Characterization of Boundary Conditions for Wedge-Lock-Mounted Printed Circuit Boards

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(ABSTRACT)

Vibration testing and analysis is becoming increasingly important in the electronics industry. It is used as a workmanship screen, as well as a way to duplicate the deployed environment in both the military and commercial avionics worlds. To minimize the effects of the vibration from testing and what will be encountered in service, the mechanical analyst must be able to accurately predict mode shapes and frequencies of in-situ PC board.

This thesis will investigate modeling the wedge locks as non-uniform translational and rotational springs. The first eight natural frequencies of a rectangular circuit board (with no components soldered to it, and with wedge locks along two edges) will be empirically determined. Eight frequencies will be used to solve for four unknowns: continuously distributed translational and rotational spring stiffnesses along the segments of board that are in contact (two unknowns) with the wedge lock, and those that are not in contact (two unknowns). A finite element model will be developed of the physical system. The translational and rotational spring stiffnesses will be optimized to minimize an error function involving the difference between the empirical and analytically predicted natural frequencies.
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and Riley, for accepting the fact that when I was on the computer doing
work either for school or for my job, I couldn’t play with them.
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Nomenclature

Roman Symbols

\[ A \] Area (in\(^2\))

\[ D \] Plate Constant (in•lbs)

\[ e \] percent strain (dimensionless)

\[ E \] Young’s Modulus (lb/in\(^3\))

\[ F \] Force (lb)

\[ F_f \] First modal frequency of a plate with two fixed edges (Hz)

\[ F_n \] First modal frequency (Hz)

\[ F_s \] First modal frequency of a plate with two simply supported edges (Hz)

\[ G \] Shear Modulus (lb/in\(^2\))

\[ k \] Stiffness (lb/in)

\[ K_k \] Stiffness (lb/in)

\[ K_r \] Rotational Stiffness (in•lbs/rad)

\[ L \] Length (in)

\[ l \] change in length (in)

\[ m \] slope

Greek Symbols

\[ \varepsilon \] strain (in/in)

\[ \lambda^2 \] Plate frequency parameter (dimensionless)

\[ \mu \] Poisson’s Ratio (dimensionless)
\( \sigma \) stress (lb/in\(^2\))
\( \tau \) shear stress (psi)
\( \gamma \) shear deformation (in/in)
\( \varepsilon_x \) strain in the X direction (in/in)
\( \varepsilon_y \) strain in the Y direction (in/in)
Chapter 1: Introduction

Vibration testing and analysis is becoming increasingly important in the electronics industry. It is used as a workmanship screen, as well as a way to duplicate the deployed environment in both the military and commercial avionics worlds. To minimize the effects of the vibration from testing and what will be encountered in service, the mechanical analyst must be able to accurately predict mode shapes and frequencies of in-situ PC board vibrations.

Far too often the best modeling of printed circuit boards is done by modifying the modeling techniques to match the frequencies found in testing. This does little good at the beginning of a project, before the first boards are produced. The analyst will know the size of the board, the mounting technique, and little else before the boards are laid out and fabricated. After the boards are fabricated, it becomes an expensive proposition to change the board for vibration concerns. This makes the modeling done early in the design phase that much more important. The analyst can, with information about the mode shapes that occur within the equipment’s operational vibration spectrum, modify the design to break up or move certain modes, or rearrange components to minimize stressing of the electrical leads.
The more difficult modeling parameter to characterize has been the boundary conditions, especially when wedge-lock card guides are used. These devices provide more than a simply supported edge, but not quite a fully fixed edge for the boards [1]*.

Some studies found in the literature [2,3] attempted to characterize the wedge locks as simply supported with continuously distributed rotational springs. The rotational springs were assumed to have the same stiffness, which was adjusted until the first modal frequency prediction matched that found empirically. The study assumed that:

- the wedge locks were infinitely stiff in translation normal to the board.
- the stiffness of the rotational spring was uniform along the edge of the board or is unknown.

Figure 1 shows the way wedge locks are segmented. There are alternating sections of the circuit board that are clamped by the wedge lock, and those sections that are pressed against the card slot, but not touching the wedge lock. Thus, the last assumption appears in question.

*Numbers in square brackets refer to references listed at the end of this thesis.
This thesis will investigate modeling the wedge locks as non-uniform translational and rotational springs. A test sample (a rectangular plate with wedge locks along two edges) will be constructed and modal testing will determine the first eight natural frequencies. Eight frequencies will be used to solve for four unknowns: continuously distributed translational and rotational spring stiffness along the segments of board that are in contact with the wedge lock, and those that are not. A finite element model will be developed of the physical model using translational and rotational springs as the boundary conditions. The translational and rotational spring stiffnesses will be modified based upon minimization of the error function between the measured and predicted frequencies.
Most of the literature deals with the wedge-lock issue in English Units. For this reason, this thesis will present its data in English units instead of Metric units.
Chapter 2: Literature Search

Much of the literature’s treatment of the wedge locks has been primarily confined to predicting the first natural frequency of a printed circuit board. Steinberg [1] states that for a board with a first mode

- below 100 Hz the wedge lock will act like a fixed edge.

- below 600 Hz the frequency will correspond to an interpolated value between the frequencies for the board with fixed and simply supported edges according to the following equation:

\[
F_n = \frac{F_{s} + 1.1(F_{f}-F_{s})}{1 + 0.001(F_{f}-F_{s})}
\]  

(1)

Barker and Chen’s results [2] discount this equation altogether, stating that the fixity provided by the wedge lock is independent of the natural frequency of the board.

Roza’s test data [4] supports Steinberg’s equation for 5 segment wedge locks using the manufacturer’s recommended torque, but not for the 3 segment version. Although Roza does provide the slot width that was used in the
testing, the width of the of the ribs that made the sides of the slots is not
given. The slots in the fixture were made to replicate a typical configuration. It
is not known if the stiffness of the sides of the slots are comparable to that
being attributed to the wedge locks. Roza's study used aluminum plates as
test samples because of the certainty of the material properties used in
modeling. However, the contact mechanics differences between the wedge
locks and the aluminum compared to wedge lock and composite PC boards
were ignored. Such contact stiffness differences could influence the optimized
stiffness values. It also only addressed optimizing the rotational spring
constants until the first mode was matched in frequency.

Barker and Chen[2] determined that there was no closed form solution for
describing the stiffnesses of the boundary conditions provided by wedge locks.
Instead, they tabulated the dimensionless frequency parameter $\Lambda^2$ as described
by Leissa[5]. This study also used aluminum plates as test samples. One
figure does indicate that a fairly rigid test fixture was used, so that the values
measured were attributable to the wedge locks.

Banks [3] attempted to find the stiffness of the bolted joint produced by the
wedge lock. In the end, it was stated that it appeared to be prohibitively
complicated to find a closed-form solution for, or even to develop a FEM of this
bolted joint. Part of the problem encountered may have been with the free-
body diagrams that provided the basis for the analysis. They did not satisfy rotational equilibrium.

If the stiffness of a plate clamped with wedge locks can have an effect on the boundary condition, then anisotropic plate material could also have an effect on the boundary condition. For this reason, it was decided that the test samples for the study in this thesis would be fabricated from materials commonly used for printed circuit boards.
Chapter 3: Reconciliation of Theoretical Predictions with Free-Free Testing

The testing for this thesis was divided into the following phases:

Phase 1: Preliminary Free-Free Testing to Validate Testing and Analysis Procedures

i. Test an unpopulated FR4 circuit board in the free-free condition.

ii. Build a FEM of the test article using material properties found in the literature. If the modeling agrees with the empirical data, go to the next phase of testing; otherwise go to step iii.

iii. Test a piece of FR4 that does contain component mounting holes in the free-free condition.

iv. Build a FEM of the test article using material properties found in the literature. If the modeling agrees with the empirical data, go to the next phase of testing; otherwise go to step v.

v. Obtain material property data for the test samples via modulus testing.

vi. Build a FEM of the test article using material properties found in the modulus testing. If the modeling agrees with the empirical data, go to next phase of testing. If the FEM predicted frequencies still do not agree with the test results, continue to refine the material property
definition.

Phase 2: Free-Free Testing Of Samples to be Used in the Restrained Testing with the Wedge Locks

i. Test the aluminum sample in the free-free condition.

ii. Build a FEM of the test article using material properties found to be most representative in the Preliminary Testing.

iii. Test the FR4 sample in the free-free condition.

iv. Build a FEM of the test article using material properties found to be the most accurate in the Preliminary Testing. If the modeling agrees with the empirical data, go to next phase of testing; otherwise go to step v.

v. Obtain material property data for the test samples via modulus testing.

vi. Incorporate the material properties found in the modulus testing into the FEM.

Phase 3: Restrained Testing Using Wedge Locks

i. Obtain modal frequency data for an aluminum plate mounted with wedge locks.

ii. Build a FEM of the aluminum test article and compare the various analytical techniques found in the literature to model the boundary conditions provided by the wedge locks.
iii. Modify the FEM to contain non-uniform translational and rotational springs to model the wedge-lock boundary conditions. Optimize the translational and rotational spring stiffnesses to minimize the error function between the FEM predicted and empirical modal frequencies.

iv. Obtain modal frequency data for the FR4 plate modeled in the free-free condition mounted with wedge locks.

v. Build a FEM of the FR4 test article and compare the various analytical techniques found in the literature to model the boundary conditions provided by the wedge locks.

vi Modify the FEM to contain non-uniform translational and rotational springs to model the wedge-lock boundary conditions. Optimize the translational and rotational spring stiffnesses to minimize the error function between the FEM predicted and empirical modal frequencies.

Phase 1, is described in sections 3.1, 3.2, and 3.3. Phase 2, is described in sections 3.4 through 3.7. Chapter 4 is devoted to Phase 3.
3.1 Phase 1: Preliminary Free-Free Testing of FR4 Printed Circuit Boards

3.1.1 Phase 1, Step i: Free-Free Testing of an Unpopulated Printed Circuit Board

Before the testing of the wedge locks began, it was decided that it would be prudent to verify that the test samples could be accurately modeled in the free-free condition. To this end, an unpopulated circuit board (manufactured from FR4*) was suspended using monofilament line and shock cords and impact tested. A PCB piezo hammer (Model 086C03) was used for the input and the output was measured using a Kistler accelerometer (Model 8618A500) which weighed only .0002 lbs (0.1 gm). This accelerometer was used because of its low mass and low profile, which would reduce the mass and mass moment loading of the test article by the instrumentation. These were both connected to an Hewlett Packard (HP) model 35665 Fast Fourier Transform (FFT) Analyzer. The characterization of the wedge locks would require a minimum of 4 frequencies, but given the early stages of the project the testing of the project sample was limited to 0 to 200 Hz. The Frequency Response Function (FRF) magnitude, coherence function, and the

*A grade of fiberglass defined by the National Equipment Manufacturer’s Association.
FRF's imaginary component obtained from this sample are shown in Figures 2, 3, and 4, respectively. The responses from five impacts were averaged.

The data was converted from HP's SDF format to ASCII and read into MathCad®* for graphical presentation. The subscript \( i \) on the graph's axis labels denotes the range variable set up in MathCad to track the number of the spectral line for each data point. For this sample, the HP was set to a resolution of 400 lines over the bandwidth of 0 to 200 Hz. Thus, \( i \) could range from 0 to 400, which in turn corresponds to a frequency range of 0-200 Hz.

Table 1 lists the resonances that were found in this data by using the peak-picking method (selecting the spectral lines where the imaginary component was maximized or minimized).

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*MathCad is a registered trademark of MathSoft, Inc., Cambridge, Mass.
Figure 2  FRF Magnitude of 12.63"x9.25"x0.06" PCB Sample, Free-Free

Figure 3  Coherence Function for FRF for 12.63"x9.25"x0.06" PCB Sample, Free-Free

Figure 4  Imaginary Component of FRF for 12.63"x9.25"x0.06" PCB Sample, Free-Free
Table 1

FEM Frequency Estimates and Modal Test Results for 12.63" x 9.25" x 0.06" PCB with Percent Deviation From Test Results

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>TEST Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40 (-5.9%)</td>
<td>42.5</td>
</tr>
<tr>
<td>2</td>
<td>44 (-24.1%)</td>
<td>58</td>
</tr>
<tr>
<td>3</td>
<td>84 (-6.7%)</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>91 (-27.8%)</td>
<td>126</td>
</tr>
<tr>
<td>5</td>
<td>113 (-26.4%)</td>
<td>153.5</td>
</tr>
<tr>
<td>6</td>
<td>127 (-34.0%)</td>
<td>192.5</td>
</tr>
<tr>
<td>7</td>
<td>171</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>186</td>
<td></td>
</tr>
</tbody>
</table>
3.1.2 Phase 1, Step ii: Finite Element Modeling of an Unpopulated Printed Circuit Board in the Free-Free Condition

A finite element model (FEM) of the board was constructed using MacNeal-Schwendler’s MSC-Pal2®*. The model consisted of 108 quadrilateral plate elements with 902 active degrees of freedom (DOF’s). Two sets of values were found for Young’s Modulus and Poisson’s ratio [1]. One set was for "epoxy glass" ($E_{\text{static}} = 2 \times 10^6$ psi, $\mu = 0.12$), and the other was for "epoxy glass with copper planes" ($E_{\text{static}} = 3 \times 10^6$ psi, $\mu = 0.18$). An average value for the mass density was calculated given the measured weight and dimensions of the board. The weight of the accelerometer was only 0.0002 lbs. This was found to have negligible effect on the behavior of the FEM, and was left out of the model. The model was attached to "ground" via a series of spring elements around the perimeter of the board. The other end of the springs were attached to a row of nodes created 0.01" away (out-of-plane of the model) from the corresponding nodes on the board’s perimeter. The springs were extremely soft ($k = 0.0001$ lb/in/node) and had stiffness in the three translational directions only. All of the nodes in the

*MSC/Pal2 is a registered trademark of The MacNeal-Schwendler Corp., Los Angeles, Ca.
model were restrained in the out-of-plane rotational direction since the quadrilateral plate elements in this program do not have stiffness in this direction. The other end of the springs were restrained in all three translational and all three rotational degrees of freedom.

A total of 14 frequencies and modes were calculated, with the first 6 being rigid-body modes with natural frequencies below 1 Hz. The fact that the frequencies of rigid-body modes were so low indicated that the springs attaching the model to ground were soft enough that they would not affect the next eight flexural modes. The frequencies predicted by the FEM are also shown in Table 1. Table 1 show that there is very poor agreement between the FEM and the test data (the values in parentheses are the errors with respect to the empirical values). The peak picking method used to find the resonances in the test data was not as accurate as using the circle-fit technique, but was considered adequate since the errors seen were well above the resolution at which the data was taken (0.5 Hz).

We could assume that the FEM incorrectly predicted the presence of modes 3 and 5. This would make the errors for modes 4, 6, 7, and 8 drop to 1.1%, +0.8%, +10.3%, and -2.1%.
3.1.3 Phase 1, Steps iii and iv Testing and Analysis of a Printed Circuit Board Sample without Component Mounting Holes in the Free-Free Condition

One possible explanation for this was the non-homogeneous properties of the board caused by the presence of all of the unfilled holes meant to be filled with solder and component leads. One 3.44" x 6.40" region of this sample board was devoid of any holes, although it did have some copper traces running across it. To test this possibility, this section of the board was cut out and impact tested, and a finite element model was created along the same lines as described above for the first test sample. For this sample, the HP 35665 was set to a resolution of 800 lines over the bandwidth of 0 to 800 Hz. The FRF magnitude, coherence function, and the FRF's imaginary component obtained for this sample are shown in Figures 5, 6 and 7, respectively. Table 2 lists the natural frequencies found in impact testing and from the FEM for the second test sample. Table 2 shows that the test data and the FEM predictions still did not agree. The FEM was constructed using 108 quadrilateral plate elements with 860 active degrees of freedom.
Figure 5 FRF Magnitude of 3.44"x6.40"x0.06" PCB Sample, Free-Free

Figure 6 Coherence Function for FRF for 3.44"x6.40"x0.06" PCB Sample, Free-Free

Figure 7 Imaginary Component of FRF for 3.44"x6.40"x0.06" PCB Sample, Free-Free
Table 2

FEM, Theoretical Frequency Estimates and Modal Test Results for 3.44"x6.40"x0.06" PCB with Percent Deviation from Test Results

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>Leissa Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150 (-17.2%)</td>
<td>158 (-12.7%)</td>
<td>181</td>
</tr>
<tr>
<td>2</td>
<td>177 (12.8%)</td>
<td>189 (-6.9%)</td>
<td>203</td>
</tr>
<tr>
<td>3</td>
<td>391 (-7.7%)</td>
<td>387 (-8.7%)</td>
<td>424</td>
</tr>
<tr>
<td>4</td>
<td>418 (-25.8%)</td>
<td>430 (-23.6%)</td>
<td>563</td>
</tr>
<tr>
<td>5</td>
<td>555 (-23.0%)</td>
<td>565 (-21.6%)</td>
<td>721</td>
</tr>
<tr>
<td>6</td>
<td>665 (-16.7%)</td>
<td>622 (-22.1%)</td>
<td>798</td>
</tr>
</tbody>
</table>

Given the poor agreement between the FEM and the test data, a third source was sought. This was found in Leissa’s Vibration of Plates [5]. This Reference contained information only to predict the first six modes, but this would still be a good check on the FEM. Table 2 also contains the first six frequencies predicted by linearly interpolating between two aspect ratios provided by Leissa.
Table 2 shows that the FEM and Leissa agree very well. The small amount of error between the two could be attributed to the linear interpolation done between the two aspect ratios. The fact that the two analytical models tracked each other, but not the empirical data, implied that something common to both analytical models was inadequate. One of the primary assumptions in both models was that the material was homogeneous. Given that the test samples were manufactured from a composite material (FR4 fiberglass) it is entirely possible that the material properties are directionally dependent.

3.1.4 Phase 1, Step v: Modulus Testing of Samples From 3.44" x 6.40" x 0.06" Printed Circuit Board Sample without Component Mounting Holes

To see if there were significant variations in the material properties, some of the 3.44" x 6.40" x 0.06" test samples were cut into 0.5" wide pieces and used for modulus testing. Three samples were cut so that the long axes of the samples were parallel to the 6.40" sides and were designated as "longitudinal" samples. Three other samples were cut so that the long axes of the samples were parallel to the 3.44" sides, and were designated as "transverse" samples. All longitudinal samples were cut from the same PCB,
and all transverse samples were all cut from another PCB.

The samples were tested using an Instron 4204, with an extensometer to measure the percent deflection in the direction of loading. The axial load and the axial strain were recorded using a MAC-based data acquisition system, and saved in ASCII format. These files were later read into MathCad so that the data could be graphed and curve fitted using MathCad curve fitting commands (see Appendix A). Table 3 shows the Young's Moduli measured for the longitudinal and transverse samples. This table shows a slight directional dependence for stiffness.

Table 3

Modulus Test Results

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Young's Modulus Parallel To Longitudinal Axis of Sample (psi)</th>
<th>Young's Modulus Parallel to Transverse Axis of Sample (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.64 x 10^6</td>
<td>3.11 x 10^6</td>
</tr>
<tr>
<td>2</td>
<td>3.01 x 10^6</td>
<td>2.95 x 10^6</td>
</tr>
<tr>
<td>3</td>
<td>2.90 x 10^6</td>
<td>3.39 x 10^6</td>
</tr>
<tr>
<td>Average</td>
<td>2.85 x 10^6</td>
<td>3.15 x 10^6</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.90 x 10^5</td>
<td>2.23 x 10^5</td>
</tr>
</tbody>
</table>
3.1.5 Phase 1, Step vi: Modeling of 3.44" x 6.40" x 0.06"

Printed Circuit Board Sample without Component Mounting Holes

Using Data From Modulus Testing

Using these values for Young’s Moduli, density derived from the board’s measured weight and size, and values from the literature for Poisson’s ratio and shear Modulus, the FEM was ran again. Table 4 shows the values obtained from this run compared to those found in test. Although the agreement is not outstanding, the results obtained by orthotropically describing the material are superior to those obtained from the model that described the material isotropically.
Table 4

FEM Frequency Estimates Using Isotropic and Orthotropic Material and Modal Test Results for 3.44"x6.40"x0.06" PCB with Percent Deviation from Test Results

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM, Isotropic Material Properties Natural Frequency, Hz (%error)</th>
<th>FEM, Orthotropic Material Properties Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150 (-17.2%)</td>
<td>195 (+7.7%)</td>
<td>181</td>
</tr>
<tr>
<td>2</td>
<td>177 (12.8%)</td>
<td>209 (+2.9%)</td>
<td>203</td>
</tr>
<tr>
<td>3</td>
<td>391 (-7.7%)</td>
<td>466 (+9.9%)</td>
<td>424</td>
</tr>
<tr>
<td>4</td>
<td>418 (-25.8%)</td>
<td>537 (-4.6%)</td>
<td>563</td>
</tr>
<tr>
<td>5</td>
<td>555 (-23.0%)</td>
<td>668 (-7.4%)</td>
<td>721</td>
</tr>
<tr>
<td>6</td>
<td>665 (-16.7%)</td>
<td>790 (-1.0%)</td>
<td>798</td>
</tr>
</tbody>
</table>

When these same orthotropic material properties were applied to the FEM of the 12.63" x 9.25" x 0.06" PCB, the response was that shown in Table 5.

As before, the agreement is not as good as would be expected with an orthotropic material, but it is significantly better than that yielded from modeling the material as isotropic.
Table 5

FEM Frequency Estimates Using Isotropic and Orthotropic Material and Modal Test Results for 12.63" x 9.25" x 0.06" PCB with Percent Deviation from Test Results

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM, Isotropic Material Properties Natural Frequency, Hz (%error)</th>
<th>FEM, Orthotropic Material Properties Natural Frequency, Hz (%error)</th>
<th>TEST Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>40 (0.0%)</td>
<td>39 (-2.5%)</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>44 (-20%)</td>
<td>49 (-10.9%)</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>84 (-6.7%)</td>
<td>93 (3.3%)</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>91 (-27.8)</td>
<td>121 (-3.9%)</td>
<td>126</td>
</tr>
<tr>
<td>5</td>
<td>113 (-27.1%)</td>
<td>138 (-10.9%)</td>
<td>155</td>
</tr>
<tr>
<td>6</td>
<td>127 (-33.2)</td>
<td>180 (-5.2%)</td>
<td>190</td>
</tr>
<tr>
<td>7</td>
<td>171</td>
<td>190</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>186</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.1.6 Free-Free Testing of a 3.44" x 6.40" x 0.06" Aluminum Plate

In an effort to verify both the testing and analytical procedures a test and modeling sequence was added that was not stated in the original plan at the beginning of this chapter. A test sample was fabricated from 5052-H32 aluminum with the same dimensions (3.44" x 6.40" x 0.06") as the second test circuit board sample. Again the sample was impact tested (looking for the first six frequencies) and the material properties in the FEM were modified to those for 5052-H32 aluminum [6]. Figures 8, 9, and 10 show the FRF magnitude, coherence function, and the FRF’s Imaginary component respectively, obtained for the aluminum plate. For this sample, the HP 35665 was set to a resolution of 800 lines over the bandwidth of 0 to 1600 Hz. Table 6 lists the frequencies of the first 6 modes found in test, those predicted by a FEM, and those predicted by Leissa. The FEM used here was the same model used for the same size FR4 PCB sample but with isotropic material description. There is good agreement among all three sets, as is shown by the error with respect to the test values shown in parentheses for the two theoretical models. This provided confidence in not only the testing procedures, but also the modeling techniques.

Thus, the differences between the test and analytical results for the
composite have to be related to the anisotropy of the composite, not the testing procedure or the finite element modeling density. This means that the orthotropic properties must be further detailed.

**Figure 8** FRF Magnitude of 3.44"x6.40"x0.06" Aluminum Sample, Free-Free

**Figure 9** Coherence Function for FRF for 3.44"x6.40"x0.06" Aluminum Sample, Free-Free
Table 6

FEM, Theoretical Frequency Estimates and Modal Test Results for a 3.44" x 6.40" x 0.06" Aluminum Plate, Free-Free with Percent Deviation from Test Results

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (% error)</th>
<th>Leissa Natural Frequency, Hz (% error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>296 (-2.6%)</td>
<td>314 (3.3%)</td>
<td>304</td>
</tr>
<tr>
<td>2</td>
<td>343 (-1.4%)</td>
<td>375 (7.8%)</td>
<td>348</td>
</tr>
<tr>
<td>3</td>
<td>761 (-2.2%)</td>
<td>766 (-1.5%)</td>
<td>778</td>
</tr>
<tr>
<td>4</td>
<td>821 (-2.7%)</td>
<td>851 (0.8%)</td>
<td>844</td>
</tr>
<tr>
<td>5</td>
<td>1096 (-4.5%)</td>
<td>1120 (-2.4%)</td>
<td>1148</td>
</tr>
<tr>
<td>6</td>
<td>1309 (-3.3%)</td>
<td>1233 (8.9%)</td>
<td>1354</td>
</tr>
</tbody>
</table>
3.2 Phase 2: Free-Free Testing and Finite Element Modeling of Samples to be Used in Wedge-Lock Testing

3.2.1 Phase 2, Steps i and ii: Free-Free Testing and Finite Element Modeling of a 5.84" x 7.06" x 0.06" Aluminum Plate

An aluminum test sample was then fabricated to fit into the wedge-lock fixture designed for this testing. The sample was 5.84" x 7.06" x 0.06". The results of impact testing, and the predictions made by a FEM and Liessa’s tables are shown in Table 7. The FEM, shown in Figure 11, was constructed using 180 quadrilateral plate elements with 1170 active degrees of freedom. The mode shapes predicted by the FEM are shown in Appendix E. Figures 12, 13, and 14 show the FRF magnitude, coherence function, and the FRF’s Imaginary component, respectively obtained for the aluminum plate. For this sample, the HP was set to a resolution of 800 lines over the bandwidth of 0 to 1600 Hz. The FEM in this case only had springs ($K_t=0.0002$ lb/in/in, $K_r=0.0002$ in•lb/rad/in) attaching it to ground along the two 5.84" edges that would have wedge locks mounted to them, if the sample were mounted in the fixture. This data shows that the FEM was able to accurately predict the first eight modes of this plate with free-free boundary conditions. This test sample would be used as a baseline against
which to measure the analysis techniques used against the FR4 samples.

Table 7

FEM, Theoretical Frequency Estimates and Modal Test Results for Al5052

Plate (5.84" x 7.06" x 0.06"), Free-Free

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>Leissa Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>194 (1%)</td>
<td>192 (0%)</td>
<td>192</td>
</tr>
<tr>
<td>2</td>
<td>255 (0.3%)</td>
<td>237 (-6.7%)</td>
<td>254</td>
</tr>
<tr>
<td>3</td>
<td>399 (-1.2%)</td>
<td>407 (0.7%)</td>
<td>404</td>
</tr>
<tr>
<td>4</td>
<td>473 (-2.7%)</td>
<td>489 (3.2%)</td>
<td>474</td>
</tr>
<tr>
<td>5</td>
<td>542 (0.7%)</td>
<td>534 (-0.7%)</td>
<td>538</td>
</tr>
<tr>
<td>6</td>
<td>750 (-0.5%)</td>
<td>739 (-1.9%)</td>
<td>754</td>
</tr>
<tr>
<td>7</td>
<td>928 (0.4%)</td>
<td></td>
<td>924</td>
</tr>
<tr>
<td>8</td>
<td>1079 (-0.4%)</td>
<td></td>
<td>1084</td>
</tr>
</tbody>
</table>
Figure 11 Finite Element Model of 5.84" x 7.06" x 0.06" Plates
Figure 12  FRF Magnitude of 5.84''x7.06''x0.06'' Aluminum Sample, Free-Free

Figure 13  Coherence Function for FRF for 5.84''x7.06''x0.06'' Aluminum Sample, Free-Free
3.2.2 Phase 2, Steps iii and iv: Free-Free Testing and Finite Element Modeling of a 5.84" x 7.06" x 0.06" FR4 Plate

A 5.84" x 7.06" x 0.06" region of an FR4 PCB was then cut from a larger board. The region was selected for the fact that it was relatively devoid of holes. Figures 15, 16, and 17 show the FRF magnitude, coherence function, and the FRF’s imaginary component, respectively obtained for the FR4 plate. For this sample, the HP was set to a resolution of 800 lines over the bandwidth of 0 to 800 Hz. Table 8 shows the comparison of the modal frequencies found empirically to those predicted by the FEM. The mode shapes predicted by the FEM are shown in Appendix E. The FEM was constructed using the same approach described before. The FEM was the same model shown in Figure 11 that was used for the same size aluminum
plate, with the exception that the material descriptor referred to the composite database within the code.

**Figure 15** FRF Magnitude of 5.84"x7.06"x0.06" FR4 Fiberglass Plate, Free-Free

**Figure 16** Coherence Function of FRF for 5.84"x7.06"x0.06" FR4 Fiberglass Plate, Free-Free
The material properties that were used in the FEM were the same ones that were used for the model of the 3.44'' x 6.40'' x 0.06'' FR4 PCB sample and are listed as average values in Table 3. Given the poor agreement between the empirical and FEM predicted modal frequencies, it is worth noting that these samples were not from the same PCB design. Also, the several investigators in the modal behavior of composite plates [7,8] having good agreement between FEM predictions and empirical values used material properties in their FEM’s that were taken from modulus test data from their samples.

The FEM predicted the first modal frequency to be closer to the second frequency found empirically. The first question that arose after looking at the test data was whether or not the first mode that appeared was an artifact of electrical noise (since it was so close to 60 Hz).
Table 8

FEM Modal Frequency Estimates Using Orthotropic Material Properties From Table 3 and Modal Test Results for FR4 Plate (5.84" x 7.06" x 0.06"), Free-

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz, (% error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>112 (+77.8%)</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>154 (+26.2%)</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>239 (+26.5%)</td>
<td>189</td>
</tr>
<tr>
<td>4</td>
<td>274 (+32.4%)</td>
<td>207</td>
</tr>
<tr>
<td>5</td>
<td>326 (+36.9%)</td>
<td>238</td>
</tr>
<tr>
<td>6</td>
<td>432 (+18.4%)</td>
<td>365</td>
</tr>
<tr>
<td>7</td>
<td>537 (+30.7%)</td>
<td>411</td>
</tr>
<tr>
<td>8</td>
<td>552 (-0.5%)</td>
<td>555</td>
</tr>
</tbody>
</table>

To check for electrical noise, the trigger level on the accelerometer channel was lowered until the accelerometer was continuously triggering. The resulting
autospectrum was inspected for any 60 Hz component. None were found. As a further validation of the modal frequencies found, two other samples were prepared and impact tested. The same modal frequencies were found on both samples.

If the 63 Hz mode were a result of electrical noise, it should have shown up in the other testing. The FR4 plate was taken down and replaced with the aluminum plate of the same size and no 63 Hz components was found.

3.2.3 Phase 2 step v: Modulus Testing of 5.84" x 7.06" x 0.06"

FR4 Plate

To improve the modeling accuracy, some PCB’s of the same design were cut into test samples for modulus testing. In the previous composite modeling, the material properties provided to the FE code were the in-plane Young’s moduli, $E_1$, $E_2$, the in-plane Poisson’s ratio, $\mu_{12}$, the in plane shear modulus, $G_{12}$, and the density, $\rho$. The values supplied to the code for $\mu_{12}$ and $G_{12}$ were estimates from the literature[1]. It was decided that for the modeling of the sample that would actually be mounted with wedge locks that these should be measured values. Three sets of three samples were produced for this series of modulus testing, one set cut parallel to each of the orthogonal
axes of the plate, and one set cut at approximately 45° to the orthogonal axes. In order to obtain data for Poisson’s ratio biaxial strain gages (Measurements Group, Inc. part number CEA-06-125UT-350) were mounted to these nine samples.

The samples were tested using an Instron 4204, with the biaxial strain gages connected to a Measurements Group, Inc. 2210 Signal Conditioning Amp.. The axial load and both strains were recorded using a MAC-based data acquisition system and saved in ASCII format. These files were later read into MathCad so that the data could be graphed and curve fitted using MathCad curve fitting commands (see Appendix B). Table 9 shows the Young’s Moduli, shear modulus and Poisson’s ratio measured for these samples.
<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Young’s Modulus and (Poisson’s Ratio) Parallel to 7.06” Edge of Sample (psi)</th>
<th>Young’s Modulus and (Poisson’s Ratio) Parallel to 5.60” Edge of Sample (psi)</th>
<th>Shear Modulus (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.51 x 10^6 (0.20)</td>
<td>No Data</td>
<td>No Data</td>
</tr>
<tr>
<td>2</td>
<td>2.48 x 10^6 (0.19)</td>
<td>3.24 x 10^6 (0.25)</td>
<td>5.97 x 10^5</td>
</tr>
<tr>
<td>3</td>
<td>2.53 x 10^6 (0.21)</td>
<td>3.17 x 10^6 (0.25)</td>
<td>5.81 x 10^5</td>
</tr>
<tr>
<td>Average</td>
<td>2.51 x 10^6 (0.20)</td>
<td>3.21 x 10^6 (0.255)</td>
<td>5.89 x 10^5</td>
</tr>
</tbody>
</table>
3.2.4 Phase 2 step vi: Incorporation into FEM of Material Property data from Modulus Testing of 5.84" x 7.06" x 0.06" FR4 Plate

Table 10 compares the test results with the FEM's predictions using the average material properties from Table 9.

Table 10

FEM Modal Frequency Estimates using the Material Properties From Table 9 and Modal Test Results for FR4 Plate (5.84" x 7.06" x 0.06"), Free-Free

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>87 (+38.1%)</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>162 (+32.8%)</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>218 (+15.3%)</td>
<td>189</td>
</tr>
<tr>
<td>4</td>
<td>242 (+16.9%)</td>
<td>207</td>
</tr>
<tr>
<td>5</td>
<td>275 (+15.5%)</td>
<td>238</td>
</tr>
<tr>
<td>6</td>
<td>452 (+23.8%)</td>
<td>365</td>
</tr>
<tr>
<td>7</td>
<td>461 (+12.2%)</td>
<td>411</td>
</tr>
<tr>
<td>8</td>
<td>522 (-5.9%)</td>
<td>555</td>
</tr>
</tbody>
</table>
Given the poor agreement of the empirical data with the model containing the new material properties, the construction of the original circuit board was revisited. If there were solid copper planes were present in the board, they could have a significant effect on the tensile stiffnesses of the samples. These copper planes would have a lesser effect on the flexural stiffness if they were near the center of the board since they would lie near the plate’s neutral axis.

The fabrication drawing and the artworks used to etch the copper planes were reviewed. The artworks revealed that there were solid power and ground planes that ran throughout the board. The fabrication drawing indicated that these planes were to be 1 ounce/ft², or 0.00135" thick and were to be the two middle planes in a six layer board.

To determine if these copper planes had a significant effect on the measured tensile moduli, it was necessary to back out the stiffnesses of both the FR4 and the copper from the measured values. The measured tensile moduli could be viewed as a result of two springs in parallel, one made of copper and one made of FR4.

\[ K_{eff} = K_{cu} + K_{FR4} \]  \hspace{1cm} (2)

Where
\[ K_{cu} = \frac{A_{cu}E_{cu}}{L_{cu}} \]  

(3)

\[ K_{cu} = \frac{A_{FR4}E_{FR4}}{L_{FR4}} \]  

(4)

and since the length of the FR4 and the copper are the same and the stiffness is the ratio of the applied force to the change in displacement caused by the applied force.

\[ \frac{F}{\Delta L} = \frac{1}{L} (A_{cu}E_{cu} + A_{FR4}E_{FR4}) \]  

(5)

which can be rewritten as

\[ \frac{FL}{\Delta L} = A_{cu}E_{cu} + A_{FR4}E_{FR4} \]  

(6)

which is proportional to the slope of the load-deflection curve generated during the tensile tests. The proportionality constant is the sensitivity of the strain gage amplifier to which the strain gages were attached. The amplifier sensitivity was 1mV/microstrain (1000 V/in/in), making Eq. (6)
\[ 1000m = A_{cu}E_{cu} + A_{FR4}E_{FR4} \] (7)

where \( m \) is the slope of the line fitted to the load-deflection curve. Since the overall thickness of the tensile modulus test specimens is known and the fabrication drawing gives the thickness of copper in the sample, the thickness of the FR4 is also known. After measuring the overall cross-sectional dimensions of each specimen the cross-sectional areas of FR4 and copper were calculated. This left the modulus of FR4 as the only unknown in Eq. (7).

The shear modulus for the FR4 could be calculated using the test data from the samples cut at 45° to the orthogonal sets above combined with Eq. (8).

\[ G_{plate}A_{plate} = G_{cu}A_{cu} + G_{FR4}A_{FR4} \] (8)

which can then be solved for the shear modulus of FR4.

Table 11 list the FR4 tensile and shear moduli corrected for the copper midplanes.
Table 11
Moduli for FR4 Corrected for Copper Midplanes

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Young’s Modulus Parallel to 7.06” Edge of Sample (psi)</th>
<th>Young’s Modulus Parallel to 5.84” Edge of Sample (psi)</th>
<th>Shear Modulus (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.00 x 10^6</td>
<td>No Data</td>
<td>No Data</td>
</tr>
<tr>
<td>2</td>
<td>1.99 x 10^6</td>
<td>2.85 x 10^6</td>
<td>3.44 x 10^5</td>
</tr>
<tr>
<td>3</td>
<td>2.05 x 10^6</td>
<td>2.72 x 10^6</td>
<td>3.20 x 10^5</td>
</tr>
<tr>
<td>Average</td>
<td>2.01 x 10^6</td>
<td>2.79 x 10^6</td>
<td>3.32 x 10^5</td>
</tr>
</tbody>
</table>

It was decided to use the corrected moduli. Thus, the material properties listed in Table 11 were entered into the composite database and the FEM was run again. Poisson’s Ratio, $\mu_{12}$ was found in the previous testing for the composite plate to be 0.2. One resource indicated that for FR4 $\mu_{12}$ might be as low as 0.147. These small variations were found to have a negligible effect on the modal frequencies predicted by the model. Poisson’s Ratio was set to .02. Table 12 shows the comparison of this FEM’s predicted modal frequencies with the test results.
Table 12

FEM Modal Frequency Estimates Using The Material Properties From Table 11 and Modal Test Results for FR4 Plate (5.84" x 7.06" x 0.06"), Free-Free

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>64 (+1.6%)</td>
<td>63</td>
</tr>
<tr>
<td>2</td>
<td>146 (+19.7%)</td>
<td>122</td>
</tr>
<tr>
<td>3</td>
<td>186 (-1.6%)</td>
<td>189</td>
</tr>
<tr>
<td>4</td>
<td>197 (-4.8%)</td>
<td>207</td>
</tr>
<tr>
<td>5</td>
<td>223 (-6.3%)</td>
<td>238</td>
</tr>
<tr>
<td>6</td>
<td>356 (-2.5%)</td>
<td>365</td>
</tr>
<tr>
<td>7</td>
<td>412 (+0.2%)</td>
<td>411</td>
</tr>
<tr>
<td>8</td>
<td>449 (-19.1%)</td>
<td>555</td>
</tr>
</tbody>
</table>

Although the agreement was still not as good as that achieved with the aluminum plate, it is a substantial improvement over the model (Table 10) that used material properties derived directly from the tensile tests. This indicates
that in an actual circuit board the tensile and flexural properties can be significantly different when solid copper planes are involved in the design.

The agreement between the FEM and the empirical results are good with the exception of the second and the eighth modes. The FEM was then checked for sensitivity to through the thickness shear moduli. The model was very insensitive to wide variations in these properties so further investigation of this effect was terminated. However, boards with other aspect ratios and different clamping methods could result in the through the thickness shear modulus being important.

3.3 Comparison of Empirical and FEM Predicted Mode Shapes

It was decided the second and the eighth mode shapes of the test sample should be compared to those predicted by the FEM. The test sample was suspended as it was for the free-free impact tests. A force transducer was attached to the test sample. A shaker was attached to the force transducer via a stinger. The test sample was excited at approximately the frequency found in impact testing (allowing for the mass loading of the test sample due to the force transducer). The response shapes were then determined by scanning the test sample with a laser vibrometer. The mode shapes found were essentially those predicted by the FEM with allowances made for the
mass loading due to the force transducer and the localized increase in
stiffness due to the force transducer’s mounting pad.

It was decided that the current model corrected for the copper midplanes
was the best representation of the test sample that the FEM would be
capable of producing in the near term. It was possible that certain modes
would be discarded when modeling of the test sample with wedge locks
mounted in place began.
Chapter 4: Phase 3: Restrained Testing Using Wedge Locks

4.1 Phase 3, Step i: Restrained Testing Using Wedge Locks of a 5.84" x 7.06" x 0.06" Aluminum Plate

It was decided to test the aluminum plate with the wedge locks since it would provide an opportunity to:

- compare results with those found in the literature
- develop the optimization techniques to find the spring constants with a homogeneous material.

Wedge locks (EG & G Birtcher part number 42-5-12-LF) were installed on the aluminum plate that was successfully modeled with free-free edge conditions. This assembly was then mounted in the test fixture shown in Figure 18. The wedge locks were torqued to 6 in-lbs in accordance with the manufacturer’s recommendation. An accelerometer was mounted to the bottom of the sample (see x’s Figure 19) at three locations and the unit was impact tested. Figures 20, 21, and 22 show the FRF, coherence function, and the FRF’s Imaginary component, respectively obtained for the aluminum plate. For this sample, the HP was set to a resolution of 800 lines over the bandwidth of 0 to 3200 Hz.
Figure 18  Fixture Used To Test Wedge Locks

Figure 19  Accelerometer and Impact Locations (See x's in Figure)
**Figure 20** FRF Magnitude of 5.84"x7.06"x0.06" Aluminum Plate Restrained with Wedge Locks Along the 5.84" Edges

**Figure 21** Coherence Function of FRF for 5.84"x7.06"x0.06" Aluminum Plate Restrained with Wedge Locks Along the 5.84" Edges
4.2 Phase 3, Step ii: Comparison of Analytical Techniques to Model Wedge-Lock Boundary Conditions for a 5.84" x 7.06" x 0.06" Aluminum Plate

As a first check of the test data against the literature, how well does Steinberg’s equation predict the first mode? Young’s[9] tables for flat plates with classical boundary conditions tells us that the first natural frequency for our test sample with two edges simply supported, and two fixed edges would be

\[ F_s = 131 \text{ Hz} \]

\[ F_f = 296 \text{ Hz} \]

Steinberg’s equation would then predict a first mode at

\[ F_f = 269 \text{ Hz} \]

This prediction is within 1.5% of the experimentally determined first natural
frequency.

The next question would be, does the practice of approximating the wedge lock as fixed in translation and flexible in rotation accurately represent the natural frequencies beyond the first?

A FEM model of the plate shown in Figure 23 was constructed using 156 flat plate quadrilateral elements with 1170 active degrees of freedom. As a first check of the model's adequacy, the 5.84" edges were pinned and the model's first resonance frequency was found and compared against that predicted by Young [9]. Next, the 5.84" edges were clamped and the model's first resonance frequency was found and compared against that predicted by Young [9]. The FEM's predictions for the plate's first resonance frequency was within 5% of those predicted by Young [9] for both the pinned and clamped edge conditions. Thus, the model's mesh density was deemed adequate.

Now, a row of nodes was created 0.01" away (out-of-plane of the model) from those on each of the 5.6" sides. The model's nodes were attached to these duplicate nodes by spring elements, and the nodes along the 5.6" sides were pinned.
Figure 23 Finite Element Model of 5.84" x 7.06" x 0.06" Plates Used for Restrained Condition (Unsupported Area Only Modeled)
The rotational spring stiffness would have to be adjusted to minimize the error between the FEM's predicted first modal frequency and that found empirically. One of the simpler search routines is Interval Halving [10]. Since only one variable (rotational spring stiffness) would be involved, and the dependent variable (first modal frequency) would be increasing with the rotational stiffness, Interval Halving was found to be an adequate method.

This method converged upon a rotational spring stiffness of 500 in•lbf/rad/node, or 1027 in•lb/rad/in. Table 13 lists the first eight modes predicted by the FEM with pinned edges and rotational springs and those found in the modal testing.

A FEM was constructed of the fixture itself to verify that the rotational stiffness it provided in series to the FR4 sample was much greater than that being associated with the wedge-locks by the modeling. The FEM indicated that the boundary condition provided by the fixture would have a rotational stiffness approximately 100 times greater than that being attributed to the wedge-locks. The details of this analysis can be found in Appendix C.
Table 13

FEM Frequency Estimates and Modal Test Results for Al5052 Plate (5.84" x 7.06" x .06"), Restrained with Wedge Locks

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM $K_r=1027$ in-lb/rad/in</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Natural Frequency, Hz (% error)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>276 (1.4%)</td>
<td>272</td>
</tr>
<tr>
<td>2</td>
<td>349 (-3.1%)</td>
<td>360</td>
</tr>
<tr>
<td>3</td>
<td>660 (-.6)</td>
<td>664</td>
</tr>
<tr>
<td>4</td>
<td>769 (1.7%)</td>
<td>756</td>
</tr>
<tr>
<td>5</td>
<td>845 (-4.4%)</td>
<td>884</td>
</tr>
<tr>
<td>6</td>
<td>1231 (0.2)</td>
<td>1228</td>
</tr>
<tr>
<td>7</td>
<td>1316 (-.6%)</td>
<td>1324</td>
</tr>
<tr>
<td>8</td>
<td>1520 (1.7%)</td>
<td>1494</td>
</tr>
</tbody>
</table>

At first, it was attempted to compare this stiffness to the values obtained by Roza[4]. Roza's rotational stiffness values for a plate with a comparable plate
constant (D=205 in\cdot lbs vs Roza's 191 in\cdot lbs), and only slightly different in size were significantly larger. More importantly, the rotational stiffnesses did not have any units listed for them. It was thought that perhaps the Roza's values were, in fact, the dimensionless rotational stiffness parameter as defined by Gorman [11]. Converting the rotational stiffnesses to the dimensionless parameter still did not cause them to look any more like those reported by Roza.

As a final attempt to correlate these findings with Roza's, a FEM was created along the same lines as described above of a plate the size used by Roza (4.65" x 6.03" x .059"). A mesh of 9 x 12 quadrilateral plate elements was used. (Another model with an 18 x 24 mesh was created and found to have the same modal response as the original model, so the original model was deemed adequate.) The material properties were set to those used by Roza. The 4.65" edges were those restrained, both with classical boundary conditions and with spring elements. The model was ran with classical boundary conditions as a verification of the model. This model had a first resonance frequency at 147 Hz when the 4.65" edges were pinned, and 341 Hz when these edges were clamped. Roza listed 140 Hz and 319 Hz for pinned and clamped edges, respectively. Then the 4.65" edges were pinned and the rotational spring stiffnesses were varied to match Roza's data. Table 14 shows the response of this model compared to that reported by Roza[4]. Even
if the values for spring stiffnesses were not per unit length of restrained edge, but the total spring stiffness these values do not agree. These differences have not yet been resolved.

**Table 14**

<table>
<thead>
<tr>
<th></th>
<th>Roza's Table 3.1</th>
<th>9 x 12 Mesh Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Mode: 4.65 Edges Simply Supported</td>
<td>140 Hz</td>
<td>147 Hz</td>
</tr>
<tr>
<td>First Mode: 4.65 Edges Fixed</td>
<td>319 Hz</td>
<td>341 Hz</td>
</tr>
<tr>
<td>K for Wedge Locks to Match Roza's Test Data</td>
<td>14235</td>
<td>623 in•lbs/rad/in</td>
</tr>
<tr>
<td>First Mode: 4.65&quot; Edges Restrained with Wedge Locks</td>
<td>287 Hz</td>
<td>294 Hz</td>
</tr>
</tbody>
</table>

**4.3 Phase 3, Step iii: Optimization of Boundary Stiffness**

**Estimates for the Aluminum Plate With Wedge Locks**

Could the agreement between theoretical and empirical be improved by accounting for the possible non-uniform boundary conditions provided by the wedge locks? To explore this possibility, it would be necessary to define an error function between the empirical data and the FEM predictions, and minimize it. The minimized error function could then be compared with that obtained by modeling the boundary conditions as uniform. The error function
was defined as the sum of the squares of the percentage error in frequency for the FEM predictions.

\[
err = \sum_{i=1}^{n} \left( (1 - \frac{f_{\text{fem},i}}{f_{\text{test},i}}) \times 100 \right)^2
\]  

(9)

Using this convention, the model of the aluminum plate with pinned edges and rotational springs (see Table 13) would have an error function value of 37.47. This would mean that the root mean square (rms) error was 2.16%.

In this analysis, the objective function is the error function between the modal frequency predictions of the FEM and the empirically determined modal frequencies. The four variables that define the design space are the rotational and translational spring constants (2 variables) where the wedge-locks contact the circuit board, and the rotational and translational spring constants (2 variables) where the wedge-locks do NOT touch the circuit board. Figure 24 shows where the spring constants that will be modeled would be located on an actual wedge lock.

The FEM predicted modal frequencies are relatively easy to evaluate for various points within the design space. Since neither an expression describing the error function in the design space nor its derivative would not be readily
available, the gradients were numerically evaluated using a forward difference approximation [10]. The minimization algorithm selected was a quasi-Newton gradient-based method using the Broyden, Fletcher, and Shanno (BFS) update method for the search vectors [10]. Since the FEA code used had no optimization capabilities, the minimization routine in this study was far from automatic. This meant that the procedure followed was:

1) the FEM was ran at a design point (using a set of trial or starting point values for the 4 spring constants)
2) each design variable was independently incremented and the FEM was re-evaluated 
3) the results were entered into MathCad for gradient and search vector evaluation 
4) the model was iteratively ran along the search vector using the interval halving search routine[10] to find the minimum along the search vector. 
5) when the minimum was found, go to step 2).

This routine was repeated until the new search vectors appeared to be directing the search back along the same direction as the previous iteration.

One of the fundamental assumptions of many minimization routines is that the object function is unimodal within the search domain. One of the properties of such a function is that no matter where the search is started within the domain, the minimization routine will always converge to the same minimum. The error function in this study was found to have many local minima that would tend to trap the search routine. This meant that the minimum found was very dependent on the starting point of the search. The lowest value for the error function was found to be 22.03 (1.66% rms). The starting point was
\[ K_{tt} = K_{t2} = 18493 \text{ lbs/in/in} \]
\[ K_{r1} = K_{r2} = 822 \text{ in-lbs/rad/in} \]

A word of explanation is required to explain why such odd values were selected as a starting point. The stiffness values that were entered into the FEM file were in units of lbs/in/node and in-lbs/rad/node. The starting points selected were not such odd values in these units (9000 lbs/in/node and 400 in-lbs/rad/node). The values listed here are in units of lbs/in/in and in-lbs/rad/in and were obtained by dividing the values input to the model by the FEM's node point spacing (0.541”).

The minimum was found where

\[ K_{tt} = 18497 \text{ lbs/in/in} \]
\[ K_{t2} = 18495 \text{ lbs/in/in} \]
\[ K_{r1} = 1219 \text{ in-lbs/rad/in} \]
\[ K_{r2} = 1054 \text{ in-lbs/rad/in} \]

Table 15 shows the comparison of the FEM's predicted modal frequencies using these boundary stiffnesses with the test results. The mode shapes predicted by the FEM are shown in Appendix E.
Table 15

FEM Frequency Estimates Using Optimized Boundary Stiffnesses and Modal Test Results for Al5052 Plate (5.84" x 7.06" x .06"), Restrained with Wedge Locks

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>278 (2.2%)</td>
<td>272</td>
</tr>
<tr>
<td>2</td>
<td>349 (-3.1%)</td>
<td>360</td>
</tr>
<tr>
<td>3</td>
<td>658 (-0.9%)</td>
<td>664</td>
</tr>
<tr>
<td>4</td>
<td>768 (1.6%)</td>
<td>756</td>
</tr>
<tr>
<td>5</td>
<td>870 (-1.6%)</td>
<td>884</td>
</tr>
<tr>
<td>6</td>
<td>1221 (-0.6%)</td>
<td>1228</td>
</tr>
<tr>
<td>7</td>
<td>1310 (-1.1%)</td>
<td>1324</td>
</tr>
<tr>
<td>8</td>
<td>1505 (0.7%)</td>
<td>1494</td>
</tr>
</tbody>
</table>

4.4 Phase 3, Step iv: Restrained Testing Using Wedge Locks of a 5.84" x 7.06" x 0.06" FR4 Plate
Wedge locks were installed on the FR4 plate that was modeled with free-free edge conditions. This assembly was then mounted in the test fixture shown in Figure 18. The wedge locks were torqued to 6 in-lbs in accordance with the manufacturer’s recommendation. An accelerometer was mounted to the side of the sample facing the test fixture (see Figure 19) at three locations and the unit was impact tested. Figures 25, 26, and 27 show the FRF magnitude, coherence function, and the FRF’s Imaginary component, respectively obtained for the FR4 plate. The HP was set to a resolution of 800 lines over the bandwidth of 0 to 3200 Hz.
Figure 25 FRF Magnitude of 5.84"x7.06"x0.06" FR4 Fiberglass Plate Restrained with Wedge-Locks Along the 5.84" Edges

Figure 26 Coherence Function of FRF for 5.84"x7.06"x0.06" FR4 Fiberglass Plate Restrained with Wedge-Locks Along the 5.84" Edges
4.5 Phase 3, Step v: Comparison of Analytical Techniques to Model Wedge-Lock Boundary Conditions for a 5.84" x 7.06" x 0.06" FR4 Plate

As was done with the aluminum plate, this test data was then compared against what would be predicted using Steinberg’s[1] and Roza’s[4] techniques. Steinberg’s[1] first modal frequency prediction requires estimates of the first modal frequencies for the same part with pinned and clamped boundary conditions. The best FEM of the composite plate was that used to produce the results in Table 13. The edges of this model where the wedge locks would be were pinned and then clamped. The first modal frequencies for these boundary conditions were found to be

\[ F_s = 75 \text{ Hz} \]

\[ F_r = 171 \text{ Hz} \]

Steinberg’s equation[1] would then predict a first mode at
\[ F_r = 165 \text{ Hz} \]

Steinberg’s prediction is approximately 20% higher than the first frequency of 137 Hz found in test. Steinberg’s prediction is in line with his criteria that if the first modal frequency for plate of this size with pinned edges is below 100 Hz the wedge-locks will look like a clamped edge. The test data does not support either this statement or the equation.

Another of the descriptive characteristics that Steinberg has defined for wedge-locks is the percent fixity

\[ P_f = \frac{f_{test} - f_s}{f_{ref} - f_s} \quad (10) \]

Steinberg’s basic position is that the percent fixity provided by the wedge locks will vary with the stiffness of the plate being clamped. The test data collected for this thesis does support this. The wedge locks provided 83% fixity for the aluminum plate and 65% fixity for the same size circuit board test sample. This is at odds with Barker and Chen’s[2] reports that the boundary condition provided by the wedge locks were independent of the plate they were mounting.
The same FEM was used to check Roza’s[4] technique with composite plates. The edges of the model where the wedge locks would be positioned were pinned and attached to ground using rotational springs. As with the aluminum plate, there was only one design variable, the rotational stiffness at the edge of the board provided by the wedge lock, so this could be treated as a line search. The line search routine would again be interval halving. This routine converged to a rotational stiffness of 106 in•lbs/rad/in. This is one eighth of the stiffness found for the same wedge-locks mounted to the same size aluminum plate. Table 16 compares the first eight modes predicted by the FEM with pinned edges and rotational springs and those found in the modal testing. As a point of comparison, the error function for this FEM is 404, where the error function for the same model in the free-free condition was 827. An error function of 404 means that the rms error was 7.1%.
Table 16

FEM Modal Frequency Estimates Using the Material Properties From Table 11 and Modal Test Results for FR4 Plate (5.84" x 7.06" x .06"), Restrained with Wedge Locks

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM $K_r=106\text{in}\cdot\text{lb/ rad/in}$ Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>141 Hz (2.9%)</td>
<td>137 Hz</td>
</tr>
<tr>
<td>2</td>
<td>157 Hz (-4.3%)</td>
<td>164</td>
</tr>
<tr>
<td>3</td>
<td>273 Hz (-10.2%)</td>
<td>304</td>
</tr>
<tr>
<td>4</td>
<td>399 Hz (5.8%)</td>
<td>377</td>
</tr>
<tr>
<td>5</td>
<td>422 Hz (1.9%)</td>
<td>414</td>
</tr>
<tr>
<td>6</td>
<td>525 Hz (-4.4%)</td>
<td>549</td>
</tr>
<tr>
<td>7</td>
<td>553 Hz (-14.3%)</td>
<td>645</td>
</tr>
<tr>
<td>8</td>
<td>765 Hz (3.7%)</td>
<td>738</td>
</tr>
</tbody>
</table>
4.6 Phase 3, Step iii: Optimization of Boundary Stiffness

Estimates for the FR4 Plate With Wedge Locks

Could this agreement be improved using the same optimization routine that was used for the aluminum plate? That is, the pinned constraint will be replaced with unknown translational stiffnesses. The first starting point was

\[ K_{t1} = K_{t2} = 2055 \text{ lbs/in/in} \]
\[ K_{r1} = K_{r2} = 62 \text{ in}^4\text{lbs/ rad/in} \]

A minimum error function of 309 was found where

\[ K_{t1} = 2055 \text{ lbs/in/in} \]
\[ K_{t2} = 2055 \text{ lbs/in/in} \]
\[ K_{r1} = 136 \text{ in}^4\text{lbs/ rad/in} \]
\[ K_{r2} = 52 \text{ in}^4\text{lbs/ rad/in} \]

When the search started at

\[ K_{t1} = K_{t2} = 1028 \text{ lbs/in/in} \]
\[ K_{r1} = K_{r2} = 62 \text{ in}^4\text{lbs/ rad/in} \]

a minimum error function of 299 was found where

\[ K_{t1} = 1029 \text{ lbs/in/in} \]
\[ K_{t2} = 1029 \text{ lbs/in/in} \]
\[ K_{r1} = 156 \text{ in}^4\text{lbs/ rad/in} \]
\[ K_{r2} = 97 \text{ in}^4\text{lbs/ rad/in} \]
Table 17 shows the comparison of the FEM’s predicted modal frequencies using these boundary stiffnesses with the test results. The mode shapes predicted by the FEM are shown in Appendix E.

Table 17

FEM Modal Frequency Estimates Using the Material Properties From Table 11 and Optimized Boundary Stiffnesses and Modal Test Results for FR4 Plate (5.84" x 7.06" x 0.06"), Restrained with Wedge Locks

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>FEM $K_r=106\text{in} \cdot \text{lb/rad/in}$ Natural Frequency, Hz (%error)</th>
<th>Test Natural Frequency, Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>142 Hz (3.6%)</td>
<td>137 Hz</td>
</tr>
<tr>
<td>2</td>
<td>158 (-3.7%)</td>
<td>164</td>
</tr>
<tr>
<td>3</td>
<td>276 (-9.2%)</td>
<td>304</td>
</tr>
<tr>
<td>4</td>
<td>396 (5.0%)</td>
<td>377</td>
</tr>
<tr>
<td>5</td>
<td>419 (1.2%)</td>
<td>414</td>
</tr>
<tr>
<td>6</td>
<td>525 (-4.4%)</td>
<td>549</td>
</tr>
<tr>
<td>7</td>
<td>573 (-11.2%)</td>
<td>645</td>
</tr>
<tr>
<td>8</td>
<td>777 (5.3%)</td>
<td>738</td>
</tr>
</tbody>
</table>
This was comparable to the rotational stiffnesses that were found from the previous starting point. This error function corresponds to an rms error of 6.12%. This is not as low as was found with the aluminum plate, but is acceptable given the agreement this model exhibited with the test data in the free-free condition. An error function of 299 would seem to be an incredible improvement over the same model’s performance in the free-free state.

However, in the free-free condition the second and eighth modes were responsible for 90% of the total error. This seemed to imply that there was still some material properties that were not being accurately modeled, and that this inaccuracy was showing up in the second and eighth modes. When the model was run with the boundary conditions representing the wedge locks the inaccuracies were no longer manifested in just two modes. The improvement in the model’s agreement with the test results says that, in the restrained condition, the boundary conditions are more significant and are able to mask the problems with the material properties.
Chapter 5: Conclusions

The testing of an aluminum plate for this thesis agreed well with

- Steinberg’s [1] estimate of the first modal frequency of a flat plate restrained by boundary conditions.
- Roza’s [4] general approach of modeling the boundary conditions provided by a wedge lock as simply supported with rotational springs.

When the testing moved on to a plate made of FR4, the agreement with Steinberg’s estimate was not as good. As for Roza’s goal of providing a set of tables listing the boundary stiffnesses of wedge locks,

- There is a difference in the performance of the wedge lock tested caused by the PCB material that is being clamped. The literature offered conflicting views on whether the boundary conditions provided by the wedge locks varied with the modulus of the clamped plate. The testing in this study supported the position that the wedge lock boundary stiffness does vary with the material modulus being clamped. This calls into question the studies in the literature that used aluminum plates to characterize the wedge lock’s boundary stiffnesses.
• The literature lists material properties that this study found is representenative the circuit board material alone. The literature does not address the orthotropic nature of the material’s properties at all. The orthotropic material properties had a significant effect on the accuracy of the FEM’s modal frequency predictions.

• Steinberg’s methods do not apply accurately to FR4 PC Boards.

• Barker and Chen’s claim that the wedge-lock boundary conditions are independent of the plate being mounted is false.

• If time and computing abilities are limited, Roza’s method of characterizing the wedge-lock boundary conditions as pinned with rotational springs is good first approximation of the actual condition. His goal of providing look-up tables for the rotational stiffnesses for the wedge-lock boundary condition was not acheived.

• In general, the \( K_{11} = K_{12} \) appears to be a good assumption but \( K_{11} = 1.5K_{12} \) is an optimal solution for the clamping of FR4.
Chapter 6: Recommendations for Future Work

This thesis addressed questions concerning how industry and the literature are modeling wedge locks in modal analysis. The work has raised many questions. Recommendations for future work have been developed to help provide direction to developing answers to these questions.

- Application of the boundary conditions found in this study to a model of a populated circuit board. This would aid in understanding if the orthotropic properties of the FR4 tested in this study would become less significant as the stiffnesses of the components were added to the system.

- Testing of other types of wedge locks. Do the different segmentation schemes of the wedge-locks offer different performance?

- Testing of this style of wedge lock in a fixture that would be more representative of an actual application. Build a FEM of both the test sample and the mounting structure. Incorporate the boundary...
conditions found in this study between the FEM of the circuit board sample and the supporting structure. This would reveal whether the stiffness of the series supporting structure had any effect on the effective rotational and translational stiffnesses provided by the wedge locks. In some cases it is envisioned that the supporting structure may be the dominant flexibility in the system.

- Testing of the wedge locks on circuit board material with larger input force amplitudes. One of the earlier studies [3] indicated that the rotational stiffness varied with input amplitude, but the testing was done using aluminum plates. This work would have the added challenge of compensating for the mass and stiffness loading by the force transducer on a test sample of relatively low mass and stiffness, a circuit board.

- The resolution of the inaccuracies in the material properties for the printed circuit board material, FR4, that caused the large errors in the second and eighth mode in the free-free condition.

- Attempt to generalize the results of wedge lock stiffness predictions.
• Develop a method to measure the through thickness shear modulus for PCB’s.
References


Appendix A
Stiffness Testing of FR4 with Strain Data from Extensometer

The force carried by the test sample in the modulus test is

\[ F = \frac{AE\Delta L}{L} \quad (A1) \]

Since \( m \) the slope of the line fitted to the load versus deflection curve is

\[ m = \frac{F}{100 \frac{l}{L}} \quad (A2) \]

Equation (A2) can be inserted in Eq. (A1), which can then be rearranged to find the Young’s modulus.

\[ E = \frac{100m}{A} \quad (A3) \]

Figures A1, A2, and A3 show excerpts from the MathCad files used to reduce the tensile test data for the three samples that were cut parallel to the long axis of modal test sample. Figures A4, A5, and A6 show excerpts from the MathCad files used to reduce the tensile test data for the three samples that were cut perpendicular to the long axis of modal test sample.
These excerpts contain

- the original load versus deflection curve with a straight-line fitted to the data.
- the calculation of the Young’s Moduli using Eq. (A3)
Load Curve for FR4 Test Sample P1, Parallel to the Long Axis of the Test Article Load(lbs) vs. Percent Elongation in the Direction of Loading

\[ A \approx 0.539 - 0.066 \]  

\[ E \approx \frac{100 \text{ m}}{A} \]  

\[ E = 2.652 \times 10^6 \text{ psi} \]  

**Figure A-1** Calculation of Young's Modulus for FR4 Test Sample P1 From Load Vs. Extensometer Deflection Data
Load Curve for FR4 Test Sample P2, Parallel to the Long Axis of the Test Article Load(lbs) vs. Percent Elongation in the Direction of Loading

\[ A := 0.534 \times 0.066 \text{ sq in: measured dimensions of test sample} \]

\[ E := \frac{100 \text{ m}}{A} \]

\[ E = 3.891 \times 10^4 \text{ psi} \]

Figure A-2 Calculation of Young's Modulus for FR4 Test Sample P2 From Load Vs. Extensometer Deflection Data
Load Curve for FR4 Test Sample P3, Parallel to the Long Axis of the Test Article
Load(lbs) vs. Percent Elongation in the Direction of Loading

$A := 0.538 \cdot 0.066 sq in$ measured dimensions of test sample

$E := \frac{100 \cdot m}{A}$

$E = 2.924 \cdot 10^6$ psi

Figure A-3 Calculation of Young's Modulus for FR4 Test Sample P3 From Load Vs. Extensometer Deflection Data
Load Curve for FR4 Test Sample T1, Perpendicular to the Long Axis of the Test Article Load(lbs) vs. Percent Elongation in the Direction of Loading

\[ A := 0.53 \cdot 0.066 \text{sq in: measured dimensions of test sample} \]

\[ E := \frac{100 \cdot m}{A} \]

\[ E = 3.026 \cdot 10^6 \text{ psi} \]

Figure A-4 Calculation of Young's Modulus for FR4 Test Sample T1 From Load Vs. Extensometer Deflection Data
Load Curve for FR4 Test Sample T2, Perpendicular to the Long Axis of the Test Article Load(lbs) vs. Percent Elongation in the Direction of Loading

\[ A := 0.522 \times 0.066 \text{sq in} \text{ measured dimensions of test sample} \]

\[ E := \frac{100 \cdot m}{A} \]

\[ E = 2.951 \times 10^4 \text{ psi} \]

**Figure A-5** Calculation of Young's Modulus for FR4 Test Sample T2 From Load Vs. Extensometer Deflection Data
Figure A-6 Calculation of Young’s Modulus for FR4 Test Sample T3 From Load Vs. Extensometer Deflection Data
Appendix B

Stiffness Testing of FR4 with Strain Data from Biaxial Strain Gages

The force carried by the test sample in the modulus test is

\[ F = \frac{AE\Delta L}{L} \]  
(B1)

Since the sensitivity of the amplifiers was 1 mV/\(\mu\)strain (1000V/in/in), the slope of the line fitted to the load-deflection curve is

\[ m = \frac{F}{1000 \Delta L/L} \]  
(B2)

Equation (B2) can be inserted in Eq. (B1), which can then be rearranged to find the Young’s modulus.

\[ E = \frac{1000m}{A} \]  
(B3)

For calculation of the shear modulus, \(G_{12}\), samples were cut at
approximately 45° to the orthogonal ply angles of the FR4. The shear modulus in an angle ply laminate is [12]

\[ G_{12} = \frac{\tau_{12}}{\gamma_{12}} \]  \hspace{1cm} (B4)

The strain in an angle ply laminate is

\[ \epsilon_{12} = (\epsilon_\beta - \epsilon_\alpha) \sin 2\phi - \epsilon_\alpha \beta \cos 2\phi \]  \hspace{1cm} (B5)

In the tension test, the samples were in a uniaxial tension which makes \( \epsilon_\alpha \) zero. Due to the way the samples were cut, \( \phi = 45° \). This makes it possible to reduce Eq. (B5) to

\[ \gamma_{12} = \epsilon_\beta - \epsilon_\alpha \]  \hspace{1cm} (B6)

where \( \epsilon_\alpha \) and \( \epsilon_\beta \) are the outputs from the strain gages. These strains will be relabeled \( \epsilon_1 \) and \( \epsilon_2 \).

Also, since the samples were in uniaxial tension,
\[ \tau_{12} = \frac{\sigma_1}{2} \]  

(B7)

where

\[ \sigma_1 = E\varepsilon_1 \]  

(B8)

This makes it possible to express Eq. (B4) in terms of properties that can be calculated from data obtained from the tensile test.

\[ G_{12} = \frac{E\varepsilon_1}{2} \frac{1}{\varepsilon_1 - \varepsilon_2} \]  

(B9)

Figures B1 through B6 show excerpts from the MathCad files used to reduce the tensile test data for the three samples that were cut parallel to the long axis of modal test sample. Figures B7 through B10 show excerpts from the MathCad files used to reduce the tensile test data for the three samples that were cut perpendicular to the long axis of modal test sample. These excerpts show, alternatively,

- the graphing of the load vs. strain curve and the calculation of the Young’s Moduli using Eq. (B3)
- the graphing of the principal strains and the calculation of Poisson’s Ratio.
Figures B11 and B12 show excerpts from the MathCad files used to reduce the tensile test data for the three samples that were cut at 45° to the long axis of modal test sample. These excerpts also show the calculation of the shear modulus using Eq. (B9)
Load Curve for FR4 Test Sample P1, Parallel to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in*.001) in the Direction of Loading

\[ A = 0.611 - 0.063 \text{ sq in: measured dimensions of test sample} \]

\[ E = \frac{1000 \cdot m}{A} \text{ Simplified formula for Young's Modulus:} \]

\[ A = \text{cross sectional area of the test sample} \]
\[ m = \text{slope of line fitted to the load curve} \]

\[ E = 2.305 \cdot 10^6 \text{ psi: Young's Modulus} \]

**Figure B-1** Calculation of Young’s Modulus for FR4 Test Sample P1 From Load Vs. Strain Gage Data
Load Curve for FR4 Test Sample P1, Parallel to the Long Axis of the Test Article
Strain(in/in*0.001) Perpendicular to Load vs. Strain(in/in*0.001) in the Direction of Loading

\[ \mu = 0.2 \]

Poisson's Ratio (Magnitude of the slope of the fitted line)

Figure B-2 Calculation of Poisson's for FR4 Test Sample P1 From Strain Gage Data
Figure B-3 Calculation of Young's Modulus for FR4 Test Sample P2 From Load Vs. Strain Gage Data
Load Curve for FR4 Test Sample P2, Parallel to the Long Axis of the Test Article
Strain (in/in \( \times 0.001 \)) Perpendicular to Load vs. Strain (in/in \( \times 0.001 \)) in the Direction of Loading

\[ \mu = 0.196 \quad \text{Poisson's Ratio (Magnitude of the slope of the fitted line)} \]

Figure B-4 Calculation of Poisson's Ratio for FR4 Test Sample P1 From Strain Gage Data
Load Curve for FR4 Test Sample P3, Parallel to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in*0.001) in the Direction of Loading

A := 0.617 - 0.063

\[ E := \frac{1000 \cdot m}{A} \]
Simplified formula for Young's Modulus:
A = cross sectional area of the test sample
m = slope of line fitted to the load curve

\[ E = 2.532 \times 10^6 \text{ psi} \]

Figure B-5 Calculation of Young's Modulus for FR4 Test Sample P3 From Load Vs. Strain Gage Data
Load Curve for FR4 Test Sample P3, Parallel to the Long Axis of the Test Article
Strain (in/in* .001) Perpendicular to Load vs. Strain (in/in* .001) in the Direction of Loading

\[ \mu = 0.205 \]

Poisson’s Ratio (Magnitude of the slope of the fitted line)

**Figure B-6** Calculation of Poisson’s Ratio for FR4 Test Sample P1 From Strain Gage Data
Load Curve for FR4 Test Sample TR2, Perpendicular to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in*.001) in the Direction of Loading

\[ A := 0.619 \times 0.0063 \text{ sq in. measured dimensions of test sample} \]

\[ E := \frac{1000 \times m}{A} \]

Simplified formula for Young's Modulus:
\( A = \) cross sectional area of the test sample
\( m = \) slope of line fitted to the load curve

\[ E = 3.241 \times 10^6 \text{ psi} \]

Figure B-7 Calculation of Young's Modulus for FR4 Test Sample TR2 From Load Vs. Strain Gage Data
Figure B-8 Calculation of Poisson’s Ratio for FR4 Test Sample TR2 From Strain Gage Data
Load Curve for FR4 Test Sample TR3, Perpendicular to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in·.001) in the Direction of Loading

\[ A := 0.632 \text{ in}^2 \quad \text{sq in: measured dimensions of test sample} \]

\[ E := \frac{1000 \cdot m}{A} \quad \text{Simplified formula for Young’s Modulus:} \]
\[ A = \text{cross sectional area of the test sample} \]
\[ m = \text{slope of line fitted to the load curve} \]

\[ E = 3.176 \cdot 10^6 \text{ psi} \]

**Figure B-9** Calculation of Young’s Modulus for FR4 Test Sample TR3 From Load Vs. Strain Gage Data
Load Curve for FR4 Test Sample TR3, Perpendicular to the Long Axis of the Test Article. Strain(in/in*\.001) Perpendicular to Load vs. Strain(in/in*\.001) in the Direction of Loading.

\[ \mu = 0.255 \] Poisson’s Ratio (Magnitude of the slope of the fitted line).

**Figure B-10** Calculation of Poisson’s Ratio for FR4 Test Sample TR3 From Strain Gage Data.
Load Curve for FR4 Test Sample D2, Cut at 45 Degrees to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in*.001) in the Direction of Loading

\[ A := 0.671 \cdot 0.061 \text{ sq in. measured dimensions of test sample} \]

\[ E := \frac{1000 \cdot m}{A} \text{ Simplified formula for Young's Modulus:} \]
\[ A=\text{cross sectional area of the test sample} \]
\[ m=\text{slope of line fitted to the load curve} \]

\[ E = 1.797 \cdot 10^6 \text{ Young's Modulus: psi} \]

\[ \gamma_{12} := \varepsilon_2 - \varepsilon_1 \text{ Shear deformation} \]

\[ \tau_{12} := \frac{\varepsilon_1 E}{2} \text{ Shear stress: psi} \]

\[ |\text{slope(}\gamma_{12}, \tau_{12})| = 5.968 \cdot 10^4 \]

Figure B-11 Calculation of Shear Modulus for FR4 Test Sample D2 From Load vs. Strain Gage Data
Load Curve for FR4 Test Sample D3, Cut at 45 Degrees to the Long Axis of the Test Article
Load(lbs) vs. Strain(in/in*.001) in the Direction of Loading

\[ A := 0.657 \cdot 0.061 \text{ sq in: measured dimensions of test sample} \]

\[ E := \frac{1000 \cdot m}{A} \text{ Simplified formula for Young's Modulus:} \]
\[ A=\text{cross sectional area of the test sample} \]
\[ m=\text{slope of line fitted to the load curve} \]

\[ E = 1.791 \cdot 10^6 \text{ Young's Modulus: psi} \]

\[ \gamma_{12} := \varepsilon_2 - \varepsilon_1 \text{ Shear deformation} \]

\[ \tau_{12} := \frac{\varepsilon_1 \cdot E}{2} \text{ Shear stress: psi} \]

\[ |\text{slope}(\gamma_{12}, \tau_{12})| = 5.813 \cdot 10^5 \text{ G}_{12}: \text{In plane shear modulus, psi} \]

Figure B-12 Calculation of Shear Modulus for FR4 Test Sample D3 From Load vs. Strain Gage Data
Appendix C

Test Fixture Finite Element Model Results and Data Reduction

A FEM was constructed of the part of the test fixture that contained the slot that contained the FR4 plate and the wedge lock (see Figure C1). The FEM was constructed using 570 nodes connected to make 342 solid hexa elements resulting in a model with 1710 active DOF’s. A load of 100 lbs/node was applied at the upper outer edge of the fixture’s slot. A load of 100 lbs/node was also applied at the lower inner corner of the slot (see Figure C2).

The model was restrained approximately at the 6 bolt locations that attached this part to the baseplate. The upright part of the fixture was attached to the fixture’s baseplate using 0.25” bolts. Classical bolt preload theory [13] would assume that the two members were preloaded up to 3 bolt diameters away from the bolt. As a first conservative run of the model, only one hexa element was restrained in translation at each bolt location. Each of the restrained regions was approximately 0.25”x0.5”.

The PAL2 output file was edited to contain only the nodes that were under load, which were those with the maximum displacement. Figure C3 shows
the MathCad file that read and reduced the data from the PAL2 output file.

Figure C4 shows the graph generated in MathCad of the fixture stiffness along the fixture length (node number). This graph shows that the fixture’s stiffness is approaching 70,000 in•lbs/rad/in. Since this was significantly stiffer than the typical values of 100 to 1000 in•lbs/rad/in assigned to the wedge locks, the fixture was deemed to have negligible effect on the stiffness values being assigned to the wedge locks.
Figure C-1 Fixture Component With Card Slot Modeled
Figure C-2 FEM Mesh and Location of Line Load
\begin{align*}
\text{temp} & := \text{READPRN} \left( \text{fix5a} \_{\text{out}} \right) \\
\text{length} & := \text{rows} (\text{temp}) \\
\text{length} & = 38 \\
i & := 0 .. \text{length} - 1 \\
\text{nodenum} & := \text{temp}^{<0>} \\
\text{dx} & := \text{temp}^{<1>} \\
\text{dy} & := \text{temp}^{<2>} \\
\text{i1} & := 0 .. \frac{\text{length}}{2} - 1 \\
\text{i2} & := \frac{\text{length}}{2} .. \text{length} - 1 \\
\text{dz1}_i & := \left( \text{temp}^{<3>} \right)_{i1} \\
\text{dz2}_{19} & := \left( \text{temp}^{<3>} \right)_{i2} \\
\text{dz} & := \text{dz2} - \text{dz1} \\
\theta_n & := \arctan \left( \frac{\text{dz}_{19}}{0.297} \right) \\
k_n & := \frac{29.7}{0.5278} \\
k_0 & := \frac{\theta_0}{0.5278} \\
k_{19} & := \frac{\theta_{19}}{0.5278} \\
\text{Set up a range variable based on the number of data points} \\
\text{Assign the first column of data into an array, node numbers} \\
\text{Assign the second and third columns of data in arrays denoting x and y displacements} \\
\text{Set up a range variable based on the number nodes along the length of the fixture} \\
\text{Assign the fourth column of data in arrays denoting x displacements of both rows of nodes that had loads applied to them.} \\
\text{find the total growth in the width of the slot} \\
\text{find the angular displacement caused by the growth of the slot. 0.297 is the depth of the slot} \\
\text{find the rotational stiffness per unit length for the applied moment} \\
\text{(100 lbs applied at 0.297 inches). 0.5278 is the distance between nodes.} \\
\end{align*}

Figure C-3 MathCad File Used to Read the Output From the Fixture FEM
Rotational Stiffness (in*lbs/rad/in) vs position along the fixture (node number)

Figure C-4 MathCad Generated Graph Showing the Stiffness of the Fixture Along The Fixture's Length
Appendix D

Optimization/Minimization Algorithms

A design point consisted of a set of values for the rotational and translational spring stiffnesses that were entered into the FEM. The stiffnesses were, one at a time, changed by some amount and the model was re-run to find the first eight modal frequencies. The sum of the errors (between the FEM predictions and the empirical frequencies) squared was calculated for the design point and the four perturbations. The gradients between the design point and the four perturbations were calculated using a forward difference approximation[12].

\[
\frac{df}{dx_{x \to x_1}} = \frac{f(x_1 + \epsilon e^{(1)}) - f(x)}{\epsilon} \quad (D1)
\]

Figures D1 and D2 show the MathCad file that was used to calculate the gradients at a design point.
column matrix containing the model frequencies found empirically

\[ f := \begin{bmatrix} 269 & 269 & 269 & 273 & 271 \\ 342 & 342 & 342 & 345 & 344 \\ 651 & 652 & 651 & 655 & 652 \\ 748 & 748 & 748 & 757 & 752 \\ 852 & 853 & 853 & 860 & 857 \\ 1204 & 1206 & 1204 & 1214 & 1205 \\ 1307 & 1307 & 1307 & 1308 & 1307 \\ 1473 & 1474 & 1473 & 1487 & 1480 \end{bmatrix} \]

Matrix containing the FEM predicted model frequencies for, by column
1. the design point
2. the design point plus some change in \( K_t1 \)
3. the design point plus some change in \( K_t2 \)
4. the design point plus some change in \( K_r1 \)
5. the design point plus some change in \( K_r2 \)

\[ f_{\text{test}} := \begin{bmatrix} 5 \\ 5 \\ 5 \\ 4.1667 \\ 4.4444 \\ 1.9578 \\ 1.9578 \\ 1.9578 \\ 1.3554 \\ 1.9072 \\ 1.0582 \\ 1.0582 \\ 1.0582 \\ 0.1323 \\ 0.5231 \\ 3.6199 \\ 3.5062 \\ 3.5062 \\ 2.7149 \\ 3.0543 \\ 1.9544 \\ 1.7915 \\ 1.9544 \\ 1.1461 \\ 1.873 \\ 1.284 \\ 1.284 \\ 1.294 \\ 1.2085 \\ 1.284 \\ 1.4056 \\ 1.3387 \\ 1.4056 \end{bmatrix} \]

Error:

\[ \text{error}_{\text{j,i}} := \left| 1 - \frac{f_{\text{test},\text{i}}}{f_{\text{j}}} \right| \times 100 \]

The percent error for each of the FEM predicted frequencies at each of the design points

**Figure D-1** Page 1 of MathCad File Used to Evaluate Gradients at a Point in the Design Space
\[ \text{objfunc}_i := \sum_j (\text{error}_{j,i})^2 \]

The objective function to be minimized, the summation of all of the errors


\[
\begin{bmatrix}
51.7172 \\
49.5502 \\
50.911 \\
29.7014 \\
38.7978
\end{bmatrix}
\]

\( \text{objfunc} = \)

\[ l := 0 \ldots 1 \]

\[ \text{gradient}_1 := \frac{\text{objfunc}_{1+1} - \text{objfunc}_0}{1000} \]

\[ l := 2 \ldots 3 \]

\[ \text{gradient}_1 := \frac{\text{objfunc}_{1+1} - \text{objfunc}_0}{100} \]

\[
\begin{bmatrix}
-0.0022 \\
-0.0008 \\
-0.2202 \\
-0.1292
\end{bmatrix}
\]

\[ \text{gradient} = \]

\textbf{Figure D-2} Page 2 of MathCad File Used to Evaluate Gradients at a Point in the Design Space

In the first iteration the search vector is taken to be the vector defined by these gradients. A line search was conducted along this vector using the interval halving technique\[12\] until a minimum in the error function was found. Figures D3, D4, and D5 show the MathCad file that was used to evaluate and graph the error function during the line search.
column matrix containing the modal frequencies found empirically

\[
f := \begin{bmatrix}
272 \\
360 \\
664 \\
756 \\
884 \\
1228 \\
1324 \\
1494
\end{bmatrix}
\]

column matrix containing the FEM predicted modal frequencies for some step length along the search vector

\[
\mathbf{f}_{\text{fem}} := \begin{bmatrix}
298 \\
364 \\
659 \\
793 \\
880 \\
1202 \\
1291 \\
1459
\end{bmatrix}
\]

\[
i := 0 \quad j := 0 \ldots 7
\]

\[
\text{error}_{i,i} := \left| 1 - \frac{\mathbf{f}_{\text{fem},i}}{\mathbf{f}_i} \right| \cdot 100
\]

calculation of the percent error for each FEM predicted frequency

\[
\text{error} = \begin{bmatrix}
9.5588 \\
1.1111 \\
0.753 \\
4.8942 \\
0.4525 \\
2.1173 \\
2.4924 \\
2.3427
\end{bmatrix}
\]

Figure D-3 Page 1 of MathCad File Used to Evaluate the Error Function at a Point in the Design Space
$$\text{objfunc}_i = \sum_{j} (\text{error}_{j,i})^2$$
calculation of the error function for this point in the design space

$$\text{objfunc} = 133.5138$$

graphing of the error function for various step lengths along the search vector

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$$\text{i} := 0 \ldots \text{rows(}\alpha\text{)} - 1$$

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Figure D-4 Page 2 of MathCad File Used to Evaluate the Error Function at a Point in the Design Space
The gradients were evaluated again at this new minimum using the same MathCad file that was used at the first design point. The search vector would be updated using the Broyden-Fletcher-Shanno (BFS) method. The new search vector is
\[ S^{(j)} = -A^{(j)} \nabla F(x^{(j)}) \]  

(D2)

Where the matrix \( A \) is the identity matrix for the first iteration, and for subsequent iterations is defined as

\[ A^{(j+1)} = (I - \frac{\Delta x^{(j)} \Delta g^{(j)}}{\Delta x^{(j)} T \Delta g^{(j)}}) A^{(j)} (I - \frac{\Delta x^{(j)} \Delta g^{(j)}}{\Delta x^{(j)} T \Delta g^{(j)}}) + \frac{\Delta x^{(j)} \Delta x^{(j)} T}{\Delta x^{(j)} T \Delta g^{(j)}} \]  

(D3)

Figures D6 and D7 contain the MathCad file that was used to calculate the new \( A \) matrix and the new search vector.
\( \delta x := \begin{bmatrix} 84 \\ 38 \\ 286 \\ 130 \end{bmatrix} \)

the change in the four design variables between the current design point and the original

\( \delta g := \begin{bmatrix} 6.82 \\ 3.212 \\ 23.03 \\ 10.532 \end{bmatrix} \)

the change in the gradients for the four design variables between the current design point and the original

\( A/ := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)

the original A vector is set to the identity matrix

\( J := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \)

Figure D-6 Page 1 of MathCad File Used to Evaluate the New Search Vector Using the Gradients at the Current and Previous Design Points
\begin{align*}
  i & := 0 \ldots 3 \quad j := 0 \ldots 3 \\
  \text{num} & := dx \cdot dg^T \quad \text{den} := dx^T \cdot dg \\
  \text{term}_{i,j} & := \frac{\text{num}_{i,j}}{\text{den}_{0,0}} \\
  \text{num}_2 & := dx \cdot dx^T \\
  \text{den}_2 & := dx^T \cdot dg \\
  \text{term}_2_{i,j} & := \frac{\text{num}_2_{i,j}}{\text{den}_2_{0,0}} \\
  A2 & := (I - \text{term}) \cdot A1 \cdot (I - \text{term}) + \text{term}_2 \\
  A2 & = \begin{bmatrix}
    1.749 & 0.338 & 2.553 & 1.16 \\
    0.339 & 1.153 & 1.155 & 0.525 \\
    2.552 & 1.15 & 9.694 & 3.95 \\
    1.16 & 0.523 & 3.952 & 2.795 
  \end{bmatrix} \\
  dt & := \begin{bmatrix}
    .104 \\
    .192 \\
    .132 \\
    .132 
  \end{bmatrix} \\
  -A2 \cdot dt & = \begin{bmatrix}
    -0.737 \\
    -0.478 \\
    -2.287 \\
    -1.112 
  \end{bmatrix}
\end{align*}

The update of the search vector

\[ A2 := (I - \text{term}) \cdot A1 \cdot (I - \text{term}) + \text{term}_2 \]

\[ A2 = \begin{bmatrix}
    1.749 & 0.338 & 2.553 & 1.16 \\
    0.339 & 1.153 & 1.155 & 0.525 \\
    2.552 & 1.15 & 9.694 & 3.95 \\
    1.16 & 0.523 & 3.952 & 2.795 
  \end{bmatrix} \]

The new matrix which with the gradient at the current design point
defines the new search vector

\[ dt := \begin{bmatrix}
    .104 \\
    .192 \\
    .132 \\
    .132 
  \end{bmatrix} \]

The gradient at the current design point

\[ -A2 \cdot dt = \begin{bmatrix}
    -0.737 \\
    -0.478 \\
    -2.287 \\
    -1.112 
  \end{bmatrix} \]

The new search vector

\textbf{Figure D-7} Page 2 of MathCad File Used to Evaluate the New Search Vector Using the Gradients at the Current and Previous Design Points
Appendix E

FEM Predicted Mode Shapes

Figures E-1 and E-2 show the FEM predicted mode shapes for the 5.86" x 7.06" x 0.06" aluminum and FR4 plates, respectively, in the free-free condition.

Figures E-3 and E-4 show the FEM predicted mode shapes for the 5.86" x 7.06" x 0.06" aluminum and FR4 plates, respectively, when restrained with wedge locks.
Figure E-1 FEM Predicted Mode Shapes for 5.84\" x 7.06\" x 0.06\" Aluminum Plate, Free-Free
Figure E-2 FEM Predicted Mode Shapes for 5.84" x 7.06" x 0.06" FR4 Plate, Free-Free
Figure E-3 FEM Predicted Mode Shapes for 5.84" x 7.06" x 0.06" Aluminum Plate, Restrained with Wedge Locks
Figure E-4 FEM Predicted Mode Shapes for 5.84" x 7.06" x 0.06" FR4 Plate, Restrained with Wedge Locks
Vita

Mr. McMurray was born in Parkersburg, West Virginia in 1962. He attended high school in Belpre, Ohio. Mr. McMurray received his Bachelor’s of Science in Mechanical Engineering from the Ohio State University in 1984. He worked for Martin Marietta Electronic Systems Center as an Electronics Packaging Design Engineer on ground mobile and airborne electro-optic targeting systems for approximately two years. He then transferred to the Structural Analysis Group where he worked on airborne systems for the next three years. He began work on his Master’s at the University of Central Florida in the fall of 1988. In 1989 he moved to Roanoke, Virginia to work for FiberCom, Inc. on local area networking hardware for the Space Station Freedom. He was responsible for the development of a electromagnetic actuator for a fiber-optic switch as well as all structural and thermal analysis for the station hardware that FiberCom supplied. He was certified as a Professional Engineer in the state of Virginia and enrolled in the graduate school at Virginia Polytechnic Institute and State University in 1992. He is currently a Senior Mechanical Engineer at Litton-FiberCom where he supervises the mechanical design of networking hardware for tactical environments and consults on programs throughout the company.

Kevin McMurray

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