

# **Weapons Control Re-Entry Simulation Enhancement**

by

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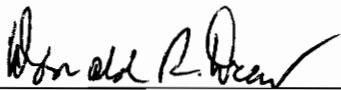
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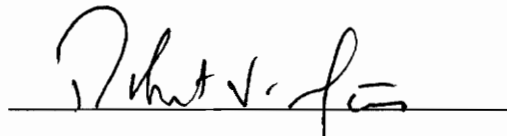
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Committee Chairman: Benjamin S. Blanchard  
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## **(ABSTRACT)**

Extending weapon systems beyond their design life to cope with the new type of threat(s) is often preferable to new weapon system development. Weapon system performance can be enhanced through hardware and/or software changes. Software changes or enhancements are required in either case. Requirements associated with strategic weapon system (SWS) for a mobile launch platform can result in the use of approximate computations. An example is the computation of the aerodynamic effects which ballistic warheads experience when they descend in the atmosphere. The accuracy of this approach, while adequate for a nuclear SWS, may not be acceptable for a conventional SWS.

This paper describes the motivation and the use of the systems engineering process in a study of the feasibility of incorporating a six-degree-of freedom (6DOF) re-entry simulation into a typical strategic weapon control system (WCS). After the problem is defined, the WCS operational requirements are described and used to guide the optimization of a typical 6DOF re-entry trajectory simulation. A recommendation for further action is based on software testing and evaluation results.

## **ACKNOWLEDGMENTS**

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## TABLE OF CONTENTS

<b>1.0 Problem Definition</b> .....	<b>1</b>
<b>2.0 Optimization Process</b> .....	<b>4</b>
<b>2.1 Weapons Control Operational Analysis</b> .....	<b>5</b>
<b>2.2 Operational Requirements For The Re-Entry</b>	
<b>Simulation Enhancement</b> .....	<b>10</b>
<b>2.3 Human Factors Requirements</b> .....	<b>13</b>
<b>2.4 The 6DOF Re-Entry Simulation Reference</b>	
<b>Model Description</b> .....	<b>15</b>
2.4.1 Logical Computation .....	15
2.4.2 Functional Components .....	17
2.4.3 Computational Time Allocation .....	22
<b>2.5 Optimization Development</b> .....	<b>23</b>
<b>2.6 Testing and Evaluation</b> .....	<b>26</b>
<b>2.7 Conclusion</b> .....	<b>28</b>
<b>2.8 Recommendation</b> .....	<b>29</b>
<b>Appendices</b> .....	<b>31</b>
Appendix A Coordinate Frames .....	32
Appendix B Numerical Integration Methods .....	34
Appendix C Searching Techniques .....	38
Appendix D Deletion of Time Derivative of Transformation Matrix From GC Frame to Missile Frame (TGC2MD) .....	39

**LIST OF TABLES**

2.1 Test results ..... 27

## LIST OF FIGURES

1.1 Overview of a typical mobile launch system .....	3
2.1 WCS functional flow .....	8
2.2 WCS functional flow .....	9
2.3 Functional components .....	18
2.4 Functional sub-components .....	19
2.5 Functional sub-components .....	20
2.6 Functional sub-components .....	21

## 1.0 PROBLEM DEFINITION

The changes imposed by the evolving world situation and the realities of defense budgeting are changing the nature of the traditional nuclear strategic deterrence mission and related operational and technical requirements. Economic factors, coupled with the growth of more diverse and smaller threats, militate against the development of either a significant weapon system upgrade or a new weapon system. The proliferation of nuclear and ballistic missile technologies, the availability of chemical (and, perhaps, to a lesser degree, biological) production facilities, and the exporting of missile and weapon systems are resulting in the development of a wide threat, albeit one with limited capability. This new threat suggests a modification to the existing multiple independently targeted re-entry vehicle (MIRV) system so that it would carry non-nuclear warheads. The kinetic energy from the re-entry vehicles (RV's) of an intercontinental range ballistic missile can destroy very hard or buried targets but better accuracy is needed.

Better accuracy can be achieved through hardware and/or software changes. Hardware changes can be a new type of RV or changes to one or more of the subsystems of a mobile ballistic missile. The Guidance, Navigation, Missile or Weapons Control (WC) subsystems can be changed individually or in conjunction with other subsystems (Figure 1.1). While such changes will improve system accuracy, the degree of improvement may not be sufficient to ensure the required level of weapon effectiveness against the hardest target types. A maneuvering re-entry vehicle (MARV) is required to achieve significant improvement in accuracy.

The Weapon Control System (WCS) software receives and provides information to the other systems and, hence, any change in another subsystem may require changes to the WCS software. Further, current WCS software can not simulate the re-entry trajectory



with sufficient fidelity or accuracy to target a MARV. It is appropriate to examine the expansion of WCS software computational capability so that the ability to accommodate the required system improvement is provided. Its enhancement can expose any weapon systems hardware limitation and, therefore, provides guidance for hardware change(s). It can be also be a cost effective alternative to make an existing system more effective.

The WCS software simulates all aspects of the missile flight from the launch preparation to RV deployment. The re-entry computation part of the WCS software is a good candidate for improvement since there is no active control of the re-entry flight after RV release and, hence, impact miss is directly related to the WCS computational accuracy. The limited computing capability of the typical WCS and the need to comply with a tightly controlled launch sequence result in the use of approximations in the re-entry computations. One such approximation is a three-degree-of-freedom (3DOF) point mass model with compensation for the effects of higher order gravity terms, climate (wind and density variation), and rigid body terms. The 6DOF re-entry simulation which covers all the aforementioned effects not only improves the weapon system accuracy but also is required to simulate a maneuvering re-entry vehicle trajectory.

The WCS operational time constraints and possible computing limitations are the main factors affecting optimization of the full-blown 6DOF re-entry simulation. WCS requirements and a knowledge of functional computations provide a background for the optimization process, which is the main focus of this paper. In the optimization process, the current 6DOF re-entry simulation model is analyzed to identify the major computational blocks. Efficient computation algorithms are then defined and applied to those blocks. The optimized model is evaluated to check whether it meets the WCS requirements.

### The First Level

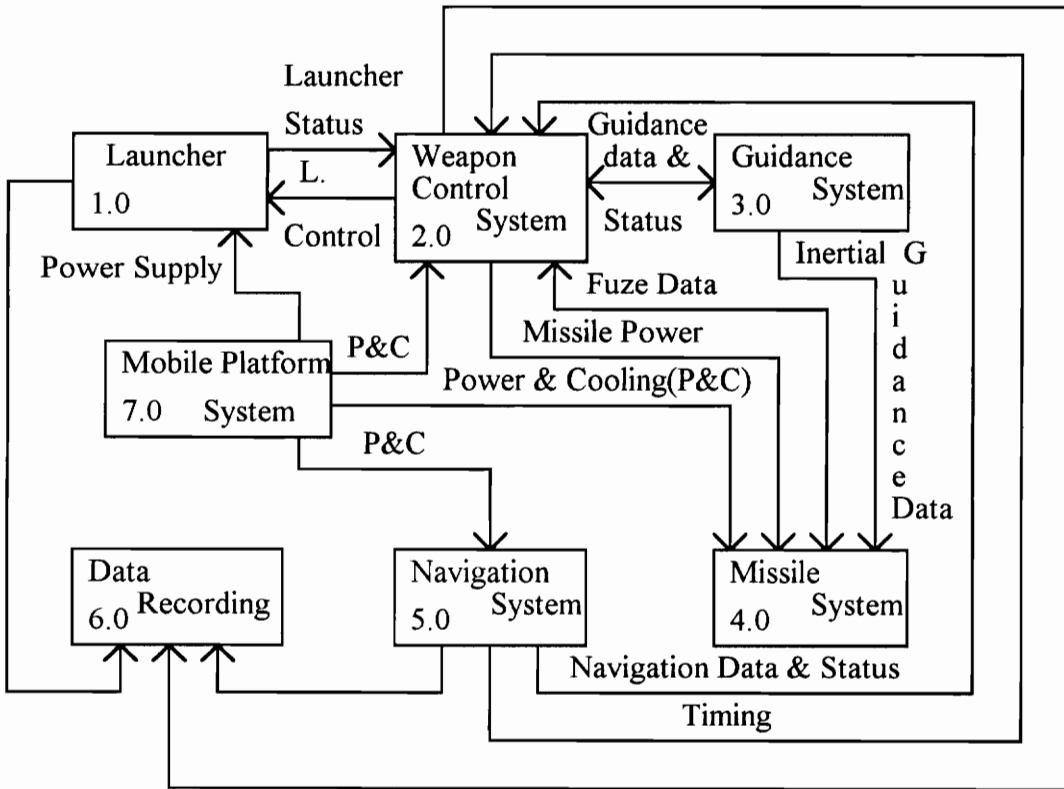


Figure 1.1 Overview of a typical mobile launch system

## **2.0 OPTIMIZATION PROCESS**

An optimized 6DOF re-entry simulation will be an integral part of WCS software. Existing inputs and outputs will be maintained to assure system integrity. Implementation into the existing WCS software must meet the system readiness time constraint and must be adaptable to future hardware changes, such as new re-entry vehicle (RV) types or new guidance technologies.

Requirements specification and operational analysis of a typical WCS identify the major computational tasks which need to be optimized and help to justify the requirement for fast computations. Testing and evaluation of the algorithm will be performed to check whether the requirements are met. The testing will be performed on a mainframe computer at the development site and assumptions will be made concerning the computing power ratio between a typical WCS computer and the mainframe.

## 2.1 WEAPONS CONTROL OPERATIONAL ANALYSIS

The WCS on a mobile launch platform computes and provides the missile guidance system with the data which satisfy system constraints and which can be used to steer the missile to assigned targets. The WCS also supports the launch sequence for all missiles through its interfaces with other subsystems. Training and testing are also done using the WCS.

The mobile launch platform operates under some constraints:

### **(1) Mobile platform constraint**

While mobile launch platforms have a high survivability, they complicate WCS computations since the mission parameters change as the platform moves, even to the extent that the mission, as planned, can not be executed. This must be regularly checked to determine whether the mission is achievable.

### **(2) Readiness time constraint**

The WCS is required to simultaneously prepare multiple missiles to be launched. The system must be able to launch a missile within a specific period of time after receipt of the execution order. This readiness time is assumed to be five (5) minutes per missile. The system must also be able to launch missiles at a specific rate after the first launch. This firing rate is assumed to be 5 seconds.

Thus, the computation must be designed to satisfy these constraints given some assumed hardware (computing, storage, and communication) configuration and capability. A typical solution for a system with a computer centered weapon control system architecture will require that computations be partitioned into phases based on criteria such as sensitivity to the knowledge of launch time or position, required targeting flexibility, and the magnitude of the particular computational task (Figure 2.1 - The third

level diagram). The target computations (Figure 2.2 - Block 2.2.2) address multiple missiles and assure that the mobile platform constraint is met. The computations performed in preparation for launch (Figure 2.2 - Block 2.2.3) are done missile by missile and provide the detailed data required to define the mission. The final phase-launch missile (Figure 2.2 - Block 2.2.4) - concludes the process for missile being launched.

### **1. Data Interface**

The WCS receives a coordinated set of data from the Navigation System at a prescribed interval. Typical data quantities include position, velocity, and attitude, gravity data, velocity and position corrections, and time of day. Navigation data are required in all launch phases to support computations.

### **2. Targeting Computations**

WCS targeting computations assure mission definition and mission verification as the mobile platform moves. They are lengthy computations and are not sensitive to the actual launch conditions (which are unknown). The purpose is to maintain targeting flexibility. Typical WCS computations during this phase are twofold: (1) compute initial conditions and keep them current, and (2) assure that all possible missions can be executed from the current position of the mobile platform. The results of all computations are stored for the next phase, where they will be tested again.

### **3. Prepare for Launch ( Figure 2.1 - The fourth-level diagram)**

The computations in this phase are somewhat sensitive to launch conditions but targeting flexibility is not an issue (i.e., a mission has been selected). The WCS software uses data provided by other subsystems (e.g., Navigation and Guidance) to perform two basic types of computations: (1) those necessary to prepare the inertial guidance system of the missile, and (2) trajectory related computations which are too lengthy to be performed

during the final phase of launch. The missile trajectories, including their re-entry simulations, are part of these computations. Proposed improvements to these missile trajectory computations (Three degree-of-freedom (3DOF) trajectory simulation and target offset computations) must meet computational time requirements.

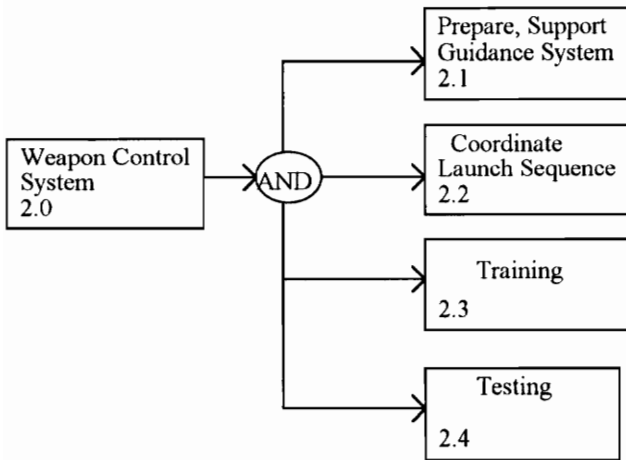
A major presetting task is the computation of re-entry effects and the modification of missile steering data. The steering equations evaluated in the guidance computer are subject to simplifying assumptions which would cause large misses if they are not accounted for in WCS computations. The WCS solves this problem by computing the re-entry effects caused by these assumptions and modifying the missile steering data accordingly. These computations rely heavily on a trajectory simulation in the WCS (Figure 2.2 - The fifth-level diagram, block 2.2.3.4 (1,2,)).

These computations use the WCS trajectory model and stored environmental data to compute the effects of unmodelled gravity and weather on the trajectory. In a similar manner, the effects on the missile trajectory of other reentry body dynamics are computed. The output of this process is a set of missile steering data which will be transmitted to guidance. These computations operate in a continuous loop until the missile is selected for launch. Readiness time requirements are applied in this phase.

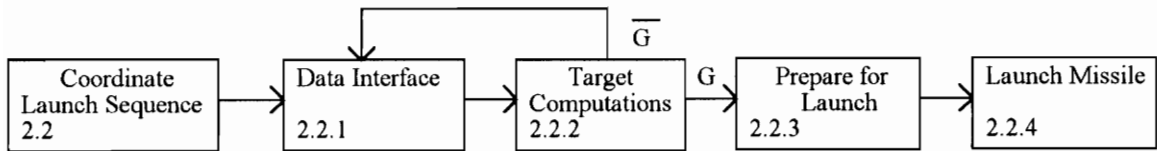
#### **4. Launch Missile**

The computations in this phase serve as the final preparation of the missile to be launched. The computations are either especially sensitive to the launch conditions, require little execution time, or are required in order support coordinated interfaces (e.g., data transmission) such as navigation, guidance and missile interfaces. The firing rate requirements are applied in this phase.

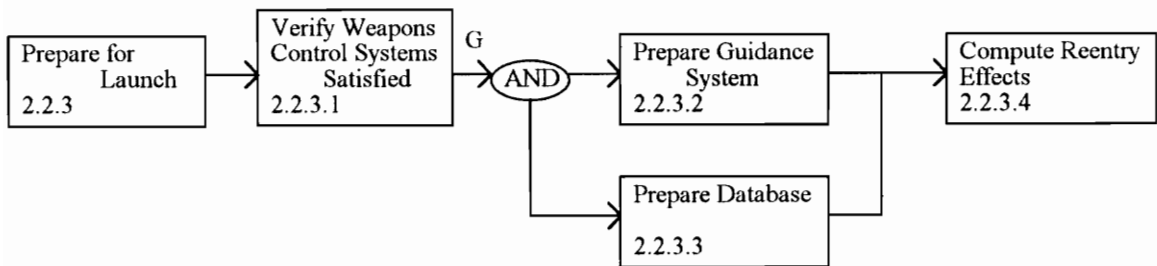
**The Second Level**



**The Third Level**

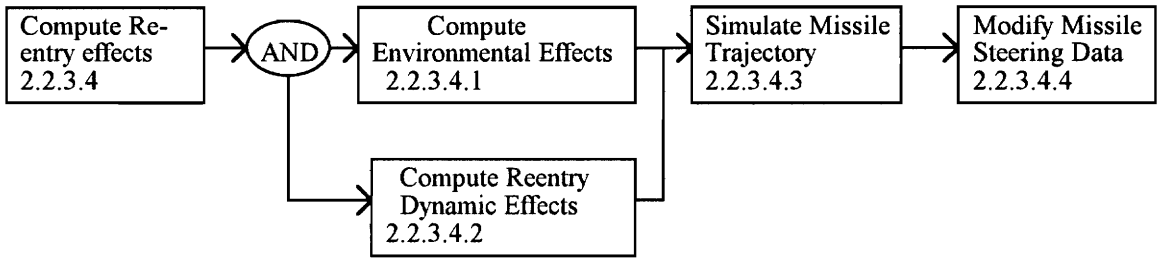


**The Fourth Level**



**Figure 2.1** WCS functional flow

**The Fifth Level**



**Figure 2.2** WCS functional flow (Cont.)



## 2.2 OPERATIONAL REQUIREMENTS FOR THE REENTRY SIMULATION ENHANCEMENT

Besides the general requirements for the WCS, the simplified 6DOF re-entry algorithm needs to meet some specific requirements. These requirements are defined in terms of degradation from a reference algorithm and based on the operational requirements of weapons control computations (readiness time, impact position accuracy and time of flight).

### 1. Deviation tolerance from the reference 6DOF re-entry model :

1.1 For position	20.0 feet
1.2 For velocity	5.0 feet/second
1.3 For time of flight	0.004 second

### 2. Computation time for one re-entry trajectory 1.0 minute

### 3. Deployment

The 6DOF ballistic re-entry simulation model is part of the Weapons Control System supporting strategic weapon platforms such as silos, conventionally armed submarine launched ballistic missiles (CSLBM), and mobile ballistic missile launchers.

### 4. Mission scenario

Data computed using the re-entry trajectories are provided to the Guidance System before the missile is launched with the assumption that the designated targets are achievable.

### 5. Reliability

Readiness time and launch are constraints to the WCS and WCS reliability is represented as: (1) 0.9999 for operational readiness reliability, (2) 0.9999 for launch reliability, and (3) 0.9998 for total WCS reliability. These reliabilities are met by WCS software using the current re-entry trajectory approximations. Replacement of these computations should maintain the same levels of reliability. The reliability for the enhanced re-entry computations is then defined as 100.0%.

For the enhanced re-entry simulation, failure occurs when the deviation from the reference model exceeds defined requirements. When the operational 6DOF is integrated in the WCS software it is also a failure if the simulated trajectory has a target miss greater than the maximum miss generated from WCS software using an approximation of the 6DOF re-entry computations. Intensive validation and verification will be performed to assure a zero failure rate which is defined as follows:

The failure rate = number of failures / number of trajectories generated  
Redundancy will be provided if needed to assure the reliability requirement. The approximate computation simulation will be used as a backup.

#### 6. Maintenance (The third-level maintenance)

Weapons Control software will record any failure. The 6DOF re-entry computation part of the Weapons Control software is disabled and approximation of the 6DOF re-entry computation will take place.

An analyst in the supporting center will rebuild the situation and analyze the problem. The cause and suggested solution will be documented for future use and WCS software improvement.

## 7. Retirement

Weapon system effectiveness can be improved by enhancement of its embedded software. Financial and political problems make extension of the current weapon systems beyond their designed lives more desirable. Thus, retirement of the 6DOF re-entry simulation model involves future upgrades to take advantage of new technologies, such as a maneuvering re-entry vehicle. The implementation of new advanced technologies adds more capabilities to the current weapon systems to meet the new threats.

## **2.3 HUMAN FACTORS REQUIREMENTS**

Optimizing the 6DOF re-entry trajectory simulation is a complex task which requires an interdisciplinary knowledge. The definition of human factors requirements often dictates a further breakdown of the optimization process to job operation, duty, and tasks. The work schedule and cost breakdown are also provided.

### **1. Job Operation**

Optimizing the 6DOF re-entry trajectory simulation reference model requires a journeyman level scientist or engineer. He or she should have a strong background in physics, mathematics and computer science. The final validation and verification of the operational model requires another journeyman level scientist or engineer.

### **2. Duty**

Personnel involved in the optimization phase will perform the duty during regular government operation time and 40 hours per week. He or she will optimize the 6DOF re-entry trajectory simulation reference model so it can be implemented into the current WCS software. He or she also does preliminary testing and evaluation on the operational model. All the work is done on the CDC 875 mainframe computer whose computation speed is assumed to be ten times faster than the computer on the mobile launch platform.

### **3. Tasks**

The process of optimization is broken down to the following tasks:

- 3.1 Define operational requirements for the operational model
- 3.2 Determine functional flow of the model.
- 3.3 Determine functional components and their allocations.
- 3.4 Determine functional components which need improvement.
- 3.5 Search for more efficient computational algorithms.

3.6 Apply the desired algorithms.

3.7 Integrate and do preliminary test on the operational model.

#### 4. Cost

##### . Work schedule

. Tasks from 3.1 to 3.4 : 3 man-months

. Tasks from 3.5 to 3.6 : 2.5 man-months

. Task 3.7 : 2.5 man-months

##### . Cost

. Personnel cost =  $\$20.0/\text{hr} * 40 \text{ hrs/week} * 32 \text{ weeks} = \$25600.$

. Computer cost =  $\$20.0/\text{hr} * 20\text{hrs/week} * 32 \text{ weeks} = \$12800.$

Total = \$38400.

## **2.4 THE 6DOF RE-ENTRY SIMULATION REFERENCE MODEL**

### **DESCRIPTION**

#### **2.4.1 Logical computations**

The simulation of a 6DOF re-entry vehicle is very dependent upon the interpolation of tabular data, particularly in the calculation of aerodynamic coefficients. The translational and angular accelerations are calculated from the aerodynamic information. Then, the accelerations are integrated numerically over a small time step to yield updated values of position, velocity, and other kinematic variables. The process is repeated until conditions for stopping the calculations are satisfied. The 6DOF re-entry simulation implemented into the Weapons Control System is operational; therefore non-operational user's options, such as round earth and its associated gravitational computation, can be eliminated. The appendices provide description of coordinate frames, linear search and the integration methods.

The 6DOF re-entry simulation is a data-driven model and, thus, the amount of data to be searched, the searching method and the interpolation methods are where the optimization can best be applied. Since the data tables are sorted in ascending order, a linear search with pointer is used to locate the specific data for computation. This linear search is more efficient than one which always starts from the beginning of data tables. If the data can not be found, interpolation is used to compute the needed data. While an efficient searching technique reduces the simulation time, the reduction of data table sizes can reduce the computer memory needed for simulation, which is also a WCS constraint. The final stage of the model is trajectory simulation which is done using a fourth-order Runge-Kutta integration technique. Formulas of the Runge-Kutta type are among the most widely used formulas for the numerical solution of ordinary differential equations

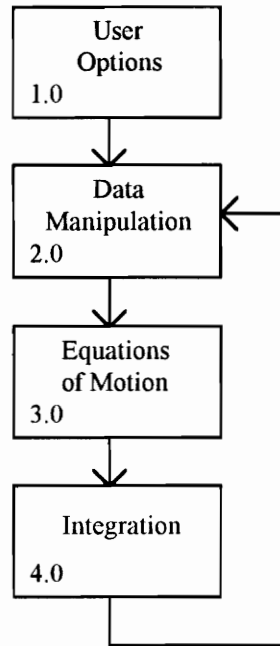
(see Appendix B for additional discussion.) Their primary disadvantages are (1) they require significantly more computer time than other methods of comparable accuracy, and (2) local error estimates are somewhat difficult to obtain. If a substantial computer investment is involved (as in production code), then it is wise to consider one of the more efficient methods such as a predictor-corrector method (Appendix B.)

### 2.4.2 Functional components

The current 6DOF re-entry simulation is not an operational model but an analysis tool. The optimization process requires a thorough understanding of the model. It is broken down into components and associated computational allocations. Each component is then examined to see whether its subcomponents can be simplified or improved by application of more efficient computational algorithms. As shown in Figure 2.3, the current 6DOF re-entry simulation is a mathematical model consisting of equations of motion of the re-entry body, integration scheme and data processing based on the user's control options. Since it serves as a tool for re-entry trajectory analysis, it provides the user a spectrum of choices of coordinate frames, gravitational models, environment models, Earth models (such as round or oblate), and output variables besides the other input values needed for simulation. Figures 2.4, 2.5, 2.6 present the aforementioned models with their options. The user can have computations done and output presented in the desired coordinate frame (Appendix A). He or she can simulate the re-entry trajectory with or without wind and atmospheric density. Wind values can be a constant input (ballistic wind) or variable input with altitude (tabular wind). Density can be a constant input or a function of altitude. If a round Earth model is used, then the associated gravity model is used; otherwise, the gravity values will be extracted from the database. Users can select a Runge-Kutta with fixed step size or variable step size to simulate the re-entry trajectory.

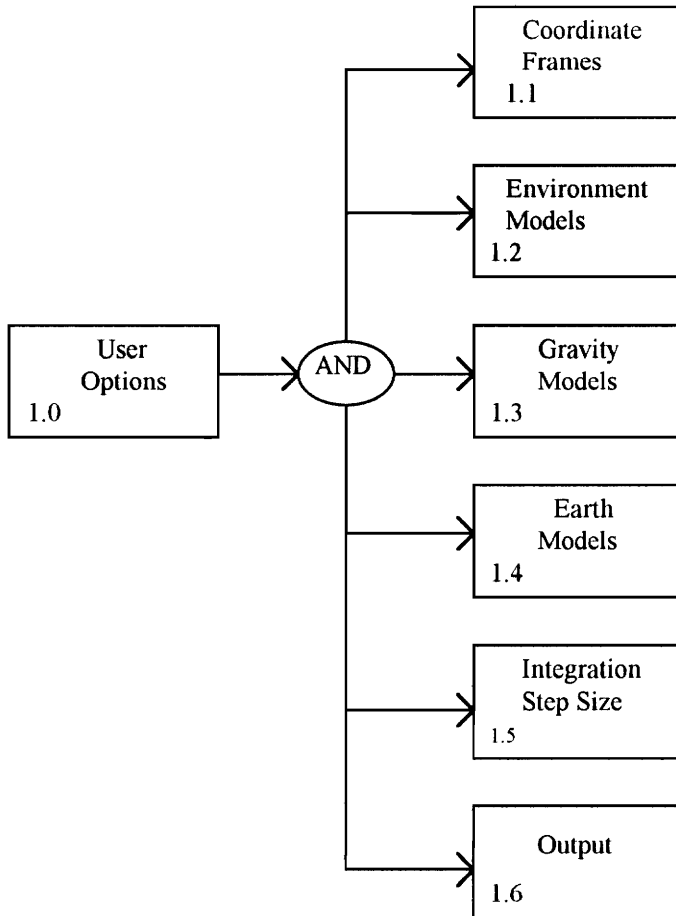


**The First Level**



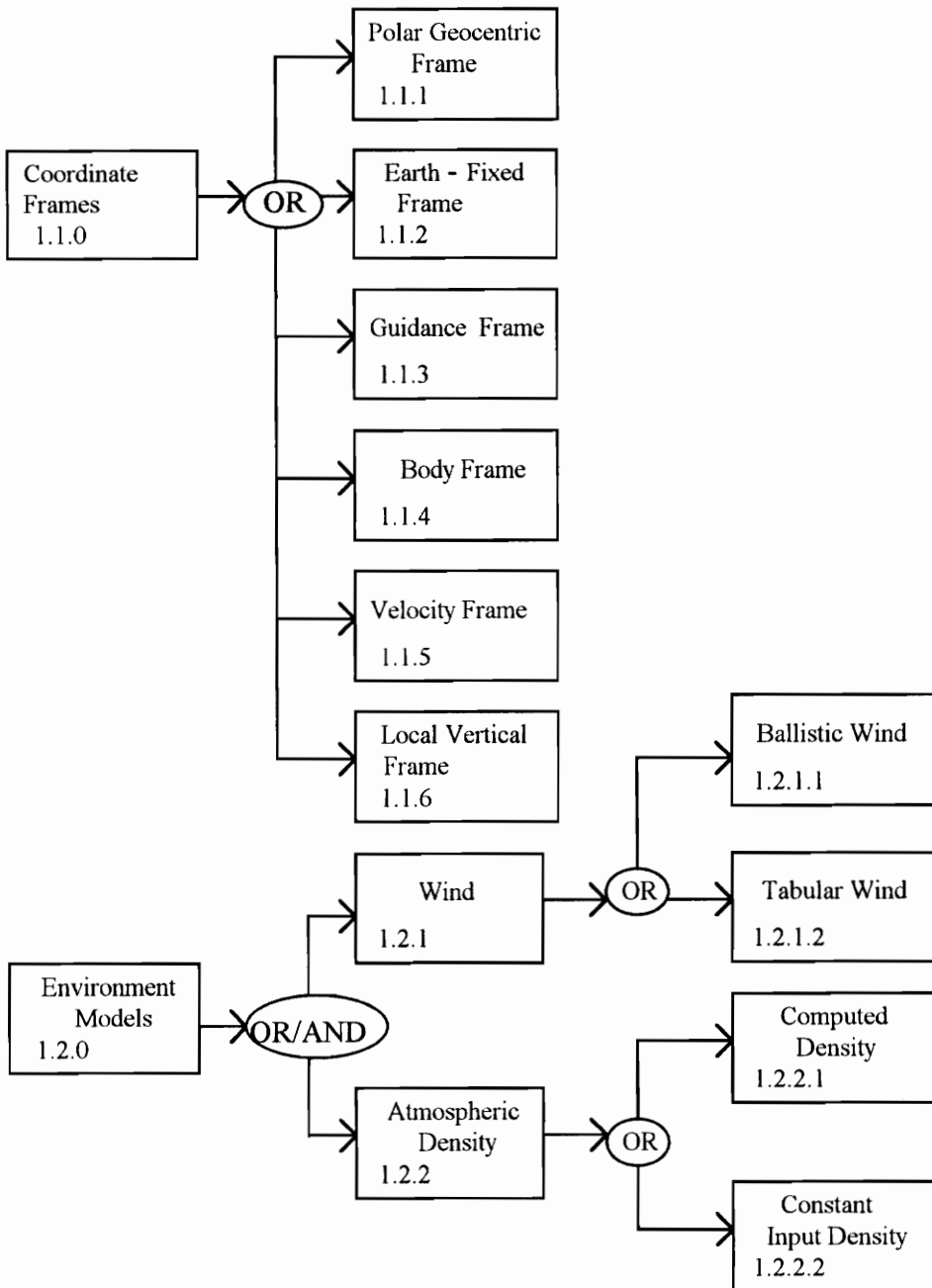
**Figure 2.3** Functional components

**The Second Level**



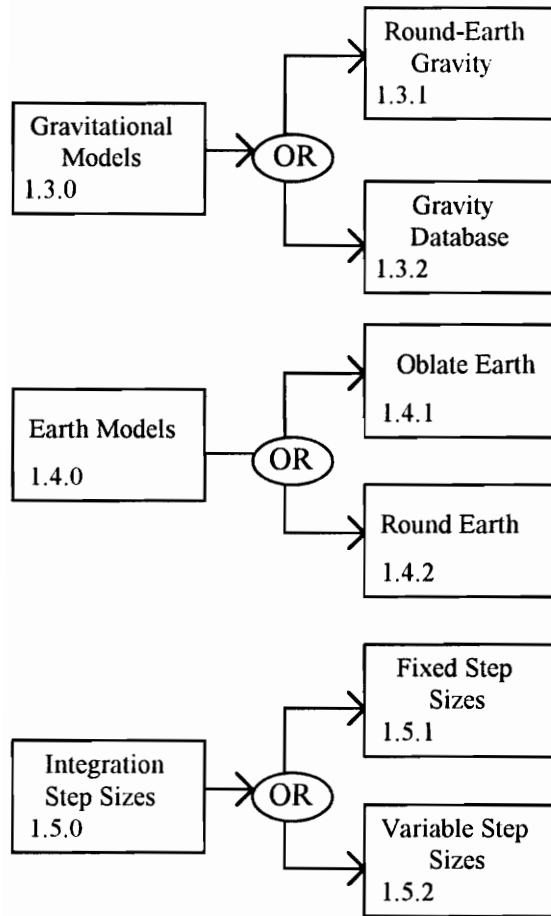
**Figure 2.4** Functional subcomponents

### The Third and Fourth Level



**Figure 2.5** Functional subcomponents

### The Third and Fourth Level



**Figure 2.6** Functional subcomponents

### 2.4.3 Computational time allocation

A Hot Spot procedure on the CDC 875 computer system is used to identify the total computational time of each program subroutine. The functional components in Figure 2.3 are shown below with their time allocation drawn from the above analysis. The improvement can occur in the major functions through simplification or by the use of more efficient computational algorithms. Note that allocation results will indicate which functions are the best candidates for improvement and the order in which they will be examined.

1. Data Manipulation : 40%
  - a. Data tables
  - b. Data search
  - c. Interpolation and extrapolation techniques
2. Integration technique : 30%
3. User's options : 20%
  - a. Coordinate frames
  - b. Gravity model
  - c. Environment model
  - d. Earth model
  - e. Output
4. Equations of motion : 10%

## 2.5 OPTIMIZATION DEVELOPMENT

Each functional component of the reference 6DOF re-entry simulation is examined to see whether the computation can be improved through simplification of algorithms or by more efficient algorithms. Functional analysis indicates that the improvement should be attempted in the data manipulation, trajectory integration and user options. The improvements made to each function are outlined below. System integration is then performed for testing.

### 1. Data manipulation

- 1.1 Reduce size of data tables in order to reduce the computer memory. Each data table is examined to see whether its size can be reduced while maintaining the accuracy of interpolation and extrapolation. Elimination of the duplicated values can satisfy both.
- 1.2 Use linear search with pointer because of sorted data tables (in ascending order.) The search direction is dependent on the characteristics of data tables. For instance, for the mass loss/altitude data table, the search direction is upward once the pointer is computed since the re-entry vehicle will experience mass loss while it descends (Appendix C).
- 1.3 Simplify mathematical equations to reduce the number of operations. For instance, in logarithmic-logarithmic interpolation,

$$Z = \log Z(j-1) + \left\{ \frac{(\log Z(j) - \log Z(j-1))}{(\log X(j) - \log X(j-1))} \right\} * (\log X - \log X(j-1))$$

can be simplified as

$$Z = \log Z(j-1) + \left( \frac{\log(X / X(j-1))}{(\log(X(j) / X(j-1)))} \right) * \log(Z(j) / Z(j-1))$$

1.4 Use more efficient mathematical built-in functions such as

$\text{sqrt}(A)$	versus	$A^{**} (0.5)$
$A*A$	versus	$A^{**2}$

## 2. Integration technique

The real-time fourth-order Adams-Moulton predictor-corrector integration technique (RTAM4) is used for altitudes above 100000 feet and the real time three-pass predictor-corrector integration technique (RT3P) otherwise. The fourth-order Runge-Kutta is used to generate values for the predictor-corrector integration technique. The RTAM4 and RT3P have been proven to be stable in experiments at the University of Michigan (8). The integration step sizes are fixed for specific altitudes. These integration step sizes are designed such that the deviation tolerance from the reference 6DOF simulation is acceptable.

## 3. Equations of Motion

Unnecessary computations are eliminated. Appendix D gives one example in which the time derivative of the transformation matrix from the Polar Geocentric frame to the missile frame (TGC2M) is eliminated.

## 4. User's options

4.1 Only Polar Geocentric, velocity and body coordinate frames are used.

These frames are necessary for the 6DOF re-entry computations that are needed in the WC computations. Details of each coordinate frame are described in Appendix A.

4.2 The oblate earth is used with only one option of wind and density. The ballistic horizontal wind is computed from its components as input by user.

The current atmosphere density is also computed from standard atmospheric density model, where density is a tabular function of altitude.

The gravitational model is an oblate gravity field.

4.3 Output contains only the necessary information.



## 2.6 TESTING AND EVALUATION

The simplified 6DOF re-entry simulation is used to run 27 test cases on the CDC 875 mainframe computer. These test cases cover all of the trajectories in which aerodynamics are expected to have great effect on the re-entry simulation.

The test results given in Table 2.1 show that the accuracy degradation is within the designed specifications. The degradation is caused by application of the predictor-corrector integration method and the larger step sizes.

### 1. Deviation tolerance :

- 1.1 Position            0.2 to 6.45 feet
- 1.2 Velocity            0.3 to 2.1 feet/second
- 1.3 Time                0.00004 to 0.0032 second

### 2. Execution time and saving :

- 1.1 Time                5.0 to 10. seconds  
(50 to 100 seconds for computers on the typical mobile launch platform)
- 1.2 Time saving        89.5 to 95.%

The failure rate from the testing process is zero. It indicates that the model meets the deviation requirements from the reference 6DOF model. However, the results indicate that the simplified 6DOF re-entry simulation still requires more computing power than offered by the assumed WCS if the system readiness time requirement is to be met.

Table 2.1 Test Results

CASE #	POSITION ( FEET )	VELOCITY ( FT/SEC )	TIME ( SEC )
1	4.471	0.703	-0.0032
2	0.168	1.3	0.0003
3	1.322	1.795	-0.0015
4	0.33	0.266	0.0001
5	0.362	0.722	0.0005
6	1.36	1.09	0.0004
7	2.884	0.675	0.0004
8	0.655	2.048	0.0001
9	3.21	0.952	0.0003
10	4.02	0.973	-0.0013
11	1.104	1.227	0.0009
12	6.45	0.42	0.0021
13	4.005	1.27	0.0004
14	0.99	1.507	0.0003
15	1.37	1.996	0.00004
16	2.337	0.332	0.0004
17	1.168	1.958	-0.0002
18	2.151	1.035	0.0017
19	3.46	0.68	0.0008
20	4.21	1.28	0.0006
21	0.922	0.935	0.0009
22	0.83	1.55	0.0005
23	0.871	1.33	0.0004
24	0.507	1.23	0.0004
25	0.359	1.002	-0.0002
26	2.66	0.783	0.0004
27	0.4	0.86	0.0005

## 2.7 CONCLUSION

Conventional strategic weapon systems take advantage of kinetic energy provided by high velocity re-entry vehicles, but a better system accuracy is required. The typical re-entry segment of a missile flight is ballistic without active control and correction. Thus, its accuracy is directly related to the accuracy of associated WCS. In current systems it is approximated in order to meet a readiness time constraint and because of computing power limitations of a typical strategic weapon control systems. While this approximate computation approach meets current weapon system requirements, it limits potential hardware or software enhancements. Thus, the re-entry simulation is a good candidate for improvement. The system engineering process is applied to a study of the feasibility of implementing 6DOF re-entry computations in lieu of approximate computations in the WCS software of a mobile weapon system. The study was finished within the prescribed time frame and cost and shows that computational accuracy requirements are met but the computing time requirement is not. This is due to the limitations of the current WCS computing hardware. If a WCS upgrade addresses expanding the computational power of the WCS, then implementation of the simplified 6DOF re-entry simulation may be feasible. It may be possible to affect the choice of architecture and processes to ensure that the implementation is feasible.

## 2.8 RECOMMENDATION

The optimized 6DOF re-entry model meets the accuracy requirements but it still requires more computing power than offered by the assumed WCS hardware technology. However it can be an important consideration in selection of the hardware for a future WCS upgrade. New computing technologies can make the implementation of the optimized 6DOF re-entry simulation feasible.

The introduction of low-cost microprocessors is pushing distributed processing, multiprocessor configuration, and parallel processing. Almost all microprocessors have the capability and standard interfaces to be interconnected into distributed processing systems. These technologies, coupled with the new commercially available powerful computing hardware and improvement in the data storage and retrieval technologies, can speed up the simplified 6DOF re-entry and make its application to the WCS feasible. The Weapons Control problem is to simulate the trajectory of each re-entry body of a missile and for all missiles on the mobile platform and, hence, there are two explicit approaches to distribution of functionality across a series of processors: (1) by functionality of algorithm (i.e., the independent tasks or parts of the algorithm are performed simultaneously by different processors and the simulation of a trajectory requires an integration of computations from all processor), or (2) simulation of a complete trajectory for a re-entry vehicle by one processor. Thus, complete trajectories for a full missile are done in parallel.

The second design is similar to the "**star**" configuration in the network architecture. In the "**star**" design all the computers are tied to a central computer, but the major processing is done at the satellite processors rather than the host. All computers, including the central one, are on an equal level with all others on the system. The host processor holds the database and the simulation models from which the other processors

will access the information. This approach (1) does not require new coding for the simplified 6DOF simulation, (2) can greatly reduce or avoid the microprocessor interaction and data integration overhead, and (3) allows the future WCS upgrade to be done locally at the individual microprocessor.

## **APPENDICES**

**Appendix A : Coordinate Frames**

**Appendix B : Numerical Integration Methods**

**Appendix C : Searching Techniques**

**Appendix D : Deletion of Time Derivative of Transformation Matrix from GC Frame to  
Missile Frame**

## APPENDIX A

### COORDINATE FRAMES (11)

The first requirement for describing a trajectory or an orbit is a suitable reference frame. The reference 6DOF re-entry model uses six different frames with the guidance frame is established from the star sightings that are convenient to position of the re-entry vehicle (RV) at the end of boost. The specific force is measured in the frame affixed to the RV while the computations are to be carried out in the inertial frame such as the guidance frame or the Earth-centered frame. The positional coordinates of the RV relative to the inertial frame are then converted into the Earth coordinates such as longitude, latitude, and altitude along the local normal to the geoid. The velocity frame along with body and Earth-centered frames are used in the study of RV motion. Note that all coordinate frames are right hand orthogonal frames.

#### 1. Polar geocentric coordinate frame (GC)

This is an Earth-centered coordinate system where the z-axis lies along the Earth's polar axis and the xy-plane contains the equator. This frame is defined precisely by fixing the direction of the x-axis.

#### 2. Earth-fixed coordinate frame (EF)

An Earth-fixed coordinate frame is one which is fixed to the Earth and thus rotates with the Earth.

Often during the trajectory computation, it is necessary to locate a position with respect to the Earth rather than some inertial frame. For example, the impact location of a ballistic missile simulation is normally located with respect to the Earth rather than inertial space. For these computations, an Earth-fixed coordinate frame is necessary. The usual practice is to perform trajectory computations in an inertial frame, and rotate to the Earth-

fixed frame when needed. It is possible to compute trajectories in an Earth-fixed frame, however, this introduces coriolis and centrifugal accelerations. For this reason, inertial frames such as the GC frame are preferred.

### **3. Guidance coordinate frame (G)**

It is an inertial frame defined with respect to the GC frame by a sequence of rotations involving the geodetic latitude and longitude, the vertical deflection and the local bearing and elevation of the star used to orient the missile inertial guidance system. The equations of motion are integrated in this frame.

### **4. Local vertical reference frame (LV)**

It is a non-inertial frame that is defined in relation to the surface of the Earth. The positive z-axis lies in the direction of the local geodetic vertical unit vector. The x- and the y-axes are positive in the direction of East, and North, respectively.

### **5. Velocity frame (V)**

The velocity frame is defined with respect to the missile frame and relative velocity vector. The positive x-axis is coincident with the x-axis of missile frame. The z-axis lies along the projection of relative velocity of missile into y-z plane of missile frame and is positive in that direction. The y-axis completes the right hand orthogonal frame. This frame is a non-inertial frame.

### **6. Missile frame (body frame - M)**

A frame affixed to the missile or re-entry body. It is in the body frame that the missile controls are fixed; also, for simulation purposes the aerodynamics loads are fundamentally given in a body frame.



## APPENDIX B

### NUMERICAL INTEGRATION METHODS

#### I. THE FOURTH-ORDER RUNGE-KUTTA METHOD

Formulas of the Runge-Kutta type are among the most widely used formulas for the numerical solution of ordinary differential equations. Their advantages include:

- (1) They are easy to program.
- (2) They have good stability characteristics.
- (3) The step size can be changed as desired without any complications.
- (4) They are self-starting. This is perhaps the most important advantage of these methods.

Their primary disadvantages are:

- (1) They require significantly more computer time than other methods of comparable accuracy.
- (2) Local error estimates are somewhat difficult to obtain.

Among the family of Runge-Kutta is the fourth-order Runge-Kutta (RK4) which is most widely used due to its ability to minimize truncation error. The RK4 requires four evaluations of derivative calculations per iteration and its formula can be found in [2] and [7] and [10].

#### Formulas

$$Y[n+1] = Y[n] + (h/8) (k_1 + 3k_2 + 3k_3 + k_4)$$

where

$$k_1 = F(t[n], y[n])$$

$$k_2 = F(t[n] + (1/3)h, y[n] + (1/3)hk_1)$$

$$k_3 = F(t[n] + (2/3)h, y[n] - (1/3)hk_1 + hk_2)$$

$$k_4 = F(t[n] + h, y[n] + h k_1 - h k_2 + h k_3)$$

$t[n]$  = current time

$h$  = integration step size

$F$  = external function computing the derivative

$y[n]$  = current value at  $t[n]$

$y[n+1]$  = value at next integration step at  $t[n+1]$

## **II. THE REAL-TIME PREDICTOR-CORRECTOR INTEGRATION METHODS**

References (7), (9)

### **1. General description of conventional predictor-corrector methods**

The efficiency of the predictor-corrector (PC) method is one of the primary reasons for their current popularity. For many problems, the most time consuming (and hence expensive) part of the numerical solution is the evaluation of the derivative  $F(y,t)$ . A typical predictor-corrector method (regardless of order) requires one evaluation of  $F$  (at  $t[n]$ ) for the predictor (all the other values of  $F$  will have been previously computed and stored) and one evaluation of  $F$  (at  $t[n+1]$ ) for each iteration of the corrector. The major concern is the stability of the PC methods. The stability of the predictor is not of concern since it is only used to provide a first estimate. The corrector, on the other hand, must have excellent stability characteristics. This requirement is satisfied by the correctors used in the fourth-order Adams method and Hamming's method. Most modern PC methods are self-starting while traditional PC methods are not self-starting which is also the disadvantage of PC methods (7).

### **2. The fourth-order Adams-Bashforth-Moulton predictor-corrector**

In the new PC method the first pass through the state equations uses an Adams-Bashforth type of predictor algorithm to compute an estimate of the state at  $(n+1/2)$  frame instead of the  $(n+1)$  frame, as is customary. This estimate is then used to compute the derivative at the  $(n+1/2)$  frame which, along with derivative at the  $n, n-1, n-2, \dots$  frames is used in the final corrector pass to calculate the state at the  $(n+1)$  frame. This new real-time fourth-order Adams-Bashforth-Moulton (RTAM4) has been proven more stable than the

conventional fourth-order Adams-Moulton (AM4) based on characteristic-root errors, sinusoidal transfer function gain and phase errors and time-domain errors (9).

**Formulas (1)**

$$YP[n+1/2] = YC[n] + (h/384)(297F[n] - 187F[n-1] + 107F[n-2] - 25F[n-3])$$

$$YC[n+1] = YC[n] + (h/30)(36FP[n+1/2] - 10F[n] + 5F[n-1] - F[n-2])$$

where

h = integration step size

YP[n+1/2] = predictor value at the next integration step t[n+1/2]

YC[n+1/2] = corrector value at t[n+1/2]

F[n] = state-variable derivative at t[n]

FP[n+1/2] = state-variable derivative estimate at t[n+1/2]

Integrator error coefficient e = 59h<sup>4</sup>/2880

**3. The real-time three-pass predictor-corrector method (RT3P)**

The RT3P requires three evaluations of the state-variable derivative F for each integration step. It yields significantly more accurate results than third-order RK (Runge-Kutta) integration.

**Formulas (1)**

$$YP[n+1/3] = YC[n] + (h/324)(137F[n] - 40F[n-1] + 11F[n-2])$$

$$YP[n+2/3] = YC[n] + (h/54)(39FP[n+1/3] - 4F[n] + F[n-1])$$

$$YC[n+1] = YC[n] + (h/4)(F[n] + 3FP[n+2/3])$$

Where

h, YP, YC, FP, F as above definitions.

Integrator error coefficient e = h<sup>3</sup>/216

## APPENDIX C

### SEARCHING TECHNIQUES (8)

Any searching technique has two input parameters, the target (or key) and the ordered or unordered list of  $n$  records being searched. The sequential search or linear search begins at one end of the list and scans down it until the desired target is found and the other end is reached. The average number of comparisons in the successful case is  $(n+1)/2$ . For the large  $n$  this many comparisons is very inefficient. To find any target in a long ordered list, there are more efficient methods such as binary search. In this method, the target is compared with the record in the center of the list and then restrict successive searches to only the first or second half of the list, depending on whether the target comes before or after the central record. In this way, the length of the list is reduced by half. Hence, in the worst case, this method requires  $\log(n)$  target comparisons to search a list.

In the linear search with a pointer, the location where the target is found or assigned from the interpolation process is used as a pointer to search the new target. In the interpolation process, if the target is found the desired value will be the value of target position (or pointer.) Otherwise, the interpolation will be performed on the two values around the target value. The position of the value less than the target value is the pointer to search the new target. The first search is the binary search if the list is long. The linear search with the pointer will be used in successive searches.

## APPENDIX D

### DELETION OF TIME DERIVATIVE OF TRANSFORMATION MATRIX FROM GC FRAME TO MISSILE FRAME (TGC2MD)

TGC2MD is the time derivative of the transformation matrix TGC2M. It is used to compute the time derivative of the relative velocity TMDVFR. The computation of TGC2MCD can be eliminated by using a different equation derived from the equation below. The new equation consists only the variables which have been computed earlier. It reduces execution time of the current 6DOF re-entry simulation since it has a fast execution time and the computation of TGC2MD requiring the evaluation of nine equations is eliminated.

$$\text{TMDVFR} = (\text{TGC2MD})(\text{VR}) + (\text{TGC2M})(\text{TDVLCM})$$

where VR = Velocity of the center of mass of the re-entry body relative to the moving atmosphere represented in the Polar Geocentric (GC) frame.

TDVLCM = Time derivative of VR

Note :

$$\text{TM2GCD} = (\text{TM2GC})(\text{A}[b,ib])$$

$$\text{TGC2MD} = (\text{A}[b,ib])(\text{TGC2M})$$

Thus  $\text{TMDVFR} = (\text{A}[b,ib])(\text{TGC2M})(\text{VR}) + (\text{TGC2M})(\text{TDVLCM})$

But  $(\text{TGC2M})(\text{VR}) = \text{VELMFR}$

Therefore  $\text{TMDVFR} = (\text{A}[b,ib])(\text{VELMFR}) + (\text{TGC2M})(\text{TDVLCM})$

Where  $A[b,ib]$  = skew-symmetric matrix of angular velocity of the body in the missile frame.

$VELMFR$  = Velocity of the center of mass of the body relative to the moving atmosphere represented in the missile frame.

$VELMFR$  is computed earlier in the simulation.

Note that :

1. The new formulation eliminates the  $TGC2MD$
2. The execution of  $(A[b,ib])(VELMFR)$  is faster than the execution of  $(TG2MD)(VR)$  since  $A[b,ib]$  is a skew-symmetric matrix.

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