NUMERICAL MODELING OF TOOL-ROCK INTERACTION
IN LAMINATED FORMATIONS

by

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(ABSTRACT)

Based on a review of the literature, a numerical model of the chip formation process in laminated rock formations has been developed. The model uses a finite element approach to simulate the anisotropic behavior of laminated rock formations.

The finite element program has been developed with the assumption of two-dimensional plane strain. Anisotropic elements and dynamic loading are used to represent the actual penetration process of a bit tooth. An iteration method, using an incremental approach, has been applied for the continuous tooth penetration process. Displacements and axial loads have been modified after each iteration.

The program provides quantitative information on stresses and displacements during the penetration process. Furthermore, bit deviations in laminated formations can be inferred. Different rock strength characteristics have been employed on each side of the bit. Bit tooth-rock interaction for various formation dips (from 0° to 90°) of the Berea sandstone have been simulated and compared with experimental results.
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I wish to dedicate this thesis to my parents for their unwavering encouragement and support.

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INTRODUCTION

Drillhole deviation has always been an area of great concern in the drilling industry. The ability to maintain a borehole in the desired direction can mean efficiency and economy. Since most rocks have laminations or foliation, the anisotropy effect has been noticed as a tendency for a drillhole to deviate perpendicular to the lamination in moderately dipping formations, and to deviate parallel to the lamination in steeply dipping formations.

There are two principal causes resulting in drillhole deviation. The first is drillstring flexibility, which is a bending force causing the drill bit to deviate from the axis of the drillhole due to the unsupported weight of the drillstring. The second is tool-rock interaction which is an unbalanced side force causing the bit-tooth to deflect toward one side of the drillhole.

Much of the past work in this area has been done on drillstring mechanics. Drillstring mechanics have been well defined and successfully used in the drilling industry. However, the tool-rock interaction remains an area not well understood. If the tool-rock interaction in laminated formations and the resulting problems can be quantified, the information can be used in the planning stage of wells and in the bit design.

The objective of this study is to develop a numerical model to simulate the penetration process of a single wedged bit-tooth in laminated formations. This can contribute to the understanding of drillhole deviation because of tool-rock interaction. A two-dimensional finite element program with plane strain assumption is used for this study. Such a program can handle the complex behavior of rocks, arbitrary geometrical configurations and boundary conditions without difficulty.
Chapter 1 presents a review of the literature related to tool-rock interaction, and some previous findings on anisotropic effects.

Chapter 2 covers the finite element method used to accomplish the simulation, and the finite element model used to simulate the penetration process of a bit tooth in the laminated formations.

Chapter 3 describes the simulation procedure of tool-rock interaction.

Chapter 4 presents the simulation results and the interpretation of the simulation results.

Chapter 5 presents some conclusions and recommendations drawn from this study.
1. LITERATURE REVIEW

A common observation in drilling is that many formations exhibit "building", "dropping", and "walking" tendencies related in some manner to dipping laminated rock formations. Drillhole deviation from the projected trajectory can be an area of major concern and expenditure in the drilling industry. Drillstring mechanics and tool-rock interaction are two causes of drillhole deviation. Although drillstring mechanics has been used to control deviation and a number of computer programs have been developed to analyze the drillstring behavior, the tool-rock interaction study is still in the research stage. In addition, most of the research which has been conducted in tool-rock interaction, was done with isotropic and homogeneous assumptions. This does not explain the deviation due to tool-rock interaction in laminated formations. The following is a selective review of some research which deals with drillhole deviation, and the effects of tool-rock interaction on this deviation.

1.1 Drillstring Mechanics

Drillstring flexibility may cause hole deviation for two reasons. First, due to formation drillability. Second, due to the drillstring's mechanical behavior. Flexible drillstring is under the influence of formation drillability, that is, when the drill bit passes from one formation into another formation having a different drilling characteristic, the bit will drill off in a slightly different direction. For example, deviation will be up-dip when drilling from a softer into a harder formation, and down-dip when drilling from a harder to a softer formation. Lubinski and Woods (1953) defined a drilling anisotropy index to give the formation drillability. They found that rock has slightly lower drillabilities parallel to the bedding or laminations
than it has perpendicular to them.

Borehole deviation is also caused, in part, by the mechanical behavior of the drillstring as it buckles, causing the resultant thrust on the drill bit to deviate from the axis of the drillhole. These forces are reasonably well understood and used to help control hole deviation. The length of the drillstring involved in the deviation process depends on a number of factors, i.e., the physical makeup of the drillstring, the geometry of the hole, and the weight on bit (Bradly, 1975).

1.2 Tool-Rock Interaction

Many theories have been introduced in tool-rock interaction studies to provide the magnitude and the direction of the deviation force and none seems capable of predicting accurate deviation behavior.

Tool-Rock interaction in different rocks under different loading conditions produces a wide range of behaviors. Yet, the progressive failure pattern observed has been similar (Evans and Murrell, 1958), i.e., sequences of radial cracking, crushing, and chipping. Reichmuth (1963) and Singh and Johnson (1967) investigated the depth of penetration after cutting the indented specimen perpendicular to the wedge. The sectioned surfaces showed that the damaged depths were greater than those observed after removing the crushed material, and tensile cracks were initiated a short distance away from the tool-rock interface. The vertical cracks ran deep into the rock while others tended to curve toward the free surface as shown in Figure 1. As the applied load increased, the cracks continued stable growth up to a certain load level. Finally, the cracks propagated to the free surfaces in a stable manner.

In laminated rocks, preferential failure causes chips of unequal size to be
FOR NARROW ANGLE WEDGE

10 MM

FOR WIDE ANGLE WEDGE

Figure 1. Fracture Patterns of Rocks (After Pang & Goldsmith, 1990)
formed on either side of the bit tooth when the strike of the bit tooth coincides with the strike of the rocks. As a result, the bit will deflect in the direction of largest chip formation. This causes a gradual deviation of the drillhole (Bradley, 1975). Also, the penetration rates, energy input, and dust generation in a given formation depends on the tool's angle and the formation dip (Jung and Khair, 1989; Kurth, Sundae and Schultz, 1975; Voltz, 1974).

Many investigators have employed the slip-line theory in the study of stress fields due to bit tooth indentation problems in isotropic materials. This theory deals with rigid, perfectly plastic materials under plane strain assumption (Corant and Friedrichs, 1948; Johnson, Sowerby, and Haddow, 1970; Cheatham, 1958; Hill, 1950; Pariseau and Fairhurst, 1967).

Hill extended the theory to the case of a plastic material using yield conditions based on the Mohr-Coulomb criterion. He showed that two families (triangle regions from either side of the bit to the open surface) of this characteristic exist, which are inclined at an angle of \( \pm \left( \frac{\pi}{4} - \frac{\phi}{2} \right) \), as shown in Figure 2, where \( \phi \) is the angle of internal friction.

Cheatham (1958) used this theory to study maximum and minimum forces necessary to penetrate a rock with a wedge-shaped tooth. He varied the confining pressure and wedge angle to determine their effects on the penetration process in terms of force-displacement analysis. Later, Pariseau and Fairhurst (1967) extended the work to include the frictional resistance at the rough tooth-rock interface. There are three models in their assumed stress field based on slip-line theory. The first model is developed for a smooth bit case, and no friction at the bit-rock interface is assumed. In this model, the stress field can be separated into three regions. The first region (tensile failure area) and third region (crushing zone) are called the
Assumed Stress Field for Smooth Bits.

Assumed Stress Field for Rough Bits.

Assumed Stress Field for False Nose Situation.

Figure 2. Diagram of the Slip-Line Theory (After Pariseau & Fairhurst, 1967).
constant state regions (crushing region), and the second region (shear failure area) is called the radial shear region. The force-penetration relationship for this model is given by

\[
\frac{F}{bh\sigma_0} = \frac{\tan\beta}{\tan\phi \tan\mu} \left[ \exp(2\beta \cdot \tan\phi) - \tan^2\mu \right]
\]

where

- \( F \) = vertical load
- \( h \) = penetration depth
- \( \sigma_0 \) = unconfined rock compressive strength
- \( b \) = the width of the bit normal to the plane of the paper
- \( \beta \) = half bit tooth angle
- \( \phi \) = internal friction angle
- \( \mu = \frac{\pi}{4} - \frac{\phi}{2} \).

In the second model, the rough bit case is used and frictional resistance is taken into account. There are only two regions left in the stress field for this model (Figure 2), because it is assumed that the bit-rock interface coincides with one of the failure planes (i.e., the rock material adjacent to the bit must be a radial shear region with one of the radials as the interface). The force-penetration relation for this model is given by

\[
\frac{F}{bh\sigma_0} = \frac{\tan\beta}{\tan\phi \tan\mu} \left[ [1 + \sin\phi(\cot\beta \tan\mu - 1)] \exp(2\xi \tan\phi) - \tan^2\mu \right]
\]

where \( \xi = \frac{\pi}{2} - \mu + \beta \).

When the half bit tooth angle is larger than 30°, a third model is used and
called the "false nose case" (Figure 2). The assumed stress field in this model is again composed of two constant state regions separated by a center fan (radial shear region). The expression for force-penetration relation is given by

\[
\frac{F}{bh\sigma_0} = \frac{\tan \beta}{\tan \phi \tan \mu} \left[ \exp(2\xi \tan \phi) - \tan^2 \mu \right]
\]

where \( \xi = \pi/2 \).

While the slip-line theory gives a good first approximation of the stress field, especially at the high confining pressure condition where rock behaves as a ductile material, the theory, with its idealized material properties, is unable to interpret the brittle penetration failure mechanism.

Paul and Sikarskie (1965) introduced a preliminary theory to describe the static penetration of a rigid wedge into brittle isotropic materials. The model they used (Figure 3) was expected to predict axial forces and corresponding penetrations during the penetration process. They assumed that fractures occurred along a plane extending from the wedge tip to the free surface at an unknown angle \( \psi \), and that the Mohr-Coulomb failure criterion was applied along the failure plane (chip surface). The force-penetration relation is given by

\[
\frac{F_v}{H} = \frac{4c \cdot \sin(\theta + \phi_f) \cos \phi}{1 - \sin(\theta + \phi_f + \phi)}
\]

where \( \phi_f = \) friction between tool-rock interface

\( \phi = \) internal friction angle
Figure 3. Incipient Chip Formation (After Sikarskie, 1965)
c = cohesive strength  
\( \theta \) = one half wedge angle  
\( F_v \) = axial force  
\( H \) = penetration depth.

The earlier model was improved by Benjumen and Sikarskie (1969) to include anisotropic material. However, only rocks with the bedding planes oriented either parallel or perpendicular to the direction of penetration were tested. They used Jaeger and Cook's (1976) modification of the Coulomb-Mohr failure criteria to describe the anisotropic case. The modified Coulomb-Mohr failure criterion is given by:

\[
| \ddot{\tau} | - \sigma \cdot \tan \phi - [S_1 - S_2 \cos 2(\psi - \gamma)] = 0
\]

where  
\( \phi \) = angle of internal friction  
\( S_1, S_2 \) = material constants (for isotropic material \( S_2 = 0 \))  
\( \ddot{\tau} \) = shear stress average along the fracture surface  
\( \psi \) = failure angle of fracture plane.

This modified theory can give the penetration depth corresponding to the vertical load, and the equation is:

\[
\frac{P_{i+1}}{d_{i+1}} = \frac{4 \sin \theta \cos \phi}{\cos^2(\theta+\phi)} \left[ \frac{S_1 \sin(\theta+\phi) - S_2 \sin(2\gamma+\theta+\phi) +}{\sqrt{([S_1 \sin(\theta+\phi) - S_2 \sin(2\gamma+\theta+\phi)]^2 + [S_1^2 - S_2^2] \cos^2(\theta+\phi))}} \right]
\]
where \( P_{i+1}^* \) = force per unit length of cutting edge necessary to remove
the \((i+1)\)st chip
\( \dot{d}_{i+1} \) = the penetration at the \((i+1)\)st chip removal
\( \theta \) = half wedge angle
\( \gamma \) = dip angle
\( \phi \) = internal friction angle.

Later, Altiero and Sikarskie (1973), using an integral method, succeeded in
predicting incipient fracture from the penetration of a wedge-shaped tool, without
considering the compressive failure of the material and the change in the stress field.
They stated that the fracture is initiated at some point in the field, and that this
fracture initiation is a function of traction distribution (Figure 4), geometry of the
wedge, and physical properties of the material. The traction distribution is given by:

\[
\begin{align*}
t_n &= 0 & \text{on DB plane} \\
t_n &= p(\xi/L)^m(1 - \cos2\pi\xi/L) & \text{on BA plane} \\
t_n &= 0 & \text{on AC plane} \\
t_s &= 0 & \text{on DB plane} \\
t_s &= \mu t_n = p\mu(\xi/L)^m(1 - \cos2\pi\xi/L) & \text{on BA plane} \\
t_s &= 0 & \text{on AC plane}
\end{align*}
\]

where \( L \) = length of the bit-rock interface
\( t_n \) = normal traction components
\( t_s \) = tangential traction components
\( \theta \) = half wedge angle
\( \xi \) = coordinate along bit-rock interface
Figure 4. The Traction Distribution from Sikarskie and Altiero's Model (1973)
\[ \mu = \text{coefficient of friction} \]

\[ m = \text{traction "form" parameter which can be obtained experimentally} \]

\[ p = \text{a scalar pressure which can be related to the vertical load as follows} \]

\[ p = \frac{P \cos \phi}{L \sin(\theta + \phi) \int_0^1 \xi^m (1 - \cos 2\pi \xi) d\xi} \]

where \( P = \text{vertical load}. \)

Qualitative agreement between the model and experimental results was shown. They found that relatively large differences in the energy required to remove unit volume of rock can occur, even for relatively small anisotropy.

Fuh (1983) has tried using both models, i.e., the plastic model and the brittle fracture model, to determine the cutter bit spacing for optimum coal mining. He found that the bit spacing derived from the plastic model tended to overestimate the true requirement of spacing, while the brittle fracture model yielded much better results. He concluded that this was not unexpected, because the mining of coal (especially hard coal) by cutter bit action has been characterized by brittle fracture rather than plastic flow (Evans, 1965 and 1972; Gottlieb and Moore, 1981; Roxborough et al., 1981).

McLamore (1971) performed a series of tests and concluded that as a wedge penetrates into rocks, energy is stored symmetrically in the rock. When this state of stress exceeds the strength of the rock, chips will form. In isotropic and homogeneous rock, equal chip volumes are formed on each side of the bit tooth and there is no deviation. On the other hand, in dipping laminated formations, larger chip volume is formed on one side of the bit tooth, and deviation occurs toward the
direction of the preferential chip formation (Figure 5). The geometry of the chip is a function of the formation dip, the anisotropic strength characteristic of the rock, and the included wedge angle. These is known as McLamore's "preferred chip theory." Later, Bradley (1975) performed wedge impact tests to verify McLamore's theory and found a close correlation between chip volumes and deviation forces at the corresponding dip angles.

McLamore and Bradley extended Sikarskie's study to include the unbalanced side force \( F_R \) on the wedged bit tooth due to the axial load at chip formation and a variable cohesive strength model was used (Figure 6). This is called the deviation force theory and the side force due to the vertical load can be given by

\[
F_R = \frac{F_v}{2\sin\beta} \tag{1}
\]

at failure

\[
F_r = \frac{\tau_0 \cdot H \cos \phi}{\sin \psi \cos (\beta + \psi + \phi)} \tag{2}
\]

where

- \( F_v \) = vertical loading on a bit tooth
- \( F_R \) = lateral force on the side of a bit tooth
- \( F_r \) = lateral side force required for chip formation
- \( H \) = bit tooth penetration depth
- \( \beta \) = included half angle of a bit tooth
- \( \psi \) = angle of failure (measured from the bottom of the hole to the surface of the rock)
- \( \phi \) = angle of internal friction along failure plane
- \( \tau_0 \) = initial shear strength along failure plane.
Figure 5. Chip Formations by a Wedge in Isotropic and Anisotropic Rocks
Figure 6. Schematic of the Deviation Force Theory (After McIamore, 1971)
It is noted that the $\psi$ and $\phi$ angle can be different at each side of the bit tooth. If the side force $F_r$ is equal on both sides of the bit tooth, no deviation will occur. If the side force $F_r$ on both sides of the bit tooth is not equal, a larger chip will be formed on the side with smaller $F_r$. As a result, the bit tooth will be deflected by the unbalanced side force toward the direction of largest chip volume removal. The failure angle at any bedding angles can be predicted by

$$\psi = \frac{90^\circ - \phi}{2}.$$  

A composite model was developed based on McLamore's deviation force theory (Karfakis and Evers, 1987). This model assigns a set of strength parameters or failure criterion to either side of the tooth based on the angle $\rho$ between the lateral force and the bedding plane. The model predicts that, for a given tooth thrust in a dipping formation, chips may be formed on either side of the tooth as long as the resultant force on the tooth is equal or greater than the critical force for chip formation. The volume of chips on either side will depend on the strength characteristics of the rock controlled by the local configuration of the tooth-rock interaction. The model also suggests that the chip volume will be proportional to $\frac{1}{(F_r^2 \tan \psi)}$. The relative chip volume $V_R$, when the bit tooth is normal to the bedding plane, is given by:

$$V_R = \frac{\hat{F}_r^2(z=0) \tan \psi(z=0)}{\hat{F}_r^2(z) \tan \psi(z)}.$$  

where $\hat{F}_r$ is the lateral force per inch of tooth penetration and the
dip angle is denoted by $\varsigma$. 
2. PURPOSE AND SCOPE OF THE INVESTIGATION

The purpose of the present investigation is to develop a finite element model capable of simulating the tool-rock interaction in laminated formations. The finite element method has been used by some researchers to study the tool-rock interaction problem, but mostly for static loading condition. It is thought that the dynamic loading condition would be more pertinent to describe the actual penetration process of a bit tooth.

Different material behavior models have been used to simulate the tool-rock interaction with the finite element method, i.e., the plasticity model, the Coulomb model, the Coulomb/tensile model, and Hoek and Brown model. All models offer an excellent explanation for some aspects of rock behavior but fail to explain others which depends on the nature of problems. The Hoek and Brown model (1980) adequately describes the response of intact rock to the full range of stress conditions that can be encountered under bit tooth loading. Therefore, the criterion is employed for this study.

The FEM model used in this study is capable of predicting the stress and displacement during the penetration process in anisotropic rocks. The progressive failure of rocks can also be analyzed using Hoek and Brown failure criterion.

2.1 Finite Element Method

The finite element method is a numerical procedure for analyzing structures and continua. It originated as a method of stress analysis. It has since been used to analyze all kinds of problems, e.g., heat transfer, fluid dynamics, etc.

This method has been used to analyze rock mechanics problems by many investigators (Desai and Abel, 1972; Zienkiewicz, 1979). Wang (1976) used the
method to simulate the behavior of isotropic rock under static loading condition. Swenson (1983) modeled drag-bit cutting and analyzed the displacement corresponding to the vertical force. Reasonable agreement was shown between the experimental tests and Swenson's finite element model. Cook et al. (1984) performed experimental tests to observe the crack growth in hard rock loaded by an indenter, and made comparisons between the results from experimentation and the finite element analysis.

The finite element method (FEM) is widely used by practicing engineers, and a large volume of literature has been written on this method during the past decade. The details of FEM are beyond the scope of this thesis. The idea of FEM is to separate a continuum into many subdomains called elements. The shape of an element can be triangle, rectangle, or quadrilateral, which depends on the problem in question. All elements within the continuum are connected at discrete points called nodes. The finite element mesh is generated by assembling each element at nodal points on the boundaries of the elements. The finite element procedure produces many simultaneous algebraic equations, which are generated and solved on a digital computer. Results are rarely exact. However, errors are decreased by processing more equations, and results accurate enough for engineering purposes are obtained.

Although everything is three-dimensional in reality, many engineering problems encountered in the field are two-dimensional in nature. There are two kinds of assumptions in the plane elasticity problem - one is called plane stress, the other is called plane strain. The study of tool-rock interaction can be considered as the plane strain type of problem. Thus, the two dimensional plane strain assumption is used for most studies in tool-rock interaction. Based on this assumption, the finite
element method of rock failure has been developed.

For plane strain problems, where the body is very thick in the z direction, the assumption is that the strains in the z direction are zero

$$
\epsilon_z = \gamma_{yz} = \gamma_{zx} = 0
$$

where

$$
\epsilon_z = \text{strain in the z direction}
$$

$$
\gamma_{yz} = \text{shear strain on the Y-Z plane}
$$

$$
\gamma_{zx} = \text{shear strain on the X-Z plane.}
$$

The equations governing the plane strain type of plane elasticity problem can be summarized below.

**EQUILIBRIUM EQUATIONS IN TERMS OF STRESSES**

$$
\begin{align*}
\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xx}}{\partial y} + f_x &= 0 \\
\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + f_y &= 0 \\
\end{align*}
$$

in \( \Omega \)

\( f_x, f_y \) are the body forces along the x and y directions; however, body forces are not considered in this study.

**STRAIN - DISPLACEMENT RELATIONS**

$$
\epsilon_x = \frac{\partial u}{\partial x} \quad \epsilon_y = \frac{\partial v}{\partial y} \quad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}
$$

**STRESS - STRAIN RELATIONS**

$$
\sigma_x = C_{11} \epsilon_x + C_{12} \epsilon_y \\
\sigma_y = C_{12} \epsilon_x + C_{22} \epsilon_y \\
\tau_{xy} = C_{33} \gamma_{xy}
$$
where \( C_{ij} \) are the elasticity constants. For an isotropic elastic body, they are given as
\[
\begin{align*}
C_{11} &= C_{22} = \frac{E}{1 - \nu^2} \\
C_{12} &= \frac{E\nu}{1 - \nu^2} \\
C_{33} &= \frac{E}{2(1 + \nu)}
\end{align*}
\]

**BOUNDARY CONDITIONS**
\[
\begin{align*}
\text{NATURAL} & \quad \sigma_x n_x + \tau_{xy} n_y = t_x \\
& \quad \tau_{xy} n_x + \sigma_y n_y = t_y \\
\text{on } \Gamma_1
\end{align*}
\]
\[
\begin{align*}
\text{ESSENTIAL} & \quad \hat{u} = u \\
& \quad \hat{v} = v \\
\text{on } \Gamma_2
\end{align*}
\]

where \( \hat{u} \), \( \hat{v} \) are the specified displacements along X and Y directions, respectively.
\( \Gamma_i \) are portions of the boundary
\( u^e \) and \( v^e \) (displacements over element \( e \)) be approximated by finite element interpolation of the form
\[
\begin{align*}
u^e &= \sum_{j=1}^{R} u^e_j \psi^e_j \\
v^e &= \sum_{j=1}^{R} v^e_j \psi^e_j
\end{align*}
\]

\[
\{ \delta^e \} = \{ u^e_1, v^e_1, \ldots, u^e_n, v^e_n \}^T
\]

**VARIATIONAL (DISPLACEMENT) FORMULATION**

\( w_1, w_2 \) test functions
\[
0 = \int_{\Omega} \left( \frac{\partial w_1}{\partial x} (C_{11} \frac{\partial u}{\partial x} + C_{12} \frac{\partial v}{\partial y}) + C_{33} \frac{\partial w_1}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) - w_1 f_x \right) dx dy - \oint_{\Gamma_e} w_1 t_x ds \quad (3)
\]
\[ 0 = \int_{\Omega} \left\{ C_{32} \frac{\partial w_2}{\partial x} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial w_2}{\partial y} \left( C_{12} \frac{\partial u}{\partial x} + C_{22} \frac{\partial v}{\partial y} \right) - w_2 f_y \right\} dxdy - \oint_{\Gamma_e} w_2 t_y ds \quad (4) \]

Substituting \( w_1, w_2 = \psi_i \) into equation (3) and (4) then

\[
\begin{align*}
[K^{11}][u_1] + [K^{12}][v_1] &= \{F^1\} \\
[K^{21}][u_2] + [K^{22}][v_2] &= \{F^2\}
\end{align*}
\]

Thus \([K] \{\delta\} = \{F\}\)

where

\[
K^{11}_{ij} = \int_{\Omega} \left( C_{11} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_i}{\partial y} + C_{33} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_i}{\partial y} \right) dxdy
\]

\[
K^{12}_{ij} = K^{21}_{ji} = \int_{\Omega} \left( C_{12} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_i}{\partial x} + C_{33} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_i}{\partial x} \right) dxdy
\]

\[
K^{22}_{ij} = \int_{\Omega} \left( C_{33} \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_i}{\partial x} + C_{22} \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_i}{\partial y} \right) dxdy
\]

\[
F^1 = \int_{\Omega} \psi_i f_y dxdy + \oint_{\Gamma_e} \psi_i t_y ds
\]

\[
F^2 = \int_{\Gamma_e} \psi_i f_y dxdy + \oint_{\Gamma_e} \psi_i t_y ds
\]

Where \( K \) and \( F \) are the stiffness matrix and force acting on the nodal point, respectively.

Powerful analytical tool, the FEM can make a good engineer better and a bad engineer dangerous (Cook, R.D., D.S. Malkus, and M.E. Plesha, 1989). Because the proper use of the FEM depends on the nature of the problem, the results from the FEM are only valuable when a reasonable interpretation is made by engineers. A large volume of solutions is generated by the finite element procedure,
which makes it better to present the solutions in a graphic format. In order to
associate the lamination effect with the finite element method, some finite element
techniques are used in the simulation. Anisotropic material properties are employed
to represent elements after tensile failure. The finite element program for this study
can be used for either eight node or four node elements. With some modification,
this program can also be used to handle triangular elements. Transient analysis is
used in the simulation of progressive strength failure of rock. Variable elasticity and
external load are applied during the process of successive penetration. The details of
programming and other features can be found in Reddy’s (1984) book.

2.2 Previous Modeling of Tool-Rock Interaction Using FEM Computer Code

Wang and Leinhoff (1976) did FEM static to simulate the force-penetration
curve for blunt point bit, sharp wedge bit, and cylindrical bit. They used an
incremental displacement approach to simulate continuous penetration with
modification of material properties and displacement on limestone.

In their study, the rock was considered to be isotropic and homogeneous
before failure of the rock. An anisotropic elasticity matrix was used to represent the
element after tensile failure with open cracks. In addition, since the stiffness of
rock decreases with displacement, the instantaneous value of Young’s moduli for a
fractured rock was modified by a simple mathematical relation as:

$$E_i = E \left(\frac{\tau_i - \tau_f}{\tau_{max} - \tau_f}\right)^c$$

where

- $E_i = \text{instantaneous stiffness}$
- $\tau_i = \text{instantaneous shear strength}$
\[ \tau_{\text{max}} = \text{maximum shear strength} \]
\[ \tau_r = \text{residual shear strength.} \]

They concluded that quantitative information on stress, displacement and material failure in the process of bit penetration can be obtained. The analytical results have shown reasonable agreement with experimental observations. They also recommended that the effects of tool shape and post failure rock strength can and should be studied using FEM Program.

Swenson (1983) simulated a single drag-bit cutting, using polycrystalline-diamond compact cutters, by FEM static analysis on Berea sandstone. He assumed that the front face of the cutter is in contact with rock, Also, because of symmetry, the calculation was made using only one-half of the rock sample size. The plasticity model and Coulomb model with tensile failure criterion were used in his research. He concluded that the indentation type loading is the major fracture forming mechanism rather than the contact between the front face of a drag bit and rocks.

2.3 Mathematical Rock Failure Model

There are several mathematical rock failure models used in the study of tool-rock interaction, namely the plasticity model, the Coulomb model, Coulomb/Tensile model, and Hoek and Brown model. The plasticity model is used to simulate materials which possess the characteristics of plastic deformation rather than brittle failure as is the case for rock. The Coulomb model assumes material failure will occur in shear and this model was originally developed for material such as soil. For material like rock, it would be easier to form a tensile crack rather than a shear fracture plane. Therefore, tensile failure criterion should be used to
describe the progressive failure of rocks. The following are the detailed explanations of these models.

2.3.1 Plasticity Model

The plasticity model assumes a yield surface, which is a surface surrounding the hydrostat in the principal stress space for the material (Figure 7). The radius of the surface around the hydrostat is taken to be a quadratic function of the mean pressure, $p$, defined as

$$p = -\frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3),$$

where $\sigma_1$, $\sigma_2$, and $\sigma_3$ are the major, intermediate, and minor principal stresses.

The plasticity model includes both deviatoric and volumetric plasticity. The volumetric plasticity defines the position of the end cap on the normally open end of the surface of revolution, as shown in Figure 7.

The yield surface can be described with two functions, one describing the parabola ($\theta_s$) and one describing a plane ($\theta_p$) which is normal to the hydrostat.

$$\theta_s = J_2 - (a_0 + a_1 p + a_2 p^2),$$

$$\theta_p = p - f,$$

where $p$ is the mean pressure (positive), $a_0$, $a_1$, and $a_2$ are material constants and $J_2$ is the second invariant of the deviatoric stresses. The quantity, $f$, is a function of the mean volumetric strain and defines the volumetric stress-strain curve for the material, as shown in Figure 8.
Figure 7. Plasticity Yield Surface in Principal Stress Space.
Figure 8. The Pressure vs. Finite Volume Strain Behavior of the Plasticity Model (After Swenson, 1983).
The strain increment, which includes corrections to account for crack opening strains, is decomposed into a mean volumetric strain increment, \( \Delta \varepsilon_v \), and a deviatoric strain increment, \( \Delta \varepsilon \).

The plasticity theories for the volumetric and deviatoric parts are now taken to be completely independent. Mean volumetric strain is updated as

\[
\varepsilon_v^{i+\Delta t} = \varepsilon_v^i + \Delta \varepsilon_v.
\]

A check is then made to see if

\[
\varepsilon_v^{i+\Delta t} > \varepsilon_u.
\]

(5)

where \( \varepsilon_u \) is the smallest volumetric strain previously experienced by the material. If equation 5 is satisfied, the step is elastic and,

\[
P^{i+\Delta t} = P^i - k_0 \Delta \varepsilon_v,
\]

where \( k_0 \) is the elastic bulk modulus for the material. If equation 5 is not satisfied,

\[
P^{i+\Delta t} = f(\varepsilon_v^{i+\Delta t}),
\]

and \( \varepsilon_u = \varepsilon_v^{i+\Delta t} \),

the detail of this plastic model can be found in Krieg's (1972) report.

2.3.2 Coulomb Model
In this model, explicit shear cracks are assumed to form at the element integration points (Figure 9). The Coulomb criterion assumes that the shear stress tending to cause failure across a plane is resisted by a cohesion of the material, $S_0$, and a linear function of the normal stress across the plane (Jaeger and Cook, 1976).

$$|\tau| > S_0 + \mu \sigma_n$$

where $\tau$ and $\sigma_n$ are the shear stress and normal stress across the failure plane. The coefficient of internal friction for the material is denoted by $\mu$. The cohesion and the unconfined compressive strength of the material, $C_0$, are related by

$$S_0 = \frac{C_0}{2[\sqrt{\mu^2+1} + \mu]}.$$

When shear failure occurs, two cracks form simultaneously in the two equally probable shear directions inclined at angles $\pm \alpha$ on either side of the major principal stress direction

$$\alpha = \pm \frac{1}{2} \tan^{-1}(1/\mu).$$

After a crack has formed, the state of stress is modified according to the Coulomb criterion and used to calculate the shear stress on the crack face.

### 2.3.3 Coulomb/Tensile Model

Because rock is weak in tension, it would be more reasonable to check for tensile failure before shear failure, especially under brittle failure conditions. Yet,
\[ \xi = -0.577 \quad \eta = 0.577 \quad \xi = 0.577 \]

*\(\xi\) and \(\eta\) are the local coordinates of an element

Figure 9. Positions of Element Integration points.
the plastic model assumes a constant strain rate after the failure of materials (Figure 10), and this is not the case during the penetration process in the brittle material. Also, note that the Coulomb model assumes only the occurrence of shear failure within materials, and this assumption is not appropriate for brittle materials such as rocks. Therefore, the Combination of Coulomb model with tensile failure criterion was introduced to interpret the brittle penetration failure mechanism of rock (Swenson, 1983).

2.3.4 Hoek and Brown Model

The empirical Hoek and Brown failure criterion can better represent the nonlinear behavior of the rock mass. The relationship between the principal stresses at failure adequately describes the response of intact rock to the full range of stress conditions encountered in practice. Furthermore, the failure criterion is capable of accounting for anisotropic strength behavior associated with the existence of plane of weakness. Failure in a biaxial tensile zone can also be predicted. The empirical relationship between the principal stresses associated with the failure of rock is given by

\[ \sigma_1 = \sigma_3 + \sqrt{m \sigma_c \sigma_3 + s \sigma_c^2} \]

where

- \( \sigma_1 \) = maximum allowable principal stress,
- \( \sigma_3 \) = minor principal stress applied to the rock sample
- \( \sigma_c \) = uniaxial compressive strength of rock sample
- \( m, s \) = material constants (obtained experimentally).
Figure 10. Post Failure Response of the Coulomb and Plasticity Models (After Swenson, 1983).
The uniaxial tensile strength of the rock sample is given by substitution of $\sigma_1 = 0$ in the above equation and by solving the resulting equation to get $\sigma_t$:

$$\sigma_t = \sigma_0 \left(1 - \sqrt{\frac{m^2 + 4s}{2}} \right)$$

where $\sigma_t = \text{uniaxial tensile strength of rock sample}$

2.3.5 Progressive Failure

The impact of a bit tooth will cause a stress distribution at and around the imprint of impact. If the stress distribution results in areas of the rock where the imposed stress exceed the strength of the material, the material will fail. Once the material fails, its strength and stiffness will be reduced, diverting the stresses to adjacent intact material. The cycle of successive stress distribution and failure will continue until the energy imparted to the bit tooth is dissipated. The progressive failure concept, utilizing the Hoek and Brown failure criterion, was used to simulate progressive failure under the bit tooth in the finite element modeling.

2.4 Algorithm of the FEM Program

Since it is very difficult to express complicated rock failure phenomena by a single criterion, a numerical model based on observations of experimental results is proposed as follows:

1. Before the state of stress exceeds the critical failure strength, rock is considered linear-elastic, isotropic, and homogeneous.

2. The failure criterion is assumed to be a linear Mohr envelope. The shape of the linear envelope can be determined by the major and minor principal
stresses from the triaxial test. The composite model for chip formation is used for this study. The model assigns one set of strength characteristics parallel to the bedding plane and a different set to the cross bedding. The model predicts that chips may be formed on either side of a bit tooth if the normal stresses along each side of a bit tooth equal or exceed the compressive strength of the rock.

3. After tensile failure, rock loses its cohesion on the crack surface while retaining its strength in the direction parallel to the fracture surface with friction being the dominant factor. An increase in load causes further stable growth of the crack.

4. After compressive failure, crushed elements can no longer sustain any load, and the neighboring elements will bear the load as the stiffness of the crushed elements continues to decrease. However, the effect of fragment reconsolidation is not treated here.

The basic algorithm for modeling tool-rock interaction can be summarized into the following steps:

1. Obtain the stress state of the rock,
2. Check for element failure,
3. Classify the failed element,
4. Assign small Young's moduli to crushed element,
5. Modify the finite element mesh according to the displacement after each iteration,
6. Modify the vertical load at corresponding time interval to simulate the dynamic loading condition,
7. Repeat steps 1-6 for each level of vertical load until the final time limit is
reached.

The detail of this algorithm is given in a flow chart (Figure 11).
Figure 11. Flow Chart of The Finite Element Program
3. SIMULATION

Numerical simulations of tooth-rock interaction were performed using the strength parameter of Berea sandstone (Long, 1988). The simulation results were compared to wedge indentation test results on the same rock. Long conducted indentation tests on the Berea sandstone to observe the chip volumes and corresponding lateral forces in anisotropic rocks. The model configurations of the finite element analysis are the same as in Long’s experiment. The size of the rock sample is 2" in width and 4" in length. The size of the bit is 0.5" x 0.375", with a half wedge angle of 30°. The maximum axial force (8000 lbs) was reached within 2.5 milliseconds.

During the simulation, information about progressive failure corresponding to time can be obtained from the changing state of stresses. This is accomplished by simulating the dynamic loading due to tool-rock interaction. Numerical techniques were used to represent an element after tensile failure (with open cracks), and to transfer the stress to the system after an element was crushed.

Tool-interaction simulation starts from the initial contact between a bit and an intact rock without pre-existing stresses and angular forces. A small incremental time interval is assigned to each iteration. With the assigned small time interval and the external load with respect to current time interval, the incremental stresses can be obtained by solving the system of equilibrium equation:

\[
[K](U) = \{F\},
\]

where \([K]\) is the global stiffness matrix, and \([U]\) and \([F]\) are the nodal
displacements and the external forces acting at the nodes.

If the time increment is sufficiently small, then each incremental solution may be considered linear and can be accomplished accurately. After each iteration, the state of stress of all elements are checked to determine their current conditions. Further modifications for stress release follow, if necessary. An indication matrix for Young's moduli has been generated to determine whether an element should release the stress acting on it or not. If crushing occurs in an element, the stress is released and taken by neighboring elements.

Before each execution of the program, the number of iterations $n$ and time intervals $\Delta t$ must be specified. With these numbers, the computer program can automatically iterate $n$ times to modify the nodal displacements at each load level and to give the stress distribution at various intensities of external load. The data of stress distribution and element conditions are recorded on a disc for plotting. Progressive failure can be shown by a subroutine which checks for failed elements. This subroutine will check element condition from the bottom to the surface of the finite element mesh. If an element is crushed, an indication matrix will record the number of that element. After the whole mesh is checked, the subroutine will have located the position of all crushed elements. According to the direction (up-dip or down-dip) of the crushed elements, different nodal points are recorded within crushed elements and the failure boundary can be located at the end of the subroutine. A list of input instructions is given in Appendix A.

3.1 Dynamic Loading of the Bit Tooth

In finite element programming for dynamic problems, the displacement functions depend on time, and the test functions depend on spatial coordinates. This
leads to two stages of the solution, both of which employ approximate methods. In
the solution of time-dependent problems, the spatial approximation is considered
first and the time approximation next. In structural dynamics problems, the equations
of motion involve the second-order time derivatives of the dependent variables. The
spatial approximation of the equations results in a matrix differential equation of the
form

\[ [A][\dot{C}] + [B][C] = [F] \quad (6) \]

There are several approximation schemes available for time derivatives. The
one used in this study is the Newmark direct integration method. In the Newmark
direct integration method the first time derivative \( \{\dot{C}\} \) and the function \( \{C\} \) itself
are approximated at the \((n+1)\)th time step \( (\Delta t_1 = \Delta t_2 = \ldots = \Delta t) \)

\[
\begin{align*}
\{\dot{C}\}_{n+1} &= \{\dot{C}\}_n + [1/(1-\alpha)][\{\ddot{C}\}_n + \alpha\{\ddot{C}\}_{n+1}]\Delta t, \\
\{C\}_{n+1} &= \{C\}_n + \{\dot{C}\}_n\Delta t + [(1/2-\beta)\{\ddot{C}\}_n + \beta\{\ddot{C}\}_{n+1}]\Delta t^2,
\end{align*}
\quad (7)
\]

where \( \alpha \) and \( \beta \) are parameters that control the accuracy and the stability of the
scheme, and the subscript \( n \) indicates that the solution is evaluated at the \( n \)th time
step. The values for \( \alpha \) and \( \beta \) are 0.5, 0.25 respectively for unconditional stable cases.

Rearranging equations (6) and (7), results in

\[ [\hat{A}][C]_{n+1} = [\hat{F}]_{n+1}, \]

where \( [\hat{A}] = [B] + a_0[A] \) and \( [\hat{F}] = [F]_{n+1} + [A](a_0[C]_n + a_1[\dot{C}]_n + a_2[\ddot{C}]_n) \).
Once the solution \{C\} is known at \(t_{n+1} = (n+1)\Delta t\), the first and second derivatives (velocity and acceleration) of \{C\} at \(n+1\) time step can be computed from

\[
\{\ddot{C}\}_{n+1} = a_0(\{C\}_{n+1}-\{C\}_n) - a_1\{\dot{C}\}_n - a_2\{\ddot{C}\}_n
\]

\[
\{\dot{C}\}_{n+1} = \{\dot{C}\}_n + a_3\{\ddot{C}\}_n + a_4\{\ddot{C}\}_{n+1}
\]

\[
a_0 = \frac{1}{\beta\Delta t^2} \quad a_1 = a_0\Delta t \quad a_2 = (1/2\beta) - 1 \quad a_3 = (1-\alpha)\Delta t \quad a_4 = \alpha\Delta t
\]

Since the intensity of external load is increasing with respect to time, for a given number of iterations the program is going to update the external load corresponding to the current time interval.

The dynamic loading condition (from 0 to 8000 lbs in 3 milli-seconds) is modified within each time step, and after each iteration a subroutine will locate the failure boundary which will be used for the next iteration to carry the side forces. After the bit tooth penetrates into the rock, side forces at either side of the bit tooth have developed. A subroutine will search for the failure boundary, to locate the side forces at these nodes along the boundary. The side forces can be calculated from equation 1, and it is assumed that the side forces are evenly distributed along the failure boundary at either side of the bit tooth.

Figure 12 presents a typical computer display of axial load versus time for brittle impact craters (Long, 1988). A wedge in the brittle crater formed along AB, and a chip formed at B. Along BC the wedge collapsed and the tooth moved to the bottom of the crater. The process was repeated with different degrees of chipping at
Figure 12. Output Display of a Wedge Impact (After Long, 1988).
D and E. The axial load is a function of loading rates, rock properties, and tooth profiles.

The force-time curve of Long's experiment on Berea sandstone reveals a jump in applied force as shown in Figure 12. Each jump indicates the change of load condition. Using transient analysis, the change of load condition can be accomplished by assigning a sufficiently small time step for the increasing load magnitude. If adequate time intervals are chosen, then the falling branch of the force-time curve can be closely followed. In addition, the coordinates of the finite element mesh are adjusted according to the displacements after each iteration, to keep the bit aligned.

3.2 Classification of the Failure Element

The conditions of the failure element can be classified into three categories: tensile, compressive, and shear failure.

3.2.1 Tensile Failure

Tensile failure will occur when the minor principal stress exceeds the critical value in tension, and a fracture surface will form parallel to the major principal stress direction. The direction of the major principal stress ($\theta$) can be determined by

$$\theta = \alpha \quad \text{if } \sigma_x > \sigma_y,$$

$$\theta = \alpha + \pi/2 \quad \text{if } \sigma_x > \sigma_y \text{ and } \tau_{xy} > 0,$$

$$\theta = \alpha - \pi/2 \quad \text{if } \sigma_x < \sigma_y \text{ and } \tau_{xy} < 0,$$
\[ \tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}. \]

where
- \(\sigma_x\) is the stress along the horizontal direction
- \(\sigma_y\) is the stress along the vertical direction
- \(\tau_{xy}\) is the shear stress at the boundary of each element
- \(\theta\) is the angle counterclockwise to the \(\sigma_x\) axis.

The newly developed surface imposes an additional boundary to the system, resulting in a significant stress redistribution in the immediate vicinity, and also a change in the structure stiffness.

It is difficult to consider the crack surface as a free boundary and to redefine the finite element mesh, because the work involved is considerable and the cost is far from practical. By using anisotropic elements (Wang and Lehnhoff, 1976) to represent tensile failure elements with open cracks, this problem can be solved easily. The method assumes that the crack plane is a principal plane for the anisotropic element, and the closure of the crack is checked to determine the shear strength of an element for later use (i.e., if the crack is opened, an element loses its cohesion). The plane strain elasticity matrix for a symmetric anisotropic element can be shown as below

\[
D = \frac{E_2}{(1+\nu_1)(1-\nu_1-2\nu_2)} \begin{bmatrix}
    n(1-n\nu_2^2) & n\nu_2(1+\nu_1) & 0 \\
    n\nu_2(1+\nu_1) & (1-\nu_1^2) & 0 \\
    0 & 0 & m(1+\nu_1)(1-\nu_1-2\nu_2^2)
\end{bmatrix},
\]

where \(n = \frac{E_1}{E_2}\) and \(m = \frac{1}{2(1+\nu_2)}\).

The constants \(E_1\) and \(E_2\) are the Young's moduli with respect to the directions.
normal and parallel to the impact surface.

If the crack is closed, then the elasticity matrix for an anisotropic element is not used. Whether the crack is closed or not is determined by comparing the current volume of an element with its volume at previous iterations. This can be accomplished by a subroutine, VOLUME, which uses the Gauss quadrature rule to calculate the volume of an element. If the current volume of the element is smaller than or equal to that of previous iterations, then the crack is closed. Whether the crack is opened or closed, the element loses its cohesion for the following iterations. Moreover, some tensile failure elements may change their classification from tensile failure to compressive failure as penetration continues.

A transformation for the elasticity matrix is required when the direction of the fracture surface (normal to the minor principal stress) does not coincide with the global coordinates.

3.2.2 Compressive Failure

When the normal stress on the predicted failure plane exceeds the compressive strength of the rock, compressive failure occurs. Elements in this category are crushed into very small fragments. When compressive failure occurs within an element, the excessive stress that the element cannot bear should be transferred to the system, i.e., excessive stress should be released from an over-stressed element to neighboring elements. As a result, Young's moduli are assigned a small value.

3.2.3. Shear Failure

When the shear stress along the failure plane exceeds the shear strength on
that plane, an element is said to be under shear failure and consequently loses its strength to a certain extent. However, an element under shear failure is still capable of sustaining the stress normal to the failure plane. If the stress normal to the failure plane is greater than the compressive strength of an element, the element is classified as a crushed element. Park and Vojtech (1989) suggested that by using a strength reduction factor $R$, the compressive strength of an element after shear failure can be computed as:

$$R = \frac{\sigma_{\text{allowable}}}{\sigma_{\text{FEM}}}$$

where $R =$ strength reduction factor

$$\sigma_{\text{allowable}} = \sigma_3 + \sqrt[4]{m \sigma_3 + \sigma_3^2}$$

$\sigma_{\text{FEM}} =$ calculated major principal stress from FEM program

and consequently the Young's moduli to the X and Y direction can be modified as:

$$E_{\text{new}} = E_{\text{old}} \times R$$

where $E_{\text{new}} =$ new Young's moduli after reduction

$E_{\text{old}} =$ Young's moduli at current iteration

3.3 Factors Controlling the Deviation Force During Tool-Rock Interaction

A number of factors would affect the magnitude and direction of the deviation force during the tool-rock interaction process. These factors are the bit tooth angle, the formation dip, and the rock strength characteristics.
3.3.1. Bit Tooth Angle

The side force $F_R$ creates lateral pressure on the side element which results in chip formation. The side force can be calculated from equation 2. However, equation 2 can only give the side force when the half included wedge angles are equal at either side of the axis of the bit tooth. If the half wedge angles are different, the side forces at either side of the bit tooth are given by

$$\frac{F_{R_1}}{\sin(-\frac{\pi}{2} - \beta_2)} = \frac{F_{R_2}}{\sin(-\frac{\pi}{2} - \beta_1)} = \frac{F_v}{\sin(\beta_1 + \beta_2)} \tag{8}$$

$F_{R_1}$: the side force at the right hand side

$F_{R_2}$: the side force at the left hand side

$F_v$: the vertical load

$\beta_1$: the half bit tooth angle at right hand side

$\beta_2$: the half bit tooth angle at left hand side

and the diagram of forces distribution is shown in Figure 13.

If the included wedge half angle becomes large enough, the rock will fail by crushing rather than chipping, and the penetration rate will be slower than that of a sharp wedge; however, the bit wearing rate will also be slower than that of a sharp wedge.

3.3.2 Formation Dip

Rocks with various formation dips are used in the penetration simulations to represent different degrees of anisotropic effect. The bit tooth is considered free to move sideways, forming either a single chip on the weaker side or two equal chips
Figure 13. The Force Distribution of A Skewed Bit Tooth.
when $F_R$ (side force) is identical for both sides.

The compressive strength of rocks depends on the angle $\rho$ between the major principal compressive stress and the formation dip (Figure 6). The rock's maximum strength occurs with $0^\circ$ and $90^\circ$ dips and is the weakest in the range of $30^\circ$ to $75^\circ$ dips. Maximum chip volume occurs between $10^\circ$ and $30^\circ$, and gradually decreases at higher dip angles.

Eight formation dips ($0^\circ$, $10^\circ$, $20^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $75^\circ$, $90^\circ$) were tested by Long. He did triaxial tests at $0^\circ$, $30^\circ$, $45^\circ$, $60^\circ$, $70^\circ$, $80^\circ$, and $90^\circ$ formation dips to obtain the strength characteristics of Berea sandstone. Unfortunately, some angles (between the axial load and the formation dip) were not tested; therefore, simulations have been done only for the formation dips which have the material properties provided in Long's report, i.e., at $0^\circ$, $30^\circ$, $60^\circ$, and $90^\circ$. These are the formation dips which have the strength parameters provided for the angles between lateral forces and formation dips.

3.3.3 Rock Strength Characteristics

The composite model for tool-rock interaction suggests that the material property at either side of the bit tooth is the same when in the isotropic rock and is different when in the anisotropic rocks. The angle $\rho$ in Figure 6, between the side force $F_R$ and the bedding plane, determines the strength of rock at either side of the bit tooth. This angle is different for each side of the bit tooth when the bedding plane is other than $0^\circ$ and $90^\circ$. Chip formation will occur when the compressive strength at the side is less than the compressive strength at the side corresponding to the particular angle $\rho$. For example, where the angle $\rho$ is $90^\circ$ in the up-dip direction, and is $30^\circ$ in the down-dip direction of $60^\circ$ formation dip (Figure 14).
Figure 14. $\beta$ Angle for 60° Formation Dip
Also, the shear strength of an element along the failure plane is a function of the angle \( \rho \).

Because of the lack of information about Young's moduli of various formation dips in Long's report, the Young's moduli used in this study are from Swenson's (1983) tests on Berea sandstone in the SAND83-0278 report. The overall size and the imposed boundary conditions of the finite element mesh are comparable to the experimental test conducted by Long, which will be compared with the analytical results. Strength characteristics of Berea sandstone from laboratory tests are shown in Table I.
Table I. Hoek and Brown Predicted Strength Characteristics From Triaxial Tests:

(After Long, 1988)

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>$C_0$</th>
<th>$m$</th>
<th>$s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>11217</td>
<td>10.80</td>
<td>1.00</td>
</tr>
<tr>
<td>80°</td>
<td>9803</td>
<td>14.18</td>
<td>.7386</td>
</tr>
<tr>
<td>70°</td>
<td>10479</td>
<td>10.99</td>
<td>.8727</td>
</tr>
<tr>
<td>60°</td>
<td>10355</td>
<td>14.30</td>
<td>.8522</td>
</tr>
<tr>
<td>45°</td>
<td>8282</td>
<td>15.34</td>
<td>.5425</td>
</tr>
<tr>
<td>30°</td>
<td>8518</td>
<td>13.39</td>
<td>.5766</td>
</tr>
<tr>
<td>15°</td>
<td>8109</td>
<td>13.63</td>
<td>.5226</td>
</tr>
<tr>
<td>0°</td>
<td>8875</td>
<td>9.41</td>
<td>1.00</td>
</tr>
</tbody>
</table>

$\beta$: angle between the axial load and the formation dip,

$C_0$: uniaxial compressive strength (psi),

$m$: material constant;

$s$: material constant.
4. RESULTS AND DISCUSSION

The initial position of the bit is shown in Figure 15. All figures of stress contours due to tool-rock interaction were plotted according to the major and minor principal stresses. The corresponding progressive failure was also plotted. The complete set of figures for stress distribution and progressive failure can be found in Appendix C.

The rock begins to fail after a small elastic deformation at both sides of the bit tooth (Figure 16b), where high stress intensity exists at either side of the bit tooth. Major principal stresses in all elements that are compressive are directed toward the direction of bit penetration (Figure 16.1). The tensile crack under the compressive failure zone starts to propagate before the side elements have developed a high enough pressure to form a chip. When the penetration reaches a certain depth, chip formation starts. Further penetration will complete the chip.

A wedge-shaped bit creates lateral forces on the side elements, which results in high lateral stresses, causing them to be crushed before the elements directly beneath the bit. This process can be observed in the progressive failure simulation by the FEM program. Lateral forces are modified after the contact boundary is developed. Each formation of the rock is assumed to be of the same orthotropic material; however, the FEM program does not model individual laminations, but assignes different strength parameters on each side of the bit tooth, depending on the local tooth jointing configurations.

The stress distributions for 0° and 90° formation dips are all symmetric because the angle $\rho$ is the same between the lateral force and the formation dip at each side of the bit, as is the case in isotropic rock. From the development of
Figure 15. The Initial Position of the Bit
Figure 16b. Penetration Process of a Wedge Impact at 0° Formation Dip (4000 lbs)
progressive failure, it is noted that side elements are crushed before elements beneath the wedge tip because of high lateral pressures, and the tensile cracks run from the wedge tip deep into the rock.

As the penetration continues, the crushed zone expands laterally, and the stiffness of the crushed elements decreases simultaneously. If the penetration is further increased, the increasing pressure on the side element will reach the point where fractures start to propagate in these elements and finally form a chip. Chip formations are observed on each side of the bit, and equal chip volume at either side of the bit tooth is observed at 0° and 90° formation dips, as shown in Figure 16d and 17d.

Although the progressive failures observed in 0° and 90° formation dips were all symmetrical at either side of the bit tooth, the crushed zone is different because the angle $\rho$ for 0° and 90° formations is different. In the 0° formation dip, the crushed zone is developed more vertically in the direction of penetration while in the 90° formation dip, the crushed zone is expanded toward the edge of the finite element mesh. However, the depth of tensile cracks at the 0° and 90° formation dips is the same.

The penetration simulation of the stress field is plotted at axial forces level of 2000, 4000, 6000, 8000 lbs. Figure 16 and 17 show the stress distribution and progressive failure for 0° and 90° formation dips. The development of progressive failure is plotted up to maximum loading (8000 lbs), because after that the axial force soon decreases and no further development of the failure elements were found.

Figure 18 shows the stress distribution and penetration process for the 30° formation dip. From the stress distribution, it can be seen that high stress concentrations are present in the down-dip direction (Figure 18.3), because elements
Figure 16d. Penetration Process of a Wedge Impact at 0° Formation Dip (8000 lbs)
Figure 17d. Penetration Process of a Wedge Impact at 90° Formation Dip (8000 lbs)
Formation Dip 30°
Axial Force 4000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 18.3 Stress Contour of a Wedge Impact
in the down-dip direction are stronger than elements in the up-dip direction, according to the composite model for a 30° formation dip. This high stress concentration creates a deviation force that can deflect the bit tooth toward the up-dip direction. Therefore, more down-dip elements were crushed at the beginning of the penetration process and crushing in the down-dip direction consumed more energy than tensile failure in the up-dip direction. This makes chip formation to occur more easily in the up-dip direction. The final stage of deviation is shown in Figure 18d. It is obvious that the bit tooth has a tendency to move in the up-dip direction at 30° formation dip. As a result, soon after a crushed zone develops in the down-dip direction, the bit will deflect to the up-dip direction and finally will end up perpendicular to the bedding plane (up-dip direction), to accomplish the deviation process. Thus, a larger crushed zone was observed in the down-dip direction in the 30° formation dip (Figure 18d).

The stress distribution and penetration process at 60° formation dip are shown in Figure 19. At the beginning of the simulation, high stress concentrations occurred in the up-dip direction (Figure. 19.3). With successive penetration, the bit eventually moved down-dip. Also, from the penetration process at 60° formation dip, a larger crushed zone developed in the up-dip direction (Figure 19d), that caused higher stress concentration (deviation force) in the up-dip direction, pushing the bit tooth to the down-dip direction. This is in agreement with field observations; that is, when the dip is more than 45°, the bit will deviate parallel to the bedding plane (down-dip).

It can be noted that at the beginning stage, as the bit tooth penetrated into the rock, the tensile failure region for a 60° formation dip is more severe than for a 30° formation dip. This can be understood intuitively, because it is easier for the
Figure 18d. Penetration Process of a Wedge Impact at 30° Formation Dip (8000 lbs)
Formation Dip 60°
Axial Force 4000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 19.3 Stress Contour of a Wedge Impact
Figure 19d. Penetration Process of a Wedge Impact at 60° Formation Dip (8000 lbs)
tensile cracks to occur in steeply dipping formations. Also, a larger crushed zone in a 60° formation dip than in a 30° formation dip was found during the simulations. Since element failures in crushing consume more energy than element failures in tension, it is more efficient to drill a borehole in a 60° formation dip. Also, crushing would generate more dust than tensile failure and the amount of crushed element is a function of the angle ρ between the formation lamination and the lateral force of the bit tooth. The strength parameter of rocks will increase if the angle ρ is increased.

Comparisons were made of the penetrations (Figure 20) and lateral forces (Figure 21) between the laboratory tests and FEM output. The results are in a reasonable agreement, especially when proper element sizes were chosen (see Table II & Table III).

These observations can be directly applied to rock cutting tools in mining applications (e.g., continuous miners), to minimize energy consumption and dust generation when cutting through a laminated formation (e.g., coal). The attack angle of the cutting bit or pick can be adjusted so that the angle ρ, between the formation lamination and the lateral force, in the direction of deviation, can be minimized.

During the simulation, some tensile failures were found at the outward edge of the finite element mesh, which might not have occurred if a semi-infinite rock surface was used. Although the FEM is written based on the composite model for tool-rock interaction, it is unrealistic to assume that laminations be present in the vicinity of the impact; consequently individual laminations are not modeled in the simulation. Instead, based on the angle ρ, different failure parameters are assigned for the up-dip and down-dip side of the bit tooth. In addition, the FEM program simulates the penetration process at 0° and 90° formation dips, as is the case for
Figure 20. Penetrations from the Laboratory Tests and FEM Output
Figure 21. Lateral Forces from the Laboratory Tests and FEM Output
Table II. Penetrations from Laboratory Tests and FEM Output:

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>0.123</td>
<td>0.176</td>
<td>0.164</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.117</td>
<td>0.157</td>
<td>0.155</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>0.127</td>
<td>0.093</td>
<td>-</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>0.103</td>
<td>0.109</td>
<td>0.091</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>0.145</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>0.142</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>0.157</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>0.107</td>
</tr>
</tbody>
</table>

$\gamma$: Dip Angle
Table III. Lateral Forces from Laboratory Tests and FEM Output:

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
</tr>
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<tbody>
<tr>
<td>0$^\circ$</td>
<td>5127</td>
<td>6341</td>
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<td>30$^\circ$</td>
<td>4667</td>
<td>9328</td>
<td>4864</td>
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<tr>
<td>60$^\circ$</td>
<td>6210</td>
<td>2711</td>
<td>-</td>
</tr>
<tr>
<td>90$^\circ$</td>
<td>1779</td>
<td>2741</td>
<td>-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\gamma$</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^\circ$</td>
<td>7951</td>
</tr>
<tr>
<td>30$^\circ$</td>
<td>8370</td>
</tr>
<tr>
<td>60$^\circ$</td>
<td>7542</td>
</tr>
<tr>
<td>90$^\circ$</td>
<td>8132</td>
</tr>
</tbody>
</table>

$\gamma$: Dip Angle
isotropic and homogeneous materials, because the angle \( \rho \) is the same at either side of the bit tooth.

A stress criterion was used for this study, which makes the solutions dependent on mesh size, because of the stress singularity at a crack tip, where the finite element program predicts an infinite stress for a sharp crack. After the mesh size is reduced, the finite element program captures the stress singularity. As a result, the cost to run the program is higher. Thus, the proper size of the mesh depends on engineering judgment.

Although the numerical model shows good qualitative agreement with the experimental observations, the input data are not quite appropriate for the dynamic impact problem. If dynamic strength properties can be used, (e.g., dynamic Young's moduli, shear strength, etc.), the actual tool-rock interaction can be simulated better. Since the penetration process of tool-rock interaction is dynamic, static strength values would overestimate the extent of failure at the area beneath the wedge tip, because rocks' strength and mechanical properties are greater (work hardening) under dynamic loading.

Previous investigators failed to simulate the dynamic and progressive failure of the bit-rock interaction. In static penetration for instantaneous loading to 8000 lbs, where progressive failure is not considered, simulation indicates that the crushed zone is restricted to an area directly below the wedge tip (Figure 22). Furthermore, the effects of lamination on fragmentation are not well represented. Modeling of dynamic and progressive failure in the FEM gives a better representation of the fragmentation process and the results are in close agreement with the experimental dynamic. For the static penetration simulation, chip formation at either side of the bit tooth was absent and the crushed zone was
Figure 22. Penetration Process of Static and Dynamic Loading of a Wedge Impact at 30° Formation Dip (8000 lbs)
restricted in the direction directly beneath the wedge tip.

Uneven half wedge angles were tried for 30° and 60° formation dip cases to determine whether deviation due to unbalance lateral forces can be minimized by varying the ρ angle and the lateral forces. The half wedge angle in the down-dip direction was set to 60° in the 30° formation dip case. By doing so, the deviation tendency has changed from the up-dip to the down-dip direction (Figure 23). Also, there are more crushed elements in the up-dip direction during the penetration process. For the 60° formation dip case, the deviation is balanced by increasing the half wedge angle in the up-dip direction to 60° (Figure 24).

Therefore, the deviation due to unbalance lateral forces can be eliminated simply by modifying the half wedge angle at either side of the bit tooth. When the stresses at either side of the bit tooth have reached the equilibrium condition during the penetration process, deviation can be minimized. The bit attack angles may also be modified to obtain the same effect.

Despite the limitations mentioned above, the model is capable of predicting the penetration process of a bit tooth in laminated rocks, and simulating dynamic loading. The results have shown a reasonable agreement with experimental observations.
Figure 23a. Penetration Processes of a Wedge Impact for Uneven and Even Half Wedge Angles at 30° Formation Dip (8000 lbs)
Formation Dip 30°
Axial Force 8000 lbs
Dipping from Right to Left
Major Principal Stress
β for Down-Dip 60°

Figure 23.1 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 8000 lbs
Dipping from Right to Left
Minor Principal Stress
β for Down-Dip 60°

Figure 23.2 Stress Contour of a Wedge Impact
Figure 24a. Penetration Processes of a Wedge Impact of Uneven and Even Half Wedge Angles at 60° Formation Dip (8000 lbs)
Formation Dip 60°
Axial Force 8000 lbs
Dipping from Right to Left
Major Principal Stress
\( \beta \) for Up-Dip 60°

Figure 24.1 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 8000 lbs
Dipping from Right to Left
Minor Principal Stress
β for Up-Dip 60°

Figure 24.2 Stress Contour of a Wedge Impact
5. CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusions

A finite element model to simulate tool-rock interaction in laminated formations has been developed. The model uses anisotropic elements to simulate dynamic loading and progressive failure. Furthermore, the tool geometry can be modified. The program provides quantitative information on stresses, displacements and rock failure during the tool penetration process.

From this research, the following conclusions can be made:

1. Progressive failure can be described by the FEM program, and deviation due to laminated formations can be inferred. The model shows good qualitative (i.e., penetration depth and lateral forces) agreement with laboratory wedge indentation tests.

2. The numerical model shows that at 30° formation dip the bit tooth is deflected in the up-dip direction and at 60° formation dip the bit tooth is deflected in the down-dip direction, as observed in wedge indentation tests.

3. Chip formation is present on both sides of the bit tooth for all simulated dip angles as predicted by the composite model, and observed in wedge indentation tests.
5.2 Recommendations

1. Further studies are needed to determine how to apply the information from the stress analysis to tool design, so that the anisotropic effect can be minimized.

2. Dynamic properties must be used in the FEM program to simulate the dynamic bit tooth penetration process.

3. More dip angles should be tested to determine rocks' dynamic strength properties and mechanical behavior so that the information for tool-rock interaction can be well established.

4. Dynamic wedge indentation tests should be conducted on rocks with more pronounced anisotropic properties (e.g., shales) to verify the numerical model.

5. Different failure criteria (e.g., fracture mechanics criterion) should be used for the mathematical model in simulating the penetration process, to see whether better results can be obtained during this process.

6. The numerical model can be improved by including angular force to the bit tooth to simulate the rotation of the bit tooth.
REFERENCES


APPENDIX A.

PROGRAM INPUT INSTRUCTIONS
In this section, the information about the input data is given. This computer program is capable of analyzing stress, displacement, and the penetration process of rock with a plane strain assumption. Incremental load and time are used to simulate dynamic loading conditions during actual drilling.

The following is a description of the input data.

1. Control Data Card (515)

<table>
<thead>
<tr>
<th>Cols.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>Element type</td>
</tr>
<tr>
<td>6-10</td>
<td>Number of nodal points per element</td>
</tr>
<tr>
<td>11-15</td>
<td>Indicator for mesh generation, 0 input by user,</td>
</tr>
<tr>
<td></td>
<td>1 Computer generates the mesh</td>
</tr>
<tr>
<td>16-20</td>
<td>Indicator for transient analysis, 1 transient analysis,</td>
</tr>
<tr>
<td></td>
<td>0 static analysis</td>
</tr>
<tr>
<td>21-25</td>
<td>Indicator for print, number of times to print the results</td>
</tr>
</tbody>
</table>

2. Strength Characteristics Data (8F10.4)

<table>
<thead>
<tr>
<th>Cols.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>Material constant, m, of the down-dip elements</td>
</tr>
<tr>
<td>11-20</td>
<td>Material constant, s, of the down-dip elements</td>
</tr>
<tr>
<td>21-30</td>
<td>Compressive strength of the down-dip elements</td>
</tr>
<tr>
<td>31-40</td>
<td>Failure Angle of the down-dip elements</td>
</tr>
<tr>
<td>1-10</td>
<td>Material constant, m, of the up-dip elements</td>
</tr>
<tr>
<td>11-20</td>
<td>Material constant, s, of the up-dip elements</td>
</tr>
<tr>
<td>21-30</td>
<td>Compressive strength of the up-dip elements</td>
</tr>
<tr>
<td>31-40</td>
<td>Failure Angle of the up-dip elements</td>
</tr>
</tbody>
</table>

3. Material Properties Data (8F10.4)

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<tr>
<th>Cols.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10</td>
<td>Young's Modulus (horizontal)</td>
</tr>
</tbody>
</table>
11-20 Young's Modulus (vertical)
21-30 Poisson's ratio
31-40 Density of the rock
41-50 Thickness of the rock
51-60 Half wedge angle in the down-dip direction
61-70 Half wedge angle in the up-dip direction
1-10 Formation Dip

4. Finite Element Mesh Data (12I5,12I5,8F10.4,8F10.4)
1-5 Number of subdivisions in the horizontal direction
6-10 Number of subdivisions in the vertical direction
1-80 Distance between nodes along the horizontal direction
1-80 Distance between nodes along the vertical direction

If User Input Data (I4,I4,16I4,8F10.4,8F10.4)

1-5 Number of elements in the mesh
6-10 Number of nodes in the mesh
1-45 Connectivity matrix of the elements
1-80 X coordinates
1-80 Y coordinates

5. Boundary Condition Data (16I5,10F10.4,10F10.4)
1-5 Number of specified displacements along the boundary
1-80 Positions of specified displacements
1-80 Values of the specified displacements
1-5 Number of specified external loads
1-80 Position of specified external loads
1-80 Values of specified external loads

6. Transient Analysis Data (3F10.4)

1-10 Time step for transient analysis
11-20 Parameter in time approximation
21-30 Final time to terminate the program
APPENDIX B.

LISTING OF THE FINITE ELEMENT PROGRAM
PROGRAM FEM

DESCRIPTION OF THE VARIABLES

******************************************************************************

ALFA  ANGLE OF THE MAJOR PRINCIPAL STRESS
       (COUNTERCLOCKWISE FORM THE X AXIS)
ANU12  POISSON'S RATIO FOR ELASTICITY PROBLEMS
ANU21  POISSON'S RATIO FOR ORTHOTROPIC MEDIUM
SM(I,J) MASS MATRIX FOR TRANSIENT ANALYSIS
C1,C2  YOUNG'S MODULI IN THE X AND Y AXES
C3  POISSON'S ROTIO (ANU12)
C4  THICKNESS OF THE ROCK
C5  DENSITY OF THE ROCK
C6  HALF WEDGE ANGLE TO THE RIGHT HAND SIDE
C7  HALF WEDGE ANGLE TO THE LEFT HAND SIDE
C8  RESIDUAL YOUNG'S MODULUS
DT  TIME STEP FOR TRANSIENT ANALYSIS
ELSTIF ELEMENT COEFFICIENT MATRIX
ELXY GLOBAL COORDINATES OF ELEMENT NODES
F(I) ELEMENT FORCE VECTOR
DP(I) SOLUTION VECTOR AT CURRENT TIME
DV(I) FIRST TIME DERIVATIVE OF THE SOLUTION
DA(I) SECOND TIME DERIVATIVE OF THE SOLUTION
FV  AXIAL FORCE
IEL  INDICATOR FOR THE TYPE OF ELEMENTS:
     IEL = 1, FORU-NODE QUADRILATERAL ELEMENT
     IEL = 2, EIGHT/NINE-NODE QUADRILATERAL ELEMENT
IFAIL(I) INDICATOR FOR THE CRUSHED ELEMENTS
ITOF(I) ARRAY CORRESPONDING TO IFAIL
IMESH INDICATOR FOR MESH GENERATION:
     IMESH = 0, USER WILL INPUT THE MESH INFORMATION
     IMESH = 1, PROGRAM WILL GENERATE THE MESH
IS(I)  INDICATOR FOR SHEAR FAILURE ELEMENTS
IT(I)  INDICATOR FOR TENSILE FAILURE ELEMENTS
ITEM  INDICATOR FOR TRANSIENT ANALYSIS
AF  ANGLE OF FAILURE
NCMAX COLUMN DIMENSION OF GSTIF IN THE
     DIMENSION STATEMENT
NDF  NUMBER OF DEGREES OF FREEDOM PER NODE
NEQ  TOTAL NUMBER OF EQUATIONS IN THE PROBLEM
NEM  NUMBER OF ELEMENTS IN THE MESH
NHBW  HALF BAND WIDTH OF THE COEFFICIENT MATRIX, GSTIF
NN  TOTAL DEGREES OF FREEDOM PER ELEMENT
NNM  NUMBER OF NODES INT FINITE ELEMENT MESH
NOD(I,J) GLOBAL NODE NUMBER CORRESPONDING TO THE
       J-th NODE OF ELEMENT I (CONNECTIVITY MATRIX)
NPE  NUMBER OF NODES PER ELEMENT
MRMAX ROW DIMENSION OF GSTIF IN THE DIMENSION
     STATEMENT
NSBF NUMBER OF SPECIFIED NONZERO BOUNDARY FORCE
NSDF NUMBER OF SPECIFIED PRIMARY DEGREES OF FREEDOM
NTIME NUMBER OF ITERATIONS IN DYNAMIC LOADING CONDITION
NTER INDICATOR FOR PRINT
NX, NY NUMBER OF ELEMENTS IN X AND Y DIRECTIONS, RESPECTIVELY
SIG1 MAJOR PRINCIPAL STRESS
SIG2 MINOR PRINCIPAL STRESS
THETA PARAMETER IN TIME APPROXIMATION:
  THETA = 0, FORWARD-DIFFERENCE
  THETA = 0.5, THE CRANK-NICOLSON SCHEME
  THETA = 2/3, THE GALERKIN SCHEME
  THETA = 1, BACKWARD-DIFFERENCE SCHEME
A0, A1, A2, A3, A4 PARAMETER IN TIME APPROXIMATION T0 MAXIMUM LIMIT ON TIME IN TRANSIENT ANALYSIS
VBDF(I) ARRAY OF THE VALUES CORRESPONDING TO THE SPECIFIED DEGREES OF FREEDOM IN ARRAY IBDF(I)
VBSF(I) ARRAY OF THE VALUES CORRESPONDING TO THE SPECIFIED NONZERO FORCES IN ARRAY IBSF(I)
VOLP(I) ELEMENT VOLUME AT PREVIOUS ITERATION
VOL(I) ELEMENT VOLUME AT CURRENT ITERATION
W0, W1, W2 ARRAYS CORRESPONDING TO DP, DV, DA
X(I) X COORDINATE OF THE GLOBAL NODE I
XM(I,J) X COORDINATE AT CENTER OF AN ELEMENT
Y(I) Y COORDINATE OF THE GLOBAL NODE I
YM(I,J) Y COORDINATE AT CENTER OF AN ELEMENT

***********************************************************************

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION GSTIF(1000,70),GF(1000),DP(1000),DV(1000),DA(1000),
*W(2,9),IBDF(130),VBDF(130),IBSF(100),VBSF(100),VOLP(300),P(2,4)
*,VOL(300),XM(30,20),YM(30,20),IFAIL(300),ITOF(300),IT(300),IS(300)
*,E1(300),E2(300),CO(300)
COMMON/MSH/NOD(500,9),X(500),Y(500),DX(30),DY(30)
COMMON/STF/ELSTIF(18,18),ELXY(9,2),F(18),W0(18),W1(18)
*,W2(18),A0,A1,A2,A3,A4
DATA NRMAX,NCMAX/1000,70/
READ *,IEL,NPE,IMESH,ITEM,NTIME,NTER
DO 5 I = 1,2
5 READ *,(P(I,J),J=1,4)
IF(IMESH .EQ. 1)GOTO 30
READ *,NEM,NNM
DO 20 N = 1,NEM
20 READ *,(NOD(N,D),D=1,NPE)
READ *,(X(I),Y(I),I=1,NNM)
GOTO 40
30 READ *,NX,NY
NXX1 = IEL*NX+1
NYY1 = IEL*NY+1
IF(IEL .EQ. 0)NXX1=NX+1
IF(IYL.EQ.0)NYY1=NY+1
READ *,(DX(I),I=1,NXX)
READ *(DY(I),I=1,NYY1)
CALL MESH(IYL,NX,NY,NPE,NNM,NEM)
40 READ *,C1,C2,C3,C4,C5,C6,C7,C8
   E1 = C1
   E2 = C2
50 READ *,NSDF
   IF(NSDF,EQ.0)GOTO 60
   READ *(IBDF(I),I=1,NSDF)
   READ *(VBDF(I),I=1,NSDF)
60 READ *,NSBF
   IF(NSBF,EQ.0)GOTO 70
   READ *(IBSF(I),I=1,NSBF)
   READ *(VBSF(I),I=1,NSBF)
70 IF (ITEM.EQ.1)READ *,DT,THETA,T0
C
END OF DATA INPUT
C
FV = -1000.0
IF (ITEM.EQ.1)THEN
   DO 71 I=1,NEQ
   DP(I)=0.0
   DV(I)=0.0
71 DA(I)=0.0
ENDIF
DO 72 I = 1,NEM
   IS(I) = 0
   IT(I) = 0
   E1(I) = C1
   E2(I) = C2
72 ITOF(I) = 0
NDF=2
NEQ=NNM*NDF
NN=NPE*NDF
PRINT 660, IYL
PRINT 530,C1,C2,C3,C4,C5
PRINT 450,NEM,NNM,NEQ
TIME = 0.0
ITER = 0
IDO = 0
C
TRANSIENT ANALYSIS BEGINS HERE
C
90 IF(TIME.GT.T0)STOP
   IDO = IDO + 1
   ITER = ITER + 1
   TIME = TIME+DT
   PRINT 430, TIME
   IF(ITEM.EQ.NTER OR ITEM.EQ.0)THEN
      IF (NSDF.GT.0)PRINT 460, NSDF,(IBDF(I),I=1,NSDF)
      IF (ITEM.EQ.0)PRINT 450, NEQ, NN, NNM, NEQ
      IF (ITEM.EQ.1)PRINT 440, NSDF, IBDF(I), VBDF(I)
      IF (ITEM.EQ.2)PRINT 440, NSBF, IBSF(I), VBSF(I)
   ENDIF
   TIME = TIME + DT
   PRINT 430, TIME
   IDO = IDO + 1
   ITER = ITER + 1
90 CONTINUE
STOP
IF (NSBF.NE.0) THEN
PRINT 640, (IBSF(I), I=1,NSBF)
PRINT 600
PRINT 480, (VBSF(I), I=1,NSBF)
ENDIF
ENDIF
NHBW=0

C COMPUTE THE HALF BAND WIDTH

C DO 110 N = 1,NEM
DO 110 I = 1,NPE
DO 110 J = 1,NPE
NW = (IABS(NOD(N,J)-NOD(N,I))+1)*NDF
110 IF (NHBW.LT. NW) NHBW=NW
IF(ITER.EQ. NTER) PRINT 590, NHBW
IF(ITEM.NE. 1) GOTO 125
BETA = 0.25*(0.5+THETA)**2
A0 = 1.0/(BETA*DT**2)
A1 = 1.0/(BETA*DT)
A2 = 0.5/BETA - 1.0
A3 = (1-THETA)*DT
A4 = DT*THETA
125 DO 130 I = 1,NEQ
GF(I) = 0.0
DO 130 J = 1,NHBW
130 GSTIF(I,J) = 0.0
IF(ITEM.EQ. 0) TIME = 0.0

C DO LOOP IN THE NUMBER OF ELEMENTS TO CALCULATE
C THE ELEMENT-MATRIXES, AND ASSEMBLY OF THE
C ELEMENT MATRICES

C DO 300 N = 1,NEM
L = 0
DO 150 I = 1,NPE
NI = NOD(N,I)
ELXY(I,1) = X(NI)
ELXY(I,2) = Y(NI)
LI = (NI-1)*NDF
DO 150 J = 1,NDF
LI = LI+1
L = L+1
W0(L) = DP(LI)
W1(L) = DV(LI)
W2(L) = DA(LI)
150 CONTINUE
CALL STIFF(NPE,NN,IEL,ITEM,C1,C2,C3,C4,C5,C6,E1,E2,CONT,ITOF,IT)
IF(TIME.LE. DT)VOLP(N) = CONT

C ASSEMBLE ELEMENT MATRIX TO OBTAIN GLOBAL MATRIX
C
DO 280 I = 1,NPE
   NR = (NOD(N,I)-1)*NDF
   DO 280 II = 1,NDF
   NR = NR+1
   L = (I-1)*NDF+II
   GF(NR) = GF(NR)+F(L)
   DO 260 J = 1,NPE
   NCL = (NOD(N,J)-1)*NDF
   DO 260 JJ = 1,NDF
   M = (J-1)*NDF+JJ
   NC = NCL+JJ+1-NR
   IF (NC) 260,260,250
250 GSTIF(NR,NC) = GSTIF(NR,NC)+ELSTIF(L,M)
260 CONTINUE
280 CONTINUE
300 CONTINUE
C
IMPLEMENTATION OF THE BOUNDARY CONDITIONS ON
PRIMARY AND SECONDARY VARIABLES
C
IRES = 0
IF(NSBF.EQ.0) GOTO 340
DO 310 I = 1,NSBF
II = IBSF(I)
310 GF(II) = VBSF(I)+GF(II)
340 IF (NSDF .EQ. 0) GOTO 350
   CALL BNDY(NRMAX,NCMAX,NEQ,NHBW,GSTIF,GF,NSDF,IBDF,VBDF)
350 CALL SOLVE(NRMAX,NCMAX,NEQ,NHBW,GSTIF,GF,IRES)
C
CALCULATE THE CENTER POINT OF ELEMENTS
C
IF(IDO .EQ. NTIME)THEN
   DO 351 I = 1,NNM
      X(I) = X(I)+GF(I+I-1)
351 Y(I) = Y(I)+GF(I+I)
   DO 372 J=1,MY+1
   DO 372 I=1,NX
      XM(J,I)=0.0
372 YM(J,I)=0.0
      K = 0
   DO 373 J=1,MY+1
   DO 373 I=1,NX
      K=K+1
      IF(K .EQ. (NX+1)*(J-1))K=K+1
      XM(J,I)=(X(K)+X(K+1))*0.5
373 YM(J,I)=(Y(K)+Y(K+1))*0.5
   DO 374 J=1,MY
   DO 374 I=1,NX
      XM(J,I)=(XM(J,I)+XM(J+1,I))*0.5
374 YM(J,I)=(YM(J,I)+YM(J+1,I))*0.5
ENDIF
IF(ITEM.EQ.1)THEN
DO 366 I=1,NEQ
DP(I) = A0*(GF(I)-DP(I))-A1*DV(I)-A2*DA(I)
DV(I) = DV(I)+A3*DA(I)+A4*DP(I)
DA(I) = DP(I)
366 DP(I) = GF(I)
ENDIF
PI = 3.1415929
NL = 0
DO 390 N = 1,NEM
VOL(N) = 0.0
DO 380 I = 1,NPE
NI = NOD(N,I)
L = NI*NDF-1
ELXY(I,1) = X(NI)
ELXY(I,2) = Y(NI)
W(I,1) = GF(L)
380 W(2,I) = GF(L+1)
CALL VOLUME(N,NPE,AREA)
VOL(N) = AREA
DF=VOL(N)-VOLP(N)
NL = NL+1
IF(NL .LE. NX/2)THEN
HM=P(1,1)
HS=P(1,2)
IF(TIME .EQ. DT)CO(N) = P(1,3)
AF = P(1,4)
ELSEIF(NL .GT. NX/2)THEN
HM=P(2,1)
HS=P(2,2)
IF(TIME .EQ. DT)CO(N) = P(2,3)
AF = P(2,4)
IF(NL .EQ. NX)NL = 0
ENDIF
390 CALL STRESS(N,NPE,ELXY,C1,C2,C3,C4,W,ITER,DF,IFAIL,
*ITOF,HM,HS,CO,NTER,TIME,DT,IT,AF,NEM,C8,E1,E2)
* IF(TIME .GE. 2.5)DT = 0.421875
IF(TIME .LT. 2.5 .AND. IDO .EQ. NTIME)FV = FV - 1000.0
IF(TIME .GE. 2.5 .AND. IDO .EQ. NTIME)FV = FV + 1000.0
IF(IDO .EQ. NTIME)IDO = 0
IF(ITER .EQ. NTER)THEN
PRINT 407,(GF(I+I-1),GF(I+I),I=1,NNM)
DO 400 J = 1,NY
400 WRITE(20,423)(XM(J,I),YM(J,I),I=1,NX)
ITER = 0
ENDIF
CALL FORCE(FV,NNM,NX,NY,NSBF,IBSF,VBSF,C6,C7,ITOF)
IF (ITEM.EQ.1)GOTO 90
406 STOP
407 FORMAT (4(2X,E15.6))
420 FORMAT (10X,'THETA=',F10.3,2X,'TIME STEP=',F10.3,2X,'MAX. TIME=',
  * E10.3)
421 FORMAT (11X,'SIG1',10X,'SIG3',9X,'TENSILE AGL')
422 FORMAT (8(F6.4,2X))
423 FORMAT (2(F9.4,2X))
430 FORMAT (/,3X,'TIME =',F10.3)
440 FORMAT (10X,2015)
450 FORMAT (10X,'ACTUAL NUMBER OF ELEMENTS IN THE MESH...=',13/,10X,
  * NUMBER OF EQUATIONS IN THE MODEL...=',13/,10X,'TOTAL
* NUMBER OF EQUATIONS IN THE MODEL...=',13/)
460 FORMAT (/,'ARRAY OF THE SPECIFIED DEGREES OF FREEDOM=',15/,
  *10X,1215/,10X,1215/,10X,1215/,10X,1215/,10X,1215/,10X,1215/,
  *10X,1215/,10X,1215)
480 FORMAT (4(2X,E14.6))
485 FORMAT (I4,F10.7,2X,F10.7)
510 FORMAT (10X,'PARAMETERS, C1,C2,C3,C4, AND C5=',15X,'C1=',
  *E10.3/,15X,'C2=',E10.3/,15X,'C3=',E10.3/,15X,'C4=',E10.3/,
  *15X,'C5=',E10.3)
520 FORMAT (1615)
530 FORMAT (10X,'MODULUS OF ELASTICITY, E1',15X='E10.3/',
  *10X,'MODULUS OF ELASTICITY, E2',15X='E10.3/,10X,
  *POISSONS RATIO, ANU12',15X='E10.3/,10X,
  *ROCK THICKNESS',15X='E10.3/,10X,
  *ROCK DENSITY',15X='E10.3)
540 FORMAT (/,3X,'SOLUTION VECTOR=',3X,'TIME =',F10.3)
550 FORMAT (5X,'A PLANE ELASTICITY PROBLEM')
580 FORMAT (8F15.7)
590 FORMAT (5X,'HALF BAND WIDTH OF GLOBAL STIFFNESS MATRIX
  =',13)
600 FORMAT (/,5X,'VALUES OF THE SPECIFIED FORCES:')
640 FORMAT (/,5X,'SPECIFIED FORCE DEGREE OF FREEDOM :',10I5)
660 FORMAT (10X,'ELEMENT TYPE',15X='I2)
700 FORMAT(3X,'GLOBAL STIFFNESS MATRIX:')
710 FORMAT(3X,'GLOBAL FORCE VECTOR:')
END
SUBROUTINE FORCE(FV,NNM,NX,NY,NSBF,IBSF,VBSF,BETAL,BETAR
  *,ITOF)
C C THE SUBROUTINE LOCATES THE FAILURE BOUNDARY DUE
C TO TOOL-ROCK INTERACTION AND RELOCATES THE
C EXTERNAL FORCES AT NODES ALONG THE BOUNDARY
C IMPLICIT REAL*8(A-H,O-Z)
DIMENSION IRSUR(500),IRSLIL(500),ITOF(300),IBSF(100)
+,VBSF(100),IANSR(300),IANSIL(300),IPV(300)
COMMON/MESH/NOD(500),X(500),Y(500),DX(30),DY(30)
IP = NX*NY-NX/2
IF(ITOF(IP) .EQ. 0 .AND. ITOF(IP+1) .EQ. 0)GOTO 130
DO 1 I = 1,NNM
  IRSUR(I) = 0
1  IRSL(I) = 0
   JP = NY
   IDOTL = 0
   DO 5 I = 1,NX/2+1
       J = 0
10  IF(ITOF(J*NX+I) .EQ. 1)GOTO 20
    J = J + 1
    IF(J .EQ. NY-1 .AND. ITOF(J*NX+I) .EQ. 0)GOTO 5
    GOTO 10
20  DO 25 II = NY-1,J,-1
    IF(ITOF(II*NX+I) .EQ. 1)THEN
        LEV = II+1
    GOTO 26
ENDIF
25  CONTINUE
26  IF(I .EQ. NX/2+1 .AND. JP .GE. J)THEN
    NCOM = NOD(NX*(JP-1)+I-1,3)
    IRSUL(NCOM) = 1
    KL = 1
    GOTO 5
    ELSEIF(I.EQ.NX/2+1 .AND. JP .LT. J)THEN
    KL = 0
    GOTO 5
    ENDIF
IF(JP .GT. J)THEN
    L = (J-1)*NX+I
    MS = J
    ELSE
    L = (JP-1)*NX+I
    MS = JP
ENDIF
   DO 30 M = MS,LEV
       IF(M+1 .EQ. LEV)GOTO 30
       IF(M.EQ.MS)GOTO 29
       IF(ITOF((M-1)*NX+I-1).EQ.1 .AND. ITOF((M-1)*NX+I) .EQ. 1
         * .AND. ITOF(M*NX+I-1).EQ.1 .AND. ITOF((M-1)*NX+I-1).EQ.1
         * .AND. ITOF(M*NX+I-1).EQ.1)GOTO 30
       IF(M.EQ.NY .AND. ITOF((M-1)*NX+I-1).EQ.1)GOTO 30
   29  1RSUL(NOD(L,4)) = 1
30    L = L+NX
    JP = J
5  CONTINUE
    JP = NY
   IDOTR = 0
   DO 45 I = NX,NX/2,-1
       J = 0
40  IF(ITOF(J*NX+I) .EQ. 1)GOTO 50
    J = J + 1
    IF(J .EQ. NY-1 .AND. ITOF(J*NX+I) .EQ. 0)GOTO 45
    GOTO 60
45  DO 55 II = NY-1,J,-1
    IF(ITOF(II*NX+I).EQ. 1)THEN
LEV = II+1
GOTO 56
ENDIF
55 CONTINUE
56 IF(I .EQ. NX/2 .AND. JP .GE. J) THEN
   NCOM = NOD(NX*(JP-1)+I+1,4)
   IRSUR(NCOM) = 1
   KR = 1
   GOTO 45
   ELSEIF(I.EQ.NX/2 .AND. JP .LT.J) THEN
   KR = 0
   GOTO 45
   ENDIF
   IF(JP .GT. J) THEN
   L = (J-1)*NX+I
   MS = J
   ELSE
   L = (JP-1)*NX+I
   MS = JP
   ENDIF
   DO 70 M = MS,LEV
   IF(MS+1 .EQ. LEV) GOTO 70
   IF(M .EQ. MS) GOTO 69
   IF(ITORF((M-1)*NX+I+1) .EQ. 1 .AND. ITOF((M-1)*NX+I) .EQ. 1 .AND. *
      ITOF(M*NX+I+1) .EQ. 1) GOTO 70
   IF(M .EQ. NY .AND. ITOF((M-1)*NX+I+1) .EQ. 1) GOTO 70
   IRSUR(NOD(L,3)) = 1
   70 L = L + NX
   JP = J
45 CONTINUE
   DO 85 I = 1,NNM
   IANSL(I) = 0
   85 IANSR(I) = 0
   DO 17 LM = 0,NNM,NX+1
   ML = LM
   DO 17 IJ = 1,NNM
   IF(ML .EQ. 0) ML = 1
   IF (IJ .EQ. ML) THEN
   IPV(IJ) = ML
   IF(ML .NE. 1 .AND. ML .NE. NNM) THEN
   ML1 = ML+1
   IPV(ML1) = ML1
   ENDIF
   ENDIF
17 CONTINUE
   DO 90 I = 1,NNM
   IF(IRSLU(I) .EQ. 1) THEN
   IF(I .EQ. IPV(I)) GOTO 90
   IANSL(I) = 1
   IDOTL = IDOTL + 1
   ENDIF
90 CONTINUE
90 CONTINUE
  IF(KL.EQ.0) IDOTL = IDOTL + 1
  DO 100 I = 1,NNM
  IF(IRSR(I).EQ.1) THEN
    IF(1.EQ.IPV(I))GOTO 100
    IANSR(I) = 1
    IDOTR = IDOTR + 1
  ENDIF
  100 CONTINUE
  IF(KR.EQ.0) IDOTR = IDOTR + 1
  FRL = SIN(1.570796-BETAR)*FV/SIN(BETAL+BETAR)
  FRR = -SIN(1.570796-BETAL)*FV/SIN(BETAL+BETAR)
  K = 0
  DO 140 I = 1,NNM
  IF(IANSI(I).EQ.I.OR.IANSR(I).EQ.I) THEN
    K = K+1
    IBF(K*2-1) = I*2-1
    IBFI(K*2) = I*2
  ENDIF
  IF(I.EQ.IANSR(I)) THEN
    IF(I.EQ.NCOM)GOTO 2300
    VB(K*2-1) = FRL/IDOTL*COS(BETAL)
    VB(K*2) = FRL/IDOTL*SIN(BETAL)
  GOTO 2400
  2300 VB(K*2-1) = FRL/IDOTL*COS(BETAL) + FRR/IDOTR*COS(BETAR)
    VB(K*2) = FRL/IDOTL*SIN(BETAL) - FRR/IDOTR*SIN(BETAR)
  2400 ENDIF
  IF(I.EQ.IANSR(I)) THEN
    IF(I.EQ.NCOM)GOTO 140
    VB(K*2-1) = FRR/IDOTR*COS(BETAR)
    VB(K*2) = -FRR/IDOTR*SIN(BETAR)
  ENDIF
  ENDIF
140 CONTINUE
  NSBF = K*2
130 RETURN
END

SUBROUTINE VOLUME(N,NPE,AREA)

THE SUBROUTINE CALCULATES THE VOLUME OF
ELEMENTS AT CURRENT ITERATION

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ELXY(9,2),GAUSS(4,4),WT(4,4),SF(9)
* GDSF(9)
COMMON/MSH/NOD(500,9),X(500),Y(500),DX(30),DY(30)
DATA GAUSS/4*0.0D0,-.7735027D0,.7735027D0,2*0.0D0,-.77459667D0,
  *0.0D0,.77459667D0,0.0D0,-.86113631D0,-.33998104D0,.33998104D0,
  *-.86113631D0/
DATA WT/2.0D0,3*0.0D0,2*1.0D0,2*0.0D0,.55555555D0,.88888888D0,
  * .55555555D0,0.0D0,.34785485D0,2*.65214515D0,.34785485D0/
AREA = 0.0
DO 20 I = 1,NPE
NI = NOD(N,1)
ELXY(1,1) = X(NI)
20 ELXY(1,2) = Y(NI)
DO 30 NI = 1,2
DO 30 NJ = 1,2
XI = GAUSS(NI,2)
ETA = GAUSS(NJ,2)
CALL SHAPE(NPE,XI,ETA,SF,GDSF,DET,ELXY)
PART = DET*WT(NI,2)*WT(NJ,2)
AREA = AREA + PART
30 CONTINUE
RETURN
END
SUBROUTINE STIFF(N,NPE,NN,IEL,ITEM,C1,C2,C3,C4,C5,C8,E1,E2,CONT
*,ITOF,IT)

THE SUBROUTINE COMPUTES THE ELEMENT COEFFICIENT
MATRICES

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION SF(9),GDSF(2,9),GAUSS(4,4),WT(4,4),SM(18,18),S(9,9)
+E1(300),SXY(9,9),SX(9,9),SY(9,9),C(3,3),ITOF(300),IT(300)
+E2(300)
COMMON/STF/ELSTIF(18,18),ELXY(9,2),F(18),W0(18),W1(18)
+,W2(18),A0,A1,A2,A3,A4
DATA GAUSS/4*0.0D0,-.57735027D0,.57735027D0,2*0.0D0,-.77459667D0,
+0.0D0,.77459667D0,0.0D0,-.86113631D0,.33998104D0,.33998104D0,
+.86113631D0/
DATA WT/2.0D0,3*0.0D0,2*1.0D0,2*0.0D0,.55555555D0,88888888D0,
+.55555555D0,0.0D0,.34785485D0,2*.65214515D0,.34785485D0/
IF(ITOF(N) .EQ. 1)THEN
C1 = C8
C2 = C8
ELSE
C1 = E1(N)
C2 = E2(N)
ENDIF
IF(IT(N) .EQ. 2)THEN
ANU21 = C3*C2/C1
RATION = C1/C2
RATIOM = 1/(2*(1+ANU21))
S0 = C2/(1+C3)*(1-C3-2*RATION*ANU21**2))
C(1,1) = RATION*(1-RATION*ANU21**2)*S0*C4
C(1,2) = RATION*ANU21*(1+C3)*S0*C4
C(2,2) = (1-C3**2)*S0*C4
C(3,3) = RATIOM*C2*C4
C(1,3) = 0.0
C(2,3) = 0.0
C(2,1) = C(1,2)
C(3,1) = C(1,3)
C(3,2) = C(2,3)
GOTO 19
ENDIF
ANU21 = C3*C2/C1
DENOM = 1.0 - C3*ANU21
S0 = (1.0-3.0*C3*ANU21-C3*ANU21*(C3+ANU21))
C(1,1) = C1*C4*DENOM/S0
C(1,2) = C1*C4*ANU21*(1.0+C3)/S0
C(2,1) = C(1,2)
C(2,2) = C2*C(1,1)/C1
C(3,3) = C2*C4/(2*(1+ANU21))
C(1,3) = 0.0
C(2,3) = 0.0
C(3,1) = 0.0
C(3,2) = 0.0
19 NDF = 2
NGP = IEL+1
DO 20 I = 1,NPE
DO 20 J = 1,NPE
S(I,J)=0.0
SX(I,J)=0.0
SY(I,J)=0.0
20 SX(I,J)=0.0
DO 30 I = 1,NN
F(I) = 0.0
DO 30 J = 1,NN
30 SM(I,J) = 0.0
CONT = 0.0
DO 40 NI = 1,NGP
DO 40 NJ = 1,NGP
XI = GAUSS(NI,NGP)
ETA = GAUSS(NJ,NGP)
CALL SHAPE(NPE,XI,ETA,SF,GDSF,DET,ELXY)
CONST = DET*WT(NI,NGP)*WT(NJ,NGP)
CONT = CONT + CONST
DO 80 I = 1,NPE
DO 80 J = 1,NPE
S(I,J) = S(I,J)+CONST*SF(I)*SF(J)*C5*C4
SX(I,J) = SX(I,J)+CONST*GDSF(1,I)*GDSF(1,J)
SY(I,J) = SY(I,J)+CONST*GDSF(2,I)*GDSF(2,J)
SXY(I,J) = SXY(I,J)+CONST*GDSF(1,I)*GDSF(2,J)
80 CONTINUE
40 CONTINUE
II=1
DO 200 I=1,NPE
JJ=1
DO 150 J=1,NPE
ELSTIF(II,JJ) = C(1,1)*SX(I,J)+C(3,3)*SY(I,J)
ELSTIF(II+1,JJ+1) = C(3,3)*SX(I,J)+C(2,2)*SY(I,J)
ELSTIF(II,JJ+1) = C(1,2)*SXY(I,J)+C(3,3)*SXY(I,J)
ELSTIF(I+1,JJ) = C(1,2)*SXJ(I,J) + C(3,3)*SXJ(I,J)
SM(I,J) = S(I,J)
SM(I+1,JJ+1) = S(I,J)

150 JJ = NDF*I+1
200 I = NDF*I+1
IF (ITEM.EQ.1) THEN
DO 340 I=1,NN
DO 340 J=1,NN
F(I) = F(I) + SM(I,J)*(A0*W0(J)+A1*W1(J)+A2*W2(J))
340 ELSTIF(I,J)=ELSTIF(I,J)+A0*SM(I,J)
ENDIF
RETURN
END

SUBROUTINE SHAPE(NPE,XI,ETA,SF,GDSF,DET,ELXY)

C C THE SUBROUTINE EVALUATES THE INTERPOLATION C FUNCTION SF(I), AND ITS DERIVATIVES WITH RESPECT TO C NATURAL COORDINATES SF(I,J), AND THE DERIVATIVES C OF SF(I) WITH RESPECT TO GLOBAL COORDINATES FOR C FOUR, EIGHT, AND NINE NODE RECTANGULAR ELEMENTS C SF(I) INTERPOLATION FUNCTION FOR NODE I OF THE C ELEMENT
C DSF(I) DERIVATIVE OF SF(I) WITH RESPECT TO XI IF I=1 C AND WITH RESPECT TO ETA IF I=2 C GDSF(I,J) DERIVATIVE OF SF(I) WITH RESPECT X IF I=1 AND C WITH RESPECT Y IF I=2 C XNODE(I,J) J-th COORDINATE OF NODE I OF THE ELEMENT C NP(I) ARRAY OF ELEMENT NODES C GJ(J) JACOBIAN MATRIX C GJINV(J) INVERSE OF THE JACOBIAN MATRIX

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION ELXY(9,2),XNODE(9,2),NP(9),DSF(2,9),GJ(2,2),GJINV(2,2),
+ SF(9),GDSF(2,9)
DATA XNODE/-1.0D0,-2.0D0,-1.0D0,0.0D0,1.0D0,0.0D0,-1.0D0,0.0D0,
+ 2.0D0,1.0D0,-1.0D0,0.0D0,1.0D0,2.0D0/
DATA NP/1,2,3,4,5,6,7,8,9/
FNC(A,B)=A*B
IF(NPE-8) 60,10,80
10 DO 40 I = 1,NPE
   NI = NP(I)
   XP = XNODE(NI,1)
   YP = XNODE(NI,2)
   XI0 = 1.0+XI*XP
   ETA0 = 1.0+ETA*YP
   XI = 1.0-XI*XI
   ETA = 1.0-ETA*ETA
   IF(LGT.4) GO TO 20
   SF(NI) = 0.25*FNC(XI0,ETA0)*(XI*XP+ETA*YP-1.0)
DSF(1,NI) = 0.25*FNC(ETA0,XP)*(2.0*XI*XP+ETA*YP)
DSF(2,NI) = 0.25*FNC(XI0,YP)*(2.0*ETA*YP+XI*XP)
GOTO 40

20 IF(1.GT.6) GO TO 30
SF(NI) = 0.5*FNC(XI1,ETA0)
DSF(1,NI) = -FNC(XI,ETA0)
DSF(2,NI) = 0.5*FNC(YP,XI1)
GOTO 40

30 SF(NI) = 0.5*FNC(ETA1,XI0)
DSF(1,NI) = 0.5*FNC(XP,ETA1)
DSF(2,NI) = -FNC(ETA,XI0)

40 CONTINUE
GOTO 130

60 DO 70 I = 1,NPE
   XP = XNODE(I,1)
   YP = XNODE(I,2)
   XI0 = 1.0+XI*XP
   ETA0 = 1.0+ETA*YP
   SF(I) = 0.25*FNC(XI0,ETA0)
   DSF(1,I) = 0.25*FNC(XP,ETA0)
70 DSF(2,I) = 0.25*FNC(YP,XI0)
GOTO 130

80 DO 120 I = 1,NPE
   NI = NP(I)
   XP = XNODE(NI,1)
   YP = XNODE(NI,2)
   XI0 = 1.0+XI*XP
   ETA0 = 1.0+ETA*YP
   XI1 = 1.0-XI*XI
   ETA1 = 1.0-ETA*ETA
   XI2 = XP*XI
   ETA2 = YP*ETA
   IF(1.GT.4) GOTO 90
   SF(NI) = 0.25*FNC(XI0,ETA0)*XI2*ETA2
   DSF(1,NI) = 0.25*XP*FNC(ETA2,ETA0)*(1.0+2.0*XI2)
   DSF(2,NI) = 0.25*YP*FNC(XI2,XI0)*(1.0+2.0*ETA2)
GOTO 120

90 IF (1.GT.6) GO TO 100
   SF(NI) = 0.5*FNC(XI1,ETA0)*ETA2
   DSF(1,NI) = -XI*FNC(ETA2,ETA0)
   DSF(2,NI) = 0.5*FNC(XI1,YP)*(1.0+2.0*ETA2)
GOTO 120

100 IF (1.GT.8) GO TO 110
   SF(NI) = 0.5*FNC(ETA1,XI0)*XI2
   DSF(1,NI) = -ETA*FNC(XI2,XI0)
   DSF(2,NI) = 0.5*FNC(ETA1,XP)*(1.0+2.0*XI2)
GOTO 120

110 SF(NI) = FNC(XI1,ETA1)
   DSF(1,NI) = -2.0*XI*ETA1
   DSF(2,NI) = -2.0*ETA*XI1
120 CONTINUE
130 DO 140 I = 1,2
   DO 140 J = 1,2
   GJ(I,J) = 0.0
   DO 140 K = 1,NPE
140 GJ(I,J) = GJ(I,J)+DSF(I,K)*ELXY(K,J)
   DET = GJ(1,1)*GJ(2,2)-GJ(1,2)*GJ(2,1)
   GJINV(1,1) = GJ(2,2)/DET
   GJINV(2,2) = GJ(1,1)/DET
   GJINV(1,2) = -GJ(1,2)/DET
   GJINV(2,1) = -GJ(2,1)/DET
   DO 150 I = 1,2
   DO 150 J = 1,NPE
   GDSF(I,J) = 0.0
   DO 150 K = 1,2
150 GDSF(I,J) = GDSF(I,J)+GJINV(I,K)*DSF(K,J)
RETURN
END
SUBROUTINE STRESS(N,NPE,ELXY,C1,C2,C3,C4,W,ITER,DF
*,IFAIL,ITOF, HM, HS, CO, NTER, TIME, DT, IT, AF, NEM, C8, E1, E2)
C SUBROUTINE COMPUTES THE GRADIENT OF SOLUTIONS
C AND STRESSES FOR QUADRILATERAL ELEMENTS AND THE
C CLASSIFICATION OF FAILURE ELEMENTS BEGINS HERE
C
C IMPLICIT REAL*8(A-H,O-Z)
DIMENSION SF(9),GDSF(2,9),C(3,3),W(2,9),IFAIL(300)
*,E1(300),E2(300),ITOF(300),IT(300),CO(300),ELXY(9,2)
IFAIL(N) = 0
UX=0.0
UY=0.0
VX=0.0
VY=0.0
XI = 0.0
ETA = 0.0
CALL SHAPE(NPE,XI,ETA,SF,GDSF,DET,ELXY)
IF(ITOF(N) .EQ. 1)THEN
   C1 = C8
   C2 = C8
ELSE
   C1 = E1(N)
   C2 = E2(N)
   IF(C1 .LT. 10.0)C1 = 10.0
   IF(C2 .LT. 10.0)C2 = 10.0
ENDIF
IF(IT(N) .EQ. 2)THEN
   ANU21 = C3*C2/C1
   RATION = C1/C2
   RATIOM = 1/(2*(1+ANU21))
   S0 = C2/((1+C3)*(1-C3-2*RATION*ANU21**2))
   C(1,1) = RATION*(1-RATION*ANU21**2)*S0*C4
C(1,2) = RATION*ANU21*(1+C3)*S0*C4
C(2,2) = (-C3**2)*S0*C4
C(3,3) = RATION*C2*C4
C(1,3) = 0.0
C(2,3) = 0.0
C(2,1) = C(1,2)
C(3,1) = C(1,3)
C(3,2) = C(2,3)
GOTO 109
ENDIF
ANU21 = C3*C2/C1
DENOM = 1.0 - C3*ANU21
S0 = (1.0-3.0*C3*ANU21-C3*ANU21*(C3+ANU21))
C(1,1) = C1*C4*DENOM/S0
C(1,2) = C1*C4*ANU21*(1.0+C3)/S0
C(2,1) = C(1,2)
C(2,2) = C2*C(1,1)/C1
C(3,3) = C2*C4/(2*(1+AUN21))
C(1,3) = 0.0
C(2,3) = 0.0
C(3,1) = 0.0
C(3,2) = 0.0
109 DO 110 I = 1,NPE
   UX = UX+W(I,1)*GDSF(1,I)
   UY = UY+W(I,1)*GDSF(2,I)
   VX = VX+W(2,I)*GDSF(1,I)
110 VY = VY+W(2,I)*GDSF(2,I)
   SX = -(C(1,1)*UX+C(1,2)*VY)
   SY = -(C(1,2)*UX+C(2,2)*VY)
   SXY = -C(3,3)*(UY+VX)
   PI = 3.1415925
   ALFA = DATAN(2*SXY/(SX-SY))
   ALFA = ALFA/2
   IF(SX .LT. SY .AND. SXY .GT. 0.0)ALFA = ALFA + PI/2
   IF(SX .LT. SY .AND. SXY .LT. 0.0)ALFA = ALFA - PI/2
   S1 = (SX+SY)/2 + SQRT(((SX-SY)/2)**2+SXY**2)
   S2 = (SX+SY)/2 - SQRT(((SX-SY)/2)**2+SXY**2)
   CH = HM*S2*CO(N)+CO(N)**2*HS
   IF(CH .GE. 0.0) THEN
      SA = S2 + SQRT(CH)
   ELSE
      SA = -1.0
   ENDIF
   SN = (SX+SY)/2 + (SX-SY)*COS(2*AF)/2 + SXY*SIN(2*AF)
   ST = CO(N)*(HM-SQRT(HM**2+4*HS))/2
   ALF = ALFA/PI*180.0
   IF(ITOF(N) .EQ. 1) GOTO 13
   IF(S2 .LE. ST) THEN
      IF(DF .LE. 0.0) THEN
         IT(N) = 1
         IF(ITER .EQ. NTER .OR. TIME .EQ. DT) WRITE(16,1)N
      ENDIF
   ENDIF

1 FORMAT(I3,2X,'TENSILE FAILURE WITH CLOSE CRACK')
GOTO 777
ELSEIF(DF .GT. 0.0)THEN
  IT(N) = 2
  IF (ITER .EQ. NTER .OR. TIME .EQ. DT)WRITE(16,2)N
2 FORMAT(I3,2X,'TENSILE FAILURE WITH OPEN CRACK')
GOTO 777
ENDIF
ENDIF
IF(SN .GE. CO(N))THEN
  IFAIL(N) = 1
GOTO 777
ENDIF
IF(CH .GE. 0.0)THEN
  IF(S1 .GE. SA .AND. SA .GT. 0.0)THEN
    IF(ITER .EQ. NTER .OR. TIME .EQ. DT)WRITE(16,4)N
4 FORMAT(I3,2X,'SHEAR FAILURE')
  IF(S1 .LT. 0.0001)THEN
    SD = 0.1
    ELSEIF(S1 .GT. 0.1)THEN
      SD = S1
  ENDIF
  R = SA/SD
  E1(N) = R*E1(N)
  E2(N) = R*E2(N)
  CO(N) = R*CO(N)
ENDIF
ENDIF
777 IF(IFAIL(N) .EQ. 1)ITOF(N)=1
13 IF(ITOF(N) .EQ. 1)THEN
  IF(ITER.EQ.NTER.OR.TIME.EQ.DT)WRITE(16,3)N
3 FORMAT(I3,2X,'CRUSHING')
ENDIF
IF(ITER.EQ.NTER.OR.TIME.EQ.DT)THEN
PRINT 411,N,51,52,SN,ALF
WRITE(21,422)CH,CO(N),ST,SA,C1,C2
422 FORMAT(F12.2,2X,F10.2,2X,F10.2,2X,F10.2,2X,F10.2,2X,F10.2,2X,0.2)
ENDIF
IF(N .EQ. NEM)WRITE(16,15)
15 FORMAT('NEXT')
CONTINUE
RETURN
END
SUBROUTINE MESH(IEL,NX, NY,NPE,NNM,NEM)
C
C THE SUBROUTINE GENERATE FINITE ELEMENT MESH
C
IMPLICIT REAL*8 (A-H,O-Z)
COMMON/MSH/NOD(500,9),X(500),Y(500),DX(30),DY(30)
IF(IEL.GT.0)GOTO 100
NEM = 2*NX*NY
NX1=NX+1
NY1=NY+1
NXX1=2*NX
NYY1=2*NYY
NNM=NXX1*NY1
NOD(1,1)=1
NOD(1,2)=2
NOD(1,3)=NX1+2
NOD(2,1)=1
NOD(2,2)=NX1+2
NOD(2,3)=NX1+1
K=3
DO 60 IY=1,NY
   L=IY*NXX1
   M=(IY-1)*NXX1
   IF(NX.EQ.1)GOTO 40
   DO 30 N=K,L,2
   DO 20 I=1,NPE
      NOD(N,I)=NOD(N-2,I)+1
   20 CONTINUE
   30 CONTINUE
   40 IF(NY.EQ.1)GOTO 60
   DO 50 I = 1,NPE
      NOD(L+1,I)=NOD(M+1,I)+NX1
   50 CONTINUE
   60 K=L+3
   70 L=0
   YC=0.0
   DO 90 I=1,NY1
      XC=0.0
      DO 80 L=1,NX1
         X(L)=XC
         Y(L)=YC
      80 XC=XC+DX(I)
      YC=YC+DY(J)
   90 RETURN
100 NEX1=NX+1
   NEY1=NY+1
   NXX=IEL*NX
   NYY=IEL*NYY
   NXX1=NXX+1
   NYY1=NYY+1
   NEM=NX*NYY
   NNM=NXX1*NYY1-(IEL-1)*NX*NY
   IF(NPE.EQ.9)NNM=NXX1*NYY1
   K0=0
   IF (NPE .EQ. 9) K0=1
   NOD(1,1)=1
   NOD(1,2)=IEL+1
NOD(1,3) = NXX1+(IEL-1)*NEX1+IEL+1
IF (NPE .EQ. 9) NOD(1,3) = 4*NX+5
NOD(1,4) = NOD(1,3)-IEL
IF(NPE.EQ.4) GO TO 200
NOD(1,5) = 2
NOD(1,6) = NXX1+(NPE-6)
NOD(1,7) = NOD(1,3)-1
NOD(1,8) = NXX1+1
IF(NPE.EQ.9) NOD(1,9) = NXX1+2
200 IF(NY .EQ. 1) GOTO 230
   M = 1
   DO 220 N = 2, NY
   L = (N-1)*NX+1
   DO 210 I = 1, NPE
   210 NOD(L,I) = NOD(M,I)+NXX1+(IEL-1)*NEX1+K0*NX
   DO 220 I = 1, NPE
   220 M = L
   230 IF(NX .EQ. 1) GOTO 270
   DO 260 NI = 2, NX
   DO 240 I = 1, NPE
   K1 = IEL
   IF (1 .EQ. 6 .OR. 1 .EQ. 8) K1 = 1+K0
   240 NOD(NI,I) = NOD(NI-1,I)+K1
   M = NI
   DO 260 NJ = 2, NY
   L = (NJ-1)*NX+NI
   DO 250 J = 1, NPE
   250 NOD(L,J) = NOD(M,J)+NXX1+(IEL-1)*NEX1+K0*NX
   DO 260 J = 1, NPE
   260 M = L
   270 YC = 0.0
   IF (NPE .EQ. 9) GOTO 310
   DO 300 NI = 1, NEY1
   I = (NXX1+(IEL-1)*NEX1)*(NI-1)+1
   J = (NI-1)*IEL+1
   X(I) = 0.0
   Y(I) = YC
   DO 280 NJ = 1, NXX
   I = I+1
   X(I) = X(I-1)+DX(NJ)
   280 Y(I) = YC
   IF(NI.GT.NY .OR. IEL.EQ.1) GOTO 300
   J = J+1
   YC = YC+DY(J-1)
   I = I+1
   X(I) = 0.0
   Y(I) = YC
   DO 290 NI = 1, NX
   K = 2*NI-1
   L = I+1
   X(I) = X(I-1)+DX(K)+DX(K+1)
   290 Y(I) = YC
   300 YC = YC+DY(J)
RETURN
310 DO 330 NI=1,NYY1
   I=NXX1*(NI-1)
   XC=0.0
   DO 320 NJ=1,NXX1
      I=I+1
      X(I)=XC
      Y(I)=YC
   320 XC=XC+DX(NJ)
330 YC=YC+DY(NI)
RETURN
END

SUBROUTINE BNDY(NRMAX,NCMAX,NEQ,NHBW,S,SL,NSDF,IBDF,VBDF)

C C SUBROUTINE USED TO IMPOSE BOUNDARY CONDITIONS
C ON BANDED EQUATIONS

C IMPLICIT REAL*8(A-H,O-Z)
DIMENSION S(NRMAX,NCMAX),SL(NRMAX)
DIMENSION IBDF(NSDF),VBDF(NSDF)
DO 300 NB = 1,NSDF
   IE=IBDF(NB)
   SVAL = VBDF(NB)
   IT=NHBW-1
   I=IE-NHBW
   DO 100 II=1,IT
      I=I+1
      IF (I .LT. 1) GOTO 100
      I=IE-I+1
      SL(I)=SL(I)-S(IJ)*SVAL
      S(IJ)=0.0
   100 CONTINUE
   S(IE,1)=1.0
   SL(IE)=SVAL
   I=IE
   DO 200 II=2,NHBW
      I=I+1
      IF (I .GT. NEQ) GOTO 200
      SL(I)=SL(I)-S(IE,II)*SVAL
      S(IE,II)=0.0
   200 CONTINUE
300 CONTINUE
RETURN
END

SUBROUTINE SOLVE(NRM,NCM,NEQNS,NBW,BAND,RHS,RES)

C C SOLVING A BANDED SYMMETRIC SYSTEM OF EQUATIONS
C
C IMPLICIT REAL*8(A-H,O-Z)
DIMENSION BAND(NRM,NCM),RHS(NRM)
MEQNS=NEQNS-1
IF (IRES .GT. 0) GOTO 90
DO 500 NPIV=1,MEQNS
NPIVOT=NPIV+1
LSTSUB=NPIV+NBW-1
IF(LSTSUB .GT. NEQNS) LSTSUB=NEQNS
DO 400 NROW=NPIVOT,LSTSUB
NCOL=NROW-NPIV+1
FACTOR=BAND(NPIV,NCOL)/BAND(NPIV,1)
DO 200 NCOL=NROW,LSTSUB
ICOL=NCOL-NROW+1
JCOL=NCOL-NPIV+1
200 BAND(NROW,ICOL) = BAND(NROW,ICOL) - FACTOR * BAND(NPIV,JCOL)
400 RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
500 CONTINUE
GO TO 101
90 DO 100 NPIV=1,MEQNS
   NPIVOT=NPIV+1
   LSTSUB=NPIV+NBW-1
   IF(LSTSUB .GT. NEQNS) LSTSUB=NEQNS
   DO 110 NROW=NPIVOT,LSTSUB
   NCOL=NROW-NPIV+1
   FACTOR=BAND(NPIV,NCOL)/BAND(NPIV,1)
110 RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
100 CONTINUE
101 DO 800 IJK=2,NEQNS
   NPIV=NEQNS-IJK+2
   RHS(NPIV)=RHS(NPIV)/BAND(NPIV,1)
   LSTSUB=NPIV-NBW+1
   IF(LSTSUB.LT.1) LSTSUB=1
   NPIVOT = NPIV-1
   DO 700 JKI=LSTSUB,NPIVOT
   NROW=NPIVOT-JKI+LSTSUB
   NCOL=NPIV-NROW+1
   FACTOR=BAND(NROW,NCOL)
700 RHS(NROW)=RHS(NROW)-FACTOR*RHS(NPIV)
800 CONTINUE
   RHS(1)=RHS(1)/BAND(1,1)
RETURN
END
APPENDIX C.

STRESS CONTOUR AND PENETRATION PROCESS AT VARIOUS FORMATION DIPS
Figure 16a. Penetration Process of a Wedge Impact at 0° Formation Dip (2000 lbs)
Figure 16b. Penetration Process of a Wedge Impact at 0° Formation Dip (4000 lbs)
Figure 16c. Penetration Process of a Wedge Impact at 0° Formation Dip (6000 lbs)
Figure 16d. Penetration Process of a Wedge Impact at 0° Formation Dip (8000 lbs)
Figure 16.1 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 2000 lbs
Minor Principal Stress

Figure 16.2 Stress Contour of a Wedge Impact
Formation Dip $0^\circ$
Axial Force 4000 lbs
Major Principal Stress

Figure 16.3 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 4000 lbs
Minor Principal Stress

Figure 16.4 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 6000 lbs
Major Principal Stress

Figure 16.5 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 6000 lbs
Minor Principal Stress

Figure 16.6 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 8000 lbs
Major Principal Stress

Figure 16.7 Stress Contour of a Wedge Impact
Formation Dip 0°
Axial Force 8000 lbs
Minor Principal Stress

Figure 16.8 Stress Contour of a Wedge Impact
Figure 17a. Penetration Process of a Wedge Impact at 90° Formation Dip (2000 lbs)
Figure 17b. Penetration Process of a Wedge Impact at 90° Formation Dip (4000 lbs)
Figure 17c. Penetration Process of a Wedge Impact at 90° Formation Dip (6000 lbs)
Figure 17d. Penetration Process of a Wedge Impact at 90° Formation Dip (8000 lbs)
Formation Dip 90°
Axial Force 2000 lbs
Major Principal Stress

Figure 17.1 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 2000 lbs
Minor Principal Stress

Figure 17.2 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 4000 lbs
Major Principal Stress

Figure 17.3 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 4000 lbs
Minor Principal Stress

Figure 17.4 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 6000 lbs
Major Principal Stress

Figure 17.5 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 6000 lbs
Minor Principal Stress

Figure 17.6 Stress Contour of a Wedge Impact
Formation Dip $90^\circ$
Axial Force 8000 lbs
Major Principal Stress

Figure 17.7 Stress Contour of a Wedge Impact
Formation Dip 90°
Axial Force 8000 lbs
Minor Principal Stress

Figure 17.8 Stress Contour of a Wedge Impact
Figure 18a. Penetration Process of a Wedge Impact at 30° Formation Dip (2000 lbs)
Figure 18b. Penetration Process of a Wedge Impact at 30° Formation Dip (4000 lbs)
Figure 18c. Penetration Process of a Wedge Impact at 30° Formation Dip (6000 lbs)
Figure 18d. Penetration Process of a Wedge Impact at 30° Formation Dip (8000 lbs)
Formation Dip 30°
Axial Force 2000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 18.1 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 2000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 18.2 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 4000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 18.3 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 4000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 18.4 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 6000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 18.5 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 6000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 18.6 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 8000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 18.7 Stress Contour of a Wedge Impact
Formation Dip 30°
Axial Force 8000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 18.8 Stress Contour of a Wedge Impact
Figure 19a. Penetration Process of a Wedge Impact at 60° Formation Dip (2000 lbs)
Figure 19b. Penetration Process of a Wedge Impact at 60° Formation Dip (4000 lbs)
Figure 19c. Penetration Process of a Wedge Impact at 60° Formation Dip (6000 lbs)
Figure 19d. Penetration Process of a Wedge Impact at 60° Formation Dip (8000 lbs)
Formation Dip 60°
Axial Force 2000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 19.1 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 2000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 19.2 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 4000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 19.3 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 4000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 19.4 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 6000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 19.5 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 8000 lbs
Dipping from Right to Left
Major Principal Stress

Figure 19.7 Stress Contour of a Wedge Impact
Formation Dip 60°
Axial Force 8000 lbs
Dipping from Right to Left
Minor Principal Stress

Figure 19.8 Stress Contour of a Wedge Impact