Chapter 2
Effective Girder Moment of Inertia

2.1 Overview

Accurate evaluation of a floor for vibrations cannot be accomplished if large errors are made in predicting the floor's stiffness. These errors result in inaccurate predictions of frequency and peak acceleration, factors which affect an evaluation. The current AISC/CISC Design Guide 11 provisions for effective moment of inertia of joist-girders and girders with joist seats as horizontal shear connections suggest that the effective moment of inertia is equal to

$$I_{eff} = I_g + (I_{comp} - I_g)/4 \tag{2.1}$$

This relationship was developed from single bay tests with K-series joists 24-30 in. on center and supported by hot-rolled girders. Refinement of Equation 2.1 is the primary purpose of the series of tests reported in this chapter. The relationship must be improved to account for recent trends in floor design.

These trends include the use of larger and/or compositely constructed joists, usually with wide spacing. Joists of this type typically have seats taller than the seats associated with smaller joists at narrower spacing. One or both of these factors may be the cause of several recent unexpected problems. For example, two recently constructed floor bays, 40 ft x 40 ft and 28 ft x 40 ft, were problematic. For both floors, LH-series joists at the girder quarter points were used to span the 40 ft direction, providing only one shear connector in each half span. The floors were probably more flexible than predicted causing them to vibrate at a lower frequency. The tall seats and wide spacing of the joists used in these floors are likely causes for the problems. Identification and correction of this type of problem will be attempted using the results from several experimental tests and finite element models.

The experimental testing included a total of eight 30 ft x 8 ft "footbridges" with girders spanning the 30 ft dimension and joists spanning the 8 ft dimension. Four footbridges had W14x22 girders and four had 14G joist-girders. Supported joist spacing was 30, 60, or 90 in. and supported joist type was K-series or LH-series. For seven footbridges, bolted, welded, and reinforced seat-to-girder connections, described in Section 2.2 and shown in

Figures 2.1 and 2.2, were tested, providing three complete sets of data for each footbridge. The order of testing was reinforced (as constructed), bolted, then welded. The eighth footbridge was a special case in that tubing was added between each pair of joist seats in an attempt to attain full composite stiffness. Table 2.1 shows the details of each test configuration with hot-rolled girders, and Table 2.2 shows each configuration with joist-girders.

Static and dynamic tests were performed on each configuration. Girder effective moment of inertia, I_{eff}, was calculated using the load-deflection data from static testing. Dynamic tests were used to measure girder frequencies. The results from these tests are compared with Equation 2.1 and FEM results to assess accuracy. In addition, tests were done on the bolted case to measure seat stiffness in the horizontal direction, one test for each joist type. This data is compared with the stiffness used for FEM seat members.

Test Identification. Test configurations are identified by girder type, joist type, joist spacing, and seat connection type – Bolted, Welded, or Reinforced. For example, K-30-B is the floor using *K*-series joists on *30* in. centers and *B*olted. Tests with added 15 psf live loading have an L added at the end of the identification.

Table 2.1—Test Matrix for W14x22 Girders spanning 30 ft and spaced 8 ft on center.

Specimen	Date Poured	Date Tested	Joist	Seat Height (in.)	Seat Connection	Joist Spacing (in.)	Slab Depth (in.)	Deck Height (in.)	Concrete Strength (ksi.)
K-30-B	7/3/01	7/29/01	8K1	2.5	Bolted	30	4	0.6	3.3
K-30-BL	7/3/01	7/29/01	8K1	2.5	Bolted	30	4	0.6	3.3
K-30-W	7/3/01	8/17/01	8K1	2.5	Welded	30	4	0.6	3.3
K-30-WL	7/3/01	8/16/01	8K1	2.5	Welded	30	4	0.6	3.3
K-30-R	7/3/01	7/24/01	8K1	2.5	Reinforced	30	4	0.6	3.3
K-30-RL	7/3/01	7/29/01	8K1	2.5	Reinforced	30	4	0.6	3.3
K-60-B	4/24/01	5/14/01	8K1	2.5	Bolted	60	4	1.5	2.8
K-60-W	4/24/01	5/23/01	8K1	2.5	Welded	60	4	1.5	2.8
K-60-WL	4/24/01	7/12/01	8K1	2.5	Welded	60	4	1.5	2.8
K-60-R	4/24/01	5/24/01	8K1	2.5	Reinforced	60	4	1.5	2.8
LH-60-B	6/4/01	6/21/01	18LH550	5	Bolted	60	4	1.5	2.7
LH-60-W	6/4/01	6/26/01	18LH550	5	Welded	60	4	1.5	2.7
LH-60-WL	6/4/01	7/12/01	18LH550	5	Welded	60	4	1.5	2.7
LH-60-R	6/4/01	7/3/01	18LH550	5	Reinforced	60	4	1.5	2.7
LH-60-RL	6/4/01	7/9/01	18LH550	5	Reinforced	60	4	1.5	2.7
LH-90-B	6/7/01	7/2/01	18LH800	5	Bolted	90	5	2	3.1
LH-90-BL	6/7/01	7/10/01	18LH800	5	Bolted	90	5	2	3.1
LH-90-W	6/7/01	7/11/01	18LH800	5	Welded	90	5	2	3.1
LH-90-WL	6/7/01	7/11/01	18LH800	5	Welded	90	5	2	3.1
LH-90-R	6/7/01	7/1/01	18LH800	5	Reinforced	90	5	2	3.1

Note: L at end of specimen designation indicates a test with added 15 psf simulated live load

Table 2. 2—Test Matrix for 14G Joist-girders spanning 30 ft and spaced 8 ft on center.

Specimen	Date Poured	Date Tested	Joist	Seat Height (in.)	Seat Connection	Joist Spacing (in.)	Slab Depth (in.)	Deck Height (in.)	Concrete Strength (ksi.)
JG-K-30-B	8/13/01	9/24/01	8K1	2.5	Bolted	30	4	0.6	2.9
JG-K-30-BL	8/13/01	9/24/01	8K1	2.5	Bolted	30	4	0.6	2.9
JG-K-30-W	8/13/01	9/28/01	8K1	2.5	Welded	30	4	0.6	2.9
JG-K-30-WL	8/13/01	9/28/01	8K1	2.5	Welded	30	4	0.6	2.9
JG-K-30-R	8/13/01	9/5/01	8K1	2.5	Reinforced	30	4	0.6	2.9
JG-K-30-RL	8/13/01	9/5/01	8K1	2.5	Reinforced	30	4	0.6	2.9
JG-LH-60-B	7/10/01	8/15/01	18LH550	5	Bolted	60	4	1.5	2.8
JG-LH-60-BL	7/10/01	8/15/01	18LH550	5	Bolted	60	4	1.5	2.8
JG-LH-60-W	7/10/01	8/17/01	18LH550	5	Welded	60	4	1.5	2.8
JG-LH-60-WL	7/10/01	8/17/01	18LH550	5	Welded	60	4	1.5	2.8
JG-LH-60-R	7/10/01	8/2/01	18LH550	5	Reinforced	60	4	1.5	2.8
JG-LH-60-RL	7/10/01	8/2/01	18LH550	5	Reinforced	60	4	1.5	2.8
JG-LH-90-B	9/12/01	10/12/01	18LH800	5	Bolted	90	4	2	3.6
JG-LH-90-BL	9/12/01	10/12/01	18LH800	5	Bolted	90	4	2	3.6
JG-LH-90-W	9/12/01	10/17/01	18LH800	5	Welded	90	4	2	3.6
JG-LH-90-WL	9/12/01	10/17/01	18LH800	5	Welded	90	4	2	3.6
JG-LH-90-R	9/12/01	10/8/01	18LH800	5	Reinforced	90	4	2	3.6
JG-LH-90-RL	9/12/01	10/9/01	18LH800	5	Reinforced	90	4	2	3.6
JG-LH-90-T	12/19/01	1/4/02	18LH800	5	Tubes	90	4	2	2.4
JG-LH-90-TL	12/19/01	1/4/02	18LH800	5	Tubes	90	4	2	2.4

Note: L at end of specimen designation indicates a test with added 15 psf simulated live load

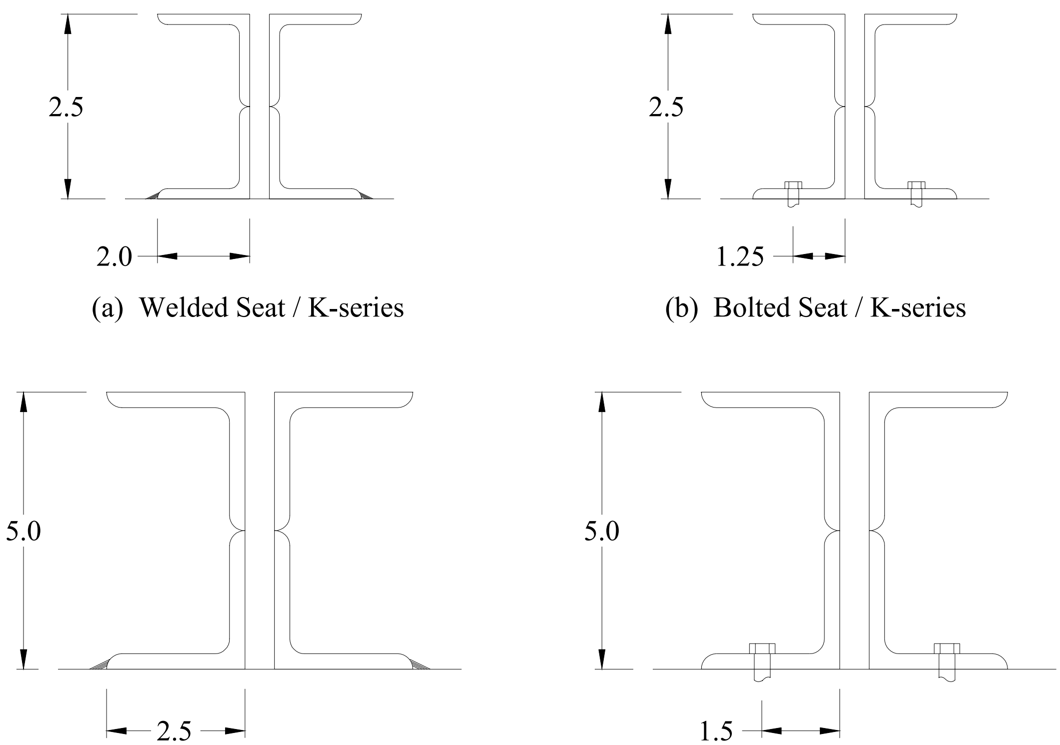
2.2 Construction of Test Specimens

The girder supports were 9 ft long, W18 or W21 sections placed perpendicular to the 30 ft span. Following girder placement, the joists were bolted in place, the horizontal bridging was welded under the joist chords, the decking was screwed to the joists, and pour stops were screwed around the decking exterior. The concrete was poured, covered with plastic, and kept moist while curing. Figure 2.2 shows a test specimen in its constructed state.

The first tests were done on the floors with reinforced seats. Figure 2.1 shows a typical reinforced seat. The bolts were also in place for these tests. For each reinforced seat connection, two flat bar reinforcements were welded at one end to the throat of the upper seat angle, and at the other end to the girder top flange. The thickness of the reinforcements was 1/4 in.+/- 1/16 in. For the second test configuration, the reinforcements were torch-cut, leaving the seats bolt-connected.

To make the third and final test configuration, the floor was jacked upward ½ in., then the seats were welded, to ensure that the welds would be active under initial deflection. At each seat, two welds were made, each 1 in. long and ¼ in. thick. The bolts and jacks were then removed for testing. Figure 2.2 shows typical bolted and welded seats.

Figure 2. 1—Reinforced Seats



(a) Welded Seat / K-series

(b) Bolted Seat / K-series

(c) Welded Seat / LH-series

(d) Bolted Seat / LH-series

Figure 2. 2—Bolted and Welded Seats

Figure 2. 3—Constructed Specimen

2.3 Experimental Testing Procedures

The methods for measuring test data are discussed in this section. Static and dynamic tests were performed on each configuration. Bolted seat stiffness tests were performed on each type of joist. The calculations made for comparing the test data to Equation 2.1 are discussed in Section 2.5.

2.3.1 Static Testing Procedures

Deflections were measured on the bare steel girders of each floor, and then on the partially composite sections, bolted, welded, and reinforced. The measurements were made with 0.001 in. precision dial gages placed three underneath each girder, one at midspan and one near each support. After placing the dial gages, weight was placed at midspan in 20 lb increments. Support and midspan deflections were recorded with 100, 200, 300, 400, 500, and 600 lb on each girder. The method of loading and midspan deflection measurement is depicted in Figure 2.4 and the dial gage location for support measurement is shown in Figure 2.5. The moments of inertia of the bare girder (I_{gm}) and the effective girder (I_{eff}) were calculated with the load-deflection data from the static testing.

A second value of I_{eff} was measured using 15 psf of distributed loading and the same method of deflection measurement as previous. For the loading, 20 lb-boxes were evenly distributed over the slab as shown in Figure 2.6.

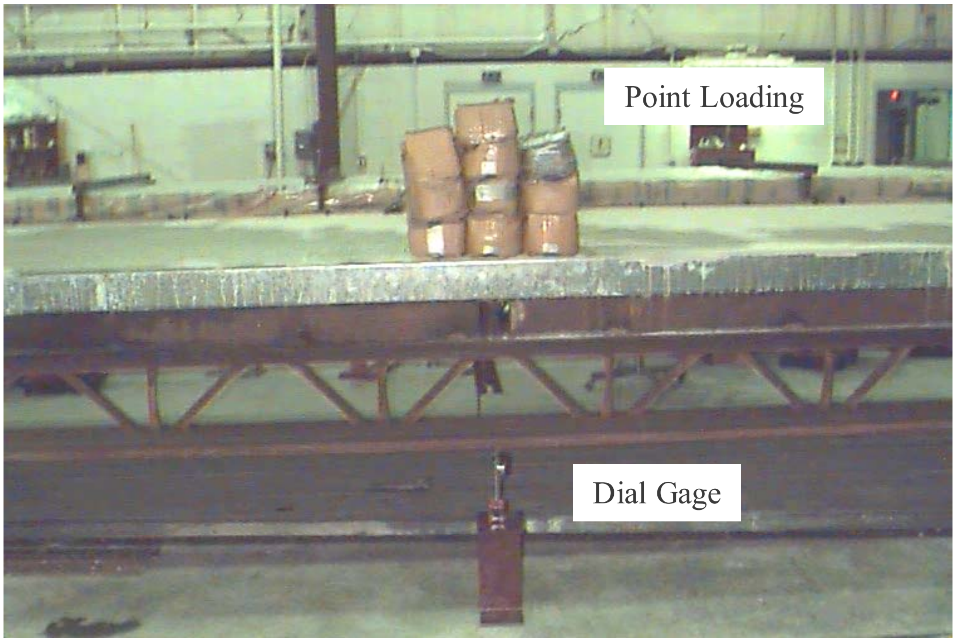


Figure 2. 4—Midspan Point Loading and Deflection Measurement

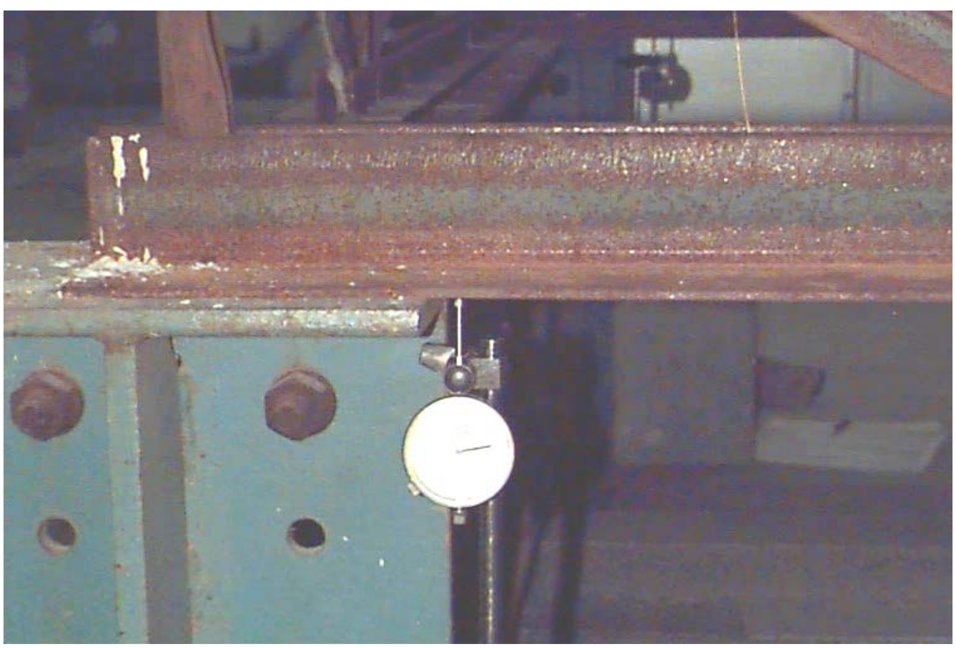


Figure 2. 5—Measurement of Support Deflections

Figure 2. 6—Distributed Loading-15 psf

2.3.2 Dynamic Testing Procedures

The measurement procedure described below was used for both live loading cases, zero psf and 15 psf. The measurements were made using a 0.25 Hz precision Ono Sokki CF-1200 Handheld FFT Analyzer and a seismic accelerometer.

Center-floor frequencies, $f_{gm'}$, were measured at the location of the accelerometer shown in Figure 2.7. Ambient, heel drop, walking, and bouncing induced frequencies were measured. Ambient movements were recorded at center-floor with no external forces acting on the floor. Heel drop induced movements were recorded at center-floor immediately following the 2.5 in. toes-to-heels drop of a 160 lb person at center-floor. Walking induced movements were recorded at center-floor while a 160 lb person walked 1.75-2 steps per second along a line parallel to and halfway between the girders. Bouncing induced movements were recorded at center-floor while a 160 lb person was bouncing on center-floor at one half of floor fundamental frequency. The average value of $f_{gm'}$ from the four tests is used in the analysis of the results.

Support frequencies, f_{su}, were measured at the location of the accelerometer shown in Figure 2.8. Support frequency is that of the slab-to-seat-to-girder-to-support, which is the load path from the test slab to the laboratory floor. Ambient and heel drop induced frequencies were measured at each corner, giving eight support measurements. Ambient movements were recorded from each corner while no external forces were acting on the floor. Heel drop induced movements were recorded from each corner immediately following the 2.5 in. toes-to-heels drop of a 160 lb person on the corner. The average value of f_{su} from the eight tests is used in the analysis of the results.

The measured girder frequency, f_{gm}, is a function of measured center-floor frequency, $f_{gm'}$, and measured support frequency, f_{su}. Calculation of f_{gm} is explained in Section 2.5.

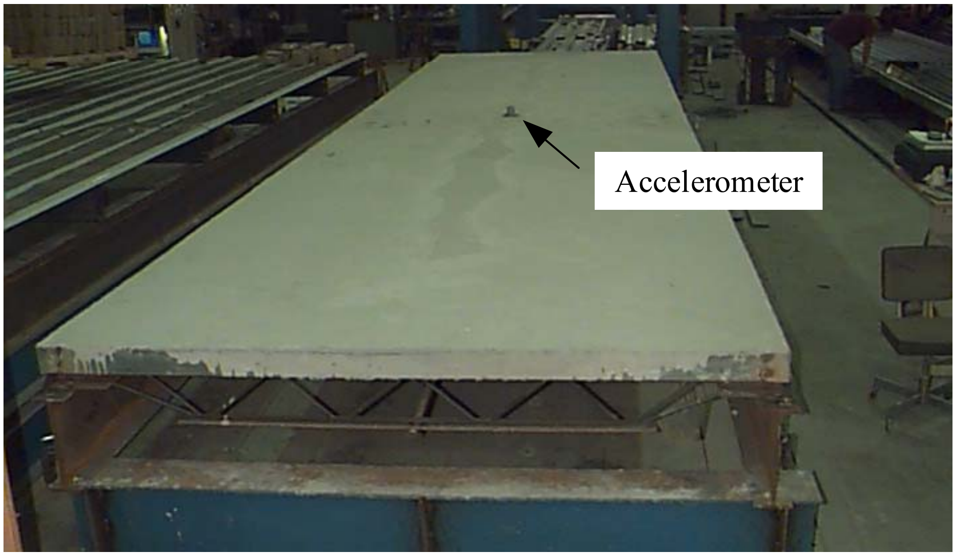


Figure 2. 7—Midspan Frequency Measurement Location

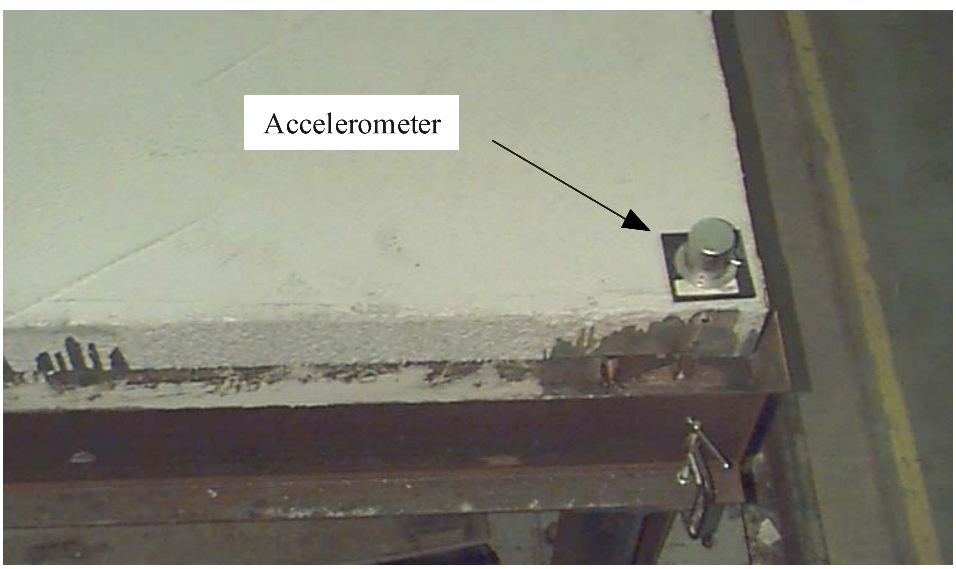


Figure 2. 8—Corner Frequency Measurement Location

2.3.3 Seat Stiffness Testing Procedures

Bolted seat stiffness in the horizontal direction was tested for each type of joist used. This data is compared with the stiffness used for FEM seat elements. Top of seat deflection measurements were made with 0.001 in. precision dial gages. Deflections were recorded from two top of seat locations with one dial gage placed 1.5 in. to either side of the seat-to-girder connection bolt line. Horizontal load was applied with a plate screwed to the seat top by six screws. Load was measured with a 2 kip capacity load cell and deflections were recorded at each 50 lb increment. The test setup is depicted in Figure 2.9.

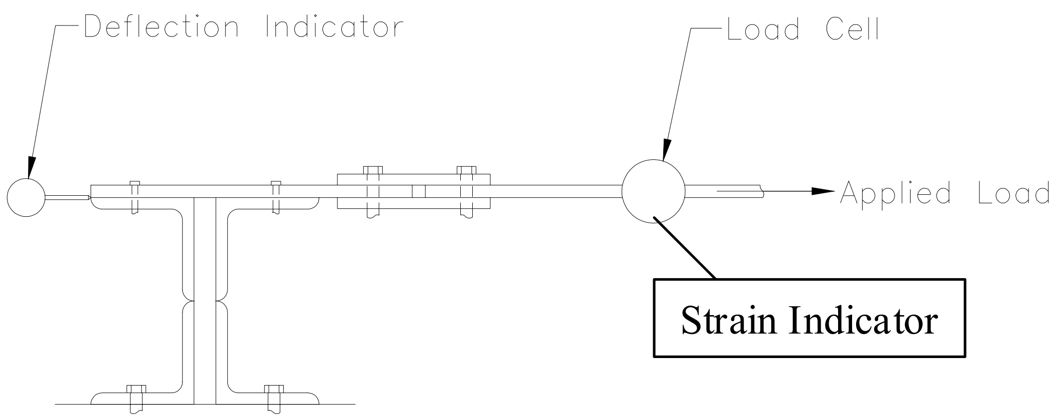


Figure 2. 9—Seat Stiffness Test Setup

2.4 Finite Element Modeling Procedures

Finite element modeling was used to help understand the behavior of the test footbridges and for comparison with the predicted and measured results. The model, which is shown in Figure 2.10, is a two dimensional representation of the footbridge that has the same geometric, stiffness, and mass properties as half of the footbridge. The cross-section of the girder model has two shell elements per flange and four shell elements for the web. The fineness of this mesh was found to provide sufficient convergence of model stiffness. The aspect ratio of each shell element is approximately one-to-one. Seats are represented by flat bar sections rising from the top girder flange to the bottom of decking (dimension d_{seat} of Figure 2.10). Rigid frame elements extend from the top of each seat element to the slab element center of gravity (dimension y of Figure 2.10). The slab consists of one frame element spanning between the tops of each pair of rigid links. The slab element properties were taken from the transformed concrete section shown in Figure 2.11.

In each of the following determinations of model moment of inertia, a vertical point load of 600 lb was applied at the joint at slab midspan (girder midspan for the girder-only model) to obtain model deflection and back-calculate model moment of inertia. As a preliminary check, the girder or joist girder section of the model was run alone and adjusted until it was found to have the same section properties as the tested girders. Next, *rigid* seat elements and the slab elements were added to the model to assure that the full composite moment of inertia was as predicted. By these two steps, it was found that the properties of the girder and slab sections were accurately incorporated in the models, allowing the effects of the joist seat to be analyzed. With negligible joist seat stiffness, the model has a moment of inertia equal to the sum of the girder and slab moments of inertia. With rigid joist seats, the model has full composite stiffness. The stiffness of the seat elements determines where, between the two extremes, the behavior of the system will be.

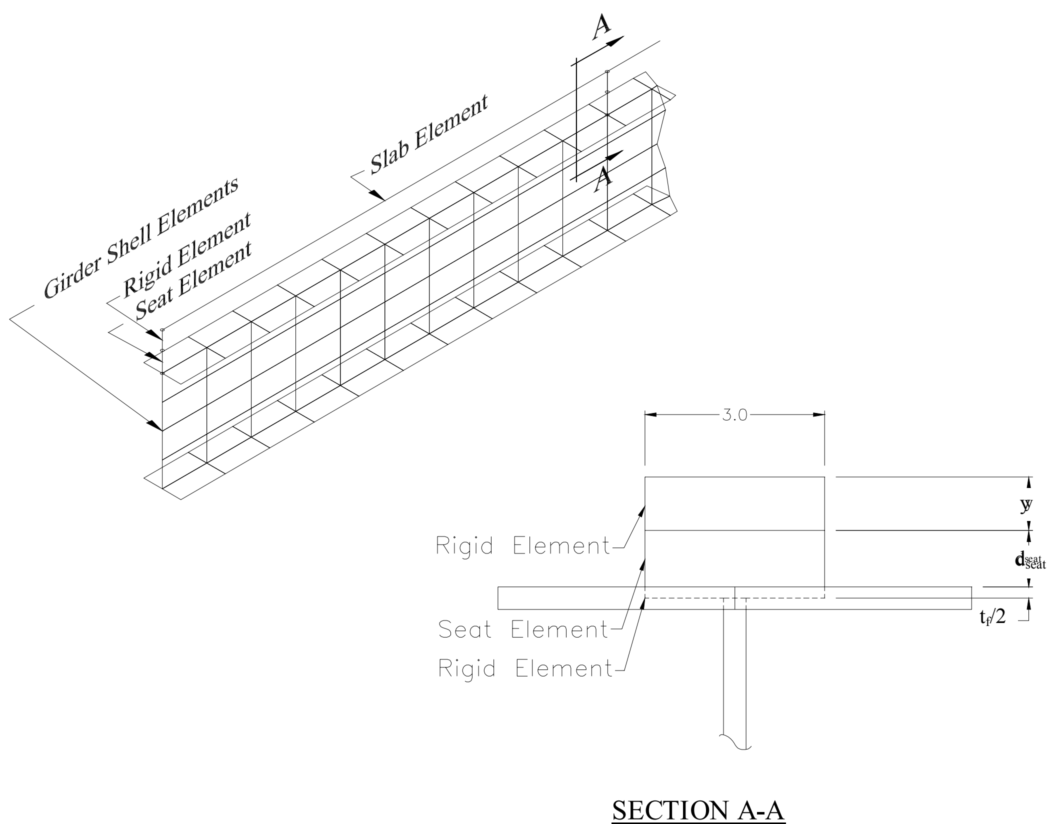


Figure 2. 10—Partial FE Model

As the final step in the model construction, the stiffness of each model was adjusted to be equal to the partially composite girder measured moment of inertia. To accomplish this, the moment of inertia of the seat elements was adjusted by an iterative process and the seat stiffness that caused the model to have the measured moment of inertia was used. The seats were modeled as rectangular elements extending 3 inches in the direction of the joists and with the thickness required to give the desired overall stiffness. For example, a seat moment of inertia of 0.0044 in^4 was needed for the K-30-B model to be as stiff as measured. This required the 2.5 in. tall and 3 in. wide seats to be 0.26 in. thick. Analyses were then run with the models and the results from them were used as discussed in Section 2.5.

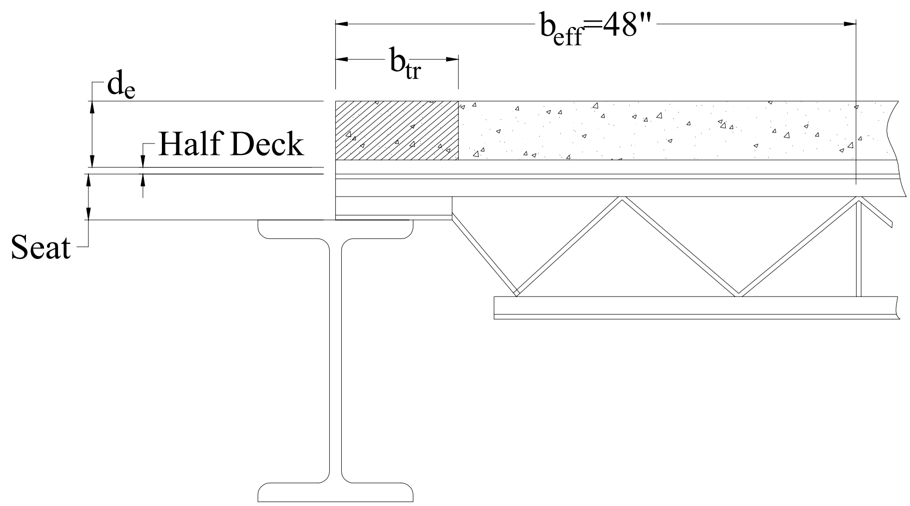


Figure 2. 11—Transformed Girder Section

2.5 Calculations and Terminology for Results

Static Testing. Girder effective moment of inertia, I_{eff}, was predicted using Equation 2.1, which is repeated here.

$$I_{eff} = I_g + (I_{comp} - I_g)/4 \tag{2.1}$$

where I_g is the bare girder moment of inertia and I_{comp} is the girder transformed moment of inertia. The measured moment of inertia of the bare girders, I_{gm}, was used for the bare girder moment of inertia of Equation 2.1. This was done so that errors in estimating I_g were eliminated from Equation 2.1. The composite moment of inertia was calculated by applying the parallel axis theorem to the steel section and the transformed concrete section, which are shown in Figure 2.11. The effective concrete width, b_{eff}, is equal to half of the joist span (48 in. for each footbridge). The transformed concrete width, b_{tr}, is equal to b_{eff} divided by the modular ratio ($n = E_s/1.35E_c$). It should be noted that the factor of 1.35 in the modular ratio is for only dynamic loading. However, the small effects were ignored in the static testing analysis to avoid confusion of data and because the resulting difference is small. The effective concrete depth, d_e, is equal to the total slab depth less half of the decking depth.

Predicted I_{eff} is compared with two measurements of I_{eff}. The first measurement of I_{eff} was calculated using the midspan point load-midspan deflection relationship

$$\Delta_{ms} = \frac{PL^3}{48EI_{eff}} \tag{2.2}$$

where D_{ms} is the net midspan deflection (in.), P is the midspan load (lb), L is the span length (in.), and E is Young's modulus (29,000,000 psi). The net midspan deflection is given by subtracting the average support deflection from the gross midspan deflection. Solving Equation 2.2 for I_{eff} gives

$$I_{eff} = \frac{PL^3}{48E\Delta_{ms}} \tag{2.3}$$

The *reported* value for measured I_{eff} is the average of the I_{eff} values calculated at each 100 lb load-deflection increment with Equation 2.3.

For the second measurement of I_{eff}, with distributed loading, the static deflection is

$$\Delta_{ms} = \frac{5wL^4}{384EI_{eff}} \tag{2.4}$$

where L, E, and Δ_{ms} are as previous, and w is the live load (15 psf). Solving Equation 2.4 for I_{eff} gives

$$I_{eff} = \frac{5wL^4}{384E\Delta_{ms}} \quad \textbf{(2.5)}$$

In addition to the first I_{eff} measurement, the value of measured I_{eff} from Equation 2.5 is *re-ported* in the results.

Further comparison of measured and Equation 2.1 values was made using only the second term of Equation 2.1. Equation 2.1 can be rewritten as

$$I_{eff} = I_g + \alpha(I_{comp} - I_g) \quad \textbf{(2.6)}$$

and then rearranged so that

$$\alpha = \frac{I_{eff} - I_g}{I_{comp} - I_g} \quad \textbf{(2.7)}$$

where α is termed the composite factor. The measured composite factor, *reported* in the results, was calculated with the average measured value of I_{eff}. This value is compared to the Equation 2.1 composite factor of 0.25 (or ¼).

Dynamic Testing. Girder fundamental natural frequency, f_g, was predicted using the Design Guide procedure described in Section 1.3.2. This frequency is

$$f_g = f_n = 0.18(g/\Delta)^{0.5} \quad \textbf{(2.8)}$$

As described in Section 1.3.2, system, or center-floor frequency, is a function of the frequency of the joist and girder members. However, by design the aspect ratio of the test footbridges was large enough that the system frequency, f_n, and girder frequency, f_g, are essentially the same. For example, the combined mode of a joist frequency of 27 Hz and a girder frequency of 5.6 Hz is 5.5 Hz. In general, predicted and measured system frequencies were 0.1 to 0.2 Hz lower than actual girder frequencies. Therefore, it was not necessary to measure joist frequency and both the measurements and the predictions of midspan frequency are accurate approximations of girder frequency.

For the prediction of f_g, Δ is the predicted deflection based on Equation 2.1. Once I_{eff} has been measured, the theoretical girder frequency, f_{gt}, can be calculated with Equation 2.8, where Δ is the deflection based on measured I_{eff}. This calculation reduces to

$$f_{gt} = f_g(I_1/I_2)^{0.5} \quad \textbf{(2.9)}$$

where I_1 is the measured I_{eff} and I_2 is the predicted I_{eff}.

In the measurements of girder frequency, it was necessary to account for support frequencies. The correction was made using Dunkerley's relationship

$$\frac{1}{f_{gm'}^{2}} = \frac{1}{f_{gm}^{2}} + \frac{1}{f_{su}^{2}} \quad \textbf{(2.10)}$$

where f_{gm} is the girder frequency, f_{su} is the support frequency, and $f_{gm'}$ is the combined mode of the two. Solving for the corrected girder frequency, f_{gm}, gives

$$f_{gm} = \left(\frac{1}{f_{gm'}^{2}} - \frac{1}{f_{su}^{2}} \right)^{-0.5} \quad \textbf{(2.11)}$$

This is the measured frequency *reported* in the results. The girder frequency measurements, f_{gm}, are compared with f_{gt} to assess the accuracy of the measurements and compared to f_g to assess the accuracy of Equation 2.1.

Finite Element Modeling. FE-models were also used for frequency comparisons. As described in Section 2.4, the models were constructed with the same geometric and mass properties as the actual girders and made as stiff as the actual girders through adjustments of seat stiffness.

The nominal FEM girder frequencies, $f_{gf'}$, required adjustment with Dunkerley's relationship to correct for the effect of joist frequencies. Because, the measured, predicted, and theoretical girder frequencies were taken as the combined mode of the joists and girders, they were lower than the actual girder frequencies by 0.1 to 0.2 Hz. The FE-models were two-dimensional and did not have joist frequencies. Therefore, the models were adjusted to include joist frequencies as follows:

$$f_{gf} = \left(\frac{1}{f_{gf'}^{2}} + \frac{1}{f_{j}^{2}} \right)^{-0.5} \quad \textbf{(2.12)}$$

where f_{gf} is the value *reported* for FE girder frequency and f_j is the theoretical joist frequency.

In addition to frequency comparisons, a goal of the finite element modeling was accurate prediction of girder I_{eff}, which requires accurate prediction of seat behavior.

Seat Stiffness Testing. Measured seat moment of inertia is compared with the moment of inertia used for FEM seat members. A linear regression of load-deflection test data was used to define the load-deflection relationship of the seats. The lower bound seat shear force for the regression range was 0 lb. The upper bound was taken as the average of the seat shear forces that resulted when a 600 lb (heel drop) midspan load was applied to the FEM girders. After defining the load-deflection relationship, seat stiffness was calculated using the cantilever deflection equation

$$\Delta = \frac{Fh^3}{3EI_s} \quad \textbf{(2.13)}$$

Rearranging for the seat stiffness, I_s, gives

$$I_s = \frac{Fh^3}{3E\Delta} \quad \textbf{(2.14)}$$

where D is the top of seat deflection, F is the top of seat load, h is the seat height, E is Young's modulus (29,000,000 psi), and I_s is seat moment of inertia. This I_s value is *reported* in the results along with the values of I_s used in the FE-models.

Summary of Terminology for Results. The terminology used for the reporting and discussion of the results is summarized in Table 2.3.

Table 2. 3—Summary of Terminology used in Evaluation

Symbol	Description
I_{eff}	Girder effective moment of inertia
a	Composite factor
I_g	Bare girder moment of inertia
I_{comp}	Girder fully composite moment of inertia
I_s	Seat moment of inertia
f_g	Predicted girder frequency. *Approximately equal to* f_n.
f_{gm}	Measured girder frequency. *Corrected for* f_{su}.
f_{gt}	Theoretical girder frequency. *Based on measured* I_{eff}.
f_{gf}	FEM girder frequency. *Based on measured* I_{eff}. *Corrected for* f_j.

2.6 Results

The results for the eight footbridges, with a subsection for each, are presented in detail in this section. The Design Guide predicted behavior using Equation 2.1 is presented first. Then, the results for static testing, dynamic testing, and seat stiffness testing are presented. The FEM frequency results are presented with the dynamic testing and the FEM seat stiffness results are presented with the seat stiffness testing. A summary of results table is included at the end of the section for each footbridge.

2.6.1 Results from K-30 Tests

The K-30 footbridge had W14x22 girders and 8K1 joists spaced 30 in. on center. The height of the joist seats was 2.5 in. The slab was 4 in. thick with 0.6 in. decking and a concrete compressive strength of 3.3 ksi. The results from the K-30 testing are summarized in Table 2.4.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 405.1 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 225.1 in^4, the composite factor, a, was 0.25, and I_{comp} was 945.1 in^4. For the tests with no live load, the predicted frequency, f_g, was 5.75 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.02 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, a, predicted by Equation 2.1 and the measured values of a calculated with Equation 2.7.

Measured I_{eff} values were 396.6 in^4, 331.6 in^4, and 449.5 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding a values were 0.24, 0.15, and 0.31 for the bolted, welded, and reinforced tests, respectively. The bolted and welded I_{eff} measurements were 2.1 % and 18.1 % lower than the Equation 2.1 I_{eff} of 405.1 in^4, or in terms of a, 4.7 % and 40.8 % lower than the Equation 2.1 a of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and a for the welded test. The reinforced configuration had 11.0 % higher I_{eff} than Equation 2.1, or, in terms of a, 24.7 % higher than Equation 2.1. The gain in I_{eff}, from the reinforcements, indicates that girder I_{eff} depends largely on seat stiffness. However, it was somewhat surprising that the reinforcements did not result in near full composite stiffness.

Table 2. 4—K-30 Results

Test Identification	a		I_{eff}		f_g			
	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
K-30-B	0.24	0.25	396.6	405.1	5.74	5.73	5.88	5.75
K-30-BL	0.26	0.25	408.7	405.1	5.35	5.21	5.19	5.02
K-30-W	0.15	0.25	331.6	405.1	5.18	5.28	5.36	5.75
K-30-WL	0.19	0.25	359.1	405.1	4.80	4.89	4.73	5.02
K-30-R	0.31	0.25	449.5	405.1	6.32	6.15	6.22	5.75
K-30-RL	0.38	0.25	497.8	405.1	6.15	5.75	5.48	5.02

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, were 5.74 Hz, 5.18 Hz, and 6.32 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.73 Hz, 5.28 Hz, and 6.15 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.88 Hz, 5.36 Hz, and 6.22 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 0.2 % lower, 9.9 % lower, and 9.9 % higher than the predicted frequency of 5.75 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, were 5.35 Hz, 4.80 Hz, and 6.15 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.21 Hz, 4.89 Hz, and 5.75 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.19 Hz, 4.73 Hz, and 5.48 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of bolted, welded, and reinforced girder frequencies were 6.5 % higher, 4.4 % lower, and 22.5 % higher than the predicted frequency of 5.02 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.0051 in^4, and from the exterior was 0.0059 in^4. The average of the two values is 0.0055 in^4. The K-30-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0044 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The K-30-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0021 in^4. The K-30-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0068 in^4.

2.6.2 Results from K-60 Tests

The K-60 footbridge had W14x22 girders and 8K1 joists spaced 60 in. on center. The height of the joist seats was 2.5 in. The slab was 4 in. thick with 1.5 in. decking and a concrete compressive strength of 2.8 ksi. The results from the K-60 testing are summarized in Table 2.5.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 390.4 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 218.0 in^4, the composite factor, a, was 0.25, and I_{comp} was 907.6 in^4. For the tests with no live load, the predicted frequency, f_g, was 5.81 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.06 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, a, predicted by Equation 2.1, and the measured values of a calculated with Equation 2.7.

Measured I_{eff} values were 366.3 in^4, 341.7 in^4, and 489.8 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding a values were 0.22, 0.18, and 0.39 for the bolted, welded, and reinforced tests, respectively. The bolted and welded I_{eff} measurements were 6.2 % and 12.5 % lower than the Equation 2.1 I_{eff} of 390.4 in^4, or in terms of a, 14.0 % and 28.2 % lower than the Equation 2.1 a of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and a for the welded test. The reinforced configuration had 25.5 % higher I_{eff} than Equation 2.1, or, in terms of a, 57.7 % higher than Equation 2.1. The gain in I_{eff}, from the reinforcements, indicates that girder I_{eff} depends largely on seat stiffness. However, it was somewhat surprising that the reinforcements did not result in near full composite stiffness.

Table 2. 5—K-60 Results

	a		I_{eff}		f_g			
Test Identification	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
K-60-B	0.22	0.25	366.3	390.4	5.51	5.77	5.77	5.81
K-60-W	0.18	0.25	341.7	390.4	5.49	5.57	5.65	5.81
K-60-WL	0.19	0.25	350.4	390.4	4.77	4.91	4.91	5.06
K-60-R	0.39	0.25	489.8	390.4	6.29	6.67	6.63	5.81

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, were 5.51 Hz, 5.49 Hz, and 6.29 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.77 Hz, 5.57 Hz, and 6.67 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.77 Hz, 5.65 Hz, and 6.63 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 5.2 % lower, 5.6 % lower, and 8.3 % higher than the predicted frequency of 5.81 Hz.

For tests with 15 psf live load, the measured girder frequency, f_{gm}, was 4.77 Hz for the welded test. Bolted and reinforced tests were not conducted. The theoretical girder frequency, f_{gt}, was 4.91 Hz for the welded test, which compares closely with the measured frequency. The FEM frequency, f_{gf}, was 4.91 Hz, for the welded test, which also compares closely with the measured frequency. The measured frequency was 5.7 % lower than the predicted frequency of 5.06 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.0044 in^4, and from the exterior 0.0050 in^4. The average of the two values is 0.0047 in^4. The K-60-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0058 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The K-60-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0049 in^4. The K-60-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0185 in^4.

2.6.3 Results from LH-60 Tests

The LH-60 footbridge had W14x22 girders and 18LH550 joists spaced 60 in. on center. The height of the joist seats was 5 in. The slab was 4 in. thick with 1.5 in. decking and a concrete compressive strength of 2.7 ksi. The results from the LH-60 testing are summarized in Table 2.6.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 472.0 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 218.0 in^4, the composite factor, a, was 0.25, and I_{comp} was 1234.0 in^4. For the tests with no live load, the predicted frequency, f_g, was 6.65 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.80 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, a, predicted by Equation 2.1 and the measured values of a calculated with Equation 2.7.

Measured I_{eff} values were 294.3 in^4, 273.4 in^4, and 318.2 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding a values were 0.08, 0.05, and 0.10 for the bolted, welded, and reinforced tests, respectively. The bolted, welded, and reinforced I_{eff} measurements were 37.6 %, 42.1 %, and 32.6 % less than the Equation 2.1 I_{eff} of 472.0 in^4, or in terms of a, 70.0 %, 78.2, and 60.6 % less than the Equation 2.1 a of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and a for the welded test. The reinforced configuration was stiffer than the others were, but even it did not reach the predicted stiffness.

Table 2. 6—LH-60 Results

Test Identification	a		I_{eff}		f_g			
	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
LH-60-B	0.08	0.25	294.3	472.0	5.13	5.36	5.48	6.65
LH-60-W	0.05	0.25	273.4	472.0	5.21	5.17	5.28	6.65
LH-60-WL	0.06	0.25	275.7	472.0	4.52	4.52	4.58	5.80
LH-60-R	0.10	0.25	318.2	472.0	5.37	5.57	5.74	6.65
LH-60-RL	0.10	0.25	321.3	472.0	4.99	4.88	4.97	5.80

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, were 5.13 Hz, 5.21 Hz, and 5.37 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.36 Hz, 5.17 Hz, and 5.57 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.48 Hz, 5.28 Hz, and 5.74 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 22.8 %, 21.6 %, and 19.2 %, lower than the predicted frequency of 6.65 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, 4.52 Hz and 4.99 Hz for the welded and reinforced tests, respectively. A bolted test was not conducted. The theoretical girder frequencies, f_{gt}, were 4.52 Hz and 4.88 Hz for the welded and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 4.58 Hz and 4.97 Hz for the welded and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of welded and reinforced girder frequencies were 22.0 % and 13.9 % lower than the predicted frequency of 5.80 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.020 in^4, and from the exterior was 0.031 in^4. The average of the two values is 0.0255 in^4. The LH-60-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0132 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The LH-60-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0081 in^4. The LH-60-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0199 in^4.

2.6.4 Results from LH-90 Tests

The LH-90 footbridge had W14x22 girders and 18LH800 joists spaced 90 in. on center. The height of the joist seats was 5 in. The slab was 5 in. thick with 2 in. decking and a concrete compressive strength of 3.1 ksi. The results from the LH-90 testing are summarized in Table 2.7.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 509.2 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 211.4 in^4, the composite factor, a, was 0.25, and I_{comp} was 1402.7 in^4. For the tests with no live load, the predicted frequency, f_g, was 6.35 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.65 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, a, predicted by Equation 2.1 and the measured values of a calculated with Equation 2.7.

Measured I_{eff} values were 319.6 in^4, 264.4 in^4, and 418.2 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding a values were 0.09, 0.04, and 0.17 for the bolted, welded, and reinforced tests, respectively. The bolted, welded, and reinforced I_{eff} measurements were 37.2 %, 48.1%, and 17.9 % lower than the Equation 2.1 I_{eff} of 509.2 in^4, or in terms of a, 63.7%, 82.2 %, and 30.5% lower than the Equation 2.1 a of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and a for the welded test. The reinforced configuration was stiffer than the others were, but even it did not reach the predicted stiffness.

Table 2. 7—LH-90 Results

	a		I_{eff}		f_g			
Test Identification	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
LH-90-B	0.09	0.25	319.6	509.2	4.83	5.09	5.25	6.35
LH-90-BL	0.08	0.25	309.1	509.2	4.31	4.46	4.66	5.65
LH-90-W	0.04	0.25	264.4	509.2	4.46	4.63	4.75	6.35
LH-90-WL	0.05	0.25	272.0	509.2	4.13	4.18	4.21	5.65
LH-90-R	0.17	0.25	418.2	509.2	5.70	5.83	5.96	6.35

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, 4.83 Hz, 4.46 Hz, and 5.70 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.09 Hz, 4.63 Hz, and 5.83 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, 5.25 Hz, 4.75 Hz, and 5.96 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 23.9 %, 29.8 %, and 10.3 % lower than the predicted frequency of 6.35 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, were 4.31 Hz and 4.13 Hz for the bolted and welded tests, respectively. A reinforced test was not conducted. The theoretical girder frequencies, f_{gt}, 4.46 Hz and 4.18 Hz for the bolted and welded tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 4.66 Hz and 4.21 Hz for the bolted and welded tests, respectively, which also compare closely with the measured frequencies. The measurements of bolted and welded girder frequencies were 23.7 % and 26.9 % lower than the predicted frequency of 5.65 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.037 in^4, and from the exterior was 0.045 in^4. The average of the two values is 0.041 in^4. The LH-90-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0202 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The LH-90-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0035 in^4. The LH-90-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.061 in^4.

2.6.5 Results from JG-K-30 Tests

The JG-K-30 footbridge had 14G12N1.95 joist-girders and 8K1 joists spaced 30 in. on center. The height of the joist seats was 2.5 in. The slab was 4 in. thick with 0.6 in. decking and a concrete compressive strength of 2.9 ksi. The results from the JG-K-30 testing are summarized in Table 2.8.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 399.5 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 247.6 in^4, the composite factor, α, was 0.25, and I_{comp} was 856.6 in^4. For the tests with no live load, the predicted frequency, f_g, was 5.79 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.14 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, α, predicted by Equation 2.1 and the measured values of α calculated with Equation 2.7.

Measured I_{eff} values were 346.9 in^4, 321.1 in^4, and 398.4 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding α values were 0.16, 0.12, and 0.25 for the bolted, welded, and reinforced tests, respectively. The bolted, welded, and reinforced I_{eff} measurements were 13.2 %, 19.6 %, and 0.3 % lower than the Equation 2.1 I_{eff} of 399.5 in^4, or in terms of α, 34.8 %, 51.7 %, and 1.0 % lower than the Equation 2.1 α of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and α for the welded test.

Table 2. 8—JG-K-30 Frequency Results

	a		I_{eff}		f_g			
	Meas.	D. G.	Meas.	D. G.	Meas.	Theo.	FEM	D. G.
					f_{gm}	f_{gt}	f_{gf}	f_g
Test Identification			(in^4)	(in^4)	(Hz)	(Hz)	(Hz)	(Hz)
JG-K-30-B	0.16	0.25	346.9	399.5	5.20	5.35	5.49	5.79
JG-K-30-BL	0.17	0.25	352.4	399.5	4.77	4.79	4.87	5.14
JG-K-30-W	0.12	0.25	321.1	399.5	5.04	5.15	5.29	5.79
JG-K-30-WL	0.14	0.25	333.3	399.5	4.70	4.66	4.69	5.14
JG-K-30-R	0.25	0.25	398.4	399.5	6.22	5.74	5.87	5.79
JG-K-30-RL	0.30	0.25	429.1	399.5	5.26	5.28	5.20	5.14

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, 5.20 Hz, 5.04 Hz, and 6.22 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.35 Hz, 5.15 Hz, and 5.74 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, 5.49 Hz, 5.29 Hz, and 5.87 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 10.1 % lower, 12.9 % lower, and 7.3 % higher than the predicted frequency of 5.79 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, 4.77 Hz, 4.70 Hz, and 5.26 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 4.79 Hz, 4.66 Hz, and 5.28 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 4.87 Hz, 4.69 Hz, and 5.20 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of bolted, welded, and reinforced girder frequencies were 7.2 % lower, 8.5 % lower, and 2.4

% higher than the predicted frequency of 5.14 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.0051 in^4, and from the exterior was 0.0059 in^4. The average of the two values is 0.0055 in^4. The JG-K-30-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0021 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The JG-K-30-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0012 in^4. The JG-K-30-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0041 in^4.

2.6.6 Results from JG-LH-60 Tests

The JG-LH-60 footbridge had 14G6N3.90 joist-girders and 18LH550 joists spaced 60 in. on center. The height of the joist seats was 5 in. The slab was 4 in. thick with 1.5 in. decking and a concrete compressive strength of 2.8 ksi. The results from the JG-LH-60 testing are summarized in Table 2.9.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 468.3 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 245.4 in^4, the composite factor, α, was 0.25, and I_{comp} was 1138.6 in^4. For the tests with no live load, the predicted frequency, f_g, was 6.74 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.90 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, α, predicted by Equation 2.1 and the measured values of α calculated with Equation 2.7.

Measured I_{eff} values were 299.6 in^4, 315.4 in^4, and 375.0 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding α values were 0.06, 0.08, and 0.15 for the bolted, welded, and reinforced tests, respectively. The bolted, welded, and reinforced I_{eff} measurements were 36.0 %, 32.7 %, and 19.9 % lower than the Equation 2.1 I_{eff} of 468.3 in^4, or in terms of α, 75.7 %, 68.7 %, and 42.0 % lower than the Equation 2.1 α of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and α for the welded test. The reinforced configuration was stiffer than the others were, but even it did not reach the predicted stiffness.

Table 2. 9—JG-LH-60 Results

	a		I_{eff}		f_g			
Test Identification	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
JG-LH-60-B	0.06	0.25	299.6	468.3	5.39	5.34	5.55	6.74
JG-LH-60-BL	0.08	0.25	317.5	468.3	4.96	4.81	4.83	5.90
JG-LH-60-W	0.08	0.25	315.4	468.3	5.42	5.48	5.63	6.74
JG-LH-60-WL	0.09	0.25	329.3	468.3	4.90	4.90	4.91	5.90
JG-LH-60-R	0.15	0.25	375.0	468.3	6.38	5.98	6.17	6.74
JG-LH-60-RL	0.18	0.25	404.0	468.3	5.62	5.43	5.38	5.90

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, were 5.39 Hz, 5.42 Hz, and 6.38 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.34 Hz, 5.48 Hz, and 5.98 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.55 Hz, 5.63 Hz, and 6.17 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 20.0 %, 19.6 %, and 5.3 % lower than the predicted frequency of 6.74 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, 4.96 Hz, 4.90 Hz, and 5.62 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 4.81 Hz, 4.90 Hz, and 5.43 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, 4.83 Hz, 4.91 Hz, and 5.38 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of bolted, welded, and reinforced girder frequencies were 16.0 %, 17.0 %, and 4.7 % lower than the predicted frequency of 5.90 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.020 in^4, and from the exterior was 0.031 in^4. The average of the two values is 0.0255 in^4. The JG-LH-60-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0086 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The JG-LH-60-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0112 in^4. The JG-LH-60-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0313 in^4.

2.6.7 Results from JG-LH-90 Tests

The JG-LH-90 footbridge had 14G4N5.85 joist-girders and 18LH800 joists spaced 90 in. on center. The height of the joist seats was 5 in. The slab was 5 in. thick with 2 in. decking and a concrete compressive strength of 3.6 ksi. The results from the JG-LH-90 testing are summarized in Table 2.10.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 507.3 in^4 for all three seat connection types. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 241.9 in^4, the composite factor, a, was 0.25, and I_{comp} was 1304.9 in^4. For the tests with no live load, the predicted frequency, f_g, was 6.43 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.73 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1 and is therefore the same for each seat connection type.

Static Testing. The static testing results are used to assess the accuracy of Equation 2.1. The first comparison is of the I_{eff} value predicted by Equation 2.1 and the measured values of I_{eff} calculated with Equation 2.3. The second comparison is of the composite factor, a, predicted by Equation 2.1 and the measured values of a calculated with Equation 2.7.

Measured I_{eff} values were 380.0 in^4, 346.1 in^4, and 457.8 in^4 for the bolted, welded, and reinforced tests, respectively. Corresponding a values were 0.13, 0.10, and 0.20 for the bolted, welded, and reinforced tests, respectively. The bolted and welded I_{eff} measurements were 25.1 %, 31.8 %, and 9.8 % lower than the Equation 2.1 I_{eff} of 507.3 in^4, or in terms of a, 48.0 %, 60.8 %, and 18.8 % lower than the Equation 2.1 a of 0.25. The welded seats have a longer bending length than the bolted seats, as shown in Figure 2.2. This explains the lower values of I_{eff} and a for the welded test. The reinforced configuration was stiffer than the others were, but even it did not reach the predicted stiffness.

Table 2. 10—JG-LH-90 Results

Test Identification	a		I_{eff}		f_g			
	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	FEM f_{gf} (Hz)	D. G. f_g (Hz)
JG-LH-90-B	0.13	0.25	380.0	507.3	5.32	5.50	5.69	6.43
JG-LH-90-BL	0.14	0.25	389.2	507.3	4.80	4.96	5.05	5.73
JG-LH-90-W	0.10	0.25	346.1	507.3	5.28	5.25	5.43	6.43
JG-LH-90-WL	0.11	0.25	357.4	507.3	4.67	4.75	4.82	5.73
JG-LH-90-R	0.20	0.25	457.8	507.3	5.68	6.04	6.24	6.43
JG-LH-90-RL	0.20	0.25	459.8	507.3	5.30	5.39	5.55	5.73

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Dynamic Testing. The measured girder frequencies (f_{gm} of Equation 2.11) are used to assess the accuracy of the predicted frequencies, which are based on the I_{eff} found with Equation 2.1. The accuracy of the measurements is verified by comparing them to theoretical frequencies (f_{gt} of Equation 2.9) and FEM frequencies (f_{gf} of Equation 2.12). Both f_{gt} and f_{gf} are based on measured I_{eff} and thus should be close to f_{gm}.

For tests with no live load, the measured girder frequencies, f_{gm}, were 5.32 Hz, 5.28 Hz, and 5.68 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 5.50 Hz, 5.25 Hz, and 6.04 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.69 Hz, 5.43 Hz, and 6.24 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of the bolted, welded, and reinforced frequencies were 17.2 %, 18.0 %, and 11.7 % lower than the predicted frequency of 6.43 Hz.

For tests with 15 psf live load, measured girder frequencies, f_{gm}, 4.80 Hz, 4.67 Hz, and 5.30 Hz for the bolted, welded, and reinforced tests, respectively. The theoretical girder frequencies, f_{gt}, were 4.96 Hz, 4.75 Hz, and 5.39 Hz for the bolted, welded, and reinforced tests, respectively, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were 5.05 Hz, 4.82 Hz, and 5.55 Hz for the bolted, welded, and reinforced tests, respectively, which also compare closely with the measured frequencies. The measurements of bolted, welded, and reinforced girder frequencies were 16.3 %, 18.6 %, and 7.5 % lower than the predicted frequency of 5.73 Hz.

Seat Stiffness Testing. The measured moment of inertia of the bolted seats (I_s of Equation 2.14) taken from the interior load-deflection data was 0.0374 in^4, and from the exterior was 0.0446 in^4. The average of the two values is 0.0410 in^4. The JG-LH-90-B FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0332 in^4.

No measurements of welded or reinforced seat stiffnesses were made. The JG-LH-90-W FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0195 in^4. The JG-LH-90-R FEM had an I_{eff} equal to the measured girder I_{eff} when FEM I_s was 0.0749 in^4.

2.6.8 Results for Special Case Footbridge, JG-LH-90-T Tests

A special case footbridge was constructed and subjected to static and dynamic testing in an attempt to attain full composite action. This was done with a floor similar to JG-LH-90, except that an HSS 5x3x5/16 section was added between each pair of joist seats, as shown in Figure 2.12.

The JG-LH-90-T footbridge had 14G4N5.85 joist-girders and 18LH800 joists spaced 90 in. on center. The height of the joist seats was 5 in. The slab was 5 in. thick with 2 in. decking and a concrete compressive strength of 2.4 ksi.

Predicted Behavior. Equation 2.1 predicted an I_{eff} of 497.8 in^4. For the calculation, the measured bare girder moment of inertia, I_{gm}, was 243 in^4, the composite factor, a, was 0.25, and I_{comp} was 1264 in^4. For the tests with no live load, the predicted frequency, f_g, was 6.37 Hz. For tests with 15 psf live load, the predicted frequency, f_g, was 5.65 Hz. The predicted frequency (Equation 2.8) is based on the I_{eff} of Equation 2.1.

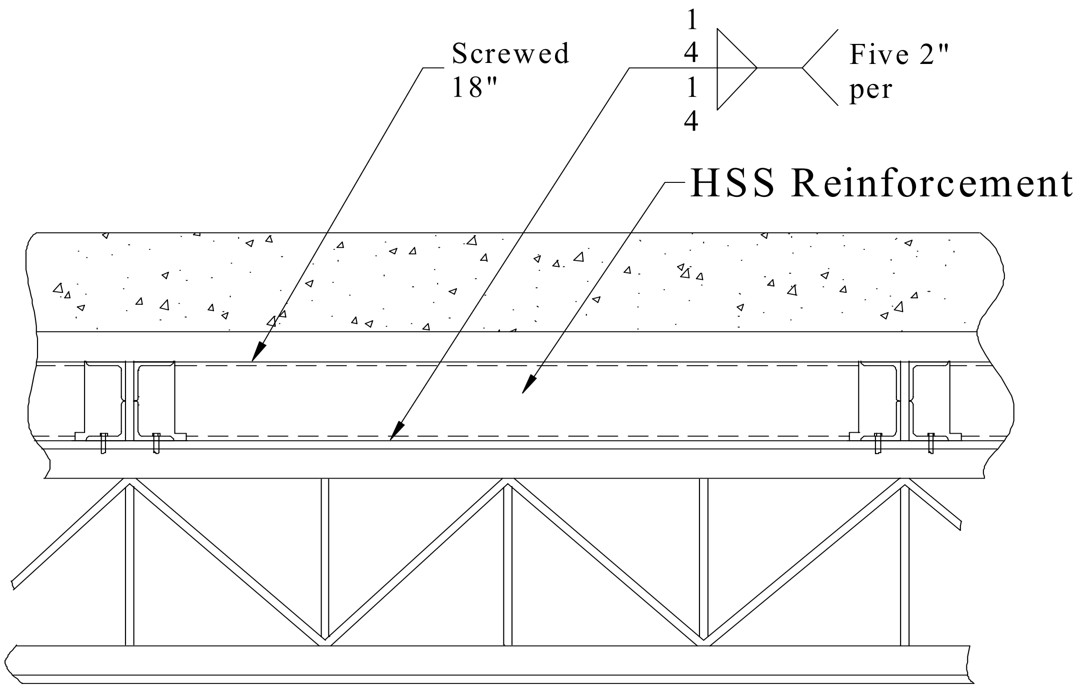


Figure 2. 12—HSS Seat Reinforcements

Static Testing. Measured girder I_{eff} was 927.2 in^4 and a was 0.67. As intended, the stiffness was much higher than predicted. However, the full composite stiffness was not reached even with the HSS reinforcement.

Dynamic Testing. The measured girder frequencies, f_{gm}, were 8.56 Hz with no live load and 8.16 Hz with 15 psf live load. The theoretical girder frequencies, f_{gt}, were 8.60 Hz with no live load and 7.71 Hz with 15 psf live load, which compare closely with the measured frequencies. The FEM frequencies, f_{gf}, were, which also compare closely with the measured frequencies.

2.7 Summary of Results

Tables 2.11 through 2.13 show the major static and dynamic results of the experimental testing, grouped by seat connection type. The static testing results are used to assess the accuracy of Equation 2.1 in Section 2.8. The first comparison is of the composite factor, a, predicted by Equation 2.1 and the measured values of a, as back-calculated with Equation 2.7. The second comparison is of the I_{eff} value predicted by Equation 2.1, and the measured values of I_{eff} calculated with Equations 2.3 and 2.5. For the dynamic testing results, the measured girder frequencies (f_{gm} of Equation 2.11), theoretical frequencies (f_{gt} of Equation 2.9), and predicted frequencies, f_g, are shown. The values of f_{gt} were based on measured I_{eff} and thus indicate accurate measurements when f_{gt} and f_{gm} are close. In addition, the ratio of f_{gt} to f_g is shown as an indication of the accuracy of f_g, which is predicted with the stiffness from Equation 2.1.

Table 2. 11—Summary of Bolted Case Results

	a		I_{eff}		f_g			
Test Identification	Meas.	D. G.	Meas. (in^4)	D. G. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	D. G. f_g (Hz)	(Theo/DG) x 100%
K-30-B	0.24		396.6	405.0	5.74	5.73	5.75	99.7%
K-30-BL	0.26		408.7	405.0	5.35	5.21	5.02	103.8%
JG-K-30-B	0.16		346.9	399.5	5.20	5.35	5.79	92.4%
JG-K-30-BL	0.17		352.4	399.5	4.77	4.79	5.14	93.2%
K-60-B	0.22		366.3	390.5	5.51	5.77	5.81	99.3%
LH-60-B	0.08	0.25	294.3	472.0	5.13	5.36	6.65	80.6%
JG-LH-60-B	0.06		299.6	470.5	5.39	5.34	6.74	79.2%
JG-LH-60-BL	0.08		317.5	470.5	4.96	4.81	5.90	81.5%
LH-90-B	0.09		319.6	509.0	4.83	5.09	6.35	80.2%
LH-90-BL	0.08		309.1	509.0	4.31	4.46	5.65	78.9%
JG-LH-90-B	0.13		380.0	513.0	5.32	5.50	6.43	85.5%
JG-LH-90-BL	0.14		389.2	513.0	4.80	4.96	5.73	86.6%

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Table 2. 12—Summary of Welded Case Results

Test Identification	a Meas.	a D. G.	I_{eff} Meas. (in^4)	I_{eff} D. G. (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g D. G. f_g (Hz)	(Theo/DG) x 100%
K-30-W	0.15	0.25	331.6	405.0	5.18	5.28	5.75	91.8%
K-30-WL	0.19		359.1	405.0	4.80	4.89	5.02	97.4%
JG-K-30-W	0.12		321.1	399.5	5.04	5.15	5.79	88.9%
JG-K-30-WL	0.14		333.3	399.5	4.70	4.66	5.14	90.7%
K-60-W	0.18		341.7	390.5	5.49	5.57	5.81	95.9%
K-60-WL	0.19		350.4	390.5	4.77	4.91	5.06	97.0%
LH-60-W	0.05		273.4	472.0	5.21	5.17	6.65	77.7%
LH-60-WL	0.06		275.7	472.0	4.52	4.52	5.80	77.9%
JG-LH-60-W	0.08		315.4	470.5	5.42	5.48	6.74	81.3%
JG-LH-60-WL	0.09		329.3	470.5	4.90	4.90	5.90	83.1%
LH-90-W	0.04		264.4	509.0	4.46	4.63	6.35	72.9%
LH-90-WL	0.05		272.0	509.0	4.13	4.18	5.65	74.0%
JG-LH-90-W	0.10		346.1	513.0	5.28	5.25	6.43	81.6%
JG-LH-90-WL	0.11		357.4	513.0	4.67	4.75	5.73	82.9%

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

Table 2. 13—Summary of Reinforced Case Results

Test Identification	a Meas.	a D. G.	I_{eff} Meas. (in^4)	I_{eff} D. G. (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g D. G. f_g (Hz)	(Theo/DG) x 100%
K-30-R	0.31	0.25	449.5	405.0	6.32	6.15	5.75	107.0%
K-30-RL	0.38		497.8	405.0	6.15	5.75	5.02	114.5%
JG-K-30-R	0.25		398.4	399.5	6.22	5.74	5.79	99.1%
JG-K-30-RL	0.30		429.1	399.5	5.26	5.28	5.14	102.7%
K-60-R	0.39		489.8	390.5	6.29	6.67	5.81	114.8%
LH-60-R	0.10		318.2	472.0	5.37	5.57	6.65	83.8%
LH-60-RL	0.10		321.3	472.0	4.99	4.88	5.80	84.1%
JG-LH-60-R	0.15		375.0	470.5	6.38	5.98	6.74	88.7%
JG-LH-60-RL	0.18		404.0	470.5	5.62	5.43	5.90	92.0%
LH-90-R	0.17		418.2	509.0	5.70	5.83	6.35	91.8%
JG-LH-90-R	0.20		457.8	513.0	5.68	6.04	6.43	93.9%
JG-LH-90-RL	0.20		459.8	513.0	5.30	5.39	5.73	94.1%

Note: D.G. refers to the AISC/CISC Design Guide recommendations.

2.8 Assessment of Design Guide Relationship and Proposed Improvements

2.8.1 Assessment of Measurements

Close correlation exists between the several types of measured data for each specimen, indicating that the measurement were accurate. For each test specimen, two measurement of I_{eff} and four of f_{gm} were made.

I_{eff} measurements. It can be inferred from the following characteristics of the results that the I_{eff} measurements were accurate:

- Distributed loading generally gave 1-3 % higher I_{eff} values than point loading. Discrepancies of this magnitude are not unreasonable when shear deformation is considered. Point loading results in greater shear deformation, which causes greater midspan deflection. Therefore, slightly less stiffness with point loading was to be expected.
- Comparing the measured and theoretical frequencies, f_{gm} and f_{gt} is a secondary method of verifying I_{eff} measurements. If discrepancies in the two frequencies are small, errors in stiffness measurements must also be small, because f_{gt} is calculated using the measured stiffness. The maximum difference between the two frequencies for the bolted and welded cases was +/- 5 % and slightly more for reinforced specimens. The average difference between f_{gm} and f_{gt} was 2.75 %. This is not unreasonable considering the precision of the instrumentation (0.25 Hz), which was approximately +/- 3 % of the frequency measurements.
- Finally, FE-model frequencies, f_{gf} were close to both f_{gm} and f_{gt}. The average difference between f_{gt} and f_{gf} was 2.04 %. The FE-models were constructed so that they were as stiff as the measured specimens. Thus, if the model frequencies were accurate, then the stiffness measurement must also have been accurate.

Frequency Measurements. It can be inferred from the following characteristics of the results that frequency measurements, f_{gm}, were accurate:

- Four measurements of girder frequency were made for each specimen. The maximum difference in a single measurement and the average of the four measurements was less than 0.5 Hz and in most cases less than 0.25 Hz. For example, ambient frequencies were typically 0.25 Hz higher than the other three measured frequencies, which were generally the same.

- As previously stated, differences in f_{gm} and f_{gt} were small, which indicated that the measured stiffnesses used to calculate f_{gt} were accurate. It follows then that the frequency measurements were also accurate.
- Likewise, the same conclusion can be drawn from the fact that FE-model frequencies were close to f_{gm} and f_{gt}.

It is evident that the measurements are sufficiently accurate for assessment of the I_{eff} relationship. However, it can be shown that comparing the theoretical girder frequency using the measured moment of inertia, f_{gt}, and not the corrected measured girder frequency, f_{gm}, to the predicted, and later the proposed, relationships will give better representation of the actual behavior of the test specimens. This is predicated first on the fact that differences in f_{gt} and f_{gm} values were small, as can be seen in the scatter plot of Figure 2.13. Second, it is assumed that the likelihood that the combined error in measuring stiffness and calculating frequency is less than the error in directly measuring frequency. This likelihood can be demonstrated by considering a typical test floor. If a floor with a measured stiffness of 300 in^4 and support frequency of 20 Hz, had a measured frequency of 5.0 Hz, the worst case instrument precision error would be +/- 0.15 Hz. For the same error in f_{gt} to result, the error in I_{eff} measurement would have to be +/- 19 in^4 or 6 %. Consistent occurrence of errors this large was not apparent in the measurements of I_{eff}. On the other hand, there is no way of identifying the amount of precision error, other than identifying its possible range. Moreover, if the comparisons were based on f_{gm}, the unmeasured variable of floor mass could result in further truncation of error. It is assured when using f_{gt} for comparison that the compared frequencies are based on the same floor mass. Therefore, in the following assessment and proposal, f_{gt} will be considered the most accurate representation of actual specimen frequency.

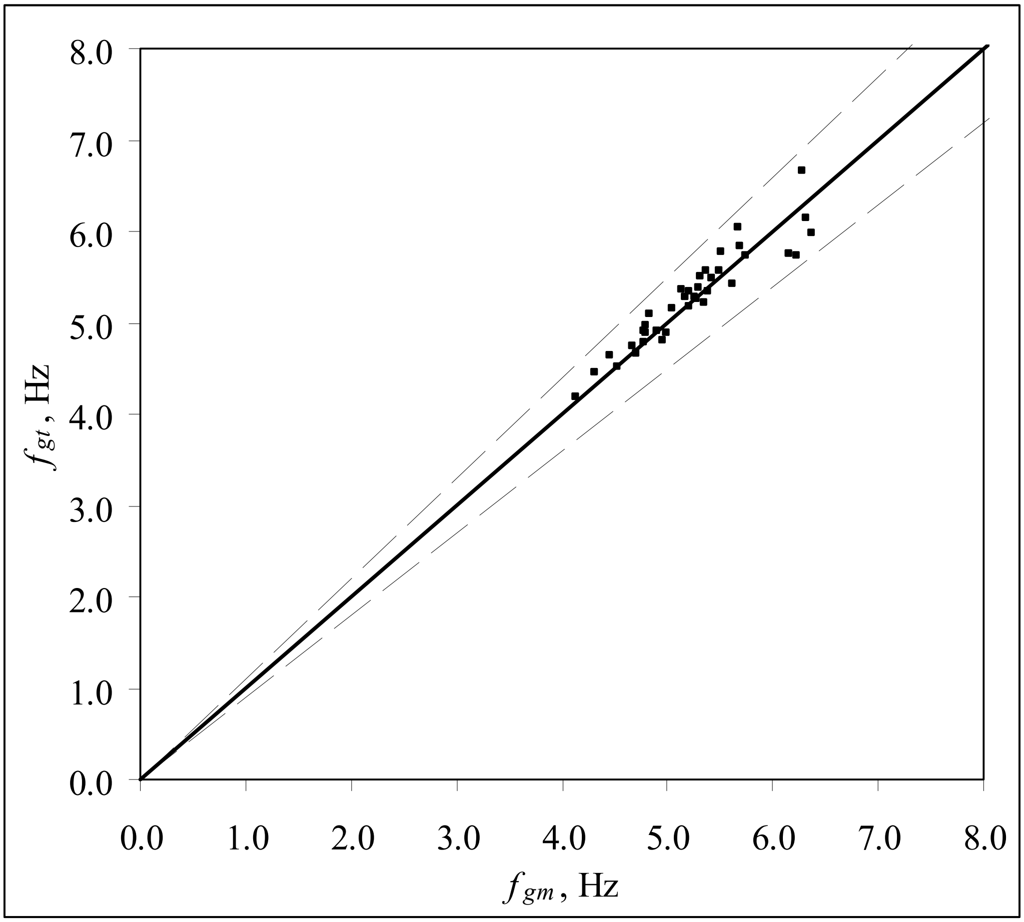


Figure 2. 13—Scatter of Measured Frequency, f_{gm}, and Theoretical Frequency, f_{gt}

2.8.2 Assessment of Design Guide Relationship

The "control floor", K-30-B, was expected to correlate with the experimental testing used to develop the current relationship and did in fact have the predicted composite effect of 25 % and predicted frequency. However, K-30-B was the only test specimen for which predictions were accurate. Surprisingly, a for the same floor with welded seats was approximately 0.17. Many LH-joist-floors had less than half of the predicted composite factor. As shown in Figure 2.14, a was much lower than predicted for the majority of tests, which was the primary cause of lower than predicted frequencies. Figure 2.15, a scatter plot of measured and predicted frequencies, shows that the frequency predictions using Equation 2.1 are too high for the majority of test specimens. The current Design Guide method of predicting I_{eff} assumes that all steel joist-supported floor systems behave the same, but large variability was found in testing the different floor types.

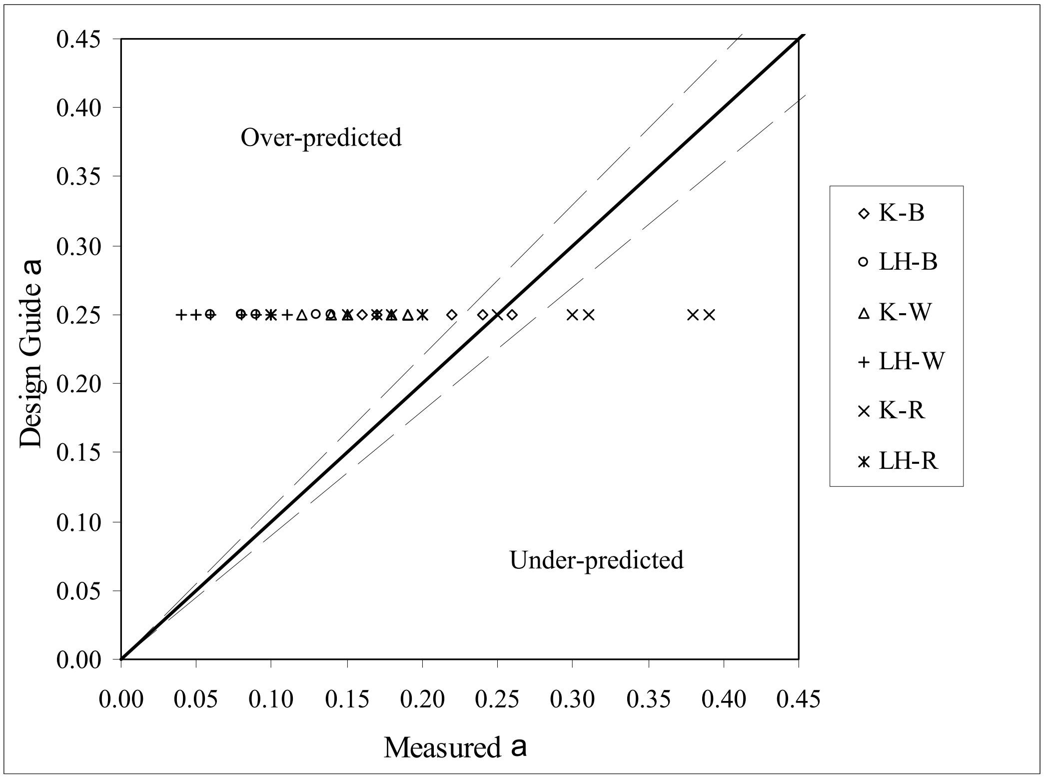


Figure 2. 14—Scatter of Design Guide a and Measured a

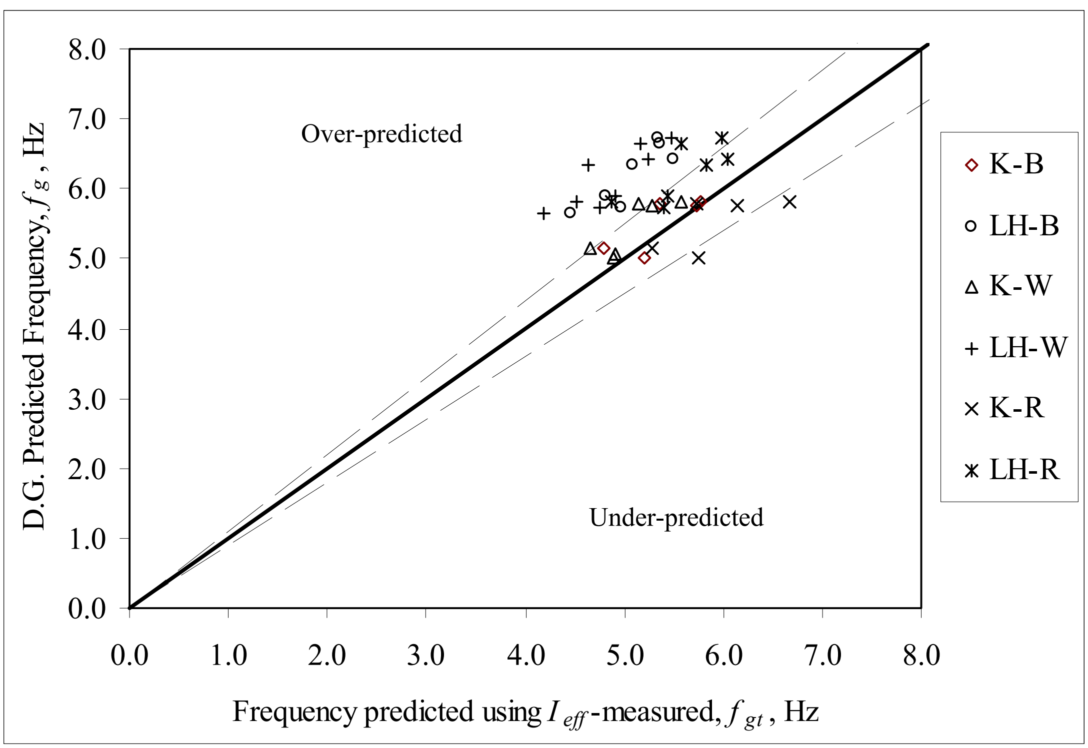


Figure 2. 15—Scatter of D. G. Frequency, f_g, and Theoretical Frequency, f_{gt}

Two comparisons will be used to show the effects of joist spacing and girder, joist, and seat connection types. For stiffness, measured a is compared to predicted a. For frequency, the ratio of f_{gt} to f_g is used for comparisons. It is noted that, if the ratio is 90 %, the actual frequency would be 10% less than predicted, thus resulting in higher than predicted peak accelerations.

Girder type and joist spacing. In several tests and FE-models, there were slight differences in the composite factor between floors with girders and with joist-girders. However, neither type was stiffer in a large majority of the tests and no trend was found with respect to frequency. Therefore, it was not possible to define a relationship between girder type and I_{eff}.

Likewise, there was no definable relationship between joist spacing and a. The finite element models did show a small decrease in stiffness when spacing was increased. However, this relationship cannot be found in the experimental results. One possible reason for this is that slab and decking properties were often varied when joist spacing was changed. The proposed improvements in Section 2.4.2 will account for the variability caused by changes in slab thickness.

Because no relationship was found between girder type or joist spacing, floors can now be placed into one of six groups with like behavior, for which the average a's and average f_{gt} to f_g frequency ratios will be used for the discussion of the effects of joist type and seat connection type. These six groups are: K-joist-floors with bolted, welded, or reinforced seats or LH-joist-floors with bolted, welded, or reinforced seats. The values for comparison of these groups are shown in Table 2.14.

Table 2. 14—Group Averages

Group	Group Average	
	a	(f_{gt}/f_g)
K-bolted	0.210	97.7%
K-welded	0.162	93.6%
K-reinforced	0.326	107.6%
LH-bolted	0.094	81.8%
LH-welded	0.073	78.9%
LH-reinforced	0.157	89.8%

Joist type and seat connection type. Equation 2.1 was more accurate for floors with K-joists than for floors with LH-joists. The average a measurements made for K-joist-floors was consistently twice the average for LH-joist-floors with the same type seat connection, as the ratios in Table 2.15 indicate. For example, bolted K-joist had an average a of 0.21, compared to 0.09 for bolted LH-joists. Similar differences in frequency resulted from the joist type. For example, K-bolted tests were approximately 98 % of predicted frequency compared to 82 % for LH-bolted tests.

Table 2. 15—Average a Values Compared With Respect to Joist Type

	a average		a ratio
	K-Joist	LH-Joist	K/LH
Bolted	0.210	0.094	2.23
Welded	0.162	0.073	2.22
Reinforced	0.326	0.157	2.08

A similar pattern can be found in comparing the behavior of the three seat connection types. With respect to bolted cases, a's of the welded cases were approximately 20 % lower and a's of the reinforced cases were approximately 60 % higher, as can be seen in Table 2.16. Similar differences in frequency result from the connection type, which can be seen in Tables 2.11 through 2.13. For example, K-bolted tests were approximately 98 % of predicted frequency compared to 94 % for K-welded and 108 % for K-reinforced floors.

Table 2. 16—Average a Values Compared With Respect to Seat Connection Type

	a average			a ratio	
	Bolted	Welded	Reinforced	W/B	R/B
K-Joist	0.210	0.162	0.326	0.77	1.55
LH-Joist	0.094	0.073	0.157	0.78	1.67

The effects of joist type and seat connection type appear to be unrelated, as evidenced by the consistent a ratios of Tables 2.15 and 2.16. Therefore, the proposed relationship will be a function of the two variables, which will be considered separately.

2.8.3 Proposed Improvement

The proposed relationship for effective moment of inertia is

$$I_{eff} = I_g + I_s + \alpha_1 (I_{comp} - I_s - I_g) \qquad \textbf{(2.15)}$$

where I_g and I_{comp} are as previous, and I_s is the slab moment of inertia, b_{tr} x $d_e^3/12$, which is in terms of steel. A composite factor a_1 is proposed for each combination of joist type and seat connection type. The slab moment of inertia, I_s, is computed for the transformed concrete section (Figure 2.11) with a width of b_{tr} and a depth of d_e.

Tables 2.17 through 2.19 show the proposed values of a_1 and the resulting stiffness and frequency. The values of a_1, shown with respect to measured a_1 in Figure 2.16, were chosen such that worst case measured frequencies of each group were approximately 97 % of the frequencies that result from the proposed relationship. By this method, measured frequencies generally range from 3 % over-predicted to 5 % under-predicted. Figure 2.17 is a scatter plot of the theoretical and proposed frequencies.

Table 2. 17—Proposed Values of a_1 for Bolted Cases

	a_1		I_{eff}		f_g			
Test Identification	Meas.	Prop.	Meas. (in^4)	Prop. (in^4)	Meas. f_{gm} (Hz)	Theo. f_{gt} (Hz)	Prop. (Hz)	(Theo/Prop.) x 100%
K-30-B	0.20		396.6	373.3	5.74	5.73	5.71	100.4%
K-30-BL	0.22		408.7	373.3	5.35	5.21	4.98	104.6%
JG-K-30-B	0.12	0.17	346.9	374.3	5.20	5.35	5.56	96.2%
JG-K-30-BL	0.13		352.4	374.3	4.77	4.79	4.94	97.0%
K-60-B	0.19		366.3	351.4	5.51	5.77	5.65	102.1%
LH-60-B	0.06		294.3	296.9	5.13	5.36	5.38	99.6%
JG-LH-60-B	0.04		299.6	317.4	5.39	5.34	5.50	97.1%
JG-LH-60-BL	0.06		317.5	317.4	4.96	4.81	4.81	99.9%
LH-90-B	0.06	0.06	319.6	318.4	4.83	5.09	5.08	100.1%
LH-90-BL	0.05		309.1	318.4	4.31	4.46	4.52	98.6%
JG-LH-90-B	0.09		380.0	348.0	5.32	5.50	5.26	104.5%
JG-LH-90-BL	0.10		389.2	348.0	4.80	4.96	4.69	105.8%

Table 2. 18—Proposed Values of a_1 for Welded Cases

Test Identification	a_1 Meas.	a_1 Prop.	I_{eff} Meas. (in^4)	I_{eff} Prop. (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g Prop. (Hz)	(Theo/Prop.) x 100%
K-30-W	0.11	0.12	331.6	338.8	5.18	5.28	5.44	97.1%
K-30-WL	0.15		359.1	338.8	4.80	4.89	4.75	103.0%
JG-K-30-W	0.08		321.1	344.9	5.04	5.15	5.34	96.5%
JG-K-30-WL	0.10		333.3	344.9	4.70	4.66	4.74	98.4%
K-60-W	0.16		341.7	317.9	5.49	5.57	5.38	103.6%
K-60-WL	0.17		350.4	317.9	4.77	4.91	4.68	104.9%
LH-60-W	0.04	0.03	273.4	267.0	5.21	5.17	5.11	101.3%
LH-60-WL	0.04		275.7	267.0	4.52	4.52	4.45	101.5%
JG-LH-60-W	0.06		315.4	290.9	5.42	5.48	5.26	104.1%
JG-LH-60-WL	0.07		329.3	290.9	4.90	4.90	4.61	106.3%
LH-90-W	0.01		264.4	283.8	4.46	4.63	4.80	96.5%
LH-90-WL	0.02		272.0	283.8	4.13	4.18	4.27	97.9%
JG-LH-90-W	0.06		346.1	317.1	5.28	5.25	5.02	104.5%
JG-LH-90-WL	0.07		357.4	317.1	4.67	4.75	4.48	106.1%

Table 2. 19—Proposed Values of a_1 for Reinforced Cases

Test Identification	a_1 Meas.	a_1 Prop.	I_{eff} Meas. (in^4)	I_{eff} Prop. (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g Prop. (Hz)	(Theo/Prop.) x 100%
K-30-R	0.28	0.26	449.5	435.3	6.32	6.15	6.16	99.8%
K-30-RL	0.35		497.8	435.3	6.15	5.75	5.38	106.9%
JG-K-30-R	0.21		398.4	427.3	6.22	5.74	5.94	96.6%
JG-K-30-RL	0.26		429.1	427.3	5.26	5.28	5.27	100.1%
K-60-R	0.38		489.8	411.8	6.29	6.67	6.12	109.0%
LH-60-R	0.08	0.10	318.2	336.8	5.37	5.57	5.73	97.1%
LH-60-RL	0.08		321.3	336.8	4.99	4.88	5.00	97.6%
JG-LH-60-R	0.13		375.0	352.7	6.38	5.98	5.80	103.2%
JG-LH-60-RL	0.16		404.0	352.7	5.62	5.43	5.07	107.0%
LH-90-R	0.15		418.2	364.5	5.70	5.83	5.44	107.2%
JG-LH-90-R	0.17		457.8	389.2	5.68	6.04	5.57	108.5%
JG-LH-90-RL	0.17		459.8	389.2	5.30	5.39	4.96	108.7%

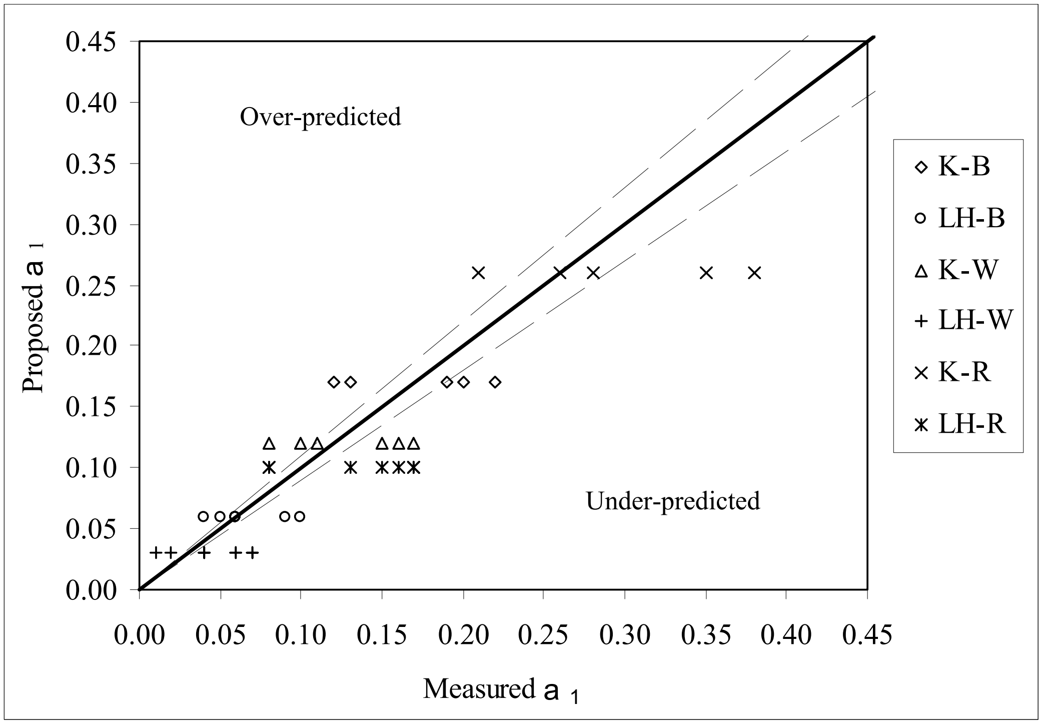


Figure 2. 16— Scatter of Proposed a_1 and Measured a_1

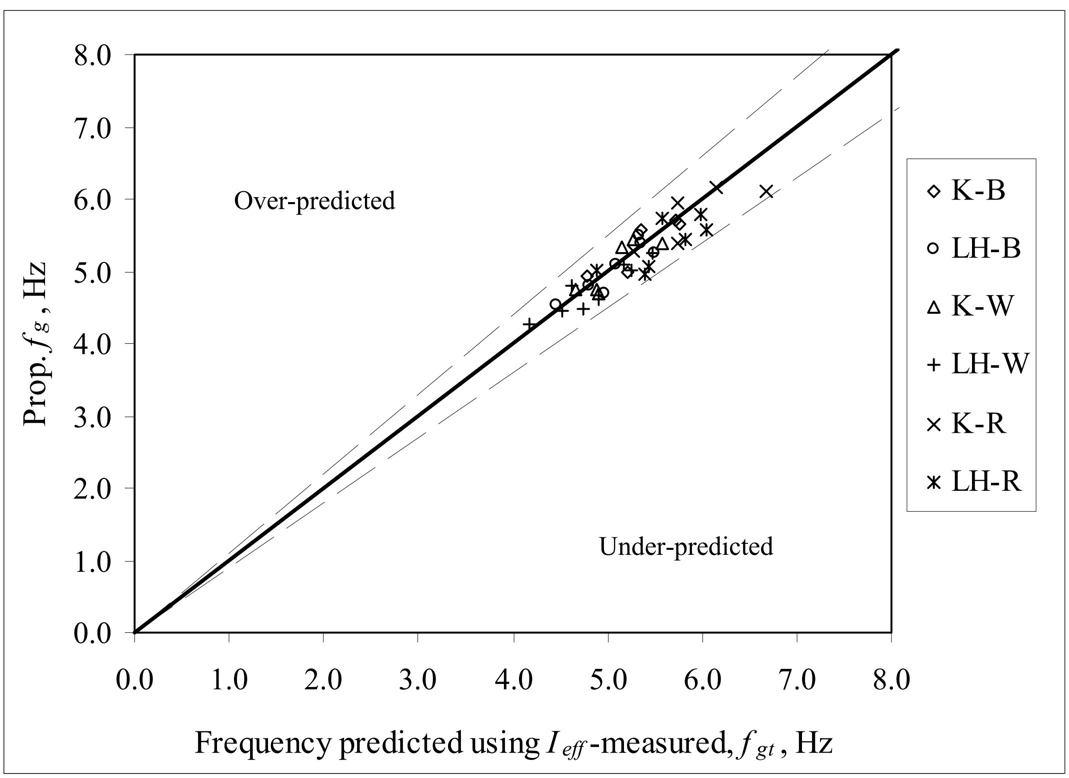


Figure 2. 17— Scatter of Proposed f_g and Theoretical, f_{gt}

2.9 Proposed Finite Element Modeling Methods

When constructed with the measured stiffness, the finite element models were accurate for predictions of frequency. This was useful for research purposes. However, for design, it is useful to predict stiffness and frequency. The prediction of stiffness requires that seat behavior be predicted. Poor correlation was found between the measured and predicted values of horizontal seat stiffness. The measured values of seat moment of inertia were approximately 1.5 times on average that required to obtain correlation with the FE-models. Two factors may have caused the difference. First, the average stiffness along the span of the joists may have been lower than the stiffness at the location of the measurements. Second, the connection of the seat testing apparatus to the top of the seat was assumed to be pinned, but may have resisted rotation to some degree. If the top connection was partially restrained, an overestimation of seat moment of inertia would result with the calculation by the cantilever equation, which assumed the seat was fixed at the bottom and pinned at the top. Thus, it is evident that the measurements should not be taken as the actual joist stiffness in the horizontal direction. However, general guidelines can be provided on a FEM results basis.

Guidelines are suggested for use with the groups described in Section 2.8.3 using the FE-modeling procedure described in Section 2.4. The curves of Figure 2.18, which represent the model stiffness that is in addition to the bare girder and slab stiffness, were developed by running the model for each footbridge with multiple seat stiffness values. The FEM I_{eff} values are found by summing the bare girder, slab, and curve values of stiffness. For each group, a seat stiffness is chosen such that worst case measured frequencies of each group were approximately 97 % of the FEM frequencies that result when the suggested seat stiffness is used. Tables 2.20 through 2.22 show the suggested seat stiffness values and the resulting stiffness and frequency comparisons.

By this method, measured frequencies generally range from 3% unconservative to 8% conservative, with exceptions of some reinforced cases that are more conservative. The bolted and welded cases give only slightly less correlation with f_{gt} than did the proposed relationship of Equation 2.15. The reinforced case suggestions under-predict frequency in more cases than with the proposed relationship.

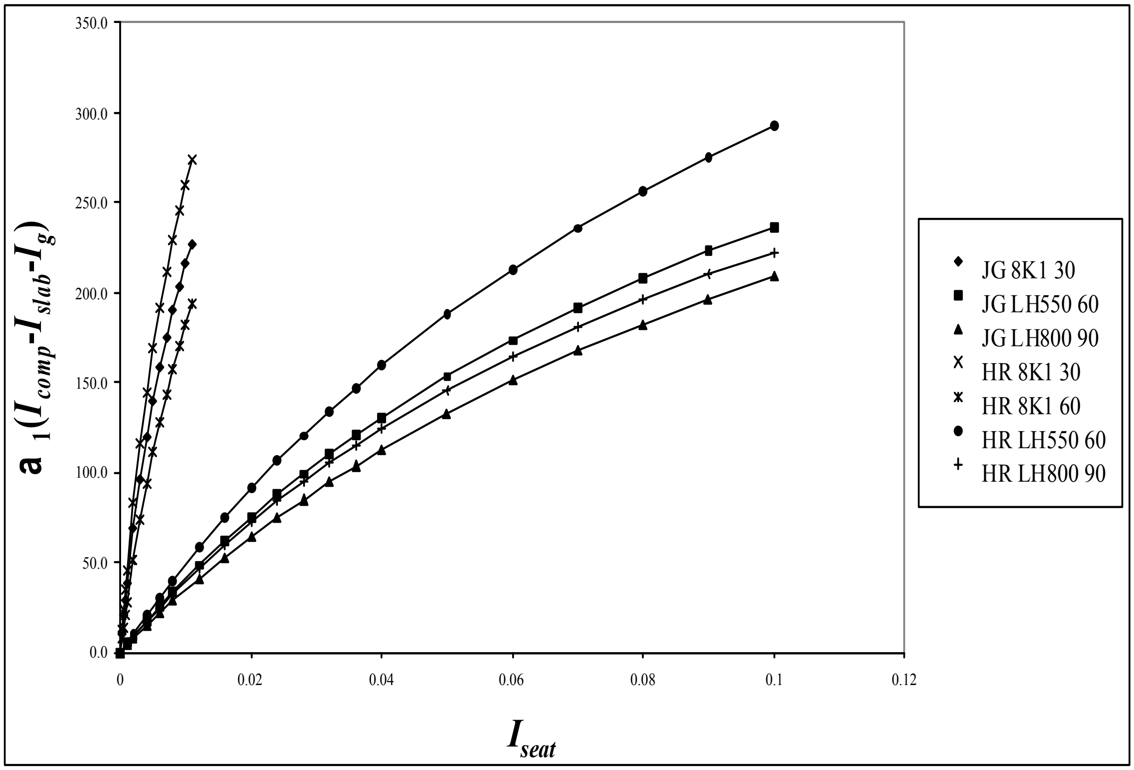


Figure 2. 18—SAP Curves for Development of Suggested Seat Stiffness

Table 2. 20—Suggested SAP Seat Stiffness for Bolted Cases

	I_{seat}	a 1		I_{eff}		f_g			
	FEM	Meas.	FEM	Meas.	FEM	Meas. f_{gm}	Theo. f_{gt}	FEM f_{gf}	(Theo/FEM)
Test Identification	(in^4)			(in^4)	(in^4)	(Hz)	(Hz)	(Hz)	x 100%
K-30-B	0.0032	0.20	0.18	396.6	379.4	5.74	5.73	5.75	99.6%
K-30-BL		0.22	0.18	408.7	379.4	5.35	5.21	5.02	103.7%
JG-K-30-B		0.12	0.17	346.9	372.1	5.20	5.35	5.54	96.5%
JG-K-30-BL		0.13	0.17	352.4	372.1	4.77	4.79	4.92	97.3%
K-60-B		0.19	0.12	366.3	319.3	5.51	5.77	5.39	107.1%
LH-60-B	0.015	0.06	0.07	294.3	307.4	5.13	5.36	5.48	97.8%
JG-LH-60-B		0.04	0.07	299.6	322.7	5.39	5.34	5.54	96.3%
JG-LH-60-BL		0.06	0.07	317.5	322.7	4.96	4.81	4.85	99.1%
LH-90-B		0.06	0.05	319.6	305.1	4.83	5.09	4.98	102.3%
LH-90-BL		0.05	0.05	309.1	305.1	4.31	4.46	4.43	100.7%
JG-LH-90-B		0.09	0.05	380.0	335.6	5.32	5.50	5.17	106.4%
JG-LH-90-BL		0.10	0.05	389.2	335.6	4.80	4.96	4.61	107.7%

Note: FEM I_{eff} and FEM f_g are the values that result when the suggested seat stiffness is used.

Table 2. 21—Suggested SAP Seat Stiffness for Welded Cases

Test Identification	I_{seat} FEM (in^4)	a_1 Meas.	a_1 FEM	I_{eff} Meas. (in^4)	I_{eff} FEM (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g FEM f_{gf} (Hz)	(Theo/FEM) x 100%
K-30-W	0.0021	0.11	0.13	331.6	342.7	5.18	5.28	5.47	96.6%
K-30-WL		0.15	0.13	359.1	342.7	4.80	4.89	4.77	102.4%
JG-K-30-W		0.08	0.12	321.1	344.3	5.04	5.15	5.33	96.6%
JG-K-30-WL		0.10	0.12	333.3	344.3	4.70	4.66	4.73	98.4%
K-60-W		0.16	0.08	341.7	293.0	5.49	5.57	5.16	107.9%
K-60-WL		0.17	0.08	350.4	293.0	4.77	4.91	4.49	109.2%
LH-60-W	0.009	0.04	0.04	273.4	281.0	5.21	5.17	5.24	98.7%
LH-60-WL		0.04	0.04	275.7	281.0	4.52	4.52	4.57	99.0%
JG-LH-60-W		0.06	0.04	315.4	300.9	5.42	5.48	5.35	102.4%
JG-LH-60-WL		0.07	0.04	329.3	300.9	4.90	4.90	4.69	104.6%
LH-90-W		0.01	0.03	264.4	284.2	4.46	4.63	4.80	96.4%
LH-90-WL		0.02	0.03	272.0	284.2	4.13	4.18	4.27	97.8%
JG-LH-90-W		0.06	0.03	346.1	316.9	5.28	5.25	5.02	104.5%
JG-LH-90-WL		0.07	0.03	357.4	316.9	4.67	4.75	4.48	106.1%

Note: FEM I_{eff} and FEM f_g are the values that result when the suggested seat stiffness is used.

Table 2. 22—Suggested SAP Seat Stiffness for Reinforced Cases

Test Identification	I_{seat} FEM (in^4)	a_1 Meas.	a_1 FEM	I_{eff} Meas. (in^4)	I_{eff} FEM (in^4)	f_g Meas. f_{gm} (Hz)	f_g Theo. f_{gt} (Hz)	f_g FEM f_{gf} (Hz)	(Theo/FEM) x 100%
K-30-R	0.0065	0.28	0.30	449.5	463.0	6.32	6.15	6.36	96.8%
K-30-RL		0.35	0.30	497.8	463.0	6.15	5.75	5.55	103.6%
JG-K-30-R		0.21	0.26	398.4	427.4	6.22	5.74	5.94	96.6%
JG-K-30-RL		0.26	0.26	429.1	427.4	5.26	5.28	5.27	100.1%
K-60-R		0.38	0.23	489.8	390.6	6.29	6.67	5.96	111.9%
LH-60-R	0.023	0.08	0.10	318.2	339.5	5.37	5.57	5.76	96.8%
LH-60-RL		0.08	0.10	321.3	339.5	4.99	4.88	5.02	97.2%
JG-LH-60-R		0.13	0.10	375.0	348.9	6.38	5.98	5.77	103.7%
JG-LH-60-RL		0.16	0.10	404.0	348.9	5.62	5.43	5.05	107.6%
LH-90-R		0.15	0.07	418.2	330.1	5.70	5.83	5.18	112.6%
JG-LH-90-R		0.17	0.07	457.8	358.2	5.68	6.04	5.34	113.1%
JG-LH-90-RL		0.17	0.07	459.8	358.2	5.30	5.39	4.76	113.3%

Note: FEM I_{eff} and FEM f_g are the values that result when the suggested seat stiffness is used.

2.10 Conclusions

The primary goal of this study was to improve the current AISC/CISC Design Guide 11 provisions for effective moment of inertia of joist-girders and girders with joist seats as horizontal shear connections. The current Design Guide relationship for the effective moment of inertia is

$$I_{eff} = I_g + (I_{comp} - I_g)/4 \quad \textbf{(2.1)}$$

where no distinctions between various types of joist supported floor systems are made. A significant relationship of I_{eff} to joist spacing or girder type was not apparent in the measured data. However, I_{eff} did depend largely on the joist type and seat connection used, which were best accounted for with the following proposed relationship:

$$I_{eff} = I_g + I_s + \alpha_1(I_{comp} - I_s - I_g) \quad \textbf{(2.15)}$$

Where, I_s, the slab moment of inertia, is separated from the composite factor, and the composite factor, a_1, is as given in Table 2.23. It was shown that, with a few higher exceptions, the measured frequencies were 97% to 105 % of frequencies predicted using the proposed relationship. By the Design Guide method, this range was approximately 75 % to 110 %, with all but the K-reinforced-floors falling below 100 %. Therefore, the proposed relationship is a substantial improvement in accuracy.

A secondary goal was to find an accurate method of predicting stiffness and thus frequency with FE modeling. As in this research, previous researchers have used FEA to accurately predict frequency for systems of known stiffness. However, only limited success has been had using the models to predict the stiffness of joist supported systems. Along with the composite factors, guidelines for FEM seat element moment of inertia are suggested in Table 2.23. It was shown that measured frequencies were approximately 97% to 107 % of frequencies for models with the suggested seat elements. The suggested models are slightly less accurate than the proposed relationship, but more accurate than the current Design Guide relationship. Good correlation of physical tests of seat moment of inertia with the FE-model seat elements was not generally found. Two factors were identified that may have caused an overestimation of seat moment of inertia in the physical testing and thus the seat test data was not used in the development of the suggested FEM methods.

Table 2. 23—Summary of Proposed and Suggested Values

Floor type	a_1	I_{seat} for FEM
K-bolted	0.17	0.0032
K-welded	0.12	0.0021
K-reinforced	0.26	0.0065
LH-bolted	0.06	0.015
LH-welded	0.03	0.009
LH-reinforced	0.10	0.023

Despite the success in proposing a more accurate method of prediction, there are aspects of the behavior of the studied systems that remain unclear. Contrary to the measured data, small FEM effects, indicated in Figure 2.18, resulted from variations in joist spacing. In the modeling, the reduction in stiffness resulting from increased joist spacing was slight, but evident nevertheless. No provision for this finding was made in the proposed relationship. This was in part because the FEM effects were small and in part because no definable effects were found in the measurements.

Similar model effects were found with respect to girder type. The joist-girder flange models rotated locally slightly more than girders when transferring seat moments, resulting in less seat stiffness, and thus less overall stiffness. This is consistent with the expected behavior of the two type girders, but could not be concluded from the measurements.

Further study of the effects of joist spacing and girder type might justify liberalization of the proposed relationship. However, it is not evident that the effect of these variables is large enough to render the proposed relationship inaccurate.

The special case test with HSS seat reinforcements had an a-value of 0.67, which was much higher than any other test footbridge. The reason for less than full composite action was not determined. However, it is clear that a joist-supported floor can be greatly stiffened by adding the reinforcements.