COMPUTER SOFTWARE TO CALCULATE THE SYSTEMATIC
COORDINATE DIFFERENCES BETWEEN TWO GEODETIC DATUMS

by

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Committee Chairman: Steven D. Johnson 
Civil Engineering

(ABSTRACT)

The high degree of accuracy now found using GPS observation techniques has led to worldwide acceptance of the geocentric datums, specifically the WGS84 datum as the mainstay for referencing in the geodetic community. Nevertheless, local datums are non-geocentric and if we want to use GPS on their positions, some disagreement will result. This report presents PC-based software to transform coordinates between any two arbitrary datums. Transformations between NAD27 and NAD83 are used as examples expanded with the development of maps which illustrate shifts between those two datums in Latitude, Longitude, and Geoidal Height. It should be stressed that these transformations are based upon the standard seven parameters (3 shifts, 3 rotations, and scale change) and changes in the semimajor axis and the flattening including second partial differentials. This software does not take into account any random distortions that may be present in the datum coordinates.
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1.0 INTRODUCTION

1.1 HISTORICAL BACKGROUND NAD27

The first geodetic surveys in the United States happened in 1816-1817 in the New York Bay area. These surveys were performed by the Survey of the Coast, which later became the United States Coast and Geodetic Survey, which is today the National Geodetic Survey (NGS). The first geodetic datum was the New England Datum which was formed in 1879 and used the Clarke Spheroid of 1866 as its reference surface. The New England Datum was defined by observations in the Northeast and mid-Atlantic states with an origin at Principio, Maryland. The New England Datum provided the basis for a transcontinental network as the country grew in size. Surveys extending down the East Coast to the Gulf of Mexico and to the Pacific Coast soon provided a rough national network. The New England Datum then became known as the U.S. Standard Datum with its origin at Meades Ranch, Kansas. Canada and Mexico tied their regional surveys to this U.S. Standard Datum in 1913 thus establishing geodetic control for this continent and forming the North American Datum. During the 1920's federal surveyor Jaspar S. Bilby made precise 1st-order surveys in the national network and adjusted these positions to form the North American Datum of 1927 (NAD27). NAD27 provided the basis for all mapping and charting on this continent for the next 60 years. By 1970, NAD27 was comprised of more than 250,000 geodetic coordinates, some of which were
suffering from distortion caused by holding original inaccurate surveys as fixed. As more points were added, the distortions became more apparent, thereby indicating a new network solution was needed to form better positional accuracy. This solution could only be achieved by utilizing today's technology to update our reference datum, thus giving birth to a new geocentric system - The North American Datum of 1983 (NAD83).

1.2 REASONS WHY A NEW DATUM AND ADJUSTMENT WAS NECESSARY

The reasons for developing a new reference datum and a network readjustment of NAD27 to NAD83 are

a) Over 100,000 new stations in the USA, Canada, and Mexico have been added (NAD27) causing a strain on the network and creating more distortions.

b) The ability to measure precise distances between observation stations occurred in NAD27.

c) Some of the 1st order azimuths (NAD27) lacked the desired accuracy.

d) More advanced equipment exists today for measuring distances and angles.

e) Major fault line areas have displayed tectonic movements as much as 5 cm a year.

1.3 DEVELOPMENT OF NAD83

From 1969 to 1981 NGS, DMA, and other federal and state
agencies began to resurvey using updated technology. This equipment consists of satellites, laser ranging, and long base line radio interferometry techniques. The goal of NAD83 is to provide an ideal geocentric datum that is developed by acquiring highly accurate measurements of azimuths, distances, and gravity readings. The observations are incorporated into a least squares adjustment method known as "Helmert Blocking" to provide accurate geodetic positioning. The National Academy of Sciences helped design the following specifications for this new datum. The specifications are listed as follows

a) This new datum should be part of a world system defined in three dimensional cartesian coordinates. The z axis shall pass through the mean pole and be referenced to the zero meridian defined by the BIH.

b) Control points in the datum would be referred to a new international ellipsoid with an equatorial radius determined to ±5 meters mean square error and polar flattening is to be determined by dynamic satellite programs. Origin of the system should be the intersection of the z axis and the xy (equatorial) plane so that there is no unique datum point such as Meades Ranch (NAD27).

c) Absolute geoidal separations should be known to within ±2 meters for most of this continent, ±3 meters for the extremities, and ±5 meters for the outermost points.
d) Spacing of the primary geodetic satellite stations should be such that no point is more than 1000 kilometers from any one of the stations.

The results of this 12-year project was the new NAD83 datum. The new adjustment has provided some interesting changes. The Washington Monument is now 94.8 feet to the northeast, the Empire State Building is 120.5 feet to the northwest, and the state capitol dome in Annapolis is 98.5 feet to the northeast. The greatest change occurs in the Hawaiian Islands with the Honolulu Judiciary Building moving 1,480.8 feet to the southeast. The changeover of datums will affect all mapping and charting projects compiled by the Defense Mapping Agency, National Ocean Service, U.S. Geological Survey and others. The new datum will provide more accurate positioning for engineers, regional planners, surveyors, scientists, geologists, the military, and commercial navigators. These new adjusted positions will be 10 to 100 times more accurate than their NAD27 counterparts, yielding an error of less than one inch per six miles. Table 1.1 shows a comparison of datum elements between NAD27 and NAD83. Rigorous transformations between NAD27 and NAD83 have been achieved recently by the development of a program called NADCON at NGS (Dewhurst 1990). This program takes into consideration not only the systematic transformations in this report (mainly 3 shifts, 3 rotations,
and scale factor) but also the local distortions of the network which are random and impossible to compute by closed equations. NADCON can convert large quantities of data and has yielded accuracies in the neighborhood of 15 centimeters for the contiguous United States. Some remote areas may experience accuracy on the order of less than 1.0 meter.

### TABLE 1.1 COMPARISON OF DATUM ELEMENTS

<table>
<thead>
<tr>
<th></th>
<th>NAD27</th>
<th>NAD83</th>
</tr>
</thead>
<tbody>
<tr>
<td>REFERENCE</td>
<td>a = 6,378,206.4 M.</td>
<td>a = 6,378,137 M.</td>
</tr>
<tr>
<td>ELLIPSOID</td>
<td>b = 6,356,583.8 M.</td>
<td>f = 1/298.2572221</td>
</tr>
<tr>
<td>DATUM POINT</td>
<td>TRIANGULATION STATION</td>
<td>NONE</td>
</tr>
<tr>
<td></td>
<td>MEADE'S RANCH</td>
<td>CENTER OF EARTH</td>
</tr>
<tr>
<td>LONGITUDE</td>
<td>GREENWICH MERIDIAN</td>
<td>GREENWICH MERIDIAN</td>
</tr>
<tr>
<td>ORIGIN</td>
<td>(BIH ZERO MERIDIAN)</td>
<td>(BIH ZERO MERIDIAN)</td>
</tr>
<tr>
<td>AZIMUTH ORIENTATION</td>
<td>FROM SOUTH</td>
<td>FROM NORTH</td>
</tr>
<tr>
<td>ADJUSTMENT</td>
<td>25K POINTS</td>
<td>250K POINTS</td>
</tr>
<tr>
<td></td>
<td>SEVERAL HUNDRED BASE LINES</td>
<td>APPROX 30K EDMI B.L.</td>
</tr>
<tr>
<td></td>
<td>SEVERAL HUNDRED ASTRO AZIM.</td>
<td>5K ASTRO AZIMUTHS</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DOPPLER POINT POS.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>VLBI VECTORS</td>
</tr>
<tr>
<td>BEST FITTING</td>
<td>NORTH AMERICA</td>
<td>WORLD-WIDE</td>
</tr>
</tbody>
</table>

### 1.4 OBJECTIVE

The objective of this project is to develop PC-based software that easily converts coordinates between two arbitrary datums when a predetermined set of seven parameters are given. The
software developed to meet this objective can transform a single coordinate or an endless data file. The software is based on equations originally presented by Soler (1976) using seven (7) parameters - 3 shifts, 3 rotations, and 1 scale factor. By setting up a data file (e.g. NAD27) at five (5) degree intervals of latitude and longitude for the United States an output of new coordinates (e.g. NAD83) with the shifts from the first datum will be produced. This output can be used to create maps of the original datum that show the shifts in latitude, longitude, and geoidal height. In the case of NAD27 and as a check of the software, these maps will be compared to existing maps produced by the National Ocean Service in 1978. As an extension of the applications, maps of the State of Virginia will also be produced with a data base set for a ten (10) minute interval in latitude and longitude. The seven (7) parameter transformations also includes 2nd partial differential equations with respect to changes in semimajor axis and flattening. The PC-based software will be altered to provide the contribution of those differentials as changes are made between various datums or ellipsoids. The results will be in tabular form.
2.0 TRANSFORMATION EQUATIONS

The following equations used for datum transformations are taken from Soler and Van Gelder (1987) and were used to develop an interactive computer program. Datum transformations are generally denoted by (D1) \[ \text{GEOD} \rightarrow \text{GEOC} \]

meaning from datum D1 to D2 or in this specific case from a Geodetic datum to a Geocentric datum. In this paper the datum transformation will be on the order of a local geodetic datum NAD27 transformed to geocentric systems such as WGS84, WGS72, and NAD83. The transformation can be represented by the notation

\[
\begin{array}{c}
\text{(GEOD)} \\
(u,v,w) \\
\Delta x, \Delta y, \Delta z, \delta \epsilon, \delta \psi, \delta \omega, \delta s
\end{array}
\rightarrow
\begin{array}{c}
\text{(GEOC)} \\
(x,y,z) \\
\end{array}
\]

The standard seven (7) parameter transformations that will be used going from a local datum to a geocentric system is

\[
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} +
\begin{bmatrix}
\Delta x \\
\Delta y \\
\Delta z
\end{bmatrix} +
\begin{bmatrix}
1 & \delta \omega & -\delta \psi \\
-\delta \omega & 1 & \delta \epsilon \\
\delta \psi & -\delta \epsilon & 1
\end{bmatrix}
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix} + \delta s
\begin{bmatrix}
u \\
v \\
w
\end{bmatrix}
\]

[2.1]

The following terms are defined for future reference:

- \( \delta s = (s_{02} - s_{01})/s_{01} \) is differential scale change
- \( \delta \bar{a} = a_{02} - a_{01} \) is change in semimajor axis
- \( \delta a = \delta \bar{a} + a_{01} \delta s = a_{02} - (1 - \delta s)a_{01} \) is total change in semimajor axis when a differential scale change \( \delta s \) is involved
- \( \delta f = f_{02} - f_{01} \) is change in flattening
\[ \Delta X, \Delta Y, \Delta Z \]

are coordinates of the origin of the Cartesian system \((u,v,w)\) of datum D1 along the frame \((X,Y,Z)\) of datum D2.

\[ \delta \epsilon, \delta \psi, \delta \omega \]

are the differential rotations around the axes \((u,v,w)\) of Datum D1 to establish parallelism with respect to datum D2.

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}
= 
\begin{bmatrix}
  (N+h) \cos \phi \cos \lambda \\
  (N+h) \cos \phi \sin \lambda \\
  [N(1-e^2)+h] \sin \phi
\end{bmatrix}
\]

geodetic coordinates

\[ N = a/W \]

principal radius of curv.
in plane of prime vertical

\[ M = a(1-e^2)/W^3 \]

principal radius of curv.
in plane of the meridian

\[ W = (1-e^2 \sin^2 \phi)^{1/2} \]

Simply stated the 7-parameter transformation equations based on three shifts, three rotations, and one scale difference are

\[
\begin{bmatrix}
  du \\
  dv \\
  dw
\end{bmatrix}
= 
\begin{bmatrix}
  \Delta X \\
  \Delta Y \\
  \Delta Z
\end{bmatrix}
+ 
\begin{bmatrix}
  0 & \delta \omega & -\delta \psi \\
  -\delta \omega & 0 & \delta \epsilon \\
  \delta \psi & -\delta \epsilon & 0
\end{bmatrix}
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}
+ \delta s
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}
\]

[2.2]

Now utilizing the 7-parameter transformations and including the transformations due to the change of semimajor axis \((a)\) and the flattening \((f)\) we develop equations to express curvilinear coordinates \((d\lambda,d\phi,dh)\). The first and second-order differences in the semimajor axis \((a)\) and the flattening \((f)\) in addition to the 7-parameter transformations are multiplied by the rotation matrix \((R)\) to define the curvilinear coordinates.
The rotation matrix (R) is a [3x3] matrix consisting of sequential rotations about the w axis and the u axis.

\[
R = R_1(90-\phi) R_3(\lambda+90) = \begin{bmatrix}
-sin\lambda & cos\lambda & 0 \\
-sin\phi cos\lambda & -sin\phi sin\lambda & cos\phi \\
cos\phi cos\lambda & cos\phi sin\lambda & sin\phi 
\end{bmatrix}
\]

The transformation equations that are based upon the differences in semimajor axis (a) and flattening (f) are

\[
\begin{bmatrix}
\Delta u \\
\Delta v \\
\Delta w
\end{bmatrix} = \begin{bmatrix}
\delta a \\
\delta f \\
\delta a, \delta f
\end{bmatrix} - \begin{bmatrix}
\frac{\partial[D]}{\partial a} \\
\frac{\partial[D]}{\partial f} \\
\delta a, \delta f
\end{bmatrix}
\]

Where the matrix [D] can be expressed as

\[
[D] = \begin{bmatrix}
\partial u/\partial a & \partial u/\partial f \\
\partial v/\partial a & \partial v/\partial f \\
\partial w/\partial a & \partial w/\partial f
\end{bmatrix}
\]

Expanding the first-order partial derivatives we find the equivalent formulas which will be used later in computer form

\[
\begin{align*}
\partial u/\partial a &= \cos\phi cos\lambda/W \\
\partial v/\partial a &= \cos\phi sin\lambda/W \\
\partial w/\partial a &= (1-e^2) \sin\phi/W \\
\partial u/\partial f &= a(1-f) \sin^2\phi \cos\phi cos\lambda/W^3 \\
\partial v/\partial f &= a(1-f) \sin^2\phi \cos\phi sin\lambda/W^3 \\
\partial w/\partial f &= (M sin^2\phi - 2N)(1-f) \sin\phi
\end{align*}
\]
Elements of the matrices \( \partial[D] \) and \( \partial[D] \) that are the second-order partial derivatives are expressed here as

\[
\begin{align*}
\partial^2 u / \partial a^2 &= \partial^2 v / \partial a^2 = \partial^2 w / \partial a^2 = 0 \\
\partial^2 u / \partial a \partial f &= \partial^2 u / \partial f \partial a = (1-f) \sin^2 \phi \cos \phi \cos \phi / W^3 \\
\partial^2 v / \partial a \partial f &= \partial^2 v / \partial f \partial a = (1-f) \sin^2 \phi \cos \phi \sin \phi / W^3 \\
\partial^2 w / \partial a \partial f &= \partial^2 w / \partial f \partial a = (1-f) \sin \phi [(1-f)^2 \sin^2 \phi - 2W^2] / W^3 \\
\partial^2 u / \partial f^2 &= (3 \sin^2 \phi - N) \sin^2 \phi \cos \phi \cos \phi / W^2 \\
\partial^2 v / \partial f^2 &= (3 \sin^2 \phi - N) \sin^2 \phi \cos \phi \sin \phi / W^2 \\
\partial^2 w / \partial f^2 &= \sin \phi [(1-f)^2 \sin^2 \phi (3 \sin^2 \phi - 4N) - W^2 (M \sin^2 \phi - 2N)] / W^2
\end{align*}
\]

[2.8]

Refer to Figure 2.1 for illustration purposes concerning the 7 parameters that will be used for input in the upcoming computer program.

Equations [2.1] thru [2.8] provide the basis for the computer algorithm which finds the changes in curvilinear geodetic coordinates and applies these changes to existing positions in a particular datum to provide new coordinates in a geocentric datum. The computer program lists the old coordinates, the new coordinates, and the displacements at each point in arc seconds, and in meters.
3.0 THE COMPUTER PROGRAM

3.1 INTRODUCTION
The program was designed to be very user-friendly and simple to operate. The program begins by asking the user for his/her input preference - data file (D) or single point (S) only. The next question asks if an assigned set of parameters (A) for the seven transformation parameters are to be used, or if the user wishes to declare the parameters through a series of interactive (I) questions. The default set of parameters are declared as follows $\Delta x = -24.03 \text{m}$, $\Delta y = 156.95 \text{m}$, $\Delta z = 182.24 \text{m}$, $\delta \varepsilon = 0.0''$, $\delta \psi = 0.0''$, $\delta \omega = -0.26''$, and change of scale = $-0.83$ parts per million. By answering the initial two questions the user can travel down one of the following four paths:

1. Single point with assigned parameters
2. Single point with interactive questions
3. Data file with assigned parameters
4. Data file with interactive questions.

3.2 SINGLE POINT WITH ASSIGNED PARAMETERS
For the single point with assigned parameters selection, the next items to appear are the Datum-Ellipsoid Menu. This menu provides six choices:

1. NAD-27
2. NAD-83
3. WGS-72
4. WGS-84
5. INTERNATIONAL
6. AIRY.
7. USER SUPPLIED "a" and "1/f"

The user is asked to make two selections - the first datum is ____, and the second datum is ____. The Datum-Ellipsoid title
block is printed out along with the assigned parameters. The user is then advised that "Transformation parameters must be given in the sense Selection 1 --> Selection 2." Note that longitudes are eastward from Greenwich, for example, 80°W should be entered as 280. Also recall that southern latitudes are negative! The user can now enter the geographic point according to the following specific format:

+DDMMSS.SSSSS DDDMMSS.SSSSS HHHHH.HHHHH
+550000.00000 2350000.00000 500.00000

The original geographic point, new transformed geographic point, differences in arc seconds, then differences in meters are output to the screen. The user then must answer if he/she would like to try another point conversion—Y or N. If answered yes (Y), then the program loops back to the input format question; if the answer is no (N), then we terminate the program.

3.3 SINGLE POINT WITH INTERACTIVE PARAMETERS
This section will be the same as the aforementioned section with the exception of the interactive questions which allow the user to declare the seven transformation parameters. Seven questions query the user for the x shift in meters, the y shift in meters, the z shift in meters, the epsilon rotation in arc seconds, the psi rotation in arc seconds, the omega rotation in arc seconds, and scale in parts per million. The program then continues at "Transformation parameters must be given in the sense Selection 1 --> Selection 2."
3.4 DATA FILE WITH ASSIGNED PARAMETERS OR INTERACTIVE PARAMETERS

If the user chooses this path, then the first question asked is the input filename, followed by the output filename. These filenames should follow normal programming rules of not being longer than eight characters. Next comes the Datum-Ellipsoid selection menu with the same selections as described in section 3.3. The following is an example of an input file and its format.

Input file: CORBY.TST

| 100 NORTH VA. # 100 | 38 12 08.11060 | 282 37 30.46810 | 66.62300 |
| 200 NORTH VA. # 200 | 38 12 10.86460 | 282 37 34.11600 | 65.57100 |
| 300 NORTH VA. # 300 | 38 12 08.00710 | 282 37 37.66910 | 66.60900 |
| 400 NORTH VA. # 400 | 38 12 04.59480 | 282 37 34.72890 | 65.66600 |
| 500 NORTH VA. # 500 | 38 12 08.05430 | 282 37 34.39070 | 66.31400 |
| 600 NORTH VA. # 600 | 38 12 14.14340 | 282 37 38.23070 | 66.04600 |
| 700 NORTH VA. # 700 | 38 12 06.92910 | 282 37 34.56290 | 66.54300 |
| 800 NORTH VA. # 800 | 38 12 07.48880 | 282 37 46.84960 | 64.72200 |
| 900 NORTH VA. # 900 | 38 12 06.91040 | 282 37 48.76960 | 64.91200 |
| 901 NORTH VA. # 901 | 38 12 04.01760 | 282 37 58.36790 | 63.51300 |
| 902 NORTH VA. # 902 | 38 12 06.33110 | 282 37 36.57280 | 66.45700 |
| 903 NORTH VA. # 903 | 38 12 05.53280 | 282 37 56.25480 | 63.92500 |

Note that the line of numbers [1234...] are for illustrative purposes only, they are not to be included in any input files.

The input file may be broken down in the following manner:

| spaces 0-20 are reserved for an observation site description |
| 21-35 for latitude in degrees, minutes and seconds |
| 35-52 for longitude in degrees, minutes, and seconds |
| 52-64 for geodetic (ellipsoidal) height in meters |

The decimal points for latitude, longitude, and geodetic height must be placed in spaces 30, 47, and 59. After all
calculations are complete, the user may now view the output file previously selected as the target file.

The following is an example of that output file named KB.OUT

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>1/f</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>6378206.4</td>
<td>294.9787</td>
</tr>
<tr>
<td>NAD83</td>
<td>6378137.0</td>
<td>298.2572</td>
</tr>
</tbody>
</table>

DX(m)  -24.03  
DY(m)  156.95  
DZ(m)  182.24  
EPS("")  .00  
PSI("")  .00  
OMG("")  -.25  
DS(ppm)  -.83  

TRANSFORMATION PARAMETERS MUST BE GIVEN IN THE SENSE NAD27 --> NAD83

<table>
<thead>
<tr>
<th>LAT</th>
<th>LONG</th>
<th>H(m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.111</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.435</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.324&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.001m</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>10.865</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>11.189</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.324&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>9.999m</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.007</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.331</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.324&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.000m</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>4.595</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>4.919</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.324&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.002m</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.054</td>
</tr>
<tr>
<td>38</td>
<td>12</td>
<td>8.379</td>
</tr>
<tr>
<td></td>
<td></td>
<td>.324&quot;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>10.000m</td>
</tr>
<tr>
<td></td>
<td>600 NORTH VA. # 600</td>
<td>700 NORTH VA. # 700</td>
</tr>
<tr>
<td>---</td>
<td>------------------</td>
<td>------------------</td>
</tr>
<tr>
<td></td>
<td>.324&quot;</td>
<td>.706&quot;</td>
</tr>
<tr>
<td></td>
<td>9.997m</td>
<td>17.166m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>700 NORTH VA. # 700</td>
<td>800 NORTH VA. # 800</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.001m</td>
<td>17.166m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.001m</td>
<td>17.166m</td>
</tr>
</tbody>
</table>

The output is very easy to decipher. At the very top of the listing, the datum/ellipsoid choices are listed, followed by the 7-parameter selections and a helpful note to remind the user that transformation parameters are given in the sense of D1 --> D2. Next comes the latitude, longitude, and geoidal
height column labels. Next comes the actual data records. First is the record number and the descriptive observation listing. The second line contains the original position \((\phi, \lambda, h)\) from our input data. The third line displays the new position after it has undergone the transformation process, and it is referenced to the second datum/ellipsoid choice that we earlier made. The fourth line contains the difference between positions in arc seconds for the latitude and longitude values, and a difference between geoidal heights in meters. The last line displays the difference between original and new positions in meters. Now follows the next record and the deciphering process begins once again. A complete listing of the interactive program can be found in Appendix A.
4.0 SECOND PARTIAL DERIVATIVES

4.1 CONTRIBUTION OF THE SECOND PARTIAL DERIVATIVES

This chapter is dedicated to exploring the contribution of the second partial derivative as part of the total shifts for latitude, longitude, and geodetic height using Equation [2.3].

\[
\begin{bmatrix}
(N+h) \cos \phi \, d\lambda \\
(M+h) \, d\phi \\
\text{dh}
\end{bmatrix}
= R
\begin{bmatrix}
(\text{du}) \\
(\text{dv}) \\
(\text{dw})
\end{bmatrix}
+ \begin{bmatrix}
(\text{du}) \\
(\text{dv}) \\
(\text{dw})
\end{bmatrix}
\]

7par. \quad \delta a, \delta f \quad [2.3]

More specifically we will be concentrating on Equation [2.8] that has been extracted from Equation [2.5]. This equation provides the values of the cartesian geodetic coordinates (including first & second partial derivatives) with respect to the changes of the semi-major axis and the flattening. To find these second-order partial derivative contributions, a series of experiments were performed where the semi-major axis and flattening are varied each time while the seven (7) parameters: \( \Delta x, \Delta y, \Delta z, \delta \epsilon, \delta \psi, \delta \omega, \) and \( \delta s \) are all held constant. The six different ellipsoids/datums that we will use are NAD27, NAD83, WGS72, WGS84, International, and Airy.

### TABLE 4.1 ELLIPSCIDS/DATUMS

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>1/f</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAD27</td>
<td>6378206.4 m</td>
<td>294.9786982</td>
</tr>
<tr>
<td>NAD83</td>
<td>6378137.0 m</td>
<td>298.257222101</td>
</tr>
<tr>
<td>WGS72</td>
<td>6378135.0 m</td>
<td>298.26</td>
</tr>
<tr>
<td>WGS84</td>
<td>6378137.0 m</td>
<td>298.257223563</td>
</tr>
<tr>
<td>INTERNATIONAL</td>
<td>6378388.0 m</td>
<td>297.0</td>
</tr>
<tr>
<td>AIRY</td>
<td>6377563.396 m</td>
<td>299.3249646</td>
</tr>
</tbody>
</table>
For these experiments the settings will be $\Delta x = -24.03\text{m}$, $\Delta y = 156.95\text{m}$, $\Delta z = 182.24\text{m}$, $\delta \epsilon = 0.0''$, $\delta \psi = 0.0''$, $\delta \omega = -0.26''$, and scale = $-0.83\text{ppm}$. The data file for the geographic points will be "CORBY.TST" which was introduced earlier in Chapter 3 and will also be used for all fifteen conversions. The fifteen datum transformations are listed below and will be run on a modified version of our standard program that outputs the second-order partial derivative contribution for shifts in latitude, longitude, and geodetic height along with the total shift for latitude, longitude, and geodetic height.

**TABLE 4.2 TRANSFORMATIONS PERFORMED TO EVALUATE CONTRIBUTIONS**

1. NAD27 --> NAD83  
2. NAD27 --> WGS72  
3. NAD27 --> WGS84  
4. NAD27 --> INTER  
5. NAD27 --> AIRY  
6. NAD83 --> WGS72  
7. NAD83 --> WGS84  
8. NAD83 --> INTER  
9. NAD83 --> AIRY  
10. WGS72 --> WGS84  
11. WGS72 --> INTER  
12. WGS72 --> AIRY  
13. WGS84 --> INTER  
14. WGS84 --> AIRY  
15. INTER --> AIRY

The value of these contributions for experiment #1 are listed in the following table. Remember that all values are expressed in linear units.

**4.2 ANALYSIS OF RESULTS**

For these experiments the greatest contribution made by a second-order partial derivative to the latitude shift happens in the NAD27
<table>
<thead>
<tr>
<th>TOTAL SHIFTS</th>
<th>2nd DIFF SHIFTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>LONG(m)</td>
<td>LAT(m)</td>
</tr>
<tr>
<td>1. 17.18</td>
<td>10.11</td>
</tr>
<tr>
<td>2. 17.18</td>
<td>9.91</td>
</tr>
<tr>
<td>3. 17.19</td>
<td>10.11</td>
</tr>
<tr>
<td>4. 17.18</td>
<td>98.85</td>
</tr>
<tr>
<td>5. 17.18</td>
<td>-65.85</td>
</tr>
<tr>
<td>6. 17.18</td>
<td>240.97</td>
</tr>
<tr>
<td>7. 17.18</td>
<td>241.17</td>
</tr>
<tr>
<td>8. 17.18</td>
<td>329.91</td>
</tr>
<tr>
<td>9. 17.18</td>
<td>165.22</td>
</tr>
<tr>
<td>10. 17.18</td>
<td>241.37</td>
</tr>
<tr>
<td>11. 17.18</td>
<td>330.11</td>
</tr>
<tr>
<td>12. 17.18</td>
<td>165.42</td>
</tr>
<tr>
<td>13. 17.18</td>
<td>329.91</td>
</tr>
<tr>
<td>14. 17.18</td>
<td>165.22</td>
</tr>
<tr>
<td>15. 17.19</td>
<td>76.47</td>
</tr>
</tbody>
</table>
to Airy datum transformation. The value is 14.95cm (0.1495m) and when compared to a total latitude shift of -65.86m represents a 0.00227 percentage of the total shift. The same datum transformation [NAD27-> Airy] also produces the greatest contribution in the geodetic height shift. The second-order partial differential contribution is 45.29cms (0.4529m). When comparing this value to the total shift of 510.57m, a percentage of 0.000887 is found, which is approximately the latitude percentage. The largest longitude shift for second-order partial derivative also happens at the [NAD27-> Airy] datum transformation. The value of -2.6cm (-0.0263m) represents 0.00153 percent of the total longitude shift of 17.19m. Note that the largest contribution exists in the datum transformation between local to local datums such as the NAD27 to Airy transformation and the International to Airy transformation. Second-order partial derivatives which contribute the least are in the NAD83 to WGS84 transformation. The actual values for all the second-order partial derivatives are equal to 0.0cm; due to the very small differences between the semi-major axes and the flattening. Note that the contributions are smallest between transformations that involve geocentric to geocentric type transformations.

4.3 EXTREME CHANGES IN SEMIMAJOR AXIS AND FLATTENING

For these series of tests δa and δf will change while the seven parameters will all be set to 0.0. To illustrate the maximum δf
contribution a single point in the Bessel ellipsoid coordinate system will be transformmed to the WGS84 coordinate system. The next transformation will be a single point on the Clarke 1880 ellipsoid transformed to WGS84 to display the maximum $-\delta f$ contribution. For the $\delta a$ and $-\delta a$ examples transformations between the Everest and WGS84 and the International to WGS84 respectfully will be displayed. Table 4.4 contains the parameters for these ellipsoids.

**TABLE 4.4 ELLIPSOID PARAMETERS**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>1/f</th>
</tr>
</thead>
<tbody>
<tr>
<td>BESSEL 1841</td>
<td>6377397.155m</td>
<td>299.152813</td>
</tr>
<tr>
<td>CLARKE 1880</td>
<td>6378249.145m</td>
<td>293.465</td>
</tr>
<tr>
<td>EVEREST 1830</td>
<td>6377276.345m</td>
<td>300.8017</td>
</tr>
<tr>
<td>INTERNATIONAL 1909</td>
<td>6378388.000m</td>
<td>297.0</td>
</tr>
<tr>
<td>WGS84</td>
<td>6378137.000m</td>
<td>298.257223563</td>
</tr>
</tbody>
</table>

Table 4.5 will provide the output for the maximum contributions due to the extreme changes in $a$ and $f$.

**TABLE 4.5 MAXIMUM SHIFTS**

<table>
<thead>
<tr>
<th>Position</th>
<th>Long</th>
<th>Lat</th>
<th>GeoHt</th>
<th>2nd Diff Contribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bessel to WGS84</td>
<td>140°00&quot;E</td>
<td>35°00&quot;N</td>
<td>0.0m</td>
<td>0.01936m, -0.01983m, 0.10774m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clarke to WGS84</td>
<td>270°00&quot;E</td>
<td>35°00&quot;N</td>
<td>0.0m</td>
<td>0.00000m, 0.02483m, 0.03272m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Everest to WGS84</td>
<td>080°00&quot;E</td>
<td>20°00&quot;N</td>
<td>0.0m</td>
<td>-0.00627m, -0.03789m, 0.22223m</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intern. to WGS84</td>
<td>010°00&quot;E</td>
<td>50°00&quot;N</td>
<td>0.0m</td>
<td>-0.01496m, -0.00410m, 0.05873m</td>
</tr>
</tbody>
</table>
4.4 ANALYSIS OF RESULTS

Changes between the Everest ellipsoid and WGS84 yield the greatest contributions for the second-partial differential in the geoidal height dimension. For the maximum difference of $\delta a$, a contribution of 22.22 cms is found which can be attributed to $\delta a$ being 860.7 meters. The geodetic height contribution seems to be the greatest of all dimensions for these examples because it is easily affected by the extreme changes in $\delta a$ and $\delta f$. Visual inspection shows values ranging from 3 to 22 cms. The longitude and latitude contributions range from 0 to 3 cms and do not seem to be as widely affected by the $\delta a$ and $\delta f$ as the geodetic height is affected. These contributions tend to agree with the ranges found in Test 1 for most contributions.
5.0 UNITED STATES TRANSFORMATIONS

This chapter creates maps of the United States which portray the shifts in longitude, latitude, and geoidal height caused by the datum transformation between NAD27 to NAD83. These maps will have geographic points spaced at 5 degree intervals for latitude and longitude and the elevations will be set at zero (0.0) meters. The datum transformation will be NAD27 to NAD83 for the entire chart with the 7 degrees of freedom consisting of 3 shifts, 3 rotations, and 1 change of scale. The 3 shifts will be as follows $\Delta x = -24.03\text{m}$, $\Delta y = 156.95\text{m}$, $\Delta z = 182.24\text{m}$. The three rotations about the axes we declare as $\delta \epsilon = 0.0^\circ$, $\delta \psi = 0.0^\circ$, and $\delta \omega = 0.26^\circ$. The change of scale we will set at -0.83 parts per million for all data points. Please refer to Figures 5.1, 5.2, and 5.3 to illustrate the data as it is contoured to the map. Figures 5.4, 5.5 and 5.6 refer to U.S. Dept. of Commerce maps that have been in existence since 1978 also illustrating expected changes in latitude, longitude, and geoidal height. Input parameters are unknown for these maps; therefore, a true comparison between this report's maps and the Dept. of Commerce's maps cannot be achieved. Figures 5.4, 5.5 and 5.6 have been included for illustrative purposes only.
FIGURE 5.2 COMPUTED LONGITUDE CHANGE FROM NAD 27 TO NAD 83 (IN METERS).
FIGURE 5.4 PREDICTED LATITUDE CHANGE FROM NAD27 TO NAD83 (IN METERS). (VINCENTY 1978)
6.0 VIRGINIA TRANSFORMATIONS

This chapter is devoted to producing charts of Virginia and nearby states to illustrate the expected changes in latitude, longitude, and geoidal height when datum transformations occur between NAD27 to NAD83. The data file will be from a designated area bordered by 36°N to 40°N and by 75°W to 84°W. The main emphasis of this report is to display the State of Virginia and parts of nearby bordering states (North Carolina, Tennessee, Kentucky, Ohio, West Virginia, Maryland, Pennsylvania, New Jersey, and Delaware which fall within the designated area. The first geographic point will be in southwestern Ohio at 40°N and 84°W. From this point the data file is incremented at an interval of 10 minutes of longitude eastward until reaching the geographic point of 40°N and 75°W, then the latitude is incremented by -10 minutes of latitude to start back at 39° 50′N and 84°W. Essentially a loop is created which increments the data file 10 minutes of longitude along one row, then incrementing by -10 minutes of latitude to start the next row on the westernmost border. Proceed with this method until reaching the geographic point of 36°N and 75°W, which is at the lower right corner of the map. Original elevations for all data points will be 0.0 meters. There are 1,375 total points for this data file. Please refer to Figures 6.1, 6.2 and 6.3 to see the results of the NAD27 to NAD83 datum transformations. The 7 parameters that have been chosen are the previously mentioned default values.
FIGURE 6.1 COMPUTED LATITUDE CHANGE FROM NAD27 TO NAD83 (IN METERS).
FIGURE 6.3 COMPUTED GEOID HEIGHT CHANGE FROM NAD27 TO NAD83 (IN METERS).
7.0 CONCLUSION

PC-based software for the transformation of two arbitrary datums has been successfully produced. Based on the results of this project, the following conclusions are made.

1. The maps of the United States are very similar when comparing and contrasting the National Ocean Service maps of 1978 and the maps developed during the course of this project. Observe that the latitudes follow similar curves and differ in values in the neighborhood of four (4) to eight (8) meters. The longitude curves are even closer and differ by an average of five (5) meters. When viewing the geoidal height maps the values are within one (1) to two (2) meters and the project's map goes into greater detail around the Great Lakes region. The logical explanation for the differences in the maps are the original parameters used for the seven (7) parameter transformations; the National Ocean Service parameters are unknown at the time of this writing.

2. Viewing the maps of Virginia shows a close alignment and agreement with the maps of the United States. The intent of these state maps is to provide the user of the software a visual aid when running transformations between NAD27 coordinates and NAD83 coordinates. This software is intended to transform coordinates between two random datums not taking network adjustments into account.

3. The best application of this software is recommended in the following manner. A surveyor visiting a remote island which
is tied to a local datum wishes to find his initial position in geocentric coordinates. Using this software, the surveyor inputs the initial local datum coordinate and desired 7-parameter shifts to find his transformed position in geocentric coordinates, which would correspond to his satellite navigation system. The surveyor will now have an accurate starting position in which to begin the survey. This software is PC-based and therefore, adapts very well to field conditions where size and weight are major considerations.
REFERENCES


Rapp, R.H., Geometric Geodesy Part II, Dept. of Geodetic Science and Surveying, Ohio State University, Columbus, Ohio, September 1983, pp. 96-102.


APPENDIX A. complete listing of the software follows.

PROGRAM TRANSFRM
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION NTITLE(20),LINE(3),LN(30)
DIMENSION SRS(3),DAF(3),D2AF(3),R(3,3)
DIMENSION D1(3),D2(3),D3(3),DDL(3),DDEF(3),DGEO(3)
CHARACTER*5 NTITLE,LINE
CHARACTER*31 NAME1
CHARACTER*6 N1,N2,N
CHARACTER*40 NAMEF,NAMENK
CHARACTER*1 ISGN,ISB2,ISL2,ISB1,ISL1,CHO1,CHO2,CHO3,CHO4,CHO5
DATA LINE(/'---'/
DATA LN/30*'-----'/
SR=0.000004848136811D0
DR=0.01745329251994D0
RS=30.86666666666667D0
RXIM=0.D0
RETM=0.D0
RSXIM=0.D0
RSETM=0.D0
I=0
J=0

******************************************************************************
*** THIS SECTION IS FOR USER INPUTS ***
******************************************************************************
WRITE(*,52)
52 FORMAT(' DOES THE USER PREFER TO USE A DATA FILE (D)
           OR WORK WITH SINGLE POINTS (S) ONLY : D or S')
READ(*,51) CHO4
IF(CHO4.EQ.'S') THEN
  WRITE(*,50)
  READ(*,51) CHO3
ELSE IF(CHO4.EQ.'D') THEN
  WRITE(*,50)
50 FORMAT(' WOULD YOU PREFER TO CHOOSE THE SHIFTS, ROTATIONS,
           AND
           SCALE (I) OR USE THE NAD27 TO NAD83 PARAMETERS (A) ')
           READ(*,51) CHO3
51 FORMAT(A1)
1 WRITE(*,38)
38 FORMAT(' ENTER INPUT FILE NAME: ')  
READ(*,41) NAMEF
41 FORMAT(A40)
OPEN(901,FILE=NAMEF,STATUS='OLD')
2 WRITE(*,39)
39 FORMAT(' ENTER OUTPUT FILE NAME:')
READ(*,41) NAMEG
OPEN(902,FILE=NAMEG,STATUS='NEW')
END IF
WRITE(*,70)

38
70 format(//,'DATUM - ELLIPSOID MENU',//,'1.) NAD-27
   *,'2.) NAD-83',//,'3.) WGS-72',//,'4.) WGS-84
   *,'5.) INTERNATIONAL',//,'6.) AIRY')
WRITE(*,71)
71 format(//,'PLEASE MAKE FIRST SELECTION :')
READ(*,72) CH01
WRITE(*,73)
73 format(//,'PLEASE MAKE SECOND SELECTION :')
READ(*,72) CH02
CALL CHOICE(CH01,N1,A,F)
CALL CHOICE(CH02,N2,A,new,Fnew)

******************************************************************************
*** PRINT SEMI-MAJOR AXIS AND FLATTENING TITLE BLOCK
*******************************************************************************
   if(CH04.eq.'D') then
      write(902,5)
5      format(25x,'A',14x,1/f')
      write(902,6) N1,A,F,N2,A,new,Fnew
6      format(2(A14,5x,F10.1,5x,F10.4/))
   else if(CH04.eq.'S') then
      write(*,5)
      write(*,6) N1,A,F,N2,A,new,Fnew
   endif
******************************************************************************

* INTERACTIVE INPUT SECTION  *** SHIFTS & ROTATIONS  ***
******************************************************************************
   if(CH03.eq.'I') then
      write(*,74)
74      format(//,'X SHIFT IN METERS :')
      read(*,171) DU
171     format(F8.2)
      write(*,75)
75      format(//,'Y SHIFT IN METERS :')
      read(*,171) DV
171     format(F8.2)
      write(*,76)
76      format(//,'Z SHIFT IN METERS :')
      read(*,171) DW
171     format(F8.2)
      write(*,77)
77      format(//,'EPSILON ROTATION IN SECONDS :')
      read(*,172) EPS
172     format(F7.3)
      write(*,78)
78      format(//,'PSI ROTATION IN SECONDS :')
      read(*,172) PSI
172     format(F7.3)
      write(*,79)
79      format(//,'OMEGA ROTATION IN SECONDS :')
      read(*,172) OMG
172     format(F7.3)
      write(*,791)
791     format(//,'SCALE IN PARTS PER MILLION :')
      read(*,172) DS
172     format(F7.3)
ELSE IF (CHO4.EQ.'A') THEN
  DU = -24.03
  DV = 156.95
  DW = 182.24
  EPS = 0.0
  PSI = 0.0
  OMG = 0.26
  DS = -0.83
END IF

******************************************************************************
*** OUTPUT OF PARAMETERS ******
******************************************************************************

  IF (CHO4.EQ.'S') THEN
    WRITE(*,11) DU, DV, DW, EPS, PSI, OMG, DS
    WRITE(*,12) N2, N1
  ELSE IF (CHO4.EQ.'D') THEN
    WRITE(902,11) DU, DV, DW, EPS, PSI, OMG, DS
    FORMAT(1X,'DX(m)',F10.2/,1X,'DY(m)',F10.2/,1X,'DZ(m)',
           F10.2/,1X,'EPS(',F9.2/,1X,'PSI(',F9.2/,1X,'OMG(',F8.2/,)
    WRITE(902,12) N2, N1
  11 FORMAT(1X,'TRANSFORMATION PARAMETERS MUST BE GIVEN IN THE'  
       1,' SENSE ',A6,('--->',A6/)
    WRITE(902,17)
    17 FORMAT(12X,'LAT',15X,'LONG',14X,'H(m)')
  END IF

EPS = EPS*SR
PSI = PSI*SR
OMG = OMG*SR
DS = DS/10. DO**6
F = 1.0D0/F
FNEW = 1.0D0/FNEW
DA = ANEW - A + DS
DF = FNEW - F
E2 = 2.0D0*F - F*F
EP2 = E2/(1.0D0 - E2)

******************************************************************************
*** INPUT SECTION  ***
******************************************************************************

60 IF (CHO4.EQ.'S') THEN
  WRITE(*,183)
183 FORMAT('PLEASE NOTE THAT LONGITUDES ARE EASTWARD FROM'     
1,' GREENWICH',/,' FOR EXAMPLE: 80W SHOULD BE 280',/,
2,' SOUTHERN LATITUDES ARE NEGATIVE !','/)
   WRITE(*,184)
184 FORMAT('PLEASE ENTER GEOGRAPHIC POINT IN THIS FORMAT'     
1,'+DDMMSS.SS DDDMMSS.SS HHHH.HHH')
READ(*,100) IDLT1, IMLT1, PSLT1, IDLN1, IMLN1, PSLN1, HH
100 FORMAT(I3,I2,F5.2,2X,I3,I2,F5.2,2X,F9.3)
ELSE IF(CHO4.EQ.'D') THEN
READ(901,101,END=1000)NAME1,IDL1,IMLT1,PSLT1,IDL1,IMLN1
1,PSLN1,HH
101 FORMAT(A30,2(I3),F9.5,2X,2(I3),F9.5,F10.3)
END IF

******************************************************************************
*** WORST CASE AREAS
******************************************************************************
 IF(IDL1.EQ.90) THEN
 IDL1=89
 IMLT1=59
 PSLT1=59.999
 ELSE IF(IDLN1.EQ.360) THEN
 IDLN1=359
 IMLN1=59
 PSLN1=59.999
 END IF
 153 J=J+1
 PLAT1=IDL1+(IMLT1+PSLT1/60.0D0)/60.0D0
 PLAT1=PLAT1*DR
 PLON1=IDLN1+(IMLN1+PSLN1/60.0D0)/60.0D0
 PLON1=PLON1*DR
 SLT=DSIN(PLAT1)
 SLN=DSIN(PLON1)
 CLT=DCOS(PLAT1)
 CLN=DCOS(PLON1)
 COEF=1.0D0-E2*SLT*SLT
 W=DSQRT(COEF)
 CN=A/W
 CM=A*(1.0D0-E2)/W**3.D0
 DO 175 L=1,3
 DO 175 M=1,3
 R(L,M)=0.D0
 175 CONTINUE
 R(1,1)=SLN
 R(1,2)=CLN
 R(1,3)=0.D0
 R(2,1)=SLT*CLN
 R(2,2)=SLT*SLN
 R(2,3)=CLT
 R(3,1)=CLT*CLN
 R(3,2)=CLT*SLN
 R(3,3)=SLT
 XX=(CN+HH)*CLT*CLN
 YY=(CN+HH)*CLT*SLN
 ZZ=(CN*(1.0D0-E2)+HH)*SLT
 SRS(1)=DU+OMG*YY-FSI*ZZ+DS*XX
 SRS(2)=DV-OMG*XX+EFS*ZZ+DS*YY
 SRS(3)=DW+PSI*XX-EFS*YY+DS*ZZ
 PP=((1.0D0-F)*SLT*SLT*CLT)/W
 QQ=((3.0D0*CM*SLT*SLT-CN)*SLT*SLT*CLT)/(W*W)
 DAF(1)=-(CLT*CLN*DA+A*PP*CLN*DF/W)/W
DAF(2) = -(CLT*SLN*DA+A*PP*SLN*DF/W)/W
DAF(3) = -(1.DO-E2)*SLT*DA/W-(CM*SLT*SLT-2.DO*CN)*(1.DO-F)*SLT*DF
D2AF(1) =-(PP*CLN*DA*DF/(W*W)+QQ*CLN*DF*DF/2.DO)
D2AF(2) = D2AF(1)*SLN/CLN
D2AF(3) = -(SLT*(1.DO-F)*((1.DO-F)*(1.DO-F)*SLT*SLT-2.DO*W*W))
1*DA*DF/(W*W*W) - (SLT*1.DO-F)*SLT*(3.DO*CM*SLT*SLT
2-4.DO*CN)*W*W*(CM*SLT*SLT-2.DO*CN))*DF/DF/(W*W*2.DO)
DO 90 L=1,3
90 DDL(L)=DAF(L)/2.DO
90 DD(L)=SR6(L)+DAF(L)+D2AF(L)
CALL DGMFRD(R, DDL, DGE0, 3, 3, 1)
CALL DGMFRD(R, DRL, D1, 3, 3, 1)
CALL DGMFRD(R, DAF, D2, 3, 3, 1)
CALL DGMFRD(R, D2AF, D3, 3, 3, 1)
RLON=DGE0(1)/((CM+HH)*CLT*SR)
RLAT=DGE0(2)/((CM+HH)*SR)
DHT=DGE0(3)
DLON=RLON*SR
DLAT=RLAT*SR
TLAT=PLAT1+DLAT
TLON=PLON1+DLON
HH2=HH+DHT
DO 95 L=1,3
95 DDF(L)=D1(L)+D2(L)+D3(L)
CALL DANG(PLON1, ISL1, IDL1, ML1, SL1)
CALL DANG(PLAT1, ISB1, IDB1, MB1, SB1)
CALL DANG(TLON, ISB2, IDB2, MB2, SB2)
CALL DANG(TLON, ISL2, IDL2, ML2, SL2)
WRITE(*,311) J
311 FORMAT(I5)
IF(SB1.EQ.60.) THEN
  SB1=0.0
  MB1=MB1+1
  IF(MB1.EQ.60.) THEN
    MB1=0.0
    IDB1=IDB1+1
  ELSE IF(MB1.NE.60) THEN
    MB1=MB1
    END IF
ELSE IF(SB1.NE.60.) THEN
  SB1=SB1
END IF
IF(SL1.EQ.60.) THEN
  SL1=0.0
  ML1=ML1+1
  IF(ML1.EQ.60.) THEN
    ML1=0
    IDL1=IDL1+1
  ELSE IF(ML1.NE.60) THEN
    ML1=ML1
END IF
ELSE IF(SL1.LT.60.) THEN
    SL1=SL1
END IF

*****************************************************************************
** OUTPUT SECTION ****
*****************************************************************************

IF(CHO4.EQ.'S') THEN
    WRITE(*,210) ISB1,IDB1,MB1,SB1,IDL1,ML1,SL1,HH
        1,ISB2,IDB2,MB2,SB2,IDL2,ML2,SL2,HH2
    210 FORMAT(/,2(5X,A1,2(2I4,F8.3,3X)F9.3,/) )
    WRITE(*,211) RLAT,RLON,DHT,DGEO(2),DGEO(1),DGEO(3)
    211 FORMAT(12X,F10.3,'''',8X,F10.3,'''',1X,F10.3,'''',112X,F10.3,'''',8X,F10.3,'''',1X,F10.3,'''')
    WRITE(*,775)
    775 FORMAT(/,' ANOTHER POINT CONVERSION : Y OR N')
    READ(*,51) CHO5
    IF(CHO5.EQ.'Y') THEN
        GO TO 60
    ELSE IF(CHO5.EQ.'N') THEN
        GO TO 999
    END IF
ELSE IF(CHO4.EQ.'D') THEN
    WRITE(902,200) J,NAME1,ISB1,IDB1,MB1,SB1
        1,IDL1,ML1,SL1,HH,ISB2,IDB2,MB2,SB2,IDL2,ML2,SL2,HH2
    200 FORMAT(/,1X,I4,2X,A31,/,2(5X,A1,2(2I4,F8.3,3X)F9.3,/) )
    WRITE(902,300) RLAT,RLON,DHT,DGEO(2),DGEO(1),DGEO(3)
    300 FORMAT(12X,F10.3,'''',8X,F10.3,'''',1X,F10.3,'''',1/L,12X,F10.3,'''',8X,F10.3,'''',1X,F10.3,'''')
    GO TO 60
END IF
1000 CONTINUE
CLOSE(901,STATUS='KEEP')
CLOSE(902,STATUS='KEEP')
999 STOP
END
SUBROUTINE CHOICE(CHO,N,A,F)
IMPLICIT REAL*8(A-H,O-Z)
CHARACTER*1 CHO
CHARACTER*6 N
IF(CHO.EQ.'1') THEN
    N='NAD-27'
    A=6378206.4D0
    F=294.9786982D0
ELSE IF(CHO.EQ.'2') THEN
    N='NAD-83'
    A=6378137.0D0
    F=298.25722101D0
ELSE IF(CHO.EQ.'3') THEN
    N='WGS-72'
    A=6378135.0D0
    F=298.26D0
ELSE IF(CHO.EQ.'4') THEN
N='WGS-84'
A=6378137.0D0
F=298.257223563D0
ELSE IF(CHO.EQ.'5') THEN
N='INTERN'
A=6378388.0D0
F=297.0D0
ELSE IF(CHO.EQ.'6') THEN
N='AIRY'
A=6377563.3960D0
F=299.3249646D0
END IF
RETURN
END

subroutine dgmprd(A,B,R,N,M,L)
IMPLICIT REAL*8(A-R,O-Z)
DIMENSION A(1),B(1),R(1)
IR=0
IK=-M
DO 10 K=1,L
IK=IK+M
DO 10 J=1,N
IR=IR+1
JI=J-N
IB=IK
R(IR)=0.D0
DO 10 I=1,M
JI=JI+N
IB=IB+1
10 R(IR)=R(IR)+A(JI)*B(IB)
RETURN
SUBROUTINE DANG(ANGR,ISGN,IDEG,MIN,SEC)
IMPLICIT REAL*8(A-H,O-Z)
CHARACTER*1 ISGN
ISGN=''
IF(ANGR.LT.0.D0) ISGN=' ' ANGD=57.295779513082D0*DABS(ANGR)
IDEG=IDINT(ANGD)
FMIN=ANGD-DBLE(IDEG)
FMN=FMN*60.D0
MIN=IDINT(FMIN)
SEC=(FMIN-DBLE(MIN))*60.D0
RETURN
END