Multiuser Detection for CDMA Systems with Convolutional Coding

by

Ning Yang

Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE
in
Electrical Engineering

APPROVED

[Signatures]

Dr. Brian D. Woerner, Chairman

Dr. Theodore S. Rappaport

Dr. Festus Gail Gray

October 17, 1995
Blacksburg, Virginia

Key Words: CDMA, Convolutional Coding, Interference Cancellation
Multiuser Detection for CDMA Systems with Convolutional Coding

by

Ning Yang

Committee Chairman: Dr. Brian D. Woerner

Bradley Department of Electrical Engineering
Virginia Polytechnic Institute and State University
Blacksburg, Virginia

Abstract

In Code Division Multiple Access (CDMA) systems, the multiple access interference limits the capacity of current systems which use a matched filter or correlation receiver. It has been shown by previous research that multiuser detection receivers employing interference cancellation techniques can significantly improve the capacity of CDMA systems. Error correction coding is also an important technique for overcoming severe channel degradation. In this project, we investigate the performance of multiuser receivers which use the combination of interference cancellation techniques and error correction codes. Specifically, we look at the combination of multistage interference cancellation and convolutional coding. Two different combination schemes are proposed and the performance of these two schemes is studied. The first scheme is a partitioned approach where the multistage interference cancellation is in front of the decoder and is performed on the coded data. The second scheme is an integrated approach where the decoded data is used to reconstruct the transmitted signals in order to do interference cancellation.

Both of these two schemes result in significant performance improvement over a receiver using either multistage interference cancellation or convolutional coding techniques only, for a reasonable range of operating points. The first scheme is recommended due to its soft-decision estimation of transmitted signal and implementation considerations. The analytical results for the first scheme are presented. Simulation results for both of these two schemes are obtained and compared.
Acknowledgments

I am sincerely grateful to Dr. Brian D. Woerner for being my advisor, and for his guidance during this thesis project and throughout my program of study at Virginia Tech. His insight and motivation are invaluable. I am also thankful to Dr. Theodore S. Rappaport and Dr. Festus Gail Gray for their suggestions and comments. I would like to thank Dr. Ira Jacobs and Dr. Jeff H. Reed for their encouragement.

I would like to thank the Bradley Department of Electrical Engineering, Center for Wireless Telecommunications, the Advanced Research Projects Agency (ARPA), the Scientific Application International Company (SAIC) for providing me the financial assistance and sponsoring this research and the projects I have been doing during my program of study at Virginia Tech.

I am fortunate to be able to work at Mobile and Portable Radio Research Group (MPRG). I would like to thank Parag Agashe, Mike Buehrer, Rick Cameron, Francis Dominique, Rong He, Varun Kapoor, Ashish Kaul, Youngmia Kim, Joe Liberii, Nitin Mangalvedhe, Rias Muhamed, Sanjy Nagpal, Paul Petrus, Stavros Striglis, T. P. Subramanian, Matt Welborn, and many others for their constant help. I would also like to thank Prabhakar Koushik for maintaining the computer network at MPRG. I appreciate the help from the MPRG staff members.

Finally, I dedicate this work to my family and thank them for their constant love, support and encouragement throughout my academic studies.
# Table of Contents

1. Introduction ........................................................................................................... 1

2. Model for Spread Spectrum Systems ................................................................... 4
   2.1 Introduction ....................................................................................................... 4
   2.2 Properties of Spread Spectrum System ............................................................ 5
   2.3 Applications of Spread Spectrum ...................................................................... 5
   2.4 Basic Types of Spread Spectrum Systems ......................................................... 7
   2.5 Principles of DS/SS Systems .............................................................................. 9
       2.5.1. DS/SS Transmitter ................................................................................... 9
       2.5.2. DS/SS Receiver ......................................................................................... 13
       2.5.3. Multiple Access Communication Systems using CDMA ...................... 14
       2.5.4. PN Sequences ........................................................................................... 16
   2.6 Chapter Summary .............................................................................................. 19

3. Error Correction Coding. .................................................................................... 20
   3.1 Introduction ....................................................................................................... 20
   3.2 Block Coding ..................................................................................................... 21
       3.2.1 Encoding of Block Codes ........................................................................ 22
       3.2.2 Decoding of Block Codes ........................................................................ 22
       3.2.3 Performance of Block Codes .................................................................... 23
   3.3 Convolutional Coding ........................................................................................ 24
       3.3.1 Encoding of Convolutional Codes .............................................................. 24
       3.3.2 Decoding of Convolutional Codes and Viterbi Algorithm ...................... 27
       3.3.3 Performance Bounds for Convolutional Codes ......................................... 29
   3.4 Chapter Summary .............................................................................................. 35

4. Multiuser Detection Techniques for CDMA Systems ............................................ 36
   4.1 Introduction ....................................................................................................... 36
   4.2 Conventional Detection Techniques ................................................................. 36
       4.2.1 Correlation Receiver ................................................................................ 37
# Multiuser Detection for CDMA Systems with Convolutional Coding

4.2.2 Rake Receiver ........................................... 37  
4.3 Optimum Multiuser Receiver .................................. 39  
4.4 Sub-Optimum Multiuser Receiver  
  4.4.1 Decorrelation Receiver .................................. 43  
  4.4.2 Decision Feedback Receiver .................................. 46  
  4.4.3 Multistage Receiver .................................. 50  
4.5 Chapter Summary ........................................... 53  

5. Analysis of Multiuser Detection for CDMA Systems  
with Convolutional Coding ........................................... 54  
  5.1 Introduction ........................................... 54  
  5.2 Approaches of Multistage Interference Cancellation Combined  
    with Convolutional Coding ........................................... 55  
  5.3 A Model of Multistage Interference Cancellation Combined  
    with Convolutional Coding ........................................... 57  
  5.4 Analytical Evaluation of Performance ........................................... 62  
  5.5 Analytical Results of Performance ........................................... 69  
  5.6 Chapter Summary ........................................... 75  

6. Simulation of Multiuser Detection for CDMA Systems  
with Convolutional Coding ........................................... 89  
  6.1 Introduction ........................................... 89  
  6.2 Simulation Approach ........................................... 90  
    6.2.1 Block Diagrams of Simulation ........................................... 91  
    6.2.2 Implementation of Transmitter ........................................... 93  
    6.2.3 Implementation of the Channel ........................................... 94  
    6.2.4 Implementation of the Detection Receiver ........................................... 95  
  6.3 Simulation Results ........................................... 98  
  6.4 Chapter Summary ........................................... 102  

7. Conclusions ........................................... 119  
  7.1 Summary ........................................... 119  
  7.2 Future Work ........................................... 122  

Table of Contents
Multiuser Detection for CDMA Systems with Convolutional Coding

References .......................................................... 124

Appendix Software Documentation ............................. 128

Vita ....................................................................... 132
List of Figures

Figure 2.1: Frequency-hop Spread Spectrum Modem ........................................... 8
Figure 2.2: BPSK DS/SS Transmitter ................................................................. 9
Figure 2.3: Signals in BPSK DS/SS Transmitter .............................................. 11
Figure 2.4: Power Spectral Density of Signals during Spreading Process ............ 12
Figure 2.5: BPSK DS/SS Receiver ................................................................. 14
Figure 2.6: DS/CDMA Communication System Model ........................................ 15
Figure 2.7: Probability of Bit Error Rate vs. Eb/No, Analytical and simulation results for conventional correlation CDMA receiver, PN spreading gain N = 31, number of users K = 10 ........................................... 17
Figure 3.1: Encoding of Block Codes ............................................................. 22
Figure 3.2: Convolutional Encoder with Υ = 1/2 and γ = 3 .............................. 26
Figure 3.3: Trellis diagram representation for encoder of Figure 3.2 ..................... 26
Figure 3.4: State diagram for the convolutional encoder of Figure 3.2 ................. 30
Figure 3.5: Probability of Bit Error vs. Eb/No, Analytical results for BPSK modulation without error correction coding and with convolutional coding for coding rate r = 1/2, constraint length L = 3 .................................................. 33
Figure 4.1: Correlation Receiver Bank for CDMA Communication System .......... 37
Figure 4.2: Structure of a single RAKE receiver ............................................. 38
Figure 4.3: Optimum Multiuser Receiver for CDMA Communication System ........ 40
Figure 4.4: Decorrelation Multiuser Receiver .................................................. 45
Figure 4.5: Decision Feedback Multiuser Receiver ........................................... 49
Figure 4.6: Multistage Multiuser Receiver ...................................................... 51
Figure 5.1: Transmitter and Receiver of Multiuser Detection for Convolutionally Encoded CDMA System ................................................................. 56
Figure 5.2: The Second Scheme of Multiuser Detection Receiver for Convolutionally Encoded CDMA System ................................................................. 57
Figure 5.3: Model of Convolutional Encoded DS/CDMA Systems ...................... 59
Figure 5.4: Block diagram of the first two stages of multistage interference cancellation ................................................................. 61
Figure 5.5: Probability of Bit Error vs. Eb/No, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, assuming perfect power control, PN processing gain N = 62, number of users K = 10 ................. 76
Figure 5.6: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length $L = 3$, number of users $K = 10$. .......................... 77

Figure 5.7: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length $L = 5$, number of users $K = 10$. .......................... 78

Figure 5.8: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length $L = 7$, number of users $K = 10$. .......................... 79

Figure 5.9: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length $L = 3$, number of users $K = 10$. .......................... 80

Figure 5.10: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length $L = 5$, number of users $K = 10$. .......................... 81

Figure 5.11: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length $L = 7$, number of users $K = 10$. .......................... 82

Figure 5.12: Probability of Bit Error vs. Number of Users, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, assuming perfect power control, PN processing gain $N = 62$, Eb/No = 10 dB .......................... 83

Figure 5.13: Probability of Bit Error vs. Number of Users, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN processing gain $N = 31$, coding rate $\gamma = 1/2$, constraint length $L = 5$, Eb/No = 10 dB .......................... 84

Figure 5.14: Probability of Bit Error vs. Eb/No, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, half of the users including the desired user are 6 dB higher in power than the other half, PN processing gain $N = 62$, number of users $K = 10$ .......................... 85
Multiuser Detection for CDMA Systems with Convolutional Coding

Figure 5.15: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, half of the users including the desired user are 6 dB higher in power than the other half, PN processing gain \( N = 31 \), coding rate \( r = 1/2 \), constraint length \( L = 5 \), number of users \( K = 10 \)........................................................................ 86

Figure 5.16: Probability of Bit Error vs. Eb/No, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, half of the users including the desired user are 6 dB lower in power than the other half, PN processing gain \( N = 62 \), number of users \( K = 10 \)........................................................................ 87

Figure 5.17: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, half of the users including the desired user are 6 dB lower in power than the other half, PN processing gain \( N = 31 \), coding rate \( r = 1/2 \), constraint length \( L = 5 \), number of users \( K = 10 \)........................................................................ 88

Figure 6.1: Block Diagram of the First Scheme of Combined Multistage Interference Cancellation and Convolutional Coding........................................................................ 91

Figure 6.2: Block Diagram of The Second Scheme of Combined Multistage Interference Cancellation with Convolutional Coding................................................................. 92

Figure 6.3: BER vs. Eb/No, Simulation results of multistage interference cancellation for DS/CDMA system without using error correction coding in AWGN channel, assuming perfect power control, PN processing gain \( N = 62 \), number of users \( K = 10 \)........................................................................ 103

Figure 6.4: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain \( N = 31 \), coding rate \( \gamma = 1/2 \), constraint length \( L = 3 \), number of users \( K = 10 \)........................................................................ 104

Figure 6.5: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain \( N = 31 \), coding rate \( \gamma = 1/2 \), constraint length \( L = 5 \), number of users \( K = 10 \)........................................................................ 105

Figure 6.6: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain \( N = 31 \), coding rate \( \gamma = 1/2 \), constraint length \( L = 7 \), number of users \( K = 10 \)........................................................................ 106

Figure 6.7: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length
$L = 3$, number of users $K = 10$ ..................................................... 107

Figure 6.8: BER vs. Eb/No, Simulation results of the first scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length
$L = 5$, number of users $K = 10$ ..................................................... 108

Figure 6.9: BER vs. Eb/No, Simulation results of the first scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length
$L = 7$, number of users $K = 10$ ..................................................... 109

Figure 6.10: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length
$L = 3$, number of users $K = 10$ ..................................................... 110

Figure 6.11: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length
$L = 5$, number of users $K = 10$ ..................................................... 111

Figure 6.12: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/2$, constraint length
$L = 7$, number of users $K = 10$ ..................................................... 112

Figure 6.13: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length
$L = 3$, number of users $K = 10$ ..................................................... 113

Figure 6.14: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
PN spreading gain $N = 31$, coding rate $\gamma = 1/3$, constraint length
$L = 5$, number of users $K = 10$ ..................................................... 114

Figure 6.15: BER vs. Eb/No, Simulation results of the second scheme, combined
multistage interference cancellation and convolutional coding for
DS/CDMA system in AWGN channel, assuming perfect power control,
Multiuser Detection for CDMA Systems with Convolutional Coding

PN spreading gain $N = 31$, coding rate $r = 1/3$, constraint length $L = 7$, number of users $K = 10$ .............................................. 115

Figure 6.16: BER vs. The number of users, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $L = 5$, $\text{Eb/No} = 10 \text{ dB}$ .................................................. 116

Figure 6.17: BER vs. The number of users, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $L = 5$, $\text{Eb/No} = 10 \text{ dB}$ .................................................. 117

Figure 6.18: Probability of Bit Error vs. Eb/No, comparation of analytical results and simulation results for the first scheme, combined multistage interference cancellation and convolutional coding, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 3$, the number of users $K = 10$. .................................................. 118
Chapter 1

Introduction

There has been rapid development in wireless communications in the past several years. In order to satisfy the increasing demand in wireless communication with limited spectrum resources, several multiple access technologies have been developed, such as Code Division Multiple Access (CDMA), Time Division Multiple Access (TDMA), and Frequency Division Multiple Access (FDMA). CDMA may be extensively used in the next generation of wireless communication systems and it has been standardized by the Telecommunication Industry Association in the IS-95 standard for cellular communication. Much research effort is now focused on understanding and developing CDMA technology. This thesis is aimed at developing new receiver technology which can increase the capacity of a CDMA system in a cellular communication application.

CDMA based on spread spectrum technology differs from TDMA and FDMA in that it allows several users to transmit using the same frequency band at the same time by assigning each user a unique spreading code sequence. Every user in the system suffers from the interference caused by the signals from the other users. Additional users add more interference to the system, which causes a higher bit error rate for all users. As a result, the performance of a CDMA system is interference limited rather than noise limited. Any reduction in interference converts directly into an increase in capacity [Gil91]. Recently, there has been considerable interest in interference cancellation techniques to improve single user performance and system capacity.

At the base station receiver of cellular CDMA systems, the received signal is the summation of the asynchronously transmitted signals of every user plus noise. The conventional CDMA receiver demodulates the desired user by correlating the received signal with a synchronized replica of its own spreading code. This enhances the signal of the desired user while suppressing the signals from the other users considered as interference or noise. However, the correlation receiver is subject to a bit error floor due to interference and is particularly vulnerable to the near-far problem, caused by interfering users which are substantially stronger than the desired user.
Multiuser Detection for CDMA Systems with Convolutional Coding

For the receiver of the base station in cellular CDMA systems, a different approach is to demodulate the signals of all users at the same time and share information among receivers of all users to form a joint estimate of information for all users. This is termed multiuser reception. The optimum multiuser receiver in [Ver86a] has shown large capacity improvements over the conventional single user correlation receiver. Although this receiver is suitable for only the Gaussian channel and its structure is too complex for practical implementation, the large capacity gain stimulates the effort to search for better receivers than the conventional single user receiver.

Subsequent efforts have focused on the development of sub-optimal multiuser receivers which can achieve significant performance gain over conventional receivers but have low complexity which allows for practical implementation. Researchers have investigated a decorrelating receiver in [Lup89], [Lup90]. A multistage receiver has been proposed in [Var90], [Var91]. A study of a decision-feedback receiver is presented in [Due93], [Due95a]. Some other types of suboptimal multiuser receivers have been proposed in [Xie90], [Xie90a]. A detailed analysis of multistage interference cancellation receivers is presented in [Kau95]. For practical CDMA applications, there has been a considerable amount of research on designing multiuser receivers for noncoherent modulation and channels with fading, multipath. A multistage RAKE receiver is designed and simulated in [Str94]. These alternative approaches to multiuser receiver design are discussed in more detail in Chapter 4.

In this thesis, we investigate the performance of a CDMA system using combined multistage interference cancellation and convolutional coding. Little work has previously been done in this area. Since both are techniques for overcoming severe channel degradations, it is important to understand the interaction between these two techniques. Typical error correction coding is able to reduce signal to background noise requirements for wireless communication systems, but are subject to the noise floor of an interference limited channel and are also vulnerable to burst errors caused by severe fading. On the other hand, interference cancellation techniques are able to effectively reduce the interference floor in interference limited situations. However, some research has shown that interference cancellation may be ineffective for low signal to noise ratio [Pat94], [Kau95]. It is possible that the combination of interference cancellation and error correction coding techniques can overcome the weaknesses of both. Likewise it is also possible that the reported capacity increases for CDMA with both interference
cancellation and error correction coding may overlap when the two techniques are considered together.

The remainder of this thesis is organized as follows. The basic concept of CDMA spread spectrum communication is introduced in Chapter 2. Chapter 3 discusses the principles and performance evaluation of error correction coding techniques. Chapter 4 presents a survey of multiuser detection techniques currently being studied. Chapter 5 proposes a structure for combined multistage interference cancellation with convolutional coding, and analyzes the system by extending an analytic method first developed in [Kau95]. Chapter 6 investigates two different approaches to the combined multistage interference cancellation and convolutional coding by simulation methods. Finally Chapter 7 concludes this thesis.
Chapter 2

Model for Direct-Sequence Spread Spectrum Systems

2.1 Introduction

The Spread Spectrum technique, with its inherent resistance to interference, has become an increasingly popular technique for use in many different systems, including the significant IS-95 standard [TIA93]. Applications range from anti-jam systems to systems designed to combat multipath propagation. In this chapter, we introduce a basic model for a spread spectrum system which we will subsequently refine using new receiver structures in later chapters. Spread spectrum techniques may be viewed as a direct result of C. E. Shannon’s channel capacity expression

\[ C = W \cdot \log_2 \left( 1 + \frac{S}{N} \right), \]  \hspace{1cm} (2.1)

where \( C \) is channel capacity in bits per second, \( W \) is occupied bandwidth in hertz, \( N \) is noise power, \( S \) is signal power. This gives the relationship between the ability of a channel to transfer error-free information as a function of signal to noise ratio, and the bandwidth used to transmit the information. We can see that for any signal to noise ratio we can increase the information carrying capacity of the channel by increasing the bandwidth used to transmit the information.

A spread spectrum system distinguishes itself by two characteristics [Zie92]. First, the bandwidth of the transmitted signal is larger than the information bandwidth and is independent of the information bandwidth. Second, demodulation must be accomplished by correlation of the received signal with a replica of the spreading signal used in the transmitter. A number of modulation techniques use a transmission bandwidth much larger than the minimum bandwidth required for data transmission but are not spread
spectrum modulation, such as low rate error correction coding and wideband frequency modulation.

2.2 Properties of Spread Spectrum Systems

Spread spectrum modulation can improve system performance dramatically compared to narrowband modulation techniques. The amount of performance improvement that is achieved through the use of spread spectrum is defined as the processing gain of the spread spectrum. In other words, processing gain is the difference between system performance using spread spectrum techniques and system performance when not using spread spectrum techniques. Processing gain is approximately the ratio of the bandwidth of spread spectrum signal to the bandwidth of the information signal.

\[ G_p = \frac{BW_{ss}}{R_{info}} \]  

(2.2)

where \( G_p \) is the processing gain, \( BW_{ss} \) is first null baseband bandwidth of the spread spectrum signal after PN sequence spreading, and \( R_{info} \) is the first null baseband bandwidth of the information signal.

Jamming margin is the ability of the spread spectrum system to overcome the interference from a narrowband signal [Dix94]. It can be expressed by

\[ M_j = G_p - \left[ L_{sys} + \left( \frac{S}{N} \right)_{out} \right] \]  

(2.3)

where \( M_j \) is the jamming margin, in dB, \( G_p \) is the processing gain, in dB, \( L_{sys} \) is the system implementation loss, in dB and \( (S/N)_{out} \) is the required signal to noise ratio at output.

2.3 Applications of Spread Spectrum
Multiuser Detection for CDMA Systems with Convolutional Coding

A spread spectrum system has the following desirable features that make it very popular in both military and commercial applications. Properties (i) and (ii) are useful for military applications and properties (iii) and (iv) are useful for commercial applications.

i. Low probability of detection/interception by other users.
ii. Resistant to jamming.
iii. Resistant to multipath fading.
iv. Graceful performance degradation in multiple access interference.

As it is described in [Vit95], spread spectrum systems can be applied in the following situations

1. A system that requires high tolerance to intentional interference or unintentional interference. Spread spectrum can suppress the interference by an amount proportional to its processing gain.

2. A system that locates the position of mobile user and estimates the velocity of mobile user. The accuracy of position location and velocity estimation is proportional to the processing gain.

3. In a secure communication system which needs low detectability of the transmitted signal by an intended receiver. The probability of being detected decreases as the processing gain increases.

4. A multiple access communication system which serves a large number of uncoordinated users sharing the same band of frequency, in the same and neighboring geographical areas. The number of users the system can accommodate simultaneously increases as the processing gain increases.

This thesis will concentrate on the multiple access communication application of spread spectrum techniques. It may not be obvious at first that multiple access by sharing a common spread spectrum channel which is very wide can be superior in spectrum efficiency to the traditional multiple access techniques, which separates users by providing every one with unique disjoint frequencies (FDMA) or time slots (TDMA). In fact, in a single cell system, with constant data rate, the capacity of CDMA is significantly lower than the capacity of an orthogonal multiple access technique. However, CDMA makes possible both universal frequency reuse and exploitation of voice activity. As a result, significant capacity improvements can result for CDMA cellular systems [Gil91].
2.4 Basic Types of Spread Spectrum Systems

There are several major types of spread spectrum systems:

1. **Direct Sequence Spread Spectrum System (DS/SS):** The transmitted information signal is first spread by pseudorandom or pseudo-noise (PN) code sequence which is implemented by modulo 2 addition of the information signal and the PN sequence. The RF carrier signal is modulated by the spread information signal to generate the direct sequence spread spectrum signal. The code rate of the PN sequence is usually high, thus the resulting bandwidth and spectral characteristics of the DS/SS signal is dominated by the PN code sequence [Dix94]. The bandwidth of a DS/SS signal is expanded to be much larger than the bandwidth of the information signal. For instance, if the information data rate is 1 kbits/sec and Binary Phase Shift Keying (BPSK) modulation with rectangular pulse shaping is used, the first null baseband bandwidth of the transmitting signal is 1 kHz. If we use a PN code sequence of 128 chips/bit to modulate the information data, the first null baseband bandwidth of the transmitted signal is 128 kHz.

2. **Frequency Hopping Spread Spectrum System (FH/SS):** The RF carrier frequency is modulated by the transmitted information signal to form a narrowband signal. The frequency hopping spread spectrum signal is obtained by changing the carrier frequency of the narrowband signal according to a PN sequence [Dix94]. The frequency hopping spread spectrum system must have a large number of frequencies usable on demand and the sequence of carrier frequencies is pseudorandom. The bandwidth dedicated to a FH/SS signal is much larger than the narrowband signal even though it may not use all of them at the same time. When the hopping rate is faster than the data rate it is called fast frequency hopping and when the hopping rate is slower than the data rate it is called slow frequency hopping. The block diagram of a frequency hopping modulator and demodulator are shown in Figure 2.1 [Zie92]. For example, if the information data rate is 1 kbits/sec and the number of frequencies with 1 kHz bandwidth which can be hopped is 128, then the bandwidth allocated to the FH/SS signal is 128 kHz.
3. **Time Hopping Spread Spectrum System (TH/SS):** In this system, the transmitter is switched on and off by the PN sequence, therefore the transmitting and no-transmitting times are pseudorandom [Dix94]. The spectrum of the transmitted signal is spread in the frequency domain. Two companies, Aetherwire and Pulson Communications are currently developing commercial time-hopped systems.
4. **Pulsed FM (Chirp) System**: A chirp signal is generated by varying the frequency of pulsed RF signal in a known way during each pulse period [Dix94]. It does use a wider bandwidth than the minimum required bandwidth for transmission but it does not necessarily employ a PN sequence code.

In this thesis, direct sequence spread spectrum is used exclusively.

### 2.5 Principles of DS/SS Systems

#### 2.5.1 DS/SS Transmitter

The most commonly used spread spectrum system is a direct sequence spread spectrum system employing binary phase-shift keying (BPSK) modulation. Ideal BPSK modulation results in instantaneous phase changes of the carrier by 180 degrees and can be mathematically represented as a multiplication of the carrier by a function \( a(t) \) which takes on the values \( \pm 1 \). The transmitter of a BPSK DS/SS for a single user is shown in Figure 2.2.

![Figure 2.2 BPSK DS/SS Transmitter](image)

As shown in the above figure, the transmitted signal \( s(t) \) is given by the expression

\[
s(t) = \sqrt{2} P b(t) a(t) \cos(\omega_c t + \theta),
\]

where \( b(t) \) is a binary data signal which can be expressed as

\[
b(t) = \sum_{i = -\infty}^{\infty} b_i P_T(t - iT),
\]
where $b_i \in \{\pm 1\}$ represents the $i$th data bit and is an independent identically distributed (i.i.d.) random variable, and $p_T(t)$ is a unit rectangular pulse with duration $T$, i.e. $p_T(t) = 1$ for $0 \leq t < T$ and $p_T(t) = 0$ elsewhere. The PN sequence $a(t)$ is given by

$$a(t) = \sum_{j=-\infty}^{\infty} a_j p_{T_c}(t - jT_c).$$  \hspace{1cm} (2.6)

where $a_j \in \{\pm 1\}$ is the $j$th chip in the PN sequence, and $p_{T_c}(t)$ is unit rectangular pulse with duration $T_c$. The signal $a(t)$ is sometimes also called the spreading signal or signature signal.

In Figure 2.2, $\omega_c$ is the common center carrier frequency, $\theta$ is the phase of the carrier and $P$ is the power of the signal. Examples of the resulting signals generated at each stage of the transmitter are shown in Figure 2.3. For this example, $N = 6$ and the spreading code repeats every data bit.

The power spectral density functions of the data sequence $b(t)$, the PN sequence $a(t)$, and the transmitted signal $s(t)$ are shown in Figure 2.4 as $B(f)$, $A(f)$, and $S(f)$ respectively. The bandwidth of the chip sequence after PN spreading is much wider than the bandwidth of the data sequence.
Figure 2.3  Signals in BPSK DS/SS Transmitter
Figure 2.4 Power Spectral Density of Signals during Spreading Process
Frequently the PN sequence \( a(t) \) has the period \( N = T/T_c \) which means it repeats for every data bit. Such a system is termed "code-on-pulse". The reverse channel of the IS-95 standard uses a very long period PN sequence which is not code on pulse. The performance evaluation developed in this research is general enough that it is not dependent on the period of the PN sequence.

### 2.5.2 DS/SS Receiver

The transmitted signal is delayed and corrupted when it reaches the receiver. The received signal at the receiver is the summation of the transmitted signal with propagation delay \( \tau \) and noise. It is represented by

\[
r(t) = \sqrt{2P}b(t-\tau)a(t-\tau)\cos(\omega_c t + \phi) + n(t),
\]

where \( \phi = [\theta - \omega_c \tau] \mod 2\pi \) is uniformly distributed on \([0, 2\pi)\), and \( n(t) \) is modeled as an additive white Gaussian noise process (AWGN) with power spectral density \( P_n(f) = N_0/2 \).

The receiver performs a complementary function to that of the transmitter. The received signal \( r(t) \) is demodulated to a baseband signal. A replica of the transmitted PN sequence which is synchronized to the received signal is used to despread the received signal in baseband. The despread signal is passed through a correlator to generate the decision statistic \( Z_i \) for the \( i \)th data bit. The decision statistic is given by

\[
Z_i = \int_{iT}^{(i+1)T} r(t)a(t-\tau)\cos(\omega_c t + \phi)dt,
\]

Then the data bit is recovered according to the rule, if \( Z_i < 0 \), \( \hat{b}_i = -1 \) and if \( Z_i > 0 \), \( \hat{b}_i = 1 \), where \( \hat{b}_i \) is the estimate of the \( i \)th transmitted data bit. This process is shown in Figure 2.5.
2.5.3 Multiple Access Communication Systems using CDMA

By assigning different PN sequences to different users in the system, CDMA provides multiple access communication which allows several users to share the same frequency band at the same time. In the cellular system, every mobile station needs to communicate with the base station; therefore the received signal at the base station receiver is the summation of the signal from every user plus the Gaussian noise. Because of the lack of coordination among the mobile stations, the signals from different mobile stations arrive asynchronously and independently. The communication channel causes delays, attenuations and phase shifts to the signals which are different for different mobile stations. The model of a multiple access communication system using DS/CDMA is shown in Figure 2.6.

The transmitter and receiver structures for every user are the same as those which are shown in Figure 2.2 and Figure 2.5. The signals from the other users transmitting in the same frequency band at the same time with the desired user are called multiple access interference. The received signal can be represented as

\[ r(t) = \sum_{k=1}^{K} s_k(i - \tau_k) + n(t) \]  

(2.9)

where \( n(t) \) is additive white Gaussian noise with two sided power spectral density \( N_0/2 \).
Multiuser Detection for CDMA Systems with Convolutional Coding

\[ s_k(t - \tau_k) = \sqrt{2P_k} a_k(t - \tau_k)b_k(t - \tau_k)\cos(\omega_c t + \phi_k), \quad P_k \text{ is the received power for the} \]
\[ \text{kth user's signal, } a_k(t) \text{ and } b_k(t) \text{ are the PN sequence and data signal of the kth user} \]
\[ \text{respectively, and } \tau_k \text{ and } \phi_k = [\theta_k - \omega_c \tau_k] \mod 2\pi \text{ are the random delay and phase shift} \]
\[ \text{of the kth user uniformly distributed over } [0, T) \text{ and } [0, 2\pi) \text{ respectively. Without loss of} \]
\[ \text{generality the users can be numbered so that } \tau_1 < \tau_2 < \ldots < \tau_K. \quad T \text{ is the signaling} \]
\[ \text{interval of every user, and } K \text{ is the number of users sharing the same frequency band} \]
\[ \text{simultaneously.} \]

![Diagram](image)

Figure 2.6 DS/CDMA Communication System Model

The receiver of the kth user correlates the received signal given in eq. (2.9) with a

synchronous replica of the kth user's PN sequence. The decision statistics of the ith bit of

the kth user can be expressed by

\[ Z_{k,i} = \int_{iT + \tau_k}^{(i + 1)T + \tau_k} r(t) a_k(t - \tau_k)\cos(\omega_c t + \phi_k) dt, \]
\[ = \xi + A_k + \sum_{\kappa = 1}^{K} I_{\kappa}, \quad (2.10) \]

where \( A_k \) is the contribution to decision statistics from the desired user, \( \xi \) is the contribution from AWGN and \( \sum_{\kappa = 1}^{K} I_{\kappa} \) is the multiple access interference which may be in the same cell or in different cell.

The average bit error rate can be evaluated if the decision statistic in eq. (2.10) is modeled as a Gaussian random variable [Pur77]. A rigorous derivation of the Gaussian approximation may be found in Appendix C of [Rap96]. The detailed analysis of eq. (2.10) will be presented in Chapter 5 where the conventional correlation receiver bank is used in the first stage of multistage interference cancellation. The probability of bit error versus \( E_b/N_0 \) for an example case, where the PN processing gain \( N = 31 \) and the number of users \( K = 10 \), is shown in Figure 2.7. Both the simulation results and analytical results are plotted. A Gaussian approximation is employed in modeling decision statistics given in eq. (2.10). We can see from these curves that the probability of bit error is subject to an error floor at high signal to noise ratio due to interference.

It is obvious from eq. (2.10) that the decision statistics of the \( k \)th user will be affected by the signals of the other users. Especially when the signals of each user are at different power levels and the desired user is weaker in power than the other users, the decision statistics of the desired user will be influenced by the other users and the performance of the receiver will be degraded. The power of one user's signal may differ from the powers of the other user's signals at the base station receiver because the distances between the base station and mobile users are different or the channel path losses of each user are different, which is defined as the near-far effect.

2.5.4 PN Sequences

The PN sequence in a CDMA system spreads the bandwidth of data bits and provides a way of distinguishing one user from the other users transmitting in the same frequency
Figure 2.7 Probability of Bit Error Rate vs. Eb/No, Analytical and simulation results for conventional correlation CDMA receiver, PN spreading gain N = 31, number of users K = 10.
band simultaneously. The first function makes the transmitted signal noiselike and random while the second function is a new mechanism for multiple access communication. The PN sequence must be generated at the transmitter as well as at the receiver and must be synchronized to coincide perfectly with the timing of the received signal.

It is very natural that a PN sequence should have good autocorrelation properties in order that a CDMA system can perform well in a multipath fading channel where different multipath components arrive at the receiver with different time delays and perform well in rapid synchronization. This property is defined as

$$\frac{1}{N} \cdot \sum_{n=0}^{N-1} a_n a_{n+m} = 0; \quad \text{for } m \neq 0$$

(2.11)

where \( N \) is the length of the PN sequence and \( a_n \) is the \( n \)th chip of the PN sequence.

When the received signal contains transmitted signals from a number of users, the PN sequences must have good cross-correlation properties in order to pick up the signal of the desired user and suppress the signals from the other users. The cross-correlation between sequences is defined by

$$\frac{1}{N} \cdot \sum_{n=0}^{N-1} a_{k,n} a_{\kappa,n+m} = 0; \quad \text{for } \kappa \neq k$$

(2.12)

where \( a_{k,n} \) and \( a_{\kappa,n+m} \) are the \( n \)th chip of the \( k \)th user and \((n+m)\) th chip of the \( \kappa \)th user respectively. Ideal autocorrelation and cross-correlation properties can not be achieved in a practical asynchronous system, leading to performance degradation. Error correction coding and improved receiver design are two techniques for improving system performance in this case.
2.6 Chapter Summary

In this chapter, we have introduced the basic model for a spread spectrum system. We have seen that the performance of CDMA systems is interference limited. One traditional technique for mitigating interference in a CDMA system is the use of error correction codes. We discuss error correction coding in Chapter 3. In later chapters we discuss interference cancellation as a technique for mitigating interference.
Chapter 3

Error Correction Coding

3.1 Introduction

Until the late 1940's, it was believed that noisy channels, such as the multiple access interference limited CDMA system described in Chapter 2, resulted in an inherent bit error rate (BER) floor. If this were true, then the only way to reduce BER to an acceptable level would be to increase the processing gain $N$, or to reduce the number of users $K$, limiting the spectral efficiency of the system. Fortunately, the BER of a noisy system can be made arbitrarily small provided that the channel capacity is not exceeded, through the use of error correction coding. The channel capacity $C$ is the maximum number of information bits per second which can theoretically be transmitted with arbitrarily low error rate over the channel. If rate of data transmission is lower than the channel capacity, an arbitrarily low error rate can be achieved; if rate of data transmission is higher than the channel capacity, an arbitrarily low error rate can not be achieved. Channel capacity is a function of channel characteristics, such as signal to noise ratio and bandwidth of the channel. For an additive white Gaussian noise (AWGN) channel, channel capacity is given by [Sha48a],[Sha48b]

$$C = B \cdot \log_2 \left(1 + \frac{P}{N_o B} \right), \quad (3.1)$$

where $C$ is the channel capacity, in bits per second; $B$ is the transmission bandwidth, in Hertz; $P$ is the received signal power, in Watts; $N_o$ is the single sided noise power spectral density, in Watts/Hertz.

From Eq. (3.1), we know that the channel capacity $C$ is proportional to the transmission bandwidth $B$ for a fixed signal to noise ratio $P/(N_o B)$, and the channel capacity $C$
monotonically increases with increasing signal to noise ratio \( P/(N_oB) \) for a fixed transmission bandwidth \( B \). On the other hand, the signal to noise ratio requirement \( P/(N_oB) \) for a fixed channel capacity \( C \) can be reduced by expanding the transmission bandwidth \( B \). The effect is measured by the system coding gain. The coding gain of a system is the difference between the \( E_b/N_o \) required to achieve a particular bit error probability without coding and the \( E_b/N_o \) required to achieve the same error probability with coding. In practice, coding gains of 2 - 9 dB can be achieved for the AWGN channel and larger coding gains can be achieved for fading channels.

Eq. (3.1) gives a theoretical limit which can be reached but does not show how to achieve the channel capacity. There has been a tremendous amount of research in the error correction coding area to discover good coding schemes which approach the theoretical limit, with reasonable decoding complexity. One way of looking at error correction coding is as sending many modulation symbols together in a block to approximate a higher dimensional signal constellation while maintaining reasonable complexity. There are two major classes of error correction coding techniques used in practice today, block coding and convolutional coding. Recently, there are several new coding techniques, like trellis coding which combines convolutional coding and modulation to conserve bandwidth [Ung87a], [Ung87b]; and turbo coding which concatenates two recursive systematic convolutional codes to achieve performance near the Shannon limit in terms of bit error rate [Ber93]. We will discuss block coding and convolutional coding techniques in the following sections.

### 3.2 Block Coding

Block coding techniques are those coding techniques which process information in blocks [Zie92]. The basic idea of a block code is to insert extra symbols into the data stream so that symbols in error may be corrected.
3.2.1 Encoding of Block Codes

A block encoder groups input symbols into blocks of \( k \) symbols and maps them into blocks of \( n \) output symbols. The coding rate of a block code is defined as \( r = k/n \) and \( r \) is less than 1. After block encoding, a total of \( n - k \) redundant symbols are added to the information symbols to form \( n \) symbols. The encoder is shown in Figure 3.1.

![Figure 3.1 Encoding of Block Codes](image)

This encoding block can be realized by matrix multiplication which is given by

\[
\mathbf{c} = \mathbf{b} \cdot \mathbf{G},
\]

where \( \mathbf{b} = (b_1, \ldots, b_k) \) is \( k \) data bits, \( \mathbf{G} \) is a generator matrix, and finite field arithmetic is employed.

3.2.2 Decoding of Block Codes

A bounded distance decoding algorithm such as the Berlekamp-Massey algorithm is commonly used for the decoding of block codes. This algorithm correctly decodes all received sequences within certain distance \( t \) of a code word [Lin83].

Let \( C = \{c_1, c_2, \ldots, c_{2^k}\} \) be the set of \((n,k)\) linear code vectors. No matter which vector in \( C \) is transmitted, there are \( 2^n \) possible received code vectors. The function of the decoder is to classify these \( 2^n \) possible received vectors into \( 2^k \) disjoint subsets \( S_i \), \( 1 \leq i \leq 2^k \), so that the received vector will fall into one of the subsets. There is a one to one correspondence between the subsets and code vectors.
3.2.3. Performance of Block Codes

In general, for constant coding rate \( r \), performance increases when the number of output bits \( n \) increases. The performance also increases when the coding rate \( \gamma \) decreases. The performance of block coding is better than the uncoded system at high signal to noise ratios. At low signal to noise ratios, block coding can actually hurt performance.

The number of bits differing between a pair of codewords is called the Hamming distance between these two codes. The minimum Hamming distance between any two codewords in the code is called the minimum distance of the code and is denoted by \( d_{H, \text{min}} \). The error correction capability of block codes is due to the fact that not all possible \( 2^n \) symbols are used by the code and it is bounded by the minimum distance of the code. A code with minimum distance \( d_{H, \text{min}} \) is guaranteed correct to \( t \) errors, where

\[
\begin{align*}
    t &= \left\lfloor \frac{d_{H, \text{min}} - 1}{2} \right\rfloor, \\
    (3.3)
\end{align*}
\]

Sometimes the code can correct more than \( t \) errors. The code can detect up to \( d_{H, \text{min}} - 1 \) errors, if no error correction is attempted.

When hard decision decoding is used, from [Zie92] the performance of bounded distance decoding is given by

\[
\begin{align*}
P_c(\varepsilon) &= \sum_{i = t + 1}^{n} \binom{n}{i} p^i (1 - p)^{n - i}, \\
    (3.4)
\end{align*}
\]

where \( P_c(\varepsilon) \) is probability of codeword error, \( n \) is the code length, \( p \) is the uncoded probability of symbol error and is determined by modulation, and \( t \) is the error correction capability of the code.

When soft decision decoding is used, the union bound is applied to evaluate the performance and the probability of codeword error is approximately bounded by

\[
\begin{align*}
P_c(\varepsilon) &\leq a_{\text{min}} \cdot Q \left( \sqrt{\frac{2E_b r d_{H, \text{min}}}{N_o}} \right), \\
    (3.5)
\end{align*}
\]
where \( a_{\min} \) is the number of codewords at distance \( d_{H,\min} \), \( r \) is coding rate, \( E_b/N_0 \) is the signal to noise ratio [Zie92].

### 3.3 Convolutional Coding

The other major type of error correction coding is convolutional coding. The structure of convolutional coding is different from the structure of block coding although many of the basic concepts of block coding can be applied to convolutional coding. In convolutional coding, a continuous sequence of input information symbols is mapped into a continuous sequence of output symbols from the encoder. This mapping is highly structured, so the convolutional coding can achieve a large coding gain. We will focus on convolutional codes because they are more widely used in wireless communication standards including the IS-95 standard. There are a number of reasons for this:

i. Viterbi decoders for decoding of convolutional decoders are widely available in commercial hardware.

ii. The Viterbi algorithm provides a straightforward implementation of soft-decision decoding while soft-decision decoding of the major classes of block codes is difficult.

iii. Convolutional codes may be decoded in real time with low delay which is important for transmission of speech and video signals.

iv. Some authors argue that convolutional codes achieve larger coding gain for a given complexity [Zie92], although this general statement is controversial.

#### 3.3.1 Encoding of Convolutional Codes

In general, convolutional codes generate \( n \) output symbols for every \( k \) input symbols; the coding rate is \( r = k/n \). In this thesis we only consider the case \( k = 1 \). Convolutional codes have memory so that the output bits are determined by the previous input bits as well as the current input bit. The constraint length \( \gamma \) is defined to be one plus the number of past inputs affecting the current outputs [Zie92]. The state of the encoder is the contents of the shift register and is completely determined by the previous input bits. The number
of states is $2^{\gamma-1}$. The output symbol sequence can be obtained by the convolution of the input symbol sequence and the generator matrix which can be obtained from generator polynomials. If we assume that $\bar{b} = [b_i]$ is the sequence of input bits and $\bar{c}_i = [c_{i1}, c_{i2}, \ldots, c_{in}]$ is the output sequence of bits corresponding to the $i$th input bit. The following relationship holds

$$c_{ij} = \sum_{m=1}^{\gamma} b_{i+1-m} \cdot g_{mj}$$

(3.6)

where $g_{mj}$ is the element of the generator matrix $G$ at the $m$th row and the $j$th column. Modulo 2 operations are assumed for binary inputs and outputs.

In communication systems, data is transmitted in packet format, so the data is not continuous but it is truncated into blocks. $\gamma - 1$ zeros are added to the end of each packet to clear the register, in a process called zero padding. The length of a packet is denoted by $B$. This will generate the last $\gamma - 1$ output bits which is called the tail of the code. Every time a packet of data is encoded, the shift register is initialized to the all zeros state and the state of the encoder will go back to the all zeros state at the end of encoding of every packet because of the $\gamma - 1$ zeros added to every packet. In the IS-95 standard, the full rate transmission consists of 184 bits per packet which includes 172 data bits and 12 CRC bits used for error detection on the decoder data. Since a rate $r = 1/3$, constraint length $\gamma = 9$ convolutional code is used for the reverse link from mobile to base station in this standard, 8 zero bits are padded to the end of each packet. The final length of the packet is 192 bits, which is input to the encoder.

The convolutional coding is easily described by example. Figure 3.2 shows a shift register logic block diagram which generates a rate $1/2$ and constraint length 3 convolutional code.

Input bit $b_i$ is clocked into the circuit from the left. The generator polynomials $g_1(D)$ and $g_2(D)$ can be written in matrix form, i.e. $g_1 = [1 \ 1 \ 1]^T$ and $g_2 = [1 \ 0 \ 1]^T$. So the generator matrix can be expressed as $G = [g_1, g_2]$. 

Chapter 3 Error Correction Coding
The convolutional encoder can be regarded as a linear finite state machine consisting of a $k$ stage shift register and $n$ linear algebraic function generators [Vit71]; thus it can be described completely by a trellis diagram or a state diagram. The convolutional encoder can also be represented by either a table form or a generator matrix. Every means of representation is advantageous for representing certain features of convolutional codes.

The trellis diagram of the convolutional encoder in Figure 3.2 is shown in Figure 3.3.
code branches produced by a 0 input bit are shown as solid lines and code branches produced by a 1 input bit are shown as dashed lines. Because we do zero padding, the trellis terminates at node $a$. The last two branches are the tail of the code.

3.3.2. Decoding of Convolutional Codes and the Viterbi Algorithm

The Viterbi algorithm provides an efficient means to implement maximum likelihood decoding (MLD) of a convolutional code [For73]. The decoder has the knowledge of the code structure, the received sequence, and the statistical characteristics of the channel. Let $C$ be the set of allowable sequences through the trellis diagram. Each code sequence $c \in C$ can be represented by a unique path through the trellis diagram. Suppose that an information sequence $\bar{b} = [b_i]$ is encoded into a code sequence $c$ and sequence $\bar{y}$ is received. The maximum likelihood decoding rule is applied to decode the convolutional code. That is, choose $\hat{c} \in C$, such that

$$\hat{c} = \arg \max \{P_r(y|c)\}, \quad c \in C \tag{3.7}$$

The function of an MLD is to find the code sequence which was most likely to have been transmitted given the received channel output sequence. If the channel is a discrete memoryless channel (DMC), an MLD chooses $\hat{c}$ as the code sequence which maximizes the log-likelihood function $\log P(y|c)$ [Lin83]. Since for DMC

$$P(y|c) = \prod_{i=0}^{B-1} P(y_i|c_i), \tag{3.8}$$

where $B$ is the length of the code word. Take logarithm on this equation, we have

$$\log P(y|c) = \sum_{i=0}^{B-1} \log P(y_i|c_i), \tag{3.9}$$

where $P$ is the channel transition probability. The log-likelihood function $\log P(y|c)$ is
called the metric associated with the path \( c \). The terms \( \log P(y_i|c_i) \) in eq. (3.9) are called branch metrics.

Setting the branch metric equal to the Hamming distance between symbols results in "hard-decision" decoding. Setting the branch metrics equal to squared Euclidean distance between symbols results in "soft-decision" decoding. We consider primarily the soft-decision case here, because it is more general and yields superior performance. Eq.(3.9) indicates that the metric of the whole path is the summation of the branch metrics along that path. This means that the metric of the whole path can be obtained step by step in the trellis by calculating the branch metric at each step. The path whose accumulated metric at a certain step is unlikely to survive can be discarded, which is the basic idea of the Viterbi algorithm.

The Viterbi algorithm is an efficient way of performing maximum likelihood decoding of convolutional codes. In practice, the Viterbi algorithm can be implemented through the following steps:

i. Initialize metrics \( M_j(0) \) for each state \( j = 0, \ldots, 2^v - 1 \) in the trellis at time \( i = 0 \). Since we start from zero state, let \( M_0(0) = 0 \) and \( M_j(0) = -\infty \) for all \( j \neq 0 \) states.

ii. Compute metrics for every path converging to state \( j \) at time \( i \) and choose the survivor path whose metric is the largest. Repeat for every state \( j \) at time \( i \).

iii. Update the accumulated metric information for every state at time \( j \).

iv. If \( i < B/r \), go back to step ii.

v. Choose the surviving path whose accumulated metric is the largest.

This rule is equivalent to finding the path through the trellis diagram whose code sequence is closest to the received sequence. Note that if the Hamming distance between the received signal and the all zero path in the trellis is used as a metric instead of the log-likelihood function, the chosen path is the one with smallest value.

The Viterbi algorithm described above assumes zero padding is implemented. If packets do not use zero padding to truncate the convolutional code into finite blocks, the number of information bits which must be stored becomes very large requiring significant memory for the path metric. This also causes a long delay before the whole packet of data can be
decoded. Therefore, it is desirable to make the decoding bit decisions prior to the zeroing of the encoder. Typically, the decoder stores all of the surviving paths for five times of the constraint length of the encoder and a decoding decision is made when the memory is filled. The MLD is violated and this process is an approximation to MLD.

The complexity of the Viterbi decoder is directly proportional to the number of states in the trellis which grows exponentially with the constraint length [Zie92]. In this project, soft decision decoding is used since it has better performance than hard decision decoding.

3.3.3. Performance Bounds for Convolutional Codes

The probability of error for linear codes can be bounded in terms of the weights of all codes which correspond to the set of distances from any one code to all the others, or equivalently to the set of distances from the all zeros path to all paths which have diverged from it. Since the convolutional code is a linear code, the performance of convolutional codes can be bounded by this method. This set of distances can be calculated with the aid of a trellis diagram or a state diagram.

In Figure 3.3, consider all paths which merge with the all zero path for the first time at arbitrary node $j$. There is one path at distance 5 from the all zero path and this path diverges from the all zero path three branches back. There are two at distance 6 from the all zero path, one which diverge four branches back and the other which diverge five branches back. The minimum Hamming distance between two code sequences associated with trellis paths which diverge from one another at some point and remerge at some later point is called free distance. The free distance is 5 for this encoding structure.

By examining the state diagram shown in Figure 3.4, we can obtain a closed form expression called the generating function whose expansion gives all distance information directly [Vit79]. Since the paths which diverge from and then converge to the all zero path are of interest, state $a$ is split and acts as both the start and the end point. The exponent of $D$ represents the Hamming distance of a branch from the all zero path. The exponent of $L$ represents the length of the path. The exponent of $I$ represents the number of "1"s in the input information sequence.
The generating function can also be regarded as the transfer function of the signal flow graph with unity input and can be computed by solving the state equations obtained from the state diagram

\[ \xi_b = D^2LI + LI\xi_c \]
\[ \xi_c = DL\xi_b + DL\xi_d \]
\[ \xi_d = DLI\xi_b + DLI\xi_d \]

\[ T(D, L, I) = D^2L\xi_c \quad (3.10) \]

where \( \xi_b, \xi_c \) and \( \xi_d \) are dummy variables for the partial paths to the intermediate nodes \( b, c \) and \( d \) respectively. The input to the node \( a \) is unity and the output is the desired generating function \( T(D, L, I) \). Solving equation (3.10), we have

\[ T(D, L, I) = \frac{D^5L^3I}{1 - DL(1 + L)I} \]
\[ = \sum_{k=0}^{\infty} D^{5+k} L^{3+k} (1 + L)^{k} l^{1+k}, \quad (3.11) \]

The structure and complexity of the convolutional code depend on the constraint length and the coding rate. Let us consider the probability of error event, \( P_e \), which is defined as the probability that the decoder chooses an incorrect path at node \( j \). Without loss of generality, we assume that the all zero path is the correct path transmitted. All potential decoding error events at node \( j \) are enumerated, and their error probabilities are calculated and summed. The summation is the union bound by which the probability of events error is bounded by the following union bound [Vit79]

\[ P_e(j) \leq \sum_{d=d_f}^{\infty} a(d) P_d, \quad (3.12) \]

where \( d_f \) is the free distance, \( a(d) \) is the number of paths at Hamming distance \( d \) from the correct path, \( P_d \) is the probability of that the decoder will choose an incorrect path at Hamming distance \( d \) from the correct path which is also called the probability of pairwise error event [Vit79].

The calculation of \( P_d \) depends on the signal to noise ratio and whether soft-decision or hard-decision decoding is used [Zie92]. For soft-decision decoding, it is given by

\[ P_d = Q \left( \frac{2E_b R d_H}{N_o} \right), \quad (3.13) \]

where \( d_H \) is the Hamming distance from the correct path. For hard decision decoding, it is given by

\[ P_d = \sum_{k=d+1 \over 2}^{d} \binom{d}{k} p^k (1 - p)^{d-k}, \quad (3.14) \]

when \( d \) is odd; and it is given by
\[ P_d = \sum_{k=\frac{d}{2}+1}^{d} \binom{d}{k} p^k (1-p)^{d-k} + \frac{1}{2} \binom{d}{d/2} p^{d/2} (1-p)^{d/2}, \quad (3.15) \]

when \( d \) is even. \( p \) is the transition probability or error probability of the binary symmetric channel.

The value \( a(d) \) can be found from eq. (3.11) by letting \( L = 1 \) and \( I = 1 \). Specifically,

\[ T(D, 1,1) = \sum_{d = d_f}^{\infty} a(d) D^d = \sum_{d = d_f}^{\infty} 2^{d-5} D^d \quad (3.16) \]

We have \( a(d) = 2^{d-5} \) which means that at \( d = 5 \), \( a(d) = 1 \); at \( d = 6 \), \( a(d) = 2 \). This also verifies the results we obtained from the trellis diagram.

The probability of bit error is the expected number of bit errors in a given sequence of received bits normalized by the total number of bits in the sequence and is denoted by \( P_b \). This expected number of errors caused by any incorrect path which diverges from the correct path at node \( j \) can be bounded by weighting each term of the union bound by the number of bit errors which occur on that incorrect path. Since the all zeros path is the correct path, this corresponds to the number of “1”s in the data sequence over the unmerged segment. Therefore the bound on the expected number of bit errors caused by an incorrect path diverging at node \( j \) is [Vit79]

\[ P_b(j) = E[n_b(j)] \leq \sum_{i=1}^{\infty} \sum_{d = d_f}^{\infty} ia(d,i) P_d, \quad (3.17) \]

where \( a(d,i) \) is the number of paths diverging from the all zero path at node \( j \) at distance \( d \) and with \( i \) “1”s in its data sequence over the unmerged segment. The coefficients \( a(d,i) \) are also the coefficients of the augmented generating function \( T(D, 1,I) \) which can be obtained from eq. (3.11) by letting \( L = 1 \), because path length does not influence the performance.
Figure 3.5, Probability of Bit Error vs. Eb/No, Analytical results for BPSK modulation without error correction coding and with convolutional coding for coding rate $r = 1/2$, constraint length $\gamma = 3$. 
\[ T(D, 1, I) = \frac{D^5 I}{1 - 2DI} = \sum_{d=5}^{\infty} 2^{d-5} D^d I^{d-4} \]  

(3.18)

Hence

\[ a(d, i) = \begin{cases} 2^{d-5} & \text{for } i = d - 4, d \geq 5 \\ 0 & \text{otherwise} \end{cases} \]  

(3.19)

Generally speaking, the structure of the state diagram of a convolutional code is very complex and it is difficult to find the generating function \( T(D, L, I) \). Under these circumstances, we need to trace the trellis diagram and count the number of paths with distance \( d \) from the all zero path and with \( i \) "1"s in its input data sequence in order to get \( a(d, i) \). In this project, a computer program is implemented to compute \( a(d, i) \) for different coding structures. The source code for this program is listed in Appendix.

In general, the augmented generating function can be expanded in the form

\[ T(D, 1, I) = \sum_{i=1}^{\infty} \sum_{d=d_f}^{\infty} a(d, i) D^d I^i \]  

(3.20)

whose derivative at \( I = 1 \) is

\[ \frac{\partial T(D, 1, I)}{\partial I} \bigg|_{I=1} = \sum_{i=1}^{\infty} \sum_{d=d_f}^{\infty} i a(d, i) D^d \]  

(3.21)

Thus, eq.(3.11) can also be expressed as

\[ P_b(j) = E\{n_b(j)\} \leq \left. \frac{\partial T(D, 1, I)}{\partial I} \right|_{I=1, D^d = P_d} \]  

(3.22)

If \( k \neq 1 \), the probability of bit error is bounded by

\[ P_b(j) = E\{n_b(j)\} \leq \frac{1}{k} \cdot \left. \frac{\partial T(D, 1, I)}{\partial I} \right|_{I=1, D^d = P_d} \]  

(3.23)
Figure 3.5 presents the probability of bit error rate for both a BPSK system with a convolutional code and for BPSK system without error correction codes. The coding rate is \( r = 1/2 \), constraint length is \( \gamma = 3 \), and the generating function is the same as the one shown in Figure 3.2. Performances of both hard-decision and soft-decision decoding are plotted. The system with convolutional coding using either hard-decision or soft-decision outperforms the system without error correction coding at high signal to noise ratio. Soft-decision decoding achieves additional performance gain over systems employing hard-decision decoding.

3.4 Chapter Summary

In this chapter we have reviewed the basic principles of error correction coding, with particular emphasis on convolutional codes which find widespread use in wireless communications. We have observed that such codes are useful in reducing the impact of noise and interference, which is typically found in a CDMA system. In fact, researchers have previously found that in a CDMA system without appropriate error correction coding, the number of users which may be supported is limited to 10-20% of the processing gain \( N \) [Kch93]. However, if sufficient error correction coding is employed, practical CDMA systems may use values of \( K \) which approach 65% of \( N \) for standard convolutional codes and 75% of \( N \) for specially designed “Trellis codes” [Kim95].

In the next chapter, we introduce interference cancellation as a complementary technique for mitigation of multiple access interference in CDMA systems. In Chapters 5 and 6, we explore the combination of interference cancellation with error correction coding through analysis and simulation.
Chapter 4

Multiuser Detection Techniques for CDMA Systems

4.1. Introduction

As we saw in Chapter 2, CDMA systems based on direct-sequence spread-spectrum are interference limited. Under the standard Gaussian approximation for the performance of a single user correlation receiver, multiple access interference from each user contributes to the noise floor of the systems. While under certain conditions, this approximation results in an accurate estimate for the performance of a correlation receiver, this approximation is not an accurate model for the physical problem. The multiple access interference from each user is not a random process, but a signal which can be estimated. A receiver with knowledge of each user’s spreading codes can exploit that shared information to form a better estimate of the received data from each user.

In this chapter, we explore the different classes of multiuser receivers. Receivers which implement multistage interference cancellation will be of particular interest. Note that while error correction coding was effective against interference which could be modeled as random noise, interference cancellation will be effective against interference which we can resolve and estimate. Therefore the two techniques should be at least partially synergistic. In subsequent Chapters 5 and 6 we explore the relationship between these two techniques.

4.2. Conventional Detection Techniques

We first review standard CDMA reception techniques which are implemented in today’s systems.
4.2.1 Correlation Receiver

For a standard correlation receiver, the base station has the knowledge of the PN sequence of the signal transmitted by the mobile station. Thus the received signal $r(t)$ is input to a matched filter receiver which is matched to the signal of desired user $s_i(t)$. The matched filter correlates the received signal with the PN sequence of the desired user, so the signal of the desired user is enhanced and the signals of the other users are suppressed according to eq.(2.10) and eq.(2.12). The receiver regards the signals of the undesired users as interference or noise. This receiver at the base station is shown in Figure 4.1 and the mathematical equation describing its operation is given by eq. (2.10).

![Correlation Receiver Bank for CDMA Communication System](image)

Figure 4.1 Correlation Receiver Bank for CDMA Communication System

4.2.2 Rake Receiver

In the wireless communication environment, the transmitted signal may reach the destination through many different paths, including a direct line of sight and reflections off of the surrounding objects. The received signal contains delayed and distorted replicas of
the original transmitted signal due to this multipath propagation. When there is multipath propagation, the received signal consists of the superposition of several signal replicas which are complex signals, each with its own delay, amplitude and phase. This can result in both constructive and destructive interference to the desired signal and causes large variations in the received signal levels. In a narrowband environment, the received signal may be Rayleigh distributed when the received signal is the summation of diffuse components and may be Ricean distributed when the received signal is the summation of diffuse components plus a specular component. For a wideband, spread spectrum system, multipath components may result in self interference and the receiver of Figure 4.1 may be severely degraded.

Different multipath components can be regarded as independent receptions of signals. The RAKE receiver structure which has $M$ branches is shown in Figure 4.2. Each branch of the RAKE receiver is a correlation receiver and can be matched to a multipath component of the signal. The decision statistics is the summation of the weighted output of these branches. The multipath signal can provide a beneficial time diversity if multipath reception is employed.

![Structure of a single RAKE receiver](image)

Figure 4.2 Structure of a single RAKE receiver

The decision statistics $Z_i$ for the $i$th bit is given by
\[ Z_i = \sum_{l=1}^{M} w_l \cdot Z_{i,l} \]  

(4.1)

4.3 Optimum Multiuser Receiver

The conventional correlation receiver of DS/CDMA system treats signals from each user separately. The performance of this kind of receiver is satisfactory under the circumstance where two conditions are satisfied, i.e., first, the cross-correlations for all possible relative delays among the data sequences transmitted by different users are approximately zero; second, the powers of received signal from all of the users are the same. However, the cross-correlations among signals can not be nearly zero due to the pseudorandom property of PN sequences and asynchronous transmission from mobile to base station. If the received signal from a user near the receiver has higher power than the signal from a user far away from the receiver, the user further away suffers degradation in performance. Even if the users are at the same distance from the receiver, the received signals can differ in power because different users pass through different channels and some users may be transmitted through a fading channel. A power control algorithm implemented in the DS/CDMA system may reduce the near-far effect but it can not completely eliminate this effect [Cam92]. Trying to design signals with more stringent cross-correlation properties will not make the PN sequences of different asynchronous users totally uncorrelated. Since the output of each correlation receiver contains a spurious component that is proportional to the amplitude of each of the interfering signals, the conventional receiver performs worse in recovering the signal transmitted from the weaker users as the signals from interfering users become stronger.
The signals from the interfering users contain information and are of interest to the base station. The basic idea of multiuser detection is to detect the signals from all of the users at the same time and use the signals from the interfering user to help to cancel the multiple access interference added to the desired user, rather than just ignoring the signals from the interfering users.

In order to improve the receiver performance, an optimum multiuser detection receiver is proposed in [Ver86a] and the block diagram of which is shown in Figure 4.3. There is no unique optimality criterion since the transmitted signals are not independently conditioned on the received signal. It is possible to select the set of symbols according to maximum likelihood sequence detection (global optimum) or to select the set of symbols according to minimum probability of error detection (local optimum). The derivation of [Ver86a] follows the first criterion. It is noted that for low noise levels the two criteria yield identical results [Ver84].

In maximum likelihood sequence detection, we want to maximize the joint posteriori probability.
Multiuser Detection for CDMA Systems with Convolutional Coding

\[ P[b \mid (r(t), t \in \mathbb{R})], \]  
(4.2)

where \( b \) is the matrix of the \( K \) transmitted bit sequences, \( r(t) \) is the received signal and is given in eq.(2.9), and \( \mathbb{R} \) is the set of real numbers. If all transmitted symbols are equiprobable, the maximum likelihood sequence detection selects the sequence that maximizes

\[ P[(r(t), t \in \mathbb{R} \mid b)], \]  
(4.3)

For an AWGN channel, this results in

\[ \hat{b} = \arg \max \left[ 2 \int_{-\infty}^{\infty} s(b)[s(b)dt + \sigma dw(t)] - \int_{-\infty}^{\infty} s^2(b)dt \right], \]  
(4.4)

where \( \hat{b} \) is the estimate of the transmitted bit matrix, \( s(b) = \sum_{i=1}^{N} \sum_{k=1}^{K} b_k^{(i)} s_k(t - iT - \tau_k), \) and \( \omega(t) \) is a Weiner process. From the Figure 4.3, \( z \) is the matrix of correlator outputs whose elements \( z_k^{(i)} \) for the \( k \)th row and \( i \)th column are given by

\[ z_k^{(i)} = \int_{iT + \tau_k}^{(i+1) + \tau_k} r(t)a_k(t - \tau_k)\cos(\omega_c t + \phi_k)dt, \]  
(4.5)

dena equation (4.4) becomes

\[ \hat{b} = \arg \max \left[ 2 \sum_{i=1}^{N} (b^{(i)})^T z^{(i)} - \int_{-\infty}^{\infty} s^2(b)dt \right], \]  
(4.6)

where \( b^{(i)} \) is the \( i \)th column of \( b \).

Even though \( z_k^{(i)} \) is not a sufficient statistic for the detection of \( b_k^{(i)} \), eq.(4.4) and eq.(4.6) indicate that the whole sequence of outputs of the bank of \( K \) matched filters \( z \) is a sufficient statistic for the selection of the most likely sequence \( b \). Optimum multiuser detection of asynchronous signals is a problem of sequence detection, so observation of the whole received signal is required to produce a sufficient statistic for any symbol decision. This implies that the optimal receiver consists of a bank of matched filters, one for each user, followed by a decision algorithm which selects the sequence \( b \) that
maximizes eq. (4.3). It is observed in [Sch79] that although the entire received sequence needs to be observed for a symbol decision, the detection of a given symbol depends only on the adjacent symbols. The optimum detection algorithm can be viewed as a linear finite state machine and is similar to a rate 1 convolutional encoder. This means that the maximum likelihood optimization algorithm can be implemented via the Viterbi algorithm. It is shown in [Ver86a] that the most efficient implementation of this algorithm has a dimensionality of state space equal to $2^{K-1}$ and a computational complexity per bit equals $O(2^K)$ for binary symbol modulation. This receiver also requires the knowledge of received signal energies and the spreading PN sequences of all the users.

This optimum multiuser receiver achieves tremendous performance gain over the conventional receiver. This receiver reaches maximum asymptotic efficiency, a measure developed in [Ver86c] which can be expressed as

$$\eta_k = \lim_{\sigma \to 0} \frac{e_k}{w_k}.$$  \hspace{1cm} (4.7)

where $\sigma^2 = N_o/2$ is the power of AWGN, $w_k$ is the energy required to obtain a given probability of error $P_e$, $e_k$ is the effective energy of the $k$th user defined as the energy required to obtain the same probability of error in a single user environment. The performance measure of $k$th user’s near-far resistance is defined as the worst case asymptotic efficiency in [Lup90]

$$\bar{\eta}_k = \inf \{\eta_k\}, \text{ for } w_j \geq 0 \text{ and } j \neq k.$$  \hspace{1cm} (4.8)

The conventional receiver has a near-far resistance of zero. This optimal receiver has a near-far resistance greater than zero, which means that it is near-far resistant. Therefore, the near-far problem is not inherent to CDMA system; instead, it results from the fact that the received signal is not processed appropriately by the conventional receiver.

It is obvious that for systems of reasonable size, the optimum multiuser receiver is too computationally intensive to implement in practical systems. This motivates the search for good sub-optimum multiuser receivers.
4.4 Sub-Optimum Multiuser Receiver

Inspired by the significant performance improvement from the optimum multiuser detection receiver, several sub-optimum multiuser detection receivers have been proposed which require less computation complexity for implementation and do not sacrifice performance too much.

4.4.1 Decorrelation Receiver

The decorrelation receiver relies on the principal that the cross-correlations between each user's spreading codes can be computed. The effects of interference can be removed by multiplying the decision statistics by the inverse of this correlation matrix or "decorrelating" the decision statistics. This receiver depends on a linear combination of the sufficient statistics for decisions which is developed in [Lup89] and [Lup90] for synchronous and asynchronous AWGN channels respectively. This is a linear sub-optimum multiuser receiver and we concentrate on the asynchronous case. The set of sufficient statistics given by eq.(4.5) can be revised in vector format

\[ z = R W b + n, \]  \hspace{1cm} (4.9)

where \( z \) is the vector form of matched filter output, \( W \) is an \( N K \times N K \) diagonal matrix containing the energies of the received signals from every users, \( N \) is the length of transmitted sequence, \( K \) is the number of users, \( W = \text{diag} \left( \left[ \sqrt{P_{1,0}/2}, \ldots, \sqrt{P_{K,0}/2}, \ldots, \sqrt{P_{1,N}/2}, \ldots, \sqrt{P_{K,N}/2} \right] \right) \), \( P_{k,i} \) is the received power of the \( i \)th bit of \( k \)th user, \( b = \left( (b^{(1)})^T, \ldots, (b^{(N)})^T \right)^T \) is the vector form of transmitted data sequence from every users where \( b^{(i)} \) is a column vector of transmitted data sequence from every users at time \( i \), \( n \) is the \( N K \times 1 \) column vector of AWGN components in each correlator output for each bit interval, and \( R \) is an \( N K \times N K \) block Toeplitz matrix that defines the dependence of a given correlation output at time \( i \) on the bits \( b_k^{(i-1)}, b_k^{(i)} \).
and $b_k^{(i+1)}$ for all $k$ due to the asynchronous nature of the channel. $R$ can be regarded as the cross-correlation matrix for the equivalent synchronous problem where the whole transmitted sequence can be considered as the result of $NK$ users transmitting during one interval of duration $NK + \tau_K - \tau_1$. $R$ can be expressed as

$$R = \begin{bmatrix}
H(0) & H(-1) & 0 & \ldots & 0 \\
H(1) & H(0) & H(-1) & \ddots & \vdots \\
0 & H(1) & H(0) & \ddots & 0 \\
\vdots & \ddots & \ddots & \ddots & H(-1) \\
0 & \ldots & 0 & H(1) & H(0)
\end{bmatrix}, \quad (4.10)$$

where $H(i)$ is a $K \times K$ matrix, the element of $k$th row and $l$th column of this matrix is defined by

$$h_{kl} = \int_{-\infty}^{\infty} a_k(t - \tau_k)a_l(t + iT - \tau_l)dt, \quad (4.11)$$

Since the modulating signals are zero outside $[0, T]$, we have $H(-1) = H^{T}(1)$, $H(i) = 0$ $\forall |i| > 1$. Because all the users are ranked as $\tau_1 < \tau_2 < \ldots < \tau_K$, $H(1)$ is an upper triangular matrix with zeros along the diagonal.

A linear detection receiver is the one that makes decisions based on a linear transformation of the sufficient statistics $z$. A block diagram of such kind of receiver is shown in Figure 4.4. Its operation can be expressed as

$$\hat{b} = \text{sgn}(Tz)$$

$$= \text{sgn}(T(RWb + n)), \quad (4.12)$$

where $T$ is a linear operator on $z$.

Using different optimization criterion can lead to different linear operator $T$. It is shown in [Lup90] that if we use near-far resistance criterion to derive this receiver the linear receiver can obtain optimal near-far resistance which is the same near-far resistance as the optimal receiver. This receiver use a linear operator which is the inverse of cross-correlation matrix $R$ to cancel the multiple access interference. That is, $T = R^{-1}$.
Thus, eq. (4.12) becomes

\[ \hat{b} = \text{sgn}(R^{-1}z) \]

\[ = \text{sgn}(Wb + R^{-1}n), \]  \hspace{1cm} (4.13)

Because the cross-correlation matrix \( R \) is normalized, the decorrelating process is independent of the energies of the received users' signals. There is significant performance gain for decorrelation receiver over conventional receiver. The performance of decorrelating receiver is invariant to the energy of interfering users. The decorrelation receiver is an optimal linear receiver for unknown signal energies and it does not need the estimates of the received user energies. The complexity is linear in the number of users, once the matrix inversion has been performed.

The disadvantages of this receiver include calculation of the inverse of the cross-correlation matrix, \( R^{-1} \), in order to obtain the decorrelation coefficients. In fact the term \( R^{-1}n \) introduces correlation to the original white Gaussian noise and the performance of the receiver can degrade in high noise environments.

It is recognized in [Ver84] that the multiple access interference cancellation is analogous...
to the equalization of intersymbol interference (ISI). The decorrelation receiver derived above is similar to the zero-forcing equalizer ISI channels. In [Xie90] the linear detectors using minimum mean squared error (MMSE) optimization criterion are proposed, which minimize $E[(b - \hat{b})^T (b - \hat{b})]$ where $E[ ]$ is the expected value and $\hat{b} = \text{sgn}(Tz)$ for linear detector. The linear operator in this receiver which use MMSE criterion is

$$T = \left(R + \frac{N_0}{2W^2}\right)^{-1}, \quad (4.14)$$

then the decision rule becomes

$$\hat{b} = \text{sgn}\left[\left(R + \frac{N_0}{2W^2}\right)^{-1} z\right]. \quad (4.15)$$

It is shown in [Xie 90] that the performance of this receiver is comparable to the decorrelation receiver using decision rule of eq. (4.13). We note from eq.(4.15) and eq.(4.13) that in low noise environment where $N_0 = 0$ these two receivers are identical. If the noise level is very high where $N_0$ is extremely large the MMSE receiver reduces to conventional type of receiver. The MMSE receiver obtains optimal near-far resistance, but it requires knowledge of the received signal energies.

Furthermore, the decorrelation approach relies on matrix inversion operation which can be numerically unstable for some values and which can be computationally complex for CDMA systems which are not "code on pulse" such as the reverse link of the commercially important IS-95 systems. Therefore we will not ultimately rely on the decorrelation approach.

### 4.4.2 Decision Feedback Receiver

The performance of the decorrelation receiver degrades in high noise environments because the inversion of cross-correlation matrix $R$ multiplying the AWGN term $n$ enhances noise. This creates a gap between the bit error probability of the decorrelation receiver and probability of bit error of single user which is used as an lower bound for probability of bit error. This problem can be solved by factorization of the
cross-correlation matrix. This approach is called decision feedback receiver which is developed in [Due93] and [Due95a] for synchronous and asynchronous CDMA systems respectively. As in last section, we will focus on the asynchronous situation.

Before the discussion of decision feedback receiver, we introduce a $D$-transform notation, where $D$ is the delay of signaling interval $T$ seconds. The $D$-transform of the input vector is

$$b(D) = b_0 + b_1 D^1 + b_2 D^2 + \ldots,$$

(4.16)

It has $K$ components

$$b(D) = (b_1(D), b_2(D), \ldots, b_K(D))^T,$$

(4.17)

The $D$-transform of the sufficient statistics given by eq.(4.5) has $K$ components

$$z(D) = (z_1(D), z_2(D), \ldots, z_K(D))^T,$$

(4.18)

and can be expressed as

$$z(D) = S(D) \cdot W \cdot b(D) + \tilde{n}(D),$$

(4.19)

where $b(D)$ is given in eq.(4.16) and eq.(4.17), $W = \text{diag} \left( \frac{P_{1,i}}{2}, \frac{P_{2,i}}{2}, \ldots, \frac{P_{K,i}}{2} \right)$ is the diagonal matrix of energies, $P_{k,i}$ is the received signal power at $i$th bit of $k$th user, $\tilde{n}(D)$ is vector of colored Gaussian noise, $S(D)$ is the matrix spectrum of the channel and is given by

$$S(D) = H_{-1} D^{-1} + H_0 + H_1 D,$$

(4.20)

Here $H_i$ is the cross-correlation matrix whose element at $k$th row and $l$th column is given by eq. (4.11).

The spectrum $S(D)$ is nonnegative definite on the unit circle (for $D = e^{i2\pi f}$, $0 \leq f < 1$) and symmetric ($S(D) = S^T(D^{-1})$). Since signature waveforms are limited to the interval of length $T$ and normalized, the spectrum $S(D)$ only has three terms and all diagonal elements are equal to one. The spectrum of the additive colored Gaussian noise vector
\( \hat{n}(D) \) is proportional to the channel spectrum

\[
S_t(D) = \sum_{i = -\infty}^{\infty} H_{\hat{n}}(i)D^i = N_0S(D), \quad (4.21)
\]

where the autocorrelation matrix \( H_{\hat{n}}(i) = E(\hat{n}_i\hat{n}_0^T) \).

From the Spectral Factorization Theorem introduced in [Due95a], the matrix spectrum \( S(D) \) can be factored as

\[
S(D) = F(D^{-1})^TF(D), \quad (4.22)
\]

where \( F(D) \) and \( F(D)^{-1} \) are both causal and stable matrix filter. The anticausal filter \( F(D^{-1})^T \) transforms a white process with spectrum \( I \) (identity) into a process with spectrum \( S(D) \). Alternatively, the anticausal inverse \( (F(D^{-1})^T)^{-1} \) is a whitening filter for a process with spectrum \( S(D) \). Thus, if \( z(D) \) is fed to the filter \( (F(D^{-1})^T)^{-1} \), the output signal is

\[
r(D) = (F(D^{-1})^T)^{-1}z(D) = F(D) \cdot W \cdot b(D) + n(D), \quad (4.23)
\]

where \( n(D) \) is white Gaussian noise with spectrum \( S_n(D) = N_0I \). Since the transformation \( F(D) \) is causal, the multiuser interference in eq.(4.23) is due to past input symbols. The decision-feedback receiver, as shown in Figure 4.5, is a symbol by symbol detector which utilizes past decisions in addition to channel outputs. It requires two linear filters: the feedforward filter \( G(D) \) fed by outputs \( z(D) \), and the feedback filter \( B(D) \) fed by past decisions \( \hat{b}(D) \). The outputs of these filters are combined at the inputs to the decision devices:

\[
\tilde{b}(D) = G(D) \cdot z(D) - B(D) \cdot \hat{b}(D), \quad (4.24)
\]

When making a decision for user \( k \) at time \( n \), \( \hat{b}_k(\cdot^n) \), we can utilize decisions made by all users in previous time frames. In addition, if user \( k \) waits until user \( j \) has made its
decision at time $n$, $\hat{b}_j^{(n)}$ can also be used. Since weaker users suffer more from multiuser interference and their benefit from feedback is greater, the receiver makes decisions in the order of decreasing signal strength, so that decisions of the stronger users made in the current time frame can be utilized by weaker users.

![Diagram](image)

**Figure 4.5 Decision Feedback Multiuser Receiver**

The objective of the receiver is to maximize the ideal signal-to-noise ratio at the input to each decision device. The feedforward and feedback filters of this receiver are chosen to eliminate multiuser interference at the inputs to the decision devices, i.e. the estimate $\hat{b}(D)$ is a noisy version of the input $b(D)$ provided all past decisions are correct. The optimal feedforward filter is the anticausal noise whitening filter

$$G(D) = (F(D^{-1})^T)^{-1},$$

(4.25)

It eliminates multiuser interference due to future symbols. The feedback filter
\[ B(D) = (F(D) - \text{diag}(F_0)) \cdot W, \]  

(4.26)

is strictly causal and attempts to cancel multiuser interference due to past symbols. The causal filter \( F(D) \) can be expressed as a one-sided power series

\[ F(D) = F_0 + F_1D + F_2D^2 + \ldots, \]  

(4.27)

where \( F_0 \) is a left lower triangular constant term.

The decision-feedback receiver is reliable and robust. It has better performance than the conventional receiver. It improves the performance of decorrelation receiver in high noise environment. It is near-far resistant and is stable in the presence of interferers with various energy distributions. The performance of weaker user is close to the single user bound when interferers are strong in power. The detection scheme is nonlinear but the complexity is linear to the number of users. The disadvantage is that it requires the estimation of the energies of users' signals.

### 4.4.3 Multistage Receiver

A multistage algorithm for multiple access interference cancellation is proposed and analyzed in [Var90]. This results in the multistage multiuser receiver whose block diagram is shown in Figure 4.6. This detector is used for asynchronous CDMA systems. The basic idea is to cancel interference stage by stage in order for the estimates to approach the real transmitted signal for each user. Based on the information the receiver has, estimate of each user's transmitted signal can be obtained in parallel. These estimates can be used to cancel multiple access interference at the next stage by subtracting the estimates of the undesired users from the received signal to get the signal of desired user. Theoretically, this can remove all of multiple access interference from the desired user's signal. In fact, due to the inaccuracy of the estimation, the interference can not be eliminated at one time. Therefore this process may be repeated for several stages, in order to get more reliable estimates of each user's signal. The higher the number of stage, the more accurate the estimates we can get.
Using the maximum likelihood sequence detection rule, the sequence which maximizes the value of eq. (4.2) is chosen. Equivalently, we can choose the sequence which maximizes the log-likelihood function

$$L[b,r(t)] = 2\int s(t,b) r(t) dt - \int s^2(t,b) dt,$$

where $b$ is a $K \times N$ matrix whose $k$th row and $i$th column element is $i$th bit of $k$th user's signal $b_k^{(i)}$, $s(t,b)$ is the asynchronous sum of $K$ users' signal, $r(t)$ is the received signal. This expression shows that the sufficient statistics can be expressed in vector form

$$z^{(i)}(0) = \eta^{(i)} + H(1)b^{(i-1)} + H(0)b^{(i)} + H(-1)b^{(i+1)},$$

where $H(i)$ is given by eq.(4.11), $b^{(i)}$ is the $i$th column of $b$, $\eta^{(i)}$ is a $K$ dimensional vector of Gaussian random noise, $z^{(i)}(0)$ is $K$ dimensional vector and can be denoted as $[z_1^{(i)}(0), z_2^{(i)}(0), ..., z_K^{(i)}(0)]^T$. 

Figure 4.6 Multistage Multiuser Receiver
Assume that we are in the stage of $m + 1$ and the $m$th stage estimates of bits $b_l^{(j)}$ be denoted as $\hat{b}_l^{(j)}(m)$ for all $l$ and $j$. The estimate of $b_k^{(i)}$ at the $m + 1$ stage is given by

$$\hat{b}_k^{(i)}(m+1) = \arg\left\{\max[L(b, r(t))] \right\},$$

$$b_k^{(i)} \in \{\pm 1\}$$

$$b_l^{(j)} = \hat{b}_l^{(j)}(m) \tag{4.31}$$

An additive decomposition of the log-likelihood function can be obtained as

$$L[b, r(t)] = \sum_i \langle b^{(i)}(0), 2z^{(i)}(0) - H(0)b^{(i)} - 2H(1)b^{(i-1)} \rangle, \tag{4.32}$$

where $\langle x, y \rangle$ denotes the inner product between vectors $x$ and $y$. The estimation at $m + 1$ stage follows the rule

$$\hat{b}_k^{(i)}(m+1) = \text{sgn}[z_k^{(i)}(m)], \tag{4.33}$$

where

$$z_k^{(i)}(m) = z_k^{(i)}(0) - \sum_{l=k+1}^K h_{kl}(0) \hat{b}_l^{(i-1)}(m)$$

$$- \sum_{l \neq k} h_{kl}(0) \hat{b}_l^{(i)}(m) - \sum_{l=1}^{k-1} h_{kl} \hat{b}_l^{(i+1)}(m), \tag{4.34}$$

The conventional detector is chosen for the first stage to obtain sufficient statistics because it is simple in concept and easy to analyze. This can be expressed as

$$\hat{b}^{(1)}(1) = \text{sgn}[z^{(1)}(0)], \quad \forall i, \tag{4.35}$$

In general, the multistage receiver can be implemented through the following procedures:

1) Obtain sufficient statistics from the received signal waveform.
II) Perform $M$ stages of processing the sufficient statistics where the $n$th ($m \geq 1$) stage processor acts on the statistics produced by the $(m - 1)$th stage. The $m$th stage consists of the following procedures: i) estimation of the unknown symbols from the $(m - 1)$th stage statistics. If $m = M$, stop; else, proceed with ii). ii) reconstruction of the multiple access interference using the estimates obtained in step i) and subtraction of the reconstructed multiple access interference from the sufficient statistics to obtain the $m$th stage statistics.

This multiuser receiver alleviates the near-far problem and its computational complexity is linear in the number of users. The performance of this multistage receiver closely tracks that of the optimum receiver. In most cases of practical interest, it achieves significant performance gains over the conventional receiver. But it requires the knowledge of signal energies of all users.

The work of Varanasi and Aazhang [Var90], [Var91] has been extended in several significant respects. Patel and Holtzman have considered a successive cancellation technique in which estimated interference is subtracted from the received signal in descending order of signal power. Striglis [Str94] has proposed the use of a multistage RAKE receiver in multipath, and Kau [Kau95] has studied a generalized model of this approach in which received signal power need not be known. The work of [Kau95] has resulted in a closed form expression for BER of a multistage interference cancellation system which we make use of in the next chapter.

In this thesis, we use multistage detection technique combined with convolutional coding technique to cancel multiple access interference.

4.5 Chapter Summary

In this chapter, we have summarized the major types of multiuser receivers which have been studied in the last few years. In the remainder of this thesis we will focus on the multistage interference cancellation in conjunction with convolutional coding. Despite the flurry of research on multiuser receiver techniques, there has been relatively little work on the performance of interference cancellation in conjunction with error correction coding.
Chapter 5

Analysis of Multiuser Detection for CDMA Systems with Convolutional Coding

5.1 Introduction

Channel coding is an important technique for overcoming performance degradations due to severe channel fading. It can reduce signal to background noise ratio requirement in wireless communication systems which is equivalent to increasing the performance of systems. But channel coding is subject to a noise floor in interference limited systems. The base station receiver in a CDMA cellular system is limited by the multiple access interference if a conventional CDMA receiver is employed. The multiuser detection techniques which were introduced in Chapter 4 can effectively reduce the noise floor caused by multiple access interference in CDMA systems. Therefore, channel coding techniques combined with multiuser detection techniques may result in better performance than using either of these two techniques alone, since each one of these techniques can compensate for the weakness of the other.

In [Vit90], powerful very low rate orthogonal convolutional codes have been used in CDMA spread spectrum multiple access systems. This paper showed that the 'coding gain' and 'processing gain' can be achieved within the same bandwidth, and that the ultimate capacity of the multiple access channel in AWGN is achievable in the limit of arbitrarily long codes if the entire spreading is dedicated to error control coding and coordinated processing is added. However, this result was developed without regard to complexity, and based on asymptotic analysis.

Multiuser reception for trellis-coded CDMA communication systems is investigated in [Faw94]. The optimum receiver and two suboptimal receivers, employing a reduced tree search algorithm and the multistage approach are used as multiuser receivers. The trellis coding structure uses the scheme proposed in [Woe90] and [Woe94]. It utilizes an
extended set of signature waveform to obtain a set of bi-orthogonal sequences. The design of encoder and the assignment of the signature sequences to its trellis is carried out such that the Euclidean distance is maximized between distinct transmitted signals. The results show that this receiver is near-far resistant. It achieves the same performance at an SNR which is more than 3 dB lower compared to the uncoded system at the same data rate and signal bandwidth. The receiver based on the multistage approach and the reduced tree search algorithm exhibit only near optimal performance, but their complexity depends linearly on the number of users $K$ whereas this relation is exponential for the receiver based on the optimum approach.

The Turbo-codes have been applied to a CDMA mobile radio system using multiuser detection and coherent receiver diversity in [Jun94]. The new receiver achieves better performance than the receiver not using error correction coding or multiuser detection. The signal processing expense is higher than for the use of convolutional coding due to the more complex structure of Turbo-Code decoder. Further discussion of Turbo-codes can be found in [Ber93].

In this project, we combine a multistage interference cancellation technique with convolutional coding and apply this new scheme to the CDMA base station receiver. Convolutional coding is commonly used based on the codes used in the IS-95 standard [TIA93]. The computational complexity is reasonable if the Viterbi algorithm is employed. We will focus on the study of performance of multistage interference cancellation combined with convolutional coding with reasonable complexity for practical implementation.

5.2 Approaches to Multistage Interference Cancellation Combined with Convolutional Coding

We assume the data sequence is convolutionally encoded and interleaved. Then PN sequence spreading is performed on the coded bit sequence. The spreading chip sequence is modulated to generate the transmitted signal.

We consider two approaches to multistage interference cancellation for convolutionally encoded signals at the receiver. The first scheme is a partitioned approach in which the
multistage interference cancellation precedes the decoder and does not utilize the decoded data. The first scheme can be summarized as:

I. Demodulate the encoded bits.
II. Perform multistage interference cancellation.
III. Decode the decision statistics by Viterbi algorithm.

The second scheme is the integrated approach in which the decoded data of the interfering users is used for interference cancellation in the signal of the desired user. The second scheme can be implemented according to the following procedures

I. Demodulate the encoded bits.
II. Decode the decision statistics by the Viterbi algorithm.
III. Re-encode and re-spread the estimated bits to reconstruct the transmitted signals.
IV. Perform interference cancellation.
V. Decode the decision statistics by the Viterbi Algorithm.
VI. Repeat steps III. to V. until the desired number of stages are completed.

![Diagram of the transmitter and receiver](image)

A. Transmitter of Multiuser Detection for Convolutionally Encoded CDMA System

B. Receiver of the First Scheme of Multiuser Detection for Convolutionally Encoded CDMA System

Figure 5.1 Transmitter and Receiver of Multiuser Detection for Convolutionally Encoded CDMA System

It is obvious that the second scheme requires more computational power than the first
scheme. Also the first scheme is more analytical tractable than the second scheme. Therefore we use the analytical method to evaluate the performance of the first scheme. In the next chapter, the simulation technique is employed to compare the performance of these two schemes. The transmitter which can be used in both of schemes is shown in Figure 5.1A. The block diagram of the first scheme used in the base station receiver is shown in Figure 5.1B. The block diagram of receiver using the second scheme is shown in Figure 5.2.

![Block Diagram](image)

Figure 5.2. The Second Scheme of Multiuser Detection Receiver for Convolutionally Encoded CDMA System

### 5.3 A Model of Multistage Interference Cancellation Combined with Convolutional Coding

In a multiple access CDMA communication system, there are $K$ active users transmitting independently. The $k$th user transmits a binary data signal
Multiuser Detection for CDMA Systems with Convolutional Coding

\[ b_k(t) = \sum_{i = -\infty}^{\infty} b_{k,i}p_{T_1}(t - iT_1), \quad (5.1) \]

where \( b_{k,i} \in \{0, 1\} \) is an independent identically distributed (i.i.d.) random variable representing the \( i \)th data bit of the \( k \)th user and \( p_{T_1}(t) \) is a unit pulse function of duration \( T_1 \) such that

\[ p_{T_1}(t) = \begin{cases} 
1, & t \in [0, T_1) \\
0, & \text{otherwise}
\end{cases} \quad (5.2) \]

The data bits are sent to the convolutional encoder. The generating functions of convolutional code are obtained from [Pro89] which generates maximal free distance convolutional codes for given coding rate and constraint length. After convolutional coding and interleaving, the encoded bits are represented by

\[ c_k(t) = \sum_{i = -\infty}^{\infty} c_{k,i}p_T(t - iT), \quad (5.3) \]

where \( c_{k,i} \in \{\pm 1\} \) is the \( i \)th code bit of the \( k \)th user which is mapped from the set \( \{0, 1\} \) to the set \( \{-1, 1\} \), and \( p_T(t) \) is unit pulse function of duration \( T \). The ratio of code duration \( T \) to bit duration \( T_1 \) is the coding rate \( r \). The value of \( c_{k,i} \) can be calculated from \( b_{k,i} \) by using eq.(3.5) and then map the result from \( \{0, 1\} \) to \( \{-1, 1\} \).

The \( k \)th user’s PN sequence is given by

\[ a_k(t) = \sum_{j = -\infty}^{\infty} a_{k,j}p_{T_c}(t - jT_c), \quad (5.4) \]

where \( a_{k,j} \in \{\pm 1\} \) is the \( j \)th chip of the \( k \)th user and \( p_{T_c}(t) \) is unit pulse function of duration \( T_c \). The ratio of code duration \( T \) to the chip duration \( T_c \) is the processing gain, i.e. \( N = T/T_c \). The \( k \)th user’s transmitted signal \( s_k(t) \) is given by

\[ s_k(t) = \sqrt{2P_k} c_k(t) a_k(t) \cos(\omega_c t + \theta_k), \quad (5.5) \]
where $P_k$ is the signal power, $\theta_k$ is the phase of the $k$th user and $\omega_c$ is the common carrier frequency of CDMA system.

![Diagram of Convolutional Encoding](image)

**Figure 5.3 Model of Convolutional Encoded DS/CDMA Systems**

The model of convolutionally coded DS/CDMA multiple access communication systems is shown in Figure 5.3. As derived previously and shown in this figure, the received signal can be expressed as

$$r(t) = n(t) + \sum_{k=1}^{K} \sqrt{2P_k} c_k(t - \tau_k) a_k(t - \tau_k) \cos(\omega_c t + \phi_k),$$

(5.6)

where $n(t)$ is an AWGN with two sided power spectral density $N_0/2$, $\tau_k$ is the delay of $k$th user which is uniformly distributed on $[0, T)$, and $\phi_k = [\theta_k - \omega_c \tau_k] \mod 2\pi$. If $\theta_k$ is uniformly distributed on $[0, 2\pi)$, then for large carrier frequencies $\omega_c T > 1$, $\phi_k$ is uniformly distributed on $[0, 2\pi)$. Without loss of generality, all delays and phases can be recorded with respect to user 1, so we have $\phi_1 = 0$ and $\tau_1 = 0$.

The detailed implementation of multistage interference cancellation for convolutionally
encoded data is shown in Figure 5.4. For analytical results, we consider the simpler method 1 approach only. We will verify through simulation in Chapter 6 that this simpler approach achieves performance comparable to the more complex second approach. The first stage of the receiver is a bank of conventional correlation receivers used for obtaining the sufficient statistics. The received signal \( r(t) \) is correlated with synchronous copy of the spreading signal. Let \( Z_{k,i}^{(s)} \) be the decision statistics for the \( i \)th bit of the \( k \)th user at stage \( s \). Then the decision statistics at stage 1, i.e. the sufficient statistics is given by

\[
Z_{k,i}^{(1)} = \int_{(i+1)T + \tau_k}^{iT + \tau_k} r(i) a_k(t-\tau_k) \cos(\omega_c t + \phi_k) dt,
\]

(5.7)

As in [Pur77], \( Z_{k,i}^{(1)} \) can be modeled as a Gaussian random variable with mean

\[
E[Z_{k,i}^{(1)}] = c_{k,i} T \sqrt{\frac{P_k}{2}},
\]

(5.8)

All subsequent stages of the receiver perform interference cancellation based on the decision statistics from the previous stage. At stage \( s \) of the receiver, the decision statistics can be used to form an unbiased estimate of the product \( c_{k,i} \sqrt{P_k} \), where

\[
\hat{c}_{k,i} = \frac{Z_{k,i}^{(s)}}{\sqrt{Z_{k,i}^{(s)}}}
\]

is an estimate of the \( k \)th user's code bits, and from eq.(5.8)

\[
\hat{P}_k = 2[Z_{k,i}^{(s)}/T]^2
\]

is an estimate of the \( k \)th user's power. Substituting these estimates into eq.(5.5), we form an estimate of user \( k \)'s signal at stage \( s \), \( \hat{s}_k^{(s)}(t) \) expressed by

\[
\hat{s}_k^{(s)}(t) = \frac{2}{T} a_k(t) \cos(\omega_c t + \phi_k) \sum_{i = -\infty}^{\infty} Z_{k,i}^{(s)} p_T(t-iT),
\]

(5.9)

As can be seen from eq.(5.9) and previous discussion, we can get the estimates \( \hat{c}_{k,i} \) and \( \hat{P}_k \) individually which requires hard decision and can lead to the loss of information due to a hard decision on the transmitted bit. Fortunately the estimated signal of user \( k \), \( \hat{s}_k^{(s)}(t) \) can be generated at each stage with only the product \( c_{k,i} P_k \). The resulting signal \( \hat{s}_k^{(s)}(t) \) is an unbiased estimate of \( s_k(t) \) and can be obtained from \( Z_{k,i}^{(s)} \) by using
Figure 5.4. Block diagram of the first two stages of multistage interference cancellation.
eq.(5.9) which provides for soft decisions in reconstructing the transmitted signals.

Interruption is cancelled by subtracting the estimated signals of the interfering users from the received signal \( r(t) \) to produce a new received signal \( r_k^{(s)}(t) \) for the \( k \)th user at stage \( s \). This procedure can be expressed as

\[
r_k^{(s)}(t) = r(t) - \sum_{\kappa = 1}^{K} s_{\kappa}^{(s)}(t - \tau_{\kappa})
\]

\[
= n(t) + \sqrt{2P_k} c_k(t - \tau_k)a_k(t - \tau_k) \cos(\omega_c t + \phi_k) + \sum_{\kappa = 1}^{K} [s_{\kappa}(t - \tau_{\kappa}) - s_{\kappa}^{(s)}(t - \tau_{\kappa})],
\]

The decision statistic \( Z_{k,i}^{(s+1)} \) at stage \( s + 1 \) for the \( i \)th code bit of the \( k \)th user is obtained by correlating the new received signal \( r_k^{(s)}(t) \) with the \( k \)th user's signature signal

\[
Z_{k,i}^{(s+1)} = \int_{iT + \tau_k}^{(i+1)T + \tau_k} r_k^{(s)}(t)a_k(t - \tau_k) \cos(\omega_c t + \phi_k)dt,
\]

Interference cancellation can be performed for an arbitrary number of stages, although diminishing returns will eventually result. The diagram shown in Figure 5.4 is a block diagram of two stage interference cancellation.

### 5.4 Analytic Evaluation of Performance

An analytical expression for the probability of bit error for uncoded DS/CDMA systems with multistage interference cancellation is derived in [Kau95]. Since the interference cancellation and convolutional coding are separated in scheme number one, the results in [Kau95] and the performance bound of probability of bit error for convolutional codes can be combined to analyze the performance of multistage interference cancellation for
convolutionally encoded DS/CDMA systems. The multiple access interference is modeled as Gaussian random process. Our analysis here follows and extends the work of [Kau95].

The first stage of our receiver is a bank of correlators and the analysis is based on the model illustrated in Figure 5.3. Substituting eq.(5.6) into eq.(5.7), the decision statistic for the kth user at stage 1 is given by

\[
Z_{k,i}^{(1)} = \int_{iT + \tau_k}^{(i+1)T + \tau_k} \left( n(t) + \sum_{k=1}^{K} \sqrt{2P_k c_k(t - \tau_k)a_k(t - \tau_k)\cos(\omega_c t + \phi_k)} \right) \cdot a_k(t - \tau_k)\cos(\omega_c i + \phi_k) dt,
\]

which can be separated into three components

\[
Z_{k,i}^{(1)} = \xi + A_k + \sum_{\kappa = 1}^{K} I_{\kappa}^{(1)},
\]

where \( \xi \) is due to the AWGN term, \( A_k \) is the contribution from the desired user's signal and \( I_{\kappa}^{(1)} \) is the multiple access interference from the \( \kappa \)th user at stage 1.

The noise term \( \xi \) can be expressed as

\[
\xi = \int_{iT + \tau_k}^{(i+1)T + \tau_k} n(t)a_k(t - \tau_k)\cos(\omega_c t + \phi_k) dt,
\]

Here \( \xi \) is a Gaussian random variable because \( n(t) \) is a zero mean Gaussian random process and integration is a linear process. The mean of \( \xi \) is

\[
E[\xi] = 0,
\]

The variance of \( \xi \) may be computed as [Pur77]

\[
Var[\xi] = \frac{N_0 T}{4},
\]

The desired user's component can be found from
\[ A_k = \int_{iT + \tau_k}^{(i+1)T + \tau_k} \sqrt{2P_k}c_k(t - \tau_k)a_k^2(t - \tau_k)\cos^2(\omega_c t + \phi_k) dt, \] 
\[ = c_k i T \sqrt{\frac{P_k}{2}}, \] 

The multiple access interference from the \( \kappa \)th user at stage 1 is given by
\[ I_{\kappa}^{(1)} = \int_{iT + \tau_k}^{(i+1)T + \tau_k} \sqrt{2P_k}c_k(t - \tau_k)a_k(t - \tau_k)\cos(\omega_c t + \phi_k)a_k(t - \tau_k)\cos(\omega_c t + \phi_k) dt \]
\[ = \sqrt{\frac{P_k}{2}} \cos(\psi_k) \int_{iT + \tau_k}^{(i+1)T + \tau_k} c_k(t - \tau_k)a_k(t - \tau_k)a_k(t - \tau_k) dt, \] 

where \( \psi_k = (\phi_k - \phi_k) \mod 2\pi \) is the relative phase between the desired and interfering signal and is uniformly distributed on \( [0, 2\pi] \). As it is shown in [Pur77] and [Kau95], \( I_{\kappa}^{(1)} \) is modeled as a Gaussian random variable with mean 0 and variance
\[ \text{Var}[I_{\kappa}^{(1)}] = \frac{NT_c^2}{6} \cdot P_{\kappa}. \] 

then the multiple access interference in eq.(5.13) is also modeled as a Gaussian random variable with mean 0 and variance
\[ \text{Var} \left[ \sum_{\kappa = 1}^{K} I_{\kappa}^{(1)} \right] = \frac{NT_c^2}{6} \cdot \sum_{\kappa = 1}^{K} P_{\kappa}. \] 

Therefore the decision statistics \( Z_{k,i}^{(1)} \) can be modeled as Gaussian random variable with mean and variance given by
\[ E[Z_{k,i}^{(1)}] = A_k = c_k i T \sqrt{\frac{P_k}{2}}, \]
\[ \text{Var}[Z_{k,i}^{(1)}] = \text{Var}[\xi] + \text{Var} \left( \sum_{\kappa = 1}^{K} \frac{P_{\kappa}^{(1)}}{4} \right) = \frac{N_0 T}{4} + \frac{N T \xi^2}{6} \cdot \sum_{\kappa = 1}^{K} P_{\kappa}^{(1)} \]  

(5.22)

In this project, soft-decision decoding of convolutional code is used, thus the probability of pairwise error event can be found from eq.(3.13), which can be expressed as

\[ P_d = Q \left( \frac{2E_b r d_H}{N_0} \right) \]

\[ = Q \left( \frac{E_b r d_H}{N_0/2} \right) \]  

(5.23)

where \( E_b \) is the energy per data bit which is equal to \( P_k T/r \) and \( N_0/2 \) is the two sided power spectral density for AWGN process. For AWGN process, the relationship between variance and \( N_0 \) can be derived from eq.(5.16) and expressed as

\[ N_0 = \frac{4}{T} \cdot \text{Var}[\xi], \]  

(5.24)

For CDMA communication system on AWGN channel with multiple access interference cancellation, the decision statistics \( Z_{k,i}^{(1)} \) is modeled as Gaussian random variable which includes Gaussian noise and multiple access interference. The probability of pairwise error event can be calculated from

\[ P_d = Q \left( \frac{E_b r d_H}{N_0'/2} \right) \]  

(5.25)

where \( N_0'/2 \) is the two sided power spectral density of Gaussian process \( Z_{k,i}^{(1)} \). According to eq.(5.24), the one sided power spectral density \( N_0' \) of the Gaussian process \( Z_{k,i}^{(1)} \) can be obtained from

\[ N_0' = \frac{4}{T} \cdot \text{Var}[Z_{k,i}^{(1)}]. \]  

(5.26)
Since the energy per code bit has the relationship with the energy per data bit given by $E_c = r E_b = r P_k T$. Substituting eq.(5.26) and eq.(5.22) into eq.(5.25), we get the probability of pairwise error event for user $k$ at the first stage

$$P_{d_k}^{(1)} = Q \left\{ \frac{2 E_b r d_H}{4} \cdot \text{Var}[Z_{k,i}^{(1)}] \right\}$$

$$= Q \left\{ \frac{2 P_k T d_H}{\frac{4}{T} \left( \frac{N_0 T}{4} + \frac{N T_c}{6} \cdot \sum_{\kappa = 1}^{K} P_{\kappa} \right)} \right\}^{1/2}$$

$$= Q \left[ d_H \right]^{1/2} \cdot \left[ \frac{1}{2(P_k T / N_0)} + \frac{1}{3N} \cdot \sum_{\kappa = 1}^{K} \frac{P_{\kappa}}{P_k} \right]^{1/2}$$

$$= Q \left[ d_H \right]^{1/2} \cdot \left[ \frac{1}{2(r E_b / N_0)} + \frac{1}{3N} \cdot \sum_{\kappa = 1}^{K} \frac{P_{\kappa}}{P_k} \right]^{1/2}, \quad (5.27)$$

where $E_{b_k}$ is the energy per data bit for user $k$.

Now let’s consider stage $s + 1$, where the received signal $r_k^{(s)}(t)$ for the $k$th user after $s$ stages of interference cancellation is given by eq.(5.10). The decision statistics $Z_{k,i}^{(s+1)}$
at stage \( s + 1 \) given by eq.(5.11) can be separated into three components as we do in eq.(5.13)

\[
Z_{k,i}^{(s+1)} = \xi_k + A_k + \sum_{\kappa = 1}^{K} I_{\kappa}^{(s+1)},
\]

(5.28)

where \( \xi \) and \( A_k \) are the same as those in eq.(5.13). \( I_{\kappa}^{(s+1)} \) is the multiple access interference from the \( \kappa \)th user remained at stage \( s + 1 \) after \( s \) stages of interference cancellation and can be found from

\[
I_{\kappa}^{(s+1)} = \int_{(iT + \tau_k)}^{(i+1)T + \tau_k} [s_{\kappa}(t - \tau_k) - \hat{s}_{\kappa}^{(s)}(t - \tau_k)] a_k(t - \tau_k) \cos(\omega_c t + \phi_k) \, dt
\]

(5.29)

\[
= I_{\kappa}^{(1)} - I_{\kappa}^{(s)},
\]

where \( \hat{s}_{\kappa}^{(s)}(t - \tau_k) \) is an unbiased estimate of the \( \kappa \)th user's transmitted signal given by eq.(5.9) and \( I_{\kappa}^{(s)} \) is the estimate of multiple access interference from the \( \kappa \)th user at stage \( s \) whose variance is given by [Kau95]

\[
\text{Var}[I_{\kappa}^{(s+1)}] = \frac{1}{3N} \cdot \text{Var}[Z_{\kappa,i}^{(s)}],
\]

(5.30)

As in the first stage, the multiple access interference is modeled as Gaussian random variable with mean 0 and variance given by

\[
\text{Var} \left[ \sum_{\kappa = 1}^{K} I_{\kappa}^{(s+1)} \right] = \frac{1}{3N} \cdot \text{Var} \left[ \sum_{\kappa = 1}^{K} Z_{\kappa,i}^{(s)} \right],
\]

(5.31)

Therefore the decision statistics \( Z_{k,i}^{(s+1)} \), which is the sum of the desired signal component, noise component and cancelled multiple access interference, can be modeled as Gaussian random variable with mean \( A_k = c_{k,i} \sqrt{P_k/2} \) and variance.
Multiuser Detection for CDMA Systems with Convolutional Coding

\[ Var[Z_{k,i}^{(s+1)}] = \frac{N_0 T}{4} + \text{Var} \left[ \sum_{\kappa=1}^{K} I_{\kappa}^{(s+1)} \right], \quad (5.32) \]

The variance of \( Z_{k,i}^{(s)} \) can be recursively computed. By mathematical induction, we have the expression

\[ Var[Z_{k,i}^{(s)}] = \frac{N_0 T}{4} \cdot \left[ 1 - \frac{(K-1)^{s}}{3N} \right] \]
\[ + \frac{T^2}{2 \cdot (3N)^s} \cdot K \left[ \frac{(K-1)^s - (-1)^s}{K} \left( \sum_{\kappa=1}^{K} P_\kappa \right) + (-1)^s \cdot P_k \right], \quad (5.33) \]

Because the decision statistics at stage \( s \) can also be modeled as Gaussian random variable which includes Gaussian noise and cancelled interference, eq.(5.26) still holds and can be modified as

\[ N'_0 = \frac{4}{T} \cdot \text{Var}[Z_{k,i}^{(s)}], \quad (5.34) \]

Substituting eq.(5.34) and eq.(5.33) into eq.(5.25), the probability of pairwise event error for user \( k \) at stage \( s \) can be computed from

\[ P_{d_k}^{(s)} = Q \left[ d_H / 2\right]^{1/2} \cdot \left[ \frac{1}{2(rE_b / N_0)} \cdot \frac{1 - \frac{(K-1)^s}{3N}}{1 - \frac{K-1}{3N}} \right]^s \]
\[ + \frac{1}{(3N)^s} \cdot \left[ \frac{(K-1)^s}{K} \cdot \left( \sum_{\kappa=1}^{K} \frac{P_\kappa}{P_k} \right) + (-1)^s \right]^{-1/2}, \quad (5.35) \]

The upper bound of probability of bit error is used to evaluate the performance of
DS/CDMA systems using multistage interference cancellation combined with convolutional coding. This upper bound can be used for any stages of interference cancellation. As it is discussed in Chapter 3, for a rate $r = 1/n$ convolutional codes, the upper bound for probability of bit error of user $k$ with $s$ stages of interference cancellation is given by

$$P_{b_k}^{(s)} \leq \sum_{i=1}^{\infty} \sum_{d=d_f}^{\infty} i a(d,i) P_{d_k}^{(s)},$$  \hspace{1cm} (5.36)$$

where $i$ is the number of "1"s in its input data sequence to the encoder, $a(d,i)$ is the number of paths diverging from the all zero path at distance $d$ with $i$ "1"s in its data sequence over the unmerged segment, and $P_{d_k}$ can be found from eq.(5.27) and eq.(5.35).

The values of $a(d,i)$ depend on the particular convolutional coding structure and they can be obtained by tracing through the trellis diagram. Specifically, $a(d,i)$ have different values for coding structures with different constraint lengths and coding rates. A computer program has been developed to compute $a(d,i)$, and it is listed in Appendix. The performance bounds for different convolutional coding structures are different because of the unique set of values of $a(d,i)$ corresponding to a particular coding structure.

### 5.5 Analytical Results of Performance

The analytic results of performance of CDMA system using combined multistage interference cancellation with convolutional coding are studied for AWGN channel when the parameters of convolutional coding and multistage interference cancellation are changed. These results are also compared with the performance of multistage interference cancellation for uncoded CDMA systems.

Figure 5.5 shows the probability of bit error versus $E_b/N_0$ for CDMA systems with multistage interference cancellation and no error correction coding. Perfect power control is assumed. Processing gain $N = 62$ and the number of users $K = 10$. The frequency expansion is 62 times of the bandwidth of data bits. The second stage receiver with
interference cancellation achieves substantial performance improvement over the first stage with conventional receiver. The performance at the third stage achieves some additional improvement. Most of the performance improvement comes from the first two stages of interference cancellation.

Figure 5.6, 5.7 and 5.8 give probability of bit error versus $E_b/N_0$ for CDMA systems with the combination of multistage interference cancellation and convolutional coding for constraint length 3, 5 and 7 respectively. The code rate is $r = 1/2$. Perfect power control is assumed, $N = 31$ and $K = 10$. The frequency expansion due to convolutional coding and PN spreading is 62 times of the data bit bandwidth. This is the same as the frequency expansion in the uncoded CDMA system. We do not take advantage by occupying additional frequency band. The receiver using multistage interference cancellation combined with convolutional coding achieves significant performance improvement over the multistage interference cancellation receiver without using error correction coding at high $E_b/N_0$. The second stage receiver which has both convolutional coding and interference cancellation has better performance than the first stage receiver which only has convolutional coding at high $E_b/N_0$. The third stage receiver achieves additional performance improvement at high $E_b/N_0$ where receiver have both combination of interference cancellation and convolutional coding. The performance improves as the constraint length of convolutional code becomes longer at high $E_b/N_0$. The curves become steeper as the constraint length becomes longer.

Figure 5.9, 5.10 and 5.11 display probability of bit error versus $E_b/N_0$ for CDMA systems using combined multistage interference cancellation with convolutional coding for constraint length 3, 5 and 7 respectively. The code rate $r = 1/3$. Perfect power control is assumed, $N = 20$ and $K = 10$. The frequency expansion due to convolutional coding and PN spreading is 60 times of the data bit bandwidth, which is roughly the same as the frequency expansion in the cases shown in Figures 5.5-5.8. The performance in Figures 5.9-5.11 achieve significant performance gain than the performance in Figure 5.5. Comparing the performance of combined multistage interference cancellation and convolutional coding with different coding rate and the same constraint length respectively, the performance in Figure 5.9 is a little bit worse than the performance in Figure 5.6; the performance in Figure 5.10 is a little bit better than the performance in
Multiuser Detection for CDMA Systems with Convolutional Coding

Figure 5.7; the performance in Figure 5.11 is worse than the performance in Figure 5.8. This means that when the total frequency expansion due to coding and PN sequence spreading is fixed, the performance with lower coding rate may not guaranteed to be better than the performance with higher coding rate using the same constraint length. The receivers at higher number of stages gain performance improvement over the receivers at lower number of stages at high $E_b/N_0$.

Figure 5.12 shows the probability of bit error versus the number of users $K$ for CDMA systems using multistage interference cancellation without error correction coding, $N = 62$ and $E_b/N_0 = 10$ dB. Perfect power control is assumed. The probability of bit error increases as the number of users increases. The performance achieves significant improvement at the second stage and third stage receivers since multistage interference cancellation plays an important role at this high signal to noise ratio $E_b/N_0 = 10$ dB.

When $K = 50$, the probability of bit error is between $10^{-4}$ and $10^{-3}$ at the third stage.

Figure 5.13 demonstrates the probability of bit error versus the number of users $K$ for CDMA systems with the combination of multistage interference cancellation and convolutional coding, $N = 31$ and $E_b/N_0 = 10$ dB. The coding rate $r = 1/2$ and constraint length $\gamma = 5$. Perfect power control is assumed. The performance at the second stage receiver is significantly better than the performance at the first stage. The performance at the third stage achieves additional improvement. The performance at all three stages are better than the performance at corresponding stages in Figure 5.12 where only multistage interference cancellation is employed. When $K = 50$, the probability of bit error is between $10^{-5}$ and $10^{-4}$ which is an order of magnitude lower than it is if only multistage interference cancellation is used. The number of users we can accommodate increases when combination of multistage interference cancellation and convolutional coding is used.

Figure 5.14 plots the probability of bit error versus $E_b/N_0$ for CDMA systems using multistage interference cancellation without error correction coding, $N = 62$ and $K = 10$. Half of the users including the desired user are 6 dB higher in power than the other half. This is the near-far situation where the signals from interfering users are lower.
in power than the desired user. There is still performance improvement in the second stage receiver over the performance of the first stage receiver. The third stage achieves additional performance improvements. The performance at the first stage is better than the performance at the first stage when the powers of signals are equal from each user shown in Figure 5.5. The performance at the third stage is almost identical to the performance at the third stage when every user have the same signal power in Figure 5.5.

Figure 5.15 shows the probability of bit error versus $E_b/N_0$ for CDMA systems using the combination of multistage interference cancellation and convolutional coding, $N = 31$ and $K = 10$. Coding rate $r = 1/2$ and the constraint length $\gamma = 5$. Half of the users including the desired user are 6 dB higher in power than the other half. This is the near-far environment where the signals from the interfering users are lower in power than the desired user. The performance for all three stage receivers are better than the corresponding performances in Figure 5.14 where only multistage interference cancellation is used at high $E_b/N_0$. The performance at the second stage receiver is better than the performance at the first stage at high $E_b/N_0$. There is additional performance improvement in the third stage receiver at high $E_b/N_0$. Compared with the situation when every user have the same signal power shown in Figure 5.7, the performance at the first stage is better than the performance at the first stage in Figure 5.7, but the performance at the third stage is almost identical to the performance at the third stage in Figure 5.7. Thus, this receiver is near far resistant.

Figure 5.16 displays the probability of bit error versus $E_b/N_0$ for CDMA systems using multistage interference cancellation without error correction coding, $N = 62$ and $K = 10$. Half of the users including the desired user are 6 dB lower in power than the other half. This is the near-far situation where the desired user is lower in power than the interfering users. There is huge performance improvement in the second stage receiver than the performance in the first stage. The third stage receiver achieves some additional performance improvement. Since there is no interference cancellation at the first stage, the performance at the first stage receiver is worse than the performance at first stage in Figure 5.14 where the signal of the desired user is stronger than the signal from the interfering users. But the performance at the third stage receivers in both Figure 5.14 and 5.16 are almost identical. Compared with the situation when every user have the same signal power.
shown in Figure 5.5, the performance at the first stage is worse than the performance at the first stage shown in Figure 5.5, but the performance at the third stage is almost identical to the performance at the third stage in Figure 5.5. The multistage interference cancellation technique is near-far resistant.

Figure 5.17 shows the probability of bit error versus $E_b/N_0$ for CDMA systems with combined multistage interference cancellation and convolutional coding. $N = 31$ and $K = 10$. Coding rate $r = 1/2$ and the constraint length $\gamma = 5$. Half of users including the desired user are 6 dB lower in power than the other half. This is the near-far situation where the signal from the desired user is lower in power than the interfering users. Because the powers of interfering users are stronger than the power of the desired user, there is significant gain in performance in the second stage receiver over the performance in the first stage receiver at high $E_b/N_0$. There is some additional gain in the third stage at high $E_b/N_0$. The performance at all three stages are better than the corresponding performance in Figure 5.16 where only the interference cancellation is used at high $E_b/N_0$. The performance at the first stage is also worse than the performance at the first stage in Figure 5.15 where the powers of the interfering users are lower than the power of the desired user. But the performance at the third stage receivers at both Figure 5.17 and 5.15 are nearly the same. Compared with the situation when every user have the same signal power shown in Figure 5.7, the performance at the first stage is worse than the performance at the first stage in Figure 5.7 but the performance at the third stage is almost identical to the performance at the third stage in Figure 5.7. The performance is identical no matter what the power levels of interfering users are. This means that the combination of multistage interference cancellation and convolutional coding is near-far resistant. The techniques yield performance gain over the scheme where only multistage interference cancellation technique or only convolutional coding is used under near-far situation.

The probability of bit error performance of CDMA system using the combination of multistage interference cancellation and convolutional coding is superior to the performance of the system using either multistage interference cancellation or convolutional coding only at high signal to noise ratio, which is in the reasonable signal to noise ratio range for practical CDMA system implementation. There is a threshold point on the probability of bit error versus $E_b/N_0$ curve, where when $E_b/N_0$ is lower than this
point, the probability of bit error of the combination of convolutional coding and multistage interference cancellation is higher than the probability of bit error of the multistage interference cancellation without error correction coding. When $E_b/N_0$ is higher than this point, the probability of bit error of the combination of convolutional coding and multistage interference cancellation is lower than the probability of bit error of the multistage interference cancellation without error correction coding. This threshold point is influenced by the convolutional coding structure.

There is another threshold point on the probability of bit error versus $E_b/N_0$ curve of combined multistage interference cancellation with convolutional coding, such that when $E_b/N_0$ is lower than this point, the probability of bit error at the second stage receiver receiver is higher than the probability of bit error at the first stage receiver, and the probability of bit error at the third stage receiver is higher than the probability of bit error at the second stage receiver. When $E_b/N_0$ is higher than this point, the probability of bit error at the second stage receiver is lower than the probability of bit error at the first stage, and the probability of bit error at the third stage receiver is lower than the probability of bit error at the second stage. This threshold point is influenced by the multistage interference cancellation.

In the curves presented in this research, both of these two threshold points are at very low $E_b/N_0$. Most of times, this threshold is below 0 dB and the maximum value observed is around 1 dB. Therefore, for practical implementation at reasonable range of $E_b/N_0$, the system using combination of multistage interference cancellation with convolutional coding has better performance than the system using either multistage interference cancellation or convolutional coding only.
5.6 Chapter Summary

In this chapter, we have formulated an analytic model for the use of multistage interference cancellation in conjunction with error correction coding. We used this model to derive analytic results for the performance of these combined systems. In Chapter 6, we study the performance of combined multistage interference cancellation and error correction coding using simulation.
Figure 5.5: Probability of Bit Error vs. Eb/No. Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, assuming perfect power control, PN processing gain $N = 62$, number of users $K = 10$. 
Figure 5.6: Probability of Bit Error vs. Eb/No, Analytical results of combined multi-stage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 3$, number of users $K = 10$. 

Chapter 5 Analysis of Multiuser Detection for CDMA Systems with Convolutional Coding
Figure 5.7: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 5.8: Probability of Bit Error vs. Eb/No. Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 7$, number of users $K = 10$. 
Figure 5.9: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 20$, coding rate $r = 1/3$, constraint length $\gamma = 3$, number of users $K = 10$. 
Figure 5.10: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 20$, coding rate $r = 1/3$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 5.11: Probability of Bit Error vs. Eb/No. Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 20$, coding rate $r = 1/3$, constraint length $\gamma = 7$, number of users $K = 10$. 
Figure 5.12: Probability of Bit Error vs. Number of Users. Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, assuming perfect power control, PN processing gain $N = 62$, $Eb/No = 10$ dB.
Figure 5.13: Probability of Bit Error vs. Number of Users, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN processing gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, Eb/No = 10 dB.
Figure 5.14: Probability of Bit Error vs. Eb/No, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, half of the users including the desired user are 6 dB higher in power than the other half, PN processing gain $N = 62$, number of users $K = 10$. 
Figure 5.15: Probability of Bit Error vs. Eb/No, Analytical results of combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, half of the users including the desired user are 6 dB higher in power than the other half, PN processing gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 5.16: Probability of Bit Error vs. Eb/No, Analytical results of multistage interference cancellation for DS/CDMA system without error correction coding in AWGN channel, half of the users including the desired user are 6 dB lower in power than the other half, PN processing gain $N = 62$, number of users $K = 10$. 
Figure 5.17: Probability of Bit Error vs. Eb/No, Analytical results of combined multi-stage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, half of the users including the desired user are 6 dB lower in power than the other half, PN processing gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, number of users $K = 10$. 
Chapter 6

Simulation of Multiuser Detection for CDMA Systems with Convolutional Coding

6.1 Introduction

In the previous chapter, we introduced two schemes for integrating convolutional coding with multistage interference cancellation, and obtains closed form analytic expressions for the probability of bit error of one of those schemes. In this chapter we investigate the performance of both schemes using simulation techniques. The transmitter of DS/CDMA system with convolutional coding, the communication channel, and receivers using either of the two detection schemes of combined multistage interference cancellation with convolutional coding are implemented in software to conduct Monte Carlo simulations. This provides a platform for studying the performance of these new techniques in different communication environments by changing the parameters in the simulations. Because of the complexity of communication systems, it is difficult for the analytical method to model all of the details of the communication systems. The convolutional coding and multistage interference cancellation techniques employed in the new detection algorithm will introduce memories and nonlinearities into the system respectively.

The simulation results of the first scheme will provide a comparison with the analytical results we obtained for the first scheme in Chapter 5. It also gives more insights than the analytical results.

In the second scheme where the multistage interference cancellation technique is more integrated with convolutional coding, the combined effect of these two techniques are more complicated than in the first scheme. Therefore, we rely on the simulation technique to evaluate the performance of the second scheme.
6.2 Simulation Approach

There are many approaches to simulate the communication systems, including RF simulation, IF simulation, and baseband simulation. Baseband simulation is used in this project, so that we do not need to simulate the high carrier frequency component which requires very high sampling rate. Generally speaking, the bandpass signal encountered in radio communication can be represented using the baseband complex envelope form of the signal in the following way.

Suppose $s(t)$ is a signal with frequency content in a band of frequencies in the vicinity of a frequency $f_c$ that can be expressed as

$$s(t) = a(t) \cos(\omega_c t + \theta(t)), \quad (6.1)$$

where $a(t)$ is the amplitude of $s(t)$, $\theta(t)$ is the phase of $s(t)$, $\omega_c = 2\pi f_c$ is the carrier frequency. This expression can be expanded to

$$s(t) = a(t) \cos \omega_c t \cos \theta(t) - a(t) \sin \theta(t) \sin \omega_c t, \quad (6.2)$$

Let $s_I(t) = a(t) \cos \theta(t)$ and $s_Q(t) = a(t) \sin \theta(t)$, then eq.(6.2) can be denoted as

$$s(t) = s_I(t) \cos \omega_c t - s_Q(t) \sin \omega_c t, \quad (6.3)$$

where $s_I(t)$ and $s_Q(t)$ are called the in-phase and quadrature components of $s(t)$ respectively. The frequency content of $s_I(t)$ and $s_Q(t)$ concentrates at low frequencies, hence these components are appropriately called lowpass signals. The representation of $s(t)$ can be written as

$$s(t) = \text{Re}[\tilde{s}(t)e^{j\omega_c t}], \quad (6.4)$$

where $\tilde{s}(t) = s_I(t) + js_Q(t) = a(t)e^{j\theta(t)}$ is defined as the complex envelope of $s(t)$. Therefore the signal $s(t)$ can be completely represented by its baseband complex envelope signal $\tilde{s}(t)$.
6.2.1 Block Diagrams of Simulation

The block diagram of simulation of the first scheme is described by Figure 6.1. The data bits are generated. It goes through the convolutional encoder and interleaver. Then the code bits are spread and modulated. This process is carried out for all of the users in the CDMA system whose signals are summed up to form the transmitted signal. After going through the channel, the received signal is demodulated at the first stage by a bank of conventional receivers to obtain the sufficient statistics. Then multistage interference cancellation is performed. The decision statistics are decoded using Viterbi decoder and the recovered bits are compared with the original data bits to calculate the bit error rate.

Figure 6.1 Block Diagram of the First Scheme of Combined Multistage Interference Cancellation and Convolutional Coding

Figure 6.2 shows the simulation block diagram of the second scheme. The implementation of the transmitter is exactly the same as the one in the first scheme. At the receiver, the sufficient statistics is obtained from a bank of conventional receivers and it is deinterleaved and decoded by Viterbi decoder. The estimated data bits are re-encoded and
respread to generate the transmitted signals of every user which are used for interference cancellation in the next step. The decision statistics is decoded by Viterbi decoder to recover the transmitted bits. This process of regenerating the transmitted signals, interference cancellation, and Viterbi decoding is repeated in a preset number of stages. Finally the recovered data bits is compared with the transmitted data bits to calculate the bit error rate.

Figure 6.2 Block Diagram of The Second Scheme of Combined Multistage Interference Cancellation with Convolutional Coding
6.2.2 Implementation of Transmitter

The random binary data bits are generated as the transmitted data bits $b_k(t)$. They are sent to the convolutional encoder to get the code bits $c_k(t)$. The generating functions of convolutional code used in simulation are the same as those used in analytical approach.

The transmitted signal of the $k$th user given in eq.(5.5) can be expressed in the complex envelope form

$$s_k(t) = Re[\sqrt{2P_k}a_k(t)c_k(t)e^{j(o_k t + \theta_k)}],$$  \hspace{1cm} (6.5)

$$= Re[\tilde{s}_k(t)e^{j(o_k t)}],$$

where $\tilde{s}_k(t)$ is the complex envelope of the transmitted signal $s_k(t)$ and it can be found by

$$\tilde{s}_k(t) = \sqrt{2P_k}a_k(t)c_k(t)e^{j\theta_k},$$  \hspace{1cm} (6.6)

$$= \sqrt{2P_k}a_k(t)c_k(t)\cos(\theta_k) + j\sqrt{2P_k}a_k(t)c_k(t)\sin(\theta_k).$$

This baseband envelope can be separated into inphase and quadrature parts which are given by eq.(6.7) and (6.8) respectively

$$\tilde{s}_{k, I}(t) = \sqrt{2P_k}a_k(t)c_k(t)\cos(\theta_k),$$  \hspace{1cm} (6.7)

$$\tilde{s}_{k, Q}(t) = \sqrt{2P_k}a_k(t)c_k(t)\sin(\theta_k),$$  \hspace{1cm} (6.8)

The data bits which is transmitted through the channel is generated randomly. After convolutional coding and interleaving, we get the code bit sequences. The PN sequence used in this simulation is unique for every user and is random binary bits which gives the average performance among the commonly used PN sequences. The PN sequence has period $N$ which repeat every data bit. The data sequence after PN spreading is modulated by BPSK modulation. The signal is sampled at a rate of $N_s = 4$ samples per chip. The data bits transmitted are grouped into frames whose length is 184 bits per frame in this simulation in order to be consistent with IS-95 standard. The signals from every users are summed up asynchronously to form the transmitted signal.
6.2.3 Implementation of the Channel

The additive white Gaussian channel is used in this simulation. Therefore the white Gaussian noise is added to the transmitted signal. The received signal is given by

\[ r(t) = s(t) + n(t), \quad (6.9) \]

\[ \tilde{r}(t) = \tilde{s}(t) + \tilde{n}(t), \quad (6.10) \]

\[ = \frac{1}{2} \cdot \sum_{k=1}^{K} \sqrt{2P_k a_k(t - \tau_k) c_k(t - \tau_k)} e^{j\phi_k} + \tilde{n}(t), \]

where \( \phi_k = [\theta_k - \omega t \tau_k] \mod 2\pi \), \( \tilde{r}(t) \) is the baseband complex envelope of the received signal, \( \tilde{n}(t) \) is the baseband complex envelope of Gaussian noise. The received signal has its inphase and quadrature parts which are given by eq.(6.11) and (6.12) respectively

\[ \tilde{r}_I(t) = \frac{1}{2} \cdot \sum_{k=1}^{K} \sqrt{2P_k a_k(t - \tau_k) c_k(t - \tau_k)} \cos(j\phi_k) + \tilde{n}_I(t), \quad (6.11) \]

\[ \tilde{r}_Q(t) = \frac{1}{2} \cdot \sum_{k=1}^{K} \sqrt{2P_k a_k(t - \tau_k) c_k(t - \tau_k)} \sin(j\phi_k) + \tilde{n}_Q(t), \quad (6.12) \]

where \( \tilde{n}_I(t) \) and \( \tilde{n}_Q(t) \) are the in-phase and quadrature parts of the baseband Gaussian noise envelope respectively.

The noise effect is quantified by the signal to noise ratio the communication system experienced. To simulate the signal to noise ratio effect, one can either change the power of the signal according to the signal to noise ratio while fixing the noise power, or one can change the noise power according to the signal to noise ratio while keeping the power of signal fixed. In this simulation, the power of the signal is normalized to unit value and the power of noise is changed according to the signal to noise ratio. The mean of the AWGN is zero. The power of noise which is equal to the variance of AWGN is given by

\[ \sigma^2 = \frac{N_0}{2} \cdot 2B_n = N_0 \cdot B_n, \quad (6.13) \]

where \( N_0/2 \) is the two sided power spectral density of the noise process, \( B_n \) is the noise
bandwidth of the system. In a digital communication system, we have the signal to noise ratio given by

\[ SNR = \frac{E_b}{N_0} = \frac{PT_bB_n}{\sigma^2}, \quad (6.14) \]

where \( P \) is the power of the data bit signal, \( T_b \) is duration of data bits. The following relation holds if there is convolutional coding, PN sequence spreading and chip sampling in the system

\[ T_b = \frac{T_{code}}{r} = \frac{N \cdot T_{chip}}{r} = \frac{N_s \cdot N \cdot T_s}{r}, \quad (6.15) \]

where \( r \) is the coding rate, \( N \) is the PN sequence processing gain, \( N_s \) is the number of samples per chip, \( T_{code} \) is the duration of code bits, \( T_{chip} \) is the duration of PN chips, \( T_s \) is the duration of the sample. The bandwidth of the system with sampling is

\[ B_n = \frac{1}{2T_s}, \quad (6.16) \]

Substituting eq.(6.16) and (6.15) to eq.(6.14), we have the variance of noise

\[ \sigma^2 = \frac{PNN_s}{2 \cdot r(E_b/N_0)}, \quad (6.17) \]

Normalizing the power of data bit signal to 1, the variance of noise is

\[ \sigma^2 = \frac{NN_s}{2 \cdot r(E_b/N_0)} \quad (6.18) \]

The change of noise variance is associated with the change of signal to noise ratio.

6.2.4 Implementation of the Detection Receiver

At the first stage, the in-phase component of the received signal \( \tilde{r}_I(t) \) and the quadrature component of the received signal \( \tilde{r}_Q(t) \) are correlated with the synchronized replica of all the user's PN sequences. It is assumed that the receiver can get the phase and delay
information through the synchronization process. The sufficient statistics are the decision statistics from a bank of conventional receivers which is given by

\[ Z_{k,i}^{(1)} = \int_{iT + \tau_k}^{(i+1)T + \tau_k} r(t) a_k(t - \tau_k) \cos(\omega_c t + \phi_k) dt \] (6.19)

\[ = \int_{iT + \tau_k}^{(i+1)T + \tau_k} [\tilde{r}_I(t) \cos(\omega_c t) - \tilde{r}_Q(t) \sin(\omega_c t)] a_k(t - \tau_k) \]

\[ \times [\cos(\phi_k) \cos(\omega_c t) - \sin(\phi_k) \sin(\omega_c t)] dt \]

\[ = \frac{1}{2} \int_{iT + \tau_k}^{(i+1)T + \tau_k} [\tilde{r}_I(t) \cos(\phi_k) + \tilde{r}_Q(t) \sin(\phi_k)] a_k(t - \tau_k) dt, \]

where the in-phase and quadrature components of the decision statistics are given by

\[ Z_{k,i,I}^{(1)} = \frac{1}{2} \int_{iT + \tau_k}^{(i+1)T + \tau_k} \tilde{r}_I(t) a_k(t - \tau_k) \cos(\phi_k) dt, \] (6.20)

\[ Z_{k,i,Q}^{(1)} = \frac{1}{2} \int_{iT + \tau_k}^{(i+1)T + \tau_k} \tilde{r}_Q(t) a_k(t - \tau_k) \sin(\phi_k) dt, \] (6.21)

The sum of in-phase and quadrature components of decision statistics forms the decision statistics

\[ Z_{k,i}^{(1)} = Z_{k,i,I}^{(1)} + Z_{k,i,Q}^{(1)}, \] (6.22)

The implementations of the two schemes are different after the sufficient statistics is obtained. For the first scheme, multistage interference cancellation on the convolutional encoded bits is conducted. At the second stage, the sufficient statistics \( Z_{k,i}^{(1)} \) are multiplied by \( 2/T \) to form an unbiased estimate of every user's signal amplitude. The quantity \( 2Z_{k,i}^{(1)}/T \) is used to reconstruct the in-phase and quadrature components of the transmitted signals of all users.
\[ \tilde{s}_{k,I}^{(1)}(t) = a_k(t) \cdot \sum_{i = -\infty}^{\infty} \frac{2Z_{k,i}^{(1)}}{T} \cdot p_T(t - iT) \cos(\theta_k), \]  
(6.23)

\[ \tilde{s}_{k,Q}^{(1)}(t) = a_k(t) \cdot \sum_{i = -\infty}^{\infty} \frac{2Z_{k,i}^{(1)}}{T} \cdot p_T(t - iT) \sin(\theta_k), \]  
(6.24)

These estimates are used in the interference cancellation which is implemented as

\[ \tilde{r}_{k,I}^{(1)}(t) = \tilde{r}_I(t) - \sum_{\kappa = 1, \kappa \neq k}^{K} \tilde{s}_{k,I}^{(1)}(t), \]  
(6.25)

\[ \tilde{r}_{k,Q}^{(1)}(t) = \tilde{r}_Q(t) - \sum_{\kappa = 1, \kappa \neq k}^{K} \tilde{s}_{k,Q}^{(1)}(t), \]  
(6.26)

The new received signal of each user is sent to a correlation receiver dedicated to the user itself at the second stage instead of a bank of receivers. The decision statistics at the second stage is obtained in the way that is similar to what is given in eq.(6.19), (6.20) and (6.21). The reconstructed signals can be established as given in eq.(6.23) and (6.24). The interference cancellation is performed at the second stage. This process is repeated until the predetermined number of stages is finished. The decision statistics from the final stage is sent to a Viterbi decoder to recover the estimates of transmitted data bits.

In the second scheme, the sufficient statistics is sent to the Viterbi decoder and get the estimates of the transmitted data bits. Then the loop of multistage implementation begins. The estimates of data bits are sent to the convolutional encoder, PN sequence spreader, and modulator in order to reconstruct the transmitted signals for each user. The interference is cancelled according to eqs.(6.25) and (6.26). The new received signal for each user is sent to the receiver at the second stage to get the new decision statistics as that is given in eq.(6.19), (6.20) and (6.21). The new decision statistics is sent to the Viterbi decoder to recover the transmitted data bits. This process is repeated until the preset number of stages is finished.

For both of these schemes, the bit error rate is calculated by comparing the recovered data bits with the original data bits.
6.3 Simulation Results

In this section, simulation results of CDMA system using combined multistage interference cancellation and convolutional coding in AWGN channel are presented. The performances of these two schemes under different communication environments are studied. The performances of systems using different coding structures are investigated. Perfect power control is implemented in all the simulations.

Figure 6.3 shows the BER versus $E_b/N_0$ for CDMA system using multistage interference cancellation without error correction coding. Processing gain is $N = 62$ and the number of users is $K = 10$. The frequency expansion is 62 times of the data bit bandwidth. The receiver at the second stage gains performance improvement over the first stage conventional receiver. The third stage receiver gets additional performance gain. It is noted that the performance at all three stage receivers agree well with the performances at corresponding stages obtained from analytical method in Chapter 5 and presented in Figure 5.5.

Figure 6.4, 6.5 and 6.6 present BER versus $E_b/N_0$ for CDMA system using the combination of multistage interference cancellation and convolutional coding with constraint lengths 3, 5 and 7 respectively. The first scheme is implemented, and the PN processing gain is $N = 31$ and the number of users is $K = 10$. The coding rate used is $r = 1/2$. The total frequency expansion is 62 times of data bit bandwidth. The performance at the second stage receiver where both multistage interference cancellation and convolutional coding are used yields significant performance improvement over the first stage receiver which only uses convolutional coding at high $E_b/N_0$. The third stage receiver achieves an additional performance gain at high $E_b/N_0$. The performance of receivers at all stages using combined multistage interference cancellation and convolutional coding are better than the performance of receivers at corresponding stages using only multistage interference cancellation at high $E_b/N_0$. The performance improves as the constraint length becomes longer at high $E_b/N_0$. The curves becomes steeper as the constraint length becomes longer. Due to the limited number of transmitted
data bits, few errors are accumulated at high $E_b/N_0$, and it is difficult to simulate BER below $10^{-5}$. The performance obtained from simulation in Figure 6.4-6.6 are very close to the performance predicted from analytical method in Figure 5.6-5.8 at signal to noise ratio above 3 dB. Since the analytical result is the upper bound of probability of bit error, the analytical results in Figures 5.6-5.8 are a little bit higher than the results of simulation in Figures 6.4-6.6. The analytical bound on probability of bit error is loose when $E_b/N_0$ belows 1 to 2 dB.

Figures 6.7, 6.8 and 6.9 display BER versus $E_b/N_0$ for CDMA system using combined multistage interference cancellation and convolutional coding with constraint lengths 3, 5 and 7 respectively. The first scheme is implemented. Coding rate is $r = 1/3$, processing gain is $N = 20$ and the number of users is $K = 10$. The receivers at higher number of stages gain performance improvement over the receivers at lower number of stages at high $E_b/N_0$. The performance in Figure 6.7 is a little bit worse than the performance in Figure 6.4; the performance in Figure 6.8 is a little bit better than the performance in Figure 6.5; the performance in Figure 6.9 is about the same as the performance in Figure 6.6. When the total frequency expansion is fixed due to error correction coding and PN spreading, the performance of lower coding rate is not guaranteed to be better than the performance of higher coding rate for the same constraint length. This confirms the conclusion we obtained from the analytical approach. The analytical results in Figures 5.9-5.11 closely bound the simulation results in Figures 6.7-6.9 when $E_b/N_0$ is above 3 dB. The analytical bound on probability of bit error rate is loose when $E_b/N_0$ belows 1 or 2 dB.

Figures 6.10, 6.11 and 6.12 present BER versus $E_b/N_0$ for CDMA system using the combination of multistage interference cancellation and convolutional coding with constraint lengths 3, 5 and 7 respectively. The second scheme is implemented. The processing gain is $N = 31$ and the number of users is $K = 10$. The coding rate is $r = 1/2$. The total frequency expansion is 62 times of data bit bandwidth. The performance at the second stage receiver where both multistage interference cancellation and convolutional coding are used yields significant performance improvement over the first stage receiver which only uses convolutional coding at high $E_b/N_0$. The third stage
receiver obtains additional performance gain at high $E_b/N_0$. The performance of receivers at all stages using combined multistage interference cancellation and convolutional coding are better than the performance of receivers at corresponding stages using only multistage interference cancellation at high $E_b/N_0$. The performance improves at high $E_b/N_0$ as the constraint length becomes longer. The curves becomes steeper as the constraint length becomes longer. Due to the limited number of transmitted data bits, few errors are accumulated at high $E_b/N_0$. Comparing the results on Figures 6.10-6.12 with the corresponding results on Figures 6.4-6.6, they are almost the same. The performance of the first scheme and the second scheme are almost identical. The performance of the first scheme at the third stage receiver is a little better than the performance of the second scheme at the third stage receiver. This is encouraging because the complexity of implementing scheme 2 which fully integrates interference cancellation and coding would be greater than the complexity of implementing scheme 1.

Figures 6.13, 6.14 and 6.15 present BER versus $E_b/N_0$ for CDMA system using combined multistage interference cancellation and convolutional coding with constraint length 3, 5 and 7 respectively. The second scheme is implemented, with $r = 1/3$, $N = 20$ and $K = 10$. The frequency expansion due to coding and PN sequence spreading is 60 times of the data bandwidth which is approximately the same as the frequency expansion of multistage interference cancellation in Figure 6.3 which is 62 times of the data bandwidth. The receiver with higher number of stages of interference cancellation obtains performance improvement over the receivers with lower number of stages of interference cancellation at high $E_b/N_0$. The performance in Figures 6.13-6.14 is a little bit better than the performance in Figures 6.10-6.11; the performance in Figure 6.15 is a little bit worse than the performance in Figure 6.12. As in the first scheme, when the total frequency expansion is fixed due to error correction coding and PN sequence spreading, the performance of lower coding rate is not guaranteed to be better than the performance of higher coding rate for the same constraint length. The results on Figures 6.13-6.15 are very close to the corresponding results on Figures 6.7-6.9, indicating that the performance of the first and second scheme are almost the same. The performance of the first scheme at the third stage receiver is a little better than the performance of the second scheme at the third stage receiver.
Figure 6.16 shows the BER versus number of users \( K \) for CDMA system using combined multistage interference cancellation and convolutional coding. The first scheme is implemented, with \( N = 31 \), \( E_b/N_0 = 10 \) dB, \( r = 1/2 \) and \( \gamma = 5 \). The performance at the second stage receiver is significantly better than the performance at the first stage. The third stage receiver obtains additional performance improvement. When \( K = 65 \), the BER is between \( 10^{-3} \) and \( 10^{-2} \) at the third stage.

Figure 6.17 shows the BER versus number of users \( K \) for CDMA system using combined multistage interference cancellation and convolutional coding. The second scheme is implemented, with \( N = 31 \), \( E_b/N_0 = 10 \) dB, \( r = 1/2 \) and \( \gamma = 5 \). The performance at the second stage receiver is significantly better than the performance at the first stage. The third stage receiver obtains additional performance improvement. When \( K = 65 \), the BER is between \( 10^{-3} \) and \( 10^{-2} \) at the third stage. The performance at all three stages are almost identical to the performance at corresponding stages in figure 6.16. The performance of the first scheme is a little better than the performance of the second scheme at the third stage receiver.

Figure 6.18 gives a comparison of analytical results and simulation results for the first scheme of combined multistage interference cancellation and convolutional coding. PN spreading gain \( N = 31 \), coding rate \( r = 1/2 \), constraint length \( \gamma = 3 \), and the number of users \( K = 10 \). For signal to noise ratio greater than 4 dB, the analytical result matches simulation result well. Since Gaussian approximation of the decision statistic is used in the analysis, the analytical results are optimistic at low bit error rate region.

The performance of CDMA system using combined multistage interference cancellation with convolutional coding is significantly better than the performance of the system using either multistage interference cancellation or convolutional coding only at high signal to noise ratio. This signal to noise ratio range where the new system achieves performance improvement is reasonable for practical CDMA system implementation. There is a threshold point on the BER versus \( E_b/N_0 \) plot, and when \( E_b/N_0 \) is lower than this point the BER of the combined multistage interference cancellation and convolutional coding is higher than the BER of the multistage interference cancellation without error correction coding. When \( E_b/N_0 \) is higher than this point the BER of the combination of multistage
interference cancellation and convolutional coding is lower than the BER of multistage interference cancellation with no error correction coding. This point is affected by the coding structure of convolutional coding.

There is second threshold point on the BER versus $E_b/N_0$ curve of the combined multistage interference cancellation and convolutional coding. When $E_b/N_0$ is lower than this point the BER of the second stage receiver stage receiver is higher than the BER of the first stage receiver; the BER of the third stage receiver stage receiver is higher than the BER of the second stage receiver. When $E_b/N_0$ is higher than this point the BER at the second stage receiver is lower than the BER of the first stage receiver, the BER at the third stage receiver is lower than the BER of the second stage receiver. This threshold is affected by the multistage interference cancellation.

In our simulation results, both of these two threshold points are at very low $E_b/N_0$, most of the time it is below 0 dB and the maximum value observed is around 1 dB. Therefore, for practical implementation at reasonable range of $E_b/N_0$, the combination of multistage interference cancellation and convolutional coding gain performance improvement over either multistage interference cancellation or convolutional coding only.

From the simulation results, the performance of the first and the second scheme is almost identical. The performance of the first scheme is a little bit better than the performance of the second scheme at the third stage receiver.

### 6.4 Chapter Summary

The simulation models of two schemes for the combined multistage interference cancellation with convolutional coding are given in this chapter. The simulation results for these schemes are obtained.
Figure 6.3: BER vs. Eb/No, Simulation results of multistage interference cancellation for DS/CDMA system without using error correction coding in AWGN channel, assuming perfect power control, PN processing gain $N = 62$, number of users $K = 10$. 
Figure 6.4: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 3$, number of users $K = 10$. 
Figure 6.5: BER vs. Eb/No. Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 6.6: BER vs. Eb/No. Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 7$, number of users $K = 10$. 
Figure 6.7: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 20, coding rate $r = 1/3$, constraint length $\gamma = 3$, number of users $K = 10$. 
Figure 6.8: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 20$, coding rate $r = 1/3$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 6.9: BER vs. Eb/No, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 20, coding rate $r = 1/3$, constraint length $\gamma = 7$, number of users $K = 10$. 
Figure 6.10: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 3$, number of users $K = 10$. 
Figure 6.11: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 6.12: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain \( N = 31 \), coding rate \( r = 1/2 \), constraint length \( \gamma = 7 \), number of users \( K = 10 \).
Figure 6.13: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 20, coding rate $r = 1/3$, constraint length $\gamma = 3$, number of users $K = 10$. 
Figure 6.14: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain $N = 20$, coding rate $r = 1/3$, constraint length $\gamma = 5$, number of users $K = 10$. 
Figure 6.15: BER vs. Eb/No, Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 20, coding rate $r = 1/3$, constraint length $\gamma = 7$, number of users $K = 10$. 
Figure 6.16: BER vs. The number of users, Simulation results of the first scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 31, coding rate $r = 1/2$, constraint length $\gamma = 5$, $E_b/N_0 = 10$ dB.
Figure 6.17: BER vs. The number of users. Simulation results of the second scheme, combined multistage interference cancellation and convolutional coding for DS/CDMA system in AWGN channel, assuming perfect power control, PN spreading gain N = 31, coding rate $r = 1/2$, constraint length $\gamma = 5$, Eb/No = 10 dB.
Figure 6.18: Probability of Bit Error vs. Eb/No, comparison of analytical results and simulation results for the first scheme, combined multistage interference cancellation and convolutional coding, PN spreading gain $N = 31$, coding rate $r = 1/2$, constraint length $\gamma = 3$, the number of users $K = 10$. 
Chapter 7

Conclusions

7.1 Summary

In an effort to increase the capacity of CDMA systems, several multiuser detection techniques have been proposed recently. A literature survey on multiuser detection receivers presented in Chapter 4 introduced the major multiuser detection schemes being proposed. Multistage interference cancellation technique is studied in this thesis.

Error correction coding is a commonly used technique to combat interference and fading in CDMA communication systems. In order to combine the benefits of convolutional coding and multistage interference cancellation, and to further improve the performance achieved in CDMA systems using multistage interference cancellation technique, a combined multistage interference cancellation and convolutional coding technique is proposed and analyzed for the AWGN channel in Chapter 5. A simulation technique is used to evaluate two different approaches to this proposed receiver for the AWGN channel in Chapter 6.

The first scheme for combined multistage interference cancellation and convolutional coding is analytical tractable. A transfer function bound on probability of bit error is used to evaluate the performance of the first scheme. The performance of CDMA system using combined multistage interference cancellation and convolutional coding is superior to the performance of CDMA system using either interference cancellation or error correction coding alone at high signal to noise ratio. There is a threshold point on the probability of bit error versus $E_b/N_0$ curves. If the $E_b/N_0$ is lower than this point, the probability of bit error of combination of multistage interference cancellation and convolutional coding is higher than the probability of bit error of multistage interference cancellation without error correction coding. On the other hand, if $E_b/N_0$ is higher than this point, the probability of bit error of the combination of multistage interference cancellation and
convolutional coding is lower than the probability of bit error of multistage interference cancellation without error correction coding. This threshold occurs at extremely low values of \( E_b/N_0 \).

The performance of a CDMA system using combination of multistage interference cancellation and convolutional coding improves as the number of stages of multistage interference cancellation increases. The performance of the second stage receiver which uses the combination of interference cancellation and convolutional coding is significantly better than the performance of the first stage receiver which uses the convolutional coding without interference cancellation at high \( E_b/N_0 \). The third stage achieves some additional performance improvement at high \( E_b/N_0 \). There is another threshold point on the probability of bit error versus \( E_b/N_0 \) curves of combined multistage interference cancellation with convolutional coding, such that when \( E_b/N_0 \) is lower than this point the probability of bit error at the second stage receiver is higher than the probability of bit error at the first stage receiver, and the probability of bit error at the third stage receiver is higher than the probability of bit error at the second stage receiver. When \( E_b/N_0 \) is higher than this point the probability of bit error at the second stage is lower than the probability of bit error at the first stage, and the probability of bit error at the third stage is lower than the probability of bit error at the second stage.

The signal to noise ratio where \( E_b/N_0 \) is higher than both of these threshold points lies in a signal to noise range which is reasonable for practical CDMA system implementation.

The performance and complexity of a CDMA system using combination of multistage interference cancellation and convolutional coding depends on the structure of convolutional encoder employed. At reasonable signal to noise ratio, the performance of a system using a longer constraint length is better than a system using a shorter constraint length for the same coding rate. When the total frequency expansion due to error correction coding and PN sequence spreading is fixed, the performance of system using lower rate coding is not guaranteed to be better than the performance of system using higher rate coding for the same constraint length. There is an optimal coding rate and associated PN spreading gain which achieves the best performance. The complexity of the system increases as the constraint length increases. The complexity of the system increases when the coding rate decreases. It is important for practical implementation to
choose a code rate and constraint length whose performance can satisfy the performance requirements with a complexity which is not too high to implement using available hardware.

The performance in terms of probability of bit error versus $E_b/N_0$ obtained from analytical method closely bounds the performance in terms of BER versus $E_b/N_0$ obtained from simulation method at reasonable $E_b/N_0$. When $E_b/N_0$ belows 1 or 2 dB, the analytical bound on probability of bit error is loose.

From the analytical and simulation results, the performance of a CDMA system using combined multistage interference cancellation and convolutional coding is better than the performance of CDMA system using interference cancellation without error correction coding at high signal to noise ratio.

From the analytical and simulation results, the performance of a CDMA system using combination of multistage interference cancellation and convolutional coding increases as the number of interference cancellation stage increases. The performance of the second stage receiver which uses the combination of interference cancellation and convolutional coding is significantly better than the performance of the first stage receiver which uses the convolutional coding without interference cancellation at high $E_b/N_0$. The third stage achieves additional performance improvement at high $E_b/N_0$.

The analytical and simulation results also show that the signal to noise ratio where $E_b/N_0$ is higher than the threshold points lies in the reasonable signal to noise ratio range for practical CDMA system implementation.

The performance in terms of BER versus $E_b/N_0$ curves obtained from simulation method of the first scheme and the second scheme of combination of multistage interference cancellation and convolutional coding are almost identical to each other. The performance of the first scheme is a little better than the performance of the second scheme at the third stage receiver. It is obvious that the complexity of the second scheme is higher than the complexity of the first scheme. Thus the first scheme is recommended for use in practical implementation. Although the interference cancellation and convolutional coding are more integrated in the second scheme than they are in the first scheme, a hard decision has to be made whenever the data bits are recovered at each stage in order to reconstruct the...
transmitted signals of each user in the second scheme. This forces the receiver to lose some information at first stage of decoding in the second scheme. While in the first scheme the soft decision multistage interference cancellation is conducted on the code bits and at the last stage of receiver we have better soft-decision statistics to feed into the Viterbi decoder. The loss of information in the second scheme may counterbalance the effect of decoding at every stage.

The combination of multistage interference cancellation and convolutional coding is near-far resistant. The receiver was tested under the first scenario where half of the users including the desired user are 6 dB lower in power than the other half and the second scenario where half of the users including the desired user are 6 dB higher in power than the other half. The first stage receiver achieves worse performance in the first scenario than it achieves in the second scenario since the signal powers from the interfering users are stronger than they are in the second scenario. The performance of the third stage receiver under these two scenario are nearly identical because there are multistage interference cancellation and convolutional coding at the third stage. The performance at the third stage receiver under both of these two situations are identical to the performance of the third stage receiver when every user have the same signal power.

7.2 Future Work

This thesis shows that the capacity of a CDMA system can be increased by combining multistage interference cancellation and convolutional coding techniques. There are several issues that need to be working on in the future based on the results of this research.

This project studies the performance of the new CDMA receiver in AWGN channel. There are other channel models which are used to model the wireless communication channels. These include the N-ray Rayleigh model, or N-ray Ricean model. In the multipath channel model, the Possion distribution of the arriving of multipath should be modeled. The effects of Doppler shift and velocity of mobile station should be taken into account. Under multipath channel, RAKE receiver should be used to combat multipath fading instead of conventional correlation receiver. The performance of combined multistage interference cancellation using RAKE receiver with convolutional coding is of interest.
The performance study in this thesis considers only the interference from the other users within the same cell as the desired user. In fact, the CDMA system consists of a network of cells instead of a single cell. Research shows that up to 50% of the interference comes from out of the cell where the desired user is. Thus the results presented in this research are optimistic. The interference cancellation technique should be able to cancel the interference from the other cells. Several models of out of cell interference have been developed [Aga95] and can be incorporated into our interference modeling process, which could make our model more close to a practical environment.

This analysis and simulation use coherent modulation/demodulation and the accurate phase information is required. In practice, the estimation of phase and delay may not be very accurate, the effects of phase error and timing error should be studied for practical implementation [Bue95]. Noncoherent demodulation should also be considered and investigated. In the IS-95 CDMA standard, the phase information on the reverse channel is not available. In the IS-95 standard, Walsh coding is used, thus the Walsh code should be implemented in conjunction with the interference cancellation technique. In order to further investigate the feasibility of our new receiver in practical implementation, the transmitted signal should be processed in the manner specified on the reverse channel of the IS-95 standard, and the combination of multistage interference cancellation and convolutional coding can be incorporated into the reverse channel receiver. Then the performance of a receiver using combined multistage interference cancellation and convolutional coding under the IS-95 standard can be studied.

When the total frequency expansion for CDMA system is fixed, the optimal coding rate and PN spreading gain need to be found which can achieve best performance among different coding rates and different PN spreading gains.

Other error correction coding techniques, including trellis codes and turbo-codes, can be combined with multistage interference cancellation to design new receivers. The performance of these systems should be investigated and compared with the results of this research to find out the most suitable coding structure for CDMA multiuser detection.
References


Multiuser Detection for CDMA Systems with Convolutional Coding


Appendix Software Documentation

The software programs used in simulation method are written in C and the programs for analytical method are written in Matlab software package. The files used in simulation method are introduced in 1. The simulation programs are presented in 2 and the analytical programs are introduced in 3.

The C programs are compiled using cc compiler by typing the following command.

```
cc -g source_filename.c -o executable_filename -lm
```

The Matlab files can be executed by typing the file name without the extention .m after the Matlab package has been executed under UNIX environment.

1. Simulation Files

There are two files for two different implementation schemes of combined multistage interference cancellation and convolutional coding. The programs for the first scheme is in the file scheme_no1.c and the programs for the second scheme can be found in scheme_no2.c.

The file msic.c is for multistage interference cancellation without error correction coding.

2. Simulation Programs

These programs are the building blocks for the two implementation schemes.

2.1. rand_g()

This function generates the white Gaussian variable with mean zero and unit variance.
2.2. *rand_bit()*

This function generates the binary random variable whose value is either zero or one with equal probability one half.

2.3. *rand_u()*

It generates the random variable uniformly distributed on \([0,1]\).

2.4. *ran2()*

It generates the random variable uniformly distributed on \([0,1]\).

2.5. *oct_to_dec()*

This routine converts a octal number into its decimal format.

2.6. *init_itl()*

This function generates the block interleaver with different number of rows and columns for different coding rate and constraint length.

2.7. *init_g()*

This function generates the generating function for different coding rate and constraint length in octal format.

2.8. *init_decode()*

This function generates the information needed by the Viterbi decoder. The generating function of convolutional code is known.
2.9. `encode_frame()`

This routine encodes the data bit frame by frame. It also performs zero padding at the end of each frame.

2.10. `hard_decode()`

This routine implements the hard decision Viterbi decoding algorithm.

2.11. `soft_decode()`

This function does convolutional decoding using soft decision Viterbi algorithm.

2.12. `bin_spread()`

This program does PN sequence spreading to signal in baseband.

2.13. `bin_despread()`

This function performs the PN sequence despreading.

2.14. `interlv()`

This routine does interleaving to the convolutional code bit.

2.15. `deinterlv()`

This function performs the deinterleaving for the convolutional code bit.

2.16. `main()`

The main function of the file performs the implementation block diagram according to
different implementation. The parameters can also be defined here.

3. Analytical Programs

The file adic.m is to compute the probability of bit error for multistage interference cancellation without error correction coding.

The file num_pa.c is to calculate the number of paths which diverges from the correct path with hamming distance d and has i “1”s in its input sequence. The files convicX.m are used for calculating the probability of bit error of combined multistage interference cancellation with convolutional coding using different coding rate and constraint length. The different cases with corresponding files are listed in the following table.

<table>
<thead>
<tr>
<th>File Name</th>
<th>Coding Rate</th>
<th>Constraint Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>convic.m</td>
<td>1/2</td>
<td>3</td>
</tr>
<tr>
<td>convic1.m</td>
<td>1/2</td>
<td>5</td>
</tr>
<tr>
<td>convic2.m</td>
<td>1/2</td>
<td>7</td>
</tr>
<tr>
<td>convic3.m</td>
<td>1/3</td>
<td>3</td>
</tr>
<tr>
<td>convic4.m</td>
<td>1/3</td>
<td>5</td>
</tr>
<tr>
<td>convic5.m</td>
<td>1/3</td>
<td>7</td>
</tr>
</tbody>
</table>
Vita

Ning Yang was born in Tianjin, China on January 27, 1967. He received the Bachelor of Science degree in Electrical Engineering from Tianjin University, China in July 1989. He joined the graduate program in Bradley Department of Electrical Engineering, Virginia Polytechnic Institute and State Institute, Blacksburg, Virginia in August 1993 and he has been a member of Mobile and Portable Radio Research Group since August 1993. His research interests include communication and signal processing. He has been focused on code division multiple access (CDMA) spread spectrum communication systems, error correction coding and multiple access interference cancellation techniques since he came to Virginia Tech. He was also involved in channel modeling, IS-95 CDMA standard modeling and simulation study.

Ning was a student member of advisory council of the Bradley Department of Electrical Engineering for the year 1994-1995. He is a member of IEEE. He served as vice president of IEEE Communication and Vehicular Technology Society, Virginia Tech, 1994-1995. He is a member of Eta Kappa Nu (HKN), Tau Beta Pi, Phi Kappa Phi honor societies.

Ning Yang