Design of the Control Systems Operations for the Iridium Satellite System

By

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(ABSTRACT)

The Iridium satellite system as proposed by Motorola, Inc. provides continuous global communications coverage by supplementing prevailing telephone services. The satellite system is comprised of 66 Low Earth Orbiting (LEO) satellites which are able to accommodate Iridium handsets or existing telephone links. These satellites are monitored and controlled by ground control facilities for approximately 6 years (the estimated life of the vehicles) and are then replaced.

The ground control facilities are required to provide software upgrades, monitor the health of the vehicles, remove a satellite from the constellation when necessary, and coordinate launch and deorbit of the satellites. The approach used to code and modulate the telemetry processed by the ground stations is analyzed in order to develop support software for a recommended data network.
configuration used at the ground stations. A Reed-Solomon coding scheme is prepared for the telemetry channel since this coding scheme corrects bit errors and protects against burst errors. The Groupe Speciale Mobile (GSM), a combination of time division multiple access, TDMA, and frequency division multiple access, FDMA, is recommended for use by Iridium. A bus topology with a fiber backbone is suggested to support the operations at each facility and a separate landline is considered to support operations between the facilities when possible. The support software is available in the appendix and is written in Fortran 77.

A data security coding algorithm is introduced; however, this is not a requirement or a concern for Motorola and is an option that may be considered by the ground facilities. The data security for a commercial satellite system is considered insignificant at this time.
ACKNOWLEDGEMENTS

I am greatly indebted to the many people who have supported me during my effort to complete my Master’s Degree. I only wish that I could thank everyone individually. I would especially like to thank my parents, Charles and Ruth Holtzman. Without their help, I never would have been able to complete this thesis.

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</tbody>
</table>
CHAPTER 1    INTRODUCTION

Motorola is planning to bring communications closer together by introducing Iridium. The Iridium constellation will consist of 66 low earth orbiting (LEO) satellites. These satellites and handsets will make it possible to communicate in virtually any part of the world. In order to accomplish this feat, the Iridium system requires a link budget design, ground stations, gateways, satellite design, handset design, link analysis, frequency allocations, launch analysis, and marketing analysis.

The system and segments provided in the report are shown in Figure 1. It is the purpose of this paper to provide the following:

(1) reliable satellite control system telemetry capable of recovering from burst errors
(2) privacy for the satellite control system telemetry
(3) an analysis of the frequency and time division used for the satellite control system telemetry and the telemetry produced from the handsets and gateways
(4) a recommendation for the network design for the main satellite control facility
(5) a program used by the satellite control facility to monitor and control the 66 satellites
FIGURE 1  CONTROL SYSTEMS OPERATIONS FOR IRIDIUM

Path A Shows the Uplink Command and Control Telemetry
Path B Shows the Downlink State-of-Health Telemetry
Path C Shows the Uplink and Downlink for the ISU
In Chapter 3, Error Correction Coding is developed for Path A and B
In Chapter 4, Data security is developed for Path A and B
In Chapter 5, the TDMA/FDMA analysis is given for Path A, B, and C
In Chapter 6, the Network design is given for the System Control Segment
In Chapter 7, the Computer program is given for the System Control Segment
By providing these recommendations, this paper in conjunction with previous papers written on link budget design and other aspects of the system may be used to suggest future improvements for Motorola's Iridium design. Certain required information concerning the project is considered proprietary; therefore, information which is not obtained through articles, texts, and interviews is determined by reasonable assumptions which are based on other ground station configurations.

These assumptions are:

- changes in orbital dynamics due to the new number of satellites will not affect the analyses in this paper unless so noted
- the network operations data (NOD) is not required to use the GSM format of the ISU
- the required BER for solar disturbances and weather is less than or equal to $1 \times 10^{-7}$
- ground stations and gateways employ forward error correction
- design considerations are partially dictated by weight considerations for the satellite
- network operations data originates as digital data in an 8-bit ASCII format
- bit representation can be reduced to 4-bits by
designating locations in the telemetry channel

The method used to provide the reliable satellite control system telemetry capable of recovering from burst errors is provided in Chapter 3. Three coding schemes are developed and compared. The first two are Reed-Solomon codes, and the third is a BCH code. The preliminary design for the Reed-Solomon codes is given in order to produce the generator polynomials for the codes. An alternative BCH code is offered as a more simplistic approach to the satellite scenario. The Viterbi algorithm is mentioned as a possible alternative. The analysis for the Viterbi algorithm is omitted since the process requires a review of the statistical probability of errors occurring along the channels for the satellites.

Privacy for the satellite command and control telemetry is discussed in Chapter 4. Motorola is not intending to provide privacy assurance for the handset data which is passed. A public key and algorithm is suggested to provide a means of data security. This data security along with the error correcting codes are given to produce reliable command and control telemetry.

In Chapter 5, an analysis of TDMA and FDMA is provided for the overall system and mission telemetry. Motorola has developed a scheme of combining both TDMA and FDMA for the 66 satellites. This section is given to determine if one
method or a combination of both would be useful for Iridium. A recommendation for the best modulation technique is determined by comparing each method based on common aspects.

The network design is for the Master Control Facility which monitors and controls the state of health of the Iridium satellites. The network choices are provided in Chapter 6. The decisions are based on overall system requirements, cost analysis, and inter-operability.

Finally, support software is designed for the operation of the satellites. The support software is created to be used in the Master and Backup control facilities. Appendix F contains the FORTRAN 77 program which is addressed in Chapter 7. An alternate programming language is considered in the discussion section.

The Iridium system as proposed by Motorola Inc. is scheduled for final deployment and operation in 1996. (18, p30) Iridium services are scheduled to be provided as soon as 1997. Table 1 contains the estimated schedule for Iridium. (10, p2) The first satellite launch, however, has slipped to December of 1995. Using the relative times from the projected schedule, the new estimated time of service will be late 1998. With this projected schedule and the introduction of replenishment satellites every 6 years, Motorola’s Iridium system will continue to be under development.
<table>
<thead>
<tr>
<th>Year</th>
<th>Event</th>
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<td>Satellite Construction Begins</td>
</tr>
<tr>
<td>1994</td>
<td>Satellite Constructed</td>
</tr>
<tr>
<td>1994</td>
<td>First Satellite Launched</td>
</tr>
<tr>
<td>1995</td>
<td>First Satellite in Service</td>
</tr>
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<td>1996</td>
<td>Last Satellite Constructed</td>
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<tr>
<td>1996</td>
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</tr>
<tr>
<td>1996</td>
<td>Iridium begins Operations</td>
</tr>
<tr>
<td>1997</td>
<td>Iridium provides Services</td>
</tr>
</tbody>
</table>
CHAPTER 2  OVERVIEW OF THE IRIDIUM SYSTEM

Geosynchronous communication satellites and cellular telephones provide a limited means of communication. Communication capabilities through geosynchronous satellites, which continue to orbit the earth today at an altitude of approximately 35,717.5 km, (5, p4) are confined by the distance of the ground transmitter and by existing telephone systems.(27, p3) By reversing the current technology so that the ground station operations are primarily performed in orbit, cellular telephone technology may reach many areas of the world without delay. Motorola, Inc. has designed a system of satellites which performs just that task, and the system is known as Iridium.

Iridium is a constellation of 66 low earth orbiting (LEO) satellites, reduced from the original 77 satellites, which increase the ground coverage for existing phone systems. These satellites are launched into 6 planes of eleven satellites each (5, pg5) at a height of 765 km (778 km) above the earth. (22, p10) The Iridium configuration evenly spaces the satellites among the 6 polar orbits to allow for constant global coverage.

The Iridium satellites process the digital signals received from existing cellular phones, hand held units
developed by Motorola, or other Iridium satellites. The hand held units take advantage of the LEO orbits which reduces the power requirement necessary to receive and transmit directly to the satellite. The cellular phones utilize the original network design and are then uplinked through a gateway station. Figure 2 (27, p20) shows the path of the Iridium subscribers phone calls. Table 3 provides a breakdown of the characteristics of the Iridium System (27, p25).

2.1 Orbital Dynamics

Many of the research papers and articles obtained on the Iridium system are based on Motorola's original proposal of 77 satellites (10, p1). Currently, the design uses 66 satellites as a result of a reduction announced in 1992 due to the high operating cost of the system. This reduction does not minimize the coverage area but does reduce the number of zones in which each satellite overlaps. Consequently, there are only slight modifications to the original plan which are necessitated by this change.

The 66 Iridium satellites are launched into six planes of eleven satellites each (5, p5). This constellation of satellites, shown in Figure 2, is designed to avoid collision over the poles and to provide maximum ground coverage. The satellites are spaced equally within each
plane. Each plane covers 360 degrees, so the distance, $\theta$, between each intraplanar satellite is given by (27, p12)

$$\theta = \frac{360^\circ}{11 \text{ satellites}} = 32.767^\circ \quad \text{Equation 2.1.1}$$
FIGURE 2 PATH OF THE IRIDIUM SUBSCRIBERS PHONE CALLS
FIGURE 3  CONSTELLATION OF SATELLITES
Each satellite orbits the earth approximately once every 100 minutes. (15, p49) The satellites travel the earth in the same direction such that "all satellites in one hemisphere travel from north to south, while all satellites in the other hemisphere travel in the opposite direction." (27, p8)

2.2 The Iridium Spacecraft

The Iridium Spacecraft or Satellite Vehicle (SV) is expected to have a minimum in orbit lifetime of 6 years with 8 years of consumables on board. (22) In order to maintain the 66 satellite configuration in orbit with this life expectancy, a supply of spare vehicles must be available. A longer life expectancy is undesirable considering the advances in technology and the changing communication needs; however, a shorter life prevents the possible financial recovery from initial operating costs.

Lockheed has currently been chosen to develop and construct the SV. Figure 4 shows a medium sized satellite constructed by Lockheed for general purposes. (22) The vehicle is similar in construction to the smaller Iridium satellite. The power system, navigation and control system, propulsion system, and antenna system are based on prior satellites.
FIGURE 4  PROPOSED IRIDIUM SATELLITE VEHICLE
2.2.1 Power System

Electrical power is supplied to the on board equipment by a combination of solar panels and nickel-hydrogen batteries. A fault isolation-distribution system dispenses the power throughout the SV. A bus voltage between 22V and 36V is maintained by bus regulators and battery charging controls. (10, p4)

Solar cells supply the power for all current communications satellites (23, p53). The solar cells are composed of GaAs/Ge (10, p4) and have a typical efficiency of 10 to 15 percent. This means that the solar cells convert about 10 to 15 percent of the energy from the sun to electrical power. As the SV ages, the efficiency of the solar cells falls. This is usually compensated for by adding approximately 15 percent extra area of solar cells. (23, p66)

The Iridium satellites have 18 solar panels which are divided among 2 wings. Since the SV is in a LEO orbit and not a GEO orbit, the panels must be sun tracking continually which is accomplished by stepper motors (10, p4). The solar panels maintain the bus voltage and charge the batteries while in the sun; however, during periods of eclipse and on the dark side of the earth, the batteries are used to continue global coverage.

14
Nickel-cadmium and recently nickel-hydrogen batteries developed by RCA are used on communication satellites. Nickel-hydrogen batteries are lighter than nickel-cadmium batteries and reduce the significant amount of weight added to the SV by the power system which translates into reduced cost at launch. Therefore, the Iridium SV uses nickel-hydrogen batteries. These NiH batteries have a life of 48 amp-hours (10, p4) and "can be safely discharged to 60% of capacity (23, p68)."

2.2.2 Navigation and Control System

The operation of an on orbit satellite is maintained by ground communications and autonomous control. The on-board logic monitors the health of the SV, performs orbital maintenance, and controls the SV equipment. The same functions are performed manually by the ground stations. The combination of these two methods illustrated in Figure 5 enables the satellites to maintain proper orbital configuration while accomplishing the main objective.

The navigation and immediate response to equipment failures is performed by logic gates and sensors which are on board the SV. Iridium satellites utilize a pair of on-board Barnes Engineering Dual Cone Scanners in order to navigate the satellite. (10, p4) These scanners position the satellite using the earth, moon, and sun reference. Control software evaluates relative position based on
various other sensors on the vehicle. In the event of equipment failure, the control software is used to initiate backup systems and report the status to the ground.

The earth station monitors telemetry, computes attitude adjustments for tracking purposes, and controls on-board equipment. SV equipment and subsystems status are reported by
FIGURE 5 TYPICAL (TT&C)
sensors through telemetry. The telemetry is evaluated for orbital corrections, tracking, and general vehicle health. Once evaluated, commands are sent to the respective equipment.

If attitude corrections are necessary or equipment is faulty, the ground is used to command the respective subsystems. Commands are sent to the SV and checked for validity and sent to the correct equipment. (23, p65). In the case of the orbital subsystem, a momentum gyroscopic wheel and a magnetic torquing mechanism are commanded for minor attitude errors. Small hydrazine powered thrusters are used for other coarse corrections such as deorbit. (10, p4) Alternatively, ground commands may be used to control the thrusters for minor adjustments if the gyroscopic wheel should fail.

2.2.3 Propulsion System

The propulsion system is used to deploy, maneuver, and deorbit the vehicle. The system which is composed of a mono-propellant hydrazine mechanism is monitored by the ground. Each SV is estimated to have 413 kg of propellant after launch. This is sufficient for the estimated life of the vehicle. (10, p4)

2.2.4 Antenna System

The satellites are equipped with two types of antennas. A cupped dipole antenna and six phased array antennas are
used for L-band transmissions for ground communications. (10, p5)

Cross Link communications between neighboring satellites is performed with the use of waveguide slot arrays. The waveguides autonomously track the satellites and provide an alternative path for routing signals in the event of a satellite failure. Transmissions between satellites use the Ka band. (10, p5)

2.3 The Ground Stations

The number of ground stations and geographical locations for the ground stations have been determined by Motorola. A tentative plan has been established for the location and number of gateways. Iridium must first provide an initial test with a few launched satellites prior to approval from investors and the global market. (4) However, the initial plan is to provide a series of gateway sites (18, p34) and two control facilities.

2.3.1 The Gateways

The Iridium gateways integrate Iridium technology into existing telephone technology and route satellite telemetry to ground control facilities. The gateways are to be scattered around the globe to supplement existing phone services but are not required to complete a call from one Iridium Subscriber Unit (ISU) to another. A greater number of gateways is suspected to be located closer to the poles
to monitor the satellites at these critical locations. (24). This will prevent loss of communication with the satellites in the event of gateway downtime or bad weather conditions by providing backup gateway sites. Similarly, the gateways employ "a minimum of two 3.3 meter tracking dish antennas that are separated by up to 20 miles...(9, p34)" and maintain reliability of the system.

The gateways convert Iridium's 4.8 KBPS protocol to the local Postal, Telephone and Telegraph Authorities (PSTN) protocol. This is necessary since the satellites are essentially acting as repeaters, transmitting and receiving data to various locations through digital switching devices, and do not accommodate each ground system. Future gateways may employ a higher quality 2.4 KBPS vocoder which will not affect the operation of the satellites (18, p34).

2.3.2 System Control Segments

The System Control Segment (SCS) or Iridium ground station performs the state of health operations required to maintain the satellites. This segment includes a Master Control Facility (MCF), a Backup Control Facility (BCF), Telemetry, Tracking, and Commanding (TTAC) Facility, and relays from the gateways. (5, p7) The planned location of these facilities are listed in Table 2.
<table>
<thead>
<tr>
<th>Segment</th>
<th>Location</th>
<th>Latitude</th>
<th>Longitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCF</td>
<td>Washington, D.C.</td>
<td>38.54N</td>
<td>77.01W</td>
</tr>
<tr>
<td>BCF</td>
<td>Chandler, Arizona</td>
<td>33.18N</td>
<td>111.50W</td>
</tr>
<tr>
<td>TTAC</td>
<td>Yellowknife, Canada</td>
<td>62.27N</td>
<td>114.21W</td>
</tr>
<tr>
<td></td>
<td>Squalit, Canada</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td></td>
<td>Waimea, Hawaii</td>
<td>21.39N</td>
<td>158.04W</td>
</tr>
</tbody>
</table>
The SCS monitors telemetry from the satellites, performs software upgrades to the satellites, maintains a working Iridium configuration, initiates station keeping maneuvers when necessary and coordinates the commanding and receipt of telemetry through the MCF. The TTAC is capable of monitoring two satellites in view and relaying the information to the MCF. The SCS is conjunctively responsible for cell activation and deactivation as well as the network clock.

2.4 Iridium Handsets

The Iridium handsets, also known as Iridium Subscriber Unit (ISU), is a handheld, portable unit used for direct communication to Iridium satellites. Unlike existing mobile phones, the ISU does not require a gateway within the immediate vicinity to uplink the signal to the satellite. The ISU is capable of operation with a minimum of 600 mWatts of power and a maximum of 7 Watts (which is the biological safety constraint). (18, p35).

The services available include paging, Global Positioning Services (GPS), and basic phone communications. The GPS capability is an option which improves the ISU communication and offers a location service. With the GPS enhancement, the ISU warmup period is reduced to less than 1 minute. The ISU does not require GPS to be fully operational. (18, p35).
2.5 Iridium Data

Two functions are performed by the Iridium ground stations:
(1) the monitoring of mission performance and (2) satellite system upkeep. The data used to support the mission is referred to as the Mission Data (MD) and Mission Control Data (MCD). System Control Data (SCD) and Network Operations Data (NOD) are processed by the ground stations to evaluate overall system performance. (5, p8)

2.5.1 Mission Data (MD) and Mission Control Data (MCD)

Mission Data, or customer information, is processed and transferred by the use of Mission Control Data (MCD). Speech, digitized computer transfers, and facsimiles are a few forms of Mission Data received and transmitted by Iridium. The origination and termination points for the information are appended to the Mission Data as MCD. MCD also handles GPS information and handoffs.

2.5.2 System Control Data (SCD) and Network Operations Data (NOD)

The system control and network operations data consists of the telemetry used to monitor the satellite hardware functions, ephemeris data, orbital velocity, and satellite position. This data is processed by the ground stations for satellite maintenance. Commanding performed by the SCS is also considered a form of SCD and NOD.
### TABLE 3 CHARACTERISTICS OF THE IRIDIUM SYSTEM

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Satellites</td>
<td>66 (reduced from 77)</td>
</tr>
<tr>
<td>number of Projection cells/satellite</td>
<td>48 from 37</td>
</tr>
<tr>
<td>cell diameter</td>
<td>360 miles</td>
</tr>
<tr>
<td>projection area/satellite</td>
<td>100,000 square miles</td>
</tr>
<tr>
<td>satellite size</td>
<td>2 meters high by 1 meter in diameter</td>
</tr>
<tr>
<td>satellite weight</td>
<td>700 lbs</td>
</tr>
<tr>
<td>number of uplink/downlink antennas</td>
<td>7</td>
</tr>
<tr>
<td>uplink/downlink frequency</td>
<td>1.610-1.6265 GHz</td>
</tr>
<tr>
<td>number of crosslink antennas</td>
<td>4</td>
</tr>
<tr>
<td>crosslink frequency</td>
<td>20 GHz</td>
</tr>
<tr>
<td>Iridium phone weight</td>
<td>25 ounces</td>
</tr>
<tr>
<td>Iridium phone output power</td>
<td>7 Watts peak</td>
</tr>
<tr>
<td>Orbital Period</td>
<td>100 minutes</td>
</tr>
<tr>
<td>manufacturer of satellites</td>
<td>Lockheed</td>
</tr>
<tr>
<td>Space Vehicle In-Orbit Life batteries</td>
<td>Nickel Hydrogen</td>
</tr>
<tr>
<td>Call complete within</td>
<td>10 seconds</td>
</tr>
<tr>
<td>Each Beam can handle 230 calls</td>
<td></td>
</tr>
<tr>
<td>Gateway Antennas up</td>
<td>2.3.3 meter tracking dishes separated by to 20 miles</td>
</tr>
</tbody>
</table>
CHAPTER 3 CODING FOR THE TELEMETRY CHANNEL

The Iridium project proposed by Motorola for the late 1990's requires constant surveillance of the 66 low earth orbiting (LEO) satellites. This requirement is to be met by several ground stations which monitor the health of the satellites through telemetry. Due to the amount of information transferred and the time in view for each satellite, the SCS telemetry channels monitored should be reliable and correct.

A Reed-Solomon code is proposed to correct random errors and protect against burst errors encountered by the received and transmitted telemetry channel. In order to develop the Reed-Solomon code for the Iridium system, several assumptions are made based on the information gathered from papers and articles. Alternative coding schemes are developed based on different assumptions for comparison purposes.

A convolution code developed with the Viterbi algorithm is an alternative to the Reed-Solomon code. The convolution code is a more powerful technique; however, "decoding convolution codes is a complex process that requires decisions on the most likely transmitted data sequence where a codeword is received in error." (23, p314-315)
information is unattainable through articles and would require random speculation.

3.1 Iridium's Communication System

The paths for Iridium's communications system, shown in Figure 6, route the various forms of Iridium data to the proper destination with minimal degradation. Degradation of a signal is the result of an increase in the bit error rate (BER) for digital signal or a decrease in the signal-to-noise ratio (S/N) for analog signals. Frequency and phase modulation improve signal strength for both analog and digital signals; however, the result is a loss in bandwidth. Digital data also lends itself to error correcting codes which improve bit error rate. This is not yet possible for analog data. (23, p281)
PATH A ISU to Vehicle
PATH B Vehicle to Vehicle
PATH C Vehicle to Gateway
PATH D Vehicle to Ground Station
PATH E Gateway to Ground Station
PATH F Gateway to Local Phone Service
PATH G Gateway to Gateway
PATH H Gateway to Other Mobile Phones not an ISU

FIGURE 6 PATHS FOR IRI DIUM'S COMMUNICATIONS SYSTEM
Analog signals are converted to digital signals at the ISU for transmission over the Iridium network in order to incorporate the signal enhancing techniques. As stated in Chapter 2.3.1, voice data is transmitted at 4.8 kbps using Pan-European Groupe Speciale Mobile (GSM). The digitized voice data employs Forward Error Correction (FEC) which enables some errors to be corrected as well as detected (5, p10).

Iridium data transfer between satellites also employs error correction (EC) techniques. The data is still encoded with the capability for immediate correction; however, the satellites may not be equipped at each antenna with the error correcting circuitry. Instead, the transmission is just repeated. This is more feasible in space since the S/N ratio is not degraded by weather or man-made interferences. Future satellites may incorporate the error correcting circuitry.

Error correcting techniques improve the chances of a clean receipt thus providing less voice channel interruption. Interruptions are "perceived as a click which degrades the quality of communication and limits the throughput of control information during a call." (13, p366)

System control data and mission data received by various gateways and ground stations use FEC to combat signal losses due to solar disturbances and weather. The
bit error rate (BER) received is maintained less than $10^{-7}$ by employing these methods which meet the criteria set for the gateway links as well as the inter-satellite cross link KA bands. (10, p5).

3.2 The Reed-Solomon Code

As stated in Section 3.1, the Iridium Satellite Vehicles (SV) process transmitted voice communication using Forward Error Correction (FEC) and Error Correction (EC). (10, p9) Since each SV must perform much of the processing of the data received and transmitted with a minimum weight desired, it is logical to assume that the circuitry required is kept to a minimum. In keeping with this assumption, the network operations data (NOD) is presumed to be integrated with the mission data (MD) when possible.

The main difference between mission data and network operations data is the format in which it is received. The network operations data is assumed to have originated in each SV as digital data; whereas, the mission data originates as analog data. Thus, the network operations data is assumed to be transmitted in a common digital form, 8-bit ASCII. The 8-bit ASCII code is commonly used to transmit computer data over telephone lines and radio links. (23, p286)

Using the above information, a Reed-Solomon Code is developed to detect and correct errors encountered during
transmission of the NOD data. A Reed-Solomon code is described by a specific Galois Field (GF), the number of correctable symbol errors (t), and a generator polynomial. The GF(p) is defined as a prime field which exhibits the following characteristics:

1. p is prime.
2. The set of integers \{0, 1, 2, ..., p-1\} is a commutative group under modulo-p addition.
3. The nonzero elements, \{1, 2, ..., p-1\} form a commutative group under modulo-p multiplication.
4. The real number multiplication is distributive over real number addition, and modulo-p multiplication is distributive over modulo-p addition. (19, p21)

A GF of particular interest to digital computer and digital data applications is the binary field, GF(2), or its extension GF(2^M). (19, p24) Using the assumption of an 8-bit ASCII format, the Reed-Solomon Code is designed with a Galois Field of GF(2^8) or GF(256).

The number of correctable symbol errors (t) is based on the probability of an undetected error, \( P_{ue} \), and the probability of a single bit error, \( p \). The \( P_{ue} \) for the system, as given in Section 3.1, is less than \( 10^{-7} \). Weather conditions and space disturbances cause the probability of a single bit error to range from \( 10^{-3} \) in bad conditions to \( 10^{-6} \) in good conditions. (23, p315) The relationship between
the $P_{ue}$ and $p$ is given by

$$P_{ue} = \binom{n}{d+1} p^{d+1} (1-p)^{n-(d+1)}$$  \text{Equation 3.2.1}

where $n$ is the code length and $d$ is the number of detectable errors.

The number of detectable errors and the number of correctable symbol errors is calculated by using the following equations:

$$MD \geq d + 1, \quad \text{Equation 3.2.2}$$

$$MD \geq 2t + 1, \quad \text{Equation 3.2.3}$$

$$MD \geq d + t + 1 \quad \text{Equation 3.2.4}$$

where $MD$ is the minimum distance which is equal to $2t + 1$ for a Reed-Solomon Code. (19, p171)

By choosing a conservative symbol error correction parameter, $t$, the $P_{ue}$ remains within the specified parameters. The number of correctable symbol errors, however, is dependent upon the $MD$ and the number of detectable errors. A variety of $t$ and $d$ values can satisfy equations 3.2.2, 3.2.3, and 3.2.4; a specific value is determined by using equation 3.2.1.

The code length, $n$, is determined from
\[ n = m \times (2^m - 1) \quad \text{Equation 3.2.5} \]

Substituting the 8-bit ASCII, into the above equation yields
\[ n = 8 \times (2^8 - 1) = 8 \times (255) = 2040 \quad \text{Equation 3.2.5a} \]

Using the results from equation 3.2.5, the worst-case conditions for \( p \), and an assortment of \( d \) figures in equation 3.2.1 yields various \( P_{ue} \) values. As shown in Table 4, \( P_{ue} \) decreases with \( d \) and increases with \( t \). Therefore, the logical choice based on this data is to use the largest \( d \) with the smallest \( t \); however, the probability of retransmission, \( P_{ret} \), is maximized in this case. Since the vehicle data for 66 satellites must be received and processed by the Master Control Facility (MCF), a large \( P_{ret} \) is unacceptable.

\[ P_{ret} \] is determined by
\[ P_{ret} = \frac{n}{t+1} \left( p^{t+1} \right) (1-p)^{n-(t+1)} \quad \text{Equation 3.2.6} \]

Using the data from Table 4, \( d \) is chosen to be 10 and \( t \) is chosen to be 10. This choice produces a MD of 21 and satisfies equation 3.2.2, 3.2.3, and 3.2.4. Unfortunately, these values yield a \( P_{ue} \) slightly higher than \( 10^{-7} \); however, the probability of retransmission is reduced. The larger \( P_{ue} \) is used since the number of gateways within a
certain region allow for alternative routes if poor weather conditions arise.
### TABLE 4 $P_{ue}$, $P_{ret}$, $P = 10^{-3}$

<table>
<thead>
<tr>
<th>d</th>
<th>t</th>
<th>$P_{ue}$</th>
<th>$P_{ret}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>19</td>
<td>2.65x10^-1</td>
<td>7.73x10^-14</td>
</tr>
<tr>
<td>1</td>
<td>18</td>
<td>2.70x10^-1</td>
<td>7.65x10^-13</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>1.84x10^-1</td>
<td>7.18x10^-12</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>9.38x10^-2</td>
<td>6.38x10^-11</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>3.82x10^-2</td>
<td>5.35x10^-10</td>
</tr>
<tr>
<td>5</td>
<td>14</td>
<td>1.30x10^-2</td>
<td>4.23x10^-9</td>
</tr>
<tr>
<td>6</td>
<td>13</td>
<td>3.78x10^-3</td>
<td>3.12x10^-8</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>9.61x10^-4</td>
<td>2.15x10^-7</td>
</tr>
<tr>
<td>8</td>
<td>11</td>
<td>2.17x10^-4</td>
<td>1.38x10^-6</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>4.42x10^-5</td>
<td>8.15x10^-6</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>8.15x10^-6</td>
<td>8.15x10^-6</td>
</tr>
<tr>
<td>10</td>
<td>9</td>
<td>8.15x10^-6</td>
<td>4.42x10^-5</td>
</tr>
<tr>
<td>11</td>
<td>8</td>
<td>1.38x10^-6</td>
<td>2.17x10^-4</td>
</tr>
<tr>
<td>12</td>
<td>7</td>
<td>2.15x10^-7</td>
<td>9.61x10^-4</td>
</tr>
<tr>
<td>13</td>
<td>6</td>
<td>3.12x10^-8</td>
<td>3.78x10^-3</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>4.23x10^-9</td>
<td>1.30x10^-2</td>
</tr>
<tr>
<td>15</td>
<td>4</td>
<td>5.35x10^-10</td>
<td>3.82x10^-2</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>6.38x10^-11</td>
<td>9.38x10^-2</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
<td>7.18x10^-12</td>
<td>1.84x10^-1</td>
</tr>
<tr>
<td>18</td>
<td>1</td>
<td>7.65x10^-13</td>
<td>2.70x10^-1</td>
</tr>
<tr>
<td>19</td>
<td>0</td>
<td>7.73x10^-14</td>
<td>2.65x10^-1</td>
</tr>
</tbody>
</table>

*Note: $P_{ue}$ and $P_{ret}$ for a $d$ of 10 and a $t$ of 10 with $p$ equal to $10^{-6}$ is $6.20x10^{-38}$ and $6.20x10^{-38}$, respectively.*
The generator polynomial for a Reed-Solomon Code with a
$GF(2^m)$ and $t$ correctable symbol errors is given by
\[ g(x) = (x + \oplus)(x + \oplus^2)\cdots(x + \oplus^{2^t}) \]
Equation 3.2.7
Substituting $t$ equal to 10 into equation 3.2.7 yields
\[ g(x)_{GF(256)_{rs}} = (x + \oplus)(x + \oplus^2)(x + \oplus^3)\cdots[\text{Equation 3.2.8}] \]
Reducing the generator polynomial requires the use of
the tables located in Appendix B. First, a systematic
multiplication of the factors of the generator polynomial is
performed to yield
\[
g(x)_{GF(256)_{rs}} = \] \[ [x^2 + (\oplus + \oplus^2)x + \oplus^3][x^2 + (\oplus^3 + \oplus^4)x + \oplus^7]\cdots [x^2 + (\oplus^{19} + \oplus^{20})x + \oplus^{39}] \]
Equation 3.2.9

Using Appendix B, $(\oplus^{19} + \oplus^{20})$ is reduced to a single term
as shown below in Table 5.
TABLE 5 REDUCTION OF $\oplus^{19} + \oplus^{20}$ USING APPENDIX B

\begin{align*}
\ominus &= \text{Appendix B Value} \\
\ominus^{19} &= 01011010 \\
\ominus^{20} &= 00101101
\end{align*}

Addition of Appendix B Values

\[
\begin{array}{c}
01011010 \\
+ 00101101 \\
01110111 = b
\end{array}
\]

Matching $b$ with the values in Appendix B

\[
b = 01110111 = \ominus^{44}
\]
Continuing to combine terms and multiply factors of the generator polynomial results in the following

\[ g(x)_{GF(256)} = x^{20} + x^{18}.19 + x^{82}.18 + x^{82}.17 + x^{189}.16 + x^{80}.15 + x^{252}.14 + x^{139}.13 + x^{168}.12 + x^{211}.11 + x^{190}.10 + x^{232}.9 + x^{237}.8 + x^{96}.7 + x^{253}.6 + x^{171}.5 + x^{180}.4 + x^{229}.3 + x^{230}.2 + x^{207} + x^{210} \]

Equation 3.2.10

The generator polynomial for a 10 error correcting Reed-Solomon Code with GF(256) is used to create the encoding and decoding circuitry required on each SV and on the ground. Several circuit designs are possible which include Chien's Searching Decoder and the Meggitt Decoder. (19, p170) Likewise, the choice of logic gates for each circuit may vary and are dependent upon cost analysis and logic design. Therefore, the circuit design is omitted for future analysis.

The code described above is capable of correcting and detecting 10 symbol errors. The maximum number of burst and random errors that are guaranteed to be corrected by the code is calculated by using \( t \) and \( m \).

Considering a code with \( t \) symbols, the maximum number of burst and random errors is derived by a logical analysis. If a burst error occurs which is larger than the \( t \) symbols,
the code is determined to be unrecoverable since there are no original bits with which the code could be rebuilt. Therefore, a burst error must be less than \( t \) symbols. Likewise, if greater than \( t \) random errors occur, the code can not be recovered.

Since each symbol is represented by \( m \) bits, a maximum burst error length and a maximum number of random errors can be determined. The Reed-Solomon code developed above uses a \( t \) of 10 and an \( m \) of 8. Hence, there are 10 correctable symbols which are represented by 80 bits. Using the analysis given in the above paragraph, the maximum number of single burst errors which are correctable must be less than 80 bits. Thus, if a single burst error of 79 bits resulted, the error could be corrected. Similarly, if only 10 random errors occur, the code is recoverable. A breakdown of some combinations of burst and random errors is given in Table 6.
TABLE 6 ERROR CORRECTING CAPABILITY

<table>
<thead>
<tr>
<th>Single Bursts of length 1 or less + random r (1,r)</th>
<th>% Correctable</th>
<th>Single Burst l</th>
<th>% Correctable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(73, 0)</td>
<td>100</td>
<td>≤ 73</td>
<td>100</td>
</tr>
<tr>
<td>(65, 1)</td>
<td>100</td>
<td>74</td>
<td>87.5</td>
</tr>
<tr>
<td>(57, 2)</td>
<td>100</td>
<td>75</td>
<td>75</td>
</tr>
<tr>
<td>(49, 3)</td>
<td>100</td>
<td>76</td>
<td>62.5</td>
</tr>
<tr>
<td>(41, 4)</td>
<td>100</td>
<td>77</td>
<td>50</td>
</tr>
<tr>
<td>(33, 5)</td>
<td>100</td>
<td>78</td>
<td>37.5</td>
</tr>
<tr>
<td>(25, 6)</td>
<td>100</td>
<td>79</td>
<td>25</td>
</tr>
<tr>
<td>(17, 7)</td>
<td>100</td>
<td>80</td>
<td>12.5</td>
</tr>
<tr>
<td>(9, 8)</td>
<td>100</td>
<td>&gt; 81</td>
<td>0</td>
</tr>
<tr>
<td>(1, 9)</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0, 10)</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Double Burst (1_1, 1_2) Correctable</th>
<th>% Correctable</th>
<th>Triple Burst (1_1, 1_2, 1_3)</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(73, 0)</td>
<td>100</td>
<td>(73, 0, 0)</td>
<td>100</td>
</tr>
<tr>
<td>(65, 1)</td>
<td>100</td>
<td>(65, 1, 0)</td>
<td>100</td>
</tr>
<tr>
<td>(57, 9)</td>
<td>100</td>
<td>(57, 1, 1)</td>
<td>100</td>
</tr>
<tr>
<td>(49, 17)</td>
<td>100</td>
<td>(49, 9, 1)</td>
<td>100</td>
</tr>
<tr>
<td>(41, 25)</td>
<td>100</td>
<td>(41, 9, 9)</td>
<td>100</td>
</tr>
<tr>
<td>(33, 33)</td>
<td>100</td>
<td>(33, 25, 1)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(33, 17, 9)</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(25, 17, 17)</td>
<td>100</td>
</tr>
</tbody>
</table>

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3.3 Alternative Reed-Solomon Code

The Reed-Solomon code developed in section 3.2 is based on several assumptions. The 8-bit ASCII representation enables the network operations data (NOD) to possess vehicle identification, vehicle part identification, and health data to be transmitted. However, if SCS telemetry channels are assigned to each vehicle and each vehicle part, the transmitted data can be reduced. Consequently a 4-bit representation for the NOD is conceivable. This representation allows the NOD to be incorporated with the mission data (MD).

The Reed-Solomon Code for a 4-bit NOD is considered over a Galios Field of GF($2^4$) or GF(16). As with the original Reed-Solomon Code, the probability of an undetected error, $P_{ue}$, for the system is less than $10^{-7}$. Therefore, the number of correctable symbol errors, $t$, may be calculated as in Section 3.2.

First the code length, $n$, is determined by substituting $m$ equal to 4 into equation 3.2.5

$$n = 4 \times (2^4 - 1) = 4(15) = 60$$  \hspace{1cm} \text{Equation 3.3.1}

With the code length determined, a comparison of $t$ and $d$ values is possible by utilizing equations 3.2.1 and 3.2.6.
Substituting the worst case single bit error, $10^{-3}$, and the code length, 60, into equation 3.2.1 yields

$$P_{ue} = \left[ \frac{60}{d+1} \right] 10^{-3}(d+1)(1-10^{-3})^{60-(d+1)}$$

Equation 3.3.2

Substituting $d$ equal to 2 into equation 3.3.2 produces

$$P_{ue} = \frac{60!}{3!(60-3)!} 10^{-9}(.999)^{60-3} = 3.2323$$

Equation 3.3.3

which is larger than the desired $P_{ue}$ of $10^{-7}$. Substituting $d$ equal to 3 into equation 3.3.2 yields

$$P_{ue} = \frac{60!}{4!(60-4)!} 10^{-12}(.999)^{60-(4)} = 4.6$$

Equation 3.3.4

which is slightly larger than the desired $P_{ue}$. However, since the number of gateways within a certain area allows for alternative routes during poor weather conditions, this value suffices.

The number of correctable symbol errors, $t$, is determined by using equations 3.2.2, 3.2.3, and 3.2.4 and the value for $d$. Since the minimum distance, MD, for a Reed-Solomon code is equal to $2t + 1$, equations 3.2.3 and 3.2.4 are set equal to each other to yield

$$d + t + 1 = 2t + 1$$

Equation 3.3.5
which reduces to

\[ t = d \]  \hspace{1cm} \text{Equation 3.3.6}

Therefore, \( t \) equals 3.

Substituting \( t \) into equation 3.2.6 yields a probability of retransmission, \( P_{\text{ret}} \), equal to \( P_{\text{ue}} \). Thus, \( P_{\text{ret}} \) is \( 4.61 \times 10^{-7} \) which is a degree better than the original code.

Another concern for the alternative Reed-Solomon code involves the reduction in code rate since the code length is much smaller than the original code length. The code rate is reduced with higher values of \( t \) since more bits are used to correct the code rather than send data.

The code rate, \( c_r \), is defined by

\[ c_r = \frac{n-2t}{n} \]  \hspace{1cm} \text{Equation 3.3.7}

Substituting \( n \) and \( t \) into equation 3.3.7 yields a code rate of .90. This code rate is within a 90 percentile which is reasonable.

Finally, the generator polynomial for the alternative Reed-Solomon Code is derived from equation 3.2.7. Substituting \( t \) equal to 3 into equation 3.2.7 yields
\[ g(x)_{GF(16)}rs = (x + \oplus)(x + \oplus^2)(x + \oplus^3) \cdots (x + \oplus^6) \text{ Equation 3.3.8} \]

Factors of the generator polynomial are multiplied together as in Section 3.2 and alpha terms, \( \oplus \), are combined using Table 7. The alpha terms, \( \oplus \), are reduced using the same method illustrated in Table 6 resulting in

\[ g(x)_{GF(16)}rs = x^6 + \oplus^{10}x^5 + \oplus^5x^4 + \oplus^2x^3 + \oplus^2x^2 + \oplus^{11}x + \oplus^{10} \text{ Equation 3.2.9} \]
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>0000</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
</tr>
<tr>
<td>1</td>
<td>0100</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
</tr>
<tr>
<td>3</td>
<td>0001</td>
</tr>
<tr>
<td>4</td>
<td>1100</td>
</tr>
<tr>
<td>5</td>
<td>0110</td>
</tr>
<tr>
<td>6</td>
<td>0011</td>
</tr>
<tr>
<td>7</td>
<td>1101</td>
</tr>
<tr>
<td>8</td>
<td>1010</td>
</tr>
<tr>
<td>9</td>
<td>0101</td>
</tr>
<tr>
<td>10</td>
<td>1110</td>
</tr>
<tr>
<td>11</td>
<td>0111</td>
</tr>
<tr>
<td>12</td>
<td>1111</td>
</tr>
<tr>
<td>13</td>
<td>1011</td>
</tr>
<tr>
<td>14</td>
<td>1001</td>
</tr>
</tbody>
</table>
The alternative Reed-Solomon code is feasible during normal operations for the Iridium satellites. However, the original assumption requiring prior knowledge of the identification of the data which is being transmitted in each channel may cause a loss of data during emergency situations. If one of the 66 satellites was deorbited or malfunctioned, the channel data would be almost completely corrupted. This is highly undesirable for a system which must be maintained for global communications to be uninterrupted. Therefore, the original code is recommended.

3.4 BCH Codes and The Viterbi Algorithm

Two powerful encoding schemes for satellite communications include Bose-Chaudhuri-Hocquenghem (BCH) codes and codes developed from the Viterbi algorithm. Both techniques possess desirable qualities for implementation in the transmission of Iridium data. The BCH codes are readily implemented with shift register and logic circuits (23, p299), and the Viterbi algorithm codes provide good resistance to interference and deliberate jamming.(23, p315)

Many BCH codes are possible for the network operations data (NOD). Figure 7 shows the relationship between $(E_b/N_0)$, which is the ratio of the energy per bit to the noise density, and the bit error rate (BER) for several BCH
codes using QPSK modulation. (23, p302) The (1023, 668) BCH code is chosen for the NOD data to conform with the previous link budget design calculated in *Link Budget Design for Iridium Satellite System* by Vijay Shah. (27, p59-66)

The error correcting BCH code is determined by the code length (1023) and the number of real data bits (668). Using this information, the number of error correcting bits, \( t_{BCH} \), is determined to be 36. (19, p144). Therefore, the remaining 329 bits are used for error detection. Combining this information, the (1023, 668) BCH code is then used to develop a generator polynomial.

The generator polynomial of (1023, 668) BCH code is derived from the factors of

\[
F(x) = x^{1023} - 1
\]

Equation 3.4.1

where each factor contains no duplicate roots. The number of factors is dependent upon the desired minimum distance, \( MD \), which for a BCH code is given by equation 3.2.3. Substituting \( t \) equal to 36 into equation 3.2.3 yields
FIGURE 7  BIT ERROR RATE VERSUS $(E_b/N_0)$ FOR VARIOUS ERROR DETECTION CODES WITH QPSK MODULATION
\[ MD \geq 2(36) + 1 = 73 \quad \text{Equation 3.4.2} \]

The minimum distance minus 1 or 72 is the number of consecutive roots which are required by the factors of equation 3.4.1. Therefore, the factors of equation 3.4.1, given in Appendix C, are listed along with their respective roots by using

\[ \overline{a}, \overline{a^2}, \overline{a^4}, \ldots, \overline{a^{42}} \quad \text{Equation 3.4.3} \]

where \( \overline{a} \) is the root of each factor, \( a \) is the number from each factor, and \( b \) is the last degree of 2 which does not cause a repeated root. Table 8 shows some of the factors and the respective roots for equation 3.4.1.

It is possible to show that the line of roots from factors 1 through 71 satisfy the minimum distance criteria. Thus, by multiplying factors 1 through 71, the generator polynomial for the \((1023,668)\) BCH is formed. The generator polynomial is given by
\[ g(x) = x + x^2 + x^3 + x^4 + x^5 + x^6 + x^{11} + x^{13} + x^{15} + x^{18} + x^{21} + x^{22} + x^{24} + x^{26} + x^{28} + x^{29} + x^{30} + x^{32} + x^{35} + x^{38} + x^{39} + x^{41} + x^{43} + x^{46} + x^{47} + x^{48} + x^{49} + x^{50} + x^{51} + x^{52} + x^{55} + x^{58} + x^{59} + x^{60} + x^{61} + x^{63} + x^{73} + x^{74} + x^{77} + x^{81} + x^{82} + x^{83} + x^{85} + x^{88} + x^{94} + x^{95} + x^{98} + x^{99} + x^{100} + x^{102} + x^{110} + x^{111} + x^{113} + x^{115} + x^{121} + x^{122} + x^{123} + x^{125} + x^{126} + x^{129} + x^{133} + x^{136} + x^{137} + x^{139} + x^{140} + x^{142} + x^{143} + x^{145} + x^{146} + x^{148} + x^{149} + x^{150} + x^{151} + x^{155} + x^{156} + x^{161} + x^{163} + x^{166} + x^{168} + x^{171} + x^{172} + x^{177} + x^{178} + x^{179} + x^{181} + x^{183} + x^{184} + x^{186} + x^{187} + x^{199} + x^{202} + x^{203} + x^{210} + x^{212} + x^{213} + x^{215} + x^{216} + x^{221} + x^{222} + x^{225} + x^{230} + x^{234} + x^{237} + x^{239} + x^{243} + x^{244} + x^{246} + x^{253} + x^{255} + x^{256} + x^{258} + x^{260} + x^{261} + x^{263} + x^{264} + x^{266} + x^{268} + x^{269} + x^{270} + x^{272} + x^{274} + x^{278} + x^{280} + x^{283} + x^{286} + x^{287} + x^{288} + x^{290} + x^{291} + x^{292} + x^{295} + x^{296} + x^{300} + x^{301} + x^{302} + x^{303} + x^{304} + x^{305} + x^{306} + x^{307} + x^{309} + x^{310} + x^{311} + x^{313} + x^{314} + x^{317} + x^{318} + x^{320} + x^{321} + x^{324} + x^{327} + x^{328} + x^{333} + x^{334} \]

Equation 3.4.4
<table>
<thead>
<tr>
<th>Factors from Appendix C</th>
<th>Roots of Each Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m(x) ) polynomial form</td>
<td>( \frac{2}{64} ), ( \frac{4}{64} ), ( \frac{8}{64} ), ( \frac{16}{64} ), ( \frac{32}{64} ), ( \frac{64}{64} ), ( \frac{128}{64} ), ( \frac{256}{64} ), ( \frac{512}{64} )</td>
</tr>
<tr>
<td>( m_{3}^{-3}(x) = 1+x^2+x^3+x^{10} )</td>
<td>( \frac{3}{192} ), ( \frac{6}{192} ), ( \frac{12}{192} ), ( \frac{24}{192} ), ( \frac{48}{192} ), ( \frac{96}{192} )</td>
</tr>
<tr>
<td>( m_{5}^{-5}(x) = 1+x^2+x^3+x^8+x^{10} )</td>
<td>( \frac{5}{320} ), ( \frac{10}{320} ), ( \frac{20}{320} ), ( \frac{40}{320} ), ( \frac{80}{320} ), ( \frac{160}{320} )</td>
</tr>
<tr>
<td>( m_{7}^{-7}(x) = 1+x^2+x^3+x^7+x^8+x^9+x^{10} )</td>
<td>( \frac{7}{448} ), ( \frac{14}{448} ), ( \frac{28}{448} ), ( \frac{56}{448} ), ( \frac{112}{448} ), ( \frac{224}{448} )</td>
</tr>
<tr>
<td>( m_{9}^{-9}(x) = 1+x^2+x^3+x^5+x^7+x^{10} )</td>
<td>( \frac{9}{288} ), ( \frac{18}{288} ), ( \frac{36}{288} ), ( \frac{72}{288} ), ( \frac{144}{288} ), ( \frac{516}{288} )</td>
</tr>
<tr>
<td>( m_{11}^{-11}(x) = 1+x^2+x^4+x^5+x^{10} )</td>
<td>( \frac{11}{352} ), ( \frac{22}{352} ), ( \frac{44}{352} ), ( \frac{88}{352} ), ( \frac{176}{352} ), ( \frac{517}{352} )</td>
</tr>
<tr>
<td>( m_{13}^{-13}(x) = 1+x^2+x^3+x^5+x^6+x^{10} )</td>
<td>( \frac{13}{416} ), ( \frac{26}{416} ), ( \frac{52}{416} ), ( \frac{104}{416} ), ( \frac{208}{416} ), ( \frac{518}{416} )</td>
</tr>
<tr>
<td>( m_{15}^{-15}(x) = 1+x^2+x^3+x^5+x^7+x^{10} )</td>
<td>( \frac{15}{480} ), ( \frac{30}{480} ), ( \frac{60}{480} ), ( \frac{120}{480} ), ( \frac{240}{480} ), ( \frac{519}{480} )</td>
</tr>
<tr>
<td>( m_{17}^{-17}(x) = 1+x^2+x^3+x^6+x^8+x^{10} )</td>
<td>( \frac{17}{544} ), ( \frac{34}{544} ), ( \frac{68}{544} ), ( \frac{136}{544} ), ( \frac{272}{544} ), ( \frac{520}{544} )</td>
</tr>
<tr>
<td>( m_{19}^{-19}(x) = 1+x^2+x^3+x^6+x^7+x^8+x^{10} )</td>
<td>( \frac{19}{608} ), ( \frac{38}{608} ), ( \frac{76}{608} ), ( \frac{152}{608} ), ( \frac{304}{608} ), ( \frac{521}{608} )</td>
</tr>
<tr>
<td>***</td>
<td></td>
</tr>
<tr>
<td>( m_{69}^{-69}(x) = 1+x^2+x^7+x^8+x^{10} )</td>
<td>( \frac{69}{162} ), ( \frac{138}{162} ), ( \frac{276}{162} ), ( \frac{552}{162} ), ( \frac{81}{162} )</td>
</tr>
<tr>
<td>( m_{71}^{-71}(x) = 1+x^2+x^3+x^7+x^9+x^{10} )</td>
<td>( \frac{71}{226} ), ( \frac{142}{226} ), ( \frac{284}{226} ), ( \frac{568}{226} ), ( \frac{113}{226} )</td>
</tr>
</tbody>
</table>

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The generator polynomial for the BCH code as with the Reed-Solomon Codes is used to develop encoding and decoding circuitry. Once again, the cost analysis and choice of logic gates require further investigation. Therefore, the design of these circuits has been omitted.

The Viterbi algorithm is designed based on a maximum likelihood decoding scheme. (19, p287) In other words, error detection and correction is fulfilled by using the probability of an error sequence occurring in a received transmission. The Viterbi algorithm requires an analysis of the probability of increased errors or minimal errors occurring in different areas of the globe. Since each of the 66 satellites is transmitting data with an large probability of a compounding error, the Viterbi algorithm is not considered for the Iridium system.

Several coding schemes are proposed for use in the Iridium satellite system. The Reed-Solomon codes which possess burst error correcting capabilities, the BCH codes which are readily implemented with shift registers and logic gates, and the Viterbi algorithm which is omitted due to the lack of information to form the maximum likelihood of an error sequence. Each of these codes contain different attributes desirable for Iridium. The Reed-Solomon code incorporates the logic circuits design of the BCH codes with the burst error correcting capability of the Viterbi
algorithm.

The 8-bit Reed-Solomon code is recommended for the Iridium satellite system. The 4-bit Reed-Solomon code assumes that the data incorporated in the 8-bit representation can be reduced by using specific channels in the SCS telemetry frame which is not as generalized as the 8-bit representation. The BCH code is sufficient for the SCS telemetry; however, Reed-Solomon coding provides better protection against burst errors. The Viterbi algorithm is also a good choice for the SCS telemetry, but the algorithm is not provided for comparison.
CHAPTER 4  DATA SECURITY FOR THE TELEMETRY CHANNEL

Signals transmitted via landlines or by satellites are vulnerable to message tampering and easy access. The data these signals carry include military information, financial information, or personal phone calls. The cost of privacy for the convenient and speedy new telecommunications resides in the methods developed for data security. All mention of data security in this paper refers to privacy protection. The term encryption or cryptography is not meant to refer to non-public access keys.

In order to protect data against passive eavesdropping or active tampering, cryptography is employed. "...In satellite communication, cryptography means encoding and decoding transmitted messages, rendering them inaccessible to unauthorized users of the system. (32, p104) Unlike error correcting codes, this coding scheme makes the signal unrecoverable for unintended users. With many Iridium users, message privacy may be a consideration for transmissions.

The number of satellites and the dynamic changes to the Iridium system makes the development of a cryptographic system a challenge. Message privacy and authentication are derived by key management, encryption method, and

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decryption method. These cryptographic techniques must be used by every ground facility, ISU, and satellite to receive the data transmitted. For this reason, Iridium has opted to let the user encrypt and decrypt the data prior to transmission and not incorporate data security into the system (4).

The telemetry used to monitor and maintain the satellites must be either encoded by Motorola or sent in clear text. At present, Motorola does not consider the data to be at risk of tampering or monitoring (4). This may be true; however, each vehicle must have an authentication code in order to determine if a command is specific to that satellite. This code could be as simple as the satellite designator, i.e. first satellite launched is 1, second satellite launched is 2, etc. or the code could be more random. Since the authentication code must be present for the Iridium satellites, initial development of a security scheme would be prudent for the future.

The system control data (SCD) received by the Iridium satellites is transmitted around the world. Encoding the data for a global communication system seems unnecessary at the moment; however, message privacy can reduce the possibility of tampering. As the home computer becomes more powerful, the insurgence of computer viruses has made even the home user more cautious. Therefore, Motorola may
consider these techniques more applicable as the Iridium system becomes active.

4.1 Key Management

A key is a cryptographic device used to authenticate and decipher information received by a transmission source. There are several types of keys which include the static key, the dynamic key, the master key, and the subkey. Each of these keys plays a different role in deciphering and authenticating the encrypted message.

The static key is a hard-coded key which is known by both systems and remains constant throughout the entire mission. Iridium satellite designators can be used as static keys. While this is a simplified cryptosystem, the predictability of a static key reduces the benefits of data security. A random key or dynamic key which changes during each session or at a specified time generates a more obscure system.

A random key for each station or satellite is known as a subkey. The combination of these subkeys is a master key which is stored by each transmission site. The communication between multiple stations requires each station to hold master keys and subkeys for each transmission. This is not plausible for a satellite system consisting of 66 satellites. Therefore, a combination of static and random keys can be employed by the system.
Normally, the key system is designed as shown in Figure 8 (32, p113). A ground station (A) generates a random number key (RNA), stores it, and transmits it to ground station (B). Ground station (B) generates a random number key (RNB), stores it, performs modulo-2 addition to RNB and RNA to get RNC, stores RNC, and transmits RNB to ground station (A). Ground station (A) adds RNA to RNB and stores RNC. Therefore, each station has a subkey (RNA or RNB) and a master key (RNC).
FIGURE 8 RANDOM NUMBER KEY GENERATION
One or two ground stations are going to be receiving and commanding the 66 satellites. Therefore, the key generation between ground stations is not necessary. Since, the satellites are considered a network operation turned upside-down (27), key generation and storage is performed by each of the satellites. A random choice of seven keys is chosen to create this security coding.

The configuration of the Iridium satellites is such that any one satellite can communicate with 6 other satellites. A random generation of keys for each satellite would require an enormous amount of memory for storage of subkeys. Therefore, a flooding technique used by computer architecture for determining flaws in a network is recommended for the distribution of the keys. The flooding technique is shown in Figure 9.
FIGURE 9 FLOODING TECHNIQUE FOR COMPUTER NETWORKS.
The basic design of the flooding technique is to send a packet of information to all nodal points in the network. The network is flooded with packets and returns a receipt packet with the nodal identifier along return lines to the initiator. Those packets not received by the initiator are construed as inoperative. In computer networks, the packets must be timed or a system response must be developed to avoid collisions which is not necessary for the Iridium satellite network.

The flooding technique as applied to Iridium and the key management works as follows. Seven satellites are considered key initiators. These seven satellites generate random keys i.e. RN(1), RN(2), RN(3), RN(4), RN(5), RN(6), and RN(7). The keys are then sent along the existing satellite network with each satellite designation (SAT1, SAT66, SAT7, SAT4, ...) or static key. Every satellite receives the random and static keys.

A non-initiator holds a designator from one of the initiators in the memory for the subkey. For example, satellite number 2 holds SAT66 in its respective subkey memory. When the non-initiator, satellite 2, receives the random key from the specified initiator, satellite 66, the non-initiator stores the random key as its subkey. Therefore, satellite 2 and 66 own
the same subkey.

As the other keys are being sent, every satellite performs modulo-2 addition and stores them as master keys. So, satellite 2 receives RN(1) through RN(7) and stores RN(1) + RN(2) + RN(3) + RN(4) + RN(5) + RN(6) + RN(7) as the master key. As a result every satellite owns the same master key and only seven different subkeys are shared. The cycle ends when each satellite receives a designator from another satellite with a completion code (7). The completion code increments with the receipt of a different random/static number.

The ground stations also receive the randomly generated keys in order to decipher the text transmitted by the satellites for telemetry and commanding purposes. For safety, half of the keys are kept at the MCF and half are kept at the BCF. Since each ground station can receive telemetry from several of the satellites, all of the keys are available. The resulting configuration is shown in Figure 10.
FIGURE 10  FINAL KEY CONFIGURATION
As shown above, each satellite must be in contact with 6 different satellites with 6 different keys in order to decipher telemetry. If a satellite fails, another satellite in the area must be able to account for the lost key or the system must be reinitialized. Therefore, there is an optimum configuration with at least one redundant key for each satellite as shown in Figure 10.

The cryptosystem can be initiated at any point in the operation of Iridium. Half of a fail safe code is kept at each of the two ground stations to initiate clear text in the event of total system failure. With this public key cryptographic (PKC) system, Motorola can provide message privacy and message authentication.

4.2 Method of Encryption and Decryption

The methods available for encryption and decryption are carefully monitored by the National Security Agency (NSA). At present, there is a controversy over the two most popular unclassified encryption techniques which are the PKC Rivest-Shamir-Adleman (RSA) algorithm and the secret key Data Encryption Standard (DES). (11, p29-35) Since the secret key cryptographic system is not a consideration at present, the debate is left for future discussion.

The RSA algorithm employs two randomly chosen large prime numbers to encrypt and decrypt a message. 512-bit keys are common for the RSA system; however, advances in the
technology have made it prudent to have larger keys. "The fact that RSA has not been knowingly compromised after more than a dozen years enhances its credibility (11, p32)."

Thus, the RSA algorithm is used for encryption of satellite commands and telemetry.

The RSA algorithm (32, p150) is:

Step 1: Select two prime numbers $p_1$ and $p_2$. Calculate $m = p_1 p_2$.

Step 2: Calculate $\phi(m) = (p_1 - 1)(p_2 - 1)$.

Step 3: Select an integer $e$ such that $\gcd(\phi(m), e) = 1$ and $1 \leq e \leq \phi(m)$.

Step 4: Using the Euclidean algorithm, calculate $d$, the inverse $e$, modulo $\phi(m)$.

Step 5: For encryption raise the message to the power of $e$ modulo $m$; for decryption raise the cryptogram to the power of $d$ modulo $m$.

Employing this algorithm in the Iridium satellite constellation means using more satellite hardware or a logic design which increases ground hardware. Since the cost of increasing the ground hardware is potentially less expensive, the latter is used. With the current configuration from section 4.1, any satellite can determine a subkey used for encryption by use of the master key and surrounding satellite subkeys. Therefore, the designed ground algorithm must employ the correct keys with each
satellite for encryption and decryption. This method establishes session keys which are specific to each contact with a different satellite.

Each satellite must recognize commands from the ground, encode telemetry, and route mission data. Since the satellite identifiers are unencoded, the mission data is unaffected by the encryption and decryption process. The commands and telemetry, however, are determined by both the satellite identifier and session key.

The RSA algorithm is realized by the following method:

1. Satellite 27 transmits to the MCF its identifier.

2. The MCF receives the transmission and performs modulo-2 addition to the master key to the surrounding satellite keys:

\[ MK + RN(1) + RN(2) + RN(3) + RN(4) + \ldots + RN(6) = RN(7) \]

3. The resulting key is the subkey held by satellite 27.

4. The MCF then transmits a random session key to satellite 27 by performing modulo-2 addition with the master key.

5. Satellite 27 receives the ground transmission and performs modulo-2 addition with the master key to receive the session key.

6. While contact is maintained the, satellite and MCF encrypt and decrypt by using the satellite key and session key.
An example of the system is given below:

1. Sat27 transmits ID#27 to the MCF:
   Sat26
   Sat21    Sat33
   Sat27------------------>MCF
   Sat22    Sat34
   Sat28

2. MCF receives ID#27 and performs modulo-2 addition
   with the surrounding satellite subkeys:

\[
MK + RN(21) + RN(22) + RN(26) + RN(28) + RN(33) + RN(34) = RN(27)
\]

\[
01100 + 010 + 10001 + 101 + 111 + 10011 + 1101 = 011
\]

3. The resulting key is the subkey held by satellite
   27:

\[
RN(27) = 011
\]

4. The MCF then transmits a Session Key, SK, to
   satellite 27 by performing modulo-2 addition with
   the master key:

\[
MK + SK = 01100 + 10111 = 11011
\]

5. Satellite 27 receives the ground transmission and
   performs modulo-2 addition with the master key to
   receive the session key:

\[
MK + 11011 = 01100 + 11011 = 10111 = SK
\]

6. While contact is maintained the, satellite and MCF
   encrypt and decrypt by using the satellite key and
   session key:

   Step 1: Calculate \( m = p_1 p_2 \).
   \[
m = RN(27) \times SK = 011 \times 10111 = 3 \times 23 = 69
   \]
   Step 2: Calculate \( \delta(m) = (p_1 - 1)(p_2 - 1) \).
   \[
   \delta(m) = (3 - 1)(23 - 1) = 44
   \]
   Step 3: Select an integer \( e \).
   \[
   e = 9
   \]
   Step 4: Calculate \( d \), the inverse \( e \), modulo \( \delta(m) \):
   \[
   d = e^{-1} \mod 44 \quad \text{because} \quad 9 \times 5 = 45 \equiv 1 \mod 44
   \]
   Step 5: For encryption, the message, 2, is raised
   to the power of 9 which yields:

\[
2^9 \mod 69 = 512 \mod 69 = 29
\]
For decryption, the received message, 32, is raised to the power of 5 which yields:

$$29^5 \mod 69 = 20511149 \mod 69 = 2$$

By maintaining a limited number of keys and establishing a session key, the Iridium system can receive and transmit using data security techniques. The system flexibility regarding data security should be a consideration for its continued operation. The RSA algorithm provides this flexibility.

4.3 Secret System

As stated in section 4.1, if one of the satellites in the Iridium constellation fails, the key must be recoverable or the system must be initialized. Initializing the system takes time. Therefore, a recoverable key system or secret sharing system is used to avoid the loss or disappearance of a key (32, p158).

Safeguarding the keys makes use of Reed-Solomon coding schemes (32, p158). The recommended method involves combining mutually orthogonal Latin squares and the Reed-Solomon technique. Since the Reed-Solomon coding schemes are given in chapter 3, only the mutually orthogonal Latin squares are developed. Due to the complexity of the square, only the alternative Reed-Solomon coding technique is used.

The alternative Reed-Solomon coding scheme uses a Galios Field of GF(16) and an irreducible polynomial of $1 +
The elements of GF(16) are given in Table 7. The orthogonal Latin squares are generated from the Reed-Solomon codes which produces a "unified method of construction(32, p169)." From these orthogonal Latin squares the secret sharing systems generated by the Shamir approach, Karnin, Greene, and Hellman approach, or the McEliece and Sarwate approach can be created.

In order to develop the squares, first the Galios Field and irreducible polynomial are defined in the paragraph above. Next, an outline of the first orthogonal Latin block is formed as shown in Figure 11.
$L_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$

**Figure 11** First Orthogonal Latin Block
By using a combination of the irreducible polynomial and the desired number of rows for \( L_1 \), the matrix is completed as follows:

The first row of \( L_1 \) is given by:

\[
\begin{align*}
L_1(1,1) &= @_1 + @_1 = 1 + 1 = 0 \\
L_1(1,2) &= @_1 + @_2 = 1 + @ = @4 \\
L_1(1,3) &= @_1 + @_3 = 1 + @_2 = 101 = @8 \\
L_1(1,4) &= @_1 + @_4 = 1 + @^3 = 1001 = @14 \\
L_1(1,5) &= @_1 + @_5 = 1 + @_4 = 0100 = @ \\
L_1(1,6) &= @_1 + @_6 = 1 + @_5 = @10 \\
L_1(1,7) &= @_1 + @_7 = 1 + @_6 = 1011 = @13 \\
L_1(1,8) &= @_1 + @_8 = 1 + @_7 = 0101 = @9 \\
L_1(1,9) &= @_1 + @_9 = 1 + @_8 = 0010 = @2 \\
L_1(1,10) &= @_1 + @_10 = 1 + @_9 = 1101 = @7 \\
L_1(1,11) &= @_1 + @_11 = 1 + @_{10} = 0110 = @5 \\
L_1(1,12) &= @_1 + @_12 = 1 + @_{11} = 1111 = @12 \\
L_1(1,13) &= @_1 + @_13 = 1 + @_{12} = 0111 = @11 \\
L_1(1,14) &= @_1 + @_14 = 1 + @_{13} = 0011 = @6 \\
L_1(1,15) &= @_1 + @_15 = 1 + @_{14} = 0001 = @3
\end{align*}
\]

The second row of \( L_1 \) is given by:

\[
\begin{align*}
L_1(2,1) &= @_2 + @_1 = @ + 1 = @4 \\
L_1(2,2) &= @_2 + @_2 = @ + @ = 0 \\
L_1(2,3) &= @_2 + @_3 = @ + @_2 = 0110 = @5 \\
L_1(2,4) &= @_2 + @_4 = @ + @_3 = 0101 = @9 \\
L_1(2,5) &= @_2 + @_5 = @ + @_4 = 1 \\
L_1(2,6) &= @_2 + @_6 = @ + @_5 = 001 = @2 \\
L_1(2,7) &= @_2 + @_7 = @ + @_6 = 1111 = @11 \\
L_1(2,8) &= @_2 + @_8 = @ + @_7 = 1001 = @14 \\
L_1(2,9) &= @_2 + @_9 = @ + @_8 = 1110 = @10 \\
L_1(2,10) &= @_2 + @_{10} = @ + @_9 = 0001 = @3 \\
L_1(2,11) &= @_2 + @_{11} = @ + @_{10} = 1010 = @8 \\
L_1(2,12) &= @_2 + @_{12} = @ + @_{11} = 0011 = @6 \\
L_1(2,13) &= @_2 + @_{13} = @ + @_{12} = 1011 = @13 \\
L_1(2,14) &= @_2 + @_{14} = @ + @_{13} = 1111 = @12 \\
L_1(2,15) &= @_2 + @_{15} = @ + @_{14} = 1101 = @7
\end{align*}
\]

The third row of \( L_1 \) is determined from:

\[
\begin{align*}
L_1(3,1) &= @_3 + @_1 = @^2 + 1 = @8 \\
L_1(3,2) &= @_3 + @_2 = @^2 + @ = @5 \\
L_1(3,3) &= @_3 + @_3 = @^2 + @^2 = 0 \\
L_1(3,4) &= @_3 + @_4 = @^2 + @^3 = 0011 = @6 \\
L_1(3,5) &= @_3 + @_5 = @^2 + @^4 = 111 = @10 \\
L_1(3,6) &= @_3 + @_6 = @^2 + @^5 = 0100 = @
\end{align*}
\]
\[
\begin{align*}
L_1(3,7) &= \theta_3 + \theta_7 = \theta_2 + \theta_6 = 0001 = \theta^3 \\
L_1(3,8) &= \theta_3 + \theta_8 = \theta_2 + \theta_7 = 1111 = \theta^{12} \\
L_1(3,9) &= \theta_3 + \theta_9 = \theta_2 + \theta_8 = 1000 = 1 \\
L_1(3,10) &= \theta_3 + \theta_{10} = \theta_2 + \theta_{10} = 0111 = \theta_{11} \\
L_1(3,11) &= \theta_3 + \theta_{11} = \theta_2 + \theta_{11} = 1100 = \theta^4 \\
L_1(3,12) &= \theta_3 + \theta_{12} = \theta_2 + \theta_{12} = 0101 = \theta^9 \\
L_1(3,13) &= \theta_3 + \theta_{13} = \theta_2 + \theta_{13} = 1101 = \theta^7 \\
L_1(3,14) &= \theta_3 + \theta_{14} = \theta_2 + \theta_{14} = 1001 = \theta^{14} \\
L_1(3,15) &= \theta_3 + \theta_{15} = \theta_2 + \theta_{15} = 1011 = \theta^{13} \\
\end{align*}
\]

The fourth row of \(L_1\) is:

\[
\begin{align*}
L_1(4,1) &= \theta_4 + \theta_1 = \theta_3 + 1 = \theta_4 \\
L_1(4,2) &= \theta_4 + \theta_2 = \theta_3 + \theta_2 = 0101 = \theta_9 \\
L_1(4,3) &= \theta_4 + \theta_3 = \theta_3 + \theta_3 = 0011 = \theta_6 \\
L_1(4,4) &= \theta_4 + \theta_4 = \theta_3 + \theta_3 = 0 \\
L_1(4,5) &= \theta_4 + \theta_5 = \theta_3 + \theta_5 = 1101 = \theta_7 \\
L_1(4,6) &= \theta_4 + \theta_6 = \theta_3 + \theta_6 = 0111 = \theta_{11} \\
L_1(4,7) &= \theta_4 + \theta_7 = \theta_3 + \theta_7 = 0010 = \theta_2 \\
L_1(4,8) &= \theta_4 + \theta_8 = \theta_3 + \theta_8 = 1100 = \theta_4 \\
L_1(4,9) &= \theta_4 + \theta_9 = \theta_3 + \theta_9 = 1000 = \theta_6 \\
L_1(4,10) &= \theta_4 + \theta_{10} = \theta_3 + \theta_{10} = 0100 = \theta_6 \\
L_1(4,11) &= \theta_4 + \theta_{11} = \theta_3 + \theta_{11} = 1111 = \theta_4 \\
L_1(4,12) &= \theta_4 + \theta_{12} = \theta_3 + \theta_{12} = 0110 = \theta_6 \\
L_1(4,13) &= \theta_4 + \theta_{13} = \theta_3 + \theta_{13} = 1110 = \theta_4 \\
L_1(4,14) &= \theta_4 + \theta_{14} = \theta_3 + \theta_{14} = 1010 = \theta_6 \\
L_1(4,15) &= \theta_4 + \theta_{15} = \theta_3 + \theta_{15} = 1000 = 1 \\
\end{align*}
\]

The fifth row of \(L_1\) is calculated to be:

\[
\begin{align*}
L_1(5,1) &= \theta_5 + \theta_1 = \theta_4 + 1 = 010 = \theta \\
L_1(5,2) &= \theta_5 + \theta_2 = \theta_4 + \theta_2 = 1 \\
L_1(5,3) &= \theta_5 + \theta_3 = \theta_4 + \theta_3 = 111 = \theta^{10} \\
L_1(5,4) &= \theta_5 + \theta_4 = \theta_4 + \theta_4 = 1101 = \theta^7 \\
L_1(5,5) &= \theta_5 + \theta_5 = \theta_4 + \theta_5 = 0 \\
L_1(5,6) &= \theta_5 + \theta_6 = \theta_4 + \theta_6 = 1010 = \theta^8 \\
L_1(5,7) &= \theta_5 + \theta_7 = \theta_4 + \theta_7 = 1111 = \theta_4 \\
L_1(5,8) &= \theta_5 + \theta_8 = \theta_4 + \theta_8 = 0001 = \theta^3 \\
L_1(5,9) &= \theta_5 + \theta_9 = \theta_4 + \theta_9 = 0110 = \theta^5 \\
L_1(5,10) &= \theta_5 + \theta_{10} = \theta_4 + \theta_{10} = 1001 = \theta^{14} \\
L_1(5,11) &= \theta_5 + \theta_{11} = \theta_4 + \theta_{11} = 0010 = \theta^2 \\
L_1(5,12) &= \theta_5 + \theta_{12} = \theta_4 + \theta_{12} = 1011 = \theta^{13} \\
L_1(5,13) &= \theta_5 + \theta_{13} = \theta_4 + \theta_{13} = 0011 = 011 \\
L_1(5,14) &= \theta_5 + \theta_{14} = \theta_4 + \theta_{14} = 0111 = \theta_{11} \\
L_1(5,15) &= \theta_5 + \theta_{15} = \theta_4 + \theta_{15} = 0101 = \theta^9 \\
\end{align*}
\]

The sixth row of \(L_1\) is determined to be:

\[
L_1(6,1) = \theta_6 + \theta_1 = \theta^5 + 1 = \theta^{10} 
\]

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\[ \begin{align*}
L_1(6,2) &= \oplus_6 + \oplus_2 = \oplus_5 + \oplus_2 = 0010 = \oplus^2 \\
L_1(6,3) &= \oplus_6 + \oplus_3 = \oplus_5 + \oplus_3 = 0111 = \oplus^11 \\
L_1(6,4) &= \oplus_6 + \oplus_4 = \oplus_5 + \oplus_4 = 1010 = \oplus^8 \\
L_1(6,5) &= \oplus_6 + \oplus_5 = \oplus_5 + \oplus_5 = 0 \\
L_1(6,6) &= \oplus_6 + \oplus_6 = \oplus_6 + \oplus_6 = 0011 = \oplus^6 \\
L_1(6,7) &= \oplus_6 + \oplus_7 = \oplus_5 + \oplus_7 = 0101 = \oplus^9 \\
L_1(6,8) &= \oplus_6 + \oplus_8 = \oplus_5 + \oplus_7 = 1011 = \oplus^13 \\
L_1(6,9) &= \oplus_6 + \oplus_9 = \oplus_5 + \oplus_8 = 1100 = \oplus^4 \\
L_1(6,10) &= \oplus_6 + \oplus_{10} = \oplus_5 + \oplus_9 = 0011 = \oplus^6 \\
L_1(6,11) &= \oplus_6 + \oplus_{11} = \oplus_5 + \oplus_{10} = 1000 = \oplus^1 \\
L_1(6,12) &= \oplus_6 + \oplus_{12} = \oplus_5 + \oplus_{11} = 0001 = \oplus^3 \\
L_1(6,13) &= \oplus_6 + \oplus_{13} = \oplus_5 + \oplus_{12} = 1001 = \oplus^2 \\
L_1(6,14) &= \oplus_6 + \oplus_{14} = \oplus_5 + \oplus_{13} = 1101 = \oplus^7 \\
L_1(6,15) &= \oplus_6 + \oplus_{15} = \oplus_5 + \oplus_{14} = 1111 = \oplus^{12}
\end{align*} \]

The final rows of \( L_1 \) are given by:

\[ \begin{align*}
L_1(7,1) &= \oplus_7 + \oplus_1 = \oplus_6 + \oplus_1 = 1011 = \oplus^{13} \\
L_1(7,2) &= \oplus_7 + \oplus_2 = \oplus_6 + \oplus_2 = 0111 = \oplus^{11} \\
L_1(7,3) &= \oplus_7 + \oplus_3 = \oplus_6 + \oplus_3 = 0001 = \oplus^3 \\
L_1(7,4) &= \oplus_7 + \oplus_4 = \oplus_6 + \oplus_4 = 0010 = \oplus^2 \\
L_1(7,5) &= \oplus_7 + \oplus_5 = \oplus_6 + \oplus_5 = 1111 = \oplus^7 \\
L_1(7,6) &= \oplus_7 + \oplus_6 = \oplus_6 + \oplus_6 = 0010 = \oplus^2 \\
L_1(7,7) &= \oplus_7 + \oplus_7 = \oplus_6 + \oplus_7 = 0 \\
L_1(7,8) &= \oplus_7 + \oplus_8 = \oplus_6 + \oplus_8 = 1110 = \oplus^{10} \\
L_1(7,9) &= \oplus_7 + \oplus_9 = \oplus_6 + \oplus_9 = 1001 = \oplus^{14} \\
L_1(7,10) &= \oplus_7 + \oplus_{10} = \oplus_6 + \oplus_9 = 0110 = \oplus^5 \\
L_1(7,11) &= \oplus_7 + \oplus_{11} = \oplus_6 + \oplus_{10} = 1101 = \oplus^7 \\
L_1(7,12) &= \oplus_7 + \oplus_{12} = \oplus_6 + \oplus_{11} = 1110 = \oplus^4 \\
L_1(7,13) &= \oplus_7 + \oplus_{13} = \oplus_6 + \oplus_{12} = 0100 = \oplus^0 \\
L_1(7,14) &= \oplus_7 + \oplus_{14} = \oplus_6 + \oplus_{13} = 1000 = \oplus^1 \\
L_1(7,15) &= \oplus_7 + \oplus_{15} = \oplus_6 + \oplus_{14} = 1010 = \oplus^8 \\
\end{align*} \]

\[ \begin{align*}
L_1(8,1) &= \oplus_8 + \oplus_1 = \oplus_7 + \oplus_1 = 0101 = \oplus^9 \\
L_1(8,2) &= \oplus_8 + \oplus_2 = \oplus_7 + \oplus_2 = 1001 = \oplus^{14} \\
L_1(8,3) &= \oplus_8 + \oplus_3 = \oplus_7 + \oplus_3 = 1111 = \oplus^{12} \\
L_1(8,4) &= \oplus_8 + \oplus_4 = \oplus_7 + \oplus_4 = 1100 = \oplus^4 \\
L_1(8,5) &= \oplus_8 + \oplus_5 = \oplus_7 + \oplus_5 = 0001 = \oplus^3 \\
L_1(8,6) &= \oplus_8 + \oplus_6 = \oplus_7 + \oplus_6 = 1011 = \oplus^{13} \\
L_1(8,7) &= \oplus_8 + \oplus_7 = \oplus_7 + \oplus_7 = 1110 = \oplus^{10} \\
L_1(8,8) &= \oplus_8 + \oplus_8 = \oplus_7 + \oplus_8 = 0 \\
L_1(8,9) &= \oplus_8 + \oplus_9 = \oplus_7 + \oplus_9 = 0111 = \oplus^{11} \\
L_1(8,10) &= \oplus_8 + \oplus_{10} = \oplus_7 + \oplus_{10} = 1000 = \oplus^1 \\
L_1(8,11) &= \oplus_8 + \oplus_{11} = \oplus_7 + \oplus_{11} = 0011 = \oplus^6 \\
L_1(8,12) &= \oplus_8 + \oplus_{12} = \oplus_7 + \oplus_{12} = 1010 = \oplus^8 \\
L_1(8,13) &= \oplus_8 + \oplus_{13} = \oplus_7 + \oplus_{13} = 0010 = \oplus^0 \\
L_1(8,14) &= \oplus_8 + \oplus_{14} = \oplus_7 + \oplus_{13} = 0110 = \oplus^5 \\
L_1(8,15) &= \oplus_8 + \oplus_{15} = \oplus_7 + \oplus_{14} = 0100 = \oplus^0
\end{align*} \]
\[
L_1(12, 2) = \oplus_{12} + \oplus_{2} = \oplus_{11} + \oplus_{1} = 0011 = \oplus_{6}
\]
\[
L_1(12, 3) = \oplus_{12} + \oplus_{3} = \oplus_{11} + \oplus_{2} = 0101 = \oplus_{9}
\]
\[
L_1(12, 4) = \oplus_{12} + \oplus_{4} = \oplus_{11} + \oplus_{3} = 0110 = \oplus_{5}
\]
\[
L_1(12, 5) = \oplus_{12} + \oplus_{5} = \oplus_{11} + \oplus_{4} = 1011 = \oplus_{13}
\]
\[
L_1(12, 6) = \oplus_{12} + \oplus_{6} = \oplus_{11} + \oplus_{5} = 0001 = \oplus_{3}
\]
\[
L_1(12, 7) = \oplus_{12} + \oplus_{7} = \oplus_{11} + \oplus_{6} = 0100 = \oplus_{6}
\]
\[
L_1(12, 8) = \oplus_{12} + \oplus_{8} = \oplus_{11} + \oplus_{7} = 1010 = \oplus_{8}
\]
\[
L_1(12, 9) = \oplus_{12} + \oplus_{9} = \oplus_{11} + \oplus_{8} = 1101 = \oplus_{7}
\]
\[
L_1(12, 10) = \oplus_{12} + \oplus_{10} = \oplus_{11} + \oplus_{9} = 0010 = \oplus_{2}
\]
\[
L_1(12, 11) = \oplus_{12} + \oplus_{11} = \oplus_{11} + \oplus_{10} = 1001 = \oplus_{14}
\]
\[
L_1(12, 12) = \oplus_{12} + \oplus_{12} = \oplus_{11} + \oplus_{11} = 0
\]
\[
L_1(12, 13) = \oplus_{12} + \oplus_{13} = \oplus_{11} + \oplus_{12} = 1000 = 1
\]
\[
L_1(12, 14) = \oplus_{12} + \oplus_{14} = \oplus_{11} + \oplus_{13} = 1100 = \oplus_{4}
\]
\[
L_1(12, 15) = \oplus_{12} + \oplus_{15} = \oplus_{11} + \oplus_{14} = 1110 = \oplus_{10}
\]
\[
L_1(13, 1) = \oplus_{13} + \oplus_{1} = \oplus_{12} + \oplus_{1} = 0111 = \oplus_{11}
\]
\[
L_1(13, 2) = \oplus_{13} + \oplus_{2} = \oplus_{12} + \oplus_{2} = 1011 = \oplus_{13}
\]
\[
L_1(13, 3) = \oplus_{13} + \oplus_{3} = \oplus_{12} + \oplus_{3} = 1101 = \oplus_{7}
\]
\[
L_1(13, 4) = \oplus_{13} + \oplus_{4} = \oplus_{12} + \oplus_{4} = 1110 = \oplus_{10}
\]
\[
L_1(13, 5) = \oplus_{13} + \oplus_{5} = \oplus_{12} + \oplus_{5} = 0011 = \oplus_{5}
\]
\[
L_1(13, 6) = \oplus_{13} + \oplus_{6} = \oplus_{12} + \oplus_{6} = 1001 = \oplus_{14}
\]
\[
L_1(13, 7) = \oplus_{13} + \oplus_{7} = \oplus_{12} + \oplus_{7} = 1100 = \oplus_{4}
\]
\[
L_1(13, 8) = \oplus_{13} + \oplus_{8} = \oplus_{12} + \oplus_{8} = 0010 = \oplus_{2}
\]
\[
L_1(13, 9) = \oplus_{13} + \oplus_{9} = \oplus_{12} + \oplus_{8} = 0101 = \oplus_{9}
\]
\[
L_1(13, 10) = \oplus_{13} + \oplus_{10} = \oplus_{12} + \oplus_{9} = 1010 = \oplus_{8}
\]
\[
L_1(13, 11) = \oplus_{13} + \oplus_{11} = \oplus_{12} + \oplus_{10} = 0001 = \oplus_{3}
\]
\[
L_1(13, 12) = \oplus_{13} + \oplus_{12} = \oplus_{12} + \oplus_{11} = 1000 = 1
\]
\[
L_1(13, 13) = \oplus_{13} + \oplus_{13} = \oplus_{12} + \oplus_{12} = 0
\]
\[
L_1(13, 14) = \oplus_{13} + \oplus_{14} = \oplus_{12} + \oplus_{13} = 0100 = \oplus_{4}
\]
\[
L_1(13, 15) = \oplus_{13} + \oplus_{15} = \oplus_{12} + \oplus_{14} = 0110 = \oplus_{10}
\]
\[
L_1(14, 1) = \oplus_{14} + \oplus_{1} = \oplus_{13} + \oplus_{1} = 0011 = \oplus_{6}
\]
\[
L_1(14, 2) = \oplus_{14} + \oplus_{2} = \oplus_{13} + \oplus_{2} = 1111 = \oplus_{12}
\]
\[
L_1(14, 3) = \oplus_{14} + \oplus_{3} = \oplus_{13} + \oplus_{3} = 1001 = \oplus_{14}
\]
\[
L_1(14, 4) = \oplus_{14} + \oplus_{4} = \oplus_{13} + \oplus_{4} = 1010 = \oplus_{8}
\]
\[
L_1(14, 5) = \oplus_{14} + \oplus_{5} = \oplus_{13} + \oplus_{5} = 0111 = \oplus_{11}
\]
\[
L_1(14, 6) = \oplus_{14} + \oplus_{6} = \oplus_{13} + \oplus_{6} = 1101 = \oplus_{7}
\]
\[
L_1(14, 7) = \oplus_{14} + \oplus_{7} = \oplus_{13} + \oplus_{7} = 1000 = 1
\]
\[
L_1(14, 8) = \oplus_{14} + \oplus_{8} = \oplus_{13} + \oplus_{8} = 0110 = \oplus_{5}
\]
\[
L_1(14, 9) = \oplus_{14} + \oplus_{9} = \oplus_{13} + \oplus_{8} = 0001 = \oplus_{3}
\]
\[
L_1(14, 10) = \oplus_{14} + \oplus_{10} = \oplus_{13} + \oplus_{9} = 1110 = \oplus_{10}
\]
\[
L_1(14, 11) = \oplus_{14} + \oplus_{11} = \oplus_{13} + \oplus_{10} = 0101 = \oplus_{9}
\]
\[
L_1(14, 12) = \oplus_{14} + \oplus_{12} = \oplus_{13} + \oplus_{11} = 1100 = \oplus_{4}
\]
\[
L_1(14, 13) = \oplus_{14} + \oplus_{13} = \oplus_{13} + \oplus_{12} = 0100 = \oplus_{8}
\]
\[
L_1(14, 14) = \oplus_{14} + \oplus_{14} = \oplus_{13} + \oplus_{13} = 0
\]
\[
L_1(14, 15) = \oplus_{14} + \oplus_{15} = \oplus_{13} + \oplus_{14} = 0010 = \oplus_{2}
\]

The last row of \( L_1 \) is given by:
\[ L_1(15,1) = 15 + 1 = 14 + 1 = 0001 = 7 \]
\[ L_1(15,2) = 15 + 2 = 14 + 2 = 1011 = 13 \]
\[ L_1(15,3) = 15 + 3 = 14 + 3 = 1000 = 1 \]
\[ L_1(15,4) = 15 + 4 = 14 + 4 = 0101 = 9 \]
\[ L_1(15,5) = 15 + 5 = 14 + 5 = 1111 = 12 \]
\[ L_1(15,6) = 15 + 6 = 14 + 6 = 1010 = 8 \]
\[ L_1(15,7) = 15 + 7 = 14 + 7 = 0100 = 4 \]
\[ L_1(15,8) = 15 + 8 = 14 + 8 = 1011 = 11 \]
\[ L_1(15,9) = 15 + 9 = 14 + 9 = 0011 = 6 \]
\[ L_1(15,10) = 15 + 10 = 14 + 10 = 1100 = 10 \]
\[ L_1(15,11) = 15 + 11 = 14 + 11 = 0111 = 11 \]
\[ L_1(15,12) = 15 + 12 = 14 + 12 = 1110 = 10 \]
\[ L_1(15,13) = 15 + 13 = 14 + 13 = 0110 = 5 \]
\[ L_1(15,14) = 15 + 14 = 14 + 14 = 0010 = 2 \]
\[ L_1(15,15) = 15 + 15 = 14 + 14 = 0 \]

Substituting the calculated rows into Figure 11 yields the first orthogonal Latin square shown in Figure 12.

Subsequent Latin squares are developed by holding the 0th row of \( L_1 \) and performing cyclic permutations on the other rows as shown in Appendix D. The fifteen mutually orthogonal Latin squares, given in Appendix E, are produced by interchanging the columns of each square while holding the first column constant.
\[ L_1 = \]

\[
\begin{array}{ccccccccccccccc}
\mathbf{e}_0 & \mathbf{e}_1 & \mathbf{e}_2 & \mathbf{e}_3 & \mathbf{e}_4 & \mathbf{e}_5 & \mathbf{e}_6 & \mathbf{e}_7 & \mathbf{e}_8 & \mathbf{e}_9 & \mathbf{e}_{10} & \mathbf{e}_{11} & \mathbf{e}_{12} & \mathbf{e}_{13} & \mathbf{e}_{14} & \mathbf{e}_{15} \\
\mathbf{e}_1 & 0 & 4 & 8 & 14 & 10 & 13 & 9 & 2 & 7 & 5 & 12 & 11 & 6 & 3 \\
\mathbf{e}_2 & 4 & 0 & 5 & 9 & 1 & 2 & 11 & 14 & 10 & 3 & 8 & 6 & 13 & 12 & 7 \\
\mathbf{e}_3 & 8 & 5 & 0 & 6 & 10 & 3 & 12 & 1 & 11 & 4 & 9 & 7 & 14 & 13 & 1 \\
\mathbf{e}_4 & 14 & 9 & 6 & 0 & 7 & 11 & 2 & 4 & 13 & 12 & 5 & 10 & 8 & 1 & 12 \\
\mathbf{e}_5 & 1 & 10 & 7 & 0 & 8 & 12 & 3 & 5 & 14 & 2 & 13 & 6 & 11 & 9 & 12 \\
\mathbf{e}_6 & 10 & 2 & 11 & 8 & 0 & 9 & 13 & 4 & 6 & 1 & 3 & 14 & 7 & 12 & 12 \\
\mathbf{e}_7 & 13 & 11 & 3 & 2 & 12 & 9 & 0 & 10 & 14 & 5 & 7 & 4 & 1 & 8 \\
\mathbf{e}_8 & 9 & 14 & 12 & 4 & 3 & 13 & 10 & 0 & 11 & 1 & 6 & 8 & 2 & 5 \\
\mathbf{e}_9 & 2 & 10 & 1 & 13 & 5 & 4 & 14 & 11 & 0 & 12 & 7 & 9 & 3 & 6 \\
\mathbf{e}_{10} & 7 & 3 & 11 & 6 & 14 & 5 & 1 & 12 & 0 & 13 & 2 & 8 & 10 & 4 & 12 \\
\mathbf{e}_{11} & 5 & 8 & 4 & 12 & 2 & 1 & 7 & 6 & 13 & 0 & 14 & 3 & 9 & 11 \\
\mathbf{e}_{12} & 12 & 6 & 9 & 5 & 13 & 3 & 8 & 7 & 2 & 14 & 0 & 1 & 4 & 10 \\
\mathbf{e}_{13} & 11 & 13 & 7 & 10 & 6 & 14 & 4 & 2 & 9 & 8 & 3 & 1 & 0 & 5 \\
\mathbf{e}_{14} & 6 & 12 & 14 & 8 & 11 & 9 & 12 & 7 & 1 & 6 & 5 & 3 & 10 & 9 & 4 \\
\mathbf{e}_{15} & 3 & 7 & 13 & 1 & 9 & 12 & 8 & 6 & 4 & 11 & 10 & 5 & 2 & 0 \\
\end{array}
\]

**FIGURE 12** FIRST ORTHOGONAL LATIN SQUARE

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CHAPTER 5  TDMA AND FDMA COMPARISON FOR TELEMETRY CHANNELS

Coordinating the communications between satellites and multiple ground stations is accomplished through the use of a multiple access method. Two typical methods are frequency division multiple access (FDMA) and time division multiple access (TDMA). FDMA divides the available frequency band among the users which can transmit at the same time. TDMA divides the time each user is allotted to use over one frequency band. The time or frequency used is either preassigned (fixed access) or assigned as needed (direct access) to the earth stations. Comparisons between the two methods are based on:

(1) satellite capacity
(2) spectrum utilization
(3) satellite power
(4) interconnectivity
(5) flexibility
(6) adaptability to different traffic mixes
(7) cost
(8) user acceptability. (23, p224)

At present, the Iridium system is scheduled to implement a combination of TDMA and FDMA techniques with
QPSK modulation and the L-band multiplexing scheme. (18, p34) Each SV projects TDMA modulation over each cell for a set amount of time while FDMA is used within each cell to support the users. (27, p9) Further, both direct access and fixed access techniques are employed to create a higher concentration of used channels in a given location.

5.1 Frequency Division Multiple Access (FDMA)

FDMA is more commonly used than TDMA. As shown in Table 9, both FDMA and TDMA techniques are used in current mobile communications. Early forms of terrestrial communication implemented FDMA methods which were simple forms of filtered analog signals. As digital data became more prominent, the benefits of TDMA modulation began to enter satellite communications. Alternate forms of access methods are also employed, but FDMA currently remains the most profitable. (23, p227)

FDMA offers a variety of advantages for analog data and narrowband systems (17, p84). Due to the nature of the system, synchronization between channels is less of a concern for FDMA than TDMA methods. Therefore, individual units are not equipped with electronics to counteract the effects of signal propagation along different paths. Compingand overdeviation are employed to increase the capacity of the analog link by reducing the intermodulation
distortion resulting in FDMA. Thus, FDMA is continuing to be used in current satellite communication systems.

The disadvantages of FDMA are similar to those experienced by other analog systems. For example, with an increased number
<table>
<thead>
<tr>
<th>Parameter</th>
<th>AMPS</th>
<th>NMT</th>
<th>GSM</th>
<th>PCN</th>
<th>IS-54</th>
<th>IRIDIUM*</th>
</tr>
</thead>
<tbody>
<tr>
<td>TX freq., MHz:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>base</td>
<td>869-894</td>
<td>935-960</td>
<td>890-915</td>
<td>1710-1785</td>
<td>869-894</td>
<td>1610-1626</td>
</tr>
<tr>
<td>mobile</td>
<td>824-849</td>
<td>890-915</td>
<td>935-960</td>
<td>1805-1880</td>
<td>824-849</td>
<td>1610-1626</td>
</tr>
<tr>
<td>Multiple access</td>
<td>FDMA</td>
<td>FDMA</td>
<td>TDMA</td>
<td>TDMA</td>
<td>TDMA</td>
<td>Combin.</td>
</tr>
<tr>
<td>Duplexing method</td>
<td>FDD</td>
<td>FDD</td>
<td>FDD</td>
<td>FDD</td>
<td>FDD</td>
<td>FDD</td>
</tr>
<tr>
<td>Channel bw, kHz</td>
<td>30.0</td>
<td>12.5</td>
<td>200.0</td>
<td>10.0</td>
<td>200.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Traffic chnl /RF chnl</td>
<td>1</td>
<td>1</td>
<td>8</td>
<td>16</td>
<td>3</td>
<td>37</td>
</tr>
<tr>
<td>Total traffic ch.</td>
<td>832</td>
<td>1999</td>
<td>125x8</td>
<td>444</td>
<td>832x3</td>
<td>260x37</td>
</tr>
<tr>
<td>Voice analo analog RELP RELP VSELP VSELP</td>
<td></td>
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</tr>
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<td>2:1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>speech rate- mod.</td>
<td>13.0</td>
<td>6.7</td>
<td>8.0</td>
<td>4.8kbps</td>
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<td>pi/4</td>
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<td>pi/4</td>
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<tr>
<td>peak dev.,±12kHz ±5kHz</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>ch.rate, kpsd</td>
<td>270.8</td>
<td>48.6</td>
<td>-</td>
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<td></td>
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<tr>
<td>Control: digtl digtl digtl digtl digtl digtl</td>
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</tr>
<tr>
<td>mod. FSK FFSK GMSK GMSK pi/4 QPSK</td>
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</tr>
<tr>
<td>bb Wavefm Manch. NRZ NRZ NRZ NRZ NRZ</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ch.rate 10.0 1.2 270.8 48.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Channel</td>
<td>BCH</td>
<td>B1</td>
<td>RS</td>
<td>RS</td>
<td>Conv.</td>
<td>BCH</td>
</tr>
<tr>
<td>base --&gt;mobile (40,28) burst (12,8) (12,8) 1/2 (1023,668)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mobile--&gt;base (48,36) burst (12,8) (12,8) 1/2 (1023,668)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Not part of original chart
of users, there is a nonlinear decrease in the number of carriers able to be supported. This is mainly due to intermodulation (IM) noise which increases with the number of carriers. In order to avoid the distortion effects, guard bands must be provided to separate the signals. Nevertheless, the system is capable of providing a high quality telephone service. (23, p227,233).

Flexibility and power utilization for FDMA can also be a disadvantage. As a result of the manner in which data is extracted, FDMA suffers from "...a lack of flexibility to take full advantage of the worldwide coverage and potential interconnectivity of a satellite network(23, p227)."

Combining FDMA and a fixed access method subsequently creates an inflexible and inefficient use of the available frequency band. In addition, FDMA method does not use transponder power efficiently(23, p227).

Access methods influence the overall carrier-to-noise ratio \((C/N)_i\) available at the input to the receiving station's FM demodulator. The \((C/N)\) ratio is defined as the ratio of the signal power to the noise power at any point in a satellite link. The overall \((C/N)_i\) ratio is influenced by the uplink \((C/N)_u\), the downlink \((C/N)_d\), and the intermodulation \((C/N)_I\) ratios. Operating in a nonlinear area of the traveling wave tube (TWT) results in intermodulation and a loss of \((C/N)_i\) ratio.
As stated above, FDMA makes poor use of transponder power. The TWT must be operated at a saturation point to produce maximum output power. The characteristics of a TWT are shown in Figure 13. In order to avoid operating the TWT in a nonlinear region of its characteristic curve, the FDMA operation is at a particular input backoff, $BO_i$. The $BO_i$ results in an output backoff, $BO_o$, and this is incorporated into the overall $(C/N)_o$ ratio, as shown in Figure 14.

FDMA alone is incapable of supporting the Iridium satellite system. FDMA requires a large number of frequencies in order to cover the globe with the cover intended for Iridium. The 66 satellites would require nonoverlapping frequencies.
Figure 13 Typical Characteristic of a Traveling Wave Tube.
FIGURE 14 CARRIER TO NOISE (C/N) TRADE-OFFS AS A FUNCTION OF THE INPUT POWER TO THE TRANSPONDER TWT AMPLIFIER
5.2 Time Division Multiple Access (TDMA)

Time division multiple access (TDMA) is best used in digital and wideband applications. One main advantage is the capability to operate the traveling wave tube (TWT) amplifier at or near saturation thus avoiding backoff and intermodulation. The technique maximizes the downlink \((C/N)_d\) ratio and improves the overall \((C/N)_i\). The overall \((C/N)_i\) ratio is improved by a better it error rate resulting from a resistance to noise and interference. (23, p235)

TDMA, unlike FDMA, also allows system flexibility. As the demands of signal traffic changes, the system is easy to reconfigure. Thus, the frequency and time is utilized in an efficient manner. A comparison of the average message delay for an FDMA and TDMA shows that TDMA performs \(T_f/2 - T_f/N\) seconds better than FDMA (23, p361), where \(T_f\) is the TDMA frame length and \(N\) is the number of users.

Signals propagating along different paths can be a problem for TDMA transmissions. Stations must transmit bursts of data in a predetermined order to avoid disrupting operations. This is accomplished through the use of frames, as shown in Figure 15 (5, p13) (23, p238), and hardware coordination on the ground. Subsequently, Iridium handsets, ground stations, gateways, and any receive facility would require electronics to accommodate the use of TDMA. Consequently, FDMA techniques are employed for user data.
FIGURE 15 THE STRUCTURE OF THE FRAME
5.3 Multiple Access for the Iridium System

A list of the characteristics for FDMA and TDMA is given in Table 10 to facilitate the evaluation of the multiple access methods for the Iridium System. By using a combination of the two techniques, the frequency reuse pattern is possible which is essential for the operation of Iridium. Without using a combination of both TDMA and FDMA techniques, too many frequencies would be required to maintain a constellation of 66 LEO satellites. The TDMA is necessary to handle the varying numbers of users in each cell. Since certain cells will cover large populated areas, the number of users will be larger than those which cover deserted areas. The present procedure allows the satellites to transmit the continuous stream of telemetry in this manner.
<table>
<thead>
<tr>
<th>Characteristic</th>
<th>FDMA</th>
<th>TDMA</th>
</tr>
</thead>
<tbody>
<tr>
<td>Satellite Capacity</td>
<td>Degraded w/Number of channels carried</td>
<td>Not affected</td>
</tr>
<tr>
<td>Spectrum Utilization</td>
<td>Degraded w/Number of channels carried</td>
<td>Not affected</td>
</tr>
<tr>
<td>Satellite Power</td>
<td>Varies but may affect signal D/L capacity</td>
<td>Max</td>
</tr>
<tr>
<td>Interconnectivity</td>
<td>Poor</td>
<td>Good</td>
</tr>
<tr>
<td>Flexibility</td>
<td>Lacks</td>
<td>Good</td>
</tr>
<tr>
<td>Adaptability to Different</td>
<td>Lacks</td>
<td>Good</td>
</tr>
<tr>
<td>Traffic Mixes</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>Simple therefore less Cost</td>
<td>Data</td>
</tr>
<tr>
<td>Synchronization</td>
<td></td>
<td>&gt; cost</td>
</tr>
<tr>
<td>User Acceptability</td>
<td>Proven Technology</td>
<td>Before 1984</td>
</tr>
<tr>
<td></td>
<td></td>
<td>not used</td>
</tr>
<tr>
<td>Recommended</td>
<td>analog data</td>
<td>digital data</td>
</tr>
<tr>
<td></td>
<td>narrowband</td>
<td>wideband</td>
</tr>
</tbody>
</table>

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CHAPTER 6 COMPUTER NETWORK ANALYSIS FOR THE SCS

The computer network is developed for the SCS in support of monitoring satellite health, monitoring mission performance, commanding SVs, and archiving data. The network must be robust in order to maintain uninterrupted operations. Likewise, the computer network must be able to process information for 66 satellites. A comparison of topologies, transmission medium, and medium access control techniques yields a computer network viable for the Iridium ground stations.

6.1 Topology

Topology refers to the means by which computer stations are connected. Four possible approaches, shown in Figure 16 (29, p65), are the star, ring, tree, and bus topologies. The setup necessary for a given system is based on reliability, expandability, and performance as suggested in Table 11.
FIGURE 16 TOPOLOGIES
The star topology uses a centralized processor for operation. Communications between computers are performed at one location reducing the amount of processing time at each terminal. As a result, the central node is rather complex and also provides a single point failure which disables the entire network.

The ring topology places the burden of the network processing on the surrounding nodes. The technique requires a unidirectional circulation of data to avoid packet collisions. Access to the ring is determined by logic contained at each terminal, thus increasing station computation. The network is severely handicapped when a repeater is lost.

The bus and tree topologies are essentially the same. The tree is a series of buses connected in parallel at a headpoint. All stations are connected to a linear transmission medium. Data from one station is sent across the entire network; therefore, the distance between the farthest terminals dictates the amount of time required to complete a data transfer. Only one station may transmit at a time to prevent data collisions as in the ring topology. Thus, each terminal must contain a form of access logic to determine its turn in the loop. (29, p64-67)
<table>
<thead>
<tr>
<th>Star Topology</th>
<th>Ring Topology</th>
<th>Bus/Tree Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>- complex network</td>
<td>- unidirectional</td>
<td>+ no switch/no repeaters (interface directly to linear transmission)</td>
</tr>
<tr>
<td>+ minimum burden on station</td>
<td>+ repeaters/simple packets &amp; access control logic at each station</td>
<td>- burden on each station</td>
</tr>
<tr>
<td>- common central switch</td>
<td></td>
<td>+ most flexible</td>
</tr>
<tr>
<td>(acts as a single point failure)</td>
<td></td>
<td>+ able to handle wide range of devices/data rates/data types</td>
</tr>
</tbody>
</table>
6.2 Transmission Medium

The material used to connect the physical path between each node in the network is the transmission medium. Transmission medium can be characterized either as guided or unguided depending on the form the media assumes. With guided media, the data travels along a tangible path such as fiber optic cable. In an unguided medium, the data is transferred through the atmosphere. In both cases the transmission medium can be described using a physical description, transmission characteristics, connectivity, geographic scope, noise immunity, and relative cost (29, p69).

The choice of transmission medium is dependent upon the following factors:

1) The capacity needed to support the network.
2) The reliability needed for the system.
3) The flexibility to support various data types and various environments.
4) The performance of the medium in conjunction with the chosen topology.

A comparison of various mediums is provided in Table 12.
<table>
<thead>
<tr>
<th>Medium</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Twisted pair</td>
<td>inexpensive, prewired buildings, bandwidth limited desirable for low-traffic, local network installation</td>
</tr>
<tr>
<td>Coaxial Cable</td>
<td>medium expense, medium bandwidth capacity room for expansion, handle considerable amount of message traffic</td>
</tr>
<tr>
<td>Fiber Optic Cable</td>
<td>higher cost, low noise susceptibility, low loss, small size, light weight</td>
</tr>
</tbody>
</table>
Coordination between the transmission medium and topology, as shown in Table 13, results in a better overall performance for the system. Cost effectiveness and traffic capacity are the main proponents in choosing a specific network.
<table>
<thead>
<tr>
<th>Medium</th>
<th>Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bus</td>
</tr>
<tr>
<td>Twisted pair</td>
<td>×</td>
</tr>
<tr>
<td>Baseband coaxial cable</td>
<td>×</td>
</tr>
<tr>
<td>Broadband coaxial cable</td>
<td>×</td>
</tr>
<tr>
<td>Optical fiber</td>
<td></td>
</tr>
</tbody>
</table>
6.4 Iridium Local Area Network

The Iridium ground stations are to be equipped with no more than 6 Human Machine Interface (HMI) devices which contain Graphic User Interface (GUI) software. The workstations are to consist of a dedicated vehicle, ground, and overview station. As the system expands, the network is desired to have a reasonable amount of flexibility. (4)

A bus topology with an ethernet backbone is suggested to support the operations at each facility and a separate landline is considered to support operations between the facilities when possible. The bus topology provides the reliability necessary to support the satellite operations and the flexibility for expansion. Likewise, the ethernet backbone which is comprised of:

- Transceiver
- Transceiver cable
- Controller
- 50-ohm coaxial cable
- 50-ohm terminators

provides the bandwidth able to support the network for a reasonable cost.

Over the long term of the Iridium system, fiber proves to be a better choice over coaxial cable and twisted pair. Fiber is more durable and allows the system to grow in
bandwidth. The cost is a factor for fiber optic cable, compatible transceivers, and compatible routers. This cost factor is reduced with time.

A bus topology is recommended to support the SCS. The star topology contains a single point of failure. During critical times, a single point of failure is unacceptable for operations where real-time command and control can prevent serious failures on the satellites. The ring topology is not recommended due to the unidirectional flow and the added burden placed on each workstation.
CHAPTER 7  THE SATELLITE CONTROL FACILITY COMPUTER ALGORITHM

The software developed for the Iridium ground station is required to fulfill two main tasks. These tasks are to retrieve and display telemetry for the satellites and to provide a method of commanding the 66 satellites. In order to accomplish these tasks, the command and control software is written in Fortran 77 and the telemetry pages are displayed from a dBase IV application.

The following assumptions were used in determining the command and control software and the telemetry pages.

(1) Each satellite is capable of transmitting and receiving commands and telemetry from the ground station at any time due to the configuration of the Iridium system.

(2) The values and number of points transmitted by the satellite to determine the state of the vehicle.

(3) The binary sequence of the command structure.

The breakdown of the satellite provided in Chapter 2 is necessary to understand the make-up of the telemetry pages. The propulsion section, solar panels, batteries, and circuitry are possible telemetry data points which are provided on each page. These values are essential for the operation of the LEO satellites since orbital maintenance is a necessity.
7.1 The Command and Control Software

The software necessary to control and command the satellites is developed separately from the telemetry pages. First, the command and control software waits for the input of the operator to request or type in a command. The command is then located in a file and returned for verification. After the command is verified, it is sent to the vehicle. If the command is accepted, a satisfactory return status is given. Otherwise, the command is retransmitted. The sequence of events used to command the vehicle are given in the flow chart in Figure 17.
FIGURE 17 FLOW CHART OF THE COMMAND/CONTROL SOFTWARE
The command and control software allows for a single command or a series of commands in order to facilitate routine command structures. A sequence of commands is assumed for orbital maintenance since different telemetry values may be monitored and various pieces of equipment may have to be turned on. To allow for a sequence of commands, a separate file is created. The sequence data file and single command data files are named by the section of the vehicle with a .DAT extension.

7.2 Telemetry Pages

The telemetry pages are designed using dBase IV. The application created by dBase reads a series of files and displays the required telemetry values by selecting the correct page. Telemetry values are updated by the update of the file being read. The telemetry pages target certain aspects of the vehicles based on the analysis provided in Chapter 1. An example of a telemetry page is given in Appendix G.

The telemetry page provided in Appendix G combines six of the 66 satellites and shows the electrical system. In order to monitor each of the satellites, the pages are developed using the satellite number as a designator. Next, the system is incorporated into the page as the second designator.
CHAPTER 8 DISCUSSION

Security coding, TDMA/FDMA comparison, and Fortran programming are provided in the previous chapters. Each of these sections are left for further study. According to reports and individuals working on the Iridium system, security coding will not be incorporated in the final system. Likewise, TDMA and FDMA separately will not be incorporated; instead, a combination will be utilized. C++ will also be a part of the final software development used to receive and monitor telemetry.

The security coding requires further analysis. Since this is the largest scale of low earth orbiting satellites ever attempted for communication purposes, the need to protect the information and satellites may be necessary in the future. Cost trade off is a concern for the implementation of the coding.

TDMA and FDMA comparison is not a concern of the Iridium network. Future analysis should be towards the understanding and improvement of the current system which implements a combination of TDMA and FDMA. Since Motorola’s system is proprietary, obtaining information about the TDMA/FDMA combination has proven to be difficult.

The program given in Appendix F is written in Fortran 77. Currently, C++ has proven to be the language of choice do to the speed of object oriented programming and the
ability to cross different platforms. Converting the program to C++ requires modification in both the development and coding. Therefore, coding in C++ is left for future study.
CHAPTER 9 CONCLUSION

The development of the Iridium Satellite system is an ongoing process. The coding, data security, modulation techniques, SCS network design, and SCS computer program are given to be used with other papers to develop a better understanding and suggest a means of operations for the system. Future papers and further analysis while the system is partially operational will improve the efficiency of Iridium.

The 8-bit Reed Solomon Code is recommended for the telemetry channel. The Reed Solomon Code provides the ability to recover from burst errors and random errors. The 4-bit Reed Solomon Code and BCH code are not considered since the 8-bit Reed Solomon Code is more generalized and provides better protection against burst errors. Future study involving the Viterbi algorithm may prove to be the method of choice.

The data security algorithm provided is suggested to protect the command and control telemetry. The public access key, based on randomly generated numbers, enables the network to be initialized and recoded in the event the code is determined. Clear text is also a possibility in an emergency or during normal operations.

The comparison of TDMA and FDMA yielded the result
already determined by Motorola. A combination of the two is a necessity for the large satellite system. The FDMA is limited by the number of available frequencies. TDMA is required to accommodate the varying number of users in each cell.

For the SCS, the network analysis determined that a bus topology with fiber suffices for the Master and Backup Control Facilities. Although the cost of fiber is much higher than coaxial cable or twisted pair, fiber allows for future system growth. The bus topology removes the possibility of a single point of failure and also allows the number of computer systems to increase.

Finally, in Appendix F, a FORTRAN77 program is given to be used in the SCS. The program allows the user to command the space craft with a single command or a series of commands by issuing a command id string. Maintenance for the commands is performed by updating files which are accessible to the user but rarely edited. A telemetry page is given in Appendix G.
REFERENCES


APPENDIX A ACRONYM LIST AND SYMBOL LIST

©
ACCH
BCF
BCH
BER
DES
EC
FACCH
FDMA
FEC
GF
GPS
GSM
ISU
LEO
MCD
MCF
MD
NiH
NOD
NSA
RN
RSA
SACCH
SCD
S/N
SV
TDMA
TTAC
WARC

alpha
Associated Control Channels
Backup Control Facility
Bose-Chaudhuri-Hocquenghem
Bit Error Rate
Data Encryption Standard
Error Correction
Fast Associated Control Channels
Frequency Division Multiple Access
Forward Error Correction
Galios Field
Global Positioning Services
Groupe Speciale Mobile
Iridium Subscriber Unit
Low Earth Orbiting
Mission Control Data
Master Control Facility
Mission Data
Nickel Hydrogen
Network Operations Data
National Security Agency
Random Number
Rivest-Shamir-Adleman
Slow Associated Control Channels
System Control Data
Signal-to-Noise Ratio
Satellite Vehicle
Time Division Multiple Access
Telemetry, Tracking, and Commanding
World Administrative Radio Conference
| GF(2^5) generated by \( p(X) = 1 + X^2 + X^3 + X^4 + X^8 \) |
|-----------------|-----------------|-----------------|
| -               | 00000000        | 44              | 01110111 | 89 | 10000111 |
| 0               | 10000000        | 45              | 10000011 | 90 | 11110111 |
| 1               | 01000000        | 46              | 11111001 | 91 | 11000101 |
| 2               | 00100000        | 47              | 11000100 | 92 | 10110101 |
| 3               | 00010000        | 48              | 01100010 | 93 | 01011001 |
| 4               | 00001000        | 49              | 00110001 | 94 | 10001110 |
| 5               | 00000100        | 50              | 10100000 | 95 | 01000111 |
| 6               | 00000010        | 51              | 01010000 | 96 | 10011011 |
| 7               | 00000001        | 52              | 00101000 | 97 | 11101010 |
| 8               | 10111000        | 53              | 00010100 | 98 | 11000010 |
| 9               | 01011100        | 54              | 00001010 | 100| 10001000 |
| 10              | 00101110        | 55              | 00000101 | 101| 01001000 |
| 11              | 00010111        | 56              | 10111010 | 102| 00100010 |
| 12              | 10110011        | 57              | 01011101 | 103| 00010001 |
| 13              | 11100001        | 58              | 10010110 | 104| 10110000 |
| 14              | 11001000        | 59              | 01001011 | 105| 01101000 |
| 15              | 01100100        | 60              | 10011101 | 106| 00101100 |
| 16              | 00110010        | 61              | 11101010 | 107| 00010110 |
| 17              | 00011001        | 62              | 01111010 | 108| 00001111 |
| 18              | 10110100        | 63              | 10000101 | 109| 10111101 |
| 19              | 01011010        | 64              | 11111010 | 110| 11001100 |
| 20              | 00101101        | 65              | 01111101 | 111| 01110011 |
| 21              | 10101110        | 66              | 10001110 | 112| 10000011 |
| 22              | 01010111        | 67              | 01000111 | 113| 11111000 |
| 23              | 10010011        | 68              | 10011101 | 114| 01111100 |
| 24              | 11111011        | 69              | 11101000 | 115| 00111110 |
| 25              | 11000100        | 70              | 01111010 | 116| 00011111 |
| 26              | 01100000        | 71              | 00111101 | 117| 10110111 |
| 27              | 00110000        | 72              | 10100110 | 118| 11100011 |
| 28              | 00011000        | 73              | 01010111 | 119| 11010001 |
| 29              | 00001100        | 74              | 10010001 | 120| 11011100 |
| 30              | 00000110        | 75              | 11110000 | 121| 01011110 |
| 31              | 00000011        | 76              | 00111000 | 122| 00110111 |
| 32              | 10111001        | 77              | 00111100 | 123| 10000111 |
| 33              | 11100100        | 78              | 00011110 | 124| 11101001 |
| 34              | 01110010        | 79              | 00001111 | 125| 11001100 |
| 35              | 00111001        | 80              | 10111111 | 126| 10010010 |
| 36              | 10100100        | 81              | 11100111 | 127| 00110011 |
| 37              | 01010010        | 82              | 11001011 | 128| 10100001 |
| 38              | 00101001        | 83              | 11011101 | 129| 11101000 |
| 39              | 10101100        | 84              | 11010110 | 130| 01110000 |
| 40              | 01010110        | 85              | 01101101 | 131| 00111010 |
| 41              | 00101111        | 86              | 10001101 | 132| 00011101 |
| 42              | 10101101        | 87              | 11111110 | 133| 10110110 |
| 43              | 11101110        | 88              | 01111111 | 134| 01010111 |

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| 135 | 10010101 | 187 | 00111011 | 238 | 11010000 |
| 136 | 11110010 | 188 | 10100101 | 239 | 01101000 |
| 137 | 01111001 | 189 | 11010100 | 240 | 00110100 |
| 138 | 10000100 | 190 | 01110101 | 241 | 00011010 |
| 139 | 01000010 | 191 | 10000100 | 242 | 00011010 |
| 140 | 00100001 | 192 | 01000010 | 243 | 10111110 |
| 141 | 10100100 | 193 | 10011000 | 244 | 01011111 |
| 142 | 01010100 | 194 | 01001100 | 245 | 10010111 |
| 143 | 00101010 | 195 | 00100110 | 246 | 11110011 |
| 144 | 00010101 | 196 | 00010011 | 247 | 11000001 |
| 145 | 10110010 | 197 | 10110001 | 248 | 11011000 |
| 146 | 01011001 | 198 | 11000000 | 249 | 01101100 |
| 147 | 10010100 | 199 | 01110000 | 250 | 00110110 |
| 148 | 01001010 | 200 | 00111000 | 251 | 00011011 |
| 149 | 00100101 | 201 | 00011100 | 252 | 10110101 |
| 150 | 10101001 | 202 | 00001110 | 253 | 11100010 |
| 151 | 01010101 | 203 | 00000111 | 254 | 01110001 |
| 152 | 10010010 | 204 | 10111011 |  |  
| 153 | 01001001 | 205 | 11100101 |  |  
| 154 | 10011100 | 206 | 11001010 |  |  
| 155 | 01001110 | 207 | 01100101 |  |  
| 156 | 00100111 | 208 | 10001010 |  |  
| 157 | 10101011 | 209 | 01000101 |  |  
| 158 | 11011011 | 210 | 10011010 |  |  
| 159 | 11001110 | 211 | 01001101 |  |  
| 160 | 01100111 | 212 | 10011110 |  |  
| 161 | 10001011 | 213 | 01001111 |  |  
| 162 | 11111101 | 214 | 10011111 |  |  
| 163 | 11000110 | 215 | 11110111 |  |  
| 164 | 01100011 | 216 | 11001011 |  |  
| 165 | 10001001 | 217 | 11010011 |  |  
| 166 | 01111110 | 218 | 11010100 |  |  
| 168 | 00111111 | 219 | 01101010 |  |  
| 169 | 10100111 | 220 | 00110101 |  |  
| 170 | 11101011 | 221 | 10100010 |  |  
| 171 | 11001101 | 222 | 01010001 |  |  
| 172 | 11011110 | 223 | 10010000 |  |  
| 173 | 01101111 | 224 | 01001000 |  |  
| 174 | 10001111 | 225 | 00100100 |  |  
| 175 | 11111111 | 226 | 00010010 |  |  
| 176 | 11000111 | 227 | 00001001 |  |  
| 177 | 11011011 | 228 | 10111100 |  |  
| 178 | 11010101 | 229 | 01011110 |  |  
| 179 | 11001010 | 230 | 00110111 |  |  
| 180 | 01101001 | 231 | 10111111 |  |  
| 181 | 10001100 | 232 | 11101111 |  |  
| 182 | 01000110 | 233 | 11001111 |  |  
| 183 | 00100011 | 234 | 11011111 |  |  

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### APPENDIX C BCH CODING TABLE

<table>
<thead>
<tr>
<th>m=10</th>
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<th>3</th>
<th>(0,1,2,3,10)</th>
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<tbody>
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<td>(0,2,4,5,10)</td>
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<td>9</td>
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124
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14 & 3 & 7 & 13 & 15 & 9 & 12 & 8 & 1 & 6 & 4 & 11 & 10 & 5 & 2 & 0 
\end{array} \]

133
APPENDIX F IRIDIUM COMMAND AND CONTROL SOFTWARE

PROGRAM IRCOM

*******************************************************************************
*                       *
* PROGRAM TO SEND COMMANDS TO IRIDIUM VEHICLE                        *
* (REFERENCE FORTRAN77 FOR ENGINEERS)                                *
*                                                                    *
* VARIABLES:                                                        *
*                                                                    *
* FILE                 LREC          RECORD LENGTH FOR A DIRECT ACCESS    *
*                      INUNIT        UNIT DESIGNATOR FOR AN OPEN FILE     *
*                      SLENGTH       NOT USED                           *
*                      MM            THE NUMBER OF COMMANDS INCLING ED   *
* BEGINNING OF A*       SEQUENCE      SEQUENCE                        *
* MARK                 A COUNTER    A COUNTER                        *
* COMMAND              THE COMMAND READ FROM THE KEYBOARD            *
* SAT                   THE SECTION OF THE COMMAND WHICH              *
*                       SPECIFIES THE SATELLITE NUMBER                 *
* SECTION              THE TYPE OF SECTION OF THE COMMAND             *
* IE                    ELECTRICAL/OR A SEQUENCE OF COMMANDS          *
* NUM                   THE SPECIFIC COMMAND NUMBER                   *
* DESCRIPT              THE COMMAND DESCRIPTION                      *
* BINC                  THE BINARY CODE USED FOR THE COMMAND STRING  *
* REPLY                 THE ANSWER TO THE VERIFICATION FOR           *
*                       CORRECT COMMAND                             *

135
* HOLD USED TO ADD ON THE SATELLITE
* IDENTIFIER *
* BINSAT THE SATELLITE NUMBER CONVERTED TO
* BINARY*

* * * * * * *

INTEGER LREC
PARAMETER (LREC = 55)
INTEGER INUNIT, SLENGTH, MM, MARK
CHARACTER*80 COMMAND, SAT*2, SECTION*20, NUM*3,

DESCRIP
CHARACTER*80 BINC(0:128), REPLY*1, HOLD*40, BINSAT*7

* INITIALIZE VARIABLES
COMMAND = ''
SAT = ''
SECTION = ''
NUM = ''
INUNIT = 0
SLENGTH = 0
BINC(0) = 0
MM = 0
MARK = 0
HOLD = ''

* MAIN PROGRAM FOR COMMANDING
*
* STEP 1:
* ASK FOR COMMAND OR COMMAND ID FOR A SERIES OF
COMMANDS TO
* BE SENT TO A SPECIFIC VEHICLE
*
* STEP 2:
* CALL A ROUTINE TO OPEN THE FILE CONTAINING THOSE
COMMANDS
* OR A SERIES OF COMMANDS
*
* STEP 3:
* SEARCH THE FILE FOR THESE SPECIFIC COMMANDS AND
RETURN THE
* BINARY COMMAND AND DESCRIPTION
*
* STEP 4:
* OUTPUT THE COMMAND DESCRIPTION AND REQUEST
VERIFICATION
* FOR PASSING THE COMMAND TO THE VEHICLE
*
* STEP 5:

136
IF VERIFIED, ADD VEHICLE IDENTIFICATION NUMBER TO THE COMMAND OR COMMANDS

* STEP 6:
* TRANSMIT TO VEHICLE THE COMMAND OR SERIES OF COMMANDS BUT
* WAIT FOR VEHICLE TO RETURN A STATUS OF RECEIPT IN A SPECIFIED TIME FRAME

* STEP 7:
* REPORT STATUS TO SCREEN AND RETRANSMIT COMMAND ONLY ONCE
* IF NOT RECEIVED

* STEP 8:
* REPORT STATUS TO SCREEN AND ASK OPERATOR TO EITHER RETRANSMIT COMMAND OR CANCEL

* STEP 9:
* CLOSE FILE AND CONTINUE UNTIL THE OPERATOR EXITS THE PROGRAM

PRINT *, ' ENTER THE COMMAND TO THE IRI DIUM VEHICLE',
PRINT *, ' FOR EXAMPLE: '
PRINT *, ' I66EL001 WITH SINGLE QUOTES AROUND IT '
PRINT *, ' IRI DIUM SATELLITE #66, ELECTRICAL SUBSYSTEM, '
PRINT *, ' COMMAND 001'
PRINT *, ' TYPE END TO QUIT'
READ *, COMMAND
PRINT *, COMMAND
PRINT *

IF (COMMAND(1:3) .EQ. 'END') GOTO 100

90          SAT = COMMAND(2:3)
              SECTION = COMMAND(4:5)
              NUM = COMMAND(6:)

CALL INIT(INUNIT, SECTION, LREC)
CALL SEARCH(INUNIT,NUM,BINC,DESCRIP,SECTION,SLENGTH,MM)

* VERIFY CORRECT COMMAND

PRINT *
PRINT *, ' IS THIS THE CORRECT COMMAND? '
PRINT *, DESCRIP
PRINT *, ' Y FOR YES AND N FOR NO '
PRINT *
READ *, REPLY

IF (REPLY .EQ. 'Y') THEN

  * ADD VEHICLE ID TO THE COMMAND

MARK = 0
IF (SAT(1:1) .EQ. '0') THEN
  IF (SAT .EQ. '01') BINSAT = '0000001'
  IF (SAT .EQ. '02') BINSAT = '0000010'
  IF (SAT .EQ. '03') BINSAT = '0000011'
  IF (SAT .EQ. '04') BINSAT = '0000100'
  IF (SAT .EQ. '05') BINSAT = '0000101'
  IF (SAT .EQ. '06') BINSAT = '0000110'
  IF (SAT .EQ. '07') BINSAT = '0000111'
  IF (SAT .EQ. '08') BINSAT = '0001000'
  IF (SAT .EQ. '09') BINSAT = '0001001'
ENDIF
IF (SAT(1:1) .EQ. '1') THEN
  IF (SAT .EQ. '10') BINSAT = '0001010'
  IF (SAT .EQ. '11') BINSAT = '0001011'
  IF (SAT .EQ. '12') BINSAT = '0001100'
  IF (SAT .EQ. '13') BINSAT = '0001101'
  IF (SAT .EQ. '14') BINSAT = '0001110'
  IF (SAT .EQ. '15') BINSAT = '0001111'
  IF (SAT .EQ. '16') BINSAT = '0010000'
  IF (SAT .EQ. '17') BINSAT = '0010001'
  IF (SAT .EQ. '18') BINSAT = '0010010'
  IF (SAT .EQ. '19') BINSAT = '0010011'
ENDIF
IF (SAT(1:1) .EQ. '2') THEN
  IF (SAT .EQ. '20') BINSAT = '0010100'
  IF (SAT .EQ. '21') BINSAT = '0010101'
  IF (SAT .EQ. '22') BINSAT = '0010110'
  IF (SAT .EQ. '23') BINSAT = '0010111'
  IF (SAT .EQ. '24') BINSAT = '0011000'
  IF (SAT .EQ. '25') BINSAT = '0011001'
  IF (SAT .EQ. '26') BINSAT = '0011010'
  IF (SAT .EQ. '27') BINSAT = '0011011'
  IF (SAT .EQ. '28') BINSAT = '0011100'
  IF (SAT .EQ. '29') BINSAT = '0011101'
ENDIF
IF (SAT(1:1) .EQ. '3') THEN
  IF (SAT .EQ. '30') BINSAT = '0011110'
  IF (SAT .EQ. '31') BINSAT = '0011111'
  IF (SAT .EQ. '32') BINSAT = '0100000'
  IF (SAT .EQ. '33') BINSAT = '0100001'
ENDIF
IF (SAT .EQ. '34')  BINSAT = '0100010'
IF (SAT .EQ. '35')  BINSAT = '0100011'
IF (SAT .EQ. '36')  BINSAT = '0100100'
IF (SAT .EQ. '37')  BINSAT = '0100101'
IF (SAT .EQ. '38')  BINSAT = '0100110'
IF (SAT .EQ. '39')  BINSAT = '0100111'
ENDIF
IF (SAT(1:1) .EQ. '4') THEN
  IF (SAT .EQ. '40')  BINSAT = '0101000'
  IF (SAT .EQ. '41')  BINSAT = '0101001'
  IF (SAT .EQ. '42')  BINSAT = '0101010'
  IF (SAT .EQ. '43')  BINSAT = '0101011'
  IF (SAT .EQ. '44')  BINSAT = '0101100'
  IF (SAT .EQ. '45')  BINSAT = '0101101'
  IF (SAT .EQ. '46')  BINSAT = '0101110'
  IF (SAT .EQ. '47')  BINSAT = '0101111'
  IF (SAT .EQ. '48')  BINSAT = '0110000'
  IF (SAT .EQ. '49')  BINSAT = '0110001'
ENDIF
IF (SAT(1:1) .EQ. '5') THEN
  IF (SAT .EQ. '50')  BINSAT = '0110010'
  IF (SAT .EQ. '51')  BINSAT = '0110011'
  IF (SAT .EQ. '52')  BINSAT = '0110100'
  IF (SAT .EQ. '53')  BINSAT = '0110101'
  IF (SAT .EQ. '54')  BINSAT = '0110110'
  IF (SAT .EQ. '55')  BINSAT = '0110111'
  IF (SAT .EQ. '56')  BINSAT = '0111000'
  IF (SAT .EQ. '57')  BINSAT = '0111001'
  IF (SAT .EQ. '58')  BINSAT = '0111010'
  IF (SAT .EQ. '59')  BINSAT = '0111011'
ENDIF
IF (SAT(1:1) .EQ. '6') THEN
  IF (SAT .EQ. '60')  BINSAT = '0111100'
  IF (SAT .EQ. '61')  BINSAT = '0111101'
  IF (SAT .EQ. '62')  BINSAT = '0111110'
  IF (SAT .EQ. '63')  BINSAT = '0111111'
  IF (SAT .EQ. '64')  BINSAT = '1000000'
  IF (SAT .EQ. '65')  BINSAT = '1000001'
  IF (SAT .EQ. '66')  BINSAT = '1000010'
ENDIF

800
HOLD(8:40) = BINC(MARK)

HOLD(1:7) = BINSAT
BINC(MARK) = HOLD

MARK = MARK + 1
MM = MM - 1
IF (MM .NE. 0) GOTO 800
ELSE
   PRINT *, 'PLEASE INPUT YOUR COMMAND AGAIN'
ENDIF

PRINT *, 'ENTER THE COMMAND TO THE IRIDIUM

VEHICLE'
PRINT *, ' FOR EXAMPLE:'
PRINT *, 'I66EL001 WITH SINGLE QUOTES AROUND

IT'
PRINT *, 'IRIDIUM SATELLITE #66, ELECTRICAL

SUBSYSTEM,'
PRINT *, '
READ *, COMMAND
PRINT *

IF (COMMAND(1:3) .NE. 'END') GOTO 90

100 END

******
**INIT******************************************************************************

** THIS SUBROUTINE OPENS THE SINGLE COMMAND FILE OR THE
COMMAND * SERIES FILE

* * * *
* VARIABLES:
* * * *
* INUNIT * SHARED VARIABLE DESIGNATING
  THE UNIT FILE
* * * *
* OPENED
* *
* SECTION FROM THE MAIN SECTION
DETERMINING * COMMAND OR SECTION IE *
* *
* LREC ALLOWABLE RECORD LENGTH FOR
DIRECT* ACCESS FILE
* *
* *
**********

SUBROUTINE INIT(INUNIT, SECTION, LREC)
CHARACTER*20 SECTION
INTEGER INUNIT, LREC

IF (SECTION .EQ. 'NF') THEN

* ONLY USED WITH A NEW FILE
   CALL OPENER(SECTION, INUNIT, 'UNFORMATTED', 'OLD',
   'SE', 0)

ELSE

   CALL OPENER(SECTION, INUNIT, 'FORMATTED', 'OLD', 'DA',
   LREC)

END IF

END

**OPENER*****************************************************************************
*****
* THIS SUBROUTINE IS USED TO OPEN A FILE AND ASSIGN IT A UNIT  *
* NUMBER.  *
* *
* SUCCESSIVE CALLS TO THIS SUBROUTINE WILL ASSIGN UNIT NUMBERS  *
* 10, 11, 12, *
* *
* ... A CALL TO OPENER WITH TYPE = 'SE' WILL OPEN A SEQUENTIAL ** FILE, WHILE A CALL WITH TYPE = 'DA' WILL OPEN A DIRECT ACCESS *
* FILE.
* *
* VARIABLE:
* *
* FNAME THE FILE NAME FROM INIT WHICH IS THE *
* SECTION PLUS THE .DAT IDENTIFIER
* *
* NUNIT THE UNIT NUMBER ASSIGNED TO THE OPENED*
* *
* FORMSP THE FORMAT SPECIFIER FOR *
* *
* STAT THE STATUS OF THE FILE: IE NEW/OLD

141
* TYPE SEQUENTIAL OR DIRECT ACCESS FILE
* LREC RECORD LENGTH OF A DIRECT ACCESS FILE
* N PARAMETER TO SET THE NUNIT

*******************************************************************************
*****
SUBROUTINE OPENER(FNAME, NUNIT, FORMSP, STAT, TYPE, LREC)

CHARACTER *(* ) FNAME, FORMSP, STAT, TYPE
INTEGER LREC, NUNIT, N
SAVE N
DATA N/10/

FNAME = FNAME(1:2)/' .DAT'
IF(TYPE .EQ. ' SE') THEN

OPEN(UNIT = N, FILE = FNAME, FORM = FORMSP, STATUS =
STAT,
+ Err = 10)

ELSE

OPEN(UNIT = N, FILE = FNAME, FORM = FORMSP, STATUS =
STAT,
+ ACCESS = 'DIRECT', RECL = LREC, ERR = 10)

END IF

NUNIT = N
N = N + 1
RETURN

10 PRINT *, FNAME, 'CANNOT BE OPENED'
STOP
END

**SEARCH******************************************************************************
*****
* THIS SUBROUTINE SEARCHES THE CONTENTS OF A FILE FOR THE
* * * SPECIFIC COMMAND OR SERIES OF COMMANDS
*
*

142
* VARIABLES:
  *
  * FILE    INUNIT  THE UNIT SPECIFIER FOR AN OPEN
  * NUM     THE NUMBER PASS BY MAIN
  * BINC    THE BINARY CODE FOR THE COMMAND
  * DESCRIP STRING
  * CHOSEN  A DESCRIPTION OF THE COMMAND
  * SECTION THE SECTION USED IN ELECTRICAL/SEQUENCE
  * SLENGTH NOT USED
  * SEQUENCE MM  THE NUMBER OF COMMANDS IN A
  * C1      NOT USED
  * SEQUENCE C2  THE NUMBER OF COMMANDS IN A
  * C3      IN A SEQUENCE, THE COMMAND LISTED
  * NUMBER  C3
  * SECTION1 NOT USED
  * MATCHES ID  THE COMMAND ID NUMBER WHICH NUM
  * ID2     NOT USED
  * EOL     NOT USED
  * FILE    BADNUM REPORTS ERRORS WHEN READING A
  * BADNM   REPORTS ERRORS WHEN READING A
  * FILE    BADNM
  * ID2     THE COMMAND ID NUMBER
  * FLAG    MARKS THE END OF THE FILE OR
  * COMMAND ID4  COMMAND NUMBER
  * ID4     LOGICAL PARAMETER FOR FINDING
  * THE FOUND COMMAND

143
SUBROUTINE SEARCH(INUNIT, NUM, BINC, DESCRIPT, SECTION, SLENGTH, MM)

INTEGER C1, C2, C3, SECTION1, MM
INTEGER SLENGTH, EOL, BADNUM, BADNM
CHARACTER*80 DESCRIPT, SECTION2, ID*3, ID2*2
CHARACTER*80 NUM*3, FLAG*2, ID4*3
CHARACTER*80 BINC(0:128)
LOGICAL FOUND

C1 = 0
II = 1
C2 = 0
ID = ' '
FOUND = .FALSE.
BADNUM = 0
ID2 = ' '
SECTION1 = 0
EOL = 9
C3 = 0
MM = 0
SECTION1 = SLENGTH

IF (SECTION .EQ. 'SR') THEN

40 IF (.NOT.FOUND) THEN
    FLAG = ' '
    READ (INUNIT, 200, IOSTAT = BADNUM) ID,
+ C2, DESCRIPT

200 FORMAT (3X,A3,4X,I2,5X,A3)

    IF (ID .EQ. NUM) THEN
        FLAG = ' '
        IF (.NOT.FOUND) MM = C2
        FOUND = .TRUE.
        BADNM = 0
        COUNT = 0

85 READ (INUNIT, 201, IOSTAT = BADNM) C3, ID2,
BINC(COUNT) 201 FORMAT (3X,I3,4X,A2,5X,A3)

    C2 = C2 - 1

144
COUNT = COUNT + 1

IF (C2 .EQ. 1) THEN
    READ (INUNIT, 202, IOSTAT = BADNM) FLAG
    FORMAT(A2)
ENDIF

IF (FLAG .NE. 'ED') GOTO 85
GOTO 40
ELSE

FLAG = '

READ (INUNIT, 201, IOSTAT = BADNM) C3, ID2, BINC(0)
C2 = C2 - 1

IF (C2 .EQ. 1) THEN
    READ (INUNIT, 202, IOSTAT = BADNM) FLAG
ENDIF

IF ((FLAG .NE. 'ED').AND.(FLAG .NE. 'ZZ')) THEN
    GOTO 600
ELSE IF (FLAG .EQ. 'ED') THEN
    GOTO 40
ELSE IF (FLAG .EQ. 'ZZ') THEN
    GOTO 5000
ENDIF

ENDIF
END IF
ELSE

IF (.NOT. FOUND) THEN
    MM = 1
    READ (INUNIT, 10) ID4, DESCRIP
    FORMAT (3X, A3, 1X, A33)
*
    IF (ID4 .EQ. NUM) THEN
        FOUND = .TRUE.
        READ(INUNIT, 20) BINC(0)
        FORMAT (A33)
    END IF
    IF (ID4 .EQ. 'ZZZ') GOTO 5000
    GOTO 30
ENDIF
ENDIF
IF (.NOT.FOUND) THEN
    PRINT *, 'INVALID COMMAND; NOT FOUND IN COMMAND FILE'
ENDIF
5000       REWIND INUNIT
            CLOSE(INUNIT)
            END
### Power System Analysis for Satellites

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<tr>
<td>Batt</td>
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VITA

Wendy A. Holtzman was born September 14, 1969 in Great Lakes, Illinois. She received her Bachelor’s Degree in Electrical Engineering from The Pennsylvania State University in December 1990. In May of 1992 while working for Lockheed Martin, she completed the A and B advanced courses.

Wendy’s work experience includes employment as a Engineer in Training for RGS Associates and a Test and Evaluation Engineer for Lockheed Martin. Wendy currently works for Science Applications International Corporation (SAIC) as a Systems Engineer. She is pursuing an interest in computer networking.

Wendy is a member of IEEE, Tau Beta Pi Engineering Honor Society, and Eta Kappa Nu Electrical Engineering Society. She has completed her Engineer-In-Training exam and is studying for her Professional Engineers Exam. She has received the Lockheed Martin Pride Award and Team Awards.

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