Heat Transfer During Pulsed Laser Cutting of Thin Sheets

by

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(ABSTRACT)

A numerical model of the temperature field during pulsed laser cutting of thin sheets (approximately $2.5 \times 10^{-5}$ m) was developed. Cutting was simulated through removal of nodes from a finite difference scheme based on sensible heating to the phase change temperature and a single value of latent heat (melting or vaporization). The pulsed laser model predicts a heat-affected zone of less than 0.02 mm for pulsed laser cutting. For comparable cutting with a continuous power laser, a heat-affected zone between 0.05 and 0.10 mm is predicted. Thermal stress levels were predicted to be an order of magnitude lower for pulsed laser cutting than for continuous power cutting. The stress levels predicted by the model also increased with cut speed. Experimentally, pulsed laser cutting yielded better cut quality, based on less cracking, than continuous power cutting. In addition, the cut quality deteriorated as the cutting speed was increased for the continuous power laser. Presently, application of pulsed laser cutting is limited by its low cutting speed, which is restricted by the energy density of the laser. The model predicts that increasing energy density will decrease the size of the heat-affected zone and increase the maximum cutting speed. Therefore, pulsed laser cutting at high speeds should be attainable without deterioration in cut quality.
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Nomenclature

\( A \) \quad \text{kerf cross-section (m}^2\text{)}

\( B_i \) \quad \text{nondimensional film coefficient (Biot number)}

\( E_{ijk} \) \quad \text{energy incident from laser on node ijk (kJ)}

\( C_p \) \quad \text{specific heat (} \frac{\text{kJ}}{\text{kgK}}\text{)}

\( d \) \quad \text{material thickness (mm)}

\( h \) \quad \text{film coefficient (} \frac{\text{W}}{\text{m}^2\text{K}}\text{)}

\( H \) \quad \text{latent heat of melting/vaporization (kJ/kg)}

\( \text{H.A.Z.} \) \quad \text{heat-affected zone (mm)}

\( k \) \quad \text{thermal conductivity (} \frac{\text{W}}{\text{mK}}\text{)}

\( \text{kerf} \) \quad \text{void separating material that is cut}

\( \text{off-time} \) \quad \text{part of pulse period that laser is off}
on-time part of pulse period that laser is on.

P absorbed power entering spot area in model (W).

P_s absorbed power entering sheet in model (W).

P_k power to form kerf (W).

P_t total power leaving laser (W).

Constant power for continuous laser.

Peak power for pulsed laser.

PULSECUT name of pulsed laser cutting model.

q power in Rosenthal's line source solution (W).

R_{st} laser spot radius (mm).

r dimensionless distance from origin in Rosenthal's model.

T temperature (K).

t time (s).

u speed of sheet (cutting speed, always in x direction) (m/s).

u' dimensionless velocity in Rosenthal's model, \( \frac{ud}{2x} \).

X_{st} discretization error in sheet motion (m).

Greek Symbols

\( \alpha \) thermal diffusivity (\( \frac{m^2}{s} \)).

Nomenclature
\( \Delta T_m \) temperature rise \((T_m - T_{\infty})\) (K).

\( \Delta t \) time increment (s).

\( \delta \tau \) nondimensional time increment.

\( \Delta x \) grid spacing, \(x\) (m).

\( \delta x \) nondimensional grid spacing, \(x\).

\( \Delta y \) grid spacing, \(y\) (m).

\( \delta y \) nondimensional grid spacing, \(y\).

\( \Delta z \) grid spacing, \(z\) (m).

\( \delta z \) nondimensional grid spacing, \(z\).

\( \varepsilon_x \) absorption efficiency \( \frac{P_x}{P_t} \).

\( \rho \) density (kg/m\(^3\)).

\( \tau \) nondimensional time \( \left( \frac{10}{d} \right) \).

\( \theta \) nondimensional temperature \( \left( \frac{T - T_{\infty}}{T_m - T_{\infty}} \right) \).

\( \theta' \) \( \frac{q}{2\pi dk} \)

**Superscripts**

\( n \) Pertaining to the current time step.

\( n + 1 \) Pertaining to the next (new) time step.

\( \cdot \) Pertaining to Rosenthal’s line source solution.

**Nomenclature**
s  Pertaining to the first time split.

ss  Pertaining to the second time split.

Subscripts

i  Shorthand for ijk when differencing in x.

ijk  General specification for any node.

i-1  Specifies one index in the -x direction.

∞  Ambient conditions.

i + 1  Specifies one index in the +x direction.

j  Shorthand for ijk when differencing in y.

j-1  Specifies one index in the -y direction.

j + 1  Specifies one index in the +y direction.

k  Shorthand for ijk when differencing in z.

k-1  Specifies one index in the -z direction.

k + 1  Specifies one index in the +z direction.

las  Pertaining to the laser spot.

m  Melt or vaporization conditions.
Introduction

Cutting with lasers offers significant advantages over traditional methods: Tool wear is eliminated; therefore a more intricate, repeatable pattern is attainable particularly in hard materials. Unfortunately when working with thin sheets of some materials, the inevitable heating associated with laser cutting causes unacceptable deterioration in material quality near the cut. This deterioration results in observable defects such as cracking and color change. The color change is representative of changes in the structure due to heating above a critical point and subsequent cooling. Causes of cracking are less certain but may be the result of thermal stresses produced by large temperature gradients, plastic deformation followed by cooling, brittleness due to material change, or a combination of thermal stress, plastic deformation, and brittleness.

Experiments have shown that when cutting thin sheets of a certain material, the zone of color change (heat-affected zone), and the degree or even presence of cracking are affected by varying the cutting parameters. Some important parameters are cutting speed, laser power, and whether a pulsed or continuous wave laser is used.
When a thin sheet of material is passed beneath a rapidly pulsed, high energy focused laser beam, the energy causes the material to be heated and in many cases will melt/evaporate and subsequently remove some material. That removal of material will result in cutting or partial cutting of the sheet. The void created by the removed material is known as the kerf.

When cutting thin sheets of material with a laser, energy is both absorbed and reflected by the material. Of that energy that is absorbed, a certain amount goes directly into forming a kerf, sensible heat plus latent heat, while the rest of the absorbed energy is left in the material. The energy left in the material is conducted to cooler parts of the material and creates the heat-affected zone. It also may be lost from the material by radiation and convection. This report summarizes a model of pulsed laser cutting that takes into account the effect on the temperature field of forming a kerf. Results from this model will be compared with pulsed laser experiments and with numerical and experimental continuous power laser results to better understand the process of laser cutting.

Problems Addressed

Continuous laser cutting at useful speeds results in poor cut quality.

Pulsed laser cutting at useful speeds has not been accomplished.

To be useful the laser cutting procedure must be fast enough to compete with traditional cutting methods. Cut quality is evaluated based on cracking and material change due
to elevation above a critical temperature next to the cut edge. If superior cut quality cannot be achieved for useful speeds, the procedure will not be industrially applicable.

Objectives of Research

1. Evaluate the temperature history along the cut edge during laser cutting.
2. Predict the size of the heat-affected zone, H.A.Z., next to the cut edge.
3. Predict the thermal stresses next to the cut edge.

By attaining the above objectives, conditions causing poor cut quality may be identified and subsequently avoided in an industrial laser cutting process.
Literature Review

A simple formulation of the conduction problem for laser cutting of thin sheets is the energy equation for a line source moving with a constant velocity through an infinite sheet. A line source is a flux distributed over a length normal to the sheet, (power/length). This assumes two dimensional conduction in the sheet and was solved by Rosenthal [15] and, more recently, by Arata and Inoue [3]. Esposito et al. [10] demonstrated that a line source model could be used to predict melt lines in penetration welding with lasers. Bunting and Cornfield [4] state that the line source model is in error near the source and suggest a disk source in a thin sheet as an improvement.

The disk source is an integration of Rosenthal's line source over a disk of specified radius. Comparison by Bunting and Cornfield [4] show that cut edges from experiments are coincident with melting point isotherms from a disk source. Their report also examines performance of line and disk sources in prediction of cutting speed for various
materials. Rosenthal's method appeared to perform well for thinner materials, for instance, $2.54 \times 10^{-5}$ m thick iron/mild steel was cut at 0.217 m/s and Rosenthal's model predicted a cut speed of 0.207 m/s.

Abrakians and Modest [1] have shown that, depending upon power density from the laser, cutting may be the result of a combination of vaporization and removal of liquid by vapor pressure. The major conclusion of their work was that laser irradiation on a material is a surface phenomenon. This would suggest that proper prediction of the shape of a kerf (whether partial cutting occurs) and temperatures near the kerf formation is better predicted with a surface flux than a line or disk source. Another conclusion drawn from Abrakians and Modest [1] and Dekumbis [8] is that a Gaussian power distribution is an improvement on either a disk or line source.

Steen and Kamalu [18] describe the analysis of laser cutting in some detail. Continuous wave laser cutting is characterized as fusion controlled cutting, and pulsed cutting is characterized as vaporization controlled. They state that vaporization cutting requires approximately ten times the energy of fusion cutting and that vaporization cutting of metals requires about $10^4$W/m².

Decker et al. [7] compared laser vaporization cutting to fusion cutting and also investigated the amount of energy lost when vapor or molten material is removed to form a kerf. Laser vaporization cutting produced a narrow kerf and excellent cut quality. The evaluation of cut quality is based on the precision of cutting and the condition of material after processing. Fusion cutting was able to cut at higher travel speeds than
vaporization cutting but resulted in a worse cut quality. Analysis which ignored conduction of energy into the material demonstrated that conduction plays a significant role in cutting. The material removed to form a kerf was also shown to consume a significant amount of energy which will not participate in conduction with the remaining sheet of material. Duley [9] states that the kerf size and the amount of energy conducted into the sheet from the area of kerf formation are strongly dependent on laser power.

Andrews and Athey [2] used a one dimensional finite difference model for surface vaporization under a pulsed laser. They note that for most metallic materials, the ratio of latent heat of fusion to latent heat of vaporization is so small that for an ablative process, the heat of fusion may be neglected. Their results indicate that as the pulse width is shortened with respect to the period length (the on to off ratio is reduced), less energy is conducted away from the surface and a higher proportion goes into heat of vaporization. Thus a smaller heat-affected zone and higher quality cutting should occur as the on to off ratio of the pulse period is decreased. Most literature indicates that if power intensity is increased by a large amount, as occurs in pulsed laser processing, the kerf size should be larger and the size of the heat-affected zone should be smaller than in continuous power cutting. If the pulsed laser processing is of high enough intensity to induce some vaporization cutting, the cut quality should be better than in continuous wave cutting.
Model Development

Capabilities

A numerical model of pulsed laser cutting is useful to improve the cutting process. By using the model to simulate actual cutting conditions for given travel speeds and laser powers and comparing these numerical results with experimental observations, factors affecting cut quality can be identified.

The present model for pulsed laser cutting simulates the addition of energy to a thin sheet of material in short periodic bursts. Spatial distribution, frequency, and power of these energy bursts are specified when using the program. Cutting is simulated by the removal of material from beneath the energy source, the laser. The cut is formed when the sheet is moved with respect to the energy source thereby forming a hole elongated
in the direction of material travel. If the full thickness of material is not removed during numerical solution, this is partial cutting, and groove is formed in the material. Results from this model predict temperatures near the edge of the cut and serve as preprocessing for finite element thermal stress analysis using SDRC, Superb.

The following capabilities made this model unique and suitable for predicting cut quality in pulsed laser cutting of thin metallic sheets. The program:

1. Allows short pulse, high energy input with either a Gaussian or square distribution.

2. Simulates cutting by removing material (forming a kerf).

3. Allows cutting by multiple laser passes (allows partial cutting).

4. Predicts the temperatures near the kerf and produces suitable input for a finite element thermal stress code.

Methodology

It is assumed that the temperature distribution in a thin sheet undergoing pulsed laser cutting can be described with proper application of the temperature form of the energy equation for heat conduction in the solid utilizing constant properties and phase change as a boundary condition.
\[ \nabla^2 \theta = \frac{\partial \theta}{\partial \tau}. \]  \hspace{1cm} (3.1)

This is the basis for solution of the pulsed laser cutting problem. \( \theta \) is nondimensional temperature and \( \tau \) is nondimensional time.

\[ \theta = \frac{T - T_\infty}{T_m - T_\infty} \]  \hspace{1cm} (3.2)

\[ \tau = \frac{t \alpha}{d} \]  \hspace{1cm} (3.3)

- \( T_\infty \)-ambient temperature (K)
- \( T_m \)-phase change temperature (K)
- \( \alpha \)-thermal diffusivity (m\(^2\)/sec)
- \( d \)-thickness of sheet (m)

The energy equation includes the conduction of energy away from the kerf in the solid material. A schematic of the domain is shown in Figure 1.

Although the material is moving beneath a stationary laser, the frame of reference for solution is the moving sheet itself, thus the convective portion (velocity terms) of the energy equation is zero. As a result, the above unsteady form of the energy equation must be differenced in time and solved for repeated pulse periods until a steady state is reached. This approach was chosen because the pulsing of the laser is periodic, and the
Figure 1. Laser, Sheet, and Coordinate System
convective quasi-steady form of the energy equation would not allow periodic node removal patterns. A continuous wave laser model can take advantage of the fact that when a node is removed upstream, all nodes downstream are removed in the steady state. However, in the pulsed laser case, this assumption cannot be made. Therefore, the transient, conduction form of the energy equation is solved, and the temperature field develops beginning with some specified ambient temperature throughout the domain.

The energy equation is solved in a semi-infinite sheet to simulate the large scale of the sheet with respect to the size of the area of interest. The actual numerical model is limited to finite size. To compensate for this, the boundary conditions are chosen in the most conservative fashion; all edges adjacent to infinite directions are modelled as insulated. The equation to be solved is parabolic in time, therefore initial conditions are required throughout the domain and boundary conditions are required on all sides. Heat transfer from the upper and lower surfaces are assumed convective and values of film coefficients are specified.

The energy equation must be written in discretized form for each control volume to formulate a finite difference solution. The domain is divided into rectangular prism control volumes of dimension \( \delta x \times \delta y \times \delta z \). Nodes are located at the center of these control volumes. Values for \( \theta \) that satisfy the energy equation and boundary conditions are found for specified values of \( \tau \) at the nodes. Solutions are found in increments of \( \delta \tau \) which are not regular but instead are adjusted based on the model's approximation of phase change.
Solution of the problem is divided into an on-time and an off-time. The on-time is that portion of a pulse period when the laser is adding energy to the domain. The off-time makes up the majority of the period. It is the portion of the pulse period when the laser is not adding energy to the domain.

Finite Difference Formulation (off-time): In general, a Crank-Nicholsen [5] discretization of the energy equation is solved by some form of Gaussian Elimination. A problem of this size, however, would require immense computer storage. For a typical grid used in this investigation, a domain of $80 \times 40 \times 4$ nodes, dimensioning of $12800 \times 12800$ arrays would be necessary. The alternating direction algorithm is therefore very attractive; it reduces the problem to one solvable with, for example, $1/12800$ of the storage requirement.

Alternating direction formulations use a split time variable for each spatial dimension. For a three-dimensional problem, this produces two intermediate time steps on the way to a third time step which is second order accurate in space and time. The formulation is also unconditionally stable. Thus a three-dimensional problem that could not be solved using standard solution methods, due to storage and time constraints, is formulated with equivalent accuracy and stability. The alternating direction formulation for each split time variable is listed.

First time split

$$\frac{\theta_{jk}^{n+1} - \theta_{jk}^{n}}{\delta \tau} = \frac{\theta_{i+1,j}^{n} + \theta_{i-1,j}^{n} - 2\theta_{i,j}^{n}}{2\delta x^2}$$

Model Development
\[
+ \frac{\theta^n_{i+1} + \theta^n_{i-1} - 2\theta^n_i}{2\delta x^2} + \frac{\theta^n_{j+1} + \theta^n_{j-1} - 2\theta^n_j}{\delta y^2} \\
+ \frac{\theta^n_{k+1} + \theta^n_{k-1} - 2\theta^n_k}{\delta z^2}
\]

and second time split

\[
\frac{\theta^{ss}_{ik} - \theta^{n}_{ik}}{\delta \tau} = \frac{\theta^s_{i+1} + \theta^s_{i-1} - 2\theta^s_i}{2\delta x^2} \\
+ \frac{\theta^s_{j+1} + \theta^s_{j-1} - 2\theta^s_j}{2\delta y^2} \\
+ \frac{\theta^s_{k+1} + \theta^s_{k-1} - 2\theta^s_k}{\delta z^2}
\]

and next actual time level

\[
\frac{\theta^{n+1}_{ik} - \theta^n_{ik}}{\delta \tau} = \frac{\theta^s_{i+1} + \theta^s_{i-1} - 2\theta^s_i}{2\delta x^2} \\
+ \frac{\theta^{ss}_{j+1} + \theta^{ss}_{j-1} - 2\theta^{ss}_j}{2\delta y^2} \\
+ \frac{\theta^{n+1}_{k+1} + \theta^{n+1}_{k-1} - 2\theta^{n+1}_k}{2\delta z^2}
\]

Model Development 13
This is the form of the finite difference equations for interior nodes. On boundary nodes they are modified by imposing the boundary conditions. The following three types of boundary conditions are utilized in solution for all time splits.

1. Specified temperature for partially evaporated nodes, \( \theta_{\nu k} = 1.0 \). The finite difference equations for adjacent nodes are reduced using this condition, and the finite difference equations at this node become \( \theta_{\nu k+1}^{n+1} = 1.0 \).

2. Convection, \( \frac{\partial \theta}{\partial z} = \pm Bi \theta \). The sign depends on whether the boundary is the top surface, +, or the bottom surface, −.

For the bottom surface the equations are simplified by substituting for \( \theta_{k+1} \) in each equation using

\[
\left( \frac{Bi}{2} - \frac{1}{\delta z} \right) \frac{Bi}{2} + \frac{1}{\delta z} \theta_k = \theta_{k+1}
\]

(3.7)

for the top surface the equations are simplified by substituting for \( \theta_{k-1} \) in each equation

\[
\left( \frac{1}{\delta z} - \frac{Bi}{2} \right) \frac{1}{\delta z} + \frac{Bi}{2} \theta_k = \theta_{k-1}
\]

(3.8)
Bi (nondimensional film coefficient) = \frac{hd}{k}.

h (film coefficient) (\frac{W}{m^2 K}).

d (thickness) (m).

k (conductivity) (\frac{W}{mK}).

3. For insulated boundaries, the following substitutions are made.

In the positive x direction \( \theta_{x+1} = \theta_i \),

In the negative x direction \( \theta_{x-1} = \theta_i \),

In the positive y direction \( \theta_{y+1} = \theta_j \),

In the negative y direction \( \theta_{y-1} = \theta_j \),

Volume centered nodes were utilized in the formulation. They yield a second order accurate formulation throughout the domain and are convenient for the formulation of flux, temperature, or mixed boundary conditions.

The laser spot actually remains stationary over a specified location along the centerline of the domain, and the domain is shifted downstream. In this way, the travel speed of the material is simulated. This shifting is accomplished by adding material upstream and
removing material downstream in finite increments. This changes the boundary conditions on some individual control volumes in between time steps. Because the sheet is moved a discrete number of control volume lengths and the pulsed laser frequency is a constant, there will be a discrepancy during each time step between the distance the sheet moves in the model and the distance the sheet would move with respect to the laser based on the cut speed. This discretization error is saved after each time step and added to the calculation for moving the sheet after the following time step. Therefore, the distance the sheet moves during each pulse period, may not be exactly the same.

As the laser spot moves with respect to the domain and the energy is added over the laser spot at a specified frequency, a temperature field will develop in the model around the laser spot. Eventually, a quasi-steady solution will develop; this solution will be identical for successive pulse periods of the laser. Thus there is a steady periodic solution with the same frequency as the driving energy source, the laser. For the velocities and domains used in this study, one to two hundred pulsed periods were needed to establish a quasi-steady solution.

**Laser pulse (on-time):** The addition of energy from the laser activates the problem; without it there would be no cutting and no temperature rise. In this problem, energy is added in short periodic bursts. The duration of these bursts, \((\text{on-time})/(\text{duration of period})\), is only \(5.6 \times 10^{-4}\) of the actual time the material is being cut. As a result, conduction of energy away from the laser spot is neglected for the small portion of the solution during the laser pulse (The energy equation is not solved during energy addition).
Thus the problem is simplified greatly; energy addition is treated as an instantaneous effect on nodes exposed to the laser. The on-time changes the initial and boundary conditions for the off-time but only affects those nodes exposed to the laser.

Energy is distributed to control volumes under the spot based on the spatial distribution chosen, Gaussian or square. The nodal temperature computed by the finite difference solution is assumed representative of each control volume. An energy balance is performed based on the specific heat, the latent heat of vaporization/melting, and the mass of the control volume. During energy addition, the only heat transfer to or from any control volume is the specified heat flux from the laser.

The energy balance is performed on the nodes receiving energy from the laser. These nodes are treated as bulk control volumes with insulated boundaries during energy addition. Thus the energy balance equates the energy from the laser at a certain position in the x,y plane with the enthalpy rise in the control volumes at that position. The laser provides a finite amount of energy to each position in the plane of the sheet. If the amount of energy at a certain position is great enough to raise the enthalpy level of the surface node through phase change then the node is removed. There will then be left over energy at that position which is added to the next control volume below (in the z-direction). If there are no more control volumes at that position, the left over energy shines through the kerf.

Material removal is based on solid material either melting or vaporizing at a single temperature. Change of phase requires the addition of latent heat to the material at the
melt/vaporization temperature. Phase change at a single, known temperature is a characteristic of pure materials.

During the on-time, nodes may be removed from the solution domain. When a node is removed, the next node below in the z direction is exposed to the laser and energy is subsequently added to it. Node removal is a two step procedure; first the node is sensibly heated to melt/vaporization temperature, $T_m$; then the latent heat of vaporization is added to the control volume which remains at $T_m$. Some nodes will remain held at $T_m$ at the end of the on-time. These nodes have an enthalpy level between solid and liquid/vapor for a control volume of the specified mass, and during solution of the energy equation, they become boundary conditions at $T_m$. After each time step, an energy balance is performed on those partially melted/evaporated nodes. When the enthalpy level for a node has returned to that of solid at $T_m$ for the mass of a control volume, the node is returned to the solution domain. This energy balance is determined from results of the finite difference solution. The on-time provides energy which drives the solution. However, it is only during the off-time that the energy equation is solved. Solution is performed over discrete time steps using a finite difference formulation and boundary conditions that may be modified after each time step.

Assumptions: Here is a review of the stated assumptions.
1. Pulsed laser heating is modeled as the addition of energy to a material in a finite spot on the surface of that material.

2. If the pulse is on for a sufficiently short duration with respect to the time it is off, conduction during the pulse is negligible.

3. A temperature field is established around the laser spot that is periodic; it will have the same frequency as the laser power.

4. Boundary nodes are sufficiently remote from the laser spot to be treated as insulated.

5. The continuous motion of the sheet is simulated by discretized motion of the domain with respect to the laser spot.

6. The steady periodic solution is approached by solving the transient, temperature form of the energy equation for successive time intervals.

7. Phase change occurs, as in a pure substance, at one temperature.

8. The process of cutting is simulated by removing nodes from the solution domain to establish a kerf.

9. Node removal is treated as a two step process: first, a node receives sensible heat; second, when the node reaches phase change temperature, it receives latent heat.

10. By removing nodes one at a time, partial cutting is simulated.
Summary: The numerical procedure for pulsed laser cutting operates as follows:

- The domain is defined with an initial temperature, $T_\infty$ throughout.

- Energy is added to the surface of the domain and nodes are removed during the on-time.

- The conduction problem is solved during the off-time based on the initial and boundary conditions prescribed during the on-time.

- The on-time and off-time are executed repeatedly until a steady periodic solution is achieved.

The convergence of the pulsed laser solution is determined by comparing the value of temperature at a certain location in the domain and a certain time in the period for all of the periods that have been previously solved. This procedure is used to allow for variations in the number of control volumes the sheet will shift for each period.
Results

Verification

The pulsed cutting model was modified to allow comparison with the analytical solution for a line source in an infinite sheet of negligible thickness. This is a two dimensional problem; the line source is situated at the origin, and the sheet moves with respect to the source with a fixed velocity, \( u \), in the negative \( x \) direction. The solution was given by Rosenthal [15].

\[
\theta' = e^{-u'x} K_0 \left[ \left( Bi' + u'^2 \right) \frac{1}{2} r' \right]
\]  
(4.1)

\[
\theta' = \frac{T - T_\infty}{\left( \frac{q}{2\pi dk} \right)} \quad \text{dimensionless temperature.}
\]
\[ u' = \frac{ud}{2\alpha} \] dimensionless velocity.

\[ x' = \frac{x}{d} \] dimensionless \( x \).

\[ Bi' = \frac{(h_t + h_s)d}{k} \] Biot Number for a very thin sheet.

\[ r' = \frac{r}{d} \] dimensionless distance from the origin, \( (r^2 = x^2 + y^2) \).

\( K_s \) is a modified Bessel Function of the second kind of order zero.

Figures 2, 3, 4 and 5 demonstrate the close agreement of the model with Rosenthal's solution for this simplified test case. The deviation from Rosenthal occurs near the origin where a line source solution abruptly approaches infinite temperature. The numerical model has been modified to distribute flux over the top surface of one control volume in approximation of a line source and, therefore, will only predict finite temperatures.

The model is in worse agreement with the line source solution at higher speeds, compare Figures 2 and 3 with Figures 4 and 5. This is caused by the discretized nature of motion.
(y' = 0.5)

Figure 2. Verification of Model in the Direction of Motion Without Cutting, u = 0.0508 m/s
Figure 3. Verification of Model Perpendicular to Direction of Motion Without Cutting, $u = 0.0508 \text{ m/s}$
Figure 4. Verification of Model in Direction of Motion Without Cutting, \( u = 0.127 \text{ m/s} \)
Figure 5. Verification of Model Perpendicular to Direction of Motion Without Cutting, $u = 0.127 \text{ m/s}$
in the numerical model. The field is shifted downstream a discrete number of control volumes in between time steps based on the velocity. Thus the higher speed results in more drastic movement of the grid between time steps. This effect is lessened if smaller time steps are taken.

The velocity for Figure 4 and 5, 0.127 m/s, is beyond the upper limit used in experimental work with the material being studied. So the pulsed cutting model has been verified for the desired operational range.

**Continuous Power Cutting**

Continuous wave laser cutting results were compared with the model for continuous power cutting (see Appendix B for details) in order to identify the proper value for absorptivity. Where the absorptivity, \( \alpha = \frac{P}{P_t} \), and \( P \) is the power the program adds directly to the laser spot. The average absorptivity was then used to determine the values for power in numerical evaluation. The observed cut width and power for a given velocity were compared with numerical results for the same velocity. An absorptivity of 0.15 was suggested by this method (see Figure 6).

Predicted temperature fields are shown for velocities of 0.127 m/s (Figure 7), for 0.371 m/s (Figure 8), and for 0.762 m/s (Figure 9) all at a laser power of 200 W. This corresponds to a value of 30 W being used in the evaluation. These figures all show a region of high temperature dragged downstream by the effect of convection. Increasing the
### Determining an Appropriate Absorptivity

<table>
<thead>
<tr>
<th>Actual Power P, (W)</th>
<th>Cut Speed u (m/s)</th>
<th>Program Power P (W)</th>
<th>Absorptivity α</th>
</tr>
</thead>
<tbody>
<tr>
<td>200.0</td>
<td>0.0635</td>
<td>41.58</td>
<td>0.210</td>
</tr>
<tr>
<td>200.0</td>
<td>0.127</td>
<td>20.79</td>
<td>0.132</td>
</tr>
<tr>
<td>400.0</td>
<td>0.254</td>
<td>41.58</td>
<td>0.10</td>
</tr>
<tr>
<td>400.0</td>
<td>0.254</td>
<td>51.97</td>
<td>0.13</td>
</tr>
<tr>
<td>400.0</td>
<td>0.762</td>
<td>72.76</td>
<td>0.18</td>
</tr>
<tr>
<td>400.0</td>
<td>0.762</td>
<td>62.36</td>
<td>0.156</td>
</tr>
</tbody>
</table>

Average: 0.15

**Figure 6. Table of Absorptivities from Continuous Model and Data**
velocity decreases the average temperature in the domain. However, increasing velocity also creates higher temperature gradients normal to the direction of motion further downstream. These temperature gradients cause thermal stresses.

The absorption efficiency is defined as the amount of power absorbed by the material during cutting divided by the amount of power emitted by the laser.

\[ \varepsilon_a = \frac{P_a}{P_i} \]  

(4.2)

When no part of the laser beam shines through the kerf, all of it strikes the surface of the sheet. This is the most efficient use of power, \( \varepsilon_a = \alpha \) (the absorptivity), and is the upper limit for \( \varepsilon_a \) but it is not necessarily the optimal cut condition. The absorption efficiency decreases from a maximum as velocity is decreased and as power is increased (Figure 10 and 11). As speed is increased, a threshold speed will be reached above which material will not be cut. Thus cutting at too high a value of \( \varepsilon_a \) will result in incomplete, partial cutting.

The power necessary to melt or vaporize the material removed to form the kerf is defined as the kerf power.

\[ P_k = \rho u A \left[ C_p (T_m - T_\infty) + H \right] \]  

(4.3)

\( \rho \) - density

\( u \) - velocity
Figure 7. Continuous Power Model Temperature Distribution, $u = 0.127 \text{ m/s}$, 208.0 W Total Power
Figure 8. Continuous Power Model Temperature Distribution, $u = 0.371 \text{ m/s}$, 208.0 W Total Power
Figure 9. Continuous Power Model Temperature Distribution, \( u = 0.762 \) m/s, 208.0 W Total Power
Figure 10. Absorption Efficiency Versus Power, Continuous Power Model
Figure 11. Absorption Efficiency Versus Speed, Continuous Power Model
$A$ - kerf cross-section

$C_p$ - specific heat

$T_m$ - melt or vaporization temperature

$T_\infty$ - initial temperature

$H$ - latent heat of melting/vaporization

The kerf power fraction of the total laser power, $\left(\frac{P_s}{P_t}\right)$ is a measure of how much energy is conducted away from the area of kerf formation and into the material. Thus a large value of kerf power fraction indicates lower temperature in the cut material. Figure 12 shows the increase in kerf power fraction with velocity and the decrease in kerf power fraction with $P_t$.

Figure 13 shows the heat-affected zone versus cut speed for the continuous model at $P_t = 208$ W and the observed heat-affected zone for 200 W and 400 W. The heat-affected zone for the numerical runs was determined for the estimated critical temperature of the study material, 450 °C, while the experimental values were determined from visible changes in the material. Although there is no definite trend in the calculated data, there is also none in the measured data. The measured values are greater than the corresponding predicted values. Experimentally the heat-affected zone is determined by looking for a visible change on the surface of the material, and the numerical results are for temperatures just below the surface; therefore one might expect that the program
Figure 12. Kerf Power Ratio Versus Speed, Continuous Power Model
would underpredict this quantity. Also, from the few data points available, it appears that the program underpredicts the observed data by a consistent margin and follows the observed tendencies.

**Pulsed Laser Cutting**

The pulsed laser results all are for a frequency of 5000 Hz, a pulse width of 110.0 ns, and a Gaussian power distribution for laser radius, \( R_{\text{av}} = 0.0127 \) mm. These parameters were used in experiments, and, for the particular laser used, Nd YAG in mode \( \text{TEM}_{\infty} \), a Gaussian distribution is a better approximation of the shape of the beam and is therefore preferred over a square one [1,8]. Also, a Gaussian distribution gave better agreement with experiments based on kerf width and heat-affected zone.

Temperature fields are shown in Figures 14, 15, 16, and 17. These figures were made from data produced by PULSECUT. Figure 14 and 16 show the predicted field at the beginning of a pulse period; the energy from the laser pulse has just been added, and no time has elapsed. Figure 15 and 17 show the field at the end of the pulse period. Comparison of Figure 14 and 15 shows the diffusion of energy during a pulse period, only 0.20 ms, for cutting at 0.0127 m/s. The maximum local rate of drop in temperature during the period is on the order of \( 10^6 \) °C/s. Figure 16 and 17 are for a cut speed of 0.0254 m/s. They also show a local rate of drop in temperature on the order of \( 10^6 \) °C/s. From these figures, one can see that increasing cut speed increases the local
Figure 13. Heat Affected Zone Versus Speed
temperature gradients but reduces the local temperatures. Note that the kerf appears as the removed portion of the grid along the x axis downstream of the origin.

Figure 18 shows the absorption efficiency, $\epsilon_a$, versus velocity for the pulsed laser. This was calculated in the same manner as for continuous laser cutting. For the highest velocity shown, 0.127 m/s, the material is not cut. The figure shows that as the velocity is increased, more power is absorbed by the material, and less is lost to the process. The values of $\epsilon_a$ represent the amount of material removed from under the laser spot. The trends for pulsed and continuous laser cutting are the same, only over different velocity ranges. This velocity range is a function of the power available.

Figure 19 shows another measure of cutting efficiency, the kerf power ratio. This was also calculated as before (Equation 4.3). This value appears to peak at the point of maximum cutting velocity for a given power. In this case, $P_v = 3546$ W, and the maximum cutting velocity is 0.0889 m/s. Note that $P_v/P_p$ is generally higher for pulsed laser cutting (Figure 19) than continuous laser cutting (Figure 12). This indicates that a higher percentage of power is utilized to form the kerf in pulsed laser cutting and therefore a smaller heat-affected zone should be formed.

Figure 20 shows the predicted heat-affected zone for pulsed and continuous laser cutting. This was calculated for a critical temperature of 450 °C. The heat-affected zone calculated from results of PULSECUT is less than 0.02 mm for all cutting parameters, far less than predicted for continuous power cutting. The main conclusion to be drawn from this figure is that the heat-affected zone for pulsed laser cutting should be within ac-
Figure 14. Temperature Field, $u = 0.0127$ m/s, Beginning of Pulse Period, 3546 W Peak Power
Figure 15. Temperature Field, \( u = 0.0127 \) m/s, End of Pulse Period, 3546 W Peak Power
Figure 16. Temperature Field, \( u = 0.0508 \text{ m/s} \), Beginning of Pulse Period, 3546 W Peak Power
Figure 17. Temperature Field, $u = 0.0508$ m/s, End of Pulse Period, 3546 W Peak Power
ceptable limits for greater velocities than have been obtained experimentally. Ultimately, the pulse frequency must also be increased in order to attain higher and higher velocities and still get an uninterrupted kerf. Otherwise too much material will pass under the laser between pulses. The only restriction on moderate increases in cutting velocity appears to be the power a pulsed laser can deliver. All of these results were based on a laser radius as defined in Chapter 3 for a Gaussian Distribution. Because the results for maximum velocity were based on the same laser radius each time, the increase in power shown in Figure 21 is equivalent to an increase in power density.

The results shown in Figure 20 for minimum $P$, and in Figure 21 were determined by making successive runs of PULSECUT to determine the threshold value of cut velocity for a given $P$. This is the maximum velocity at which PULSECUT predicts cutting for a given power. Figure 21 shows that this maximum velocity should increase as power increases, as expected.
Figure 18. Absorption Efficiency Versus Speed, Pulsed Laser Model, 3546 W Total Power
Figure 19. Kerf Power Ratio Versus Speed, Pulsed Laser Model, 3546 W Total Power
Figure 20. Heat-Affected Zone Versus Speed, Pulsed and Continuous Laser Models
Figure 21. Maximum Cutting Velocities, Pulsed Laser Model
Stress Analysis

Thermal stress analysis was performed using the IDEAS [19,20] package finite element solver and post-processor (Figures 22, 23, 24, 25, and 26). Temperature fields determined by finite difference solution using the continuous power model and the pulsed laser model, PULSECUT, were used as temperature loading for the stress analysis calculation using IDEAS. The temperature codes were essentially pre-processors for the finite element stress analysis. The stress model used was two dimensional, ignoring variation in the direction normal to the thickness of the sheet. Constant properties were assumed throughout the sheet. The results represent the elastic, planar response of the sheet with room temperature properties. Only the x-normal component of stress is shown because it is in the direction of motion; this should be the stress component causing failure normal to the edge of the kerf. Regions of high stress might fall within the heat-affected zone and could be responsible for micro-cracking normal to the cut edge.

The elements used were linear quadrilaterals (plane stress) and were supported as shown in Figure 22. The loading consisted of a specified temperature at each node. The product of this model is an estimate of stress for a fixed stress element mesh for continuous and pulsed laser cutting.

Continuous Power Laser Results
Figure 22. Finite Element Mesh and Restraints
In Figures 23 and 24 a region of high compressive (negative) stress is predicted along and parallel to the edge of the kerf (the gap on the bottom of the contour plots). It is on the order of magnitude of the yield strength for steel at room temperature. In addition, there is also a region of tensile (positive) stress in the center of the model that could be described as a reaction to the compressive stress fields set up by thermal expansion. The greater the cutting speed, the more this area of tensile stress is extended downstream and the narrower and more intense the area of compressive stress becomes. If it is assumed that when the material is heated to a certain temperature (yield strength will decrease) it will no longer respond elastically and begin to deform plastically (to relieve increased stress), material adjacent to the cut edge would expand and relieve thermal stress. When the material cools, this plastically deformed portion could not contract as much as the elastically responding portion. At a certain point in the cooling process, there would be two regions of residual stress: Compressive stress in that portion of material that had responded elastically during the cutting process and tensile stress in the material along the edge of the kerf which had responded plastically. The plastically responding material will have been heated above the critical temperature and therefore be more brittle when it cools. These two regions of stressed material create an area where fracture might occur. This region of possible failure is indicated on the stress contour plots (Figures 23 and 24). The higher levels of stress predicted by the model at higher cutting speeds do appear to correlate with the experimentally observed increase in cracking in the heat-affected zone.

Pulse Cutting Results
Figure 23. Stress Field (kPa), Continuous Power (200 W), $u = 0.127 \text{ m/s}$
Figure 24. Stress Field (kPa), Continuous Power (200 W), $u = 0.762$ m/s
There are areas of high stress variation within single stress elements on the contour plots for pulsed laser cutting (Figures 25 and 26). This indicates that greater mesh refinement in those areas is necessary. The linear elements used for stress analysis are incapable of representing a stress variation across any one element. Consequently, the compressive regions that are shown within the size of one stress element should be discounted. Moreover, any stress values at convex corners would theoretically approach infinity and at concave corners would approach zero; so stress values in corners must also be discounted. The high temperature gradients experienced in pulsed laser cutting require more stress element mesh refinement than in continuous power laser cutting in order to achieve good results.

The calculated stress values for pulsed cutting, however, are an order of magnitude less than the continuous results. This supports the observation that pulsed cutting yields cut quality superior to continuous cutting. Comparing Figure 25 to Figure 26 shows that, as in continuous cutting, the stress values are higher for increased cut speed.

**Discussion**

Laser cutting has produced micro-cracking along the cut edge. The temperature fields in the sheet induce stresses and structural changes that may cause micro-cracking. If the situation that causes cracking can be predicted, perhaps it can be avoided. Thus nu-
Figure 25. Stress Field (kPa), Pulsed Laser (3546 W), u = 0.0127 m/s
Figure 26. Stress Field (kPa), Pulsed Laser (3546 W), $u = 0.0508$ m/s
Numerical predictions and some observations have been presented for laser cutting of thin sheets. Continuous power cutting can now be compared to pulsed cutting based on the results and the literature review. The processes of continuous power cutting and pulsed cutting are controlled by the mechanism of kerf formation. In continuous power cutting, formation of the kerf apparently consumes less energy per unit volume than pulsed cutting. It is assumed that the process of kerf formation can either be induced by sensible heating and melting, sensible heating and vaporization, or sensible heating and some combination of melting and vaporization. Continuous laser cutting involves lower energy densities than pulsed cutting and in general has been assumed to be fusion controlled cutting [17]. Therefore, pulsed laser cutting should be controlled by either sensible heating and vaporization, or sensible heating, melting and vaporization. The results in Figures 13 and 20 indicate that vaporization controlled cutting with the parameters given (f = 5000 Hz and pulse width = 110.0 ns) would result in a heat-affected zone an order of magnitude smaller than is observed in pulsed cutting. The most consistent agreement of numerical predictions with the observed phenomena was obtained when the latent heat of fusion and the melt temperature were used instead of the latent heat of vaporization and the vaporization temperature. Also, Von Allmen [22] observed that up to ninety percent of material ejected during pulsed laser drilling is liquid. Therefore, pulsed cutting probably involves some combination of fusion and
vaporization that requires more energy per volume of kerf formed than continuous laser cutting but is dominated by fusion.

From the numerical results, it is difficult to establish a relationship between cut speed and heat-affected zone. However, it is obvious that the heat-affected zone is smaller for pulsed laser cutting than continuous from both numerical and observed data. The real difference between the two forms of laser cutting is the energy densities at which they operate, on the order of $10^9 \frac{W}{m^2}$ for continuous and $10^{11} \frac{W}{m^2}$ for pulsed. The higher energy density results in higher gradients and causes more energy to go into formation of the kerf and less into conduction. Another factor that limits the amount of energy going into conduction is the fraction of the kerf that is vaporized. The vaporized material consumes energy itself and, if only some kerf material is vaporized, some liquid and solid material will leave as ejecta due to vapor pressure [1,22] (a phenomena that cannot be included in this model). This further reduces the amount of energy that is lost to conduction.

The stress predictions for continuous power cutting (Figures 23 and 24) appear physically reasonable. Free edges show low stress levels, and the highest compressive (negative) stress region is under the laser spot as expected. For pulsed laser cutting, the stress plots (Figures 25 and 26) indicate too much unexpected stress variation within stress elements around the laser spot area and also indicate an unexpected stress pattern overall.
Conclusions

The numerical model demonstrated two effects that are generally neglected in studying the heat transfer of laser cutting. Material removed to form a kerf carries a significant portion of energy away from the process. Energy also is lost when the kerf forms under the laser and the beam shines through the void.

The experimental and numerical results have all indicated that increasing cut velocities results in increasing states of stress in the sheet. A relationship between stress fields and fracture has been suggested for continuous power cutting. From this it appears that if the cause of fracture in the processing of the material is thermal stress, there will be a maximum practical cut speed for either pulsed or continuous laser cutting based on cut quality.

The cause of fracture may be structural change due to elevated temperature in the heat-affected zone. The heat-affected zone appears to be relatively unaffected by velocity. It is more dependent on the energy density of the laser. The heat-affected zone is larger for continuous cutting than pulsed cutting. Thus if cut quality is acceptable for pulsed laser cutting at low velocities, it should be acceptable for higher velocities.

Higher velocities may be achieved by increasing the energy density of the laser (increase the power in the same size beam). The gain in velocity for a given increase in energy
density will eventually need to be augmented by an increase in frequency to achieve an uninterrupted cut.

Pulsed laser cutting is superior to continuous laser cutting because it provides a greater energy density during the on time of the pulse. The greater energy density in pulsed cutting tends to melt through material more completely and less energy is conducted into the remaining sheet (Probably some material is actually vaporized). This leads to a smaller heat-affected zone and lower stress.

**Recommendations**

Improvements can be made on the current pulsed cutting evaluation. The following might clarify the conclusions drawn here:

- Use quadratic stress elements and coarsen the mesh away from the cut edge. This would require changes in the finite element analysis, not in the pulsed cutting model.

- Increase the number of nodes within the heat-affected zone. This will provide a more reliable prediction of H.A.Z. size, a better description of the kerf geometry, and more information to construct a finite element stress model.

- Include some effect of the vapor pressure on material removal.
Devise a better method of creating a sufficiently large domain to simulate an infinite sheet.

Further improvement would require another method of solution of the fundamental equations. One possible method for phase change problems is an integral method such as boundary elements.


Appendix A. Pulsed Laser Model

The basis for the model is solution of the energy equation for successive time steps until convergence is reached. Solution is achieved using the alternating direction implicit algorithm, A.D.I. [5]. The boundary conditions for each node are determined by the location of the node in the domain. The nodes are volume centered; therefore any specified flux, specified temperature, or mixed boundary condition is made second order accurate by utilizing imaginary nodes surrounding the domain. These imaginary nodes are eliminated from the finite difference form of the conduction equation for nodes within the domain by utilizing the boundary conditions on the domain. Each node is given a node type that identifies its boundary conditions. These node types are determined before each energy addition to the domain and before each time step.

Formulating Second Order Boundary Conditions: It will be assumed that by describing the formulation of a second order mixed boundary condition utilizing an imaginary node
that the reader can deduce the formulation of any similar boundary condition involving volume centered nodes.

The boundary condition \( a\theta_{\text{surface}} = b\left(\frac{\partial \theta}{\partial x}\right)_{\text{surface}} + c \) is applied to the expression \( \left(\frac{\partial^2 \theta}{\partial x^2}\right) \).

For convenience, an imaginary node outside the domain will be utilized and then eliminated from the finite difference form of the expression for \( \left(\frac{\partial^2 \theta}{\partial x^2}\right) \). Thus the boundary condition will have been employed to reduce the number of unknowns from the finite difference equation at the node, \( i \).

Assume that the subscript \( i \) denotes the node within the domain just inside the surface, the subscript \( i + \frac{1}{2} \) denotes the location of the surface, and the subscript \( i + 1 \) denotes an imaginary node just outside of the surface. The formulation is in one dimension (this doesn’t change anything) and the node \( i \) is separated from node \( i + 1 \) by a distance \( \delta x \).

The following second order \([5]\) substitutions are made.

\[
\left(\frac{\partial^2 \theta}{\partial x^2}\right)_i = \frac{\theta_{i+1} + \theta_{i-1} - 2\theta_i}{\delta x^2} + O[\delta x^2] \quad (A.1)
\]

\[
\theta_{i+\frac{1}{2}} = \frac{\theta_i + \theta_{i+1}}{2} + O[\delta x^2] \quad (A.2)
\]

\[
\left(\frac{\partial \theta}{\partial x}\right)_{i+\frac{1}{2}} = \frac{\theta_{i+1} - \theta_i}{\delta x} + O[\delta x^2] \quad (A.3)
\]

The boundary condition is rewritten, truncating the second order terms.

\[
\theta_{i+1} = \frac{b}{\delta x} + \frac{a}{2} \theta_i + \frac{c}{\left(\frac{b}{\delta x} - \frac{a}{2}\right)} \quad (A.4)
\]

Appendix A. Pulsed Laser Model 65
And finally the second order form of the expression, utilizing the second order boundary condition, is written, once again truncating the second order terms.

\[
\left( \frac{\partial^2 \theta}{\partial x^2} \right)_i = \frac{\theta_{i-1} - 2\theta_i}{\delta x^2} + \frac{1}{\delta x^2} \left[ \left( \frac{b}{\delta x} + \frac{a}{2} \right) \theta_i + \frac{1}{\delta x^2} \left( \frac{b}{\delta x} - \frac{a}{2} \right) \right]
\]  \hspace{1cm} (A.5)

**Energy Addition:** The spatial distribution of energy is based on distance from the center of the laser spot either for a square distribution or for a Gaussian distribution. Energy is added to the domain based on this distribution.

The addition of energy is based on the position of a node in the x,y plane. Each position receives a finite amount of energy. The top node at the x,y position will either receive all the energy or be removed if the amount of energy at the x,y position is great enough. The amount of energy a node can receive is based on the two step process of node removal (Chapter 3).

\[
E_{jk} = \rho \Delta x \Delta y \Delta z (C_p \Delta T_m + H)
\]  \hspace{1cm} (A.6)

where

\[
\Delta T_m = T_m - T_{jk}
\]  \hspace{1cm} (A.7)

\[T_{jk} - \text{temperature at end of off-time.}\]

\[T_m - \text{phase change temperature.}\]
$H$ — latent heat of phase change.

$\Delta x\Delta y\Delta z$ — volume of control volume.

$\rho$ — density.

If the energy from the laser at the $x,y$ position is greater than $E_{ij}$ for the top node, the next node below in the $z$ column receives the spare energy. This process continues until either all nodes in this column have been removed, or the energy, at the position, from the laser, has been exhausted.

**Motion of Sheet:** In between time steps, the domain is shifted based on the velocity and duration of the last time step. Motion of the sheet with respect to the laser is simulated by actually moving the domain discretely in time. $X_{\text{int}}$ is the difference between the position of the sheet in the model and the position it would have if it moved continuously. When $X_{\text{int}} \geq n\Delta x$, where $n$ is an integer greater than 0, the field is shifted downstream $n$ control volumes; the temperatures of the previous time step are shifted, and those temperatures introduced from upstream of the domain are set at $T_{\text{in}}$. The value of $X_{\text{int}}$ is updated after every time step with the relation $X_{\text{int}} = X_{\text{int}} + u\Delta t$ ($\Delta t$ is the duration of the time step and $u$ is the cut speed). When the field is shifted downstream, the value of $X_{\text{int}}$ is reset with the relation $X_{\text{int}} = X_{\text{int}} - n\Delta x$.

**Convergence:** Convergence of the solution domain is determined by comparing the temperature at a specified node at the end of a pulse period with the temperature at the end of previous pulse periods for that node. A minimum number of pulse periods are
solved, and successive solutions are then compared with each other. Due to the discretized nature of motion, the latest solution is compared with all solutions after the minimum number of periods has been solved. This acknowledges the fact that by only moving the sheet an integer number of control volumes in the numerical model, the sheet may move a different number of control volumes in successive periods. Convergence is satisfied when the latest value of temperature at the end of the pulse period at the comparison node is within a specified tolerance of any previous value at that node.

**Partially Evaporated Nodes:** Also, in between time steps, an energy balance is performed on partially removed nodes. The energy balance insures that energy is conserved for each node. When node \(i,j,k\) has reached vaporization/melt temperature, \(T_{v,k} = T_m\), it no longer may be treated as part of the solution domain; it begins to evaporate/melt. In the model, adjacent nodes see this partially evaporated/melted node as a constant \(T_{v,k} = T_m\) boundary condition. This node is at an enthalpy level somewhere in between a solid at phase change temperature and a melted/evaporated node. A reservoir of energy for latent heat of vaporization is associated with this node. This node remains at the \(T_{v,k} = T_m\) boundary condition until either the energy accumulated in this reservoir is equal to latent heat of vaporization/melting for the control volume or the energy in the reservoir is exhausted by conduction. When a node is removed, adjacent nodes receive boundary conditions appropriate to the new geometry. If the net amount of energy in any reservoir of latent heat has become negative based on the energy balance, the enthalpy level is actually below that for a solid at phase change temperature. The time step in the conduction solution is shortened, and the domain is solved again. This avoids improperly elevating temperatures and violating the first law of thermodynamics.
Figure 27. PULSECUT Flowchart
Continuous power laser cutting is a quasi-steady conduction problem. The constant velocity form of the convection equation describes the temperature field.

\[ u \frac{\partial T}{\partial x} = \alpha \nabla^2 T \]  

(A.1)

A finite difference model utilizing node removal to simulate continuous power cutting of thin metallic sheets was used; see Glass [11] for details.

The numerical model for continuous laser cutting does not include latent heat. Instead the nodes are removed based on melt temperature alone. To counter the effect of not including latent heat, nodes are removed in columns normal to the thickness of the sheet (That is, in the z direction). Whether a column is eligible for removal is determined by examining the temperatures of adjacent upstream and cross-stream columns. The crite-
ria for removal of a column of nodes are that the neighboring upstream node (-x direction), on the bottom surface, and the neighboring cross-stream node (+y direction), on the bottom surface, are above melt temperature. In the case of the convective form of the energy equation, the solution is quasi-steady. Thus when a node is removed, all nodes downstream are also removed. When this group of nodes is removed, the domain must be re-solved utilizing the new boundary conditions.

The original form of the model [21] only allowed node removal downstream of the laser spot. This resulted in unrealistically large kerf widths because the possibility of laser energy shining through the kerf was ignored. Several modifications were made to the continuous power model in order to allow node removal under the laser spot. (The laser spot is assumed circular in the continuous model.) In the model used, nodes may be removed from the domain downstream from the leading edge of the laser spot. This means that the furthest upstream node under the laser spot in each x direction row cannot be removed. This scheme for node removal is a compromise allowing the the proper amount of energy addition to the material within the limits of discretization. Initially the quasi-steady temperature field is determined using the finite difference temperature code. Then a search is made, beginning from the centerline of the laser beam, for a column of nodes (in the z direction) which meets the criteria for removal. When such a column is found, all nodes downstream, with the same y value, are removed from the solution domain. All of the nodes through the thickness of the material are removed together at any given location. The temperatures are re-solved using the newly defined domain and more nodes are subsequently removed and temperatures re-solved until all nodes meeting the criteria have been removed.
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