ADAPTIVE OUT OF STEP RELAY ALGORITHM

by

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(ABSTRACT)

A peninsular power company's extra high voltage (EHV) transmission grid and the rest of the country behave as a two machine system for the following two types of disturbances

- loss of a large generator in the southern region of the peninsular power company
- faults on the 500 kV interconnections between the two systems

Whether the two systems will remain stable relative to each other or go unstable depends on the following three factors

- severity of the disturbance
- loading on the peninsular power company's EHV transmission grid
- amount of power imported from the rest of the country

For stable oscillations the two systems *must* remain coupled at their 500 kV interconnections. For separations the two systems should be immediately *isolated* from one another at their 500 kV interconnections.

Since these two systems behave as a two machine system for these two types of disturbances the extended equal area criterion(EEAC) is used to make an extremely quick and accurate prediction of the relative stability between them. For stable oscillations following a disturbance, circuit breakers at the 500 kV interconnections are blocked from tripping. For separations these circuit breakers are tripped.

EEAC requires synchronized voltage phasor measurements at two specific locations within the overall electrical power system. The two sites are substations located on opposite sides of the electrical center of the two systems. The voltage angle at each location's electric bus will swing with respect to its equivalent machine. This information is constantly recorded to monitor the relative stability of the overall system. When a disturbance does occur, a prediction is made and the appropriate control actions are issued.

To Ann, Ling, Ming, Sophie, Mia, and Percy

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I wish to thank my advisor, Dr. Arun G. Phadke, for introducing me to the fascinating world of power system protection and for providing me with the opportunity to realize my dreams.

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CHAPTER 1

INTRODUCTION

1.1 The Problem

A peninsular power company (PPC) has experienced stability problems with their extra high voltage (EHV) transmission grid (see figure 1) for over the past decade. Their network is confined to a peninsular configuration due to the inherent geography of their service territory. PPC's only EHV interconnections with the rest of the country occur at their system interface (see figure 2).

A large number of separations occurred due to the sudden loss of large generation in PPC's southern region. A separation between two systems is termed an out of step (OS) condition. These instabilities were accompanied by transient power swings whose electrical centers passed through the system interface [1]. Before 1985, PPC used *out of step tripping* to separate their system from the rest of the country for this type of disturbance. Once they were separated, under-frequency relays tripped feeders off-line to balance internal generation and load within their own system as a function of PPC's declining frequency. Such events occurred twice in 1984, once when PPC lost two units in the southern region and once when a neighboring utility lost four units [2].

To supplement rapid load growth in their service territory, a 500 kV corridor was constructed to import power from the rest of the country via a northern electric utility (NEU). This low impedance path increased the transient stability margin to the extent that it is now possible for PPC to enter a new equilibrium point after a loss of as much as 1,200 MW of generation. For this condition it is necessary to *block* circuit breakers at the 500 kV interconnections during the ensuing transient power swing that occurs after a sudden loss of generation in their southern region. PPC no longer goes unstable due to the low impedance connections [3].

During heavy import of power from NEU, faults on either incoming 500 kV tie line can produce transient stable power swings severe enough to produce a zone 1 or zone 2 trip of distance relays at either of NEU's two substations (Hoover and Tyler) that look into PPC's EHV transmission grid. For stable oscillations it is necessary to *block* circuit breakers at the 500 kV interconnections. It is also possible for these faults to cause separations in which case it necessary to *trip* these circuit breakers.

On Tuesday, November 3, 1987 at 1607 EST a "B" phase-to-ground fault on one of the 500 kV tie lines initiated a sequence of events that caused a separation because the other

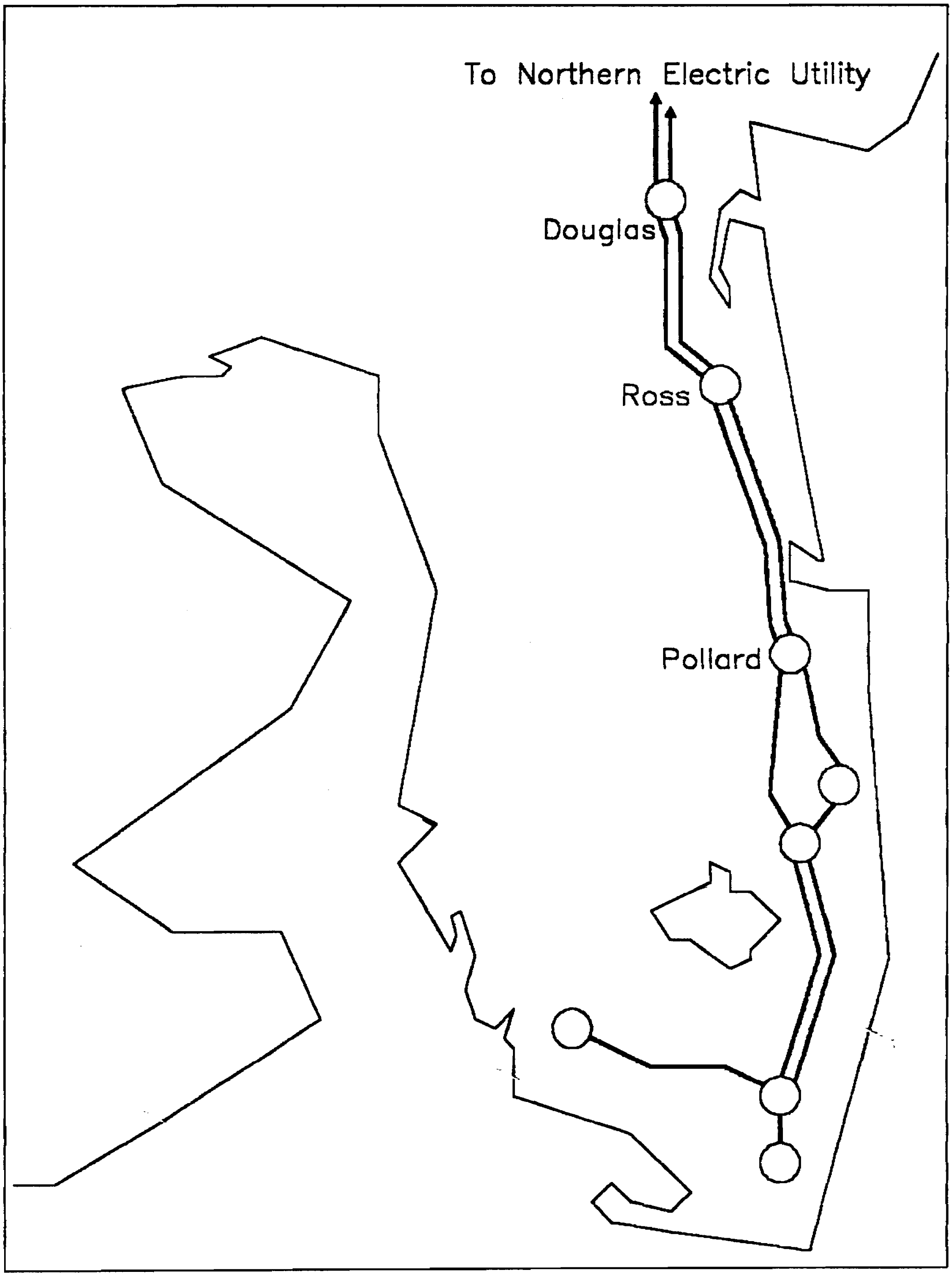


Figure 1. **PPC 500 kV Transmission Grid**

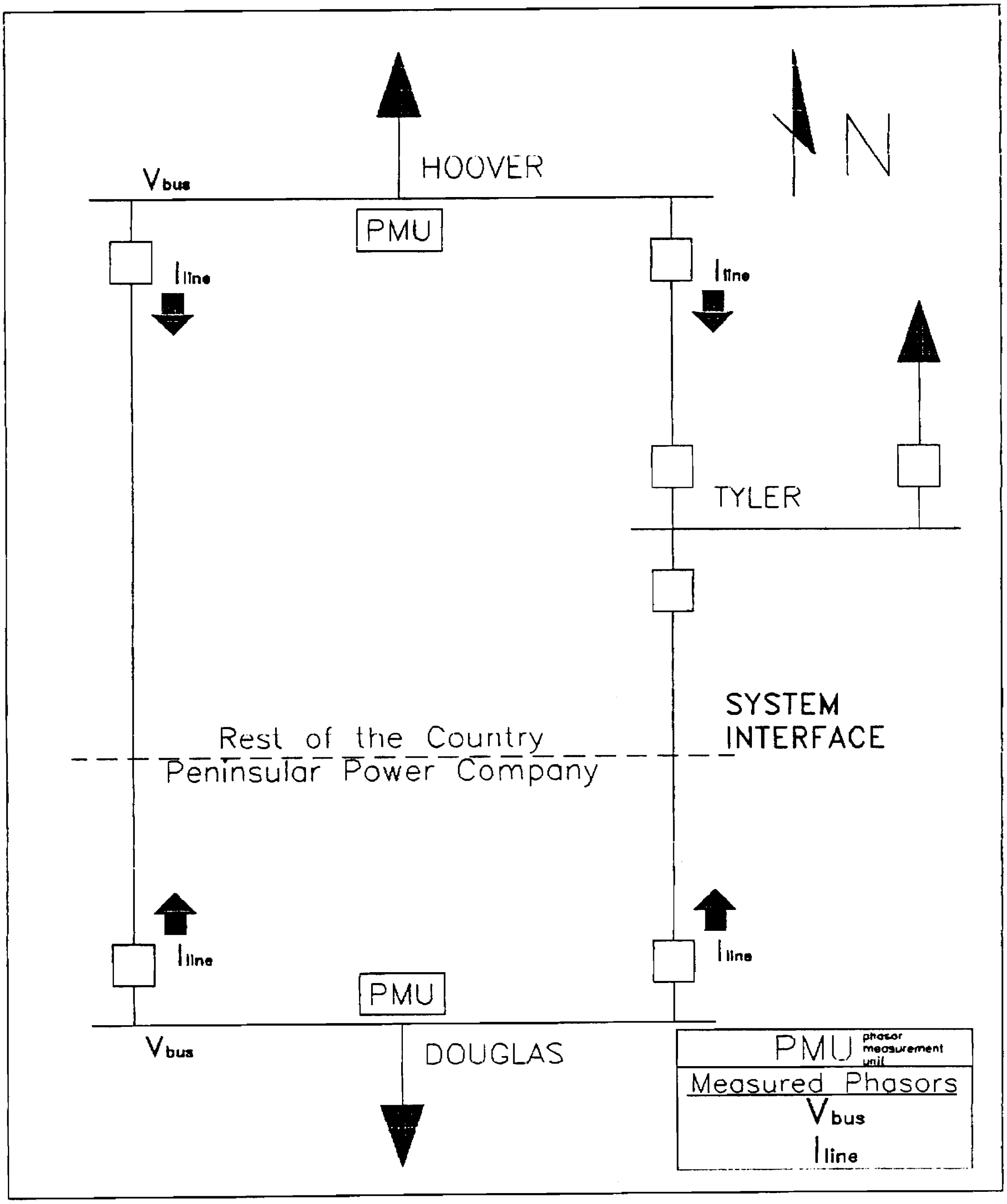


Figure 2. System Interface 500 kV Interconnections

500 kV tie line coincidentally tripped when a relay malfunctioned at the Hoover substation [4]. If the circuit breakers on this line had been properly blocked, the separation would not have occurred.

On Sunday, August 20, 1989 at 1611 EST a phase-to-phase fault occurred when a NEU switch that was used to connect a shunt reactor to the Douglas-Hoover 500 kV tie line at the Hoover substation failed. The fault occurred after the switch was opened. As a result, the ensuing transient stable power swing caused a zone 2 trip of the distance relay at the Douglas terminal of the Douglas-Tyler 500 kV tie line [5]. With both tie lines out of service, no synchronizing power flowed between the two systems and they separated. If the circuit breakers on the Douglas-Tyler 500 kV tie line had been blocked, the resultant separation would not have occurred.

1.2 Existing Solutions

Traditional OS blocking and tripping schemes are compromised because off-line stability studies are employed to determine their settings. It is not possible for all post disturbance systems to be accurately predicted based on off-line studies. There is always a contingency or combination of contingencies that can occur after the protection engineer or planner has determined the worst case on which to base the study. For example, the addition or removal of generators or transmission lines during normal operation often alter the response of distance relays to power swings. On occasion, the hidden contingency is a malfunction within the protection system. It is these events that the traditional OS schemes cannot account for.

One method presently employed for blocking is the concentric circle scheme (see figure 3). 21I represents one zone of a mho distance relay protecting line A-B at terminal A. 21O is the blocking relay's characteristic that encircles 21I. The change in impedance for a fault's trajectory is essentially instantaneous and the difference between times t_0 and t_1 is negligible. During a stable oscillation, the swing of the apparent impedance is seen by the blocking relay well ahead of the mho distance relay. Therefore the difference between times t_0 and t_1 is how this scheme distinguishes between a fault and a stable power swing [6]. This scheme will misoperate for extremely rapid swings that were unaccounted for by the protection engineer or planner.

A method used for tripping during an OS condition is the blinders scheme (see figure 4). Two tilted reactance relays divide the R-X diagram into three areas. As the apparent impedance moves through these regions, this scheme recognizes an OS condition and trips its circuit breaker. As the impedance moves from region one to region two, a timer is started at time t_0. When the impedance crosses the boundary between region two and region three, the timer is stopped at time t_1. The difference between times t_0 and t_1 is how the scheme recognizes the fast swing associated with a separation. If the swing does not pass through the characteristics of the phase distance relays protecting the line at that terminal, this scheme still sees the change in impedance [7].

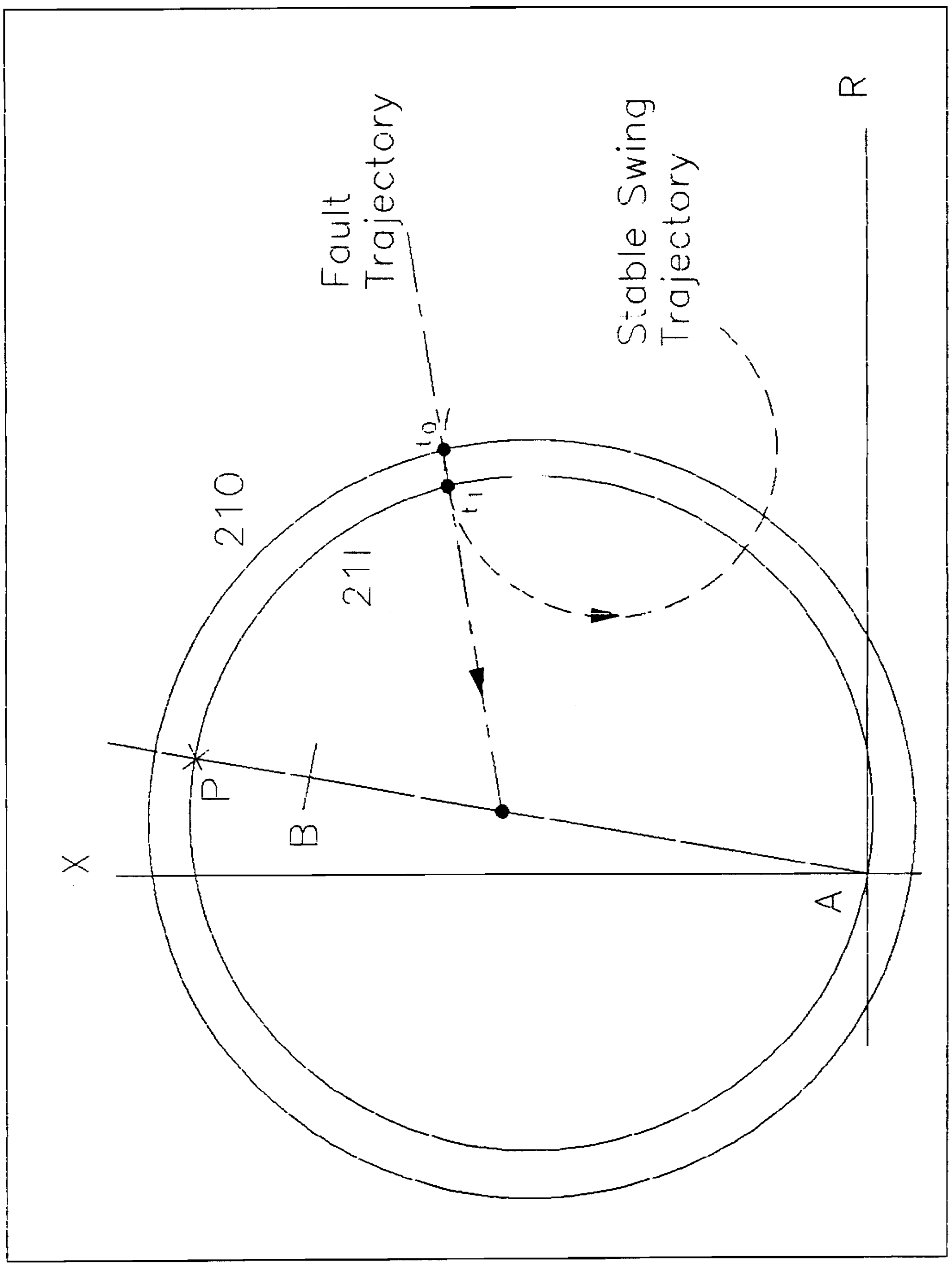


Figure 3. Concentric Circle OS Scheme

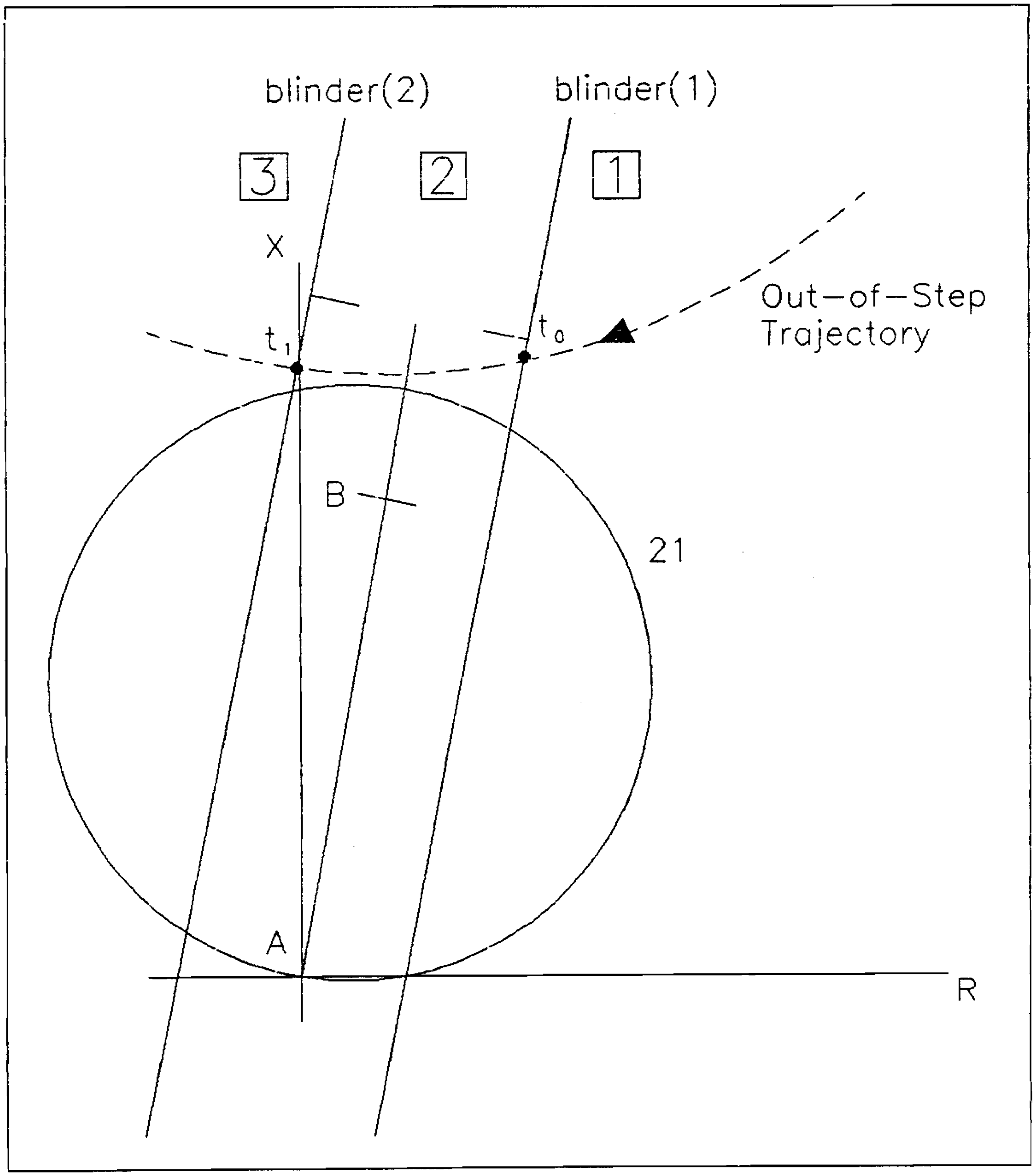


Figure 4. **Blinders OS Scheme**

1.3 Present Solution

PPC has a quick method of detecting a system disturbance that will sheds a constant 800 MW of load, regardless of the system load, to maintain stability. This scheme is the Fast Acting Load Shedding (FALS) program. This protective relay updates its observations every two seconds in order to recognize a variety of stable disturbances across the system that may result in any of the following consequences

- transmission lines operating above their ratings
- low system voltages
- heavy reactive power demands on generators

When any of these actions are identified, FALS drops 800 MW of load to stop a cascading loss of lines or generators that could result in a blackout.

One critical function of FALS is to distinguish between generation losses above and below 1200 MW. This function is responsible for large generation losses within any neighboring utility. The algorithm can determine the exact amount of lost generation within a quarter of a second.

After FALS has determined that a large loss of generation has occurred and the system is in trouble, a trip signal is sent to its load shedding program. A small number of transmission level circuit breakers are tripped to take entire distribution stations off-line. The security of the program is increased by the use of under-frequency relays that allow permissive tripping at the chosen substations. A normally open contact of the under-frequency relay is in series with the load shedding program's tripping contact. If the program incorrectly executes a trip command, the under-frequency relay prevents tripping. This whole process must be accomplished within 20 seconds.

FALS was enhanced with a time delay feature which was incorporated after a large number of switched capacitors were added to the transmission system. These capacitors may stabilize the system after they are switched on-line. Therefore, FALS must wait for their effect to take place, in which case it is no longer necessary to shed load.

There are several problems associated with FALS due to its nonadaptive nature. Because of its two second scan rate, it cannot detect rapid swings due to severe oscillations that occur after two large generating units are lost. It cannot provide protection when power import capabilities are reduced after one of the 500 kV tie lines is out of service. Also, FALS may shed load when the maximum power transfer from NEU is exceeded and a single generation unit is lost [8]. The algorithm is not affected by any of these scenarios.

In addition to FALS, out of step relays are also used to control separations. Information from off-line simulation programs and Continuous Monitoring Fault Recorders is used to determine the settings for these relays so that they are capable of detecting power swings severe enough to initiate a separation [9]. The problems associated with these types of protection schemes have already been described.

1.4 Proposed Solution

The algorithm accomplishes two goals. First, it can detect the loss of a large generating unit and determine whether or not the post disturbance system will remain stable within a quarter of a second. Because it updates its observations at a rate of one to five cycles (depending upon the communications medium), it can detect the most rapid swing that could possibly occur on the system. Depending on whether the post disturbance system will remain stable or separate from the rest of the country, this algorithm can initiate control actions quickly enough to accomplish effective blocking or tripping of circuit breakers at the 500 kV interconnections.

Secondly, it can instantaneously determine whether or not PPC will remain stable or separate for faults on the 500 kV interconnections. Again, the algorithm is quick enough to initiate the desired control actions.

Because this new algorithm is adaptive, it will always make the correct decision by accounting for the following parameters

- the configuration of the electric power system around the relay
- the load on PPC's electric power system
- power imported from NEU
- observed nature of the actual swing

1.4.1 Loss of Generation

When PPC drops a large generator in their southern region, the machines in the vicinity of the system interface swing against the machines in the southern area. Since the overall system behaves like a two machine system, this is an excellent application for the Extended Equal Area Criterion (EEAC).

Before the large generator is dropped, the mechanical power delivered to the machines is operating at some value which corresponds to equilibrium with the electric power of the transmission grid. The moment the unit is lost, the mechanical power jumps to a new value, which is greater by the amount of generation lost, since this is equivalent to an equal amount of sudden demand.

When the southern generator is dropped, a quick calculation can be performed to determine if the kinetic energy injected into the power system can be balanced by the grid's potential energy. If the potential energy is greater than or equal to the injected kinetic energy, then the system will remain stable. If the kinetic energy is greater, then PPC will separate from the rest of the country. The kinetic energy added to the electric grid corresponds to area one of the EEAC. The potential energy is represented by area two (see figure 5).

As previously stated, the mechanical power input to the machines changes instantaneously when the generating unit is dropped. Before the generator was lost, the mechanical power was equal to the electric power of the grid. After the unit is lost, it takes a small amount of time for the electrical power to regain equilibrium with the mechanical power. This corresponds to a greater separation between the rotor angles of the equivalent machine for PPC and the equivalent machine representing the rest of the country. By making an extremely quick prediction of this new angular separation between the two machines, and the relative magnitudes of the accelerating and decelerating areas, it can easily be determined if they will remain stable or go out of step [10].

1.4.2 Faults on the 500 kV Interconnections

PPC imports 3000 MW of electric power from NEU during maximum power transfer. Approximately 2800 MW of this imported power flows through the two 500 kV tie lines (see figure 6). During this period, the transient stability margin is at a minimum. Note that the reduced two machine model used at Virginia Polytechnic Institute for load flow studies matched those of the multi-machine model used by PPC's planning department.

For the months of November, 1987 and August, 1989, faults occurred on the 500 kV interconnections that drove the two systems out of step due to incorrect protection actions. The OS condition occurred in both cases because the low impedance path between the two systems was lost. Therefore, the corresponding transfer impedance between the two equivalent machines increased dramatically and synchronizing power was unable to flow from the rest of the country to PPC.

A reduced equivalent two machine model of the two systems, that retains the 500 kV busses of Hoover and Douglas, was developed so that the adaptive out of step relay algorithm can utilize the EEAC. This algorithm will predict whether the circuit breakers of the 500 kV tie lines should be blocked or tripped as soon as the fault is detected. For the faults of 1987 and 1989, the algorithm would have blocked the appropriate circuit breakers and PPC would have remained synchronized with the rest of the country after the original power swings had subsided.

When a fault occurs on one of the three 500 kV transmission lines that connects PPC to NEU, EEAC determines whether the overall system will remain stable or if a separation will occur, since it behaves like a two machine system (see figure 7). Before the fault occurs, the system is at equilibrium; i.e., the mechanical power (P_m) input to the machines is equal to the electrical power (P_e^0) of the grid and the operating point corresponds to δ_0. When the fault occurs, the electrical power of the grid drops to the new value (P_e^1) of the faulted network. As a result, the angular separation between the rotor angles of the two equivalent machines begins to accelerate. Five cycles later, the fault is cleared by tripping the faulted circuit off-line for 30 cycles to extinguish the fault's arc. During this period the electrical power of the grid is restored to an intermediate value (P_e^2) and the angular separation between PPC and the rest of the country continues to accelerate. When the transmission line is reclosed, the electrical power of the grid is restored to its initial value.

During this sequence of events, three areas are created. Area one represents the kinetic energy injected into the grid during the fault. Area two and area three, respectively, represent the kinetic energy of the network after the fault is cleared and when the transmission line is reclosed. If the sum of area two and area three is greater than or equal to area one, then the overall system remains stable [11]. Note that if area two is greater than or equal to area one it is not necessary to determine area three. Therefore, it is possible to predict the stability of the system the moment the disturbance occurs.

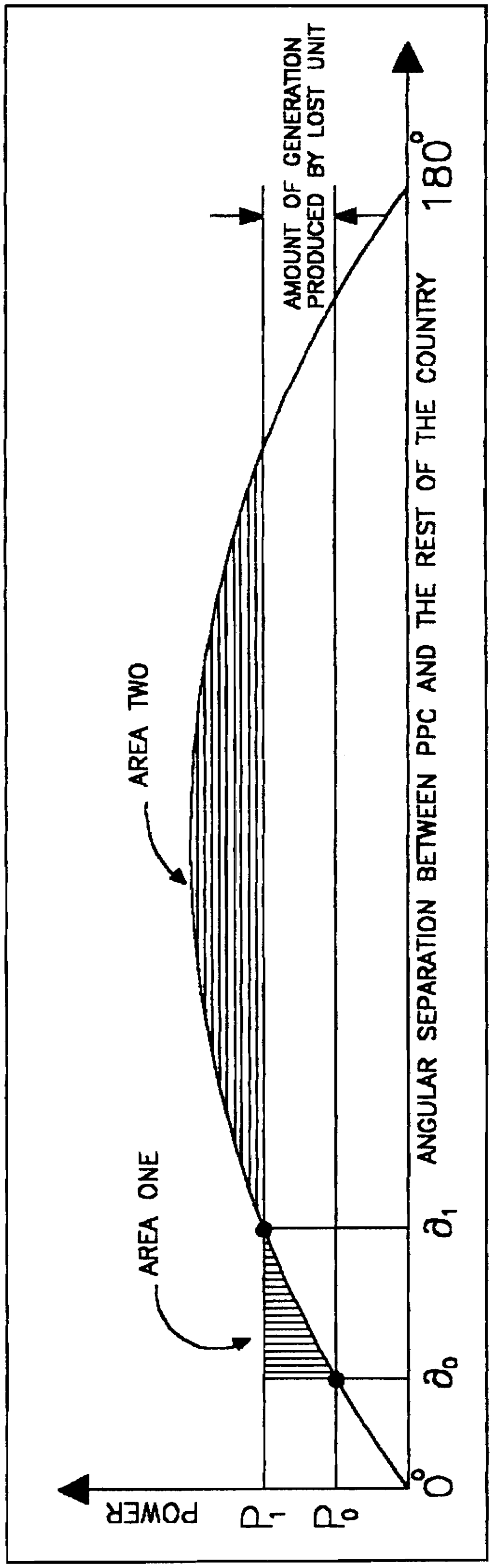
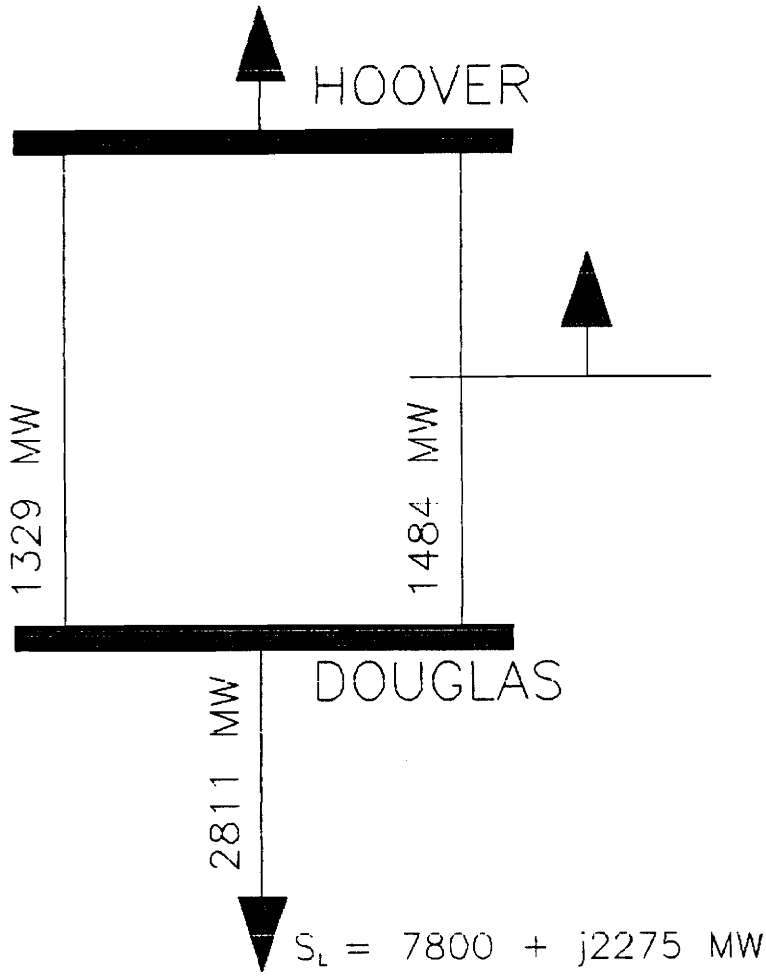


Figure 5. EEAC for Dropped Generator in Southern Region



With Hoover taken as the reference:

	PPC model		VPI model	
	|V|	Θ	|V|	Θ
Hoover	1.04	0	1.03	0
Douglas	1.03	−18.6	1.03	−19.4
	$P_{500\ kV}$		$P_{500\ kV}$	
	2800 MW		2811 MW	

Figure 6. Maximum Power Transfer

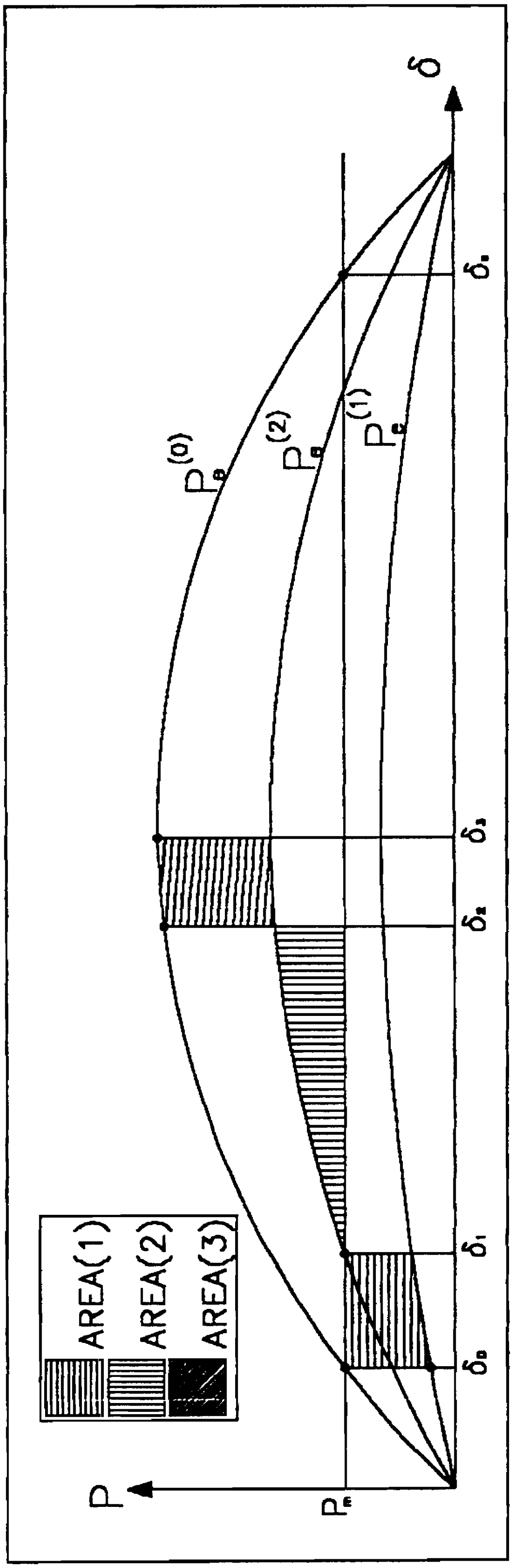


Figure 7. EEAC for Faults on the 500 kV Interconnections

1.5 Justifications

For a two machine system, instability cannot occur if the first swing is stable. Sometimes, in a multi-machine system, one of the machines may stay in step on the first swing and then go out of step on the second swing because the other machines are in different positions and react differently on the first swing.

For the equivalent two machine system under the assumptions of

- constant input
- zero damping
- constant voltage behind the transient reactance

the angular separation between the two machines either increases indefinitely or, after all the disturbances have occurred, oscillates with constant amplitude.

Even though the assumptions for the classical model are not strictly true, it does not falsify the EEAC. When the input to the generators is changed by the action of governors, this effect is generally negligible until after the first swing, at which time it acts to aid the stability of the system. The presence of damping slightly reduces the amplitude of the first swing and even more so for the subsequent swings. If the machines have voltage regulators, then it is possible to preserve stability, even in some instances when the system would have gone unstable.

From the previous discussion it is realized that if the two machine system does not lose synchronism after the first swing, then it is very probably a stable system [12].

CHAPTER 2

EXTENDED EQUAL AREA CRITERION

2.1 Equivalent Two Machine Model

Since the algorithm uses the EEAC to predict the transient stability of the overall system, it was necessary to represent PPC's EHV system and the rest of the country as an equivalent two machine system. Two models were needed.

The first model retains the 500 kV buses at Hoover, Tyler, and Douglas, and the 500 kV tie lines connecting them. This model was used to generate all the load flow and transient stability studies necessary to simulate the various disturbances of interest [13].

The second model is a reduced version of the first one. The only actual parameters retained are the 500 kV buses at Hoover and Douglas, since this is where the apparatus will be installed to measure voltage and current phasors in real-time.

2.1.1 Transfer Impedance between the Two Equivalent Machines

PPC's system protection department conducted two short circuit studies. The first study determined the transfer impedance for the rest of the country by placing a balanced three phase fault on Douglas' 500 kV bus with all circuits into the peninsular system disconnected. The transfer impedance of this system was determined directly from the total fault current at the bus. The second study determined the transfer impedance of the PPC system by placing a balanced three phase fault on Douglas' 500 kV bus with all circuits into the rest of the country disconnected (see figure 8).

The direct axis transient reactance of the PPC equivalent machine was computed by taking the parallel combination of the direct axis transient reactances of all the major generating units within the PPC system (see table 1) [14]. Since the rest of the country is heavily interconnected, the direct axis transient reactance of its equivalent is that of an extremely large machine. Its direct axis transient reactance was determined as follows.

Because the total capacity of the entire country is approximately 400,000 MW and the direct axis transient reactance for a four pole generator is approximately 0.24 per unit on its own machine kVA rating, the direct axis transient reactance for one lumped equivalent machine on a 100 MVA base is 0.00006 per unit [15].

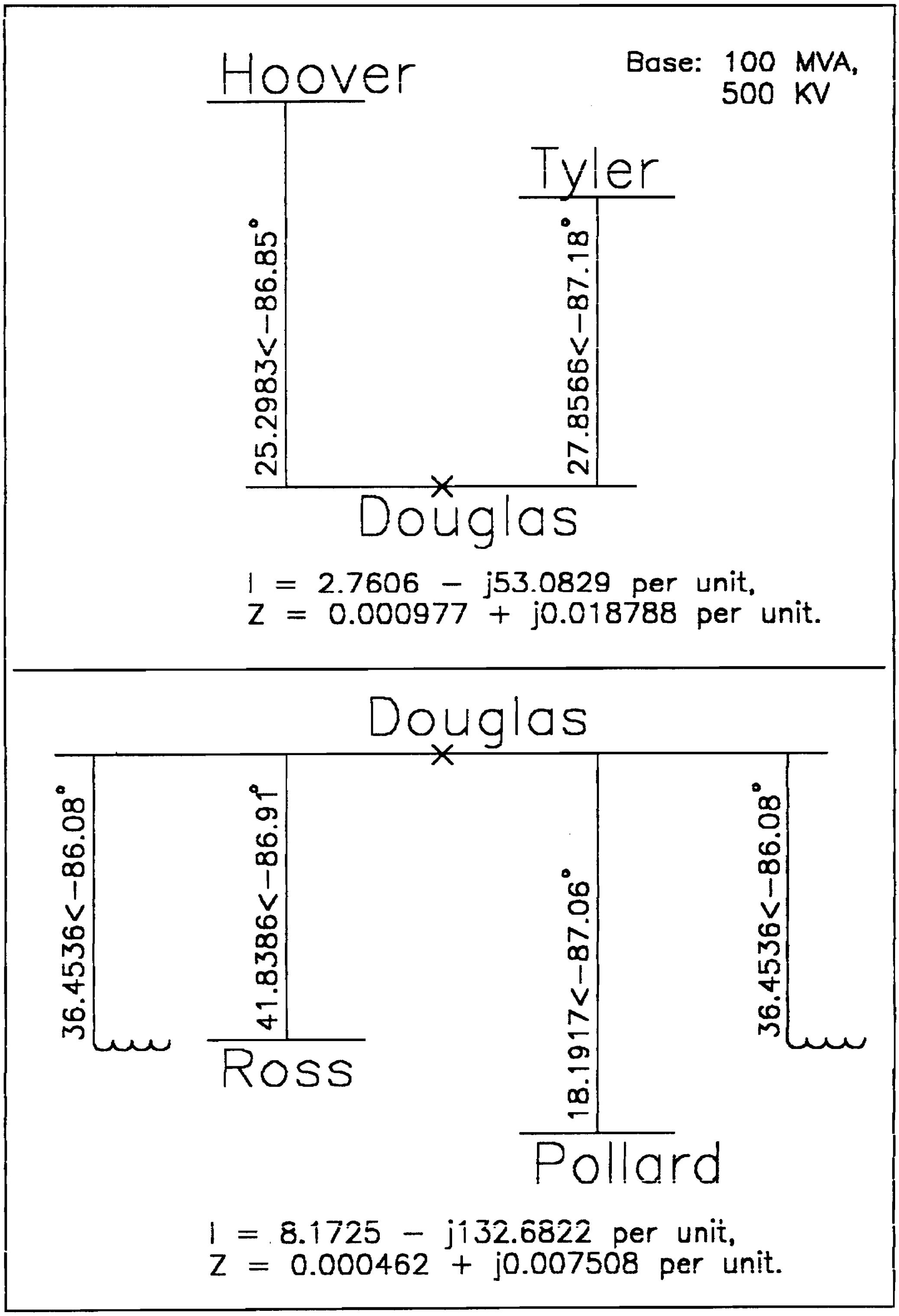


Figure 8. **Transfer Impedance of the Equivalent Two Machine System**

When this equivalent two machine system is reduced to an one machine-infinite bus (OMIB) system, the load on both machines is included in the total transfer impedance.

2.1.2 Inertia Constants of the Two Equivalent Machines

The inertia constant for the rest of the country's equivalent machine was determined to be 10,000 seconds on a 100 MVA base since the average inertia constant of a four pole generator is approximately 2.5 seconds on its own machine kVA rating [16].

The inertia constant of the PPC equivalent machine was determined by the synchronizing power coefficient formula [17] which states

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\omega_s P_{max} \cos\delta_0}{2H_t}} \tag{2.1}$$

where

f_n = frequency of oscillation,
ω_s = synchronous angular frequency,
= 377 radians/second.

$$P_{max} = |E_1||E_2||y_{12}| \tag{2.2}$$

E_i = internal node voltage of an individual machine,
y_{12} = transfer admittance between the two machines,
δ_0 = pre disturbance angular separation between the two machines.

$$H_t = \frac{H_1 H_2}{H_1 + H_2} \tag{2.3}$$

H_i = inertia constant of an individual machine.

PPC's planning department determined that, at maximum power transfer, the undamped frequency of oscillation is 0.8333 hertz for disturbances on the 500 kV interconnections. The angular separation between the two machines and their internal node voltages at maximum power transfer were determined from a load flow study. The synchronizing power coefficient formula was solved for the inertia constant of the PPC equivalent machine and found to be 212 seconds.

2.1.3 Reduced Two Machine Model

Figure 9 is an impedance diagram of the first model. The line constants for the 500 kV tie lines were obtained from PPC's planning department. Table 2 contains the positive-sequence deck for the model, results of a load flow case for maximum power transfer, and data necessary for transient stability analysis.

2.1.4 Two Machine Computer Model

This model is stored in the algorithm (see figure 10). The buses retained from the first model are the two equivalent generators' terminal buses and the actual 500 kV buses for Hoover and Douglas. Kron reduction is used to derive the computer model as follows

$$Y_{old} = \left[\begin{array}{c|c} Y_{ii} & Y_{ie} \\ \hline Y_{ei} & Y_{ee} \end{array}\right] \tag{2.4}$$

where

$$Y_{ii} = \begin{bmatrix} y_{55} & y_{56} & y_{51} & y_{53} \\ y_{65} & y_{66} & y_{61} & y_{63} \\ y_{15} & y_{16} & y_{11} & y_{13} \\ y_{35} & y_{36} & y_{31} & y_{33} \end{bmatrix}, \qquad Y_{ie} = \begin{bmatrix} y_{52} & y_{54} \\ y_{62} & y_{64} \\ y_{12} & y_{14} \\ y_{32} & y_{34} \end{bmatrix},$$

$$Y_{ee} = \begin{bmatrix} y_{22} & y_{24} \\ y_{42} & y_{44} \end{bmatrix}, \qquad Y_{ei} = \begin{bmatrix} y_{25} & y_{26} & y_{21} & y_{23} \\ y_{45} & y_{46} & y_{41} & y_{43} \end{bmatrix}.$$

$$Y_{new} = Y_{ii} - Y_{ie} Y_{ee}^{-1} Y_{ei} \tag{2.5}$$

Therefore

$$Y_{new} = \begin{bmatrix} y_{55} & y_{56} & y_{51} & y_{53} \\ y_{65} & y_{66} & y_{61} & y_{63} \\ y_{15} & y_{16} & y_{11} & y_{13} \\ y_{35} & y_{36} & y_{31} & y_{33} \end{bmatrix}.$$

There are actually four possible computer models. One model is for all three of the 500 kV tie lines in service and the other three are for the cases of any individual tie line removed from service.

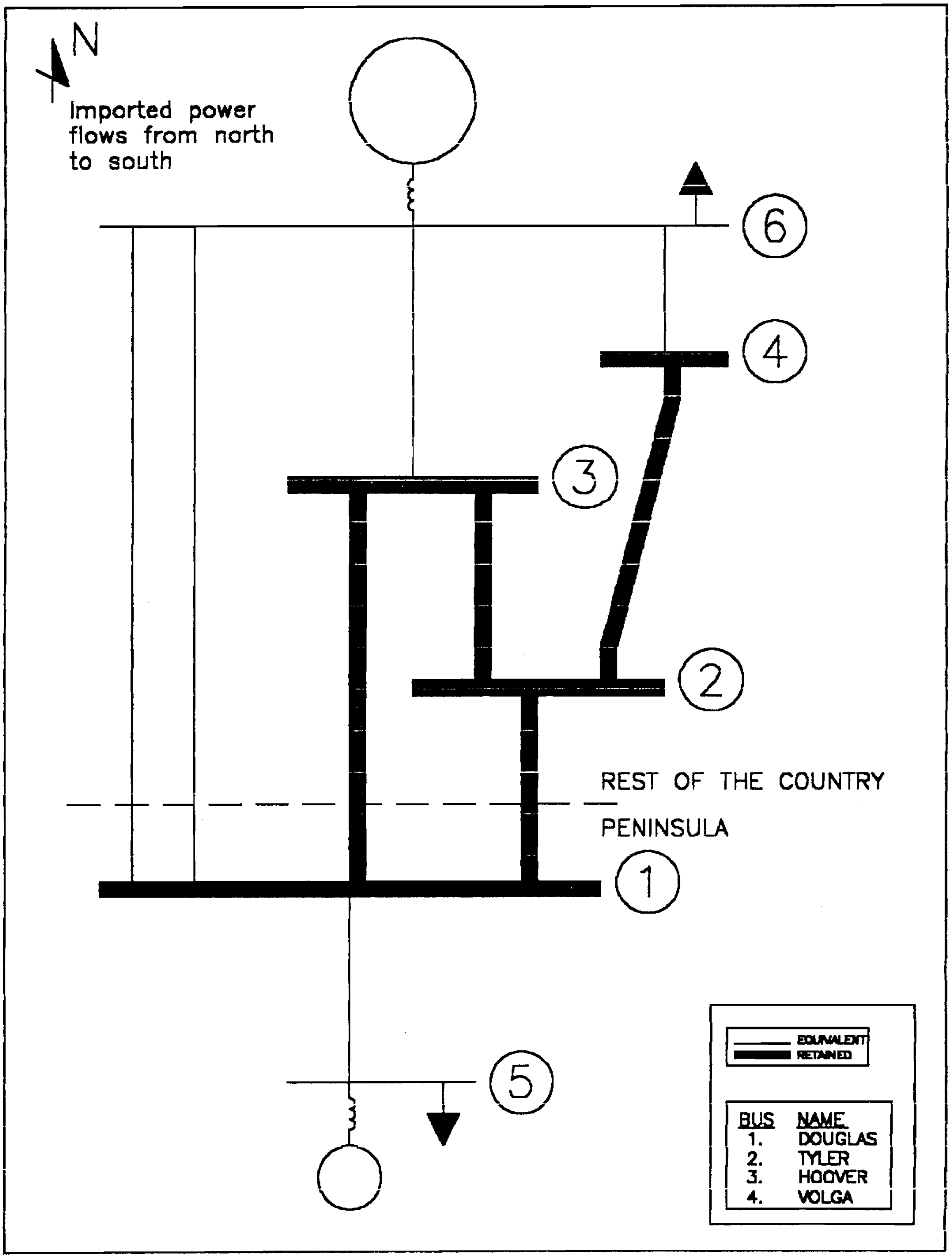


Figure 9. Model (1)

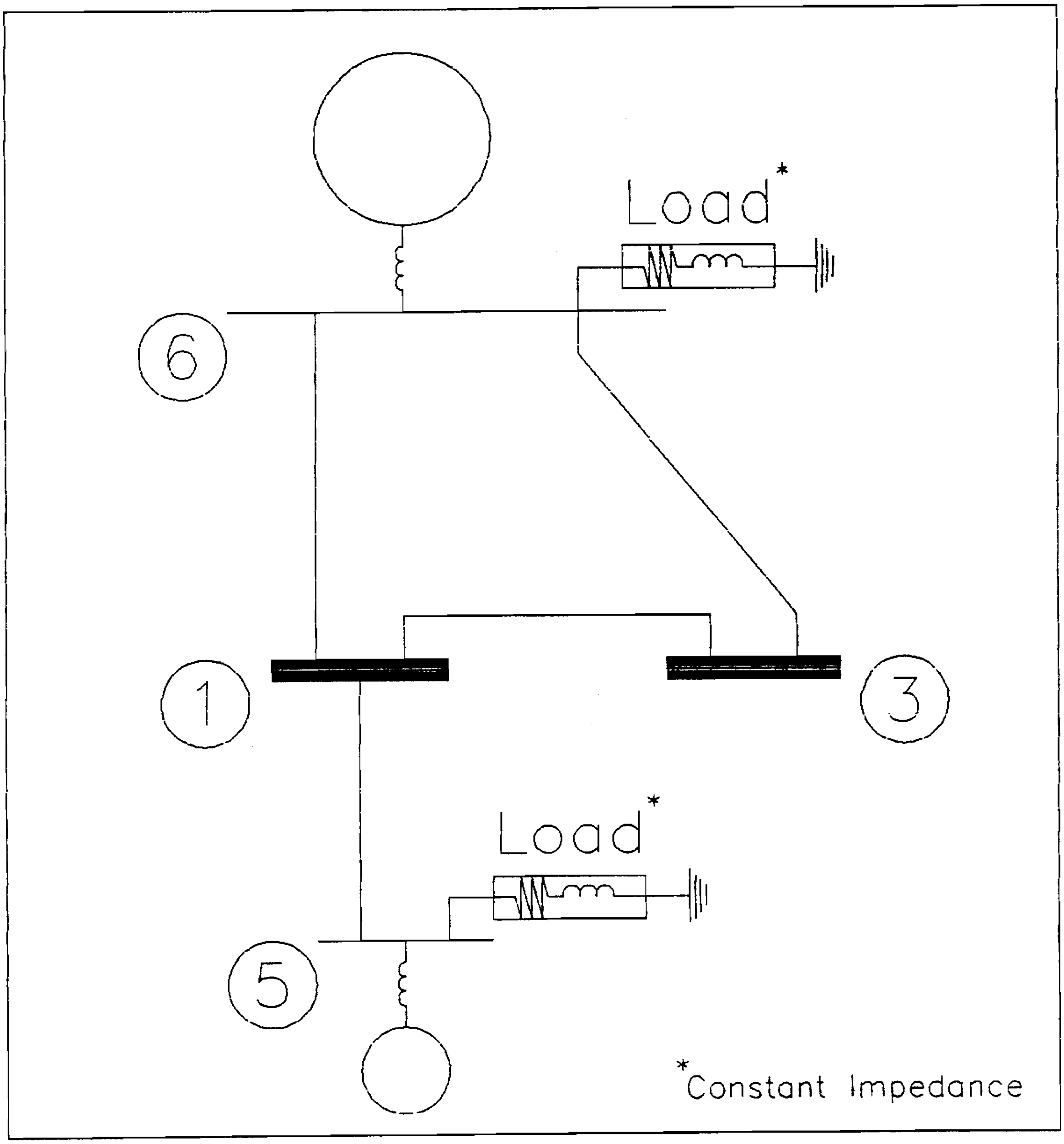


Figure 10. **Two Machine Model**

2.2 The Algorithm

There is one and only one algorithm that decides whether to *block or trip* for faults on the 500 kV tie lines or for generators dropped in the southern region of PPC's service area. For the sake of clarity, the method of prediction for the second case will be described first since it is easier to understand the mechanics of the algorithm when it is separated into two sections.

2.2.1 The Method of Prediction for Generator Dropping

Throughout this section, all numerical examples will be for the case of maximum power transfer with NEU unless otherwise specified. This case was chosen because the transient stability margin is at a minimum when the mechanical power input to the machines is at its maximum. If the prediction is correct for the worst case, then it will be correct for all others. A system base of 100 MVA was chosen.

2.2.1.1 STEP 1.

The first four steps occur whenever one of the 500 kV tie lines is taken out of service or when one is placed back in service.

If any of the 500 kV tie lines are taken out of service or placed back in service, then the appropriate elements of the admittance matrix for the reduced two machine model, Y_{old}, are updated and Kron reduction is used to reduce this to the computer two machine model, Y_{new}.

Appendix A is a numerical example of this reduction for the case of all lines in service.

2.2.1.2 STEP 2.

Buses 1 and 3 are eliminated to form another admittance matrix that is used to estimate the transfer impedance of the two machine system. Again Kron reduction is used.

$$Y_1 = Y_{ii} - Y_{ie} Y_{ee}^{-1} Y_{ei},$$

where

$$Y_{ii} = \begin{bmatrix} y_{55} & y_{56} \\ y_{65} & y_{66} \end{bmatrix}, \quad Y_{ie} = \begin{bmatrix} y_{51} & y_{53} \\ y_{61} & y_{63} \end{bmatrix},$$

$$\mathbf{Y_{ei}} = \begin{bmatrix} y_{15} & y_{16} \\ y_{35} & y_{36} \end{bmatrix}, \quad \mathbf{Y_{ee}} = \begin{bmatrix} y_{11} & y_{13} \\ y_{31} & y_{33} \end{bmatrix}.$$

Therefore

$$\mathbf{Y_1} = \begin{bmatrix} y_{55} & y_{56} \\ y_{65} & y_{66} \end{bmatrix}.$$

For the elements listed in appendix A

$$\mathbf{Y_1} = \begin{bmatrix} 3.2476 - j48.9617 & -2.8690 + j44.5577 \\ \cdot & 3.6893 - j52.0996 \end{bmatrix}.$$

2.2.1.3 STEP 3.

The load on each equivalent machine and its direct axis transient reactance are added to the corresponding self-admittance element of $\mathbf{Y_1}$. The total load on PPC's EHV system is determined by periodically referencing their SCADA system.

Since loads are modeled as constant admittances for the classical transient stability model, the following formula [18] is used

$$y_L = \frac{P_L - jQ_L}{|V_L|^2} \tag{2.6}$$

where $|V_L| \approx 1.04$.

If the PPC equivalent machine is taken to be machine 1 and the equivalent machine representing the rest of the country is taken to be machine 2, then

$$\tilde{y}_{55} = y_{55} + y_{L_1} + \frac{1}{jX'_{d_1}} \tag{2.7a}$$

$$\tilde{y}_{66} = y_{66} + y_{L_2} + \frac{1}{jX'_{d_2}} \tag{2.7b}$$

The updated admittance matrix is

$$\mathbf{Y_2} = \begin{bmatrix} \tilde{y}_{55} & y_{56} \\ y_{65} & \tilde{y}_{66} \end{bmatrix}.$$

From table 2, the loads on the two equivalent machines at maximum power transfer are

$(P_L + jQ_L)_1 = 78 + j22.75$ per unit,
$(P_L + jQ_L)_2 = 1000 + j291.66$ per unit.

By equation (2.6)

$y_{L1} = 72.1154 - j21.0337$ per unit,
$y_{L2} = 924.5562 - j269.6653$ per unit.

Therefore, from equations (2.7a) and (2.7b)

$\tilde{y}_{55} = 75.363 - j386.4451$ per unit,

$\tilde{y}_{66} = 928.2455 - j16988.4316$ per unit,

since

$(X_{d1}')^{-1} = 316.4557$ per unit,
$(X_{d2}')^{-1} = 16{,}666.6667$ per unit.

2.2.1.4 STEP 4.

The admittance matrix $\mathbf{Y_2}$ is used to reduce the computer two machine model to an internal node model for the two equivalent generators. Therefore, the following intermediate matrix is formed

$$\mathbf{Y_3} = \left[\begin{array}{cc|cc} y'_{d_2} & 0 + j0 & 0 + j0 & -y'_{d_2} \\ 0 + j0 & y'_{d_1} & y'_{d_1} & 0 + j0 \\ \hline 0 + j0 & -y'_{d_1} & \tilde{y}_{55} & y_{56} \\ -y'_{d_2} & 0 + j0 & y_{65} & \tilde{y}_{66} \end{array}\right].$$

Application of Kron reduction and reordering the resulting elements yields the internal node admittance matrix for the two equivalent machines

$$\mathbf{Y_4} = \begin{bmatrix} y_{11} & y_{12} \\ \cdot & y_{22} \end{bmatrix},$$

where nodes 1 and 2 represent the internal nodes of the two equivalent machines (see figure 11). This notation represents a symmetric matrix.

For the values computed in step (3), the internal node admittance matrix is

$$\mathbf{Y_4} = \begin{bmatrix} 48.7085 - j66.7359 & 6.3967 + j34.5789 \\ \cdot & 891.5944 - j359.5537 \end{bmatrix}.$$

2.2.1.5 STEP 5.

The next three steps are performed every 5 cycles when a satellite link serves as the communications medium, and once every cycle when a fiber optic link is used [19].

The admittance matrix, $\mathbf{Y_{new}}$, is used to estimate the voltage phasors at the terminal buses of the two equivalent machines since the actual voltage phasors are available at Hoover and Douglas (buses 1 and 3).

$$\mathbf{I} = \mathbf{Y_{new}}\,\mathbf{V} \tag{2.8}$$

where

$$\mathbf{I} = \begin{bmatrix} 0 \\ I_G \end{bmatrix}, \; 0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \; I_G = \begin{bmatrix} I_5 \\ I_6 \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} V_B \\ V_T \end{bmatrix}, \; V_B = \begin{bmatrix} V_1 \\ V_3 \end{bmatrix}, \; V_T = \begin{bmatrix} V_5 \\ V_6 \end{bmatrix},$$

$$\mathbf{Y_{new}} = \begin{bmatrix} Y_A & Y_B \\ Y_C & Y_D \end{bmatrix}.$$

The sub matrices of $\mathbf{Y_{new}}$ are

$$\mathbf{Y_A} = \begin{bmatrix} y_{11} & y_{13} \\ y_{31} & y_{33} \end{bmatrix}, \quad \mathbf{Y_B} = \begin{bmatrix} y_{15} & y_{16} \\ y_{35} & y_{36} \end{bmatrix},$$

$$\mathbf{Y_C} = \begin{bmatrix} y_{15} & y_{16} \\ y_{35} & y_{36} \end{bmatrix}, \quad \mathbf{Y_D} = \begin{bmatrix} y_{55} & y_{56} \\ y_{65} & y_{66} \end{bmatrix}.$$

The terminal bus voltages are estimated as follows

$$0 = \begin{bmatrix} Y_A & Y_B \end{bmatrix} \begin{bmatrix} V_B \\ V_T \end{bmatrix},$$

$$0 = \mathbf{Y_A V_B} + \mathbf{Y_B V_T},$$

$$\mathbf{Y_B V_T} = -\mathbf{Y_A V_B},$$

$$\mathbf{V_T}^{est} = -\mathbf{Y_B}^{-1}\, \mathbf{Y_A V_B} \qquad (2.9)$$

Therefore

$$\begin{bmatrix} V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \end{bmatrix},$$

where

$$-\mathbf{Y_B}^{-1}\, \mathbf{Y_A V_B} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

So

$$V_5 = aV_1 + bV_3 \qquad (2.9a)$$
$$V_6 = cV_1 + dV_3 \qquad (2.9b)$$

For the example of maximum power transfer

$$\begin{bmatrix} 0 \\ 0 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} 311.69\angle -84.82^\circ & 64.74\angle 93.38^\circ & 228.71\angle 95.80^\circ & 13.92\angle 93.63^\circ \\ \cdot & 203.72\angle -87.31^\circ & 0\angle 0^\circ & 135.10\angle 92.28^\circ \\ \cdot & \cdot & 228.71\angle -84.20^\circ & 0\angle 0^\circ \\ \cdot & \cdot & \cdot & 152.84\angle -87.44^\circ \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_5 \\ V_6 \end{bmatrix},$$

$$-Y_B Y_A^{-1} = \begin{bmatrix} 1.3920\angle -0.64^\circ & 0.3748\angle 177.74^\circ \\ 0.4793\angle -178.90^\circ & 1.5084\angle 0.41^\circ \end{bmatrix}.$$

So for

$$V_B = \begin{bmatrix} 1.032\angle -28.71^\circ \\ 1.031\angle -9.30^\circ \end{bmatrix},$$

$$V_T^{est} = \begin{bmatrix} 1.075\angle -35.65^\circ \\ 1.0982\angle -0.58^\circ \end{bmatrix}.$$

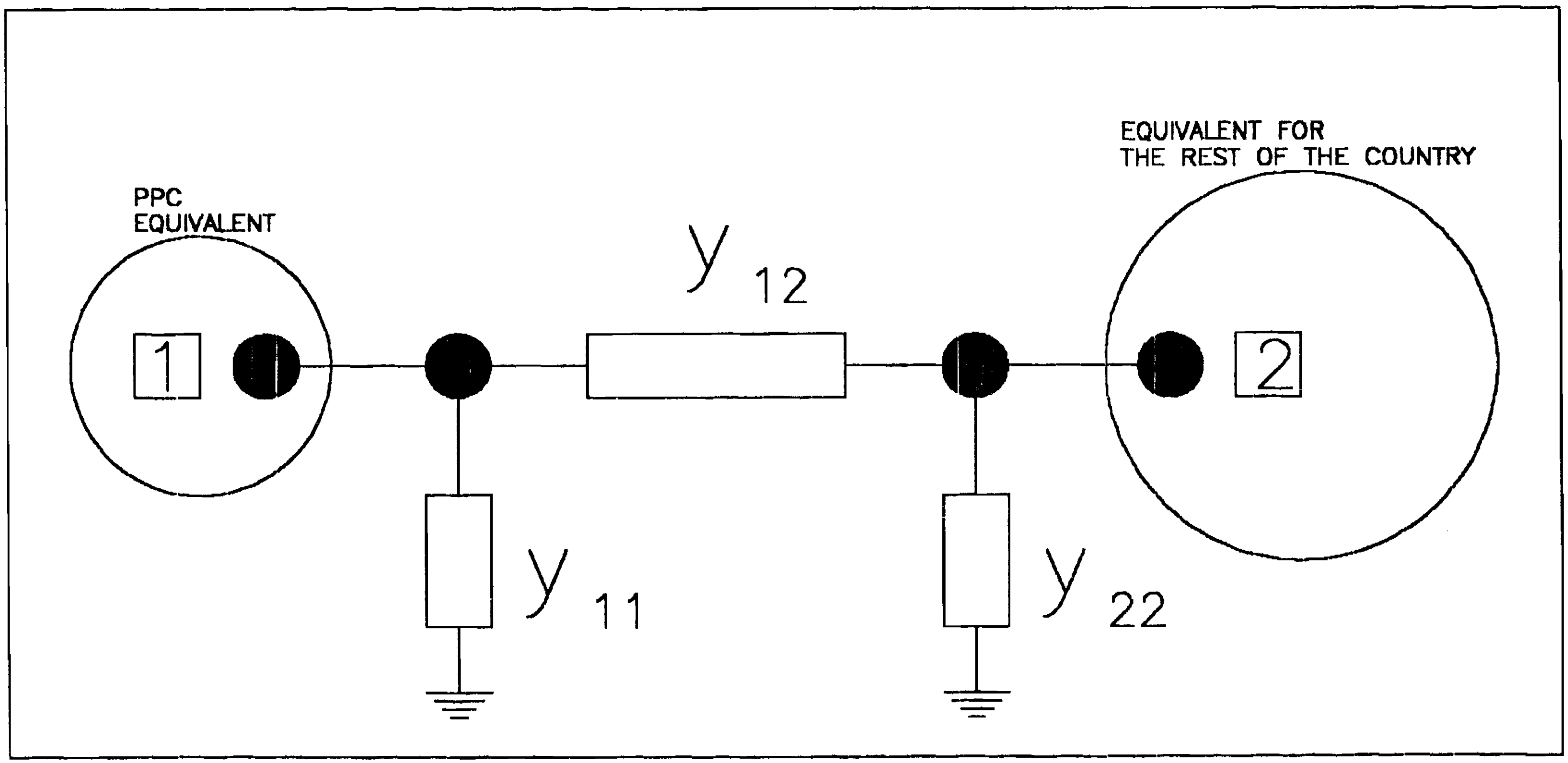


Figure 11. Internal Node Equivalent Two Machine Model

2.2.1.6 STEP 6.

The admittance matrix, Y_{new}, is used next to estimate the current flowing through the direct axis transient reactances of the two equivalent machines. To obtain the most accurate estimate possible, the self-admittance elements y_{55} and y_{66} must be modified to account for the load on the machines.

$$\hat{y}_{55} = y_{55} + y_{L_1} \tag{2.10a}$$

$$\hat{y}_{66} = y_{66} + y_{L_2} \tag{2.10b}$$

$$I_G = \begin{bmatrix} Y_C & \hat{Y}_D \end{bmatrix} \begin{bmatrix} V_B \\ V_T \end{bmatrix} \tag{2.11}$$

where

$$\hat{Y}_D = \begin{bmatrix} \hat{y}_{55} & y_{56} \\ y_{65} & \hat{y}_{66} \end{bmatrix}.$$

Equation (2.11) can also be expressed as

$$I_5 = [\, y_{51} \;\; y_{53} \;\; \hat{y}_{55} \;\; y_{56} \,] \begin{bmatrix} V_1 \\ V_3 \\ V_5 \\ V_6 \end{bmatrix} \tag{2.11a}$$

$$I_6 = [\, y_{61} \;\; y_{63} \;\; y_{65} \;\; \hat{y}_{66} \,] \begin{bmatrix} V_1 \\ V_3 \\ V_5 \\ V_6 \end{bmatrix} \tag{2.11b}$$

Returning to the example

$\hat{y}_{55} = 228.71\angle -84.20^\circ + 72.12 - j21.03,$

$\hat{y}_{55} = 266.19\angle -69.04^\circ.$

$\hat{y}_{66} = 152.84\angle -87.44^\circ + 924.56 - j269.67^\circ,$

$\hat{y}_{66} = 1022.66\angle -24.39^\circ.$

$$Y_C = \begin{bmatrix} 228.71\angle 95.80^\circ & 0\angle 0^\circ \\ 13.92\angle 93.63^\circ & 135.60\angle 92.28^\circ \end{bmatrix}, \hat{Y}_D = \begin{bmatrix} 266.19\angle -69.04^\circ & 0\angle 0^\circ \\ 0\angle 0^\circ & 1022.66\angle -24.39^\circ \end{bmatrix}.$$

For the previous set of actual bus voltage phasors and estimated terminal bus voltage phasors, the estimated generator current phasors are

$I_5 = 62.46\angle -71.97^\circ, I_6 = 1090.11\angle -17.24^\circ.$

2.2.1.7 STEP 7.

The internal node voltages of the equivalent machines are estimated (see figure 12).

$$E_1' = V_5 + \left(jX_{d_1}'\right) I_5 \tag{2.12a}$$

$$E_2' = V_6 + \left(jX_{d_2}'\right) I_6 \tag{2.12b}$$

Since

$E_i' = |E_i|\angle\delta_i,$

the angular separation between the two systems is

$$\delta = \delta_1 - \delta_2 \tag{2.13}$$

At maximum power transfer

$E_1' = 1.1393\angle -28.5^\circ, E_2' = 1.0585\angle 3.22^\circ.$

Therefore

$\delta = -31.64^\circ.$

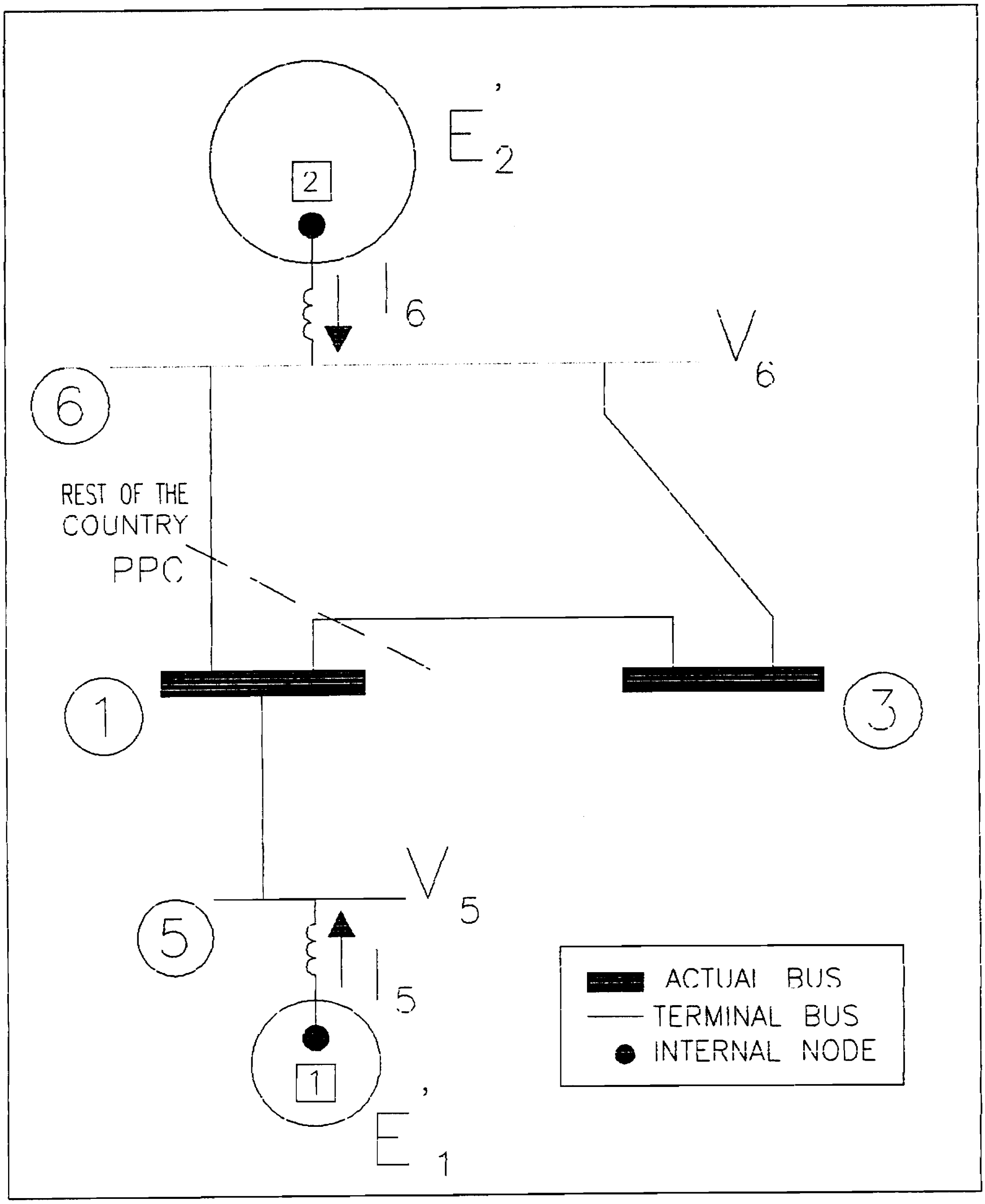


Figure 12. Estimation of the Internal Node Voltages

2.2.1.8 STEP 8.

The present value of δ is compared with its previous value. If the absolute value of the difference between $\delta^{(n)}$ and $\delta^{(n-1)}$ is greater than or equal to a predetermined index, ε, then steps (9) through (11) are performed. If the absolute value of this difference is less than the index, then the algorithm returns to step (5).

If

$$\varepsilon \leq \delta^{(n)} - \delta^{(n-1)} \tag{2.14}$$

then a disturbance has occurred. Therefore a check is required to determine whether or not the two systems will remain stable or go out of step.

2.2.1.9 STEP 9.

The internal node admittance matrix, Y_4, and the following equations are required to convert the equivalent two machine system to an OMIB [20].

$$Y_4 = \begin{bmatrix} y_{11} & y_{12} \\ \cdot & y_{22} \end{bmatrix},$$

where

$$y_{11} - g_{11} + jb_{11},$$
$$y_{12} = y_{21} = g_{12} + jb_{12} = |y_{12}| \angle \theta_{12},$$
$$y_{22} = g_{22} + jb_{22}.$$

$$P_C = \frac{H_2 |E_1^{'}|^2 g_{11} - H_1 |E_2^{'}|^2 g_{22}}{H_1 + H_2} \tag{2.15}$$

$$P_M = \frac{|E_1^{'}||E_2^{'}||y_{12}|\sqrt{H_1^2 + H_2^2 - 2H_1 H_2 \cos\theta_{12}}}{H_1 + H_2} \tag{2.16}$$

$$\gamma = -\ ARCTAN \left\{ \frac{H_1 + H_2}{H_1 - H_2} TAN\ \theta_{12} \right\} - 90^{\circ} \tag{2.17}$$

$$P_E = P_C + P_M \sin(\delta - \gamma) \tag{2.18}$$

The term P_C represents the ohmic losses (transmission lines and loads) of the OMIB. P_M is similar to the term P_{max} of equation (2.2) for the two machine system. It depicts the maximum electric power that the OMIB is capable of transmitting. γ is the resultant phase shift due to the conductances in the computer model. P_E is the electrical power of the OMIB at that particular instant in time. It is represented by the operating point on the OMIB's power curve. During equilibrium, P_E is equal to the mechanical power input to the single machine of the OMIB. Please refer to figure 13 for a graphical illustration of the OMIB.

The effects of the conductances can be eliminated by manipulating equation (2.18) as follows

$$\hat{P}_E = P_M \sin\hat{\delta} \tag{2.19}$$

where

$$\hat{P}_E = |P_E - P_C| \tag{2.20}$$

$$\hat{\delta} = |\delta - \gamma| \tag{2.21}$$

Equations (2.20) and (2.21) are expressed as absolute values so that the point of operation on the OMIB's power curve is confined to the first quadrant. Figure 14 illustrates the *adjusted* OMIB.

For maximum power transfer the following parameters were determined from the load flow study of table 2

$y_{12} = 36.1494 \angle 79.81°$,
$g_{11} = 48.7085$, $g_{22} = 891.5944$
$H_1 = 212$ seconds, $H_2 = 10{,}000$ seconds
$|E_1'| = 1.1393$, $|E_2'| = 1.0585$.

Applying these values to equations (2.15) through (2.17) yields the operating point on the power curve of the OMIB at maximum power transfer

$$P_E = 41.1723 + 42.3486 \sin(\delta_0 + 10.0542°),$$

where

$\delta_0 = -31.64°$,
$P_C = 41.1723$,
$P_M = 42.3486$,
$\gamma = -10.0542°$.

Therefore P_E is equal to 25.5925.

The electrical power of the *adjusted* OMIB given by equation (2.19) is

$$\hat{P}_E = 42.3486 \sin \hat{\delta}_0,$$

where

$$\hat{\delta}_0 = 21.59^\circ.$$

So $\hat{P}_E$ equals 15.5827.

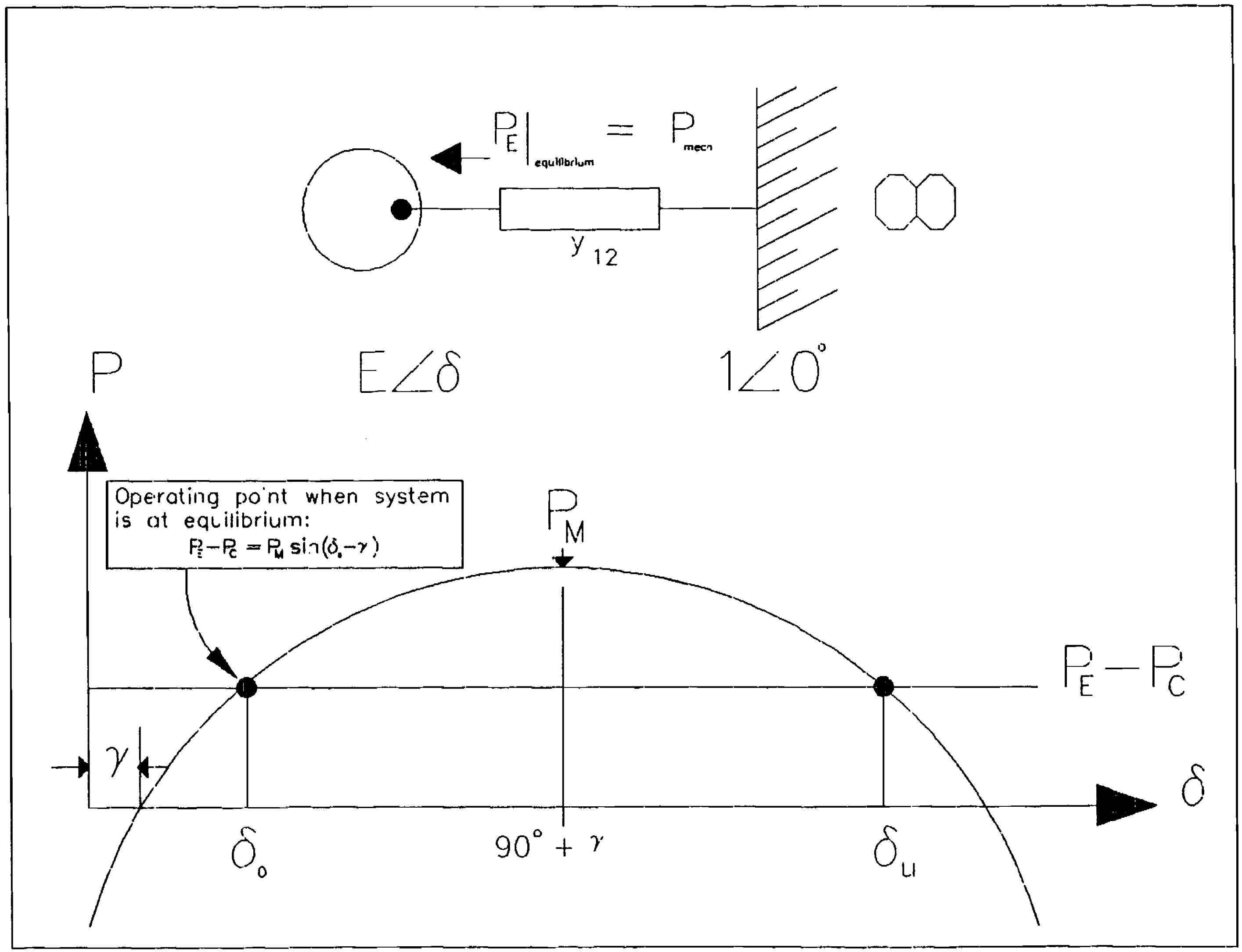


Figure 13. Graphical Illustration of the OMIB

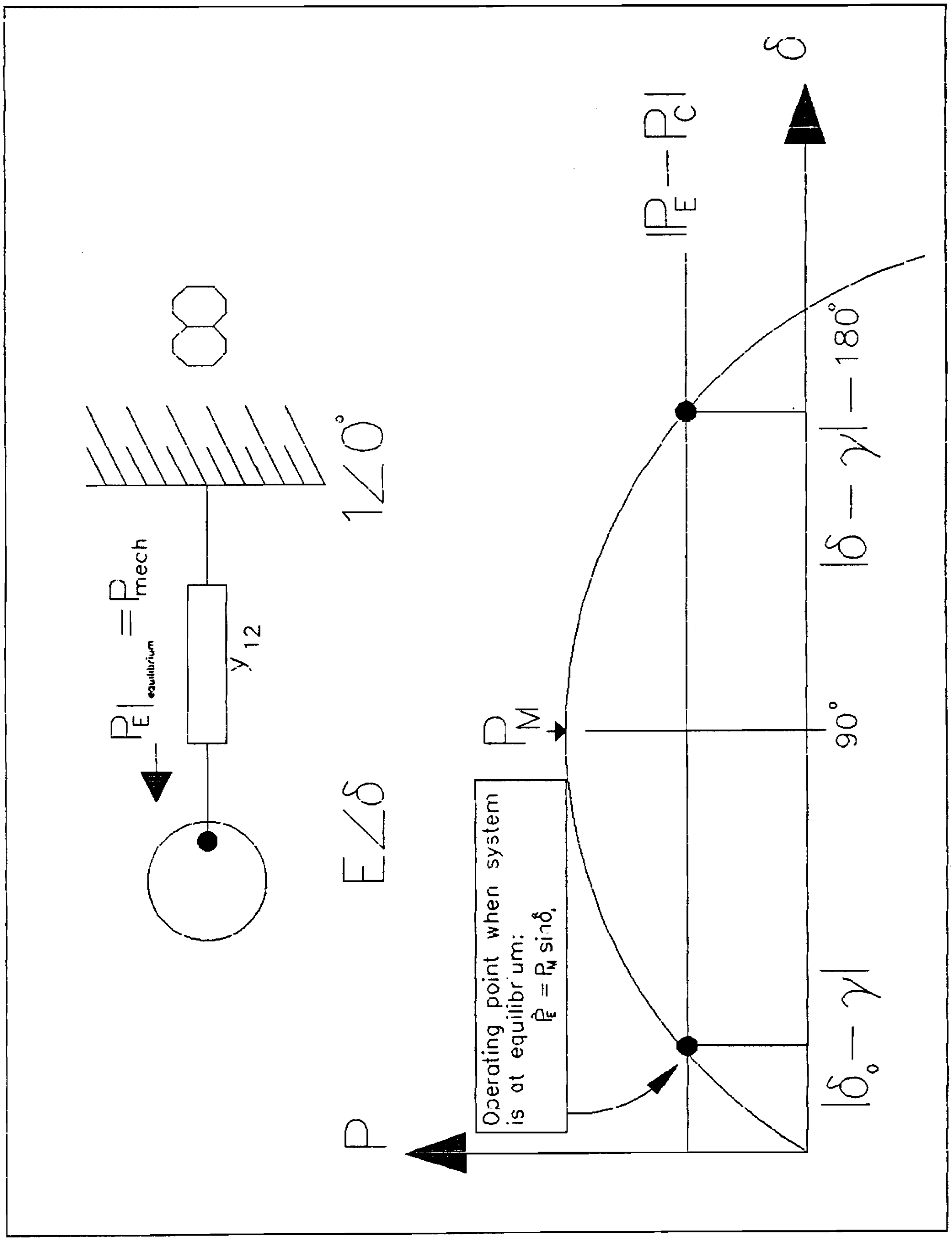


Figure 14. Graphical Representation of the Adjusted OMIB

2.2.1.10 STEP 10.

A deflection of the angular separation between PPC and the rest of the country as detected by equation (2.14), is a direct indication that a disturbance has occurred somewhere within the equivalent two machine system. This type of disturbance will occur when a large generating unit has been dropped within PPC's EHV system.

Before the large generator is dropped, the mechanical power delivered to the machines is operating at some value which corresponds to equilibrium with the electric power of the system. The moment the unit is lost, the mechanical power jumps to a new value, which is greater by the amount of generation lost since this is equivalent to an equal amount of sudden demand.

The mechanical power input to the machines changes instantaneously when the generating unit is dropped. Before the generator was lost, the mechanical power was equal to the electric power of the system. After the unit is lost, a small amount of time is required for the electrical power to regain equilibrium with the mechanical power, if possible. This corresponds to a greater separation between the rotor angles of the equivalent machine for PPC and the equivalent machine representing the rest of the country. Otherwise the two systems will go out of step and the angular separation between them grows unbounded.

Three cases are possible. For the first case, a new point of equilibrium will occur after momentary oscillation and the two equivalent machines remain stable. For the second case, a new operating point will be established, but the kinetic energy injected into the system during the disturbance is greater than its potential energy and the two equivalent machines separate. For the third case, the new value of the mechanical power input to the machines overshoots the maximum electrical power that the system can transmit and the two equivalent machines go unstable. Please refer to figure 15.

If the post disturbance operating point, $\hat{\delta}_1$, on the power curve of the *adjusted* OMIB was known, then it would be a simple task to predict whether or not the overall system would remain stable. To accomplish this task, the power curve for the OMIB is approximated as piece-wise linear between measurements and a least-squares estimate [21] of $P_E^{(1)}$ is determined by

$$\tilde{P}_E^{(1)} = \frac{2}{\tau^2\left(\nu^T\nu\right)}\left[\nu^T\delta - \nu^T 1\,\delta^{(0)}\right] + \frac{1}{\left(\nu^T\nu\right)}\left[\nu^T M P + \tfrac{1}{3}\nu^T\overline{P}\right] \qquad (2.22)$$

where

$$v = \begin{bmatrix} 1 \\ 4 \\ 9 \\ \cdot \\ \cdot \\ \cdot \\ n^2 \end{bmatrix}, \quad P = \begin{bmatrix} P_E^{(0)} \\ P_E^{(1)} \\ P_E^{(2)} \\ \cdot \\ \cdot \\ \cdot \\ P_E^{(n-1)} \end{bmatrix}, \quad \overline{P} = \begin{bmatrix} P_E^{(1)} - P_E^{(0)} \\ P_E^{(2)} - P_E^{(0)} \\ P_E^{(3)} - P_E^{(0)} \\ \cdot \\ \cdot \\ \cdot \\ P^{(n)} - P^{(0)} \end{bmatrix}, \quad \delta = \begin{bmatrix} \delta^{(1)} \\ \delta^{(2)} \\ \delta^{(3)} \\ \cdot \\ \cdot \\ \cdot \\ \delta^{(n)} \end{bmatrix}, \quad 1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ \cdot \\ \cdot \\ \cdot \\ 1 \end{bmatrix},$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 2 & 2 & 0 & 0 & \cdot & \cdot & \cdot & 0 \\ 3 & 4 & 2 & 0 & \cdot & \cdot & \cdot & 0 \\ 4 & 6 & 4 & 2 & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ n & \cdot & \cdot & \cdot & \cdot & 6 & 4 & 2 \end{bmatrix},$$

$\tau \equiv$ measurement interval (time).

The estimate and prediction must be made within a quarter of the period of the fastest possible swing on the system so that the control actions will be effective. By equation (2.1) it was found that the fastest swing occurs during minimum loading of the system. Appendix B is the calculation of this period.

Since the fastest swing is approximately one second and phasor measurements are made once every 5 cycles when a satellite link is the communications medium, a good estimate of $P_E^{(1)}$ must be made within 15 cycles. The set of measurements that indicated a disturbance has occurred serves as the initial conditions. Three subsequent sets of measurements are then needed. Appendix C is a comparison of the estimate of $P_E^{(1)}$ for measurement intervals of one and 5 cycles. Next, the post disturbance electrical power of the *adjusted* OMIB is determined and then equation (2.19) is solved for the corresponding angular separation.

Once all the necessary parameters are known, the EEAC can be applied. For cases one and two of figure 15 the following two equations are used to determine areas one and two

Area (1) =

$$
\begin{aligned}
&= \int_{\hat{\delta}_0}^{\hat{\delta}_1} \left(\hat{P}_E^{(1)} - P_M \sin\hat{\delta} \right) d\hat{\delta}, \\
&= \hat{P}_E^{(1)} \int_{\hat{\delta}_0}^{\hat{\delta}_1} d\hat{\delta} - P_M \int_{\hat{\delta}_0}^{\hat{\delta}_1} \sin\hat{\delta}\, d\hat{\delta}, \\
&= \hat{P}_E^{(1)} \left(\hat{\delta}_1 - \hat{\delta}_0 \right) + P_M \left(\cos\hat{\delta}_1 - \cos\hat{\delta}_0 \right).
\end{aligned}
\tag{23}
$$

Area (2) =

$$
\begin{aligned}
&= \int_{\hat{\delta}_1}^{\hat{\delta}_u} \left(P_M \sin\hat{\delta} - \hat{P}_E^{(1)} \right) d\hat{\delta}, \\
&= P_M \int_{\hat{\delta}_1}^{\hat{\delta}_u} \sin\hat{\delta}\, d\hat{\delta} + \hat{P}_E^{(1)} \left(\hat{\delta}_1 - \hat{\delta}_u \right), \\
&= P_M \left(\cos\hat{\delta}_1 - \cos\hat{\delta}_u \right) + \hat{P}_E^{(1)} \left(\hat{\delta}_1 - \hat{\delta}_u \right).
\end{aligned}
\tag{24}
$$

If area one is less than or equal to area two, then the system is stable. Otherwise, the two equivalent machines will separate.

For case 3, if $\hat{P}_E^{(1)}$ is greater than P_M, the system is unstable.

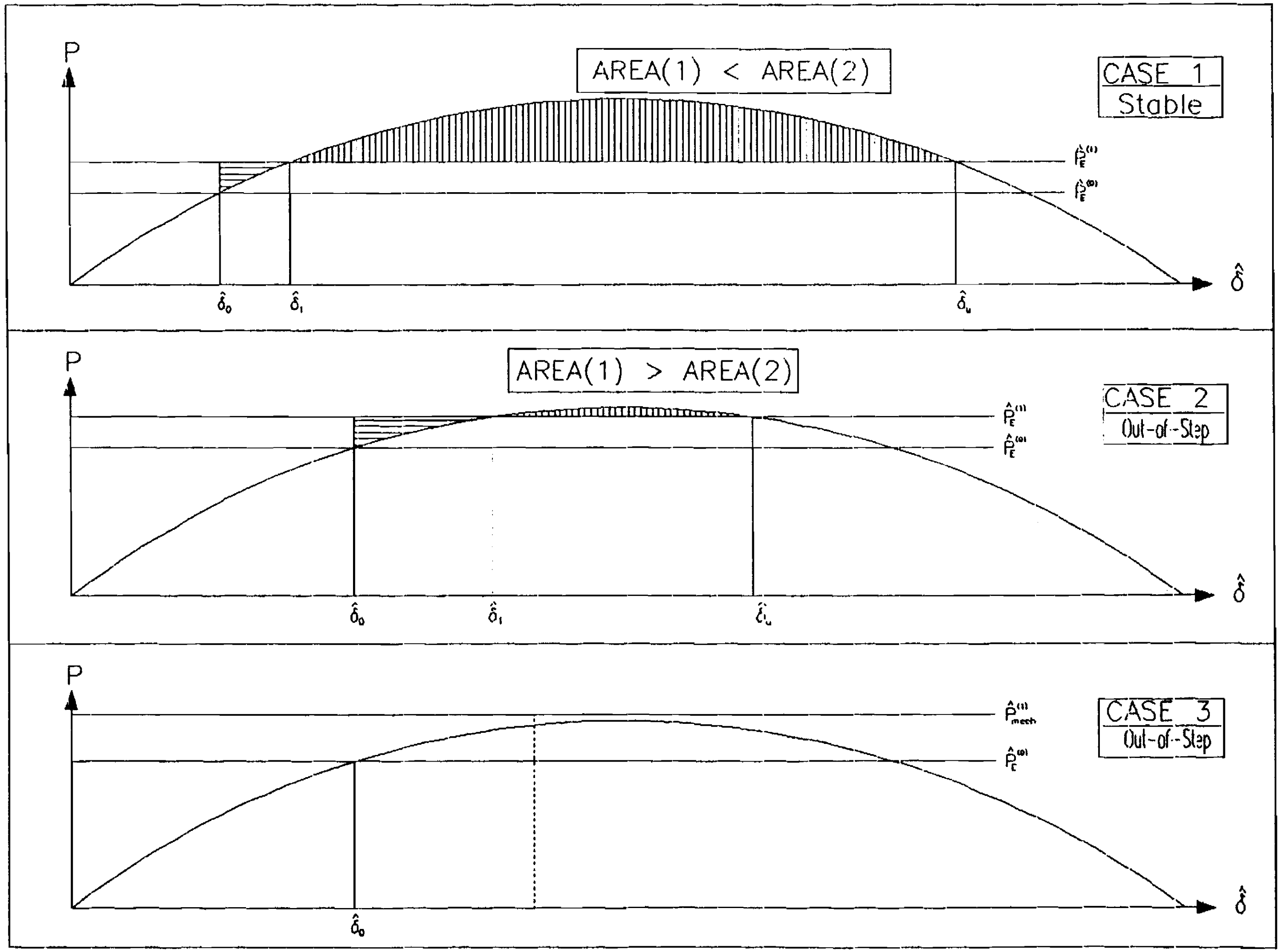


Figure 15. Three Possible Cases

2.2.1.11 STEP 11.

Once the state of the post disturbance system has been determined, it is necessary to issue the appropriate control actions needed to block circuit breakers on the 500 kV tie lines for stable oscillations or trip them for ensuing separations. Figure 16 illustrates this strategy.

This completes the method of prediction for generator dropping. Figure 17 is a flow chart of the entire procedure. Appendix D is the documented computer program, written in FORTRAN, that performs the simulations of this method.

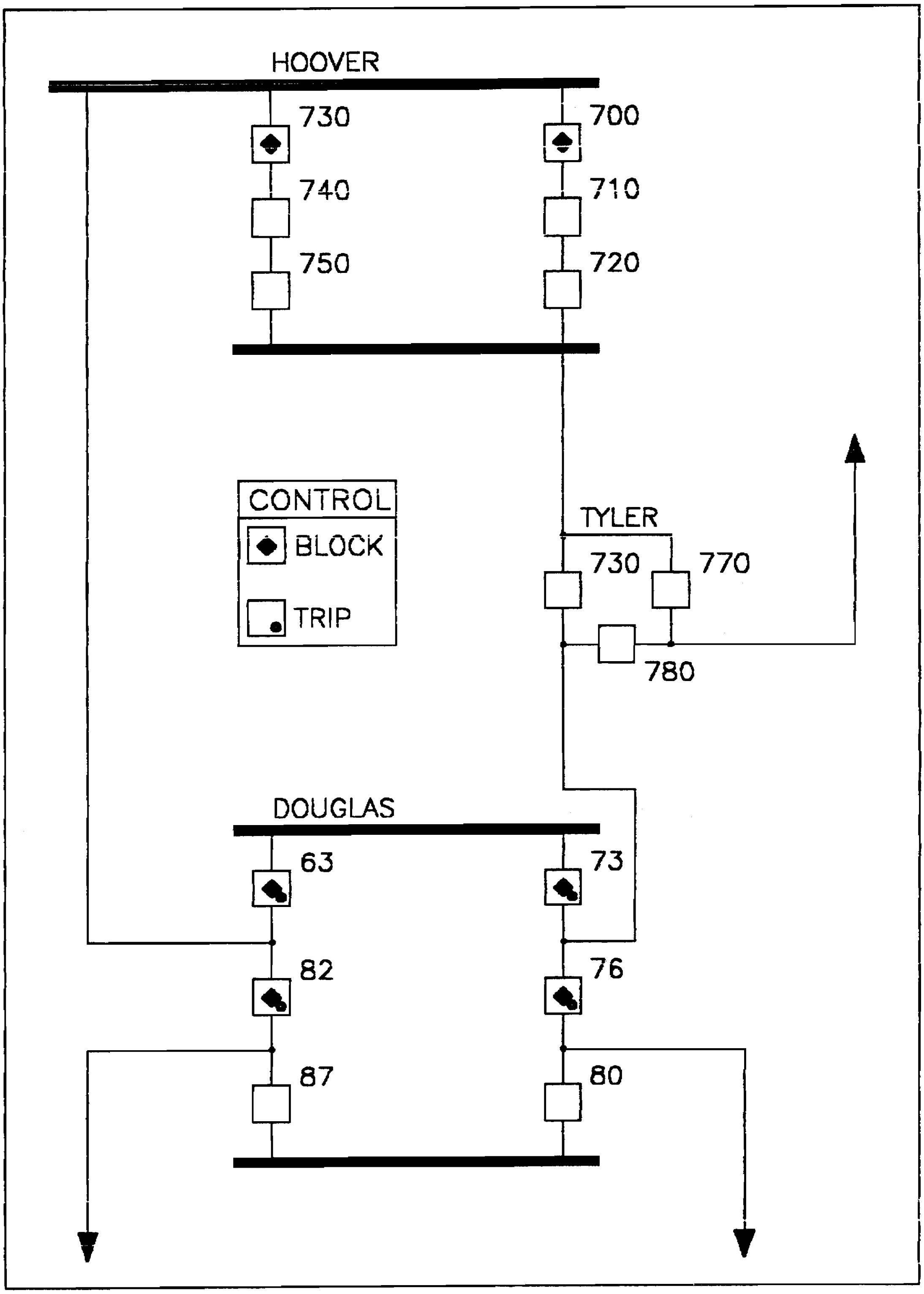


Figure 16. Control Actions

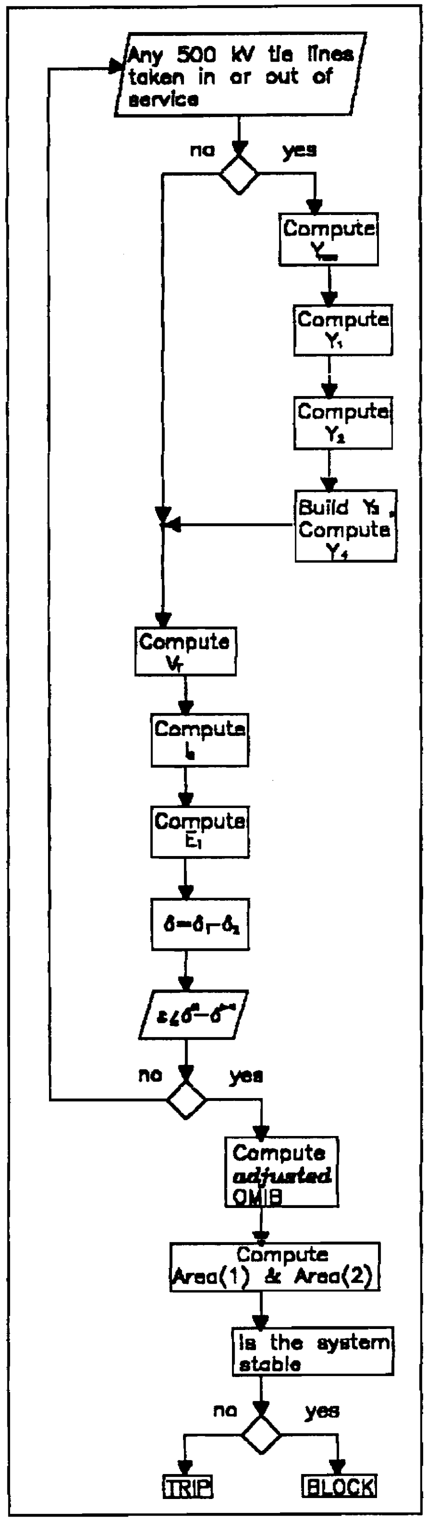


Figure 17. **Flow Chart - Generator Dropping**

2.2.2 The Method of Prediction for Faults on the 500 kV Tie Lines

Whereas the previous method was described through a step by step basis, this method is conceptually demonstrated since the same computations that were used in the previous method are also used here.

Normally, when a fault occurs, on one of the 500 kV tie lines, the following sequence of events occur

- $t = t_0$, fault is initiated on a tie line;
- $t = t_1 = t_0 + 5$ cycles, faulted line is removed from service
- $t = t_2 = t_0 + 35$ cycles, faulted line is reclosed.

It is very important to note that there are several situations when this will not happen. For example, during live line maintenance on a tie line the reclosing relays are blocked. Also, if the line recloses and the fault has not been extinguished, then the line locks out. Likewise, there is always the possibility that a breaker failure might occur.

The EEAC for a fault cleared with high speed reclosing is illustrated in figure 18 [22]. When the sum of areas two and three are greater than or equal to area one, then the system remains stable, otherwise a separation occurs. Note that the *adjusted* OMIB is used. The terminology is defined below

δ = angular separation between PPC and the rest of the country,
δ_0 = initial operating point when the system is at equilibrium (pre fault),
δ_1 = angular separation at the moment the fault is cleared (post fault),
δ_2 = angular separation at the moment the faulted line is reclosed (post fault),
$\delta_3 = 180^o - \delta_0$,

P_E = electrical power of the *adjusted* OMIB,
$P_E^{(0)}$ = initial electrical power of the *adjusted* OMIB (pre fault),
$P_E^{(1)}$ = electrical power of the faulted *adjusted* OMIB (fault),
$P_E^{(2)}$ = electrical power of the *adjusted* OMIB, faulted line removed (post fault),

P_{mech} = mechanical power input to the machines,

Area (1) = kinetic energy injected into *adjusted* OMIB during fault,
Area (2) = potential energy of *adjusted* OMIB, faulted line is out of service (post fault),
Area (3) = potential energy of *adjusted* OMIB when the faulted line is reclosed (post fault).

The mechanical power remains constant throughout the entire sequence of events. Note that this value is also modified for the *adjusted* OMIB and is determined by the following formula

$$\hat{P}^{(i)}_{mech} = \left| P_E(\delta_0) - P_C^{(i)} \right|, \; for\ i = 0, 1 \tag{2.25}$$

Normally the fault only lasts for a period of 5 cycles. Therefore, the difference between δ_0 and δ_1 is very small and area one can be neglected. This allows the use of the following approximation. Area one is due to the reduction of the system's electrical power because the faulted line is removed from service (see figure 19). Four possible cases can occur for a fault when this approximation is applied (see figure 20)

- faulted line is cleared then reclosed, $P_{mech} < P_M^{(2)}$,
- faulted line is cleared then reclosed, $P_{mech} > P_M^{(2)}$,
- faulted line is locked out, $P_{mech} < P_M^{(2)}$,
- faulted line is locked out, $P_{mech} > P_M^{(2)}$.

While it is obvious that the fourth case always results in an unstable system, calculation of the areas must be performed for the other three cases to determine the post fault state of the system. For the first two cases, it is necessary to wait an entire 30 cycles for the high speed reclosure after the fault is cleared to observe the value δ_2 (δ_1). This delay is unacceptable because when the two catastrophic faults occurred in 1987 and 1989, the faulted tie line was cleared and then the healthy tie line's circuit breakers tripped two cycles later. Therefore, it would appear that the EEAC can only be applied to cases where the areas can be determined the moment the fault occurs.

This reasoning would leave only the last two cases as applicable for the EEAC until the following logic is considered. Since it is necessary to keep the healthy interconnection in service when a fault occurs on the other, the algorithm need only be applied when both the Hoover-Douglas and Tyler-Douglas tie lines are in service; i.e., the state of the Hoover-Tyler line does not matter (see figure 21). It has been determined that during *normal* operation, when both of these two critical tie lines are in service and a fault occurs on either one, if the fault is properly cleared within 5 cycles, then a reclosure operation is not necessary to keep the post fault system stable (see figure 22). This is due to the fact that at maximum power transfer, when the transient stability margin is at its minimum, area three is not needed to absorb the kinetic energy injected into the system during the actual fault.

Therefore, when both of the two critical tie lines are in service and a fault occurs on one of them, *it is only necessary to compare area one with area two for the case of a line taken out of service (lock out) to determine whether the post fault system will remain stable.* This calculation can be performed each measurement interval, for a separate fault on each of the three tie lines so that the transient stability margins are known for an occurrence of any one fault. Thus, when a fault occurs the control action can be issued instantaneously.

This part of the algorithm can only be used to block circuit breakers on the 500 kV tie lines. This is due to the fact that if an assessment of areas one and two for one of the cases of lock-out indicated an out of step condition existed for the occurrence of a fault, then it would be necessary to calculate area three for the actual reclosure operation to be sure that the prediction was correct. Figure 23 illustrates the logic needed to implement this blocking scheme at one terminal of a critical tie line. Figure 24 is the flow chart.

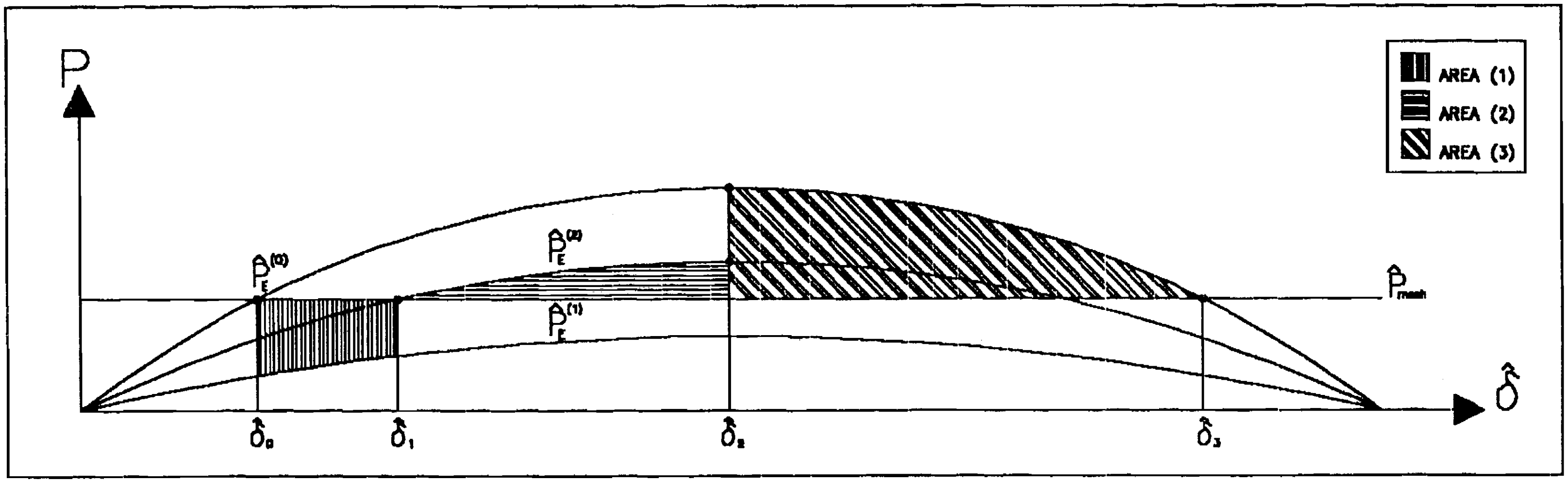


Figure 18. EEAC for Fault Cleared with Hi-Speed Reclosing

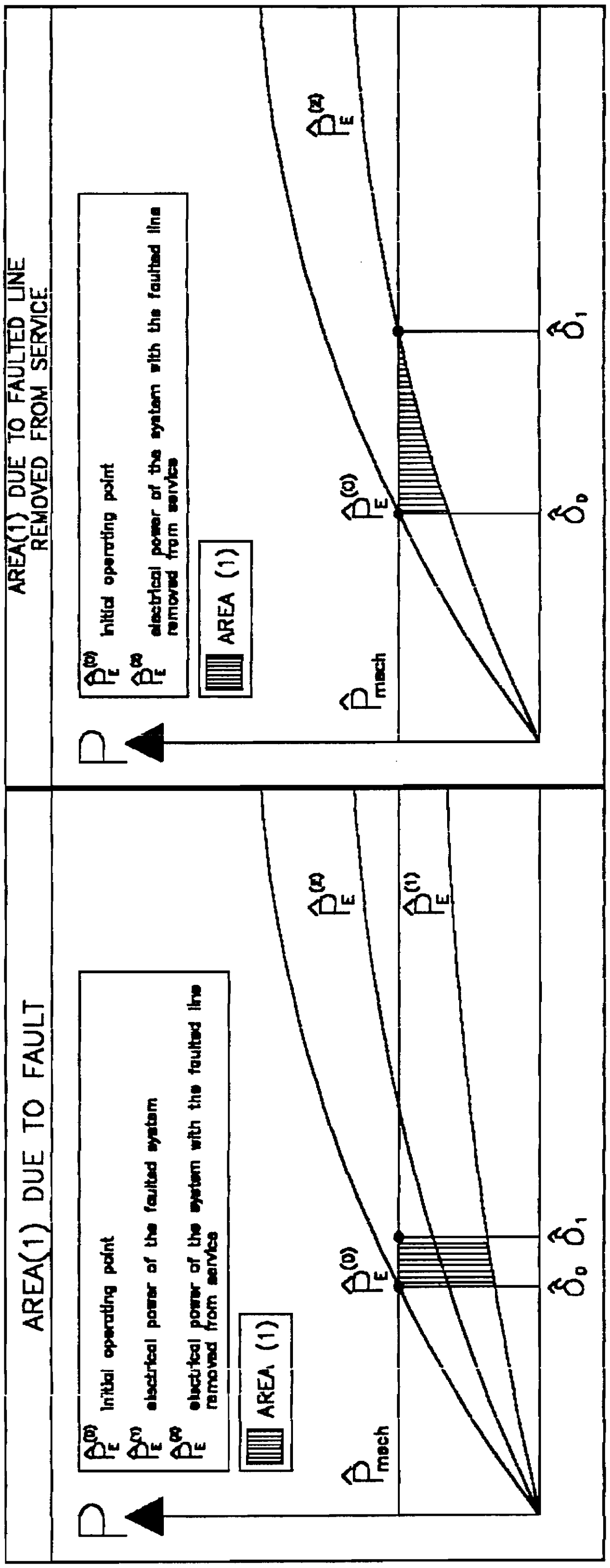


Figure 19. Approximation of Area One

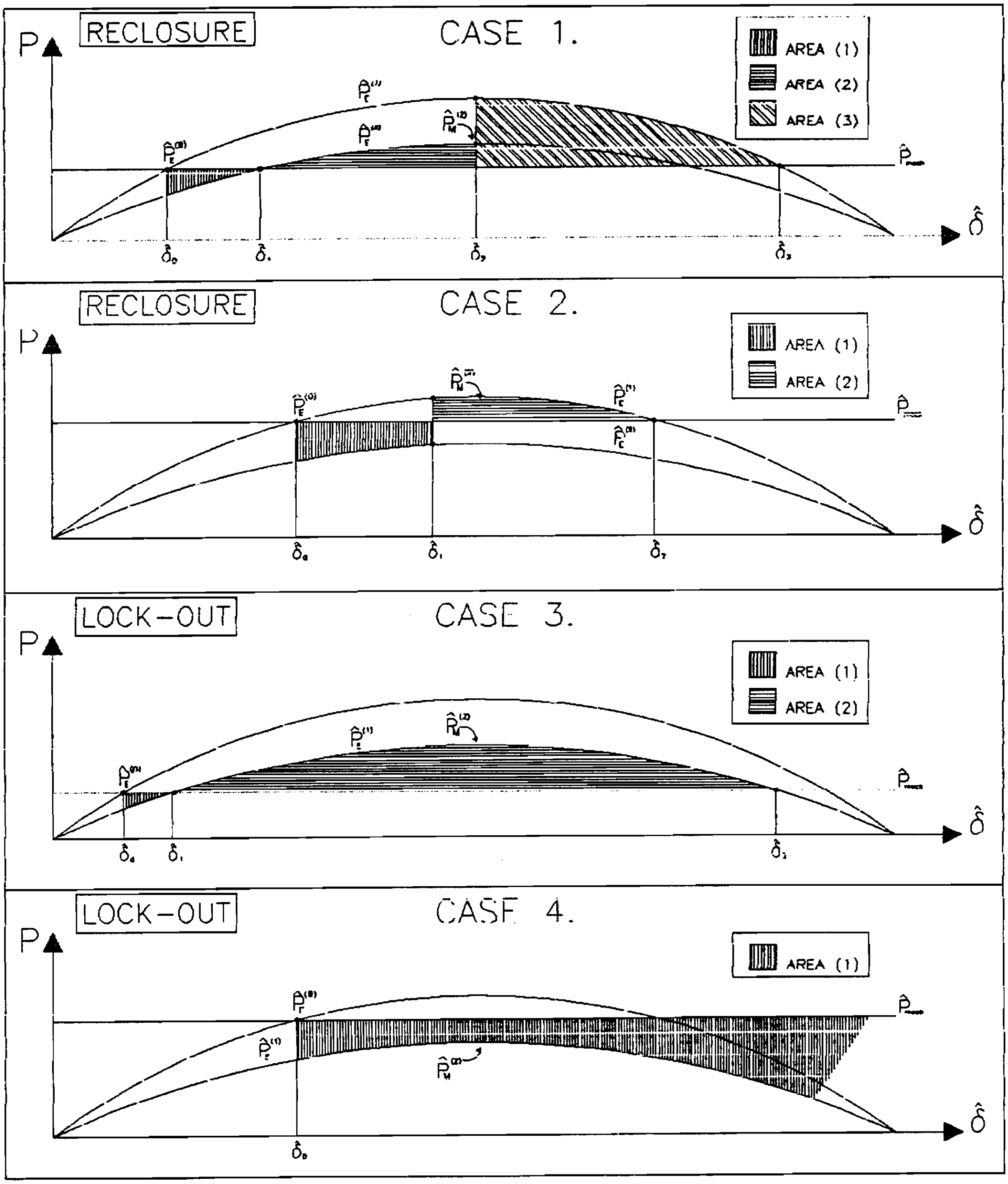


Figure 20. **Approximate EEAC for Faults on the Interconnections**

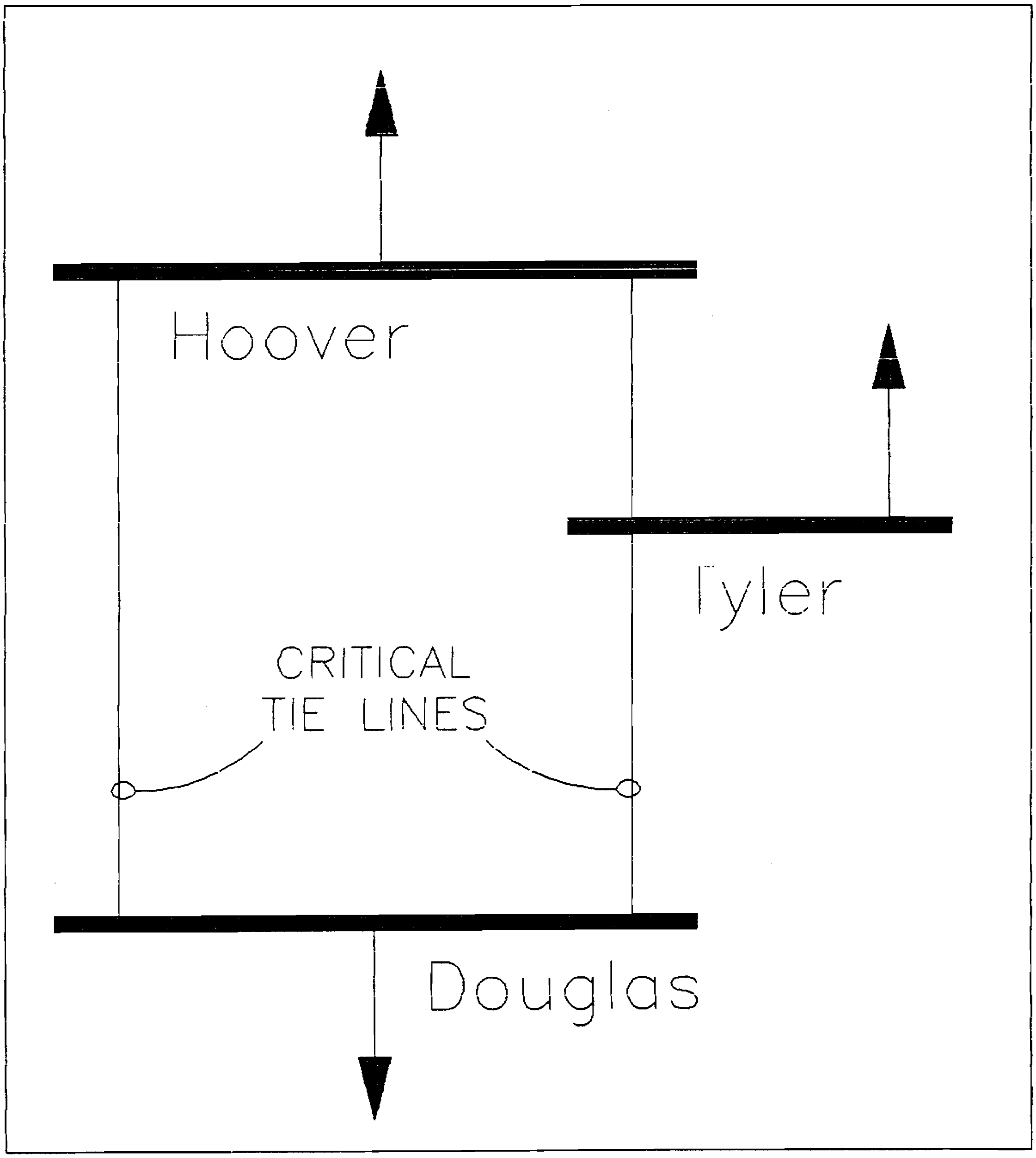


Figure 21. **500 kV Interconnections**

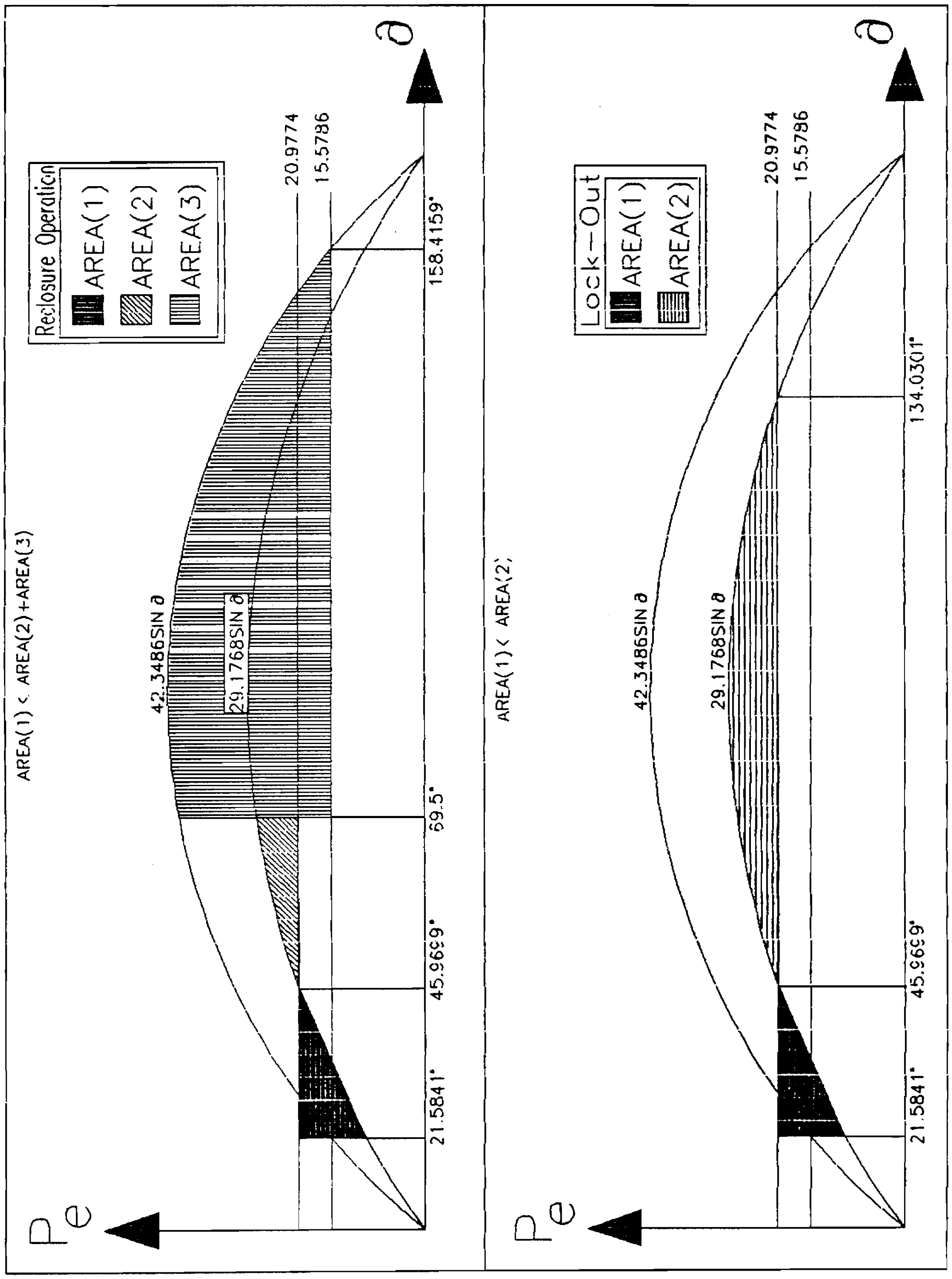


Figure 22. EEAC for Fault on Tyler–Douglas Tie Line

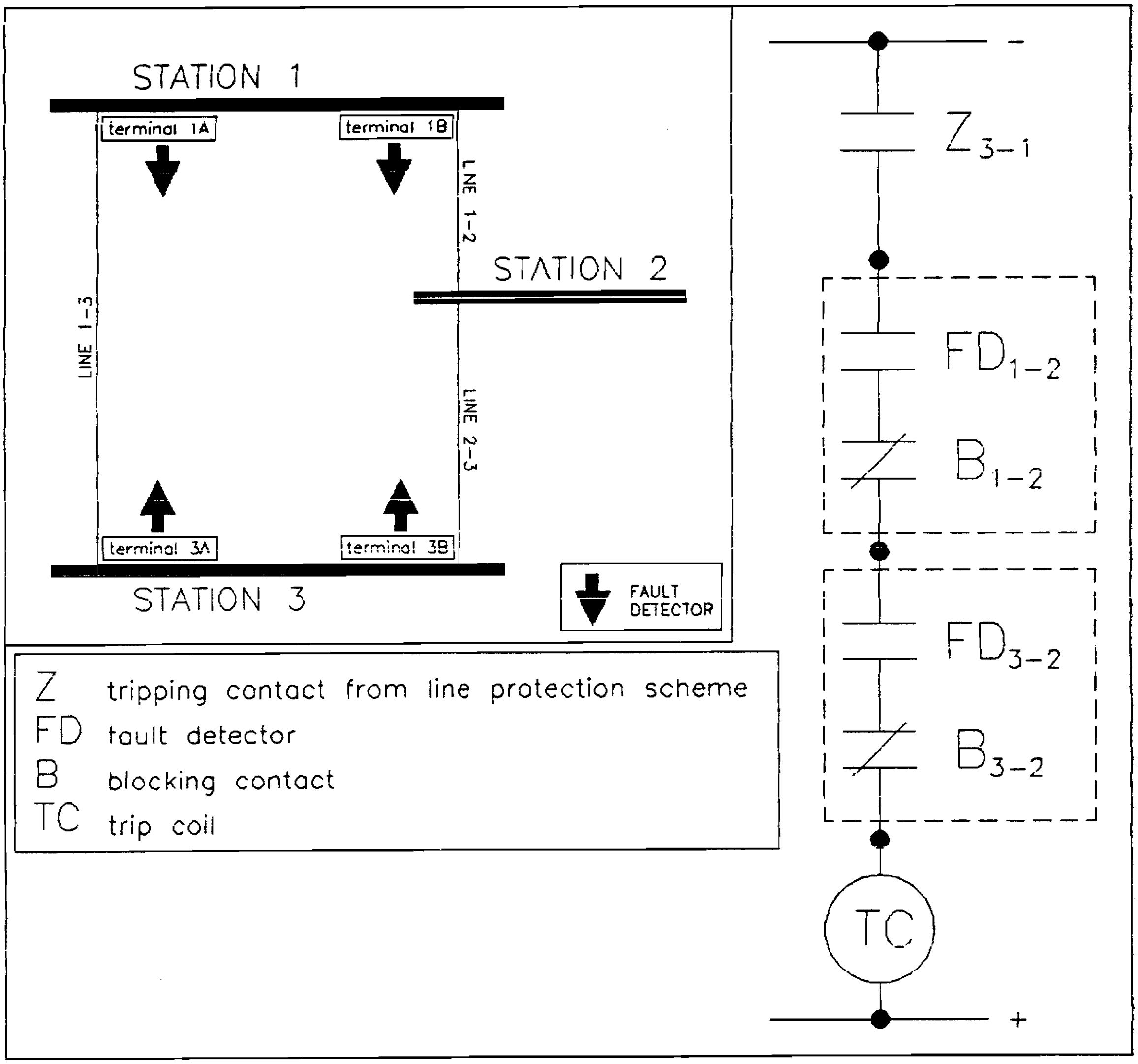


Figure 23. **Blocking Logic at Terminal 3A**

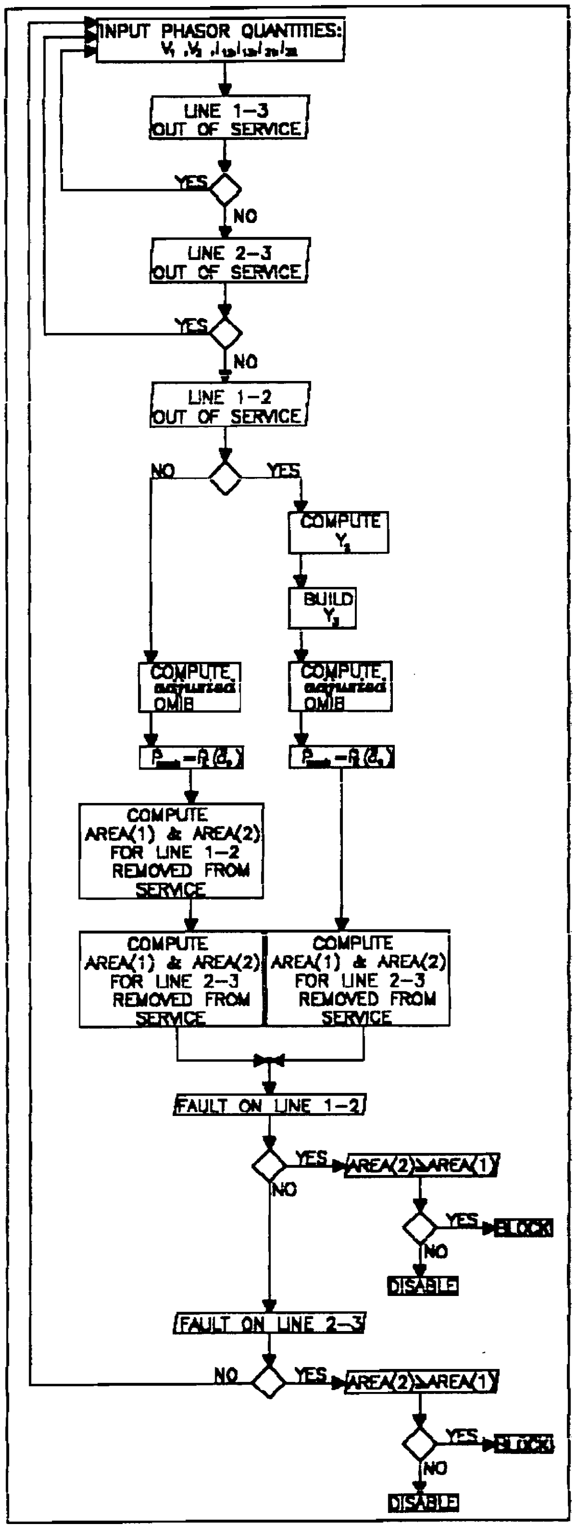


Figure 24. **Flow Chart - Blocking Scheme**

CHAPTER 3

RESULTS

3.1 Generator Dropping

Two cases were simulated to determine the both the accuracy of the algorithm's predictions and its speed. The first event is a loss of generation after which the post disturbance system remains stable. The post disturbance system suffers a separation during the second event.

Error is then introduced to the transducers of the phasor measurement units for the second case to determine its effect upon the algorithm.

3.1.1 Stable Post Disturbance System

This case occured at maximum power transfer, with all 500 kV tie lines in service, when a 800 MW generator was dropped in the southern region of PPC's service area. Figure 25 shows the swing between the two equivalent machines.

The algorithm correctly predicted that PPC would remain stable in a quarter of a second.

3.1.2 Out of Step Condition

This case also occurred at maximum power transfer, but the Douglas-Tyler tie line was out of service when the 800 MW generator was dropped. This event is actually a double contingency since one of the interconnections is out of service and then the 800 MW generator is lost before NEU's generators could be redispatched. Figure 26 depicts that PPC's equivalent machine separated from the rest of the country.

The algorithm correctly predicted that PPC would go out of step in a quarter of a second.

3.1.2.1 Error Analysis

The second case was chosen to examine the effect of error in the phasor measurement units' transducers because the transient stability margin is less at maximum power transfer when a tie line is removed from service.

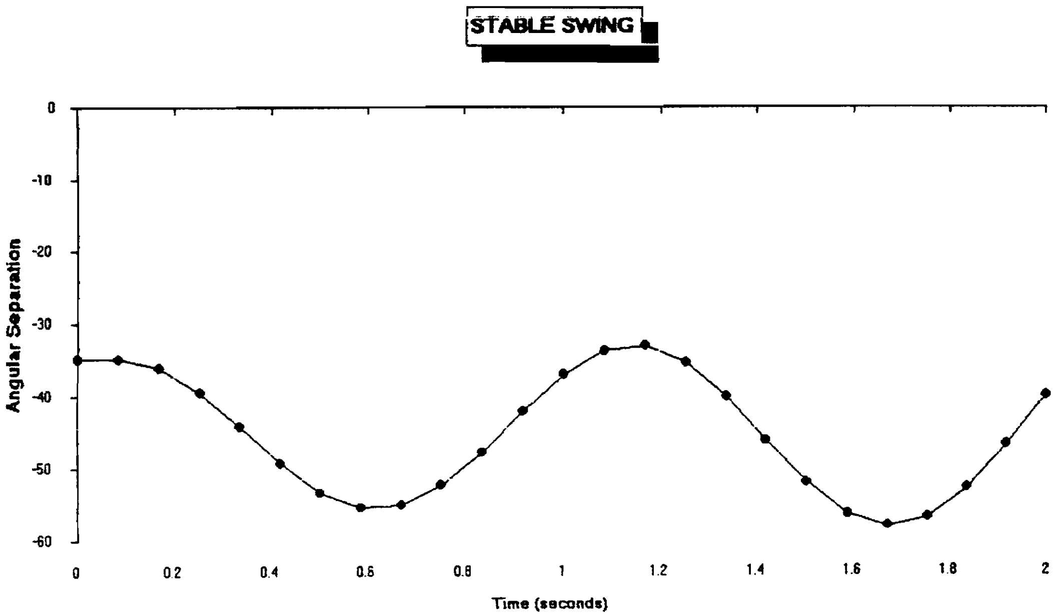


Figure 25. **Stable Swing**

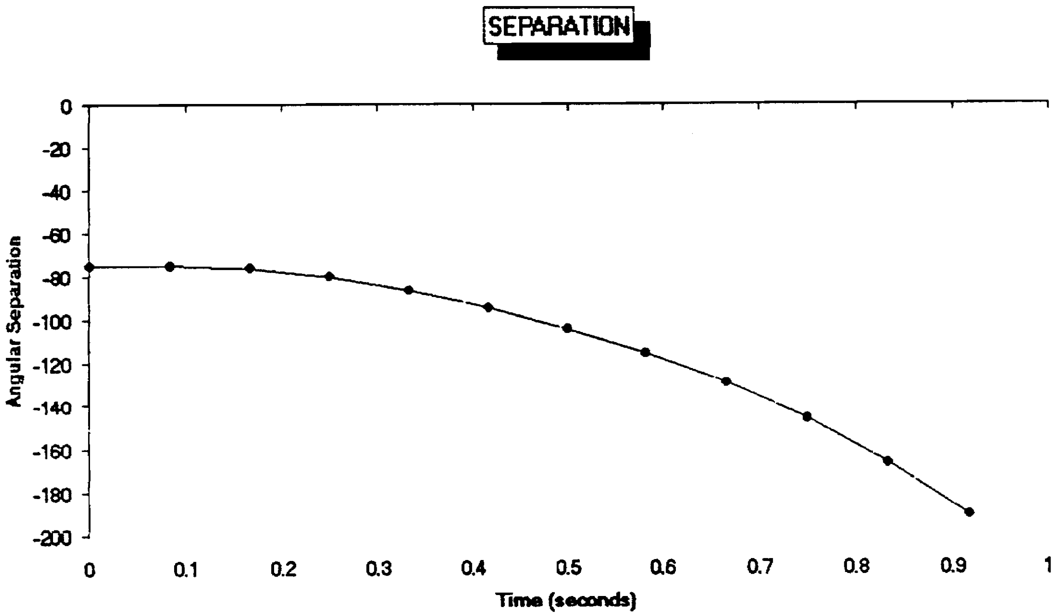


Figure 26. **Separation**

3.1.2.1.1 Zero Error

When no noise was added to the transducers, the following decision was made

The mechanical input exceeded the maximum electrical power available to the network.

Pmax = 31.9537
Pmech = 35.7121

The control actions to trip were issued in 0.25 seconds.

3.1.2.1.2 Unbiased Error

When a random distribution of -2.5° to 2.5° of error was added to the angles of the voltage phasors measured at Douglas and Hoover, the following decision was made

The mechanical input exceeded the maximum electrical power available to the network.

Pmax = 32.9739
Pmech = 33.7268

The control actions to trip were issued in 0.25 seconds.

3.1.2.1.3 Biased Error

When a random distribution of 0° to 5° of error was added to the angles of the voltage phasors measured at Douglas and Hoover, the following decision was made

Area One $<$ Area Two

$\delta(0) = 73.36°$
$\delta(1) = 69.26°$
$\delta(2) = 110.74°$

Pmax = 34.0347
Pmech = 31.8301

The control actions to *block* were issued in 0.25 seconds.

Figure 27 illustrates these three cases.

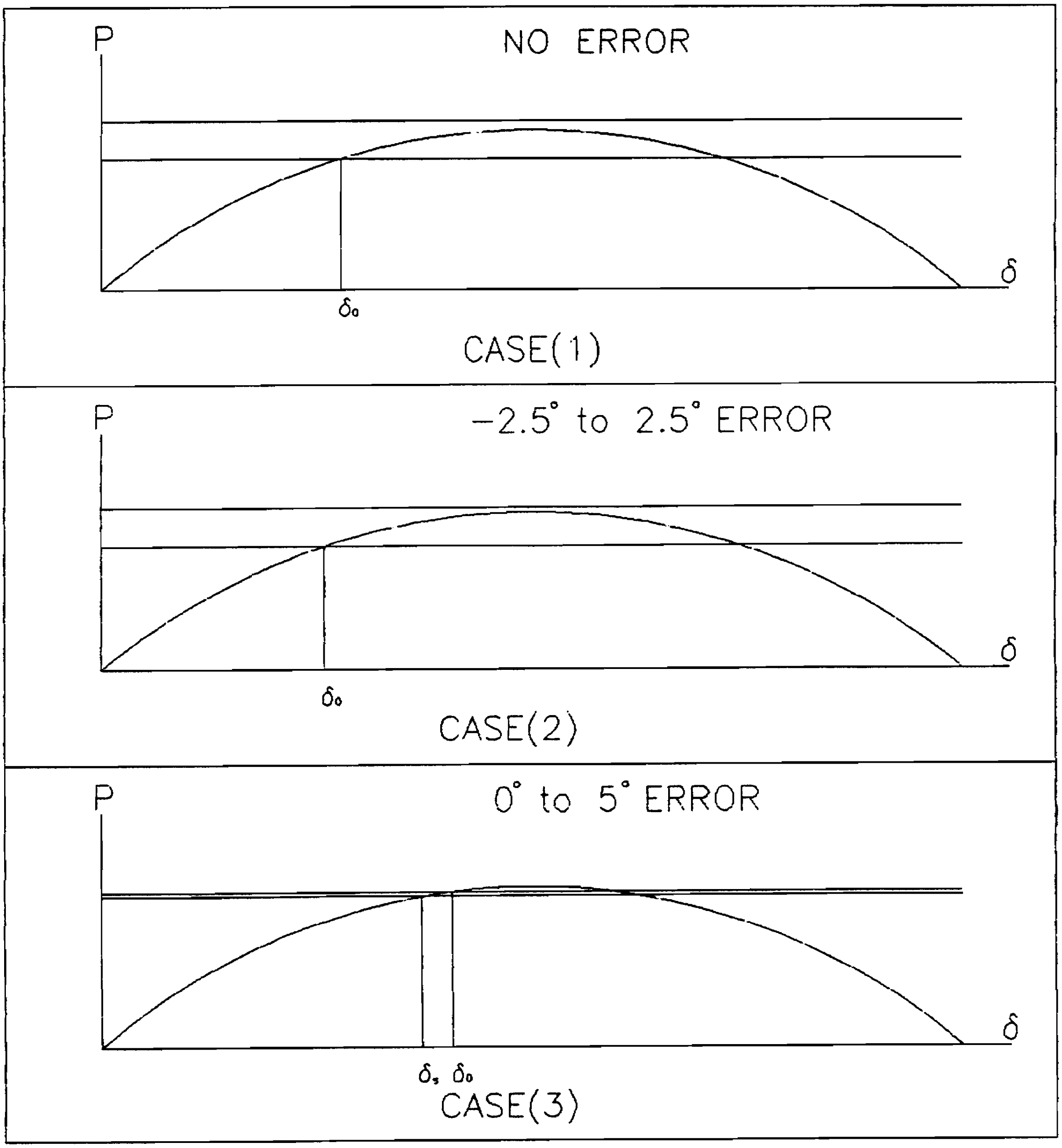


Figure 27. Error Analysis for Generator Dropping

3.2 Faults on the 500 kV Interconnections

A numerical illustration is used to confirm the results of this method of prediction.

Numerical Example

This example is for the case of all 500 kV interconnections in service during maximum power transfer when a fault occurs on the Douglas-Tyler tie line.

At maximum power transfer with all 500 kV interconnections in service (pre fault)

$$P_E(\delta_0) = 41.1723 + 42.3486 \sin(\delta + 10.0542°) \text{ where } \delta_0 = -31.64°.$$

Therefore

$$P_{mech} = 25.5937 \text{ (remains constant).}$$

Determine the electrical power (post fault) of the system with the Douglas-Tyler tie line out of service

$$Y_{old} = \left[\begin{array}{cccc|cc} 25.8842 - j272.41 & -2.22 + j37.18 & -23.12 + j227.54 & -0.54 + j5.45 & 0 & 0 \\ \cdot & 11.40 - j236.52 & 0 & -4.99 + j124.80 & -4.19 + j72.22 & 0 \\ \cdot & \cdot & 23.12 - j227.54 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & 10.69 - j216.56 & 0 & -5.15 + j84.43 \\ \hline \cdot & \cdot & \cdot & \cdot & 6.00 - j105.24 & -1.18 + j30.85 \\ \cdot & \cdot & \cdot & \cdot & \cdot & 6.97 - j117.33 \end{array}\right],$$

$$Y_{new} = \left[\begin{array}{cc|cc} 25.88 - j272.41 & -2.22 + j37.18 & -23.12 + j227.54 & -0.54 + j5.45 \\ \cdot & 8.41 - j182.82 & 0 & -5.61 + j141.31 \\ \hline \cdot & \cdot & 23.12 - j227.54 & 0 \\ \cdot & \cdot & \cdot & 6.78 - j150.73 \end{array}\right],$$

$$Y_1 = \begin{bmatrix} 2.0877 - j32.0623 & -1.9598 + j29.3799 \\ \cdot & 3.0160 - j37.1026 \end{bmatrix}.$$

Since

$$y_{L5} = 72.1154 - j21.0337, \quad y_{L6} = 924.5562 - j269.6653,$$
$$X_{d1}' = 0.00316, \quad X_{d2}' = 0.00006,$$

$$Y_2 = \begin{bmatrix} 75.2031 - j369.5517 & -1.9598 + j29.3799 \\ \cdot & 927.5722 - j16,973.4346 \end{bmatrix},$$

$$Y_3 = \left[\begin{array}{cc|cc} -j16,666.6667 & 0 & 0 & j16,666.6667 \\ \cdot & -j316.4557 & j316.4557 & 0 \\ \hline \cdot & \cdot & 75.2031 - j369.5517 & -1.9598 + j29.3799 \\ \cdot & \cdot & \cdot & 927.5722 - j16,973.4346 \end{array}\right],$$

$$Y_4 = \begin{bmatrix} 892.0721 - j347.7760 & 4.5463 + j23.7984 \\ \cdot & 52.9642 - j56.2091 \end{bmatrix}.$$

So

$$y_{12} = 24.2288 \angle 79.1849°,$$

$g_{11} = 52.9642$, $\quad g_{22} = 892.0721$,
$H_1 = 212$ seconds, $\quad H_2 = 10{,}000$ seconds,
$|E_1'| = 1.1393$, $\quad |E_2'| = 1.0585$ (voltages remain constant).

Therefore

$P_C = 46.5711$,
$P_M = 29.1768$,
$\gamma = -10.3760°$,

and

$$P_E(\delta_1) = 46.5711 + 29.1768 \sin(\delta + 10.3760°).$$

Since $\gamma^{(0)} \approx \gamma^{(1)}$, it is an excellent approximation to ignore the phase shift.

The *adjusted* OMIB for the pre fault system is

$$P_{mech} = P_E(\delta_0),$$

$$25.5937 = 41.1723 + 42.3486 \sin \delta,$$

$$-15.5786 = 42.3486 \sin \delta.$$

Using absolute values to restrict the *adjusted* power curve to the first quadrant yields

$$15.5786 = 42.3486 \sin \delta,$$

where

$$P_{mech}^{(0)} = 15.5786,$$

$$P_M^{(0)} = 42.3486.$$

The *adjusted* OMIB for the post fault system is

$$P_{mech} = P_E(\delta_1),$$

$$25.5937 = 46.5711 + 29.1768 \sin \delta,$$

$$-20.9774 = 29.1768 \sin \delta.$$

Using absolute values to restrict the *adjusted* power curve to the first quadrant yields

$$20.9774 = 29.1768 \sin \delta,$$

where

$$P_{mech}^{(1)} = 20.9774,$$

$$P_M^{(1)} = 29.1768.$$

Note that the mechanical power for the *adjusted* OMIB's actually changes by an amount of 5.3988 due to the fact that P_C, the ohmic losses of the system, increases when the tie line is taken out of service.

Figure 28 illustrates the EEAC for this case.

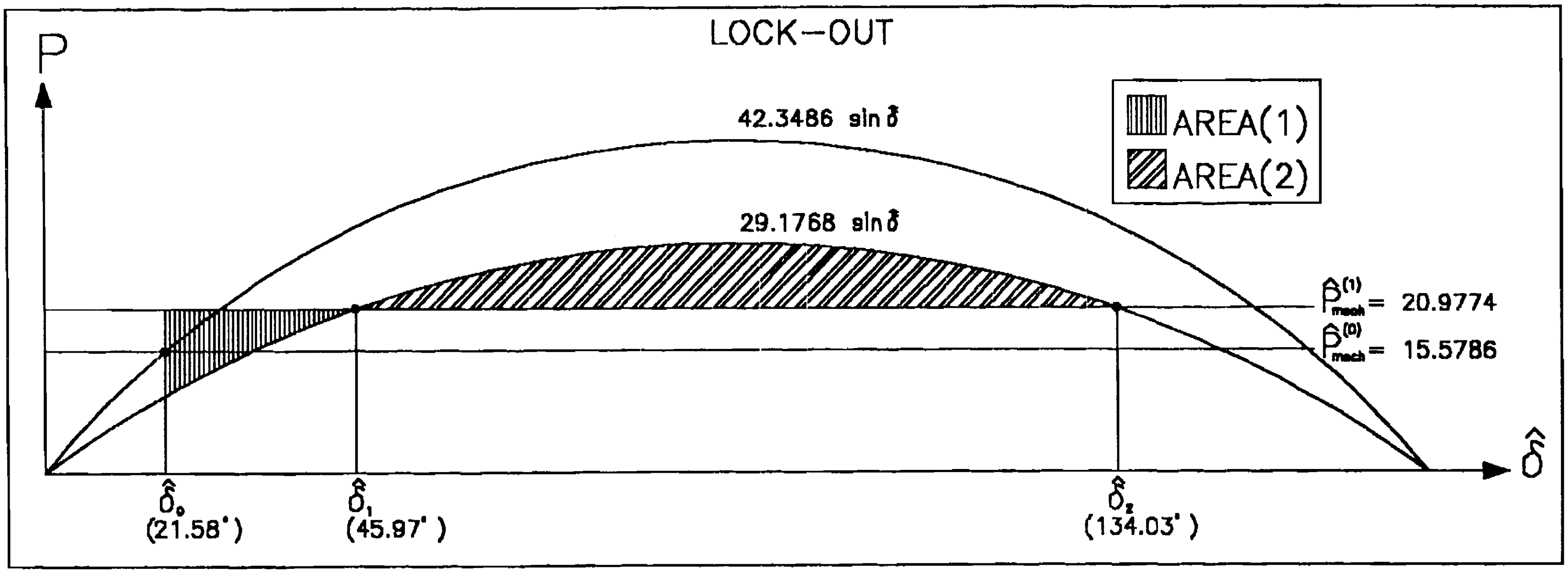


Figure 28. EEAC Approximation for Fault on Douglas-Tyler Tie Line

To determine area one, the following integral is evaluated

$$
\begin{aligned}
Area(1) &= \int_{\delta_0}^{\delta_1} \{20.9774 - 29.1768 \sin\delta\} d\delta, \\
&= 20.9774 \int_{\delta_0}^{\delta_1} d\delta - 29.1768 \int_{\delta_0}^{\delta_1} \sin\delta d\delta, \\
&= 20.9774[\delta_1 - \delta_0] + 29.1768[\cos\delta_1 - \cos\delta_0], \\
&= 8.928 - 6.852, \\
&= 2.076.
\end{aligned}
$$

To determine area two, the following integral is evaluated

$$
\begin{aligned}
Area(2) &= \int_{\delta_1}^{\delta_2} \{29.1768 \sin\delta - 20.9774\} d\delta, \\
&= 29.1768 \int_{\delta_1}^{\delta_2} \sin\delta \, d\delta - 20.9774 \int_{\delta_1}^{\delta_2} d\delta, \\
&= 29.1768[\cos\delta_1 - \cos\delta_2] + 20.9774[\delta_1 - \delta_2], \\
&= 40.5579 - 32.2423, \\
&= 8.3156.
\end{aligned}
$$

Since

$$2.076 \langle 8.3156$$

the post fault system remains stable.

Control actions are issued the moment the fault is detected.

CHAPTER 4

CONCLUSIONS

4.1 Discussion

The algorithm has been proven as a superior means of out of step blocking and tripping for this particular system when it behaves as an equivalent two machine system. While conventional means of out of step blocking and tripping are based on a *worst case* basis, that leaves the system unprotected for unconceived contingencies, the adaptive out of step algorithm has the special ability to adjust for the following changes to the system

- the configuration of the electric power system around the relay
- the load on PPC's electric power system
- power imported from NEU

A problem occurs when transmission lines south of the 500 kV interconnections are taken out of service, because this alters elements in the admittance matrices. Since the 500 kV network was built adjacent to the existing 230 kV network within PPC's high voltage network, they are electrically parallel to each other. Therefore, the loss of service of 230 kV transmission lines will not have a great effect on the value of the element representing the transfer admittance of PPC's EHV system because the 500 kV network has a much lower impedance than the 230 kV network. Also, PPC will try to keep their 500 kV network in service at all times because the bulk of their power flows through these transmission lines. Live line maintenance is performed on the 500 kV transmission lines whenever possible.

Another difficulty arises when a southern generator is lost, since its direct axis transient reactance is removed from the transient reactance of the equivalent machine representing PPC. Fortunately, the equivalent direct axis transient reactance is composed of fourteen large generating units combined in parallel, so the loss of one unit does not greatly affect its value.

It has also been shown that unbiased error in the transducers of the phasor measurement units does not cause an incorrect prediction to be made. The case used to demonstrate this fact was at the threshold for the system being either stable or unstable after the disturbance occurred.

Another 500 kV tie line is planned to be installed towards the end of this decade. When it becomes operational, it will be necessary to include it in the admittance matrices of the equivalent two machine models [23].

4.2 Future Work

As more phasor measurement units are placed around the system and other work is completed at the university, the following goals will be accomplished

- perform adaptive out of step tripping and blocking for multi-machine behavior
- dynamic models of the system and equivalent machines
- visual display of the *dynamic/transient* stability margin within PPC's EHV system
- prediction of area three (reclosure operation) for faults on the 500 kV interconnections

The algorithm presented in this paper uses a two machine model. Therefore, it is only valid for the cases where all of PPC's generators swing together after the occurrence of a disturbance. The system has demonstrated multi-machine behavior for disturbance south of the 500 kV interconnections.

Two more models are being cultivated. One model is that of an equivalent three machine system, while the other is for a four machine system. For both models, the northernmost machine represents the rest of the country. The three machine model is used for cases where southwestern machines swing against southeastern machines within the PPC EHV system after a disturbance. The four machine model splits the southeastern machine into separate northern and southern machines.

Another algorithm will decide how transfer impedances and loads affect these models during line outages. An off-line computation will recalculate the elements of the admittance matrices of the models when a line is removed from service. Transmission lines with large sensitivity factors are the most important to monitor. When their status is incorporated into the models, then accurate predictions can be made.

In order to block or trip for multi-machine oscillations, two methods are being investigated. The first method will integrate the swing equations to determine the outcome of the post disturbance systems. The second method will develop some type of equal area (*volume/hyper-volume*) criterion for disturbances that create oscillations between three, and possibly four, equivalent machines. For these cases, it will be necessary to relay the control actions to remote locations form the computer relay.

In order to determine area three created by reclosure operations after faults have occurred on the 500 kV interconnections, it may be possible to estimate the position of the angular separation at the instant of reclosure by taking a Taylor series approximation of δ.

TABLE 1

Computation of the Direct Axis Transient Reactance of the PPC Equivalent Machine

GENERATING STATION	MW	X_d' *
A	2080	0.022115
B	1568	0.029337
C	1566	0.029374
D	1566	0.029374
E	1553	0.029620
F	1122	0.040998
G	1126	0.040853
H	734	0.062670
I	544	0.084559
J	448	0.102679
K	861	0.053426
L	204	0.225490
M	1250	0.036800
TOTAL	14622	0.003160

* *The direct axis transient reactance for a 4 pole generator was taken as 0.46 per unit on its own nameplate.*

TABLE 2

Positive-Sequence Deck for Model (1)

FROM BUS	TO BUS	R	X	B/2
1	6	0.0375000	0.3780000	0.5005000
1	6	0.0350000	0.3499110	0.4633080
1	5	0.0004420	0.0043500	0.0000000
1	3	0.0016000	0.0268000	1.2781500
1	2	0.0010000	0.0166000	0.7953000
2	3	0.0008000	0.0138000	0.6584500
2	4	0.0019000	0.0323000	1.5043000
3	6	0.0003200	0.0080000	0.3816000
4	6	0.0007200	0.0118000	0.5380560

Load Flow Case for Maximum Power Transfer

BUS	TYPE	V. (PU)	(°)	P_G (MW)	Q_G	P_L (MW)	Q_L
1	1	1.032	-28.71	0	0	0	0
2	1	1.038	-15.47	0	0	0	0
3	1	1.031	- 9.30	0	0	0	0
4	1	1.050	- 4.18	0	0	0	0
5	2	1.040	-35.85	4800	2960	7800	2275
6	3	1.040	0.00	103,131	29,104	100,000	29,166

Type 1 - Load Bus
Type 2 - Generator Bus
Type 3 - Slack Bus

Transient Stability Data

BUS	H (seconds)	X_d' (per unit)
1	212	0.00316
6	10000	0.00006

All values are on a 100 MVA base.
The power factor of the EHV system is taken to be 0.96, lagging.

APPENDIX A

ADMITTANCE MATRIX FOR THE REDUCED TWO MACHINE MODEL

$$
\mathbf{Y_{old}} = \left[\begin{array}{cccc|cc}
y_{11} & y_{13} & y_{15} & y_{16} & y_{12} & y_{14} \\
y_{31} & y_{33} & y_{35} & y_{36} & y_{32} & y_{34} \\
y_{51} & y_{53} & y_{55} & y_{56} & y_{52} & y_{54} \\
y_{61} & y_{63} & y_{65} & y_{66} & y_{62} & y_{64} \\
\hline
y_{21} & y_{23} & y_{25} & y_{26} & y_{22} & y_{24} \\
y_{41} & y_{43} & y_{45} & y_{46} & y_{42} & y_{44}
\end{array}\right],
$$

where

element	admittance(per unit)
y_{11}	29.4983 - j333.2265
y_{13}	-2.2198 + j37.1809
y_{15}	-23.1197 + j227.5359
y_{16}	-0.5429 + j5.4493
y_{12}	-3.6159 + j60.0231
y_{14}	0 + j0
y_{33}	11.3985 - j236.5205
y_{35}	0 + j0
y_{36}	-4.992 + j124.8003
y_{32}	-4.1867 + j72.2211
y_{34}	0 + j0
y_{55}	23.1197 - j227.5359
y_{56}	0 + j0
y_{52}	0 + j0
y_{54}	0 + j0
y_{66}	10.6867 - j216.5638
y_{62}	0 + j0
y_{64}	-5.1517 + j84.4314
y_{22}	9.6175 - j166.0552
y_{24}	-1.1819 + j30.8530
y_{44}	6.9666 - j117.3268

Application of Kron reduction yields

$$\mathbf{Y_{new}} = \left[\begin{array}{cc|cc} y_{11} & y_{13} & y_{15} & y_{16} \\ y_{31} & y_{33} & y_{35} & y_{36} \\ \hline y_{51} & y_{53} & y_{55} & y_{56} \\ y_{61} & y_{63} & y_{65} & y_{66} \end{array}\right],$$

where

element	admittance(per unit)
y_{11}	28.1188 - j310.4163
y_{13}	-3.8174 + j64.6267
y_{15}	-23.1157 + j227.5359
y_{16}	-0.8819 + j13.8872
y_{33}	9.5513 - j203.4973
y_{35}	0 + j0
y_{36}	-5.3769 - j134.9525
y_{55}	23.1197 - j227.5359
y_{56}	0 + j0
y_{66}	6.8177 - j152.6849

These two symmetric matrices are for all 500 kV tie lines in service.

Note that

$$\mathbf{Y_{new}} = \mathbf{Y_{ii}} - \mathbf{Y_{ie}}\,\mathbf{Y_{ee}}^{-1}\,\mathbf{Y_{ei}},$$

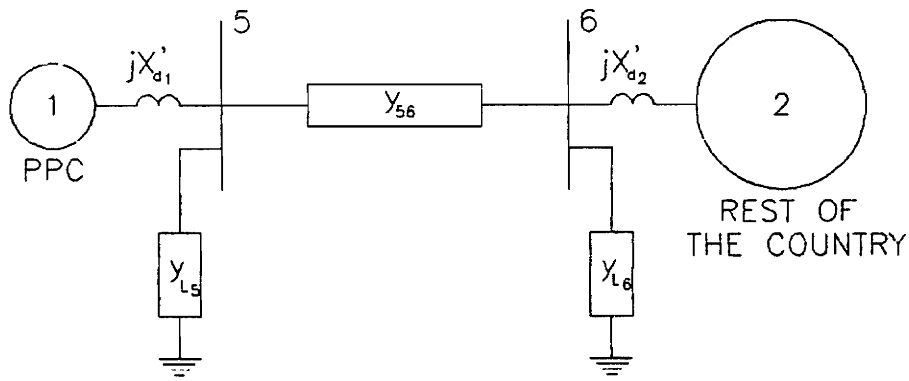
where

$$\mathbf{Y_{old}} = \left[\begin{array}{c|c} Y_{ii} & Y_{ie} \\ \hline Y_{ei} & Y_{ee} \end{array}\right].$$

APPENDIX B

DETERMINATION OF THE FASTEST POSSIBLE SWING

Minimum load on PPC's system is equal to forty percent of maximum load. During minimum load, PPC imports 1000 MW from NEU through the 500 kV tie lines interconnecting the two systems.



$X_{d1}' = 0.00316$, $\quad y_{L5} = (52 - j15.167)/(1)^2 = 52 - j15.167$,
$X_{d2}' = 0.00006$. $\quad y_{L6} = (1000 - j291.67)/(1)^2 = 1000 - j291.67$.

Therefore

$$Y_3 = \left[\begin{array}{cc|cc} -j316.4557 & 0 & j316.4557 & 0 \\ 0 & -j16,666.6667 & 0 & j16,666.6667 \\ \hline j316.4557 & 0 & 54.869 - j376.1807 & -2.896 + j44.558 \\ 0 & j16,666.6667 & -2.869 + j44.558 & 1002.8689 - j16740.3917 \end{array}\right].$$

From which

$$y_{12} = 5.1542 + j36.5302, \quad |y_{12}| = 36.892.$$

From a load flow study for minimum loading

$V_5 = 1 \angle -13.06°$, $\quad S5 = 42.0 + j16.968$,
$V_6 = 1 \angle 0°$. $\quad S6 = 1010.141 + j292.171$.

$I_5 = S_5^*/V_5^* = 45.298 \angle -35.06°$,
$I6 = S_6^*/V_6^* = 1,051.5459 \angle -16.13°$.

$E_1 = V_5 + jX_{d1}' I_5 = 1.0619 \angle -5.88°$,
$E2 = V6 + jXd2' I6 = 1.0193 \angle 3.41°$.

By equation (2.1)

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\left(377\frac{rad}{sec}\right)(1.0619*1.0193*36.892)\cos 9.2893°}{(2)(208sec)}}$$

Since $T_n = 1/f_n$,

$T_n = (0.9511\ Hz)^{-1} = 1.0514$ seconds.

APPENDIX C

LEAST-SQUARES ESTIMATE OF $P_E^{(1)}$

The first estimate is for a measurement interval of 5 cycles, while the second estimate is for a measurement interval of one cycle. Both estimates are the post disturbance value of the electrical power of the OMIB after a 800 MW generator is dropped in the southern region of PPC during maximum power transfer with NEU.

1. $\tau = 5$ cycles

Since the estimate must be made within a quarter of the shortest period

$n = 3.$

By equation (22)

$$\tilde{P}_E^{(1)} = \frac{2}{\Delta^2 \nu^T \nu}\left[\nu^T \delta - \nu^T 1 \delta^{(0)}\right] + \frac{1}{\nu^T \nu}\left[\nu^T M P - \tfrac{1}{3}\nu^T \bar{P}\right],$$

where

$$\nu = \begin{bmatrix} 1 \\ 4 \\ 9 \end{bmatrix},\ P_E = \begin{bmatrix} P_E^{(0)} \\ P_E^{(1)} \\ P_E^{(2)} \end{bmatrix},\ \bar{P}_E = \begin{bmatrix} P_E^{(1)} - P_E^{(0)} \\ P_E^{(2)} - P_E^{(0)} \\ P_E^{(3)} - P_E^{(0)} \end{bmatrix},\ \delta = \begin{bmatrix} \delta^{(1)} \\ \delta^{(2)} \\ \delta^{(3)} \end{bmatrix},\ 1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix},\ M = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 2 \end{bmatrix},$$

$\Delta = 5/60.$

Therefore

$\upsilon^T\upsilon = 98,$ $\qquad$ $(\upsilon^T\upsilon)^{-1} = 0.0102.$

$\Delta^2\upsilon^T\upsilon = 0.680556,$ $\qquad$ $2(\Delta^2\upsilon^T\upsilon)^{-1} = 2.9388.$

$\upsilon^T\delta = \delta^{(1)} + 4\delta^{(2)} + 9\delta^{(3)},$ $\quad$ $\upsilon^T 1\delta^{(0)} = \delta^{(0)} + 4\delta^{(0)} + 9\delta^{(0)} = 14\delta^{(0)}.$

$\upsilon^T\delta - \upsilon^T 1\delta^{(0)} = \delta^{(1)} + 4\delta^{(2)} + 9\delta^{(3)} - 14\delta^{(0)}.$

$$MP = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 2 & 0 \\ 3 & 4 & 2 \end{bmatrix} \begin{bmatrix} P_E^{(0)} \\ P_E^{(1)} \\ P_E^{(2)} \end{bmatrix} = \begin{bmatrix} P_E^{(0)} \\ 2P_E^{(0)} + 2P_E^{(1)} \\ 3P_E^{(0)} + 4P_E^{(1)} + 2P_E^{(2)} \end{bmatrix}.$$

$\upsilon^T \mathbf{MP} = [\ 1\ \ 4\ \ 9\ ]\ \mathbf{MP} = P_E^{(0)} + 8P_E^{(0)} + 8P_E^{(1)} + 27P_E^{(0)} + 36P_E^{(1)} + 18P_E^{(2)},$

$\upsilon^T \mathbf{MP} = 36P_E^{(0)} + 44P_E^{(1)} + 18P_E^{(2)}.$

$\upsilon^T \bar{P} = [\ 1\ \ 4\ \ 9\ ]\ [\ (P_E^{(1)} - P_E^{(0)})\ \ (P_E^{(2)} - P_E^{(0)})\ \ (P_E^{(3)} - P_E^{(0)})\ ]^T,$

$\upsilon^T \bar{P} = P_E^{(1)} - P_E^{(0)} + 4P_E^{(2)} - 4P_E^{(0)} + 9P_E^{(3)} - 9P_E^{(0)} = P_E^{(1)} + 4P_E^{(2)} + 9P_E^{(3)} - 14P_E^{(0)}.$

$(1/3)\upsilon^T \bar{P} = 0.3333P_E^{(1)} + 1.3333P_E^{(2)} + 3P_E^{(3)} - 4.6667P_E^{(0)}.$

$\upsilon^T \mathbf{MP} + (1/3)\upsilon^T \bar{P} = 31.3333P_E^{(0)} + 44.3333P_E^{(1)} + 19.3333P_E^{(2)} + 3P_E^{(3)}.$

$$\tilde{P}_E^{(1)} = \begin{bmatrix} 0.3197 & 0.4524 & 0.1973 & 0.0306 & \vdots & -41.1429 & 2.9389 & 11.7551 & 26.4490 \end{bmatrix} \begin{bmatrix} P_E^{(0)} \\ P_E^{(1)} \\ P_E^{(2)} \\ P_E^{(3)} \\ \cdots \\ \delta^{(0)} \\ \delta^{(1)} \\ \delta^{(2)} \\ \delta^{(3)} \end{bmatrix}$$

Values for $P_E^{(i)}$ are in per unit and values for $\delta^{(i)}$ are in radians

For values determined from the simulation of this case, $\tilde{P}_E^{(1)}$ was found to be 17.0538. The actual value of $P_E^{(1)}$, as determined by a load flow study for the post-disturbance system, was 17.5733. Therefore, the percent error for this estimate is 2.96 %.

2. $\tau = 1$ cycle

$n = 15.$

Again, application of equation (22) yields

$$\tilde{P}_E^{(1)} = a_0\delta^{(0)} + \sum_{i=1}^{15} a_i\delta^{(i)} + b_0 P_E^{(0)} + \sum_{i=1}^{15} b_i P_E^{(i)},$$

where

$[\, a_0 \ a_1 \ a_2 \ a_3 \ \cdots \ a_{15} \,] =$

[-50.0695 0.0404 0.1615 0.3634 0.6461 1.0095 1.4536 1.9786 2.5842 3.2707 4.0379 4.8858 5.8145 6.8240 7.9142 9.0852],

$[\, b_0 \ b_1 \ b_2 \ b_3 \ \cdots \ b_{15} \,] =$

[0.0784 0.1476 0.1337 0.1199 0.1061 0.0926 0.0793 0.0664 0.0541 0.0425 0.0319 0.0223 0.0141 0.0076 0.0029 0.0004].

Values for $P_E^{(i)}$ are in per unit and values for $\delta^{(i)}$ are in radians.

For values determined from the simulation of this case $\tilde{P}_E^{(1)}$ was found to be 17.1696. The actual value of $P_E^{(1)}$, as determined by a load flow study for the post-disturbance system, was 17.5733. Therefore the percent error for this estimate is 2.30 %.

APPENDIX D

Source Code to Simulate the Algorithm for Generator Dropping

```
      PROGRAM OMIB! One Machine-Infinite Bus Conversion Kit & Stability Detector
c
c        The original admittance matrix is for line 1-2 out of service
c
c
c
c        VARIABLES
      CHARACTER*64 FILENAME1
c
      REAL mV1, a1, mV3, a3! voltage magnitudes and angles
      REAL d, dp! angular separation, past and present
      REAL Pc, Pm, GAMMA, H1, H2, G11, G22, mY12, aY12! parameters of the OMIB
      REAL E1, E2! internal node voltages
      REAL dd(4), PPe(4)! angular separation and electrical power for LSM estimate
c
      COMPLEX Yl(2), Xd(2)! load and direct axis transient reactance
      COMMON Yl, Xd
      COMPLEX Ybus1(6,6)! admittance matrix
      Yii1(4,4), Yie1(4,2), Yei1(2,4), Yee1(2,2)! sub matrices
      COMPLEX Ybus02(4,4), Trap1(2,4), Trap2(4,4)! sub matrices
      COMPLEX Ybus2(4,4), Yii2(2,2), Yie2(2,2), Yei2(2,2), Yee2(2,2)1 sub matrices
      COMPLEX Ybus3(2,2)! matrix
      COMPLEX Ybus4(2,2), Yii3(2,2), Yie3(2,2), Yei3(2,2), Yee3(2,2)! sub matrices
      COMPLEX Y11(2,2), Y12(2,2), Y21(2,2), Y22(2,2), X(2,2)! sub matrices
      COMPLEX V1, V3, V5, V6, E(2), I5, I6! voltage and current phasors
c
      WRITE (*,'(A\)') ' input file? '
      READ  (*,'(A\)') FILENAME1
      OPEN (747, FILE = FILENAME1 )
c
c
c        Inertia Constants
      H1 =   212.0
      H2 = 10000.0
c
c        load at the terminal buses
      Yl(1) = (072.1154,-021.0337)
      Yl(2) = (924.5562,-269.6653)
```

```
c       direct axis transient reactances
     Xd(1) = (0.0,0.00006)
     Xd(2) = (0.0,0.00316)
c
c       Original Admittance Matrix(6x6)
c       row 1
     Ybus1(1,1) = (0025.8824,-271.9252)
     Ybus1(1,2) = (-002.2198,0037.1809)
     Ybus1(1,3) = (-023.1197,0227.5359)
     Ybus1(1,4) = (-000.5429,0005.4493)
     Ybus1(1,5) = (-00000000,000000000)
     Ybus1(1,6) = (000000000,000000000)
c
c       row 2
     Ybus1(2,1) = Ybus1(1,2)
     Ybus1(2,2) = (0011.3985,-236.5205)
     Ybus1(2,3) = (000000000,000000000)
     Ybus1(2,4) = (-004.9920,0124.8003)
     Ybus1(2,5) = (-004.1867,0072.2211)
     Ybus1(2,6) = (000000000,000000000)
c
c       row 3
     Ybus1(3,1) = Ybus1(1,3)
     Ybus1(3,2) = Ybus1(2,3)
     Ybus1(3,3) = (0023.1197,-227.5359)
     Ybus1(3,4) = (000000000,000000000)
     Ybus1(3,5) = (000000000,000000000)
     Ybus1(3,6) = (000000000,000000000)
c
c       row 4
     Ybus1(4,1) = Ybus1(1,4)
     Ybus1(4,2) = Ybus1(2,4)
     Ybus1(4,3) = Ybus1(3,4)
     Ybus1(4,4) = (0010.6867,-216.5638)
     Ybus1(4,5) = (000000000,000000000)
     Ybus1(4,6) = (-005.1517,0084.4314)
c
c       row 5
     Ybus1(5,1) = Ybus1(1,5)
     Ybus1(5,2) = Ybus1(2,5)
     Ybus1(5,3) = Ybus1(3,5)
     Ybus1(5,4) = Ybus1(4,5)
     Ybus1(5,5) = (0006.0015,-104.7539)
     Ybus1(5,6) = (-001.1819,0030.8530)
```

```
c        row 6
    Ybus1(6,1) = Ybus1(1,6)
    Ybus1(6,2) = Ybus1(2,6)
    Ybus1(6,3) = Ybus1(3,6)
    Ybus1(6,4) = Ybus1(4,6)
    Ybus1(6,5) = Ybus1(5,6)
    Ybus1(6,6) = (0006.9666,-117.3268)
c
c
c        Step 1. Eliminate nodes 2 and 4 from the original admittance matrix
    DO 1   i = 1, 4
     DO 2 j = 1, 4
c
     Yii1(i,j) = Ybus1(i,j)
c
  2   CONTINUE
  1 CONTINUE
c
c
c
    DO 3   i = 1, 4
     DO 4 j = 1, 2
c
    Yie1(i,j) = Ybus1(i,4+j)
c
  4   CONTINUE
  3 CONTINUE
c
c
c
    DO 5   i = 1, 2
     DO 6 j = 1, 4
c
    Yei1(i,j) = Ybus1(4+i,j)
c
  6   CONTINUE
  5 CONTINUE
c
c
c
    DO 7   i = 1, 2
     DO 8 j = 1, 2
c
    Yee1(i,j) = Ybus1(4+i,4+j)
c
  8   CONTINUE
  7 CONTINUE
```

```
      CALL INVERT(Yee1)
c
c
c
      DO 9   i = 1, 2
       DO 10 j = 1,4
c
      Trap1(i,j) = Yee1(i,1)*Yei1(1,j) + Yee1(i,2)*Yei1(2,j)
c
   10  CONTINUE
    9 CONTINUE
c
c
c
      DO 11   i = 1, 4
       DO 12 j = 1, 4
c
      Trap2(i,j) = Yie1(i,1)*Trap1(1,j) + Yie1(i,2)*Trap1(2,j)
c
   12  CONTINUE
   11 CONTINUE
c
c
c
      DO 13   i = 1, 4
       DO 14 j = 1, 4
c
      Ybus02(i,j) = Yii1(i,j) - Trap2(i,j)
c
   14  CONTINUE
   13 CONTINUE
c
c
c        Step 2. Eliminate nodes 1 and 3 from the new admittance matrix
      DO 15   i= 1, 2
       DO 16 j = 1, 2
c
      Ybus2(i,j)       = Ybus02(2+i,2+j)
      Ybus2(2+i,2+j) = Ybus02(i,j)
      Ybus2(2+i,j)     = Ybus02(i,2+j)
      Ybus2(i,2+j)     = Ybus02(2+i,j)
c
   16  CONTINUE
   15 CONTINUE
```

```
      CALL SORT(Ybus2,Yii2,Yie2,Yei2,Yee2)
c
      CALL REDUCE(Ybus3,Yii2,Yie2,Yei2,Yee2)
c
c
c        Step 3. Add load and transient reactances to the self-admittance elements
      CALL ADDER(Ybus3)
c
c
c        Step 4. Reduce the admittance matrix back to its internal nodes
      CALL INTERNAL(Ybus3,Yii3,Yie3,Yei3,Yee3)
c
      CALL REDUCE(Ybus4,Yii3,Yie3,Yei3,Yee3)
c
      CALL SORT2(Ybus4)
      call view(ybus4,2,2)
c
c
c        Step 5. Estimate the terminal bus voltage phasors
      CALL SORT(Ybus02,Y11,Y12,Y21,Y22)
c
      CALL INVERT(Y12)
c
      CALL NEGATIVE(Y12)
c
      CALL MULT(X,Y12,Y11)
c
      CALL ADDER2(Y22)
c
c
      DO 101 k = 1, 1000
       dp = d
c
       READ   (747,77) mV1, a1, mV3, a3!  input voltage phasors from actual buses
   77  FORMAT (11x,f6.4,2x,f6.2,2x,f6.4,2x,f6.2)
       OPEN (96, file='c:\apsa\e_ang')!  add error to the voltage phasors' angles
       READ (96,69) e_ang1, e_ang3
   69  FORMAT (2x,F9.4,2x,F9.4)
       a1 = a1 + e_ang1
       a3 = a3 + e_ang3
c
      CALL VOLTAGES(V1,V3,V5,V6,mV1,a1,mV3,a3,X)
c
c
c        Step 6. Estimate the generator current phasors
      CALL CURRENTS(I5,I6,V1,V3,V5,V6,Y21,Y22)
```

```
c        Step 7. Estimate the internal node voltage phasors
      CALL node_E(E,V5,V6,I5,I6)
      CALL DELTA(d,E)
      write(*,*) 'delta(0)= ',(180.0/(4.0*ATAN(1.0)))*d
c
c
c        Step 9. Build the OMIB
      E1 = CABS(E(1))
      E2 = CABS(E(2))
c
      G11 = REAL(Ybus4(1,1))
      G22 = REAL(Ybus4(2,2))
c
      mY12 = CABS(Ybus4(1,2))
      aY12 = ATAN2(AIMAG(Ybus4(1,2)),REAL(Ybus4(1,2)))
c
      CALL Losses(Pc,H1,H2,G11,G22,E1,E2)
      write(*,*) 'Pc= ', Pc
      CALL Pmax(Pm,H1,H2,mY12,aY12,E1,E2)
      write(*,*) 'Pmax= ', Pm
      CALL Pshift(GAMMA,H1,H2,aY12)
      write(*,*) 'Gamma= ', (180.0/(4.0*ATAN(1.0)))*GAMMA
      write(*,*) ' '
      CALL Pelect(Pe,Pc,Pm,d,GAMMA)
c
c
c        Step 8. Check for a disturbance
      diff = ABS(d-dp)
c
c
c        Step 10. Perform a LMS estimate to determine the new angular separation
      IF (k.GT.1) THEN
                IF (diff.GE.0.001745) GOTO 1000
             ELSE
                GOTO 101
      END IF
c
 1000 dd(1) = d
      PPe(1) = Pe
c
         DO 1001 kk = 1, 3
c
         READ (747,77) mV1, a1, mV3, a3

         CALL VOLTAGES(V1,V3,V5,V6,mV1,a1,mV3,a3,X)
         CALL CURRENTS(I5,I6,V1,V3,V5,V6,Y21,Y22)
         CALL node_E(E,V5,V6,I5,I6)
```

```
        CALL DELTA(d,E)
c
        dd(kk+1) = d
        CALL Pelect(PPe(kk+1),Pc,Pm,dd(kk+1),GAMMA)
c
 1001     CONTINUE
c
c
      CALL LSM(Pm1,dd,PPe)
c
      c1 = ABS(Pc-Pm1)
c
      d0 = ABS(dd(1)-GAMMA)
c
      IF (c1.GT.Pm) THEN
            CALL REPORT2(c1,Pm,REAL(kk)*(5.0/60.0))
            CLOSE (747)
            STOP
      END IF
c
      ds = ASIN(c1/Pm)
      du = (4.0*ATAN(1.0)) - ds
c
c       Compute area one and area two
      A1 = c1*(ds-d0) + Pm*(COS(ds)-COS(d0))
      A2 = Pm*(COS(ds)-COS(du)) + c1*(ds-du)
c
      Top = REAL(kk)*(5.0/60.0)
c
      CALL REPORT(A1,A2,d0,ds,du,c1,Pm,Top)
c
      CLOSE (747)
      STOP
c
c
 101 CONTINUE
c
      CLOSE (747)
      STOP
      END
```

```
c       SUBROUTINES
        SUBROUTINE ADDER(A)
c       Adds load and transient reactance to the self-admittance elements
        COMPLEX Yl(2), Xd(2)
        COMMON  Yl,   Xd
        COMPLEX A(2,2)
c
        a(1,1) = a(1,1) + Yl(1) + ((1.0,0.0)/Xd(2))
        a(2,2) = a(2,2) + Yl(2) + ((1.0,0.0)/Xd(1))
c
        RETURN
        END

c
c
c
        SUBROUTINE ADDER2(A)
c       Adds load and transient reactance to the self-admittance elements
        COMPLEX Yl(2), Xd(2)
        COMMON  Yl,   Xd
        COMPLEX A(2,2)
c
        a(1,1) = a(1,1) + Yl(1)
        a(2,2) = a(2,2) + Yl(2)
c
        RETURN
        END
c
c
c
        SUBROUTINE CURRENTS(I5,I6,V1,V3,V5,V6,Y21,Y22)
c       Computes the generator current phasors
        COMPLEX I5, I6, V1, V3, V5, V6, Y21(2,2), Y22(2,2)
c
        I5 = y21(1,1)*V1 + y21(1,2)*V3 + y22(1,1)*V5 + y22(1,2)*V6
        I6 = y21(2,1)*V1 + y21(2,2)*V3 + y22(2,1)*V5 + y22(2,2)*V6
c
        RETURN
        END
c
c
c
        SUBROUTINE DELTA(d,E)
c       Computes the angular separation between the two equivalent machines
        REAL d, d1, d2
        COMPLEX E(2)
```

```
      d1 = ATAN2(AIMAG(E(1)),REAL(E(1)))
      d2 = ATAN2(AIMAG(E(2)),REAL(E(2)))
c
       d = d2 - d1
c
      RETURN
      END
c
c
c
      SUBROUTINE INTERNAL(A,B,C,D,E)
c         Builds the 4x4 matrix from which the internal node matrix is derived
      COMPLEX YI(2), Xd(2)
      COMMON  YI,   Xd
      COMPLEX A(2,2), B(2,2), C(2,2), D(2,2), E(2,2)
c
      DO 1 i = 1, 2
      DO 2 j = 1, 2
c
      e(i,j) = a(i,j)
c
    2 CONTINUE
    1 CONTINUE
c
      b(1,1) = (1.0,0.0)/Xd(1)
      b(1,2) = (0.0,0.0)
      b(2,1) = (0.0,0.0)
      b(2,2) = (1.0,0.0)/Xd(2)
c
      c(1,1) = (0.0,0.0)
      c(1,2) = (-1.0,0.0)/Xd(1)
      c(2,1) = (-1.0,0.0)/Xd(2)
      c(2,2) = (0.0,0.0)
c
      d(1,1) = (0.0,0.0)
      d(1,2) = (-1.0,0.0)/Xd(2)
      d(2,1) = (-1.0,0.0)/Xd(1)
      d(2,2) = (0.0,0.0)
c
      RETURN
      END
```

```
      SUBROUTINE INVERT(X)
c         Inverts a 2x2 matrix
      COMPLEX X(2,2), Y(2,2), D
c
      Y(1,1) = X(1,1)
      Y(1,2) = X(1,2)
      Y(2,1) = X(2,1)
      Y(2,2) = X(2,2)
c
         D = (1.0,0.0)/(Y(1,1)*Y(2,2) - Y(2,1)*Y(1,2))
c
      X(1,1) = D*Y(2,2)
      X(1,2) = D*(-1.0,0.0)*Y(1,2)
      X(2,1) = D*(-1.0,0.0)*Y(2,1)
      X(2,2) = D*Y(1,1)
c
      RETURN
      END
c
c
c
      SUBROUTINE Losses(Pc,H1,H2,G11,G22,E1,E2)
c         Computes the ohmic losses of the OMIB
      REAL Pc, H1, H2, G11, G22, E1, E2, D1, D2
c
      D1 = H2*G11*(E1**2.0)
      D2 = H1*G22*(E2**2.0)
c
      Pc = (D1-D2)/(H1+H2)
c
      RETURN
      END
c
c
c
      SUBROUTINE LSM(Pm1,dd,PPe)
c         Performs the Least Squares Mean Estimate
      REAL Pm1, dd(4), PPe(4), a(4), b(4), D(4)
c
      a(1) = 0.319725
      a(2) = 0.452377
      a(3) = 0.197277
      a(4) = 0.030612
c
      b(1) = -41.142864
      b(2) =   2.938776
      b(3) =  11.755104
```

```
      b(4) =  26.448984
c
      DO 1 i = 1, 4
c
        D(1) = a(i)*PPe(i)
        D(2) = D(2) + D(1)
c
        D(3) = b(i)*dd(i)
        D(4) = D(4) + D(3)
c
    1 CONTINUE
c
      Pm1 = D(2) + D(4)
c
      RETURN
      END
c
c
c
      SUBROUTINE MULT(A,B,C)
c         Multiplies a 2x2 matrix by another 2x2 matrix
      COMPLEX A(2,2), B(2,2), C(2,2)
c
      DO 1 i = 1, 2
      DO 2 j = 1, 2
c
      a(i,j) = b(i,1)*c(1,j) + b(i,2)*c(2,j)
c
    2 CONTINUE
    1 CONTINUE
c
      RETURN
      END
c
c
c
      SUBROUTINE NEGATIVE(A)
c         Multiplies a 2x2 matrix by the scalar (-1)
      COMPLEX A(2,2)
c
      DO 1 i = 1, 2
      DO 2 j = 1, 2
c
      a(i,j) = (-1.0,0.0)*a(i,j)
c
    2 CONTINUE
    1 CONTINUE
```

```
      RETURN
      END
c
c
c
      SUBROUTINE node_E(E,V5,V6,I5,I6)
c         Computes the internal node voltage phasors
      COMPLEX YI(2), Xd(2)
      COMMON  YI,   Xd
      COMPLEX E(2), V5, V6, I5, I6
c
      E(1) = V6 + Xd(1)*I6
      E(2) = V5 + Xd(2)*I5
c
      RETURN
      END
c
c
c
      SUBROUTINE Pelect(Pe,Pc,Pm,d,GAMMA)
c         Computes the electrical power of the OMIB
      REAL Pe, Pc, d, GAMMA
c
      Pe = Pc + Pm*SIN(d-GAMMA)
c
      RETURN
      END
c
c
c
      SUBROUTINE Pmax(Pm,H1,H2,mY12,aY12,E1,E2)
c         Computes the maximum power that the OMIB can transmit
      REAL Pm, H1, H2, mY12, aY12, E1, E2, D1, D2
c
      D1 = SQRT(H1**2.0 + H2**2.0 - 2.0*H1*H2*COS(2.0*aY12))
      D2 = E1*E2*mY12
c
      Pm = (D1*D2)/(H1+H2)
c
      RETURN
      END
```

```
      SUBROUTINE Pshift(GAMMA,H1,H2,aY12)
c         Computes the phase-shift of the OMIB due to transfer conductances
      REAL GAMMA, H1, H2, aY12, D
c
      D = ((H1+H2)*TAN(aY12))/(H1-H2)
c
      GAMMA = (-1.0)*ATAN(D) - 2.0*ATAN(1.0)
c
      RETURN
      END
c
c
c
      SUBROUTINE REDUCE(A,B,C,D,E)
c         Reduces a matrix back to its internal nodes
      COMPLEX A(2,2), B(2,2), C(2,2), D(2,2), E(2,2)
      COMPLEX X(2,2)
c
      CALL INVERT(E)
c
      DO 1 i = 1, 2
      DO 2 j = 1, 2
c
      x(i,j) = (c(i,1)*e(1,1)+c(i,2)*e(2,1))*d(1,j) +
     $        (c(i,1)*e(1,2)+c(i,2)*e(2,2))*d(2,j)
c
      a(i,j) = b(i,j) - x(i,j)
c
    2 CONTINUE
    1 CONTINUE

      RETURN
      END
c
c
c
      SUBROUTINE REPORT(A1,A2,d0,ds,du,c1,Pm,Top)
      CHARACTER*64 FILENAME2
      REAL A1, A2, d0, ds, du, c1, Pm, Top
c
      WRITE (*,'(A\)') ' output file? '
      READ  (*,'(A)') FILENAME2
      OPEN (10, FILE = FILENAME2 )
c
      unit = 180.0/(4.0*ATAN(1.0))
c
      d0 = unit*d0
```

```
      ds = unit*ds
      du = unit*du
c
      WRITE  (10,01) d0, ds, du
   01 FORMAT (5x,'delta(0) = ',f10.4/,5x,'delta(s) = ',f10.4/
     $        5x,'delta(u) = 'f10.4//)
c
      WRITE  (10,02) Pm
   02 FORMAT (5x,'Pmax = ',f10.4//)
c
      WRITE  (10,03) c1
   03 FORMAT (5x,'Pmech = ',f10.4//)
c
      WRITE  (10,04) A1, A2
   04 FORMAT (5x,'Area(1) = ',f10.4,2x,'Area(2) = ',f10.4//)
c
      IF (A1.LE.A2) THEN
                WRITE (10,05)
              ELSE
                WRITE (10,06)
      END IF
c
   05 FORMAT (5x,'*** BLOCK ***'//)
   06 FORMAT (5x,'*** TRIP! ***'//)
c
      WRITE  (10,07) Top
   07 FORMAT (5x,'This decision was made in ',f10.8,' seconds.'//)
c
      CLOSE (10)
c
      RETURN
      END
c
c
c
      SUBROUTINE REPORT2(c1,Pm,Top)
      CHARACTER*64 FILENAME3
      REAL c1, Pm, Top
c
      WRITE (*,'(A\)') ' output file? '
      READ  (*,'(A\)') FILENAME3
      OPEN (11, FILE = FILENAME3 )
c
      WRITE  (11,01)
   01 FORMAT (5x,'The mechanical input has exceeded the maximum power')
      WRITE  (11,02)
   02 FORMAT (5x,'available to the network.'//)
```

```
      WRITE  (11,03) Pm, c1
   03 FORMAT (5x,'Pmax = ',f10.4,/,5x,'Pmech = ',f10.4//)
      WRITE  (11,04)
   04 FORMAT (5x,'*** TRIP ***'//)
      WRITE  (11,05) Top
   05 FORMAT (5x,'This decision was made in ',f10.8,' seconds.'//)
c
      CLOSE (11)
c
      RETURN
      END
c
c
c
      SUBROUTINE SORT(A,B,C,D,E)
c          Reorders the elements of a matrix
      COMPLEX A(4,4), B(2,2), C(2,2), D(2,2), E(2,2)
c
      DO 1   i = 1, 2
       DO 2 j = 1, 2
c
      B(i,j) = A(i,j)
      C(i,j) = A(i,2+j)
      D(i,j) = A(2+i,j)
      E(i,j) = A(2+i,2+j)
c
    2   CONTINUE
    1 CONTINUE
c
      RETURN
      END
c
c
c
      SUBROUTINE SORT2(A)
c          Reorders the elements of a matrix
      COMPLEX A(2,2), B(2)
c
      b(1) = a(1,1)
      b(2) = a(2,2)
c
      a(1,1) = b(2)
      a(2,2) = b(1)
c
      RETURN
      END
```

```
      SUBROUTINE VOLTAGES(V1,V3,V5,V6,mV1,a1,mV3,a3,X)
c         Computes the terminal voltage phasors of the equivalent machines
      REAL mV1, a1, mV3, a3
      COMPLEX V1, V3, V5, V6, X(2,2)
c
      a1 = (4.0*ATAN(1.0)/180.0)*a1
      a3 = (4.0*ATAN(1.0)/180.0)*a3
      V1 = CMPLX(COS(a1)*mV1,SIN(a1)*mV1)
      V3 = CMPLX(COS(a3)*mV3,SIN(a3)*mV3)
c
      V5 = x(1,1)*V1 + x(1,2)*V3
      V6 = x(2,1)*V1 + x(2,2)*V3
c
      RETURN
      END
c
c
c
      SUBROUTINE VIEW(A,i,j)
      INTEGER i, j
      COMPLEX A(i,j)
c
      OPEN (13, FILE = 'C:\APSA\uVIEW')
c
      DO 1 k1 = 1, i
      DO 2 k2 = 1, j
c
        WRITE(13,12) k1, k2, a(k1,k2)
   12 FORMAT(5x,'element(',i2,',',i2,')= ',f10.5,2x,f10.5)
   02 CONTINUE
   01 CONTINUE
c
      CLOSE(13)
c
      RETURN
      END
```

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VITA

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