Volume and Taper Equations For Loblolly Pine Trees Using Dimensional Analysis

by

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(ABSTRACT)

A dimensional analysis approach was applied to derive a general volume equation for a tree. A taper equation compatible with the general volume equation was developed. Data from loblolly pine trees grown in natural stands in the Coastal Plain of North Carolina and the Coastal Plain and Piedmont areas of Virginia were used to estimate the parameters for these volume equations.

A dimensionally compatible volume equation is shown to be a better equation for estimation of the volume of loblolly pine trees and can be applied for the estimation of total volume. The taper equation can be applied to predict the diameter for any specified height and to predict height for any top diameter limit. The volume between any two points on the tree bole can be calculated by integration of the taper equation.

This work is dedicated to my late mother,

HEMAKALA DEVI SHARMA

and late grandmother,

REWATI DEVI SHARMA

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INTRODUCTION

Accurate information on the volume of individual trees is required for harvesting and utilization of timber. The demand for forest products is increasing at a rate substantially often greater than the potential for tree growth. Consequently, the value of flexible and accurate methods for determining the volume of individual trees to optimize harvesting is increasing.

If the shapes of the trees were perfectly cylindrical, conic or paraboloid, their volume could be easily obtained by squaring their diameters (or radii) and multiplying them by heights and a constant. The shape of a tree can be approximated into three parts: (i) the lower part as truncated neiloids, (ii) the middle part as truncated paraboliod, and (iii) the upper part as conoid (Avery and Burkhart, 1994). But the actual tree taper differs from species to species and even from tree to tree.

A number of different types of volume equations (combined variable, constant form factor, comprehensive, Schumachar, etc.) have been developed to predict the volume of a tree, accounting for different geographic regions and types of regeneration. Researchers have not been using underlying principles for the selection of these different volume equations. Methods have tended to be ad hoc with researchers combing predictor variables in various ways. Then regressing predictors on the dependent variable (volume) to find the best fit using least-squares regressions.

Theoretical information on the underlying relationships between dependant and independent variables can be incorporated to improve the accuracy of regression estimations for the dependent variables.

Similarly, many different approaches have been used to estimate the upper stem diameter and/or to calculate the merchantable volume of a tree (segmented polynomials, volume ratio, development of compatible and noncompatible taper equations). In addition, the shape of the trees has been described by utilizing variable-form taper functions. The estimation of parameters in these models and the calculations of desired variables need the intensive use of computers. In addition, some of these models are so complex that calculations are impossible without high level computer programming techniques and are subject to logic errors.

Simple methods that yield accurate predictions of the upper stem diameter of a tree would be extremely useful in forest inventory, where skilled personnel and advanced computer facilities are not available. The objectives of this study are to;

(i) Develop dimensionally compatible and cubic-foot volume equations for loblolly pine,

(ii) Derive taper equations which are compatible with volume equations, and

(iii) Investigate various taper equations where tree shape is described as a paracone.

Chapter II

LITERATURE REVIEW

2.1 Volume Equations

Construction of volume tables date back to the nineteenth century. Henric Cotta published the first volume table for beech in 1804 and a set of standard volume tables in 1817. Behre (1927) proposed a hyperbolic equation that relates the ratio of diameter at a certain height to DBH. Schumacher and Hall (1933) proposed logarithmic volume equation in the form;

$$\ln V = a + \ln D + \ln H \qquad (2.1.1)$$

Where,

V, D, and H represent total volume outside bark (ob), diameter at breast height (DBH) outside bark, and total height of the tree respectively. For the remainder of this paper, the above definitions will be used unless otherwise specified.

Several other potential volume equations were examined including those listed in Clutter et al. (1983, 7-9 p). Gevorkiantz and Olsen (1955) presented an equation, known as constant form factor volume equation, to predict total cubic-foot (cuft) volume inside bark (ib) for a composite of species in the Lake States as;

$$V = b_1 D^2 H \qquad (2.1.2)$$

Bennett et al. (1959) developed an equation (combined variable volume equation) to predict merchantable-stem volume for old-field slash pine plantations in the Georgia middle Coastal plain and the Carolina Sandhills. Their equation was;

$$V = b_0 + b_1 D^2 H \tag{2.1.3}$$

Romancier (1961) investigated a volume equation for loblolly pine in the lower Piedmont of Georgia. This equation, known as the generalized combined variable volume equation, was of the form;

$$V = b_0 + b_1 D^2 + b_2 H + b_3 D^2 H \tag{2.1.4}$$

Honer (1965) used an equation (Honer transformed variable volume equation) to predict total stem cubic-foot volume (ib) for red pine in Canada. His equation was;

$$V = D^2/(b_0 + b_1 H^{-1}) \tag{2.1.5}$$

Newnham (1967) presented a volume equation (Generalized logarithmic volume equation) for red pine in eastern Canada as;

$$V = b_0 + b_1 D^{b2} H^{b3} \tag{2.1.6}$$

Bruce et al. (1968) derived an estimating equation for red alder, expressing the ratio of upper stem diameter inside bark (dib) to squared D as a function of D, H, and to the 3/2, 3rd, 32nd, and 40th powers of relative height. He also calculated the total cubic-foot volume (ib) above stump by using an equation;

$$V = 0.00545415 * D^2 * (H - 4.5) * F \tag{2.1.7}$$

where,

F is a form factor.

Evert (1969) presented a form-class stand volume equation which only needed two sweeps with a relaskop (one at breast height and the other at a fixed upper height) together with summations of the heights of the trees selected in each sweep. The total volume of the stand was;

$$V = r(a \Sigma h_1 + b_2 \Sigma h_2 + b_3 \Sigma h_3^2) \qquad (2.1.8)$$

where,

r = the basal area factor of the relaskop,

Σh_1 = the sum of the heights of the trees selected with the relaskop sweep at breast height, and

Σh_2 = the sum of the heights of the trees selected with relaskop sweep at a fixed upper height,

Moser and Beers (1969) took the nonlinear form of volume equation as;

$$V = a D^b H^c \qquad (2.1.9)$$

originally proposed by Schumacher and Hall and transformed it into a logarithmic form i.e.

$$\ln V = a' + b * \ln D + c * \ln H \qquad (2.1.10)$$

They used a weighting procedure to reduce the effect of heterogeneous variance and found that weighted non-linear regression was a feasible method for fitting the tree volume equation proposed by Schumacher and Hall (1933).

Parker (1972) developed equations to predict merchantable volume in board feet, using Girard form class. Those equations were in the form;

$$V = b_0 + b_1 D^2 H F \qquad (2.1.11)$$

Where,

F = Girard form class expressed as a proportion and

H = merchantable height in number of 16 ft logs and half-logs.

Smith (1976) developed product-form volume equations which needed a third predicting variable in addition to dbh and height. He compared the following volume equations;

1. $V = b_0 + b_1D^2$ (volume/diameter equation) (2.1.12)

2. $V = b_2 + b_3D\,L + b_4D^2L + b_5L$ (standard) (2.1.13)

3. $V = b_6 + b_7F_x + b_8D\,F_x$ (product form) (2.1.14)

4. $V = b_6 + b_7D\,F\,L + b_8D^2\,F\,L + b_9D^2F^2L^2$ (form class) (2.1.15)

Where,

$F = d/D$,

$F_x = D\,F\,L$, and

L, & d are total height and diameter at certain height of a tree respectively.

Burkhart (1977) investigated equations to estimate the cubic-foot volume of a tree to any desired top diameter limit. After comparing the following three models,

$$V = \beta_{01} + \beta_{11}\,D^2\,H \qquad (2.1.16)$$

$$V = \beta_{02} + \beta_{12}\,D^{\beta 22}\,H^{\beta 32} \qquad (2.1.17)$$

$$V = D^2/(\beta_{03} + \beta_{13}/H) \qquad (2.1.18)$$

he found that the combined-variable function (2.1.16) was the best one for loblolly pine grown in plantations and in natural stands. He also developed a model for predicting the ratio of merchantable to the total stem volume;

$$R = 1 + \beta_1 (D_t^{\beta 2}/D^{\beta 3}) \qquad (2.1.19)$$

Where,

D_t = top diameter (ob or ib) in inches,

R = merchantable cubic feet volume (ob or ib) to top diameter D_t/total stem volume (ob or ib) in cuft

Evert (1983) used a form factor weighting approach to overcome the heterogeneous variables problem. His stand volume equation for black spruce was;

$$V = b_0 G\, h_L + b_1\, G\, /d_g^{\,2} + b_2\, G\, d_g^{\,2}\, h_L^{\,2} \qquad (2.1.20)$$

where,

V = total stand volume (cubic meter),

G = stand basal area (square meter)

d_g = quadratic mean diameter at 1.3 m in cm, and

h_L = mean stand height

McTague and Bailey (1987) investigated a method of simultaneously estimating the coefficients of a total and merchantable ratio volume equation. Their volume equation for loblolly pine was;

$$V_m = a_0 + D^{a1}H^{a2} + a_3\, (2D_m^{\,4}/D^2 - D_m^{\,5})\, (H - 1.3)$$
$$+ a_4\, (2D^4_m/D^2 - D_m^{\,5}/D - D^3_m/D)\, (H - 1.3) \qquad (2.1.21)$$

where,

V_m = merchantable volume (ob) in cubic meter to an outside upper stem diameter D_m in cm,

D = dbh in cm, and

H = total height in meter

when $D_m = 0$, $V_m = V_T = a_0 + D^{a1} H^{a2}$ = total volume

Amateis and Burkhart (1987) developed equations to predict total and merchantable volumes for loblolly pine trees grown in cutover, site-prepared plantations. Volume equations proposed were;

$$V_t = a_0 + a_1 D^2 H \tag{2.1.22}$$

$$V_d = V_t(1 + b_1 d^{b2}/D^{b3}) \tag{2.1.23}$$

$$V_h = V_t[1 + c_1(p^{c2}/H^{c3})] \tag{2.1.24}$$

where,

V_t = total volume (cuft) ob or ib,

V_d = volume (cuft) to top diameter d (ob or ib),

V_h = volume (cuft) to height h,

d = top diameter (in.)

P = H - h, distance from the top of the tree,

They also used equations (2) and (3) to predict height (h) for a given upper stem diameter (d) or vice versa and developed a taper relationship between the two equations.

Rustagi and Loveless (1991) presented prediction equations using stem height at a predetermined fraction of DBH outside bark, or the ratio of this height to total height, to predict cylindrical form factor and total stem volume for Douglas fir. Their volume equation was of the form;

$$V = F * \{K\, D^2\, H\} \qquad (2.1.25)$$

where,

F = cylindrical stem form factor

$= V/(KD^2H)$

They measured the additional height of the tree at two-thirds of breast height diameter and found that a model of the form;

$$V = a_1 D^2 H + a_2 D^2{}_{0.67D} \qquad (2.1.25)$$

predicted volume more accurate.

2.2 Taper Equations

Many equations (models) have been suggested for describing the form and the taper of a tree. Behre (1927) proposed a hyperbolic equation, the first taper equation, in the form;

$$D = L/(AL+B) \qquad (2.2.1)$$

where,

D = tree diameter at any height h, expressed in percentage of the basal diameter,

L = length to the point of diameter measurement from the tip expressed in percentage of the total tree height, and

A and B are constants, such that $A + B = 1$

Bruce et al. (1968) derived an estimating equation for red alder, expressing the squared ratio of upper stem diameter to DBH as a function of DBH, total height, and with exponents of 3/2, 3rd, 32nd, and 40th power of relative height. Their equation was;

$$d^2/D^2 = b_1(x^{3/2})\,10^{-1} + b_2(x^{3/2} - x^3)\,D\,10^{-2} + b_3(x^{3/2} - x^3)\,H\,10^{-3}$$
$$+ b_4(x^{3/2} - x^{32})\,HD10^{-5} + b_5(x^{3/2} - x^{32})\sqrt{H}\,10^{-3}$$
$$+ b_6(x^{3/2} - x^{40})H^2 10^{-6} \qquad (2.2.2)$$

where,

$x = (H - h)/(H - 4.5)$,

h = height of measurement, and

d = upper stem diameter

Kozak et al. (1969) presented an equation to describe the taper of many species in British Columbia as;

$$d^2/D^2 = b\,(h/H - 1) + c\,(h^2/H^2 - 1) \qquad (2.2.3)$$

which was obtained from the parabolic equation,

$$d^2/D^2 = a + b\,h/H + c\,h^2/H^2$$

by conditioning that $h = H$ when $d = 0$.

where,

d = diameter inside bark (ib) at height h, and

h = height above ground

Demaerschalk (1973) introduced the idea that the volume equations can be converted into compatible taper equations. For this, he assumed that if the general form of volume and taper equation are;

$$V = f_1 (D, H) \text{ and} \tag{2.2.4}$$

$$d = f_2 (D, l, H) \quad \text{respectively,} \tag{2.2.5}$$

then the total volume of a tree given by equation (2.2.4) can be obtained by integration of taper equation (2.2.5) over the total height, and any volume between length l_1 and l_2 can be calculated by the integration over the section between these lengths. He also converted most of the existing volume equations (combined variable, logarithmic, Honer's equation, and volume over basal area equation) into their compatible taper equations.

Ormerod (1973) proposed a taper equation in the form of;

$$d = D\,[(H - h)/(H - k)]^p, \; p > 0 \tag{2.2.6}$$

where,

d = diameter at height h,

D = diameter at height k (D = dbh when k = 1.37m)

Goulding and Murray (1976) presented a taper equation of the form;

$$d^2 = (V/kH)f(l/H)$$

where,

$f(l/H)$ is a polynomial in (l/H) such that coefficients of the polynomial of degree n (b_i) satisfy

$$\sum_{i=0}^{n} \frac{b_i}{i+1} = 1 \tag{2.2.7}$$

and estimated 4 coefficients to give a 5^{th} degree polynomial for radiata pine.

Max and Burkhart (1976) developed a segmented polynomial regression model to describe the three different geometric shapes of the bole. Three submodels relating to three shapes were combined to form a single segmented equation. The model they used for loblolly pine grown in plantation and natural stands was;

$$d^2/D^2 = b_1(h/H - 1) + b_2(h^2/H^2 - 1) + b_3(a_1 - h/H)^2 I_1 + b_4(a_2 - h/H)^2 I_2 \tag{2.2.8}$$

where,

$I_i = 1,\ h/H \le a_i;\ i = 1$ and 2

$= 0,\ h/H > a_i;$ and a_i's are join points

this model has been found to be superior to the models used earlier to describe the stem taper.

Clutter (1980) investigated outside bark and inside bark taper equations from variable-top merchantable volume equation;

$$V_m = V\,(1 - b_1\, D_m^{b2}\, D^{b3}) \tag{2.2.9}$$

by assuming, $D_m^2 = f(T)$ and $T + M = H$,

where,

T = distance from the top of the stem to the merchantability limit D_m

and M = the merchantable height

The resulting equation was of the form;

$$D_m = a_1 D^{a2} H^{a3} (H - M)^{a4} \quad (2.2.10)$$

Nagashima et al. (1980) derived theoretical stem taper curve by assuming,

$$x(t) = H(1 - e^{-kt}) \text{ and} \quad (2.2.11)$$

$$y(t) = D(1 - e^{-lt}) \quad (2.2.12)$$

where,

x(t) and y(t) are height and diameter respectively at time t,

H and D are upper asymptotes, and

k and l are intrinsic rates of growth.

The final equation they reached was,

$$y(h,x) = D[1-(\frac{H-h}{H-x})^{l/k}] \quad (2.2.13)$$

where,

y(h,x) = stem diameter at height x of a tree of total height h.

This taper equation was fitted to 50 jack pine trees and it was found that the fit was superior to existing empirical curves. However, the estimated parameters were relatively larger than the biologically reasonable expected values.

Reed and Byrne (1985) suggested a method to calculate the value of p in Ormerod's taper equation to describe the paracone shape of the tree recommended by

Forslund (1982). The value of p they suggested for northern conifer species was;

$$p=1-\frac{\frac{H}{D}-30}{120},30<\frac{H}{D}<90$$

(2.2.14)

p = 1; i.e. (H/D =30) indicates a cone and

p = 1/2; i.e. (H/D = 90) indicates a parabola.

Newberry and Burkhart (1986) used a model of the form,

$$d = \alpha D [(H - h)/(H - 4.5)]^{\beta} \qquad (2.2.15)$$

where,

α = slope parameter,

β = form parameter, and

d = upper stem diameter at height h

to describe the taper and form of loblolly pine. This model was obtained by combining two models as;

$$d = \alpha (H - h)^{1/2} \quad \text{and} \qquad (2.2.16)$$

$$d = D [(H - h)/(H - 4.5)]^{\beta} \qquad (2.2.17)$$

They interpreted α as accounting for the tree taper not accounted for by the tree DBH. Their study showed that in most of the cases, the tree characteristics and stand conditions were better correlated with the form parameter than with the slope parameter (Newberry and Burkhart, 1986).

Ormerod (1986) proposed a diameter-point method to describe the taper of tree boles. According to him, the diameter at some point between two taper-change points can be predicted by this approach and that three points can be used to calculate the coefficient of one parameter curve. He used the following form of Behre (1927) hyperbola curve to describe section profiles and combined three submodels that join at the taper change points to obtain a whole stem profile.

$$d = (A - B)t/(T - s(T - t)) + B \qquad (2.2.18)$$

where,

A and B are section end diameters inside bark,

d = diameter at distance t from the top diameter B, and

T = distance from B to the bottom diameter A

Using 32 species-age-location group data to develop and analyze the stem profile curve, he found that this method was superior to other whole stem methods used in British Columbia (Ormerod, 1986).

Alemdag (1988) derived a taper equation from a volume ratio equation which was in the form;

$$d = D\,[\{\exp(b_1(1\text{-}h/H)^{b2}) - 1\}/b_3]^{1/b4} \qquad (2.2.19)$$

where,

d = diameter at height h from the ground

Kozak (1988) took a new approach in which one continuous function describes the shape of the bole rather than the function joining three submodels. The function itself has an exponent which changes from the ground to the top to account for different shapes of the stem. He assumed the function as;

$$y = x^c \tag{2.2.20}$$

where,

$y = d_i/DI$,

$x = (1 - \sqrt{(h_i/H)})/(1 - \sqrt{p})$,

d_i = diameter inside bark at height h_i,

h_i = height from the ground,

DI = diameter inside bark at the inflection point,

$p = (HI/H)100$, and

HI = height of the inflection point from the ground

His expression for c was;

$$c = b_0 + b_1 z + b_2 z^2 + b_3/z + b_4 \ln(z + 0.001) + b_5 \sqrt{z} + b_6 e^z + b_7(D/H) \tag{2.2.21}$$

where,

$z = h_i/H$

After the analysis of above expression for a 33 species group, the best subset of the variables he found for these species was;

$$c = b_2 z^2 + b_4 \ln(z + 0.001) + b_5 \sqrt{z} + b_6 e^z + b_7(D/H) \tag{2.2.22}$$

and the relationship between D and DI was in the form of,

$$DI = a_0 D^{a1} a_2^D \tag{2.2.23}$$

Perez et al. (1990) used slightly modified form of taper equation, in addition to the one presented by Kozak (1988) to express the upper stem diameter of pinus oocarpa schiede in central Honduras. Their equation was;

$$d = b_0 D^{b1} x^c \qquad (2.2.24)$$

where,

$$c = b_2 z^2 + b^4 \ln(z + 0.001) + b_7(D/H),$$

$$z = h/H, \text{ and}$$

$$h = \text{height above the ground}$$

They concluded from this study that Kozak's model was the best predictor based on the standard deviation of the differences and the total squared error but the above model was superior with regard to mean bias (Perez et al., 1990).

Newnham (1992) assumed that the shape of any solid of revolution can be described by the function;

$$y^k = g\,x \qquad (2.2.25)$$

where,

$$y = \text{diameter of the solid at height } x,$$

$$x = (H - h)/(H - 1.3),$$

$$H = \text{total height in meter},$$

$$h = \text{height above ground in meter, and}$$

$$g = \text{constant depending on the system of units}$$

He further assumed that instead of having a fixed value within each section of a stem, k would vary continuously with height and would be a function of x, D/H, and 1/h. i.e.

$$k = f(x, D/H, 1/h) \quad (2.2.26)$$

where,

D = DBH in cm

Fitting four models,

i) the variable form model, $k = f(x, D/H, 1/h)$ of Newnham,

ii) the variable-exponent model of Kozak (1988),

iii) a modification of Newnham model, $\ln(k) = f(x, D/H, 1/h)$, and

iv) the Max-Burkhart segmented polynomial model (Max and Burkhart, 1976) for Alberta tree species, he found that the estimation of upper stem diameter can be improved by using model (iii). The exact form of this model was;

$$\begin{aligned}\ln(d/D_i) = {} & b_0\ln(x) + b_1\ln(x)x + b_2\ln(x)(D/H) + b_3\ln(x)x(D/H) \\ & + b_4\ln(x)(D/H)/\sqrt{h} + b_5\ln(x)(H/\sqrt{h}) + b_6\ln(x)(H^2/h) \\ & + b_7\ln(x)(D/H)/\sqrt{h} + b_8\ln(x)(D)(H/h) \quad (2.2.27)\end{aligned}$$

where,

D_i = DBH inside bark (cm) and

d = stem diameter inside bark at height h (cm)

Allen (1992) modified the taper model developed by Real & Moore (1988) to estimate the upper stem diameter of <u>pinus</u> <u>caribaea</u> grown in Queensland, Australia. The modified form of the model was;

$$\begin{aligned}d^2 = {} & DBHub^2\{x^2 + b_1(x^{5.0 - 0.12H + 0.033DBHub} - x^2) + b_2(x^5 - x^2) \\ & + b_3(x^8 - x^2) + b_4(x^{40} - x^2)\} \quad (2.2.28)\end{aligned}$$

where,

DBHub = DBH under bark,

x = (H - h)/(H - 1.3), and

h = height from base of tree

Flewelling and Raynes (1993) presented a system of equations that allows shape to change as a function of DBH and total height (H). In their system, three segments were utilized and the overall system was constrained so that the first derivatives of the diameter from the different segments agree at both join points. The equations for three segments were,

(i) $y = c_2 x + c_1(x^2/2) + c_1(x^3/6),$

$$x = (1 - rh)/(1 - rh_c) \qquad (2.2.29)$$

(ii) $y = b_0 + b_4 x - b_2(x^{b1+2})/\{(b_1 + 1)(b_2 + 2)\} + b_2(x^3/6),$

$$x = (rh - rh_i)/(rh_c - rh_i) \qquad (2.2.29)$$

(iii) $y = a_0 + (a_4 + a_2/a_3)x + a_2/(2a_3^2)x^2 + a_1x^3,$

$$x = (rh_i - rh)/rh_i \qquad (2.2.30)$$

where,

y = unscaled estimate of inside bark diameter,

rh = h/H (relative height),

rh_c = the value of rh that separates the middle segment from upper segment, and

rh_i = the value of rh that separates the lower segment from middle segment

Bailey (1994) while working with a compatible volume-taper model based on the Schumachar and Hall generalized constant form factor volume equation for slash pine trees in unthinned site-prepared plantations in Southeastern Coastal plain found the following taper equation;

$$D_m = \{\alpha(H - h)^{1-\theta 2} + \{(H - h)/(H - 4.5)\}^{\theta 2(2/\theta 1-1)} [\, D^{\theta 1-2} - \alpha(H - 4.5)^{1-\theta 2}]^{1/(\theta 1-2)} \tag{2.2.31}$$

where,

D_m = stem diameter above ground,

h = height above ground,

$\alpha = k(\theta_1 - 2)/\{(\theta_1 - 2\theta_2)F(size)\}$, and

F(size) = form factor

Chapter III

MODEL DEVELOPMENT

A typical approach to develop a model is to identify variables that affect the dependent variable, determine the appropriate functional relationship between the dependent and independent variables, and then develop a model that expresses the functional relationships in a mathematical form (Amaties and McDill, 1989). However, an appropriate theoretical approach is needed to identify the functional relationship between dependent and independent variables, then available data can be used to estimate the parameters and the results can be validated.

Dimensional analysis methods have been proved efficient in solving mechanical and physical problems and are independent of the specific properties of these problems (Khil'mi, 1957). Hence, the dimensional analysis approach is a completely justified methodological technique to identify the variables and determine their functional relationships.

3.1 Dimensional Analysis

Dimensional analysis is a method by which information about a phenomenon is deduced from a single premise and the phenomenon can be described by a dimensionally correct equation among certain variables. A partial solution to any functional form problem can be easily obtained by applying dimensional analysis.

However, a complete solution of the inner mechanisms of a phenomenon can not be revealed by dimensional analysis (Langhaar 1951).

The application of dimensional analysis to a practical problem is based on the hypothesis that the solution of the problem is expressible by means of a dimensionally homogeneous equation in terms of specified variables. The equation should contain all the variables that would appear in an analytical derivation of the equation.

An equation is said to be dimensionally homogeneous if the form of the equation does not depend on the fundamental units of measurement. In other words, the dimension of each side of an equation should be the same if the equation is dimensionally homogeneous. Thus, an equation;

$$x = a + b + c + \ldots\ldots \tag{3.1.1}$$

is dimensionally homogeneous if, and only if, the variables x, a, b, c, ... all have the same dimension.

Thus, the first step of dimensional analysis for a problem is to decide which variables should enter the problem. If variables are omitted that logically may influence the phenomenon, an incomplete or erroneous result will be obtained (Langhaar, 1951). Even though some variables are essentially constants, they may be essential because they interact with other active variables to form dimensionless products (Langhaar, 1951). Expressions of the type;

$$y = x_1^{k1} x_2^{k2} \ldots\ldots x_n^{kn} \tag{3.1.2}$$

are called "products" and the product y is dimensionally homogeneous if, and only if, the exponents (k_1, $k_2 \ldots\ldots k_n$) are a solution of the linear equations;

$$a_1k_1 + a_2k_2 + \ldots\ldots\ldots + a_nk_n = a \tag{3.1.3}$$

$$b_1k_1 + b_2k_2 + \ldots\ldots\ldots + b_nk_n = b \tag{3.1.4}$$

$$c_1k_1 + c_2k_2 + \ldots\ldots\ldots + c_nk_n = c \tag{3.1.5}$$

where,

a, b, and c are dimensional exponents of left hand side of equation (3.1.2) and a_1, a_2, ..., a_n are constants.

Similarly, expressions like,

$$y^{k1}y^{k2}\ldots\ldots y^{kn} = \text{constant} \tag{3.1.6}$$

are called dimension-less products where the exponents (k_1, $k_2 \ldots k_n$) are the solution of the linear equations;

$$a_1k_1 + a_2k_2 + \ldots\ldots\ldots + a_nk_n = 0 \tag{3.1.7}$$

$$b_1k_1 + b_2k_2 + \ldots\ldots\ldots + b_nk_n = 0 \tag{3.1.8}$$

$$c_1k_1 + b_2k_2 + \ldots\ldots\ldots + b_nk_n = 0 \tag{3.1.9}$$

If we denote the dimensionless product by π, equation (3.1.6) can be written as,

$$\psi(\pi) = 0 \tag{3.1.10}$$

where, ψ is a function. Then the results of dimensional analysis can be generalized as,

$$\psi(\pi_1, \pi_2, \pi_3, \ldots\ldots\ldots, \pi_n) = 0 \tag{3.1.11}$$

where, π_1, π_2, ...π_n are dimensionless products.

The number of these products depends on the number of quantities (variables), n, to be related and the number of independent indicial equations, m. The total number of dimensionless products is equal to (m - n). Normally, there will be one indicial equation for each fundamental dimension and hence the minimum number of products equal to (n - k), where k is the number of fundamental dimensions. This leads to Buckingham's Pi theorem which is stated in the form (Buckingham, 1914),

"A compatible dimensionally homogeneous equation relating n physical quantities which are expressible in terms of k fundamental quantities can be reduced to a functional relationship between (n - k) dimensionless products."

Thus, if the quantities $q_1, q_2, \ldots\ldots, q_n$ are related by the complete dimensionally homogeneous equation,

$$f(q_1, q_2, \ldots\ldots\ldots, q_n) = 0 \tag{3.1.12}$$

then according to Buckingham's Pi theorem, this relationship can be reduced to the form,

$$\psi(\pi_1, \pi_2, \ldots\ldots, \pi_{n-k}) = 0 \tag{3.1.12}$$

where, $\pi_1, \pi_2, \ldots\ldots, \pi_{n-k}$ are dimensionless products formed by the combinations of any k quantities with the remaining (n - k) quantities in turn.

3.2 Units And Dimensions

In mechanics, there are usually three fundamental units. These are the units of length, unit of mass, and unit of time. Thus, in the MKS system of units, these are meter, kg, and second. These are foot, pound, and second in FPS system. These are called fundamental units, because one can not be expressed in terms of the others. For example, mass cannot be expressed in terms of length and/or time, time cannot be expressed in terms of length and/or mass, and length cannot be expressed in terms of mass and/or time.

All other measurement units are expressed in terms of these three fundamental units by means of the physical laws which exist between them and the fundamental quantities. For example, the unit of volume needs only one fundamental unit (length) to be expressed, but the unit of force, defined as the product of mass and acceleration, is derived by a combination of all of these fundamental units.

The dimension then, of any quantity 'a', is designated as [a] and is expressed in terms of these fundamental units, where the square brackets denote "dimension of." The fundamental units are symbolized by the letters, L for length, M for mass, and T for time. Thus,

$$[L] = L \qquad (3.2.1)$$

$$[M] = M \qquad (3.2.2)$$

$$[T] = T \qquad (3.2.3)$$

and dimensions of other quantities (units) are expressed as $M^a L^b T^c$, where a, b, and c are rational numbers that may be positive, negative, or zero. For example,

$$\text{Dimension of area } [A] = L^2 = M^0 L^2 T^0, \qquad (3.2.4)$$

$$\text{Dimension of volume } [V] = L^3 = M^0 L^3 T^0, \qquad (3.2.5)$$

$$\text{Dimension of force } [F] = M L T^{-2}, \text{ etc.} \qquad (3.2.6)$$

3.3 Volume Equation For a Tree

Assume that the volume (V) of a tree is expressed as,

$$V = \alpha + \beta D^{\gamma} H^{\delta} M^{\theta} T^{\phi} \tag{3.3.1}$$

Where,

D = diameter at breast height (DBH),

H = total height,

M = mass,

T = time (age) of the tree, and

α, β, γ, δ, θ, and ϕ are parameters to be determined. D and H can be in any system of units.

Now, Dimension of volume = L^3

Dimension of D = L

Dimension of H = L

Dimension of M = M

Dimension of T = T

There are no dimensions of α and β.

Hence, dimensions of L. H. S. of equation (3.3.1) = $L^3 = L^3 M^0 T^0$

and the dimensions of R. H. S. of the same equation = $L^{\gamma+\delta} M^{\theta} T^{\phi}$.

Then the dimensional form of volume equation (3.3.1) is written as

$$L^3 M^0 T^0 = L^{\gamma+\delta} M^{\theta} T^{\phi} \tag{3.3.2}$$

This equation is valid only when $\gamma + \delta = 3$, $\theta = 0$, and $\phi = 0$.

This shows that there is no contribution of mass and age of a tree to its volume. The analysis also indicates that the specific values of γ and δ are indeterminate, however, the sum of γ and δ must be equal to three. These values are related to the taper of the tree. If the shape of the tree is exactly cylindrical, conic, or paraboloid, then $\gamma = 2$ and $\delta = 1$. Otherwise, these may be different from 2 and 1 and can take fractional numbers.

Then, the final general volume equation of a tree is given as;

$$V = \alpha + \beta D^{\gamma} H^{\delta} \tag{3.3.3}$$

This equation with the term α yields cubic-foot or cubic-meter volume equation and the equation without α yields dimensionally compatible volume equation.

3.4 Taper Equations

The taper equation compatible with both cubic-foot and dimensionally compatible volume equations is given as;

$$d^2 = D^{\gamma} a \left(1 - \{h/H\}\right) h^{\delta-1} \tag{3.4.1}$$

where,

d = upper stem diameter at height h,

a = constant for conditioning $d = D$ when $h = 4.5$ feet, and

D = DBH

This equation is compatible with the volume equations in the sense that its integration over the total height (from $h = 0$ to $h = H$) gives the total volume of the tree, i.e.

for the dimensionally compatible volume equation;

$$V=\int_0^H kd^2dh=\int_0^H kaD^\gamma(1-h/H)h^{\delta-1}dh+c_1$$

(3.4.2)

where, c_1 is an integration constant.

When D = 0 and H = 0, V = 0 ($V = \beta D^\gamma H^\delta$) $\Rightarrow$ $c_1 = 0$

the result is,

$$V=\frac{ka}{\delta(\delta+1)}D^\gamma H^\delta=\beta D^\gamma H^\delta$$

(3.4.3)

where

$$k=\frac{\beta(\delta+1)\delta}{a}$$

(3.4.4)

and for the cubic-foot volume equation;

$$V=\int_0^H kd^2dh=\int_0^H kaD^\gamma(1-h/H)h^{\delta-1}dh+c_2$$

(3.4.5)

where, c_2 is an integration constant.

When D = 0 and H = 0, V = α ($V = \alpha + \beta D^\gamma H^\delta$) $\Rightarrow$ $c_2 = \alpha$

the result is,

$$V=\alpha+\frac{ka}{\delta(\delta+1)}D^{\gamma}H^{\delta}=\alpha+\beta D^{\gamma}H^{\delta}$$

(3.4.6)

where,

$$k=\frac{\beta(\delta+1)\delta}{a}$$

(3.4.7)

Assuming that the exponents of diameter and height in an equation describing the paraboloid shape of the tree should resemble the corresponding exponents in the volume equation, the simplest forms of taper equations describing the paracone shape of the tree are give as;

$$d = D\, b[(1 - h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}], \qquad (3.4.8)$$

$$d = D/2[c(1 - h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}], \text{ and} \qquad (3.4.9)$$

$$d^2 = D^2 e(1 - h/H)(1 - \{h/H\}^{\delta})^{1/\gamma} \qquad (3.4.10)$$

where,

d = upper stem diameter at height h,

h = height from the ground ($0 \leq h \leq H$),

D = DBH, and

b, c, and e are constants obtained by conditioning $d = D$ when $h = 4.5$ feet.

γ and δ are exponents of DBH and total height (H) respectively in the volume equation.

The first term, (1- h/H), describes the cone and the second term , $(1-\{h/H\}^{\delta})^{1/\gamma}$, describes the paraboloid. These two terms collectvely desrcibe the tree shape between a cone and a parabola. Since the diameter of the a tree is measured at breast height (4.5 feet above the ground), a constant is necessary to obtain d = D when h = 4.5 feet.

Chapter IV

MATERIALS AND METHODS

4.1 Data

The data used in this study were obtained from the biometrics section of the Department of Forestry at Virginia Polytechnic Institute and State University at Blacksburg, Virginia. The data were collected by field crews from several industrial forestry organizations from natural stands of loblolly pine in the Piedmont and Coastal plain areas of Virginia and in the Coastal plain of North Carolina. Burkhart et al. (1972) used this data to estimate the per acre yields for natural stands of loblolly pine. In total, 209 trees were used in this study.

Among these 209 trees, 156 trees were separated randomly to estimate the parameters for the volume equation and the remaining 53 trees were used to evaluate the resulting volume and taper equations. Summary statistics (DBH, total height, and total volume inside and outside bark) for the 209 loblolly pine sample trees have been presented in table 1.

4.2 Estimation of Parameters

Least-squares linear and nonlinear regression techniques were used to estimate the best values for the parameters defined in the volume equations. In the case of linear least-square regression, those values of the parameters were chosen as the best

Table 1. Summary statistics (DBH, total height, and total volume) for 209 loblolly pine trees.

	dbh (ob) (inches)	dbh(ib) (inches)	total height (feet)	total volume(ob) (cuft)	total volume(ib) (cuft)
minimum	4.60	3.60	32.40	2.27	1.62
mean	8.11	6.83	58.31	11.67	9.22
maximum	13.20	11.80	88.50	40.76	34.69
std dev	1.92	1.74	11.19	7.46	6.33

values for which R^2 was maximum and mean square error (MSE) was minimun. In the case of nonlinear regression, those estimates were taken as the best values where the convergence was occured.

DBH and total height, both in feet were applied to fit the regressions for both outside and inside bark volume equations. Outside bark DBH was used for outside bark volume equation and inside bark DBH was applied for inside bark volume equations. Since the exponents of DBH and H are highly correlated to each other, nonlinear regressions were performed by supplying different initial values of γ (1.0, 1.5, 2.0, and 2.5). The point of convergence for the volume equation was insensitive to the initial values of γ.

Linear regressions were fitted by applying all possible values of γ starting from 1.0 and increasing by 0.1 in every step. The value of γ was extended to second, third, and fourth decimal places in the interval from where R^2 started to decrease and mean square error (MSE) started to increase. The value of δ was restricted by conditioning $\gamma + \delta = 3$ for each value of γ. The optimal values for γ and δ were chosen on the basis of where the R^2 was maximum and the mean square error (MSE) was minimum.

The optimal values of α, β, γ, and δ from linear and nonlinear regressions using the above data are given in table 2. Values of α's for both outside and inside bark volume equations were found to be insignificant at 5% level. The equations for estimating the cubic-foot volume of loblolly pine trees are as follows,

Table 2. Summary statistics for fitting cuft volume equations.

Volume equations	α	β	γ	δ	R^2	MSE
Outside bark						
linear	-0.10409	0.43123	2.02020	0.97980	0.9801	1.17821
nonlinear	-0.10404	0.43131	2.02024	0.97976		
Inside bark						
linear	0.05747	0.57025	2.06350	0.93650	0.9853	0.63168
nonlinear	0.05748	0.57029	2.06352	0.93648		

$$V(ob) = -0.104093 + 0.431231\, D^{2.0202}\, H^{0.9798} \qquad (4.2.1)$$

$$V(ib) = 0.057473 + 0.5702252\, D^{2.0635}\, H^{0.9365} \qquad (4.2.2)$$

4.3 Dimensionally Compatible Volume Equations

Since the constant term α (as expected in both outside and inside bark volume equations was insignificant) was estimated by using data in English system units, these equations are applicable only in english system of units. However, dimensionally compatible volume equations for loblolly pine were sought and this was achieved by regressing volume on D^a and H^b without the intercept. Estimated values of γ and δ are given in table 3. The intercepts for these values of β, γ, and δ were also insignificant at 5% level.

These equations are given as;

$$V(ob) = 0.444876\, D^{2.0287}\, H^{0.9713} \qquad (4.3.1)$$

$$V(ib) = 0.549905\, D^{2.0546}\, H^{0.9454} \qquad (4.3.2)$$

4.4 Volume Above DBH

To determine the extent to which the butt swell part of the tree (volume between ground to DBH) is influencing the values of the parameters in the volume equations, separate linear and nonlinear regressions were fitted for the truncated trees above DBH. The height above DBH is used as total height and the volume between DBH and tip of the tree was used as total volume to estimate the parameters for

Table 3. Summary statistics for fitting dimensionally compatible volume equations.

Volume equations	β	γ	δ	R^2	MSE
Outside bark					
linear	0.44487	2.02870	0.97130	0.9941	1.1735
nonlinear	0.44485	2.02868	0.97132		
Inside bark					
linear	0.54990	2.05460	0.94540	0.9951	0.62851
nonlinear	0.54985	2.05458	0.94542		

outside and inside bark volume equations. These equations are as follows;

$$V(ob) = -0.806701 + 0.224380\, D^{1.8908}\, H^{1.1092}$$

$$\text{with } R^2 = .974 \text{ and } MSE = 1.14494 \qquad (4.4.1)$$

$$V(ib) = -0.478242 + 0.323827\, D^{1.9544}\, H^{1.0456}$$

$$\text{with } R^2 = .982 \text{ and } MSE = 0.59823 \qquad (4.4.2)$$

The intercepts for these truncated volumes were significant at 5% level.

4.5 Evaluation of the Volume Equations

Since the combined variable volume equation (with $\gamma = 2$ and $\delta = 1$) was found to have an excellent fit and predictive ability for loblolly pine by Burkhart (Burkhart, 1977; Amateis and Burkhart, 1987), these results are used for comparison with the outside and inside bark volume equations developed in this paper. The estimates of parameters (α and β) for the combined variable volume equations using the same data are given in table 4.

Evaluation was performed by using the 53 loblolly pine trees' data that were not used for parameter estimation. For comparison purposes, measure of bias was calculated using ,

$$D=\sum_{i=1}^{53} \frac{D_i}{53}$$

(4.4.1)

Table 4. Summary statistics for fitting combined variable volume equations

volume equations	α	β	R^2	MSE
Outside bark				
with intercept	-0.12497	0.39503	0.9801	1.17964
without intercept		0.39207	0.9940	1.17658
Inside bark				
with intercept	-0.01128	0.42923	0.9851	0.63966
without intercept		0.42888	0.9951	0.63557

Where, $D_i = Y_i - \hat{Y}_i$, and Y_i and $\hat{Y}_i$ are measured and predicted volumes respectively.

And the measure of precision was calculated using,

$$S_D=\sqrt{\frac{\sum_{i=1}^{53} D_i^2-\frac{(\sum_{i=1}^{53} D_i)^2}{53}}{52}}$$

(4.4.2)

Measure of bias and precision were combined into a mean square type of measure given below;

$$MS=(\bar{D})^2+S_{\bar{D}}^2$$

(4.4.3)

Where,

$$S_{\bar{D}}^2=\frac{S_D^2}{53}$$

(4.4.4)

and

$$\bar{D}=\sum_{i=1}^{53}\frac{D_i}{53}$$

(4.4.5)

Then $\sqrt{(MS)}$ was calculated.

4.6 Evaluation of Taper Equations

In the case of taper equations, there is only one constant which is a necessary condition, when height (h) = 4.5 feet, the diameter (d) must be equal to DBH (D)of the tree. There are no other parameters or constants imposed that would require estimation by least square techniques.

Hence, the taper equation compatible with volume equations was compared with the three taper equations describing paracone shape of the tree. For this, diameters for every four feet from the stump, using data from 10% of the 53 randomly selected trees were estimated. Measure of bias and mean absolute difference of measured and estimated diameters at these heights were calculated for each taper equation.

Chapter V

RESULTS AND DISCUSSIONS

5.1 Volume Equations

Results from regressions showed that the intercept term, α, was not different from zero even at the 50% significance level for both inside and outside bark volume equations. Even for combined variable volume equations (outside and inside bark), it was insignificant at the 40% level. In addition, these intercepts were found negative in the cases of outside bark volume equation developed here and combined variable (both outside and inside bark) volume equations. The negative values of α's are not consistent with expectations and indicate their insignificance.

Due to the facts described above, only dimensionally compatible volume equations (outside bark and inside bark) were compared with the corresponding combined variable volume equations without intercepts. Table 5 summarizes the values of bias, measure of precision, a combined measure of bias and precision, and standard error of the estimates.

The magnitude of the measure of bias in the case of dimensionally compatible volume equations (out side and inside bark) is smaller than the corresponding values obtained from the combined variable volume equations (outside bark and inside bark). However, the measure of precision in the case of combined variable volume equations is smaller than for dimensionally compatible volume equations. Critically the

Table 5. Summary statistics of measure of bias, measure of precision, combined measure of bias and precision, and standard error of estimates.

	D	S_d	$\sqrt{MS}$	$S_{y.x}$
Dimensionally compatible volume equations;				
V(ob)	-0.11150	0.93847	0.17044	1.08328
V(ib)	-0.07317	0.91925	0.14594	0.79279
Combined variable volume equations (without intercept)				
V(ob)	-0.12006	0.92113	0.17442	1.08470
V(ib)	-0.09107	0.88246	0.15161	0.79723

combined measure $\sqrt{(MS)}$ in the case of dimensionally compatible volume equations are superior to the corresponding $\sqrt{(MS)}$ for the combined variable volume equations. Further, standard error of estimates for dimensionally compatible volume equations were also less than those for combined variable volume equations.

These results showed that the dimensionally compatible volume equations (outside bark and inside bark) have a better fit and predictive ability than combined variable volume equations for loblolly pine. The dimensionally compatible volume equations developed in this paper are simple and are applicable using any system of units to estimate volume of an individual tree or the volume of a stand.

5.2 Truncated Tree Volumes

The results of linear and nonlinear regressions to estimate the volume of trees truncated from DBH, showed that the butt swell part of the tree has a great influence on the parameters α, β, γ, and δ. In addition, α was found to be significant for both outside and inside bark volume equations at 5% significance level which was not expected. Generally, the intercept term is interpreted as the content of volume that accounts for the portion of the volume below DBH. Another noticeable change is the value of the exponent of the diameter (γ) which was expected to be close to 2.0, however, this value for truncated trees was less than 2.0. The amount of decrease for the outside bark volume equation was greater than that for the inside bark volume equation.

5.3 Taper Equations

Taper equation (3.4.1) which is compatible with both the dimensionally compatible and the cubic-foot volume equations yields $d = 0$ when $h = H$. But the value of d for $h = 0$, depends on the value of δ. There are three possible cases;

Case 1,

If $\delta < 1$, d tends to be very large compared to DBH as h tends to zero which is logical and a very important characteristic of this taper equation.

Case 2,

If $\delta = 1$, the function assumes the shape of the tree is purely parabolic. With the condition that $d = \text{DBH}$ when $h = 4.5$ feet, stump diameter obtained will still be greater than DBH.

Case 3,

If $\delta > 1$, the equation will still yield a total volume equal to the volume of the tree obtained by integrating over the total height. The diameters predicted below DBH will be smaller than DBH and tends to zero when h approaches to zero.

From these three different cases discussed above, it is obvious that the value of δ for describing the shape of the trees should be less than or equal to one. In the case of the dimensionally compatible volume equation for loblolly pine trees, it is equal to 0.9713 ($\gamma = 2.0287$ and $\delta = 0.9713$).

Tables 6 and 7 summarize the measure of bias (average difference) in estimating diameters at different heights. Mean absolute differences between

Table 6. Average difference (in inches) between measured and predicted outside bark diameters at different heights.

Height (feet)	Equation (3.4.1)	Equation (3.4.8)	Equation (3.4.9)	Equation (3.4.10)
0.5	2.78	2.89	2.88	2.94
4.5	0.00	0.00	0.00	0.05
8.5	-0.32	-0.26	-0.25	-0.20
12.5	-0.30	-0.14	-0.13	-0.09
16.5	-0.23	0.02	0.05	0.09
20.5	-0.28	0.07	0.10	0.14
24.5	-0.18	0.27	0.31	0.35
28.5	-0.16	0.38	0.42	0.48
32.5	-0.17	0.45	0.50	0.57
36.5	-0.18	0.52	0.57	0.65
40.5	-0.31	0.45	0.51	0.61
44.5	-0.44	0.36	0.41	0.54
48.5	-0.57	0.24	0.29	0.45
52.5	-0.46	0.45	0.51	0.70
56.5	-0.43	0.66	0.72	0.89
60.5	-0.52	0.56	0.62	0.83
64.5	-0.93	0.11	0.17	0.41

Equation (3.4.1), $d^2 = D^\gamma a (1- \{h/H\}) h^{\delta-1}$

Equation (3.4.8), $d = D b[(1- h/H) + \{1 - (h/H)^\delta\}^{1/\gamma}]$

Equation (3.4.9), $d = D/2[c(1- h/H) + \{1 - (h/H)^\delta\}^{1/\gamma}]$

Equation (3.4.10), $d^2 = D^2 e(1- h/H) (1 - \{h/H\}^\delta)^{1/\gamma}$

Table 7. Average difference (in inches) between measured and predicted inside bark diameters at different heights.

Height (feet)	Equation (3.4.1)	Equation (3.4.8)	Equation (3.4.9)	Equation (3.4.10)
0.5	2.07	2.38	2.38	2.38
4.5	0.00	0.00	0.00	0.00
8.5	-0.04	-0.02	-0.01	-0.02
12.5	0.10	0.17	0.18	0.17
16.5	0.23	0.37	0.39	0.38
20.5	0.29	0.50	0.52	0.52
24.5	0.35	0.63	0.66	0.66
28.5	0.34	0.70	0.74	0.75
32.5	0.32	0.75	0.79	0.81
36.5	0.31	0.81	0.85	0.89
40.5	0.13	0.68	0.72	0.79
44.5	-0.01	0.57	0.61	0.70
48.5	0.02	0.40	0.45	0.56
52.5	0.01	0.69	0.74	0.88
56.5	0.06	0.89	0.95	1.10
60.5	-0.13	0.71	0.77	0.95
64.5	-0.54	0.27	0.32	0.54

Equation (3.4.1), $d^2 = D^\gamma a (1- \{h/H\}) h^{\delta-1}$

Equation (3.4.8), $d = D b[(1- h/H) + \{1 - (h/H)^\delta\}^{1/\gamma}]$

Equation (3.4.9), $d = D/2[c(1- h/H) + \{1 - (h/H)^\delta\}^{1/\gamma}]$

Equation (3.4.10), $d^2 = D^2 e(1- h/H) (1 - \{h/H\}^\delta)^{1/\gamma}$

measured and predicted diameters (outside and inside bark) at different heights for four taper equations are given in tables and 8 and 9 and figure 1 and 2.

The estimates of diameters between DBH and the tip of the tree are shown to be more accurate by equation (3.4.1) than the estimates by the other three taper equations. For some trees, the differences between observed diameters and the diameters predicted by equation (3.4.1) are in the range of 0.0 - 0.2 inches. Equation (3.4.1) slightly overestimated the outside bark diameters above DBH. The other three equations underestimated these diameters to a greater extent than equation (3.4.1) overestimated them. In the case of inside bark diameters, all four taper equations were underestimating the diameters between DBH and the tip of the tree by a slight amount.

Stump diameters (both outside and inside bark) were shown to be poorly estimated by all four taper equations. The taper equation (3.4.1) which is compatible with the dimensionally compatible volume equation is shown to have a slightly better predictive ability than the other three taper equations (3.4.8), (3.4.9), and (3.4.10) for the estimation of stump diameters. However, some estimates of diameters at the tip of the trees using these three equations were relatively better than the estimation using equation (3.4.1).

On average, the measure of bias and the mean absolute difference for equation (3.4.1) is smaller than those for equations (3.4.8), (3.4.9), and (3.4.10) in estimating both outside and inside bark diameters. The bias in predicting inside bark diameters

Table 8. Mean absolute difference (in inches) between measured and predicted outside bark diameters at different heights.

Height (feet)	Equation (3.4.1)	Equation (3.4.8)	Equation (3.4.9)	Equation (3.4.10)
0.5	2.78	2.89	2.88	2.94
4.5	0.00	0.00	0.00	0.06
8.5	0.34	0.30	0.29	0.26
12.5	0.30	0.18	0.17	0.13
16.5	0.29	0.24	0.25	0.20
20.5	0.30	0.26	0.28	0.23
24.5	0.19	0.37	0.40	0.36
28.5	0.19	0.50	0.52	0.50
32.5	0.35	0.61	0.64	0.63
36.5	0.23	0.61	0.65	0.65
40.5	0.32	0.51	0.55	0.61
44.5	0.46	0.42	0.45	0.54
48.5	0.57	0.40	0.39	0.45
52.5	0.46	0.60	0.62	0.70
56.5	0.43	0.66	0.72	0.89
60.5	0.52	0.56	0.62	0.83
64.5	0.93	0.11	0.17	0.41

Equation (3.4.1), $d^2 = D^{\gamma} a (1- \{h/H\}) h^{\delta-1}$

Equation (3.4.8), $d = D b[(1- h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}]$

Equation (3.4.9), $d = D/2[c(1- h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}]$

Equation (3.4.10), $d^2 = D^2 e(1- h/H) (1 - \{h/H\}^{\delta})^{1/\gamma}$

Table 9. Mean absolute difference (in inches) between measured and predicted inside bark diameters at different heights.

Height (feet)	Equation (3.4.1)	Equation (3.4.8)	Equation (3.4.9)	Equation (3.4.10)
0.5	2.07	2.38	2.38	2.38
4.5	0.00	0.00	0.00	0.00
8.5	0.15	0.13	0.13	0.13
12.5	0.11	0.17	0.18	0.17
16.5	0.25	0.37	0.39	0.38
20.5	0.37	0.51	0.53	0.53
24.5	0.35	0.63	0.66	0.66
28.5	0.39	0.70	0.74	0.75
32.5	0.43	0.75	0.79	0.81
36.5	0.41	0.81	0.85	0.89
40.5	0.24	0.68	0.72	0.79
44.5	0.34	0.57	0.61	0.70
48.5	0.35	0.44	0.47	0.56
52.5	0.43	0.69	0.74	0.88
56.5	0.06	0.89	0.95	1.10
60.5	0.13	0.71	0.77	0.95
64.5	0.54	0.27	0.32	0.54

Equation (3.4.1), $d^2 = D^{\gamma}\, a\, (1- \{h/H\})\, h^{\delta-1}$

Equation (3.4.8), $d = D\, b[(1- h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}]$

Equation (3.4.9), $d = D/2[c(1- h/H) + \{1 - (h/H)^{\delta}\}^{1/\gamma}]$

Equation (3.4.10), $d^2 = D^2\, e(1- h/H)\, (1 - \{h/H\}^{\delta})^{1/\gamma}$

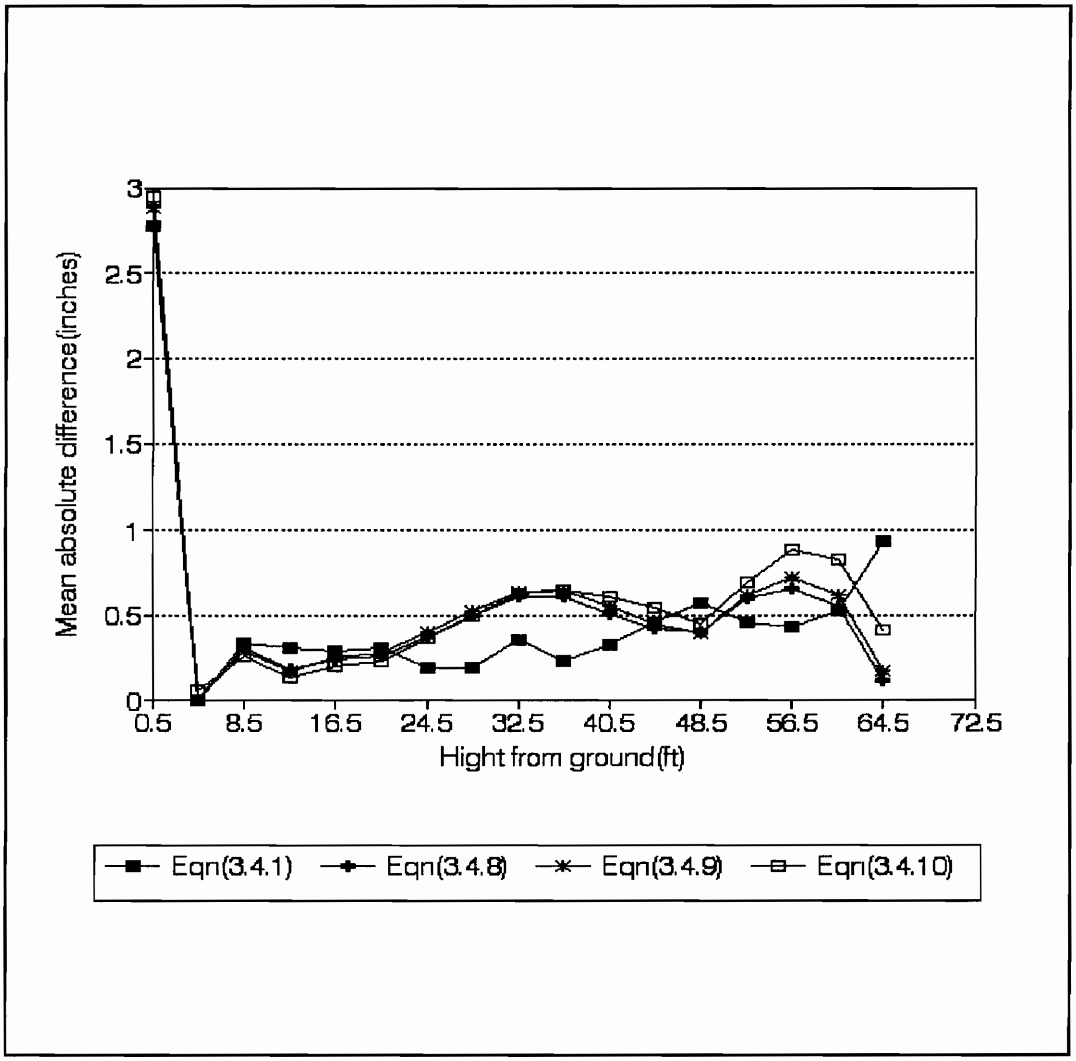

Figure 1. Mean absolute difference between measured and predicted outside bark diameters at different heights.

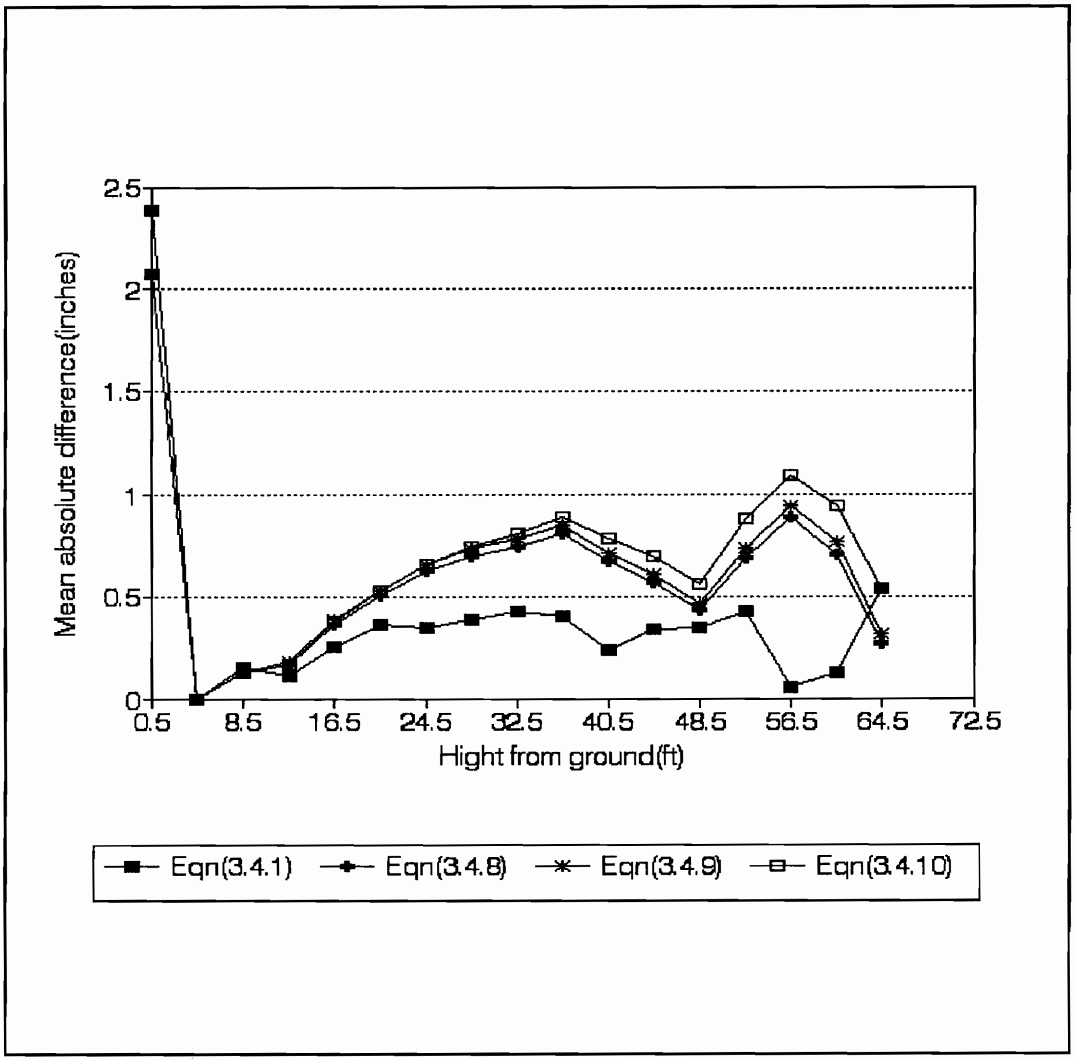

Figure 2. Mean absolute difference between measured and predicted inside bark diameters at different heights.

is less than the bias in predicting outside bark diameters by equation (3.4.1).

From these results it can be concluded that the taper equation compatible with volume equations is superior to the other three taper equations for estimation of the upper stem diameters of loblolly pine. Moreover, merchantable volume of a tree for any upper limit diameter and to any specified height can be calculated by using equation (3.4.1) but the other three taper equations cannot be used to estimate volume they can only be used to estimate upper stem diameters.

Chapter VI

CONCLUSIONS

According to Khil'mi (1957), in the study of living forest phenomenon, no new mathematical approach is applied. Scientific forestry has employed empirical curves and statistical methods extensively but methods based on a theoretical approach for understanding forestry phenomenon have not been extensively used. Statistical methods only represent the results of particular numbers obtained through empirical observations which limits their applications. The information obtained in this way has only mathematical descriptive value for limited cases of the particular tree stands and does not have any general significance. It is inappropriate to use such scientific information for locations other than the specific locations where the data was obtained.

This study using a theoretical approach shows that the developed volume equation form is appropriate for all trees, variations are captured through parameter changes. The values of the constants (exponents of DBH and height, intercept and the constant factor of DBH and height) may differ from tree to tree for trees of different taper. The values of the exponents for the dimensionally compatible volume equation need not be exactly 2 and 1, they can vary by the taper of the tree. If the taper from the bottom to the top of the tree is not regular, these exponents can take values different from 2 and 1. But for the dimensionally valid volume equation their sum must be equal to 3.

The concept that the exponents can differ from 2 and 1 and their sum must be equal to 3, facilitates the development of the taper equation compatible with the volume equation. The most remarkable characteristics of the proposed taper equation are: (1) it has theoretical basis, and (2) it is the simplest form of currently available taper equations. Since it does not contain any extraneous parameters and complexity in form, the diameters to any point on the tree bole can be calculated with only the use of a hand calculator.

The taper equation developed in this paper is shown to efficiently and accurately predict tree diameters (outside and inside bark) between DBH and the tip of the tree. This taper equation has only a minor drawback in that it does not estimate the butt swell of the stem very well. However, this is not critical since the diameters to any point on the tree bole above DBH and the merchantable volume to any upper stem diameter or height of a tree can be calculated accurately by using this taper equation. This is the information that concerns field foresters and to which they should have easy access.

The volume and taper equations presented here are developed for naturally regenerated loblolly pine trees. These equations can be adapted for loblolly pine trees grown in different sites and for other species by fitting the developed general volume equation to data specific from those sites in the United States and other countries.

This study represents the beginning of an effort to determine the theoretical proper relationships between volume and its predictor variables and in developing taper equations compatible with volume equations. Development of a more flexible taper equation for predicting accurate stump diameters was out of the scope of this study.

Dimensional analysis is not the only way to derive equations or develop models. The model developed by this analysis does not necessarily represent the best way to incorporate theoretical information. However, it is shown superior to existing approaches in this paper and is a partial solution of the volume estimation problem. Efforts to develop proper volume equation and taper equations that accurately describe the shape of trees should continue using methods based on the fundamental laws of mathematics.

In summary, volume and taper equations presented here can be applied in any system of units to estimate total and merchantable volume of loblolly pine trees grown in natural stands within the geographic area that the data were collected. Diameters to any point on the bole or height to any diameter limit can be calculated by using the taper equation. Models based only on statistical assumptions can yield accurate results only when applied within the range of observed data. However, the equations developed in this paper yield consistent results even applied to ranges beyond the observed data.

Finally, I hope that this procedure that resulted from my research will be useful in the teaching and research at the Institute of forestry, Nepal. In addition, it will facilitate the update of research and timber management currently used in Nepal.

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VITA

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