Behavior of wood under transverse compression

by

Bohumil Kasal

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
Master of Science
in
Wood Science and Forest Products

APPROVED:

Frederick A. Kamke, Co-chairman

David A. Dillard, Co-chairman

Christen Skaar

January, 1989
Blacksburg, Virginia
Behavior of wood under transverse compression

by

Bohumil Kasal

Frederick A. Kamke, Co-chairman
David A. Dillard, Co-chairman

Wood Science and Forest Products

(ABSTRACT)

The increasing demand on wood and wood products, and the simultaneously decreasing quality of wood as a raw material leads to the increasing significance of wood-based composites such as particleboard or flakeboard. The resulting mechanical and physical properties are to the large extent dictated by the densification of the wood component. To be able to predict the density of the material, the behavior of structural elements must be known. A theory developed for rigid plastic foams was modified and applied to the deformation of wood in transverse compression. A testing procedure for high strain compression over a range of temperatures was developed. In addition, a stochastic model for prediction of high strain behavior was developed. Wood of yellow poplar (Liriodendron tulipifera) was used as the experimental material.
Acknowledgements

I express my sincere appreciation to my advisors Dr. F.A. Kamke and Dr. D.A. Dillard for their support throughout the course of this investigation and the critical review of the presented work. I wish to thank Dr. C. Skaar for his careful review of this text. My thank is due to Dr. G. Ifju for his support and personal understanding which helped me to solve many problems.

My colleagues, and assisted me in numerous ways. I owe them and many others my thanks and appreciation.

Finally, my love and gratitude are due to my wife, for her support and patience which made this study easier and helped me to overcome all the critical stages of this research.
# Table of Contents

- **Introduction** ............................................................ 1  
- **Objectives** ............................................................. 3  
- **Literature review** .......................................................... 4  
  - Compression properties of wood ......................................... 4  
  - Theories developed for foams applicable to transverse compression of wood ........................................... 5  
  - Effect of temperature and moisture content on mechanical properties of wood ........................................ 10  
- **Material and Methods** ..................................................... 13  
  - Test specimens ........................................................... 13  
  - Compression apparatus ................................................... 15  
  - Test procedure ........................................................... 18  
- **Results and discussion** ................................................... 23  
  - Effect of specimen height ................................................ 23  
  - Effect of surface quality and load eccentricity ....................... 30  
  - Effect of density ........................................................ 30  

Table of Contents iv
Effect of the load direction ................................................ 33
Temperature and moisture content effect ............................ 45
Lateral expansion during the compression. ......................... 45
Testing of flakes ........................................................ 55
Foam model applied to thin specimens .............................. 59
  Sensitivity study .................................................... 59
  Quality of the model ................................................ 66

Stochastic Model ........................................................ 73
Theoretical basis ....................................................... 75
Monte Carlo simulation ............................................... 77

Conclusions ........................................................... 87

Literature cited ......................................................... 89

Appendix A. Two dimensional model of cellular material ............. 94

Appendix B. Geometrical approach to the derivation of the expansion ratio for cellular material 104

Appendix C. Computer program for Monte Carlo simulation .......... 110

Vita ................................................................ 120

Table of Contents
List of Illustrations

Figure 1. Schematic model of the regular hexagon. .............................................. 6
Figure 2. Theoretical transverse compression stress-strain curve. ............................ 9
Figure 3. Schematic diagram of compression testing apparatus. .............................. 16
Figure 4. Compression testing apparatus. ............................................................. 17
Figure 5. Clip gage for measuring of the expansion ratio. ..................................... 19
Figure 6. Calibration curve for the clip gage. ..................................................... 20
Figure 7. Normalized Young's modulus in compression as a function of specimen height for yellow poplar and poly(methyl methacrylate) ................................................ 24
Figure 8. Yield stress and strain as a function of specimen height for yellow poplar ...... 25
Figure 9. Crack development in 3 mm specimen of yellow poplar in radial direction. .... 27
Figure 10. Electron micrograph of the specimen surface before and after compression . 28
Figure 11. Electron micrograph of the flake compressed to 60% strain. ...................... 29
Figure 12. Model of compression specimen with (a) surface roughness and (b) non-parallel mating platen-surfaces. ................................................................. 31
Figure 13. Predicted ratio of apparent modulus (Eapp) to actual modulus (E) .............. 32
Figure 14. Density cubed - yield stress relationship for transverse compression of yellow poplar in radial direction. ................................................................. 34
Figure 15. Densities for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table 2. ......................................................... 40
Figure 16. Modulus for 20 mm yellow poplar cubes in radial direction for the treatments listed in Table 2. ......................................................... 41
Figure 17. Modulus for 20 mm yellow poplar cubes in tangential direction for the treatments listed in Table 2. ......................................................... 42
Figure 18. Yield stress for 20 mm yellow poplar cubes in radial direction for the treatments listed in Table 2. .............................................. 43

Figure 19. Yield stress for 20 mm yellow poplar cubes in tangential direction for the treatments listed in Table 2. .............................................. 44

Figure 20. Normalized yield stress for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table 2. 46

Figure 21. Normalized modulus for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table 2 47

Figure 22. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x20 mm. ........................................ 49

Figure 23. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x5 mm. ........................................ 50

Figure 24. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x3 mm. ........................................ 51

Figure 25. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x1 mm. ........................................ 52

Figure 26. Influence of the non-zero expansion coefficient on the density without surface restraint 54

Figure 27. Influence of the non-zero expansion coefficient on the density with surface restraint 56

Figure 28. Relative density as a function of strain and expansion ratio. ............... 57

Figure 29. Prediction of compression behavior for yellow poplar for treatments listed in Table 2 .......................................................... 58

Figure 30. Influence of the modulus on the proposed model 63

Figure 31. Influence of relative density on the proposed model 64

Figure 32. Influence of the yield stress on the proposed model 65

Figure 33. Influence of the expansion ratio on the proposed model 67

Figure 34. Calculation of the error of prediction. 69

Figure 35. Density-yield stress relationship from the Monte Carlo simulation. 78

Figure 36. Density-E relationship from the Monte Carlo simulation. 79

Figure 37. Modulus-yield stress relationship from the Monte Carlo simulation. 80

Figure 38. Histogram and probability density function for density 81

Figure 39. Histogram and probability density function for yield stress 82

Figure 40. Histogram and probability density function for modulus 83
Figure 41. Expected stress-strain relationship for yellow poplar as a result of Monte Carlo simulation ................................................... 84

Figure 42. Schematic model of a regular hexagon. ........................................ 95

Figure 43. Elastic bending ................................................ 97

Figure 44. Elastic buckling ................................................ 99

Figure 45. Plastic collapse ............................................... 101

Figure 46. Schematic model of lateral expansion of the hexagon. .................... 105

Figure 47. Model of cylindrical cell ......................................... 107

Figure 48. Expansion ratio as a function of the axial strain for different cell models ...... 109
List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>LSD test on differences between $p$ for radial and tangential direction for yellow poplar 20 mm cubes ($\alpha = 0.05$).</td>
<td>35</td>
</tr>
<tr>
<td>2.</td>
<td>Modulus and yield stress at different moisture content and temperatures for yellow poplar 20 mm cubes.</td>
<td>36</td>
</tr>
<tr>
<td>3.</td>
<td>LSD test on differences between moduli for radial and tangential direction for yellow poplar 20 mm cubes ($\alpha = 0.05$).</td>
<td>37</td>
</tr>
<tr>
<td>4.</td>
<td>LSD test on differences between normalized moduli for radial and tangential direction for yellow poplar 20 mm cubes.</td>
<td>38</td>
</tr>
<tr>
<td>5.</td>
<td>LSD test on differences between normalized yield stress for radial and tangential direction for yellow poplar 20 mm cubes.</td>
<td>39</td>
</tr>
<tr>
<td>6.</td>
<td>Yield stress and density for yellow poplar flakes.</td>
<td>60</td>
</tr>
<tr>
<td>7.</td>
<td>LSD test on differences between normalized yield stress in radial compression of yellow poplar flakes ($\alpha = 0.05$).</td>
<td>61</td>
</tr>
<tr>
<td>8.</td>
<td>Modulus of the cell wall and $c_2$ coefficient.</td>
<td>71</td>
</tr>
<tr>
<td>9.</td>
<td>LSD test on differences between $c_2$ for radial and tangential direction ($\alpha = 0.05$).</td>
<td>72</td>
</tr>
<tr>
<td>10.</td>
<td>Results from Monte Carlo simulation.</td>
<td>86</td>
</tr>
</tbody>
</table>
Introduction

The increasing demand on wood and wood products, and simultaneously decreasing quality of wood as a raw material, leads to the application of wood composite materials, such as particleboard or flakeboard. The mechanical properties of wood composites are to a large extent determined by the density (Geimer et al. 1985). Density can be controlled during the manufacturing process. Thus knowing the mechanisms which are responsible for density formation provides the necessary tool to significantly control the resulting properties of the materials. This approach of tailoring composite materials based on the known properties of its components has been used for the design of synthetic fiber reinforced composites (Jones 1975, Askhenazi and Ganov 1980, Malmeister et al. 1980).

Wood, being a natural material, possesses a rather high variability of properties; as a result, the situation here is even more difficult than in the case of more uniform synthetic materials (e.g. carbon fibers and epoxy matrix). During the production of composite materials, wood is subjected to high pressure under extreme temperature and moisture content conditions. When the resulting thickness of the board is achieved, the wood is compressed far beyond the proportional limit and is highly densified. Thus the process of pressing changes the structure and mechanical properties of the wood components. The mechanism of flake deformation during the pressing process is basic information which can be used for the prediction of material properties.
Wood can be qualitatively compared with the man-made cellular foams, which have similar structure. Rather intensive research efforts have been dedicated to the development of theories describing foam deformation characteristics (Meinecke, Clark 1973, Gibson et al. 1981, Menges and Knipschild 1982 etc.). Those foam theories gave rather promising results for wood (Easterling et al. 1982), mainly for lower density species, such as balsa (*Ochroma lagopus*). Ashby et al. (1985) modelled wood as a system of hexagonal cells. A number of different materials including wood species of different densities were tested. Although, no analysis of the results was presented, a good qualitative agreement between theory and data was found.
**Objectives**

The objectives of the research are:

1. Apply an existing theory developed for foams to explain behavior of wood in transverse compression.

2. Examine the effect of a non-zero expansion coefficient on the performance of the foam theory.

3. Develop an experimental technique to measure wood behavior under high strain in transverse compression.

4. Measure the qualitative effect of temperature and moisture content on transverse compression.

5. To use Monte Carlo simulation to predict the transverse compression behavior.
Literature review

Compression properties of wood

There are not many works dedicated to the study of transverse compression. Most of the research has been conducted to study the properties up to the proportional limit, whereas little has been done in high strain regions. Schneeweiss (1961) studied the influence of specimen dimensions on yield stress. He found a non-linear relationship between the yield stress and volume of the specimen. Since the specimens were large (cubes from 45 mm to 150 mm) it was difficult to retain the same angle of annual rings. Also, the moisture content was different for each compared dimension (from 16% to 29.5%). Bodig (1963) studied the influence of the specimen height on modulus of elasticity (E) and yield stress (σy). Whereas there was no difference between the values of σy, E increased with increased height (heights 1, 1.5, 2.0, and 3.0 inches were tested). He explained this phenomena by the structure of wood assuming non-uniform strain distribution. According to this theory, weak earlywood layers take up most of the deformation and an additional height does not totally control the overall deformation. Thus the strain decreases with the height of the specimen. If this assumption is valid, then this phenomena should not occur in homogeneous materials. Bodig (1965) also studied the failure modes in transverse compression by defining progressive failure
of cell wall rows as a controlling phenomena. He indicated a buckling type of failure. Kunesh (1966) studied properties of Douglas Fir (*Pseudotsuga menziensis*) and Western Hemlock (*Tsuga heterophylla*) on 0.5, 1.0, and 2.0 inch thick specimens and different cross-sectional areas (1, 4, and 36 square inch). An increase of E and slight decrease of the proportional limit with increasing thickness was reported. Easterling et al. (1982) studied the behavior of balsa under high strains. A foam theory, developed by Gibson et al. (1982) was used to explain high strain behavior. Ashby et al. (1985) did similar tests as Easterling, but studied several different species. Quite good agreement between theory and experiments was found. Wingate-Hill and Cunningham (1986) and Wingate-Hill and Crowes (1987) studied the moisture content change of several species during compression up to 40% strain. However, no stress-strain relationship was presented.

**Theories developed for foams applicable to transverse compression of wood**

Wood can be considered a cellular material composed of a system of cells of different sizes. Dimensions of those structural elements are random variables; however, if one is interested in overall response of the material to the applied load or strain, the whole system can be modeled in terms of an average cell. This approach was applied by Gibson et al. (1981) who modelled cellular foam as hexagons. Easterling (1982) and Ashby (1983, 1984) have shown the applicability of this theory to wood. A brief description follows:

A regular hexagon of wall thickness \( t \) and side length \( l \) is used to model an individual cell. Then, relative density of a cellular material can be expressed as a ratio of the wall area and the total area of the hexagon (see Figure 1). The linear elastic behavior is assumed to be controlled by the bending of the cell wall.
Figure 1. Schematic model of the regular hexagon.
Under linear elastic behavior a linear stress-strain relationship is understood and terms such as $E$ can be defined; then, from the simple geometrical calculation and the elastic beam theory, the modulus of elasticity can be expressed in terms of the cell wall $E_s$ and a relative density.

$$E = c_1 E_s \rho_{rel}^3$$  \hspace{1cm} [1]

where

$$\rho_{rel} = \frac{\rho}{\rho_s}$$  \hspace{1cm} [2]

where:

$\rho$ = density of the foam
$\rho_s$ = density of the cell wall

Note: Easterling et al (1982) suggested $c_1 = 1.0$, because $E = E_s$ if $\rho_{rel} = 1.0$.

The onset of non-linear elastic behavior is derived from the assumption of the elastic buckling of the cell wall:

$$\sigma_y = c_2 E_s \rho_{rel}^3$$  \hspace{1cm} [3]

Where:

$c_2$ = constant

The non-linear elastic behavior is based on the elastic buckling theory, where the first buckling mode is assumed. This equation is derived in the original work by Gibson et al (1981), where the elastic buckling of the regular hexagon was used as a model (see Timoshenko, Gere 1961). If elastic buckling occurs first, and is immediately followed by plastic deformations, the value of the yield...
stress $\sigma_y$ is very close to critical stress resulting from the critical buckling load. Thus, $\sigma_y$ can be determined from Eq. [3]. The non-linear elastic behavior starts at point 1 in Figure 2.

If yielding of the cell wall occurs, then the foam yield stress is a function of a cell wall yield stress and relative density:

$$\sigma_y = c_3 \sigma_{yw} \rho_{rel}^{3/2} \quad [4]$$

where $c_3$ is a constant.

If non-linear elastic behavior is assumed to be responsible for further plastic collapse of the cell wall Eq. [5] can be derived (Ashby et al 1985).

$$\sigma = E_s \rho_{rel}^3 \left[ \frac{1 - \rho_{rel}^{1/3}}{1 - \left( \rho_{rel} \frac{1}{1 - \epsilon} \right)^{1/3}} \right]^3 \quad [5]$$

The existing theory assumes that there is no Poisson’s effect on the stress-strain relationship. However, Papirno, Mescall and Hower (1983) show, that the cross-sectional area change is an important factor in calculating stress for metallic materials subjected to high strains. Guess and Ericksen (1983) considered the Poisson’s ratio effect not to be negligible for Kevlar-Epoxy composites with porosity up to 20 %. They also found, that the Poisson’s ratio changes as a function of the strain. However, no theoretical explanation was given in their work. Casey (1987) measured the thickness and width change of yellow poplar flakes isolated from compressed flakeboard. A 38 % change in thickness and only 0.48 % change in width (measured after recovery) was reported.
Figure 2. Theoretical transverse compression stress-strain curve.
Effect of temperature and moisture content on mechanical properties of wood

Temperature and moisture content have a significant influence on wood properties. The relationships are discussed in many citations in classical wood literature (Kollmann 1952, Skaar 1972, Panshin and de Zeeuw 1980, Bodig and Jayne 1982 etc.). Generally, the modulus and strength of wood decrease with increasing moisture content up to the fiber saturation point, where curves describing this relationship became flat. A temperature increase causes a decrease of both elastic and strength properties. Zadorina and Tschernova (1965) studied the properties of Sibirian larch (Larix spp.) at 8, 15, 20, and 30% moisture content and 20, 50, 80 and 100°C. A 78% decrease in bending strength from conditions 20°C and 10% moisture content to 100°C and 10% moisture content was found. A similar relation was found for shearing strength. A linear relationship between temperature and those properties was proposed. Schneider (1971) studied properties of pine sapwood (Pinus spp.) and beechwood (Fagus silvatica) after temperature treatment up to 200°C. Modulus of elasticity of both species was considerably reduced after heating above 150°C. Similar results were obtained for crushing strength parallel to the fibers. A thermal degradation expressed as a loss of wood substance was reported to be higher for small specimens. Erinsch and Kulkevitsa (1981) studied the properties of birch (Betula spp.) and pine (Pinus spp.) after plasticization in boiling water and subsequent compression to 50% strain and drying at 20, 105 and 160 °C. No significant influence of post-compression drying temperature on the amount of 'spring-back' was found for birch. Skhirando et al (1983) conducted a chemical analysis of fiberboards pressed at 9.0, 30.2, and 68.1% moisture content of fibers and 190°C. The best mechanical characteristics were obtained for boards manufactured at 30.2% moisture content of fibers. At this moisture content, lignin contained the fewest number of hydroxylic groups. As a result, this lignin kept its thermosetting character and was involved in fiber bonding. The change of the mechanical properties can be irreversible depending upon environment conditions and time (Sumi 1978). Östman

Literature review 10
(1985) tested spruce (*Picea spp.*) in tension parallel to the grain up to 250°C. Moisture content varied from 0 to 30% when temperature was below 100°C and 0% for higher temperatures. Both modulus and strength decreased with increasing temperature for any given moisture content. From the rate of the decrease of E and strength Östman estimated the glass transition temperature to be between 200 and 250°C for dry wood. Ganowicz et al. (1980) studied the elastic properties of pine (*Pinus spp.*) up to 60°C under different moisture content conditions (0, 15 and 30%). A linear relationship between the E in bending and temperature with the correlation coefficients about R = -0.90 was found. Melcer (1985) assumed lignin and hemicellulose to be responsible for the change of the wood properties as a result of imposed temperature and moisture. The glass transition temperature of cellulose was assumed to be above 230°C and thus its contribution to wood softening was low. According to this theory the lignin-hemicellulosis matrix can become a viscous fluid independently of the celluloses behavior. Hillis and Rosza (1985) applied torsion to relate the torsional stiffness to changing temperature up to 80°C and moisture content 30%. For *Pinus radiata* three regions of accelerated rates of change in twist angle under a constant torque were found. The first softening point was defined at the temperature T = 60-70°C and was attributed to the hemicelluloses; the second point was at T = 90-110°C and was attributed to lignin. They also defined a third point between 70-90°C where the rate of the change of the twist angle decreased. Differences in glass-transition temperature for wood chemical components were found responsible for those points. Babicki et al. (1977) found that temperatures up to 50°C did not produce any significant changes in mechanical properties of beech (*Fagus spp.*) wood heated in water. Scharr (1986) applied torsion to spruce (*Picea spp.*) and oak (*Quercus spp.*) at a temperature range from 20 - 70 °C and moisture content 6 - 9 %. Temperature had a significant influence on creep compliance in torsion. Khmelidze (1986) tested pine (*Pinus spp.*) and larch (*Larix spp.*) in tension and compression parallel to the fibers at temperatures up to 230°C. It was found that wood behaved as an elastic material independently of applied temperature.

Ladomersky and Pajtik (1987) considered the slope of the plateau of the stress-strain relationship (see Figure 2, point 1-2) as a quantitative measure of the degree of plasticization. The influence of temperature and different chemicals (including water) on transverse compression characteristics
was studied. However, slope of the plateau region had a high variability (7-33%), possibly because no density corrections were taken.
Material and Methods

Test specimens

To study the effect of specimen height on transverse compression, yellow poplar specimens, conditioned to 12% moisture content, were cut with a cross-section of 20x20 mm and heights of 3, 6, 10, 20, 30 and 40 mm. Commercially obtained poly(methyl methacrylate) (PMMA) samples were cut 20x20 mm in cross-section and 6, 10 and 18 mm high. These two materials represent a non-isotropic cellular material and a homogeneous, isotropic solid for comparison purposes.

To investigate the influence of temperature and moisture on $E$ and $\sigma_y$, specimens 20x20x20 mm were used. Samples were cut from the mature wood region of a quarter-sawn board. This enabled the suppression of the variability across the diameter. To eliminate the influence of the density, pairs of radial (RD) and tangential (TG) specimens were randomly assigned to groups. The number of specimens required was calculated using the following formula (Walpole and Myers 1985):

$$n = \left(\frac{s}{\Delta}\right)^2(z_{\alpha/2} + z_{\beta})^2$$  \[6\]

Where:
\( n = \) number of observations

\( s = \) estimated standard deviation

\( \Delta = \) error of the observed parameter

\( \alpha = \) probability of type I error

\( \beta = \) probability of type II error

\( z = \) value from the standard normal distribution

\( \alpha \) means the probability that the zero hypothesis will be rejected, although it is true. \( \beta \) represents the probability that zero hypothesis will be accepted, although it is false (see Walpole and Myers 1985).

Based on the results in the literature (Kollman 1952, Bodig and Jayne 1982) the following values were assumed:

- \( s = 8 \)
- \( \Delta = 12 \)
- \( \alpha = 0.05 \)
- \( \beta = 0.05 \)

From the standard normal distribution tables and Eq. [6] we obtain \( z_{a/2} = 1.96 \), \( z_{\beta} = 1.65 \) and \( n = 6 \). Thus 6 replications were used for each group. This implies that for a property having a mean value of 50 units and 16% variability, the prediction of the population mean based on 6 observations will be accurate within ±12 units (assuming \( \alpha \) and \( \beta \) errors are 5%). A goal of this study is a qualitative assessment rather than quantitative, therefore, we assume that \( n = 6 \) gives sufficient information.
To test the high strain behavior of thin specimens, flakes of approximately 1 mm in thickness and 20x20 mm in cross-section were cut and conditioned to the required moisture content. To suppress the density influence on the properties, flakes from different locations within a growth increment were randomly assigned to groups.

For estimating the density effect on yield stress, specimens of different thickness were cut along a bar of 20x20 mm cross-section in such a way that there was always different amounts of early and latewood present. This yielded a wide variation of densities from the same tree. This technique eliminated the influence of variability in wood structure, but allowed a variation in specimen thickness.

Note: In this work the density is expressed as a ratio of the dry weight and the wet volume. This ratio is equivalent to specific gravity using the SI systems of units (Siau 1971). We will use the term density with units g/cm³.

**Compression apparatus**

The compression testing apparatus is shown in Figures 3 and 4. A universal hydraulic testing machine (MTS) was used for compression tests. Heated platens (4) were firmly attached to the load cell (1) and machine table (7). The loading block consisted of heated aluminum (4) which was directly in contact with the specimen. The surface of this plate was ground smooth to within 0.025 mm. A thermocouple was placed beneath the surface of the heated block to monitor the temper-
1 - load cell
2 - cooling manifold
3 - steel block
4 - heated platen
5 - specimen
6 - steel block
7 - machine table
8 - strain measurement reference
9 - LVDT

Figure 3. Schematic diagram of compression testing apparatus.
Figure 4. Compression testing apparatus.
ature. To get a uniform heat distribution, three heating cartridges were located at the upper surface of the aluminum block. A steel block (3) was placed between the heated and cooled zone (2). An aluminum cooling manifold (2) enabled the load cell to remain within the calibrated temperature range. A stiff aluminum arm was placed between the cooling manifold and the load cell. A linear variable differential transformer (LVDT) was attached to the arm and placed against the strain measurement reference (8). Thus, the deformation of the load frame was excluded from the measurement. The horizontal and parallel position of the platens was adjusted with an accuracy of 0.05 mm.

A clip gage (see Figure 5) was designed to monitor the transverse deformation of the samples. Two steel arms (3) were soldered to the elastic bronze ribbon (2). A strain gage (1) was bonded to the tension and compression side. Sharp tips (4) on the arm ends prevented gage slippage on the specimen.

An HP 2000 Data Acquisition System was used to collect data. Time intervals between the readings were automatically adjusted with respect to the stress rate. The calibration curve for the clip gage is in Figure 6.

Temperature of the platens was electronically controlled with an accuracy ±1°C and time step 1 second. Two independent temperature control units, one for each plate, were used.

**Test procedure**

The strain rate was 6% per minute. This rate has been used by many researchers for tests on foams as well as wood (Easterling et al 1982, Ashby et al 1985). For the height effect study, samples were tested between 25x25x6 mm steel plates that were ground flat and parallel to within 0.025 mm.

To eliminate the error introduced by the inherent flexure in the testing apparatus, a compression test was conducted on the steel plates with no specimen present. The load-deflection relationship of the testing apparatus was found to be linear. Thus, the deflection attributable to the apparatus...
Figure 5. Clip gage for measuring of the expansion ratio.
Figure 6. Calibration curve for the clip gage.
could be easily subtracted from the overall deflection. This error is not significant for high strains beyond the proportional limit. However, it introduces a significant error in the measured deflection within the elastic region of thin specimens (Wolcott et al 1988).

For tests on yellow poplar 20 mm, cubes at different temperature and MC conditions, no steel plates were used and specimens were in direct contact with the heated surfaces. As the test time was approximately 10 minutes, there was a possibility of a moisture content change, especially at higher temperatures. In order to maintain a non-zero equilibrium moisture content above 100°C the environment surrounding the specimen must be pressurized. To retard the moisture escape, specimens were sealed into poly (vinylidene chloride) foil (SARAN Wrap 560) 0.15 mm thick. Using this procedure a moisture content of 8% at 115°C can be maintained. As higher temperatures and moisture content levels require higher pressures, they cannot be reached using this technique. Before the tests, specimens were equilibrated to the desired temperature and moisture content, when possible. For the temperature and moisture content of 225°C and 0% MC, dry specimens were preheated in an oven for 15 minutes at 225°C. A thermocouple was placed in the center of several specimens to verify that 15 minutes was sufficient time for the center of a 20 mm specimen to reach 225°C. To prevent heat conduction through the thermocouple wire, silicon rubber was used to insulate the part of the wire exposed to the heat.

The temperature and moisture combinations studied are shown in Table 2. As no quantitative relationship was sought, combinations where a glass transition temperature region could be expected were investigated.

The expansion ratio (which is equivalent to the Poisson's ratio in the densification region) was measured at 30°C and 12% moisture content on 3, 5, and 20 mm specimens (loaded in the radial direction). Flakes were placed between two 20x20 mm aluminum plates to avoid the contact of the gage arms with the surface of the heated platens.

Flake behavior at high strains was tested using the same procedure as the 20 mm specimens. As the polymer foil deformation can represent a significant error for the thin specimens, an aluminum foil was used to retard moisture escape. Specimens were wrapped in three layers of 0.05 mm thick foil. The deformation of the foil was neglected. This procedure was sufficient for temperatures
below 100°C as no pressurization was necessary. Most of the flake tests were conducted at 30°C or 0% MC.
Results and discussion

Effect of specimen height

The effect of specimen height was determined using an analysis of variance. A least significance difference test (LSD) was employed to determine the differences between treatment means at the 99% confidence level.

Despite corrections made for machine flexure, the apparent modulus ($E_{app}$) increased with specimen height for both yellow poplar and PMMA (Figure 8). For PMMA samples, $E_{app}$ increased for each specimen height tested. The $E_{app}$ of yellow poplar increased with heights less than 6 mm. No significant difference was noted for $E_{app}$ of specimens 12 mm and higher.

In addition to $E_{app}$, the yield stress ($\sigma_y$) and yield strain ($\epsilon_y$) were studied for yellow poplar. The effect of specimen height on $\sigma_y$ was statistically significant as determined through analysis of variance. However, no significant differences were noted among treatment means with the LSD test. In addition, no trend between the two variables was apparent (Figure 8). Whereas the effect of height on $\sigma_y$ may be statistically significant, the effect may, in reality, be small enough to ignore. The percent difference between the extreme treatment means was approximately 15%.
Figure 7. Normalized Young's modulus in compression as a function of specimen height for yellow poplar and poly(methyl methacrylate). Error bars denote 99% confidence interval.
Figure 8. Yield stress and strain as a function of specimen height for yellow poplar. Error bars denote 99% confidence interval.
Similar relationships of E with specimen height have been shown for wood, PMMA, and foams (Menges and Knipschild 1982). These corroborative findings suggest that this relationship is an anomaly of the testing technique and not a true material property unique to wood. However, from the findings above, no statement can be made on the origin of the phenomenon.

Under the higher strains (more than 20%), the tall specimens tend to buckle because of the non-uniform structure. The growth rings have a finite curvature which results in shear stress and moment development. As a result, no pure compression stress-state can be achieved and thus there is a non-uniform deformation. When high strain is imposed the specimen center tends to expand, whereas the restrained surface remains fixed. This leads to the additional shear and tensile stress development in the edge zone and consequently cracks occur. The tendency for crack development was observed even in 3 mm thick specimens (see Figure 9). The collapse of the cell wall can probably contribute to crack initiation. Electron micrographs were taken from the transverse surfaces of 20mm cubes and 1mm thick flakes. No change was observed on the cube before and after compression to $\sigma$, (see Figure 10). This coincides with the assumption of Gibson (1982) that the elastic buckling of the cell wall controls yielding of the specimen. Thus Eq. [1] can be used to express the $\sigma$. In Figure 11 a cross-section of the flake compressed to 60% strain at 0% moisture content and 125 °C is shown. One can recognize that fracturing of the cell wall occurs. This may not be the case at higher moisture contents or temperatures, when the cell wall may be highly plasticized.
Figure 9. Crack development in 3 mm specimen of Yellow poplar in radial direction.
Figure 10. Electron micrograph of the specimen surface before and after compression to $\sigma_y$: yellow poplar 20mm cube in radial direction.
Figure 11. Electron micrograph of the flake compressed to 60% strain.
Effect of surface quality and load eccentricity

This effect was discussed in the paper of Wolcott et al (1988). A simple model was proposed, where surface roughness or non-parallelism are modelled as triangular (Figure 12). Surface quality may have a significant influence on E. For example, for a specimen with a height of 5 mm, a roughness of only 0.05 mm gives a 45% drop in E. Due to the natural structure of wood it is apparent that the influence of the surface quality cannot be eliminated. In Figure 13 the effect of surface roughness on E for different thicknesses is shown. One can recognize that the trends are similar to the experimental results presented in Figure 7. The asperities are totally collapsed when the stress is equal to $\sigma_y$. Therefore, the $\sigma_y$ exhibited by the specimen is independent of the specimen height as is seen in the experimental results. This also implies that $\varepsilon_y$ will vary with the inverse of the E. Thus, if thin specimens or flakes are tested in compression, the values of E and $\varepsilon_y$ do not reflect the real material property, however $\sigma_y$ is not affected by the specimen height.

Effect of density

Density has a significant influence on transverse compression properties. In Figure 14, $\sigma_y$ is plotted against $\rho^3$ in the radial direction for yellow poplar. This result was obtained from the test described above, where samples were cut along the bar. When influences such as moisture content or structural differences are suppressed, there is a high correlation ($R^2 = 0.99$) between those two properties. This shows a very good agreement with the proposed theory. As E does not have a significant

Results and discussion
Figure 12. Model of compression specimen with (a) surface roughness and (b) non-parallel mating platen-surfaces.
Figure 13. Predicted ratio of apparent modulus (E_{app}) to actual modulus (E): expressed as a function of specimen height (h) for different surface roughnesses (r).

Results and discussion
influence on the foam model at high strain (this will be discussed further), the relationship between 
ρ and E was not studied and results from the literature were employed (Bodig and Jayne 1982).  
The analysis of variance was used to detect the statistical differences between the densities for indi-
vidual treatments means. Results are summarized in Tables 1 to 5 and Figures 15-19.  
The asterisk under the values in Tables 1, 3-5 means, that there in no statistical difference between 
the treatments. As the radial and tangential specimens were paired, the rank of the values has the 
same order for both directions except treatments 2 and 3. There is, however, no significant differ-
ence between the radial and tangential densities.  
The total average for all treatments and directions was ρav = 0.456 g/cm³ with the COV = 6 % 
which coincides with values reported in the literature for mature wood (Core et al 1978) and is 
somewhat higher than the average values taken for the whole tree (Bodig and Jayne 1982, Haygreen 
and Bowyer 1982).

Effect of the load direction

The influence of the load direction on E and σ, for all treatments was studied. A significant dif-
ference between the radial and tangential directions was found for all treatments tested (see Table 
2). A t-test on an α = 0.01 level was employed. Mean values of E and σ, for the tangential direction 
were significantly lower. This is possibly caused by the difference in the wood structure. Rays 
running in the radial direction represent longitudinal elements which cause higher stiffness in this 
direction (Bodig and Jayne 1982). The influence of the rays probably offsets the fact that in the 
radial direction the weaker earlywood zones deform more (earlywood and latewood are in series) 
which would imply less stiffness in the radial direction.
Figure 14. Density cubed - yield stress relationship for transverse compression of yellow poplar in radial direction.
Table 1. LSD test on differences between $\rho$ for radial and tangential direction for yellow poplar 20 mm cubes ($\alpha = 0.05$).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>2</th>
<th>8</th>
<th>5</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ [g/cm$^3$]</td>
<td>0.479</td>
<td>0.475</td>
<td>0.468</td>
<td>0.468</td>
<td>0.460</td>
<td>0.437</td>
<td>0.437</td>
<td>0.425</td>
</tr>
</tbody>
</table>

Radial direction

<table>
<thead>
<tr>
<th>Treatment</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
<th>6</th>
<th>4</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ [g/cm$^3$]</td>
<td>0.480</td>
<td>0.473</td>
<td>0.470</td>
<td>0.468</td>
<td>0.467</td>
<td>0.430</td>
<td>0.429</td>
</tr>
</tbody>
</table>

Tangential direction

Asterisks in the line mean that there is no statistical difference between treatments.
Table 2. Modulus and yield stress at different moisture content and temperatures for yellow poplar 20mm cubes.

<table>
<thead>
<tr>
<th>Trt.</th>
<th>n</th>
<th>MC</th>
<th>T</th>
<th>Dir</th>
<th>$\sigma_y$</th>
<th>S</th>
<th>E</th>
<th>$\rho$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>-</td>
<td>%</td>
<td>°C</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>0.468</td>
<td>0.022</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>8</td>
<td>30</td>
<td>rd</td>
<td>8.99</td>
<td>1.20</td>
<td>680</td>
<td>91.3</td>
<td>0.468</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>8</td>
<td>30</td>
<td>tg</td>
<td>4.81</td>
<td>0.51</td>
<td>377</td>
<td>37.7</td>
<td>0.468</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>60</td>
<td>rd</td>
<td>8.70</td>
<td>1.15</td>
<td>550</td>
<td>96.0</td>
<td>0.479</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>60</td>
<td>tg</td>
<td>4.87</td>
<td>0.61</td>
<td>250</td>
<td>34.5</td>
<td>0.480</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>100</td>
<td>rd</td>
<td>6.11</td>
<td>0.50</td>
<td>404</td>
<td>50.7</td>
<td>0.460</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>100</td>
<td>tg</td>
<td>3.54</td>
<td>0.37</td>
<td>274</td>
<td>30.7</td>
<td>0.473</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12</td>
<td>30</td>
<td>rd</td>
<td>6.42</td>
<td>0.51</td>
<td>478</td>
<td>43.8</td>
<td>0.437</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12</td>
<td>30</td>
<td>tg</td>
<td>3.55</td>
<td>0.28</td>
<td>227</td>
<td>27.3</td>
<td>0.429</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>rd</td>
<td>7.27</td>
<td>0.54</td>
<td>537</td>
<td>72.0</td>
<td>0.468</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>tg</td>
<td>4.02</td>
<td>0.31</td>
<td>225</td>
<td>44.7</td>
<td>0.470</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>20</td>
<td>30</td>
<td>rd</td>
<td>4.79</td>
<td>0.39</td>
<td>376</td>
<td>63.9</td>
<td>0.438</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>20</td>
<td>30</td>
<td>tg</td>
<td>2.54</td>
<td>0.21</td>
<td>143</td>
<td>27.8</td>
<td>0.430</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>20</td>
<td>60</td>
<td>rd</td>
<td>3.04</td>
<td>0.25</td>
<td>240</td>
<td>10.6</td>
<td>0.425</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>20</td>
<td>60</td>
<td>tg</td>
<td>1.68</td>
<td>0.29</td>
<td>98</td>
<td>22.8</td>
<td>0.428</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0</td>
<td>225</td>
<td>rd</td>
<td>2.36</td>
<td>0.50</td>
<td>254</td>
<td>28.9</td>
<td>0.475</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0</td>
<td>225</td>
<td>tg</td>
<td>1.47</td>
<td>0.24</td>
<td>131</td>
<td>16.3</td>
<td>0.467</td>
</tr>
</tbody>
</table>
Table 3. LSD test on differences between $E$ for radial and tangential direction for yellow poplar 20 mm cubes ($\alpha = 0.05$).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>2</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>6</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>E [MPa]</td>
<td>680</td>
<td>550</td>
<td>537</td>
<td>478</td>
<td>404</td>
<td>376</td>
<td>254</td>
<td>240</td>
</tr>
</tbody>
</table>

Radial direction

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>3</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>E [MPa]</td>
<td>377</td>
<td>274</td>
<td>250</td>
<td>227</td>
<td>226</td>
<td>143</td>
<td>131</td>
<td>98</td>
</tr>
</tbody>
</table>

Tangential direction

Asterisks in the line mean that there is no statistical difference between treatments.
Table 4. LSD test on differences between normalized moduli for radial and tangential direction for yellow poplar 20 mm cubes ($\alpha = \ldots$

<table>
<thead>
<tr>
<th>Radial direction</th>
<th>Treatment</th>
<th>1</th>
<th>4</th>
<th>5</th>
<th>2</th>
<th>6</th>
<th>3</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E/p^3$</td>
<td>6659</td>
<td>5755</td>
<td>5242</td>
<td>5108</td>
<td>4506</td>
<td>3951</td>
<td>3133</td>
<td>2380</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tangential direction</th>
<th>Treatment</th>
<th>1</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>8</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E/p^3$</td>
<td>3777</td>
<td>2872</td>
<td>2621</td>
<td>2297</td>
<td>2174</td>
<td>1812</td>
<td>1311</td>
<td>1265</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

Asterisks in the line means that there is no statistical difference between treatments.
Table 5. LSD test on differences between normalized yield stress for radial and tangential direction for yellow poplar 20 mm cubes.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_p / \rho^3 )</td>
<td>87.4</td>
<td>79.4</td>
<td>77.5</td>
<td>71.2</td>
<td>64.8</td>
<td>57.3</td>
<td>39.7</td>
<td>22.0</td>
</tr>
<tr>
<td>Radial direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Treatment</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>5</th>
<th>3</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_p / \rho^3 )</td>
<td>48.8</td>
<td>44.7</td>
<td>44.1</td>
<td>38.8</td>
<td>33.6</td>
<td>32.4</td>
<td>21.7</td>
<td>14.4</td>
</tr>
<tr>
<td>Tangential direction</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>*</td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
<td></td>
</tr>
</tbody>
</table>

Asterisks in the line mean that there is no statistical difference between treatments.
Figure 15. Densities for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table 2.
Figure 16. Modulus for 20 mm yellow poplar cubes in radial direction for the treatments listed in Table 2.
Figure 17. Modulus for 20 mm yellow poplar cubes in tangential direction for the treatments listed in Table 2.
Figure 18. Yield stress for 20 mm yellow poplar cubes in radial direction for the treatments listed in Table 2.
Figure 19. Yield stress for 20 mm yellow poplar cubes in tangential direction for the treatments listed in Table 2.
Temperature and moisture content effect

The effect of moisture content and temperature was studied separately for both directions. Results are summarized in Table 2. A least square difference test on means was employed and results are given in Tables 3-5 and Figures 18 and 19. Again, an asterisk under the mean value means that there is no statistical difference for the tested level. One can recognize that there is an identical trend in the moisture and temperature effect for both directions. It can be also noticed that some treatments have the same effect. In general, a high moisture content has a similar effect as a high temperature (e.g. for $E/\rho^3$ MC = 20% and T = 60°C gave statistically not different results from the treatment with MC = 0% and T = 225°C). This can be explained by the effect of bound water which causes the plasticization of the cell wall. Bound water interacts with the secondary linkage between adjacent polymer molecules and as a result, the wood becomes softer. High temperature causes greater molecular motion of chemical components in wood which causes further softening. The moisture and temperature influence is generally known and has been studied mostly for properties along the fibers (Kollmann 1969, Bodig and Jayne 1982). Results presented coincide with findings of other researchers for directions parallel to the fibers.

The influence of different treatments on $\sigma_f$ and modulus is shown in Figures 20 and 21.

Lateral expansion during the compression.

As mentioned in the literature review, there are rather contradictory opinions on the expansion of materials subjected to high strains. To clarify this problem, several tests on different specimen thicknesses were conducted and expansion was measured. Typical relationships are shown in Figures 22-25. The solid line represents the stress-strain relationship, whereas the dashed line shows
Figure 20. Normalized yield stress for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table: Yield stress divided by the density cubed.
Figure 21. Normalized modulus for 20 mm yellow poplar cubes in radial and tangential direction for the treatments listed in Table 2: Modulus divided by the density cubed
the expansion coefficient as a function of strain. In Figure 22 a typical test result for a 20 mm cube is shown. One can recognize that the expansion ratio is approximately 0.4-0.45 in the linear region, which is the value that is usually reported for the Poisson’s ratio. However, after $\sigma_y$ was reached, the rate of the strain in the load direction was much higher than in the lateral direction, which caused a rapid decrease of the expansion coefficient. Tests on 20 mm specimens could not be conducted beyond the strains of 0.2, because of the cracks mentioned earlier.

In Figures 23 and 24 typical curves for 5 mm and 3 mm specimen are shown. A similar shape as for the 20 mm specimen can be observed. However, the surface restraint caused a faster decrease and a lower value of the expansion ratio in the elastic region. For high strains, the value of the expansion ratio levels off and becomes approximately constant. The initial decrease of the expansion ratio was most likely caused by the surface roughness of the specimen as well as nonparallel surfaces. Thus the initial measured deformation in the load direction was too high as a result of the surface effects. As load increased, the specimen started to deform in the lateral direction. This can be observed as an increase of the Poisson’s ratio in the linear region. This increase, however could not take place in thin specimens, because of the strong influence of the surface restraint. The constant value of the expansion ratio means that there is a proportional relationship between axial and transverse strain. A curve for a flake is shown in Figure 25. This low value of the expansion ratio, which remained constant during the entire test period was most likely caused by the surface restraint, which did not allow the surface of the specimen to deform in the lateral direction. Since the thickness is very low the whole cross-section is under the influence of the surface restraint and the resulting deformation is small.

Measurements of the thickness and width of the flakes immediately after opening of the compression apparatus indicate a value of the expansion ratio between 0.1-0.15. This shows that flakes recover more in the thickness direction. Similar results were obtained for 3 mm radial specimens, where the post-test expansion ratio was between 0.2 and 0.25, whereas values obtained during compression testing were below 0.1.
Figure 22. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x20 mm.
Results and discussion

Figure 23. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x5 mm.
Figure 24. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x21 mm.
Figure 25. Strain-expansion ratio relationship for yellow poplar in radial direction for specimen measuring 20x20x1 mm.
The situation during the hot pressing of flakeboard, however, may be more complex. At the beginning of the pressing procedure the lateral expansion of a flake can be relatively high, because of low frictional forces caused by the normal stress. Also, the expansion ratio in the elastic region (Poisson's ratio) is high. When load increases, the friction increases and results in higher restraining forces. Thus, a decrease in the expansion ratio can be expected. As can be noticed from Figures 22-25, the load causing the yielding is high enough to suppress the specimen expansion. Therefore, if this load is achieved during a hot pressing process, a similar response can be expected, providing that friction between flakes will not be substantially different from friction of wood on steel.

Based on the observations above, Eq. [5] can be modified to incorporate the width change, which influences the volume and density change. The expansion change can be incorporated into the model using a simple approach of volume change. The model is in Figure 26. The lateral expansion in the fiber direction is neglected. Thus, volume change can be expressed as the area change.

The relative density \( \rho'_{\text{rel}} \) after axial strain \( \varepsilon \) is:

\[
\rho'_{\text{rel}} = \frac{\rho_{\text{rel}}}{1 - \varepsilon}
\]

The lateral expansion causes the increase of the area by the quantity \( \mu \varepsilon \), which expresses the volume increase due to the expansion. Plates movement causes reduction of the area by \( \varepsilon_1 \varepsilon_2 \). Since \( \mu = \varepsilon_1 / \varepsilon_2 \), \( \varepsilon_1 \varepsilon_2 = \varepsilon^2 \mu \). Then Eq. [5] becomes:

\[
\sigma = \sigma_y \left[ \frac{1 - \rho_{\text{rel}}^{1/3}}{\rho_{\text{rel}}^{1/3}} \right]^{1/3} \left[ 1 - \left( \frac{\rho_{\text{rel}}^{1/3}}{1 - \varepsilon + \mu \varepsilon - \mu \varepsilon^2} \right)^{1/3} \right]
\]

\[
\sigma_y = E_s \rho_{\text{rel}}^3
\]

Where

\( \mu = \) expansion ratio

Results and discussion 53
Figure 26. Influence of the non-zero expansion coefficient on the density without surface restraint: load in the direction of $\varepsilon_1$. 

Results and discussion
The model in Figure 26 can be improved by assuming that there is no lateral displacement at the boundary of the specimen which is in contact with the platens (see Figure 27). A parabolic shape of the displacement is assumed:

\[ \sigma = \sigma_y \left[ \frac{1 - \rho_{rel}^{1/3}}{1 - \left( \frac{\rho_{rel}}{1 - \varepsilon + 2/3\mu \varepsilon - \mu \varepsilon^2} \right)^{1/3}} \right]^3 \]  \[7\]

The influence of different values of the expansion ratio on the relative density is shown in Figure 28. One can recognize that the non-zero expansion coefficient suppresses the rate of the change of the relative density. Based on the previous discussion we assume the expansion ratio is not negligible.

In Figure 29 Eq. [7] is used for prediction of the stress-strain relationship for treatments listed in Table 2. Average values were taken for the prediction. Differences in curve shapes demonstrate the influence of the moisture content and temperature. Curves are plotted for constant value of density for all treatments.

**Testing of flakes**

The results for flakes tested in the radial and tangential directions are in Table 6. Despite the random assignment to groups, there was a significant difference between densities in the tangential flakes for different treatments. Since the flake thickness was smaller than the annual ring thickness, there is a high variance in the flake density and a larger sample size is required to account for the differences in the density. Radial flakes, however, were consistent in the density because of ap-
Figure 27. Influence of the non-zero expansion coefficient on the density with surface restraint: load in the direction of $\epsilon_1$.  

Results and discussion
Results and discussion

Figure 28. Relative density as a function of strain and expansion ratio.
Figure 29. Prediction of compression behavior for yellow poplar for treatments listed in Table 2: density is constant for all curves.
proximately the same amount of earlywood and latewood present. To suppress the density influence, a statistical comparison of \( \sigma_3 \) was carried out for the ratio \( \sigma_3/\rho^3 \). Results for radial compression (tangential flakes) are presented in Table 7. One can recognize that as for cubes, there is no significant difference between several treatments, which implies the interaction between temperature and moisture content. The total average density was \( \rho = 0.417 \text{ g/cm}^3 \) with the standard deviation \( s = 0.049 \text{ g/cm}^3 \).

Since the value of \( E \) calculated from the tests of thin specimens has no meaning (because of surface effects), no evaluation was conducted on this quantity. A typical stress-strain curve for a flake is shown in Figure 25. The low slope in a linear region expresses the influence of the surface quality and other factors mentioned earlier.

**Foam model applied to thin specimens**

**Sensitivity study**

The sensitivity of the proposed model to different parameters was studied. As wood exhibits a high variability of properties, knowledge of the influence of different parameters on model behavior helps to determine which property should be studied closer. The influence of \( E \), relative density \( (\rho_{rel}) \), \( \sigma_y \), and \( \mu \) on the model was investigated. One parameter was varied while the others remained constant. Based on the literature and previous results, the following constant values were chosen:

- \( \rho_{rel} = 0.3 \)
- \( E = 680 \text{ MPa} \)
- \( \sigma_y = 9.0 \text{ MPa} \)
Table 6. Yield stress and density for yellow poplar flakes.

<table>
<thead>
<tr>
<th>Trt.</th>
<th>n</th>
<th>MC</th>
<th>T°C</th>
<th>Dir</th>
<th>$\sigma_s$</th>
<th>s</th>
<th>$\rho$</th>
<th>g/cm$^3$</th>
<th>s</th>
<th>g/cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>12</td>
<td>30</td>
<td>rd</td>
<td>6.65</td>
<td>1.86</td>
<td>0.393</td>
<td>0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>10</td>
<td>30</td>
<td>rd</td>
<td>11.36</td>
<td>3.23</td>
<td>0.450</td>
<td>0.054</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>rd</td>
<td>7.75</td>
<td>2.45</td>
<td>0.483</td>
<td>0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>tg</td>
<td>5.03</td>
<td>1.28</td>
<td>0.447</td>
<td>0.038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>20</td>
<td>30</td>
<td>rd</td>
<td>5.05</td>
<td>0.97</td>
<td>0.382</td>
<td>0.048</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>20</td>
<td>30</td>
<td>tg</td>
<td>3.83</td>
<td>0.33</td>
<td>0.462</td>
<td>0.040</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>02</td>
<td>125</td>
<td>rd</td>
<td>7.47</td>
<td>3.13</td>
<td>0.383</td>
<td>0.055</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0</td>
<td>225</td>
<td>rd</td>
<td>4.98</td>
<td>0.677</td>
<td>0.492</td>
<td>0.032</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>0</td>
<td>225</td>
<td>tg</td>
<td>3.11</td>
<td>1.19</td>
<td>0.458</td>
<td>0.010</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7. LSD test on differences between normalized yield stress in radial compression of yellow poplar flakes (α = 0.05).

<table>
<thead>
<tr>
<th>Treatment</th>
<th>5</th>
<th>2</th>
<th>1</th>
<th>4</th>
<th>3</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_x/\rho^3$</td>
<td>129.0</td>
<td>124.1</td>
<td>110.6</td>
<td>92.5</td>
<td>90.4</td>
<td>41.8</td>
</tr>
</tbody>
</table>
| Asterisks in the line mean that there is no statistical difference between treatments.
The influence of E is given in Figure 30. The range of the values tested was 100-800 MPa with 100 MPa steps. One can recognize that E does not have a significant influence on the non-linear part of the curve. This is a consequence of the model, because the \( \sigma_y \) (which is identical to the critical buckling stress) controls the high strain behavior. The linear region is obviously controlled by the E (see Figure 30).

The relative density was varied from 0.1 to 0.5 g/cm\(^3\). The higher the density the shorter the plateau region. Since the value of the relative density in the model is to the third power, the influence is logically high (see Figure 31).

The \( \sigma_y \) (Figure 32) also significantly changes the length of the plateau. Lower \( \sigma_y \) causes a longer plateau and higher slope in the densification region, whereas higher \( \sigma_y \) acts in the opposite direction. If \( \sigma_y \) is high, there is a higher stress required to collapse the cell wall and the stress-strain curve becomes steeper in the plateau region. For a low \( \sigma_y \) value, most of the cell walls collapse at a lower stress and a greater portion of the densification takes place in the plateau region. Thus, the slope becomes steeper because the relative density approaches unity.

As mentioned earlier, the expansion ratio causes shifting of the stress-strain curve towards high strains, because the rate of the increase of the relative density decreases with increasing \( \mu \) (Figure 33). There is no large difference between \( \mu = 0.0 \) and \( \mu = 0.1 \), however, neglecting this value can cause a difference in the prediction. It remains a question, what expansion ratio should be applied in a real situation (e.g. flakeboard manufacturing). Based on our tests, we estimate \( \mu \) to be less than 0.1.

When the high variability of wood properties is taken into account, as well as the fact that those properties are not independent random variables, a high scatter of the stress-strain curves can be obtained from experiments. As the density and \( \sigma_y \) are the most dominant controlling factors and
Results and discussion

Figure 30. Influence of E on the proposed model. E varies between 100 and 800 MPa.
Figure 31. Influence of relative density on the proposed model: $\rho_{rel}$ varies between 0.1 and 0.5 g/cm$^3$. 
Figure 32. Influence of the yield stress on the proposed model: $\sigma_y$ varies between 5 and 15 MPa.
as there is a strong correlation between them, the shape of the curves can be remarkably different even within a species. Also, if thin specimens are tested (flakes), the variability in the density and $\sigma_y$ is higher because of different amounts of earlywood and latewood. The influence of these parameters on the predictive force of the model was studied as a stochastic process.

Quality of the model

To study the predictive quality of the model, theoretical stress-strain diagrams were compared with the results obtained from flake testing. Each measured point on the experimental curve was compared with the prediction based on Eq.[7]. Values of $\mu = 0.05$ and $\rho_s = 1.5$ g/cm$^3$ were taken as constant in Eq. [7]. The error of the prediction was calculated using the formulas:

\[
Error_1 = \sum_{i=1}^{N} \left| \frac{\sigma_{y,\text{theor}}(i) - \sigma_{y,\text{exp}}(i)}{\sigma_{y,\text{exp}}(i)} \right| \times 100
\]

\[
Error_2 = \sum_{j=1}^{m} \left| \frac{\varepsilon_{y,\text{theor}}(j) - \varepsilon_{y,\text{exp}}(j)}{\varepsilon_{y,\text{exp}}(j)} \right| \times 100
\]

Where:

$\sigma_{y,\text{exp}}(i)$ = experimental value of $\sigma_y$ for i-th point

$\sigma_{y,\text{theor}}(i)$ = predicted value of $\sigma_y$ for i-th point

$\varepsilon_{y,\text{exp}}(j)$ = experimental value of strain at yield stress for j-th point

Results and discussion
Figure 33. Influence of the expansion ratio on the proposed model: \( \mu \) varies between 0.0 and 0.5.
\[ \varepsilon_{y_{\text{theor}}}(j) = \text{predicted value of strain at yield stress for j-th point} \]

Eq. [8] was used when the slope of the stress-strain curve was less than 1.0, whereas Eq. [9] was used when the slope was greater than 1.0. Errors were calculated for each point on the experimental curve. A typical stress-strain diagram, with prediction, is shown in Figure 34. The portion predicting the elastic behavior is based on the results from testing the 20 mm cubes (see Table 2). The dashed line running parallel to the x-axis represents the amount of the strain associated with a thin specimen. The prediction begins at the yield stress. There Error1 and Error2 are equal to zero. Error1 represents the average vertical shifting and Error2 represents the horizontal shifting of the theoretical curve against the experimental stress-strain relationship.

The results for the 60 specimens tested under conditions described in Table 6 were used for the comparison. Error1 was 5.6% with a 62.0% coefficient of variation, Error2 was 11.7% with a 58.0% coefficient of variation. These results can be considered as very encouraging with respect to the initial assumptions made for the model. Except for the value of the density of the cell wall, all other values were determined by test and used for prediction. The average E value from Table 2 was used for the linear part. As was shown in the sensitivity study, E has negligible influence on the prediction beyond the yield stress. The possible variance in the density of the cell wall may have a significant effect on the difference between the experiment and prediction. The mean cell wall density \( \rho_s \) is reported to be from 1.44 to 1.56 g/cm³ (Panshin and de Zeeuw 1980, Haygreen and Bowyer 1982). Since the \( \rho_{rel} \) is raised to the third power, a small change in \( \rho_s \) causes a relatively large change in the prediction (see chapter on sensitivity study). For example Error1 = 17.4% and Error2 = 11.8% for \( \rho_s = 1.54 \) was suppressed to 8.9% and 3.3% using \( \rho_s = 1.44 \). This shows the large influence of the \( \rho_s \) on prediction. As the values reported for the cell wall density are mean values, the individual measurements can vary even more. However, \( \rho_s \) is almost always an unknown quantity. Thus, the generally accepted value of 1.5 g/cm³ is recommended. Another source of error can by the presence of cracks in the cell wall during high strain deformation, a phenomenon that is not treated by the model.
Figure 34. Calculation of the error of prediction.
As mentioned earlier, the coefficient in Eq. [1] is assumed to be equal to 1.0. Then the coefficient $c_2$ can be expressed from the known $E$ and $\sigma_y$ as the strain at the yield point. The calculated values of $c_2$, as well as $E$, are given in Table 8. Since $E$ is a function of the moisture content and temperature conditions, the same is true for $E$, (see Eq. [1]). However, $c_2$ does not have to follow the same logic. A test of differences between $c_2$ for different treatments is shown in Table 9. One can recognize that a number of differences were detected. Duncan's multiple range test (Walpole 1985), however, did not detect any differences except for treatments 3 and 8 for the radial direction and 2 and 8 in the tangential and radial direction together. In the tangential direction Duncan's test gave the same result as Tukey's (LSD) test.

If $c_2$ is considered a constant independent of temperature, moisture content and direction, it has the value of 0.015 with a coefficient of variation 24%. The constant $c_2$ for the radial and tangential directions was 0.014 and 0.016 with a coefficient of variation 26% and 14%, respectively.
Table 8. Modulus of the cell wall and $c_2$ coefficient.

<table>
<thead>
<tr>
<th>Trt.</th>
<th>n</th>
<th>MC</th>
<th>T</th>
<th>Dir</th>
<th>$E_i$</th>
<th>s</th>
<th>$c_2$</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>-</td>
<td>%</td>
<td>°C</td>
<td>-</td>
<td>MPa</td>
<td>MPa</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>8</td>
<td>30</td>
<td>rd</td>
<td>22 472</td>
<td>3 183</td>
<td>0.0133</td>
<td>0.0016</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>8</td>
<td>30</td>
<td>tg</td>
<td>12 746</td>
<td>1 510</td>
<td>0.0128</td>
<td>0.0010</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>60</td>
<td>rd</td>
<td>17 240</td>
<td>4 321</td>
<td>0.0162</td>
<td>0.0033</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>8</td>
<td>60</td>
<td>tg</td>
<td>7 754</td>
<td>1 652</td>
<td>0.0196</td>
<td>0.0024</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>100</td>
<td>rd</td>
<td>13 334</td>
<td>5 040</td>
<td>0.0191</td>
<td>0.0095</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>8</td>
<td>100</td>
<td>tg</td>
<td>8 846</td>
<td>1 528</td>
<td>0.0129</td>
<td>0.0019</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12</td>
<td>30</td>
<td>rd</td>
<td>19 422</td>
<td>2 731</td>
<td>0.0135</td>
<td>0.0019</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12</td>
<td>30</td>
<td>tg</td>
<td>9 692</td>
<td>1 717</td>
<td>0.0157</td>
<td>0.0013</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>rd</td>
<td>17 692</td>
<td>2 314</td>
<td>0.0137</td>
<td>0.0012</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>12</td>
<td>60</td>
<td>tg</td>
<td>7 336</td>
<td>1 395</td>
<td>0.0183</td>
<td>0.0032</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>20</td>
<td>30</td>
<td>rd</td>
<td>15 208</td>
<td>2 656</td>
<td>0.0129</td>
<td>0.0016</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>20</td>
<td>30</td>
<td>tg</td>
<td>6 116</td>
<td>1 281</td>
<td>0.0182</td>
<td>0.0026</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>20</td>
<td>60</td>
<td>rd</td>
<td>10 575</td>
<td>1 075</td>
<td>0.0127</td>
<td>0.0009</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>20</td>
<td>60</td>
<td>tg</td>
<td>4 268</td>
<td>1 100</td>
<td>0.0174</td>
<td>0.0015</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0</td>
<td>225</td>
<td>rd</td>
<td>8 032</td>
<td>1 032</td>
<td>0.0092</td>
<td>0.0014</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>0</td>
<td>225</td>
<td>tg</td>
<td>4 424</td>
<td>1 012</td>
<td>0.0114</td>
<td>0.0026</td>
</tr>
</tbody>
</table>
Table 9. LSD test on differences between $c_2$ for radial and tangential direction ($\alpha = 0.05$).

<table>
<thead>
<tr>
<th></th>
<th>Radial direction</th>
<th>Tangential direction</th>
<th>Tangential and radial direction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Treatment</strong></td>
<td>3 2 5 4 1 6 7 8</td>
<td>2 5 6 7 4 3 1 8</td>
<td>2 5 3 7 4 1 8</td>
</tr>
<tr>
<td>$c_2$</td>
<td>0.0191 0.0162</td>
<td>0.0196 0.0183</td>
<td>0.0175 0.0162</td>
</tr>
<tr>
<td></td>
<td>0.0137 0.0135</td>
<td>0.0182 0.0174</td>
<td>0.0160 0.0156</td>
</tr>
<tr>
<td></td>
<td>0.0133</td>
<td>0.0157</td>
<td>0.0150</td>
</tr>
<tr>
<td></td>
<td>0.0129</td>
<td>0.0129</td>
<td>0.0145</td>
</tr>
<tr>
<td></td>
<td>0.0127</td>
<td>0.0128</td>
<td>0.0131</td>
</tr>
<tr>
<td></td>
<td>0.0092</td>
<td>0.0114</td>
<td>0.0100</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

Asterisks in the line means that there is no statistical difference between treatments.
Wood as a natural material exhibits high variability of properties. This variability is controlled by many factors starting from the variability between species to the variability within species, trees, position in the tree, etc. (Panshin and de Zeeuw, 1980). Whereas the variability between species can be eliminated by defining properties for each species, the lower sources of variability cannot be neglected. As has been shown by a number of researchers (Kollmann 1951, Panshin and de Zeeuw 1980, Bodig and Goodman 1972, Bodig and Jayne 1982) there is a relationship between the mechanical properties and the density, or between the mechanical properties mutually. In other words, these properties are not statistically independent random variables. This means that any particular variable cannot take an arbitrary value, but must be correlated with others. Bodig and Goodmann (1972) related the elastic properties of different woods to the density. Bodig and Jayne (1982) report the relationship for strength properties in the following form:

\[ Y = a \rho^b \]  \[10\]

Where:

- \( Y \) = property as a dependent variable
- \( \rho \) = density at dry weight and wet volume
Whereas $b$ is close to 1.0 for $E$ in bending (linear relationship), $b = 2.25$ was found for $\sigma_y$. A correlation coefficient $R^2 = 0.96$ for $\sigma_y$ and $R^2 = 0.965$ for $E$ were reported. Bendtsen and Galligan (1979) studied the properties in transverse compression for different hardwood and softwood species. An $R^2 = 0.89$ for $\sigma_y$ was reported. Although some skewness was found in the data, a normal model for the analysis was used.

Contrary to these results, Schniewind and Gammond (1978) found very low correlation between density and mechanical properties for diger pine ($Pinus spp.$) ($R^2 = 0.05-0.38$). However, experiments were performed on green wood and the coefficient of variation of moisture content was between 23 and 63%. This obviously overlapped the influence of the density despite the fact that most of the tests were conducted above the fiber saturation point.

In recent studies of mechanical properties of wood, a log-normal or Weibull (Weibull 1955) distribution were applied. However, the normal distribution approach still gives a good prediction for many applications. In fact, most of the data available are described by the first two moments (mean value and standard deviation) which fully defines only a normal distribution. Some studies also indicate no substantial differences between distributions applied to the data (O’Halloran et al 1988). It is understood that the normality assumption does not have to fully reflect the reality. However, it has been shown that a normal distribution gives satisfactory results for the properties discussed (Bodig and Jayne 1982).

To account for the natural variability, a stochastic model is introduced to simulate the scatter of the data. Thus, the expected reliability of the prediction can be studied. The simulation procedure can be used, not only to predict the single specimen behavior, but also to estimate the parameters for the population.

Stochastic Model
Theoretical basis

The Monte Carlo procedure described by Rubinstein (1981) was used to simulate the data and the stress-strain relationship. A polar method (Knuth 1969) was involved to generate random numbers. The algorithm is as follows:

1. Generate two sets of independent random numbers $U_1, U_2$, and $V_1, V_2$

   \[ V_1 = 2U_1 - 1 \]
   \[ V_2 = 2U_2 - 1 \]

   $V_1$ and $V_2$ are uniformly distributed between -1 and +1 (random points uniformly distributed inside the unit circle). $U_1$ and $U_2$ are distributed between 0 and 1.

2. Calculate a square of the magnitude of the random vector with components $V_1$ and $V_2$

   \[ S = V_1^2 + V_2^2 \]

   if $S \geq 1$ then repeat steps 1 to 2

3. Compute two normally distributed variables

   \[ X_1 = V_1 \sqrt{\frac{-2 \ln S}{S}} \]
   \[ X_2 = V_2 \sqrt{\frac{-2 \ln S}{S}} \]

4. To obtain the dependent normally distributed random variables the following formulae are used:

   \[ Y_1 = \bar{x}_1 + s_1 X_1 \]
\[ Y_2 = \bar{x}_2 + s_2 \left( R X_1 + \sqrt{1 - R^2} X_2 \right) \]  

Where:
- \( \bar{x} \) = mean value (mean estimate)
- \( s \) = standard deviation
- \( R \) = correlation coefficient

This procedure can be used to generate any number of correlated normally distributed random variables (see Knuth 1969). Here we assume that \( E \) and \( \sigma \) are positively correlated with the specific gravity. Thus the procedure can be easily modified to generate the third statistically dependent variable. The listing of the simulation program is enclosed in Appendix B.

As a result of the simulation procedure, a set of values of \( E(I), \sigma(I), \rho(I) \) is obtained (I is an I-th variable). For each set a theoretical stress-strain curve can be plotted and a statistical evaluation can be carried out. Thus, if statistical parameters for those three variables are known (mean value, standard deviation and coefficient of correlation) prediction characteristics can be studied. This enables an estimate of the statistical characteristics describing the response of wood to the applied strain. Therefore, the average densification stress and densification strain, as well as distribution characteristics and confidence intervals, can be predicted.
Monte Carlo simulation

For the Monte Carlo simulation the following parameters of random variables were taken as inputs:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Mean</th>
<th>s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>g/cm³</td>
<td>0.420</td>
<td>0.050</td>
</tr>
<tr>
<td>σ₂</td>
<td>MPa</td>
<td>9.0</td>
<td>1.2</td>
</tr>
<tr>
<td>μ</td>
<td></td>
<td>0.1</td>
<td>const.</td>
</tr>
</tbody>
</table>

These values represent the estimate of parameters for 30°C and 12% moisture content tests. The correlation coefficient between the density and σ₂ was 0.9, and between density and E it was 0.7. A thousand values of ρ, σ₂, and E were generated. Scattergrams for mutual relationships between variables are shown in Figures 35-37.

The resulting correlation coefficient for the E-σ₂ relationship was 0.62. The probability density functions are plotted along with the histograms representing two random variables are in Figures 38-40. A statistical analysis of the means as well as a regression analysis, were performed.

The impact of the variability on the prediction of the transverse compression behavior is illustrated in Figure 41. The solid line represents the average expected response for 1000 measurements.

The two dashed lines nearest to the solid line represent the 99% confidence interval for the means. This indicates that if random samples of 1000 replications were drawn from the population, then 99% of the means would lie between these two lines. A single measurement, however, can fall into a much broader range, which is represented by the two dashed lines furthest from the solid line.
Figure 3.5. Density-yield stress relationship from the Monte Carlo simulation.
Figure 39. Histogram and probability density function for yield stress: 1000 values.
Figure 40. Histogram and probability density function for modulus: 1000 values.
Figure 41. Expected stress-strain relationship for yellow poplar as a result of Monte Carlo simulation: 1000 values.
(99% confidence interval for a single measurement). The values for the 99% confidence intervals are in Table 10.

The Monte Carlo procedure can be used to simulate the uniformity of the densification across the density gradient in a flakeboard. One can recognize, that for a given level of stress some flakes will be highly densified, whereas some can still be elastically deformed. Thus this procedure offers, not only the qualitative image about the high strain deformation mechanism, but also the possibility of quantifying the expected density gradient. However, the vertical and horizontal flake distributions should be studied, which is beyond the scope of this work.
Table 10. Results from Monte Carlo simulation.

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1000</td>
<td>0.29</td>
<td>0.42</td>
<td>0.55</td>
</tr>
<tr>
<td>E</td>
<td>1000</td>
<td>403</td>
<td>682</td>
<td>961</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1000</td>
<td>5.33</td>
<td>8.96</td>
<td>12.59</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Min</th>
<th>Mean</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density</td>
<td>1000</td>
<td>0.415</td>
<td>0.420</td>
<td>0.423</td>
</tr>
<tr>
<td>E</td>
<td>1000</td>
<td>675</td>
<td>682</td>
<td>690</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>1000</td>
<td>8.86</td>
<td>8.96</td>
<td>9.06</td>
</tr>
</tbody>
</table>
Conclusions

Compression behavior of small yellow poplar specimens in the transverse direction was studied. A testing procedure for thin wood specimens was developed and the application of the theory developed for rigid plastic foams was studied. Based on the analysis of the experimental results, the following conclusions can be drawn:

1. The modulus of elasticity in transverse compression is dependent upon specimen height and increases with the specimen height. This is a geometrical phenomenon rather than material.

2. The yielding of wood under transverse compression is most likely caused by initial buckling of the cell wall and does not show any trend relating to the specimen height.

3. There is a lateral expansion during the high strain deformation. The expansion coefficient decreases rapidly at the yield point and remains approximately constant. The value of \( \mu \) is slightly larger for thicker specimens, presumably because the constraint at the surfaces has an influence.

4. A modified foam theory gives good agreement with the experimental data for the temperature and moisture contents studied. Yield stress, density and \( E \) must be known as input parameters.
The model is quite robust against variation in $E$, but sensitive to $\rho$ and $\sigma_y$ variation. Both quantities are relatively easy to measure.

5. Yield stress and $\rho$ are random statistically dependent variables. A Monte Carlo procedure can be used to simulate the transverse compression behavior.


Appendix A. Two dimensional model of cellular material

Here a theory developed by Gibson et al (1982) is presented in abbreviated form. A regular hexagon is assumed.

The two dimensional model is based on the assumption that a foam can be modelled using an average cell representing the overall foam response to loading. The derivation is based on the geometry in Figure 42. The relative density of the hexagon can be expressed as the ratio of the area of the cell wall and the total area of the hexagon:

\[
\frac{\rho}{\rho_s} = \frac{(2 + h/l) t/l}{2 \cos \theta (h/l + \sin \theta)}
\]  

[A1]

where: 
- \(h\) = height of the hexagon side 
- \(l\) = length of the hexagon side 
- \(t\) = thickness of the cell wall 
- \(\theta\) = angle between the hexagon wall and the horizontal 
- \(\rho\) = foam density 
- \(\rho_s\) = wall density
Figure 42. Schematic model of a regular hexagon.
For a regular hexagon with $\theta = 30^\circ$ and $h=1$ Eq. [A1] becomes:

$$\rho_{rel} = \frac{\rho}{\rho_s} = \frac{2}{\sqrt{3}} \frac{l}{l}$$

Where: $\rho_{rel} = \text{relative density}$

The linear elastic response is modelled as the elastic bending of the cell wall. Shear deformations and axial extension or compression is neglected and small deformations are assumed.

If a stress $\sigma_1$ is applied, then from the equilibrium condition the vertical reaction $C = 0$ (see Figure 43) and the bending moment is:

$$M = \frac{1}{2} P l \sin \theta$$

Where:

$$P = \sigma_1 (h + l \sin \theta) b$$

The cell wall deflects by:

$$\delta = \frac{P l^3 \sin \theta}{12E_s l}$$

Strain from the component $\delta \sin \theta$ parallel to the direction $l$ is:

$$\varepsilon_1 = \frac{\delta \sin \theta}{l \cos \theta} = \frac{\sigma_1 (h + l \sin \theta) b l^2 \sin \theta^2}{12E_s l \cos \theta}$$

Then the Young’s modulus is:

Appendix A. Two dimensional model of cellular material
Figure 43. Elastic bending
\[ E_1 = \frac{\sigma_1}{\varepsilon_1} \quad [A7] \]

For a regular hexagon with \( \theta = 30^\circ \), moduli for directions 1 and 2 are the same. Then from Eq. [A7] we get:

\[ E_1 = E_2 = E = \frac{4}{\sqrt{3}} E_s \frac{l^3}{I^3} = \frac{3}{2} \rho_{rel}^3 \quad [A8] \]

The elastic buckling of the cell wall allows further large deformations at almost constant load. It has been observed that the buckling mode is similar to that illustrated in Figure 44 (Gibson et al 1982). Beam BE acts as an Euler column constrained by rotational springs at both ends. For the critical buckling load we get:

\[ P_{cr} = \frac{n^2 \pi^2 E_s}{l^2} \quad [A9] \]

Where: \( n^2 \) = the end constraint factor, which is a function of the stiffness of the rotational spring

The buckling stress can be then expressed:

\[ \sigma_{el} = \frac{P_{cr}}{2 lb \cos \theta} = \frac{E_s t^3 n^2 \pi^2}{6 lh^2 \cos \theta} \quad [A10] \]

For a regular hexagon with \( h = 1 \) and \( n = .343 \) Eq. [A10] becomes:

\[ \sigma_{el} = c_2 E_s \rho_{rel}^3 \quad [A11] \]
Figure 44. Elastic buckling

Appendix A. Two dimensional model of cellular material
If the material of the cell wall exhibits a plastic yield point, the structure will collapse plastically if a plastic moment is reached. Since we assume a regular hexagon, we can take any of the two directions for the derivation. If the loading in the direction 1 is taken (see Figure 45) then the work done by the force is:

\[ P = \sigma_1 (h + l \sin \theta) b \]  \hspace{1cm} [A12]

Plastic work done at hinges A,B,C,D equals:

\[ 4M_p \phi \geq 2\sigma_1 (h + l \sin \theta) \phi l \sin \theta \]  \hspace{1cm} [A13]

Where:

- \( \theta \) = plastic rotation
- \( M_p \) = fully plastic moment of the cell wall in bending

\[ M_p = \frac{1}{4} \sigma_y b t^2 \]

\( \sigma_y \) = yield stress of the cell wall material

\[ (\sigma_1)_{pl} \leq \frac{\sigma_y t^2}{2 l (h + l \sin \theta) \sin \theta} \]  \hspace{1cm} [A14]

Calculating the maximum moment in the beam and equating it to \( M_p \) we get:

\[ M_{\text{max}} = \frac{1}{2} \sigma_1 (h + l \sin \theta) b l \sin \theta \]  \hspace{1cm} [A15]

from which:
Figure 45. Plastic collapse
\[(\sigma_1)_{pl} \geq \frac{\sigma_y l^2}{2 l (h + l \sin \theta) \sin \theta}\]  [A16]

Then from Eq. [A14] and [A16] we obtain:

\[(\sigma_2)_{pl} = \frac{\sigma_y l^2}{2 l^2 \cos \theta^2}\]  [A17]

Again, for regular hexagon:

\[(\sigma_1)_{pl} = \frac{2}{3} \frac{l^2}{l^2} \sigma_y\]  [A18]

\[\sigma_{pl} = c_3 \sigma_{rel}^{3/2}\]

Ashby et al (1985) postulated that the elastic buckling was controlling phenomenon in foam non-linear behavior. The relative density after a nominal compressive strain \(\varepsilon\) is expressed as:

\[\frac{\rho(\varepsilon)}{\rho_s} = \frac{\rho}{\rho_s} \left( \frac{1}{1 - \varepsilon} \right)\]  [A19]

Where: \(\frac{\rho}{\rho_s}\) = initial relative density

Densification is complete, when \(\rho(\varepsilon)/\rho_s = 1\) and when the strain is:

\[\varepsilon_f = 1 - \frac{\rho}{\rho_s}\]

Length of the edges which are about to buckle is given by:

\[l(\varepsilon) = l_0 \frac{1 - (\frac{\rho(\varepsilon)}{\rho_s})^{1/3}}{1 - (\rho/\rho_s)^{1/3}}\]  [A20]
Eq. [A20] is based on a distribution of cell edge-lengths and angles and no closer explanation is given by the authors. Using Eq.[A19], Eq.[A20] can be expressed:

$$l(e) = l_0 \frac{1 - (\frac{\rho_{rel}}{(1 - e)})^{1/3}}{1 - \rho_{rel}^{1/3}}$$  \[A21\]

Appendix B. Geometrical approach to the derivation of the expansion ratio for cellular material

Here a simple geometrical approach is used to derive the expansion ratio as a function of the axial strain. Geometry of the regular hexagon is shown in Figure 46. To keep the derivation simple, plastic hinges are assumed. When the hexagon is loaded in the direction 1, the axial displacement can be expressed as:

$$\delta_1 = \ell \left( \frac{1}{2} - \sin \alpha \right)$$  \[A22\]

Lateral displacement as:

$$\delta_2 = \ell \left( \cos \alpha - \frac{\sqrt{3}}{2} \right)$$  \[A23\]

and strains $\varepsilon_1$ and $\varepsilon_2$ as:

$$\varepsilon_1 = \frac{-\delta_1}{l}$$  \[A24\]
Figure 46. Schematic model of lateral expansion of the hexagon.
From which the expansion ratio $\mu$ can be obtained:

$$
\mu = \frac{2(\cos \alpha - \frac{\sqrt{3}}{2})}{\sqrt{3} \left(\frac{1}{2} - \sin \alpha\right)} \tag{425}
$$

This approach defines $\mu$ up to $\varepsilon_1 = 0.5$. At this strain the hexagon becomes a quadrilateral and further assumptions should be made.

If the hexagon is loaded in the direction 2, following derivation can be carried out:

$$
\varepsilon_1 = -\frac{\delta_1}{L\sqrt{3}/2} \tag{426}
$$

$$
\alpha = \arcsin\left(\frac{h}{l}\right) = \arcsin\left(\frac{\sqrt{3}}{2} (1 - \varepsilon_1)\right) \tag{427}
$$

Lateral strain $\varepsilon_2$ is:

$$
\varepsilon_2 = \frac{2(l\cos \alpha - l/2)}{2 \frac{l}{2} + 2} = \cos \alpha - \frac{1}{2} \tag{427}
$$

and expansion coefficient is:

$$
\mu = -\frac{\varepsilon_2}{\varepsilon_1} \tag{428}
$$

For a cylindrical cell we assume no cell wall elongation during the deformation (no change in circumference). An elliptical shape after deformation is assumed. The circumference of the ellipse can be expressed by approximate formula (Rektorys 1981):

Appendix B. Geometrical approach to the derivation of the expansion ratio for cellular material
Figure 47. Model of cylindrical cell
\[ o \approx \pi \left[ 1.5(a + b) - \sqrt{ab} \right] \] \hspace{1cm} [A29]

Where:

- \( a \) - semi-major axis
- \( b \) - semi-minor axis

Let \( a = b = 1.0 \) before deformation and apply strain \( \varepsilon_1 \) in the direction of \( b \)-axes. The new axis length is:

\[ b' = 1 - \varepsilon_1 \] \hspace{1cm} [A30]

and circumference after straining is:

\[ o' \approx \pi \left[ 1.5(a' + 1 - \varepsilon_1) + \sqrt{a'(1 - \varepsilon_1)} \right] \] \hspace{1cm} [A31]

Eq. [A31] is solved numerically for \( a' \) under the condition \( o' = o \). Results for different models are shown in Figure 49. If a buckling of the ring is assumed first, the axial and radial deflections can be expressed (Timoshenko, Gere 1961) as:

\[ \Delta b = P \frac{b^3}{4EI} \left( \pi - \frac{8}{\pi} \right) \] \hspace{1cm} [A32]

\[ \Delta a = P \frac{a^3}{2EI} \left( \frac{4}{\pi} - 1 \right) \]

which for \( a = b = 1.0 \) leads to \( \mu = 0.918 \).

Appendix B. Geometrical approach to the derivation of the expansion ratio for cellular material
Figure 48. Expansion ratio as a function of the axial strain for different cell models

Appendix B. Geometrical approach to the derivation of the expansion ratio for cellular material
Appendix C. Computer program for Monte Carlo simulation
This program uses Monte Carlo procedure to simulate the compression behavior of wood. Density, yield stress and MOE are dependent random variables. To use this program following quantities must be known:

1. Mean values for all three variables
2. Standard deviations for all three variables
3. Correlation coefficient between density and yield stress and density and MOE.
Normal distribution of the variables is assumed.

Program is based on:


Appendix C. Computer program for Monte Carlo simulation

```
! SIMULATION OF TRANSVERSE COMPRESSION BEHAVIOR OF WOOD

! Bohumil Kasal    November 1988    UPI&SU

! This program uses Monte Carlo procedure to simulate the compression behavior of wood.
! Density, yield stress and MOE are dependent random variables.
! To use this program following quantities must be known:
! 1. Mean values for all three variables
! 2. Standard deviations for all three variables
! 3. Correlation coefficient between density and yield stress and density and MOE.
! Normal distribution of the variables is assumed.
! Program is based on:

*--------------------------------------------------------------*
* RANDOMIZE
* DIM C$[3],N$[60]
*--------------------------------------------------------------*
* READING DATA FOR THE SIMULATION
*--------------------------------------------------------------*
Seed=RND+10000*(10+RND)
RANDOMIZE Seed
DATA 10,.7   ! N=number of runs Rho=correlation coeff. between the density and MOE
READ N,Rho
DATA .42,.042   ! Mean1=density Sd1=standard deviation
READ Mean1,Sd1
DATA .98   ! Mean2=MOE Sd2=standard deviation
READ Mean2,Sd2
DATA 9,1.2,.9   ! Mean3=Yield stress Sd3=standard deviation
READ Mean3,Sd3,Rho3   ! Rho3=correlation coeff. between the density and yield stress
GOTO 458

*--------------------------------------------------------------*
Appendix C. Computer program for Monte Carlo simulation
```
IF (N<1) OR (Rho<=0) OR (Rho>=1) THEN RETURN

CALL Set_up(N,Seed,Mean1,Sd1,Mean2,Sd2,Rho,Mean3,Sd3,Rho3)
DISP "PROGRAM COMPLETE."
END

SUB Set_up(N,Mean1,S1,Mean2,S2,Rho)
RQD
QLLOCQTE Y1(1:N),Y2(1:N)
Repeat: Repeat=3
FOR I=1 TO Repeat
Seed=RND*2^31-1
CALL Random_norm2(N,Mean1,Mean2,S1,S2,Rho,Y1(*),Y2(*))
Mean(N,Y1(*),Mean)
CALL Standard_dev(N,Y1(*),Sd)
PRINT USING 535;Seed,Mean,Sd
IMAGE 1,"Seed","12D," Mean1":","3D.3D," Std.Dev1":","3D.3D
CALL Mean(N,Y2(*),Mean)
CALL Standard_dev(N,Y2(*),Sd)
PRINT USING 555;Mean,Sd
IMAGE 14X," Mean2":",3D.3D," Std.Dev2":",3D.3D
CALL Correlation(N,Y1(*),Y2(*),CcoeFP)
PRINT USING 570;CcoeFP
IMAGE 7X,"Correlation Coefficient":",3D.3D
NEXT I
SUBEND
SUB Mean(N,X(*),Mean)
RQD
Mean=0
FOR I=1 TO N
Mean=Mean+X(I)
NEXT I
Mean=Mean/N
SUBEND
SUB Variance(N,Mean,X(*),Variance)
RQD
Variance=0
FOR I=1 TO N
Variance=Variance+(X(I)-Mean)^2
NEXT I
Variance=Variance/(N-1)
SUBEND
SUB Standard_dev(N,X(*),Sd)
RAD

Appendix C. Computer program for Monte Carlo simulation
690 IF N<2 THEN SUBEXIT
695 Temp_=_0
700 Sumx=Temp_
705 Sumxx=Temp_
710 FOR I=1 TO N
715 Xi=X(I)
720 Sumx=Sumx+Xi
725 Sumxx=Sumxx+Xi*Xi
730 NEXT I
735 Sd=SQR((Sumxx—Sumx*Sumx/N)/(N·1))
740 SUBEND
745 SUB Random_normal(N,X(*))
755 RAD
765 Baddata=N MOD 2 OR (N<1)
770 IF NOT Baddata THEN Begin
775 Error: PRINT FNLin$(2);"ERROR IN SUB Random_normal."
780 IF N MOD 2 THEN PRINT "N MUST BE EVEN."
790 PRINT "N: ";N
791 CONTROL 2,2;0
795 PAUSE
796 CONTROL 2,2;1
800 GOTO 765
805 Begin: FOR I=1 TO N/2
806 RANDOMIZE
810 P1: U1=RND
815 U2=RND
820 U1=2*U1—1
825 U2=2*U2-1
830 P2: S=U1*U1+U2*U2
835 P3: IF S>=1 THEN P1
840 P4: Temp=SQR(-2*LOG(S)/S)
845 X(I*2-1)=U1*Temp
850 X(I*2)=U2*Temp
855 NEXT I
860 SUBEND
865 SUB Random_norm2(N,Mu1,Mu2,Mu3,Sigma1,Sigma2,Sigma3,Rho,Rho3,Y1(*),Y2(*),Y3(*))
875 RAD
880 DIM Temp_(1:2)
885 IF (Sigma1>0) AND (Sigma2>0) THEN 910
890 PRINT FNLin$(2);"ERROR IN Random_norm2. SD<=0.";FNLin$(2)
891 CONTROL 2,2;0
895 PAUSE
896 CONTROL 2,2;1
900 GOTO 885

Appendix C. Computer program for Monte Carlo simulation 113
910 IF NOT (N MOD 2) THEN 945
915 IF N<>1 THEN CALL Random_normal(N-1,Y1(*))
920 IF N<>1 THEN CALL Random_normal(N-1,Y2(*))
921 IF N<>1 THEN CALL Random_normal(N-1,Y3(*))
925 CALL Random_normal(3,Temp_(*))
930 Y1(N)=Temp_(1)
935 Y2(N)=Temp_(2)
936 Y3(N)=Temp_(3)
940 GOTO 960
945 CALL Random_normal(N,Y1(*))
950 CALL Random_normal(N,Y2(*))
951 CALL Random_normal(N,Y3(*))
953 Temp=SQR(1-Rho*Rho)
954 Temp1=SQR(1-Rho3*Rho3)
956 FOR I=1 TO N
957 Y2(I)=Nu2+Sigma2*(Rho*Y1(I)+Temp*Y2(I))
958 Y3(I)=Nu3+Sigma3*(Rho3*Y1(I)+Temp1*Y3(I))
965 NEXT I
975 FOR I=1 TO N
980 Y1(I)=Sigma1*Y1(I)+Mu1
985 NEXT I
990 SUBEND
995 SUB Correlation(N,X(*),Y(*),Coeff)
1000 RND
1010 Temp_=0
1015 Sumx=Temp_
1020 Sumy=Temp_
1025 FOR I=1 TO N
1030 Sumx=Sumx+X(I)
1035 Sumy=Sumy+Y(I)
1040 NEXT I
1045 Xmean=Sumx/N
1050 Ymean=Sumy/N
1055 Temp_=0
1060 Sumx=Temp_
1065 Sumy=Temp_
1070 Sum=Temp_
1075 FOR I=1 TO N
1080 Sumx=Sumx+(X(I)-Xmean)^2
1085 Sumy=Sumy+(Y(I)-Ymean)^2
1090 Sum=Sum+(X(I)-Xmean)*(Y(I)-Ymean)
1095 NEXT I
1100 Coeff=Sum/(SQR(Sumx)*SQR(Sumy))
1105 SUBEND
1110 SUB Set_up1(N,Seed,Hean1,Sd1,Mean2,Sd2,Rho,Mean3,Sd3,Rho3)
1120 RND
1125 !
1130 ALLOCATE X(1:N), Y(1:N), Z(1:N)
1131 Seed=(RND*1000)^(RND*10)
1135 RANDOMIZE Seed
1145 PRINT " PRINTER IS 1"
1150 CALL Rand0m_norm2(N,Mean1,Mean2,Mean3,Sd1,Sd2,Sd3,Rho,Rho3,X(*),Y(*),Z(*))
1151 CALL Foamplot(N,Mean1,Mean2,Mean3,Sd1,Sd2,Sd3,Rho,Rho3,X(*),Y(*),Z(*))
1155 PRINT " PRINTER IS 1"
1160 CALL Print_file(N,X(*),Y(*),Z(*))
1165 SUBEND
1170 SUB Print_file(N,X(*),Y(*),Z(*))
1175 PRINT " PRINTER IS 1"
1180 CALL Print_file(N,X(*),Y(*),Z(*))
1185 DIM CS(3),File$(20)
1190 LINPUT " WOULD YOU LIKE TO SAVE THE DATA SET? (Y/N)";CS
1195 CALL Yesno(CS,Ynflag)
1200 IF Ynflag=2 THEN SUBEXIT
1205 IF Ynflag=3 THEN 1435
1210 LINPUT " ENTER FILE NAME. E.G.: File:INTERNAL ?";File$
1215 ON ERROR GOTO Recovery
1220 ASSIGN @File1 TO File$
1225 OFF ERROR
1230 GOTO 1515
1235 Recovery: IF ERRN<>56 THEN Error
1240 CREATE BDAT File$,3*N*10/256+1
1245 ASSIGN @File1 TO File$
1250 OFF ERROR
1255 GOTO 1515
1260 LINPUT " FILE ALREADY EXISTS. OK TO WRITE ON IT? (Y/N)";CS
1265 CALL Yesno(CS,Ynflag)
1270 IF Ynflag=2 THEN 1435
1275 IF Ynflag=3 THEN 1515
1280 ON ERROR GOTO Error1
1285 OUTPUT @File1;X(*)
1290 OUTPUT @File1;Y(*)
1295 OUTPUT @File1;Z(*)
1300 ASSIGN @File1 TO *
1305 OFF ERROR
1310 DISP " DATA HAS BEEN STORED SUCCESSFULLY. ";
1315 SUBEXIT
1320 Error: PRINT FNline$(2);"ERROR";ERRN;"IN SUB Print_file."
1325 OFF ERROR
1330 GOTO 1435
1335 Error1: IF ERRN<>59 THEN Error
1340 PRINT FNline$(2);"SORRY, FILE IS NOT LARGE ENOUGH."
1635 PRINT "I SUGGEST THAT YOU CHOOSE A NEW NAME AND I WILL CREATE THE FILE FOR
YOU.";FNLin$(2)
1640 GOTO 1435
1645 SUBEND
1650 DEF FNLin$(INTEGER X)
1655 INTEGER I
1660 IF X=0 THEN RETURN CHR$(13)
1665 EO1$=CHR$(13)&CHR$(10)
1670 IF X<0 THEN EO1$=CHR$(10)
1675 ALLOCATE Rs(X*LEN(EO1$))
1680 Rs=""
1685 FOR I=1 TO X
1690 Rs=Rs&EO1$
1695 NEXT I
1700 RETURN Rs
1705 FNEND!
1710 DEF FNSpa$(INTEGER X)
1715 INTEGER I
1720 ALLOCATE Rs[(X+NXT X)]
1725 Rs=""
1730 FOR I=1 TO X
1735 Rs=Rs&CHR$(32)
1740 NEXT I
1745 RETURN Rs
1750 FNEND!
1760 SUB Yesno(C$,YnFlag)
1770 IF (C$[1,1]="Y") OR (C$[1,1]="y") THEN
1780 YnFlag=1
1790 ELSE
1800 IF (C$[1,1]="N") OR (C$[1,1]="n") THEN
1810 YnFlag=2
1820 ELSE
1830 YnFlag=3
1840 END IF
1841 END IF
1850 SUBEND
1860 SUB Foamplot(N,Mean1,Mean2,Mean3,Sd1,Sd2,Sd3,Rho,Rho3,X(*),Y(*),Z(*))
1870 !
1880 !
2000 !******************************************************************************
2001 !***** FOAMPLOT ***********
2002 !******************************************************************************
2005 !
2006 ! A foam theory developed by Gibson and Ashby is used to predict the!
2007 ! wood behavior under the perpendicular to the fibers direction. !
Stress-strain curve is derived as a function of the density.
Modulus of elasticity and yield stress can be also predicted, but it is more accurate to use these parameters as an input.
Unlike the original theory, this model considers the Poisson's Ratio non-zero value. Poisson's Ratio is expressed as a function of the strain (curve 1 and 2), constant (curve 3) and zero (4).
This is a significant difference from the original theory where this ratio was considered to be zero.

APPENDIX C.

Computer program for Monte Carlo simulation
LORG 6
MOVE 0,50
LDIR 90
LORG 6
LABEL "Stress (MPa)"
LDIR 0
MOVE 65,5
LORG 4
LABEL "Strain"
CSIZE 4,.5
MOVE 65,95
LORG 6
VIEWPORT 15,120,15,90
FRAME
WINDOW 0,Strain_max,0,Stress_max
AXES .01,Stress_max/10,0,0,5,2,3
CSIZE 4,.4
LORG 9
VIEWPORT 3,120,15,90
FOR I=0 TO Stress_max STEP Stress_max/10
MOVE .10,I
LABEL I
NEXT I
VIEWPORT 15,120,10,90
LORG 4
CSIZE 3.5,.5
FOR I=0 TO Strain_max STEP .10
MOVE I,0
LABEL I
NEXT I
VIEWPORT 15,120,15,90
!
!*****************************************************************
! PLOTTING
!*****************************************************************
Mu=.2
FOR I=1 TO N
St=.01
PRINT "DENS=";X(I),"Y.S.=";Z(I),"MOE=";Y(I)
MOVE 0,0
DRAW Z(I)/Y(I),Z(I)
X(I)=X(I)/1.5
Eps1=(Z(I)/Y(I))
K=0
FOR Eps=(Z(I)/Y(I)) TO 1 STEP St

Appendix C. Computer program for Monte Carlo simulation
2984       K=K+1
3023       Sigma=Z(I)*((1-X(I)^((1/3))/(1-X(I)^((1/3))/(1-Eps+(2/3)*Eps*Mu-
       Mu+Eps^2)^((1/3)))^3
3024       IF Sigma>=35 THEN St=.001
3026       IF K=1 THEN Delta=Z(I)-Sigma
3027       Sigma=Sigma+Delta
3028       IF Sigma>=50 OR Sigma<0 THEN Next
3033       DRAW Eps,Sigma
3043       NEXT Eps
3053       NEXT I
3075   !oooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooooo
3076       RETURN
3078       SUBEND
The vita has been removed from the scanned document.