USE OF PRIOR DISTRIBUTIONS FROM AERIAL PHOTOGRAPHS IN FOREST INVENTORY

by

Mary F. Bleier

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APPROVED:

R. G. Oderwald, Chairman

H. E. Burkhart

J. L. Smith

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Committee Chairman: Richard G. Oderwald
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(ABSTRACT)

Bayesian estimates of gross cubic-foot volume per acre were computed for four stand types (plantation pine, natural pine, hardwood, and mixed wood stands) using aerial photo volume tables as the prior information source. Aerial photographs provided a reliable source of information even though most photographs were nearly five years old.

For a given level of precision within a particular stand, Bayesian methods reduced the required field sample size up to 50% using all or half of the prior information available. Those priors which utilized a regression or a regression/topographic correction in the estimation of photo heights required less field information for the given precision level than those priors which used uncorrected or topographic corrected photo heights.

In order to obtain meaningful gains in sample size reduction, corrections to the estimated photo heights should be made. Although the uncorrected prior produced generally less biased estimates, the reduction in sample

size was not as large as that observed using other prior types. Greater gains were attributed to the better accuracy of the prior distribution.

Although Bayesian methods are biased, it appeared that these methods tempered severely biased prior distributions. In the hardwood stand for example, the average bias present in the photo volume data amounted to -140%. After combining the prior with the field sample, the greatest average bias was -50%.

Bayesian methods performed better than the traditional estimation methods in terms of precision. In a one to one comparison, the Bayes standard error was consistently less than its non-Bayes counterpart. The one exception to this trend was the regression prior from the hardwood stand. The poor performance of the prior was due to the weak height regression correction equation.

Modal priors utilized were not subject to the extreme input values for prior distributin development as their conservative empirical prior counterparts were. Less overall variation was observed in the estimated values. Under the conditions for mode selection set forth in this project, modal priors provided another good source of prior information.

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INTRODUCTION

Prior information of forested areas is available from LANDSAT imagery, growth and yield models, previous inventories, and aerial photography. Traditionally this prior information has only been utilized for inventory design. However, prior information could directly influence inventory estimates through the use of Bayesian procedures. Bayesian methods have the potential to augment current inventory methods in areas where sufficient prior information exists. In this investigation, aerial photographs were evaluated as the prior information source.

In general, Bayesian methodology has not been utilized to a great extent in forest inventory. This lack of use revolves around the uncertainties faced in prior specification and the formidable calculations involved to obtain Bayesian estimates.

With the revival of interest in Bayesian estimation and with the availability of prior information, it appears advantageous to examine the possible applications that Bayesian methods have in forest inventory. Specifically, there is a need for further investigation into the required accuracy of the prior distributions used in Bayesian estimation for forestry use. The primary objective of this investigation was to determine the effect of prior distribution specification on the precision of Bayesian methodology using aerial photography within different forest types.

Aerial photo volume tables were examined as the information source for the Bayesian prior distributions. Intrinsic in the utilization of the aerial photographs for the prior distribution source is the type of data input into the aerial photo volume equations and the errors associated with these inputs.

In addition to the accuracy of prior specification, there is also the question of estimate improvement. It is possible that once a prior has been correctly specified, Bayesian estimates will have the same precision as those obtained from traditional estimation methods while requiring fewer sample plots. A smaller sample size means the Bayesian methods would cost less. The converse may also be true - Bayesian estimates based on the same sample size as traditional estimates may have improved precision. An objective of this study was to compare the overall performance of Bayesian methodology with traditional estimation procedures.

Objectives

In summary, the specific objectives of this project were:

1. To determine the effect of prior distribution specification on the accuracy and precision of Bayesian methodology within different forest types.
2. To compare the overall performance of Bayesian methodology with traditional estimation methods.
3. To evaluate aerial photographic volume tables as the prior information source for Bayesian calculations.

LITERATURE REVIEW

Bayesian Methodology

Since Thomas Bayes first introduced his theorem, circa 1763, interest in decision making under uncertainty has recurringly waxed and waned. With the revival of interest in this technique and the availability of prior information, Bayesian theory has an intuitive appeal to forest inventory applications (Ek and Issos, 1978).

Bayesian methods are not theoretically controversial. However, in practical application, there is a source of dispute. Bayes theorem is stated as follows (Box and Tiao, 1973):

$$p(\theta|y) = c*f(y|\theta)*p_o(\theta) \qquad (1)$$

where:

θ = parameter of interest

y = vector of n observations

$f(y|\theta)$ = probability distribution of y for a given population mean, θ

$p_o(\theta)$ = prior distribution of θ

$p(\theta|y)$ = posterior distribution of θ for the given y data

$1/c = \int f(y|\theta)*p_o(\theta)*d\theta$

Criticism of Bayesian methods arises from the specification of the prior distribution used in equation 1. Savage (1959) commented that the precise measurements deemed necessary for prior specification were very rarely obtained. He also observed that subjective priors were erroneous in areas dealing with out-lying data and "empirical surprises". In a more forestry related context, Swindel (1972) made similar criticisms of Bayesian methods, stressing the lack of utility for Bayesian estimates when only limited or inaccurate records were used for the prior distribution.

Within the forestry realm, there exists a tremendous amount of information that may be used in prior specification. Growth and yield models can be used for this purpose. Other types of information that exist and could be used in prior specification include regression estimates, updated inventories, and aerial photography. Public and private forest concerns generally keep current records of stand information, such as basal area per acre, stand age and number of trees per acre. Many of the larger operations also use current aerial photography in their forest inventories. With such a vast amount of information available in forestry, there should be little difficulty developing an empirical prior for whatever stand characteristic is desired.

Since prior specification is such a source of dispute, it is important to examine the different methods developed for specifying prior distributions. The subjective prior, suggested by T. R. Bayes himself, is a distribution that is developed simply by using personal intuition (Box and Tiao, 1973). A non-informative or locally uniform prior assumes that the probability of an observation is as likely as the next over some specified minimum and maximum values (Schmitt, 1969).

Another method, and one that has received considerable attention in forestry applications, is known as an empirical prior. First introduced by Robbins in 1955, the empirical prior is based on previous samples from the population of interest (Burk and Ek, 1980). Ek and Issos (1978) found that using the empirical prior and the computer software developed for Bayesian estimation, reliable estimates with smaller confidence intervals than those calculated with traditional estimation methods could be obtained. Burk and Ek (1980) found similar results using James-Stein estimators.

Several other facts were observed in research involving Bayesian methods. First, as the sample size increases, the effect of the prior distribution decreases and hence the biasedness caused by an incorrectly specified prior decreases (Ek and Issos, 1978). It was also observed that estimates improved when prior distributions were more refined (Ek and Issos, 1976).

Green and Strawderman (1985) used Bayesian estimators to develop an individual tree volume equation. An informative prior was used for the slope coefficient and non-informative priors were utilized for the error term and the intercept. Good results were attributed to more precisely specified priors. Reductions in sample size could be realized with the use of Bayesian methods.

In addition to the recent work done on Bayesian methods, previous researchers have suggested the use of Bayesian theory in forestry. Bayesian estimation was suggested for use in stratified sampling (Ericson, 1965). With the consideration of optimality in mind, both Neyman allocation and Bayesian decision theory were examined extensively using multivariate normal distributions.

Lin and Minter (1976) developed an algorithm which uses Bayesian theory to classify a single cell type from satellite data. With the single-class classifier and the Bayesian algorithm, only training samples for the class of interest are required. This reduces the amount of ground data required for verification of the system.

Dane (1965) used Bayesian estimation for estimating the expected costs of logging operations based on soil conditions, logging method, and rainfall. Considerations are made within the context of the problem for the acquisition of additional information. Using the Bayesian methods, expected costs and payoffs are computed for several alternatives available to the forest engineer. These values may then be used to aid the engineer in the decision making process.

Aerial Photography for Volume Estimation

Aerial photography has not been extensively investigated as a source of prior information for use with Bayesian methods. Pope (1949) and Spurr (1946) both reported that aerial volume tables could be derived through two approaches. The first approach is to estimate individual tree volumes from photo measurements of crown diameter and of tree height. This approached has been used extensively by the Canadian Department of Forestry (Aldred and Kippen 1967, Aldred and Sayn-Wittgenstein 1972). Estimating volume per acre based on average stand height and percent crown closure is the second approach.

Meyer and Worley (1957) found that in the development of aerial volume tables, areas of greater homogeneity - with respect to species composition and stand age - required fewer photo plots than other, more heterogeneous stands. In addition, both ground and photo measurements are required in order to detect systematic errors and to determine the accuracy of the volume table (Pope 1962).

Past research in the development of photo volume tables has been conducted for many forest types. Avery (1958) developed a composite volume table for mixed Southern pine and hardwoods using average crown diameter, average stand height, and percent crown closure as the independent input variables. Average crown diameter, average stand height, and percent crown closure were also used by Hanks and Thomson (1964) to estimate cubic-foot volume for hardwoods in Iowa. Their models were not accurate estimators of volume, however, since average errors often exceeded 65% on test plots.

Despite the poor findings of Hanks and Thomson (1964), it is still possible to obtain reasonable volume estimates from aerial photo volume tables. A volume table developed to estimate cubic-foot volume/acre from crown diameter, crown closure and stand height for central hardwoods was found to have average errors of approximately 17% (Moessner et. al., 1951). Errcrs averaging about 20% were found with Avery's composite volume table (1958). In a volume table developed for upland oak stands in Pennsylvania, Gingrich and Meyer (1955) found the average error to range from 20% to 25%. This particular volume table only used crown closure and average stand height to estimate volume in cubic-feet per acre. In research conducted by Pope (1962), average crown diameter was found insignificant in the presence of crown closure and average stand height.

METHODS AND MATERIALS

Study Area

Four general forest types were used in this study - hardwood, plantation southern pine, natural southern pine, and mixed hardwood and pine. The stands selected for this study are located in Appomattox, Amherst, and Campbell counties in the state of Virginia and are owned by Westvaco Corporation.

The plantation pine stand, located on the Holiday Lake 7.5' Quadrangle in Appommattox County, is a 25 year-old loblolly pine (*Pinus taeda*) and Virginia pine (*Pinus virginiana*) plantation with a site index of 55 (base age 25 years for loblolly pine). Total basal area per acre is 79 square feet with pine and hardwood comprising 60 square feet and 19 square feet of basal area per acre, respectively. Loblolly and Virginia pine diameters ranged from 6 to 14 inches at breast height. The main canopy was almost entirely pine. The cruised area was approximately 26 acres in size.

The natural pine stand used in this study is located on the Castle Craig 7.5' Quadrangle in Campbell County. The 45 year-old stand was comprised mainly of loblolly and Virginia pine. Total basal area per acre was 50 square feet with pine and hardwood basal areas per acre being 38 square feet and 12 square feet, respectively. Diameters at breast height ranged from 6 to 16 inches on the 20 acres cruised.

The mixed pine and hardwood stand, located in a very hilly section of Amherst County on the Big Island 7.5' Quadrangle, was comprised mainly of yellow-poplar (*Liriodendron tulipifera*), eastern white pine (*Pinus strobus*), and eastern hemlock (*Tsuga canadensis*). The stand, although very rugged, was on a good site since the average height of the trees is 90 feet in a stand that is 75 years-old. Total basal area per acre is 82 square feet with pine and hardwood comprising 42 square feet per acre and 40 square feet per acre, respectively. On the 17 acres cruised, the diameters at breast height ranged from 6 to 23 inches.

The final forest type selected, the hardwood stand, was a 65 year-old stand located in the NE-1/4 of the Appomattox 15' Quadrangle in Appomattox County. Red oak (*Quercus rubra*) and sugar maple (*Acer saccharum*) were the dominant species on the plots on the 20 acres cruised. Very little pine existed in the stand. Total basal area per acre was 55 square feet with pine and hardwood contributing 3 square feet per acre and 52 square feet per acre, respectively. Tree diameters ranged from 5 to 24 inches.

Selection of these particular stands was based on the availibility of stereo aerial photography, a stand size of at least 15 contiguous acres, forest type, and stand age. In order to make the necessary photo measurements, a stereo pair of the stand was required. A minimum stand size was required in order to obtain enough data for the various field and photo sample sizes. One area of each general stand type was desired in order to examine the effect stand type had on the accuracy of prior specification. Finally, a minimum stand age was specified in order to obtain a field volume that would be comparable to the photo volume calculated for the plot. A minimum age was specified since a negative photo volume was not desired.

The aerial photographs of the previously described stands were provided by Westvaco. The color infrared stereo pairs were approximately 5 years old. The photographs were in a 9-inch by 9-inch format and had an average scale of 1:15,840. The photographs were taken under clear conditions in early spring before the hardwoods leafed out.

Measurements

Field

For each stand, tenth acre plots were located on a 2 by 3 chain grid. Plots were located using a hand compass and pacing. For each tree on the plot total height, dbh, and tree species group, either hardwood or softwood, were recorded. Suunto clinometers were initially used to check the occular estimates of total tree height. After sufficient training, only occular estimates of tree height were necessary. Total tree height was estimated to the nearest five feet for all trees 6 inches dbh (outside bark) and greater on the plot. For the hardwood stand, all trees greater than 5 inches dbh (outside bark) were measured. Diameter at breast height was estimated to the nearest inch using a Biltmore stick and diameter tape. Plots that were located less than .5 chain from a road or clearing were not included in the field cruise since matched photo plots did not include the road.

Photo

Photo plots were randomly selected from an overlaid 2 by 3 chain grid set up for each stand under consideration. The number of photo plots selected depended upon the size of the area to be cruised. Measurements were made on one photo plot, 1/5 acre in size, per acre in the stand.

The photo measurements required for volume determination included average height of the trees on the plot or average height of the four tallest trees on the plot (to the nearest foot), percent crown closure, and in one case, average crown diameter (to the nearest foot).

Tree heights were estimated from a differential parallax equation of the form shown in equation 2 (Wolf, 1983):

$$HT = FH*dP/(dP + B) \qquad (2)$$

where:

HT = height in feet of the object in question

FH = flying height in feet above the base of the object

dP = difference in absolute stereoscopic parallax between the base and the top of the object, in millimeters

B = average distance between the principal point and conjugate principal point of the stereo pair, in millimeters

Differential parallax was measured with the aid of an Old Delft stereoscope and a parallax bar. Differential parallax readings were taken for each of the four tallest trees on the plot. These four readings were averaged and this average dP was used in equation 2.

Each plot was located on a topographic map. The reference point used at each plot was recorded and also located on the topographic map. The difference in the elevation of the plot and the elevation of the reference point is the topographic correction that was used in a simple topographic correction of the heights.

If measurements of tree height were suspect, for example, too high or too low, the plots were re-examined. If the new height measurements were very different from the old values, the new values were used. Otherwise, all measurements were used to estimate the average height.

Crown closure was measured on each plot using a fifth acre circle guide. This circle guide contained five, one-fifth acre circles arranged in a square. The circle guide was utilized in order to obtain a better estimate of average plot crown closure. The center circle was placed over the plot center and then, examining the areas defined by the circles under the stereoscope, the percent of each circle covered by relevant tree crowns was estimated for all circles falling within the stand. The circle guide was situated in the stand such that at least two circles fell into the stand. The averaged percent crown closure for each plot was used in the photo volume equation for that stand.

In one case, average crown diameter was needed in order to use the photo volume equation for that stand. Crown diameter was estimated using a 7X magnifier and a single photograph from the stereo pair for that stand.

The photo with the stand closest to its principal point was chosen for examination since distortion was least likely to affect measurements made on that photo. Four readings were taken per plot, averaged, and then converted from photo inches to actual feet in diameter using photo scale.

Volume Calculations

Field

Field volumes were calculated using the cruise data. Individual tree volume was estimated using equation 3 (McDonald et. al., 1963) for softwoods or equation 4 (Clark, 1982) for hardwoods. These equations are:

$$VOL = -3.438 + .84195*D - .05096*D^2 + .00001*D*H + .00266*D^2*H \qquad (3)$$

$$R^2 = .9768 \qquad S_{y.x} = 1.596$$

$$VOL = .00062*(D^2*H)^{1.12612} \qquad (4)$$

$$R^2 = .9900 \qquad S_{y.x} = 0.0543$$

where:

VOL = total cubic-foot volume of a tree with height, H and dbh, D

D = diameter at breast height, in inches

H = total height of the tree, in feet

Estimates of the gross total cubic-foot volume per acre were then made from the individual tree volumes on each plot.

Photo

Gross cubic-foot volume per acre was determined by aerial volume equations for each of the particular stand types. These equations were selected for their ease of use and similarity to the stand types under consideration. None of the aerial volume equations were specifically developed in the study areas, however.

For the hardwood stand, the aerial photo volume table developed for upland oak by Gingrich and Meyer (1955) using equation 5 below was utilized.

$$Y = -3.465*CC + 11.68*CC*HT/100 \quad (5)$$

$$R^2 = 0.85 \quad S_{y.x} = 108 \quad CV = 25\%$$

where:

Y = gross cubic-foot volume per fifth acre of all trees 5 inches dbh or greater (outside bark)

CC = percent crown closure

HT = average height of the 4 tallest trees measured on the photo, in feet

For the natural southern pine stand, an aerial photo volume table developed in FOR 5970 - Aerial Volume Table Study using equation 6 (Bleier, 1985) was used. Also used to determine cubic-foot volume per acre was an aerial volume equation 7 that did not require corrections to the height measurements. In both of the pine stands, a volume equation developed specifically by the photo interpreter was utilized. This interpreter specific volume equation did not require the use of corrected heights and may even reduce the amount of estimation error (Smith, et. al., 1986).

$$Y = -303.473 + 0.179442*HT^2 + 0.454072*CC*HT \qquad (6)$$

$$R^2 = 0.75 \quad S_{y.x} = 625 \quad CV = 48\%$$

$$Y = 1073.79 + .00575225*HT^2*CC \qquad (7)$$

$$R^2 = 0.4236 \quad S_{y.x} = 1097 \quad CV = 48\%$$

where:

Y = gross cubic-foot volume per acre of all trees 6 inches dbh or greater (outside bark)

HT = average photo measured height of the 4 tallest trees on the plot, in feet

CC = percent crown closure

For the plantation southern pine stand, the aerial photo volume tables developed from equations 6 and 7 were used.

Finally, for mixed pine and hardwood stand, an aerial volume table developed by Avery (1958) using photo measurements of percent crown closure, average crown diameter and average stand height was used. A regression equation was fitted to the data in Avery's table in order to slightly extend the range of the table's utility. All the variables in the equation were significant at the 5% level:

$$Y = -32.8 + 2.82*HT - 9.03*CD + 18.60*CC - 269.28*CC/HT + .0086*HT*CD*CC + .288*CC*HT/CD \quad (8)$$

$$R^2 = 0.9919 \qquad S_{y.x} = 71 \qquad CV = 5.1\%$$

where:

Y = gross cubic-foot volume per fifth acre of all trees 6 inches dbh or greater (outside bark)

CC = percent crown closure

HT = average photo measured height in feet

CD = average crown diameter in feet

Procedures

In order to determine how accurately a prior distribution must be specified, various types of height data were used as input into the photo volume equations. Photo measured heights were the only variables corrected since measurement of crown closure and crown diameter were more accurately determined from photographs. In all, four variations of photo height data were used in each photo volume equation: uncorrected, regression corrected, topographic corrected, and regression/topographic corrected.

An uncorrected prior was developed by using uncorrected height data in the appropriate photo volume equation. This prior type was examined since the raw height data was the easiest to obtain and to use.

Regression corrections for the photo height data were developed by matching the photo plots to their corresponding field plots. The average height of the four tallest trees in the field plot was regressed against the average height of the four tallest trees in the photo plot. This regression equation was an attempt to remove interpreter bias from the height estimation. The regression corrected heights were used in the appropriate photo volume equations to yield the regression prior type.

The topographic prior was developed by using topographically corrected heights in the appropriated photo volume equations. The topographic height correction was accomplished by estimating the difference in the vertical distance between the perceived reference point elevation for photo height measurement and the actual plot elevation. The topographic correction was necessary in most of the stands since the reference point was not at the same elevation as the base of the tree.

For the regression/topographic correction, the topographically corrected heights were regressed against field measured height in the manner of the regression corrected heights described above. These regression/topographic corrected heights were utilized in the appropriated photo volume equation to develop the regression/topographic prior.

The equations developed for the regression corrections and the regression/topographic corrections are:

a) hardwood stand regression correction:

$$HT = 57.34 + 0.0214*h \qquad (9)$$

$R^2 = 0.0013$ $\quad S_{y.x} = 9.88$ $\quad CV = 16.7\%$

b) hardwood stand regression/topographic correction:

$$HT = 75.71 - 0.2155*(h+t) \qquad (10)$$

$R^2 = 0.1448$ $\quad S_{y.x} = 9.18$ $\quad CV = 15.4\%$

c) natural pine stand regression correction:

$$HT = 61.01 + 0.0901*h \qquad (11)$$

$R^2 = 0.0432$ $\quad S_{y.x} = 9.84$ $\quad CV = 14.7\%$

d) natural pine stand regression/topographic correction:

$$HT = 48.43 + 0.3092*(h+t) \qquad (12)$$

$R^2 = 0.2265$ $\quad S_{y.x} = 8.84$ $\quad CV = 13.2\%$

e) plantation pine stand regression correction:

$$HT = 61.85 - 0.0963*h \tag{13}$$

$R^2 = 0.0954$ $S_{y.x} = 3.21$ CV = 5.82

f) plantation pine stand regression/topographic correction:

$$HT = 59.62 - 0.0679*(h+t) \tag{14}$$

$R^2 = 0.0672$ $S_{y.x} = 3.26$ CV = 5.9%

g) mixed pine and hardwood stand regression correction:

$$HT = 78.50 + 0.1221*h \tag{15}$$

$R^2 = 0.0625$ $S_{y.x} = 12.0$ CV = 13.6%

h) mixed pine and hardwood stand regression/topographic correction:

$$HT = 63.16 + 0.3147*(h+t) \tag{16}$$

$R^2 = 0.3392$ $S_{y.x} = 10.06$ CV = 11.4%

where:

HT = corrected average height from photo in feet

h = raw, average height of 4 tallest trees on the plot in feet

t = topographic correction in feet

The height data for the different stands was summarized in Table 1. The actual field height values, uncorrected photo height values, ranges of field heights, and differnce measures were examined. On the average, most heights were overestimated from the photos. On the most rugged terrain, however, these heights were underestimated.

Prior Distribution Development

Conservative empirical Bayesian prior distributions, based on a beta probability density function, were developed from the photo volume per acre information for each stand with each prior type. Varying the sample size of the photo cruise approximated various degrees of prior sampling intensity. The amount of photo information used for prior distribution development was 100%, 50%, and 25%.

TABLE 1. DIFFERENCES IN ESTIMATED PHOTO HEIGHTS AND FIELD HEIGHTS IN FEET

stand[1]	average height field	average height photo	average diff	average |diff|	range field height
PP	69.1	55.2	-13.9	14.8	50 - 60
NP	66.6	67.0	-0.5	19.0	45 - 80
HD	87.6	59.2	-28.4	28.5	40 - 70
MX	79.7	88.2	+8.5	18.4	60 - 105

(1) - The stands used in this table were:
PP -- plantation pine stand
NP -- natural pine stand
HD -- hardwood stand
MX -- mixed wood stand

A prior distribution developed from the specification of only the minimum, maximum, and modal volume per acre values was also examined using the uncorrected and regression corrected priors in each stand. Modal, minimum, and maximum values were converted to mean and variance values using the algorithms from PERT (Levin, Kirkpartick and Rubin, 1982):

$$\text{MEAN} = (\text{MIN} + 4*\text{MODE} + \text{MAX})/6 \qquad (17)$$

$$\text{VAR} = (\text{MAX} - \text{MIN})^2/36 \qquad (18)$$

where:

VAR, MEAN = variance and mean, respectively

MIN, MAX = minimum and maximum, respectively

MODE = modal value

The modal value was chosen from the photo volume data as a true mode or as a midpoint of the largest class in a frequency distribution. The minimum and maximum values are the actual minimum and maximum values from the prior data in question. Only modal prior developed from all prior information available were considered since selction of the mode would be complex and tedious for less than the full information prior.

Input Samples for Posterior Distribution Development

Independent volume per acre samples were taken from ground cruises in each of the four forest types. Since some of the ground plots were used in the height correction equations, the full gound field sample was never examined in order to minimize the possibility of non-independence between the prior and the posterior. The field sample sizes used were representative of different sampling intensities. These field volumes were the input "samples" for Bayesian calculations. From the data collected, 50%, 25%, 12.5%, and 6.25% of the plots were used. A diagram of the break down of the various data combinations is contained in Figure 1.

The following calculations were performed for the various ground samples within each of the forest type-prior accuracy-prior type combinations:

a) Bayesian estimation of average volume/acre and variance: The information acquired from each photo cruise-forest type-prior type combination was used as a prior distribution for Bayesian calculations (based on a computer program by Ek and Issos, 1976). The mean and variance from the posterior distribution were compared to the original, considered true, volume per acre field data from which the sample came. The mean and

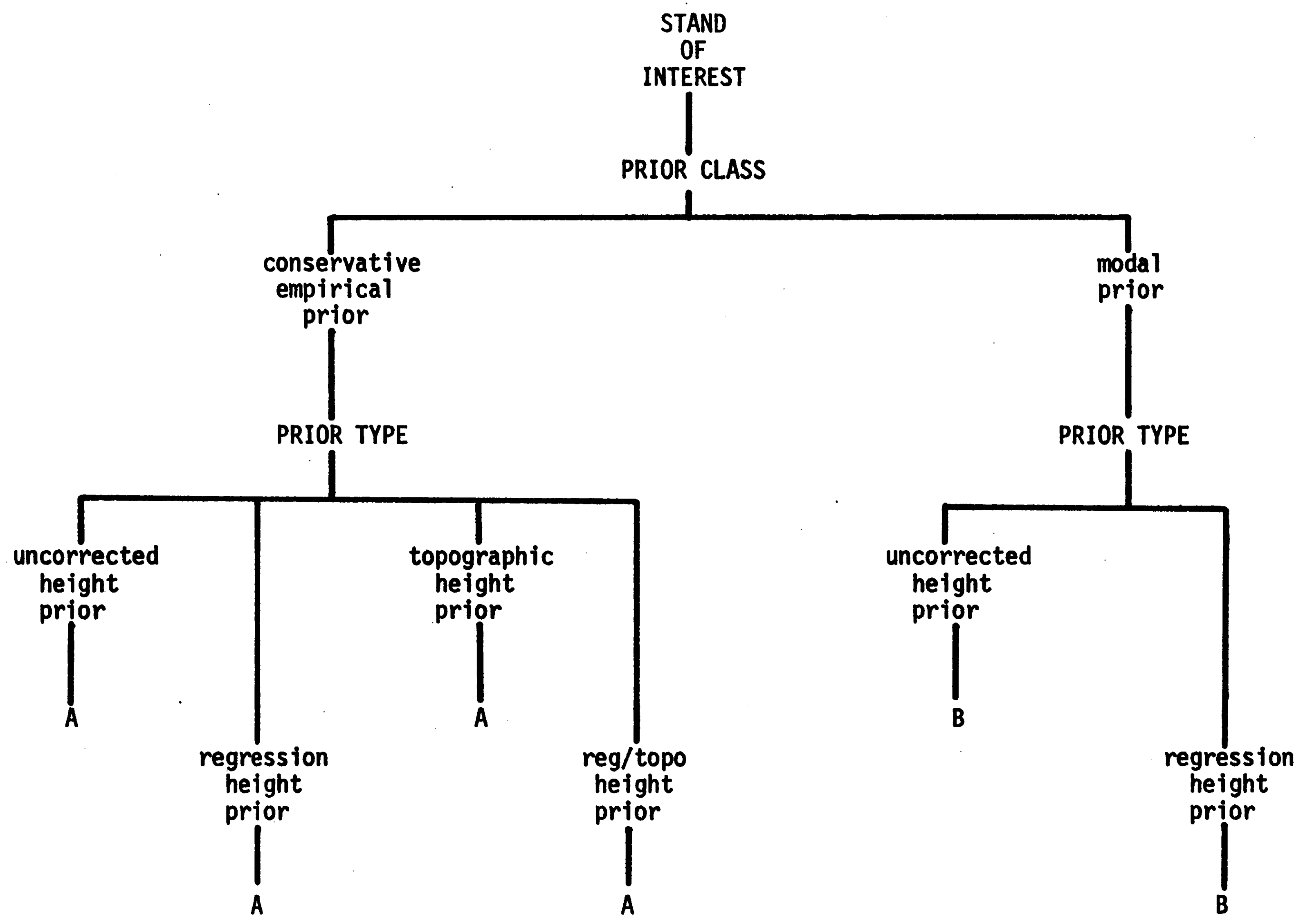

FIGURE 1a. DATA INPUT FOR BAYESIAN ESTIMATES FROM STAND TYPE TO PRIOR TYPE

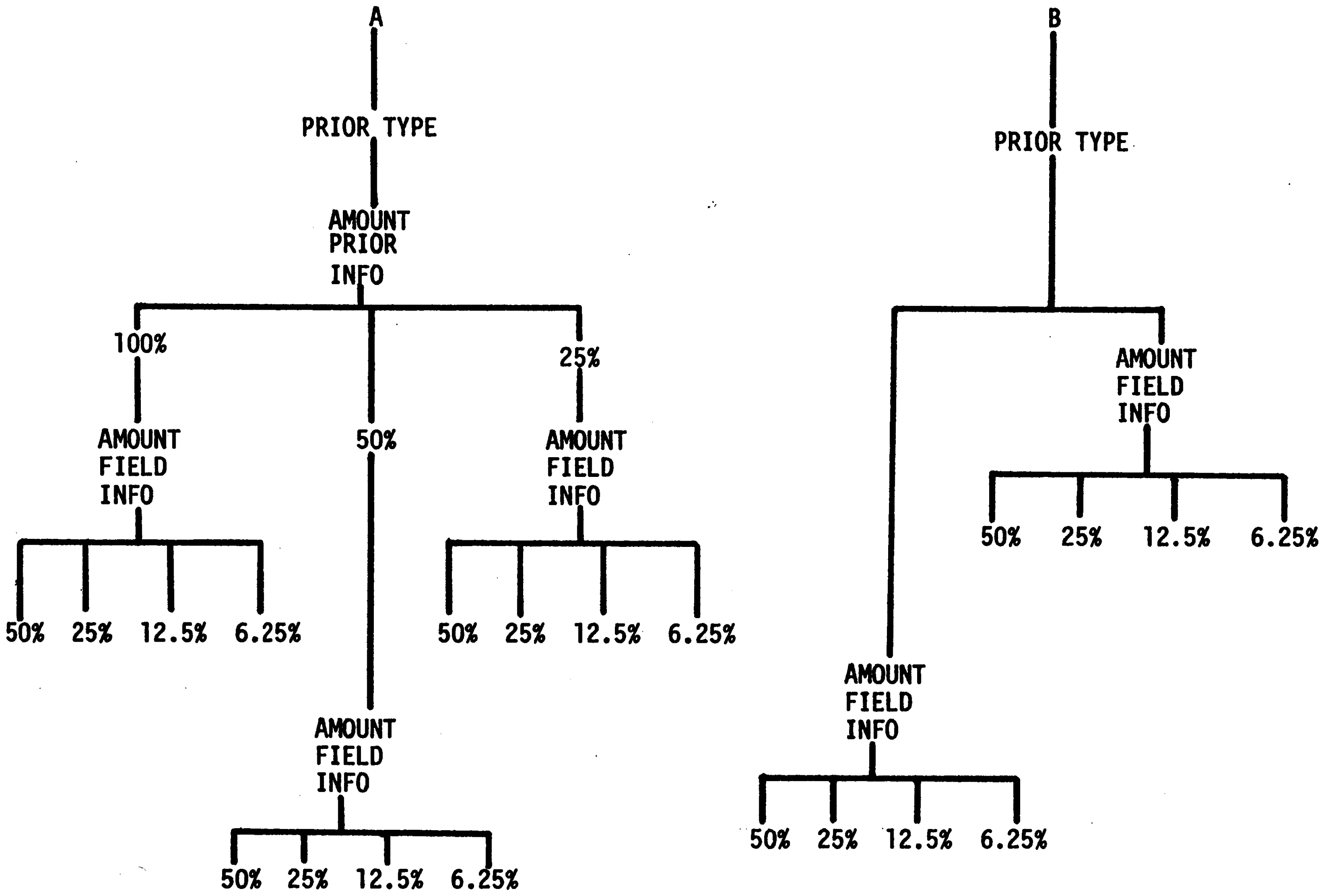

FIGURE 1b. DATA INPUT FOR BAYESIAN ESTIMATES FROM PRIOR TYPE TO AMOUNT OF FIELD INFORMATION USED

variance of the entire field sample were used as standards since these were the best estimates available.

b) Traditional estimation in the form of a simple random sample of the average volume per acre and variance: These estimates were based only on the ground sample.

The simulation process was repeated 1000 times with each combination of forest type, prior accuracy, prior type and sampling intensity. These simulations created the effect of repeated sampling. The FORTRAN program developed by Ek and Issos (1976) was modified to accommodate the changes in the program's running. These changes included the addition of the PERT equations and a random number generator in order to select observations for the various sample sizes. The simple random sampling subroutine in IMSL, GGSRS, was accessed in order to obtain the various field and/or photo samples required.

Computer Analysis Using Bayes Theorem

The computer program developed by Ek and Issos (1976), which was used to generate the various data described, based calculations of the prior and posterior distributions on a beta density function. The mean and variance of the distributions were found through an iterative algorithm approximating a method of moments estimation. Several options for the type of data input were available to the user of the program. In the course of this investigation, prior data consisted of either random observations or modal values. The refinement process, the development of a posterior distribution, consisted solely of independent observations.

Comparison Criteria

The estimates of gross cubic-foot volume per acre averaged over the 1000 iterations for each stand-prior type-prior data-field data combination were examined with respect to the proximity to the mean from the complete ground data set. To quantify the difference between the standard mean and the estimated mean, a standardized percent bias was computed for each of the 1000 iterations and then averaged over these iterations. For each of the iterations, the standardized percent bias was defined as:

$$\text{bias} = \frac{(\text{standard mean} - \text{estimated mean})*100}{(\text{standard mean})} \qquad (19)$$

where:

bias = percent bias in estimated value

standard mean = average gross cubic-foot volume per acre estimated from all field plots

estimated mean = average gross cubic-foot volume per acre estimated using Bayesian methods

Since the variability of an estimate can be as important as the estimate itself, the standard error for each of the previously mentioned combinations was examined. Lower standard errors were considered desireable.

The ratio of the Bayes 95% confidence interval to its non-Bayes counterpart was used as the comparison between the variation of the Bayesian estimate to the non-Bayes estimate's variation. A smaller ratio of the confidence iterval lengths indicated lower variation using Bayesian methods.

Another comparison was effective sample size (Bleier, et. al., 1986). Effective sample size was defined as the sample size needed with traditional estimation methods to obtain the same precision as that obtained with Bayesian estimation. Effective sample size was a function of the field sample size used and the confidence interval ratio of the Bayes 95% confidence interval to its traditional, non-Bayes counterpart. That is:

$$N^{*} = N/CIL^{2} \qquad (20)$$

where:

N^{*} = effective sample size

N = field sample size currently under consideration

CIL = ratio of the 95% Bayes confidence interval length to the 95% non-Bayes confidence interval length

Equation 20 is derived from the definition of the confidence interval length ratio. If one assumed that the Bayes standard deviation was equivalent to the non-Bayes standard deviation, then the only difference in the two standard errors would be the sample size used. One may then use this assumption to calculate the effective sample size. Expressed in an equation form,

$$N^{*} = (S/S_{o})^{2} \qquad (21)$$

where:

S = an estimate of the non-Bayes standard deviation

S_o = Bayesian standard error

N^* = sample size required to obtain S_o from S

S and S_o are calculated as:

S_o = (CI length ratio of Bayes to non-Bayes) * (standard error of the traditional sample)

S = (field sample size) * (standard error of the non-Bayes sample)

Equation 21 will reduce to equation 20 under the assumptions of normality of the observations and that the two standard deviations are equal.

RESULTS AND DISCUSSION

Results Using Conservative Empirical Priors

Table 2 contains the Bayesian means for the four stands examined. Although not shown in the table, the variation in the estimated values increased with decreasing field and photo sample size.

In the plantation pine stand, the Bayesian estimates of the mean volume per acre varied depending upon the prior type being used. The average mean value estimated by the interpreter specific prior was farther from the entire field sample mean than was the average mean for the uncorrected prior. The estimated means for all prior types were relatively close at the largest field sample sizes. This implies that less prior information can be used in uniform stands without affecting the estimate severely. However, as the field sample size decreased, the interpreter specific prior departed farther from the standard mean than the other priors. The regression, topographic, and regression/topographic priors were relatively close across all field and photo sample sizes.

For the natural pine stand, the mean of the uncorrected prior type was closer to the true value than the means of any other prior type for all field sample sizes. The regression and regression/topographic prior

TABLE 2a. PLANTATION PINE STAND - AVERAGE[1] BAYESIAN MEAN

sample size		prior type[2]				
field	photo	UC	REG	ISP	TOPO	R/T
25	26	1525.3	1522.5	1559.7	1503.9	1521.6
	13	1527.5	1531.3	1563.3	1505.5	1529.2
	6	1536.2	1547.2	1577.6	1512.8	1545.2
13	26	1575.6	1554.9	1639.0	1536.3	1553.6
	13	1579.1	1561.8	1646.2	1537.6	1559.7
	6	1593.1	1578.0	1673.2	1544.2	1575.2
6	26	1669.4	1588.8	1806.7	1589.7	1587.3
	13	1677.7	1596.1	1824.1	1593.6	1593.7
	6	1690.2	1601.2	1848.2	1599.8	1598.6
3	26	1814.4	1620.4	2035.6	1692.0	1619.1
	13	1824.2	1626.0	2054.5	1697.1	1624.1
	6	1839.2	1629.1	2074.1	1707.0	1626.5

* - The gross cubic foot volume per acre estimated from all field plots is 1483.95.

TABLE 2b. NATURAL PINE STAND - AVERAGE[1] BAYESIAN MEAN

sample size		prior type[2]				
field	photo	UC	REG	ISP	TOPO	R/T
17	20	1236.6	1455.8	1245.9	1269.2	1598.8
	10	1246.6	1877.0	1269.0	1274.7	1627.4
	5	1275.4	1922.2	1322.9	1297.5	1701.4
8	20	1256.7	1645.6	1278.8	1321.2	1877.9
	10	1275.3	2142.0	1321.7	1330.9	1899.8
	5	1321.0	2145.9	1406.8	1352.7	1927.2
4	20	1324.0	1878.1	1375.1	1407.8	2128.3
	10	1348.3	2290.8	1442.4	1421.4	2135.2
	5	1418.9	2294.4	1561.7	1467.9	2154.0
2	20	1451.6	2067.1	1559.5	1520.8	2269.3
	10	1497.5	2366.4	1673.4	1548.2	2277.9
	5	1582.3	2358.9	1794.6	1586.1	2273.7

* - The gross cubic foot volume per acre estimated from all field plots is 1218.87.

TABLE 2c. HARDWOOD STAND - AVERAGE[1] BAYESIAN MEAN

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	19	990.5	1063.3	1005.3	1038.2
18	10	1024.6	968.4	1008.4	1041.8
	5	1050.0	974.1	1023.4	1048.8
	19	1035.9	1099.6	1035.7	1073.1
9	10	1079.7	943.7	1039.8	1076.1
	5	1106.6	937.1	1048.9	1071.4
	19	1160.6	1129.6	1101.8	1105.8
4	10	1213.8	822.9	1116.0	1105.9
	5	1284.2	837.7	1142.3	1106.8
	19	1383.9	1150.5	1221.3	1134.8
2	10	1424.9	751.4	1246.2	1134.0
	5	1506.4	792.8	1286.1	1135.7

* - The gross cubic foot volume per acre estimated from all field plots is 990.45.

TABLE 2d. MIXED WOOD STAND - AVERAGE[1] BAYESIAN MEAN FOR FIRST PHOTO VOLUME EQUATION TYPE[3]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	17	2595.9	2572.8	2578.6	2571.2
16	9	2578.9	2563.7	2567.0	2566.3
	4	2538.7	2544.3	2532.6	2552.8
	17	2501.2	2502.0	2478.4	2499.5
8	9	2482.5	2492.3	2465.3	2493.2
	4	2454.7	2483.5	2444.9	2490.2
	17	2404.8	2446.7	2381.0	2443.4
4	9	2391.6	2451.0	2369.7	2448.0
	4	2370.2	2435.5	2365.3	2441.1
	17	2335.4	2405.5	2309.5	2401.3
2	9	2327.0	2403.1	2302.6	2401.4
	4	2249.9	2367.0	2249.6	2368.2

* - The gross cubic foot volume per acre estimated from all field plots is 2734.18.

TABLE 2e. MIXED WOOD STAND - AVERAGE[1] BAYESIAN MEAN FOR SECOND PHOTO VOLUME EQUATION TYPE[4]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	17	2721.3	2775.8	2719.0	2766.1
16	9	2713.2	2784.2	2714.6	2768.1
	4	2700.7	2803.3	2706.1	2783.9
	17	2704.2	2801.0	2700.5	2789.0
8	9	2690.9	2810.0	2692.8	2789.4
	4	2689.4	2848.6	2693.4	2810.9
	17	2672.0	2831.3	2668.2	2817.9
4	9	2656.8	2848.8	2657.8	2824.3
	4	2652.4	2878.2	2668.9	2857.3
	17	2650.3	2871.2	2646.0	2852.6
2	9	2639.7	2877.7	2634.5	2854.4
	4	2549.4	2861.4	2560.5	2828.5

* - The gross cubic foot volume per acre estimated from all field plots is 2734.18.

(1) - values averaged over 1000 iterations
(2) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation
- ISP -- interpreter specific equation using using uncorrected photo heights
- TOPO -- topographic corrected photo heights
- R/T -- appropriate regression equation with topographic corrected photo heights

(3) - photo volumes from Avery (1958)
(4) - photo volumes from a percent hardwood composition estimate and photo volume equations 5 and 6

types estimated mean values that were consistently several hundred cubic-feet higher than the standard.

For the hardwood stand, the regression prior type led to some atypical results. In general for these data, the Bayesian mean departed farther from the actual mean as the photo sample size or the field sample size decreased. The magnitude of the over/under estimation increased as the sample sizes decreased. However, for the regression prior in the hardwood stand, the Bayesian mean began as an overestimation of the actual mean, then became an underestimation with the 50% prior sample size, and then approached the standard mean at the 25% prior information level. This was attributed to the weak height correction equation.

The mixed wood stand presented several unique problems. The stand was on an obviously superior site with high volume per acre. The photo volume table used was developed for mixed woods in southern Mississippi which were comprised of shortleaf pine, loblolly pine, and upland hardwoods, species not found in the study area. A volume equation also had to be fitted to the original volume table in Avery (1958) using regression techniques in order to accommodate the larger tree sizes.

Using Avery's volume table for the mixed stand, all prior types underestimate the mean volume per acre from all field sample plots. At the 50% field sample size, the Bayesian estimators are approximately the same across the three photo sample sizes. The regression and regression/topographic priors, however, were closer to the standard mean under the smaller field sample sizes.

An alternative to Avery's volume table in the form of a ratio estimator was examined. An estimate of hardwood volume was made using the percent main canopy occupied by hardwood crowns as measured on the aerial photographs. This estimate of hardwood volume was then combined with photo volume equations 5 and 6 to obtain an estimate of gross cubic-foot volume per acre.

Using the ratio estimator in the mixed wood stand, the regression and regression/topographic prior types overestimated the mean while the other prior types underestimated the mean. For a given sample size and prior type, the estimated values digressed farther from the standard mean as photo sample size decreased.

Table 3 contains the average Bayesian standard errors for all the stand and empirical prior types. The Bayesian standard errors are at least equal to, if not greater than, the standard error obtained with the full field sample. However, a more valid comparison may be made with the average standard errors at each sampling level. In most cases, the average Bayes standard error is less than the non-Bayes counterpart.

The general trends shown in Table 3 were similar to

TABLE 3a. PLANTATION PINE STAND - AVERAGE[1] BAYESIAN STANDARD ERROR

sample size field	sample size photo	prior type[2] UC	REG	ISP	TOPO	R/T
25	26	140.7	126.0	140.5	141.7	126.3
	13	140.4	123.9	140.3	141.5	124.7
	6	140.1	119.7	140.5	140.7	120.5
13	26	193.7	158.1	197.6	195.0	158.9
	13	193.1	155.3	197.7	194.8	156.4
	6	196.7	149.8	203.1	196.4	151.4
6	26	292.0	195.5	321.0	288.7	197.3
	13	292.1	192.3	321.7	288.2	194.4
	6	291.0	181.3	318.3	284.3	183.9
3	26	396.0	215.6	444.4	390.5	218.6
	13	397.2	211.4	444.7	390.0	214.6
	6	396.3	204.4	436.2	391.0	207.7

* - The standard error for the entire field sample is 98.95 cubic feet/acre.

TABLE 3b. NATURAL PINE STAND - AVERAGE[1] BAYESIAN STANDARD ERROR

sample size field	sample size photo	prior type[2] UC	REG	ISP	TOPO	R/T
17	20	215.3	200.2	215.0	207.7	207.8
	10	213.9	204.3	213.6	207.1	208.1
	5	211.5	191.9	212.2	203.6	199.9
8	20	325.5	302.4	327.2	303.2	309.4
	10	323.7	246.6	328.5	302.8	300.7
	5	315.7	233.5	322.6	295.3	279.6
4	20	478.0	401.7	501.7	419.9	360.0
	10	475.0	256.2	504.0	420.2	350.0
	5	467.9	243.2	494.8	410.7	324.8
2	20	668.0	439.2	747.0	545.6	370.4
	10	666.4	255.7	738.2	541.7	358.0
	5	654.4	249.8	698.4	527.3	346.4

* - The standard error for the entire field sample is 147.79 cubic feet/acre.

TABLE 3c. HARDWOOD STAND - AVERAGE[1] BAYESIAN STANDARD ERROR

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
18	19	120.3	87.4	115.3	96.0
	10	115.5	119.5	115.2	94.0
	5	117.0	119.7	115.2	90.1
9	19	180.0	105.3	169.4	121.0
	10	174.7	192.6	169.8	118.5
	5	179.7	189.5	169.6	109.4
4	19	331.2	120.0	278.6	144.9
	10	326.2	331.5	281.7	140.1
	5	327.2	322.5	285.4	132.6
2	19	547.4	125.0	408.9	155.1
	10	518.0	418.8	413.0	149.1
	5	487.2	424.5	401.7	144.2

* - The standard error for the entire field sample is 79.90 cubic feet/acre.

TABLE 3d. MIXED WOOD STAND - AVERAGE[1] BAYESIAN STANDARD ERROR FOR FIRST PHOTO VOLUME EQUATION TYPE[3]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
16	17	303.3	264.2	298.8	264.5
	9	298.9	260.4	292.9	261.4
	4	281.2	249.2	276.8	250.8
8	17	393.8	316.7	383.7	317.2
	9	386.5	311.7	374.8	313.7
	4	363.7	297.0	353.1	300.8
4	17	466.4	355.1	449.8	355.7
	9	452.2	350.1	434.7	351.0
	4	437.1	336.1	414.1	339.2
2	17	512.6	387.0	494.0	387.7
	9	498.7	378.9	479.7	382.0
	4	456.7	357.7	443.8	362.7

* - The standard error for the entire field sample is 230.02 cubic feet/acre.

TABLE 3e. MIXED WOOD STAND - AVERAGE[1] BAYESIAN STANDARD ERROR FOR SECOND PHOTO VOLUME EQUATION TYPE[4]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	17	335.7	285.0	333.5	300.4
16	9	332.8	279.4	331.2	295.7
	4	325.5	264.2	324.0	278.4
	17	484.9	355.7	476.3	387.0
8	9	477.7	348.5	471.7	380.3
	4	466.6	330.7	460.0	358.7
	17	658.6	411.8	635.6	460.3
4	9	647.4	404.2	633.1	455.9
	4	641.5	379.8	613.6	422.9
	17	754.7	443.8	721.7	502.2
2	9	748.9	430.6	717.5	489.7
	4	722.4	406.1	705.1	466.0

* - The standard error for the entire field sample is 230.02 cubic feet/acre.

(1) - values averaged over 1000 iterations
(2) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation
- ISP -- interpreter specific equation using using uncorrected photo heights
- TOPO -- topographic corrected photo heights
- R/T -- appropriate regression equation with topographic corrected photo heights

(3) - photo volumes from Avery (1958)
(4) - photo volumes from a percent hardwood composition estimate and photo volume equations 5 and 6

the trends from Table 2. Important to note was the trend that with decreasing field sample size, standard error increased -- very similar to traditional statistical theory. An interesting trend shown in Table 3 that encompassed all prior types was with decreasing prior sample size, the average Bayesian standard error decreased also. This may be explained by considering that as the amount of prior information used decreased, the inclusion of outlying observations became less frequent and hence, standard errors were smaller.

In all stands, lower standard errors were achieved with the regression and regression/topographic priors. In the natural pine stand and in the hardwood stand, however, as the sample sizes decreased, the regression/topographic prior produced much lower standard errors than the regression prior counterparts. This indicates that the topographic correction when combined with the regression correction in a prior can improve the results of a Bayesian estimate.

The interpreter specific prior type performed as well as the uncorrected and topographic prior types in the plantation pine stand. However, the regression and regression/topographic prior types had smaller average standard errors.

In the natural pine stand, the interpreter specific prior performed worse that the other prior types within the smaller prior and field sample sizes. Lower average standard errors were calculated using the regression and regression/topographic prior types. With the larger field sample size, the topographic prior type was comparable to the regression and regression/topographic prior types.

For the hardwood stand, both the regression prior types had smaller computed standard error. The regression/topographic had smaller values for the smaller prior sample sizes than for the larger photo sample sizes.

Using the photo volume table from Avery (1958) for the mixed wood stand, the regression and regression/topographic prior types yielded lower standard errors than the uncorrected and topographic prior types. Topography did not seem either to enhance or to detract from the regression prior types.

Using the ratio estimator in the mixed wood stand, the regression prior type had consistently smaller average standard errors. Even the regression/topographic prior type did not approach the regression prior type. In this case, the topography component of the regression/topographic prior type appeared to lessen the estimation ability of the methodology.

In order to better utilize the information given in Table 3, plots were constructed from the average standard error and field sample sizes. For each prior sample size and stand type, graphs were drawn of the average standard error from each of the prior types against the field sample size. It was found that within each stand type the three plots separated on amount of prior information used had the same general shape. The only difference among these three prior size plots was the y-intercept, or rather the worst possible average standard error calculated. Since these curves were so similar, only the 50% prior information curves were included for interpretation. From these curves, the field sample size was determined for the various prior types for a given "desireable" standard error.

For example, in the plantation pine stand, if a standard error of 175 cubic-feet per acre was desired, using an uncorrected or topographic prior type, eighteen field plots were required to obtain that level of precision. With the regression or regression/topographic prior types only ten plots were necessary.

For the plantation pine stand (Figure 2a), the lines for the regression and regression/topographic prior types are practically coincident. The steeper sloped lines of the uncorrected, interpreter specific, and topographic prior types vary slightly at the smaller field sample sizes but then became coincident as the field sample size increases. If the lines were coincident, then for the

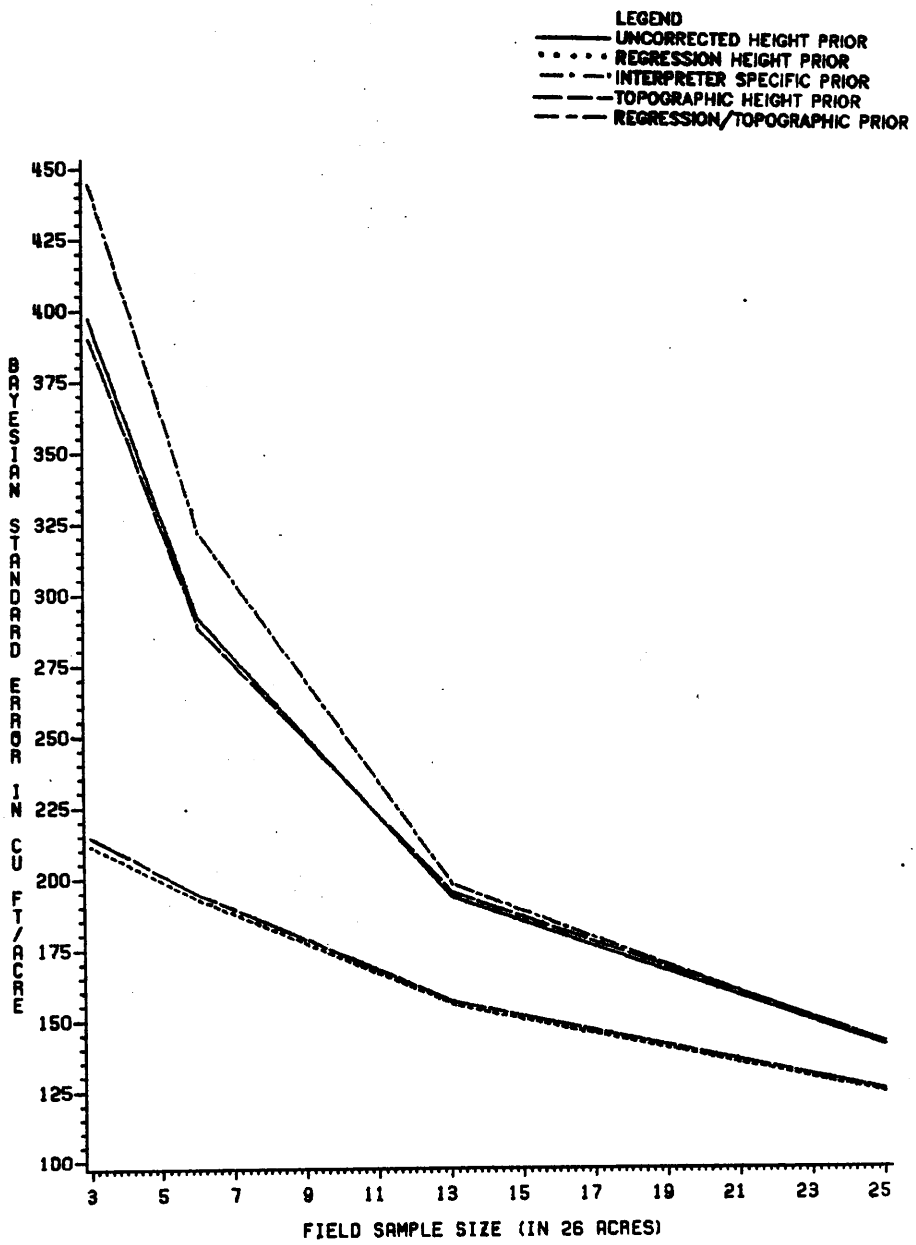

FIGURE 2a. PLANTATION PINE STAND - 50% PRIOR INFORMATION
BAYESIAN STANDARD ERROR VS. FIELD SAMPLE SIZE

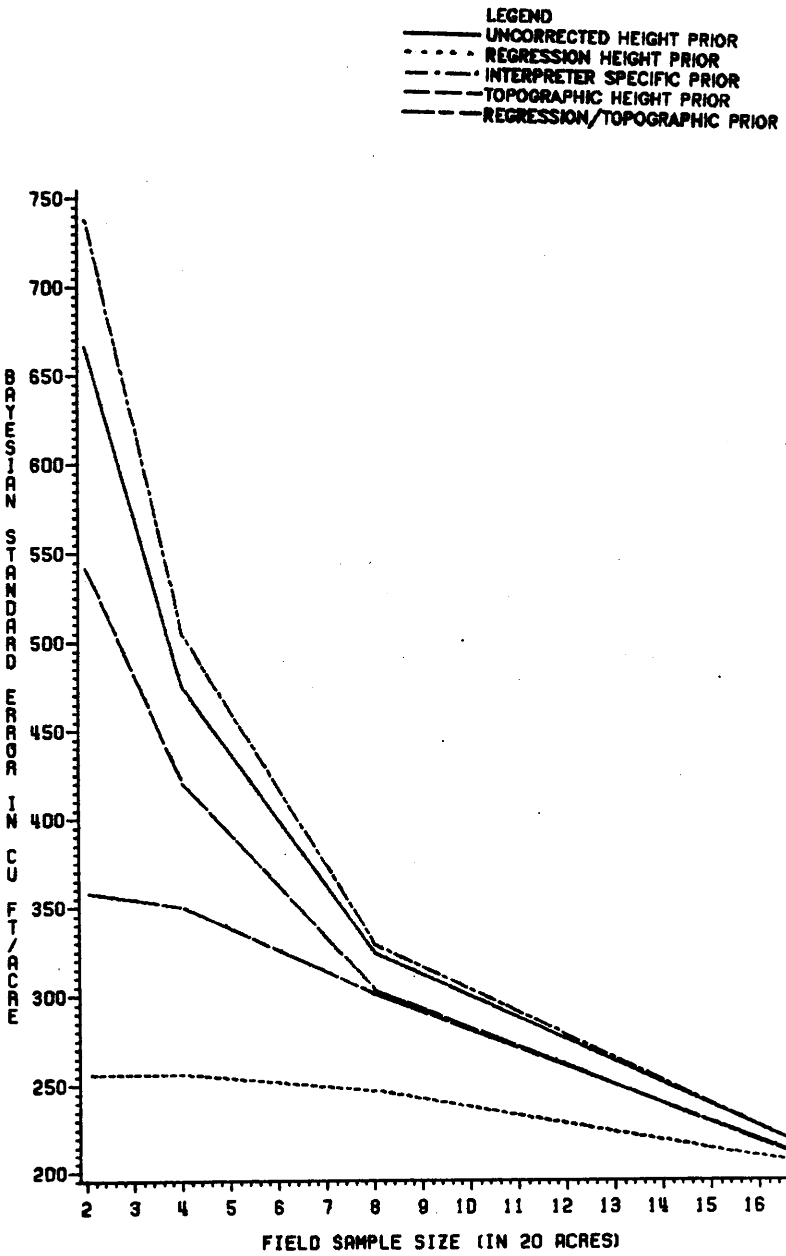

FIGURE 2b. NATURAL PINE STAND - 50% PRIOR INFORMATION
BAYESIAN STANDARD ERROR VS. FIELD SAMPLE SIZE

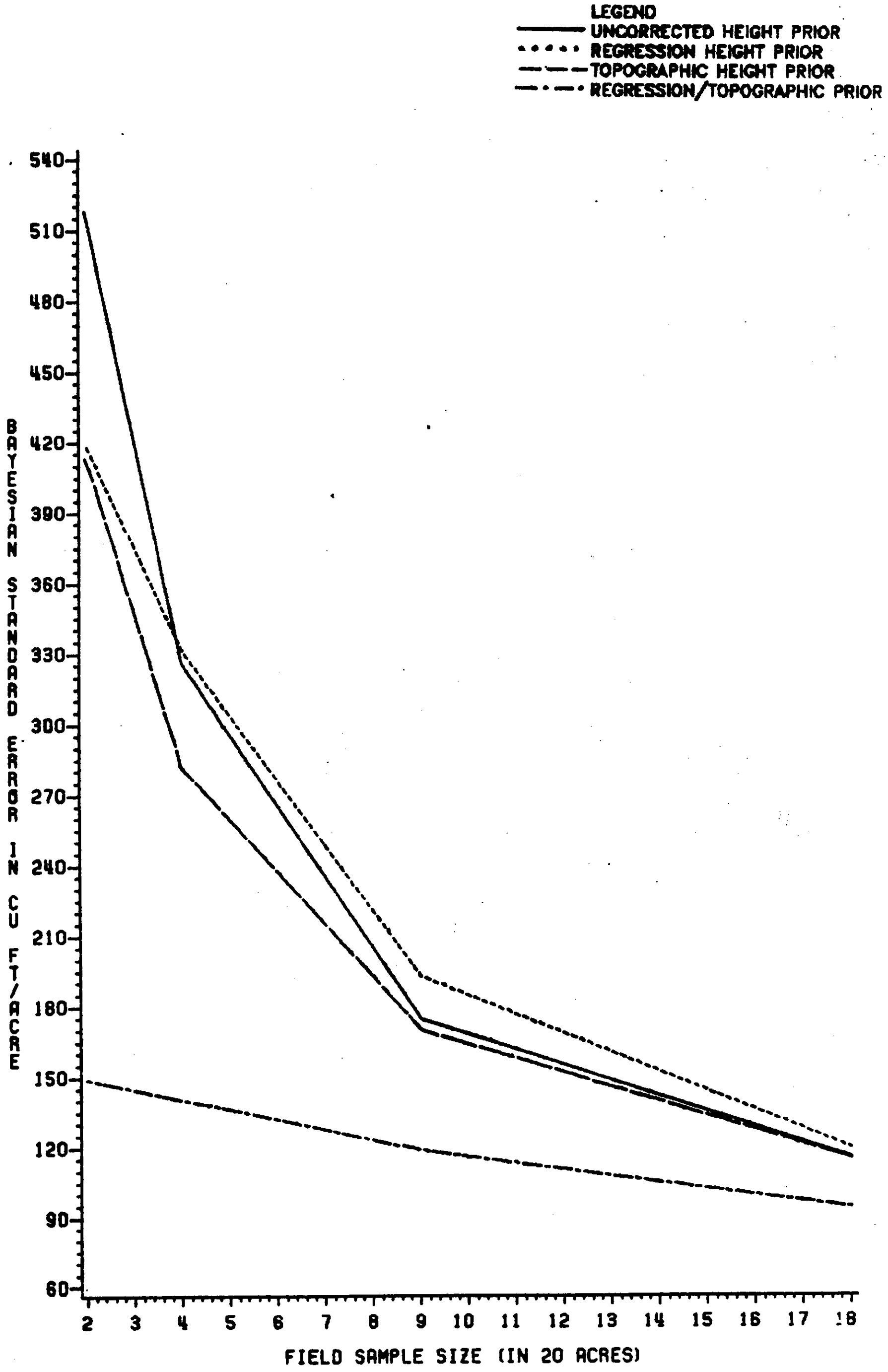

FIGURE 2c. HARDWOOD STAND - 50% PRIOR INFORMATION
BAYESIAN STANDARD ERROR VS. FIELD SAMPLE SIZE

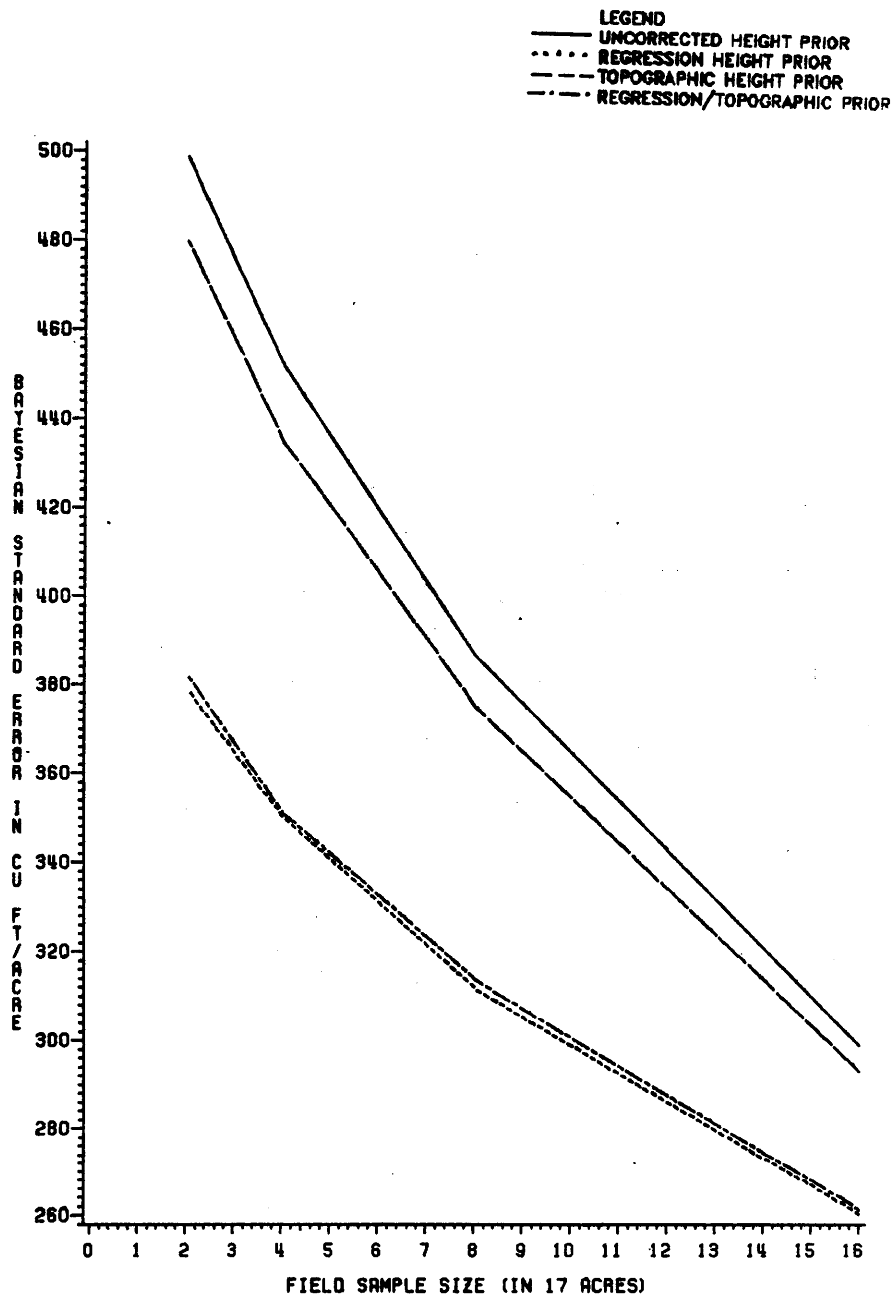

FIGURE 2d. MIXED WOOD STAND - 50% PRIOR INFORMATION
FIRST VOLUME EQUATION TYPE
BAYESIAN STANDARD ERROR VS. FIELD SAMPLE SIZE

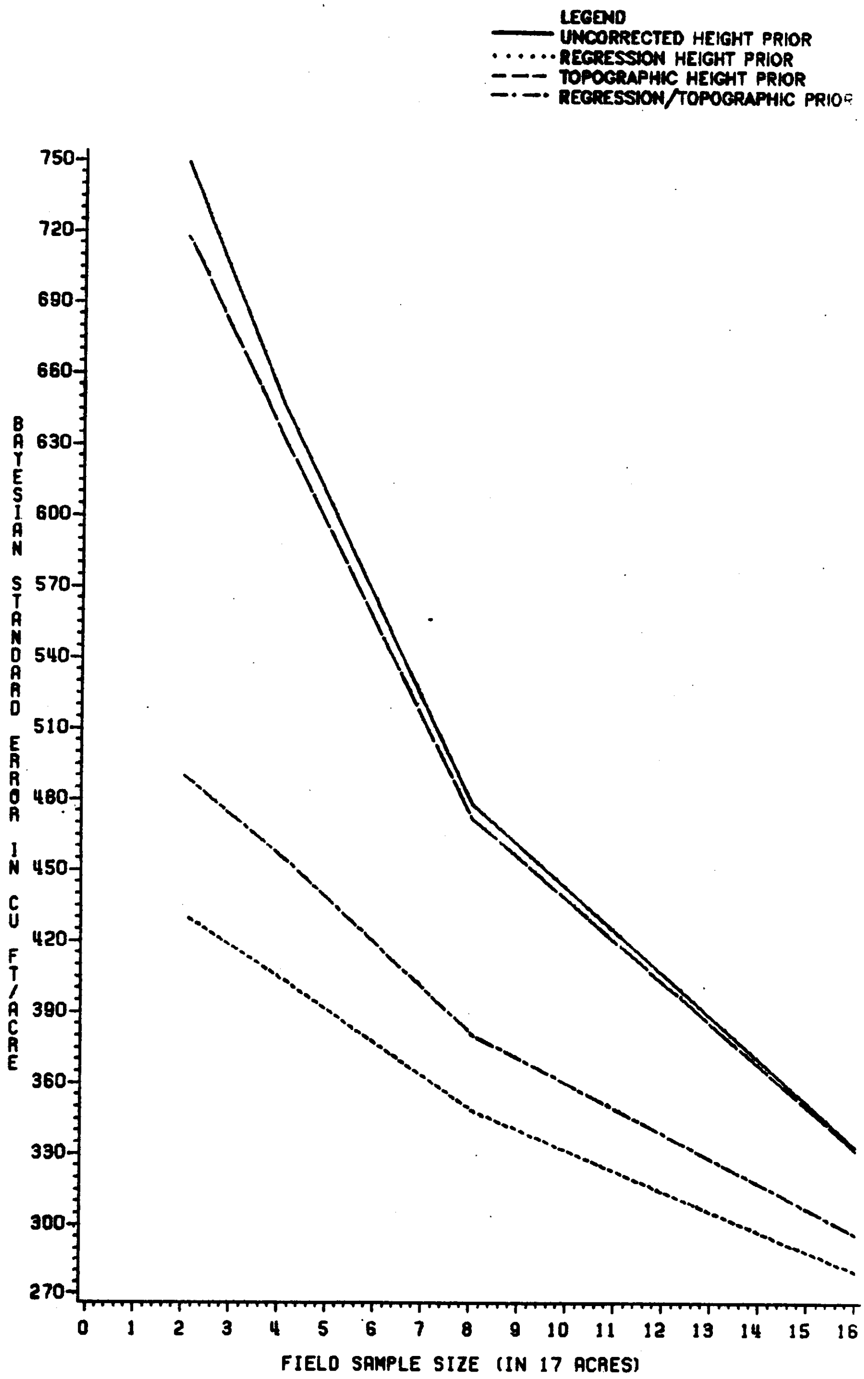

FIGURE 2e. MIXED WOOD STAND - 50% PRIOR INFORMATION
SECOND VOLUME EQUATION TYPE
BAYESIAN STANDÅRD ERROR VS. FIELD SAMPLE SIZE

same standard error, the more easily obtained information could be used to determine cubic-foot volume per acre with the same standard error.

For the natural pine stand (Figure 2b), the type of prior selected had a great effect on the precision of the estimate using the smaller field sample sizes. As the field sample size increased, the interpreter specific and uncorrected prior types and the topographic and regression/topographic prior types became coincident. The regression prior type had the least slope and was consistently lower than the other prior types.

In the hardwood stand (Figure 2c), the regression/topographic prior type was much lower than the other three prior types. There was very little variation in the regression/topographic prior type's average standard error.

Using Avery's photo volume table for the mixed wood stand (Figure 2d), the lines for the regression and regression/topographic prior types are nearly coincident. There was a noticeable difference between the lines for the uncorrected and topographic prior types. However, the intercepts for these latter two lines were much higher than for the former two lines.

Using the ratio estimator in the mixed wood stand (Figure 2e), the two lines for the uncorrected and topographic prior types are practically coincident. The regression and regression/topographic prior types plotted different lines, although, both are less than the former two prior types. The regression prior had lower average standard error over the possible field sample sizes.

Table 4 shows the ratio of confidence interval lengths for all stand and empirical prior types. As the sample size decreased so did the estimate of the average ratio of the confidence interval length, indicating that the average Bayesian standard error was less than the average sample standard error. This implied that the influence of the prior was greater at the smaller sample sizes and had a reducing effect on the sampling error.

Upon examination of the values in Table 4, it was also evident that the various prior types influenced the precision of the Bayesian estimator. In most cases, the regression corrected prior produced smaller confidence intervals than their counterparts in the other prior types.

TABLE 4a. PLANTATION PINE STAND - AVERAGE[1] RATIO OF CONFIDENCE INTERVAL LENGTHS[2]

sample size		prior type[3]				
field	photo	UC	REG	ISP	TOPO	R/T
	26	.967	.866	.965	.976	.870
25	13	.966	.854	.965	.974	.859
	6	.965	.826	.967	.970	.831
	26	.934	.769	.949	.945	.774
13	13	.934	.758	.951	.944	.763
	6	.933	.717	.958	.934	.725
	26	.849	.590	.922	.847	.596
6	13	.851	.581	.928	.846	.587
	6	.854	.558	.933	.838	.565
	26	.610	.361	.707	.593	.365
3	13	.616	.356	.717	.594	.360
	6	.605	.337	.689	.583	.342

TABLE 4b. NATURAL PINE STAND - AVERAGE[1] RATIO OF CONFIDENCE INTERVAL LENGTHS[2]

sample size		prior type[3]				
field	photo	UC	REG	ISP	TOPO	R/T
	20	.984	.951	.983	.948	.939
17	10	.978	.929	.975	.945	.945
	5	.966	.873	.967	.929	.906
	20	.937	.754	.939	.869	.915
8	10	.932	.740	.943	.868	.887
	5	.919	.710	.936	.857	.838
	20	.775	.510	.801	.703	.707
4	10	.774	.501	.822	.708	.684
	5	.768	.484	.820	.698	.638
	20	.319	.269	.360	.334	.359
2	10	.347	.258	.409	.336	.347
	5	.410	.277	.486	.361	.386

TABLE 4c. HARDWOOD STAND - AVERAGE[1] RATIO OF CONFIDENCE INTERVAL LENGTHS[2]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
18	19	.981	.742	.981	.815
	10	.981	1.016	.979	.798
	5	.988	1.015	.976	.764
9	19	.955	.592	.944	.677
	10	.964	1.074	.945	.664
	5	.994	1.065	.946	.618
4	19	.874	.381	.797	.452
	10	.907	.918	.805	.438
	5	.942	.918	.829	.420
2	19	.546	.217	.444	.252
	10	.553	.516	.451	.244
	5	.761	.538	.495	.267

TABLE 4d. MIXED WOOD STAND - AVERAGE[1] RATIO OF CONFIDENCE INTERVAL LENGTHS[2] FOR FIRST PHOTO VOLUME EQUATION TYPE[4]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
16	17	.884	.769	.871	.770
	9	.871	.758	.853	.761
	4	.819	.726	.806	.730
8	17	.757	.612	.738	.613
	9	.744	.602	.721	.606
	4	.694	.568	.674	.574
4	17	.542	.432	.527	.433
	9	.527	.426	.514	.428
	4	.507	.410	.487	.409
2	17	.281	.290	.287	.291
	9	.301	.289	.283	.290
	4	.270	.256	.248	.263

TABLE 4e. MIXED WOOD STAND - AVERAGE[1] RATIO OF CONFIDENCE INTERVAL LENGTHS[2] FOR SECOND PHOTO VOLUME EQUATION TYPE[5]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
16	17	.981	.831	.974	.876
	9	.972	.814	.967	.862
	4	.950	.770	.945	.812
8	17	.935	.683	.918	.745
	9	.920	.673	.909	.733
	4	.888	.632	.876	.687
4	17	.742	.476	.716	.526
	9	.731	.467	.713	.521
	4	.719	.436	.692	.482
2	17	.291	.258	.281	.256
	9	.293	.247	.283	.257
	4	.283	.239	.280	.242

(1) - average value over 1000 iterations
(2) - ratio of Bayes 95% CI to non-Bayes 95% CI
(3) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation
- ISP -- interpreter specific equation using using uncorrected photo heights
- TOPO -- topographic corrected photo heights
- R/T -- appropriate regression equation with topographic corrected photo heights

(4) - photo volume from Avery (1958)
(5) - photo volume from a percent hardwood composition estimate and photo volume equations 5 and 6

Table 5 shows the effective sample size values for all stands and empirical prior types. The influence of the prior was small at the full prior information level and larger field sample sizes. It was also evident that the stands with greater homogeneity, such as the plantation pine stand or even the hardwood stand, benefited more from Bayesian estimation methods. The gains in these homogeneous stands were greater than those in the heterogeneous stands (the natural pine stand and the mixed wood stand).

The type of prior chosen also had an effect on the gains in sample size. From Table 5, it was evident that a regression corrected prior would lead to larger gains in sample size reduction or in precision reduction.

In the plantation pine stand, effective sample size was greatest for the regression prior type. The interpreter specific prior was either poorer or equal to the other prior types involved.

For the natural pine stand, the interpreter specific photo prior type performed as well as the uncorrected prior type. However, for the smaller field sample sizes, the uncorrected had more noticeable gains.

For largest gains in sample size in the hardwood stand, the regression/topographic prior type was the best. Trends suggested that as field or photo sample size decreased, the effective sample size increased.

TABLE 5a. PLANTATION PINE STAND - AVERAGE[1] EFFECTIVE SAMPLE SIZE[2]

sample size		prior type[3]				
field	photo	UC	REG	ISP	TOPO	R/T
25	26	26.8	33.4	26.9	26.3	33.1
	13	26.8	34.7	26.9	26.3	34.3
	6	26.9	38.2	26.8	26.6	37.8
13	26	14.9	22.4	14.5	14.6	22.1
	13	14.9	23.7	14.4	14.6	23.3
	6	15.1	28.1	14.5	15.0	27.5
6	26	8.6	19.3	7.4	8.5	18.9
	13	8.6	20.8	7.4	8.6	20.2
	6	8.8	26.4	7.8	9.1	25.8
3	26	11.6	41.2	10.6	11.1	40.1
	13	12.0	47.7	11.1	11.4	45.6
	6	13.7	62.4	13.4	12.9	59.6

TABLE 5b. NATURAL PINE STAND - AVERAGE[1] EFFECTIVE SAMPLE SIZE[2]

sample size		prior type[3]				
field	photo	UC	REG	ISP	TOPO	R/T
17	20	17.5	19.7	17.6	18.9	19.5
	10	17.8	20.8	17.9	19.1	19.6
	5	18.4	25.6	18.4	20.1	23.7
8	20	9.1	17.5	9.1	10.6	11.6
	10	9.3	19.1	9.1	10.8	12.5
	5	9.9	26.3	9.7	11.5	18.0
4	20	6.9	28.8	6.4	8.9	15.6
	10	7.3	31.6	6.8	9.2	17.5
	5	8.5	46.6	8.4	11.5	29.2
2	20	43.0	448.0	35.6	75.1	227.7
	10	53.0	503.2	50.0	82.7	265.2
	5	74.2	730.2	79.6	119.8	458.3

TABLE 5c. HARDWOOD STAND - AVERAGE[1] EFFECTIVE SAMPLE SIZE[2]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
	19	18.7	32.9	18.7	27.2
18	10	18.7	17.4	18.7	29.0
	5	18.6	17.5	19.0	34.3
	19	9.9	27.0	10.1	20.2
9	10	9.7	7.8	10.1	22.2
	5	9.4	8.0	10.1	30.3
	19	5.4	38.4	6.5	25.6
4	10	5.3	5.9	6.4	30.6
	5	5.5	5.8	6.6	41.1
	19	26.3	515.9	38.3	319.9
2	10	29.3	76.7	39.4	401.6
	5	47.4	71.7	58.0	603.4

TABLE 5d. MIXED WOOD STAND - AVERAGE[1] EFFECTIVE SAMPLE SIZE[2] FOR FIRST PHOTO VOLUME EQUATION TYPE[4]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
	17	20.5	27.3	21.2	27.2
16	9	21.4	28.5	22.5	28.2
	4	30.1	37.7	28.0	17.8
	17	14.4	22.7	15.2	22.7
8	9	15.6	24.2	16.8	23.8
	4	27.4	41.9	24.3	35.1
	17	17.7	32.4	19.4	32.3
4	9	20.4	35.0	23.1	34.5
	4	36.7	53.0	37.1	58.5
	17	221.2	438.3	247.4	437.0
2	9	262.3	472.5	294.9	461.8
	4	822.9	728.5	544.4	723.5

TABLE 5e. MIXED WOOD STAND - AVERAGE[1] EFFECTIVE SAMPLE SIZE[2] FOR SECOND PHOTO VOLUME EQUATION TYPE[5]

sample size		prior type[3]			
field	photo	UC	REG	TOPO	R/T
	17	16.6	23.2	16.7	20.7
16	9	17.0	24.7	17.1	21.9
	4	18.4	34.3	18.4	43.7
	17	9.2	17.6	9.5	14.7
8	9	9.6	19.1	9.9	15.8
	4	11.5	26.2	11.5	31.3
	17	7.7	22.3	8.4	17.3
4	9	8.6	24.7	8.9	18.8
	4	11.5	42.2	12.1	46.6
	17	70.1	272.9	78.0	199.7
2	9	82.7	335.4	89.7	248.3
	4	154.3	656.4	160.5	864.9

(1) - average value over 1000 iterations
(2) - actual field sample size divided by the square of the appropriate confidence interval length ratio
(3) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation
- ISP -- interpreter specific equation using using uncorrected photo heights
- TOPO -- topographic corrected photo heights
- R/T -- appropriate regression equation with topographic corrected photo heights

(4) - photo volume from Avery (1958)
(5) - photo volume from a percent hardwood composition estimate and photo volume equations 5 and 6

Atypical results utilizing the regression prior type were observed in this stand.

For the mixed wood stand using Avery's (1958) photo volume table, consistently larger gains were obtained using the regression/topographic prior. For the mixed wood stand using the ratio estimator, the regression priors had the largest gains.

Results Using Modal Priors

Table 6 contains the values of the Bayesian mean, Bayesian standard error, confidence interval length ratio and effective sample size for an uncorrected and regression corrected modal prior. Many of the trends that were discussed earlier are contained within this table. It is interesting to note that with these modal priors, although not shown, the variations in the Bayesian mean and Bayesian standard error were not as great as in the empirical prior cases. In addition to the reduced variation, the Bayesian standard errors are smaller than their counterparts in the empirical prior types. The regression corrected modal prior performs better than its uncorrected counterpart.

In the plantation pine stand, the average mean using a regression prior type was closer to the standard mean than the uncorrected prior type. The largest gain

TABLE 6a. PLANTATION PINE STAND - MODAL INFORMATION PRIOR

field sample	prior type[1]	average comparison values[2] BM	BS	CIL	EFS
25	UC	1746.7	114.4	.756	44.1
	REG	1529.7	124.5	.854	34.4
13	UC	1883.3	179.0	.820	19.5
	REG	1560.4	150.1	.727	25.4
6	UC	2130.4	299.2	.860	9.5
	REG	1585.6	174.5	.524	26.1
3	UC	2345.0	367.3	.648	18.6
	REG	1604.7	183.6	.311	65.5

TABLE 6b. NATURAL PINE STAND - MODAL INFORMATION PRIOR

field sample	prior type[1]	average comparison values[2] BM	BS	CIL	EFS
17	UC	1342.6	194.3	.873	22.4
	REG	2072.8	105.5	.433	92.1
8	UC	1468.2	296.6	.808	12.3
	REG	2153.4	141.4	.387	61.1
4	UC	1687.6	485.5	.787	8.0
	REG	2205.0	158.0	.286	84.2
2	UC	2016.4	699.5	.521	60.1
	REG	2234.9	165.0	.153	1201.4

TABLE 6c. HARDWOOD STAND - MODAL INFORMATION PRIOR

field sample	prior type[1]	average comparison values[2] BM	BS	CIL	EFS
18	UC	1409.7	49.8	.325	170.7
	REG	1048.5	62.3	.468	88.5
9	UC	1456.6	100.6	.416	53.2
	REG	1067.6	74.4	.372	73.0
4	UC	1565.7	222.6	.511	20.2
	REG	1080.7	82.8	.237	107.0
2	UC	1696.3	348.3	.837	108.4
	REG	1090.2	85.7	.107	1456.7

TABLE 6d. MIXED WOOD STAND - MODAL INFORMATION PRIOR USING FIRST PHOTO VOLUME EQUATION TYPE[3]

field sample	prior type[1]	average comparison values[2] BM	BS	CIL	EFS
16	UC	2560.3	259.6	.749	28.7
	REG	2460.9	200.6	.574	49.3
8	UC	2473.7	311.6	.592	24.0
	REG	2400.1	217.9	.414	51.6
4	UC	2399.9	348.7	.416	34.3
	REG	2359.8	226.5	.276	87.6
2	UC	2333.9	370.4	.180	1312.7
	REG	2329.4	227.1	.286	470.3

TABLE 6e. MIXED WOOD STAND - MODAL INFORMATION PRIOR USING SECOND PHOTO VOLUME EQUATION TYPE[4]

field sample	prior type[1]	average comparison values[2]			
		BM	BS	CIL	EFS
16	UC	2657.6	331.3	.967	17.1
	REG	2887.3	261.0	.753	28.7
8	UC	2579.9	465.4	.897	10.0
	REG	2942.0	297.7	.559	28.2
4	UC	2475.3	591.7	.664	10.5
	REG	2988.7	318.2	.357	45.1
2	UC	2408.2	650.5	.274	123.6
	REG	3026.3	322.1	.215	650.8

(1) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation

(2) - comparison values averaged over 1000 iterations defined as:
- BM -- Bayesian mean of the gross cubic foot volume per acre
- BS -- Bayesian standard error of the mean
- CIL -- ratio of the Bayes 95% confidence interval to its non-Bayes counter-part
- EFS -- the actual field sample size divided by the square of the appropriate confidence interval length ratio

(3) - photo volumes from Avery (1958)

(4) - photo volumes from a percent hardwood composition estimate and photo volume equations 5 and 6

in effective sample size was obtained using the regression prior type. The ratio of the confidence interval lengths was generally smaller than the empirical prior counterparts.

The uncorrected prior type in the natural pine stand approached and then departed from the standard mean. The regression prior type mean was several hundred cubic-feet greater than the standard mean. Effective sample size was greater in modal priors than in empirical prior counterparts. Greater gains were obtained with the smaller field sample sizes.

For the hardwood stand, the Bayesian mean varied little over the 1000 iteratons for the regression prior type. This regression prior type was closer to the standard mean. The average standard error for the regression prior type was lower than its empirical counterpart. In the previous discussion, it was stated that the observed trend for ratio of confidence interval lengths was one that decreased with decreaing sample sizes. However in this stand, the uncorrected prior type created a ratio that increased with increasing sample size.

In the mixed wood stand that utilized Avery's photo volume table (1958), both priors underestimated the standard mean. Generally, the same values were obtained for both the modal and empirical priors. Effective sample size increased with decreasing field sample size.

Using the ratio estimator in the mixed wood stand, the uncorrected prior type underestimated while the regression prior type overestimated the standard mean. Larger gains were obtained with the smaller field sample sizes. The standard error using the modal priors was comparable to the empirical counterparts.

Input and Output Biases

Attempts were made to remove tree height bias through the use of topographic corrections and regression corrections. By using only a regression correction, many of the photo heights were fixed to a constant. In all cases, the regression equations fit to the data were not very strong. Theoretically in a simple linear regression of the photo heights against the field heights, the y-intercept should have been close to zero and the coefficient of the photo height should have been approximately one. In the case of the hardwood stand, the photo height coefficient was .02 which was not significantly different from zero. This may account for the erratic results using the regression corrected prior in the hardwood stand.

Using both the topographic correction and a regression equation, the correlation between the photo heights and actual field heights increased. In several cases, the coefficient of photo height increased to .30. This indicated that the choice of a reference point played a role in the estimation of tree height from photographs.

The choice of volume equation can cause error in the prior information. Since all of the photo volume equations were developed outside of the study area, there was potential for severe errors in volume estimation. The age of the photographs and terrain could also cause errors.

In the plantation pine stand, the average error in the height measurement was -13.9 feet (t = -6.5) or approximately a 25% overestimation. Combining the photo measured height with the topographic correction gave an average error of -10.0 feet (t = -3.7) which was approximately 19% overestimation.

As far as volume equation errors in the plantation pine stand, the range was from -100 to -1109 cubic-feet per acre. Using equation 6 with either regression or regression and tographic corrected heights yields unbiased volume estimates. Uncorrected, topo corrected, and the interpreter specific volume estimates were biased.

For the natural pine stand, the same four variations of height were used as inputs to equation 6. The volume errors range from -1378 to -1005 cubic-feet per acre. All volume estimates were biased and the appropriateness of this volume equation for this stand was in question.

The appropriateness of equation 6 for the natural pine stand is even more questionable when the heights for the stand are examined. The average error in the measured photo heights is very close to zero for both the raw and topographic corrected heights.

In the hardwood stand, the average error in height estimation is -28 feet ($t = -6.7$) or approximately a 52% overestimation for the raw heights. When the topographic correction is accounted for, this average error is reduced to -17 feet ($t = -3.4$) or 35% overestimation.

The average photo volume errors for the hardwood stand ranged from -241 to -1410 cubic-feet per acre (or -379% to -131%) using the various height corrections in equation 5. The photo volume equation may be biased for the stand in the study area since the equation 5 assumes a Girard form class of 77. The estimates could also be biased in the crown diameter measurements. Since the leaves had not budded out, determination of the crown edges was difficult and most likely underestimated.

With respect to the photo height errors of the mixed wood stand, both the raw and topo corrected heights were unbiased.

The volumes generated from the different corrected and uncorrected heights used in equation 8 were also unbiased. These volumes, however, tended to underestimate the actual volume per acre.

The alternative to using equation 8 for the mixed wood stand provided volume estimates that for the various height corrections were unbiased. However, the estimates of average volume per acre were closer to the mean value from all field plots.

A comparison of the field sample mean to the Bayesian mean was examined for bias in each of the prior type-prior size-field sample size combinations (Tables 7 and 8). In almost all cases, the average error was significantly different from zero. However, considering the resulting average errors, these errors maybe acceptable given the errors and biases associated with the prior information. One thing to note, however, was all volume inputs for the mixed wood stand were statistically unbiased. As expected, the first photo volume equation used gave biased results since it was not near the actual mean.

For the plantation pine stand, the interpreter specific prior type was more biased than the other prior types considered. The bias increased as the sample sizes decreased. All values within Table 7a are biased.

The natural pine stand (Table 7b), all values were biased. The uncorrected prior type was least biased. In

TABLE 7a. PLANTATION PINE STAND - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEAN

sample size field	photo	prior type[2] UC	REG	ISP	TOPO	R/T
	26	-2.8	-2.6	-5.1	-1.3	-2.5
25	13	-2.9	-3.2	-5.3	-1.4	-3.0
	6	-3.5	-4.3	-6.3	-1.9	-4.1
	26	-6.2	-4.8	-10.4	-3.5	-4.7
13	13	-6.4	-5.2	-10.9	-3.6	-5.1
	6	-7.4	-6.3	-12.8	-4.1	-6.1
	26	-12.5	-7.1	-21.8	-7.1	-7.0
6	13	-13.1	-7.6	-22.9	-7.4	-7.4
	6	-13.9	-7.9	-24.5	-7.8	-7.7
	26	-22.3	-9.2	-37.2	-14.0	-9.1
3	13	-22.9	-9.6	-38.4	-14.4	-9.4
	6	-23.9	-9.8	-39.8	-15.0	-9.6

- all values biased at the 1% level

TABLE 7b. NATURAL PINE STAND - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEANS

sample size field	photo	prior type[2] UC	REG	ISP	TOPO	R/T
	20	-1.5	-19.4	-2.2	-4.1	-31.2
17	10	-2.3	-54.0	-4.1	-4.6	-33.5
	5	-4.6	-57.7	-8.5	-6.5	-39.6
	20	-3.1	-35.0	-4.9	-8.4	-54.1
8	10	-4.6	-7.6	-8.4	-9.2	-55.8
	5	-8.4	-76.1	-15.4	-10.9	-58.1
	20	-8.6	-54.1	-12.8	-15.5	-74.6
4	10	-10.6	-87.9	-18.3	-16.6	-75.2
	5	-16.4	-88.2	-28.1	-20.4	-76.7
	20	-19.0	-69.6	-27.9	-24.7	-86.2
2	10	-22.9	-94.1	-37.3	-27.0	-86.9
	5	-29.8	-93.5	-47.2	-30.1	-86.6

- all values biased at the 1% level

TABLE 7c. HARDWOOD STAND - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEAN

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	19	0.0*	-7.4	-1.5	-4.8
18	10	-3.4	2.2	-1.8	-5.2
	5	-6.0	1.7	-3.3	-5.9
	19	-4.6	-11.0	-4.6	-8.3
9	10	-9.0	4.7	-5.0	-8.6
	5	-11.7	5.4	-5.9	-8.2
	19	-17.2	-14.0	-11.2	-11.6
4	10	-22.5	16.9	-12.7	-11.7
	5	-29.7	15.4	-15.3	-11.7
	19	-39.7	-16.2	-23.3	-14.7
2	10	-43.9	24.1	-25.8	-14.5
	5	-52.1	20.0	-29.8	-14.7

* - value unbiased at the 5% level, all other values biased at the 1% level

TABLE 7d. MIXED WOOD STAND - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEAN FOR FIRST PHOTO VOLUME EQUATION TYPE[3]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
	17	5.1	5.9	5.7	6.0
16	9	5.7	6.2	6.1	6.1
	4	7.2	6.9	7.4	6.6
	17	8.5	8.5	9.4	8.6
8	9	9.9	8.8	9.8	8.8
	4	10.2	9.2	10.6	8.9
	17	12.0	10.5	12.9	10.6
4	9	12.5	10.4	13.3	10.5
	4	13.3	10.9	13.5	10.7
	17	14.6	12.0	15.5	12.2
2	9	14.9	12.1	15.7	12.2
	4	17.7	13.4	17.7	13.4

- all values biased at the 1% level

TABLE 7e. MIXED WOOD STAND - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEAN FOR SECOND PHOTO VOLUME EQUATION TYPE[4]

sample size		prior type[2]			
field	photo	UC	REG	TOPO	R/T
16	17	0.5*	-1.5	0.6*	-1.2
	9	0.8	-1.8	0.7	-1.2
	4	1.2	-2.5	1.0	-1.8
8	17	1.1	-2.4	1.2	-2.0
	9	1.6	-2.8	1.5	2.0
	4	1.6	-4.2	1.5	-3.2
4	17	2.3	-3.6	2.4	-3.1
	9	2.8	-4.2	2.8	-3.3
	4	3.0	-5.3	2.4	-4.5
2	17	3.1	-5.0	3.2	-4.3
	9	3.5	-5.2	3.6	-4.4
	4	6.8	-4.7	6.4	-3.5

* - value unbiased at the 5% level, all other values biased at the 1% level

(1) - mean of the percent bias over the 1000 iterations. Percent bias was computed as (average volume from entire field sample - Bayesian estimate of volume) *100 / average volume from entire field sample.
(2) - prior types defined refer to the type of photo height data used in the appropriate photo volume equations:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation
- ISP -- interpreter specific equation using using uncorrected photo heights
- TOPO -- topographic corrected photo heights
- R/T -- appropriate regression equation with topographic corrected photo heights

(3) - photo volumes from Avery (1958)
(4) - photo volumes from a percent hardwood composition estimate and photo volume equations 5 and 6

TABLE 8. MODAL INFORMATION PRIOR - AVERAGE PERCENT BIAS[1] IN BAYESIAN MEANS USING VARIOUS STAND TYPES

field sample	prior type[2]	stand type[3] PP	NP	HD	MX1	MX2
50%	UC	-17.7	-10.2	-42.3	6.4	2.8
	REG	-3.1	-70.1	-5.9	10.0	-5.6
25%	UC	-26.9	-20.5	-47.1	9.5	5.6
	REG	-5.2	-76.7	-7.8	12.2	-7.6
12.5%	UC	-43.6	-38.5	-58.1	12.2	9.5
	REG	-6.9	-80.1	-9.1	13.7	-9.3
6.25%	UC	-58.0	-65.4	-71.3	14.6	11.9
	REG	-8.1	-83.4	-10.1	14.8	-10.7

(1) - Average percent bias is the mean of the percent bias over the 1000 iterations. Percent bias was computed as (average volume from entire field sample - Bayesian estimate of volume)*100 divided by average volume from entire field sample.

(2) - The prior types defined in this table refer to the type of photo height data that was used in the appropriate photo volume equations. These are:
- UC -- raw, uncorrected photo heights
- REG -- average tree height obtained by using a height correction equation

(3) - The stand types represented in this table are:
- PP -- plantation pine stand
- NP -- natural pine stand
- HD -- hardwood stand
- MX1 -- mixed wood stand using Avery's aerial photo volume table (1958)
- MX2 -- mixed wood stand using percent hardwood composition and photo volume equations 5 and 6

the hardwood stand, an unbiased estimate of the average Bayesian mean was obtained using 50% of the field data and all prior information.

In the mixed wood stand (Table 7d), all values underestimated the standard mean. All values were biased, however, for a given field sample size, this bias appeared to be stable across prior types. In Table 7e, the regression prior types overestimated the mean. All the observations were biased.

For the modal priors (Table 8), the uncorrected prior type was more biased for the plantation pine. For the natual pine stand, the prior biases were reversed. For the second mixed wood table, the uncorrected prior type underestimated the standard value where as the regression prior type overestimated the value. Greater bias occured with the smaller field sample sizes.

CONCLUSIONS

Prior distribution specification has long been a source of controversy in the use of Bayesian methodology. Opponents have argued that the prior must be exactly known while proponents have not been so rigid.

Four general stand types, miscellaneous types and sizes of prior input and various field sampling regimes were considered. In most cases, the uncorrected prior proved to be inferior to other types of prior input within a stand. This implies that in order to obtain any meaningful reductions in field sample sizes, the prior must be modified in some way.

Secondly, although a prior can be statistically unbiased for the parameter of interest, the Bayesian estimate of this parameter may not be unbiased, as was the case with the mixed wood stand.

Even though the prior information may be biased for the parameter of interest, the bias that was present in the Bayesian estimates was not as great as the bias from the original input data. This implies that Bayesian methods can reduce the magnitude of error present in prior information. This gives justification for further research into Bayesian methodology in forestry.

With respect to the amount of information needed to specify a prior, more homogeneous stands needed less information than the heterogeneous stands which required more information. The 20% photo sample used in this study seemed to perform well so long as the field sample was not reduced substantially.

One final observation to comment on is the trends in sample sizes. As the sample size decreased, both in photo plots and field plots, the magnitude of the over/under estimation increased. The over/under estimation path seems to be set at the 50% field sample size with the full information prior. Bayesian estimation is still a possible estimation alternative when sufficient prior information exists. In the estimation of average volume per acre using aerial photographs for the information source, almost all the estimates were biased. However, many of these biased estimates may be considered within tolerable levels. The type of information going into the Bayesian estimation plays a role in the accuracy of the final results. The conditions surrounding the parameter of interest as well as the limits of tolerable error will dictate how accurately the prior must be specified.

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