INVESTIGATION OF INDUCED STRAIN ACTUATOR PATCHES
IMPLEMENTING MODELING TECHNIQUES AND DESIGN
CONSIDERATIONS TO REDUCE CRITICAL STRESSES

by

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Abstract

One of the major problems with surface-mounted or embedded induced strain actuator (ISA) patches are the considerably high stress gradients introduced at the edges of the actuator patches when an electric field is applied. These excessive stress gradients initiate debonding of the actuators from the substrate, thus affecting the mechanical reliability of the structure.

This thesis is begun by investigating existing theoretical models of induced strain actuated structures, and will later use these to compare with the finite element analysis. The finite element analysis is used to explore the stress concentrations located at the edges of the actuators and begins by refining the mesh areas of the same structure focusing in on the ends of the ISA’s. This preliminary analysis is conducted on a structural configuration
with a perfectly bonded actuator and proceeds to one with a finite bonding layer.

After completion of the mesh refinement investigation several modifications in the design and implementation of the induced strain actuators are examined to reduce the stress concentrations at the edges of the actuators. In the finite element analysis two separate modeling considerations are examined:

1) The actuator is perfectly-bonded to the substrate.

2) A finite adhesive layer is incorporated between the actuator and the substrate.

With each modeling consideration several design modifications are considered in this thesis including employing partial electrodes on the induced strain actuator surface regions instead of fully electroded surfaces, examining an actuator with a chamfered end, and using caps to reduce the stress concentrations and possibly increase the performance of the structure by allowing the induced strain actuators to utilize their piezoelectric strain coefficient in the thickness direction, $d_{33}$. The design modifications and different modeling techniques help to alleviate the critical stresses in the structure while gaining a better understanding of causes them.
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<td>(\tau)</td>
<td>Shear Stress</td>
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<td>(\Omega)</td>
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<td>(\Psi)</td>
<td>Stiffness Ratio ((E_oT_o)/(E_aT_a))</td>
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<td>Cap</td>
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<td>(_{BE})</td>
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<td>(_\circ)</td>
<td>Middle-Surface</td>
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<td>(_{PF})</td>
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<td>Surface of Substrate</td>
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<td>Max. Stress Magnitude</td>
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Chapter 1

Introduction

1.1 Intelligent Materials and Structures

Engineers now find themselves challenged by requirements to develop, analyze and control advanced materials and structures as the world of science and technology continues to progress and push forward into the twenty-first century. The advanced structures of today must not only be stronger and lighter than those of yesteryear, but also able to provide "life" functions of sensing, actuation, control, and intelligence to materials and structures. The intelligent systems under investigation are asked to take on human-like qualities, being able to sense their surroundings and react to external stimuli. By taking on such characteristics, structures will be able to operate in an optimum manner and maximize their performance, thus defining new capabilities for structures and material systems. Although intelligent materials and structures will inevitably be used in the design and construction of automobiles, buildings, bridges, highways, and pipelines, recent efforts have focused on potential aerospace applications furthering the development of advanced aircraft, large space structures, and launch
vehicles.

Potential applications for intelligent structures include (Rogers et al., 1988):

- Active Vibration control and acoustic suppression for submarines, robot manipulators, propeller aircraft, large flexible structures, etc.

- Failure detection/prevention of structures (i.e., bridges, walkways, phone and electrical cable, and mechanical components).

- Active control of helicopter rotor blades.

- Thermal expansion balancing.

- Robot manipulators (fingers).

- Thermally activated valves, ducts, and switches.

- Structural dimension adjustment and environment adaption for large antennas.

The introduction and implementation of these type of materials could have a major influence on a variety of technological fields.

The intent of this chapter is to provide an overview of intelligent structures while discussing the three major components of a structural configuration that make it intelligent (the sensors, the actuators, and the control systems), and giving background research that has been performed and areas to which the field of intelligent structures is heading.
1.1.1 Sensors

"The concept of smart materials is based on the integration of sensors into preexisting or existing structural configurations, so that the structure has its own 'nervous system' making it capable to communicate with outside intelligence" (Rogowski et al., 1988). The sensors are used to monitor the structure and can be used to determine the necessary actuation needed to modify the system response through the use of control algorithms. The system may be comprised of discrete or distributed sensors depending on the particular application. Accelerometers and microphones are essentially discrete sensors used to measure sound pressures and vibration responses. Piezoceramic, polyvinylidene fluoride (PVDF), and optical fibers are new distributed sensors which will result in electrical output signals in response to external stimuli.

Sensors for intelligent structures may be fully integrated into the structure by locating them internally, or may be placed externally on the structure. Internal sensors are used to monitor the interior of the structure measuring vibration, strain, temperature, and the often invisible signs of cracking and bending of the structure due to excessive loadings and overheating. Externally mounted sensors are used to detect bending of the structure, measure strains, but are primarily used to detect electromagnetic energy. Some of the most common sensors used in intelligent structures are listed and described below.
1.1.1.1 Optical Fiber Sensors

The advantages that fiber optics can provide make them the most versatile and the most promising type of sensor. Fiber optics perform a dual role being capable of performing as the sensor as well as transmitting the sensor’s signal. The potential advantages of such sensors are their small size, low weight, EMI and EMP immunity, geometrical flexibility, large bandwidth, low power multiple multiplexing options, and all-dielectric profile (Claus, McKeeman, May, and Bennett, 1988). Fiber optics have the capability of measuring various physical quantities which are listed in Table 1.1.

There are a wide range of applications that are inherent to fiber optics including in situ measurements of composite cure, in-service monitoring of structural components, damage detection and evaluation, and many more. The future of fiber optics is indeed exciting and more sophisticated sensors are sure to be developed (Measures, 1990).

1.1.1.2 Piezoelectric Sensors

During the past few years other sensors as well fiber optics have been incorporated into intelligent structures, and one of the most prevalent is the piezoelectric ceramic and polymer materials. The sensors made from a ceramic material called PZT (lead zirconium titanate) produce a voltage on the order of $3\mu$V/Pa when mechanically
Table 1.1: Fiber Optic Sensing (Main, 1985, Mann, 1985)

<table>
<thead>
<tr>
<th>Variable</th>
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<td>Force</td>
<td>Induces birefringence</td>
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<td>Pressure</td>
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<td>Electric Current</td>
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<td>Magnetic Field</td>
<td>Magneto-optical Effect, Farraday Effect</td>
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<td>Temperature</td>
<td>Thermal change in refractive index, absorptive properties, or fluorescence, thermoluminescence</td>
</tr>
<tr>
<td>Photoelectric Emission</td>
<td>Fiber defects leading to alteration in refractive index &amp; absorptive properties</td>
</tr>
<tr>
<td>X-rays, Gamma rays</td>
<td>Radiation-induced luminescence</td>
</tr>
<tr>
<td>Changes in Chemical Composition</td>
<td>Changes in absorption and refractive owing to chemical effects, chemoluminescence</td>
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deformed. These type of sensors demonstrate many advantages; they are chemically inert, exhibit reasonably high sensitivity, and can be used over a wide range of pressures without serious non-linearity. The major drawback to this type of sensor is its fragility; like all ceramics (Rogers, 1990).

A partial solution to the brittle nature of the piezoceramic sensors was brought forth in the 7th Symposium on Ultrasound Electronics in Kyoto, where a paper presented by four NTK researchers described a new composite material which consisted of piezoelectric ceramic material embedded in a synthetic rubber, Chlorobren. The composite structure maintained excellent piezoelectric properties while remaining completely flexible. One major result of this investigation was the introduction of a piezoelectric coaxial cable, shown in Figure 1.1, similar in appearance to a normal 50Ω coaxial cable, but with the insulating layer between the screen and center conductor made of piezoelectric rubber composite designated PR-302 (Rogers, 1990).

Another alternative to the piezoelectric ceramic is the use of a piezoelectric polymer material, polyvinylidene fluoride (PVDF or PVF₂), which outperforms many sensors in its mechanical strength and its high sensitivity to pressure changes. Being a polymer PVDF can be formed into very thin sheets and adhere to almost any surface (Rogers, 1990).
Figure 1.1 Piezoelectric Coaxial Cable
1.1.1.3 Nitinol Sensors

Nickel-Titanium shape memory alloys demonstrate similar characteristics as that of piezoelectric materials in that they perform sensing as well as actuation functions. The nitinol fiber sensor is currently being used to measure strain in a structure, and is currently being researched to measure temperature and strain simultaneously.

1.1.2 Control Systems

For an intelligent structure to properly utilize the integrated sensors and actuators, it must be able to receive the signals from the sensors and interpret them through some type of controller which in turn designates actuation to alter the response of the structure. Several controllers are being considered for the responsibility, and one that is gaining considerable attention is the neural network control system, which takes its inspiration from the human biological system. The neural network could be invaluable to intelligent structures, due to its ability to learn from prior experiences and modify its control patterns as time progresses. Neural networks are made of simple processors interconnected by variable memory elements whose weights are adjusted by experience, thus initiating the learning process (L. N. Cooper, 1989). There are many other types of control systems such as the feedback control algorithm developed by Meirovitch and Thangjitham (1990), and the LMS adaptive feedforward control algorithm used in active
structural acoustic control (ASAC) by Fuller et al. (1989).

1.1.3 Induced Strain Actuators

In order for a structural configuration to be considered as "smart" or "intelligent" it must have the potential to actuate and change its physical geometry along with its mechanical and electrical properties when exposed to an external excitation. Actuation materials which are able to induce noticeable forces and strains through nonmechanical stimuli (thermal, electrical, or magnetic) are usually classified as "induced strain actuators", and are capable of controlling a material/structural geometrical configuration, mechanical properties, and internal stress-strain characteristics. Examples of induced strain actuators include piezoelectrics, electrostrictors, magnetostrictors, and shape memory alloys.

When an induced strain actuator is subjected to an external stimuli it demonstrates the following constitutive relationship:

\[ \varepsilon = \frac{\sigma}{E} + \Lambda , \]  

(1.1)

where, \( \sigma/E \) is the mechanical strain and \( \Lambda \) is called the actuation strain, which is analogous to the thermal strain in conventional structural analysis.
Induced strain actuators may be assembled into systems either by surface-bonding or embedding the actuators within the structures themselves, depending on the requirements imposed on the actuator or on the ease of manufacture. As the induced strain actuator, be it a piezoceramic, shape memory alloy, or electrostrictor, is activated through a nonmechanical stimulus, a strain differential between the actuator and the host material structure is introduced resulting in an internal state of stress in the composite structure. By altering the activation mechanism, the induced strain actuator can directly influence and control the strain, curvature, and the strain energy of the geometrical configuration.

The most common induced strain actuators are discussed in the following sections, along with relevant research that has been performed.

1.1.3.1 Piezoelectric Actuators

It has been over one hundred and ten years since the Curie brothers discovered the piezoelectric effect, and while it was not used for many practical applications for almost a third of a century it found its first purpose at the end of World War I. Paul Langevin attempted to develop a means for locating submarine vessels using piezoelectric crystals (quartz) to generate acoustic waves. This device was, in fact, never used for submarine detection, but was later employed as a depth finder. Soon after World War I Professor G.W. Pierce of Harvard used the quartz transducer to produce a ultrasonic
interferometer, which allowed the measurement of the velocity and attenuation of many gases as a function of composition and temperature (W.P. Mason, 1981). Before proceeding forward and examining the Curie brother's effect on the research of today, piezoelectricity and the piezoelectric effect should properly be discussed.

Piezoelectrics are materials that exhibit the piezoelectric effect, i.e., materials that generate an electric charge in response to mechanical deformations, or conversely physically deform in the presence of an electric field. Piezoelectricity is a result of spontaneous separation of charge within certain crystal structures under the proper conditions. This phenomenon, spontaneous polarization, is caused by a displacement of the electron clouds in relation to their individual atoms, as well a displacement of the positive ions relative to the negative ions within their crystal cells. This situation produces an electric dipole. There are a variety of materials that exhibit this type of behavior to some degree, polycrystalline ceramic being one of the most active piezoelectric materials. Polycrystalline ceramic is composed of randomly oriented minute crystallites, and at this stage still lacks the piezoelectric behavior, but the material can be forced to exhibit macroscopic polarization in any direction through the application of a strong electric field, \( \approx 50-60 \text{ volts/mil} \), across electrodes on the outer surfaces of the material at a temperature above its Curie point. Once the polarization is induced, see Figure 1.2, the materials are then termed "ferroelectric" (Piezo Systems, 1987).
Before Polarization

After Polarization

Figure 1.2 Induced Macroscopic Polarization
As a result of the polarization the material becomes permanently elongated in the poling field and shortened in the transverse direction, and demonstrates the "piezoelectric effect", i.e., when an external compressive force is applied in the poling direction or a tensile force in the transverse direction, a voltage will be generated in the poling direction. Alternatively a voltage can be generated opposite to the poling direction by applying a tensile force in the poling direction or a compressive force in the transverse direction.

This material also can be used as an actuator by displaying the "converse piezoelectric effect", i.e., when an electric field is applied in the poling direction the material will further expand in the poling axis and contract in the transverse direction. When the external stimulus is removed the material will return to its original state. Conversely, when the electric field is applied opposite to the poling direction, the material will contract along the poling axis and expand in the transverse direction. Once again the material will revert to its initial state when the electric field is removed.

If too large a voltage is applied opposite to the poled direction, the original polarization will be degraded, or may even flip 180° and repole the piece in the opposite direction. The coercive field, $E_c$, is the term given to the maximum antipoling field a piece can withstand without experiencing depolarization. This value is highly dependent on whether the piece is being activated in a static or dynamic mode (Piezo Systems, 1987).
A great deal of research has been done utilizing piezoceramic type actuators because they are easily incorporated into induced strain actuator/substrate systems, they are simple, light weight, compact, and exhibit strong electromechanical coupling (Lazarus and Crawley, 1989). Although piezoceramics have a limited stroke (200-400 μstrain) they are extremely useful because of their wide operating bandwidth (greater than 20 KHz), and their capability to expand and contract.

Another piezoelectric material that is frequently implemented into a structural configuration for induced strain actuation purposes is the piezoelectric polymer. Polyvinylidene fluoride, PVDF or PVF₂, usually occurs in film form and unlike the more brittle piezoceramic’s, PVDF can be used on more complex geometries. In order to polarize the PVDF it is necessary to expose the film to an unconfined stretch during crystallization, followed by the application of an electric field perpendicular to the surface of the film. PVDF also exhibits the piezoelectric and the converse piezoelectric effect, but compared to piezoceramics it cannot transfer as much strain energy for a given field, and also has a lower Curie Point, which makes it less suitable for high temperature applications.

1.1.3.2 Electrostrictive Actuators

Electrostrictive actuators are like piezoelectric actuators in that they are ferroelectric
materials, but unlike piezoelectric materials, electrostrictives exhibit an electrostrictive effect, which is a second order effect, i.e., the strain induced by an electric field is proportional to the square of the electric field. Thus, when an electric field, positive or negative, is applied to the electrostrictive material it induces an electric polarization and causes the material to expand.

The most common electrostrictive actuators are the electrostrictive ceramics with the compound lead-magnesium-niobate (PMN), which have a stroke limit of about 1000 μstrain and require high voltages. Electrostrictive actuators exhibit less hysteresis compared to piezoelectrics and also demonstrate no signs of creep since no poling in necessary for electrostrictives, making them highly attractive to induce repetitive and exact actuation (Uchino, 1986).

1.1.3.3 Shape Memory Alloy (SMA) Actuators

Shape memory alloys (SMA’s) are a type of thermal actuator which demonstrate the shape-memory effect (SME). The shape memory effect can be described as follows: an object, in the low-temperature martensitic condition, when plastically deformed and the external stresses removed, will regain its original (memory) shape when heated, see Figure 1.3. This phenomenon is a result of a martensitic transformation taking place during the heating (Jackson et al.,1972; Schetky, 1979; Wayman and Shimizu, 1972).
Figure 1.3 Transformation of the SMA
There are many alloys that possess this shape recovery effect including several copper alloy systems, but the most common is the nickel-titanium (NiTi, Nitinol) alloy. The U.S. Naval Ordinance Laboratory was the first to discover this shape memory effect in NiTi, and hence the name (Nitinol), from Ni (Nickel) - Ti (Titanium) - NOL (Naval Ordinance Laboratory). The shape recovery effects in Nitinol are enormous, being able to recover plastic strains of typically 6% - 8% through heating. Restraining the material from regaining its "memory" shape can generate stresses on the order of 100,000 psi. It is these force and displacement capabilities that are exploited for electromechanical actuators (Liang and Rogers, 1990).

1.2 Intelligent Materials Research Applications

Many of the problems facing design engineers (nondestructive evaluation, damage detection and control, vibration and acoustic control, etc.) can be solved through the incorporation of intelligent systems. A brief review of relevant work done in these areas is presented.

1.2.1 Vibration Control

The concept of using piezoelectrics for vibrational control was first studied by Olsen in the mid 1950's (Olsen, 1956), but only in the last twenty to twenty-five years has it been
extensively investigated. Bailey and Hubbard (1985) and Crawley and de Luis (1987) introduced the use of piezoceramic materials incorporated into beam structures for vibrational control. In Bailey and Hubbard's work it was theoretically shown that all vibration modes could be controlled by applying the proper voltage to the distributed actuator to create a bending moment that opposed the angular velocity of the tip of the beam. Dimitriadis, Fuller, and Rogers (1991) extended the work done by Crawley and de Luis to a two-dimensional rectangular plate problem with piezoceramic patches bonded symmetrically to the top and bottom surfaces. It was shown in their work that properly configured piezoceramic patches could initiate out-of-plane displacements of the structure enabling the control of plate vibrations as well as sound radiation.

1.2.2 Acoustic Control

Along with controlling the vibration of a system, research engineers have investigated the use of Active Structural Acoustic Control (ASAC) implementing induced strain actuators as a means of controlling low frequency noise. W.R. Saunders, H.H. Robertshaw and C.A. Rogers (1990) investigated the active control of sound radiation from a clamped, baffled, composite beam using Shape Memory Alloy (SMA) fibers. The activation of the embedded SMA fibers made it possible to modify the sound radiated from the structure by changing the frequencies and mode shapes. The success of the experiments conducted relied implicitly on the Active Strain Energy Tuning
(ASET) control method for SMA composite structures. Active strain energy tuning takes advantage of the SMA fibers characteristic shape recovery effect, which manifests itself in a large restoring force and/or recovery of a tremendous amount of plastic strain. A large volume fraction of SMA fibers were placed in the graphite/epoxy matrix in the experimental lamina, and were plastically elongated and constrained in the fixture in order to prevent contraction during the consolidation of the composite material. Once the lamina was completed it was placed into a residual state of strain by heating the embedded fibers. The resulting change in energy balance modified the modal response of the structure. Using the ASET technique described above, Saunders et al. provided a means for the control of subsonic radiation through modification of SMA composite structure’s dynamic response. Bor-Tsuen Wang, E.K. Dimitriadis and C.R. Fuller (1990) analytically investigated the control of structurally radiated noise using multiple piezoelectric actuators. In their research Wang et al. harmonically exited a rectangular elastic panel through the use of multiple piezoelectric actuators and suggested that the use of multiple independently controlled piezoelectric actuators should greatly enhance the control effectiveness by reducing the spillover of control energy into the residual modes. In the study conducted by Wang several parameters were examined including the number, size and location of the actuators. From their analysis it was shown that effective sound radiation control could be achieved by appropriately tailoring the location and size of piezoelectric actuators.
1.2.3 Adaptive Structures

The use of smart material systems has also been incorporated into adaptive structures, which are defined as a structure that can purposefully vary its physical and mechanical properties. K.B. Lazarus and E.F. Crawley (1990) studied the feasibility of using a representative box wing adaptive structure for static aeroelastic control. In the aeroelastic analysis deformable type sections were utilized and activated with induced strain actuators to alter the lifting surface by changing the curvature and twist of the airfoil to produce an optimum configuration. The potential benefits of incorporating such a design for aeroelastic control compared to conventional articulated control surfaces were examined, and it was found that employing the induced strain actuators gave greater control authority along with a lower weight penalty for various wing designs. Lazarus and Crawley demonstrated that the strain actuated adaptive wings could be utilized instead of conventional lifting surfaces to provide better performance characteristics while reducing weight, decreasing loads in critical areas, improving the radar cross section, and maximizing the lift to drag ratio for many flight conditions.

T. Sato, H. Ishida, and O. Ikeda (1980), examined the usefulness of piezoelectric devices in deformable optics and mirrors. The thin devices can help correct aberrations and control the phases of light waves. Intelligent systems used with adaptive structures have an extremely important role for future space development, and are now being considered
to correct the focusing problems with the Hubble Space Telescope. This type of system can be incorporated into deployable structures, used for shape control of truss beams etc.

1.2.4 Damage Detection and Control

Intelligent materials and structures have an important role in health monitoring and active damage control. The following examples of intelligent materials currently imaginable were recently listed in a recent report published by the Science & Technology Agency;

- Material which restrains the advance of cracks by producing compression stress around them through volume change from stress-induced transformation at the tip of the crack, when the cracks are produced in strong parts due to repeated stress.

- Materials which recognize the load speed of stress and generate a large force against the shock stress by discriminating whether it is a shock stress or a static stress.

- Materials which give a warning and suppress the advance of generated deformation, damage, etc. or repair themselves in time.

- Materials which are useable over a wide range of temperatures with suitable change in composition by transformation on chemical reaction according to the environment.

Research has been performed at Virginia Polytechnic Institute & State University in the area of active damage control by C.A. Rogers, C. Liang and S. Li (1991). In their work several induced strain actuators were utilized, including Shape Memory Alloys (SMA’s),
piezoelectric, electrostrictive and magnetostrictive actuators, to control the propagation of cracks and fatigue damage within hybrid composite and metallic material systems. Significant reductions in the stress intensity factors were shown theoretically and experimentally. It was also illustrated that the actuators can generate a stress-strain field to counteract the external loading, thus reducing the cyclic stress amplitude and increasing the life-span of the material system.

1.3 Scope and Objectives

This investigation focuses on reducing the stress concentrations, which cause the actuators to peel away from the substrate in operational use (see Figure 1.4), located at the critical regions of the structure in order to improve the reliability and possible enhance the effectiveness of the induced strain actuator patches. In the finite element analysis conducted only a beam structure is analyzed, but the analysis can easily be extended to incorporate plate structures. In the structural configuration the actuators are symmetrically mounted to the top and bottom of the substrate and excited both in and out of phase producing extensional and bending modes respectively.

This thesis investigates existing theoretical models of induced strain actuated structures, and uses these to compare with the finite element analysis. The finite element analysis is used to explore the stress concentrations located at the edges of the actuators and
begins by refining the mesh areas of the same structure focusing in on the ends of the ISA’s. This preliminary analysis is conducted on a structural configuration with a perfectly bonded actuators and proceeds to those with a finite bonding layer.

After completion of the preliminary finite element analysis, several modifications in the design and implementation of the induced strain actuators are examined to reduce the stress concentrations at the edges of the actuators. In the finite element analysis two separate modeling considerations are examined:

1) The actuator is perfectly-bonded to the substrate.

2) A finite adhesive layer is incorporated between the actuator and the substrate.

With each of the above consideration in mind several design modifications are considered in this thesis including employing partial electrodes on the induced strain actuator surface regions instead of fully electroded surfaces, examining an actuator with a chamfered end, and modeling an actuator with structural caps. The design modifications and different modeling techniques are shown to have a significant impact of the stress concentrations in the critical regions in the structure, while also changing the performance characteristics of the structure.
Figure 1.4 Peeling Effects of the ISA’s
Chapter 2

Mechanics Model of Induced Strain Actuation of Beam Structures

To fully understand induced strain actuators, a description of the coupling behavior between the actuator and the substructure is needed, and a variety of models have been developed which do just that. The first models, such as the pin force model (Bailey and Hubbard, 1985; Crawley and Anderson, 1990), were relatively simple but provided excellent insight into the physics and mechanics of this type of actuation. With these models serving as a base, more encompassing analyses were investigated including the consistent plate model (Lazarus and Crawley, 1989), and models proposed by Dimitriadis and Fuller (1989), and Lee (1990). The more advanced models such as Crawley’s consistent plate model are more accurate than the simpler approaches but lack the insight into the mechanics of interaction between the actuator and the substrate. More recently models have been examined that are even more inclusive accounting for transverse shear deformations. A couple of the more extensive structural representations include that of Pai, Nayfeh and Oh (1992), which account for nonlinearities, and the linear shear stress variation model (LSSVM) developed by Lin and Rogers (1992), which assumes a shear
stress distribution in the structure, and from this determines the resulting stresses and strains along with the structural response. In the following sections the pin-force, the enhanced pin-force, and the Bernoulli-Euler models are developed and discussed.

2.1 Pin-Force Model

As mentioned earlier the pin-force model was developed from a mechanics point of view and proves to be extremely helpful in understanding and analyzing the physics of this type of actuation. In the next sections the analytical solution for this model is provided for symmetrically mounted induced strain actuator (ISA) patches as seen in Figure 2.1. In the analysis the actuators are activated both in and out of phase with the same electric field producing extension/compression and bending modes respectively.

The pin-force concept can be theorized in the following way. If a bonding layer of finite stiffness is present between the actuators and the substrate, then the forces generated by the actuators are transmitted by the means of shear stresses, which vary along the length of the actuators, but concentrate in infinitesimal regions near the ends. In the case of a perfect bond between the actuators and the substrate, the forces produced by the actuators are concentrated at points at the end of the actuator, and can be modeled as "pins" (Anderson, 1989). The pin-force idealization represents the actuators and the substrates as separate structures, connected only by pins at the edges of the actuators. The idea of
Figure 2.1 Symmetrically Mounted ISA's
this type of modeling is demonstrated in Figures 2.2 and 2.3. By connecting the structures with pins it allows the strain fields in each entity to be modeled independently, as long as compatibility is satisfied at the interface (i.e., at the pin locations), and imposes that all of the shear stress be concentrated at the ends of the actuators.

In the subsequent analysis the subscripts 'a' and 'b' are used to refer to the actuators and beam respectively.

2.1.1 Extension/Compression Mode

When using the pin-force concept (under in phase activation of the actuators) to determine the static response of the structure, the following assumptions are made:

a) The strain distribution is uniform through the thickness of both actuators.

b) The strain distribution is uniform through the thickness of the substructure.

These assumptions are illustrated in Figure 2.2.

The stress-strain relationships for the actuators and the substructure are represented in equations (2.1) and (2.2). In equation (2.1) \( \sigma_c/E_s \) represents the mechanical strain and \( \Delta \) is the free induced strain.
Figure 2.2 Pin-Force Model in Compression
\[ \varepsilon_a = \frac{\sigma_a}{E_a} + \Lambda , \]  
\[ \varepsilon_b = \frac{\sigma_b}{E_b} . \]  

Assuming that the actuators are both in compression, the stress in the actuators and beam are governed by the following relationships:

\[ \sigma_a = \frac{F}{b t_a} , \]  
\[ \sigma_b = -\frac{2F}{b t_b} . \]  

The force F in equations (2.3) and (2.4) is generated from a thermal load, \( P_\Lambda \). The two quantities are not equal, but are related by the following equation, which is analogous to thermal effects experienced in composite analysis:

\[ F = (EA)_a \varepsilon_a - P_\Lambda . \]  

It is clear from the above representation of F, that it is the actual resultant mechanical force and only equals \( P_\Lambda \) for the case of a perfect restraint (i.e., \( \varepsilon_a = 0 \)).

Enforcing the condition of strain compatibility at the interface requires:

\[ \varepsilon_a = \varepsilon_b . \]  

30
Setting equations (2.1) and (2.2) equal while utilizing equations (2.3) and (2.4) provides the following relationships:

\[ \frac{F}{b} = -\frac{E_b t_b}{2 + \Psi} \Lambda, \quad (2.7) \]

\[ \varepsilon_a = \varepsilon_b = \frac{2}{2 + \Psi} \Lambda, \quad (2.8) \]

where \( \Psi \), also known as the stiffness ratio, is defined by the following:

\[ \Psi = \frac{E_b t_b}{E_a t_a}. \quad (2.9) \]

Examining equation (2.9) reveals that an increase in strain may be obtained through an increase in the free induced strain, \( \Lambda \), or by incorporating a more compliant substrate.

### 2.1.2 Bending Mode

When the actuators are activated out of phase producing a bending mode the following assumptions in the pin-force model are employed:

a) The strain distribution remain constant through the thickness of the actuator.

b) The strain distribution varies linearly through the thickness of the substrate.

The preceding assumptions are illustrated in Figure 2.3.
Figure 2.3 Pin-Force Model in Bending
Incorporating the first approximation of a constant strain distribution through the actuator is a good one as long as the substrate is much thicker than the actuators.

In the analysis for pure bending the same stress-strain relationships used for the extension/compression case are still valid.

The moment-curvature expression for the beam is:

\[ M = F t_b = (EI)_b \kappa, \]  
(2.10)

where, \((EI)_b\) represents the beams flexural stiffness, and \(\kappa\) is the curvature.

Invoking the condition of strain compatibility at the interface requires:

\[ \varepsilon_b \big|_{z=\frac{z}{2}} = \varepsilon_a \big|_{\text{consi.}}. \]  
(2.11)

The following curvature and equivalent moment relationships are obtained by incorporating equations (2.1), (2.3), (2.8), and (2.10) along with Bernoulli-Euler beam theory (i.e., \(\varepsilon_b(z=\frac{z}{2}) = -(\varepsilon_b \kappa)/2\)) substituted into equation (2.11):

\[ \kappa = \frac{12}{t_b (6 + \Psi)} \Lambda, \]  
(2.12)

\[ M_{eq} = (EI)_b \kappa = \frac{b t_b^2 E_b}{(6 + \Psi)} \Lambda. \]  
(2.13)
As will be shown later in a comparison analysis, the pin-force model incorrectly predicts the structure surface strains approaching the actuation strains for small thickness ratios, \( t_s/t_u \).

### 2.1.3 Surface-Mounted Model with a Bonding Layer

This analysis is taken from work performed by Crawley and de Luis (1987), and a brief review of their analysis follows along with results for the extension and bending models which include a finite-stiffness bonding layer between the actuators and the substrate (see Figure 2.4). In the following review the same strain distributions as were postulated for the pin-force model are incorporated into this analysis (see Figures 2.2 and 2.3). Because of the strain assumptions, Crawley and de Luis were able to treat the bonded extension and bending cases in the same manner.

In all cases it is advantageous to include as stiff a bonding layer as possible in order to minimize losses due to shear lag. The increasing losses due to shear lag are caused by incorporating a more compliant or a thicker adhesive layer.

In Figure 2.5, two symmetrically mounted ISA's are bonded by a finite-stiffness bonding layer to an elastic substrate, and it is assumed that the shear stress remains constant through the thickness of the adhesive layer. The structure may be deformed in both
Figure 2.4 Structure with a Bonding Layer
Figure 2.5 Transfer of Shear Stress Through the Bonding Layer
extension/compression or bending, depending on the activation of the ISA's.

To derive the governing equations, the equilibrium of a differential element in the actuator substructure region must be analyzed. Assuming that the bonding layer is under one-dimensional shear and that the ISA's and the substructure experiences only extensional strain, the strain displacements relationships are:

\[ \varepsilon_a = \frac{du_a}{dx} = u_a', \quad (2.14) \]

\[ \varepsilon_b = \frac{du_b}{dx} = u_b', \quad (2.15) \]

\[ \gamma = \frac{u_a - u_b}{t_{bl}}, \quad (2.16) \]

where, \( t_{bl} \) is the bonding layer thickness, the superscript 's' on the displacement terms refers to the surface of the substrate, and \( \gamma \) represents the shearing strain.

Given the assumed strain distribution for extension and bending along with the uniform strain assumption in the induced strain actuators, the equilibrium equations are:

\[ \frac{d\sigma_a}{dx} - \frac{\tau}{t_a} = 0, \quad (2.17) \]

\[ \frac{d\sigma_b}{dx} + \frac{\alpha \tau}{t_b} = 0. \quad (2.18) \]
where $\alpha$ is a constant depending on the assumed beam strain distribution, and $\tau$ is the applied shear stress ($\tau = G_{bl} \gamma$). For a pure extension/compression case $\alpha=2$ (Figure 2.2); for a linear Bernoulli-Euler strain distribution $\alpha=6$ (Figure 2.3).

The above equations along with the stress-strain relationships (equations 2.1 and 2.2), and the correlation between the shear stress and the shear strain provide a system of eight equations with eight unknowns ($\sigma_a$, $\sigma_b$, $\tau$, $\varepsilon_a$, $\varepsilon_b$, $\gamma$, $u_a$, and $u_b$). Substituting equation (2.16) into the relationship between the shear stress and shear strain ($\tau = G_{bl} \gamma$), inserting the result into equations (2.17) and (2.18), and differentiating and utilizing equations (2.1), (2.2), (2.14), and (2.15) yield two coupled second order differential equations, which are reduced to an uncoupled set of fourth order differential equations:

$$\varepsilon_{b}^{iv} - \Gamma^2 \varepsilon_{b}^{iii} = 0 \ ,$$

$$\varepsilon_{a}^{iv} - \Gamma^2 \varepsilon_{a}^{iii} = 0 \ ,$$

where differentiation is with respect to a non-dimensional coordinate $\bar{x}$ ($\bar{x}=2x/L$). The shear lag parameter ($\Gamma$) is a function of the actuator length and the material properties of the actuator, substrate, and the bonding layer and is given by:

$$\Gamma^2 = \left[ \frac{G_{bl}}{E_a} \right] \left[ \frac{l_{bl}}{l_a} \right]^2 \left[ \frac{\alpha + \Psi}{\Psi} \right] \left[ \frac{l_{bl}}{L} \right] ,$$

(2.21)
Even though the fourth order equations are uncoupled, their solutions are coupled due to the coupled relationship of the second order equations. Solving equations (2.19) and (2.20) for the strain distributions in the induced strain actuators and the substrate and applying these coupling constraints gives:

\[
\begin{bmatrix}
\varepsilon_a \\
\varepsilon_b^* \\
\end{bmatrix} = 
\begin{bmatrix}
1 \\
1 \\
\end{bmatrix} B_1 + 
\begin{bmatrix}
1 \\
1 \\
\end{bmatrix} B_2 \bar{x} + 
\begin{bmatrix}
\Psi \\
\alpha \\
1 \\
\end{bmatrix} B_3 \sinh \Gamma \bar{x} + 
\begin{bmatrix}
-\Psi \\
\alpha \\
1 \\
\end{bmatrix} B_4 \cosh \Gamma \bar{x},
\]

(2.22)

By applying the four strain boundary conditions, the four unknown constants (B₁, B₂, B₃, and B₄) can be determined. An induced strain actuator with free ends is stress free at the ends, and thus by equation (2.1) the actuation strain at the free ends must equal the free induced strain, \( \Lambda \). The points on the substrate directly underneath the ends of the actuator may have a non-zero strain due to loading or deformation caused by the actuation of the induced strain actuators. The four boundary conditions resulting from these assumptions are:

\[ \bar{x} = +1: \quad \varepsilon_a = \Lambda, \quad \varepsilon_b^* = \varepsilon_b^{+} \quad , \]

and at

\[ \bar{x} = -1: \quad \varepsilon_a = \Lambda, \quad \varepsilon_b^* = \varepsilon_b^{-} \quad , \]

where \( \varepsilon_b^{+} \) and \( \varepsilon_b^{-} \) are known substructure strains at locations to the left (-) and right (+) of the induced strain actuator.
Substituting these conditions into equation (2.22), the unknown constants are found to be:

\[
B_1 = \frac{\Psi}{\Psi + \alpha} \left( \frac{\varepsilon_b^{*+} + \varepsilon_b^{*-}}{2} + \frac{\alpha \Lambda}{\Psi} \right), \quad (2.23a)
\]

\[
B_2 = \frac{\Psi}{\Psi + \alpha} \left( \frac{\varepsilon_b^{*+} + \varepsilon_b^{*-}}{2} \right), \quad (2.23b)
\]

\[
B_3 = \frac{\alpha}{(\Psi + \alpha) \sinh \Gamma} \left( \frac{\varepsilon_b^{*+} - \varepsilon_b^{*-}}{2} \right), \quad (2.23c)
\]

\[
B_4 = \frac{\alpha}{(\Psi + \alpha) \cosh \Gamma} \left( \frac{\varepsilon_b^{*+} + \varepsilon_b^{*-}}{2} - \Lambda \right). \quad (2.23d)
\]

The shear stress in the bonding layer, \( \tau \), can be determined by substituting equations (2.22) and (2.23) into equations (2.14) and (2.15) to obtain expressions for the first derivatives of \( u_a \) and \( u_b^* \). Integration of these results can be used with equation (2.16) and the shear stress-strain relationship (\( \tau = G_b \gamma \)) to provide the following expression for the shear stress in the bonding layer:

\[
\frac{\tau}{E_b} = \frac{G_{bl} L}{E_b t_{bl}} \left[ \frac{\varepsilon_b^{*+} - \varepsilon_b^{*-}}{2} \frac{\cosh \Gamma \bar{\gamma}}{\sinh \Gamma} + \left( \frac{\varepsilon_b^{*+} + \varepsilon_b^{*-}}{2} - \Lambda \right) \frac{\sinh \Gamma \bar{\gamma}}{\cosh \Gamma} \right]. \quad (2.24)
\]

The complete solution for the strains in the ISA's and the substructure along with the stress in the bonding layer have now been represented. All quantities are seen to be dependent on the voltage applied to the ISA's through the free induced strain (\( \Lambda \), and
on the strains in the substrate at the ISA boundaries. The terms dependent on \( e_b^+ \) and \( e_b^- \) are representations of additional passive stiffness added by bonding a reinforcement to the surface (Crawley and de Luis, 1897). The terms dependent on the free induced strain (\( \Lambda \)) represent the ability of the ISA’s to apply stress or strain to the substrate. In order to illustrate these effects more explicitly, the applied strain boundary conditions at the end of the substrate are set to zero, and equations (2.22) and (2.24) reduce to:

\[
\begin{bmatrix}
\varepsilon_d \\
\varepsilon_b^+
\end{bmatrix}
= \begin{bmatrix}
\Lambda \\
\Lambda
\end{bmatrix}
\frac{\alpha}{\Psi + \alpha} - \begin{bmatrix}
-\frac{\Psi \Lambda}{\alpha} \\
\Lambda
\end{bmatrix}
\frac{\alpha \cosh \Gamma \bar{x}}{(\Psi + \alpha) \cosh \Gamma},
\]

(2.25)

\[
\frac{\tau}{E_b} = -\frac{G_{bl}L}{E_b t_{bl}} \left( \frac{\sinh \Gamma \bar{x}}{\cosh \Gamma} \right) \Lambda.
\]

(2.26)

Equation (2.25) is plotted in Figure 2.6 for various values of \( \Gamma \) and for \( \Psi = 14.5 \) and \( \alpha = 6 \), typical for a piezoelectric ceramic exiting bending with an aluminum substrate 10 times its thickness. The upper three curves represent the strain in the actuator, and the lower three are the strain in the substrate, which illustrate the reduction in strain near the actuator edges. The perfect bond representation predicts a constant normalized strain of 0.2927 along the actuator length. The parameter \( \Gamma \) is a direct indicator of the effectiveness of the shear transfer between the ISA’s and the substrate, and is influenced heavily by the thickness and stiffness of the bonding layer. Minimizing losses due to shear lag can be accomplished by increasing the shear modulus, \( G_{bl} \), or decreasing the
Figure 2.6 Strains in the Structure for Various Values of $\Gamma$
thickness of the bonding layer, and this will effectively help to transfer the shear in a smaller zone near the ends of the induced strain actuators.

This model effectively reduces to that of the perfectly bonded pin-force model as the stiffness of the bonding layer approaches infinity, i.e., the shear lag parameter ($\Gamma$) approaching infinity. In this situation a sharp gradient in the shear stress is present at the ends of the ISA's indicating that the strain is transferred between the ISA’s and the substructure in an infinitesimal region near the ends of the induced strain actuators.

2.2 Enhanced Pin-Force Model

In order to incorporate the basic theories of the pin-force concept and approach better solutions for thinner structures, an improvement on the mechanics of the formulation is needed, and is provided by the enhanced pin-force model. Only out of phase activation resulting in bending is considered due to the fact that only in this type of induced strain does the conventional pin-force model break down for thin structures. The enhanced pin-force theory was first developed by Chaudhry and Rogers (1992) and allows for a correction in the traditional pin-force approach. The first point to be made is that the discrepancy in the pin-force model for small thickness ratios is not due to the uniform strain assumption in the actuators, as previously thought, but because of the incorrect moment-curvature expression (equation 2.9). In the expression for the moment, only
bending of the beam is accounted for. Thus, in order to accurately describe the system, the bending of the actuators must be duly represented by incorporating their bending stiffness into the moment-curvature expression.

The enhanced pin-force philosophy theoretically steps on its own feet so to speak. The assumption of a constant strain in the actuators actually implies that the actuators do not bend, and therefore no moments are applied to them. In the formulation of the enhanced pin-force model moments are applied to the actuators to induce bending, and these moments are then superimposed onto the substrate to incorporate the bending stiffness of each actuator into the moment-curvature relationship.

The modified moment-curvature relation incorporating the actuator stiffness is:

\[ M = Ft_b - 2 M_a = (EI)_b \kappa, \quad (2.27) \]

where, \( M_a \) is a moment applied to each actuator and subtracted from the moment \( (F^*t_b) \) applied to the beam (see Figure 2.7). Equation (2.27) can be rewritten in the following form by replacing \( M_a \) with its corresponding curvature relation (i.e., \( M_a = (EI)_a \kappa \)):

\[ Ft_b = [(EI)_b + 2 (EI)_a] \kappa, \quad (2.28) \]

where the flexural stiffnesses of the beam and the actuators are:
Figure 2.7 Enhanced Pin-Force Model
\[(EI)_b = E_b \frac{b l_b^3}{12}, \quad (2.29)\]
\[(EI)_a = E_a \frac{b l_a^3}{12}. \quad (2.30)\]

The force \( F \) in equations (2.27) and (2.28) is the equivalent force caused by the actuators, and was defined previously in equation (2.5).

The reasoning behind incorporating the additional flexural stiffness is rather obvious. By actuating the induced strain actuators out of phase the actuators on the top and the bottom of the structure not only expand and contract, but also bend along with the substrate. If however, the actuators were mounted on the beam with pin connections free to rotate, then they would not experience any bending deformation. In the actual structure the actuators are bonded to the structure and will bend along with the rest of the configuration, and thus their bending stiffness should be accounted for.

In order to compare the enhanced pin-force concept with the other models it is necessary to find the expression for the curvature, or the equivalent moment. To accomplish this it is first necessary to invoke the strain compatibility condition (i.e., \( \varepsilon_b(\varepsilon = \ell_b/2) = \varepsilon_a \)) utilizing Bernoulli-Euler beam theory (i.e., \( \varepsilon_a(\varepsilon = \ell_a/2) = -[\ell_b \kappa]/2 \)) and equations (2.1) and
(2.3) to obtain:

\[-\frac{t_b}{2}\kappa = \frac{\sigma_a}{E_a} + \Lambda = \frac{F}{b t_a E_a} + \Lambda. \tag{2.31}\]

Solving equation (2.27) for F, and substituting that expression into equation (2.31) the following curvature and equivalent moment relations are obtained:

\[
\kappa = \frac{12}{t_b \left[ 6 + \Psi + \frac{2}{T^2} \right]} \Lambda, \tag{2.32}
\]

\[
M_{eq} = \frac{b t_b^2 E_b}{\left[ 6 + \Psi + \frac{2}{T^2} \right]} \Lambda, \tag{2.33}
\]

where, T is the thickness ratio defined by:

\[T = \frac{t_b}{t_a}. \tag{2.34}\]

In previous work (Im and Atluri, 1989) the structural configuration was treated as a beam-column problem, including the effects of the in-plane axial forces on the out-of-plane displacements of the substrate. But in the case where the actuators are allowed to bend with the substrate there is no column action, and the only coupling between the in-plane forces and out-of-plane displacements is through the actuator offset distance (Chaudhry and Rogers, 1991).
2.3 Bernoulli-Euler Model

The derivation of the Bernoulli-Euler model proceeds on the exact same lines as the pin-force model with the exception that the entire cross-section, both the substrate and the actuators, undergo consistent Bernoulli-Euler strains, uniform for extension, and linear with \( z \) for bending. In the first part of each section the conventional derivation is provided, and this is followed by the mechanics based approach. It is important to note that the Bernoulli-Euler model provides the correct strain and curvature expression for all thickness ratios (Crawley and Anderson, 1990).

Once again a symmetric arrangement of the actuators is considered (see Figure 2.1), and the actuators are assumed to be perfectly bonded to the substructure. The Bernoulli-Euler concept is illustrated in Figures 2.8 and 2.9.

In conventional beam notation, the assumed Bernoulli-Euler strains are:

\[
\varepsilon = \varepsilon_o - z \kappa , \tag{2.35}
\]

where \( \varepsilon_o \) represents the middle-surface strain. The constitutive relations are:

\[
E(z) \varepsilon(z) = \sigma(z) + E(z) \Lambda(z) . \tag{2.36}
\]

Substituting equation (2.35) into (2.36) and integrating with respect to \( z \) gives:
Figure 2.8 Bernoulli-Euler Model in Compression
Figure 2.9 Bernoulli-Euler Model in Bending
\[(EA)_{\text{total}} \varepsilon_\phi - (EI)_{\text{total}} \kappa = P_\lambda , \quad (2.37)\]

or putting equation (2.35) into (2.36), premultiplication by \( z \), and integration yields:

\[(ES)_{\text{total}} \varepsilon_\phi - (EI)_{\text{total}} \kappa = M_\lambda , \quad (2.38)\]

where,

\[(EA)_{\text{total}} = \int_z E(z) \ b(z) \ dz , \quad (2.39)\]

\[(ES)_{\text{total}} = \int_z E(z) \ b(z) \ z \ dz , \quad (2.40)\]

\[(EI)_{\text{total}} = \int_z E(z) \ b(z) \ z^2 \ dz , \quad (2.41)\]

\[P_\lambda = \int_z E(z) \ \Lambda(z) \ b(z) \ dz , \quad (2.42)\]

\[M_\lambda = \int_z E(z) \ \Lambda(z) \ b(z) \ z \ dz , \quad (2.43)\]

are the total axial stiffness, the first and second moment of inertia, and the internal force and moment caused by the actuators respectively.

### 2.3.1 Extension/Compression Mode

When the actuators are activated in phase producing pure extension/compression, the
curvature, \( \kappa \), is zero, and equation (2.37) becomes:

\[
(EA)_{\text{total}} \varepsilon_o = P_{\Lambda} .
\]  
(2.44)

Utilizing equations (2.39) and (2.42) with \( b \) remaining constant gives:

\[
(EA)_{\text{total}} = \int_{-\frac{l}{2}}^{\frac{l}{2}} E(z) b \, dz = b(2E_a t_a + E_b t_b) ,
\]  
(2.45)

\[
P_{\Lambda} = \int_{-\frac{l}{2}}^{\frac{l}{2}} b e_a \Lambda \, dz + \int_{-\frac{l}{2}}^{\frac{l}{2}} b E_a \Lambda \, dz = 2b E_a t_a \Lambda .
\]  
(2.46)

Employing equation (2.44) with equations (2.45) and (2.46) provides the following expression for the strain field:

\[
\varepsilon_o = \frac{2}{(2 + \Psi)} \Lambda .
\]  
(2.47)

As should be expected, the strain field found using the Bernoulli-Euler approach is identical to that established employing the pin-force philosophy, since both types of modeling assume the same strain distribution.

Now the expression is derived from a mechanics approach. This method proceeds along the same lines as the pin-force approach, and begins by providing a representation for the strains in the structure as a function of the force, \( F \).
\[ \varepsilon_a^o = \frac{\sigma_a}{E_a} + \Lambda = -\frac{F}{b \; t_a}, \quad (2.48) \]

\[ \varepsilon_b^o = \frac{\sigma_b}{E_b} = \frac{2 \; F}{b \; t_b}, \quad (2.49) \]

where \( \varepsilon_a^o \) and \( \varepsilon_b^o \) represent the mid-surface strain of the actuators and beam respectively.

Enforcing the condition of strain compatibility at the interface requires:

\[ \varepsilon_a^o = \varepsilon_b^o. \quad (2.50) \]

Utilizing the above expression results in the same expression for the strain field:

\[ \varepsilon_a = \varepsilon_b = \frac{2}{(2 + \Psi)} \Lambda. \quad (2.51) \]

### 2.3.2 Bending Mode

Under the state of bending, equation (2.38) of the conventional approach reduces to:

\[ -(EI)_{total} \kappa = M_\Lambda, \quad (2.52) \]

and \((EI)_{total}\) and \(M_\Lambda\) are given by the following relationships:

\[ (EI)_{total} = \int z \; E(z) b z^2 \, dz = \frac{b E_a}{12} \; t_a \; t_b^2 \left[ 6 + \Psi + \frac{12}{T} + \frac{8}{T^2} \right], \quad (2.53) \]
\[ M_A = \int_{-\frac{L}{2}}^{\frac{L}{2}} E_a b \Lambda z dz + \int_{\frac{L}{2}}^{\frac{L}{2}+h} E_a b \Lambda z dz = -b E_a \left[ t_a t_b \left( 1 + \frac{1}{T} \right) \right] \Lambda , \quad (2.54) \]

where \( T \), known as the thickness ratio is defined by equation (2.34).

Substituting equations (2.53) and (2.54) into (2.52) gives the Bernoulli-Euler expression for the curvature, which is then used to find the representation for the equivalent moment:

\[ \kappa = \frac{12 \left( 1 + \frac{1}{T} \right)}{t_b \left( 6 + \Psi + \frac{12}{T} + \frac{8}{T^2} \right)} \Lambda , \quad (2.55) \]

\[ M_{eq} = (ED)_b \kappa = \frac{b E_a t_b^2 \left( 1 + \frac{1}{T} \right)}{\left( 6 + \Psi + \frac{12}{T} + \frac{8}{T^2} \right)} \Lambda . \quad (2.56) \]

Now, the above expressions for the moment and curvature relations are derived based on mechanics considerations. The resulting equations are found in a similar manner as was performed in the pin-force modeling, except now the strain in the actuators are no longer constant, but linear in \( z \). Therefore, the resulting strain fields in the beam and the actuators are dictated by the following relations:

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\[ \varepsilon_a = \varepsilon_a^0 - z \kappa , \quad (2.57) \]
\[ \varepsilon_b = \varepsilon_b^0 - z \kappa . \quad (2.58) \]

Utilizing the stress-strain relationships, the mid-surface strains can be expressed in terms of the force \( F \):

\[ \varepsilon_a^0 = \frac{F}{(EA)_a} + \Lambda , \quad (2.59) \]
\[ \varepsilon_b^0 = 0 . \quad (2.60) \]

The moment-curvature equation is:

\[ M = F (t_a + t_b) = [(EI)_b + 2(EL)_a] \kappa . \quad (2.61) \]

Note that the moment arm now accounts for the actuators by including half an actuator thickness on each of the two actuators, resulting in a total moment arm of \((t_a + t_b)\).

Insuring continuity of strain at the interface is accomplished similarly as was done in section 2.1.2 while accounting for the variation of the strain within the actuators:

\[ -\frac{t_b}{2} \kappa = \varepsilon_a^0 + \frac{t_a}{2} \kappa . \quad (2.62) \]

Solving equation (2.59) for \( F \) and substituting this into equation (2.62) gives an
expression for mid-surface strain as a function of the curvature, and if this resultant relation is put into equation (2.62) the following curvature and equivalent moment equations are obtained:

\[
\kappa = \frac{12 \left( 1 + \frac{1}{T} \right)}{t_b \left( 6 + \Psi + \frac{12}{T} + \frac{8}{T^2} \right)} \Lambda ,
\]

\[
M_{eq} = (EI)_{b} \kappa = \frac{bE_b t_b^2 \left( 1 + \frac{1}{T} \right)}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} \Lambda ,
\]

which are the exact expressions derived from the conventional Bernoulli-Euler model.

2.3.3 Surface-Mounted Model with a Bonding Layer

When the actuators are activated in phase producing an extensional strain, the Bernoulli-Euler and the pin-force models are identical, and therefore the strains in the actuators and the substrate have the same form as equation (2.25):

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y
\end{bmatrix} = \begin{bmatrix}
\Lambda \\
\Lambda
\end{bmatrix} \begin{bmatrix}
\frac{2}{2 + \Psi} \\
-\frac{\Psi \Lambda}{2}
\end{bmatrix} \begin{bmatrix}
\frac{2 \cosh(\Gamma \tilde{x})}{(2 + \Psi) \cosh(\Gamma \tilde{x})}
\end{bmatrix},
\]

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where \( \bar{x} \) is once again a non-dimensional parameter, which varies from -1 to +1 along the length of the actuators. The shear lag parameter for stretching (\( \Gamma_s \)) is:

\[
\Gamma_s^2 = \left[ \frac{G_{bl}}{E_a} \right] \left[ \frac{t_{bl}}{t_a} \right] \left[ \frac{2 + \Psi}{\Psi} \right] \left( \frac{t_{bl}}{L} \right)^2.
\]  

(2.66)

When the actuators are activated out of phase producing a state of bending it is assumed that the bonding layer does not contribute to the bending stiffness of the structure and is sufficiently thin as to not change the moment arm to the neutral axis. The overall effect of the bonding layer is to reduce the amount of curvature and add a component of pure extensional strain to the actuator (Crawley and Anderson, 1989). The strains are inferred to be:

\[
\frac{\varepsilon_x}{\Lambda} = \left( \frac{12}{t_b} \right) \frac{z + \cosh(\Gamma_b \bar{x})}{\cosh(\Gamma_b)} \left( \frac{1 + \frac{1}{T}}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} \right) \left[ 1 - \frac{\cosh(\Gamma_b \bar{x})}{\cosh(\Gamma_b)} \right],
\]

(2.67)

\[
\frac{\varepsilon_y}{\Lambda} = -\frac{z t}{\Lambda} = \left( \frac{12}{t_b} \right) \frac{z}{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}} \left[ 1 - \frac{\cosh(\Gamma_b \bar{x})}{\cosh(\Gamma_b)} \right],
\]

(2.68)

where the shear lag parameter for bending (\( \Gamma_b \)) is defined to be:
\[
\Gamma_b^2 = \left( \frac{G_{bl}}{E_a} \right) \left( \frac{t_{bl}}{t_a} \right) \left( \frac{t_{bl}}{L} \right)^2 \left( \frac{6 + \Psi + \frac{12}{T} + \frac{8}{T^2}}{\Psi \left(1 + \frac{1}{T}\right)} \right).
\]

(2.69)

The Bernoulli-Euler model effectively reduces to that of the perfectly bonded case as the stiffness of the bonding layer approaches infinity, i.e., the shear lag parameter (\(\Gamma_b\)) approaching infinity. Once again the shear lag parameter is a direct indicator of the effectiveness of the shear transfer between the ISA's and the substrate, and is influenced heavily by the thickness and stiffness of the bonding layer. Losses are minimized by increasing the shear modulus, \(G_{bl}\), or decreasing the thickness of the bonding layer, thus effectively helping to transfer the shear in a smaller zone near the ends of the induced strain actuators. In the Bernoulli-Euler strain distribution a sharp gradient in the shear stress is present at the ends of the ISA's indicating that the strain is transferred in an infinitesimal region near the ends of the ISA's. Equations (2.67) and (2.68) are not represented graphically, but display a similar pattern as those seen in Figure 2.6.

2.4 Finite Element Analysis

The finite element model developed is the most encompassing of the three previous concepts used to describe the interaction between the actuators and the substrate. Only the perfectly bonded case is examined in this section and the results for both extension...
and bending are compared to the analytical approaches in the subsequent section.

In the finite element analysis (FEA) it is only necessary to analyze half of the structure due to the symmetric arrangement and excitation of the actuators. A detailed two-dimensional model is first generated using I-DEAS (Integrated Design Engineering Analysis Software), and this model data is then transferred to ABAQUS to perform the analysis. Each entity of the structure is composed of two-dimensional plane stress linear isoparametric elements. For each thickness ratio the actuators are modeled with 240 (6x40) elements and the beam structure with 900 or more depending on the thickness of the beam. The mesh is refined near the edges of the actuators to account for the large stress gradients in the area and to highlight the shear lag. To represent the free-free model, the left edge of the structure is restrained from movement in the x-direction, and point P is constrained in both the x and z directions. The basic finite element arrangement is shown in Figure 2.10.

The same model is used for both induced extension and bending. The structure is modeled with a steel beam (E₀ = 29.6 msi), and Piezoelectric Products G-1195 piezoceramic actuators. To simplify the finite element analysis the induced strain actuators are modeled with an isotropic thermoelastic material type. Due to these simplifications in the representation, a single value for the Young's modulus of the actuator had to be specified. In the isotropic element a value of 9.13 msi, corresponding
Figure 2.10 Finite Element Model with Symmetric Actuators
to the inplane modulus, is used. Furthermore, a single value for the coefficient of thermal expansion (CTE) is allowed, specifying that $\alpha_x = \alpha_z = \alpha$. The actuator thickness is held constant at 10 mils and the beam thickness is allowed to vary representing the changing thickness ratio. The length of each actuator is 0.25 inches and the length of the entire structure is 0.325 inches.

Extension is induced by specifying a common CTE to the actuators while giving the beam a CTE of zero, and then applying a temperature differential to the entire model. Pure bending of the structure is accomplished in the same manner except that the CTE of one of the actuators has an equal magnitude but opposite sign causing one actuator to expand while the other contracts when a temperature change is imposed on the structure.

2.5 Model Comparisons

Three different analytical models and a finite element investigation of induced strain actuation has been presented. The results for the pin-force, the enhanced pin-force, the Bernoulli-Euler, and the finite element models are now compared for induced extension and induced bending.

For the case of induced extension/compression, the amount of strain produced in the structure is of primary interest. In the pin-force and the enhanced pin-force models it
is assumed that the actuators strain uniformly, while in the Bernoulli-Euler model the actuators bend as well as extend. For the situation where the actuators are activated in phase producing an extension mode, the formulas for induced strain in all three models are identical:

\[ \varepsilon_{\text{PF}}^{\text{PP}} = \varepsilon_{\text{EPF}}^{\text{PP}} = \varepsilon_{\text{BE}}^{\text{PP}} = \frac{2\Lambda}{2 + \Psi} \].

(2.70)

For the finite element model the strain is computed away from the edges of the actuator. The analytical prediction and the finite element results of the induced strain is depicted in Figure 2.11 for a range of thickness ratios, T. The modulus ratio, \( E_y/E_a \), is taken to be 3.24, which is typical for a steel structure and G-1195 actuators. In Figure 2.11, the four points representing the finite element results coincide with the three curves representing the analytical predictions, and the difference between the results is less that 0.2 %.

For the case of induced bending, the three analytical models are inherently different. The predicted induced curvatures with surface-bonded actuators are:

\[ \kappa_{\text{PF}}^{\text{PP}} = \frac{12\Lambda}{t_b(6 + \Psi)} \],

(2.71)

\[ \kappa_{\text{EPF}}^{\text{PP}} = \frac{12\Lambda}{t_b \left( 6 + \Psi + \frac{2}{T^2} \right)} \].

(2.72)
Figure 2.11 Comparison of Induced Extension Strains
\[ \kappa^{BE} = \frac{12 \left( 1 + \frac{1}{T} \right) \Lambda}{t_b \left( 6 + \psi + \frac{12}{T} + \frac{8}{T^2} \right)} \]  

(2.73)

The additional term in the denominator of the enhanced pin-force curvature expression is due to the addition of the flexural stiffness of the two actuators. The 1/T term in the numerator of the Bernoulli-Euler curvature equation represents the additional moment arm from the structure surface (where the "pin forces" of the pin-force model are located) to the midpoint of the actuators. The extra terms in the denominator of the Bernoulli-Euler curvature expression are due to the inclusion of the full bending stiffness of the actuators. In the finite element analysis the curvature is not directly obtainable from the computed results. Therefore, the curvature at each x location is obtained by using the stresses in the x-direction found from ABAQUS, and then taking moments about the neutral axis. The curvature is then found from the conventional moment curvature relationship \( M = (EI)_b \kappa \). The three analytical curvature expressions and the finite element results are plotted in Figure 2.12 for a range of thickness ratios and a modulus ratio of 3.24. Once again the curvatures found from the FEA are those obtained away from the edges of the actuator. It can be seen from the graph that the pin-force model breaks down for smaller thickness ratios, and deviate from the remaining models. Also seen from Figure 2.12 is that the Bernoulli-Euler results compare extremely well, and differ by less than 1.8 %, with those obtained from the finite
Figure 2.12 Comparison of Induced Bending Strains
element analysis. This is expected since the Bernoulli-Euler model provides the correct curvature expressions for all thickness ratios (Crawley and Anderson, 1990). The enhanced pin-force model does not suffer from the same effects as the more simplistic pin-force model, and better estimates the curvature for all thickness ratios.

It also interesting to examine the distribution of the equivalent moment in the x-direction. All three of the analytical models represent the moment as remaining constant as x increases or decreases until $\bar{x} = \pm 1$ where the equivalent moment suddenly drops to zero. The finite element results however are somewhat different, giving a more realistic description of the moment distribution. As seen in Figure 2.13 the equivalent moment found from the finite element analysis begins to drop off at around 80 % of the actuator length, and actually spills over past the end of the actuator before zeroing out. In Figure 2.13 each moment expression is normalized by the moment at $\bar{x} = 0$ found from the finite element results, $M^*_{eq}$. 
Figure 2.13  Effective Moments on the Beam along the x-axis
Chapter 3

Reduction of Critical Stresses

The finite element method is used in all subsequent analyses, and is performed on a free-free structure as seen in Figure 3.1, whether the structure has perfectly bonded actuators, or ISA's which incorporate a finite-stiffness adhesive layer. The dimensions of various parameters change throughout the analysis, and are so noted, but the basic free-free configuration remains the same. Due to the symmetric arrangement of the actuators and the type of induced modes, there is only a need to model the right half of the structure. The appropriate boundary conditions needed to represent this structure are described in section 2.4. Once again in this section the actuators are modeled as being isotropic, for simplicity. Later in the more encompassing analysis the actuators are modeled as having orthotropic properties to provide a more accurate representation of ISA's.

As is previously mentioned in the opening sections of my thesis, there is a need to investigate incorporating various modeling techniques and design considerations to alleviate the peeling tendencies of the ISA's, see Figure 1.4. In the next sections these modifications to the original structural configuration are examined.
Figure 3.1 Free-Free Structural Arrangement
3.1 Modeling Techniques

In most investigations the induced strain actuator (ISA) patches are modeled as being perfectly bonded to the substrate, thus defining the critical regions as those at the edges of the actuator patches, since this is the weakest part of the structure. However, once an adhesive layer is incorporated into the model it immediately becomes the region of critical interest, because it is in this region that the structure is most likely to fail in operation. In the first part of this section the structure is represented as having perfectly bonded induced strain actuator patches, and then a finite-stiffness adhesive layer is added to the model, and the differences between the modeling techniques are pointed out.

3.1.1 Perfect Bonded ISA's

The first finite element model used in the analysis is shown in Figure 3.2, and uses symmetry to model the right half of the structure. Figure 3.2 is a simpler picture than the actual finite element model, and is provided for a clearer representation of the structure. In the present configuration each part of the structure (actuators and substrate) are composed of two-dimensional plane stress linear isoparametric elements. The plane stress elements are implemented to provide an accurate representation of the 2-D structure. If one were to incorporate plane strain elements in to the analysis, disregarding the strain in the thickness direction, then additional constraints would be
added to the model and provide an inaccurate representation of the structure.

Once again a coefficient of thermal expansion (CTE) is assigned to each actuator, while the beam is given a CTE of zero. With the CTE's assigned to each actuator an appropriate temperature differential is imposed on the entire model, thus prescribing the free induced strain (Δ) used to actuate the structure. In this study a free induced strain of 100 μstrain is employed for the actuation. This value is chosen in order to not impose any nonlinearities into the analysis, such as large displacements, large rotations, or cause the actuators to deform outside of their prescribed elastic limit.

3.1.1.1 Extensional Mode

In the extensional mode each actuator is given a positive coefficient of thermal expansion, and a positive temperature differential is applied to the structure causing both actuators to expand. The deformed geometry for a thickness ratio of 6 is shown in Figure 3.3, where the blue elements represent the deformed configuration and the red lines correspond to the original boundaries of the undeformed structure. The deformed elements are all initially rectangular, but when the actuators are activated there is evidence of shearing, see Figure 3.3. The deformed elements illustrated exaggerate the displacements in both the x and z directions. The FEA investigation makes evident the finite shear in the actuators and the substrate, which reduces the effectiveness of the
Figure 3.2 FEA Model with Perfectly Bonded ISA's
actuators. This shear lag effect is not included in any of the analytical investigations, and is most prevalent near the edges of the actuators where most of the shear stress is concentrated. Away from the edges the strain across the structure is nearly uniform as is predicted by the analytical models.

The stresses (axial, normal, and shearing) in the structure are plotted in Figures 3.4, 3.5, and 3.6.

For the extensional mode it is observed that both actuators are in a state of compression, and this might seem a bit odd, but remember that the free induced strain is similar to a thermal strain. Restating equation (1.1) which provides the following relationship:

\[ \varepsilon = \frac{\sigma}{E} + \Lambda. \]  \hspace{1cm} (3.1)

Thus, the stress found in the actuators should be equal to the negative of the free induced strain times the Young's modulus, and examining Figure 3.4 it is seen that this is indeed the case. The stress in the ISA's remain nearly constant up to about 90% of the actuators length, at which point certain edge effects start to influence the solution. Near the ends of the actuators the compressive axial stresses start to drop off (i.e., become more positive), and the reason for this is found by examining the boundary conditions that must be satisfied at the edges. Theoretically the axial stress at the edges of the
Figure 3.3 Deformed Geometry, (Extension)
actuators must be zero due to the stress free conditions in this area. The FEA satisfies these boundary conditions for the outer parts of the actuators (i.e., the upper part of the top actuator, and the lower part of the bottom actuator). However, the FEA fails to satisfy these boundary conditions near the actuator-substrate interface and this is due to several factors which cause the stress concentrations in this region. The most important reasons are the 90° intersection between the actuator and the substrate, which in effect creates a crack tip, and the abrupt change in material properties, which causes differential straining between the two materials.

The axial compressive stress introduced to the ISA’s then induce a nearly constant tensile stress in the substrate up to approximately 65% of the total length of the beam. At this point the edge effects of the actuators start to influence the stress distribution in the beam, causing a overall reduction in the axial stress until it zero’s out just past the ends of the actuators.

The peeling stress in the extensional case is approximately zero throughout the entire structure, except at the highly stressed ends of the actuators, see Figure 3.5. It should also be mentioned that the peeling stress also satisfies its outer boundary conditions by forcing $\sigma_z$ to zero at $z = t_a + t_b/2$ and $z = - (t_a + t_b/2)$. Near the actuator edges it is noticed that the normal (peeling) stresses are actually compressive, causing the actuators to dig into the substrate. This occurrence is a good one, because the compressive peeling
Figure 3.4 Axial Stress, $\sigma_x$ (Extension)
Figure 3.5  Normal Stress, $\sigma_z$  (Extension)
Figure 3.6 Shear Stress, $\tau_{xz}$ (Extension)
stresses will not tend to pull the actuators away from the substrate. However, if the actuators are activated in a compressional mode then the peeling stresses would be tensile, causing the actuators to tear away from the beam structure. This result could initiate debonding of the actuators from the substrate if the structure is cyclically loaded over a period of time. The normal stresses take on a more significant meaning when a bonding layer is introduced into the analysis.

The shear stress distribution, shown in Figure 3.6, displays a similar pattern to the normal stress distribution, with the shearing stress remaining nearly zero over almost the entire structure, giving exception to the region near the ends of the actuators. In this critical area it is observed that there is a shear stress concentration, with the top being positive in shear, and the bottom being negative. These shear stresses cancel each other out across any cross-sectional cut, thus eliminating any net transverse shearing stresses. These stresses have a significant influence on the structure in each localized region.

3.1.1.2 Bending Mode

The bending mode is initiated by providing each actuator with the same magnitude of free induced strain ($\Delta$), but opposite in sign. In the bending deformation illustrated in Figure 3.7 the top actuator is given a positive $\Delta$, while the bottom actuator is given a negative $\Delta$. The deformed geometry in Figure 3.7 is represented by the blue elements,
Figure 3.7 Deformed Geometry, (Bending)
and the undeformed structure is symbolized with the red lines. The deformed configuration is magnified in both the x and z directions to fully demonstrate the bending effect. Also illustrated by the vertical lines in Figure 3.7 is the Kirchhoff hypothesis, utilized in many theoretical works, which states that plane sections remain plane. More simply stated the Kirchhoff hypothesis states that normal lines do not deform, they simply translate and rotate as a consequence of the deformation.

The stresses (axial, normal, and shearing) in the structure are plotted in Figures 3.8, 3.9, and 3.10.

For the bending deflection shown in Figure 3.7 the top actuator, which is under extensional strain, is in a state of axial compression, and the bottom actuator, which is in compressional strain, exhibits a near constant tensile axial stress, see Figure 3.8. The stresses in both the top and bottom ISA’s remain almost constant for about 90% of the actuator length, and those in the outer regions taper off to nearly zero to satisfy the axial boundary conditions at the edges. However, the axial stresses at the edge, near the actuator-substrate interface do not approach zero in the FEA due to the modeling factors mentioned in the previous section. The equal but opposite axial stresses in the symmetrically mounted ISA’s transfer those stresses into the substrate causing the stress distribution in this region to be anti-symmetric about the neutral axis.
Figure 3.8 Axial Stress, $\sigma_x$ (Bending)
Figure 3.9 Normal Stress, $\sigma_z$ (Bending)
Figure 3.10 Shear Stress, $\tau_{xz}$ (Bending)
The peeling stresses ($\sigma_z$), illustrated in Figure 3.9, is only significant in the regions near the actuator edges, and these stresses too are anti-symmetric about the neutral axis. The positive peeling stress will tend to initiate debonding of the actuators over a period of time as the structure is continuously loaded. The shear stress ($\tau_{xy}$), shown in Figure 3.10, also concentrates at the highly stressed ends of the actuators, and remains symmetric about the neutral axis approaching a maximum at the actuator-substrate interface.

3.1.2 Inclusion of an Adhesive Layer

A direct effect of including an adhesive layer into the finite element analysis is that the bonding layer reduces the effectiveness of the actuators, and the amount of this loss depends on the thickness and the stiffness of the adhesive. This shear lag effect is covered in detail in section 2.1.3. The two bonding layer models in the finite element investigation are modeled as having linear isotropic material properties, with thicknesses one tenth and two tenths that of the ISA's (i.e., $t_{bl} = 0.001$ in., and $t_{bl} = 0.002$ in.), and each having a Young modulus approximately one tenth that of the ISA's (i.e., $1 \times 10^6$ psi). The FE models are constructed similarly to the one shown in Figure 3.2, except that there is a finite-stiffness bonding layer included between both actuators. The mesh of the first adhesive layer model (0.001 in. thick adhesive layer) uses two elements in the z direction and the matching number of elements as the actuators in the x direction, and
the second adhesive model (0.002 in. thick bonding layer) uses four elements in the z direction to provide a similar representation of the bonding layers. The influence of incorporating the bonding layer can directly be seen by examining the beam deflection, see Figure 3.11. The deflections obtained in Figure 3.11 are taken at the centerline of the substrate (z = 0) and normalized by the free induced strain. Adhesive Layer I and Adhesive Layer II refer to models which incorporate bonding layers that are 0.001 in. and 0.002 in. thick respectively. The three curves measuring the beam deflection in Figure 3.11 are extremely close, and the Adhesive Layer I and Adhesive Layer II curves only differ from the perfect bond case by only 0.75% and 1.08% respectively at the tip of the beam (x = 1). There are several factors contributing to the closeness of the three curves. The bonding layers implemented into the FEA are relatively thin and stiff and help to minimize the losses as the stresses are transferred from the actuators to the substrate. The bonding layers themselves also increase the moment arm between the actuators and the neutral axis, thus allowing the actuators to generate larger bending moments.

The stress distributions similar to those shown in Figures 3.4 - 3.6 and 3.8 - 3.10 are not provided for the two adhesive layer cases for the sake of brevity. The distributions for each stress case looks very similar to those given for the perfect bond case, and only show variations near the edges of the actuator.
Figure 3.11 Beam Deflection Comparison (z = 0)
In the next section the stresses at the critical regions of each model are provided for structures that have perfectly bonded actuators and compared with the models which include adhesives layers between the actuators and the substrate.

3.2 Model Comparison

The critical regions in the structure with perfectly bonded actuators are the interfaces between the actuators and the substrate, and the important areas to examine in the model which includes adhesive layers between the actuators and the substrate are the interfaces between the actuators and the bond layers and the interfaces between the substrate and the adhesive layers. The stresses found at the Gauss points in these zones are analyzed and compared to point out the differences encountered when a bonding layer is incorporated into the investigation.

3.2.1 Critical Regions in the Extensional Mode

The stresses in each part of the structure are found at the Gauss points using the ABAQUS finite element software solving package. The stresses in the critical regions of each part of the structure are normalized by the corresponding maximum magnitude \((\sigma_v^*, \sigma_z^*, \tau_{xz}^*)\) from the perfect bond model, and are shown as they vary along the axial
direction, see Figures 3.12 - 3.15. In each of the figures it is interesting to point out that there exists a singular nature in all of the stresses at $\bar{x} = 1$. In all three models there are several reasons for the singularity of the stresses, and range from the $90^\circ$ intersection of the actuators or adhesive layers with the substrate, or the abrupt change in material properties in the z direction, to certain edge effects similar to those studied in composite laminates, which tend to initiate delaminations. The stresses at the bottom of the top actuator are plotted in Figure 3.12 as they vary in the x direction. It is seen that the adhesive layers tend to reduce the stresses at the edge of the top actuator, and as one would expect the thicker adhesive layer provides a larger stress reduction compared to the thinner bonding layer. The amount of the reductions are provided in Table 3.1, where $\sigma^m_x, \sigma^m_z, \tau^m_{xz}$ are the maximum stresses at or near $\bar{x} = 1$. The main reason for the stress reductions with the additions of the adhesive layers has to do with the soft more compliant adhesive layer that tends to dampen out the edge effects to some degree. The stresses at the top of the substrate are given in Figure 3.13, and they too show evidence of stress alleviation with the addition of a bonding layer, see Table 3.1. The stresses in the top adhesive layer for both thicknesses, see Figures 3.14 and 3.15, are given at the actuator-adhesive and the adhesive-substrate interfaces. It is in these areas that the models which include the adhesive layers are most likely fail since this is the weakest link of the structure. It is noticed from Figures 3.14 and 3.15 that all of the stresses are transferred in a small region near the edge of the bonding layer, and that the stresses in the thicker adhesive are less than those in the thinner layer. It is also
Figure 3.12 Top Actuator Stresses at the Bottom (Extension)
(- → Perfect Bond, o → Adhesive Layer I, + → Adhesive Layer II)
Figure 3.13 Substrate Stresses at the Top (Extension)
(- → Perfect Bond, o → Adhesive Layer I, + → Adhesive Layer II)
Figure 3.14 Top Adhesive Layer (Actuator-Adhesive Interface)
(x → Adhesive Layer I, o → Adhesive Layer II)
Figure 3.15  Top Adhesive Layer (Substrate-Adhesive Interface)
(◦ → Adhesive Layer I, o → Adhesive Layer II)
Table 3.1: Adhesive Layer Effects (Extension)

<table>
<thead>
<tr>
<th></th>
<th>Axial Stress $\sigma_x^m/\sigma^*_x$</th>
<th>Normal Stress $\sigma_\varphi^m/\sigma^*_z$</th>
<th>Shear Stress $\tau_x^m/\tau^*_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Actuator</td>
<td>Top of Beam</td>
<td>Top Actuator</td>
</tr>
<tr>
<td>Perfectly Bonded Actuators</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Adhesive Layer I (0.001 in.)</td>
<td>0.183</td>
<td>0.869</td>
<td>0.281</td>
</tr>
<tr>
<td>Percentage of Stress Reduction</td>
<td>81.7%</td>
<td>13.1%</td>
<td>71.9%</td>
</tr>
<tr>
<td>Adhesive Layer II (0.002 in.)</td>
<td>0.131</td>
<td>0.715</td>
<td>0.237</td>
</tr>
<tr>
<td>Percentage of Stress Reduction</td>
<td>86.9%</td>
<td>28.5%</td>
<td>76.4%</td>
</tr>
</tbody>
</table>
interesting to note that the stresses at the bottom of the top adhesive layer are slightly larger than those at the top, and this is most likely due to the fact that the stress concentration due to the 90° intersection between the adhesive and the substrate is greater than that caused by the free edge effects at the actuator-adhesive interface.

The stresses on the top of the structure are the only ones provided due to the symmetry of the model. In order to visualize the critical stresses on the opposite side of the neutral axis it is only sufficient to know that in the extensional mode the axial and the normal stresses are symmetric about the neutral axis, while the shear stress remains antisymmetric.

3.2.2 Critical Regions in the Bending Mode

The stresses in the critical regions of each part of the structure in the bending mode are also normalized by the corresponding maximum magnitude (σ*, σ*, τ*) from the perfect bond model, and are shown as they vary along the axial direction, see Figures 3.16 - 3.19. The singular nature of the stresses prevalent in the extensional mode also appears in the bending mode for the same reasons as mentioned previously.

The stresses at the bottom of the top actuator and at the top of the substrate are plotted in Figures 3.16 and 3.17 respectively as they vary in the x direction. Once again it is
Figure 3.16 Top Actuator Stresses at the Bottom (Bending)
(- → Perfect Bond, o → Adhesive Layer I, + → Adhesive Layer II)
Figure 3.17 Substrate Stresses at the Top (Bending)
(- → Perfect Bond, o → Adhesive Layer I, + → Adhesive Layer II)
Figure 3.18 Top Adhesive layer (Actuator-Adhesive Interface)
(x \rightarrow \text{Adhesive Layer I}, \circ \rightarrow \text{Adhesive Layer II})
Figure 3.19  Top Adhesive layer  (Substrate-Adhesive Interface)
(x → Adhesive Layer I, o → Adhesive Layer II)
observed that the adhesive layers tend to reduce the stresses at and around $\overline{x} = 1$, and
the amount of this reduction is shown in Table 3.2, where once again $\sigma^m_2, \sigma^m_3, \tau^m_{23}$ are the
maximum stresses at or near $\overline{x} = 1$. The main reason for the reduction in stresses is due
to the compliant adhesive layer that tends to dampen out the stress concentration effects.
The stresses in the top adhesive layer, see Figures 3.18 and 3.19, are given at the
actuator-adhesive and the adhesive-substrate interfaces, and it is in this region that the
structure is most likely fail in the bending mode. It is noticed that all of the stresses are
transferred near the edge of the bonding layer, and that the stresses at the bottom of the
adhesive layer are slightly larger than those at the top for reasons explained in section
3.2.1.

Again the stresses on the upper part of the structure only are provided due to the
symmetry of the model. In order to visualize the stresses on the opposite side of the
neutral axis it is only necessary to know that in the bending mode the axial and the
normal stresses are antisymmetric about the neutral axis, while the shear stress remains
symmetric.

3.2.3 Conclusions

Reviewing the results from section 3.2.1 and 3.2.2 it is evident that the incorporation of
adhesives layers into the finite element analysis is necessary to obtain an accurate
### Table 3.2: Adhesive Layer Effects (Bending)

<table>
<thead>
<tr>
<th></th>
<th>Axial Stress $\sigma_2^m/\sigma_x^*$</th>
<th>Normal Stress $\sigma_2^p/\sigma_2^*$</th>
<th>Shear Stress $\tau_{xy}^m/\tau_{xy}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Top Actuator</td>
<td>Top of Beam</td>
<td>Top Actuator</td>
</tr>
<tr>
<td>Perfectly Bonded Actuators</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>Adhesive Layer I (0.001 in.)</td>
<td>0.188</td>
<td>0.960</td>
<td>0.281</td>
</tr>
<tr>
<td>Percentage of Stress Reduction</td>
<td>81.2%</td>
<td>4.02%</td>
<td>71.9%</td>
</tr>
<tr>
<td>Adhesive Layer II (0.002 in.)</td>
<td>0.137</td>
<td>0.779</td>
<td>0.237</td>
</tr>
<tr>
<td>Percentage of Stress Reduction</td>
<td>86.3%</td>
<td>22.1%</td>
<td>76.4%</td>
</tr>
</tbody>
</table>
representation of the critically stressed regions in the structure. The high stress gradients that are prevalent for the perfect bond case are significantly altered through the addition of the adhesive layers, giving more favorable stress distributions. Examining Tables 3.1 and 3.2 reinforces the fact that the thicker adhesives layers tend to dampen out the stress concentration effects existing at and around $x = 1$. Even though it was not shown in the analysis it is apparent that the inclusion of a softer more compliant bonding layer will further reduce the stresses at the critical regions in the structure, but as a tradeoff, these physical changes in the adhesive layers degrade the performance of the actuators by increasing the shear lag effects, and thus affecting the transfer of stresses from the actuators to the substrate.

3.3 Design and Modeling Considerations

With the incorporation of the adhesive layers into the FEA, there is a necessity for the reduction of the high stress gradients at the edges of the bonding layers. In order to meet this need, certain design and implementation modifications in the induced strain actuator patches essential. Three design considerations are examined with the first employing partial electrodes to the ISA’s instead fully electroded surfaces. The second modifications examines ISA’s with tapered ends. The final design factor to be incorporated into the investigation looks at adding structural caps to the ISA’s to reduce the stress concentrations and to also try and take advantage of the piezoelectric strain
coefficient in the $z$ direction ($d_{33}$) to enhance the performance of the structure.

### 3.3.1 Incorporation of Partial Electrodes

To polarize and use induced strain actuators, such as piezoceramic and electrostrictive actuator patches, it is necessary to apply metal electrodes that act as conductors to the outer surfaces of the actuators. Each pair of electrodes have potentials of equal magnitude, but opposite sign and allow an electric field to be applied across the ISA to initiate extension or compression of the actuator depending on the direction of the field.

The first method investigated for minimizing the high stress gradients found in the localized region near $\bar{x} = 1$, examines employing partial electrodes on the actuator surfaces instead of fully electroded surfaces. Partial electroding the ISA's affects the stress and strain patterns in the actuator, thus affecting the stress, strain, and performance characteristics of the rest of the structure. In the next sections two partial electrode configurations are examined. In each model only the cases which include adhesive layers into the analysis are examined, since this is the most realistic model.

#### 3.3.1.1 Partial Electrode Configuration A

The first partial electrode configuration (PEA) examines the situation where the electrode
at the actuator-adhesive interface is placed exactly below the outer surface counterpart. The effect of such an arrangement ideally assumes that the electric field remains constant in that portion of the actuator that is electroded on both sides, and the electric field immediately drops to zero in the electrodeless part of the ISA. The impact in the FEA, which takes a thermal approach to the problem, assumes ideally that the ISA’s suddenly change material properties were the electrodes end. In actuality there will be some spillage of the electric field into the electrodeless portion of the actuator, but the effect is concentrated in the localized regions at the ends of the electrodes. The portion of the ISA’s with electrodes on each side can be referred to as the activating part of the actuator, while the electrodeless portions of the actuators is referred to as being the non-activating part of the ISA. The idea of this type of modeling is demonstrated in Figure 3.20.

With this type setup several partial electrode lengths are examined, and range from 50 - 97.5% of the original actuator length. Each of these cases are examined determining the amount of stress reduction in each critical region, while also examining the performance of the structure both in the extensional and the bending modes. The performance of the structure is evaluated by comparing the change in deflection at the centerline of the substrate of the partially electroded models to that of the fully electroded model.
Figure 3.20 Partial Electrode Configuration A
3.3.1.1.1 Extensional Mode

Once again the extensional mode is induced in the FEA by assigning like CTE's to the activating part of the actuators, while the remaining parts of the structure are given a CTE of zero.

Each case is set up using I-DEAS for the pre- and post-processing and the stresses, strains, and displacements are calculated using the ABAQUS software package. The partial electrode cases are compared to the fully electroded case by examining the stresses in the top adhesive layer at the actuator-adhesive and the substrate-adhesive interfaces. Due to the similar nature of the stresses for the two different bonding layer cases only the first case (i.e., $t_{bl} = 0.001$ in.) is used for the analysis.

Figure 3.21 provides the resulting changes in the maximum stresses in the adhesive layers, along with the impact on the performance characteristics of the structure for the actuators activated in-phase. Examining Figure 3.21 it is realized that the partial electrode configuration does have a significant impact on the critical stresses in the adhesive layers. It should also be pointed out at this juncture that the adhesive layers perform best in the shearing mode, and it is the axial, but more importantly the normal (peeling) stresses that initiate failure within the structure. Referring back to Figure 3.21 it is seen that as the electrode length is continually shortened, the amount of stress
Figure 3.21 Partial Electrode Configuration A (Extension)
alleviation in the adhesive layers continues to increase up to partial electrode lengths about 75% of the length of the actuator, where a limiting value is reached. As is expected though, when the electrodes are increasingly shortened, the performance characteristics of the structure start to fall off dramatically. Thus it is realized that with the first partial electrode configuration that in order to improve the reliability of the structure a trade-off must be made with the effectiveness of the actuators.

3.3.1.1.2 Bending Mode

Once again the bending mode is induced in the FEA by assigning CTE’s of equal magnitude, but opposite sign to the activating part of the top and bottom actuators, while the remaining portions of the structure are given a CTE of zero.

The same partial electrode cases that were investigated in the extension mode are examined in the bending mode, and the results are compared to the fully electroded model by once again examining the stresses in the adhesive layers, and the deflection of the centerline of the substrate. Figure 3.22 provides the percentage of stress alleviation in the critical regions of the adhesive layers, while also giving the resulting changes in the out-of-plane deflection of the structure.

The first partial electrode configuration in the bending mode displays the same patterns
Figure 3.22 Partial Electrode Configuration A (Bending)
in terms of the amount of stress relief, and in the effectiveness of the actuators as is seen in the extensional mode.

3.3.1.2 Partial Electrode Configuration B

The second partial electrode model (PEB) investigates the condition where the electrodes at the actuator-adhesive interface cover the entire length of the actuator, while the electrodes on the outer surface of the ISA’s extends out only over a prescribed portion of the total actuator length. It is ideally assumed that the electric field remains constant in that portion of the actuator that is electroded on both sides, and that the electric field follows a straight line decline from the end of the outer surface electrode to the corner of the ISA at the actuator-adhesive interface. The concept of this type of modeling and the FE representation is shown in Figure 3.23.

With this type arrangement several partial electrode lengths are examined and range from 50 - 97.5% of the length of the actuator. Each of theses cases are analyzed determining the percentage of stress relief in each critical regions of the adhesive layers, while also examining the performance of the structure both in the extensional and the bending modes of activation.

The resulting reductions in the critical stresses, and changes in the performance
Figure 3.23  Partial Electrode Configuration B
characteristics for both modes of activation are seen in Figures 3.24 and 3.25.

The second partial electrode configuration demonstrates similar stress and performance characteristics as the first arrangement (PEA), providing the best stress relief and the worst performance characteristics for the shorter electrode lengths, and the benefits, in terms of stress alleviation, again reach a limiting value at electrode lengths approximately equal to 75% of the length of the actuator.

The second partial electrode arrangement does outperform the first by providing slightly larger reductions in the maximum stresses, while the reduction in the effectiveness of the actuators is about three times less for both modes of activation than that found in PEA.

3.3.2 ISA's with Chamfered Ends

The relatively thin ISA's most commonly used in practice pose a difficult problem when trying to taper their ends in an effort to reduce the stresses in the adhesive layers. Therefore, due to the practicality of the problem only two cases are examined in the FEA, and those models have chamfers located on each end of the ISA's with $\Theta = 45^\circ$ and $\Theta = 60^\circ$, see Figure 26. The FE models for the chamfer cases are very similar to those examined in the second partial electrode case, except that there is no actuator material on the outside of the chamfers.
Figure 3.24 Partial Electrode Configuration B (Extension)
Figure 3.25  Partial Electrode Configuration B (Bending)
Figure 3.26 ISA's With Chamfered Ends
The amount of alleviation found by chamfering the ends of the ISA’s, and the resulting changes in the effectiveness of the actuators are seen in Figures 3.27 and 3.28.

The two chamfer models do provide some stress relief in both modes of activation, but not on the same order as is seen for the two partial electrode configurations. The performance of the structure remains nearly unchanged compared to the full actuator model (i.e., $\Theta = 90^\circ$).

Due to the unsatisfactory results obtained in the chamfer analysis, and the feasibility of constructing such a model due to the relative thinness of most ISA’s, this design modification is not recommended for implementation.

### 3.3.3 ISA’s Implementing Structural Caps for Stress Alleviation

Another alternative considered to help reduce the critical stresses in the adhesive layers and possibly provide increased performance characteristics is to mount structural caps on the ISA’s. In this section two separate cases are examined with the first using caps on surface-mounted actuator patches, see Figures 3.29 and 3.30. It should be noted that a gap between the actuator patches and the caps is maintained along the x-axis to allow the ISA to expand in the axial direction, and that the cap thickness ($t_c$) remains constant in the x and z directions. With these configurations supplying a restraint in the z direction
Figure 3.27  Chamfer Model (Extension)
Figure 3.28 Chamfer Model (Bending)
Figure 3.29  Surface-Mounted Actuators with Edge Caps
Figure 3.30 Surface-Mounted Actuators with Full Caps
the ISA’s are able to take advantage of their piezoelectric strain coefficient in the z direction, $d_{33}$, along with the normally used piezoelectric strain coefficient along the x-axis, $d_{31}$. The piezoelectric strain coefficient (PSC) $d_{33}$ relates the ratio of strain along the z-axis to the electric field applied along the z-axis, and PSC $d_{31}$ relates the excitation in the z direction to the strain along the x-axis. Although the caps help the ISA’s to utilize more of their material parameters, they also increase the structural stiffness of the model, which deters from the performance of the structure.

The second model examines the situation where the induced strain actuator patches are embedded flush with the surface of the substrate, and the edge caps are mounted to the ends of the actuators and to the substrate, see Figure 3.31. This configuration eliminates some of the structural stiffness of the substrate, but also decreases the moment arm of the actuators, thus the actual effect on the performance of the structure in undetermined.

With each of the above modeling considerations, the finite element analysis, which had previously used isotopic actuator patches, had to be somewhat modified. The actuators are now modeled as being transversely isotropic having the same dimensions as shown in Figure 3.1, with material properties associated with G-1195 piezoceramic actuators, see Table 3.3. Once again the strain in the actuators is induced by applying a thermal load to the model and assigning the appropriate CTE’s to the actuator patches. In the isotropic model $\alpha_x = \alpha_z$, but now in the transversely isotropic model $\alpha_x \neq \alpha_z$. 

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Figure 3.31  Embedded Actuators with Edge Caps
Table 3.3 Material Properties Of The Constituents

<table>
<thead>
<tr>
<th>G-1195 PZT Patch</th>
<th>Steel Substrate</th>
<th>Steel Caps</th>
<th>Aluminum Caps</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_x = 9.13$ (ksi)</td>
<td>$E_x = 29.6$ (ksi)</td>
<td>$E_x = 29.6$ (ksi)</td>
<td>$E_x = 10.0$ (ksi)</td>
</tr>
<tr>
<td>$E_x = 7.11$ (ksi)</td>
<td>$E_x = 29.6$ (ksi)</td>
<td>$E_x = 29.6$ (ksi)</td>
<td>$E_x = 10.0$ (ksi)</td>
</tr>
<tr>
<td>$G_{xx} = 3.19$ (ksi)</td>
<td>$G_{xx} = 11.5$ (ksi)</td>
<td>$G_{xx} = 11.5$ (ksi)</td>
<td>$G_{xx} = 3.77$ (ksi)</td>
</tr>
<tr>
<td>$\nu_x = \nu_y = 0.3$</td>
<td>$\nu_x = \nu_y = 0.3$</td>
<td>$\nu_x = \nu_y = 0.3$</td>
<td>$\nu_x = \nu_y = 0.3$</td>
</tr>
<tr>
<td>$d_{31} = -65.4 \times 10^{-10}$ (in/V)</td>
<td>$\Lambda = 0.0$</td>
<td>$\Lambda = 0.0$</td>
<td>$\Lambda = 0.0$</td>
</tr>
<tr>
<td>$d_{33} = 141.7 \times 10^{-10}$ (in/V)</td>
<td>$t_y = 6t_x$</td>
<td>$t_y = 0.3t_x \rightarrow 2t_x$</td>
<td>$t_y = 0.3t_x \rightarrow 2t_x$</td>
</tr>
</tbody>
</table>
Therefore, a relationship must be developed between the piezoelectric strain coefficients, \( d_{33} \) and \( d_{31} \), and the CTE's, \( \alpha_x \) and \( \alpha_z \).

The strain in the x direction is related to \( d_{31} \) by the following equation:

\[
\frac{\Delta L}{L} = \frac{V}{t_a} d_{31} = \alpha_x \Delta T ,
\]

where, \( L \) and \( \Delta L \) are the original and changed axial lengths of the actuator, \( V \) is the voltage, \( t_a \) is the actuator thickness, \( \alpha_x \) is the CTE in the x direction, and \( \Delta T \) is the change in temperature applied to the model.

The strain in the thickness direction is expressed in the subsequent equation:

\[
\frac{\Delta t_a}{t_a} = \frac{V d_{33}}{t_a} = \alpha_z \Delta T .
\]

Solving equation (3.2) for the voltage as a function of the CTE in the x direction, the change in temperature, \( d_{31} \), and the actuator thickness, and substituting this into equation (3.3) the following relationship between the CTE's in the x and z directions can be found:

\[
\alpha_z = \frac{\alpha_x d_{33}}{d_{31}} .
\]

Thus, once one of the coefficients of thermal expansion is prescribed the other is

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automatically defined.

3.3.3.1 Caps on Surface-Mounted Actuators

The models for the surface-mounted actuators incorporating caps into their configuration are shown in Figures 3.29 and 3.30. Both arrangements seem viable for reducing the critical stresses in the adhesive layers mainly because most of the stresses are transferred in a very small region near the ends of the ISA's. The full cap arrangement helps to better utilize the strain components in the thickness direction to provide increased performance qualities. This configuration also significantly increases the axial and bending stiffnesses of the structure, which adversely affect the effectiveness of the actuators. The edge cap arrangement on the other hand also incorporates the effects of the piezoelectric strain coefficient in the z direction, but only moderately affects the stiffness of the structure, and should alleviate the unwanted tearing stresses prevalent near the ends of the ISA's.

3.3.3.1.1 Full Cap Arrangement

Figures 3.32 - 3.37 are the resulting stress alleviation patterns and performance characteristics for the full cap model. For comparison both steel and aluminum caps are incorporated into the analysis.
Figure 3.32 Full Cap in Extension ($\sigma_x$, $u_x$)
Figure 3.33 Full Cap in Extension ($\sigma_z$, $u_x$)
Figure 3.34 Full Cap in Extension ($\tau_{xz}$, $u_x$)
Figure 3.35  Full Cap in Bending ($\sigma_x$, $w_x$)
Figure 3.36  Full Cap in Bending ($\sigma_z, w_z$)
Figure 3.37 Full Cap in Bending ($\tau_{xz}$, $w_z$)
The full cap configuration is seen to demonstrate excellent alleviation of the critical stresses in the adhesive layers, as is expected, but in the performance category this type of structural arrangement causes the actuators to suffer serious performance degradation in both modes of activation. Therefore, the full structural cap model is deemed unacceptable.

3.3.3.1.2 Edge Cap Arrangement

Figures 3.38 - 3.43 are the resulting stress alleviation patterns and performance characteristics for the edge cap model. Once again both steel and aluminum caps are incorporated into the analysis.

The edge ap arrangement is seen to demonstrate excellent alleviation of the critical stresses in the adhesive layers, and also displays moderate gains in the performance characteristics of the structure in both modes of activation. Examining Figures 3.38 - 3.43 it is realized that the steel edge caps are more effective in reducing the stress concentrations in the adhesive layers up to cap thicknesses around 50 - 60% of the actuator thickness, and as the cap thickness is increased beyond this point the more compliant aluminum caps become more beneficial in providing adhesive layer stress relief. It is intuitively difficult to understand why the more compliant aluminum caps start to demonstrate an advantage in alleviating the ISA's peeling tendency at larger cap
Figure 3.38  Edge Cap in Extension ($\sigma_x$, $u_x$)
Figure 3.39  Edge Cap in Extension ($\sigma_z$, $u_z$)
Figure 3.40  Edge Cap in Extension ($\tau_{xz}$, $u_x$)
Figure 3.41  Edge Cap in Bending ($\sigma_x$, $w_z$)
Figure 3.42  Edge Cap in Bending ($\sigma_z$, $w_z$)

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Figure 3.43  Edge Cap in Bending ($\tau_{xz}$, $w_z$)
thicknesses, but it has to do with the complex interaction between the edge caps and the ISA’s.

As far as the effectiveness of the actuators is concerned, the analysis reveals that the stiffer steel caps are more effective for all cap thicknesses, and the performance characteristics continually increase up to cap thicknesses about 1.5ta. As the cap thickness gets thicker than one and a half times the thickness of the actuator, the effectiveness of the ISA’s start to gradually decline.

3.3.3.2 Edge Caps on Embedded Actuators

Due to the performance losses exhibited by the full cap model, only the edge cap configuration is examined with the embedded actuator model, see Figure 3.31. The ISA patches are embedded flush with the substrate surface, and a gap between the end of the actuators and the substrate is maintained to allow the actuators to expand in the x direction. Again, both aluminum and steel caps (te = 0.3ta) are incorporated into the analysis for completeness.

In the preliminary working of the model it was hoped that the reduced axial and flexural stiffness caused by embedding the actuators would out weigh the losses in decreasing the moment arm of the ISA’s, but after completion of the work it was seen that this was not
the case, and the performance characteristics fall off by over 30%.

Tables 3.4 and 3.5 list the amount of stress alleviation experienced with the implementation of the edge caps, and the change in the performance parameters (note: the parameters in Tables 3.4 and 3.5 are compared to the embedded configuration with no edge caps). It is evident that the edge caps serve their purpose in reducing the stress concentrations in the adhesive layers, with the steel edge caps being more effective. The performance of the structure remains nearly the same (compared the embedded model with no Edge caps), with only a slight increase in structural deflections.

### 3.3.4 Edge Cap - Partial Electrode Combination

Realizing that the two best configurations are the partial electrode and the edge cap configurations, it seems natural to combine the two to achieve the best results. This new crossbreed is then analyzed using the first partial electrode configuration for various electrode lengths (87.5%, 95%, and 97.5% of the length of the actuator) and the steel edge caps ($t_e = 0.3t_s$). The results are presented in Tables 6 and 7. After constructing other models with increasing cap thicknesses and reviewing the results from the following tables it is concluded that the two modifications together do not provide a larger relief for the critical stresses, or provide any improvements in the actuator effectiveness, than when the two modifications are implemented independently from one another.
Table 3.4: Maximum Stress Changes
Embedded Actuator - Edge Cap Configuration ($t_c = 0.3t_a$)

<table>
<thead>
<tr>
<th></th>
<th>Axial Stress $\sigma^m_a/\sigma_i^{*(bl)}$</th>
<th>Normal Stress $\sigma^m_x/\sigma_i^{*(bl)}$</th>
<th>Shear Stress $\tau^m_{xz}/\tau_i^{*(bl)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Steel Caps</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Reduction in Stress <em>(Extension)</em></td>
<td>35.0%</td>
<td>76.1%</td>
<td>75.5%</td>
</tr>
<tr>
<td>% Reduction in Stress <em>(Bending)</em></td>
<td>(4.22%)</td>
<td>65.8%</td>
<td>66.2%</td>
</tr>
<tr>
<td><strong>Aluminum Caps</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% Reduction in Stress <em>(Extension)</em></td>
<td>44.2%</td>
<td>65.8%</td>
<td>82.3%</td>
</tr>
<tr>
<td>% Reduction in Stress <em>(Bending)</em></td>
<td>15.4%</td>
<td>72.4%</td>
<td>74.6%</td>
</tr>
</tbody>
</table>
Table 3.5: Maximum Deflection Comparisons
Embedded Actuator - Edge Cap Configuration ($t_r = 0.3t_a$)

<table>
<thead>
<tr>
<th>% Change in Deflection</th>
<th>Steel Caps $\Delta u_x^m$</th>
<th>$\Delta w_x^m$</th>
<th>Aluminum Caps $\Delta u_x^m$</th>
<th>$\Delta w_x^m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Edge Caps</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>Edge Caps</td>
<td>+ 1.59%</td>
<td>+ 0.61%</td>
<td>+ 1.45%</td>
<td>+ 0.57%</td>
</tr>
</tbody>
</table>
Table 3.6: Maximum Stress Changes
Edge Cap ($t_c = 0.3t_a$) - Partial Electrode Combination

<table>
<thead>
<tr>
<th>Steel Caps</th>
<th>Axial Stress $\sigma_x^m/\sigma_x^{(bl)}$</th>
<th>Normal Stress $\sigma_x^m/\sigma_x^{(bl)}$</th>
<th>Shear Stress $\tau_{xx}^m/\tau_{xx}^{(bl)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Extension</strong></td>
<td>Actuator Adhesive Interface</td>
<td>Substrate Adhesive Interface</td>
<td>Actuator Adhesive Interface</td>
</tr>
<tr>
<td>97.5% Electroded % Reduction in Stress</td>
<td>(69.2%)</td>
<td>12.3%</td>
<td>5.93%</td>
</tr>
<tr>
<td>95.0% Electroded % Reduction in Stress</td>
<td>(19.3%)</td>
<td>53.2%</td>
<td>45.4%</td>
</tr>
<tr>
<td>87.5% Electroded % Reduction in Stress</td>
<td>(7.57%)</td>
<td>72.1%</td>
<td>48.4%</td>
</tr>
<tr>
<td><strong>Bending</strong></td>
<td>Actuator Adhesive Interface</td>
<td>Substrate Adhesive Interface</td>
<td>Actuator Adhesive Interface</td>
</tr>
<tr>
<td>97.5% Electroded % Reduction in Stress</td>
<td>(108.0%)</td>
<td>11.3%</td>
<td>1.49%</td>
</tr>
<tr>
<td>95.0% Electroded % Reduction in Stress</td>
<td>(76.3%)</td>
<td>58.6%</td>
<td>35.3%</td>
</tr>
<tr>
<td>87.5% Electroded % Reduction in Stress</td>
<td>(42.0%)</td>
<td>69.5%</td>
<td>38.6%</td>
</tr>
</tbody>
</table>
Table 3.22: Maximum Deflection Comparisons
Edge Cap ($t_c = 0.3t_e$) - Partial Electrode Combination

<table>
<thead>
<tr>
<th></th>
<th>Steel Caps</th>
<th>No Caps</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta u^m_x$</td>
<td>$\Delta w^m_z$</td>
</tr>
<tr>
<td>Fully Electroded % Change in Deflection</td>
<td>+ 2.20%</td>
<td>+ 1.00%</td>
</tr>
<tr>
<td>97.5% Electroded % Change in Deflection</td>
<td>+ 0.53%</td>
<td>+ 0.25%</td>
</tr>
<tr>
<td>95.0% Electroded % Change in Deflection</td>
<td>- 0.39%</td>
<td>- 0.21%</td>
</tr>
<tr>
<td>87.5% Electroded % Change in Deflection</td>
<td>- 6.00%</td>
<td>- 3.18%</td>
</tr>
</tbody>
</table>
3.3.5 Results and Conclusions

The variety of design and implementation considerations can easily be evaluated and compared by examining the figures and tables of sections 3.3.1 - 3.3.4.

Reviewing the results from the partial electrode configurations it is clear that the larger stress reductions are associated with the models incorporating the shortest electrode lengths. As is expected though, the models incorporating the shorter electrodes suffer from reductions in performance characteristics for both modes of activation. Comparing the two partial electrode models it is observed that the second configuration, Figure 3.23, provides, on average, larger stress reductions and demonstrates better performance characteristics.

The chamfer model examined does demonstrate a fair amount of stress alleviation, while the actuator effectiveness remains nearly the same as with the full actuator model. The chamfer model does not provide the magnitude of stress reductions found in the other modeling configurations considered. This type of design modification is also somewhat impractical due to the thinness of most ISA's. Thus, this type of design consideration does not seem to be a viable option for implementation.

Of the models implementing the caps to the surface mounted actuator arrangement to
provide stress alleviation and possibly enhance performance parameters, the analysis reveals that only the edge cap model demonstrates acceptable results. The edge cap configuration provides excellent stress reduction properties, while actually increasing the performance of the structure. The full cap model on the other hand provided excellent alleviation for the stress concentrations, but suffered seriously in the performance category.

It was stated in the previous section that the embedded actuator configuration without the edge caps exhibits excessive performance losses. Adding the edge caps to the embedded actuator model does provide a large reduction in the maximum stresses and also enhances the effectiveness of the actuators. Thus, if this type of model is already in service it would be beneficial to incorporate the edge caps to improve the reliability and performance of the structure, but it is not recommended to embed the actuators to start with.

Comparing the viable models to each other it can be seen that the second partial electrode configuration (at electrode lengths < 87.5% of the actuator length) demonstrates the largest stress alleviation followed closely by the first partial electrode arrangement (at electrode lengths < 87.5% of the actuator length), the edge cap configurations follows, and then the combination partial electrode edge cap model. In the performance category the steel edge cap and the aluminum edge cap configurations display the best
performance characteristics, followed by the crossbreed model, and then the two partial electrode configurations.

On an overall basis the edge cap arrangement provides the best results, since it supplies the total package of enhanced reliability and moderate performance increases. The partial electrode configurations also contain many distinct advantages, and since implementing edge caps is not applicable for all uses of the ISA’s, the partial electrode arrangements should be incorporated to improve the reliability of the structure.
Chapter 4

Conclusions and Recommendations

The effectiveness of induced strain actuators for structural control has been previously established (Bailey and Hubbard, 1985; Crawley and de Luis, 1987; Rogers, 1990b). It is now time to consider certain design and implementation modifications to enhance the reliability and improve the effectiveness of the induced strain actuators.

4.1 Conclusions

Reviewing the results from the various finite element models, the following conclusions are reached:

- Each design modification and implementation technique provided excellent alleviation of the critical stresses in the adhesive layers, giving a more reliable structure.
- Incorporating edge caps and partial electroding the actuator surfaces proved to be...
the most beneficial modifications.

- The two partial electrode configurations (at partial electrode lengths < 0.875 L) yielded the largest amounts of stress alleviation, followed closely by the edge cap arrangements.

- The steel cap configurations demonstrated the best performance characteristics, showing moderate gains in structural deflection in both modes of activation.

- On an overall basis the edge cap configuration demonstrates the best results, giving excellent alleviation of the critical stresses in the adhesive layers, while improving the effectiveness of the actuators.

- Incorporating edge caps might not be applicable to all uses of ISA’s, and in those situations the partial electrode configurations become extremely attractive since they also did an excellent job of improving the reliability of the structure while only moderately reducing the performance characteristics of the structure.

4.2 Recommendations

In order to insure the acceptance of these design and implementation techniques a variety of actual fatigue testing procedures should be performed to ensure the feasibility of the design and implementation modifications and to also identify any potential problems.
References


Rogers, C.A., 1990b, "Active Vibration and Structural Acoustic Control of Shape Memory Alloy Hybrid Composites: Experimental Results," Proceedings, International
Congress on Recent Developments in Air- and Structure-Bourne Sound and Vibration, Auburn University, 6-8 March 1990, pp. 695-708.


VITA

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John Walker