ($\tau = 1200$ ms, $p = 600 \mu s/m$).
Figure 10. Forward $\tau$-$p$ transform of synthetic data: This data is the $\tau$-$p$ transform of the synthetic data shown in Figure 5. The range of the ray parameters is defined by $p_{\text{min}} = 0 \mu$s/m, $p_{\text{max}} = 1200 \mu$s/m. The number of $p$ traces is $n_p = 192$. These ray parameters ensure a $\Delta p$ which satisfies the sampling criteria of $\tau$-$p$ transform while $p_{\text{max}}$ is greater than the optimum one. It is evident that high frequency noise is introduced by high $p_{\text{max}}$, especially in high $p$ region.

Chapter 3. Amplitude Balancing in $\tau$-$p$ Domain
Figure 11. Inverse τ-p transform of synthetic data: This data is the inverse τ-p transform of the τ-p data shown in Figure 10 on page 23. This is a better reconstruction of the original data (Figure 5) when it is compared with Figure 9 since a smaller Δp was used. However it is not as good as the data reconstructed shown in Figure 7 because the choice of a too big p_{max} produces some noise at small offset.
Figure 12. Synthetic shot record with 36 dB pseudo random noise: The shot gather is generated from the subsurface model shown in Figure 4. 36 dB pseudo random noise was added to the section shown in Figure 5 during modeling. Number of channels is 96. Receiver interval is 20 m. Nearest offset is 0 m and maximum offset is 1900 m.
Figure 13. Forward $\tau$-$p$ transform of synthetic data with pseudo random noise: This data is the $\tau$-$p$ transform of the synthetic data with pseudo random noise shown in Figure 12. The range of the ray parameters is defined by $p_{\text{min}} = 0$ $\mu$s/m, $p_{\text{max}} = 600$ $\mu$s/m. The number of $p$ traces is $n_p = 96$. The linear streaks observed in Figure 6 are buried in pseudo random noise and become less visible. The long tails of ellipses are shortened due to cancellation of pseudo random noise. Compare with Figure 6 to see the effects of pseudo random noise in $\tau$-$p$ transformation.
\( \mu s/m \). The gain values for the other \((\tau, p)\) coordinates were obtained by a linear interpolation. The interpolated gain values defined for all \((\tau, p)\) coordinates are multiplied with the amplitudes of the traces at these coordinates. The data after this gain manipulation is shown in Figure 14 where the amplitudes in the noise region are attenuated when compared with the data as shown in Figure 13. After this amplitude adjustment, the data (Figure 14) was input to an inverse \(\tau-p\) transformation and the result of this process is shown in Figure 15. Comparison of Figure 12 and Figure 15 indicates that the hyperbolic trajectories are recovered satisfactorily while the random noise level is reduced. This example illustrates that the amplitude balancing in \(\tau-p\) domain improves signal-to-noise ratio. Application of the method to the real data is discussed in the next chapter.

Table 2. 2-D gain function for amplitude balancing in \(\tau-p\) domain for modeled data

<table>
<thead>
<tr>
<th>(\tau) (ms)</th>
<th>0 ((\mu s/m))</th>
<th>600 ((\mu s/m))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>24</td>
<td>12</td>
</tr>
<tr>
<td>1100</td>
<td>12</td>
<td>0</td>
</tr>
<tr>
<td>1200</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 14. Amplitude balanced synthetic $\tau$-$p$ data: This is the synthetic $\tau$-$p$ data shown in Figure 13 on page 26 after amplitude balancing. The straight line defined by points ($\tau = 1100$ ms, $p = 0$ μs/m) and ($\tau = 0$ ms, $p = 600$ μs/m) is the approximated boundary separating the signal and noise region in $\tau$-$p$ domain as shown the figure. Compare with Figure 13 to see amplitude change after gain in $\tau$-$p$ domain.
Figure 15. Inverse $\tau$-$p$ transform of synthetic data with amplitude balancing in $\tau$-$p$ domain applied. This data is the inverse $\tau$-$p$ transform of the $\tau$-$p$ transformed data with amplitude balancing in $\tau$-$p$ domain applied shown in Figure 14. After amplitude balancing in $\tau$-$p$ domain, the hyperbolic trajectories are recovered satisfactorily while the random noise level is reduced. Compare with Figure 12 to see the improvement of signal-to-noise ratio.
Chapter 4. Reflection Seismic Data

The amplitude balancing in $\tau$-$p$ domain was applied to reflection seismic data acquired on the Atlantic Coastal Plain sediments. The method was applied to the real data sets different reasons. The lines used are: NRC line 2 recorded near Richmond, Virginia by the Regional Geophysics Laboratory vibroseis system of VPI&SU in November 1980; and SRP line 2EXP recorded near Aiken, South Carolina by Conoco in 1987. On NRC line 2, the method was used to enhance the reflections from within the Atlantic Coastal Plain sediments by attenuating high amplitude noise. On SRP line 2EXP, the method was tested to recover and enhance the weak reflections from within the Dunbarton Triassic basin beneath the Coastal Plain sediments. The $\tau$-$p$ domain processing was included as part of a conventional seismic data processing sequence before stacking the CMP sorted data.

NRC line 2

The acquisition parameters and other information are tabulated in Table 5 in Appendix 1. A typical shot record is displayed in Figure 16 (A). In this figure, event B is interpreted to be the
reflection from the top of the basement. Event GR is the large-amplitude surface waves (ground-roll) with an apparent velocity of \( \approx 580 \text{ m/s} \) and event AW is the air wave with an apparent velocity of \( \approx 330 \text{ m/s} \). The vibroseis whitening (Çoruh and Costain, 1983) was applied to NRC line 2 with AGC window length of 1000 ms to improve the signal-to-noise ratio and increase seismic resolution. Figure 16 (B) demonstrates the improvement of signal-to-noise ratio by vibroseis whitening. The basement reflection is enhanced and the surface waves and air wave are attenuated.

For a dipping reflector with depth \( h \) below shotpoint and dip angle \( \alpha \), the axis of symmetry of the traveltme curve is at \( x = -2h \sin \alpha \) (where \( x \) is offset) instead of the t-axis (Robinson and Çoruh, 1988, Figure 4-12). So in a shot gather, the apex of a reflection hyperbola is located at \( x \) for a dipping reflector. The hyperbola \( t = \frac{1}{V} \sqrt{x^2 + h^2} \) transforms to the ellipse \( \tau = h \sqrt{V^2 - p^2} \) by \( \tau-p \) transform. From the delay property of Radon transform, the hyperbola \( t = \frac{1}{V} \sqrt{(x - a)^2 + h^2} \) transforms to the ellipse \( \tau = h \sqrt{V^2 - p^2 - pa} \) (Durrani and Bisset, 1984). However, in a CMP gather, the apex of the reflection hyperbola is at zero-offset. Because of the limitation of \( \tau-p \) transform algorithm on DISCO, there is no negative \( \tau \) can be used during slant stack and therefore the \( \tau-p \) transform should be applied to CMP data rather than for shot data.

Turner (1990) suggested that the spacing of seismic traces \( \Delta x \) before forward \( \tau-p \) transform should satisfy equation (2.8). The lowest horizontal phase velocity was obtained as \( \approx 900 \text{ m/s} \). The maximum frequency in the data is 100 Hz. According to equation (2.8), the spacing between traces should be less than 4.5 m to avoid spatial aliasing. A CMP sorting using a linear geometry gives 24-fold data where the trace spacing is 9.6 m. So there is a chance of aliasing the data in forward \( \tau-p \) transform, especially for high frequency components. To avoid the spatial aliasing problem, two adjacent CMP gathers were combined into one gather and therefore the spacing between traces in CMP domain was only 4.8 m which satisfies equation (2.8). Figure 17 shows three CMP gathers (after vibroseis whitening) at three different locations of NRC line 2 after combining two adjacent CMP gathers together. The surface waves (GR) and air wave (AW) are still visible after the vibroseis whitening application. The same CMP gathers without vibroseis whitening are displayed in Figure 18. After the comparison between CMP gathers with and without vibroseis whitening, it is concluded that the data with vibroseis whitening shown in Figure 17 have better
Figure 16. Shot data from NRC line 2 with and without vibroseis whitening: The left seismogram is the original shot record. The right seismogram is the same shot data after vibroseis whitening. Event B is interpreted to be the reflection from the top of the basement beneath the Atlantic Coastal Plain sediments. Event GR is the large-amplitude surface wave with an apparent velocity of $\approx 580$ m/s and event AW is the air wave with an apparent velocity of $\approx 340$ m/s.
signal-to-noise ratio and higher resolution and therefore vibroseis whitening was applied to NRC line 2 for further processing.

Noise attenuation by the application of amplitude balancing in $\tau$-$p$ domain was first tested on three CMP gathers shown in Figure 17. The forward $\tau$-$p$ transform was performed on each CMP gather using a range of the ray parameters defined by $p_{\min} = 0 \ \mu s/m$, and $p_{\max} = 5000 \ \mu s/m$ with the number of $p$ traces of $n_p = 200$. Figure 19, Figure 20, and Figure 21 are the results of forward $\tau$-$p$ transform of NRC line 2 data for the three CMP gathers shown in Figure 17.

By limiting the ray parameter range $(p_{\min}, p_{\max})$ during the reconstruction of $x$-$t$ data by inverse $\tau$-$p$ transform, events with slow velocities can be attenuated or even removed, and this is called dip filtering in $\tau$-$p$ domain which is nearly equivalent to $f$-$k$ dip filtering (Yilmaz, 1987). The purpose of this study is not to apply the dip filtering in $\tau$-$p$ domain but to examine the effect due to amplitude balancing in $\tau$-$p$ domain only. By choosing a large $p$ range, original data can be recovered by inverse $\tau$-$p$ transform with no additional process applied in $\tau$-$p$ domain. Events with slow horizontal phase velocities are preserved and the smearing effect of slant stack is avoided and thus the effect of amplitude balancing in $\tau$-$p$ domain is isolated from other processes. The reason of choosing a large number of $p$ traces $(n_p)$ is to have sufficient sampling in the $p$ direction. For NRC line 2, the range of offset $x$, is 226 m and the highest frequency in the data is 100 Hz. From equation (2.9), $\Delta p$ should be less than 44 $\mu s/m$. A $p$ range of of 5000 $\mu s/m$ with a $n_p$ of 200 gives a $\Delta p$ of 25 $\mu s/m$ which satisfies equation (2.9).

A 2-D scalar, in decibels, was formed (Table 3) after extensive testing to apply the amplitude balancing in $\tau$-$p$ domain to NRC line 2. Different 2-D gain functions representing amplitude change between 0 and 36 dB with 6 dB increment were used to define the optimum gain parameter. The amplitude balancing in $\tau$-$p$ domain was applied to the data shown in Figure 19, Figure 20, and Figure 21 and the results are shown in Figure 22, Figure 23, and Figure 24.

<table>
<thead>
<tr>
<th>$\tau$ (ms)</th>
<th>$p$ ($\mu s/m$)</th>
<th>$p$ (900 $\mu s/m$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>1000</td>
<td>18</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 17. Three typical CMP gathers for NRC line 2 with vibroseis whitening: Three CMP gathers (#128, #420, #702) were obtained by combining two adjacent CMP gathers of 24-fold. The number of fold is 48 after the combining. The trace interval is 4.8 m. The reflections from the top of the basement are labeled B. The surface waves GR are still visible with an apparent velocity of $\approx 587$ m/s even though vibroseis whitening was applied. Compare with Figure 18 to examine the signal-to-noise ratio improvement by vibroseis whitening.
Figure 18. Three typical CMP gathers for NRC line 2 without vibroseis whitening: Three CMP gathers (#128, #420, #702) are obtained by combining two adjacent CMP gathers of 24-fold. The number of fold is 48 after the combining. The trace interval is 4.8 m. The reflections from the top of the basement are labeled B. The surface waves (GR) are dominant in the section with an apparent velocity of $\approx 587$ m/s.
Figure 19. Forward t–p transform of CMP #128 of NRC line 2 data. This data is the t–p transform of the CMP #128 shown in Figure 17. The range of the ray parameters is defined by $f_{\text{min}} = 0$ ps, $f_{\text{max}} = 5000$ ps. The number of p traces is $n_p = 200$.  

Chapter 4. Reflection seismic data
Figure 20. Forward $\tau$-$p$ transform of CMP #420 of NRC line 2 data: This data is the $\tau$-$p$ transform of the CMP #420 shown in Figure 17. The range of the ray parameters is defined by $p_{\text{min}} = 0 \ \mu s/m$, $p_{\text{max}} = 5000 \ \mu s/m$. The number of $p$ traces is $n_p = 200$. 

Chapter 4, Reflection seismic data
Figure 21. Forward $\tau$-$p$ transform of CMP #702 of NRC line 2 data: This data is the $\tau$-$p$ transform of the CMP #702 shown in Figure 17. The range of the ray parameters is defined by $p_{\text{min}} = 0 \ \mu$s/m, $p_{\text{max}} = 5000 \ \mu$s/m. The number of $p$ traces is $n_p = 200$. 
Figure 22. Amplitude balanced τ-p data (CMP #128): This is the τ-p data shown in Figure 19 after amplitude balancing. The straight line defined by points (τ = 0 ms, p = 900 µs/m) and (τ = 1000 ms, p = 600 µs/m) is the approximated boundary separating the signal region on the left and the noise region on the right in τ-p domain. Compare with Figure 19 to see the amplitude changes after gain in τ-p domain.
Figure 23. Amplitude balanced \( \tau-p \) data (CMP \#420): This is the \( \tau-p \) data shown in Figure 20 after amplitude balancing. The straight line defined by points \((\tau = 0 \text{ ms}, p = 900 \mu s/m)\) and \((\tau = 1000 \text{ ms}, p = 600 \mu s/m)\) is the approximated boundary separating the signal region on the left and the noise region on the right in \( \tau-p \) domain. Compare with Figure 20 to see the amplitude changes after gain in \( \tau-p \) domain.
Figure 24. Amplitude balanced $\tau$-$p$ data (CMP #702): This is the $\tau$-$p$ data shown in Figure 21 after amplitude balancing. The straight line defined by points ($\tau = 0$ ms, $p = 900$ $\mu$s/m) and ($\tau = 1000$ ms, $p = 600$ $\mu$s/m) is the approximated boundary separating the signal region on the left and the noise region on the right in $\tau$-$p$ domain. Compare with Figure 21 to see the amplitude changes after gain in $\tau$-$p$ domain.
The $x$-$t$ domain data were reconstructed by inverse $\tau$-$p$ transforming the data shown in Figure 22, Figure 23, and Figure 24 and the results are displayed in Figure 25. Compared with the $x$-$t$ data without amplitude balancing in $\tau$-$p$ domain displayed in Figure 17, the $x$-$t$ data with amplitude balancing in $\tau$-$p$ domain show the attenuated surface waves and enhanced reflections from the basement top in addition to the arrivals from within the basement. The improvement of signal-to-noise ratio is significant.

For further signal-to-noise ratio improvement, predictive deconvolution in $\tau$-$p$ domain (Alam and Austin, 1981) was applied. A preliminary choice of gap and prediction operator length was determined from the autocorrelogram of $p$ traces. A two-step test was performed to get the optimum parameters for gap and operator length for a short CMP range in a similar way given by Yilmaz (1987) for deconvolution in $x$-$t$ domain:

1. Testing for an optimum gap:
   - While keeping the preliminary operator length constant, deconvolve CMP gathers in $\tau$-$p$ domain with different gaps.
   - Inverse $\tau$-$p$ transform the deconvolved CMP gathers back to $x$-$t$ domain.
   - Stack the data in $x$-$t$ domain and display them as panels.
   - Determine the optimum gap by examining the displayed panels.

2. Testing for an optimum operator length:
   - Using the optimum gap determined previously, deconvolve CMP gathers in $\tau$-$p$ domain with different operator length.
   - Inverse $\tau$-$p$ transform the deconvolved CMP gathers to $x$-$t$ domain.
   - Stack the CMP data in $x$-$t$ domain and display them.
   - Pick the optimum operator length by examining the quality of the stacked data.

The two-step deconvolution test in $\tau$-$p$ domain led to a gap of 8 ms and an operator length of 600 ms. Then the data in CMP domain were deconvolved in $\tau$-$p$ domain and the results are shown in Figure 26. Deconvolution in $\tau$-$p$ domain further improved the data quality.
Figure 25. Inverse $\tau-p$ transform of NRC line 2 data after amplitude balancing in $\tau-p$ domain: This $x-t$ data is the inverse $\tau-p$ transform of the $\tau-p$ transformed data after amplitude balancing in $\tau-p$ domain. The reflections from the basement top (B) are enhanced, the surface waves (GR) and air waves (AW) are attenuated, and the improvement of signal-to-noise ratio is obtained.
Figure 26. CMP gathers after amplitude balancing in τ-p domain and deconvolution in τ-p domain: Predictive deconvolution with a gap of 8 ms and an operator length of 600 ms was applied after amplitude balancing. Compare this figure with Figure 25 to see the effect of deconvolution in τ-p domain.
The NRC line 2 has a very small spread length of 226 m. The amount of normal moveout is equal to sampling rate of 2 ms at 274 m offset for a two-way travel time of 0.8 s. Therefore no velocity resolution is expected for deeper part of the data. After picking preliminary velocity functions for the line, the CMP data is NMO corrected and brute stacked. The near-surface irregularities deviate seismic arrival times of reflection from regular hyperbolic moveout. Surface-consistent residual statics estimations were used to minimize this effect. The assumption about surface-consistent statics means velocity of the shallow top layer (weathering layer, usually) is extremely slow so that raypaths under the surface are near vertical within the low velocity zone. Therefore, the static shifts depend only on the location of the surface independent from the depth of reflection. The input to the surface-consistent statics computation module is windowed CMP gathers with good signal-to-noise ratio. The traces were corrected for normal moveout and events in the window were adjusted for structural variations before estimations were performed. The estimated statics using crosscorrelation were decomposed into residual statics for the source and receiver locations. For NRC line 2, a 300 ms window and a maximum time shift of 8 ms were used to estimate the residual statics. Four passes, each with four iterations and a new velocity pick, were performed on the data with pilot traces formed from 7 to 21 CMP gathers. A 20-80 Hz band-pass filter was applied to the CMP data before crosscorrelation for residual static calculation. The stack section after residual statics correction showed a significant improvement in the continuity of reflections.

Figure 27 shows four stack sections obtained by different processes: (A) is the stack section with conventional x-t domain processing only; (B) is the stack section with amplitude balancing in \( \tau-p \) domain; (C) is the stack section with conventional x-t domain processing, including a predictive deconvolution before stack. The predictive gap is 12 ms and the operator length is 550 ms; (D) is the stack section with amplitude balancing in \( \tau-p \) domain and deconvolution in \( \tau-p \) domain. From these figures, it is noticed that the improvement observed on the CMP data by amplitude balancing in \( \tau-p \) domain is not that apparent after the stack. Combination of the amplitude balancing in \( \tau-p \) domain and deconvolution in \( \tau-p \) domain gives better result than deconvolution in x-t domain for this test. Deconvolution in \( \tau-p \) domain improves the vertical and horizontal resolution for reflections above 350 ms, especially for the horizon at about 150 ms.
Figure 27. Four stack sections with different processing steps: (A) is the stack section with conventional $x-t$ domain processing only; (B) is the stack section with amplitude balancing in $\tau-p$ domain; (C) is the stack section with conventional $x-t$ domain processing, including a predictive deconvolution before stack. (D) is the stack section with amplitude balancing in $\tau-p$ domain and $\tau-p$ domain deconvolution.
For further signal-to-noise ratio improvement, the following poststack processing steps were applied to NRC line 2: a 35-100 Hz band-pass filtering, a predictive deconvolution with 6 ms gap and 600 ms operator length, and an AGC with 390 ms window. The final processed section is shown in Figure 28. The section to be compared is shown in Figure 29 which is processed by the same processing sequence and the same processing parameters except the application of \( \tau-p \) domain processing. From these two sections, it is evident that the steps in \( \tau-p \) domain improves the signal-to-noise ratio and is applicable to some data.

**SRP line 2EXP**

The signal-to-noise ratio enhancement obtained by the amplitude balancing in \( \tau-p \) domain motivated the implementation of the method to improve weak reflections from within the Dunbarton Triassic basin in South Carolina, beneath the Atlantic Coastal Plain sediments using SRP line 2EXP (W. J. Domoracki in prep). The acquisition parameters are tabulated in Table 5 on page 67 in Appendix 1. SRP line 2EXP was processed by W. J. Domoracki using a conventional seismic data processing sequence. During this test, the \( \tau-p \) domain amplitude balancing process was embedded in that sequence to examine the improvement in the quality of data. Three CMP gathers from the line are displayed in Figure 30. The high amplitude reflections at about 400 ms were interpreted as the reflections from the top of unweathered basement rock (W. J. Domoracki, personal communications). For CMP #1673, the top of the unweathered basin fill is at about 400 ms. The forward \( \tau-p \) transform was performed on these CMP gathers using a ray parameter range of \( p_{\text{min}} = 0 \) \( \mu s/m \) and \( p_{\text{max}} = 4000 \) \( \mu s/m \) with the number of \( p \) traces of \( n_p = 400 \). Figure 31, Figure 32 and Figure 33 are the results of the forward \( \tau-p \) transform of SRP line 2EXP data.

The amplitude balancing in \( \tau-p \) domain was applied to the data using the scaling factors displayed in Table 4. The CMP gathers after amplitude balancing in \( \tau-p \) domain are shown in
Final stacked section of NRC line 2 with amplitude balancing in $t-p$ domain and deconvolution in $t-p$ domain. The amplitude balancing in $t-p$ domain and deconvolution in $t-p$ domain were imbedded in a conventional processing sequence to enhance the data quality. 

Figure 28.
Figure 29. NRC line 2 stack section without τ-p domain application: This stack section was processed by the same processing sequence and the same processing parameters except the application of τ-p domain processing. Compare with Figure 28 to examine the difference.
Figure 34, Figure 35 and Figure 36. These figures are to be compared with Figure 31, Figure 32 and Figure 33 to examine the amplitude change in $\tau$-$p$ domain by amplitude balancing in $\tau$-$p$ domain. The $x$-$t$ domain data were recovered by the inverse $\tau$-$p$ transform and the results are displayed in Figure 37. Compared with the $x$-$t$ data without amplitude balancing in $\tau$-$p$ domain shown in Figure 30, the reconstructed $x$-$t$ data after amplitude balancing in $\tau$-$p$ domain shown in Figure 37 have better definition of reflection hyperbolae and higher signal-to-noise ratio.

**Table 4.** 2-D gain function for amplitude balancing in $\tau$-$p$ domain for SRP line 2EXP.

<table>
<thead>
<tr>
<th>$\tau$ (ms)</th>
<th>$0$ ((\mu)s/m)</th>
<th>$900$ ((\mu)s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>12</td>
<td>6</td>
</tr>
<tr>
<td>1000</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

To minimize the effect of velocity selection, constant velocity panels were used to examine the effect of the amplitude balancing in $\tau$-$p$ domain on SRP line 2EXP. Figure 38 and Figure 39 are the constant velocity panels with and without amplitude balancing in $\tau$-$p$ domain. From these two figures, the signal-to-noise ratio improvement is barely discernible. Further tests using velocity spectrum analysis performed on CMP #1673 suggest that amplitude balancing in $\tau$-$p$ domain did help to enhance the reflections below 400 ms, however, the coherency values from the reflections below 400 ms within the Dumbarton Triassic basin were quite small compared with the coherencies of reflections above 400 ms which is within Atlantic Coastal Plain sediments. This indicates that the smoothing effect of the stacking processing is filtering the effect of the amplitude balancing in $\tau$-$p$ domain. Figure 40 is the velocity spectrum of CMP #1673 data shown in Figure 30 with the maximum and minimum coherencies displayed on the top and coherency contour displayed on the bottom. The range of coherency values for the reflections above 400 ms are relatively high when compared with the range of coherency values for the reflections below 400 ms. Figure 41 is the velocity spectrum of the same CMP data with amplitude balancing in $\tau$-$p$ domain applied. The range of coherencies is overall increased. Weak reflections below 400 ms were enhanced, giving higher coherency values for the data below 400 ms. However, the coherency values are still relatively small for the deeper part of the data (below 400 ms) when they are compared with the shallower part. From the comparison of the $x$-$t$ data before and after amplitude balancing in $\tau$-$p$
Figure 31. Forward $\tau$-$p$ transform of CMP #1400 of SRP line 2EXP data: This data is the $\tau$-$p$ transform of the CMP #1400 shown in Figure 30. The range of the ray parameters is defined by $p_{\text{min}} = 0 \mu s/m$, $p_{\text{max}} = 4000 \mu s/m$. The number of $p$ traces is $n_p = 400$. 
Figure 32. Forward τ-p transform of CMP #1500 of SRP line 2EXP data: This data is the τ-p transform of the CMP #1500 shown in Figure 30. The range of the ray parameters is defined by $\tau_{\min} = 0 \mu s/m$, $\tau_{\max} = 4000 \mu s/m$. The number of p traces is $n_p = 400$. 
Figure 33. Forward $p_n$ transform of CMP #1673 of SRR line 2EXP data. This data is the $p_n$ transform of the CMP #1673 shown in Figure 30. The range of the $p_n$ transform is defined by $P_{min} = 0$ m/s, $P_{max} = 400$ m/s. The number of $p_n$ trances is $N_p = 400$. 

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Figure 34. Amplitude balanced -p data (CMP #14000): This is the "p" data shown in Figure 31 after amplitude balancing. Compare with Figure 31 to see the amplitude changes after gain in "p" domain.
Figure 35. Amplitude balanced r-p data (CMP #1500): This is the r-p data shown in Figure 32 after amplitude balancing. Compare with Figure 32 to see the amplitude change after gain in r-p domain.
Figure 36. Amplitude balanced $\tau$-$p$ data (CMP #1673): This is the $\tau$-$p$ data shown in Figure 33 after amplitude balancing. Compare with Figure 33 to see the amplitude change after gain in $\tau$-$p$ domain.
Figure 37. Inverse $\tau$-$p$ transform of SRP line 2EXP data after amplitude balancing in $\tau$-$p$ domain: This $x$-$t$ data is the inverse $\tau$-$p$ transform of the $\tau$-$p$ transformed data after amplitude balancing in $\tau$-$p$ domain. The reflections are enhanced, and signal-to-noise ratio is improved (compare with Figure 30 to see the data quality enhancement by amplitude balancing in $\tau$-$p$ domain).
domain and the comparison of the velocity spectra before and after amplitude balancing in \( \tau-p \) domain, it is concluded that amplitude balancing in \( \tau-p \) domain did help to enhance the weak reflection signals from within the Dunbarton Triassic basin to a limited extent.
Figure 38. Constant velocity analysis panels for SRP line 2EXP with amplitude balancing in $\tau$-$p$ domain: Unstacked, CMP #1660-#1690 from SRP line 2EXP were processed by amplitude balancing in $\tau$-$p$ domain and then input to a constant velocity analysis module. Stacked CMPs are plotted together in one panel for the velocity shown on the top of the panel.
Figure 39. Constant velocity analysis panels for SRP line 2EXP without amplitude balancing in τ-p domain: The same CMP data were input to a constant velocity analysis module without being processed by amplitude balancing in τ-p domain first.
Figure 40. Velocity spectrum of CMP #1673: This is the velocity spectrum of CMP gather #1673 shown in Figure 30 with coherency function plotted on the top and coherency contour plotted on the bottom. The coherency values are small for the data below 400 ms.
Figure 41. Velocity spectrum of CMP #1673 with amplitude balancing in $\tau-p$ domain applied: This is the velocity spectrum of CMP gather #1673 with amplitude balancing in $\tau-p$ domain applied shown in Figure 37. The enhanced coherencies for the data below 400 ms are obtained. However, they are still relatively small compared with the coherency values above 400 ms.
Chapter 5. Conclusions

The amplitude balancing in $\tau$-$p$ domain introduced in this study is a new approach to enhance the quality of seismic data. During forward and inverse $\tau$-$p$ transformation, special attention should be paid concerning the aliasing problem. The choice of $\Delta p$ is more critical than the choice of $p_{\text{max}}$ provided that the $p$ range is large enough to include low velocities for reconstruction of $x$-$t$ domain data. A good reconstruction of $x$-$t$ data using inverse $\tau$-$p$ transform only needs the $\tau$-$p$ data located within the signal region as defined in this study. Random noise observed on the original $x$-$t$ data can be canceled by two-way $\tau$-$p$ transformations. Tests performed on synthetic data showed the improved signal-to-noise ratio by amplitude balancing in $\tau$-$p$ domain. Because of the limitations of the $\tau$-$p$ transform algorithms of DISCO package, only positive zero-offset time can be used. It is therefore preferable to apply a $\tau$-$p$ transform to CMP data rather than to shot data.

The application of amplitude balancing in $\tau$-$p$ domain to NRC line 2 data showed enhanced reflections and partially attenuated noise, including low-speed air wave and dispersive ground-roll, without using the dip filter effect of $\tau$-$p$ transform by limiting the $p$ range. Improvements by amplitude balancing in $\tau$-$p$ domain observed on CMP gathers were more apparent than after stacking the data. The deconvolution performed in $\tau$-$p$ domain was applied to NRC line 2 data and the comparison between the deconvolution in $x$-$t$ domain and deconvolution in $\tau$-$p$ domain suggests that it is preferable to apply deconvolution in $\tau$-$p$ domain. Tests performed for SRP line 2EXP data suggested that the amplitude balancing in $\tau$-$p$ domain improves the signal-to-noise ratio. Velocity
spectra showed the enhanced signal for deeper part of the data. However, because of the small coherency values for deeper part of the data, constant velocity analysis did not show the enhanced signal for the data below 400 ms because of the smoothing effect of stacking.

Further study of the method could be done by testing the method on more seismic data sets, especially on marine seismic data with long spread length and small group interval.
Appendix A. Seismic Reflection Data Parameters

Field parameters

Acquisition parameters and field information for NRC line 2 and SRP fine 2EXP line are listed in Table 5 on page 67.
Table 5. Acquisition parameters and field information for NRC line 2 and SRP line 2EXP

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NRC2</th>
<th>SRP2EXP</th>
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<tbody>
<tr>
<td>Year Acquired</td>
<td>1980</td>
<td>1987</td>
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<tr>
<td>Sweep Length (s)</td>
<td>20</td>
<td>8</td>
</tr>
<tr>
<td>Sweep Frequency (Hz)</td>
<td>100-20</td>
<td>30-150</td>
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<tr>
<td>Taper Length (s)</td>
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<td>.50</td>
</tr>
<tr>
<td>Record Length (s)</td>
<td>21</td>
<td>10</td>
</tr>
<tr>
<td>Data Length (s)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Recording Filter (Hz)</td>
<td>15-125, in</td>
<td>30-125, in</td>
</tr>
<tr>
<td>Sample Rate (ms)</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Station Spacing (m)</td>
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<td>6.096</td>
</tr>
<tr>
<td>Source Interval (m)</td>
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<td>6.096</td>
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<tr>
<td>Recording Channels</td>
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<tr>
<td>CDP Fold</td>
<td>24</td>
<td>48</td>
</tr>
<tr>
<td>Near Offset (m)</td>
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<td>15</td>
</tr>
<tr>
<td>Far Offset (m)</td>
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<td>300</td>
</tr>
<tr>
<td>Number of Stations</td>
<td>1-483</td>
<td>651-1051</td>
</tr>
<tr>
<td>Number of Shots</td>
<td>1-483</td>
<td>2-402</td>
</tr>
<tr>
<td>Number of CDPs</td>
<td>2-966</td>
<td>1253-2153</td>
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<td>Highest Elevation (m)</td>
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<td>Lowest Elevation (m)</td>
<td>7.8</td>
<td>64.92</td>
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<tr>
<td>Datum Elevation (m)</td>
<td>20</td>
<td>80</td>
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<tr>
<td>Replacement Velocity (m/s)</td>
<td>1200</td>
<td>900</td>
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<tr>
<td>Line Geometry</td>
<td>Straight line</td>
<td>Crooked line</td>
</tr>
</tbody>
</table>
References


Domoracki, W. J., 1994, Ph.D dissertation in progress: A geophysical investigation of geologic structure and regional tectonic setting at the Savannah River Site, South Carolina.


Vita

Mu Guo was born on November 8, 1962 in Shanghai, China. After completing his elementary, middle school education, he entered Tongji University in the fall of 1980 with a major of geophysical exploration. After earned the degree of bachelor of engineering in July 1984, he became an assistant teacher in Tongji University. In the fall of 1991, he began his master’s degree program in the Department of Geological Sciences, Virginia Polytechnic Institute and State University and majored in geophysics.

He is a member of the Society of Exploration Geophysicists, the American Geophysical Union, and the Honor Society of Phi Kappa Phi.