DYNAMO Systems Model of the Roll-response of Semisubmersibles

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(ABSTRACT)

DYNAMO (DYNAmic MOdels) is a computer program used to evaluate system dynamics models. This analysis determines the utility of the DYNAMO tool in the development of a roll-response predictor for an offshore drilling platform. The project consists of two major phases, including:

1. development of the theoretical model for the roll-response and definition of the application within the systems context,

and

2. implementation of the theoretical model using DYNAMO.

The results of the analysis demonstrate the success of DYNAMO application to the prediction of semisubmersible roll motion in regular sea states. The application successfully models the roll-response as a second-order differential equation of motion of a negative feedback system. The model is verified at semisubmersible heading angles of zero, forty-five, and ninety degrees with respect to the incident sea state.

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Thanks to James S. McMahon, Sr.. You were, and always will be, my greatest inspiration.

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CHAPTER 1: INTRODUCTION

1.1 PETROLEUM EXPLORATION AND PRODUCTION

Petroleum buried within the earth exhibits no physical or chemical property that currently permits its detection from the earth surface. Geological, geophysical, and geochemical techniques only serve as crude indicators of the oil reserves. Actual verification of oil reserves requires drilling by trial and error -- an expensive proposition. The prohibitive cost of exploration was so great that it was not until the early years of the sixties that the world demand for new oil reserves made it economically feasible to explore using semisubmersible platforms.

Much of the literature devoted to offshore development assumes a modest rate of increase in oil and gas demands until the end of the century. Despite the expected modest increase in demand, the world energy need will only be satisfied if new offshore fields are developed. This statement is supported by the reality that production from a majority of existing fields is dropping significantly.[1]

The period of offshore development until the mid-eighties was characterized by a philosophy that the most economic exploration strategy entailed the location of so-called "supergiant" oil fields. As years passed, this philosophy changed substantially, primarily due to the decline in discovery of these supergiants. The current exploration philosophy addresses smaller fields that can be developed economically and swiftly. Table 1 illustrates this shift in exploration philosophy for one of the most predominant areas of exploration -- the United Kingdom North Sea region.

In addition to the shift from few large fields to many small fields in existing regions of oil exploration, the exploration envelope is expanding to include heretofore undeveloped areas such as the Arctic and deep sea regions. What began as a small industry in the Gulf of Mexico has blossomed into an international operation in increasingly hostile environmental conditions.[1] Table 2 illustrates the expected increase in the number of drill islands in Arctic waters -- an increase from 5 in 1990 to 10 by 1995. More importantly, the table illustrates the increase in recoverable oil from 100 million cubic meters to 449 million cubic meters due to the expanded Arctic operational envelope.

The expansion of exploration efforts into deeper waters and Arctic envirionments drives a number of technological changes in the evolution of offshore platform design. These changes are now briefly addressed in an overview of current semisubmersible technology.

1.2 SEMISUBMERSIBLE TECHNOLOGY

In recent years the semisubmersbile has become the most common type of drilling rig. By the mid-eighties, the

Table 1. Oil exploration in the United Kingdom region.[1]

Size of Field (Million bbl)	Fields Under Development (1986)	Future Development (1986 + 15 years)
under 50	--	50
50 - 100	5	20
100 - 200	6	10
200 - 500	10	5
500 - 1000	3	1
over 1000	4	--
TOTAL	28	86

Table 2. Arctic oil field developments.[4]

Field	Discovery Date	Recoverable Oil (million cu.m.)	Island Number	First Production Date
Tarsiut	1979	100	1	1986
			2	1987
			3	1988
			4	1989
			5	1990
Koakoak	1981	285	1	1989
			2	1992
			3	1996
Issungnak	1981	64	1	1992
			2	1993

semisubmersible technology exceeded all other offshore rig architectures as far as number of new construction projects, with thirty semisubmersibles, two drillships, and fourteen jackup rigs under construction.[5] Several factors contribute to the recent trend towards semisubmersibles. The first is their inherent stability. As shown in Figure 1, they are column stabilized vessels which, due to large roll and pitch gyradii and small waterplane area, have longer natural periods of roll and pitch than other vessels. The small waterplane area also contributes to improved seakeeping performance, as wave-induced forces are reduced and violent motions subsequently minimized.[6]

Figure 2 illustrates the relationships that govern the stability of a floating body. Three distances are of primary concern. KB is the distance from the reference baseline to the center of vessel buoyancy. BM is the distance from the center of buoyancy to the metacenter point -- the point about which rotation occurs for small angles of roll. KG is the distance from the reference baseline to the center of vessel gravity. Each of these three distances is combined to determine the metacentric height, GM, of the vessel. As shown in Figure 3, the value of the metacentric height determines one of three static stability conditions for the vessel -- positive, neutral, and negative stability. Positive stability is the desired condition.

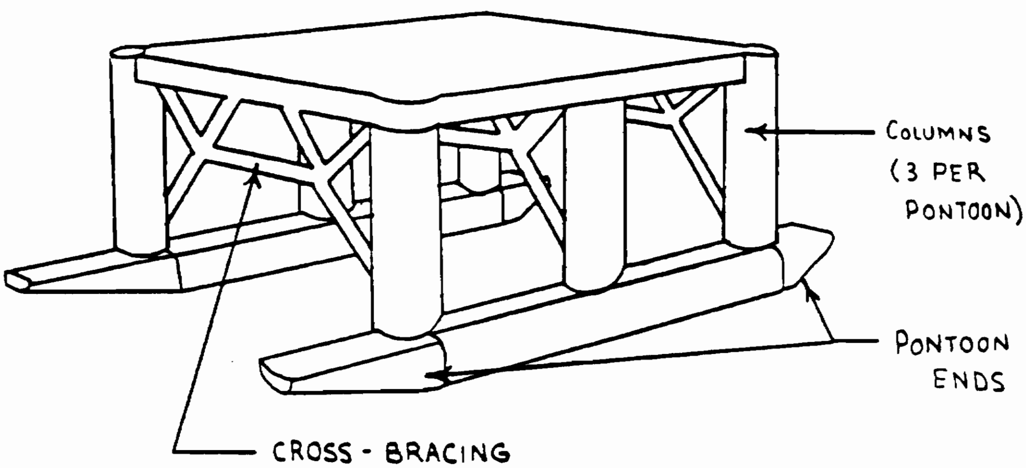


Figure 1. The column-stabilized semisubmersible design.

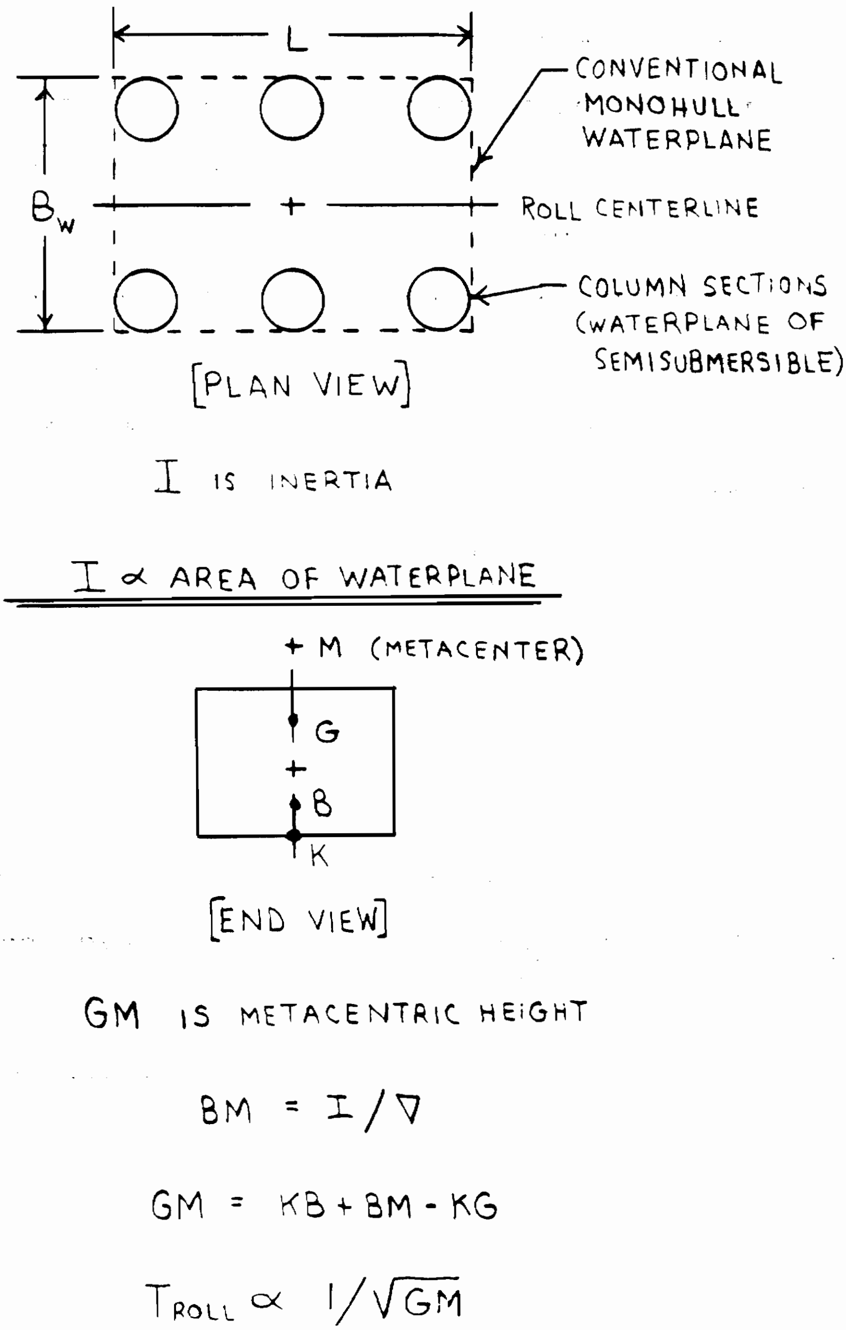


Figure 2. Determination of vessel static stability.

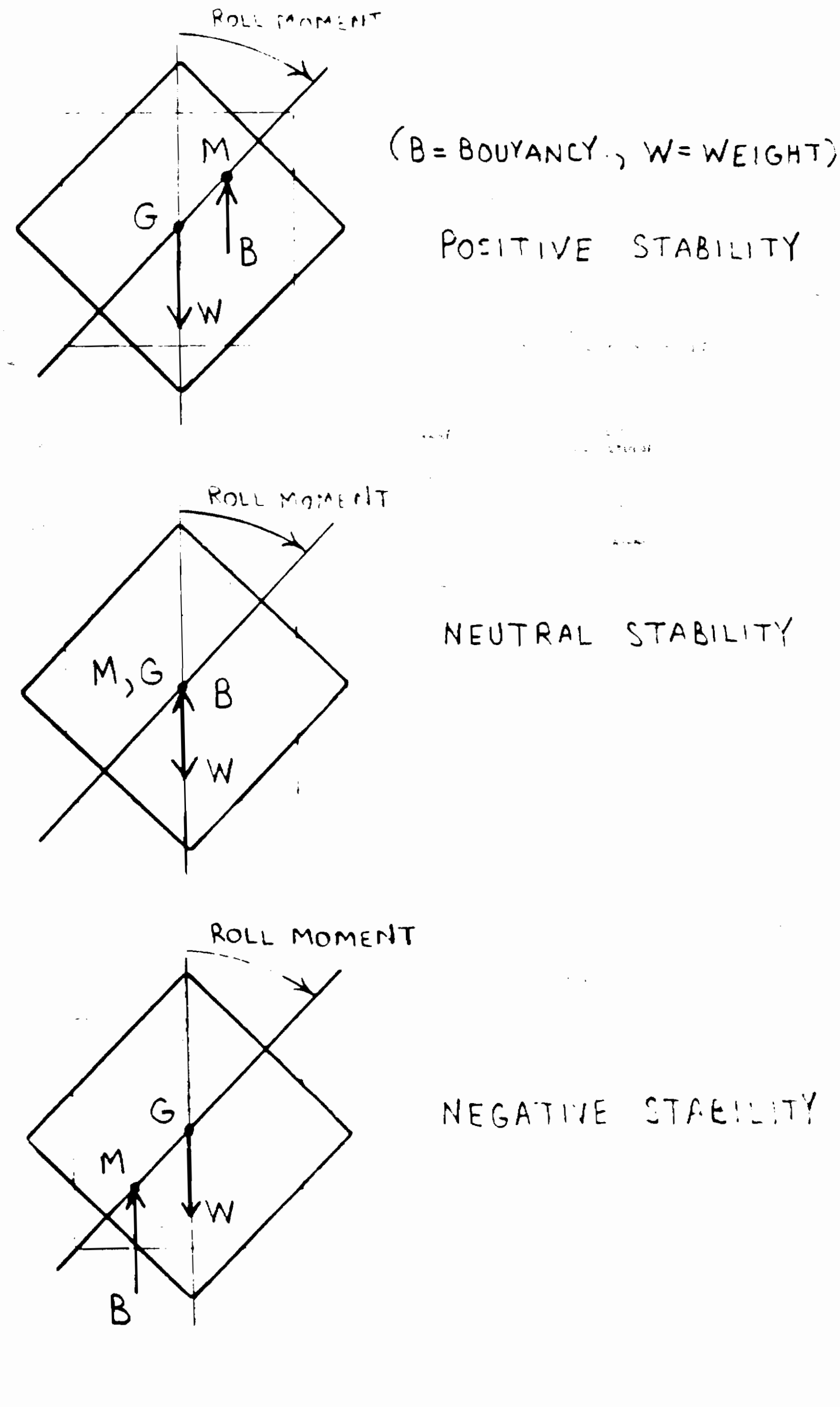


Figure 3. Stability conditions based on metacentric height.

An excess of positive stability, however, is not a desirable condition. With reference to Figure 2, the roll period of the vessel, TR, is inversely related to the square root of the metacentric height, GM. If the GM is exceptionally high, the roll period is relatively small. As described in the literature[7], the greater the GM the more stable the ship, the shorter the roll period, and the more rapid the motion. Vessels with short periods are referred to as 'stiff'. From the human factors perspective, human performance is less affected by long periods than by short, 'stiff' periods. This fact drives the desirable design GM down.

The semisubmersible attempts to achieve the optimum balance between a large GM for positive static stability and a low GM for a softer, longer roll period. With reference again to Figure 2, semisubmersibles have less waterplane area. As a result, the waterplane inertia, I, is reduced and the BM subsequently reduced. The lowered BM value yields a lower GM, thereby yielding a design better suited for hostile sea states because of its lengthened roll period.

The motion sensitivity of semisubmersibles becomes an increasingly important design issue as the exploration objectives described in Chapter 1.1 evolve. A number of critical operating motion constraints are imposed on new semisubmersible design efforts due to the characteristics of new operational environments. The technological challenge exists to meet the motion damping requirement of deep sea exploration and the stationkeeping and maneuverability requirements in Arctic regions of heavy ice flows.

1.3 PURPOSE AND OBJECTIVES OF THE STUDY

With the extensive use of semisubmersibles in motion sensitive drilling operations, vessel motion response prediction is a critical issue. This project develops a theoretical model to predict the roll-response of a six-column, twin-hull semisubmersible. The roll-response is predicted for a typical incident wave frequency at three heading angles in regular sea states.

The theoretical model is implemented by the system dynamics modeling tool referred to as DYNAMO. The model accounts for the characteristic geometry of the vessel, determines the forces on the vessel due to the given sea state, and solves the differential equation of motion for the vessel roll. Several expressions for hydrodynamic forces and geometric coefficients were drawn from the work of J. Dalzell, who explored the heave response of semisubmersibles at Stevens Institute of Technology.[8] The resulting theory permits roll-response prediction at any vessel draft, vertical center of gravity, and heading.

In addition to the primary model development goal of the study, this project achieves two secondary goals. The problem and model are defined in the systems engineering context, including within both the system life-cycle development process and the system dynamics modeling process. Finally, recommendations for future research are made based on the results of this study.

CHAPTER 2: DEFINITION WITHIN THE SYSTEMS CONTEXT

2.1 SYSTEM LIFE-CYCLE DEVELOPMENT

This section of the report defines the modeling problem within the system life-cycle development process. The framework of the system life-cycle process is shown in Figure 4. The entire process begins with a definition of need for the particular system. The need is based on a want or desire for something based on a real, or perceived, deficiency.[9]

The definition of need for the overall systems development process includes a well-defined statement of the existing deficiency, the date by which the new system is required, the magnitude of resources available for system investment, and the priority of the new capability. The defined need drives down into the rest of the entire life-cycle process.

Though this study does not generate the life-cycle process for a semisubmersible design, it is clear that the thoughts developed in Chapter 1 can be used to form the basis for a definition of need. The deficiency is due to the lack of available technology in order to meet the need for energy exploration in new environments. The deficiency extends to platform survivability in regions of deep water and ice flows.

The definition of system need is also derived from economic and ecologic deficiencies in addition to technologic deficiencies. The declining performance of existing oil

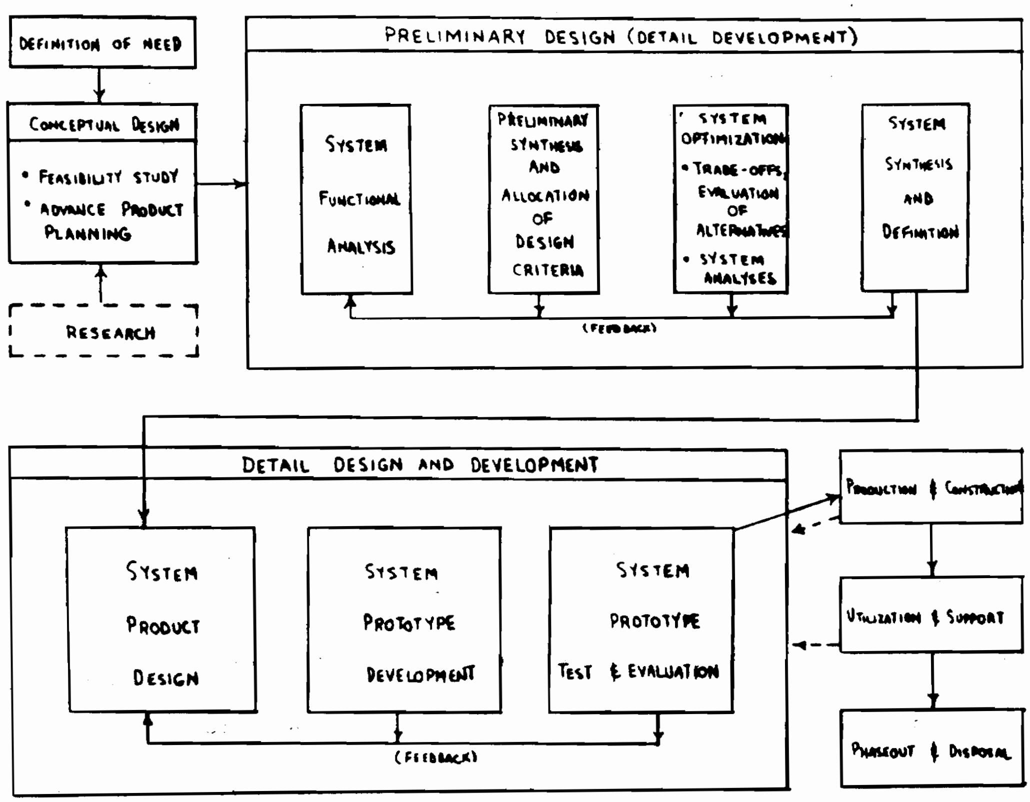


Figure 4. The system life-cycle process.

fields is described in Chapter 1, and is the economic consideration that helps define the definition of need. The literature also documents the need for drilling systems that will preserve the fragile ecologic balance in the Arctic.[10] Platform operating noise, for example, has been identified as disruptive to the natural behavior of marine wildlife. This ecologic concern drives the need for platforms with noise reducing equipment, including shrouded propellers to reduce cavitation noise and compressed air curtains around pile drivers and drills.

Clearly the deficiency of a semisubmersible design that includes each of the technologic, economic, and ecologic characteristics described herein is a feasible origin from which a system definition of need may be derived. The established definition of need would be the basis for the execution of the other system life-cycle processes, including the formulation of system operational requirements, development of the maintenance concept, and evolution of each of the conceptual, preliminary, and detail design phases shown in Figure 4.

This project is not intended to fully develop the system life-cycle of a semisubmersible system design. However, the definition of need described justifies the assumption that such a system development process is feasible. This project proposes that the application of the system modeling process described herein is a useful tool in the system optimization phase of platform preliminary design. Figure 4 contains a process described as 'system and subsystem trade-offs and evaluation of alternatives'. This process is advanced by the application of modeling tools.

The developed model of roll-response observes the systems analysis definition of a model in a number of ways. Models are described in the systems context as a tool or aid in the decision making process.[11] Such models should simplify the complexity facing the decision maker, thereby allowing the designers to consider many design alternatives. These models must also provide a means of comparing design alternatives on an equivalent basis. There are a variety of model types, one of which -- the mathematical model -- symbolically represents the principles of the situation being studied. The mathematical model represents an abstraction of reality and identifies a control variable upon which the various design alternatives are equivalently compared.

The roll-response model is a mathematical model that identifies the roll angle as the control variable by which the various platform alternatives are equivalently compared. The roll-response model also qualifies as a model in the systems context since it simplifies the complexity of roll motion analysis. The designers are afforded a means of quantifying one aspect of seaworthiness of design alternatives, considering complex factors such as damping and inertial effects.

However, the roll-response model is not strictly a pure application of the systems mathematical model. For example, the literature descirbes the mathematical model as one that incorporoeates probabilistic elements to explain the random behavior of systems.[12] Since the roll-response model is a simple application of physical laws, no elements of probability are required. If the model had been expanded in complexity to account for the multiple frequencies of irregular sea states, rather than sea spectrums of single frequencies, probability would have played an integral role in the model. The model would have then adhered more faithfully to the systems context of a mathematical model.

The literature cautions that abstract models involve many assumptions about the operational components of the system and about the nature of the operating environment.[13] This statement implies that all model results must be considered with regard to the assumptions applied during the decision making process. The roll-response model is built around a number of constraining assumptions, including:

1. roll motion is uncoupled with other platform motions,

2. the design alternative is symmetric about the transverse and longitudinal axes, and is of the

twin-pontoon, six-column variety, and

3. damping and restoring actions behave linearly, as described in Chapter 3.

As long as these constraints are considered when comparing the roll-response results of the various design alternatives, the application qualifies as a valid modeling according to the systems process definition.

2.2 SYSTEM DYNAMICS MODEL

In A Systems View of Development, by Drew and Hsieh, system dynamics is described as an evolution of the work begun by Forrester at the M.I.T. School of Industrial Management. It is described as a methodology for analyzing complex dynamic systems, and as a means of examining how system structure and decision policies affect the system in terms of mathematical equations.

The steps in the development of the system dynamics model include the formulation of the mental model in terms of a verbal description, the development of a flow diagram for this verbal description, and the generation of difference equations from the flow diagram.[14] These equations detail the system in terms of two types of variables, levels and rates. The level variables represent system states at discrete times and the rates effect changes in the level variables.[15] Rates are assumed constant during time intervals. Table 3, taken from Reference 7, further details the steps in the system dynamics approach.

The system dynamics approach described by Table 3 has been applied to natural ecological systems, engineering industrial systems, and social systems. This project examines the utility of the system dynamics model in a specific engineering application -- the prediction of roll-response in various sea states for an offshore drilling platform. The project applies the nine steps of Table 3 in modified form. Specifically, steps 2, 3, and 4 are replaced by the development of a theoretical model for the roll-response from known hydrodynamic forces.

The theoretical model that is developed yields a linear second-order differential equation of motion that describes the roll of the semisubmersible drilling platform. The equation represents a damped second-order negative feedback system dynamics model. The system structure is represented in the equation by the components of the system, including the vessel geometry and wave characteristics. The system policies, or rationales by which decisions are reached, are represented by the physical laws that govern the differential equation.

Many applications of the system dynamics approach have been successfully used to solve complex metaproblems, including

Table 3. Steps in the System Dynamics Approach.

STEP	ACTION
1.	Identify the problem.
2.	Isolate the factors that appear to interact to create the problem.
3.	Trace the cause-and-effect feedback loops that link decisions to actions.
4.	Formulate policies that describe how decisions result from information streams.
5.	Construct a simulation model.
6.	Generate the system behavior through time according to the model.
7.	Compare results against knowledge about the system and revise the model if necessary.
8.	Redesign the system within the model to find the changes which improve system behavior.
9.	Modify the real system.

Forrester's model of unchecked global growth.[16] Since metaproblems are, by definition, transdisciplinary problems, the system dynamics approach provides a means of understanding the various complex relationships involved. Though the developed equation of motion is not as transdisciplinary or as broad in scope as Forrester's World Model, it does convey the complexity of the roll model, and it does approach the simulation as a transdisciplinary analysis of the interaction between the ocean environment and the platform design.

CHAPTER 3: DEVELOPMENT OF THE ROLL THEORY

3.1 OVERVIEW

The basis of the theoretical model for the roll-response of the semisubmersible is the development of the moment terms attributable to the hydrodynamic forces acting on the submerged pontoons and columns of the design. The analysis assumes roll is independent of the other five degrees of motion, since coupled motion is difficult to accurately model and beyond the scope of the project.

The development of moment terms and the principles described by the laws of angular motion result in the differential equation of motion detailed in Table 4. The inertial function, $m(\ddot{u})$, reflects the behavior of a body as it accelerates through a fluid. The measured moment-causing force is found to be greater than the product of the vessel mass and acceleration. This behavior is due to the added, or hydrodynamic, mass of the body.[17] The actual measured force is proportional to the virtual mass of the body, which is given as the sum of the actual platform mass and the added mass. Dalzell relates the added mass of the platform to the geometry of the pontoons[18] as described by

$$C_z = W_p / H_p \qquad \text{(eq. 2)}$$

Table 4. The components of the equation of motion.

EQUATION OF MOTION

$$m(\ddot{u}) + c(\dot{u}) + k(u) = F(t) \qquad \text{(eq. 1)}$$

COMPONENT	DESCRIPTION
u	ROLL ANGLE
$m(\ddot{u})$	INERTIAL FUNCTION
$c(\dot{u})$	DAMPING FUNCTION
$k(u)$	RESTORING FUNCTION
$F(t)$	EXCITATION
t	TIME

where C_z is the added mass coefficient, W_p is the pontoon width, and Hp is the pontoon height.

The second term of equation 1, $c(\dot{u})$, represents the system damping function. This equation component varies with the roll velocity, $\dot{u}$, of the system, and accounts for the lag of the semisubmersible response behind the excitation force, F(t).

The restoring force is represented by k(u), which is a function of the vessel roll position. This force derives from the hydrodynamic forces on the columns, and from the restoring moment that results due to the difference in the centers of buoyancy and gravity of the vessel.

The literature suggests that the damping and restoring terms behave nonlinearly at large angles of motion and are described by

$$c(\dot{u}) = \sum_{j=1,3...} c_j \dot{u}^j \qquad \text{(eq. 3)}$$

and

$$k(u) = \sum_{j=1,3,...} k_j u^j \ . \qquad \text{(eq. 4)}$$

However, it is also recommended that a linear assumption of damping and restoring is considered satisfactory at small angular motions.[19] For the purpose of this analysis, linearity is assumed for the damping and restoring functions since roll angles for semisubmersibles are usually limited to twenty degrees or less.

Equation 1 represents the classic vibration problem of a mass, spring, and damper system. The equation is referred to as nonhomogeneous, since there is a forcing term that is not a function of the dependent variable. The solution of this equation is comprised of a transient and steady-state solution. The transient solution is the solution to the homogeneous form of the equation and is not calculated in this analysis since this solution only satisfies initial conditions and dies out due to system damping.[20] This study concerns itself with the steady-state response only.

Reference 8 describes the phenomena of feedback as a system response that either contributes to or counteracts system instability. When the feedback provides change in the direction of system stability, it is referred to as negative feedback.[21] Since the forces included in equation 1 act to restore the vessel to its rest position, equation 1 represents a negative feedback system. Negative feedback systems are described as goal seeking -- in this case, the goal is to restore the platform to its zero roll datum.

With a general overview of the platform motion complete, the analysis turns to the development of each of the terms of equation 1.

3.2 DEFINITION OF GEOMETRY

In the determination of the terms in the second-order differential equation of roll, a number of moment-causing forces are considered. Five dynamic forces are of concern, including the hydrodynamic force on the columns, an exciting force on the pontoons due to the wave amplitude, a pontoon damping force, the static hydrodynamic force on the pontoons and the added mass moment of inertia per pontoon. Each of these forces has been discussed by Dalzell,[11] and each results in a roll-causing moment due to its line-of-action distance from the axis of roll. A sixth force is also considered in order to account for the difference in the centers of gravity and buoyancy of the semisubmersible.

Since the roll moment attributable to each of these forces depends on the vessel orientation with respect to the wave direction, the theory development begins with a definition of geometry conventions. Figure 5 depicts the platform plan view below the waterline. Each of the columns has been identified for tracking purposes during the theory development, and each of the pontoons is visible. The X-Y axis is the fixed coordinate system representing the direction of wave propagation.

Figure 6 shows the vessel at a heading angle γ with respect to the fixed axis of the wave direction. This angle is defined as the angle between the vessel longitudinal

Vessel Particulars

Number of Columns	6	
Column Diameter	30	ft
Pontoon Length	390	ft
Distance between Pontoon Hulls, W_c	215	ft
Pontoon Height, H_P	24	ft
Pontoon Width, W_P	42	ft
Distance between Outer Columns, L_c	260	ft
Distance 1st Column CL to Front	50	ft
Drilling Draft	60	ft
Drilling Displacement	22812	Ltons

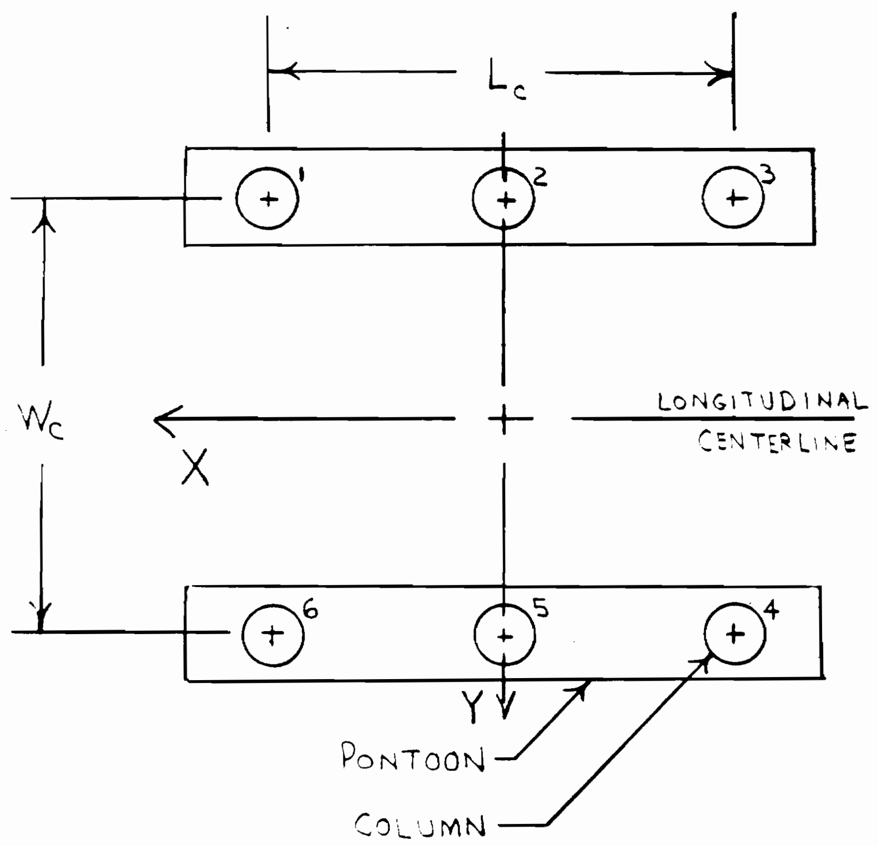
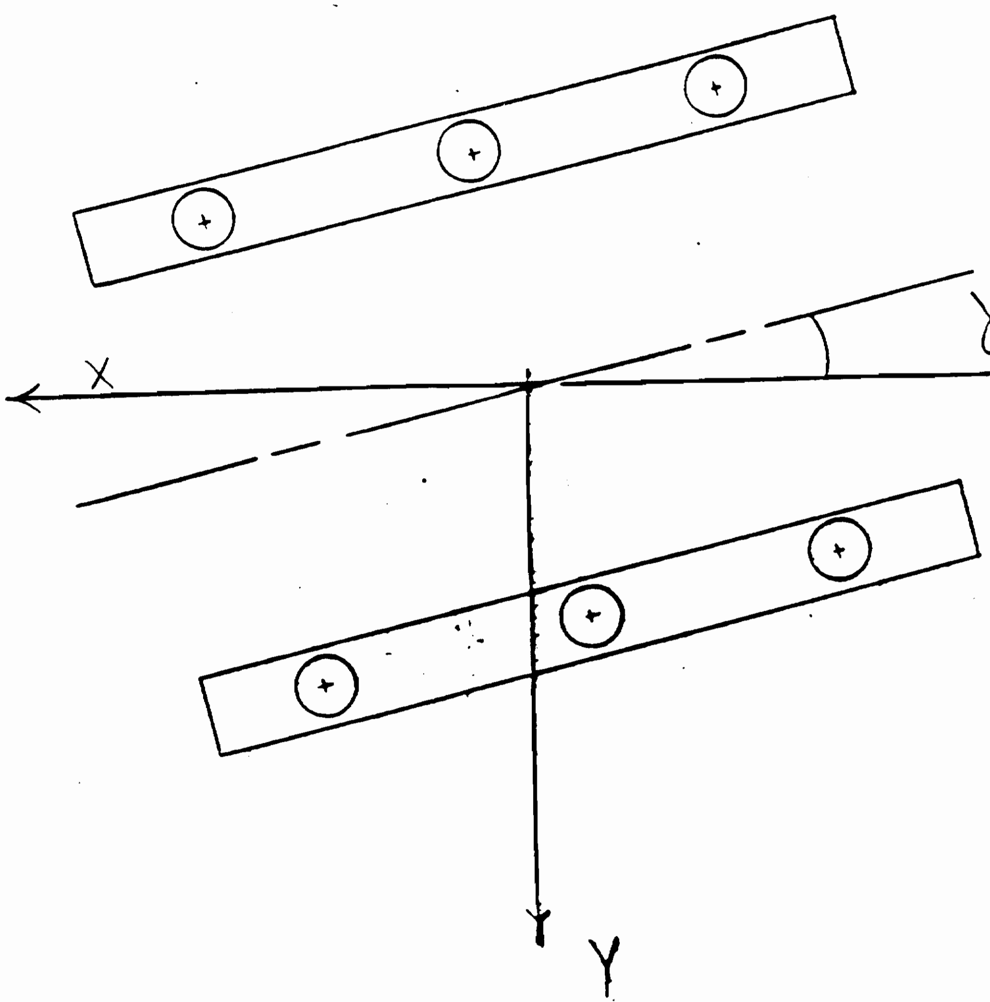


Figure 5. The platform plan view below the waterline.



γ - ANGLE BETWEEN CRAFT LONGITUDINAL CENTERLINE AND X-AXIS OF WAVE PROPAGATION.

Figure 6. Definition of the vessel heading angle.

centerline and the x-axis of wave propagation.

Figure 7 shows the column angle for the first column. The column angle is defined as the angle between the positive x-axis of wave propagation and the column location. The column angle, ϕ , is determined for each column in terms of the heading angle γ . The column angles for each of the values are given by

$$\phi_1 = 2\pi - \tan^{-1}\frac{W_c}{L_c} + \gamma , \qquad \text{(eq. 5)}$$

$$\phi_2 = \frac{3\pi}{2} + \gamma , \qquad \text{(eq. 6)}$$

$$\phi_3 = \pi + \tan^{-1}\frac{W_c}{L_c} + \gamma , \qquad \text{(eq. 7)}$$

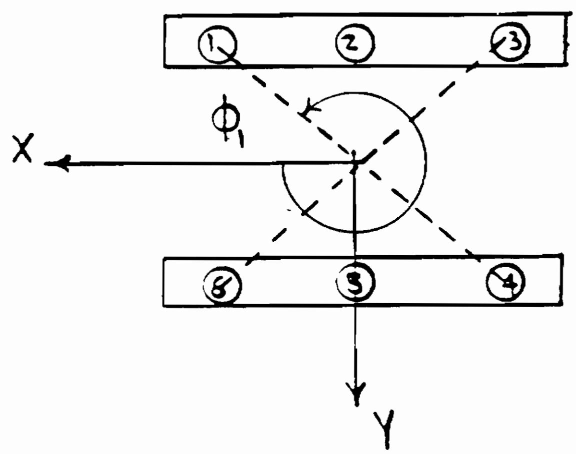
$$\phi_4 = \pi - \tan^{-1}\frac{W_c}{L_c} + \gamma , \qquad \text{(eq. 8)}$$

$$\phi_5 = \frac{\pi}{2} + \gamma , \qquad \text{(eq. 9)}$$

and

$$\phi_6 = \tan^{-1}\frac{N_c}{L_c} + \gamma . \qquad \text{(eq. 10)}$$

Each incident wave has a component that causes a roll moment on each column. This component is given by the cosine of each column angle, which yields



ϕ_i - ANGLE BETWEEN ⊕ X-AXIS OF WAVE PROPAGATION AND THE RADIUS VECTOR OF THE i-TH COLUMN.

<u>Figure 7. Definition of the column angle geometry.</u>

$$\cos\phi_1 = \cos\left[\tan^{-1}\frac{W_c}{L_c}\right]\cos\gamma + \sin\left[\tan^{-1}\frac{W_c}{L_c}\right]\sin\gamma, \qquad \text{(eq. 11)}$$

$$\cos\phi_2 = \sin\gamma, \qquad \text{(eq. 12)}$$

$$\cos\phi_3 = -\cos\left[\tan^{-1}\frac{W_c}{L_c}\right]\cos\gamma + \sin\left[\tan^{-1}\frac{W_c}{L_c}\right]\sin\gamma, \qquad \text{(eq. 13)}$$

$$\cos\phi_4 = -\cos\left[\tan^{-1}\frac{W_c}{L_c}\right]\cos\gamma - \sin\left[\tan^{-1}\frac{W_c}{L_c}\right]\sin\gamma, \qquad \text{(eq. 14)}$$

$$\cos\phi_5 = -\sin\gamma, \qquad \text{(eq. 15)}$$

and

$$\cos\phi_6 = \cos\left[\tan^{-1}\frac{W_c}{L_c}\right]\cos\gamma - \sin\left[\tan^{-1}\frac{W_c}{L_c}\right]\sin\gamma \qquad \text{(eq. 16)}$$

by application of the identity

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B\ . \qquad \text{(eq. 17)}$$

This step concludes the definition of the vessel geometry.

3.3 DETERMINATION OF THE COLUMN FORCES AND MOMENTS

Figure 8 describes the column hydrodynamic force equation found in the literature.[23] Equation 18 is used to determine Fc, which includes a first-order vertical column force and a

$$F_{c_i} = \left\{ H_c - \left[z_o + \alpha a_i \sin\theta_i - \beta a_i \cos\theta_i \right] - e^{-kH_c} \eta_o \sin[k a_i \cos(\phi_i) - \frac{\omega g}{2} k \frac{\pi D^2}{4} e^{-2kH_c} \left(\ddot{z}_o + a_i [\dot{\alpha} \sin\phi_i - \dot{\beta} \cos\phi_i] \right) \right\} \rho g \frac{\pi D^2}{4} \qquad \text{(eq. 18)}$$

TERM	DESCRIPTION
H_c	SUBMERGED COLUMN HEIGHT
z_o	HEAVE AT THE ORIGIN
α	ROLL ANGLE
a_i	RADIUS ARM TO COLUMN CENTERLINE
β	PITCH ANGLE
k	WAVE NUMBER , $2\pi/\lambda$
λ	WAVELENGTH
η_o	INCIDENT WAVE AMPLITUDE
ω	WAVE FREQUENCY
t	TIME
g	GRAVITATIONAL ACCELERATION
D	COLUMN DIAMETER
$\ddot{z}_o$	HEAVE ACCELERATION AT THE ORIGIN
ρ	FLUID DENSITY

Figure 8. The column hydrodynamic force equation.

radiation damping term. Since the analysis assumes uncoupled roll motion, all pitch terms are eliminated.

The total roll moment due to the Fc terms is given by

$$Mr_c = \left(\sum_{c=1}^{3} Fc_i - \sum_{c=4}^{6} Fc_i \right) * Wc/2 \,, \qquad \text{(eq. 19)}$$

where Wc is the span between the columns measured transversely along the Y-axis. Table 5 summarizes the total force on columns 1, 2, and 3. Table 6 summarizes the total force on columns 4, 5, and 6. Table 7 summarizes the total force on all the columns, and Table 8 determines the roll moment due to the hydrodynamic column forces.

3.4 DETERMINATION OF PONTOON FORCES AND MOMENTS

Several hydrodynamic effects occur on the pontoons. The first effect discussed is the damping force per pontoon. Figure 9 defines the pontoon geometry applied in the development of this force. The literature describes the pontoon damping force as[24]

$$F_{P/D} = \frac{\omega k^2}{\rho} e^{-2kH_h} 2\nabla p4ab(1+Cz)^2 (-\dot{\alpha} Wc/2) \,. \qquad \text{(eq. 20)}$$

In order to generate a roll moment from this force, equation 20 is multiplied by the roll moment arm, Wc/2, to yield

Table 5. The hydrodynamic force terms for columns 1, 2, & 3.

$$F_{c_1} = \left\{ H_c - z_o - \alpha a_1 \sin\left(\tan^{-1}\frac{W_s}{L_c}\right) - e^{-kH_c}\,\eta_o \sin\left[ka_1\left(\cos\left(\tan^{-1}\frac{W_s}{L_c}\right)\cos\gamma + \sin\left(\tan^{-1}\frac{W_s}{L_c}\right)\sin\gamma\right) - \omega t\right] - \frac{\omega k \pi D^2}{8g}\, e^{-2kH_c}\left(\ddot{z}_o + a_1\dot{\alpha}\sin(\tan^{-1})\right)\right\} \frac{\rho g \pi D^2}{4}$$

$$F_{c_2} = \left\{ H_c - z_o - \alpha a_2 - e^{-kH_c}\,\eta_o \sin\left[ka_2 \sin\gamma - \omega t\right] - \frac{\omega k \pi D^2}{8g}\, e^{-2kH_c}\left(\ddot{z}_o + a_2\dot{\alpha}\right)\right\} \rho g \frac{\pi D^2}{4}$$

$$F_{c_3} = \left\{ H_c - z_o - \alpha a_3 \sin(\tan^{-1}) - e^{-kH_c}\,\eta_o \sin\left[ka_3\left((-\cos(\tan^{-1}))\cos\gamma + \sin(\tan^{-1})\sin\gamma\right) - \omega t\right] - \frac{\omega k \pi D^2}{8g}\, e^{-2kH_c}\left(\ddot{z}_o + a_3\dot{\alpha}\sin(\tan^{-1})\right)\right\} \rho g \frac{\pi D^2}{4}$$

SINCE a_3 EQUALS a_1, THE FORCE SUMMATION FOR COLUMNS 1, 2, 3 IS GIVEN BY:

$$\sum_{i=1}^{3} F_c = \Big\{ 3H_c - 3z_o - 2\alpha a_1 \sin\left(\tan^{-1}\frac{W_s}{L_c}\right) - \alpha a_2 - e^{-kH_c}\,\eta_o \sin\left[ka_2\sin\gamma - \omega t\right]$$

$$- e^{-kH_c}\,\eta_o \sin\left[ka_1\left(\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma\right) - \omega t\right] - e^{-kH_c}\,\eta_o \sin\left[ka_1\left(-\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma\right) - \omega t\right]$$

$$- \frac{\omega k \pi D^2}{8g}\, e^{-2kH_c}\left(\ddot{z}_o + a_2\dot{\alpha}\right) - \frac{\omega k \pi D^2}{4g}\, e^{-2kH_c}\left(\ddot{z}_o + a_1\dot{\alpha}\sin(\tan^{-1})\right)\Big\} \rho g \frac{\pi D^2}{4}$$

NOTE: $\tan^{-1}\dfrac{W_c}{L_c}$ IS WRITTEN AS $(\tan^{-1})$

Table 6. The hydrodynamic force terms for columns 4, 5, & 6.

$$F_{c_4} = \left\{ H_c - z_o + \alpha a_4 \sin(\tan^{-1}) - e^{-kH_c} \eta_o \sin\left[k a_4 (-\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma) - \omega t \right] - \frac{\omega k \pi D^2}{8g} e^{-2kH_c} \left(\ddot{z}_o - a_4 \dot{\alpha} \sin(\tan^{-1}) \right) \right\} \rho g \frac{\pi D^2}{4}$$

$$F_{c_5} = \left\{ H_c - z_o + \alpha a_5 - e^{-kH_c} \eta_o \sin\left[k a_5 \sin\gamma - \omega t \right] - \frac{\omega k \pi D^2}{8g} e^{-2kH_c} \left(\ddot{z}_o - a_5 \dot{\alpha} \right) \right\} \rho g \frac{\pi D^2}{4}$$

$$F_{c_6} = \left\{ H_c - z_o + \alpha a_3 \sin(\tan^{-1}) - e^{-kH_c} \eta_o \sin\left[k a_6 (\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma) - \omega t \right] - \frac{\omega k \pi D^2}{8g} e^{-2kH_c} \left(\ddot{z}_o - \dot{a}_6 \dot{\alpha} \sin(\tan^{-1}) \right) \right\} \rho g \frac{\pi D^2}{4}$$

SINCE a_4 AND a_6 EQUAL a_1, AND a_5 EQUALS a_2, THE FORCE SUMMATION IS GIVEN BY:

$$\sum_{i=4}^{6} F_c = \Big\{ 3H_c - 3z_o + 2\alpha a_1 \sin\left(\tan^{-1} \frac{W_s}{L_c}\right) + \alpha a_2 + e^{-kH_c} \eta_o \left(\sin[k a_2 \sin\gamma] \cos\omega t \right)$$

$$+ e^{-kH_c} \eta_o \left(\cos[k a_2 \sin\gamma] \sin\omega t \right) - e^{-kH_c} \eta_o \sin\left[k a_1 (-\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma) - \omega t \right]$$

$$- e^{-kH_c} \eta_o \sin\left[k a_1 (\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma) - \omega t \right] - \frac{\omega k \pi D^2}{8g} e^{-2kH_c} \left(\ddot{z}_o - \dot{\alpha} a_2 \right)$$

$$- \frac{\omega k \pi D^2}{4g} e^{-2kH_c} \left(\ddot{z}_o - \dot{\alpha}_1 a_1 \sin(\tan^{-1}) \right) \Big\} \rho g \frac{\pi D^2}{4}$$

NOTE: $\tan^{-1} \frac{W_c}{L_c}$ IS WRITTEN AS $(\tan^{-1})$

Table 7. The summation of the hydrodynamic column forces.

TERMS IN THE TOTAL FORCE EQUATION $(F_{COL\,1} + F_{COL\,2} + F_{COL\,3} - F_{COL\,4} - F_{COL\,5} - F_{COL\,6})$

$$
\begin{aligned}
\Big\{ & -4\alpha a_1 \sin(\tan^{-1}) - 2\alpha a_2 - 2e^{-kH_c}\eta_o(\sin[ka_2 \sin\gamma]\cos\omega t) \\
& - e^{-kH_c}\eta_o \sin[ka_1(\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma)]\cos\omega t \\
& - e^{-kH_c}\eta_o \cos[ka_1(\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma)]\sin\omega t \\
& + e^{-kH_c}\eta_o \sin[ka_1(\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma)]\cos\omega t \\
& - e^{-kH_c}\eta_o \cos[ka_1(\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma)]\sin\omega t \\
& - e^{-kH_c}\eta_o \sin[ka_1(\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma)]\cos\omega t \\
& + e^{-kH_c}\eta_o \cos[ka_1(\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma)]\sin\omega t \\
& + e^{-kH_c}\eta_o \sin[ka_1(\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma)]\cos\omega t \\
& + e^{-kH_c}\eta_o \cos[ka_1(\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma)]\sin\omega t \\
& - \frac{\omega k}{g}\frac{\pi D^2}{4} e^{-2kH_c} a_2\dot{\alpha} - \frac{2\omega k}{g}\frac{\pi D^2}{4} e^{-2kH_c} a_1\dot{\alpha}\sin(\tan^{-1}) \Big\} \rho g \frac{\pi D^2}{4}
\end{aligned}
$$

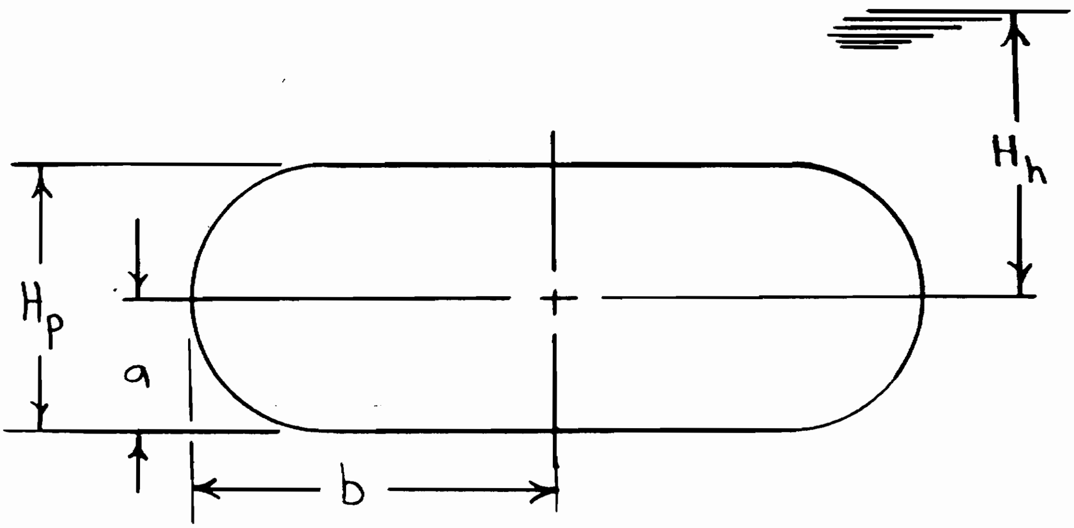
NOTE: $\tan^{-1}\frac{W_c}{L_c}$ IS EXPRESSED AS $(\tan^{-1})$

Table 8. The roll moment due to the column forces.

TERMS IN THE COLUMN ROLL MOMENT EQUATION

$$
\begin{aligned}
\Big\{ & -4\alpha a_1 \sin(\tan^{-1}) - 2\alpha a_2 \\
& -2e^{-kH_c}\eta_0 \left(\sin[ka_2 \sin\gamma]\cos\omega t\right) \\
& +2e^{-kH_c}\eta_0 \sin\left[ka_1\left(\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma\right)\right]\cos\omega t \\
& -\frac{\omega}{g} k\pi D^2 \frac{1}{4} e^{-2kH_c} a_2 \dot{\alpha} \\
& -\frac{\omega}{g} k\pi D^2 \frac{1}{2} e^{-2kH_c} a_1 \dot{\alpha} \sin(\tan^{-1}) \Big\} \rho g \frac{\pi D^2}{8} W_c
\end{aligned}
$$

NOTE: $\tan^{-1}\frac{W_c}{L_c}$ IS EXPRESSED AS $(\tan^{-1})$



H_h - DEPTH TO PONTOON CENTERLINE

a - HALF-HEIGHT OF THE PONTOON

b - HALF-WIDTH OF THE PONTOON

H_p - PONTOON HEIGHT

∇_p - VOLUME PER PONTOON

Figure 9. Pontoon geometry used to determine damping force.

$$M_{P/D} = -\frac{\omega k^2}{\rho} e^{-2kH_h} \nabla p (4ab)(1+Cz)^2((Wc)^2/2)\dot{\alpha} \;. \qquad \text{(eq. 21)}$$

The second hydrodynamic effect on the pontoons is the sectional hydrodynamic force. This effect represents the wave-induced pontoon exciting force, which is a function of the pontoon length. The literature expresses the sectional force at each column as[25]

$$F_{\eta_i} = \rho \nabla p (1+Cz)\omega^2 \eta_o e^{-kH_h} \sin(kx_i - \omega t) \;. \qquad \text{(eq. 22)}$$

Each column x value is given by

$$x_1 = a_1 \cos(\tan^{-1}\frac{W_c}{L_c})\cos\gamma \;, \qquad \text{(eq. 23)}$$

$$x_2 = a_2 \sin\gamma \;, \qquad \text{(eq. 24)}$$

$$x_3 = -a_1 \cos(\tan^{-1}\frac{W_c}{L_c})\cos\gamma \;, \qquad \text{(eq. 25)}$$

$$x_4 = -a_1 \cos(\tan^{-1}\frac{W_c}{L_c})\cos\gamma \;, \qquad \text{(eq. 26)}$$

$$x_5 = -a_2 \sin\gamma \;, \qquad \text{(eq. 27)}$$

and

$$x_6 = a_1 \cos(\tan^{-1}\frac{W_c}{L_c})\cos\gamma \;. \qquad \text{(eq. 28)}$$

Since a_2 equals a_5 and a_1 equals a_3, a_4, and a_6, the application of

$$\sin(kx_i - \omega t) = \sin kx_i \sin\omega t - \cos kx_i \cos\omega t \qquad \text{(eq. 29)}$$

permits the determination of the sine terms of equation 22 for each column, as tabulated in Table 9.

Equation 22 is rewritten in order to determine the total wave-induced pontoon exciting force as

$$\sum_{i=1}^{6} F_{\eta_i} = \rho \nabla p (1+Cz)\omega^2 \eta_0 e^{-kH_h} \sum_{i=1}^{6} \sin(kx_i - \omega t) \; . \qquad \text{(eq. 30)}$$

The summation of all the sine terms described in Table 9 permits the simplification of equation 30 as

$$F_{\eta} = \rho \nabla p (1+Cz)\omega^2 \eta_0 e^{-kH_h}(2\sin[ka_2 \sin\gamma]\cos\omega t) \; . \qquad \text{(eq. 31)}$$

Equation 31 represents the pontoon wave-induced excitation force.

The roll moment due to the pontoon exciting force of equation 31 is given by

$$Mr_{p/E} = \frac{Wc}{2}\rho \nabla p (1+Cz)\omega^2 \eta_0 e^{-kH_h}(2\sin[ka_2 \sin\gamma]\cos\omega t) \; . \qquad \text{(eq. 32)}$$

As the vessel rolls, the pontoons develop an added mass

Table 9. Tabulation of the sine terms of equation 22.

COLUMN	EQUATION 22 SINE TERM
1	$\sin[ka_1 \cos(\tan^{-1})\cos\gamma]\cos\omega t - \cos[ka_1 \cos(\tan^{-1})\cos\gamma]\sin\omega t$
2	$\sin[ka_2 \sin\gamma]\cos\omega t - \cos[ka_2 \sin\gamma]\sin\omega t$
3	$-\sin[ka_1 \cos(\tan^{-1})\cos\gamma]\cos\omega t - \cos[ka_1 \cos(\tan^{-1})\cos\gamma]\sin\omega t$
4	$-\sin[ka_1 \cos(\tan^{-1})\cos\gamma]\cos\omega t - \cos[ka_1 \cos(\tan^{-1})\cos\gamma]\sin\omega t$
5	$-\sin[ka_2 \sin\gamma]\cos\omega t - \cos[ka_2 \sin\gamma]\sin\omega t$
6	$\sin[ka_1 \cos(\tan^{-1})\cos\gamma]\cos\omega t - \cos[ka_1 \cos(\tan^{-1})\cos\gamma]\sin\omega t$
TOTAL (1+2+3-4-5-6)	$2\sin[ka_2 \sin\gamma]\cos\omega t$

NOTE: $\tan^{-1}\frac{W_c}{L_c}$ EXPRESSED AS $(\tan^{-1})$

effect. This added mass term is a function of the pontoon geometry and volumetric displacement, and is given by

$$m_{A/p} = (Cz)\nabla p\,\rho . \qquad \text{(eq. 33)}$$

Figure 10 displays the mechanics involved in the calculation of the added mass force. As shown in this figure, the roll acceleration, $-\ddot{\alpha}$, is converted to a local vertical acceleration, $\ddot{z}_o$, by the equation

$$\ddot{z}_o = -\ddot{\alpha}(Wc)/2 \quad . \qquad \text{(eq. 34)}$$

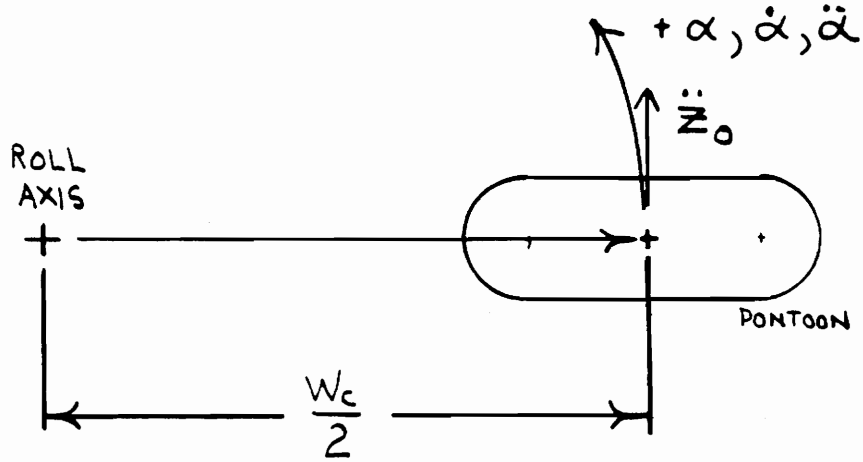
Since force is equal to the product of mass and acceleration, the added mass force is given by

$$F_{M_{A/p}} = -(Cz)\nabla p\rho\ddot{\alpha}(Wc)/2 \ . \qquad \text{(eq. 35)}$$

The multiplication of $F_{M_{A/p}}$ by the roll moment arm Wc/2, and by a factor of two to account for added mass at both pontoons, yields the added mass pontoon moment given by

$$Mp_{M_{A/p}} = -(Cz)\nabla p\rho\ddot{\alpha}(Wc)^2/2 \ . \qquad \text{(eq. 36)}$$

The final force attributable to the pontoons is the pontoon static buoyant force. However, since the pontoons are



ROLL DECELERATES $(\alpha\uparrow, \dot{\alpha}\downarrow, \ddot{\alpha}\downarrow)$

$$\ddot{Z}_o = (\text{ANGULAR ACC.})(\text{ARM})$$

$$\text{ANGULAR ACC.} = -\ddot{\alpha} \quad (\text{VESSEL ROLL SLOWS})$$

$$\therefore \ddot{Z}_o = -\ddot{\alpha}\,\frac{W_c}{2}$$

Figure 10. Mechanics of local acceleration at the pontoon.

equally buoyant and are at opposite moment arms with respect to the roll centerline, no net roll moment results. Therefore the summation of equations 21, 32, and 36 yields the total roll moment due to the pontoon forces as

$$M_{r_p} = -(Wc)^2 \dot{\alpha} \frac{\omega k^2}{\rho} e^{-2kH_h} (\nabla p 4ab(1+(Cz))^2)$$
$$+ \frac{(Wc)}{2} \rho \nabla p (1+(Cz)) \omega^2 r_o e^{-kH_h} (2\sin[ka_2 \sin\gamma] \cos\omega t)$$
$$- \frac{(Wc)^2}{2} (Cz) \nabla p \rho \ddot{\alpha}. \qquad \text{(eq. 37)}$$

3.5 FORMULATION OF THE EQUATION OF MOTION

The total semisubmersible roll moment is given by

$$Mr,\ total = Mr_p + Mr_c \qquad \text{(eq. 38)}$$

where Mr_p is determined using equation 37 and Mr_c is determined using equation 19 and the forces detailed in Tables 5, 6, and 7. This total roll moment equates to the vessel roll moment of inertia, I, and the angular roll acceleration, $\ddot{\alpha}$ by Newton's second law of motion[26] which yields

$$\sum Mr,\ total = I\ddot{\alpha} \quad . \qquad \text{(eq. 39)}$$

The roll moment of inertia is a measure of the vessel radial mass distribution about the roll axis through the center of gravity. This term is estimated using empirically determined relationships between I, the semisubmersible mass and the spacing between the pontoons. The relationship is expressed in the literature as[27]

$$I = .60ms^2 \qquad \text{(eq. 40)}$$

where s is the pontoon transverse spacing and m is the mass of the vessel. For the semisubmersible of this study, the pontoon spacing is Wc and equation 40 is rewritten as

$$I = .60m(Wc)^2 \ . \qquad \text{(eq. 41)}$$

The substitution of I and each of the column and pontoon moment forces into equation 39 now permits the development of the differential equation of motion.

Table 10 tabulates each of the column roll moment terms from Table 8, each of the pontoon roll moment terms from equation 37, and the inertial term from equation 41. Table 10 classifies each term as dependent on the roll angle, α, the roll velocity, $\dot{\alpha}$, the roll acceleration, $\ddot{\alpha}$, or independent of the three terms. Each term is labelled for reference. Each labelled term is gathered according to its function of roll to yield the form of equation 39 given by

$$(.60m(Wc)^2 + J)\ddot{\alpha} + (F+G+H)\dot{\alpha} + (A+B)\alpha = (E+I-C-D) \ . \qquad \text{(eq. 42)}$$

Table 10. Tabulation of terms in the equation of motion.

EQUATION 42 TERM	EQUIVALENT EXPRESSION
A	$W_c \rho g \pi \frac{D^2}{2} a_1 \sin(\tan^{-1} \frac{W_c}{L_c})$
B	$W_c \rho g \pi \frac{D^2}{4} a_2$
C	$W_c \rho g \pi \frac{D^2}{4} e^{-kH_c} \eta_o (\sin[ka_2 \sin\gamma] \cos\omega t)$
D	$W_c \rho g \pi \frac{D^2}{4} e^{-kH_c} \eta_o (\sin[ka_1 ((\cos(\tan^{-1})\cos\gamma + \sin(\tan^{-1})\sin\gamma)] \cos\omega t$
E	$W_c \rho g \pi \frac{D^2}{4} e^{-kH_c} \eta_o (\sin[ka_1 (\cos(\tan^{-1})\cos\gamma - \sin(\tan^{-1})\sin\gamma)] \cos\omega t$
F	$W_c \rho \omega \pi^2 \frac{D^4}{32} k\, e^{-2kH_c} a_2$
G	$W_c \rho \omega \pi^2 \frac{D^4}{16} k\, e^{-2kH_c} a_1 \sin(\tan^{-1})$
H	$W_c^2 \frac{\omega k^2}{2\rho} e^{-2kH_h} (\nabla_p 4ab (1+C_z)^2)$
I	$W_c \rho \nabla_p (1+C_z) \omega^2 \eta_o e^{-kH_h} (\sin[ka_2 \sin\gamma]) \cos\omega t$
J	$\frac{W_c^2}{2} C_z \nabla_p \rho$

Equation 42 is the preliminary differential equation of motion.

3.6 EQUATION ADJUSTMENT DUE TO STATIC LOADS

Figure 11 details the geometry of the upsetting moment attributable to the difference between the center of buoyancy, KB, and the center of gravity, KG. The literature describes the upsetting moment as[28]

$$Mu = \Delta(KG - KB)\sin\alpha \qquad \text{(eq. 43)}$$

where Δ is the displacement of the semisubmersible in pounds. For small angles of roll, equation 43 is rewritten as

$$Mu = \Delta(KG - KB)\alpha \qquad \text{(eq. 44)}$$

which indicates that the restoring term of equation 42 is reduced by this upsetting moment since it also is a function of the roll angle α.

Equation 45 incorporates this upsetting moment in equation 42 to yield the final form of the differential equation of motion,

$$(.6m(Wc)^2 + J)\ddot{\alpha} + (F+G+H)\dot{\alpha} + (A+B-Mu)\alpha = (E+I-C-D). \qquad \text{(eq. 45)}$$

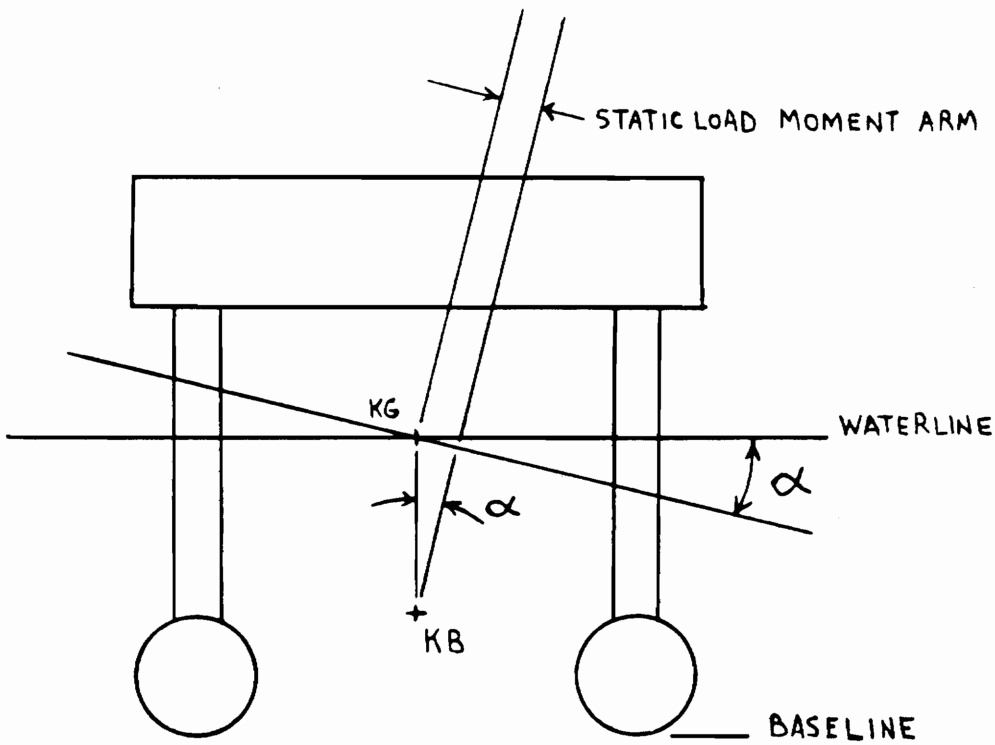


Figure 11. Geometry of the static load upsetting moment.

Table 11 equates the mass, damping, and restoring coefficients of equation 1 with the coefficients of equation 45.

Table 11. Comparison of the equation 45 and equation 1 terms.

EQUATION 1 TERM	EQUATION 45 TERM
MASS, m	$.6 m W_c^2 + J$
ACCELERATION, $\ddot{u}$	ROLL ACCELERATION, $\ddot{\alpha}$
DAMPING, c	$F + G + H$
VELOCITY, $\dot{u}$	ROLL VELOCITY, $\dot{\alpha}$
RESTORING, k	$A + B - M_u$
DISTANCE, u	ROLL ANGLE, α
FORCING TERM, $F(t)$	$E + I - C - D$

CHAPTER 4: DYNAMO THEORY IMPLEMENTATION

4.1 OVERVIEW

The system dynamics approach investigates the feedback characteristics of dynamic systems to show how structure, policies, decisions, and delays interact to influence system growth and stability.[29] System dynamics is based on information feedback theory. Specifically, how is information flow used to effect system control, and how can mathematical simulation models be applied to predict system steady-state and stability characteristics?

The complex relationships of the system are usually difficult to judge over time. Mathematical simulation helps improve the judgement of system response. The equation of motion developed in Chapter 3 is used as the basis for a DYNAMO model simulation, in order to better judge the response of the semisubmersible design in roll motions.

DYNAMO (DYnamic MOdels) is a computer program that is used to model the behavior of level variables in complex system dynamics environments. DYNAMO is designed to be an aid in simulating level variables that depend on aggregate flows, rather than discrete events.[30] Rate variables describe how the level variables change between discrete instances in time.

DYNAMO implements a postscript convention to distinguish between variables at time 't', 't-1', and 't+1'. This postscript notation permits the translation of dynamic mathematical differential equations into the DYNAMO difference equations. Figure 12, taken from Reference 8, details the notation convention of the DYNAMO tool. The roll-response study treats the angle of roll as the primary level variable of concern.

4.2 DERIVATION OF THE DYNAMO EQUATION

The roll angle, α, of equation 45 is the level variable, L, in question. The roll angular velocity, $\dot{\alpha}$, and the roll angular acceleration, $\ddot{\alpha}$, may be expressed in derivative form as

$$\dot{\alpha} = \frac{dL}{dt} \qquad \text{(eq. 46)}$$

and

$$\ddot{\alpha} = \frac{d^2L}{dt^2} \qquad \text{(eq. 47)}$$

The substitution of equations 46 and 47 into equation 45 yields

$$(.6m(Wc)^2 + J)\frac{d^2L}{dt^2} + (F+G+H)\frac{dL}{dt} + (A+B-Mu)L = (E+I-C-D). \qquad \text{(eq. 48)}$$

The right side of equation 48 is the system forcing function,

SUMMARY OF DYNAMO POSTSCRIPT CONVENTION

	TYPE OF VARIABLE						
	Dependent (Left-Hand Side)	Independent (Right-Hand Side)					
Equation Type		Level	Rate	Auxiliary	Constant	Initial	Table Name
L: Level	.K	.J	.JK	.J	none	none	n.p.
R: Rate	.KL	.K	.JK	.K	none	none	n.p.
A: Auxiliary	.K	.K	.JK	.K	none	none	none
C: Constant	none	n.p.*	n.p.	n.p.	n.p.	n.p.	n.p.
N: Initial	none	none	none	none	none	none	n.p.
T: Table Name	none	n.p.	n.p.	n.p.	n.p.	n.p.	n.p.

* n.p. = not permitted

K OCCURS AT t

L OCCURS AT t+1

J OCCURS AT t-1

Figure 12. Summary of DYNAMO postscript notation.

which is a function of cos ωt and is expressable as $F_o \cos \omega t$, where F_o is expressable using the terms in Table 11.

The integration of equation 48 with respect to time results in

$$(.6m(Wc)^2 + J)\frac{\partial L}{dt} + (F+G+H)L + (A+B-Mu)\frac{L^2}{2} = \frac{F_o}{\omega}\sin \omega t, \quad \text{(eq. 49)}$$

with the sine term the result of the integrated cosine forcing function. Equation 49 is further simplified by the substitution of

$$\frac{dL}{dt} = \frac{L_t - L_{t-1}}{dt}, \quad \text{(eq. 50)}$$

which is also expressable as

$$\frac{dL}{dt} = \frac{L.K - L.J}{dT}, \quad \text{(eq. 51)}$$

and where K is the DYNAMO notation for the present time and J is the DYNAMO notation for the previous time, t-1.

Equation 49 now appears as

$$(.6m(Wc)^2 + J)\frac{(L.K-L.J)}{\partial T} + (F+G+H)L.K + (A+B-Mu)\frac{(L.K)^2}{2}$$

$$= \frac{F_o}{\omega}\sin \omega t. \quad \text{(eq. 52)}$$

Rearrangement of terms in equation 52 yields

$$L.K = L.J + dT\frac{[\frac{F_o}{\omega}\sin\omega t - ((F+G+H)L.K + \frac{L.K^2}{2}(A+B-Mu))]}{(.6m(Wc)^2 + J)} \quad \text{(eq. 53)}$$

which is a form that can be compiled and executed by DYNAMO. It is noted that F in equation 53 equals (E+I-C-D) divided by $\cos \omega t$.

The form of equation 53 is described in the literature as that describing a damped, second-order negative feedback system.[31] The response of this type of system is described as one tending towards a return to equalibrium, rather than sustained oscillation. Since the feedback is negative the system does not proceed toward instability.

In the physical sense, this definition aptly describes the roll-response of drilling platforms. The excitation force of the sea causes a roll angle, roll velocity, and roll acceleration. The damping and restoring characteristics of the platform system cause the roll to slow, stop, and reverse until the vessel ultimately reaches the zero-roll rest state. With sustained input force, the roll motion is sinusoidal in response to the forcing function.

If the system response is modeled correctly by the DYNAMO model, several aspects of the roll-response should be evident. First, at heading angles of zero degrees the roll-response should be zero since the platform is oriented longitudinally to the oncoming waves. Due to this orientation, no roll moment arm exists, and no roll should occur.

Secondly, the roll-response should increase in magnitude, for a given wave height and frequency, as the platform heading angle changes from zero to ninety degrees. At ninety degrees the roll amplitude should be a maximum, since the platform is subject to beam seas, and the roll moment arm is therefore greatest.

Thirdly, the roll-response curve should lag behind the forcing function curve. This time lag is expected due to the damping effects of the design.

Each of these three response characteristics are expected as indicators that the DYNAMO roll-response model is functioning reasonably. The DYNAMO model of equation 53 is shown in Figure 13.

4.3 DESCRIPTION OF THE MODEL RUNS

Table 12 details the matrix of model runs. Three runs were made, representing three different semisubmersible heading angles. The frequency of encounter used represents a wave period of 10.4 seconds, which equates to a frequency of .602 radians per second. These values represent a typical wave period and frequency encountered in the open ocean.

The model is executed at three different semisubmersible heading angles. Heading angles of zero, forty-five, and ninety degrees are assumed, representing the expected no roll,

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NOTE MODEL OF SEMISUBMERSIBLE ROLL-RESPONSE
NOTE ***************************************************************
NOTE THE FOLLOWING EQUATIONS DESCRIBE THE THEORETICAL ROLL FORCES FOR
NOTE A SEMISUBMERSIBLE DRILLING PLATFORM.  THE EQUATIONS HAVE BEEN
NOTE DERIVED FOR DYNAMO3 IMPLEMENTATION IN SECTIONS ONE AND TWO OF THE
NOTE SUBMITTED PROJECT.  THE TERMINOLOGY IS DEFINED AS FOLLOWS:
NOTE    L    : VESSEL ROLL RESPONSE, RADIANS
NOTE    DT   : TIME DIFFERENTIAL, SECONDS
NOTE    RI   : TOTAL ROLL FORCING FUNCTION
NOTE    RO   : COMBINED ROLL DAMPING AND RESTORING TERMS
NOTE    N1   : INITIAL ROLL
NOTE    MU   : ROLL UPSETTING MOMENT DUE TO KB AND KG DIFFERENCE, FT-LB
NOTE    KG   : VESSEL VERTICAL CENTER OF GRAVITY, FROM BASELINE, FT
NOTE    KB   : VESSEL VERTICAL CENTER OF BUOYANCY, FORM BASELINE, FT
NOTE    MASS : VESSEL MASS, DRY+BALLAST, LB MASS
NOTE    WC   : TRANSVERSE DISTANCE BETWEEN COLUMN CENTERLINES, FT
NOTE    LC   : LONGITUDINAL DISTANCE BETWEEN FORE AND AFT COLUMNS, FT
NOTE    RHO  : STANDARD DENSITY OF SALT WATER AT 59 DEG F, LB-SEC^2/FT^4
NOTE    OMG  : FREQUENCY OF THE SEA STATE, RAD/SEC
NOTE    LAM  : WAVELENGTH, FT
NOTE    GRV  : GRAVITATIONAL ACCELERATION, FT/SEC^2
NOTE    PER  : WAVE PERIOD, SEC
NOTE    PI   : RATIO OF CIRCLE CIRCUMFERENCE TO CIRCLE DIAMETER, RAD
NOTE    DIA  : DIAMETER OF THE COLUMNS OF THE SEMISUBMERSIBLE, FT
NOTE    KK   : WAVE NUMBER, RAD/FT
NOTE    A2   : DISTANCE BETWEEN LONGITUDINAL CL AND MIDDLE COLUMNS, FT
NOTE    A1   : DISTANCE BETWEEN VESSEL CL AND EXTREME COLUMNS, FT
NOTE    HC   : SUBMERGEDCOLUMN HEIGHT, FT
NOTE    HP   : PONTOON HEIGHT, FT
NOTE    DFT  : VESSEL DRAFT, FT
NOTE    TAN  : ATAN OF WC/LC RATIO, RAD
NOTE    HH   : DISTANCE TO PONTOON VERT CENTER BELOW THE WATERLINE, FT
NOTE    VP   : VOLUMETRIC DISPLACEMENT PER PONTOON, FT^3
NOTE    AP   : PONTOON HALF-HEIGHT, FT
NOTE    BP   : PONTOON HALF-WIDTH, FT
NOTE    WP   : PONTOON WIDTH, FT
NOTE    CZ   : ADDED MASS COEFFICIENT FOR PONTOON CROSS-SECTION, RAD
NOTE    NO   : INCIDENT WAVE AMPLITUDE, FT
NOTE    ALPH : HEADING ANGLE WITH RESPECT TO SEA STATE, RAD
NOTE    AR   : AMPLITUDE RESPONSE RATIO, RATIO OF ROLL TO WAVE HEIGHT
NOTE ***************************************************************
NOTE *                     SYSTEM EQUATIONS                        *
NOTE ***************************************************************
L L.K=L.J+(DT)(RI.JK-RO.JK)
N L=LN
C LN=0.000000
R RO.KL=(L.K*((F.K+G.K+H.K)+(L.K/2)*(A.K+B.K-MU)))/((.6*MASS*WC*WC)+J.K)
N MU=MASS*(KG-KB)
C KG=54.70
C KB=18.42
C MASS=1586922
C WC=215
C LC=260
A F.K=(WC*RHO*OMG*PI*PI*DIA*DIA*DIA*DIA*KK)*A2*(EXP(-2*KK*HC))/32
C RHO=1.9905
C OMG=.602
N LAM=(2*PI*GRV)/(OMG*OMG)
C GRV=32.174
```

Figure 13. The DYNAMO roll-response model.

```
N PER=SQRT(2*PI*LAM/GRV)
C PI=3.14159
C DIA=30
N KK=OMG*OMG/GRV
N A2=HC/2
N HC=DFT-HP
C HP=24
C DFT=50
A G.K=(HC*RHO*OMG*PI*PI*DIA*DIA*DIA*DIA*KK)*A1*(EXP(-2*KK*HC))*
X (SIN(TAN))/16
N A1=SQRT((HC*HC+LC*LC)/4)
C TAN=.69094
A H.K=((HC*HC*OMG*KK*KK*EXP(-2*KK*HH))/(2*RHO))*
X (VP*4*AP*BP*(1+CZ)*(1+CZ))
N HH=HC+(HP/2)
C VP=322865
N AP=HP/2
N BP=NP/2
C HP=42
N CZ=NP/HP
A A.K=HC*RHO*GRV*PI*DIA*DIA*.5*A1*SIN(TAN)
A B.K=HC*RHO*GRV*PI*DIA*DIA*.25*A2
A J.K=HC*HC*CZ*VP*RHO/2
R RI.KL=(E.K+I.K-C.K-D.K)*(SIN(OMG*TIME.K))/(OMG*(.6*MASS*HC*HC+J.K))
A E.K=HC*RHO*GRV*PI*.25*DIA*DIA*HO*EXP(-KK*HC)*
X SIN(KK*A1*(COS(TAN)*COS(ALPH)-SIN(TAN)*SIN(ALPH)))
C HO=25
C ALPH=1.5708
A I.K=HC*RHO*VP*(1+CZ)*OMG*OMG*HO*EXP(-KK*HH)*SIN(KK*A2*SIN(ALPH))
A C.K=HC*RHO*GRV*PI*DIA*DIA*.25*EXP(-KK*HC)*HO*SIN(KK*A2*SIN(ALPH))
A D.K=HC*RHO*GRV*PI*.25*DIA*DIA*HO*EXP(-KK*HC)*
X SIN(KK*A1*(COS(TAN)*COS(ALPH)+SIN(TAN)*SIN(ALPH)))
A AR.K=L.K/(PI*HO/LAM)
NOTE ******************************************************************
SPEC DT=.1/LENGTH=10/PLTPER=.1
PLOT AR=R/RI=F
RUN
```

Figure 13. The DYNAMO roll-response model. (con't.)

Table 12. Matrix of DYNAMO model runs.

RUN	OPERATING DRAFT (ft)	WAVE AMPLITUDE (ft)	FREQUENCY OF ENCOUNTER (rad/sec)	HEADING ANGLE (deg)
1	50	25	.6021	0 [FOLLOWING SEAS]
2	50	25	.6021	45
3	50	25	.6021	90 [BEAM SEAS]

typical roll, and maximum roll orientations.

An operational draft of sixty feet is used, and a maximum wave height of 25 feet is assumed. The model test plan defined by Table 12 is not presented as a typical, thorough analysis plan that would be executed during the Preliminary Design system optimization phase shown in Figure 4. Rather, Table 12 simply illustrates a rational approach to initial validation of the DYNAMO roll-response model. This approach is used to verify or dispel the expected model performance postulated in the previous section.

The output of the model described in Figure 13 is a plot of the input forcing function, RI, and the amplitude response ratio, AR. The input forcing function is a rate of change in roll angle, which results in units of radians per second. The amplitude response ratio is given in units of radians per radian by the expression

$$AR = \frac{\alpha}{\pi \frac{NO}{LAM}} . \qquad \text{(eq. 54)}$$

where α is the roll angle, NO is the maximum wave amplitude, and LAM is the wavelength. The denominator of equation 54 is a common means of expressing vessel motion in terms of the incident wave profile and is known as the wave slope.

The units of RI and AR do not permit a direct comparison of the magnitude of each of these terms. However, the respective behavior of each as a function of time is comparable and can be used to address the expected model performance. More specifically, as the forcing function, RI, varies over time, the motion amplitude amplitude response ratio may be determined.

CHAPTER 5: SUMMARY

5.1 RESULTS OF THE DYNAMO ROLL MODEL

The results are tabulated in Table 13, and the plots of the model output are included in Figures 14, 15 and 16. Figure 14 includes the output plot for the heading of zero degrees. Figure 15 includes the output for the forty-five degree heading angle. Figure 16 includes the output for the ninety degree heading angle.

The results detailed in Table 13 and shown in Figures 14, 15, and 16 support the model performance expectations described on pages 54 and 55 . The roll-response is zero degrees at heading angles of zero. As expected, the lack of a roll moment arm results in no roll exciting moment.

Also as expected, the roll magnitude increases as the semisubmersible heading angle is modified from zero to ninety degrees. At ninety degrees, the roll moment arm is greatest which is demonstrated by the fact that the largest angle of roll results at this heading.

The third expected performance result is illustrated by Figure 15 and Figure 16. Evident in these two output plots is the lag in response of the vessel roll amplitude response ratio compared to the occurrence in time of the excitation force. For example, in Figure 15 the maximum input force occurs at approximately 2.8 seconds after initial wave train

Table 13. Tabulation of the model test run results.

RUN	HEADING ANGLE (deg)	MAXIMUM APLITUDE RESPONSE RATIO (rad)	MAXIMUM ROLL ANGLE (rad, deg)
1	0	0	0 , 0
2	45	.75	.11 , 6.1
3	90	.85	.14 , 8.1

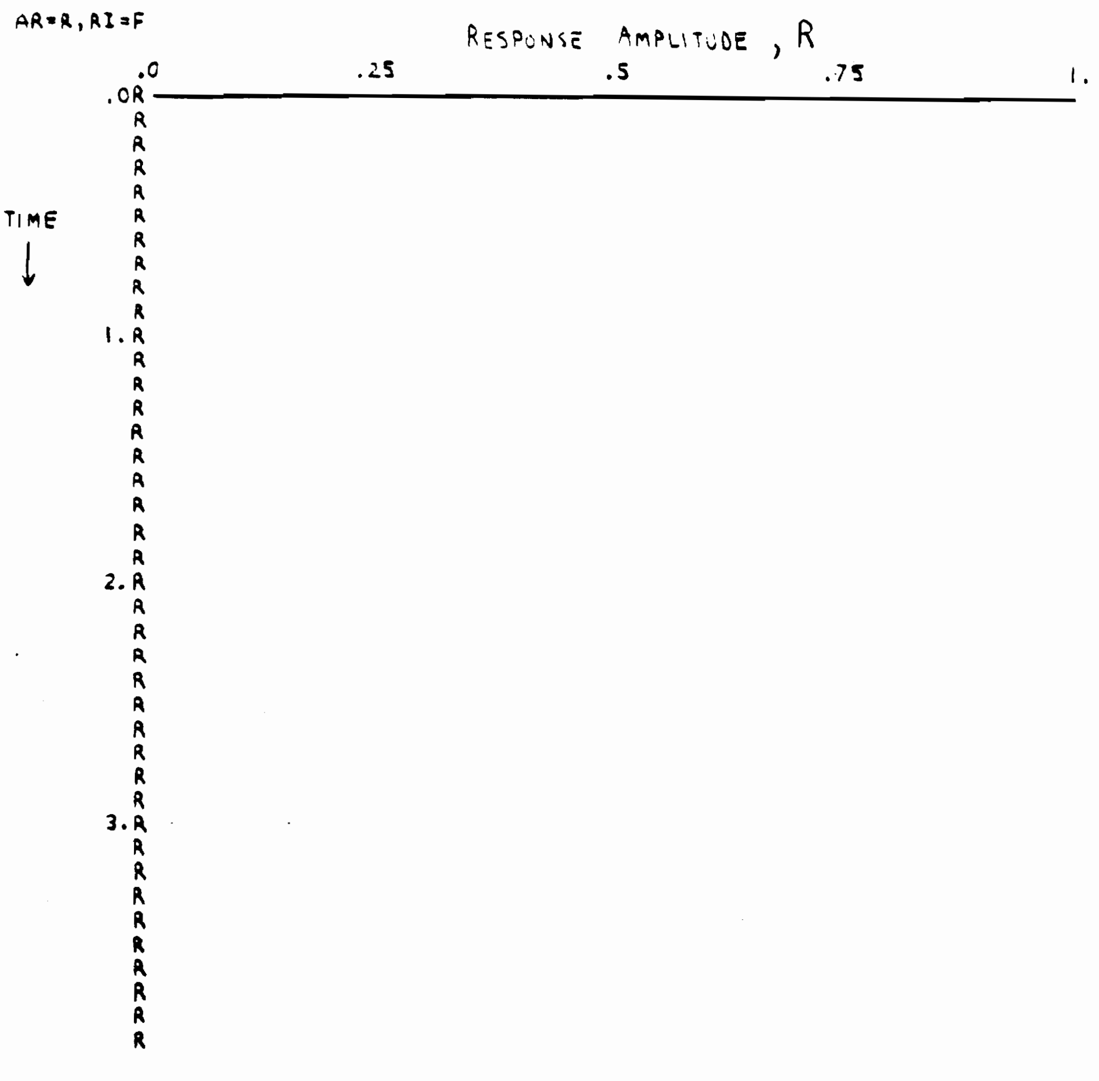

Figure 14. Amplitude response output, zero degree heading.

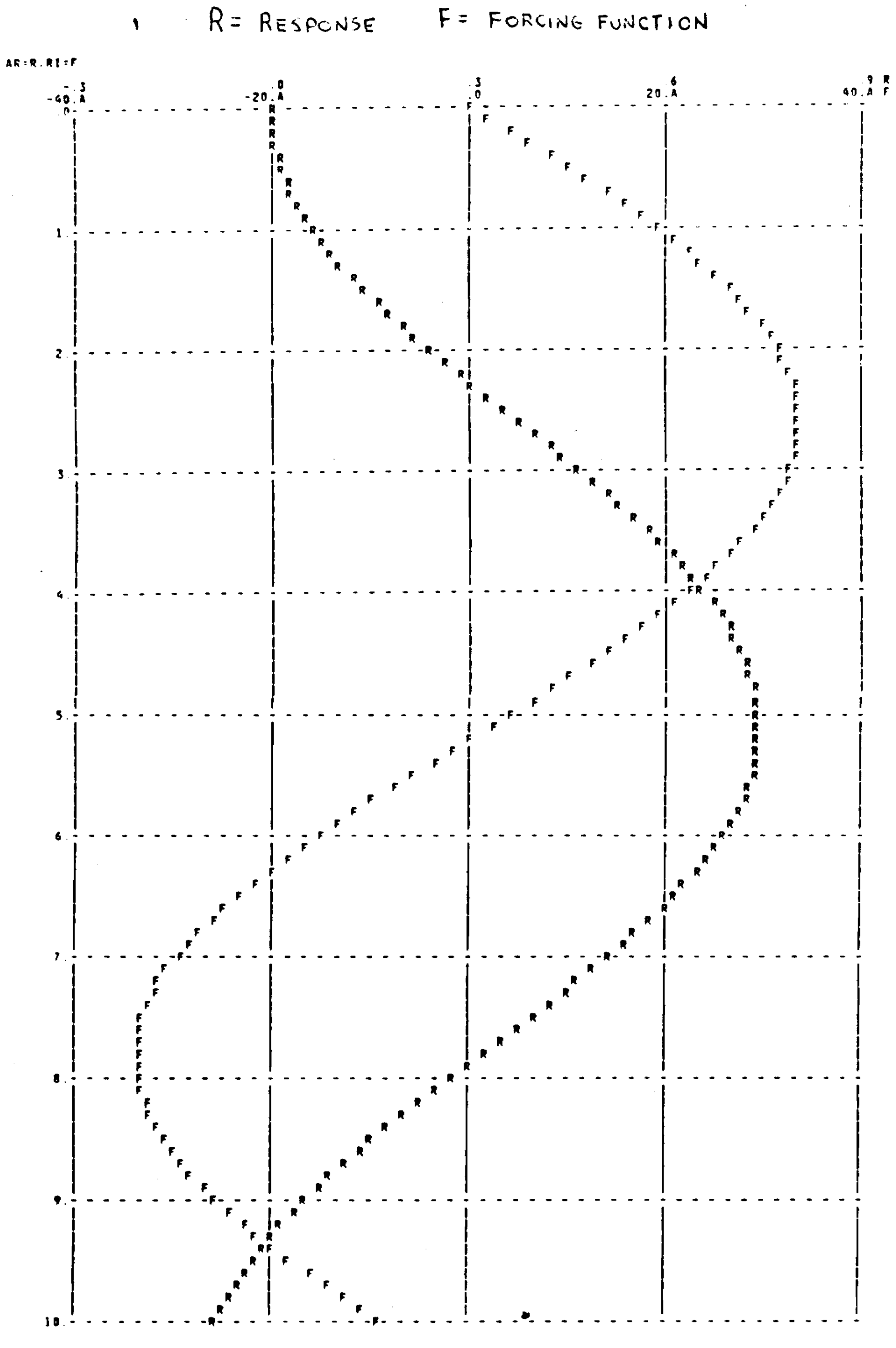

Figure 15. Amplitude response output, 45 degree heading.

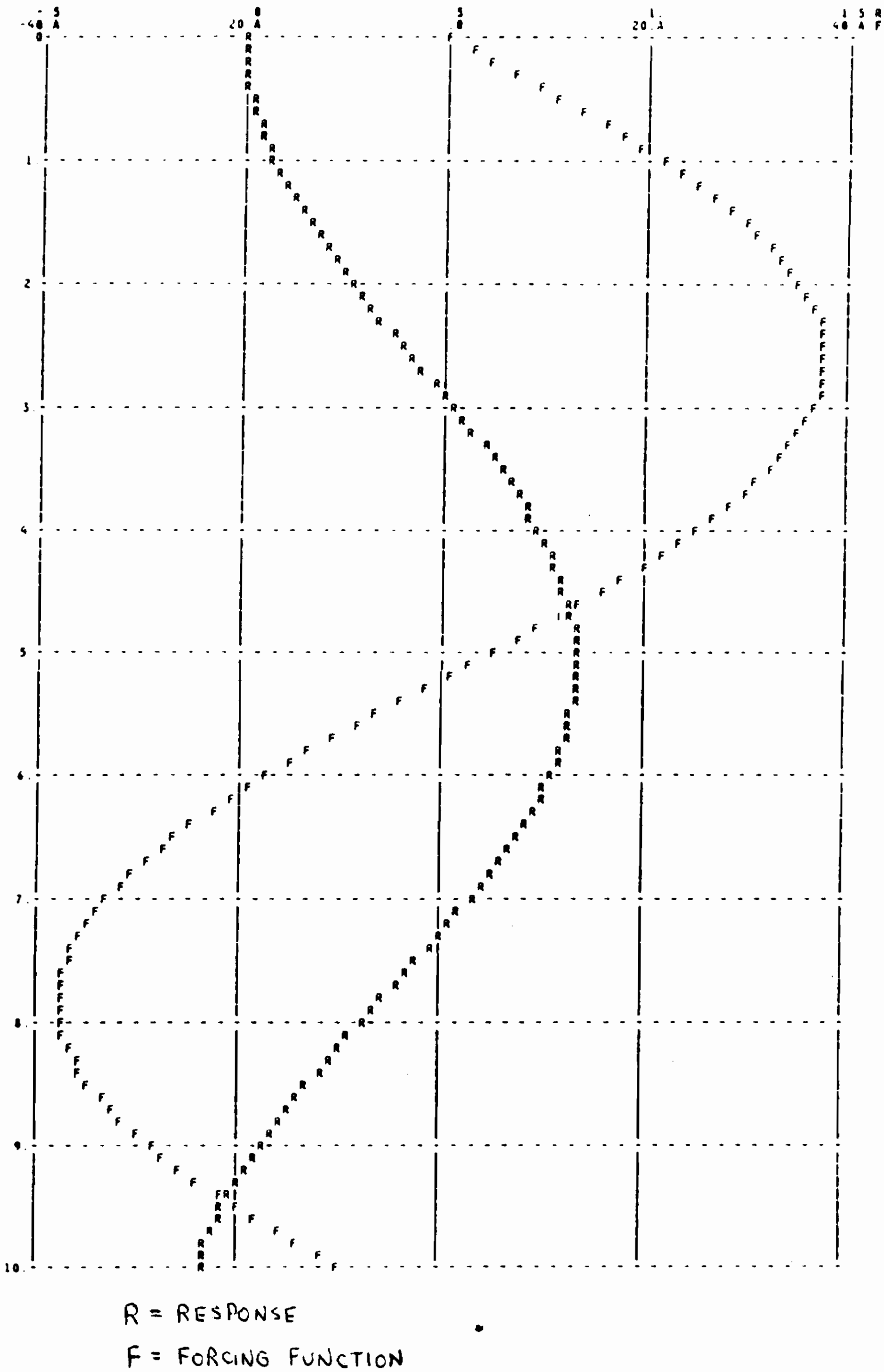

Figure 16. Amplitude response output, 90 degree heading.

encounter. The maximum roll amplitude response does not occur until 5.0 seconds, which is over 2 seconds after the maximum exciting force is encountered. The initial model appears to successfully include damping and inertial effects when determining roll-response.

5.2 CONCLUSIONS AND RECOMMENDATIONS

The design and operation of models has been discussed in the literature.[32] Model manipulation is suggested in order to reduce the misfit between the model and the real world. This process of model validation is described as a three step process, including the initial postulation of the model, model tests for comparison to measurements and observations, and modification of the model to reduce discrepancies.

This suggested application of the DYNAMO tool has achieved the model initialization step and has briefly visited the model test phase. Actual model validation is a continuous effort and would require more extensive model runs than presented in this report.

Table 14 suggests an extensive model test matrix in order to refine the DYNAMO roll-response predictor. The wide range of frequencies, headings, wave heights, and vessel geometry defined by this test plan permits the determination of model performance under several questionable circumstances, including:

Table 14. Suggested model validation matrix.

DESIGN BEAM , W_c __________ (ft)
DESIGN LENGTH , L_c __________ (ft)

HEADING (deg)	WAVE HEIGHT (ft)	[KG-KB] (ft)	FREQUENCIES OF ENCOUNTER (rad/sec): f_1 f_2 f_3 f_4 .... f_n
0	h_1	0	
		10	
		20	
		30	
		40	
	h_2	0	
		10	
		20	MAXIMUM
		30	
		40	
	⋮		AMPLITUDE
	h_n	10	
		20	
		30	RESPONSE
		40	
⋮			RATIOS
90	h_1	10	(rad/rad)
		20	
		30	
		40	
	⋮		
	h_n	10	
		20	
		30	
		40	

1. how does the model predict roll states near the natural frequency of the semisubmersible?

2. how does the model respond to variations in vessel centers of gravity and buoyancy, and how does vessel loading effect roll performance?

3. how does the height of the incident waves effect roll performance?

and

4. how do the variations in vessel beam and length design characteristics effect roll performance?

Though the model in its current state requires a number of validation tests and possible refinements, the completion of this analysis leads to the conclusion that DYNAMO can be successfully applied to the modeling of semisubmersible roll-response. The substitution of theory development for steps 2, 3, and 4 of the system dynamics approach detailed in Table 3 seems to be a viable method of modeling the complex roll behavior of the platform.

The report concludes with three recommendations for future research. The first recommendation is that probabilistic elements be incorporated into the model in order to represent the random occurrence of irregular sea states. The second recommendation is that the existing model be validated according to the plan dictated by Table 14. The final recommendation is that the model be modified based on a comparison of Table 14 validation results with both model tank test data and actual motions of similar existing platforms.

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VITA

Jim McMahon was born on March 02, 1963, in Rockville Centre, New York, to James and Barbara McMahon. In 1981 Jim graduated from Sanford H. Calhoun High School in Merrick, New York, and began his studies of naval architecture at Webb Institute of Naval Architecture. By 1985 he had earned a Bachelor of Science degree in Naval Architecture and Marine Engineering. Jim spent the first three-year period of his career at NKF Engineering, where he was a chief participant in the development of the U.S. Coast Guard Heritage Class Patrol Boat. He left NKF in September of 1988 and began working as an engineer for the General Electric Company in Springfield, Virginia.

In January of 1990 Jim met the girl of his dreams. Jim and Linda Courts will marry in September of 1992.

Jim McMahon