EXPERIMENTAL VERIFICATION OF OPTIMAL EXPERIMENTAL DESIGNS FOR THE ESTIMATION OF THERMAL PROPERTIES OF COMPOSITE MATERIALS

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(ABSTRACT)

The need to simultaneously estimate thermal properties stems from the desire to analyze complex structures which do not have the flexibility to be experimentally tested in multiple configurations. In order to produce reliable and accurate thermal property estimates, the experiments must be carefully developed. A carefully designed experiment maximizes the sensitivity of the temperature distribution with respect to the unknown thermal properties, as well as providing minimum correlation between the estimated properties.

Two objectives were set forth in this research. First to apply existing predicted optimal experimental designs developed by Moncman (1994) to simultaneously estimate the two-dimensional thermal properties of the carbon-fiber/epoxy-matrix composite, AS4/3502. Due to the anisotropic nature of the composite, the effective thermal conductivities through the thickness and in the plane of the composite needed to be estimated along with the volumetric heat capacity. After simultaneously estimating the properties, the second objective was to verify that the predicted optimal designs provided the most accurate estimates.
In accomplishing both objectives, the research plan developed in three distinct stages. As a starting point, the one-dimensional analysis was performed to gain confidence in the experimental setup and procedure. Due to the successful estimation of the one-dimensional properties, the experiments were expanded into a two-dimensional analysis. This analysis attempted to simultaneously estimate all three thermal properties from one optimal, transient, temperature measurement. But due to correlation problems invoked by experimental errors, it was unsuccessful. Therefore, the research focused on the estimation of the in-plane thermal conductivity and the volumetric heat capacity. After successfully estimating the properties, the optimal designs were verified through additional testing with perturbations applied to the optimal settings.

Complete success was not accomplished in this study due to partially satisfying the first objective. All three thermal properties were estimated for the anisotropic composite but due to near correlation between the thermal conductivities, they could not be determined from a single, optimal experiment. Therefore in an attempt to uncorrelate the thermal properties it is recommended that the experiments be performed with multiple sensors. In addition, alternative boundary conditions should be considered on their ability to provide more sensitive information on the thermal properties and practicality to experimentally maintain.

The second objective, verifying the optimal designs, was completely successful in demonstrating that the optimal parameter settings did produce the most accurate thermal property estimates. Therefore, an optimally designed experiment using multiple sensors should allow for the accurate and simultaneous estimation of the thermal properties for complex structures.
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Table of Contents

List of Tables ............................................................. viii
List of Figures ............................................................. ix
Nomenclature ............................................................. xiv

1. Introduction ......................................................... 1
   1.1 Goals and Objectives ........................................... 3

2. Literature Review ................................................. 5
   2.1 Experimental Determination of Thermal Properties of Composite
       Materials .......................................................... 5
   2.2 Gauss Linearization Method ................................... 8
   2.3 Optimal Experimental Designs ................................. 10
      2.3.1 Verification of Optimal Experimental Designs ......... 12

3. Theoretical Considerations ....................................... 15
   3.1 Mathematical Models ........................................... 16
      3.1.1 One-Dimensional Numerical Model ....................... 17
      3.1.2 Two-Dimensional Numerical Model ....................... 21
   3.2 Parameter Estimation .......................................... 25
      3.2.1 Correlation Matrix ......................................... 28
      3.2.2 Confidence Intervals ....................................... 33
   3.3 Optimization of Experimental Designs ....................... 34

4. Experimental Methods .............................................. 39
   4.1 Experimental Setup ........................................... 40
      4.1.1 One-Dimensional Experimental Apparatus ............... 42
      4.1.1.1 Advantages of a Symmetric Experimental Apparatus 48
      4.1.2 Two-Dimensional Experimental Apparatus ................ 49
      4.1.2.1 Configuration 1 Experimental Apparatus .............. 50
      4.1.2.2 Configuration 2 Experimental Apparatus .............. 53
   4.1.3 Thermocouples ............................................. 53
      4.1.3.1 Thermocouple Calibration ............................... 58
   4.1.4 Electric Resistance Heaters ............................... 60
   4.1.5 Data Acquisition System .................................. 62
      4.1.5.1 Hardware for the National Instruments Data Acquisition System .......................................................... 63
4.1.5.2 Software for the National Instruments Data Acquisition System ........................................ 66
4.2 Experimental Procedures ......................................................... 71
  4.2.1 Apparatus Assembly and Installation ................................. 71
  4.2.2 Experimental Testing Procedures .................................... 76
  4.2.3 Chronological Order of Experiments ................................ 78
4.3 Experimental Data ............................................................... 80

5. Results and Discussion .......................................................... 87
  5.1 One-Dimensional Analysis .................................................. 88
    5.1.1 One-Dimensional Optimal Experimental Design .................. 88
    5.1.2 One-Dimensional Thermal Property Estimates ................... 91
  5.2 Simultaneous Estimation of Three Thermal Properties ............. 94
    5.2.1 Optimal Two-Dimensional Experimental Designs for
          Configurations 1 and 2 ........................................... 94
    5.2.2 Analytical and Experimental Results ................................ 102
  5.3 Simultaneous Estimation of Two Thermal Properties ................ 104
    5.3.1 Optimal and Non-Optimal Experimental Designs for
          Configurations 1 and 2 .......................................... 105
    5.3.2 Comparison of Configurations 1 and 2 ............................ 117
    5.3.3 Analytical Verification of Optimal Experimental Designs ........ 120
    5.3.4 Thermal Property Estimates for Optimal and Non-Optimal
          Experimental Designs of Configurations 1 and 2 ................ 122
          5.3.4.1 Comparison of the Optimal Designs of
                 Configurations 1 and 2 .................................... 123
          5.3.4.2 Effects of Heating Time on Estimated Properties .......... 126
          5.3.4.3 Effects of Sensor Location on Estimated Properties .......... 132
          5.3.4.4 Effects of Heating Area on Estimated Properties .......... 140

6. Conclusions and Summary ..................................................... 144
  6.1 One-Dimensional Analysis ................................................. 145
  6.2 Two-Dimensional Analysis for the Simultaneous Estimation of
      Three Thermal Properties ............................................ 146
  6.3 Two-Dimensional Analysis for the Simultaneous Estimation of
      Two Thermal Properties ............................................. 146

7. Recommendations ............................................................... 149

Bibliography ................................................................. 152

Appendix A. EAL Property Estimation Program: ldepvbc .................. 156
<table>
<thead>
<tr>
<th>Appendix B.</th>
<th>EAL Property Estimation Program: 2dpe3vbc</th>
<th>170</th>
</tr>
</thead>
<tbody>
<tr>
<td>Appendix C.</td>
<td>Optimization Program for Configuration 1: optc1.for</td>
<td>190</td>
</tr>
<tr>
<td>Appendix D.</td>
<td>Optimization Program for Configuration 2: optc2.for</td>
<td>197</td>
</tr>
<tr>
<td>Appendix E.</td>
<td>Coefficients for a Type E Thermocouple Polynomial</td>
<td>205</td>
</tr>
<tr>
<td>Appendix F.</td>
<td>Thermocouple Fabrication Procedure</td>
<td>207</td>
</tr>
<tr>
<td>Appendix G.</td>
<td>Data Acquisition Program: Total.c</td>
<td>211</td>
</tr>
<tr>
<td>Vita</td>
<td></td>
<td>219</td>
</tr>
</tbody>
</table>
List of Tables

Table 4.1. Parameter Settings of the Non-Optimal Experiments Performed, Used to Verify the Two-Dimensional Optimal Experimental Designs of Configurations 1 and 2. ................................. 80

Table 5.1. One-Dimensional Thermal Property Estimates with 95% Confidence Intervals. ......................................................... 93

Table 5.2. Phase One, Optimal Experimental Designs for the Simultaneous Estimation of Three Thermal Properties for Configurations 1 and 2. ......................................................... 97

Table 5.3. Optimal Experimental Designs for Simultaneously Estimating Three Thermal Properties with Configurations 1, 2, and Monkman's (1994) Configuration 2. ................................. 100

Table 5.4. Approximate Correlation Matrix for the Simultaneous Estimation of Three Thermal Properties with Configuration 1. ............... 103

Table 5.5. Phase One Results for the Optimal and Non-Optimal Designs for the Simultaneous Estimation of Two Thermal Properties for Configurations 1 and 2. ................................. 108

Table 5.6. Optimal and Non-Optimal Experimental Designs for the Simultaneous Estimation of Two Thermal Properties for Configurations 1 and 2. ......................................................... 116

Table 5.7. Ratio of Confidence Intervals (Rci(k_{y-eff}) and Rci(C_{eff}) Using Non-Optimal Experimental Parameters for Configurations 1 and 2. ......................................................... 121

Table 5.8. Estimated Thermal Properties from the Optimal Experimental Designs of Configurations 1 and 2. ................................. 125

Table E.1. Coefficients for a Type E Thermocouple Polynomial. ............... 206
## List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>One-Dimensional Boundary Conditions</td>
<td>18</td>
</tr>
<tr>
<td>3.2</td>
<td>Boundary Conditions of Configuration 1</td>
<td>22</td>
</tr>
<tr>
<td>3.3</td>
<td>Boundary Conditions of Configuration 2</td>
<td>22</td>
</tr>
<tr>
<td>3.4</td>
<td>Location of Known Side Temperature Histories with respect to EAL Nodal Grid used in the Two- Dimensional Finite Element Model</td>
<td>24</td>
</tr>
<tr>
<td>3.5</td>
<td>Flow Chart for the Two-Dimensional Finite Element Estimation Program, 2dpe3vbc</td>
<td>29</td>
</tr>
<tr>
<td>3.6</td>
<td>Typical Dimensionless Sensitivity Coefficients, $(X_{k_r,g})^<em>$, $(X_{C_g})^</em>$, and $(X_{k_r,g})^*$</td>
<td>31</td>
</tr>
<tr>
<td>3.7</td>
<td>Ratio of Sensitivity Coefficients, $(X_{k_r,g})^<em>/(X_{k_r,g})^</em>$ to Check for Linear Dependence</td>
<td>32</td>
</tr>
<tr>
<td>4.1</td>
<td>Experimental Setup</td>
<td>41</td>
</tr>
<tr>
<td>4.2</td>
<td>Final Assembly of the Experimental Apparatus with Thermocouples for the One-Dimensional Experimental Design</td>
<td>43</td>
</tr>
<tr>
<td>4.3</td>
<td>Detailed View of the Electric Resistance Heater, Thermocouple Location, and Modifications to the Composite and Heat Sink</td>
<td>46</td>
</tr>
<tr>
<td>4.4</td>
<td>Final Assembly of the Experimental Apparatus with Thermocouple (T/C's) for Configuration 1</td>
<td>52</td>
</tr>
<tr>
<td>4.5</td>
<td>Final Assembly of the Experimental Apparatus with Thermocouple (T/C's) for Configuration 2</td>
<td>54</td>
</tr>
<tr>
<td>4.6</td>
<td>Equivalent Electrical Circuit of a Thermocouple in the Experimental Setup</td>
<td>56</td>
</tr>
<tr>
<td>4.7.a</td>
<td>Typical Raw Experimental Data (Not-Offset) of Temperature Histories from a Heated and Constant Temperature Boundary Condition</td>
<td>59</td>
</tr>
</tbody>
</table>
Figure 5.7. Phase One, Experimental Optimization of Configuration 1 for the Simultaneous Estimation of Two Thermal Properties. ... 107

Figure 5.8. Phase One, Experimental Optimization of Configuration 2 for the Simultaneous Estimation of Two Thermal Properties. ... 107

Figure 5.9. Maximum Determinant, $D_{\text{max}}^+$, for Various Sensor Locations, $y^*$, of Configuration 1, Calculated Using Optimal Experimental Parameters of $L_{p,1}^* = 0.25$, $x^* = 0.0$, While $t_h^+$, Varied. ... 111

Figure 5.10. Maximum Determinant, $D_{\text{max}}^+$, for Various Sensor Locations, $y^*$, of Configuration 2, Calculated Using Optimal Experimental Parameters of $L_{p,2}^* = 0.11$, $x^* = 0.0$, While $t_h^+$, Varied. ... 112

Figure 5.11. Maximum Determinant, $D_{\text{max}}^+$, for Various Heating Times, $t_h^+$, of Configuration 1, Calculated Using Optimal Experimental Parameters of $L_{p,1}^* = 0.25$, $x^* = 0.0$, and $y^* = 0.13$. ... 113

Figure 5.12. Modified Dimensionless Determinant, $D^+$, Used to Determine the Dimensionless Optimal Experimental Time, $t_n^+$, of Configuration 1. ... 115

Figure 5.13. Comparison of Dimensionless Determinants, $D^+$, for the Optimal Experimental Designs of Configurations 1 and 2 for the Simultaneous Estimation of Two Thermal Properties. ... 118

Figure 5.14. Comparison of Dimensionless Sensitivity Coefficients, $(X_{k_{\text{eff}}})^+$ and $(X_{C_{\text{eff}}})^+$, for the Optimal Experimental Designs of Configurations 1 and 2. ... 119

Figure 5.15. Comparison of Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, from the Optimal Experimental Designs of Configurations 1 and 2. ... 124

Figure 5.16. Comparison of In-Plane Thermal Conductivity Estimates, $k_{\text{y-eff}}$, from the Optimal Experimental Designs of Configurations 1 and 2. ... 124

Figure 5.17. Dimensionless Determinants, $D^+$, for Optimal and Non-Optimal Heating Times, $t_h^+$, of Configuration 1. ... 128

Figure 5.18. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Times, $t_h^+$, of Configuration 1. ... 129
Figure 5.19. In-Plane Thermal Conductivity Estimates, $k_{x_{\text{eff}}}$, for Optimal and Non-Optimal Heating Times, $t_n^*$, of Configuration 1. .129

Figure 5.20. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Times, $t_n^*$, of Configuration 2. .130

Figure 5.21. In-Plane Thermal Conductivity Estimates, $k_{y_{\text{eff}}}$, for Optimal and Non-Optimal Heating Times, $t_n^*$, of Configuration 2. .130

Figure 5.22. Dimensionless Sensitivity Coefficients, $(X_{k_{x_{\text{eff}}}})^*$ and $(X_{C_{\text{eff}}})^*$, for Optimal and Non-Optimal Heating Times, $t_n^*$, of Configuration 1. .131

Figure 5.23. Dimensionless Sensitivity Coefficients, $(X_{k_{y_{\text{eff}}}})^*$ and $(X_{C_{\text{eff}}})^*$, for Optimal and Non-Optimal Heating Times, $t_n^*$, of Configuration 2. .131

Figure 5.24. Dimensionless Determinants, $D^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1. .134

Figure 5.25. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1. .135

Figure 5.26. In-Plane Thermal Conductivity Estimates, $k_{y_{\text{eff}}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1. .135

Figure 5.27. Dimensionless Sensitivity Coefficients, $(X_{k_{y_{\text{eff}}}})^*$ and $(X_{C_{\text{eff}}})^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1. .136

Figure 5.28. Dimensionless Determinants, $D^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2. .138

Figure 5.29. Dimensionless Sensitivity Coefficients, $(X_{k_{y_{\text{eff}}}})^*$ and $(X_{C_{\text{eff}}})^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2. .138

Figure 5.30. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2. .139

Figure 5.31. In-Plane Thermal Conductivity Estimates, $k_{y_{\text{eff}}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2. .139
Figure 5.32. In-Plane Thermal Conductivity Estimates, $k_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,1^*}$, of Configuration 1. .... 141

Figure 5.33. In-Plane Thermal Conductivity Estimates, $k_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,2^*}$, of Configuration 2. .... 141

Figure 5.34. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,1^*}$, of Configuration 1. .... 143

Figure 5.35. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,2^*}$, of Configuration 2. .... 143

Figure 7.1. Boundary Conditions for a Proposed Third Configuration. .... 151

Figure 7.2. Boundary Conditions for a Proposed Fourth Configuration. ... 151
Nomenclature

$A$  Scalar used in the Box-Kanemasu method

$b$  Estimated parameter vector containing effective thermal conductivity and effective volumetric heat capacity values

$C$  Capacitance Matrix

$C_{ef}$  Effective volumetric heat capacity (MJ/m$^3$ K) or (Btu/ft$^3$ °F)

$c_p$  Specific Heat (J/ kg K) or (Btu/lb$m$ °F)

$D$  Determinant of $X^T X$

$d_{ij}$  Time average of the sensitivity coefficients ($i, j = 1, 2$)

$D_{max}$  Maximum determinant value

$F$  Boundary condition matrix

$G$  Scalar used in the Box-Kanemasu method

$h$  Scalar interpolation function used in the Box-Kanemasu method

$l$  Electrical current (A)

$j$  Joint location

$k$  Thermal Conductivity (W/m K) or (Btu/hr ft °F)

$k_{x,ef}$  Effective thermal conductivity (W/m K) (Btu/hr ft °F) perpendicular to the carbon fibers (through the thickness of the composite)

$k_{y,ef}$  Effective thermal conductivity (W/m K) or (Btu/hr ft °F) parallel to the carbon fibers (in the plane of the composite)

$k_{xy}$  Ratio of effective thermal conductivities ($k_{y,ef}/k_{x,ef}$)

$K$  Stiffness Matrix
$L_p$  Thickness on the y axis where the heat flux is applied (m) or (ft)

$L_x$  Thickness in the x direction (perpendicular to the fibers) (m) or (ft)

$M$  Number of temperature sensors

$N$  Number of experimental temperature measurements

$p$  Number of parameters estimated

$P$  Vector equal to $(X'X)^{-1}$

$q''$  Heat Flux (W/m²) or (Btu/hr ft²)

$s$  Standard deviation of the parameter estimates

$S$  Least squares function

SCXI  Single Channel eXtensions for Instruments

$S_0$  Sum of squares value at zero

$S_\alpha$  Sum of squares value at $\alpha$

$t$  Time (sec)

$t_h$  Total heating time (sec)

$t_N$  Total experimental time (sec)

$T$  Temperature (K) or (°F)

$T_{max}$  Maximum Temperature between the beginning and end of the experiment (K) or (°F)

$T_{\alpha,x}$  Temperature at the boundary where $x = 0.0$ (K) or (°F)

$T$  Calculated temperature vector

$V$  Measured thermocouple voltage (volts)

$x$  Position along the x axis (m) or (ft)
\( x \)  
Sensitivity coefficient matrix

\( X_{C_{\text{eff}}} \)  
Sensitivity coefficient for the effective volumetric heat capacity

\( X_{k_{\text{r-eff}}} \)  
Sensitivity coefficient for the effective thermal conductivity perpendicular to the carbon fibers

\( X_{k_{\text{f-eff}}} \)  
Sensitivity coefficient for the effective thermal conductivity parallel to the carbon fibers

\( y \)  
Position along the y axis (m) or (ft)

\( Y \)  
Measured temperature vector (K) or (°F)

**Greek**

\( \alpha \)  
Scalar used in the Box-Kanemasu method, Seebeck Coefficient, or thermal diffusivity \((m^2/s)\) or \((ft^2/s)\)

\( \beta \)  
Weighting factor

\( \beta \)  
Parameter vector containing the thermal conductivity and volumetric heat capacity values

\( \eta \)  
Calculated temperature vector

\( \rho \)  
Density \((kg/m^3)\) or \((lbm/ft^3)\)

\( \Delta \theta \)  
Vector defined in the Box-Kanemasu method

\( \Delta t \)  
Time step size

**Superscripts**

\( k \)  
Iteration number

\( T \)  
Transpose

\( + \)  
Dimensionless
CHAPTER 1

Introduction

Accurate knowledge of a material's thermal properties is necessary for the proper design of aerospace vehicles. Due to the extreme environmental conditions that an aerospace vehicle is subject to, large heat loads may impinge on the surface. From the heat loads, the surface material is thermally stressed which causes deformation, and if the loads are excessive, the material will fail. To prevent this, it is necessary to understand how a material reacts to an applied heat flux. A thermal stress analysis can be performed, but it requires prior knowledge of the temperature distribution of the material. Before the temperature distribution can be determined, the thermal properties of the material must be known. Therefore, the first step in producing better designs is to accurately determine the thermal properties of the material.

Prior research has demonstrated that there are multiple ways to estimate thermal properties, but as materials become more complex and structures become larger, traditional methods of estimating individual properties are no longer efficient. This study utilizes a modified Box-Kanemasu approach to simultaneously estimate the thermal
properties. The method requires both calculated and measured temperatures generated from an impinged heat flux. Both required temperature histories are dependent on the thermal properties and the experimental parameters such as sensor location, heating time, and heated area. The accuracy of the estimated properties is directly related to the sensitivity of the temperature distribution with respect to the thermal properties and can be manipulated by the experimental parameters. The combination of experimental settings which provide the most information on the thermal properties, as well as minimize the correlation between the properties is referred to as the optimal experimental design. Optimal experimental designs have multiple advantages from providing insight into physical phenomena to economizing experiments. Additionally, they provide the most accurate estimates and allow for the greatest probability of simultaneously estimating multiple thermal properties.

This research focuses on verifying that optimal experimental designs used in estimating the thermal properties of a carbon-fiber/epoxy-matrix composite do indeed produce the most accurate estimates. It is the second stage of a multi-stage process in improving the designs of aerospace vehicles. The first stage was performed by Moncman (1994) who developed the optimal experimental designs for both a one- and two-dimensional analysis of a similar carbon composite.
1.1 Goals and Objectives

The ultimate goal of this study was to verify that the two-dimensional, experimental design developed by Moncman (1994) was truly the optimal design for providing the most accurate estimates of thermal properties. To verify the designs, they were applied to a carbon-fiber/epoxy-matrix composite, AS4/3502, which is an anisotropic material, possessing different thermal properties in different directions. Due to the carbon fibers, which are more conductive than the epoxy resin, the material maintains a larger in-plane thermal conductivity than conductivity through the thickness. Although the composite consisted of a \([(+45^\circ_2/-45^\circ_2/0^\circ_2)_2 \ldots /90^\circ /+45^\circ_2/-45^\circ_2/0^\circ_2/-45^\circ_2/0^\circ_2/-45^\circ_2/0^\circ_2/90^\circ)]_2\) 46 plies layup, the overall effective thermal conductivities were only sought in two directions, through the thickness and in the plane of the composite. Moncman (1994) initially developed a one-dimensional optimal experiment for the estimation of the through the thickness thermal conductivity and the volumetric heat capacity and expanded into two-dimensions for the additional estimation of the in-plane thermal conductivity. The focus of this research was to experimentally verify the predicted, two-dimensional optimal design.

The first objective was to estimate the effective two-dimensional thermal properties simultaneously using the optimal design. Once the properties were estimated the experimental settings were verified. Verification was performed by comparing the confidence intervals around the estimated properties from both optimal designs and designs with perturbations in the optimal settings, referred to as non-optimal experimental
designs. The most accurate estimates would maintain the smallest confidence intervals. In summary the two objectives were:

- First, to simultaneously estimate the two-dimensional effective thermal properties: the thermal conductivity through the thickness of the composite, perpendicular to the carbon fibers, the in-plane thermal conductivity, parallel to the fibers, and the volumetric heat capacity (density times specific heat).
- Second, to verify the optimal designs by comparing confidence intervals of the estimates generated from optimal and non-optimal experimental designs.

The experiments consisted of applying a known heat flux to a portion of the composite surface for a specified time, specifying the remaining boundary conditions and recording the resultant temperature history. To maximize the sensitivity of the temperature history with respect to the thermal properties the experiments were optimized for five parameters: the sensor location parallel and perpendicular to the fibers, the heating time of the required heat flux, the heating area, and the length of time for the experiment. Although five parameters were optimized, only three were verified experimentally with non-optimal parameter settings: sensor location in the axis parallel to the fibers, heating time, and heating area. In addition to verifying the experimental designs, the research also analyzed the effects of two different boundary conditions on the estimated properties. To verify the importance of an optimal experiment, both optimal and non-optimal designs were tested with each set of boundary conditions.
CHAPTER 2

Literature Review

Prior research has been in the areas of experimental methods for estimating thermal properties, the Gauss linearization method, and the optimization of experiments. Since the focus of this study incorporates all three aspects, each of these topics is reviewed.

2.1 Experimental Determination of Thermal Properties of Composite Materials

Previous experimental methods used in the estimation of thermal properties have been broken into two parts, steady state and transient. Steady state experimental approaches have been limited solely to the estimation of thermal conductivity. Generally the experiment is expensive and time consuming due to the necessity for the material to reach steady state. Ziebald (1977) performed steady state experiments which incorporated both absolute and relative measurements. The absolute measurement method, where the thermal property is determined directly from the measurements, proved to be extremely accurate. In this analysis the specimen was heated with a guarded hot plate, and the
temperature measurement was recorded at the interface. The thermal conductivity was estimated from the absolute temperature measurement. The second approach by Ziebald (1977) was a relative measurement process. This method determines the thermal property of a material from the known thermal conductivity of a reference substance. The need for prior information was the main disadvantage of this approach.

Additional steady state experiments have been performed by Dickson (1973), where the thermal conductivity of insulation was estimated with the use of heat flow sensors. This was a simple experiment requiring the heat flux, the temperature (from the heat flow sensors) and the thickness of the test specimen. This research was expanded by Penn et al. (1986) to incorporate small composite samples. As a result of the smaller samples, experimental time was reduced to within a few hours. Harris et al. (1982) estimated the thermal conductivities perpendicular and parallel to the lengths of Kevlar 49 fibers as a function of temperature using a two plate apparatus. Additionally, the effects of fiber direction on the estimated property were analyzed by Havis et al. (1989) for a fibrous composite material.

Transient approaches allow for the estimation of the thermal diffusivity which incorporates the ratio of the thermal conductivity to the volumetric heat capacity (a time dependent property). One of the earliest transient experiments, the laser-flash method, was utilized by Parker et al. (1961). This method exposes the front surface of a test sample to a burst of radiant energy from a laser. The resulting temperature response on the back surface is recorded and the thermal diffusivity is determined from the time required to reach half of the maximum temperature rise. From the maximum temperature
rise, the specific heat, \( c_p \), can be determined and the thermal conductivity can be calculated by multiplying the thermal diffusivity, specific heat, and density together, \( k = \alpha \rho c_p \). Transient experiments incorporating the laser-flash method allow for smaller samples and shorter experimental times than the steady state approach. The advantages of the laser-flash method were realized and widely utilized in determining the thermal properties of composite materials by Taylor et al. (1985), Lee and Taylor (1975), Taylor and Kelsic (1986), and Mottram and Taylor (1987a and b).

Subsidiaries of the laser-flash approach were developed. Brennan et al. (1982) used the composite method to measure the thermal conductivity and diffusivity of silicon carbide fibers. These properties were indirectly determined by applying the laser-flash technique to the composite and the matrix, free of fibers. From the diffusivity and the Rule-of-Mixtures, which is a mathematical model describing the effective thermal conductivity of a composite, the thermal properties of the fiber are calculated. Welsh et al. (1987, 1990) also used a modified version of the laser-flash method to determine the thermal diffusivity. The method consisted of applying a pulsed heat flux on the surface of the specimen and then measuring the temperature response on the same surface. This method is at a disadvantage since only the ratio of the thermal conductivity to the volumetric heat capacity can be determined.

A final transient experimental method utilizes periodic hot-wire heating. This method was implemented by Fukai et al. (1991) to simultaneously estimate the thermal conductivity and diffusivity. From the amplitude and phase lag of the temperature response, both the conductivity and diffusivity were estimated. To confirm the estimates
of the experiments they were compared with conventional methods.

2.2 Gauss Linearization Method

Alternative methods were developed to simultaneously estimate multiple thermal properties of composite materials. One commonly used technique is the Gauss linearization method. This method is an iterative procedure that minimizes the least squares objective function by setting the derivative of the function with respect to the unknown properties equal to zero. It is one of the simplest and most effective methods used for seeking minima as long as the initial estimates are in the general region of the minima, as described by Beck and Arnold (1977). However, if poor initial estimates are applied or if the model is highly non-linear, large oscillations may occur and lead to non-converged estimates. To avoid this, Box and Kanemasu (1972) made a modification to the step size of the Gauss linearization method. In addition, Bard (1970) modified the Gauss method by including a check to ensure that the function reduced at each iteration. If the solution did not decrease, the non-linear step size introduced by Box and Kanemasu was reduced in half until an appropriate step size was determined.

Beck (1966) used the Gauss linearization method to simultaneously determine the thermal conductivity and specific heat of nickel from a transient temperature measurement. He also used the Gauss method to determine the thermal contact conductance for both steady state and transient conditions with a periodic contact (1988). Scott and Beck (1992a) utilized this method to simultaneously estimate the thermal
conductivity and volumetric heat capacity for a carbon composite as a function of
temperature and fiber orientation. Additional tests were performed by Scott and Beck
(1992b) to develop an estimation procedure for a thermoset composite material during
curing based on the Gauss method.

A two-dimensional analysis was performed on a anisotropic carbon composite by Loh
and Beck (1991) to simultaneously estimate the effective thermal conductivities parallel
and perpendicular to the fibers along with the volumetric heat capacity. Experimental
designs were first optimized to determine the optimal heating and experimental time. The
experimental design was symmetric with respect to an electric resistance heater which
covered 50% of the composite surface. Three thermocouples were placed between the
heater and the composite and an additional three thermocouples were placed on the
backside of the composite. Another composite was placed against the bottom of the first
and an additional three thermocouples along with an aluminum heat sink were added to
complete the setup. The heat sink was used to apply a constant temperature boundary
condition. Therefore from the nine different temperature measurements and the optimal
heating and experimental times, the thermal conductivities parallel and perpendicular to
the fibers and the volumetric heat capacity were simultaneously estimated. The thermal
conductivity parallel to the fibers was estimated to be seven times larger than the thermal
conductivity perpendicular to the fibers. The estimated properties were shown to be in
good agreement with results obtained from previously performed one-dimensional
experiments. Additional experiments were performed in the simultaneous estimation of
thermal conductivity and volumetric heat capacity by Jurkowski et al. (1992), Garnier et
al. (1992), and Moncman (1994). Pfah and Mitchel (1970) used the Gauss minimization technique to simultaneously estimate six thermal properties of a charring carbon-phenolic material. Again, the property values were shown to be in good agreement with the results from conventional tests.

From previous research it is clear that the advantage of the Gauss linearization method offers the ability to estimate multiple properties simultaneously and to accommodate different geometries and boundary conditions. What distinguishes this study from previous research is the use of the Gauss Method to demonstrate that the experimental parameters have an impact on the accuracy of the estimates. As materials become more complex and the analysis of larger structures becomes necessary, this method along with careful experimental designs will provide for an efficient process to simultaneously estimate the thermal properties.

2.3 Optimal Experimental Designs

Accurate thermal property estimates are extremely important in the use of composite materials. The accuracy of the estimates is improved if the experiment is designed properly. A carefully designed experiment is one in which there is minimum correlation between the estimated properties, as well as maximum sensitivity of the measured experimental variables to changes in the properties being estimated (Beck and Arnold, 1977). In addition, optimal experiments can give insight into the physical phenomenon being analyzed, reduce the number of experiments required, economize the cost of
experiments, provide ideal conditions for competing models, and improve the accuracy of the estimates.

In order to produce an optimal design, the criterion used to optimize the experiments must be determined. One of the most common criterion is to maximize the determinant of the product of the sensitivity coefficients and their transpose, $|X^TX^*|$, referred to as the D-optimal criterion. The sensitivity coefficients contained in $X^*$ represent the change in temperature due to a perturbation in the unknown thermal property. The D-optimal criterion was chosen in this study because it has the effect of minimizing the confidence intervals. Additional criteria exist based on variations of the D-optimal such as the A-, C-, E-, and L-optimal criteria. In the A-optimal criterion, the trace of the $X^TX^*$ matrix is minimized, and in the E-optimal criterion, the maximum diameter of the confidence region ellipsoid is minimized. After determining the optimal criterion, the second step of the optimization procedure is to determine the conditions to optimize. For example, in designing an experiment to estimate thermal properties of a material using the D-optimal criterion, the experimental parameters, such as boundary conditions and geometry of the material (which can be adjusted by the experimenter), are chosen to maximize the expression $|X^TX^*|$ (Scott and Haftka, 1995).

Optimal experimental designs using the D-optimal criterion have been developed by Beck (1969) and implemented into the experiments for the simultaneous estimation of thermal conductivity and specific heat. Taktak et al. (1991) estimated the thermal conductivity and volumetric heat capacity of a semi-infinite and finite thickness composite material with the following optimal parameters; heating time of the impinged heat flux,
number of sensors and sensor location. Monkman et al. (1994) also developed optimal experimental designs for a one and two-dimensional analysis of a carbon-fiber/epoxy-matrix composite for the simultaneous estimation of the thermal properties. Although two-dimensional designs were developed, only the optimal one-dimensional designs were tested Monkman (1994). The one-dimensional analysis resulted in the accurate, simultaneous estimation of the thermal conductivity perpendicular to the carbon fibers and the volumetric heat capacity. The two-dimensional designs, not implemented, were optimized for five parameters: sensor location in the plane and perpendicular to the plane of the fibers, heating area of an applied heat flux, heating time, and experimental time for two different sets of boundary conditions. This study is a continuation of Monkman's research where the optimal two-dimensional designs are implemented and experimentally verified.

2.3.1 Verification of Optimal Experimental Designs

Optimal designs are necessary for reliable and accurate estimates of thermal properties. Although optimization in this study focuses on the experimental designs used to estimate thermal properties it is not limited to any particular field. Optimization can be applied to biomedical applications, extrusion processes, structural designs, and any process or design that can be improved. These designs need to be verified to ensure that they actually produce the best results. When an analytical model is optimized it is possible to design past the applicability of the model resulting in unreliable designs. Thompson and Supple (1973) optimized a structure against buckling where the
optimization produced larger imperfection sensitivity. This was due to the optimization of a limited analytical model. The analytical model did not account for all characteristics of the physical model such as the unavoidable imperfections in the material due to manufacturing.

Verification can be accomplished by applying small perturbations to the optimal designs and comparing the results with those produced by optimal designs or by comparing with conventional results. Kaufmann and Stone (1987) used the Taguchi orthogonal arrays to develop an optimal experimental design for shear bond strength of a lap joint. First they developed the optimal designs and then they verified them with additional tests. Fox and Lee (1990) performed a similar analysis where the Taguchi method was applied to the optimization of metal injection molding. Again, additional tests were performed to demonstrate that the optimal design provided improved performance. Knight et al. (1992) also confirmed the performance of an optimal air cooled aluminum fin. The optimal design was tested along with two non-optimal designs maintaining fewer and greater fins. From the analysis, the optimally designed heat sink produced the lowest thermal resistance, therefore creating the coolest operating temperatures. Adelman (1992) sighted ten studies based on structural optimization methods where the optimal designs were developed and tested against baseline designs. In all cases the optimal designs satisfied the criterion and produced better results.

A concluding reason for the validation of experimental designs is for the acceptance of a new approach. Whitman (1991) proposed a new method for determining experimental designs which improved upon the Taguchi method. The new method was
applied to the optimization of the strength of an outboard motor propeller by varying six parameters with 27 experiments. Verification of the optimal designs demonstrated that the new approach was valid.

In conclusion, previous studies have demonstrated the importance of validating optimal designs. Validation has uncovered errors in analytical models, demonstrated improved performance over traditional methods, and helped acceptance of a new concept. The importance of optimization is starting to be realized by the scientific and industrial communities, as more designs are developed, the need to verify them becomes increasingly more important. Verification provides enhanced confidence in the final results.
CHAPTER 3

Theoretical Considerations

This chapter focuses on the theoretical considerations necessary to simultaneously estimate the thermal properties of an anisotropic material and the optimization procedure used to create an optimal experimental design. Due to the inlaid unidirectional carbon-fibers of the composite analyzed in this study, different thermal characteristics are resultant in different directions. A modified Box-Kanemasu method was used to simultaneously estimate the in-plane and through the thickness thermal conductivities along with the volumetric heat capacity. This method requires both calculated and measured temperature histories of the composite. The following section describes the mathematical models used to generate the calculated temperature history while the experimental methods used to obtain the measured temperature history are discussed in Chapter 4. The second section briefly describes how the modified Box-Kanemasu method is used to estimate the thermal properties from the two temperature histories. For a more in-depth discussion on the modified Box-Kanemasu method refer to Beck and Arnold (1977). The chapter concludes with a description of the optimization procedure used to
create an optimal experimental design.

3.1 Mathematical Models

A predicted temperature history is necessary for the simultaneous estimation of the thermal properties using the modified Box-Kanemasu method. The goal in using the mathematical model to generate this temperature history was to replicate the temperature history of the composite in the experimental tests. In order to accurately replicate the temperature history, the boundary conditions of the experiment had to be implemented into the mathematical model. The better the mathematical model approximated the experimental setup, the more valid the estimated properties became.

The composites were subjected to three different sets of boundary conditions. The first set allowed for the one-dimensional thermal properties to be estimated. Although the focus of this study is on estimating the two-dimensional thermal properties, the one-dimensional analysis was a good starting point. Two different sets of boundary conditions were applied in the two-dimensional analysis. The purpose of using the different boundary conditions was to demonstrate the influence of boundary conditions on the estimated properties.

There were multiple ways to mathematically model the composites and the boundary conditions of the experimental tests. Since the composites had a simple geometry, analytical temperature solutions could be developed if the boundary conditions were assumed to be constant. Moncman (1994) calculated the analytical temperature
distributions for the one-dimensional and both two-dimensional analyses based on the conservation of energy using Green's Function Method. One problem with analytical solutions is that they are limited to simple geometries and exact boundary conditions. Often the specified boundary conditions could not be maintained experimentally; therefore, the validity of the estimates were jeopardized and other mathematical models were considered.

In this study, finite element (numerical) models were developed to replicate the composites and boundary conditions. The numerical models offered the flexibility to incorporate complex geometries with varying boundary conditions. The following two subsections describe the one and two-dimensional numerical models used to replicate the experimental setups.

3.1.1 One-Dimensional Numerical Model

A one-dimensional analysis was used to estimate the effective thermal conductivity through the thickness of the composite, $k_{\text{eff}}$, and the effective volumetric heat capacity, $C_{\text{eff}}$. In order to estimate both thermal properties, a numerical model was required to generate a temperature history that simulated the temperature history of the one-dimensional experiment. To form an accurate model, the geometry and boundary conditions of the setup were incorporated.

The boundary conditions of the one-dimensional design are shown in Figure 3.1. To simulate one-dimensional heat transfer, the sides of the composite were insulated while an imposed heat flux was applied across the entire top surface and the bottom surface was...
Figure 3.1. One-Dimensional Boundary Conditions.
held at a constant temperature. One of the requirements for the simultaneous estimation of thermal properties is the incorporation of a heat flux boundary condition. This boundary condition introduces an additional equation containing the heat flux and the thermal conductivity. If the heat flux were not present, only the ratio of the thermal properties, the thermal diffusivity, could be estimated. In addition, a transient solution was needed for the estimation of the volumetric heat capacity ($\rho C_p$), which will be discussed in greater detail in Section 3.3, Optimization of Experimental Designs.

At both the heated and constant temperature boundaries, temperature sensors were placed to measure the temperature histories. The temperature history at the heated boundary was used in the observation matrix of the modified Box-Kanemasu method to estimate the thermal properties. The temperature history at the constant temperature boundary was incorporated into the numerical model as the boundary condition. In theory this temperature history should consist of a single constant value but due to the physical limitations of the experimental setup a slight temperature rise occurred at this boundary. Due to the flexibility of the numerical model, this temperature rise could be accounted for and the accuracy of the model improved.

Engineering Analysis Language (EAL), version 325.14 produced by Engineering Information System Incorporated, Whetstone (1983), was used to develop the finite element model. The function of the code was to numerically determine the temperature distribution which was approximated in EAL by

$$[C + \beta \Delta t K]T_{i+1} = [C - (1-\beta)\Delta t K]T_i + F\Delta t + F\beta \Delta t^2$$

(3.1)

where $T_i$ is the temperature vector at time $t_i$, $T_{i+1}$ is the temperature vector at time $t_{i+1}$, and
\( \Delta t \) is the time step size, \( C \) is the capacitance matrix, \( K \) is the stiffness matrix, \( F \) is the matrix containing the boundary conditions, and \( \beta \) is the weighting function. (Whetstone, 1983). The weighted value, \( \beta \), was set to 0.5 in all numerical models, incorporating the Crank-Nicolson algorithm.

As previously stated, the one-dimensional numerical model required information from the experiment to specify the boundary conditions. The finite element program, \textit{ldpevbc} (one-dimensional property estimation with a variable boundary condition), listed in Appendix A, required an input file. The input file, \textit{in1op}, consisted of the measured time, temperature histories from the heated and the constant temperature boundary conditions, and the imposed heat flux from the experiment. Both the measured heat flux and temperature history from the constant temperature boundary were used as the boundary conditions in the finite element model. EAL was then used to predict a temperature history at the heated surface of the composite using these conditions. The calculated temperature history was then compared with the measured temperature history at the heated surface in the modified Box-Kanemasu method to simultaneously estimate the thermal properties. It is important to realize that part of the experimental data were used to specify the boundary conditions of the model, while part of the experimental data were used in the estimation procedure.

A concern with the numerical model, \textit{in1op} was whether the grid allowed for a converged solution. Only eight elements were necessary to allow for adequate estimates. Insignificant changes, past the accuracy of the temperature measurement, occurred in the estimated properties by adding a ninth element. Therefore accurate estimates were
obtained and the program converged faster with eight elements.

3.1.2 Two-Dimensional Numerical Model

The two-dimensional analysis provides estimates for the effective thermal conductivities in the plane and through the thickness of the composite along with the effective volumetric heat capacity. Again, a calculated temperature history was required for the simultaneous estimation of the thermal properties and was achieved with a finite element model. Similar to the one-dimensional model, the two-dimensional model required the boundary conditions of the experiment to generate the required temperature history.

Two different sets of boundary conditions were analyzed to determine their effects on the estimated properties. The first, Configuration 1, had three constant temperature boundaries and a heat flux across a portion of the fourth, as shown in Figure 3.2. The second setup, Configuration 2, had one constant temperature, two insulated, and one partially heated boundary condition; see Figure 3.3. Similar to the one-dimensional analysis, both configurations required a transient heat flux for the simultaneous estimation of the properties, although the heat flux was not required to cover the entire boundary. Actually, the area that the heater covers is one of the optimal parameter settings that is determined in the optimization process.

The finite element program, 2dpe3vbc (two-dimensional property estimation with three variable boundary conditions) listed in Appendix B, was used to calculate the temperature distribution of the composite. Similar to the one-dimensional model, the two-
Figure 3.2. Boundary Conditions of Configuration 1.

Figure 3.3. Boundary Conditions of Configuration 2.
dimensional program required the temperature measurements from the boundaries of the experiment. But in the two-dimensional experiments, temperature sensors were placed along the sides of the composite in addition to the sensors located at the bottom surface to record the temperature histories of the boundary conditions. Again, the only purpose of the sensors located on the sides and bottom surface was to specify the boundary conditions which were used in the finite element model.

The nodal grid used in the numerical model is shown in Figure 3.4. The boundary nodes were specified as constant temperature, heat flux, or insulated. If a boundary condition was not specified, EAL would assume that the node was insulated. The finite element model consisted of eight elements in the $x$ direction and forty in the $y$ direction, shown in Figure 3.4. Therefore, a total of nine nodes existed along each side with thirty nine nodes across the top and bottom surfaces, all of which needed to be specified. Since temperature sensors were placed along the boundaries of the composite, there were known temperature histories at specific locations. Three sensors were placed on each side of the composite, one close to the heated surface, one in the middle, and one near the bottom constant temperature boundary. These three sensor locations matched up with the nodal locations labelled A, B, and C in Figure 3.4. Since there were a total of nine nodes per side, the remaining six nodal temperature histories were extrapolated from a quadratic fit through the three known temperatures.

The remaining two boundaries, the bottom and top surfaces, also had to have specified boundary conditions. All nodes along the bottom boundary were assumed to have the same temperature as the sensors located on the bottom surface of the composite.
Figure 3.4. Location of Known Side Temperature Histories with respect to EAL Nodal Grid used in the Two-Dimensional Finite Element Model.
The top boundary which was partially heated and partially insulated had to incorporate both boundary conditions. In EAL, to incorporate a heat flux boundary, two-dimensional source heating elements had to be super-imposed over the top elements of the numerical model. These source heating elements had an infinite conductance, no density and no specific heat. The remaining unheated nodes on the top boundary were undeclared, therefore, they were assumed to be insulated. This completed the boundary conditions for the two-dimensional model. EAL could then be used to predict a temperature history on the heated surface based on the boundary conditions. The flexibility of the numerical model, $2dpe3vbc$, allowed for the temperature history to be calculated with experimental data from either Configuration 1 or 2.

The eight by forty element grid of the model provided adequate grid refinement to allow for converged solutions. Solutions from a model containing forty and fifty elements in the $y$ direction were compared and no significant difference was noticed. Forty elements in the $y$ directions was excessive, converged solutions could have been obtained with fewer elements, but since the concern of the study was to obtain converged estimates it remained.

3.2 Parameter Estimation

The modified Box-Kanemasu interpolation method is a subsidiary of the Gauss linearization method and is presented in brevity in this section. For a more in-depth discussion refer to Beck and Arnold (1977). As previously mentioned the modified Box-
Kanemasu method allows for the simultaneous estimation of thermal properties. In this study it has been utilized to simultaneously estimate the effective thermal conductivities in two directions along with the volumetric heat capacity.

The goal of the estimation procedure is to minimize the least squares objective function. The least squares objective function, $S$, is presented as

$$ S = [Y - \eta(\beta)]^T[Y - \eta(\beta)] $$

(3.2)

where $Y$ is the observation vector (measured temperature history), $\eta$ is the corresponding modeled vector (calculated temperatures) based on the thermal properties contained in $\beta$, which is not the same $\beta$ used in Eq. (3.1). Note that the objective function could be used with multiple sensors, although only one was used in this study.

The least squares objective function, Eq. (3.2), is minimized by taking the derivative with respect to the unknown thermal properties and setting it equal to zero:

$$ \Delta_\beta S = 2[-\Delta_\beta \eta^T(\beta)]^T[Y - \eta(\beta)] = 0. $$

(3.3)

The derivative of the calculated function, $\eta$, with respect to the thermal properties, $\beta$, is the sensitivity matrix, denoted by, $X(\beta)$,

$$ X(\beta) \equiv [\Delta_\beta \eta^T(\beta)]^T. $$

(3.4)

This is an important matrix in that it contains the sensitivity coefficients for each unknown parameter. The sensitivity coefficients indicate the sensitivity of the measured variable, i.e. temperature, to changes in the unknown properties.

Due to the nonlinearities of the heat conduction process, the thermal properties can not be easily solved for from Eq. (3.3). Two assumptions are required to simplify the solution. First, the exact value of $\beta$ is replaced with an estimated value denoted as $b$ and
second, the first two terms of the Taylor series are used to approximate the function \( \eta(\mathbf{B}) \).

With these two assumptions and Eq. (3.4), Eq. (3.3) becomes,

\[
b^{(k+1)} = b^{(k)} + P^{(k)}X^{T(k)}(Y - \eta^{(k)})
\]  

(3.5 a)

where,

\[
P^{-1(k)} \equiv X^{T(k)}X^{(k)}
\]  

(3.5 b)

which is the Gauss linearization equation. Notice that the procedure is iterative with \( k \) as the iteration number and that initial estimates of \( b \) are required. The solution for the estimates are obtained when the property values do not change significantly from iteration to iteration. If the experiment is designed carefully and the initial estimates are near the minima, the solution should converge within about seven iterations.

The Gauss linearization equation was modified by Box and Kanemasu (1972). Due to the linear approximation of \( \eta \), in some nonlinear cases the corrections can oscillate with increasing amplitude and not converge. The Box-Kanemasu interpolation method modified the step size of the Gauss equation by including \( h^{(k+1)} \) into Eq. (3.5),

\[
b^{(k+1)} = b^{(k)} + h^{(k-1)} \Delta_b^{(k)}
\]  

(3.6 a)

where,

\[
\Delta_b^{(k)} = P^{(k)}X^{T(k)}(Y - \eta^{(k)})
\]  

(3.6 b)

and where \( h \) is a non-linear step function.

The Box-Kanemasu method was also modified to ensure that the sum of squares value decreased from iteration to iteration. With the Box-Kanemasu method, it is possible to over step the minima. To avoid this a check is added to ensure that the sum of squares value, \( S \), decreased with each iteration. If the sum of squares increases, the non-linear
step increment, \( h^{(k+1)} \) is cut in half. This procedure continues until a suitable step size is found or if the step size falls below 0.01 and the program is terminated. When this occurs there is usually a problem with correlated or incorrect sensitivity coefficients. Figure 3.5 demonstrates how the modified Box-Kanemasu method was implemented into the two-dimensional finite element property estimation program, \textit{2dpe3vbc}.

### 3.2.1 Correlation Matrix

In order to estimate the thermal properties simultaneously, the sensitivity coefficients cannot be correlated. If the sensitivity coefficients are linearly dependent the least squares function, \( S \), has no unique minimum. To determine if the coefficients were correlated they were first non-dimensionalized and plotted against each other. Recall, the sensitivity coefficients are the derivatives of the temperature distribution with respect to the unknown thermal properties. The three non-dimensional sensitivity coefficients, \( X^* \), were

\[
X_{k_x}^* = \frac{k_{x-eff}}{q''/L_x/k_{x-eff}} \frac{\partial T}{\partial k_{x-eff}}
\]

(3.7 a)

\[
X_{C_{eff}}^* = \frac{C_{eff}}{q''/L_x/k_{x-eff}} \frac{\partial T}{\partial C_{eff}}
\]

(3.7 b)

\[
X_{k_y}^* = \frac{k_{y-eff}}{q''/L_x/k_{x-eff}} \frac{\partial T}{\partial k_{y-eff}}
\]

(3.7 c)

where \( q'' \) is the heat flux, \( L_x \) is the thickness through the composite, \( T \) is the temperature
Figure 3.5. Flow Chart for the Two-Dimensional Finite Element Program, 2dpe3vbc.
distribution, and \( k_{\text{eff}} \), \( C_{\text{eff}} \) and \( k_{\text{v-eff}} \) are the effective thermal properties.

Typical non-dimensional sensitivity coefficients are plotted in Figure 3.6. From the figure it is clear that there is no linear dependence between the volumetric heat capacity and either of the thermal conductivities. Although, it is more difficult to determine if linear dependence exists between the thermal conductivities. To obtain a better understanding if correlation exists between the conductivities, the ratio, \( X_{k_{\text{v-eff}}} / X_{k_{\text{eff}}} \), is plotted in Figure 3.7. From the figure, there does not appear to be linear dependence between the thermal conductivities.

Another approach to determine the linear dependence between the unknown properties is to calculate the approximate correlation matrix. The correlation matrix comes directly from the \( P \) matrix of Eq. (3.5 b) which contains the product of the sensitivity coefficients and their transpose. The approximate correlation matrix, \( r \), is given by Beck and Arnold (1977) as

\[
r_{ij} = P_{ij} (P_{jj} P_{ii})^{-1/2}
\]

where the diagonal elements of \( r \) are all unity and the off-diagonal elements are between -1 and 1. Since three thermal properties are estimated, \( r \) is a 3 x 3 matrix. The matrix quantifies the correlation between the unknown thermal properties. As the coefficients approach unity, the properties approach linear dependence. Therefore all of the diagonal coefficients are unity, representing that a single thermal property is 100% correlated with itself. As a general guide line, if the off-diagonal terms exceed 0.90, difficulty is encountered estimating the properties independently and accurately.
Figure 3.6. Typical Dimensionless Sensitivity Coefficients, \((X_{k_{\text{eff}}})^*\), \((X_{C_{\text{eff}}})^*\), and \((X_{k_{y_{\text{eff}}}})^*\).
Figure 3.7. Ratio of Sensitivity Coefficients, \((X_{k_y})^*/(X_{k_x})^*\), to Check for Linear Dependence.
3.2.2 Confidence Intervals

The second objective of this research is to demonstrate that the optimal experimental designs produce the most accurate estimates. To determine the accuracy of an estimate a confidence interval with 95% certainty is placed around the estimate. The smaller the confidence interval the more accurate the estimate.

The confidence interval for each property, $\beta_k$, is calculated from the $k$th diagonal term of the $P$ matrix along with the Student-$t$ distribution. To calculate the interval it is assumed that the measurement errors are additive, have a zero mean, and are normally distributed. With these assumptions, the confidence interval given by Walpole and Myers (1978) is estimated from

$$\beta_k = b_k \pm \left[ \frac{P_{kk}}{N - p} \right]^{1/2} t_{1 - \alpha/2}(N - p)$$

(3.9)

where $S$ is the sum of squares value, $N$ is the number of data points acquired, $p$ is the number of parameters estimated, and $t$ is the Student-$t$ distribution. Since the confidence intervals incorporated the $P$ matrix, it is therefore dependent on the sensitivity coefficients. A large sensitivity coefficient will produce a small confidence interval. This agrees with the definition of a sensitivity coefficient, a large coefficient indicates that a tremendous amount of information is available on the thermal property. Therefore, more information leads to more accurate estimates.

To simplify Eq. (3.9), the value of $t_{1 - \alpha/2}(N - p)$ can be assumed to be 1.96. This allows for a confidence interval with 95% certainty when more than fifteen temperature readings are acquired. Due to the considerable number of data points taken in each experiment (up
to 500) this was a valid assumption. Note that the confidence intervals are defined for each experiment.

3.3 Optimization of Experimental Designs

The accuracy of estimated properties is enhanced if an experiment is designed properly. A carefully designed experiment is one in which there is minimum correlation between the estimated properties, as well as maximum sensitivity of the measured experimental variables to changes in the properties being estimated (Beck and Arnold, 1977). In addition, optimal experiments can give insight into the physical phenomenon being analyzed, reduce the number of required experiments, economize the cost of experiments, and provide ideal conditions for competing models.

The governing criterion for the optimization technique used in this study was to maximize the determinant of the product of the sensitivity matrix and its transpose, $|X^TX|$. Recall that the sensitivity coefficients contained in $X$, indicate the sensitivity of the temperature to changes in the unknown property. Since the temperature distribution is a function of the experimental parameters, it can be manipulated to provide more or less information on the unknown properties by their settings. Therefore, the goal is to find the settings which provide the most information, i.e., when the determinant of the $X^TX$ matrix is a maximum. As a result of this criterion the confidence intervals are minimized.

After defining the criterion the second step of the optimization technique is to
identify the experimental parameters which are to be optimized. The parameters should be chosen based on their ability to strongly influence the temperature distribution. In this study, five parameters were chosen, the sensor location in the $x$ and $y$ direction (see Figures 3.2 and 3.3), the heated surface area of the composite, the applied heating time of the heater, and the total duration of the experiment. Optimal experimental design was obtained for these five parameters for both two-dimensional configurations.

The third step is to calculate the sensitivity coefficients; see Eqs. (3.5 a,b,c). Once the non-dimensional coefficients are calculated it is helpful to graph them. As discussed in Section 3.2.1, the plots can give an indication of whether properties are linearly dependent. They are also useful in providing insight into the experiment. Refer to the plot of typical non-dimensional sensitivity coefficients, Figure 3.6. Notice how the non-dimensional sensitivity coefficient for the volumetric heat capacity approaches zero as the experiment reaches steady state ($t_r^* = 1.37$) while the thermal conductivities reach their largest magnitude. Therefore to obtain more sensitive information on the volumetric heat capacity at the expense of sacrificing information on both thermal conductivities, experiments were performed with a transient heat flux. In addition to providing insight, the coefficients indicate whether an experiment can provide adequate information on a property. As a general rule, if the magnitude of a non-dimensional sensitivity coefficient does not exceed 0.10 there is inadequate information to accurately estimate the property.

Step four of the optimization procedure is to calculate the determinant of the $X^TX$ matrix. Due to estimating three properties in the two-dimensional analysis, $|X^TX|$ is a $3 \times 3$ matrix. Since the experiments performed were transient, each coefficient was
time averaged. Therefore, the dimensionless determinant is given as

\[
D_{2-D}^* = \begin{vmatrix}
\dot{d}_{11}^* & \dot{d}_{12}^* & \dot{d}_{13}^* \\
\dot{d}_{21}^* & \dot{d}_{22}^* & \dot{d}_{23}^* \\
\dot{d}_{31}^* & \dot{d}_{32}^* & \dot{d}_{33}^*
\end{vmatrix}
\]

(3.10 a)

\[
D_{2-D}^* = \dot{d}_{11}^*(d_{22}^*d_{33}^* - d_{23}^*) - \dot{d}_{12}^*(d_{12}^*d_{33}^* - d_{13}^*d_{23}^*) + \dot{d}_{13}^*(d_{12}^*d_{23}^* - d_{13}^*d_{22}^*)
\]

(3.10 b)

where \(d_{ij}^*\) was found from (Beck and Arnold, 1977)

\[
d_{ij}^* = \left[ \frac{1}{T_{max}^2} \right] \left[ \frac{1}{M \cdot T_N^i} \right] \sum_{p=1}^{M} \int_0^{t^*} X_i^i(t^*) X_j^j(t^*) \, dt^*
\]

(3.11)

In this equation, \(M\) is the number of temperature sensors, (for all experiments, \(M = 1\)) and \(t^*, T_{max}^*, \text{ and } T_N^*\) are defined as

\[
T_{max}^* = \frac{T_{max} - T_{o,s}}{q_x L_x / k_{x,eff}} \quad t^* = \frac{k_{x,eff} t}{C_{eff} L_x^2} \quad T_N^* = \frac{k_{x,eff} T_N}{C_{eff} L_x^2}
\]

(3.12 a,b,c)

where \(T_{max}\) is the temperature achieved at steady state conditions, \(T_{o,s}\) is the initial surface temperature, and \(T_N\) is the total experimental duration. The integration within Eq. (3.11) is performed numerically, being approximated by a summation. It is evident from Eq. (3.11) that the determinant matrix, Eq. (3.10 a), is symmetric; i.e., \(d_{ij}^* = d_{ji}^*\), \(d_{13}^* = d_{31}^*\), and \(d_{23}^* = d_{32}^*\) since the multiplication of vectors is commutative. The use of symmetry helps increase the speed of the optimization program by eliminating three repetitive equations.

The final step of the optimization procedure is to maximize the determinant. There are various methods to achieve the maximum determinant. The analytical solution would
be to take the derivative of the determinant with respect to each of the experimental parameters and set them equal to zero. This would lead to five independent equations with five unknowns. Optimal solutions could then be obtained by solving the equations simultaneously. Due to the complexity of the math involved in taking the derivative of the determinant with respect to the parameters and the tedious process of solving multiple simultaneous equations, a parametric study was developed.

The parametric analysis used in this study calculated the determinant at every possible combination of the five parameters within a finite range and increment size. There were two disadvantages with the parametric study. First, the procedure was time consuming and second, if the increment size was too large, the maximum determinant could be missed. To help reduce the time consuming computations, the analysis was separated into two phases. Phase one used a fairly coarse increment size. When the maximum determinant was obtained from this phase, the increment size for all five parameters was reduced around the maximum determinant. Phase two was a repetition of phase one with smaller increments.

Optimizing an experimental design is a delicate balance between the sensitivity coefficients. Different designs would be developed for different combinations of the unknown thermal properties. Moncman (1994) had developed optimization programs for Configurations 1 and 2 based on the analytical temperature solutions and for estimating three properties. Appendix C and D list modified versions of Moncman's optimization programs, optc1.for and optc2.for, which optimize the experimental designs for only the in-plane thermal conductivity and volumetric heat capacity. In both cases the optimal
designs determine the parameter settings which allow for the most accurate estimates and minimum correlation between the properties.
CHAPTER 4

Experimental Methods

This chapter describes the experimental methods used in estimating the one- and two-dimensional thermal properties of the carbon-fiber/epoxy-matrix composite, AS4/3502. There were two fundamental criteria used for the simultaneous estimation of thermal properties. First, the Box-Kanemasa Minimization method, described in Section 3.2, required a measured temperature history of the composite. Second, the simultaneous estimation of the properties required a transient heat flux boundary condition across a portion of the composite surface. These two criteria, along with the optimal experimental designs, were combined to form the experimental designs described in this chapter.

The first section concentrated on the experimental setup while the procedures used to install and operate the experiment are described in the second section. The final section is focused on the data obtained from the experiments and how it was combined with the finite element property estimation programs.
4.1 Experimental Setup

The purpose of the experimental setup was to provide an environment where a transient heat flux could be applied to a composite sample while the resulting temperature history was being recorded. In this section, a description is given of the hardware and how it was installed into the experimental setup shown in Figure 4.1. The components of the setup consisted of the one- or two-dimensional apparatus, a thermocouple junction box, an ice bath, two high resolution multimeters, a DC power supply, and a data acquisition system.

The experimental setup had a modular design to allow different experimental apparatuses to be analyzed. The setup was centered around the one- or two-dimensional apparatus containing two composite samples, a heater, and up to twenty two thermocouples. The thermocouples from the apparatus were attached to a thermocouple junction box using thermocouple plug connectors. The purpose of the box was to simplify the processes of adding and removing apparatuses. The thermocouple junction box was hardwired to an ice bath, which provided the necessary reference temperature for the thermocouples. The connecting wires contained in the ice bath were then hardwired to the data acquisition system which recorded and converted the thermocouple voltages into temperatures.

In addition to recording the temperature history, the experimental setup was used to apply and monitor a heat flux to the composites. A heater was sandwiched between the composite samples within the one- and two-dimensional apparatuses and placed in series.
Figure 4.1. Experimental Setup.
with a DC power supply. To monitor the heat flux, two high resolution digital multimeters were placed in series and parallel with the heater. The current and voltage drop across the heater were measured by the multimeters and sent to the data acquisition system through serial ports on a personal computer using RS-232. This completed the experimental setup which focused on obtaining measured temperatures from a heated composite.

The one- and two-dimensional experimental apparatuses are described in the following two subsections. In subsection three the focus is on the thermocouples used in the apparatuses, while the emphasis in subsection four is on the difficulties encountered with electric resistance heaters. The experimental setup section concludes with a description of the hardware and software used by the data acquisition system.

### 4.1.1 One-Dimensional Experimental Apparatus

In the one-dimensional analysis, the effective thermal conductivity perpendicular to the carbon fibers, \( k_{x\text{eff}} \), and the effective volumetric heat capacity, \( C_{\text{eff}} \), were estimated. The experimental apparatus used to estimate these thermal properties was based on the one-dimensional, predicted, optimal experimental design developed by Moncman (1994).

The one-dimensional experimental apparatus consisted of an electric resistance heater, two rectangular composite plates, two aluminum blocks, eight thermocouples, thermal grease, insulation, and a grounded brace. The setup was symmetric with respect to the heater. On either side of the heater were placed two thermocouples, the composite plate, two additional thermocouples and finally the aluminum block, as shown in Figure 4.2.
Figure 4.2. Final Assembly of the Experimental Apparatus with Thermocouples (T/C's) for the One-Dimensional Experimental Design.
When the entire setup was assembled, insulation was wrapped around the outer surface of the composite plates and a grounded brace was installed.

The electric resistance heater used in the setup (part #, HK5466R5.7L12A, Minco Products Inc.) was placed between the two rectangular, thin, composite plates to supply a constant, uniform heat flux. It was necessary to apply a heat flux over a portion of a boundary in order to simultaneously estimate the thermal properties. In this case, the one-dimensional model, the entire surface of the composite had to be heated. A problem with the electric resistance heater was that the heat flux was not uniform, since the heater consisted of a foil heating element covered by a thin layer of Kapton, hot spots developed over the heating element when power was supplied.

The non-uniform heat flux created two problems. First, the finite element property estimation program assumed the heat flux was uniform. Second, the optimal designs placed the sensor at the heated surface of the composite. The temperature distributions would vary significantly depending on whether or not the sensor was located over or near the heating element. The optimal sensor location represents the location on the composite where the temperature change with respect to both the thermal conductivity through the thickness and the volumetric heat capacity was the most sensitive. Therefore, an accurate temperature history was necessary at the heated surface in order to produce valid thermal property estimates.

To negate the effects of the non-uniform heat flux, multiple thermocouples were used to take an average temperature measurement over the surface. Four thermocouples were used, two on either side of the heater, t/c 3, 4, 5, and 6 in Figure 4.2. The thermocouples
were placed approximately 3.18 mm (0.125 in) off center with the first thermocouple directly over the heating element and the second over a gap in the heating element, as shown in Figure 4.3. Therefore, the single temperature history at the heated surface was determined by averaging all four thermocouple measurements.

The rectangular composite plates that were placed on either side of the heater had to be modified to allow space for thermocouples and the leads of the heater. It was important to ensure good contact between the heater and the composite to allow the entire heat flux, generated by the heater, to enter the composite. Since the leads of the heater were internally attached, a 6.35 mm x 12.7 mm x 3° (0.25 in x 0.50 in x 3°) notch was added to the edge of the composite, as shown in Figure 4.3.

Aluminum blocks were placed on either side of the composite-heater-composite sandwich to provide a constant temperature boundary condition by acting as heat sinks. Since the aluminum blocks were not perfect heat sinks, there were unavoidable temperature rises on the adjoining surfaces of the composites. These temperature rises, above ambient room temperature, were measured with the thermocouples placed in the groves on the bottom surfaces of the composites. The groves were 0.051 mm x 76.2 mm x 0.051 mm (0.002 in x 3.0 in x 0.002 in), centered 6.35 mm (0.25 in) apart on the surface of the composite. Thermocouples were placed in these groves which allowed for the composites to be in direct contact with the aluminum blocks. To help reduce contact resistance through out the experimental setup, a limited number of thermocouples were used and the temperature rise across all boundary conditions were assumed to be uniform. In addition to diminish the effects of contact resistance which helped minimize the
temperature rises, the aluminum blocks were polished. Although copper would have been a better material for the heat sinks, aluminum was used because it was cheaper, readily available, and easier to machine.

Thermocouples were placed between the composites and the aluminum blocks to monitor the boundary condition. Temperature histories from these thermocouples were averaged together and used in the finite element estimation program only to specify the boundary condition. The purpose of these thermocouples should not be confused with the function of the thermocouples located at the optimal temperature sensor on the heated surface of the composite.

The final boundary condition was created by applying pipe insulation around the outside of the composite. This boundary condition was necessary for the one-dimensional assumption and not difficult to maintain. The insulation, produced by Thermwell Products, consisted of 3.18 mm (0.125 in) thick vinyl foam with adhesive on one side. Since the thermal conductivity of the insulation was 0.231 W/(m K) (0.133 Btu/(hr ft °F)), only 2.5 mm (0.1 in) was necessary to maintain 99% of the heat flux generated by the heater. Although only one layer of insulation was necessary, several more layers were wrapped around to ensure good insulation.

The final concern of the one-dimensional setup was the thermal contact resistance between the composites and all boundary conditions. Contact resistance was due to the surface roughness of the adjoining materials, creating contact spots interspersed with air gaps. Since air is a good insulator, the contact resistance was not negligible. To reduce these insulation effects, thermal paste and a brace were added to the experimental
apparatus. The purpose of the thermal paste was to fill in the air gaps created by the surface roughness with a high conductive material. A thin layer of the paste (OMEGATHERM Thermally Conductive Silicone Paste, part # OT201, Omega Engineering Inc.) was applied between all adjoining materials. The thermal paste maintained a thermal conductivity of 2.3 W/(m K) (16 Btu in/ (hr ft² °F)) and was used sparingly. As mentioned, its purpose was to allow for the heat to escape from the composite and enter the heat sink. Excess thermal grease would be counterproductive, since the aluminum heat sinks have a much larger thermal conductivity, the shorter the distance between the composite and the heat sink the faster the heat can escape. In addition to the thermal paste, good contact was aided by pressure applying to the setup with a brace. The brace consisted of two, 3.18 mm (0.125 in) steel plates, four bolts and a grounding wire which completed the one-dimensional apparatus, refer back to Figure 4.2 for the complete one-dimensional assembly.

4.1.1.1 Advantages of a Symmetric Experimental Apparatus

The accuracy of the thermal property estimates was improved with a symmetric experimental apparatus for two reasons. First, the entire heat flux generated by the heater could be assumed to enter the adjoining composites. Second, multiple thermocouples could be used to measure the same temperature history. To determine the temperature history at the heated surface of the composite, as shown in Figure 4.2, readings from thermocouples 3, 4, 5 and 6 were averaged together. Likewise, the temperature measurements from thermocouples 1, 2, 7 and 8 were averaged to produce the
temperature history at the boundary condition. Being able to average multiple thermocouple readings reduced the amount of noise in the thermocouple measurement.

The use of symmetry was validated by the similar temperature readings obtained on opposing sides of the heater. Although there was a slight offset between all thermocouples, due to systematic errors, opposing thermocouples had the same slopes and magnitude after the linear offset. Therefore, heat conduction was symmetric through the setup.

4.1.2 Two-Dimensional Experimental Apparatuses

Two-dimensional apparatuses were used to estimate three effective thermal properties, the effective thermal conductivity in plane and through the thickness of the composite in addition to the effective volumetric heat capacity. The apparatuses were similar to the one-dimensional apparatus described in Section 4.1.1. The similarities were a result of the same two governing principles: a heat flux had to be applied over a portion of a boundary and the temperature history of the composite had to be measured.

The two-dimensional apparatuses were designed to account for multiple boundary conditions and various temperature sensor locations. Apparatuses were designed to estimate the thermal properties with optimal and non-optimal parameter settings. The analyzed parameters were the sensor location in the y direction, \( y^* \), the heating area, \( L_p^* \), and the heating time, \( t_h^* \). In addition, two separate apparatuses were used to satisfy the different boundary conditions of Configurations 1 and 2, as shown in Figures 3.2 and 3.3.

The two-dimensional apparatuses satisfied all the necessary criteria for
simultaneously estimating thermal properties, but were restricted to certain parameter settings due to practical considerations. Not all heating areas could be analyzed due to limited sizes of manufactured heaters. In addition, the optimal sensor location could not be varied in the x direction. All optimal sensors were limited to locations along the heated surface of the composite at \( x = 0 \). Although, the final two parameters, the sensor location in the y direction and heating time, were not limited. The following two subsections describe the components of both two-dimensional apparatuses.

### 4.1.2.1 Configuration 1 Experimental Apparatus

Configuration 1 was the first two-dimensional apparatus to be analyzed. The configuration had three constant temperature, two insulated, and one heat flux boundary conditions. To implement these boundary conditions on two rectangular composite samples, four aluminum blocks were used and monitored with sixteen thermocouples.

Similar to the one-dimensional analysis, an unavoidable temperature rise occurred on all surfaces where a constant temperature boundary condition was implemented. In order to determine the temperature rise at these boundaries, three additional groves were added to each side of the composite, as shown in Figure 4.3. Thermocouples were placed in these groves to measure the temperature histories along the sides of the composite.

Although the temperature rises at the constant temperature boundaries were undesirable, they were accounted for in the finite element property estimation program, \( 2dpe3vbc \), listed in Appendix B. The purpose for the three thermocouples on each side of the composite and the two on the bottom surface was to record the temperature history
of the boundary conditions as discussed in Section 3.1.2. These thermocouples were used to specify the temperature histories for the boundary conditions of the finite element model; they have a different function than the thermocouples placed in the optimal and non-optimal sensor locations.

Six thermocouples were placed on the electric resistance heater representing the optimal and non-optimal sensor locations. Two of the six thermocouples were placed at the optimal sensor location in the y direction, one on each side of the heater, while the remaining four thermocouples were placed at non-optimal locations as shown in Figure 4.4. To maintain an exact location, all thermocouples were attached to the heater with super glue. Similar to the one-dimensional apparatus, the thermocouples had to be staggered over the heated surface to account for the non-uniform heat flux.

A second brace was added to the setup to ensure good contact between the side aluminum heat sinks and the sides of the composite. The brace was constructed from 6.35 mm (0.25 in) carbon steel and contained eight drilled and tapped, 12.7 mm (0.50 in) holes which allowed hex bolts to be threaded through. The brace would fit around the circumference of the apparatus and the hex bolts would be used to apply pressure to the side heat sinks. The circumferential brace also helped stabilize the setup, by preventing the composites from sliding over the heat sinks.

The final modification to the setup was milling the top and bottom heat sinks 0.25 mm (0.01 in) under the length of the composite plates, as shown in Figure 4.3. This was done to guarantee that the bottom and top heat sinks would not interfere with the contact of the side heat sinks and the sides of the composite.
Figure 4.4. Final Assembly of the Experimental Apparatus with Thermocouples (T/C's) for Configuration 1.
The remainder of the apparatus was identical to the one-dimensional setup. The composites, thermal paste, and the grounded vertical brace remained the same. See Figure 4.4 for the complete assembly of Configuration 1.

4.1.2.2 Configuration 2 Experimental Apparatus

Configuration 2 was the second experimental apparatus used to estimate the two-dimensional thermal properties. In addition to estimating the thermal properties, the setup validated whether or not the accuracy of the estimates depends on the boundary conditions.

Configuration 2 was similar to Configuration 1, except for the side boundary conditions and the location of the optimal and non-optimal parameters. The side boundary conditions were insulated using Thermwell Products pipe insulation (the same insulation used in the one-dimensional analysis). Similar to Configuration 1, Configuration 2 was symmetric, requiring a heater, two composite plates, and twenty two thermocouples. Both configurations had the same thermocouple pattern; sixteen boundary thermocouples, two optimally located thermocouples and four thermocouples in non-optimal sensor locations. See Figure 4.5 for a cross sectional view of the complete assembly of Configuration 2.

4.1.3 Thermocouples

In this subsection, the use and implementation of the thermocouples in the overall experimental setup is described; refer to Appendix F for the fabrication procedure.
Figure 4.5. Final Assembly of the Experimental Apparatus with Thermocouples (T/C's) for Configuration 2.
Type E, AWG 40, thermocouples made from chromega and constantan wires were chosen as the temperature sensors for all experimental apparatuses. Type E thermocouples were ideal because they produced the largest emf voltage output of any thermocouple. Even with the Type E thermocouples, the output for room temperature experiments was only between one and two millivolts. The 40 gauge wires on the thermocouples were necessary to be unobtrusive, which helped reduce the contact resistance between the layers of materials containing thermocouples. The final advantage of the thermocouples was their ability to measure a temperature at a specific location, which was required for the two-dimensional analysis.

In the experimental setup, thermocouples were not directly attached to the data acquisition system. The governing principle on thermocouples is that when dissimilar metals are brought into contact, a voltage is produced proportional to the temperature gradient. This voltage is equal to the Seebeck coefficient multiplied by the temperature gradient through the leads:

\[ V = \alpha \Delta T \]  \hspace{1cm} (4.1)

where \( \alpha \) is the Seebeck coefficient. Therefore, if the chromega and constantan leads of the thermocouple were attached to the copper terminals of the data acquisition system, two additional voltages would be created. To negate these additional voltages, copper extension wires were run from the data acquisition system and attached to an ice bath. At the ice bath they were joined with the chromega and constantan leads of the thermocouples. The equivalent electrical schematic for the setup is shown in Figure 4.6.
Figure 4.6. Equivalent Electrical Circuit of a Thermocouple in the Experimental Setup.
\[ V_{\text{data acquisition}} = V_2 + V_1 - V_3 \]  

(4.2)

where \( V_{\text{data acquisition}} \) was the voltage read by the data acquisition and \( V_{1,2,3} \) were the voltages created by the contact of dissimilar metals at junctions one, two and three. Substituting Eq. (4.1) into Eq. (4.2) produces

\[ V_{\text{data acquisition}} = \alpha \left( T_{j_2} + T_{j_1} - T_{j_3} \right) \]  

(4.3)

where \( T_{j_{1,2,3}} \) were the temperatures at junctions one, two and three. Since junctions two and three occurred in an ice bath, their temperatures were zero degrees Celsius, which reduced Eq. (4.3) to

\[ V_{\text{data acquisition}} = \alpha \ T_{j_1} . \]  

(4.4)

Because the Seebeck coefficient was nonlinear with temperature, the equation used to determine the temperature at junction one was a ninth order polynomial. The following polynomial was specific to type E thermocouples:

\[ T_{j_1} = a_0 + a_1V + a_2V^2 + a_3V^3 + \ldots + a_9V^9 \]  

(4.5)

where \( V \) was the voltage measured with the data acquisition system and the coefficients \( a_{0-9} \) are listed in Appendix E.

The experimental setup was designed to allow for the experimental apparatuses to be modified or replaced. To allow this the ends of the Type E thermocouples were connected to subminiature male connectors (part #, SMP-E-M, Omega Engineering Inc.) which could be plugged into a thermocouple junction box. The thermocouple junction box could accept up to 32 thermocouples. Type E extension wire was run from the 32 channels on the thermocouple junction box and were hardwired to copper extension wires from the data acquisition system. The 32 junctions created between the thermocouple
junction box and the extension wires were submerged in eight oil filled, sealed vials. The vials were then placed in an ice bath that consisted of approximately 90% ice and 10% water. Oil filled vials were used to maintain the same reference temperature over both the chromega-copper and the constantan-copper junctions for all thermocouples. This experimental setup allowed for different apparatuses to be analyzed with the ease of simply plugging them in.

4.1.3.1 Thermocouple Calibration

The accuracy of a Type E thermocouple was ± 1.7 °C or 5% of the temperature measurement, which ever was greater. Although the experiment only took place within a three or four degree Celsius temperature range, it was still practical to use thermocouples. The property estimation procedure was based on the difference between temperature measurements, not the actual value. Therefore, to estimate the thermal properties accurately the measurements did not need to be accurate, but precise and repeatable. A typical thermocouple, with proper grounding would fluctuate ±0.08 °C, which was adequate for the estimation procedure. To account for the ±1.7 °C inaccuracy, an offset had to be applied during post processing to bring all temperature measurements together at steady state. See Figures 4.7,a and b for typical measured temperature histories before and after being offset, respectively.

To gain confidence in the thermocouples readings, a thermocouple was compared with the temperature measurements from an accurate fluoroptic thermometer. The
Figure 4.7.a. Typical Raw Experimental Data (Not-Offset) of Temperature Histories from a Heated and Constant Temperature Boundary Condition.

Figure 4.7.b. Typical Modified Experimental Data (Offset) of Temperature Histories from a Heated and Constant Temperature Boundary Condition.
Luxtron 790 fluoroptic thermometer had been calibrated in an ice bath with an accuracy of ±0.1 °C at the calibration point, and ±1.0 °C within ±100 °C of the calibration point. A thermocouple was attached to the fluoroptic thermometer and submerged into an oil filled, sealed vial. The vial was placed in a beaker filled with ice and allowed to reach steady state. The beaker with the vial inside was slowly heated to a hundred degrees Celsius.

It was encouraging that the thermocouple and the fluoroptic thermometer response had the same slope. As expected, the thermocouple remained within the ±1.7 °C accuracy range at lower temperatures and was less accurate at higher temperatures. This indicated that the thermocouple would be capable of accurately measuring the three or four degree temperature difference in the experiments.

4.1.4 Electric Resistance Heaters

All of the electric resistance heaters used in the experimental apparatuses were produced by Minco Products Incorporated. The heaters consisted of a foil heating element with a specified electric resistance, encased in a thin layer of Kapton with 22 AWG insulated leads centered on one side; see Figure 4.3. Since all heaters used in the setups were stocked by Minco, they were readily available and inexpensive. (Custom made heaters were approximately fifteen times the cost of stocked heaters and required an additional three to four months to produce and ship.)

There were three criteria for determining which heater to order: the size of the heater, the electrical resistance of the heating element, and the location of the leads. The
first criteria, the size of the heater, was limited to the size of available, manufactured heaters. Because the composites were 76.2 mm x 101.6 mm (3.0 in x 4.0 in), all of the purchased heaters were rectangular with one 76.2 mm (3.0 in) side. The other dimension depended on the required heated area for the experiment. Due to the limited quantities of stocked, rectangular heaters with a 76.2 mm (3.0 in) side, the experiments were designed around available heaters. The second criteria for determining a heater was the electrical resistance of the foil heating element. For the fastest warm up time and the most constant heat flux, the heaters with the lowest electric resistance were chosen. The final criteria was lead location. Whenever possible, the heaters were purchased with the leads attached to the 76.2 mm (3.0 in) side of the heater. Again, there were a finite number of choices on the stocked heaters and occasionally, the ideal heater could not be found.

The electric resistance heaters did not produce a uniform heat flux. Since the heater consisted of a heating element encased in a layer of Kapton, hot spots formed over the heating element when power was applied. The non-uniform heat flux from the heater could have been diminished by adding a thin layer of aluminum to the surface of the heater or by redesigning the heating element. Although an aluminum coated heater would have improved the uniformity of the heat flux, it would have also generated a larger heat flux loss through the sides to the surroundings. In addition, the aluminum coated heater would create a delay in time from when power was applied to the heater to when the heat flux entered the composite.

Another problem with the electric resistance heaters was the internally attached leads.
When the heater was sandwiched between the composite plates, it was important that all the generated heat entered the composite plates. Due to the internally attached leads, the heaters could not make good contact with the composites. To avoid this problem, notches were added to the composites to provide room for the leads. Since different heaters had different lead locations, multiple notches had to be added to the composites to satisfy all possible lead locations.

One possible solution to the problems of a non-uniform heat flux and internally attached leads is use a custom designed heater. As previously mentioned, these heaters were not used due to extreme costs and production time. The custom designed heaters should be considered in future experiments because they can be designed with so that the heating element covers a greater surface area, which would make the heat flux more uniform. In addition, a custom designed heater could be tailored to a specific heated area and externally attached leads could be added.

4.1.5 Data Acquisition System

The National Instruments data acquisition system was used to acquire all experimental data. There were two purposes of the data acquisition system. First, the system had to record the voltages of thermocouples and convert the readings into temperatures. Second, the current and voltage measurements from the high resolution digital multimeters had to be recorded. The hardware and the software that were necessary to perform these two operations are described in the following two subsections.
4.1.5.1 Hardware for the National Instruments Data Acquisition System

A brief description of the National Instruments data acquisition system begins with a AT-MIO-16F-5 data acquisition board which was located in a 386 Blue Star personal computer. The board was attached to the SCXI-1100 module which was housed in the SCXI-1000 chassis. To complete the setup, the SCXI-1300 terminal block was attached to the front of the SCXI-1100 module. See Figure 4.8 for the complete data acquisition assembly with an attached thermocouple.

The AT-MIO-16F-5 data acquisition board, located in the personal computer, was the heart of the system. The board controlled the analog, digital, and timing input/output operations. Therefore, all measured signals had to go through the AT-MIO-16F-5 board. The board had eight differential channels, with the ability to amplify the signal from each channel. Although there were eight channels, only one channel, channel zero, was used in the experiments. Channel zero on the AT-MIO-16F-5 was connected to the SCXI-1100 module through a 50 pin cable ribbon.

The SCXI-1100 module acted as an extension of the AT-MIO-16F-5 board by increasing the number of usable channels from eight to thirty two. Similar to the data acquisition board, the SCXI-1100 module had the capability to amplify a signal with an on board programmable gain instrumentation amplifier. Therefore, a signal entering the SCXI-1100 module could have been amplified twice, once by the module and then again by the data acquisition board. The SCXI-1100 module was installed into an SCXI-1000 chassis with the original factory defaults.
Figure 4.8. Complete National Instruments Hardware Assembly with an Attached Thermocouple.
Aside-

There was an error in the SCXI-1100 reference manual. The manual clearly states that the jumper configuration for W1 (jumper pins) should be placed in a floating source setting for thermocouple readings. This was found to be erroneous by experimentally measuring thermocouple voltages with the jumper in the factory and the floating source setting. The floating source setting gave sporadic readings and was latter confirmed by National Instruments to be a faulty setting.

The purposes of the SCXI-1000 chassis were to provide power to the SCXI-1100 module and to house the module in a noise free environment. The chassis had a power supply with the ability to generate up to five volts that could be output through one of the SCXI-1100 channels. Originally, the power supply was used to create a voltage which was sent to the Hewlett Packard DC power supply. The DC power supply then amplified the signal which powered the heater. The problem with this setup was that when the chassis operated its own power supply, the incoming voltages from the thermocouples would be disturbed. Therefore, the chassis power supply was not used and the heater was powered directly from the Hewlett Packard DC power supply. Additionally, the chassis also acted as a ground for the SCXI-1100 module to provide a noise free environment for signal conditioning. The module was grounded to the chassis indirectly through a SCXI-1300 terminal block which was attached to both the chassis and the front sixty four pins of the SCXI-1100 module.

The main purpose of the SCXI-1300 terminal block was to provide access to the 32 differential channels of the SCXI-1100 module through screw terminals. A temperature
sensor was mounted on the terminal block, referred to as MTEMP, next to the screw in terminals. In order to use a thermocouple, the temperature at one end must be known. The intended purpose of the temperature sensor, MTEMP, was to provide this reference temperature, allowing for thermocouples to be directly screwed into the terminals on the terminal block. Since the MTEMP sensor had an accuracy of ±1 °C from 0 °C to 55 °C and a higher accuracy could be obtained with an ice bath, MTEMP was not used. The terminal block was the final piece of hardware in the National Instruments data acquisition system.

To provide a general overview of the setup, the path of the thermocouple voltage can now be traced from the bead of the thermocouple to the screen of the personal computer, refer to Figure 4.8. To begin, the thermocouple was attached to the experimental setup and plugged into the thermocouple junction box. The box contained extension wires that connected the thermocouples to copper extension wires in an ice bath. The copper extension wires were screwed into the terminal block which was attached to the SCXI module. The module through a 50 pin ribbon was connected to the AT-MIO-16F-5 data acquisition board. The board was housed in the computer which was manipulated by the software to read the thermocouple voltage.

4.1.5.2 Software for the National Instruments Data Acquisition System

This subsection describes some of the basic functions of the software used to manipulate the National Instruments hardware. The software, LabWindows for Dos, Version 2.2, produced by National Instruments, was a development system controlled by
programs written in a pseudo C language. The listing of the code used to control the experiments, Total.c, is presented in Appendix G and the flow chart is shown in Figure 4.9. The two functions of the program were to take thermocouple voltage readings and the current and voltage measurements from the multimeters.

The first goal of the program, Total.c, was to record the voltage from up to twenty two thermocouples. At the beginning of the program, the variables are declared and defined. Notice in the program, Total.c, listed in Appendix F, that there are two gain specifications, the MIOSCXIGAIN and the SCXIGAIN. The MIOSCXIGAIN represented the gain on the AT-MIO-16F-5 board and the SCXIGAIN was the gain on the SCXI module.

To maintain the resolution on a small signal it was best to amplify the value as large and as soon as possible. The hardware could amplify a signal brought in through the SCXI board twice. Since the voltage signal came through the terminal block, the first amplification of the signal took place with the amplifier on the SCXI-1100 module. The SCXIGAIN was set to 2000, which was the largest possible amplification gain setting for the module. The incoming voltage from a thermocouple at room temperature was in the range of one to two millivolts. Therefore, a gain of 2000 produced a signal between two and four volts. Since the limitation of the data acquisition system was five volts, no additional gain was applied with the AT-MIO-16F-5 data acquisition board.

A disadvantage of the data acquisition system was that the gain settings for all thirty-two channels on the SCXI-1100 module had to be the same. The AT-MIO-16F-5 data acquisition board had the capability of having different gain settings for different
Figure 4.9. Flow Chart for the Data Acquisition Program, Total.c
channels. If the data acquisition system needed to record the voltage of a small and large signal together, it is recommended to bring the small signal in through the SCXI-1100 module and the large signal directly into the data acquisition board.

After the variables had been defined and declared in the program, the hardware was configured. Once the system completed hardware identification, the offset value for the channels on the SCXI module were determined. Due to imperfect grounding of the SCXI module, stray voltages existed and appeared in the measurements. In an attempt to negate these additional voltages, a thousand values were averaged from a grounded channel on the module. This value was then used as an offset and subtracted from all voltage readings on the SCXI module.

Next, double-buffered multiplexing was performed to read the voltages from all of the thermocouples. Multiplexing represents a multiple channel scanned data acquisition operation which allows the data acquisition system to read a specified number of voltages from multiple channels. A buffer is simply an array of data points and a double-buffered operation refers to a circular buffer. When the end of the buffer is reached, the board returns to the beginning of the buffer and continues to write. Therefore, with double buffering, the board can effectively take data forever. When it is combined with multiplexing, the data acquisition system can continuously take data over multiple channels.

After a sufficient number of voltage readings were taken from all channels, the current and voltage were acquired from the digital multimeters. RS-232 was used to send the measurements from the meters to the computer through serial ports. There was a
certain protocol, referred to as handshaking, for the computer and multimeters to work together. First a signal from the computer was sent to the multimeters requesting a measurement. Next, after a real time delay of 0.01 seconds, measurements were sent back to computer.

The overall concept of handshaking was relatively simple, but the act of having two machines communicate proved to be challenging. An RS-232 breakout tester was extremely useful in determining which lines of the cable connecting the meter to the computer were activated. The breakout tester plugged into the serial port of the computer and attached to the cable of the meter. When a signal was sent from the computer, a light on the tester would illuminate distinguishing which line was activated. In addition to the breakout tester, a modem, talk only, software was used. This helped break down the protocol specific to LabWindows and allowed the computer to freely communicate with the meters.

After the thermocouple voltages were recorded and the voltage and current readings were received from the meters, the program was repeated for the next reading. Since the program was set in a loop, measurements were taken continuously until the program was terminated with a key hit. Therefore, the program satisfied all of the necessary requirements needed to measure the temperature histories of the composite while a heat flux was applied.
4.2 Experimental Procedures

There were two objectives in this study, to estimate the one- and two-dimensional thermal properties and then to demonstrate that optimized experiments produced the most accurate estimates. In this section, the experimental procedures used to obtain these two goals are described.

Subsection one focuses on the assembly and installation of the one- and two-dimensional apparatuses into the overall experimental setup. Subsection two concentrates on the procedures used in running the experiments to obtain the necessary data. The section concludes with a description of experiments performed to verify that the optimal designs do provide the most accurate estimates.

4.2.1 Apparatus Assembly and Installation

The assembly and installation procedures for both the one- and two-dimensional apparatuses were similar, the only differences were the number of thermocouples to install and the boundary conditions. The step-by-step procedures are given below.

Assembly and Installation Procedures

Step 1 The assembly procedure began with producing eight thermocouples for the one-dimensional apparatus and twenty-two thermocouples for the two-dimensional apparatus. Refer to Appendix F for the procedure to fabricate a thermocouple. A majority of the experimental setup time was involved in this step.
**Step 2.** The thermocouples were then installed into the composites with super glue. For the one-dimensional analysis, four of the eight thermocouples were placed into the two grooves on the bottom surface of each composite. For the two-dimensional apparatuses, twelve additional thermocouples were added to the side grooves. The first step in installing a thermocouple was to hold the thermocouple in the groove while placing the bead in the center of the composite.

The thermocouple was then checked to ensure none of the insulation over the chromega or constantan wires was on the surface of the composite. A small amount of super glue was then added, being careful not to cover the bead of the thermocouple. The super glue usually took three to five seconds to harden and attach the thermocouple to the composite. It was important not to use excessive super glue which would insulate the composite from the adjoining material. An advantage of using super glue over other adhesives, such as epoxies, was that a thermocouple could be removed from the composite without being destroyed. The thermocouple could be removed directly by pulling it out through the super glue or by adding acetone to dissolve the glue.

**Step 3.** After installing the thermocouples, their resistance was checked with a digital multimeter. If the multimeter measured an infinite resistance, the thermocouple was broken. Time was saved by continuously checking the resistance of the thermocouples through out the assembly procedure.

**Step 4.** The polished surface of each aluminum block was cleaned and a thin layer of thermal paste was applied. Between experiments it became important to remove old layers of thermal paste which collected dust and other small particles when exposed to the air. After the paste was applied to the surfaces, excess was removed with a straight edge. The paste was smoothed until a uniform, thin layer, void of bare spots, was produced.
Step 5 Thermal paste was applied to the bottom surface of each composite. Care was taken in removing the excess paste not to disturb the inlaid thermocouples.

Step 6 The bottom, greased, surface of each composite was placed onto the greased surface of the aluminum blocks. The composites were adjusted to align them directly over each aluminum block. Pressure was applied, by hand, to the top surface of each composite. Due to the viscosity of the paste, the composites had a tendency to slide when pressure was applied. If this occurred, the composites were realigned.

Step 7 Thermal paste was applied to the top surface of each composite. Again, the excessive grease was removed with a straight edge forming a thin, uniform layer.

Step 8 The remaining four thermocouples of the one-dimensional apparatus were super glued to the surface of the 76.2 mm x 101.6 mm (3.0 in x 4.0 in) heater. Two thermocouples were placed on each side of the heater, with their beads located in the center. To negate the effects of the non-uniform heat flux of the heater, two thermocouples were super glued directly over the heating element while the other two were glued over the gap. For the two-dimensional apparatus, six thermocouples were super glued to a heater, placing three on each side. Various size heaters were used in the two-dimensional experiments depending on the optimal or non-optimal heated area designs. One thermocouple, on each side of the heater, was located in the optimal temperature sensor location while the remaining two were placed in non-optimal locations. Again, to account for the non-uniform heat flux of the heater, three thermocouples were located directly over the heating element and the other three in the gap.
Step 9 One side of the heater was coated with thermal paste. This was the most difficult surface to coat because the thermocouples were on top of the surface and not in groves. Extra caution was taken to prevent the thermocouples from ripping when the grease was applied. Three, unavoidable bumps, on each side of the heater appeared in the thermal grease due to the thermocouples. The size of these bumps were reduced when pressure was applied to the setup causing the thermocouples to sink into the outer Kapton layer of the heater.

Step 10 The greased surface of the heater was placed onto the greased, top surface of a composite. The heater was aligned by placing its leads in the notch on the composite. Pressure was then applied to the heater to ensure good contact with the composite.

Step 11 The face up side of the heater was greased and the second greased composite/aluminum block was laid on top of the heater. Again, the notch in the composite was aligned with the leads of the heater.

Step 12 The entire setup was then placed between a brace. The bolts on the brace were tightened by hand resulting in slight pressure to the assembly. This pressure and the thermal grease caused the composite plates to slide a little over the aluminum blocks. The plates were then realigned and the bolts were further tightened with a wrench. Enough pressure was applied to the apparatus to cause thermal grease to squeeze out on each side.

Step 13 The boundary conditions on the sides of the composites were then applied. The thermal grease, that squeezed out between the layers of materials, was spread over the edges of the composites with a small paint brush. For the one-dimensional apparatus and Configuration 2, pipe insulation was wrapped
around the outside of all four sides of the composites. To ensure good contact between the insulation and the composites, rubber bands were placed around the outside of the insulation to apply pressure. For Configuration 1, two heat sinks were placed on the sides of the composites. One of the heat sinks had a 3.18 mm (0.125 in) drilled hole to allow the leads of the heater to go through. A second brace was then placed horizontally around the apparatus. The screws on the brace were tightened forcing the aluminum blocks against the sides of the composites. The remaining two sides were then insulated with pipe insulation. The completion of the boundary conditions finished the assembly of the apparatuses.

**Step 14** After completing the apparatus assembly, it was installed into the overall experimental setup. The eight thermocouples from the one-dimensional analysis and the twenty-two thermocouples from the two-dimensional analysis were plugged into the thermocouple junction box. The positive lead of the heater was attached to both the negative lead of the ammeter and the positive lead of the voltmeter. The negative lead of the heater was connected to both the negative lead of the power supply and the negative lead of the voltmeter. As a result, the ammeter was in series and the voltmeter was in parallel with the heater.

**Step 15** To complete the installation a grounding wire was attached from the brace to the data acquisition system. An old thermocouple was used which was wrapped around one of the bolts on the brace and plugged into the thermocouple junction box.

At this point the experimental apparatuses were completely assembled and installed. All apparatuses were prepared in this manner regardless of optimal and non-optimal
4.2.2 Experimental Testing Procedures

After completing the assembly and installation of the experimental apparatuses the system was ready to perform an experiment. In this subsection, a description of the necessary experimental procedure to execute the experiment for either the one- or two-dimensional apparatuses is given. The procedures, shown below were identical for both apparatuses.

**Experimental Testing Procedures**

**Step 1** The power supply and two multimeters that were attached to the electric resistance heater were turned on. The multimeters were used to record the current and voltage drop across the heater. The voltage output from the power supply was adjusted until the heat flux from the heater was approximately 380 W/m²,

\[
\text{heat flux} = \frac{\text{current} \times \text{voltage}}{\text{area of the heater}} = 380 \frac{W}{m^2}.
\] (4.6)

The positive lead on the ammeter was detached from the power supply. This effectively turned the heater off since this meter was in series with the power supply and heater.

**Step 2** While the heater and apparatus were cooling, the sealed, oil vials, which attached the thermocouple junction box and the terminal block of the data acquisition system, were placed in an ice bath. The ice bath consisted of 90% ice and 10% water. The experiment was delayed approximately twenty minutes to allow the oil to cool to 0° C.
Step 3  While waiting for the oil to cool, the computer was turned on and the National Instruments LabWindows software was loaded. The program, Total.c, was opened and the input variables were set: the number of channels to be read, the starting channel number, and the number of points to average per thermocouple.

Step 4  To complete the setup the digital multimeters had to be plugged into the serial ports on the Blue Star personal computer.

Step 5  When the setup was ready to take data, a stop watch was started and the program, Total.c, was compiled and executed. Data was taken for a hundred seconds, measuring the setup at steady state room temperature. This initial hundred seconds of measurements was necessary to determine the thermocouple offsets performed during post processing.

Step 6  After a hundred seconds the stop watch was stopped and cleared. The positive lead of the ammeter was connected to the positive lead of the power supply. The stop watch was restarted. When the positive leads were connected power was applied to the heater.

Step 7  The positive lead of the digital multimeter was removed after completing the required heating time of the experiment. While taking temperature measurements, the experimental setup was allowed to cool to room temperature.

Step 8  Once the temperatures of all the thermocouples reached the steady state room temperature, the program, Total.c, was terminated.
This completed the experimental procedure used to take the necessary data to estimate the one- or two-dimensional thermal properties.

4.2.3 Chronological Order of Experiments

In this subsection, a description of the experiments performed to verify that the optimal designs provided the most accurate estimates is given. To produce a good statistical basis to make conclusions about the estimated thermal properties, every experimental design was repeated nine times. Early in the research, it was noticed that the estimated thermal properties fluctuated from experimental setup to experimental setup. Therefore, when an experimental apparatus was assembled, it was tested three times. Then the apparatus was torn down and put back together and retested another three times. This was performed a third time to produce a total of nine sets of results for every experimental design. Hopefully, the repetition averaged out any experimental errors that might have been performed in any one experimental setup.

The experiments began with estimating the one-dimensional thermal properties. Since the focus of this study was not on the one-dimensional analysis, the experiments were performed with Moncman's (1994) optimal design. Nine experiments were executed with three different setups.

For both configurations of the two-dimensional analysis, the experiments were optimized for five experimental parameters. Only three of the parameters were verified with non-optimal experimental settings; the sensor location in the y direction, the heating area, and heating time. But for each of these parameters, two non-optimal settings were
determined. For example, if the optimal heating time was 1.37, then non-optimal heating times of 1.0 and 0.5 were tested. Recall, to verify the optimal parameter settings, the thermal properties were estimated with both optimal and non-optimal parameters. The parameter settings which provide the smallest confidence intervals were the best design.

To provide a controlled study, only one parameter was modified per experimental design. Experiments were first performed with the optimal designs for Configuration 1. Then the heating time was adjusted to its first non-optimal time while the sensor location and heating area remained at their optimal settings. After completing the tests with the first non-optimal heating time, the second (a worse) non-optimal heating time was tested. Experiments then progressed into testing the non-optimal sensor locations. Since it was possible to place multiple thermocouples on the surface of the heater both non-optimal sensor locations were tested in the same experiments. While the sensors were in non-optimal locations, the tests were performed with the optimal heating area and heating time. After completing the non-optimal sensor locations, the non-optimal heating areas were tested. A similar procedure was performed where the non-optimal heating areas were tested with both optimal sensor location and heating time. Remember that there are two different configurations; therefore, these test were repeated for the second configuration. Table 4.1 lists the order of the experiments performed for both configurations. To reduce the effect of the experimenter's error, each design (or row in Table 4.1) was tested three times with three different setups. Therefore, a total of ninety experiments with eighteen original setups were performed to verify the optimal designs of both configurations.
Table 4.1. Parameter Settings of Experiments Performed, Used to Verify the Two-Dimensional Optimal Experimental Designs of Configurations 1 and 2.

<table>
<thead>
<tr>
<th>Optimal Experimental Designs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st non-optimal heating time with optimal heating area and sensor location</td>
</tr>
<tr>
<td>2nd non-optimal heating time with optimal heating area and sensor location</td>
</tr>
<tr>
<td>Both non-optimal sensor locations with optimal heating area and heating time</td>
</tr>
<tr>
<td>1st non-optimal heating area with optimal sensor location and heating time</td>
</tr>
<tr>
<td>2nd non-optimal heating area with optimal sensor location and heating time</td>
</tr>
</tbody>
</table>

4.3 Experimental Data

The focus of this section is to describe the experimental data and demonstrate how it is implemented into the finite element estimation program. Figure 4.10 demonstrates a typical data set obtained from the optimal design of Configuration 1. The plot represents the temperature histories of the heated surface, the bottom and side surfaces, which were in contact with aluminum heat sinks. The most notable feature of the plot is the exponential rise and fall of the temperature history at \( y^* = 0.86 \) on the heated surface. This corresponds with the applied heat flux. A heat flux of 380 W/m² (120 Btu/(h ft²)) was applied for 130 seconds and then it was removed and the composites were allowed to cool back to room temperature. This temperature history was the observation vector used in the modified Box-Kanemasu method. There are four additional temperature histories on the plot that were recorded from the sides and bottom surface of the composite. Notice that there were unavoidable temperature rises in all histories.
Figure 4.10. Experimental Temperature Histories from Configuration 1 with the Optimal Experimental Design.
despite their location next to aluminum heat sinks. These four temperature histories were implemented into the finite element program as a specified known temperature boundary condition.

Experimental data obtained at the optimal and non-optimal sensor locations for Configuration 1 with the optimal heating area and heating time is shown in Figure 4.11. Notice that the temperature history at \( y^* = 0.93 \) did not reach the same temperature as the optimal sensor location. This is because the sensor was closer to the side heat sink. The exact opposite is true for the sensor located at \( y^* = 0.76 \). Again, the sensors located along the constant temperature boundary conditions demonstrated a slight temperature rise.

Experimental data with the optimal and non-optimal heating times for Configuration 1 with the optimal heating area and sensor location is shown in Figure 4.12. The goal of the optimal heating time is to allow the composite to come close to reaching steady state. Therefore, the two non-optimal heating times were determined at shorter heating times. The temperature histories shown in Figure 4.12 were not obtained from the same experiment but from three different tests and their result were superimposed onto the same plot. As expected, all three temperature histories had the same initial temperature rise but the two non-optimal heating times did not allow the composite to reach its steady state temperature.

A typical data set obtained from Configuration 2 with its optimal experimental design is plotted in Figure 4.13. The largest temperature rise represents the temperature history at the heated surface at the optimal temperature sensor location. While the lower three
Figure 4.11. Experimental Temperature Histories from Configuration 1 at Optimal and Non-Optimal Sensor Locations, $y^+$. 

Temperature Histories at the Heated Surface

- $y^+ = 0.76$ Non-Optimal Sensor Location
- $y^+ = 0.86$ Optimal Sensor Location
- $y^+ = 0.93$ Non-Optimal Sensor Location
Figure 4.12. Experimental Temperature Histories from Configuration 1 at Optimal and Non-Optimal Heating Times, $(t_h)^*$. 
Figure 4.13. Experimental Temperature Histories from Configuration 2 with the Optimal Experimental Design.
temperature histories represent the temperature of the left side of the composite; close to the heater, in the middle, and close to the heat sink. The increased rise in the side temperature histories was due to the insulated boundary conditions of Configuration 2. Since the optimal heating area for Configuration 2 was only eleven percent, the temperature history along the right side and bottom surface of the composite remained constant. Configuration 2 generated non-optimal parameter plots similar to Configuration 1 but with different side temperature histories.

This concludes the experimental methods used to obtain the required measured temperature history for the modified Box-Kanemasu method. The described experiments were performed and the resulting estimated properties are discussed in the next chapter.
CHAPTER 5

Results and Discussion

The optimization procedure described in Section 3.3 for estimating two-dimensional thermal properties was applied to both experimental apparatuses, Configurations 1 and 2. From the procedure, the optimal parameter settings were determined and are presented along with the resulting estimated thermal properties. Initial success was not obtained for the simultaneous estimation of all three thermal properties due to correlation, and the focus of the research changed to optimizing the configurations for only two thermal properties, the effective in-plane thermal conductivity, $k_{\text{eff}}$, and volumetric heat capacity, $C_{\text{ep}}$. Since it was possible to estimate two thermal properties, non-optimal experimental designs were developed around the optimal parameter settings for both configurations. The purpose of the non-optimal designs is to demonstrate that the properties are less accurately estimated than with the optimal design. To perform this analysis, a parametric approach was implemented where only one of the three parameters per experiment was varied. This discussion of these results concludes with the estimated thermal properties from optimal and non-optimal designs.
5.1 One-Dimensional Analysis

Although the focus of this research was on the two-dimensional analysis of a carbon-fiber/epoxy-matrix composite, the research began with a one-dimensional analysis. Similar analyses had been performed by Moncman (1994) on carbon composites. The experiments were repeated in this study to gain confidence in the experimental setup and procedure and to verify previous results.

In addition to the confidence, the one-dimensional analysis provided a check for the two-dimensional study. The thermal properties of a composite do not change due to the applied analysis technique. Therefore, the one-dimensional thermal properties, the effective thermal conductivity through the thickness, $k_{\text{eff}}$, and the effective volumetric heat capacity, $C_{\text{eff}}$, should be reproducible in the two-dimensional analysis. The following discussion describes the optimal one-dimensional experimental design and the resulting estimated thermal properties.

5.1.1 One-Dimensional Optimal Experimental Design

The goal of the optimization procedure was to determine the parameter settings which provided the largest determinant of the $X^TX$ matrix, thereby creating the smallest confidence intervals. For the one-dimensional analysis, the optimized experimental parameters were the sensor location, heating time, and experiment time. Moncman (1994) had developed the optimal one-dimensional experimental designs in non-dimensional quantities. The optimal sensor location occurred at

88
\[ x' = \frac{x}{L_x} = 0 \]  

(5.1)

where \( x \) is the distance from the heater and \( L_x \) is the thickness of the composite. Therefore, the optimal sensor location occurred at the heated surface. The remaining two parameters, the optimal heating time and experiment time were calculated from

\[ t_h = \frac{L_x^2 C_{eff} t_h^*}{k_{x-eff}} \quad t_N = \frac{L_x^2 C_{eff} t_N^*}{k_{x-eff}} \]  

(5.2 a,b)

where \( t_h^* \) and \( t_N^* \) were the non-dimensional heating time and experiment duration time, respectively. From Moncman's one-dimensional study the non-dimensional heating time, \( t_h^* \), and overall experiment time, \( t_N^* \), occurred at 2.2 and 5.0 respectively. Notice that both of the equations required the effective thermal properties, \( k_{x-eff} \) and \( C_{eff} \), which were being estimated in the analysis. Therefore an initial estimate for the properties, based on previous known composites, was applied. If the actual estimates from the experiments were found to be completely different, the optimal parameter settings would have been recalculated and the experiment redesigned.

It is important to graph the sensitivity coefficients of the properties of an experimental design. Graphs can give insight into the thermal properties, demonstrate correlation, and determine if adequate information is available for accurate estimates. The non-dimensional sensitivity coefficients created from the one-dimensional optimal design are shown for both thermal properties in Figure 5.1. Notice that the sensitivity coefficient for the thermal conductivity through the thickness, \( k_{x-eff} \), was larger in magnitude than the coefficient for the volumetric heat capacity, \( C_{eff} \). Therefore, it was assumed that the
Figure 5.1. Dimensionless Sensitivity Coefficients, $(X_{k_{df}})^*$ and $(X_{C_{df}})^*$, for the Optimal One-Dimensional Experimental Design.
experiment provided more information for the effective thermal conductivity than the volumetric heat capacity, which should result in smaller confidence intervals.

5.1.2 One-Dimensional Thermal Property Estimates

One-dimensional thermal property estimates along with their 95% confidence intervals are shown in Table 5.1 and graphically represented in Figure 5.2. The 95% confidence interval represents the range of values which the actual thermal property lies within for a particular experiment. Figure 5.2 represents the results from nine different experiments obtained from three original setups with three repetitions per setup. All experiments were performed with the same two composite samples which were obtained from the same manufactured batch. The tenth case is the average of the nine experiments with a 95% confidence interval placed around the average. This confidence interval was calculated from

\[
\text{ConfidenceInterval}_{\text{Mean}} = \pm \frac{t_{\alpha/2} \cdot s}{\sqrt{N}}
\]

(5.3)

where \(s\) is the standard deviation, \(N\) is the number of experiments (9), \(t\) is the student \(t\) distribution for \(N-1\) degrees of freedom, and \(\alpha/2\) is the confidence range (Walpole and Myers, 1978).

The estimated properties were very consistent as shown in Figure 5.2. Superimposed on the figure were the average values of both thermal properties (represented as the horizontal lines). Notice that all but two of the estimated thermal conductivities and three of the volumetric heat capacities held the mean value within their 95% confidence
Figure 5.2.a. One-Dimensional Through the Thickness Thermal Conductivity Estimates, $k_{\text{eff}}$, with 95% Confidence Intervals.

Figure 5.2.b. One-Dimensional Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, with 95% Confidence Intervals.
regions. Recall, that the 95% confidence interval for an experiment is given in Eq. (3.9) which is influenced by the sensitivity coefficients. It is different than the 95% confidence interval calculated in Eq. (5.3), which is an interval placed on the variance of the mean values. In addition, notice from Table 5.1 that there was less fluctuation in the estimated thermal conductivity from experiment to experiment. This was a result of better sensitivity on the thermal conductivity as predicted by the sensitivity coefficients.

The one-dimensional experiments provided a good basis from which to begin the two-dimensional analysis. By producing consistent estimates, which closely matched the properties predicted by Moncman (1994) for a similar composite, the experimental setup

Table 5.1. One-Dimensional Thermal Property Estimates with 95% Confidence Intervals.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>$k_{\text{eff}}$ (W/m K)</th>
<th>$C_{\text{eff}}$ (MJ/m$^3$ K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.618 ± 0.003</td>
<td>1.51 ± 0.02</td>
</tr>
<tr>
<td>2</td>
<td>0.618 ± 0.002</td>
<td>1.51 ± 0.01</td>
</tr>
<tr>
<td>3</td>
<td>0.613 ± 0.002</td>
<td>1.52 ± 0.02</td>
</tr>
<tr>
<td>4</td>
<td>0.616 ± 0.002</td>
<td>1.53 ± 0.01</td>
</tr>
<tr>
<td>5</td>
<td>0.620 ± 0.002</td>
<td>1.52 ± 0.01</td>
</tr>
<tr>
<td>6</td>
<td>0.614 ± 0.002</td>
<td>1.54 ± 0.01</td>
</tr>
<tr>
<td>7</td>
<td>0.618 ± 0.002</td>
<td>1.53 ± 0.02</td>
</tr>
<tr>
<td>8</td>
<td>0.614 ± 0.003</td>
<td>1.55 ± 0.02</td>
</tr>
<tr>
<td>9</td>
<td>0.616 ± 0.002</td>
<td>1.53 ± 0.01</td>
</tr>
</tbody>
</table>

| Mean       | 0.616 ± 0.002            | 1.53 ± 0.01                 |
and procedure were verified. In addition to verification, the one-dimensional analysis provided estimates for two of the three thermal properties which can also provide a check for the two-dimensional analysis.

5.2 Simultaneous Estimation of Three Thermal Properties

After completing the one-dimensional analysis, the objective was to design an experiment that allowed enough sensitivity from one temperature history to estimate the effective thermal conductivity in two directions, $k_{x_{\text{eff}}}$ and $k_{y_{\text{eff}}}$, and the effective volumetric heat capacity, $C_{\text{eff}}$, simultaneously. The discussion begins with a description of the optimal experimental designs used to estimate all three properties for both configurations. After completing the optimal designs, they were tested in an ideal case, where the three thermal properties were independently estimated with exact data. Finally, the section concludes with the estimated thermal properties obtained from Configuration 1.

5.2.1 Optimal Two-Dimensional Experimental Designs for Estimating Three Thermal Properties

Configurations 1 and 2, both two-dimensional apparatuses, were optimally designed for the simultaneous estimation of three thermal properties. The optimization procedure described in Section 3.3 was developed by Moncman (1994) based on analytic temperature distributions. Unlike the optimization of the one-dimensional experiment, the two-dimensional analysis had no non-dimensional equations to determine the optimal
settings. Therefore, the optimization programs developed by Moncman (1994) for Configurations 1 and 2 had to be tailored them to the specific experiments. Two quantities were required, the thickness of the composite, \( L_x \), and the ratio of the thermal conductivities, \( k_{xy} = k_{y,\text{eff}} / k_{x,\text{eff}} \). Since the ratio of the thermal conductivities, \( k_{xy} \), was unknown, an initial estimate of 5.0 was used, based on previously reported data for similar composites. If this guess proved to be inaccurate the procedure would have been repeated with a better estimate.

The optimization procedure for both configurations was a parametric process which had two phases. Phase one was performed with a coarse grid over all parameters while phase two used a refined grid specifically around the settings predicted by phase one. In the first phase of the program, four out of the five parameters were allowed to vary; the experiment time \( (t_N^+) \), the heating time \( (t_h^+) \), the sensor location in the \( y \) direction \( (y^+) \), and the heating area \( (L_h^+) \). The fifth parameter, sensor location in the \( x \) direction \( (x^+) \), was fixed to zero due to the physical limitations of the experiment. Figures 5.3 and 5.4 represent the results of phase one for both configurations. Each line on these figures was produced from a fixed heating area while the sensor location, heating time and experiment time varied. When the combination of the experiment and heating time produced the largest determinant of the \( X^+TX^+ \) matrix at a given sensor location, the determinant and parameters were recorded. This was then repeated for all of the heating areas from 5% to 100% in increments of 5%. From the figures, the maximum determinant for Configuration 1 occurred with a heating area of 100%, and 15% for Configuration 2. The remaining parameters determined from phase one for both configurations are listed in
Figure 5.3. Phase One, Experimental Optimization of Configuration 1 for the Simultaneous Estimation of Three Thermal Properties.

Figure 5.4. Phase One, Experimental Optimization of Configuration 2 for the Simultaneous Estimation of Three Thermal Properties.
Table 5.2. Again, these values were not exact because phase one incorporates a coarse grid size.

One of the disadvantages of performing a parametric analysis was the possibility of overlooking the maximum determinant. When Moncman (1994) performed the optimization analysis on Configuration 2, she started with a heating of 20%. Therefore the optimal heating area was found to be around 90%, as demonstrated in Figure 5.4. In this research the configuration was evaluated with an optimum heating area starting at 5%. Within the range of 5% to 20% a new set of parameters was determined that produced a larger determinant than the optimal design previously developed by Moncman. This new predicted optimal design makes physical sense for two reasons. First, a smaller heating area allows for greater heat conduction through the sides. Second, the sensor location at the heated corner, next to the insulation is the warmest location in the apparatus. Therefore at \( x = y = 0.0 \) the most sensitive information is obtained.

Table 5.2. Phase One, Optimal Experimental Designs for the Simultaneous Estimation of Three Thermal Properties for Configurations 1 and 2.

<table>
<thead>
<tr>
<th>Experimental Parameter</th>
<th>Configuration 1</th>
<th>Configuration 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Location, ( x^* )</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sensor Location, ( y^* )</td>
<td>0.15</td>
<td>0.00</td>
</tr>
<tr>
<td>Heating Time, ( t_h^* )</td>
<td>1.40</td>
<td>1.40</td>
</tr>
<tr>
<td>Heating Area, ( L_p^* )</td>
<td>1.00</td>
<td>0.15</td>
</tr>
<tr>
<td>Max. Determinant ( \times 10^7 )</td>
<td>5.38</td>
<td>5.10</td>
</tr>
</tbody>
</table>
After completing phase one of the analysis, phase two began by reducing the increment size around the estimated optimal parameter settings predicted by phase one. The heating area for Configuration 1, was reduced from a 5% increment over the entire surface to 1% between the areas of 90% and a 100% of the composite surface. The remaining three parameters, the sensor location in the y direction, the heating time, and experiment time were allowed to vary with their original coarse grid. For Configuration 1, the optimal heating area remained at 100% of the surface but for Configuration 2 the 15% area predicted by phase one was reduced to 14%. These two heating areas, 100% and 14%, became the optimal heating areas for the remainder of the optimization procedure.

After the heating areas were fixed, the second step of phase two allowed the sensor location in the y direction to vary with a smaller increment. Once the sensor location was determined with a fine grid, it also became a fixed parameter in the program. The procedure was repeated with the heating time and again for the experiment time. Results for both optimal experimental designs are shown in Table 5.3 which also includes the optimal experimental design produced by Moncman (1994) for Configuration 2. It should be noted that these optimal designs are for the specific thickness of the analyzed composite, $L_z = 0.006$ m (0.240 in) and ratio of thermal conductivities $k_{xy} = k_{y\text{eff}}/k_{x\text{eff}} = 5.0$. For additional optimal designs for various thicknesses and ratios refer to Moncman (1994).

The resulting maximum determinants obtained for each optimal experimental design shown in the table are also graphically represented in Figure 5.5. Configuration 1
Figure 5.5. Comparison of the Dimensionless Determinants, $D^*$, for Configuration 1, 2, and Moncman's (1994) Configuration 2.
achieves a slightly larger determinant than Configuration 2, therefore the estimates from Configuration 1 should be slightly more accurate. The fact that the maximum determinant from the new Configuration 2 is closer to the maximum determinant for Configuration 1 than originally predicted by Moncman is important. The insulated boundary conditions of Configuration 2 were chosen because they were easy to apply and maintain experimentally. Although Configuration 1 has a larger determinant than Configuration 2, the resultant difference in accuracy will not be significant.

The non-dimensional sensitivity coefficients for the optimal experimental design of Configuration 1 were plotted to give insight into the properties and are shown in Figure 5.6. Since the coefficients for Configuration 2 were virtually identical, they have been omitted for clarity. Notice that the largest coefficient was the thermal conductivity through the thickness of the composite. While the smallest non-dimensional sensitivity coefficient was the in-plane thermal conductivity with a magnitude of 0.12. As a general


<table>
<thead>
<tr>
<th>Experimental Parameter</th>
<th>Conf. 1</th>
<th>Conf. 2</th>
<th>Moncman's Conf. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Location, x⁺</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sensor Location, y⁺</td>
<td>0.86</td>
<td>0.00</td>
<td>0.76</td>
</tr>
<tr>
<td>Heating Time, tₜ⁺</td>
<td>1.36</td>
<td>1.41</td>
<td>1.58</td>
</tr>
<tr>
<td>Heating Area, Lₚ⁺</td>
<td>1.00</td>
<td>0.14</td>
<td>0.88</td>
</tr>
<tr>
<td>Max. Determinant x 10⁷</td>
<td>5.38</td>
<td>5.25</td>
<td>4.28</td>
</tr>
</tbody>
</table>
Figure 5.6. Dimensionless Sensitivity Coefficients, \((X_{k_{\text{eff}}})^*\), \((X_{C_{\text{eff}}})^*\), and \((X_{k_{y_{\text{eff}}}})^*\), for the Optimal Experimental Design of Configuration 1.
rule, a magnitude 0.10 is required for adequate information in the accurate estimation of the thermal property. Therefore, the designs appeared feasible and the only concern was possible correlation between the two effective thermal conductivities.

5.2.2 Analytical and Experimental Results

The optimal experimental parameters presented in Table 5.3 for Configuration 1 and 2 were first tested with a numerical model prior to performing the two-dimensional experiments. A temperature history at the optimal temperature sensor location was obtained from a finite element model with specified two-dimensional thermal properties. This temperature history, was then used as the observation vector in the modified Box-Kanemasu method. From the temperature history all three original thermal properties were backed out by the property estimation program. This verified that it was possible, with exact data, to estimate all three thermal properties independently and that the two-dimensional estimation program was working properly. Due to this initial success, experiments were performed with Configuration 1.

The optimal experimental parameters for estimating three thermal properties were implemented into Configuration 1. The data obtained from the experiments were processed and run through the two-dimensional finite element property estimation program, 2dpe3vbc. After nine iterations it became apparent that the program was not going to converge to a unique solution. Hoping that the cause of the problem was poor initial estimates, the program was rerun several times with different initial estimates. Similar, non-converged solutions were obtained which indicated a correlation problem.
The modified Box-Kanemasu method is a fairly lenient estimation procedure that allows for poor initial estimates and still converges within seven iterations. After the ninth iteration, the program was terminated and the final, non-converged estimates were

\[ k_{x\text{-eff}} = 0.679 \frac{W}{m\ K}, \quad C_{\text{eff}} = 1.47 \frac{MJ}{m^3\ K}, \quad k_{y\text{-eff}} = 1.11 \frac{W}{m\ K}. \]

Recall from the one-dimensional analysis that the conductivity through the thickness, \( k_{x\text{-eff}} \), was 0.616 ± 0.002 W/(m K) (0.356 ± 0.001 Btu/(h ft °F)) and the volumetric heat capacity, \( C_{\text{eff}} \), was 1.53 ± 0.01 MJ/(m³ K) (22.8 ± 0.2 Btu/(ft³ °F)). The disagreement in the estimates can be explained by the approximate correlation matrix shown in Table 5.4. Since the matrix is symmetric only half is displayed. The diagonal elements of the matrix are all unity, which indicates that each thermal property is correlated with itself, which makes sense. All of the off-diagonal coefficients have an absolute value greater than 0.90. This indicates that the effective thermal properties were highly correlated with each other. As a general guideline, if the coefficients exceed 0.90, it becomes difficult to estimate the thermal properties independently and accurately. Although the high

<table>
<thead>
<tr>
<th></th>
<th>( k_{x\text{-eff}} )</th>
<th>( C_{\text{eff}} )</th>
<th>( k_{y\text{-eff}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_{x\text{-eff}} )</td>
<td>1.000</td>
<td>-0.986</td>
<td>-0.998</td>
</tr>
<tr>
<td>( C_{\text{eff}} )</td>
<td></td>
<td>1.000</td>
<td>0.985</td>
</tr>
<tr>
<td>( k_{y\text{-eff}} )</td>
<td></td>
<td></td>
<td>1.000</td>
</tr>
</tbody>
</table>
correlation was not desired, it helped to clarify why the two-dimensional analysis could not converge to one solution. In addition it explains the disagreement between the Table estimates from the one- and two-dimensional analysis. The high correlation also explains why it was possible to estimate the thermal properties with exact data and not with experimental data containing noise.

Configuration 1 was tested a few more times with the optimal experimental designs, each set resulting in similar uncorrected estimates. This approach was terminated and the focus of the research switched to estimating two thermal properties.

5.3 Simultaneous Estimation of Two Thermal Properties

Due to the failure to estimate all three thermal properties from one optimal experimental temperature history, the experiments were redesigned for estimating two thermal properties. The focus was to simultaneously estimate the effective in-plane thermal conductivity, \( k_{\text{eff}} \), and the volumetric heat capacity, \( C_{\text{eff}} \). Although the volumetric heat capacity had already been determined from the one-dimensional analysis, it was estimated again to gain confidence in the in-plane thermal conductivity estimate. If the volumetric heat capacity was estimated with similar values as the one-dimensional analysis, it could be assumed that the in-plane thermal conductivity was also estimated correctly. This assumption is based on mild dependence between the thermal properties, the accurate estimate of one property is necessary for the accurate estimate of the remaining property. This idea is justified with the experimental results presented in
Section 5.3.4. After estimating the two thermal properties with the optimal experimental designs, the emphasis shifted to verifying the optimal designs. These designs were verified by estimating the thermal conductivities with non-optimal parameter settings and comparing the confidence intervals and are discussed in the final subsections.

5.3.1 Optimal and Non-Optimal Experimental Designs for Configurations 1 and 2

The same procedure that was used to optimize the experiment for three properties was repeated in the optimization of two properties. Both optimizations had the same goal, to find the parameter settings which created the largest determinant of the $X^*T X^*$ matrix, thereby creating the smallest confidence intervals. In addition to the optimal designs, non-optimal parameters were determined as the settings which produced a portion of the maximum determinant. The experimental parameters verified with the non-optimal settings were the sensor location in the y direction ($y^*$), the heating area ($L_p^*$), and heating time ($t_h^*$). For each of these three parameters, two non-optimal settings were determined.

One modification in the optimization program written by Moncman (1994) was necessary for the optimization of two thermal properties. The sensitivity coefficient for the thermal conductivity through the thickness of the composite, $X_l^*$ in Eq. (3.11) was removed. Therefore, all $d_{ij}^*$ terms from Eqs (3.10 a, b) were lost, resulting in the following non-dimensional determinant,

$$D_{2-D}^* = \begin{vmatrix} d_{22}^* & d_{23}^* \\ d_{32}^* & d_{33}^* \end{vmatrix} \quad D_{2-D}^* = d_{22}^* d_{33}^* - d_{23}^* d_{32}^* \quad (5.4 \text{ a,b})$$

This modification had an added advantage of accelerating the optimization program by
reducing the number of calculations. This was not trivial, since the parametric study was a time consuming process and required a day to complete on a 486 personal computer.

Again, the optimization procedure began with phase one. Phase one was used to determine a general optimal design by allowing all of the parameters to vary within a coarse increment size. Therefore, from phase one, a general heating area was determined. This general heating area was then compared to the available, stocked heaters from Minco Products Inc. The heater that came closest to the optimal heating area was chosen and the remainder of the optimization procedure was centered around that available heater.

The results from phase one for both configurations are shown in Figure 5.7 and 5.8. Configuration 1's optimal heating area moved from 100% in the estimation of three properties to only 30% for the estimation of two properties. This indicated that the in-plane thermal conductivity was very sensitive to the heating area. Although a thirty percent heating area would have created the largest determinant for Configuration 1, the closest available heater produced by Minco Products Inc. was a 25%, 0.0762m x 0.0254m (3" x 1") heater. Since their maximum determinants were close this was used as the optimal heater area. Minco also produced a 50%, 0.0762m x 0.0508m (3’ x 2") and a 100%, 0.0762m x 0.1016m (3" x 4") heater which were used as the non-optimal heated areas for Configuration 1. Notice in Figure 5.7 that they produce smaller determinants then both the 25% and the optimal 30%.

Similar to Configuration 1, Configuration 2 had to use a heating area that was different than the optimized area. The largest determinant from phase one was produced with a heating area of 13% and the closest Minco heater was 11%, 0.0762m x 0.0112m
Figure 5.7. Phase One, Experimental Optimization of Configuration 1 for the Simultaneous Estimation of Two Thermal Properties.

Figure 5.8. Phase One, Experimental Optimization of Configuration 2 for the Simultaneous Estimation of Two Thermal Properties.
(3" x 0.44"). Again, note the 11% maximum determinant was close to the maximum determinant of the optimal heater, as shown in Figure 5.8. The two non-optimal heating areas, 25% and 75%, were also chosen due to their availability from Minco and smaller maximum determinants. The parameter settings and the maximum determinant, from phase one, at the optimal and non-optimal heating areas for both configurations are listed in Table 5.5.

Table 5.5. Phase One Results for the Optimal and Non-Optimal Designs for the Simultaneous Estimation of Two Thermal Properties for Configurations 1 and 2.

<table>
<thead>
<tr>
<th>Configuration 1</th>
<th>Parameter</th>
<th>Ideal $L_{p,1}^{**} = 0.30$</th>
<th>Optimal $L_{p,1}^{**} = 0.25$</th>
<th>Non-Optimal $L_{p,1}^{**} = 0.50$</th>
<th>Non-Optimal $L_{p,1}^{**} = 1.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Loc., $x^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sensor Loc., $y^*$</td>
<td>0.15</td>
<td>0.15</td>
<td>0.15</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>Heating Time, $t_h^*$</td>
<td>1.45</td>
<td>1.35</td>
<td>1.65</td>
<td>1.65</td>
<td></td>
</tr>
</tbody>
</table>
| Max. Det. x $10^4$  | 4.50               | 3.70                        | 2.77                        | 2.45                            |                                 | $L_{p,1}^{**}$ = Non-dimensional heating area for Configuration 1

<table>
<thead>
<tr>
<th>Configuration 2</th>
<th>Parameter</th>
<th>Ideal $L_{p,2}^{**} = 0.13$</th>
<th>Optimal $L_{p,2}^{**} = 0.11$</th>
<th>Non-Optimal $L_{p,2}^{**} = 0.25$</th>
<th>Non-Optimal $L_{p,2}^{**} = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Loc., $x^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
</tr>
<tr>
<td>Sensor Loc., $y^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.10</td>
<td>0.65</td>
<td></td>
</tr>
<tr>
<td>Heating Time, $t_h^*$</td>
<td>1.65</td>
<td>1.60</td>
<td>1.90</td>
<td>1.90</td>
<td></td>
</tr>
</tbody>
</table>
| Max. Det. x $10^4$  | 2.40               | 2.20                        | 1.05                        | 0.82                            |                                 | $L_{p,2}^{**}$ = Non-dimensional heating area for Configuration 2

108
Phase one predicted a maximum determinant for Configuration 1 twice the magnitude of the determinant produce by the optimal design of Configuration 2. Recall, that the optimal experimental designs for estimating all three thermal properties had approximately the same maximum determinant for both configurations. This indicates that the in-plane thermal conductivity is very sensitive to the boundary conditions. In addition, it suggests that the in-plane thermal conductivity effect was diminished in the optimal experimental designs for estimating three properties by the dominating thermal conductivity through the thickness of the composite. It is important to remember that the optimal designs, for both configurations, in the estimation of three thermal properties provided adequate information for all properties. The reason why a unique solution could not be obtained was due to correlation between the thermal conductivities, not because of inadequate information. The optimal designs for estimating two properties simply allow for the better information to be obtained for the in-plane thermal conductivity.

Phase two of the optimization procedure was applied to refine the sensor location in the y direction, the heating time, and the experiment time. This phase was separated into three steps, each step refining the experimental design until the optimal parameters were converged upon. This procedure was applied to both configurations for the optimal and non-optimal heating areas. For the remaining two parameters, the sensor location and heating time, non-optimal designs were developed around the optimal heating area, to perform a controlled analysis of their effects.

The second phase of the optimization process began with the optimal 25% heating area of Configuration 1. In the program optc1.for listed in Appendix C, the heating area
was fixed and a smaller grid size was applied around the 0.15 sensor location in the y direction as predicted by phase one. The heating time and experiment time were allowed to vary and a maximum determinant was produced at a sensor location of 0.13. Non-optimal sensor locations were determined by taking twenty percent of the maximum determinant as shown in Figure 5.9. Since the curve was bell shaped the non-optimal sensor locations were determined on either side of the optimal sensor location at 0.047 and 0.232 of the composite surface. It was necessary to maintain the precise locations of all three sensors to avoid a large loss in the determinant due to the steep slopes of the curve.

This first step was repeated with the optimal 11% heater for Configuration 2. Similar results were obtained and are shown in Figure 5.10. Since the optimal sensor location occurred at the edge of the composite, $y^* = 0.0$, non-optimal sensors could not be placed on either side. The first non-optimal sensor location was determined by taking 20% of the maximum determinant, which placed the sensor at $y^* = 0.09$ while the second sensor was placed past the 11% heater at a location of 0.15. The second sensor, hardly received any information on the thermal properties and was chosen to demonstrate the effects of poor experimental designs.

The second step of phase two for Configuration 1 was to fix the sensor location at 13% with a 25% heated area. The heating time was allowed to vary with a smaller increment while the experiment time varied with its normal coarse increment. The results from this step are shown in Figure 5.11 which produced a maximum determinant at $t^*_h = 1.37$. Non-optimal heating times were determined by taking 20% and 80% of the
Figure 5.9. Maximum Determinant, $D_{\text{max}}^*$, for Various Sensor Locations, $y^*$, of Configuration 1, Calculated Using Optimal Experimental Parameters of $L_{p,t}^* = 0.25$, $x^* = 0.0$, While, $t_i^*$, Varied.
Figure 5.10. Maximum Determinant, \( D_{\text{max}}^* \), for Various Sensor Locations, \( y^* \), of Configuration 2, Calculated Using Optimal Experimental Parameters of \( L_{p,2}^* = 0.11 \), \( x^* = 0.0 \), While, \( t_h^* \), Varied.
Figure 5.11. Maximum Determinant, $D_{max}^*$, for Various Heating Times, $t_h^*$, of Configuration 1, Calculated Using Optimal Experimental Parameters of $L_{p,t}^* = 0.25$, $x^* = 0.0$, and $y^* = 0.13$. 
maximum determinant resulting in non-dimensional heating times of 0.75 and 0.29 respectively.

The heating time curve shown in Figure 5.11 had a more gradual slope than the sensor location plot demonstrated in Figure 5.10. Therefore it was not as critical to maintain the heating time as precisely as the sensor location. After determining the optimal and non-optimal heating times for Configuration 1, the procedure was repeated for Configuration 2 resulting in a similar solution with an optimal heating time of 1.58 and non-optimal times of 0.88 and 0.32.

The third and final step of phase two calculated the overall experiment time by removing the time averaging, $I/T_{\text{max}}$, from the determinant in Eq. (3.11). All of the remaining parameters were fixed to their optimal settings and the experiment time was allowed to increase. The results for Configuration 1 are shown in Figure 5.12. The optimal experiment time was determined when the determinant no longer changed significantly. For Configuration 1 this was obtained at a non-dimensional experiment time, $t_N^*$, of 3.05. This was a conservative value and not strictly followed. The experiment time was actually determined to be the time required for the heated surface of the composite to cool down to its initial temperature. Once again, after Configuration 1 was analyzed, step three was repeated for Configuration 2 resulting in a similar experiment time of 3.00.

These three optimization steps were repeated with both non-optimal heating areas of both configurations and the results are summarized in Table 5.6. It is important to notice that the resulting optimal experimental designs make physical sense. The heating time
Figure 5.12. Modified Dimensionless Determinant, $D^*$, Used to Determine the Dimensionless Experimental Time, $t_{R*}$, of Configuration 1.
allows for the samples to almost reach steady state, the sensor is placed in the warmest location, and the heating area allows for maximum two-dimension heat conduction. These optimal settings could have been determined intuitively, but as structures become more complex, it will be harder to predict the optimal settings.

Table 5.6. Optimal and Non-Optimal Experimental Designs for the Simultaneous Estimation of Two Thermal Properties for Configurations 1 and 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimal Design</th>
<th>Non-Opt. $y^*$</th>
<th>Non-Opt. $t_h^*$</th>
<th>Non-Opt. $L_{p,1}^* = 0.50$</th>
<th>Non-Opt. $L_{p,1}^* = 1.00$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Loc., $x^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sensor Loc., $y^*$</td>
<td>0.13</td>
<td>0.05, 0.23</td>
<td>0.13</td>
<td>0.14</td>
<td>0.87</td>
</tr>
<tr>
<td>Heating Area, $L_{p,1}^*$</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.50</td>
<td>1.00</td>
</tr>
<tr>
<td>Heating Time, $t_h^*$</td>
<td>1.37</td>
<td>1.37</td>
<td>0.29, 0.75</td>
<td>1.68</td>
<td>1.62</td>
</tr>
<tr>
<td>Exp. Time, $t_n^*$</td>
<td>3.05</td>
<td>3.05</td>
<td>3.05</td>
<td>3.10</td>
<td>3.70</td>
</tr>
<tr>
<td>Max. Det. x $10^4$</td>
<td>3.79</td>
<td>0.76, 0.76</td>
<td>0.76, 3.03</td>
<td>2.79</td>
<td>2.44</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Optimal Design</th>
<th>Non-Opt. $y^*$</th>
<th>Non-Opt. $t_h^*$</th>
<th>Non-Opt. $L_{p,2}^* = 0.25$</th>
<th>Non-Opt. $L_{p,2}^* = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensor Loc., $x^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Sensor Loc., $y^*$</td>
<td>0.00</td>
<td>0.09, 0.15</td>
<td>0.00</td>
<td>0.13</td>
<td>0.64</td>
</tr>
<tr>
<td>Heating Area, $L_{p,2}^*$</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>Heating Time, $t_h^*$</td>
<td>1.58</td>
<td>1.58</td>
<td>0.32, 0.88</td>
<td>1.92</td>
<td>1.97</td>
</tr>
<tr>
<td>Exp. Time, $t_n^*$</td>
<td>3.00</td>
<td>3.00</td>
<td>3.00</td>
<td>3.20</td>
<td>3.90</td>
</tr>
<tr>
<td>Max. Det. x $10^4$</td>
<td>2.29</td>
<td>0.46, 0.09</td>
<td>0.46, 1.83</td>
<td>1.05</td>
<td>0.82</td>
</tr>
</tbody>
</table>

116
5.3.2 Comparison of Configurations 1 and 2

The purpose of using two configurations in this analysis was to demonstrate the impact of the boundary conditions on the estimated thermal properties. The constant temperature boundary conditions of Configuration 1 were chosen because they allowed the greatest heat transfer to the sides of the composite, therefore allowing for more information on the in-plane thermal conductivity. The insulated boundary conditions of Configuration 2 were chosen because of their ease to experimentally implement and maintain.

Recall the maximum determinants obtained for estimating three thermal properties for both configurations shown in Figure 5.5. There was not much difference between the two optimal designs due to the dominant sensitivity coefficient of the thermal conductivity through the thickness. Both configurations supplied approximately the same amount of information on the thermal conductivity through the thickness due to similar boundary conditions of the top and bottom surfaces of the composite. Since this value was large any additional information supplied by Configuration 1's constant temperature side boundary conditions was effectively averaged out.

When the configurations were redesigned for only estimating two thermal properties the advantages of Configuration 1 were apparent. The maximum determinant for Configuration 1 was 65% larger than the maximum determinant for Configuration 2, as shown in Figure 5.13, even though both configurations have approximately the same sensitivity coefficient for the volumetric heat capacity. The non-dimensional sensitivity coefficients for the optimal designs of both configurations are shown in Figure 5.14. The
Figure 5.13. Comparison of Dimensionless Determinants, $D^*$, for the Optimal Experimental Designs of Configurations 1 and 2 for the Simultaneous Estimation of Two Thermal Properties.
Figure 5.14. Comparison of Dimensionless Sensitivity Coefficients, \((X_{k,g})^*\) and \((X_{C,g})^*\), for the Optimal Experimental Designs of Configurations 1 and 2.
only significant difference between the coefficients was the larger in-plane thermal conductivity coefficient for Configuration 1. Therefore the advantages of the constant temperature boundary of Configuration 1 should allow for more accurate in-plane thermal conductivity estimates.

5.3.3 Analytical Verification of Optimal Experimental Designs

Prior to experimentally verifying the optimal experimental designs, an analytical study was performed to validate the designs and to demonstrate which parameters had the largest impact on the accuracy of the properties. Recall the analytical model discussed in Section 5.2.2, Analytical and Experimental Results for the Simultaneous Estimation of Three Thermal Properties, where the properties were estimated with the temperature history generated from a finite element program. This model, although based on the optimal designs for estimating three properties, was modified for the estimation of two thermal properties, \( k_{y \text{-eff}} \) and \( C_{\text{eff}} \).

To verify the optimal designs and to demonstrate which experimental parameters had the largest impact on the accuracy of properties, perturbations were applied to the optimal setting. The thermal properties were estimated and the ratio of the confidence intervals from the optimal and non-optimal design were determined from

\[
Rci(k_{y \text{-eff}}) = \frac{ci(k_{y \text{-eff}}) \text{ non-optimal}}{ci(k_{y \text{-eff}}) \text{ optimal}} \quad Rci(C_{\text{eff}}) = \frac{ci(C_{\text{eff}}) \text{ non-optimal}}{ci(C_{\text{eff}}) \text{ optimal}} .
\] (5.5)

If the optimal parameter settings did produce the most accurate estimates, the confidence intervals would be the smaller and the ratio would be greater than unity.
The tests were incrementally performed on four of the five parameters, the heating time \((t_h^+)\), the sensor location in the \(x\) and \(y\) directions \((x^+\) and \(y^+)\), and the heating area \((L_p^+)\). To determine the effect of only one parameter, three of the parameters remained at their optimal settings while the fourth was modified \(+25\%\) or \(-25\%\). The ratio of confidence intervals for the tests on each configuration are shown in Table 5.7.

Table 5.7. Ratio of Confidence Intervals \((Rci(k_{y-eff})\) and \(Rci(C_{eff})\)) Using Non-Optimal Experimental Parameters for Configurations 1 and 2.

<table>
<thead>
<tr>
<th>Experimental Parameter</th>
<th>% Optimal value</th>
<th>Rci((k_{y-eff}))</th>
<th>Rci((C_{eff}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_h^+)</td>
<td>75</td>
<td>1.0+</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>0.9</td>
<td>1.0+</td>
</tr>
<tr>
<td>(y^+)</td>
<td>75</td>
<td>1.0+</td>
<td>1.4</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>1.6</td>
<td>1.2</td>
</tr>
<tr>
<td>(x^+)</td>
<td>25(^*)</td>
<td>52,000</td>
<td>3,100</td>
</tr>
<tr>
<td>(L_{p,1^+})</td>
<td>75</td>
<td>1.7</td>
<td>2.4</td>
</tr>
<tr>
<td>(t_h^+)</td>
<td>75</td>
<td>0.9</td>
<td>1.0-</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>(y^+)</td>
<td>75</td>
<td>2.6</td>
<td>0.9</td>
</tr>
<tr>
<td></td>
<td>125</td>
<td>4.7</td>
<td>2.3</td>
</tr>
<tr>
<td>(x^+)</td>
<td>25(^*)</td>
<td>1,000</td>
<td>2,300</td>
</tr>
<tr>
<td>(L_{p,2^+})</td>
<td>75</td>
<td>3.6</td>
<td>5.1</td>
</tr>
</tbody>
</table>

a. The optimal value for \(x^+\) was 0 (at the heated surface). Therefore a distance of 25\% of the composite thickness was used as the non-optimal value.

\(^+/\) Used to signify a slight increase or decrease in the value that was not apparent in two significant digits.
There were two important results obtained from this analysis. First, the accuracy of the estimates was very sensitive to the sensor location in the x direction. With only a 25% change from the optimal location, the confidence interval grew over a thousand times greater than the confidence interval obtained at the heated surface. Therefore it was critical that the sensor be located precisely at the heated surface of the experiment. The second result, was that the optimal design did not always produce the most accurate estimate for an individual property. For example, when Configuration 1 had a heating time at 75% of the optimal value, slightly smaller confidence bounds were produced for the volumetric heat capacity. This suggested two ideas, first the heating time did not affect the accuracy of the estimated property. With a 25% change in optimal value there was hardly a change in the confidence interval. Second, the optimal design was not concerned with the individual estimate of the volumetric heat capacity but the combined accuracy of the three estimates. Notice that the in-plane thermal conductivity was slightly larger than unity. This indicated that although one estimate was less accurate the combined accuracy of the estimates was greater at the optimal settings.

5.3.4 Thermal Property Estimates for Optimal and Non-Optimal Experimental Designs of Configurations 1 and 2

The optimal and non-optimal experimental parameters determined for estimating two thermal properties were implemented into Configurations 1 and 2. Experiments were performed and the estimated parameters along with their 95% confidence intervals are presented in this discussion. For each experiment there were two estimated effective
properties: the in-plane thermal conductivity, $k_{y,\text{eff}}$, and the volumetric heat capacity, $C_{\text{eff}}$.

In order to completely understand the estimated properties it was important to graph the sensitivity coefficients and the maximum determinant. The maximum determinant gave a general description on whether the confidence intervals would be large or small. The sensitivity coefficients were used to determine if enough information was available to accurately estimate the thermal properties. Although there was never a problem of simultaneously estimating the thermal properties due to correlation, the properties have some dependence on each other. From the approximate correlation matrix the properties had off-diagonal coefficients around 0.30. Therefore, if there was not enough information to accurately estimate one thermal property, the accuracy of the other estimate was partially effected.

The remainder of this discussion demonstrates the effects of the different boundary conditions and various parameters on the estimated thermal properties. Initially, the differences between the boundary conditions of the optimal designs are analyzed. Following the boundary conditions the effects of the non-optimal parameters: the heating time, sensor location, and the heating area, on the estimated properties are described.

5.3.4.1 Comparison of the Optimal Designs of Configurations 1 and 2

The first experiments performed were with the optimal designs for both configurations. The results demonstrate that the optimal designs from Configuration 1 do indeed produce smaller confidence intervals than the estimates from Configuration 2. Figures 5.15 and 5.16 show the estimated volumetric heat capacity and the thermal
Figure 5.15. Comparison of Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, from the Optimal Experimental Designs of Configurations 1 and 2.

Figure 5.16. Comparison of In-Plane Thermal Conductivity Estimates, $k_{\text{eff}}$, from the Optimal Experimental Designs of Configurations 1 and 2.
conductivity, along with their 95% confidence intervals, for both configurations, respectively. To provide a baseline for the estimated properties, the volumetric heat capacities, estimated from the one-dimensional analysis are superimposed on Figure 5.15 and the average in-plane thermal conductivity from Configuration 1, represented as the horizontal line, is provided in Figure 5.16. Recall, that there were nine experiments performed from three separate experimental setups each repeated three times. The dotted vertical lines in the figures represent the distinction between different setups.

It was apparent by comparing both designs that the boundary conditions do affect the estimates. Table 5.8 displays the average value for the thermal properties from both configurations. Notice that combined confidence intervals about the mean, calculated from Eq (5.3) for Configuration 1 were smaller than the estimates from Configuration 2.

It was disappointing that the designs did not share a common conductivity value, although the confidence intervals were close: 1.85 - 1.93 W/(m K) (1.07 - 1.11 Btu/(h ft °F)) for Configuration 1 and 1.96 - 2.04 W/(m K) (1.18 - 1.13 Btu/(h ft °F)) for Configuration 2. Since Configuration 1 offered the most information for the in-plane thermal conductivity the mean value of 1.89 ± 0.04 W/(m K) (1.09 ± 0.02 Btu/(hr ft °F)) was assumed to be closer to the correct estimate. Although the confidence intervals for

<table>
<thead>
<tr>
<th></th>
<th>Configuration 1</th>
<th></th>
<th>Configuration 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_{y,eff}$ (W/m K)</td>
<td>$C_{eff}$ (MJ/m$^3$ K)</td>
<td>$k_{y,eff}$ (W/m K)</td>
</tr>
<tr>
<td>Mean</td>
<td>1.89 ± 0.04</td>
<td>1.50 ± 0.01</td>
<td>2.00 ± 0.04</td>
</tr>
</tbody>
</table>

Table 5.8. Estimated Thermal Properties from the Optimal Experimental Designs of Configurations 1 and 2.
the thermal conductivities did not overlap, both of the estimated volumetric heat capacities shared common values with the one-dimensional estimate of $1.53 \pm 0.01 \text{ MJ/(m}^3 \text{ K)}$ ($22.8 \pm 0.2 \text{ Btu/(ft}^3 \text{ °F)}$). Therefore this confirmed the estimated value of the volumetric heat capacity was correct and that the in-plane thermal conductivity was the best estimate possible with the information available.

The estimated properties also demonstrate that the confidence intervals were not solely related to the magnitude of an individual sensitivity coefficient. Both of the thermal properties estimated from Configuration 1 had smaller confidence intervals than the estimates from Configuration 2. Smaller confidence intervals were present around the volumetric heat capacity for Configuration 1 even though the sensitivity coefficients for this property in both configurations were approximately the same. Since the confidence intervals were based on the $P$ matrix which incorporate the inverse of the transpose of the sensitivity matrix multiplied by itself, individual intervals are dependent on the information available for both properties.

In conclusion, the comparison confirmed that the boundary conditions did affect the accuracy of the estimates and that the properties are mildly dependent on each other. Although, the confidence intervals for Configuration 2 were larger, the boundary conditions are easier to experimentally obtain and for future experiments may be suitable depending on the necessary accuracy of the estimates.

5.3.4.2 Effects of Heating Time on Estimated Properties

The first experimental parameter to be verified was the heating time. Non-optimal
heating times are determined by taking 20% and 80% of the maximum determinant of the $X^*X^*$ matrix created by the optimal designs. The resulting maximum determinants for the optimal and non-optimal heating times are shown in Figure 5.17 for Configuration 1, similar (but smaller) results were obtained for Configuration 2. From the figure it was expected that the optimal designs should produce the smallest confidence intervals because it had the largest determinant. A heating time of $t_h^* = 0.75$ should create slightly larger confidence intervals and the smallest heating time of $t_h^* = 0.29$ should produce the largest confidence intervals.

The heating time had similar effects on both of the estimated thermal properties. The estimated volumetric heat capacities and in-plane thermal conductivities for Configuration 1 are shown in Figures 5.18 and 5.19 and repeated for Configuration 2 in Figures 5.20 and 5.21. Superimposed on the thermal conductivity estimates was the averaged value determined from the optimal design of Configuration 1. All four figures indicate that the confidence intervals grow with larger deviations from the optimal design. Also, as expected, by comparing the confidence intervals from Configurations 1 and 2 for either property estimate, smaller intervals were obtained with Configuration 1.

Estimates created with the shortest heating time ($t_h^* = 0.29$ for Configuration 1 and 0.32 for Configuration 2) not only produce the largest confidence intervals but were not centered around the baseline estimates. This inaccuracy can be explained by the sensitivity coefficients. The coefficients for all three heating times for Configurations 1 and 2 are shown in Figures 5.22 and 5.23, respectively. For the shortest heating times, the sensitivity coefficients for the in-plane conductivity never exceed 0.10. Therefore
Figure 5.17. Dimensionless Determinants, $D^*$, for Optimal and Non-Optimal Heating Times, $t_h^*$, of Configuration 1.
Figure 5.18. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Times, $t_h^\ast$, of Configuration 1.

Figure 5.19. In-Plane Thermal Conductivity Estimates, $k_{\text{eff}}$, for Optimal and Non-Optimal Heating Times, $t_h^\ast$, of Configuration 1.
Figure 5.20. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Times, $t_h^+$, of Configuration 2.

Figure 5.21. In-Plane Thermal Conductivity Estimates, $k_{\text{y-eff}}$, for Optimal and Non-Optimal Heating Times, $t_h^+$, of Configuration 2.
Figure 5.22. Dimensionless Sensitivity Coefficients, \( (X_k)^* \) and \( (X_{C_{eff}})^* \), for Optimal and Non-Optimal Heating Times, \( t_h^* \), of Configuration 1.

Figure 5.23. Dimensionless Sensitivity Coefficients, \( (X_k)^* \) and \( (X_{C_{eff}})^* \), for Optimal and Non-Optimal Heating Times, \( t_h^* \), of Configuration 1.
there was insufficient information to accurately estimate the thermal conductivity. This lack of information also affected the volumetric heat capacity estimates. Notice how the estimates for both thermal conductivity in Figures 5.19 and 5.20 deviate from the baseline.

In summary, varying the heating time was an ideal example for demonstrating the importance of optimizing an experiment. First, it demonstrated that the optimal designs produce the smallest confidence intervals. Second, the importance of the boundary conditions was again demonstrated: Configuration 1 consistently produced smaller confidence intervals at the non-optimal locations than Configuration 2. Finally, it emphasized that if an experiment was designed poorly, not only will the confidence intervals grow but the estimates might be incorrect.

5.3.4.3 Effects of Sensor Location on Estimated Properties

The second parameter analyzed was the sensor location along the $y$ direction. Like the heating time parameter, the accuracy of the estimates was extremely sensitive to the sensor location for both configurations. If the sensor location varied too far from the optimal design, the majority of the information for either the in-plane thermal conductivity or volumetric heat capacity would be lost. The optimal sensor location represented the only position where reliable estimates for both properties could be determined. The effects of the sensor location on the estimated properties is broken into two parts. Initially Configuration 1 is analyzed then the focus shifts to Configuration 2.

The first step in analyzing the estimated parameters from Configuration 1 was to
graph the maximum determinants for the optimal and non-optimal sensor locations, shown in Figure 5.24. Although the maximum determinants are generally insightful, they can be misleading. From the figure it might be assumed that the thermal properties at both non-optimal sensor locations would be estimated with similar accuracy due to their similar maximum determinants. However, the estimated volumetric heat capacities, shown in Figure 5.25, and the estimated in-plane thermal conductivities, shown in Figure 5.26, present completely different accuracies in the estimates depending on which non-optimal sensor was analyzed.

The sensitivity coefficients shown in Figure 5.27 for the optimal and non-optimal sensor locations, explain the apparent inconsistent accuracies. Both non-optimal sensor locations exceed a magnitude of 0.10 for the non-dimensional thermal conductivity sensitivity coefficient; therefore, there was enough information for reasonable estimates. But notice from Figure 5.26 that slightly smaller confidence intervals are obtained from the sensor located at \( y^* = 0.23 \) than at 0.047 even though less information was obtained. This discrepancy was due to the inadequate information available in the volumetric heat capacity at \( y^* = 0.047 \). Notice that in Figure 5.27 the volumetric heat capacity sensitivity coefficient never exceeded 0.10 and therefore produced unreliable volumetric heat capacity estimates as shown in Figure 5.25. Therefore, due to the mild dependence between the properties, incorrect volumetric heat capacity estimates affected the accuracy of the thermal conductivity estimates.

The sensor location parameter in Configuration 1 exemplified the balance of the optimization procedure. As the sensor moved toward the constant temperature boundary
Figure 5.24. Dimensionless Determinants, $D^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1.
Figure 5.25. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1.

Figure 5.26. In-Plane Thermal Conductivity Estimates, $k_{\text{eff}}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 1.
Figure 5.27. Dimensionless Sensitivity Coefficients, \( (\chi_{x_k} \cdot y^*_{\text{opt}}) \) and \( (\chi_{x_{\text{C}_{\text{opt}}}} \cdot y^*_{\text{opt}}) \), for Optimal and Non-Optimal Sensor Locations, \( y^* \), of Configuration 1.
condition ($y^* = 0.047$) more information was obtained on the in-plane thermal conductivity and less on the volumetric heat capacity (although the optimal sensor location for estimating the in-plane thermal conductivity was not at the constant temperature boundary condition). The available information reversed for the thermal properties as the sensor moved away from the constant temperature boundary. Therefore, the maximum determinants shown in Figure 5.24 for each non-optimal sensor location maintained approximately the same magnitude. One non-optimal sensor obtained more information on the conductivity and less on the volumetric heat capacity and vise versa for the other non-optimal sensor location.

Configuration 2, like Configuration 1 was also sensitive to the sensor location but for the in-plane thermal conductivity and not the volumetric heat capacity estimates. The maximum determinants along with the sensitivity coefficients are shown in Figures 5.28 and 5.29 for the optimal and non-optimal sensor locations. Notice the extremely small determinant in Figure 5.28 and the inverted sensitivity coefficient at $y^* = 0.15$. Recall that due to the limited size of the optimal heating area, this sensor was located past the length of the heater. The thermal conductivity sensitivity coefficients for both non-optimal sensor locations never exceeded a magnitude of 0.10, as shown in Figure 5.29. Configuration 2 offered so little information on the in-plane conductivity that any deviation from the optimal solution resulted in inadequate information and unreliable thermal conductivity estimates, although all three designs provided enough information for reliable volumetric heat capacity estimates. The estimated volumetric heat capacities and thermal conductivities are displayed in Figures 5.30 and 5.31, respectively. As
Figure 5.28. Dimensionless Determinants, $D^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2.

Figure 5.29. Dimensionless Sensitivity Coefficients, $(X_{X, a})^*$ and $(X_{C, a})^*$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2.
Figure 5.30. Volumetric Heat Capacity Estimates, $C_{eff}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2.

Figure 5.31. In-Plane Thermal Conductivity Estimates, $k_{r-eff}$, for Optimal and Non-Optimal Sensor Locations, $y^*$, of Configuration 2.
predicted, the thermal conductivity estimates grossly deviated from the baseline while the volumetric heat capacity estimates were reliable for both non-optimal sensor locations.

In conclusion, the sensor location was highly important for the accurate estimate of the in-plane thermal conductivity for Configuration 2. With a slight deviation from the optimal location the thermal conductivity could not be accurately estimated. On the contrary, the sensor location had more of an effect on the volumetric heat capacity estimate from Configuration 1. Therefore for accurate estimates of both thermal properties it is imperative to remain at the optimal location.

5.3.4.4 Effects of Heating Area on Estimated Properties

The final parameter verified was the optimal heating area. Like the previous two properties, the optimal heating area was verified by estimating the thermal properties with two non-optimal heating areas. These non-optimal heating areas were determined by available heaters produced by Minco Product Inc. To perform a controlled study on the effects of the heater, the non-optimal heating areas were optimized for the heating time and sensor location. Therefore, the only non-optimal parameter in the experiments was the heating area.

The in-plane thermal conductivity estimates were not surprising. Both optimal designs produced more accurate estimates than the non-optimal settings. In addition, Configuration 1 produced more accurate estimates than Configuration 2. Due to the lack of available information for the in-plane thermal conductivity in Configuration 2 both non-optimal heating areas could not produce accurate estimates. See Figure 5.32 and 5.33
Figure 5.32. In-Plane Thermal Conductivity Estimates, $k_{r,\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,1^*}$, of Configuration 1.

Figure 5.33. In-Plane Thermal Conductivity Estimates, $k_{r,\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,2^*}$, of Configuration 2.
for the estimated thermal conductivities of Configurations 1 and 2. Notice how the estimates from Configuration 1 have less variance in the average values than the estimates from Configuration 2. Sporadic estimates for Configuration 2 were a result of the in-plane thermal conductivity sensitivity coefficients well below 0.10.

The balance of the optimization procedure was again demonstrated with the estimated volumetric heat capacity estimates shown in Figures 5.34 and 5.35, for Configurations 1 and 2 respectively. Neither of the optimal designs for Configuration 1 or 2 produced the most accurate volumetric heat capacity estimates. Actually, the optimal heating areas provided the largest confidence intervals for both configurations. More information was gathered for the volumetric heat capacity as the heating area approached one hundred percent. Therefore with the 75% heater in Configuration 2, the smallest confidence intervals were produced. Although the confidence intervals for this heating area were the smallest, the estimates were not reliable due to the lack of information for the thermal conductivity. Therefore the results of the optimal heating area demonstrated that the goal of the optimal design was to estimate both rather than any one property accurately.

In conclusion, the non-optimal heating areas supported two claims of this analysis. First, the optimization procedure balanced the accuracy of the thermal properties, possibly producing lesser accuracy for one property estimate but maintaining the most accurate combined estimates. Second it stressed the importance of the optimal design of Configuration 2. Accurate estimates from Configuration 2 could be obtained at the optimal design but any deviation from those parameters may produce inaccurate results.
Figure 5.34. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,1}$, of Configuration 1.

Figure 5.35. Volumetric Heat Capacity Estimates, $C_{\text{eff}}$, for Optimal and Non-Optimal Heating Areas, $L_{p,2}$, of Configuration 2.
CHAPTER 6

Summary and Conclusions

Two objectives were set forth at the beginning of this research: to simultaneously estimate the effective thermal properties of an anisotropic material and to verify that the optimal experimental designs provide the greatest accuracy. Moncman (1994) had previously developed one- and two-dimensional experimental designs for a similar carbon-fiber/epoxy-matrix composite. These designs were tailored and implemented into the specific experiments of this research.

To verify these designs, the research developed through three distinct stages. First, the one-dimensional analysis was performed to gain confidence in the experimental setup and procedure. After estimating the one-dimensional properties, the experiments were expanded to incorporate the estimation of the in-plane thermal conductivity. Although due to the limited amount of distinct information for each thermal conductivity, it became difficult to estimate all properties simultaneously and accurately. Finally, the emphasis shifted to estimating only the effective in-plane thermal conductivity and the volumetric heat capacity. Since the original optimal designs were developed for estimating three
properties, both two-dimensional configurations were reoptimized. This optimization placed a greater emphasis on the in-plane conductivity. Therefore from the final designs and the one-dimensional analysis, all three thermal properties were estimated. To complete the second objective, the experiments were performed with non-optimal parameter settings. From the non-optimal designs, new estimates were determined which verified that the optimal settings provided more accurate estimates.

6.1 One-Dimensional Analysis

From the one-dimensional analysis the effective conductivity through the thickness of the composite and the volumetric heat capacity for the carbon-fiber/epoxy-matrix composite, AS4/3502 were estimated to be

\[ k_{x\text{-eff}} = 0.616 \pm 0.002 \frac{W}{m \ K} \quad C_{\text{eff}} = 1.53 \pm 0.01 \frac{MJ}{m^3 \ K}. \]

Since no prior knowledge existed on the thermal properties they were compared with the estimates of similar known composites and were in good agreement.

In addition to the estimated properties the one-dimensional analysis gave insight to the experimental setup. The sensitivity coefficients indicated that the experiment provided the most information for the thermal conductivity and that no linear dependence existed between the properties. In conclusion, the one-dimensional analysis was a good starting point because it gave confidence to the experimental setup and procedure which were used in the two-dimensional analysis.
6.2 Two-Dimensional Analysis for the Simultaneous Estimation of Three Thermal Properties

Originally in the two-dimensional analysis an attempt was made to estimate the in-plane and through the thickness thermal conductivities along with the volumetric heat capacity from one optimal experimental temperature history. The optimal designs appeared to be a feasible solution since they provided adequate information for all three thermal properties and could be estimated simultaneously from an analytical case. When the designs were implemented into an experiment, the resulting temperature history produced unconverged and inaccurate estimates. It was shown that the thermal properties were highly correlated and with the induction of experimental error, the process of simultaneously estimating all three properties became futile.

6.3 Two-Dimensional Analysis for the Simultaneous Estimation of Two Thermal Properties

Due to the failure of simultaneously estimating all three thermal properties, only the in-plane conductivity and the volumetric heat capacity were estimated. The final unknown thermal property, the in-plane thermal conductivity was estimated to be

\[ k_{y\text{-eff}} = 1.89 \pm 0.04 \frac{W}{mK}. \]

With this estimate, phase one of the research was completed, and the three thermal properties of the carbon fiber/epoxy matrix composite, AS4/3502 were known.

The second half of the research focused on verifying that the optimal designs did
provide the most accurate estimates. Three optimal experimental parameters were verified by comparing the confidence intervals around the estimates from optimal and non-optimal settings. In addition to these three parameters the effects of the boundary conditions were also analyzed. Six conclusions were drawn from the experimental verification of the optimal designs:

- **Optimal designs did provide the most accurate combined estimates.** An individual property might be estimated with greater accuracy at a non-optimal setting but the combination of properties maintained a higher accuracy at the optimal settings.

- **Boundary conditions did impact the accuracy of the estimates.** The constant temperature boundary conditions of Configuration 1 provided more information for the in-plane thermal conductivity and consistently estimated both properties with greater accuracy than Configuration 2.

- **The optimal designs for Configuration 2 provided reasonable estimates but with less accuracy than Configuration 1.** Since Configuration 2 offered so little information on the in-plane conductivity, any deviation from the optimal design created inaccurate estimates.

- **Both thermal properties were mildly dependent on each other.** Between the in-plane conductivity and the volumetric heat capacity the off diagonal of the approximate correlation matrix was 0.30. Therefore if adequate information was unavailable for one property, the accuracy of both properties was affected.
• It was imperative to plot the sensitivity coefficients. The coefficients give insight into the experimental design, correlation between properties, and indicated whether enough information was available for the accurate estimation of the property.

• The most sensitive parameter verified in this study was the sensor location. Since the slope of the sensor location versus the maximum determinant was steep any deviation from the optimal location produced inaccurate results.

From this analysis the importance of an optimal experimental design was observed. Optimal designs provided reliable estimates with smaller confidence intervals for both configurations. If an experiment neglected optimization not only would the confidence intervals grow but the design might provide insignificant information to produce reliable estimates.
CHAPTER 7

Recommendations

Not only did the optimal designs provide the smallest confidence intervals they also ensured significant information for reliable estimates. Although the importance of the optimization was clear, it was not possible to simultaneously estimate all three properties of an anisotropic material from one optimal experimental transient temperature history. The following are recommended to aid in the simultaneous estimation of all three thermal properties:

• Use multiple thermocouples to uncorrelate the thermal properties. The optimization procedure described in Section 3.3 could be used to determine the optimal number of sensors to use and where to locate them. Since the sum of squares objective function used in the modified Box-Kanemasu method allows for multiple sensors, the additional measurement simply needs to be implemented into the observation matrix.

• An alternative configuration that maximizes the sensitivity coefficient for the in-plane conductivity should be considered. In order to produce the most accurate estimates, large sensitivity coefficients are required. Large sensitivity
coefficients are the result of steep temperature gradients with respect to the unknown thermal properties. Therefore, a third configuration which utilizes this concept should consist of a heat flux centered over a portion of the composite away from three constant temperature boundary conditions. See Figure 7.1 for a schematic of the boundary conditions. With this design, more information might be obtained on the in-plane conductivity than produced by Configuration 1. In addition, the use of symmetry can be applied, resulting in multiple thermocouples measuring the same temperature history, therefore reducing measurement errors. Although the third configuration will provide more information on the in-plane conductivity it will be as difficult to maintain the constant temperature boundary conditions as it was for Configuration 1.

- A fourth configuration based on the experimental ease of applying and maintaining the boundary conditions should be considered. The fourth configuration should have a partially heated surface with the remaining surfaces insulated, see Figure 7.2 for the boundary conditions. This configuration does not allow for a steady state condition to be obtained. Since steady state is not desirable for the estimation of the volumetric heat capacity adequate information should be available. Although, the information for the thermal conductivities becomes more sensitive as the material reaches steady state, therefore it may be difficult to estimates the conductivities accurately. If the estimates from this optimal design are reliable, configuration four would be useful as the geometries and materials become more complex.
Figure 7.1. Boundary Conditions for a Proposed Third Configuration.

Figure 7.2. Boundary Conditions for a Proposed Fourth Configuration.
BIBLIOGRAPHY


154


Appendix A

EAL Property Estimation Program: Idpevbc

The finite element program Idpevbc (one-dimensional property estimation with variable boundary conditions) written in Engineering Analysis Language (EAL) was used for the simultaneous estimation of the effective one-dimensional thermal properties; thermal conductivity through the thickness of the composite and the volumetric heat capacity. A unique feature of this program is that it allows for a variable temperature and heat flux boundary conditions to be implemented.

An input file titled in1op, is required to execute the program. The form of the file must be:

\[ J = n : t, \quad T_{HS}, \quad T_{CTBS}, \quad q'' \]

where the \( J \) is a specification required by EAL, \( n \) is the data number and should range from 1 to the number of data points taken, \( t \) is the time, \( T_{HS} \) is the temperature history at the heated surface in either Metric or English units, \( T_{CTBS} \) is the temperature history at the constant temperature boundary surface, and \( q'' \) is the heat flux.
$ 1dpevbc
$
$ produced by Joseph P. Hanak on 2/19/95
$
$ The name of this program is an acronym for 1 dimensional property estimation
$ with a variable boundary condition.
$
$ This program estimates the thermal conductivity and volumetric heat
$ capacity of a one dimensional material whose temperature history is known
$ for both the heated and boundary nodes. The sides of the elements are
$ assumed to be insulated. This program searches for the input file: "in"
$ which must contain the time, the temperature history both heated and boundary
$ nodes, and the heat flux in the following form:
$ $ J = 1 : time(1), temp1(1), temp2(2), flux(1)$
$ $ . . . . . .$
$ $ . . . . . .$
$ $ J = n : time(n), temp1(n), temp2(n), flux(n)$
$
$*******************************************************************************
$ *XQT U1
$ *CM=200000
$
$*******************************************************************************
$
$ subroutine INPT - Define the input values
$
$*******************************************************************************
$
*$ (29 INPT VARB) INPT
$
$ !LY = 0.006096 $Height of the composite (m)
$ !NDS = 336 $The number of input data points. Don't forget the end points
$ !NY = 16 $Number of elements
$ !TIMH = 135.0 $Heating time
$
$ $ Enter initial parameter estimates
$
$ !AI1 = .5 $Initial estimate for eff. thermal conductivity
$ !AI2 = 1.5 $Initial estimate for eff. volumetric heat capacity
$
* RETURN
* INPT
$
$*******************************************************************************
$
$ subroutine TRAN - TRANSFERS the input file into appropriate tables
$
$*******************************************************************************
$
*$ (29 TRAN INPT) TRAN
$
$ $ Copy the INPT DATA into IN
$ DEFINE IN = INPT DATA
$
$ $ The time from the input table, TEMP TIME corresponds with APPL TEMP

157
TABLE(NI=1,NJ="NDS"): TEMP TIME
TRANSFER(SORCE=IN,SBASE=0,SSKIP=3,ILIM=1,CLIM=1,"NDS",OPER=XSUM)

$ The time from the input table is again needed, JUNK TIME is used to create
$ SOUR TIME in the subroutine TABL
TABLE(NI=1,NJ="NDS"): JUNK TIME
TRANSFER(SORCE=IN,SBASE=0,SSKIP=3,ILIM=1,CLIM=1,"NDS",OPER=XSUM)

$ The temperature history for the heated node
TABLE(NI=1,NJ="NDS"): HTSF TEMP
TRANSFER(SORCE=IN,SBASE=1,SSKIP=3,ILIM=1,CLIM=1,"NDS",OPER=XSUM)

$ The temperature history for the boundary node
TABLE(NI=1,NJ="NDS"): APPL TEMP
TRANSFER(SORCE=IN,SBASE=2,SSKIP=3,ILIM=1,CLIM=1,"NDS",OPER=XSUM)

$ The heat flux, INPT FLUX is used to create SOUR K21 2 in the subroutine TABL
TABLE(NI=1,NJ="NDS"): INPT FLUX
TRANSFER(SORCE=IN,SBASE=3,SSKIP=3,ILIM=1,CLIM=1,"NDS",OPER=XSUM)

*RETURN
*TRAN

$ Subroutine VARB - defines variables used in the program
$ reads in the input file
$ NOTE: Variable names can only be four
$ letters long!

$*(29 VARB DEFL) VARB

$ NY1=NY+1 $Total number of nodes
BN=FLOAT(NY1) $Boundary node as a real #, used in TEMP NODE
!N1=1 $Beginning node for matl 1 (only 1 matl in this analysis)

$ This program is limited to taking the initial starting time at time £.GE. 0.0
$ not always at time .EQ. 0.0 (when the heater actually turns on).

!TIMI= DS 1 1 1 (1 JUNK TIME) $Initial (starting) time (sec)
!TMI= DS 1 "NDS" 1 (1 JUNK TIME) $Final stopping time (sec)

*RETURN
*VARB

$ Subroutine INTP - Defines the initial temperatures of the nodes
$ A linear temperature distribution is assumed between
$ the initial temp of the heated surface node and the
$ boundary node.

$**************************************************************************
$ *(29 INIT TEMP) INTP $ 

!TH= DS 1 1 1 (1 HTSF TEMP) $Initial temperature from the heated node 
!TB= DS 1 1 1 (1 APPL TEMP) $Initial temperature from the boundary node 

$ !DTMP=(ITB-ITH)/NY $Delta temperature 

$ INTP TABL contains the initial temperatures of each node 

TABLE[NI=1,NJ="NY1"]; INTP TABL;DDATA="DTMP":J=1,"NY1":"ITH" 

$ *XQT DCU PRINT 1 INTP TABL $ 

$ *RETURN $ 

$ *INTP $ 

$******************************************************************************$ 

$ $ Subroutine NODE - defines the nodal positions $ 

$******************************************************************************$ 

$ *(29 NODE GEN)NODE $ 

*XQT TAB 

START "NY1" $ Define the total number of nodes 

JLOC $ Give the location of the nodes (set up the mesh) 

$ In the next statement, FORMAT=1 is used for rectangular coordinates; 
$ N1 is the number to start the node locations at (in this case, 1); 
$ 0,0,0 are the coordinates of N1; LY,0,0 are the coordinates 
$ for the boundary node of the mesh; NY1 is the total number of nodes 

FORMAT = 1: "N1", 0., 0., 0.,"LY", 0., 0.,"NY1" 

*RETURN $ 

*NODE $ 

$******************************************************************************$ 

$ $ Subroutine ELEM - defines the element connectivity $ 

$******************************************************************************$ 

$ *(29 ELEM GEN)ELMT $ 

*XQT ELD $ 

RESET NUTED = 1 

K21 
GROUP = 1 $Composite material 
NMAT = 1 $Material 1 
J1 = N1 $The left node of element 1 
J2 = N1 + 1 $The right node of element 1 
"J1" "J2",1,"NY" $ 

$ The point heat source is also defined as a K21 element. Note the
$ thermal conductivities and the specific heat must be set to zero in
$ COND PROP 2. In addition the two ends must have the same node number.
$
$GROUP = 2 $Source heat flux node
$NMAT = 2 $material 2
$J1=N1
$J2=J1
"J1 " "J2"
$
*RETURN
*ELMT
$

**************************************************************
$
$ Subroutine TABL - Defines the tables needed for TRTB (implicit transient
$ analyzer). COND PROP 1 is not included in this
$ subroutine but placed in the subroutine UPDA
$

**************************************************************
$
*(29 TABL GENE)TABL
*QRT AUS
TABLE(NI=1,NJ=1): K AREA: J=1: 1. The area for all K21 elements (unit area)
$
$ The next table sets the properties for the K21 heat source node to zero
TABLE(NI=9,NJ=1): COND PROP 2. 1 = 3 4 5 6: J=1: 0..0..0..0.
$
$ These tables define the time and the heat flux applied to the top
$ surface. An additional block is added to SOUR TIME at 10^-6 seconds
$ following the termination of the heater. This additional block represents
$ the time at which the heater is turned off. To correspond with this
$ additional block in SOUR TIME an additional block needs to be added
$ to SOUR K21. The value used for SOUR K21 2 is the duplicate of the
$ heat flux immediately following the termination of the heater.
$
$ The next table sets the properties for the K21 heat source node to zero.
$ First determine where the additional block needs to be added. Once the
$ time is greater than or equal to the heater time, TIMH, the block should
$ be added.
$
!CTR=0
*LABEL 987
!CTR = CTR + 1
!TYME=DS 1 "CTR" 1 (1 JUNK TIME 1 1)
*IF("TYME LE "TIMH"): *GOTO 987
!BLKN=CTR
!CTR=BLKN-1
!END=ND T=CTR
$
!SMAL = 1.E-8 $value added to TIMH to define when the heat flux turns
!THDL = TIMH + SMAL $off, this is needed due to the discontinuity at TIMH
$
$ Create a table to hold the additional time step value: THDL=TIMH+SMAL
TABLE(NI=1,NJ=1):1 THDL TABLE;J=":"THDL"
$ DEFINE A = THDL TABLE
$
160
DEFINE B = JUNK TIME $The initial time history
$
$ Create the table: SOUR TIME to hold the initial time history plus the additional time step THDL
NDS1 = NDS + 1
TABLE(NI=1,NJ=1: "NDS1"): SOUR TIME
TRANSFER(SOURCE=B,SBASE=0,SSKIP=0,ILIM=1,JLIM=1,OPER="CTR",OP="XSUM")
TRANSFER(SOURCE=A,SBASE=0,SSKIP=0,ILIM=1,JLIM=1,OPER="XSUM")
TRANSFER(SOURCE=B,SBASE="CTR",DBASE="BLKN",SSKIP=0,ILIM=1,JLIM=1,OPER="XSUM")
$
FLX=DS I "BLKN" 1 (1 INPT FLUX) $The duplicate flux following the heater turn
$
$ Create the table to hold the additional FLX
TABLE(NI=1,NJ=1: "FLX")
DEFINE C = FLUX TBLE
$
DEFINE D = INPT FLUX $Original heat flux history
$
$ Create SOUR K21 2 including the original flux history and the additional FLX
TABLE(NI=1,NJ=1: "NDS1"): 1 SOUR K21 2
TRANSFER(SOURCE=D,SBASE=0,SSKIP=0,ILIM=1,JLIM=1,OPER="CTR",OP="XSUM")
TRANSFER(SOURCE=C,SBASE=0,SSKIP=0,JLIM=1,JLIM=1,OPER="XSUM")
TRANSFER(SOURCE=D,SBASE="CTR",DBASE="BLKN",SSKIP=0,JLIM=1,JLIM=1,OPER="XSUM")
$
$ The next table defines the nodes that have a prescribed temperature (The boundary node in this analysis). Here, J is the number of nodes, not the node number!
$
TABLE(NI=1,NJ=1): TEMP NODE: J=1: "BN"
$
$ The table APPL TEMP, generated in the subroutine TRAN, defines the temperature history for the boundary node.
$
$ The corresponding times for the known temperatures are held in TEMP TIME which was created in subroutine TRAN while transferring the input data
$
XQT DCU
TOCC (1 SOUR K21 2):NJ=1,NINJ=1
TOCC(1 APPL TEMP):NJ=1,NINJ=1
$ SOUR K21 2 and APPL TEMP must be in the for of NDS 1x1 blocks
$ PRINT 1 SOUR TIME
*$XQT AUS
$ PRINT 1 SOUR TIME
$
*RETURN
*TABL
$
*******************************
$ Subroutine UPDA - Updates thermal property values
$
 *******************************
$ *(29 TABL UPDA) UPDA
*XQT AUS

$ $ The following table gives the thermal conductivity in the x and y direc. $ $ NI=9 indicates that nine variables can be entered to determine k (T, rho, $ $ c, kxx, kyy, kzz, kxy, kzy, kxz) however, NJ=1 indicates that k is $ $ temperature independent. I = 4 5: correspond to kxx and kyy inputs (i.e. $ $ the thermal conductivities in the x and y directions). NOTE: all non-zero $ $ conductivities must be specified; there are no default values. To define $ $ isotropic properties, i.e. 1D, identical values for kxx, kyy, and kzz must $ $ be entered. Note that a value of 1.0E+6 was given for the density. This $ $ is used as a scaling factor. Therefore, the estimate for the volumetric $ $ specific heat must be multiplied by (1.0E+6). $ TABLE(NI=9,NJ=1); COND PROP I: I = 2,3,4,5,6 $ OPERATION=XSUM $ J=1: 1.E+6,"A2","A1","A1","A1" $ *RETURN $ *UPDA $ $******************************************************************************* $ *(29 BILD TS) XXXX $ $ Bring surface temperature data from TRAN TEMP multi-block data set using $ $ XSUM and TRANSFER. $ $******************************************************************************* $ !NTN = N1-1 $The surface node - 1 (necessary for SBASE in TRANSFER) $*XQT AUS $DEFINE E = 1 TRAN TEMP 1 1 1 "NDS" $blocks 1 through NDS are defined in E $TABLE(NI=1,NJ="NDS"); 2 TS AUS "N3" 1 $TRANSFER(SOURCE=E,SBASE="NTN", ILIM=1,OPERATION=XSUM) $ $*$ XQT DCU $ $ PRINT 2 TS AUS "N3" 1 $*$ XQT AUS $ $ PRINT 2 TS AUS "N3" 1 $*RETURN $*XXXX $ $******************************************************************************* $ $ $ Subroutine INVIH - Minimization Procedure $ $******************************************************************************* $ *(29 INV HEAT?) INVIH $ $*XQT AUS $!CRIT = 1.4 $Criteria used to terminate estimation process $!EPS=1.0E-4 $ $Convergence criteria used in (b-b0)/(b0-EP) $!NEPS=1 $Used to determine if No. of iterations exceeds NEMX. $!NEMX=10 $Maximum number of iterations
!IT1 = 0 $Value used in determining if ests. are still changing signific.
!IT2 = 0 $Value used in determining if ests. are still changing signific.
!LOOP=0 $Used in sequential process
$ $INCO=NDST=1
$ $TABLE (NI=4,NJ="NEMX"): 4 CONV HIST 1 1 $Table that stores sequential est.
$ $A1=A1I $Set A1 and A2 back to the initial estimates.
!A2=A2I $A1 = A1I $AS1 and AS2 are previous iteration, final estimate holders.
!AS2 = A2I $ $LABEL 4000 $Begins the loop process
!A10=A1 $ $A20=A2
$ $ $Derivative calculations (Used to calculate the sensitivity coefficients
!N4=1 $ $N3=2
!NTAB=0 $ $DCALL(29 TRAN ANAL)
$ $The above call stmt. calculates temps. at the initial parameter estimates for
$ $the first iteration and at the final estimates of the previous iteration for
$ $the 2nd, 3rd, ... NEMX iterations.
!A1=1.001*A10 $Estimate at A1+dA1
!DA1=0.001*A10 $Step used to numerically differentiate
$ $N3=3 $ $DCALL(29 TRAN ANAL) $Calculates temps. at (A1+dA1)
!A1=A10 $Set A1 back to initial estimate
$ $A2=1.001*A20 $Estimate at A2+dA2
!DA2=0.001*A20 $ $N3=4 $ $DCALL(29 TRAN ANAL) $Calculates temps. at (A2+dA2)
!A2=A20 $Set A2 back to initial estimate
$ $ ** INVERSE HEAT TRANSFER BEGINS HERE ****
$ $ The parameters are initially estimated using the Gauss Method. These
$ $estimated values are then modified using the Box-Kanemasu Method.
$ $ $XQT AUS
INLIB = 2 $Identifies the source library
OUTLIB = 2 $Identifies the destination library for output datasets
DEFINE TM = 1 HTSF TEMP $Experimental Temperatures (Y)
DEFINE TO = 2 TS AUS 2 1 $Calc. temps. at initial parameter est. (ETA)
DEFINE TA1 = 2 TS AUS 3 1 $Temps at A1 + DA1
DEFINE TA2 = 2 TS AUS 4 1 $Temps at A2 + D2A
$ $A1NV = 1.0/DA1 $The following 4 statements are used in finding
!A1N2 = -1.0/DA1 $The sensitivity coefficients. (The derivative of
!A2NV = 1.0/DA2 $The temperature with respect to the parameter.
!A2N2 = -1.0/DA2 $
$ Di = \text{SUM ("AINV" TA1, "AIN2" TG) $Delta Ti (T1@A1+DA1-T1@A1)}$
$ This statement sums the derivatives for the thermal conductivity$
$ $Meaning: \ AINV \cdot TA1 + AINV \cdot TO$
$ D2 = \text{SUM ("A2INV" TA2, "A2INV" TO) $Delta Ti (T1@A2+DA2-T1@A2)$$This statement sums the derivatives for the volumetric heat capacity}$
$ N1 = \text{SUM (TM,-1, TO) $Gives the matrix (Y - ETA); the Residuals}$$Builds up the X matrix using vectors containing$
$ the derivatives$
$ SENS MATRIX = UNION(D1,D2) $Joins D1 and D2 into a new dataset$
$ D1 and D2 must have the same block length.
$ $DEFINe S=SENS MATRIX l l $Defines the matrix X, i.e., the Sens. Coeffs.$
$ ERR = XTY(S,N1) $Calculates XT \cdot (Y - ETA)$
$ STS = XTY(S,S) $Calculates XTX
$ STS1 = RINV(STS) $Calculates the INVERSE of (XTX)
$ DA = RFROD(STS1,ERR) $Calculates INV(XTX) \cdot (XTX) \cdot (Y - ETA)$
$ NTN = XTY(N1,N1) $Calculates the Sum of Squares, \((Y - ETA) \cdot (Y - ETA))$
$ TTY = XTY(TM,TM) $Calculates YTY
$ !DA1 = DS 1,1,1 (2, DA AUS, 1,1)$
$ !DA1 is the perturbation for the new estimate (thermal conductivity)$
$ !DA2 = DS 2,1,1 (2, DA AUS, 1,1)$
$ !DA2 is the perturbation for the new estimate (volumetric heat capacity)$
$ !SYS = DS 1,1,1 (2, NTN AUS, 1,1) $The sum of squares value$
$ $The following (A1 & A2) are the estimates obtained with only the Gauss Method$
$ !A1 = DA1+!A1 $New parameter estimate for the thermal conductivity$
$ !A2 = DA2+!A2 $New parameter estimate for the volumetric heat capacity$
$ $** END BASIC LOOP-BEGIN BOX-KANEMASU MODIFICATION ***$
$ This section of the program takes the estimated parameter values found
$ using the Gauss Method and modifies them using the Box-Kanemasu method.$
$ This method may allow for convergence of the parameters when the Gauss
$ method does not. It uses the direction provided by the Gauss method but
$ modifies the step size by introducing a scalar interpolation factor (H).$
$ The final parameter values are calculated using the Box-Kanemasu method.$
$ For a detailed explanation of this method, see 'Parameter Estimation' by
$ !AG1 = A1 $Fixes the Gauss estimates$
$ !AG2 = A2
$ !ASS1 = AS1 $Fixes the initial estimate for that iteration
$ !ASS2 = AS2
$ !ALPH = 2.0 $Used in finding the parameter estimates
$ !AA = 1.1 $Used to calculate H
$ *LABEL 620
$ *IF $ALPH LT 0.01);*JUMP 4001 $Alpha is too small, estimates aren't converging.$
$ !ALPH = ALPH2.0 $Alpha starts out as 1.0.$
$ !DIF1 = A1 - !ASS1 $Diff btw. Gauss & final est. of previous iteration.$
$ !DIF2 = A2 - !ASS2
$ !ASS1 = ASS1 + ALPH*DIF1 $Est. using the modified step-size
$ !ASS2 = ASS2 + ALPH*DIF2

164
ALPHA

DATR = RTRAN(DA) $Transpose (XTX\textsuperscript{-1})XT(Y - ETA)

G = RPROD(DATR,ERR) $Used in calc: H, it's the slope of the Sum of Squares

$ vs. H. By defn., it should always be a positive scalar

XVAL = DS 1,1,1 (2, G AUS, 1, 1) $Gives the scalar value found for G

!A1 = AS1

!A2 = AS2

!N3 = 3

!N4 = 1

*DCALL(29 TRAN ANAL)

$The above call stmt. calculates the temp. at the estimates obtained using ALPH

*XQT AUS

INLIB = 2

OUTLIB = 2

$ EXIT

DEFINE TOG = 2 TS AUS 5 1 $Temperatures at the Gauss est. + Step Size(alpha)

DEFINE TM = 1 HTSF TEMP $Experimental temperatures

NSS = SUM(TM,-1, TOG) $New (Y-ETA) using TOG temperatures.

SYP = XTY(NSS,NSS) $New sum of squares

ISSYP = DS 1,1,1 (2, SYP AUS, 1, 1) $Gives the sum of squares value

$ *

IF("SSYP" GT "SYS"):*JUMP 620

$ *

$ The above statement is a check to see if the sum of squares is decreasing

$ If it's not, alpha is decreased by 1/2. This process continues until the

$ above if statement is not true or until alpha is < 0.01, in which

$ case the program is terminated.

$ *CHEK = SYS - ALPH*GVAL*(2.0 - (1/AA)) $This is a check used to determine H

H = ALPH*AA $Initially set the step-size, H equal to alpha*AA.

$ *

IF SSYP > CHEK, H is given a new value; see following IF stmt.

$ *

*IF("SSYP" GT "CHEK"):H = (ALPH*ALPH*GVAL)\textsuperscript{2}(SSYP-SYS+(2.0*ALPH*GVAL))

$ *

Calculate the modified parameter values using the obtained step-size (H).

!A1 = ASS1 + H*(AG1 - ASS1) $Parameter estimates obtained using B-K method.

!A2 = ASS2 + H*(AG2 - ASS2)

$ *

Calculate the following ratios, if RAT1 and RAT2 are < CRIT (0.0001), then

$ the change in the estimated parameters is insignificant and the iterative

$ process is terminated.

$ *

!RAT1 = (A1 - ASS1)/(ASS1 + EPS)

!RAT1 = ABS(RAT1)

!RAT2 = (A2 - ASS2)/(ASS2 + EPS)

!RAT2 = ABS(RAT2)

$ *

!LOOP=LOOP+1 $Next iteration

*XQT AUS

$ *

$ Updates the table of the sequential estimates

TABLE, U(TYPE=-2): 4 CONV HIST 1 1

J="LOCP": "A1","A2","AG1","AG2"

*XQT DCU

165
PRINT 4 CONV HIST 1 1
*XQT AUS
$
$Set this iterations final estimates equal to the initial estimates for
$the next iteration.
!AS1 = A1
!AS2 = A2
$
*IF("RAT1" LE "CRIT"):!IT1 = 1
*IF("RAT2" LE "CRIT"):!IT2 = 1
!ITER = IT1 + IT2
$Determines if the change in both ests. is insignif
*IF("ITER" EQ 2):*JUMP 4001 $If ests. no longer change, stop iterating.
!NPS=NEPS+1 $Go to next iteration
*IF("NPS" GE "NEXM"):*JUMP 4001
$If the parameters don't converge before the max. No. of iters., end process
*XQT DCU
$ PRINT 2 TS AUS 2 1
$ PRINT 2 N1 AUS 1 1 $Prints out the residuals for each iteration.
PACK 1
ERASE 2
*JUMP 4000 $Est. haven't converged yet, go to next iteration
*LABEL 4001 $To end iteration process
*$XQT DCU
$ PRINT 1 HTSF TEMP
$ PRINT 4 CONV HIST 1 1
$ PRINT 1 TRAN TEMP 1 1
$ PRINT 1 TRAN TIME 1 1
$The above libraries are only printed for the final iteration. (4 TS AUS 1
$5s the for each iteration; experimental temperatures).
*RETURN
* INVH
$
$*******************************************************************************/
$
$ $Subroutine TRAN - Solves direct problem using TRTB processor
$
$*******************************************************************************/
$
*(29 TRAN ANAL)TRAN
*DCALL(29 TABL UPDA) $Update the thermal properties (estimates)
$
*XQT TGEO $Element geometry processor; it computes local coordinates
$and performs element geometry checks. The user MUST
$execute TGEO after each execution of ELD.
$
!INTS = (TIMF-TIMI)/(NDS-1) $Integral time step used in TRTB, it should be
$equal to the time step of the input data
$
$
*XQT TRTB $Transient analysis processor - Implicit with C.N. code
RESET T1="TIMI" $Initial starting time
RESET T2="TIMF" $Final experiment time
RESET DT="INTS" $Integral time step
RESET PRINT=0 $No print out
RESET MXNADT=100000 $Maximum number of iterations
RESET BETA=0.5 $Crank Nicols method
TEMP =1 INTP TABL 1 1 1 $Initial temperatures for all nodes
TSAVE=1 TEMP TIME 1 1 $Records at the same times as the input data
$
$*XQT DCU
$PRINT 1 TRAN TEMP
$*XQT AUS
$PRINT 1 TRAN TEMP
$!INCOU=NDS-1
*$DCALL (29 BILD TS)
$
$*XQT DCU
$ PRINT 1 HTSF TEMP $Experimental temperature history at the heated surface
$ PRINT 2 TS AUS 2 1 $Calculated temperature history at the heated surface
*RETURN
*TRAN
$
$*****************************************************************************
$
$ Subroutine CONF CORR - calculates the confidence intervals on the estimated
$ parameters and the correlation matrix
$
$*****************************************************************************
$
* (29 CONF CORR) CFRCR
* XQT AUS
$
$ Calculate the confidence intervals, see Beck 1977, sec 6.8.1, pg 290
$! PMS = 3 $# of estimated parameters
! PMS = 2 $# of estimated parameters
! TALPHA = 1.96 $Student T value for NDS degrees of freedom
! SNP = SSYP(NDS - PMS) * TALPHA * TALPHA
$
! PMS
! TALPHA
! SSYP
! NDS
! SNP
$
INLIB = 2
OUTLIB = 2

CF = SQRT(“SNP” STSI)
! CF1 = DS 1 1 1 (2 CF AUS 1 1)
! CF2 = DS 2 2 1 (2 CF AUS 1 1)
! CF3 = DS 3 3 1 (2 CF AUS 1 1)
$
! CF1 $Confidence interval for B1
! CF2 $Confidence interval for B2
! CF3 $Confidence interval for B3
$
$
$ Calculate the correlation matrix, Beck 1977, sec 7.7.2, pg 379
$
! P11 = DS 1 1 1 (2 STSI AUS 1 1) $Coefficients needed for the Pii and
! P22 = DS 2 2 1 (2 STSI AUS 1 1) $Pij matrices
$! P33 = DS 3 3 1 (2 STSI AUS 1 1)
$ ! P11
! P22
$! P33
$
$ ! P11 = 1.0/SQRT("P11")
! P22 = 1.0/SQRT("P22")
$! P33 = 1.0/SQRT("P33")
$
$ ! P11
! P22
$! P33
$
$ Create a p x p matrix to hold the above elements:
$
$ $ Pii = Pij = i P11 0 0 1$
$ i 0 P22 0 1$
$ 1 0 0 P33 1$
$
$ NOTE: EAL assigns NI to the number of rows and NJ to the columns
$
$ $ TABLE(NI="PMS",NJ="PMS"): 2 PI1 PIJ: I= 1, 2, 3$
$ J=1: "P11" 0.0 0.0$
$ J=2: 0.0 "P22" 0.0$
$ J=3: 0.0 0.0 "P33"$
$
$ TABLE(NI="PMS",NJ="PMS"): 2 PI1 PIJ: I= 1, 2$
$ J=1: "P11" 0.0$
$ J=2: 0.0 "P22"$
$
$ DEFINE IJJI = 2 PI1 PIJ
DEFINNE PIJ = 2 STSI AUS$
$
$ The Pij elements are held in the table 2 STSI AUS 1 1
$
$ $ PP = RPROD(IJJI, PIJ) $The product of Pii and Pij
PPT = RTRAN(PP) $The transpose [Pii x Pij]^T
PPPT = RPROD(IJJI, PPT) $The product of Pii x [Pii x Pij]^T
CORM = RTRAN(PPPT) $The correlation matrix: [Pii x [Pii x Pij]^T]^T
$
$ * XCT DCU
PRINT 2 PI1 PIJ
PRINT 2 STSI AUS
PRINT 2 PP
PRINT 2 PPT
PRINT 2 CORM
$
$ * RETURN
* CFRC
$
$*****************************************************************************
$
$ $ Main program
$
$ Set RACM = 0 to use Fortran logic in all subroutines
*RACM = 0
$
*DCALL (29 INPT VARB)
*XQT AUS
$ Note: all data read into the program must occur in the main program, NOT
$ in a subroutine
*TF OPEN 1'in1op
$Read in the temperature distribution for the top surface
TABLE (NI=4,NJ=“NDS”);INPT DATA;i=1,2,3,4
*TF READ 1
*DCALL (29 TRAN INPT)
*DCALL (29 VARB DEFI)
*DCALL (29 INIT TEMP)
*DCALL (29 NODE GENE)
*DCALL (29 TABL GENE)
*DCALL (29 ELEM DEFI)
*DCALL (29 INV HEAT)
*DCALL (29 CONF CORR) $Calculates the confidence intervals and the correlation
*XQT EXIT
Appendix B

EAL Property Estimation Program: 2dpe3vbc

The finite element program 2dpe3vbc (two-dimensional property estimation with three variable boundary conditions) is written in Engineering Analysis Language for the simultaneous estimation of the effective in-plane thermal conductivity and volumetric heat capacity. Due to the flexibility of the program, with the capability of accepting variable temperature boundary conditions, thermal property estimates are obtained from data from either Configuration 1 or 2.

Three input files; in1op, in2, and in3 are used to specify the boundary conditions and are necessary to execute the program. The form of input file in1op is

\[ J = n : t, \quad T_{HS}, \quad T_{CTBS}, \quad q'' \]

refer to Appendix A for a description of the variables. Input files in2 and in3 have identical forms and are used to specify the temperatures along the left and right surfaces of the composite, respectively.

\[ J = n : T_{upper}, \quad T_{center}, \quad T_{bottom} \]

where \( T_{upper} \) is the temperature history closest to the heat flux, \( T_{center} \) is the temperature
history at the center, and $T_{\text{bottom}}$ is the temperature history closest to the heat sink.

$2$dpe3vbc

$produced$ $by$ $Joseph$ $P.$ $Hanak$ $on$ $3/18/95$

$This$ $program$ $was$ $derived$ $from$ $l2$dpe3vbc.dat but estimates the ratio of
thermal conductivity perpendicular to thermal conductivity parallel by
fixing the thermal conductivity perpendicular to the fibers to a constant.

$This$ $program$ $estimates$ $the$ $thermal$ $properties$ $for$ $the$ $following$ $boundary$
conditions:

$\text{a)}$ specified heating area over the top surface
$\text{b)}$ the left, right, and bottom surfaces are allowed to vary

$This$ $program$ $has$ $been$ $hard$ $coded$ $for$ $8$ $elements$ $in$ $the$ $y$ $direction.$ $This$
$\text{is}$ $appropriate$ $due$ $to$ $thermocouple$ $location$ $in$ $the$ $actual$ $experiment$ $matches$
$up$ $with$ $these$ $vertical$ $nodes.$

$In$ $addition$ $the$ $user$ $must$ $change:$
$\text{a)}$ variables in subroutine VARB
$\text{b)}$ TABLE HFLX in subroutine BOUNDARY for appropriate number of heated
$\text{ elements}$ ('H')
$\text{c)}$ input file names in MAIN program

$**********************************************************$
$*
$* XQT U1
$* CM = 200000
$*
$**********************************************************$
$Subroutine$ VARB - defines variables used in the program
$\text{NOTE:}$ Variable names can only be four
$\text{letters}$ long!

$**********************************************************$
$*(29$ VARB DEFL) VARB
$Define$ geometry for 2-D plate

! $X = 0.1016$ $\text{Length of plate (m)}$
! $Y = 0.006096$ $\text{Height of plate (m)}$

! $\text{HEX} = 13$ $\text{Number of heated elements}$
! $\text{EX} = 13$ $\text{Number of elements in X direction}$

$In$ case the optimal temperature sensor does not land on a node this program
will linearly interpolate between the known temperatures of the surrounding
nodes to determine the temperature history of the optimal temperature sensor
location

! $XOP \approx .84$ $\text{Optimal temperature sensor location percent base on Y}$
$ NODE = 123   $The node prior to the optimal temperature sensor location
$ $ Enter initial parameters estimates
$ $ A11 = 0.5   $Est. for eff. k perpendicular to fibers, this value will
$ $ never change through the program
$ $ A21 = 1.5   $Initial estimate for eff. volumetric heat capacity
$ $ A31 = 3.5   $Initial est. for eff. k parallel to fibers
$ $ NDS = 101   $The number of input data points, don't forget the end pts
$ $ TIMH = 25.0 $Time the heat flux is applied.
$
$ * RETURN
$ * VARB
$
$******************************************************************************
$ $ Subroutine TRAN SFER - pulls the required data out of the input file
$ $******************************************************************************
$
$* (29 TRAN SFER) TRNS
$ *
$ XQT AUS
$
$ $ Copy IN1 DATA to IN1
$ $ DEFINE IN1 = IN1 DATA
$ $ $ Pull the time, heated node and bottom node's temperature history, and the
$ $ $ heat flux from IN1
$ $ $ The time table
$ $ TABLE(NI=1,NJ="NDS"):TEMP TIME
$ $ TRANSFER(SOURCE=IN1,ILIM=1,JLIM="NDS",SSKIP=3,DSKIP=0,OPER=XSUM)
$ $
$ $ The heated node's temperature history
$ $ TABLE(NI=1,NJ="NDS"):4 TS AUS 1 1
$ $ TRANSFER(SOURCE=IN1,SBASE=1,ILIM=1,JLIM="NDS",SSKIP=3,DSKIP=0,OPER=XSUM)
$ $
$ $ The bottom node's temperature history
$ $ TABLE(NI=1,NJ="NDS"):CNST TEMP
$ $ TRANSFER(SOURCE=IN1,SBASE=2,ILIM=1,JLIM="NDS",SSKIP=3,DSKIP=0,OPER=XSUM)
$ $
$ $ The heat flux history
$ $ TABLE(NI=1,NJ="NDS"):HEAT FLUX
$ $ TRANSFER(SOURCE=IN1,SBASE=3,ILIM=1,JLIM="NDS",SSKIP=3,DSKIP=0,OPER=XSUM)
$ $
$ * XQT DCU
$ $ PRINT 1 TEMP TIME
$ $ PRINT 4 TS AUS 1 1
$ $ PRINT 1 CNST TEMP
$ $ PRINT 1 HEAT FLUX
$
$ * RETURN
$ * TRNS

172
* Subroutine CONS TANT - defines the constants used throughout the program

* (29 CONS TANT) CNST

* XQT AUS

$ Define number of elements and nodes in each direction

! NX = EX + 1 $Number of nodes in X direction
! EY = 8 $Number of elements in Y direction
! NY = EY + 1 $Number of nodes in Y direction
! NT = NX * NY $Total number of nodes in mesh
! N1 = 1 $Beginning node

$ Determine the end points for the linear interpolation of the optimal
temperature sensor location

! XOPL = XOP * X $Optimal temperature location in meters
! SNX = X - (NT - NODE)*X/EX $X location of NODE
! GNX = SNX + X/EX $X location of the node past the opt temp sens loc
! GN = NODE $Node # for the node past the opt temp sens loc
! NODE = NODE - 1 $necessary for TRANSFER

$ This program is limited to taking the initial starting time at a time greater
than or equal to zero not always at time equal to zero (when the heater
actually turns on)

! TIMI = DS 1 1 1 (1 TEMP TIME) $The starting time
! TIMF = DS 1 "NDS" 1 (1 TEMP TIME) $The stopping time of the experiment
! TIMI
! TIMF

$ TEmi = DS 1 1 1 (4 TS AUS) $The initial temperature

$ RETURN
* CNST

$ Subroutine NODE - defines the nodal positions

$ (29 NODE GENE) NODE

* XQT TAB
START "NT" $ Define the total number of nodes
JLOC $ Give the location of the nodes (set up the mesh)

$ In the next statement, FORMAT=1 is used for rectangular coordinates;
$ N1 is the number to start the node locations at (in this case, 1);
$ 0,0,0 are the coordinates of N1; LX,0,0 are the coordinates
for the bottom right corner of the mesh; NX1 is the number of nodes
in the x direction; 1 is the increment in the node number in the x
direction; and NY1 is the total number of nodes in the y direction.
For the next line, NX1 is jump used in the y direction; 0, LY, 0 are
the coordinates of the upper left node; and LX, LY, 0 are the
coordinates of the upper right node.

FORMAT = 1: "NI", 0., 0., 0., "X", 0., 0., "NX", 1, "NY"
         "NX", 0., "Y", 0., "X", "Y", 0.

* RETURN
* NODE

******************************************************************************

$ Subroutine TABL - Defines the thickness of the elements

******************************************************************************

* (29 TABL GENE) TABL
* XQT AUS

TABLE(NI=1,NJ=1): K THIC: J=1: 1. $The thickness of K41 elements
TABLE(NI=1,NJ=1): K AREA: J=1: 1. $The area of K21 elements

$ The next table sets up the 0 value properties for the K21 element
   TABLE(NI=9,NJ=1): COND PROP 2: I = 3 4 5 6: J=1: 0.,0.,0.,0.,

* RETURN
* TABL

******************************************************************************

$ Subroutine ELEM - defines the element connectivity

******************************************************************************

* (29 ELEM DEFI) ELMT
* XQT ELD

$ Define K41 elements
$ K41 signifies a conductive, 4 node element.

$RESET NUTED=1
K41
GROUP = 1 $Group 1
NMAT=1 $Material 1
! J1 = N1 $J1 is the number of the bottom LF node in an element
! J2 = N1 + 1 $J2 is the number of the bottom KT node in an element
! J3 = NX + 2 $J3 is the number of the top KT node in an element
! J4 = NX + 1 $J4 is the number of the top LF node in an element

"J1" "J2" "J3" "J4", 1, "EX" "EY"

$ The above line sets up the nodal positions of each element. 1 is the
$ node increment and EX and EY signify the number of elements in the x and y directions, respectively.
$ 
$ Define K21 elements
$ K21 is used to represent the heat flux
$ Note: since we're using this element to model the heat flux, the thermal conductivities and specific heat must be zero (reason for Mat'1 2)

K21
GROUP = 1
NMAT = 2
! J1 = EY * NX + 1
! J2 = J1 + 1
$
"J1", "J2", 1, "HEX"
$
* RETURN
* ELMT
$
******************************************************************************
$
$ Subroutine UPDA - Updates thermal property values
$
******************************************************************************
$
* (29 TABL UPDA) UPDA
* XQT AUS
$
$ The following table gives the thermal conductivity in the x and y direc.
$ Ni=9 indicates that nine variables can be entered to determine k (T, rho, c, kxx, kyy, kzz, kxy, kzy, kzx) however, Nj=1 indicates that k is
$ temperature independent. I = 4 5: correspond to kxx and kyy inputs (i.e.
$ the thermal conductivities in the x and y directions). NOTE: all non-zero
$ conductivities must be specified; there are no default values. To define
$ isotropic properties, identical values for kxx, kyy, and kzz must be entered.
$ Note that a value of 1.0E+6 was given for the density. This is used as a
$ scaling factor. Therefore, the estimate for the volumetric specific heat
$ must be multiplied by (1.0E+6).
$
TABLE(NI=9,NJ=1): COND PROP 1: i = 2,3,4,5,6
J=1: 1.E+6,"A2","A3","A4","A5"
$
* RETURN
* UPDA
$
******************************************************************************
$
$ Subroutine TIME FLUX - generates the time and heat flux tables used in the
$ following subroutine BOUNDARY
$
******************************************************************************
$
* (29 TIME FLUX) TFUX
* XQT AUS
$
$ The next tables define the time and the heat flux applied to the top surface.
$ An additional block needs to be added to both SOUR TIME and cycle 21 due to the discontinuity with a temperature measurement on the heater.
$ termination. The additional block to SOUR TIME is equal to the heater turn off time plus a small amount. The extra value added to SOUR 21 is the duplicate value of the heat flux measured at the time step after the heater
$ is turned off.
$
$ First determine where the additional block needs to be added. This occurs when the time is greater than or equal to the heater time, TIMH.
$
!CTR = 0
* LABEL 987
!CTR = CTR + 1
!TUME = DS 1 "CTR" 1 (TEMP TIME)
*IF ("TUME" LE "TIMH"):*GOTO 987
!BLKN = CTR
!CTR=BLKN-1
!END = NDS - CTR
$
$DFLX = 1.E-8 $Small value added to time to define heat flux value
!THDL = TIMH + DFLX $This is needed due to the discontinuity at time
$
$ Create a table to hold the additional time step value:THDL.
TABLE(NI=1,NJ=1):THDL, TABLEJ=1:"THDL"
DEFINE A = THDL TABLE
$
$ DEFINE B = TEMP TIME
$
$ Create SOUR TIME to hold the initial time history plus the additional time
$ step THDL
!NDS1 = NDS + 1
TABLE(NI=1,NJ="NDS1"): SOUR TIME
TRANSFER(SOURCE=B,ILIM=1,JLIM="CTR",OPER=XSUM)
TRANSFER(SOURCE=A,DBASE="CTR",ILIM=1,OPER=XSUM)
TRANSFER(SOURCE=B,SBASE="CTR",DBASE="BLKN",ILIM=1,JLIM="END",OPER=XSUM)
$
$ Create a table to hold the heat flux plus one additional value to correspond
$ with the additional time step
$
!FLX = DS 1 "BLKN" 1 (HEAT FLUX) $Duplicate flux following the heater
$
$ Create a table to hold the additional FLX
TABLE(NI=1,NJ=1):FLUX TABLEJ=1:"FLX"
DEFINE C = FLUX TABLE
$
$ DEFINE D = HEAT FLUX $Original heat flux history
$
TABLE(NI=1,NJ="NDS1"): FLUX SOUR
TRANSFER(SOURCE=D,ILIM=1,JLIM="CTR",OPER=XSUM)
TRANSFER(SOURCE=C,DBASE="CTR",ILIM=1,OPER=XSUM)
TRANSFER(SOURCE=D,SBASE="CTR",DBASE="BLKN",ILIM=1,JLIM="END",OPER=XSUM)
$
*XQT DCU
$ PRINT 1 SOUR TIME

176
$ PRINT 1 FLUX SOUR
$ * RETURN
* TFLX
$
$******************************************************************************
$ $ Subroutine BOUN DARY - specifies the boundary conditions
$******************************************************************************
$ $ (29 BOUN DARY) BDRY
* XQT AUS
$ $ The heat flux across the top surface
$ TABLE((Nf="HEX",NJ="NDS1"): HFLX
! CTR3 = 0
* LABEL 1013
! CTR3 = CTR3 + 1
! H = DS 1 "CTR3" l (1 FLUX SOUR)
TABLE,U(TYPE=2): HFLX: J="CTR3"
"H" "H" "H" "H" "H" "H" "H" "H" "H" "H"
$ IF ("CTR3" LT "NDS1"): * GOTO 1013
$
$ Rename HFLX to SOUR K21 1
$ DEFINE G = HFLX
* HPTS = NDS1*HEX
TABLE(Nf="HEX",NJ="NDS1"): SOUR K21 1
TRANSFER(SOURCE=G,ILIM=1,JLIM="HPTS",OPER=XSUM)
$
$ The next table defines the nodes that have a prescribed temperature,
the bottom and side surfaces in this analysis. DDATA is a counter; Note:
$ it must be a REAL value (not an integer). Here, J is the number of nodes, NOT
$ the node number! Only three temperatures are known, the two side temps and
$ the bottom temp. All nodes on these surfaces are fixed to one temperature,
$ the left and right surfaces have the same temperature but the bottom may
$ vary.
$
$ Define which nodes have prescribed temperatures, this includes the 8 nodes
$ on each side plus one node off of the bottom surface
$
! FN! = FLOAT(Nf) Node 1 is used to fix the bottom surface temp
$
! LSC = N1 + NX :! LSC = FLOAT(LSC) :! LSC 82nd vertical node, left side
! LSIC = LSC + NX :! LSIC = FLOAT(LSIC) :! LSIC 83rd
! LSBC = LSIC + NX :! LSBC = FLOAT(LSBC) :! LSBC 84th
! LSB = LSBC + NX :! LSB = FLOAT(LSB) :! LSB 85th
! LSAB = LSB + NX :! LSAB = FLOAT(LSAB) :! LSAB 86th
! LSIA = LSAB + NX :! LSIA = FLOAT(LSIA) :! LSIA 87th
! LSA = LSIA + NX :! LSA = FLOAT(LSA) :! LSA 88th
! LSAA = LSA + NX :! LSAA = FLOAT(LSAA) :! LSAA 89th
$
! RSAA = LSAA + EX :! RSAA = FLOAT(RSAA) :! RSAA 99th vertical node, right side
! RSA = LSA + EX :! RSA = FLOAT(RSA) :! RSA 88th
RSIA = LSIA + EX :: RSIA = FLOAT(RSIA) :: RSIA $7th$
RSAB = LSAB + EX :: RSAB = FLOAT(RSAB) :: RSAB $5th$
RSB = LSB + EX :: RSB = FLOAT(RSB) :: RSB $5th$
RSBC = LSBC + EX :: RSBC = FLOAT(RSBC) :: RSBC $4th$
RSIC = LSC + EX :: RSIC = FLOAT(RSIC) :: RSIC $3rd$
RSC = LSC + EX :: RSC = FLOAT(RSIC) :: RSC $2nd$

$TABLE(N1=1,NJ=17):TEMP NODE: J=1,17: "LSA" "LSB" "LSC" "RSA" "RSB" "RSC" "P1"
"LSAA" "LSA" "LSAB" "LSBC" "LSC" "RSA" "RSAB" "RSBC" "RSIC"

$TABLE(N1=1,NJ="NDS"):LTT TABL
TRANSFER(SOURCE=IN2,SBASE=0,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE LTT = LTT TABL

$TABLE(N1=1,NJ="NDS"):LMT TABL
TRANSFER(SOURCE=IN2,SBASE=1,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE LMT = LMT TABL

$TABLE(N1=1,NJ="NDS"):LET TABL
TRANSFER(SOURCE=IN2,SBASE=2,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE LBT = LBT TABL

$TABLE(N1=1,NJ="NDS"):RTT TABL
TRANSFER(SOURCE=IN3,SBASE=0,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE RTT = RTT TABL

$TABLE(N1=1,NJ="NDS"):RMT TABL
TRANSFER(SOURCE=IN3,SBASE=1,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE RMT = RMT TABL

$TABLE(N1=1,NJ="NDS"):RBT TABL
TRANSFER(SOURCE=IN3,SBASE=2,JLIM=1,JLIM="NDS","SKIP=2,OPER=XSUM)
DEFINE RBT = RBT TABL

$Through Newtons interpolating polynomial a quadratic was developed to estimate
the temperatures at the remaining eight nodes. The quadratic is:

$T(Y) = [LBT] + 8/3*LY \cdot ([LMT]-[LBT]) \cdot (Y-(1/8)*LY) +$

$32/(9*LY^2) \cdot ([LTT]+[LBT]-2*[LMT]) \cdot (Y-(1/8)*LY) \cdot (Y-(1/2)*LY)$

$Where LY is the vertical height of the plate and Y is the vertical location

178
$ of the node. $

! CY = -1.
LMLB = SUM(LMT,"CY" LBT) $ [(LMT) - [LBT]] for the left side
RMRB = SUM(RMT,"CY" RBT) $ for the right side

$ LTBM = SUM(LTLB,"CY" LMT) $ [(LTT) + [LBT] -2*[LMT]] for the left side
RTBM = SUM(RTRB,"CY" RMT) $ for the right side

$ ! CY = -2.

$ This program has been hard coded for 8 elements in the y direction
$ therefore the constants in the quadratic have already been determined
$ and hard coded in.

! LAA = 5/3. :: LAB = 5.9. SLAA = 8/3(Ly) * (Y-(1/8)*Ly) for Y = 6/8 Ly
! RAA = LAA :: RAB = LAB SLAB = 32/9(Ly^2)*Y-(1/8)*Ly)(Y-(1/2)*Ly) for
$ Y = 6/8 Ly RAA is the right side equivalent of LAA
$ ! LBA = 4/3. :: LBB = 2.8. $ Y = 5/8 Ly
! RBA = LBA :: RBB = LBB
$ ! LCA = 2/3. :: LCB = -1.9. $ Y = 3/8 Ly
! RCA = LCA :: RCB = LCB
$ ! LDA = 1/3. :: LDB = -1.9. $ Y = 2/8 Ly
! RDA = LDA :: RDB = LDB
$ ! LEA = 7/3. :: LEB = 14.9. $ Y = 8/8 Ly
! REA = LEA :: REB = LEB

LAS = SUM("LAA" LMLB, "LAB" LTBM)
LBS = SUM("LBA" LMLB, "LBB" LTBM);
LCS = SUM("LCA" LMLB, "LCB" LTBM)
LDS = SUM("LDA" LMLB, "LDB" LTBM)
LES = SUM("LEA" LMLB, "LEB" LTBM)

RAS = SUM("RAA" RMRB, "RAB" RTBM)
RBS = SUM("RBA" RMRB, "RBB" RTBM)
RCS = SUM("RCB" RMRB, "RDB" RTBM)
RDS = SUM("RDA" RMRB, "RDB" RTBM)
RES = SUM("REA" RMRB, "REB" RTBM)

$ L1T = SUM(LBT, LAS) $Temp history at Y = 6/8 Ly on the left side
L2T = SUM(LBT, LBS) $ Y = 5/8 Ly
L3T = SUM(LBT, LCS) $ Y = 3/8 Ly
L4T = SUM(LBT, LDS) $ Y = 2/8 Ly
L5T = SUM(LBT, LES) $ Y = 8/8 Ly
$ R1T = SUM(RBT, RAS) $Temp history at Y = 6/8 Ly on the right side
R2T = SUM(RBT, RBS) $ Y = 5/8 Ly
R3T = SUM(RBT, RCS) $ Y = 3/8 Ly
R4T = SUM(RBT, RDS) $ Y = 2/8 Ly

179
\[ RST = \text{SUM}(RBT, RES) \]  \[ Y = 8/8 \text{ Ly} \]

$\$
$Combine$ $all$ $temperatures$ $into$ $one$ $table$. $Remember$ $the$ $order$ $of$ $the$
$temperatures$ $must$ $correspond$ $with$ $the$ $node$ $numbering$ $in$ $TEMP$ $NODE$
$\$
$DEFINE$ $BN = \text{CNST TEMP}$ $\$Need$ $for$ $the$ $temp$ $history$ $for$ $the$ $bottom$ $surface$
$\$
$TABLE(NI=17,NJ="NDS");APPL TEMP$
TRANSFER(SOURCE=IN2, ILIM=3, JLIM="NDS", DSKIP=14, OPER=XSUM)
TRANSFER(SOURCE=IN3, DBASE=3, JLIM="NDS", DSKIP=14, OPER=XSUM)
TRANSFER(SOURCE=BN, DBASE=6, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=L1T, DBASE=7, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=L1T, DBASE=8, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=L2T, DBASE=9, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=L3T, DBASE=10, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=L4T, DBASE=11, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=R5T, DBASE=12, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=R1T, DBASE=13, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=R2T, DBASE=14, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=R3T, DBASE=15, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
TRANSFER(SOURCE=R4T, DBASE=16, JLIM=1, JLIM="NDS", DSKIP=16, OPER=XSUM)
$\$
$Assign$ $all$ $the$ $temperature$ $histories$ $of$ $the$ $bottom$ $surface$ $nodes$ $equal$ $to$ $the$
$measured$ $temperature$ $history$ $from$ $one$ $location$ $on$ $the$ $bottom$ $surface.$
$\$
$TABLE(NI=2,NJ="EX");SAME TEMP:I=1,2;DDATa=1,3=1,"EX":2,1.
$\$
$*$ QXT DCU
TOCC(1 SOUR K21 1);NJ="HEX",NJ=17,NINJ=17
TOCC(1 APPL TEMP);NJ=17,NINJ=17
$ PRINT 1 SOUR K21 1
$ PRINT 1 APPL TEMP 1 1 1,15 1,1 11,91
$* XQT AUS
$ PRINT SOUR TIME
$ *
* RETURN
* BDRY
$**************
$ Subroutine BILD TS - pulls the temperature history for the optimal
$ temperature sensor location from TRAN TEMP multi
$ block data set.
$ ****************
$ *
(29 BILD TS) XXXX
$ *
* XQT AUS
$ Note, this transfer table is only valid for the number of control volumes
specified in this problem. This occurs because temperatures at the node
(42) need to be transferred into a separate table to be used in the Box-
Kanemasu method. This node number corresponds to the optimal temperature
sensor location found in the optimization procedure. If you want to
$ change the number of control volumes used, you need to change "NTN" to
$ the correct value that corresponds to this location so the right temps.
$ are transferred.
$
$$ \text{DEFINE } A = \text{TRAN TEMP}$$
$
$ The temperature history for the node prior to the optimal temperature sens loc
TABLE(NI=1,NJ="NDS"):U1
TRANSFER(SOURCE=A,SBASE="NODE",ILM=1,OPER=XSUM)
$$ \text{DEFINE } U1 = U1$$
$
$ The temperature history for the node past the optimal temperature sens loc
TABLE(NI=1,NJ="NDS"):U3
TRANSFER(SOURCE=A,SBASE="GN",ILIM=1,OPER=XSUM)
$$ \text{DEFINE } U3 = U3$$
$
$ Linearly interpolate between the known temperatures to find the temperature of
$ the optimal temperature sensor location
$
$$ ! \text{FACT} = (XOPL \cdot SNX)/(SNX-GNX) \text{SUB = SUM(U1, -1, U3)} \text{LINT = SUM("FACT" SUB, U1)}$$
$
$ copy LINT into 2 TS AUS "N3" 1
TABLE(NI=1,NJ="NDS"): 2 TS AUS "N3" 1
TRANSFER(SOURCE=LINT, ILM=1, JLM=1, OPER=XSUM)
$
$
$* XQT DCU
$ PRINT 1 U1
$ PRINT 1 U3
$ PRINT 2 TS AUS "N3" 1
*$ XQT AUS
$ PRINT 2 TS AUS "N3" 1
$
$* RETURN
* XXXX
$
$*******************************************************************************
$
$ Soboutine INVH - Minimization Procedure
$
$*******************************************************************************
$
*(29 INV HEAT) INVH
$
$*XQT AUS
$ ! CRIT = 1.E-3 $Criteria used to terminate estimation process
$ ! ITI = 0 $Value used in determining if est. are still changing signific.
$ ! IT2 = 0 $Value used in determining if est. are still changing signific.
$ ! IT3 = 0
$ ! LOOP=0 $Used in sequential process
$ ! EPS=1.0E-6 $Convergence criteria used in (b-b0)/(b0-EPS)
$ !NEPS=1 $Used to determine if No. of iterations exceeds NEMX.
NEMX=10 $Maximum number of iterations
*QOT AUS
*TIN1=DS 1.1,1 (4,TS AUS,1,1) $Defines the initial exp. temperature
TABLE (NI=7,NJ="NEMX") 4 CONV HIST 1 1 $Table that scores sequential est.
$TABLE (NI=1,NJ="NCOU") 4 RES HIST 1 1
!A1=A11 $Set A1's back to the initial estimates.
!A2=A21
!A3=A31
$!AS1 = A11 $AS1's are previous iteration, final estimate holders.
!AS2 = A21
!AS3 = A31
$
*LABEL 4000 $Begins the loop process
$!A10=A1
!A20=A2
!A30=A3
$
$Derivative calculations (Used to calculate the sensitivity coefficients
*TIN1 = DS 1,1,1 (4,TS AUS,1,1)
!N=1
!N3=2
!NTAB=0
*DCALL(29 TRAN ANAL)
$The above call stmt. calculates temps. at the initial parameter estimates for
$the first iteration and at the final estimates of the previous iteration for
$the 2nd, 3rd, ... NEMX iterations.
$!A1=1.001*A10 $Estimate at A1+dA1
$!DA1=0.001*A10 $Step used to numerically differentiate
$!N3=3
$*DCALL(29 TRAN ANAL) $Calculates temps at (A1+dA1)
$!A1=A10 $Set A1 back to initial estimate
$
$!A2=1.001*A20 $Estimate at A2+dA2
$!DA2=0.001*A20
$!N3=4
*DCALL(29 TRAN ANAL) $Calculates temps at (A2+dA2)
$!A2=A20 $Set A2 back to initial estimate
$
$!A3=1.001*A30 $Estimate at A3+dA3
$!DA3=0.001*A30
$!N3=5
*DCALL(29 TRAN ANAL) $Calculates temps at (A3+dA3)
$!A3=A30 $Set A3 back to initial estimate
$
$**** INVERSE HEAT TRANSFER BEGINS HERE ****
$
$The parameters are initially estimated using the Gauss Method. These
$estimated values are then modified using the Box-Kanemasu Method.
$
*XQT AUS
INLIB = 2 $Identifies the source library
OUTLIB = 2 $Identifies the destination library for output datasets
DEFINE TM = 4 TS AUS 1 1 $Experimental Temperatures (Y)
DEFINE TO = 2 TS AUS 2 1 $Calc. temps. at initial parameter est. (ETA)
$DEFINE TA1 = 2 TS AUS 3 1 $Temps at A1 + DA1

182
DEFINE TA2 = 2 TS AUS 4 1  $Temps at A2 + DA2
DEFINE TA3 = 2 TS AUS 5 1  $Temps at A3 + DA3

$A1N1V = 1.0/DA1  $The following 4 statements are used in finding
$A1N2 = -1.0/DA1  $the sensitivity coefficients. (The derivative of
$A2N2 = 1.0/DA2  $the temperature with respect to the parameter).
$A3N1V = -1.0/DA2
$A3N2 = 1.0/DA3

$ D1 = SUM ("A1N1V" TA1, "A1N2" TO)  $Delta Ti (Ti@A1+DA1-Ti@A1)
$ This statement sums the derivatives for k perp. to fibers
$Meaning:  A1N1V * TA1 + A1N2 * TO

D2 = SUM ("A2N2" TA2, "A2N2" TO)  $Delta Ti (Ti@A2+DA2-Ti@A2)
$This statement sums the derivatives for the volumetric heat capacity
D3 = SUM ("A3N1V" TA3, "A3N2" TO)  $Delta Ti (Ti@A3+DA3-Ti@A3)
$This statements sums the derivatives for k parallel to fibers

N1 = SUM (TM-.1: TO)  $Gives the matrix (Y - ETA); the Residuals
$Build up the X matrix using vectors containing
$ the derivatives

SENS MATRIX = UNION(D2,D3)  $Joins Di's into a new dataset
SENS MATRIX = UNION(D1,D2,D3)  $Joins Di's into a new dataset

Di's must have the same block lengths.

DEFINE S=SENS MATRIX 1 1  $Defines the matrix X, i.e., the Sens. Coeffs.
ERR = XTY(S,N1)  $Calculates XT (Y - ETA)
STS = XTY(S,S)  $Calculates XTX
STSI = RINV(STS)  $Calculates the INVERSE of (XTX)
DA = RPROD(STSI,ERR)  $Calculates INV(XTX)*(XT)*(Y - ETA)

N1N1 = XTY(N1,N1)  $Calculates the Sum of Squares, (Y-ETA)T(Y-ETA)

TTT = XTY(TM,TM)  $Calculates YTY

$DA1 = DS 1,1,1 (2, DA AUS, 1,1)
$DA1 is the perturbation for the new estimate of k perp. to fibers
$DA2 = DS 1,1,1 (2, DA AUS, 1,1)
$DA2 is the perturbation for the new estimate of volumetric heat capacity
$DA3 = DS 2,1,1 (2, DA AUS, 1,1)
$DA3 is the perturbation for the new estimate of k parallel to fibers
$YS = DS 1,1,1 (2, N1N1 AUS, 1, 1)  $The sum of squares value

$The following (A1, A2, & A3) are the estimates obtained with only the Gauss Method
$A1 = DA1+A1  $New parameter estimate for k perp. to fibers
$A2 = DA2+A2  $New parameter estimate for the volumetric heat capacity
$A3 = DA3+A3  $New parameter estimate for k parallel to fibers

$*** END BASIC LOOP-BEGIN BOX-KANEMASU MODIFICATION ***

$This section of the program takes the estimated parameter values found
$ using the Gauss Method and modifies them using the Box-Kanemasu method.
$This method may allow for convergence of the parameters when the Gauss
$ method does not. It uses the direction provided by the Gauss method but
$ modifies the step size by introducing a scalar interpolation factor (H).
$The final parameter values are calculated using the Box-Kanemasu method.
$ For a detailed explanation of this method, see 'Parameter Estimation' by
$AG1 = A1 $F1xes the Gauss estimates
$AG2 = A2
$AG3 = A3
$ASS1 = AS1 $F1xes the initial estimate for that iteration
$ASS2 = AS2
$ASS3 = AS3
$ALPH = 2.0 $Used in finding the parameter estimates
$AA = 1.1 $Used to calculate H
$*LABEL 620
$*IF("ALPH" LT 0.01):*JUMP 4001 $Alpha is too small, estms. aren't converging.
$ALPH = ALPH/2.0 $Alpha starts out as 1.0
$DIF1 = A1 - ASS1 $Diff btw. Gauss & final est. of previous iteration.
$DIF2 = A2 - ASS2
$DIF3 = A3 - ASS3
$ASS1 = ASS1 + ALPH*DIF1 $Est. using the modified step-size
$ASS2 = ASS2 + ALPH*DIF2
$ASS3 = ASS3 + ALPH*DIF3
$ALPHA
$DATR = RTRAN(DA) $Transpose (XTX)^(-1)XT(Y - ETA)
$G = RPROD(DATR,ERK) $Used in calc. H, it's the slope of the Sum of Squares
$ vs. H. By defn., it should always be a positive scalar
$GVAL = DS 1,1,1 (2, G AUS, 1, 1) $Gives the scalar value found for G
$AA1 = AS1
$AA2 = AS2
$AA3 = AS3
$IN3 = 6
$IN4 = 1
$*DCALL(29 TRAN ANAL)
$The above call stmt. calculates the temp. at the estimates obtained using ALPH
$XQT AUS
$INLIB = 2
$OUTLIB = 2
$*EXIT
$DEFINE TOG = 2 TS AUS 6 1 $Temperatures at the Gauss est. + Step Size(alpha)
$DEFINE TM = 4 TS AUS 1 1 $Experimental temperatures
$NSS = SUM(TM,-1, TOG) $New (Y-ETA) using TOG temperatures.
$SYP = XTY(NSS,NSS) $New sum of squares
$SSYP = DS 1,1,1 (2, SYP AUS, 1, 1) $Gives the sum of squares value
$*IF("SSYP" GT "SYS"):*JUMP 620
$The above statement is a check to see if the sum of squares is decreasing
$If it's not, alpha is decreased by 1/2. This process continues until the
$above if statement is no longer true or until alpha is < 0.01, in which
$case the program is terminated.
$CHEK = SYS - ALPH*GVAL*(2.0 - (1/AA)) $This is a check used to determine H
$H = ALPH*AA $Initially set the step-size, H equal to alpha*AA.
$If SSYP > CHEK, H is given a new value; see following IF stmt.
$*IF("SSYP" GT "CHEK"):H = (ALPH*ALPH*GVAL)/(SSYP-SYS+(2.0*ALPH*GVAL))
$Calculate the modified parameter values using the obtained step-size (H).
$A1 = ASS1 + H*(AG1 - ASS1)  $Parameter estimates obtained using B-K method.
$A2 = ASS2 + H*(AG2 - ASS2)
$A3 = ASS3 + H*(AG3 - ASS3)
$
$Calculate the following ratios, if $< CRIT (0.0001), then
$the change in the estimated parameters is insignificant and the iterative
$process is terminated.
$
$!RAT1 = (A1 - ASS1)/(ASS1 + EPS)
$!RAT1 = ABS(RAT1)
$!RAT2 = (A2 - ASS2)/(ASS2 + EPS)
$!RAT2 = ABS(RAT2)
$!RAT3 = (A3 - ASS3)/(ASS3 + EPS)
$!RAT3 = ABS(RAT3)
$
$!LOOP=LOOP+1  $Next iteration

*XQT AUS
$
$ Updates the table of the sequential estimates
TABLE.U(TYPE=2): 4 CONV HIST 1 1
J="LOOP": "A1","A2","A3","AG1","AG2","AG3","SSYP"
*XQT DCU
PRINT 4 CONV HIST 1 1
$ PRINT 1
*XQT AUS
$ PRINT SOUR TIME
$
$Set this iterations final estimates equal to the initial estimates for
$the next iteration.
$!AS1 = A1
$!AS2 = A2
$!AS3 = A3
$
$*IF("RAT1" LE "CRIT"):!IT1 = 1
*IF("RAT2" LE "CRIT"):!IT2 = 1
*IF("RAT3" LE "CRIT"):!IT3 = 1
$!ITER = IT1 + IT2 +IT3  $Determines if the change in the ests. is insignif
!ITER = IT2 +IT3  $Determines if the change in the ests. is insignif
$*IF("ITER" EQ 3):*JUMP 4001 $If ests. no longer change, stop iterating.
*IF("ITER" EQ 2):*JUMP 4001 $If ests. no longer change, stop iterating.
!NEPS=NEPS+1  $Goes to next iteration
*IF("NEPS" GE "NEMX"):*JUMP 4001
$If the parameters don't converge before the max. No. of iter., end process
*XQT DCU
$ PRINT 2 TS AUS 2 1
$ PRINT 2 N! AUS 1 1  $Prints out the residuals for each iteration.
PACK 1
ERASE 2
*JUMP 4000  $Est. haven't converged yet, go to next iteration
*LABEL 4001  $To end iteration process
*XQT DCU
$ PRINT 4 TS AUS 1
$ PRINT 4 CONV HIST 1 1
$ PRINT 1 TRAN TEMP 1 1

185
$ PRINT 1 TRAN TIME 1 1
$The above libraries are only printed for the final iteration. (4 TS AUS 1
$in the for each iteration; experimental temperatures).
*RETURN
* INVH
$
$*****************************************************************************$
$ $Subroutine TRAN - Solves direct problem using TRTB processor
$*****************************************************************************$
$*(29 TRAN ANAL)TRAN
$ $DCALL(29 TABL UPDA) $U$ pdate the thermal properties (estimates)
$ ! DELT= (TIMF-TIMI)/(NDS-1) $T$ ime step for transient solution
! DELT
*XQT TGEO $E$ lement geometry processor; it computes local coordinates
$ and performs element geometry checks. The user MUST
$ execute TGEO after each execution of ELD.
*XQT TRTB $T$ransient analysis processor - Implicit with C.N. code
RESET PTV=0.00001 T1="TIMI" T2="TIMF" DT= "DELT" PRINT=0 MXNDT=10000
TEM="TEMI" TSAVE="DELT"
$
$**XQT DCU
$ $PRINT 1 TRAN TEMP 1 1 1,126 1,1 51,51
$**XQT AUS
$ $PRINT 1 TRAN TEMP 1 1
$DCALL (29 BILD TS)
$
$**XQT DCU $S$ Processor that performs an array of database utility
$ functions (see Manual, Section 12-1)
DISABLE 1 EKS B
*RETURN
*TRAN
$
$*****************************************************************************$
$ $Subroutine CONF COXR - calculates the confidence intervals on the estimated
$ parameters and the correlation matrix
$*****************************************************************************$
$ *
* (29 CONF CORR) CFCR
* XQT AUS
$ $Calculate the confidence intervals, see Beck 1977, sec 6.8.1, pg 290
$! PMS = 3 $# of estimated parameters
! PMS = 2 $# of estimated parameters
! TALPHA = 1.96 $Student T value for NDS degrees of freedom
! SNE = SSYP/(NDS - PMS) * TALPHA * TALPHA
$
! PMS

186
TALPHA
SSYP
NDS
SNP
INLIB = 2
OUTLIB = 2
CF = SQRT("SNP" STSI)
CF1 = DS 1 1 1 (2 CF AUS 1 1)
CF2 = DS 2 2 1 (2 CF AUS 1 1)
CF3 = DS 3 3 1 (2 CF AUS 1 1)
$ CF1 $Confidence interval for B1
$ CF2 $Confidence interval for B2
$ CF3 $Confidence interval for B3
$
$ Calculate the correlation matrix, Beck 1977, sec 7.7.2, pg 379
$ P11 = DS 1 1 1 (2 STSI AUS 1 1) $Coefficients needed for the Pii and
P22 = DS 2 2 1 (2 STSI AUS 1 1) $Pij matrices
P33 = DS 3 3 1 (2 STSI AUS 1 1)
$
P11
P22

P11 = 1.0/SQRT("P11")
P22 = 1.0/SQRT("P22")
P33 = 1.0/SQRT("P33")
$
P11
P22

P11
P22

Pii = Pij = [ P11 0 0 ]
[ 0 P22 0 ]
[ 0 0 P33 ]

NOTE: EAL assigns NI to the number of rows and NJ to the columns
$ TABLE(NI="PMS",NJ="PMS"): 2 PII PJJ: I= 1, 2, 3
J=1: "P11" 0.0 0.0
J=2: 0.0 "P22" 0.0
J=3: 0.0 0.0 "P33"

TABLE(NI="PMS",NJ="PMS"): 2 PII PJJ: I= 1, 2
J=1: "P11" 0.0
J=2: 0.0 "P22"

DEFINE IJ = 2 PII PJJ
DEFINE PJ = 2 STSI AUS
$
The Pij elements are held in the table 2 STSI AUS 1 1

PP = RPROD(IJJ, PII) $The product of Pii and Pij
PPT = RTRAN(PP) $The transpose [Pii x Pij]^T
PPPT = RPROD(IJJ, PPT) $The product of Pii x [Pii x Pij]^T
CORM = RTRAN(PPT) $The correlation matrix: [Pii x [Pii x Pij]^T]^T

* XQT DCU
PRINT 2 PII PJJ
PRINT 2 STSI AUS
PRINT 2 PP
PRINT 2 PPT
PRINT 2 CORM

RETURN
*CFCR

*******************************************************************************

Main program

*******************************************************************************

Set RACM = 0 to use Fortran logic in all subroutines
*RACM = 0

* XQT AUS
*DCALL (29 VARB DEFI)

Read in the time, temp at the heated node, temp at the bottom surface, and
heat flux

* TF OPEN 1 inlop
  TABLE(NI=4, NJ="NDS"): IN1 DATA: I=1,2,3,4
* TF READ 1

Read in the temp history for the top, middle, and bottom nodes on the left
surface

* TF OPEN 2 in2
  TABLE(NI=3, NJ="NDS"): IN2 DATA: I=1,2,3
* TF READ 2

Read in the temp history for the top, middle, and bottom nodes on the right
surface

* TF OPEN 3 in3
  TABLE(NI=3, NJ="NDS"): IN3 DATA: I=1,2,3
* TF READ 3

* XQT DCU
PRINT 1 IN1 DATA
PRINT 1 IN2 DATA
PRINT 1 IN3 DATA

188
*DCALL(29 TRAN SFER) $Pull the required data out of the input file
*DCALL(29 CONS TANT) $Defines the constants used through out the program
*DCALL(29 NODE GENE) $Generate the nodes used in the mesh
*DCALL(29 TABL GENE) $Generate tables needed in analysis
*DCALL(29 ELEM DEFI) $Defines the elements (Cond., Conv., Heat Source, etc)
*DCALL(29 TIME FLUX) $Generates the time and heat flux tables
*DCALL(29 BOUN DARY) $Specifies the given boundary conditions
*DCALL(29 INV HEAT) $Performs the Gauss and Box Kanematsu estimation
*DCALL(29 CONF CORR) $Calculates the confidence intervals and the correlation matrix

*XQT EXIT
Appendix C

Optimization Program for Configuration 1: optc1.for

optc1.for, is a fortran code that determines the optimal settings for the experimental parameters used in Configuration 1. It is based on the analytical temperature distribution derived from the conservation of energy equation using assumed ideal boundary conditions of Configuration 1. The program was originally developed by Moncman (1994) and then modified to optimize the experimental parameters for the estimation of only two thermal properties. Two input values are required to tailor the optimization program to the specific experimental design; the ratio of thermal conductivities \( K = \frac{k_y}{k_x} \) and the thickness of the sample \( L_r \), (m).

```fortran
PROGRAM optc1.for

REAL K, Lr, BETAN, BETAN2, EXPON, TBXM1, TXM2, TERM2, TT4, X2T

C DOUBLE PRECISION SUM, SUMC, SUMKY, upper
C DOUBLE PRECISION FF1, XKY, XC, FF2, CONST, INCRKY, INCRC
C DOUBLE PRECISION EXPONTM, EXPONTIL, SSUMT, SSINCRT
C DOUBLE PRECISION SSSUMKY, SSINCRKY
C DOUBLE PRECISION TERM4, X4, Yp, TERM5, TMAX
C DOUBLE PRECISION X2T, X3T, TXT22
C DOUBLE PRECISION TXT23, TXT33, DET, D, DMAX, THOPT
```
DOUBLE PRECISION TIME,TIMEH,TTIME,DELTA,TIMET,
INTEGER M,N

OPEN(UNIT=20,FILE='opt1.dat',STATUS='UNKNOWN')

OPEN(unit=97,FILE='dv30.dat',status='unknown')
OPEN(unit=98,FILE='dvh30.dat',status='unknown')
OPEN(unit=99,FILE='dvyp.dat',status='unknown')
OPEN(unit=51,FILE='dvtn30.dat',status='unknown')
OPEN(unit=52,FILE='dv30.dat',status='unknown')
OPEN(unit=40,FILE='sens30.dat',status='unknown')

*************** Input parameters ***************
DELT A = .05D0
TTIME = 6.0D0
K = 5.0D0
Lr = .06D0

PI = DACOS(-1.0D0)
SSSUMT = 0.0D0
WRITE(*,*)PLEASE DO NOT TOUCH!!!

C Calculate Tmax (a normalizing value)

DO 2, M = 1, 3000
   Y = 0.5D0
   Yp = 1.0D0
   X = 0.0D0

TERM1 = DSIN(M*PI*Y)
TERM2 = 1.0D0 - DCOS(M*PI*Yp)
IF(TERM1.EQ.0.OR.TERM2.EQ.0.) GOTO 2
TERM3 = M**2*PI**2*Lr**2*K
DO 3, N = 1, 3000
   BETAN = PI*(N-0.5D0)
   BETAN2 = BETAN*BETAN
   TERM4 = TERM3 + BETAN2
   TERM5 = DCOS(BETAN*X)
   IF(TERM5.EQ.0.) GOTO 3

SSINCRT=TERM1*TERM2*TERM5*(1.0D0/(M*TERM4))

SSSUMT = SSSUMT + SSINCRT
3 CONTINUE
2 CONTINUE
   Tmax = SSSUMT*(4.0D0/PI)
   write(*,*)'tmax',tmax

DMA X = 0.0D0

* Begin overall loops
* Start with the steady state solution

191
DO 150, Yp = 0.26D0, .3401D0, 0.01D0
Yp = .30D0
upper = yp + .00901
DO 125, X = 0.0D0, 0.9D0, 0.25D0
DO 100, Y = 0.10D0, .20001, 0.01D0
dmax = 0.0
X = 0.0d0
Y = 0.15d0
SSSUMT = 0.0D0
SSSUMKX = 0.0D0
SSSUMKY = 0.0D0
DO 400, M = 1, 3000
   TERM1 = DSIN(M*PI*Y)
   TERM2 = 1.0D0 - DCOS(M*PI*Yp)
   IF(TERM1.EQ.0.0.OR.TERM2.EQ.0.) GOTO 400
   TERM3 = M**2*PI**2*Lh**2*K
DO 300, N = 1, 3000
   BETAN = PI*(N-0.5D0)
   BETAN2 = BETAN*BETAN
   TERM4 = TERM3 + BETAN2
   TERM5 = DCOS(BETAN*X)
   IF(TERM5.EQ.0.) GOTO 300
   SSSINCRT=TERM1*TERM2*TERM5*(1.0D0/(M*TERM4))
   SSSINCRKX = TERM1*TERM2*DCOS(BETAN*X)*(1.0D0/(M*TERM4))
   + *(TERM3/TERM4)
   SSSINCRKY = TERM1*TERM2*DCOS(BETAN*X)*(1.0D0/(M*TERM4))
   + *(TERM3/TERM4)
SSSUMT = SSSUMT + SSSINCRT
SSSUMKX = SSSUMKX + SSSINCRKX
SSSUMKY = SSSUMKY + SSSINCRKY
300 CONTINUE
400 CONTINUE

* Begin the transient solution

DO 650, TIMEH = delta, 3.001, .01
   TIMEH = 1.47d0
DO 630 XTX11 = 0.0d0
   XTX12 = 0.0d0
   XTX13 = 0.0d0
   XTX22 = 0.0d0
   XTX23 = 0.0d0
   XTX33 = 0.0d0
   SUMT = 0.0D0
   SUMC = 0.0D0

192
c SUMKX = 0.D0
SUMKY = 0.D0

c set Dmax = 0 for the determinant vs parameter cases (phase II)
c **************  DMAX = 0.D0  ********
c DMAX = 0.D0
DO 500, TIME = delta, TTIME, DELTA
dmax = 0.0

DO 500, M = 1, i0000, 1
TERM1 = DSIN(M*PI*Y)
TERM2 = 1.D0 - DCOS(M*PI*Yp)
IF(TERM1.EQ.0..OR.TERM2.EQ.0.)GOTO 500
TERM3 = M**2*PI**2*Lr**2*K
DO 600, N = 1, 1000, 1
BETAN = PI*(N-0.5D0)
BETAN2 = BETAN*BETAN
TERM4 = TERM3 + BETAN2
TERM5 = DCOS(BETAN*X)
IF(TERM5.EQ.0.)GOTO 600
EXPON = TERM4
EXPONTM = EXPON*TIME

IF(TIME.LE.TIMEH) THEN

IF(EXPONTM.LT.225.) THEN
   FF1 = DEXP(-EXPONTM)
ELSE
   FF1 = 0.D0
ENDIF

TX = FF1
XX = BETAN2*TIME*FF1-(TERM3/EXPON)-1.D0)*FF1
XXY = TERM3*TIME*FF1 + ((TERM3/EXPON)*FF1)
XC = -EXPON*TIME*FF1

ELSE

EXPONTH = (EXPON*(TIME-TIMEH))
IF(EXPONTH.LT.225..AND.EXPONTM.LT.225.)THEN
   FF1 = DEXP(-EXPONTM)
   FF2 = DEXP(-EXPONTH)
ELSE IF(EXPONTH.LT.225..AND.EXPONTM.GE.225.) THEN
   FF2 = DEXP(-EXPONTH)
   FF1 = 0.D0
ELSE IF(EXPONTH.GE.225..AND.EXPONTM.LT.225.) THEN
   FF1 = DEXP(-EXPONTM)
   FF2 = 0.D0
ELSE
   FF1 = 0.D0
   FF2 = 0.D0
ENDIF
TX = FF2 - FF1

XX = ((TERM3/EXPON)-1.D0)*(FF2-FF1) +
   BETAN2*TIME*FF1 - BETAN2*(TIME-TIMEH)*FF2

193
XKY = (-TERM3*EXPON)*(FF2-FF1) + TERM3*TIME*FF1
   + TERM3*(TIME-TIMEH)*FF2
XC = EXPON*(TIME-TIMEH)*FF2 - EXPON*TIME*FF1
ENDIF

CONST = TERM1*TERM2*TERM5*(1.D0/(M*TERM4))

INCR = TX*CONST
INCR = XC*CONST
  c INCR = X*K*CONST
INCR = XCY*CONST

IF(SUMT.NE.0.,AND.SUMK.NE.0.,AND.
   SUMC.NE.0.)THEN
   IF(Abs(INCR.T/SUMT).LE.1.D-20,AND.
   + Abs(INCRK/STUMK).LE.1.D-20,AND.Abs
   + (INCR/STUMC).LE.1.D-20) THEN
      GO TO 410
   ENDIF
ENDIF

SUMT = SUMT + INCR
SUMC = SUMC + INCR
  c SUMK = SUMK + INCRK
SUMCY = SUMCY + INCRKY

600 CONTINUE
410 IF(N.EQ.1)THEN
   IF(Abs(INCRKY).LT.1.D-20
   + .AND.ABS(INCR).LT.1.D-20,.AND.ABS(INCR),LT.
   + 1.D-20)THEN
      GO TO 450
   ENDIF
ENDIF

500 CONTINUE

450 IF(TIME.LE.TIMEH) THEN
   TEMP = (4.DOPI1)*(SSSUMT-SUMT)
   X3T = (4.DOPI1)*SUMC
  c X1T = (4.DOPI1)*(SSSUMK + SUMK)
   X2T = (4.DOPI1)*(SUMK + SSSUMKY)
ELSE
   TEMP = (4.DOPI1)*SUMT
   X3T = (4.DOPI1)*SUMC
  c X1T = (4.DOPI1)*SUMK
   X2T = (4.DOPI1)*SUMKY
ENDIF

write(40,14)yp,time,x2t,x3t
format(1x,f5.2,3e13.5)

  c X1T = X1T/TMAX
X2T = X2T/TMAX
X3T = X3T/IMAX

c XTX11 = XTX11 + X1T*X1T
c XTX12 = XTX12 + X1T*X2T
c XTX13 = XTX13 + X1T*X3T
XTX22 = XTX22 + X2T*X2T
XTX23 = XTX23 + X2T*X3T
XTX33 = XTX33 + X3T*X3T

c DET = XTX11*(XTX22*XTX33 - XTX23*XTX23) - XTX12*(XTX12*XTX33 -
    XTX13*XTX23) + XTX13*(XTX12*XTX23 - XTX13*XTX22)

c DET = XTX22*XTX33 - XTX23*XTX23

D = (1.D0/(TIME/DELTA))**2*DET

Use the following determinant equation for determining the overall experimental
Cu.me. Basically, it does not perform the time averaging thus adding subsequent
c determinant together.

c D = DELTA**2*DET

IF(D.GE.DMAX) THEN
  DMAX = D
  THOPT = TIMEH
  TIMET = TIME
  YOPT = Y
  YPOPT = YP
  write(*,*) ' latest max '
  write(*,411)ypopt,yp,opt,thopt,timet,dtmax
  write(20,411)ypopt,yp,opt,thopt,timet,dtmax
ENDIF

SUMT = 0.D0
SUMC = 0.D0

c SUMKXX = 0.D0
SUMKY = 0.D0

C end of the time loop
write(52,*) timet, dmax
c write(51,*) timet, dmax
200 CONTINUE

c write(*,409)yp,y,ámeh

C end of the heating time loop
650 CONTINUE

c write(*,412) yp,y,thopt,dmax
write(97,412) yp,y,thopt,dmax
c end of the y sensor location loop
100 CONTINUE

c WRITE(97,*) ' M M M M'
c WRITE(97,*) ' M M M M'
c end of the x sensor location loop
125 CONTINUE

c WRITE(99,*) Yp,dmax
end of the heating area loop
150 CONTINUE

write(*,*) 'optimized parameters'
write(*,111) yopt,yopt,thopt,tmopt,dmax

409 format(3(2x,f4.2))
411 format(4(2x,f4.2),e13.6)
412 FORMAT(3(2X,F4.2),2X,E15.6)

STOP
END
Appendix D

Optimization Program for Configuration 2: optc2.for

This fortran program, optc2.for, is used to calculate the maximum determinant value for Configuration 2, in the estimation of the in-plane thermal conductivity and volumetric heat capacity, and to determine the corresponding optimal experimental parameters. The program is based on the analytical temperature distribution derived from the conservation of energy equation and the ideal boundary conditions of Configuration 2. In order to tailor the program to the specific experimental conditions the ratio of thermal conductivities, $K = \frac{k_{x_{-}}}{k_{z_{-}}} \text{ and the sample thickness, } L_r, (\text{m})$ must be specified.

```
PROGRAM optc2.for

  DOUBLE PRECISION PI,SUMC,SUMKX,actemp,htflux,kxeff,io,ix
  DOUBLE PRECISION Lr,K,BETAN,BETAN2,TERM1,T1
  DOUBLE PRECISION FF1,XX,XP,FF2,CONST,INCRKX,INCRK
  DOUBLE PRECISION TERM2,TERM3,SUMT,TEMP,INCR,T1CN
  DOUBLE PRECISION EXPON,Y,XX,XP,TMAX,TEMP1,TEMP2,STEMP1
  Double precision upper
  DOUBLE PRECISION X1T,X3T,XTX11,XTX13
  DOUBLE PRECISION XTX33,DET,D,DMAX,THOPT,SSUMTN
  DOUBLE PRECISION TIME,TIMEH,TIME1,TIME2,T,T1,T2,T3,T4
```

197
INTEGER M,N

OPEN(UNIT=20,FILE='opt.dat',STATUS='UNKNOWN')
OPEN(UNIT=97,FILE='dv1.dat',STATUS='UNKNOWN')
OPEN(UNIT=98,FILE='dv2.dat',STATUS='UNKNOWN')
OPEN(UNIT=99,FILE='dv3.dat',STATUS='UNKNOWN')
OPEN(UNIT=51,FILE='dvn.dat',STATUS='UNKNOWN')
OPEN(UNIT=52,FILE='dvt.dat',STATUS='UNKNOWN')
OPEN(UNIT=40,FILE='sens.dat',STATUS='UNKNOWN')

*************** Input Parameters ***************

PI = DACOS(-1.0D0)
DELTA = 0.05D0
TTIME = 6.0D0
K = 5.0D0
LR = .06D0

c parameters to calculate the dimensional temperature
LX = .006096x10
htflux = 380.d0
kxeff = .6164x10
To = 20.6d0
ITRIP = 1
ITRIP2 = 1
ITRIP3 = 1

SSSUMT = 0.D0
SSUMTN = 0.D0
WRITE(*,*)'PLEASE DO NOT TOUCH!!'

DO N = 1, 3001,1
Y = 0.0D0
Xp = 1.0D0
X = 0.5D0

BETAN = PI*(N-0.5D0)
BETAN2 = BETAN*BETAN

INCRT = DCOS(BETAN*Y)/BETAN2
SSUMTN = SSMUTN + INCRT

ENDDO

SSTEMPI = 2.D0*Xp*SSUMTN

DO 2, M = 1, 3000
Y = 0.0D0
Xp = 1.0D0
X = 0.5D0

TERM1 = DCOS(M*PI*X)
TERM2 = DSIN(M*PI*Xp)

198
if(term1.eq.0.or.term2.eq.0.) goto 2
TERM3 = M**2*pi**2*lr**2*K
DO 3, N = 1, 3000, 1
   BETAN = PI*(N-0.5D0)
   BETAN2 = BETAN*BETAN
   TERM4 = TERM3 + BETAN2
   term5 = dcot(betan+y)
   if(term5.eq.0.) goto 3
   SSINCR = TERM1*TERM2*term5*(1.0D0/(M*TERM4))

SSSUMT = SSSUMT + SSINCR

3 CONTINUE
2 CONTINUE
SSTEMP = 4.0D0/PI*SSSUMT
TMAX = SSTEMP + SSTEMP
WRITE(*,*)TMAX,TMAX

DO 150, Xf = 0.0601D0, 1.001D0, 0.05D0
   dmax = 0.0
C DO 125, Y = 0.0D0, 0.9D0, 0.25D0
   Xp = 0.1D0
   upper = xp + .900001
   DO 100, X = 0.0D0, upper, 0.01D0
   X = 0.0D0
   Y = 0.0D0
   DMAX = 0.0D0

SSSUMT = 0.0D0
SSSUMKX = 0.0D0

DO 400, M = 1, 3000
   TERM1 = DCOS(M*PI*X)
   TERM2 = DSIN(M*PI*Xp)
   if(term1.eq.0.or.term2.eq.0.) goto 400
   TERM3 = M**2*pi**2*lr**2*K
   DO 300, N = 1, 3000, 1
      BETAN = PI*(N-0.5D0)
      BETAN2 = BETAN*BETAN
      TERM4 = TERM3 + BETAN2
      term5 = dcot(betan+y)
      if(term5.eq.0.) goto 300
      SSINCR = TERM1*TERM2*term5*(1.0D0/(M*TERM4))
      SSINCRKX = TERM1*TERM2*term5*(1.0D0/(M*TERM4))
      *(+TERM3/TERM4)
      SSSUMKX = SSSUMKX + SSINCRKX

SSSUMT = SSSUMT + SSINCR

300 CONTINUE
400 CONTINUE
C WRITE(*,*)SSSUMT,SSSUMT,SSSUMKX,SSSUMKX
DO 650, TIMEH = delta, 3.001, .05
C   DMAX = 0.0D0
C   timeh = 1.58d0

XTX11 = 0.0D0
XTX13 = 0.0D0
XTX33 = 0.0D0
SUMT = 0.000
SUMC = 0.0D0
SUMXX = 0.0D0

DO 200, TIME = DELTA, TTIME, DELTA
C   dmax = 0.0
DO N = 1, 500000, 1
   BETAN = PI*(N-0.5D0)
   BETAN2 = BETAN*BETAN
   EXPONTM = BETAN2*TIME
   IF(TIME.LE.TIMEH) THEN
      IF(EXPONTM.LE.225.) THEN
         FF1 = DEXP(-EXPONTM)
      ELSE
         FF1 = 0.0D0
      ENDIF
      T1 = (1.0D0 - FF1)
      XCP = TIME*FF1
      ELSE
      EXPONTM = BETAN2*(TIME-TIMEH)
      IF(EXPONTM.LE.225..AND.EXPONTM.LE.225.) THEN
         FF1 = DEXP(-EXPONTM)
      ELSE IF(EXPONTM.GT.225..AND.EXPONTM.LE.225.)THEN
         FF1 = DEXP(-EXPONTM)
      ELSE IF(EXPONTM.LE.225..AND.EXPONTM.GT.225.) THEN
         FF1 = 0.0D0
      ENDIF
      FF2 = 0.0D0
      ELSE IF(EXPONTM.LE.225..AND.EXPONTM.GT.225.) THEN
         FF1 = 0.0D0
      ELSE IF(EXPONTM.GT.225..AND.EXPONTM.GT.225.) THEN
         FF1 = 0.0D0
         FF2 = 0.0D0
      ENDIF
      T1 = FF2 - FF1
      XCP = TIME*FF1 - (TIME-TIMEH)*FF2
   ENDIF

INCRT = T1*DCOS(BETAN*Y)/BETAN2
INCRC = XCP*DCOS(BETAN*Y)

If(SUMT.NE.0..AND.SUMC.NE.0.) THEN
   IF(ABS(INCRT/SUMT).LE.1.0D-10..AND.+
     ABS(INCRC/SUMC).LE.1.0D-10) THEN
      TTRIP = 0
C   WRITE(*,*)************** TRANSIENT SOLN CONVERGED**********
GOTO 13

200
ENDIF
ENDIF

SUMT = SUMT + INCRT

SUMC = SUMC + INCRC

ENDDO

IF (ITKIP.EQ. i) THEN
  write(*,*)'TRANSIENT SOLN DID NOT CONVERGE ',N
ENDIF

13  TEMP1 = 2.00*Xp*SUMT
    CPN = -2.00*Xp*SUMC

SUMT = 0.00
SUMC = 0.00

DO 500, N = 1, 10000, 1
  TERM1 = DCOS(M*PI*X)
  TERM2 = DSIN(M*PI*Xp)
  IF (TERM1.EQ.0.00 OR TERM2.EQ.0.00) GO TO 500
  TERM3 = M**2*PI**2*LR**2*K
  DO 600, N = 1, 1000, 1
    BETAN = PI*(N-0.5D0)
    BETAN2 = BETAN*BETAN
    TERM4 = TERM3 + BETAN2
    TERM5 = DCOS(BETAN*Y)
    IF (TERM5.EQ.0.00) GO TO 600
    EXPONTM = TERM4*TIME
    IF (TIME.LE.TIMEH) THEN
      IF (EXPONTM.LT.225.) THEN
        FF1 = DEXP(-EXPONTM)
      ELSE
        FF1 = 0.00
      ENDIF
    T1 = FF1
    XXX = (TERM3/TERM4)*FF1 + TERM3*TIME*FF1
    XCP = -(TERM4*TIME*FF1)
  ENDIF
ELSE
  EXPONTH = TERM4*(TIME-TIMEH)
  IF (EXPONTH.LE.225..AND.EXPONTM.LE.225.) THEN
    FF1 = DEXP(-EXPONTM)
    FF2 = DEXP(-EXPONTH)
  ELSE IF (EXPONTH.GT.225..AND.EXPONTM.LE.225.) THEN
    FF2 = 0.00
    FF1 = DEXP(-EXPONTH)
  ELSE IF (EXPONTH1.LE.225..AND.EXPONTM.GT.225.) THEN
    FF1 = 0.00
    FF2 = DEXP(-EXPONTH)
  ELSE IF (EXPONTH.GT.225..AND.EXPONTM.GT.225.) THEN
    FF1 = 0.00
    FF2 = 0.00
  ENDIF
ENDIF

201
ENDIF
T1 = FF2 - FF1
XXK = (-TERM3/TERM4)*(FF2-FF1) + TERM3*TIME*FF1
+ - TERM3*(TIME-TIMEH)*FF2
XCP = TERM4*(TIME-TIMEH)*FF2 - TERM4*TIME*FF1
ENDIF

CONST = TERM1*TERM2*term5*(1.D0/(M*TERM4))

INCR = T1*CONST
INCRXX = XXK*CONST
INCRC = XCP*CONST

IF(SUMT.NE.0.,AND.SUMXX.NE.0.,AND.
+ SUMC.NE.0.)THEN
+ 1.D-20)AND.ABS
+ (INCRC/SUMC).LT.1.D-20;THEN
ITRI2 = 0
GO TO 410
ENDIF
ENDIF

SUMT = SUMT + INCRT
SUMC = SUMC + INCRC
SUMXX = SUMXX + INCRXX

600 CONTINUE

IF(ITRI2.EQ.1) THEN
WRITE(*,*) ' NOT CONVERGED'
ENDIF

410 IF(N.EQ.1)THEN
IF(ABS(INCRXX).LT.1.D-20,AND.
+ 1.D-20)THEN
ITRI3 = 0
GO TO 450
ENDIF
ENDIF

500 CONTINUE

IF (ITRI3.EQ.1) THEN
WRITE(*,*)'ITRI 3 NOT CONVERGED'
ENDIF

450 IF(TIME.LE.TIMEH) THEN
TEMP2 = (4.D0/PI)*(SSSUMT - SUMT)
X1T = (4.D0/PI)*(SSSUMKXX + SUMXX)
X3T = CPN + (4.D0/PI)*SUMC
ELSE
TEMP2 = (4.D0/PI)*SUMT
X1T = (4.D0/PI)*SUMXX

202
\[ X_{3T} = CPN + (4.00/\xi)^{n+1} \times \text{SUMC} \]

ENDIF

\[ \text{TEMP} = \text{TEMP1} + \text{TEMP2} \]

\[ c \quad \text{WRITE}(\ast, \ast) \text{ temp,htflux,lx,kxeff,lo} \]
\[ \quad \text{actemp} = \text{temp*htflux*lx/kxeff} + \text{To} \]
\[ c \quad \text{WRITE}(40,14) \text{ TIME,actemp,}\xi, lx3t \]
\[ 14 \quad \text{format}(4e13.5) \]

\[ X_{1T} = X_{1T}/\max \]
\[ X_{3T} = X_{3T}/\max \]

\[ X_{TX11} = X_{TX11} + X_{1T}^{} \times X_{1T}^{} \]
\[ X_{TX13} = X_{TX13} + X_{1T}^{} \times X_{3T}^{} \]
\[ X_{TX33} = X_{TX33} + X_{3T}^{} \times X_{3T}^{} \]

\[ \text{DET} = X_{TX11}^{} \times X_{TX33}^{} - X_{TX13}^{} \times X_{TX13}^{} \]

\[ D = (1.00/(\text{TIME}/\delta t))^{**2} \times \text{DET} \]

\[ c \quad D = \text{DET} \times \delta t^{**2} \]

IF(D.GE.DMAX) THEN
\[ \text{DMAX} = D \]
\[ \text{TMOPT} = \text{TIMEH} \]
\[ \text{TMOPT} = \text{TIME} \]
\[ x_{OPT} = x \]
\[ x_{OPT} = x \]
\[ c \quad \text{WRITE}(20,411) x_{OPT}, x_{OPT}, \text{THOPT}, \text{TMOPT}, \text{DMAX} \]
ENDIF

\[ \text{SUMT} = 0.0D0 \]
\[ \text{SUMC} = 0.0D0 \]
\[ \text{SUMKX} = 0.0D0 \]

\[ c \quad \text{WRITE}(\ast, \ast) \text{ sumt, dmgt} \]
\[ c \quad \text{WRITE}(52, \ast) \text{ sumt}, \text{dmgt} \]
\[ c \quad \text{WRITE}(51, \ast) \text{ sumt}, \text{dmgt} \]
200 CONTINUE

\[ c \quad \text{WRITE}(412) x_p, x_t, \text{THOPT, dmgt} \]
\[ c \quad \text{WRITE}(98, \ast) \text{ timet}, \text{dmgt} \]
650 CONTINUE

\[ c \quad \text{WRITE}(\ast, \ast) \text{ }^{*} \text{ WRITE}(412) x_p, x_t, \text{THOPT, dmgt} \]
\[ \text{WRITE}(97,412) x_p, x_t, \text{THOPT, DMAX} \]
160 CONTINUE

\[ c \quad \text{WRITE}(\ast, \ast) \text{ }^{*} \text{ WRITE}(99, \ast) x_p, dmgt \]
125 CONTINUE

\[ c \quad \text{WRITE}(99, \ast) x_p, dmgt \]
write(*,*) 'optimized parameters'
write(*,411)xopt,xopt,ibapt,lint,dmax

409 format(3(2x,f4.2))
411 FORMAT(4(2X,F4.2),E13.6)
412 format(3(2x,f4.2),2x,e13.6)

STOP
END
Appendix E

Coefficients for a Type E Thermocouple Polynomial

A ninth order polynomial is required to convert a voltage reading from a Type E, chromel/constantan thermocouple to a temperature measurement. The form of the polynomial is

\[ T = a_0 + a_1V + a_2V^2 + a_3V^3 + \ldots + a_9V^9 \]  

(E.1)

where \( T \) is the temperature in degrees Celsius, \( V \) is the voltage in volts, and \( a_{0-9} \) are the coefficients listed in the Table E.1. In performing the calculation it is imperative to include all of the coefficients for an accurate conversion of the voltage to a temperature. To help reduce the computation time required to perform the exponential function it is recommended to code the equation in the following form

\[ T = a_0 + V(a_1 + V(a_2 + V(a_3 + V(a_4 + V(a_5 + V(a_6 + V(a_7 + V(a_8 + V(a_9)))))))) \]  

(E.2)
Table E.1. Coefficients for a Type E Thermocouple Polynomial.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.104967246</td>
</tr>
<tr>
<td>$a_1$</td>
<td>17189.45282</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-282639.0850</td>
</tr>
<tr>
<td>$a_3$</td>
<td>12695339.5</td>
</tr>
<tr>
<td>$a_4$</td>
<td>-4448703084.6</td>
</tr>
<tr>
<td>$a_5$</td>
<td>1.10866E10</td>
</tr>
<tr>
<td>$a_6$</td>
<td>-1.76807E11</td>
</tr>
<tr>
<td>$a_7$</td>
<td>1.71842E12</td>
</tr>
<tr>
<td>$a_8$</td>
<td>-9.19278E12</td>
</tr>
<tr>
<td>$a_9$</td>
<td>2.06132E13</td>
</tr>
</tbody>
</table>
Appendix F

Thermocouple Fabrication Procedure

There are multiple ways to fabricate a thermocouple. The junction between two dissimilar wires of a thermocouple may be joined with solder, silver-soldered, welded, etc. All thermocouples used in the experimental apparatus were welded together from duplex insulated chromega/constantan, 40 gage wire (part #, TT-E-40, Omega Engineering Inc.). The wire consisted of chromega and constantan wires, individually wrapped in their own insulation and held together by an outer layer of duplex coating. Chromega, the positive lead, was covered in a purple Teflon insulation and constantan, the negative lead, in red Neoflon. The chromega and constantan wires were welded together using an argon thermocouple welder (model #, 116SRL, Tigtech Inc.). This appendix describes the basic steps used to fabricate a thermocouple from the beginning wire to the completed thermocouple.

Step 1  The thermocouple production began by cutting 1.2 m (4 ft) from the Type E thermocouple wire spool. From one end of the wire, 25 mm (1 in) was removed from
the outer duplex coating. The insulation covering the wires was thin and could be removed with a razor blade. To expose the bare wires, 13 mm (0.5 in) was removed from the purple Teflon and red Neoflon insulation. Care was taken to only remove the insulation, this operation often resulted in severing the wire.

**Step 2** The bare chromega (+) and constantan (-) wires were attached to the respective terminals of a Type E, subminiature male thermocouple connector (part #, SMP-E-M, Omega Engineering Inc.).

**Step 3** From the other end of the wire, 0.3 m (9 in) of the outer duplex coating was removed. The purple Teflon and red Neoflon insulation were stripped back 40 mm (1.5 in) to expose the chromega and constantan wires.

**Step 4** The wires were prepared for welding by twisting the last 13 mm (0.5 in) of each wire together and placing them in an argon welder clamp. The final twist of the wires was placed just below the flush surface of the clamp. The extended twisted wires were snipped, allowing for only a sixteenth inch to protrude past the end of the clamp.

**Step 5** The argon welder was set by placing the weld time and current settings to minimum. Attached to the welder was a pressurized tank of argon, whose regulator was set between 35 and 48 kN/m² (5 and 7 psi). After the welder was turned on, the
connecting hose was purged of any extraneous air that may have been trapped by compressing the purge button for five second.

**Step 6** The cover over the welding chamber was removed and the clamp with the sixteenth inch protruding wires was placed into the hole. Once in the hole the weld button was pressed until the arc light turned on. This represented that the welder had just filled the cavity with argon gas and released a spark. The argon combusted and melted the extended wires until the surface of the clamp.

**Step 7** The clamp from the welding chamber was removed and the weld was checked. If the wires were welded together the procedure continued onto step 8, congratulations! Steps four through six were repeated if success was not obtained. The welder/user had about a fifty percent success rate due to the fine gauge wires.

**Step 8** If the weld held the wires together, the thermocouple was tailored for the experimental setup. First, the thermocouple head was flattened with a pair of needle nose pliers. Second, the length of the bare wires was checked. The thermocouple needed to cover 76 mm (3 in) over the composite surface, free of insulation. If the bare wires were not long enough, the extra insulation was carefully stripped back with a razor blade.

**Step 9** After the thermocouple was completed the argon welder was shut down. The
regulator was closed and any existing argon was purged from the hose by pressing the purge button. The cover was replaced over the welding chamber and the welder was turned off.

The thermocouple produced was then ready to be implemented into the experimental apparatus and used as a temperature measurement device.
Appendix G

Data Acquisition Program: Total.c

The program, Total.c, is used to manipulate the National Instruments data acquisition system to record the time, temperature measurements from multiple thermocouples, and the current and voltage from two high resolution multimeters. A psuedo type C computer language specific to National Instruments has been used to code the program.

/*
  Total.c
  produced by Joseph P Hanak on 1/15/95

  This program reads the voltage signal from up to 32 thermocouples, and the voltage and current from two dmm's using handshaking. Both meters should be set in handshake mode 1 data format 0
*/

/*------------------------ variables ------------------------*/

#define SCXISTARTCHN 0 /* the channel to start the SCXI scan */
#define NUMSCXICHANS 8 /* # of channels that are read */
#define PTSPERCHAN 1000 /* half this # is the # of data points that are acquire per channel */
#define RATE 1 /* # of pts taken per second */
#define SAMPLETIMEBASE -1
#define SAMPLEINTERVAL 2000
#define MIOSCXIGAIN 1 /* the mio gain used for the scxi chassis */
#define SCXIGAIN 2000 /* the SCXI-1100 gain (the gain for the chassis) */
#define NUMMODULES 1 /* # of modules to be scanned */

211
/* Constants needed for array sizes */
#define NUMPOINTS (NUMSCIXCHANS * PTSPerCHAN) /* total # of points to acquire */
#define HALFNUMPOINTS (NUMPOINTS/2) /* the # of points taken by the half buffer */
#define HALFPTSPerCHAN (PTSPerCHAN/2)

main()
{
    int modulelist[NUMMODULES], numscixchans[NUMMODULES],
        scxistartchan[NUMMODULES], miochanvector[NUMSCIXCHANS],
        mfgainvector[NUMSCIXCHANS], buffer[NUMPOINTS],
        halfbuffer[HALFNUMPOINTS], scxibuffer[HALFNUMPOINTS],
        double average[NUMSCIXCHANS], voltarray[HALFNUMPOINTS],
        sam[NUMSCIXCHANS], scxivoltarray[HALFNUMPOINTS],
        pinned[NUMSCIXCHANS], temp[NUMSCIXCHANS],
        diff[NUMSCIXCHANS];

    int fh1and, fh2and, err, i, j, mioboardid, mioscixchan, scxichassisid,
        binoffset, halfready, numnumbrds, sampletimebase, sampleinterval,
        scantimebase, scantimeinterval, pfstr, daqstepped, retrieved,
        moduleslot, scxigain, scxioffchan, heatchan, on, split,
        double miogainadjust, origtime, time, scxioffset, voltage,
        refvolt, x, a, a1, a2, a3, a4, a5, a6, a7, a8, a9,
        heatstart, heatertime, heatvoltt, totalheattime, off,
        disableheater;

    /* rs232 variables */
    char remote[100], local[100], in_string1[100], in_string2[100],
        asking[100];

    int rs232err, port1, baudrate, parity, databits, stopbits,
        iqsiz, opsize, length, n, bytes, port2, in_len, disablers232;
    double value1, value2;

c1s();

    mioscixchans = 0;
    mioboardid = 1;
    scxichassisid = 1;
    moduleslot = 2;
    scxioffchan = 31; /* channel 31 should be grounded */
    numnumbrds = 0;
    heatchan = 1;
    split = 16;

    a = .104967248;
    a1 = 17189.45282;
    a2 = -282639.0850;
    a3 = 12655339.5;
    a4 = -448703084.6;
    a5 = 1.10866E10;
    a6 = -1.76807E11;
    a7 = 1.71842E12;
    a8 = -9.19278E12;
    a9 = 2.06132E13;

    off = 0.0;
    on = 0;
disablers232 = 1; /* 0 = no rs232 1 = use rs232 */
disableheater = 0; /* 0 = no heater 1 = use heater */

heaterstart = 5.0; /* the time when the heater turns on */
heattime = 50.0; /* the length of time the heater is on */
heattvol = .625; /* This value should never exceed 5 */

totalheattime = heaterstart + heattime;

/* arrays needed to specify the mioboard specifications */
for (i=0; i<NUMSCXICHANS; i++) {
  /* mio channel scan sequence */
  miochanvec[i] = miosexichan; /* the mio channel for the scxi */
}

/* mio channel gains */
for (i=0; i<NUMSCXICHANS; i++){
  miogainvec[i] = MIOSCXIGAIN; /* the mio gain applied to the scxi */
}

/* in case of multiple modules these arrays need to be set */
for (i=0; i<NUMMODULES; i++){
  modulelist[i] = moduleslot; /* the chassis slot for each module */
  numsexichans[i] = NUMSCXICHANS; /* # of channels to read on each module */
  scxistartchan[i] = SCXISTARTCHN; /* the scxi start channel for each module */
}

/* open a data file: note the second digit determines if the data
should be added on (1) or written over (0) */
  fhand = OpenFile("TMP.out",0,0,1);

/* incase of more than "split" te's the remaining data should be copied to
a second file */
if (NUMSCXICHANS > split)  
  fhand2 = OpenFile("TMP1.out",0,0,1);

/* This command is necessary for all scxi data acquisition */
  err = SCXI_Load_Config(scxichassisid);
  if (err != 0) 
    FmtOut("\n SCXI_Load_Config error = %d", err);

/* As a precaution the scxi is reset to it's default values */
  err = SCXI_Reset (scxichassisid, moduleslot);
  if (err != 0) 
    FmtOut("\n SCXI_Reset error = %d", err);

/* set the gain for the scxi */
  err = SCXI_Set_Module_Gain (scxichassisid, moduleslot, SCXIGAIN);
  if (err != 0) 
    FmtOut("\n SCXI_Set_Module_Gain error = %d", err);

/* SCXI Offset ------------------------- */
/* Define the offset channel */
  err = SCXI_Single_Chap_Setup (scxichassisid, moduleslot, sexioffchan, mioboardid);
  if (err != 0) 
    FmtOut("\n SCXI_Single_Chap_Setup error = %d", err);

/* Turn the SCXI calibration on to determine the offset */
err = SCXI_Calibrate_Setup (scxchassisid, moduleslot, 1);
if (err != 0)
    FmtOut ("an SCXI_Calibrate_Setup turn on error = %d", err);

/* Determine the offset by averaging a thousand values */
scxioffset = 0.0;
for (i=0; i<1000; i++)
    voltage = 0.0;

    /* Read the offset voltage */
    err = AI_VRead (mioboardid, mioscxican, MIOSCXIGAIN, &voltage);
    if (err != 0)
        FmtOut ("an AI_VRead offset error = %d", err);

    scxioffset = scxioffset + voltage;
} /* end for loop */
scxioffset = scxioffset/1000.0;

/* Turn the SCXI calibration off */
err = SCXI_Calibrate_Setup (scxchassisid, moduleslot, 0);
if (err != 0)
    FmtOut ("an SCXI_Calibrate_Setup turn off error = %d", err);

if (disables232 == 1)

/* ----------------------------- Configure the Ports ----------------------------- */
port1 = 1;  /* Mouse port */
port2 = 2;
baudrate = 9600;
parity = 0;  /* 0 = no parity, 1 = odd, 2 = even */
databits = 8;
stopbits = 1;
iqsize = 100;
opsize = 100;

/* open and set port parameters for port #1*/
OpenComConfig (port1, baudrate, parity, databits, stopbits, iqsize, opsize, 0, 0);
if (rs232err != 0)
    FmtOut ("OpenComConfig error = %iu", rs232err);

/* open and set port parameters for port #2*/
OpenComConfig (port2, baudrate, parity, databits, stopbits, iqsize, opsize, 0, 0);
if (rs232err != 0)
    FmtOut ("OpenComConfig error = %iu", rs232err);

/* clear importing and exporting queue's. This is probably overkill but dead is dead */
FlushInQ (port1);
if (rs232err != 0)
    FmtOut ("FlushInQ error = %iu", rs232err);

FlushOutQ (port1);
if (rs232err != 0)
    FmtOut ("FlushOutQ error = %iu", rs232err);
FlushInQ (port2);
    if (rs232err != 0)
        FmtOut ("FlushInQ error = \%d", rs232err);

FlushOutQ (port2);
    if (rs232err != 0)
        FmtOut ("FlushOutQ error = \%d", rs232err);

    /* set into remote control mode */
    n = Fmt (remote,"%s%cR%c%c",27,13,10);
    bytes = ComWrt(port1,remote,4);
    if (rs232err != 0)
        FmtOut ("ComWrt remote port1 error = \%d", rs232err);

    /* set into remote control mode */
    n = Fmt (remote,"%s%cR%c%c",27,13,10);
    bytes = ComWrt(port2,remote,4);
    if (rs232err != 0)
        FmtOut ("ComWrt remote port2 error = \%d", rs232err);

delay (.0001);
}

} /* end disabling the rs232 */

/* ----------------------------- Multiplexing ----------------------------- */

/* initializes mio circuitry for a scanned data acquisition */
    err = SCAN_Setup (mioboardid, NUMSCXICHANS, miochanvector, miogainvector);
    if (err != 0)
        FmtOut ("\n SCAN_Setup error = \%d", err);

/* Setup the SCXI chassis for multiplexing */
    err = SCXI_SCAN_Setup (ssexichassisid, NUMMODULES, modulelist, numscxichans, sxixstartchan,
                        mioboardid, 0);
    if (err != 0)
        FmtOut ("\n SCXI_SCAN_Setup error = \%d", err);

/* enable double buffering */
    err = DAQ_DB_Config (mioboardid, 1);
    if (err != 0)
        FmtOut ("\n DAQ_DB_Config error = \%d", err);

/* initiates a multiple channel scanned data acquisition operation */
    err = SCAN_Start (mioboardid, buffer, NUMPOINTS,SAMPLETIMEBASE,SAMPLEINTERVAL,
                    scanbase, scaninterval);
    if (err != 0)
        FmtOut ("\n SCAN_Start error = \%d", err);

origtime = Timer();

while(KeyHit() != 1)

    halfready = 0;
    while(halfready == 0)

        time = Timer() - origtime;

    if (disableheater == 1)
 tbsp;
if (heaterstart <= time && totalheattime > time && on == 0)
err = AQ_VWrite(mioboardid, heaterchan, heatervolt);
FmtOut("%n %n Heater turn on %n Heater turn on
");
if (err != 0)
FmtOut("%n Heater turn on = %d", err);

on = 1;
/* turn the heater off */
if (totalheattime < time && off == 0.0)
err = AQ_VWrite(mioboardid, heaterchan, off);
FmtOut("%n %n Heater turn off
");
if (err != 0)
FmtOut("%n Heater turn off = %d", err);
off = 1.0;
} /*end disable heater */

/* check to see if the next half buffer of data is available */
err = DAQ_DB_HalfReady (mioboardid, &halfready, &daqstopped);
if (err != 0)
FmtOut("%n DAQ_DB_HalfReady error = %d", err);

/* Transfers half of the data from the buffer to another buffer */
err = DAQ_DB_Transfer (mioboardid, halfbuffer, &ptstrf, &daqstopped);
if (err != 0)
FmtOut("%n DAQ_DB_Transfer error = %d", err);

/* sort the multiplexing buffer into respective channels */
err = SCAN_Demux (halfbuffer, HALFNUMPNTS, NUMSCXICHANS, nummuxbrds);
if (err != 0)
FmtOut("%n SCAN_Demux error = %d", err);

/* Convert the binary voltage readings for the scxibuffer into an actual voltage */
err = DAQ_VScale(mioboardid,miosclchan,MIOSCXIGAIN,miogainadjust,binoffset,HALFNUMPNTS,halfbuffer,scxivoltarray);
if (err != 0)
FmtOut("%n DAQ_VScale error = %d", err);

/* sum the voltages per channel inorder to determine an average voltage reading */
for(i=0; i< NUMSCXICHANS; i++) {/* sum over each channel */
sum[i] = 0.0;
for(j=(i*PTS PERCHAN/2); j<(i+1)*PTS PERCHAN/2); j++) {/* add up all the points */
sum[i] = sum[i] + scxivoltarray[j];}
FmtOut("%n%f[p1] ",time);
FmtFile(thand,"ur%f[p1] ",time);
if(NUMSCXICHANS > split)
FmtFile(thand2,"n");
/* average the sums and include the scxi offset, also determine if the voltage is pinned */
for(i=0; i< NUMSCXICHANS; i++) {/* sum over each channel */
average[i] = (sum[i]/HALFPPTS PERCHAN - scxioffset)/SCXIGAIN;
pinned[i] = (SCXIGAIN * average[i] + sxxoffset) * MIOSCXIGAIN;

/* determine the actual voltage of the thermocouples */
for(i=0;i<NUMSCXICHANS;i++){

/* determine the thermocouple temperature from the actual voltage */
x = average[i];
temp[i] = a1+x*(a1+x*(a2+x*(a3+x*(a4+x*(a5+x*(a6+x*(a7+x*(a8+x*x9))))))));
}

if(disables232 == 1){

/* read the current and voltage */
/* send a signal from the computer asking for a data point */
  n = Fmt (asking,"%s%cD%c%e",27,13,10);
  bytes = ComWrt(port1,asking,4);
  if (rs232err != 0)
    FmtOut ("ComWrt asking port1 error = %n",rs232err);

/* send a signal from the computer asking for a data point */
  n = Fmt (asking,"%s%cD%c%e",27,13,10);
  bytes = ComWrt(port2,asking,4);
  if (rs232err != 0)
    FmtOut ("ComWrt asking port2 error = %n",rs232err);
}

/* Output */
for(i=0;i<NUMSCXICHANS;i++){
  FmtOut("%f",temp[i]);
  if (i < split)
    FmtFile(fh, "%f",temp[i]);
  else
    FmtFile(fh2, "%f",temp[i]);
}

if (disables232 == 1) {

/* Current and Voltage */
/* read the incoming data from port1*/
in_len =15;
bytes = ComRdTerm (port1, in_string1, in_len, 10);
  if (rs232err != 0)
    FmtOut ("ComRdTerm port1 error = %n",rs232err);

/* read the incoming data from port2*/
in_len =15;
bytes = ComRdTerm (port2, in_string2, in_len, 10);
  if (rs232err != 0)
    FmtOut ("ComRdTerm port2 error = %n",rs232err);

/* scan the incoming data and pull out the numeric value */
bytes = Scan (in_string1, "%s>%s%d%f", &value1);
  if (rs232err != 0)
    FmtOut ("Scan in_string1 error = %n",rs232err);

/* scan the incoming data and pull out the numeric value */
bytes = Scan (in_string2, "%s>%s%d%f", &value2);

217
if (rs232err != 0)
    FmtOut ("Scan in_string2 error = %i\n",rs232err);

FlushInQ (port1);
FlushInQ (port2);

FmtOut("\t%f[p8] \t%f[p6] \t,%value1,value2");
if (NUMSCXICHANS< split)
    FmtFile(fh,\t%f[p8] \t%f[p5] \t,%value1,value2);
else
    FmtFile(fh2,"\t%f[p8] \t%f[p5] \t,%value1,value2");
} /* end disabling for rs232 */

} /* end while loop */

/** to ensure the power supply is always turned off */
err = AO_Write(mioboardid, heaterchan, 0.0);

/** return to local control mode */
    n = Fmt (local,"%s<%c1.%c%e",27,13,10);
    bytes = ComWrt(port1,local,4);
    if (rs232err != 0)
        FmtOut("ComWrt local error = %i\n",rs232err);

/** return to local control mode */
    n = Fmt (local,"%s<%cL%c%e",27,13,10);
    bytes = ComWrt(port2,local,4);
    if (rs232err != 0)
        FmtOut("ComWrt local error = %i\n",rs232err);

delay(.0001);

CloseCom (port1); /* close port #1 */
CloseCom (port2); /* close port #2 */

} /* end main program */
Vita

The author, Joseph P. Hanak was born on April 21, 1971 in Corning, New York where he was raised by Helen and Joseph Hanak. He began his formal education at Corning East High School and graduated in July of 1989. His education continued for the following two years at Corning Community College where he obtained his Associate of Science Degree in Engineering Science. Immediately following, he transferred to Virginia Tech and completed the final requirements for a Bachelor of Science degree in Mechanical Engineering in May of 1993. Remaining at Virginia Tech, he performed his graduate studies under the direction of Dr. Elaine P. Scott. With the completion of this thesis he had fulfilled the requirements for a Masters of Science degree in Mechanical Engineering in the summer of 1995.

Joseph P. Hanak