ACTIVE CONTROL OF BROADBAND ACOUSTIC
RADIATION FROM STRUCTURES

by

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(ABSTRACT)

Active Structural Acoustic Control (ASAC) has been previously demonstrated for systems excited by single and multiple frequency disturbances. This work is an extension of ASAC techniques to the control of sound radiation from structures excited by a disturbance with broadband frequency content. An adaptive, multi-input multi-output (MIMO), feedforward broadband acoustic control system has been developed. The control approach is the least mean squares (LMS) algorithm. The compensators are adaptive finite impulse response (FIR) digital filters. The system identification of the control loop transfer functions were implemented with infinite impulse response (IIR) digital filters. The control inputs were implemented with piezoelectric ceramic actuators (PZT). Both far-field microphones and polyvinylidene fluoride (PVDF) structural sensors designed to optimally control the efficient acoustic radiating modes were used as error sensors. The disturbance was band-limited zero mean white noise and was input with a point force shaker. In the control of harmonically excited systems, satisfactory attenuation is possible with a single-input single-output (SISO) controller. In contrast, for
systems excited with broadband disturbances, a MIMO controller is necessary for
significant acoustic attenuation. Experimental results for the control of two simply-
supported plates are presented. Aspects addressed include the evaluation of the
microphone and PVDF error sensors, optimization of sensors and actuators, FIR
compensator size, controller sample rate, and convergence time. Thus this work provides
a methodology for controlling broadband acoustic radiation from a structure with regard
to the practical aspects of ASAC.
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Chapter 1

Introduction

The tendency of humankind to adapt lifestyle through technological advancement has resulted in many fascinating inventions and discoveries. Often enough however, these so-called "technological advancements" generate adverse effects which are undesirable and often detrimental to the environment. Noise is one such type of undesirable technological by-product. Noise, or unwanted sound, is often a result of systems with moving machinery parts, explosive chemical reactions such as combustion, and structures subject to force excitations. In order to reduce the noise emitted by a system without altering its function, the radiated noise can be controlled by applying additional systems which are not an inherent part of the original system.
1.1 Noise Control Approaches

Noise control approaches can be categorized as either passive or active. Passive noise control techniques usually consist of either modifying the original system or constructing chambers in which to place the original system. Modifications to the system include adding mass, stiffness, and/or damping so that the acoustic emission of the structure is reduced by directly modifying its response characteristics. Noise reduction chambers include high transmission loss containers often equipped with heavy walls and acoustic-absorbing materials. Although these techniques work sufficiently in some instances, the available noise reduction is extremely limited in the low frequency range.

Noise suppression using active control techniques have received increasingly more attention in recent years. The more traditional approach to noise suppression with active control involves the use of acoustic sources such as loudspeakers as the control inputs. These methods are termed Active Noise Control (ANC) since acoustic control sources are used to generate an "anti-sound" field that cancels the undesired noise field. The technique has been successfully applied in one-dimensional sound fields such as ducts [1,2]. ANC often requires many acoustic control sources to achieve attenuation throughout a noise field, especially if the noise field is not uniform [3]. Thus ANC methods are often inefficient in achieving global noise attenuation.
1.2 Active Structural Acoustic Control (ASAC)

Another method of active noise control known as Active Structural Acoustic Control (ASAC) was first demonstrated by Fuller [4,5,6]. The principle of active structural acoustic control (ASAC) utilizes the cause-effect relationship that exists between vibration and sound radiation. Since structural vibration is a cause of sound radiation, it is possible to control sound radiation from a structure by controlling its vibration. The technique of ASAC involves the implementation of "secondary" external (control) input forces to a structure vibrating under the excitation of the "primary" disturbance force input. The control inputs are applied such that the total acoustic response, which is the sum of the radiation due to the primary and secondary inputs, is minimized. Thus the control inputs serve to "cancel" the effect of the primary disturbance input on the structure, with the result being attenuated acoustic radiation. Hansen, Snyder and Fuller performed a study which compared ASAC to ANC for a rectangular plate subject to harmonic disturbances, and found that ASAC was a more effective approach for reducing the radiated sound power [7]. ASAC has also been found to reduce the required dimensionality of the controller, i.e., reduce the number of control inputs necessary to obtain significant noise reduction [8,9].

An ASAC system involves sensors to provide signals corresponding to the acoustic radiation of the structure, a computer capable of generating the control input signals, and actuators to input the control forces. The basic idea is not new, since when one grabs and
holds a vibrating structure to stop it from making noise, they are essentially performing active control with their ears, brain and hands. However, recent advances in high speed digital signal processing have made it possible to implement active control with microphones and vibration sensors, computers, and electrical-mechanical actuators. In this sense ASAC is in essence a form of artificial intelligence.

The technique of ASAC has been implemented using both feedback and feedforward control approaches. Meirovitch used a standard LQR feedback control design to attenuate sound radiation from plates [10]. Baumann et al. developed a state-space feedback method which implemented the acoustic radiation dynamics directly into the control compensator [11]. Saunders et al. theoretically implemented a multi-state feedback control algorithm to narrowband radiation control from a plate [12].

ASAC has also been demonstrated using adaptive feedforward control techniques for structures excited by single frequency disturbances [13]. Fuller and others have shown that using feedforward control approaches, global attenuation of sound transmission and radiation from harmonically excited panels can be achieved with only one or two control inputs [14,15]. Structurally mounted vibration error sensors have also been successfully developed and implemented for harmonic ASAC [9].

Substantial control of sound radiation from structures subject to harmonic
disturbances both on and off-resonance can usually be achieved without optimally designing the sensors and control actuators [13]. In cases of control of radiation due to a harmonic disturbance, the analysis of the residual structural response exhibits a forced modification. The structural response modification can be characterized as "modal suppression" or "modal restructuring" [5,16,17]. In on-resonance harmonic disturbance excitation, where the disturbance frequency corresponds to a natural mode of the structure, the structural response is usually dominated by that mode, or a few well-coupled modes, depending on the degree of modal density. In these cases, the controlled system often exhibits "modal suppression" identified by an after control decrease in the vibrational amplitude of the dominating mode(s). In the control of off-resonant harmonic disturbances, the controlled system generally shows an increase in the vibrational amplitudes of the inefficient radiating modes [18,19]. Thus, the residual response is not attenuated but rather the structure is forced to behave like an inefficient radiator, i.e. "modal restructuring". In modal restructuring, the vibrational energy is spilled over into inefficient radiating modes. In the case of broadband excitation, all modes on the bandwidth are being excited both on-resonance and off-resonance simultaneously, since the disturbance contains content at all frequencies on the bandwidth. This implies that the control system configuration is critical in the control of broadband radiation.

ASAC has been repeatedly demonstrated for structures excited by harmonic (single frequency) disturbance forces. Work in the field of broadband radiation control (where
the disturbance contains a broad range of frequencies) has not been so manifest. However, many structures that make undesirable noise are subject to disturbances that are random and contain more than one frequency. Thus control of systems subject to broadband inputs is a viable problem, and transcends the concept of single-frequency disturbance control.

### 1.3 Actuators and Sensors

The advent of piezoelectric polymer materials has changed actuator and sensor technology immensely in the past fifty years. The fact that certain materials and composites generate an electrical charge when under application of a force or pressure, and the reciprocal effect, allows these piezoelectric materials to be effectively used as electrical transducers (sensors) and force actuators. Many of these materials are more consistent with the idea of "smart" or "adaptive" structures in that the sensors and actuators can become a more inherent part of the structure to be controlled [20]. Actuators and sensors made of these materials can often be flush mounted or embedded in typical structures, and can be considered as part of the structure itself.

The polymer implemented as control actuators in this work, lead zirconium titanate (PZT), was discovered in 1954 by Jaffee [21,22]. This PZT material stretches and contracts when an alternating voltage signal is applied across it, and has a large operating range of frequencies, making it useful as a force actuator. The PZT patches are an
effective and unobtrusive way to input forces into a structure, and they have been successfully applied as control actuators in many instances [8,9,13]. The PZT’s require negligible additional space when mounted directly on the structure, and can very nearly be considered as part of the structure itself.

Obviously microphones are suitable as error sensors for acoustic control as they supply a signal directly proportional to the radiated sound pressure. Structurally mounted vibration sensors are more compatible with the concept of "smart" structures and are desirable since microphones are often obtrusive and impractical to implement. The concept of "radiation filtering" involves modifying the output of a structurally mounted sensor so that acoustic attenuation is the result of minimizing the filtered sensor signal [23]. It is even more desirable to design the filtering effect into the sensor itself, so that additional signal processing of the error signal is not necessary. One type of piezoelectric polymer that lends itself well to such an application is polyvinylidene fluoride (PVDF) films.

The piezoelectric characteristics of PVDF polarized film were discovered in 1969 by Kawai [24]. The piezoelectric constant of PVDF film was found to be three times greater than that of nine other polymers tested at that time. In addition, the piezoelectric effect was found to be consistent over a time span of several months. Since PVDF film is 30 times more compliant and 4 times less dense than PZT, it has little effect on the
dynamic response of structures when mounted upon them. The fact that the PVDF film can be designed in shape and location makes it well suited as a structurally mounted distributed error sensor applicable for acoustic control [25].

1.4 Overview of Thesis

The objective of this work is to demonstrate that broadband radiation control can be achieved with adaptive feedforward algorithms, and to investigate different aspects of the control approach, as they apply to practical implementation. Some of the control aspects addressed include: the number of control channels, form of system identification, control compensator size, convergence characteristics, microphone error evaluation (use and location), and the use of PVDF distributed structural error sensors. The approach enlisted here is somewhat of an industrial one, since the algorithms which have been previously applied to harmonic control are extended to investigate the potential of controlling broadband radiation from practical structures.

This thesis is an experimental study into the potential of controlling broadband acoustic radiation from structures using an adaptive feedforward control approach. The experimental test structure is a simply-supported plate with the disturbance bandwidth encompassing the first five natural modes of the plate. Sound pressure level data demonstrating the effectiveness of the controller was gathered for different control configurations. The control inputs are implemented with piezoceramic actuators, and both
far-field microphones and PVDF structural sensors are used as error transducers for acoustic control. The control approach used the Filtered-x LMS algorithm and is presented in Chapter 2. Chapter 3 contains a description of the experimental setup and procedures. Experimental results for the test structure are presented for single-input single-output (SISO) and multiple-input multiple-output (MIMO) control systems for microphone error sensors and PVDF structural error sensors in Chapter 4. Chapter 5 discusses the results with some remarks on the control aspects investigated. Finally, in Chapter 6, conclusions are drawn on some of the aspects of controlling broadband acoustic radiation.
Chapter 2

Control Algorithm

In recent years, the advances in high speed digital signal processing have made digital control algorithms realizable for many applications. With the advent of fast and affordable digital signal processing chips, adaptive digital control algorithms have been successfully applied to inverse modelling and noise control, plant modelling and system identification, prediction, equalization, and in general, the generation of digital filters with desired frequency response characteristics [26]. The ability of adaptive control systems to 'adapt' or change in response to changes in their environment make them useful in a wide range of applications which require a robust control system.

2.1 Adaptive Finite Impulse Response (FIR) Filters

An adaptive FIR filter is a linear combiner, the output of the filter being a linear
combination of the input sequence and a number of adaptive filter coefficients. A schematic of an FIR filter of order L in the form of a single-input transverse adaptive filter is shown in Figure 2.1, where $x_k$ represents the input sequence, $y_k$ is the output sequence, $W_{jk}$ is the $j^{th}$ filter coefficient at time step $k$, and $z^{-1}$ represents a delay of one time step. Since the transfer function of an FIR filter contains only zeros and no poles, an FIR filter is inherently stable [26,27]. The issue of stability is important when the FIR filter is to be implemented as an adaptive compensator. However, the ability of the FIR to accurately model a transfer function with pole characteristics such as sharp resonances is limited with filters of low order, and will be further discussed in section 4.4.

2.2 Adaptive Feedforward Filtered-x LMS

The algorithm implemented in this study is a feedforward filtered-x LMS control approach. The feedforward filtered-x LMS control algorithm has been successfully applied to control of broadband structural vibration [28,29]. A block diagram of the control system for the single-input, single-output (SISO) case is shown in Figure 2.2. The control signal time sequence $u_k$ is obtained by filtering a reference signal, which is coherent to the disturbance signal $x_k$, through an adaptive finite impulse response (FIR) filter, $W(z)$. It is assumed that the reference signal is obtained by directly tapping into the disturbance signal and that there is no feedback of the control input into the reference signal. That is,
Figure 2.1 Adaptive linear combiner.
Figure 2.2 Block diagram of SISO adaptive feedforward control system.
\[ u_k = \sum_{j=0}^{L} W_{j,k} x_{k-j} \]  \hspace{1cm} (2.1)

where \( W_{j,k} \) is the \( j \)th filter coefficient at the \( k \)th time step, \( L \) is the order of the FIR filter and the subscript \( k \) indicates a signal sample at time \( t_k \). The filter output can also be expressed in convolution form as

\[ u_k = W(z) * x_k \]  \hspace{1cm} (2.2)

where \(*\) denotes a convolution. The disturbance path and control path transfer functions through the plant are \( T_d(z) \) and \( T_c(z) \) respectively.

The error signal sequence (from Figure 2.2) can be expressed as

\[ e_k = d_k + y_k \]  \hspace{1cm} (2.3)

and expressing \( y_k \) as a convolution of the control input \( u_k \) and the control path transfer function \( T_c(z) \) yields

\[ e_k = d_k + T_c(z) * u_k \]  \hspace{1cm} (2.4)

Substituting the convolution form of Equation 2.1 into equation 2.3 yields an expression for the error as a function of the compensator weight vector.
2.2.1 The Cost Function and Gradient Search Technique

The task of adaptive control algorithms in general is to generate inputs such that some measurable characteristic of the system response can be controlled. The desired system behavior can often be defined as the minimization of a particular system response or output. One approach involves the formation of a performance or cost function which is representative of the quantity to be minimized, and determination of the control input parameters that minimize the cost function. Here the cost function is a function of the filter weights of the FIR compensator. Generating the optimal control input involves finding the optimal values for the coefficients of the FIR filter. The compensating filter weights can either be designed "off-line", or in "real time". Off-line design involves obtaining data from the system and calculating the compensator coefficients from that data. An off-line procedure for designing the optimal compensator coefficients assumes stationarity in the reference and error signals and involves use of the autocorrelation function of the filter input and the cross correlation function between the filter input and the desired response [26,27]. Off-line calculation of the desired control input parameters is generally not desirable if the cost function must be estimated from error signals corrupted with noise. Real time compensator design involves adaptive control, as the compensator coefficients are adapted "on-line", i.e. while the controller is operating.
Adaptive control provides a more practical algorithm and involves making small changes in the control input parameters, stepping along the surface of the cost function in an iterative process until the control parameters that minimize the cost function are converged upon. Slow adaption is more applicable than single-step adaption in practical applications since it provides a filtering process that ameliorates the effects of gradient and measurement noise [26].

One method of stepping along the cost function towards the solution in the opposite direction of the gradient is known as the Method of Steepest Descent [26]. Since the adaptive parameters of the control system at hand are the filter weights of the FIR compensator, the algorithm involves adjusting the filter weights in the opposite direction of the cost function gradient vector at each time step. From its definition, the method of steepest descent can be expressed in terms of a filter coefficient update equation as follows,

$$\mathbf{W}_{j+1} = \mathbf{W}_j + \mu (-\nabla C_j) \quad (2.6)$$

where $C$ is the cost function, $\nabla C$ is the cost function gradient, and $\mu$ is a parameter which controls the rate of convergence and stability of the minimization process.

In practical applications, the cost function is often formed from the output of a
number of error sensors. In the following formulation, it is assumed that error transducers which provide signals respective of the acoustic radiation (such as microphones) can be implemented. For the SISO case, the cost function is formed as the mean square value of the error signal, i.e.,

$$C(W_{j,k}) = E[e_k^2]$$  \hspace{1cm} (2.7)

where $E[.]$ is the expected value operator. Formation of the cost function $C$ from the mean squared value of the estimation error (MSE) yields a cost function that is second order dependant on the unknown coefficients of the compensating filter [27]. This implies that the cost function has a single distinct minimum that uniquely defines the optimal design of the compensating filter coefficients [27]. Substituting the expression for the error from Equation 2.5 into equation 2.7 yields

$$C(W_{j,k}) = E[(d_k + T_{ac}(z) * W_{j,k} * x_{k-1})^2]$$  \hspace{1cm} (2.8)

Differentiating the cost function with respect to the filter weights gives

$$\frac{\partial C}{\partial W_{j,k}} = 2E[e_k \frac{\partial e_k}{\partial W_{j,k}}] = 2E[e_k (T_{ac}(z) * x_{k-1})]$$  \hspace{1cm} (2.9)

Letting the convolution of the disturbance signal through the control path transfer function be termed as the filtered-x signal $\hat{x}_k$, i.e.,
\hat{x}_k = T_\alpha(z) \ast x_k \quad (2.10)

then yields the cost function gradient as

\[ \frac{\partial C}{\partial W_{j,k}} = 2 \, E[e_k \, \hat{x}_{k-1}] = 2 \, e_k \, \hat{x}_{k-1} \quad (2.11) \]

In the method of steepest descent known as the Least Mean Square (LMS) algorithm, the cost function gradient is approximated with the instantaneous values of the error and filtered-x signals as shown in equation 2.11 [26,30,31]. Substituting this expression for the cost function gradient into the filter update equation 2.5 yields the final filter weight update equation, known as the Widrow-Hoff LMS algorithm [30,31].

\[ W_{j,k+1} = W_{j,k} - 2 \mu \, e_k \, \hat{x}_{k-1} \quad ; \quad j = 0, ..., L \quad (2.12) \]

Equation 2.11 shows the filter update equation for the SISO case, where \( \mu \) is a parameter that controls the rate and stability of the convergence process. Thus, the LMS algorithm requires a knowledge of both the error and the filtered-x signal. An estimate of the filtered-x signal is obtained by filtering the disturbance through a model of the control path transfer function \( T_\alpha(z) \). Section 2.3 addresses the issue of modelling and system identification of the control loop transfer function. The convergence parameter \( \mu \) can be estimated from the power of the filtered-x signal, and for a constant rate of convergence,
μ must increase as $e_k$ decreases [26,32].

For a multiple-input multiple-output (MIMO) controller, the reference signal is fed forward to an array of FIR adaptive filters whose outputs drive $N_c$ control actuators. A block diagram of the 3I3O feedforward filtered-x LMS controller is shown in Figure 2.3. The filter coefficients are updated to minimize a cost function that is now defined as the sum of the mean square value of the error signals. That is

$$C = \sum_{s=0}^{N_e} E[(e_s)_k^2]$$

(2.13)

where $(e_s)_k$ is the $s$th error sensor signal, and $N_e$ is the number of error sensors.

The update equation for the filter coefficients can be shown to be as follows [33]

$$W_{r,i,k+1} = W_{r,i,k} - 2\mu \sum_{s=1}^{N_e} (e_s)_k (\hat{x}_r)_k$$

$$j=0,1,\ldots,L \quad r=1,\ldots,N_c$$

(2.14)

where $(\hat{x}_r)_k$ is the disturbance signal filtered by the transfer function between the $r$th control actuator and the $s$th error sensor and $W_{r,i,k}$ is the $j$th coefficient of the $r$th control channel at the time step $k$. Thus, it is clear from equation (2.14) that there are $N_x N_c$ control path transfer functions that need to be modelled in order to obtain the necessary
Figure 2.3 Block diagram of 3I3O adaptive feedforward control system.
filtered-x signals.

## 2.3 System Identification of the Control Loop Transfer Functions

$T_{ae}(z)$

The filtered-x signal $\tilde{x}_k$ is the predicted response of the structure to the disturbance signal echoed through the control path transfer function $T_{ae}(z)$. That is,

$$\tilde{x} = \hat{T}_{ae}(z) \ast x_k \tag{2.15}$$

where $\hat{T}_{ae}(z)$ is an estimate of $T_{ae}(z)$.

Thus an estimate of the control path transfer function $T_{ae}(z)$, i.e., a "system identification" is necessary to obtain the filtered-x signal. Lightly damped structural control path transfer functions often exhibit frequency responses characteristic of a system with both poles and zeros, i.e. sharp, high quality resonances and anti-resonances. These aspects allow infinite impulse response (IIR) filters to be effectively implemented as models of the control path transfer functions. In fact, the high quality resonances exhibited by most lightly damped structures require IIR filters to obtain an accurate model of reasonable filter size. Once identified, if the structure does not change, the control path transfer functions remain constant, so that they do not need to be updated during control.
2.3.1 The IIR Filter as a Transfer Function Model

An IIR filter is a recursive filter, in that the filter output is a function of both the input sequence and the output sequence. A block diagram for an IIR filter is shown in Figure 2.4. The IIR filter is expressed as

\[
T(x(z)) = \frac{B(z)}{1 + A(z)} = \frac{b_0 + b_1 z^{-1} + \ldots + b_m z^{-m}}{1 + a_1 z^{-1} + \ldots + a_p z^{-p}}
\]  

which has m zeros and p poles. The IIR filter is also known as an autoregressive moving-average or ARMA model. The moving average section refers to the numerator of the IIR, which is essentially an FIR filter as it operates only on the filter input sequence. The coefficients of the numerator, the \(b_n\)'s, correspond to polynomial coefficients in the z domain, the roots of which are the transfer function zeros. The denominator polynomial of the IIR filter is known as the autoregressive portion, since it is a function of the previous output sequence of the filter. The roots of the denominator polynomial, governed by the \(a_n\) coefficients, are the poles of the filter transfer function. Since an IIR filter is capable of modelling a transfer function with both poles and zeros, it can more accurately model systems with frequency responses containing sharp resonances and anti-resonances, like those characteristic of lightly damped structures. An FIR transfer function model that needs hundreds of coefficients can often be modelled...
Figure 2.4 Block diagram of an IIR filter.
more accurately with an IIR containing only a few coefficients [34,35]. The reduction in necessary coefficients between an FIR model and an IIR model can save valuable computational time. A disadvantage of IIR filters is that they are subject to instability if any of the poles are unstable. However, the fact that the system identification filters do not need to be updated during control eliminates the potential for an unstable controller if the system identification filters are checked to ensure stability before implementation. The stability issue will be addressed further in Section 2.3.2.

2.3.2 Method of Digital Filter Design

In order to implement the adaptive feedforward filtered-x LMS algorithm, the filtered-x signal must be obtained. Thus it is necessary to identify the frequency response characteristics of each of the control path transfer functions, and design a digital filter with the same frequency response. The method adopted here is to measure each of the control path transfer functions in the frequency domain, and use a curve-fit procedure to determine the coefficients of an IIR that closely models the frequency response. In practice, the control path transfer function is measured directly from the system. The control actuator is driven with white noise and the transfer function between the control input and the error sensor output is measured in the frequency domain. The IIR filter is designed using a curve-fit technique which solves a system of equations for the coefficients $a_\star$ and $b_\star$ of the IIR filter [36]. The technique used is based on minimization of the error in a least squared ratio error norm and implements the MATLAB function
Since the IIR filter has both poles and zeros, potential for instability exists if any of the poles are located outside of the unit circle in the z-plane. In the event that the IIR filter design yields unstable poles, the poles are "reciprocated" across the unit circle into the stable region [36]. Reciprocation of poles involves replacing each unstable pole with its inverse. The process of reciprocation for a complex conjugate pair of roots is depicted in Figure 2.5. Pole reciprocation distorts the phase and magnitude response of the model somewhat, but inversion of a small number of slightly unstable poles in a large model still yields a model with a satisfactory match of both magnitude and phase response.

2.4 A Note on Causality

The causality issue of the broadband feedforward adaptive control system was addressed by Vipperman [28]. It should be noted that the optimal compensating filter is acausal when delays exist in the control path. Here the entire system will be referred to as acausal since the propagation time through the control path is in general longer than the propagation time through the disturbance path (see Figure 2.2). Thus for a given sample of the reference signal, a delay exists between the computed control input and the disturbance input. The delay of the control signal with respect to the disturbance is due primarily to the A/D and D/A converters and the computations performed in the DSP controller.
Figure 2.5 Root locus plot depicting unstable complex conjugate pair pole reciprocation in the z-plane.
The performance of the control system was found to increase when the system was made more causal by implementing a delay in the disturbance path. The delay between the control path and disturbance path was measured by sending an impulse through each path independently, and measuring their respective propagation times. The difference in the propagation times yielded an estimate of the delay. When the measured delay was implemented in the disturbance path using an additional DSP, the total error attenuation using a single microphone error sensor was found to increase from 3.1 dB to 5.5 dB. The topic of causality will not be further addressed in this work since for practical applications in general, it may not be possible to implement a delay in the disturbance path. The study performed by Vipperman provides a tool to predict the degradation of control performance for a given amount of system 'acausality' or control delay time [28].
Chapter 3

Experimental Setup And Procedures

The control algorithm described in Chapter 2 was implemented experimentally to evaluate its potential for controlling acoustic radiation from a structure excited by a broadband disturbance. The experimental approach allowed investigation of some of the more practical advantages and limitations of the feedforward filtered-x LMS algorithm. Measurement noise and other forms of uncorrelated content are present in an actual signal, and often their effects can not be accurately accounted for by merely performing simulations of the control algorithm. Signal noise and slight changes in an actual plant can not only reduce the predicted performance of a system but can often cause a control system that worked in simulation to be unstable. Real time implementation accounts for control constraints dictated by limitations in sample rate, imperfect equipment, the margin of error involved in sensor and actuator design, and other unforeseeable imperfections in
the entire actual system. Only a practical implementation of a control system on a real plant can truly examine the robustness of the system and evaluate its performance.

3.1 The Experimental Test Structures

The structure used for the experiments was a simply-supported steel plate of dimensions 380 x 300 x 2 mm. Thin, flexible metal shims connected the edges of the plate to a heavy support stand to provide the simply-supported boundary conditions. A simply-supported plate was chosen since it emulates an elemental section of many two-dimensional engineering structures, such as a wall, an airplane fuselage, structural components of a building, etc. Two different plates with the same dimensions were used for the experiments, since the optimal configurations of sensors and actuators tended to overlap, and a single plate could not be constructed for optimal SISO control, optimal 2I2O control, and optimal 3I3O control simultaneously. Table 3.1 shows the theoretically predicted natural frequencies for the plates as determined using Euler-Bernoulli theory vs. the experimentally determined natural frequencies [19]. The well known convention for the modal indices (m,n) is adopted here, where m-1 and n-1 are the number of node lines present in the horizontal and vertical directions of the plate, respectively. Figure 3.1 shows the location of the node lines corresponding to the first five resonant frequencies. As can be seen from Table 3.1, the simply-supported boundary conditions were simulated within 2.1% with the experimental configurations. Appendix A contains information on the calculations involved in computing the structural response parameters.
Figure 3.1 Node lines for the first five plate modes.
Table 3.1 Comparison of theoretical vs. experimental unmodified plate natural frequencies.

<table>
<thead>
<tr>
<th>mode (m,n)</th>
<th>Theoretical Natural Frequency (Hz) (Euler Theory)</th>
<th>Plate 1 Experimental Natural Frequency (Hz)</th>
<th>Plate 2 Experimental Natural Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>87.71</td>
<td>88</td>
<td>88</td>
</tr>
<tr>
<td>(2,1)</td>
<td>188.7</td>
<td>187</td>
<td>188</td>
</tr>
<tr>
<td>(1,2)</td>
<td>249.8</td>
<td>247</td>
<td>248</td>
</tr>
<tr>
<td>(2,2)</td>
<td>350.9</td>
<td>347</td>
<td>348</td>
</tr>
<tr>
<td>(3,1)</td>
<td>357.1</td>
<td>352</td>
<td>350</td>
</tr>
</tbody>
</table>
Table 3.2 Experimentally determined natural frequencies of plates with shaker attached.

<table>
<thead>
<tr>
<th>mode ((m,n))</th>
<th>Plate 1 Frequency (Hz)</th>
<th>Plate 2 Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1,1)</td>
<td>86</td>
<td>88</td>
</tr>
<tr>
<td>(2,1)</td>
<td>182</td>
<td>183</td>
</tr>
<tr>
<td>(1,2)</td>
<td>248</td>
<td>243</td>
</tr>
<tr>
<td>(2,2)</td>
<td>350</td>
<td>325</td>
</tr>
<tr>
<td>(3,1)</td>
<td>331</td>
<td>347</td>
</tr>
</tbody>
</table>
The experimentally measured natural frequencies for the first five modes of the plates with a point force shaker actuator attached are presented in Table 3.2. Some of the plate resonant frequencies changed after mounting the shaker. This is believed to be mainly due to the added dynamics of the shaker stinger and armature. Not only did the frequencies change but note that the order of (2,2) and (3,1) modes for Plate 1 changed after the shaker was attached. This was believed to be due to differences in the shaker mountings and the effective mass loading of the shaker. The bandwidth of the disturbance signal used to excite the plate dictated the number of modes attempted to be controlled. Thus a bandwidth of 0 to 400 Hz encompassed and excited the first five plate modes. A disturbance with random frequency content between 0 and 800 Hz allowed excitation of the first eight modes. The number of modes attempted to be controlled had considerable impact on the number of control channels necessary and the amount of attenuation achieved as will be discussed in Chapter 5.

3.2 The Controller

The control algorithm presented in Chapter 2 was implemented on a Texas Instruments TMS320C30 digital signal processing (DSP) board. Expansion boards were also used so that multiple-input, multiple-output (MIMO) control could be implemented. The boards were resident in an IBM 80386 PC host computer. The codes used to operate the DSP were written in "C" language with embedded "ASSEMBLY" code instructions.
The user interface codes implemented MATLAB to allow the user to execute or halt the controller, set the sampling rate, adjust the convergence parameters, and obtain access to the control parameters such as the converged filter weights.

3.3 The Control Actuators

The control input signals were applied by co-located pairs of G1195 piezoelectric ceramic patches (PZT), each patch measuring 38.0 x 32.0 x 0.2 mm. Each pair of PZT actuators were oppositely mounted, one on each side of the plate, and actuated 180° out of phase. The forcing function associated to this arrangement is a line moment along the actuator edges due to the applied voltage [38]. The PZT patches were attached to the structure with M-Bond 200 strain gauge glue. An electrical lead was attached to each side of each PZT patch so that the control voltage signal could be applied. The electrical lead to the side of each PZT touching the plate was attached via a thin brass shim, to allow flush mounting of the patch to the plate.

As will be seen later, the locations of the actuators on the plate were critical in the amount of control achievable. The actuator locations dictated the relative controllability of each of the plate modes and likewise the amount of broadband reduction obtainable. The locations of the actuators were designed using an optimization algorithm developed for single frequency and multiple frequency disturbances [39]. In this algorithm, the actuator locations are optimized to minimize the total radiated sound power.
at a number of simultaneous excitation frequencies. The actuator and sensor locations were optimized for a disturbance with frequency content corresponding to the natural plate modes with the highest radiation efficiency [40]. This provided a computationally efficient procedure for determining optimal actuator locations applicable for broadband disturbances. An examination of the radiation efficiency of each of the plate modes yielded a ranking of the modes in their order of importance to be controlled. The radiation efficiency for the first 5 modes of the plate are plotted in Figure 3.2. The procedure used for calculating the radiation efficiency as investigated by Wallace [40] is outlined in Appendix A.

As can be seen in Figure 3.2, the (1,1) and (3,1) modes are the most efficient radiators well below the critical frequency [40]. Therefore, for Plate 1, the system was assumed to be excited at 88 Hz and 331 Hz, and the optimal locations of the PZT actuators for SISO and 2I2O control were determined with the optimization procedure as they are shown in Figure 3.3. In Figure 3.3, A1 is the PZT actuator optimally designed for a SISO control system. The rectangles labeled A2 and A3 in Figure 3.3 show the locations of the PZT actuators optimally designed for 2I2O control. Three simultaneous disturbance frequencies were employed for Plate 2, for which the actuator locations were optimized to minimize sound power with a 3I3O control configuration. For Plate 2, the disturbance was assumed to contain frequencies corresponding to the (1,1), (2,1) and (3,1) modes (see Table 3.2) with the resulting optimal locations for the PZT actuators labeled
Figure 3.2 Radiation efficiency for the first 5 modes of the plate.
Figure 3.3 PZT actuator and PVDF sensor configuration for Plate 1.
A1, A2 and A3 in Figure 3.4.

Although it will be discussed in more detail in Chapter 6, it should be mentioned here that designing the control actuators without attempting to control all of the plate modes may result in an actuator configuration that yields the ignored modes uncontrollable. If any of the radiating modes on the bandwidth of interest are uncontrollable, the amount of broadband reduction integrated across the bandwidth could be insignificant, making the actuator locations unacceptable for broadband control.

3.4 Error Sensors

The selection and design of appropriate transducers to supply the error signal for minimization is an important part of any acoustic control system. Thus the error transducers must supply a signal related to the acoustic cost, i.e., supply an output signal such that upon the minimization of that signal, the result is acoustic attenuation. In the age of "smart" materials and structures, it is desirable to develop and implement structurally mounted error sensors for acoustic attenuation. The two types of error sensors addressed in this study are microphones and sections of polyvinylidene fluoride (PVDF) film, optimally designed in location and dimension.

3.4.1 Microphones

Since the goal of this work is to attenuate the acoustic radiation, the obvious choice for error transducers are microphones placed in various locations in the far-field.
Figure 3.4 PZT actuator and PVDF sensor configuration for Plate 2.
For this work, Brüel & Kjær type 4134 microphones were used as error transducers. The microphone signals were conditioned with a Brüel & Kjær multi-channel type 228 power supply. In addition to the low pass filter on the error signal paths as explained earlier, the microphone signals were also filtered through a high pass Ithaco type 4302 24 dB/octave filter with the cutoff set to 80 Hz, to remove the low frequency content from the signals.

3.4.2 Polyvinylidene Fluoride (PVDF) Error Sensors

Often microphones located in the far-field are not a practical choice of error transducers. In many instances, there is not sufficient space in which to locate microphones. Furthermore, using microphones in a reverberant field will destroy the coherence between the reference and the error signals and likewise the attenuation. Structural error sensors have an advantage over microphones in that the reverberant field does not effect the coherence between the reference and the error signals. In addition, structurally mounted error sensors that serve to represent the acoustic cost are more consistent with the idea of "adaptive" or "smart" structures, since structurally mounted error sensors are a more inherent part of the adaptive structure. Although vibration control is related to acoustic control, minimizing vibration in a general sense may or may not result in acoustic control. In many cases, after achieving some degree of vibration control, the acoustic radiation is found to increase. Likewise, after acoustic control is achieved in many situations, the after control modal analysis exhibits an increase in some
of the modal amplitudes of vibration [18,19]. The goal is to develop distributed structural error sensors that are characteristic of the acoustic radiation dynamics.

PVDF sensors provide an output signal proportional to the integral of the rate of strain over the surface of application. Thus a structurally mounted film of PVDF material provides a "window" which observes the vibration over the area of the structure on which the PVDF is mounted. This means that when PVDF is used as an error sensor, minimizing the signal from the PVDF will result in minimization of the vibration mode(s) observable in the window of the PVDF. The mode(s) observable by the PVDF sensor are those which have a non-zero value for rate of strain integrated over the portion of the structure to which the PVDF is attached. Thus some desired filtering characteristics can be designed into the PVDF film by designing the PVDF in shape and location. Simply minimizing the signal from a randomly shaped and located PVDF sensor may not result in significant acoustic attenuation. The development of optimal PVDF error sensors for control of broadband acoustic radiation is a topic of current research. The geometry and location of the PVDF error sensors used in these experiments were designed for optimal reduction of sound power for an excitation containing a finite number of frequencies [41]. The PVDF sensors on Plate 1 were designed to achieve optimum control for an excitation containing the frequencies of the (1,1) and (3,1) modes. Figure 3.3 shows the resulting size and locations of the PVDF sensors on Plate 1, where S1 was designed for SISO control, and S2 and S3 were designed for 2I2O control. The PVDF sensors on Plate 2
were designed for optimal 3I3O control of sound radiation for a disturbance with frequency content at the (1,1), (2,1), and (1,2) modes. Note that this is a different approach since the 3I3O PVDF sensor configuration was not designed to optimally control the (3,1) mode, as the actuators were. Also note that in Figure 3.2, the (2,1) and (1,2) modes have higher radiation efficiencies than the (3,1) mode above about 350 Hz. Figure 3.4 shows the size and locations of the PVDF sensors on Plate 2.

3.5 Experimental Setup

A schematic of the experimental setup used for SISO control is shown in Figure 3.5. MIMO control used the same setup with additional control and error channels. For these experiments, the reference signal was generated with the Random Noise function of a Brüel & Kjær type 2032 Dual Channel Signal Analyzer, and filtered through a low pass Ithaco type 4302 24 dB/octave filter. The reference signal required low pass filtering to restrict the significant frequency content to the bandwidth of interest. The frequency range of the B&K was usually larger than the bandwidth of interest since the frequency range of the B&K needed to include the nyquist frequency for measurement of the system identification transfer functions \( f_{\text{nyquist}} = \frac{1}{2} f_{\text{sample}} \). The voltage of the reference signal input to the DSP was limited to a maximum of 3 volts to protect the input D/A converters of the DSP.

The reference signal was amplified with a Type 2600A NAD variable gain power amplifier (AMP) before being input to a Ling Dynamic Systems shaker actuator as the
Figure 3.5 Schematic of SISO experimental setup.
disturbance signal. Figure 3.6 contains a plot of the disturbance input spectra for a bandwidth of 0 to 400 Hz. The shaker provided the disturbance excitation as a point force input to the structure via a titanium stinger attached to the plate. The location of the disturbance input was $x=0.075 \text{ m}$, $y=0.065 \text{ m}$, and is shown in both Figures 3.3 and 3.4. The shaker was located so as to excite the first five modes of the plate. The location of the disturbance input had considerable impact on the radiated sound pressure and the modal response of the plate, which in turn affected the amount of control obtainable. The voltage being applied to the shaker was monitored with a FLUKE Type 8050A digital voltmeter.

Each of the control inputs were filtered through Frequency Device Type 9002 low pass filters (LPF), and amplified with variable gain NAD amplifiers. The cutoff frequency of the low pass filters was set to the highest frequency on the bandwidth of interest. The control signals were low pass filtered to "smooth" the digital output from the DSP and to remove the high frequency content from the signals. The digital output from the DSP produced high frequency harmonics in the control signals if the signals were not filtered. The voltage of the control signals were stepped up with 17:1 transformers (T) before being applied to the control actuators.

Each of the error signals were also filtered with Frequency Device Type 9002 low pass filters to remove any high frequency content outside the controllable bandwidth of
Figure 3.6 Spectrum of disturbance input, 0 to 400 Hz bandwidth.
interest. The cutoff frequency of the filters were set to the highest frequency on the bandwidth of interest, consistent with the control signal low pass filter cutoff frequency. Although filtering the error signal was not as important as filtering the control input signals, the most attenuation was achieved when the frequency content of the error signals were restricted to the controllable bandwidth of interest. Each error signal was amplified with a channel of an Ithaco model 455 amplifier. It was important to restrict the peak value of the error signals to 3 volts at all times, since that is the maximum allowable input of the DSP. For this reason, the error signals were carefully monitored on an oscilloscope to protect the DSP board. Since it is desirable to utilize the maximum dynamic range of the DSP input A/D converters to obtain the best signal resolution, the error signals were adjusted so that they were at their maximum allowable voltage before control.

All data was taken in an anechoic chamber of dimensions 4 x 2.7 x 2.4 m with a cutoff frequency of 250 Hz. Each plate was mounted in a baffle that extended between the walls of the anechoic chamber, in order to simulate an infinite baffle. The purpose of the baffle was to isolate the acoustic radiation of the rear side from the sound field produced by the front side of the plate. Figure 3.7 shows an overhead view of the plate and baffle as they were oriented in the anechoic chamber.
Figure 3.7 Overhead view inside the anechoic chamber.
3.6 Experimental Procedures

An overview of the procedures necessary to implement the broadband active control system developed here is in order. This section outlines the steps that need to be taken once the structure is configured with actuators and sensors, and the control and system identification digital modelling codes are developed. The procedure is described for a general MIMO system with \( N_c \) control actuators and \( N_e \) error sensors.

Before the control implementation can begin, some prior knowledge about the system and the disturbance is helpful to determine some important aspects of the control system. First, knowing how many modes of the structure are desired to be controlled dictates the number of control channels necessary. Furthermore, if the coupling between the modes is understood, a minimum number of control channels can be determined [43]. A comprehension of the system dynamics can be utilized to construct an analytical model, and optimization techniques can be applied to configure the plate with optimally designed sensors and actuators [19,39,40].

Upon setup of the experimental system as shown in the schematic in Figure 3.5, an appropriate sample rate must be determined, and must be kept consistent throughout the entire procedure. The sampling frequency must be at least twice that of the highest frequency to be controlled, to prohibit aliasing of any frequencies on the bandwidth of interest. A general rule for sample rate selection based on harmonic disturbance control
is that the optimal sample rate should be between 4 and 6 times the frequency of the respective harmonic to be controlled [42]. Although this guideline does not apply directly to broadband disturbances, this concept can be applied considering the highest frequency on the bandwidth of interest. The sample rate used for most of the experimental data presented here was 1600 Hz. This corresponds to 4 times the highest frequency on the 0 to 400 Hz bandwidth of interest. Obviously, the selection of an appropriate sample rate also requires some prior knowledge of the system and the disturbance.

System identification of the control path transfer functions require a code that echoes the reference signal out each of the control paths independently, at the prescribed sample rate. A control system with $N_c$ control actuators and $N_e$ error sensors has $(N_c \times N_e)$ control-to-error transfer functions that need to be measured and modelled with digital filters. As explained in Chapter 2, IIR filters are designed so that their frequency response matches that of the measured control-to-error transfer functions. Each of these transfer functions is measured on a Brüel & Kjær type 2034 Dual Channel Signal Analyzer, and the real and imaginary parts of the Frequency Response H1 spectra are downloaded to a PC via a GPIB-PCIII board. The transfer functions are measured between the points A and B in Figure 3.5. Measuring the system identification between these two points includes the delay of the analog to digital (A/D) and the digital to analog (D/A) converters present at the inputs and outputs of the DSP as well as the delays due to the filters and amplifiers. The frequency range of the system identification spectra
needs to be large enough to include the Nyquist frequency. The design of the digital IIR filters to match a given frequency response is specific for a particular sample rate and requires transfer function data out to the Nyquist frequency.

Once the control path transfer functions are all downloaded and stored in a PC, the curve-fit procedure to design the IIR coefficients is implemented. The digital IIR design implements the MATLAB function "INVFREQZ.M" [37]. Note that it is important that the sample rate be kept consistent throughout the entire system identification and control procedures since the IIR filter designs are specific for a given sample rate. Since the nyquist frequency is usually higher than the bandwidth of interest, an error weighting function can be applied to the measured FRF’s in the frequency domain to obtain a more exact match on the bandwidth of interest. The error on the frequencies inside the bandwidth of interest are weighted heavier than those outside the bandwidth of interest so that the frequency response matchup in phase and magnitude is more exact on the range with the heavier error weighting. Applying the error weighting function can also help in the prevention of unstable poles in the IIR filter designs, since the filters are less concerned with matching the frequency response on a part of the spectrum that is noisy due to the low pass filtering of the control and error signals.

The IIR filter designs must be stable, i.e., all of the poles must be located within the unit circle in z-plane. To check stability, the moduli of the roots of the denominator
polynomial of the IIR in the z-domain are calculated to see if any exceed the value 1 in magnitude. In the event that there are unstable poles, the poles are inverted to reflect them back into the stable region of the z-plane. A z-plane root locus plot for a typical system ID is shown in Figure 3.8, where the filter poles are plotted as X’s and the filter zeros are plotted as O’s. If the IIR filter design yields unstable poles, and the unstable poles are inverted to make them stable, the matchup between the measured and IIR filter frequency response functions needs to be checked for severe phase mismatch on any part of the spectrum. Inaccurate phase modelling in the system ID can result in an unstable control system, since the phase of the control input energy determines whether the plate energy will cancel with the out of phase control energy or actually increase with in phase control energy.

Comparison of magnitude and phase plots of the measured transfer functions to those of the designed IIR filters allows evaluation of the IIR designs. Figures 3.9 and 3.10 contain magnitude and phase plots comparing some typical measured and digitally modelled transfer functions. The selection of the number of coefficients to be designed in the IIR filters can be estimated from an examination of the control path transfer functions. The number of modes and the quality of the resonances (i.e., sharpness of the peaks) in these transfer functions can be used as a guideline to estimate the number of coefficients that should be implemented in the IIR’s. Approximating the number of plant poles and the poles of the equipment in the control path and summing them gives an
Figure 3.8 Root locus in z-plane for IIR model of system ID transfer function; x = poles, o = zeros.
The approximate starting minimum number of IIR filter poles. Obviously as the number of coefficients increases, the tighter the match between the measured and modelled FRF's. Underestimation of the number of IIR coefficients can lead to an inadequate model, whereas attempting to implement IIR's with an excessive number of coefficients can constrain the sample rate of the controller. The latter is especially true with MIMO, due to the number of calculations required to compute all of the filtered-x signals. Figure 3.9 and 3.10 contain plots comparing the magnitude and phase of a typical system identification performed between control actuator A1 and a microphone located at 30° in the far-field for Plate 2 (see Figures 3.4 and 3.6). In this plot and throughout the experiments, the IIR's were designed with 50 numerator coefficients (bₖ's) and 49 denominator coefficients (aₖ's), corresponding to 50 zeros and 50 poles. This yielded an excellent match of both phase and magnitude response and still allowed operation of the controller at the desired sample rate.

To run control, the IIR filter coefficients and sample rate are first loaded into the controller. A suitable convergence parameter needs to be estimated before control can be initiated. Stability limits on the value of μ can be computed from the filtered-x signal as follows [26].

\[
\mu < \frac{1}{E[\hat{x}^2]} \left( L + 1 \right)
\] (3.1)
Figure 3.9 Comparison of measured vs. modelled system ID magnitude.

--- measured, ------ modelled
Figure 3.10 Comparison of measured vs. modelled system ID phase.

--- measured, ------ modelled
where \( E[x^2] \) is the power of the filtered-x signal. Thus the value of the convergence parameter \( \mu \) can be estimated from the power of the filtered-x signals [26,32]. A safe approach is to choose an extremely small value for the convergence parameter at first, then increase its value once control execution begins, while keeping close monitor of the control and error signals so that the system does not go unstable. The convergence parameter used for most of the experiments was on the order of \( 1 \times 10^{-6} \).

The next parameters to be determined are the number of coefficients in the FIR control compensating filters. Since the compensators are FIR filters and thus contain only zeros and no poles, stability of the FIR compensators is ensured. Determining the size of suitable FIR filters again depends on the bandwidth of interest and the number of modes to be controlled. For a harmonic disturbance, two coefficients are necessary in the compensating FIR [19,28]. The response of a linear system to a harmonic input is a harmonic output at the same frequency, changed only in magnitude and phase. A two coefficient FIR has two degrees of freedom and thus allows both the magnitude and phase of the reference signal to be properly adjusted to create the control input. Thus the number of FIR coefficients dictate the number of frequencies that can be properly modified in both magnitude and phase. The FIR compensator size was varied in some experiments and the topic will be further addressed in Chapter 4.

Using the B&K, it was possible to evaluate the reduction in the error spectra at
approximately 30 second intervals while the controller was converging. The controller was allowed to converge until the reduction of the error signals ceased to increase over a time period of about one minute. This resulted in typical convergence times of approximately three to six minutes. Longer FIR compensating filters require more time to converge than FIR’s with fewer coefficients.

Upon convergence of the controller, the convergence parameters could be reset to 0 to halt the adaption of the controller. The control program allowed the controller to be held and unheld so that before and after control data could be obtained. The coefficients of the FIR compensators could then be stored and their frequency responses analyzed.

A outline summary of the experimental procedures follows:

**Summary of Experimental Procedures for Broadband Control**

A. Preliminary determination of control parameters (requires knowledge of plant and disturbance)

1. Determine the number of control channels.
2. Determine suitable control actuators and locations.
3. Determine suitable error sensors and locations.
4. Determine the controller sample rate.
B. System identification of control loop transfer functions.

1. Echo reference through TMS320C30 controller, each control channel independently, and set error gains so that the controller A/D converter input limits are not exceeded.

2. Measure transfer functions and convert to frequency domain, at least up to the nuquist frequency \( f_{\text{nuquist}} = \frac{1}{2} f_{\text{sample}} \). For \( N_c \) actuators and \( N_s \) error sensors, there will be \( N_c \times N_s \) control-to-error transfer functions.

3. Solve for the coefficients of the IIR filters to model each control-to-error transfer function. Stability of these filters must be ensured before implementation.

4. Load IIR coefficients and sample rate into the TMS320C30.

C. Control execution

1. Determine convergence parameter(s).

2. Determine size of FIR compensator(s)

2. Begin convergence of controller (monitor to ensure stability.)

3. Adjust convergence parameter(s) if necessary.

4. Halt adaption upon convergence (reset convergence parameters to 0.)
Chapter 4

Experimental Results

This chapter presents the experimental data obtained for the two simply supported plates described in Chapter 3. Experiments were conducted for SISO, 2I2O, and 3I3O control systems using both far-field microphones and PVDF structural sensors as error transducers. Extensive data was collected and put into different formats for presentation. Error signal power spectral density plots before and after control showed the ability of the controller to reduce the error signals, and revealed information about modal controllability and observability. Plots of total radiation directivity, obtained by integrating the sound pressure spectra over the bandwidth of interest, provided a representation of the global reduction obtained. They were used as a basis of comparison to evaluate the performance of each experimental configuration. Radiation directivity plots before and after control at each of the plate resonant frequencies revealed the effect
of control on each of the individual modes. To quantify with a single numerical estimate the performance of the controlled system, the sound power reduction in the plane was estimated by integrating the intensity in the horizontal plane passing through the center of the plate. All sound pressure levels are reported in decibels with reference pressure of 20 μPa.

This chapter serves as a presentation of some of the most interesting data obtained. The implications of the data with respect to many of the control system aspects that were investigated will be addressed more closely in Chapter 5. Note that all of the figures for the experimental results are contained at the end of each section of this chapter to make them easy to reference and compare.

4.1 Comments on Quantifying Broadband Attenuation

With a harmonic disturbance, the reduction is easily quantified since the system responds at a single frequency. A 20 dB reduction in a harmonic tone is clearly understood as the total reduction obtained. The attenuation of a broadband signal is more difficult to quantify since an integration over the bandwidth in the frequency domain is required to quantify the total attenuation of the signal. Since the summation needs to be performed on a linear scale, and most signal attenuation results are given in terms of a logarithmic decibel scale, the total attenuation of a broadband signal can be misleading. Consider the spectra in Figure 4.1 which represent signals containing spectral content at
Figure 4.1 Three typical spectra.
six distinct frequencies. Figure 4.1(a) could be considered as the spectrum before control while Figures 4.1(b) and 4.1(c) could represent after control spectra. For simplicity, let the spectral resolution of these FRF's be 1 Hz with zero content at frequencies other than the six spectral lines. In Figure 4.1(b), the first spectral line is attenuated by 30 dB, but the total reduction integrated over the spectrum is only 4.5 dB. In Figure 4.1(c), five out of six of the frequencies were significantly attenuated by upwards of 30 dB, but the total reduction integrated across the bandwidth is only 7.2 dB. Thus the total reduction integrated across the bandwidth in dB can understate the attenuation achieved at individual frequencies.

In a system FRF, the peaks in the spectra correspond to natural modes, or resonances of the system. Thus if only one of the modes can be controlled, or if all of the modes except one can be controlled, the total attenuation integrated across the bandwidth may seem insignificant. This example demonstrates the importance of controlling all of the modes of a system in order to obtain a significant reduction across the bandwidth.

### 4.2 Results with Microphone Error Sensors

Microphones located in the far-field at approximately 1.8 m from the plate (see Figure 3.6) were implemented as error transducers for SISO, 2I2O, and 3I3O control configurations. Since the output of a microphone is proportional to the acoustic pressure
at the point where it is located, the location of the error microphones in the far-field had considerable impact on the control system observability as explained in Chapter 3.

4.2.1 Disturbance Bandwidth of 0 to 400 Hz

SISO Control System

The first experiment involved the SISO controller on Plate 1, with a disturbance bandwidth of 0 to 400 Hz. The sample rate used was 1500 Hz, and the FIR compensating filter contained 100 coefficients. The error microphone was located at 0° in the far-field (see Figure 3.6), and the control signal was input with the piezoceramic actuator A1 (see Figure 3.3), which was located to optimally control the radiation of the (1,1) and (3,1) modes. Figure 4.2 shows the error microphone spectra before and after control. The reduction in overall sound level at the error microphone was 5.95 dB. It can be seen in the spectra that the second peak at 182 Hz exhibits little reduction after control since it is not significantly observed by the error microphone at 0°. The directivity plots for the total radiation integrated across the bandwidth, and the directivity plots at each of the plate resonant frequencies are shown in Figure 4.3. Note that the total attenuation in Figure 4.3(a) tapered off in the directivity field moving away from the location of the error microphone. The total sound power reduction in the horizontal plane was only 1.96 dB. The radiation at 182 Hz, corresponding to the (2,1) mode is not controlled, and there is actually an increase (spill over effect) in the acoustic radiation at this frequency, as shown in Figure 4.3(c). The directivity at this frequency is dominated by the radiation
of the (2,1) mode that is similar to that of a dipole with relatively little radiation at 0° (see Figure 4.3(c). The error microphone at 0° is a poor location for observation of the radiation due to the (2,1) mode and therefore control at this frequency is not achieved. An error sensor that ignores the radiation of a particular mode can allow the radiation of that mode to increase after control, without a significant increase in the cost function. Therefore, although the controller minimizes the cost function, it is blind to the fact that the radiation of that particular unobserved mode actually increases. The radiation at each of the other plate modes was attenuated as shown in Figures 4.3 (b),(d),(e) and (f).
Figure 4.2 Error microphone spectra @ 0° with SISO control.

--- Before control, ······· After control
Figure 4.3 Directivity patterns using SISO with error microphone @ 0°.
To more effectively observe all radiating modes in the horizontal plane, the error microphone was moved to -30° in the far-field and the SISO experiment repeated. Figure 4.4 contains a plot of the error microphone spectra before and after control. The total sound reduction at the error microphone was 5.56 dB. Note that the after control spectrum shows that about 4 dB of attenuation was achieved at the (2,1) mode. The directivity plots of the total radiation and at each of the plate resonant frequencies are shown in Figure 4.5. Since all modes are now observed by the error microphone, global sound reduction at all the resonant frequencies is observed.

The sound radiation of the (1,1) mode in Figure 4.5(b) shows the best reduction of up to 19.3 dB at points in the far-field, while the radiation of the (2,1) mode in Figure 4.5(c) shows the least reduction with between 1 and 7 dB of attenuation at the far-field measurement points. Total reductions ranging from 3 to 6 dB were obtained at all directions in the far-field, even though the total reduction at the error microphone at -30° was slightly less than with the error microphone at 0°. This is because the radiation from all the modes was observed and controlled. A total sound power reduction of 2.97 dB was achieved in the horizontal plane with the error microphone at -30°. This demonstrates the importance of observing all modes on the bandwidth, good and poor radiators, in order to achieve global reduction with broadband inputs.

Even though the frequency of the (2,1) mode was not included in the optimization
routine for the actuator design, meaning the actuator A1 was not located to optimally control the (2,1) mode, the (2,1) mode was controllable to some extent with actuator A1 as demonstrated in the first two experiments. It should be noted that if any mode is ignored in the optimization routine, the resulting design should be checked to ensure some amount of controllability for all of the modes.
Total Reduction = 5.56 dB

**Figure 4.4** Error microphone spectra @ -30° with SISO control.

--- Before control, ··········· After control
Figure 4.5 Directivity patterns using SISO with error microphone @ -30°.
2120 Control System

The results with the 2120 control configuration were significantly better than with the SISO configuration. The 2120 algorithm was implemented on Plate 1, using the piezoceramic actuators A2 and A3 in Figure 3.3, which were located to optimally control the radiation of the (1,1) and (3,1) modes. The sample rate was 1500 Hz, and 100 coefficients were used in each of the FIR compensating filters. The two error microphones were placed at -30° and +30° in the far-field (see Figure 3.7). The error microphone spectra before and after control are shown in Figures 4.6 and 4.7, and showed reductions of 8.90 dB and 7.91 dB, respectively. The total radiation directivity and the radiation directivity at each of the plate modes are plotted in Figure 4.8. These figures show that considerable improvement in sound reduction was obtained, especially in the radiation due to the (2,1) and (3,1) modes (see Figures 4.8(c) and 4.8(e)). The 2120 system yielded more global reduction as evident by comparing Figures 4.5(a) and 4.8(a). The total sound power in the horizontal plane was reduced by 5.07 dB. The increase in the number of control channels resulted in a better observability and controllability of all the plate mode radiation. This is an important issue in broadband radiation control because to obtain significant global sound reduction, attenuation of all radiating modes is necessary. This should be contrasted to the control of harmonic disturbances where only the efficient acoustic modes need to be controlled to obtain global attenuation.

It should be noted that the 2120 control configuration was also implemented with
the error microphones located at 0° and -30° in the far-field. The attenuation observed was similar to but slightly less than with the microphones located at -30° and +30°, and the results were omitted for brevity. Again this demonstrates the effect of the microphone locations on the observability, and likewise on the attenuation achievable.
Figure 4.6 Error microphone spectra @ -30° with 2120 control.

--- Before control, ~ After control
Figure 4.7 Error microphone spectra @ +30° with 2120 control.

--- Before control, ------ After control

Total Reduction = 7.91 dB
Figure 4.8 Directivity patterns using 2I2O control with error microphones @ -30° and +30°.
3I3O Control System

The 3I3O control system was implemented on Plate 2 with actuators A1, A2 and A3 (see Figure 3.4). The actuators were optimally located to control the radiation of the (1,1), (2,1), and (3,1) modes, with a 3I3O controller. The sample rate used was 1600 Hz, and 50 coefficients were used in each of the FIR compensating filters. The controller board was not computationally fast enough to implement 100 coefficients in the FIR compensators. The error microphones were located in the far-field at -30°, 0°, and -30°. The before and after control error microphone spectra are shown in Figures 4.9, 4.10, and 4.11. The error microphone signals were reduced by 13.0 dB, 14.9 dB, and 13.8 dB, respectively. The directivity plots before and after control are contained in Figure 4.12. The total sound power attenuation in the plane was 7.69 dB, with total attenuations integrated across the bandwidth ranging from 10 to 14 dB at points in the far-field. Radiation at the frequency of the first mode was attenuated by 20 to 25 dB at points in the far-field as shown in Figure 4.12(b). Attenuation at points in the far-field at the frequency of the second mode were on the order of 12 to 25 dB, as shown in Figure 4.12(c). The radiation at the frequency of the (3,1) mode in Figure 4.12(f) exhibited attenuation ranging from 10 to 22 dB. Significant attenuation was also observed at the (1,2) and (2,2) modes, showing that although these modes were not included in the actuator optimization routine, that they were controllable.

Again, a considerable increase in attenuation was observed with the 3I3O control
system as compared to the 2I2O and SISO control systems. This is due to the fact that the 3I3O control system has more degrees of freedom, allowing more modes to be more effectively controlled, and that the actuator location design procedure included three frequencies in the optimization routine. A plot showing a comparison of the total attenuation achieved with the SISO, 2I2O, and 3I3O control systems is shown in Figure 4.13, and the increase in control performance with increasing number of control channels is evident. The optimal SISO configuration used in conjunction with the optimal 2I2O configuration on Plate 1 provided a sub-optimal 3I3O configuration. The total reduction obtained with this 3I3O sub-optimal actuator configuration is also plotted in Figure 4.13. Note that there was little increase in attenuation between the optimal 2I2O configuration and the sub-optimal 3I3O configuration. This demonstrates the importance of the actuator optimization in conjunction with the number of channels of the control system.
Figure 4.9 Error microphone spectra @ -30° with 313O control, 0 to 400 Hz bandwidth.

— Before control, ———— After control
Figure 4.10 Error microphone spectra @ 0° with 3130 control, 0 to 400 Hz bandwidth.

--- Before control, ······ After control
Total Reduction = 13.8 dB

Figure 4.11 Error microphone spectra @ +30° with 3I30 control, 0 to 400 Hz bandwidth.

--- Before control, ******* After control
Figure 4.12 Directivity patterns using 3I3O control with error microphones @ -30°, 0° and +30°, 0 to 400 Hz bandwidth.
Figure 4.13 Directivity plot comparing total SPL reduction using microphone error sensors, 3I3O: Mic@ -30°,0°,+30°; 2I2O: Mic@ -30°,+30°; SISO: Mic@ -30°.
4.2.2 Disturbance Bandwidth of 0 to 800 Hz

The disturbance bandwidth was extended to a frequency range of 0 to 800 Hz. The low pass filters in the experimental setup were adjusted to have cutoff frequencies at 800 Hz. The sample rate used was 1600 Hz, with 50 coefficients in each of the compensating FIR filters. The system ID's again had 50 numerator coefficients and 49 denominator coefficients, corresponding to 50 poles and 50 zeros. The 800 Hz bandwidth excited 8 modes in the plate, and the microphone spectra now had 8 peaks, although the resonances were not distinct. An impact hammer test performed without the shaker attached revealed the presence of 8 distinct modes on the 0 to 800 Hz bandwidth. Although these 3 additional modes were not precisely identified with a modal analysis, theoretical predictions and acoustic directivity patterns revealed that they were most likely the (1,3), (3,2), and the (2,3) modes, corresponding to 474 Hz, 500 Hz, and 564 Hz, respectively.

The 3130 control system configuration was implemented on Plate 2 for the 0 to 800 Hz disturbance bandwidth, with the error microphones located in the far-field at -30°, 0°, and +30°. The resulting attenuations at each of the error microphone signals integrated over the 0 to 800 Hz bandwidth were 5.82 dB, 6.16 dB, and 4.28 dB as shown in the microphone spectra plots in Figures 4.14, 4.15, and 4.16. The microphone error spectra show that there was spillover (increased radiation levels after control) on the high end of the frequency range of these spectra, especially at about 720 Hz. Figure 4.17
shows the total radiation directivity plot and the directivity plots at each of the first five modes. Figure 4.18 contains the directivity plots at each of the additional modes present with the expanded bandwidth. The attenuation in total sound power integrated across the bandwidth from 0 to 800 Hz was 2.82 dB with total attenuations ranging between 3 to 7 dB at the field points in the plane. The radiation at the frequency of the (1,1) mode in Figure 4.17(b) exhibited attenuation at points in the far-field ranging from 21 to 25 dB, which was more attenuation at that mode than was achieved with the 0 to 400 Hz bandwidth. It is evident that 8 modes of the plate were controlled with only 3 control inputs and 3 error sensors. The attenuation achieved at the higher modes was not as significant as the reduction obtained at the lower modes since the actuators were not designed to optimally control these higher modes.
Figure 4.14 Error microphone spectra @ -30° with 3I3O control, 0 to 800 Hz bandwidth.

--- Before control, ··············· After control
Figure 4.15 Error microphone spectra @ 0° with 3130 control, 0 to 800 Hz bandwidth.

— Before control, -------- After control

Total Reduction = 6.16 dB
Total Reduction = 4.28 dB

Figure 4.16 Error microphone spectra @ +30° with 3I3O control, 0 to 800 Hz bandwidth.

--- Before control, ········· After control
Figure 4.17 Directivity patterns using 3130 control with error microphones @ -30°, 0° and +30°, 0 to 800 Hz bandwidth.
Figure 4.18 Directivity patterns using 3130 control with error microphones @ -30°, 0° and +30°, 0 to 800 Hz bandwidth.
4.3 Results with PVDF Error Sensors

4.3.1 Disturbance Bandwidth of 0 to 400 Hz, SISO Control System

The SISO control algorithm was implemented on Plate 1 (see Figure 3.3) and used piezoceramic actuator A1 in conjunction with PVDF error sensor P1, both of which were designed to optimally control the radiation of the (1,1) and (3,1) modes. The disturbance bandwidth was 0 to 400 Hz, the sample rate used was 1500 Hz, and 100 coefficients were used in the FIR compensating filter. The spectra of the PVDF signal before and after control are shown in Figure 4.19. The total reduction of the PVDF signal was 7.09 dB, although the PVDF signal content at 350 Hz shows considerable increase after control. The directivity plots for the total acoustic radiation integrated over the bandwidth and at each of the plate resonant frequencies are shown in Figure 4.20. Total sound attenuations ranging from 0.5 dB to 2.4 dB were observed at points in the far-field as shown in Figure 4.20(a) with a sound power reduction in the plane of only 0.689 dB.

Since the SISO PVDF was designed to optimally control the radiation at the frequencies of the (1,1) and (3,1) modes, the PVDF weighted these frequencies heavily, as shown in the spectra of Figure 4.19. The (2,1) and (2,2) modes (at 182 Hz and 350 Hz, respectively) were given considerably less weighting, and since these modes were less observable, the sound radiation of these modes were not controlled as evident in Figures 4.20(c) and 4.20(f). The radiation at 350 Hz actually showed a significant increase after control. This experiment again demonstrates the need for observing all of the radiating
modes included in the disturbance bandwidth in order to obtain global sound attenuation.
Figure 4.19 Error PVDF S1 spectra with SISO control.

--- Before control, ········ After control
Figure 4.20 Directivity patterns using SISO control with PVDF error sensor S1.
**212O Control System**

The 212O control system achieved considerably more attenuation than the SISO system. The 212O configuration was implemented on Plate 1 using piezoceramic actuators A2 and A3 in conjunction with PVDF sensors S2 and S3 in Figure 3.3. These actuators and sensors were designed to optimally control the radiation of the (1,1) and (3,1) modes with a 212O controller. The disturbance bandwidth was 0 to 400 Hz, the sample rate used was 1500 Hz, and 100 coefficients were used in each of the FIR compensators. Figure 4.21 and Figure 4.22 contain the before and after control PVDF error spectra. The PVDF error sensors S2 and S3 exhibited reductions of 11.5 dB and 7.45 dB, respectively. Even though PVDF sensor S2 does not effectively observe the (2,2) mode and PVDF sensor S3 does not heavily weight the (2,1) mode, when both PVDF error sensors are used in conjunction, all modes are observable and heavily weighted. This resulted in global control of all of the radiating modes on the bandwidth as shown in the directivity plots in Figure 4.23. The total power reduction in the plane was 2.93 dB, with total attenuation at points in the far-field ranging from 4 to 8 dB. The 212O control system allowed more modes to be observed and controlled, which resulted in increased global attenuation.
Figure 4.21 Error PVDF S2 spectra with 2I2O control.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>Volts RMS (dB re 1e-6 v)</td>
<td>Total Reduction = 11.5 dB</td>
</tr>
</tbody>
</table>

---

Before control, After control
Figure 4.22 Error PVDF S3 spectra with 2I2O control.

--- Before control, -------- After control
Figure 4.23 Directivity patterns using 2120 control with PVDF error sensors S2 and S3.

(a) Total SPL  
(b) mode (1,1) 86 Hz  
(c) mode (2,1) 182 Hz  
(d) mode (1,2) 248 Hz  
(e) mode (3,1) 331 Hz  
(f) mode (2,2) 350 Hz
3I3O Control System

The 3I3O control system yielded more total attenuation than the 2I2O system as expected, but the improvement was only marginal. The disturbance bandwidth was 0 to 400 Hz, the sample rate was 1600 Hz, and 50 coefficients were used in each of the three FIR compensating filters. The 3I3O system was implemented on Plate 2, and used piezoceramic actuators A1, A2 and A3 in conjunction with PVDF error sensors S1, S2, and S3. The actuators were designed for 3I3O optimal control of the (1,1), (2,1), and (3,1) modes, while the PVDF sensors were designed for optimal control of the (1,1), (2,1) and (1,2) modes. Figures 4.24, 4.25, and 4.26 contain before and after control spectra for PVDF error sensors S1, S2, and S3, respectively. The spectra show that there is significant weighting given to all five modes on the bandwidth. The total reductions achieved for each of the PVDF signals were 15.1 dB, 12.2 dB, and 10.3 dB. Figure 4.27 contains the directivity plots for the total radiation and the radiation at each of the plate resonant frequencies. The total attenuation observed at points in the far-field ranged from 4 to 10 dB, with a power reduction in the horizontal plane of 3.92 dB.

The increase in total attenuation going from the 2I2O system to the 3I3O configuration was not as significant as that observed with the microphone error systems. In fact, the amount of attenuation achieved at each of the modes with the 3I3O PVDF configuration was lower than that achieved with the 2I2O PVDF control system. This is because the 3I3O PVDF sensor configurations were not designed to optimally control the
most efficient acoustic modes on the bandwidth of 0 to 400 Hz. Although the attenuation of the total radiation increased with the 3I30 PVDF system, the PVDF sensor designs did not implement a modal weighting function for optimal control across the bandwidth, as the microphones would automatically. Figure 4.28 contains a directivity plot comparing the total reductions obtained with the PVDF error sensors for SISO, 2I2O, and 3I3O control. It is evident that optimal sensor and actuator design for the most efficient modes is necessary for the maximum reduction, but all modes need to be observable and controllable. Still the control performance was good compared with the 2I2O case and the results with the microphone error sensors.
Figure 4.24 Error PVDF S1 spectra with 3I3O control, 0 to 400 Hz bandwidth.

--- Before control, ---------- After control
Figure 4.25 Error PVDF S2 spectra with 3I3O control, 0 to 400 Hz bandwidth.

--- Before control, ........ After control
Total Reduction = 10.3 dB

**Figure 4.26** Error PVDF S3 spectra with 3I3O control, 0 to 400 Hz bandwidth.

---

Before control, ———— After control
Figure 4.27 Directivity patterns using 3I3O control with PVDF error sensors S1, S2 and S3, 0 to 400 Hz bandwidth.
Figure 4.28 Directivity plot comparing total SPL reduction using PVDF error sensors.

--- 3I3O, ------ 2I2O, ------- SISO
4.3.2 Disturbance Bandwidth of 0 to 800 Hz

The disturbance bandwidth was extended to include frequency content on the range from 0 to 800 Hz. The 3I3O actuator and PVDF sensor configuration on Plate 2 was implemented using a sample rate of 1600 Hz. The FIR compensating filters each had 50 coefficients, and the IIR system ID filters were again designed with 50 poles and 50 zeros. The extended disturbance bandwidth now excited eight plate modes as observed by the PVDF and the peaks in the far-field microphone spectra. The additional higher modes were at 474 Hz, 500 Hz, and 564 Hz, which most likely corresponded to the (1,3), (3,2), and (2,3) modes, respectively.

The total reduction of each of the PVDF signals was 11.1 dB, 7.80 dB, and 10.3 dB as shown in their respective spectra in Figures 4.29, 4.30 and 4.31. The directivity plots of the total acoustic radiation and the radiation at each of the first five modes are contained in Figure 4.32. The directivity plots at each of the three higher modes are shown in Figure 4.33. The total attenuation integrated across the extended bandwidth at points in the far-field ranged between 5 and 5.7 dB, which corresponded to a total power reduction in the plane of 3.42 dB.

More total sound attenuation was achieved with the 3I3O PVDF configuration than with the 3I3O microphone configuration, across the bandwidth of 0 to 800 Hz. Figure 4.34 shows a comparison between the total reductions obtained with the 3I3O microphone
configuration and the 3I3O PVDF configuration with the disturbance bandwidth of 0 to 800 Hz. The sound reduction observed at each of the plate resonant frequencies was higher with the 3I3O PVDF system than with the 3I3O microphone system at every mode except the (3,1) and the (1,3) modes for the 0 to 800 Hz bandwidth. Note that above about 350 Hz, the radiation efficiency of the (2,1) and (1,2) modes is higher than that of the (3,1) mode (see Figure 2.2). Thus for a disturbance with frequency content above about 350 Hz, the 3I3O PVDF design is optimized for the most efficient radiating modes. Note that the modes with an even second modal indice, as opposed to an odd second modal indice, tend to dominate the radiation in the directions toward the edges of the directivity field (away from the perpendicular direction at 0°). Since the PVDF design included optimal control of the (1,2) mode rather than the (3,1) mode, the PVDF configuration excelled in the amount of attenuation achieved at the edges of the directivity field as shown in Figure 4.34, and resulted in more power reduction in the horizontal plane. Thus the design of the PVDF sensors to optimally control the (1,1), (2,1), and (1,2) modes yielded a modal observability weighting that was more proficient for controlling a disturbance bandwidth of 0 to 800 Hz than for a bandwidth of 0 to 400 Hz.
Figure 4.29 Error PVDF S1 spectra with 3I3O control, 0 to 800 Hz bandwidth.

--- Before control, ········· After control
Figure 4.30 Error PVDF S2 spectra with 3I3O control, 0 to 800 Hz bandwidth.

—— Before control, ———— After control

Total Reduction = 7.80 dB
**Figure 4.31** Error PVDF S3 spectra with 3I3O control, 0 to 800 Hz bandwidth,

--- Before control, :::::::: After control

Total Reduction = 10.3 dB
Figure 4.32 Directivity patterns using 3I3O control with PVDF error sensors S1, S2 and S3, 0 to 800 Hz bandwidth.
Figure 4.33 Directivity patterns using 3I3O control with error PVDF S1, S2 and S3, 0 to 800 Hz bandwidth.
Figure 4.34 Directivity plot comparing total SPL reduction using microphone and PVDF error sensors, 0 to 800 Hz bandwidth. —— 3I3O microphone, ------- 3I3O PVDF
4.4 FIR Compensator Size Study

The number of coefficients in the FIR compensating filter(s) and the sample rate affected the frequency response of the control input, and the amount of attenuation achieved. The 3i30 control configuration was implemented on Plate 2 with the PVDF error sensors to examine the effects of changing the size of the FIR compensators. Experiments using four different FIR filter sizes were performed, with an equal number of coefficients in each of the three control channels. The disturbance bandwidth was 0 to 400 Hz, the sample rate was 1600 Hz, and all four experiments implemented the same system identification. The four experiments implemented FIR filters with 10, 25, and 50 coefficients. A convergence parameter of $1 \times 10^{-6}$ was used for the experiments. Each configuration was allowed to converge until a time span of 30 seconds yielded no further attenuation of the error signals. When each experiment was considered converged with the above criterion (after a time span ranging from 2 to 6 minutes), the convergence was halted by resetting the convergence parameters to 0. The reduction at each of the PVDF error sensors were then recorded and the converged FIR filter coefficients stored. The transfer functions of each of the FIR compensators could then be determined and compared for each experiment.

The first experiment implemented 50 coefficients in the FIR filters and required a convergence time of about 6 minutes to satisfy the convergence criterion. The resulting attenuation at the PVDF error sensors S1, S2, and S3 was 12.7 dB, 16.6 dB, and 11.0 dB,
respectively. The second experiment used 25 coefficients in each of the compensators and required a convergence time of approximately 4 minutes. The attenuation of the PVDF error signals S1, S2, and S3 were observed to be 12.5 dB, 16.2 dB, and 10.6 dB, respectively. The third experiment used 10 coefficients in each of the FIR compensators and resulted in attenuations of 12.1 dB, 15.4 dB, and 10.1 dB at the PVDF error sensors S1, S2, and S3, respectively. The required convergence time was about 1 minute. The fourth experiment used only 5 coefficients in each of the compensators and required a convergence time of approximately 1 minute. The attenuation of the PVDF error signals S1, S2, and S3 were 10.2 dB, 13.3 dB, and 9.2 dB, respectively. Table 4.1 summarizes the required convergence times and the reduction obtained for each experiment. Figure 4.35 shows a comparison of the resulting converged magnitude responses for control channel 3 of each 3I3O configuration. A comparison of the phase responses for the converged filters is contained in Figure 4.36.

It is clear from Figures 4.35 and 4.36 that the frequency resolution increases with increasing compensator size, and thus a more accurate match between the optimal compensator and the converged compensator is possible with FIR filters of higher order. This means that more frequencies on the bandwidth can be optimally modified in magnitude and phase. Since a two-coefficient FIR has two degrees of freedom and can precisely modify the magnitude and phase of a single frequency, it follows that an FIR with 50 coefficients has 50 degrees of freedom and can provide optimal magnitude and
phase modification for 25 frequencies on the bandwidth.
Table 4.1 Summary of FIR compensator size experiments

<table>
<thead>
<tr>
<th>FIR Size (# weights)</th>
<th>Convergence Time</th>
<th>Total Reduction PVDF S1</th>
<th>Total Reduction PVDF S2</th>
<th>Total Reduction PVDF S3</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>6 minutes</td>
<td>12.7 dB</td>
<td>16.6 dB</td>
<td>11.0 dB</td>
</tr>
<tr>
<td>25</td>
<td>4 minutes</td>
<td>12.5 dB</td>
<td>16.2 dB</td>
<td>10.6 dB</td>
</tr>
<tr>
<td>10</td>
<td>1 minute</td>
<td>12.1 dB</td>
<td>15.4 dB</td>
<td>10.1 dB</td>
</tr>
<tr>
<td>5</td>
<td>1 minute</td>
<td>10.2 dB</td>
<td>13.3 dB</td>
<td>9.2 dB</td>
</tr>
</tbody>
</table>
Figure 4.35 Converged FIR filter magnitude response comparison.

- --- 50 Coeffs, ------ 25 Coeffs, ------- 10 Coeffs, ---- 5 Coeffs
Figure 4.36 Converged FIR filter phase response comparison.

- 50 Coeffs, 25 Coeffs, 10 Coeffs, 5 Coeffs
The frequency resolution of an FIR compensator, or the minimum spacing between frequencies that can be accurately modelled by the FIR, is maximized by using an FIR of the largest size possible in conjunction with the slowest possible sample rate. An increase in compensator size allows more frequencies on the bandwidth to be properly modified. A slower sample rate increases the time span of the FIR, and lowers the width of the controllable bandwidth, but increases the frequency resolution on that bandwidth. It was noted that structures with small frequency separation between modes in the frequency domain need finer resolution in the FIR compensator(s). Different disturbance locations changed the frequency separation between (2,2) and (3,1) modes of the plate, and when the separation between these two modes was less than the resolution of the FIR compensators, control of both modes could not be simultaneously achieved and the total reduction decreased.

A tradeoff exists between increasing the frequency resolution of the control input and the time required for convergence as can be seen in Table 4.1. In these experiments, the loss in reduction was insignificant as compared with the time required for convergence until the compensator size was reduced below 10 coefficients. This is believed to be because the plate has five modes on the bandwidth, and even the 10 coefficient FIR has the ability to effectively compensate 5 frequencies on the bandwidth. Vipperman noticed an asymptotic trend in the amount of attenuation achieved with increasing filter order in an SISO configuration [28]. The number of coefficients dictates
the controllable bandwidth whereas the number of control channels dictates the number of modes controllable.
Chapter 5

Discussion of Results

The experimental results show that ASAC using the feedforward filtered-x LMS control algorithm can be used to effectively reduce the sound radiation from a structure excited by a disturbance with broadband frequency content. Total reductions ranging from 10 to 14 dB were obtained at points in the far-field for a disturbance bandwidth of 0 to 400 Hz. Attenuations at the loudest plate resonances were on the order of 20 to 25 dB. The sound radiation from the plate before control exhibited dominant tones at the plate resonances over a broadband background noise. The application of control primarily reduced the peaks in the sound spectrum, which removed the tonal properties and changed the quality of the sound. The radiation after control sounded like white noise of reduced loudness, without the shrill tones caused by the loud radiation levels at the plate resonant frequencies. This type of sound reduction implies that the controller is essentially
cancelling the energy of the disturbance into the modes, which reduces the radiation at
the frequencies at each of the modes. Thus the controller is utilizing modal suppression,
which is more effective for broadband ASAC than modal restructuring (restructuring the
energy into inefficient radiating modes). Reduction at all of the radiating modes, both the
efficient and inefficient radiators, is required for significant reduction across the
bandwidth.

The experimental results indicate that PZT piezoceramic patches are effective
actuators for broadband ASAC. The controllability of the actuators on the structural
dynamics was governed by their location on the structure. The adopted design procedure
of locating the actuators to optimally minimize the structural sound power assuming
disturbance excitation content at the most efficient radiating modes on the bandwidth
resulted in an effective design that yielded all the modes on the bandwidth controllable.
It is important that the disturbance is assumed to be located so that all modes on the
bandwidth are excited in the optimization procedure, in order to yield a design with all
the modes controllable.

The method of supplying the acoustic cost for minimization was investigated using
both microphones and PVDF transducers as error sensors. The experimental results
demonstrated that the observability of the sensors with respect to each of the modes on
the bandwidth effected the amount of control achievable.
Microphone transducers in the far-field were found to be effective error sensors for broadband radiation control. The observability of the microphone error sensors was dictated primarily by the location of the microphone(s) in the directivity field, due to the radiation directivity patterns at the frequencies of each of the modes. The results show the importance of locating the microphones to observe all the modes on the bandwidth, i.e., positions where the radiation levels of all modes are significant with respect to each other. If any of the modes has a relatively low observability with respect to the other modes on the bandwidth, that radiation of that unobserved mode can increase after control without a considerable increase in the cost function. This could result in insignificant attenuation across the bandwidth. It should be noted that although the sound attenuation data was taken only in the horizontal plane where the error microphones were located, sound reduction out of the horizontal plane was also observed in general. However, it is entirely possible with microphone error sensors that control spillover could result in increased radiation levels in directions other than where the error microphones were located.

The results also show that PVDF films are effective transducers for broadband sound radiation control when designed to optimally control the structural sound power for a disturbance containing content at a number of frequencies. The observability of the PVDF sensors is governed by their size and location on the plate. The importance of designing the PVDF sensor(s) to minimize the radiation of all of the structural modes on
the bandwidth is important to ensure reduction of the radiation of all the modes. Ignoring modes in the optimization routine, especially with SISO, can result in a PVDF configuration that yields the ignored modes unobservable, and thus the resulting attenuation can be insignificant due to spillover into the uncontrolled modes. Design of the PVDF sensors to optimally control the most efficient modes, or with respect to the majority of modes on the bandwidth is proficient for control of broadband radiation as long as all of the modes on the bandwidth are observable to some extent. Design of the PVDF sensors in conjunction with the actuators for optimal control of the same modes yields a more effective control system design. A system with structurally mounted PVDF error sensors designed to optimally reduce sound power is more likely to result in global attenuation throughout the sound field than a system with microphone error sensors located in the horizontal plane.

Increasing the number of control channels increases the number of modes that can be controlled. Expanding the controller from SISO to MIMO resulted in significant increase in attenuation. The results demonstrate that ASAC can reduce the required dimensionality of the controller for broadband control, since the number of modes that can be controlled is greater than the number of control channels. In the 0 to 800 Hz disturbance bandwidth experiments, 8 modes were controlled with a 3130 system.

Other control system aspects irrespective of the number of control channels and
design of the actuators and sensors were also found to effect characteristics of the control. The frequency resolution of the control input is dictated by the compensator size and controller sample rate. A larger filter size in conjunction with a smaller sample rate increases the resolution of the control input which allows optimal compensator magnitude and phase modification at more frequencies on the bandwidth. However, decreasing the sample rate also decreases the controllable bandwidth, and larger filter sizes require longer times for convergence of the filters.
Chapter 6

Conclusions And Recommendations

Active Structural Acoustic Control (ASAC) using the feedforward filtered-x LMS control algorithm was implemented experimentally on simply-supported plate structures excited by disturbances with band-limited broadband random frequency content. Both far-field microphones and PVDF transducers were used as error sensors. PZT ceramic actuators were used as the control inputs, and both the PVDF sensors and control actuators were optimally designed and located to control the most efficient radiating modes of the plates.

The objectives of this work were to demonstrate that ASAC with the feedforward filtered-x LMS control algorithm can be used to control the sound radiation from structures excited by broadband disturbances; to implement and evaluate both
microphones and PVDF transducers as error sensors; and to investigate several aspects of the control system as they apply to broadband radiation control, such as the number of control channels, system identification, control compensator size, and convergence characteristics.

Active Structural Acoustic Control (ASAC) using the feedforward filtered-x LMS control algorithm was shown to be an effective means of controlling sound radiation from a structure under excitation of a band-limited broadband random disturbance. Observability and controllability were determined to be important parameters effecting the amount of attenuation achieved. Thus the sensor and actuator designs were critical in developing an effective broadband acoustic control system. The optimization routine used for design of the sensors and actuators also governed system observability and controllability. Other aspects of the control algorithm such as compensator size and sample rate were also found to dictate the control system effectiveness.

A summary of the main conclusions follows:

1. ASAC utilizing the feedforward filtered-x LMS control algorithm has been demonstrated as an effective technique for controlling sound radiation from structures subject to broadband disturbances.

2. Both microphones and PVDF transducers can be effectively implemented as error sensors for broadband control.
3. PZT piezoceramic patches are effective actuators for broadband radiation control.

4. Design of sensors and actuators to optimally minimize the sound power of a structure assuming multiple frequency excitation with content at frequencies corresponding to the most efficient radiating modes on the disturbance bandwidth yields designs applicable for broadband disturbances with frequency content across the bandwidth. However, it is important that all modes on the bandwidth are observable and controllable to some extent. The reduction obtained using the PVDF error sensors were comparable to the attenuation achieved with microphones.

5. The compensator size and sample rate dictate the frequency resolution, the controllable bandwidth, and the time required for convergence.

6. The number of control channels dictate the number of modes that can be controlled and the amount of attenuation achieved. Increasing the number of control channels allows for more modes to be controlled.

7. If ASAC is implemented on the frequency range below the critical frequency, effective control can be achieved with less control channels than the number of modes to be controlled.

A list of recommendations for future work are as follows:

1. If the dynamics of the structure are approximately measured or modelled, a theoretical analysis can be used to generate an initial guess for the compensator
coefficients instead of starting the adaption from zero. This should improve the time
required for convergence.

2. Expansion of the actuator and sensor optimization routine to optimally locate
the actuator(s) and sensor(s) for control of broadband disturbances, rather than multiple
frequency disturbances will result in designs more effective for broadband control. This
will also eliminate the need to verify that all modes on the bandwidth are observable and
controllable.

3. In order to accurately identify the control mechanism and its effects,
measurement of the plate response before and after control is necessary. A modal
analysis before and after control should be performed to see what happens to the plate
vibration response.

4. The broadband control system can be extended to more complex structures such
as cylinders, plates with discontinuities and three-dimensional structures.

5. Further parametric studies on aspects of the control system design such as
variations in the convergence parameter and the compensator sizes between control
channels could be performed for further understanding of some of the more practical
aspects of ASAC applied to broadband control.
Bibliography


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APPENDIX A

Structural and Acoustic Response, Radiation Efficiency

This Appendix summarizes the computation of the structural and acoustic response of the plate, and the radiation efficiency.

A.1 Structural Response

The structural response for the simply-supported plate can be expressed as a linear combination of the modes as follows:

\[ w(x,y,z) = \sum_{n=1}^{N} q_n(\omega) \phi_n(x,y) \]  \hspace{1cm} (A.1)

where \( q_n(\omega) \) is the \( n \)-th generalized modal coordinate and \( \phi_n(x,y) \) is the \( n \)-th eigenfunction.

The modal displacement can be written as follows

\[ q_n(\omega) = f_n F(\omega) H_n(\omega) \]  \hspace{1cm} (A.2)

where \( f_n \) is the \( n \)-th modal disturbance force, \( F(\omega) \) is the Fourier transform of the disturbance input, \( H_n(\omega) \) is the \( n \)-th modal frequency response function,
\[ H_n(\omega) = \frac{1}{\omega_n^2 - \omega^2 + j2\zeta_n \omega_n \omega} \quad (A.3) \]

where \( \omega_n, \zeta_n \) are the \( n \)th natural frequency and modal damping ratio, respectively, and \( j \) is the imaginary number. The natural frequencies are given by

\[ \omega_n = \sqrt{\frac{D}{\rho_n h}} (\gamma_x^2 + \gamma_y^2) \quad (A.4) \]

where

\[ \gamma_x = \frac{\pi n_x}{L_x}, \quad \gamma_y = \frac{\pi n_y}{L_y} \quad D = \frac{E h^3}{12(1-\nu^2)} \quad (A.5) \]

where \((n_x, n_y)\) are the modal indices, \( h \) is the plate thickness, \( \nu \) is Poisson’s ratio, and \( D \) is the flexural rigidity. For a simply-supported plate, the \( n \)th eigenfunction normalized with respect to the mass distribution is [43]

\[ \phi_n(x,y) = K \sin(\gamma_x x) \sin(\gamma_y y) \quad (A.6) \]

where
The radiated pressure can also be expressed as a linear contribution of the modes as

\[
p(\mathbf{r}, \omega) = \sum_{n=1}^{N} q_n(\omega) p_n(\mathbf{r}, \omega) \tag{A.8}
\]

where \( \mathbf{r} \) is the position vector to the observation point in the acoustic field, and for light fluids

\[
p_n(\mathbf{r}, \omega) = \frac{j \omega \rho_0}{2\pi} \int_{A} j \omega \phi_n(x, y) e^{-ikR} \frac{e^{-ikR}}{R} dA \tag{A.9}
\]

is known as the Rayleigh integral for the radiated pressure distribution given by the \( n^\text{th} \) mode with surface velocity \( j \omega \phi_n(x, y) \). In the Rayleigh integral, \((x_n, y_n)\) are the coordinates of the elemental surface dA, \( A \) is the area of the radiator, \( R \) is the magnitude of the distance from the elemental source to the observation point, \( \rho_0 \) is the density of the acoustic fluid, and \( k \) is the acoustic wavenumber.

Assuming the observation point is in the far-field \((kL_x \gg 1)\)
\[ p_\omega(r, \omega) = \frac{-\omega^2 \rho_0 L_x K}{2\pi^2 n_x n_y} \frac{e^{-ikr}}{r} \left[ (-1)^n e^{-ik\alpha} - 1 \right] \left[ (-1)^n e^{-ik\beta} - 1 \right] \] 

\[ \alpha = kL_x \sin(\theta) \cos(\phi) \quad \beta = kL_y \sin(\theta) \sin(\phi) \]  

A.3 Radiation Efficiency

The radiation efficiency is defined as the ratio of the radiated sound power from one side of the plate to the velocity averaged over the plate. The radiated sound power of the \( n \)\textsuperscript{th} mode is given by

\[ \Pi_\omega(\omega) = \int_0^{2\pi} \int_0^{\pi} \frac{P_n P^*_n}{\rho_0 c} r^2 \sin(\theta) \, d\theta \, d\phi \]  

which was computed numerically.

The radiation efficiency is given by
\[ S_s(\omega) = \frac{1}{L_x L_y \rho_0 c} \frac{\Pi_s(\omega)}{\langle V_s(\omega) \rangle} \]  \hspace{1cm} (A.13)

where

\[ V_s(\omega) = \frac{\omega^2}{2 L_x L_y} \int_0^{L_x} \int_0^{L_y} \phi_s^2(x,y) \, dx \, dy \]  \hspace{1cm} (A.14)

\[ = \frac{\omega^2}{2 L_x L_y} K^2 \frac{1}{8} 2 L_x L_y = \frac{\omega^2 K^2}{8} \]  \hspace{1cm} (A.15)

The numbers used were for a steel plate as follows:

\[ \rho_s = 7833 \text{ N-s/m}^4 \text{ (plate density)}, \]

\[ E = 2.0 \times 10^{11} \text{ N/m} \text{ (Young's Modulus)}, \]

\[ \nu = 0.3 \text{ (Poisson's Ratio)}, \]

\[ h = 0.002 \text{ m (plate thickness)}, \]

\[ L_x = 0.38 \text{ m (x-dimension of plate)}, \]

\[ L_y = 0.3 \text{ m (y-dimension of plate)}, \]

\[ \xi_s = 0.01 \text{ (1\% damping ratio)}, \]

\[ N = 5 \text{ (number of modes)}. \]
Integration Of Spectra

In this Appendix, the integration over a spectrum is presented. To quantify the attenuation of a broadband signal from the before and after control autospectra, an integration over the spectra is necessary.

The integration over a two-sided autospectrum $G(\omega)$ is given by:

$$\sigma = \int_{-\infty}^{\infty} G(\omega) \, d\omega \quad (A.1)$$

For a continuous, finite, single-sided autospectrum, the integral is

$$\sigma = 2 \int_{0}^{\infty} G(\omega) \, d\omega \quad (A.2)$$

For a discrete autospectrum, the integral becomes a summation as follows:

$$\sigma = 2 \sum_{i=0}^{\infty} G(\omega_i) \, \Delta \omega \quad (A.3)$$
For an autospectrum in Hz:

$$\Delta f = \frac{\Delta \omega}{2\pi} \tag{A.4}$$

which gives an expression for the total as

$$\sigma = 4\pi \sum_{f_0} G(f_j) \Delta f \tag{A.5}$$

where $\Delta f$ is the frequency resolution of the autospectrum.
APPENDIX C

Calculation Of Sound Power In The Traverse Plane

To quantify the total acoustic reduction with a single numerical estimate, the sound power reduction per unit length in the plane of the traverse microphone was computed.

Figure A.1 shows a diagram of the 11 traverse microphone measurement points in the far field. The before and after control pressure autospectra were recorded at each of the 11 points, which allowed calculation of the power in the horizontal plane before and after control. The total power per unit length in the plane is computed as

\[
\Pi_{\text{tot}} = \sum_{i=1}^{11} \frac{p_i^2}{\rho c} A_i
\]  

(C.1)

for 11 measurement points, where \(p_i\) is the total pressure at measurement point \(i\) (integrated across the pressure spectrum as explained in Appendix B), \(\rho\) is the density of the acoustic fluid (air), \(c\) is the speed of sound, and \(A_i\) is the area associated with measurement point \(i\). For power per unit length in the plane, the area \(A_i\) is the arclength associated with measurement point \(i\). For 11 points in the directivity field, \(A_i = \pi r/10\) for the center points (points 2 through 10), where \(r\) is the radius (1.8 m). For the edge points (the points next to the baffle), \(A_i = \pi r/20\).
Figure C.1 Overhead view in anechoic chamber showing the 11 traverse microphone measurement points.
Vita

Jerome Peter Smith was born on February 18, 1969 in Wilkes-Barre Pennsylvania. He lived the first seventeen years of his life in Alexandria, Virginia, and entered the College of Engineering at Virginia Tech in 1986. During his final year as an undergraduate in the Department of Mechanical Engineering, he worked part-time on two acoustic consulting projects for Virginia Tech. Upon obtaining a B.S. in Mechanical Engineering in 1991, he immediately began graduate studies at Virginia Tech, and was employed as a research assistant in the Vibration and Acoustics Laboratories in the Department of Mechanical Engineering. He completed his M.S. degree in August, 1993 and accepted a one year Research Associate position at Virginia Tech, working on active control of broadband noise radiated from jet engines.

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