DESIGN AND TESTING OF A NONLINEAR MECHANICAL ADVANTAGE
DEMONSTRATION MECHANISM

by

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A four-bar linkage with prescribed nonlinear mechanical advantage has been synthesized and tested. This tool is intended to improve cognition of linkage synthesis theory through interactive demonstrations and has both industrial and educational applications. Industrial applications include use as a proof-of-concept demonstrator and sales device. Academic use will focus on interactive classroom instruction.

In accordance with current educational theory, this tool requires active participation on the part of the student, whether in academia or industry. Interactive methods have repeatedly demonstrated increased effectiveness in developing cognition and retention of technological subjects.

Synthesis of the linkage is based on four-precision-point Burmester theory. Transformation of the given prescribed mechanical advantage problem into classical synthesis forms, i.e., function generation and body guidance, is
necessary during the procedure. The precision points are chosen from the flexion/extension strength curve of the forearm.

Unlike most strength training equipment, this linkage exploits unused muscular capability by allowing variable resistance. Users are capable of dramatic increases in maximum resistance and efficiency over constant mechanical advantage methods. By allowing the full potential of the muscles to be used, the mechanism achieves a work output per repetition that approaches the maximum theoretical work.
Dedication

In a very real way, this work would never have been completed without the help of some very special people. The constant encouragement and support provided by my family has allowed me to pursue this degree. JoAnn, who supplied cajolery in the face of reticence, is to thank for my motivation to complete this thesis. I owe you all more than I can ever repay.
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Nomenclature

Greek Symbols:

\(\alpha_n\)  
Input rotation, \(n = 1,2,3,4\)

\(\beta\)  
Input angle

\(\beta_n\)  
Precision points, \(n = 1,2,3,4\)

\(\delta_n\)  
Vector connecting body positions \(n\) and \(n+1\)

\(\delta_{\text{out}}\)  
Virtual angular displacement

\(\vartheta_n\)  
Loop closure coefficient, \(n = 1,2,3,4\)

\(\Phi\)  
Angle of weight offset link with horizontal

\(\phi_n\)  
General vector rotation

\(\eta\)  
Cam rotation

\(\mu\)  
Mechanical Advantage

\(\Theta_i\)  
Angle of ground link with horizontal

\(\Theta_2\)  
Angle of input link with horizontal

\(\Theta_3\)  
Angle of coupler link with horizontal

\(\Theta_4\)  
Angle of output link with horizontal/output angle

\(\Theta_{in}\)  
Angle between input offset link and input link

\(\Theta_{w}\)  
Angle between weight offset link and output link

\(\Theta_3\)  
Angular velocity of input link

\(\Theta_4\)  
Angular velocity of output link

\(\sigma_{\text{all}}\)  
Allowable Stress

\(\tau_m\)  
Mean Shear Stress

\(\tau_a\)  
Alternating Shear Stress

\(\tau\)  
Transmission angle

\(\Psi_n\)  
Coupler rotation due to input rotation, \(n = 1,2,3,4\)

\(\Omega_n\)  
Loop closure exponent, \(n = 1,2,3,4\)

\(\omega_{\text{out}}\)  
Output angular velocity
Roman Symbols:

\( A \)  Cross-sectional Area
\( A_{sc} \)  Area under the strength curve
\( \delta_t \)  Time derivative
\( dy \)  Virtual linear displacement
\( D_n \)  Cofactor Matrix, \( n=1,2,3,4 \)
\( E \)  Difference function
\( e \)  Exponential
\( F_{in} \)  Input Force
\( F_{out} \)  Output Force
\( k_i \)  Marin correction factors, \( i=a,b,c,d,e,f \)
\( L_n \)  Scaled link length
\( l_i \)  Input offset link
\( l_w \)  Weight offset link
\( l_1 \)  Ground link
\( l_2 \)  Input Link
\( l_3 \)  Coupler Link
\( l_4 \)  Output Link
\( M \)  Vector representing the coupler link
\( M' \)  Vector representing coupler link in function generation space
\( n \)  Factor of Safety
\( P_{dl} \)  Allowable Load
\( P \)  Cam surface Vector
\( R \)  Resistance curve
\( r \)  Matrix rank
\( S \)  Percentage of maximum strength
\( S_{se} \)  Corrected endurance strength
\( S_{se} \)  Uncorrected endurance strength
\( T_{in} \)  Input torque
\( v_{in} \)  Input velocity
\( W \)  Resistance
\( X \)  General vector in body guidance space
\( X' \)  General vector in function generation space
\( y \)  Linear displacement
\( Z \)  Transmission angle variable
\( Z \)  Vector representing the input link
\( Z' \)  Vector representing input link in function generation space
Chapter 1 Introduction and Background

1.1 Motivation

During his inaugural lecture at the University of Cambridge, James Clerk Maxwell said, "Science appears to us with a very different aspect after we have found out that it is not in lecture rooms only, and by means of the electric light projected on a screen, that we may witness physical phenomena, but that we may find illustrations of the highest doctrines of science in games and gymnastics, in traveling by land and by water, in storms of the air and of the sea, and wherever there is matter in motion" (Tricker, 1967). In an exceedingly verbose manner, Maxwell is giving credence to the cliché, "Experience is the best teacher."

The statement points out that deep understanding of many technological subjects is usually achieved after leaving academia, through time spent in
industry and research. Bernard Shaw echoed this sentiment when he said, “Teach a man anything, he will never learn” (Carnegie, 1936). While experience in a field should and will remain an important factor in expertise, the onus is on the academic community to continually improve and better prepare students for their chosen fields.

There is a practical way to improve cognition and retention of technological information presented in the classroom. This may be accomplished through application of new tools and implementation of proposed changes in educational techniques. Application of these methods in kinematics and mechanism design courses will have an immediate impact given the often complex nature of these fields.

Promoting comprehension of physical properties in conjunction with rigorous theoretical preparation will greatly enhance problem-solving skills. Students will enter industry well prepared with theoretical background and the skill to ably apply what they have learned. Note that the term students does not restrict the field of users to those enrolled in an engineering curriculum. This mechanism may find application in both the classroom and in industry. In academia, this tool may be used to inculcate students with a thorough
understanding of synthesis and analysis techniques. Industrial applications include use as a proof-of-concept demonstration unit or as a sales device.

1.2 Background

It is difficult to understate the importance of preparing the student with the skills required in industry. Unfortunately, concentration on theory often prevents the student from developing a holistic understanding of the tangible results of mechanism synthesis. The tool developed in this work may be used during instruction to encourage college students to develop hands-on understanding which supplements and reinforces analytical skills.

Engineering education is beginning to progress from the standard lecture format to an interactive forum which functions only through constant student input. Section 1.4.5 summarizes the published research which points out the benefits of interactive learning environments. Current testing indicates that education incorporating a wealth of interactive learning tools provides an environment much more conducive to learning than those without (Bösser, 1994 and Anderson, 1983). The current educational system is not geared toward this atmosphere, however the level of interest and experimentation is rising. This interest underscores the need for tools such as those described above.
Classroom time constraints force educators to take a pragmatic approach and cover only topics most useful in industry. It is unrealistic to expect a future increase in available instruction time. These time constraints must not be allowed to diminish understanding. It is possible to present the necessary material with the given time constraints and increase student retention and cognition. This may be accomplished by structuring courses to require active student participation.

It is obviously impractical to attempt to develop a tool for instruction in every aspect of mechanism design. The scope of this work is limited to the development of a tool which promotes understanding of prescribed nonlinear mechanical advantage linkages. This type of mechanism receives emphasis in kinematics coursework and also has several popular industrial applications. For these reasons, an immediate and positive impact in both the educational and industrial arenas should be realized.

Kinematics coursework often presents students with the opportunity to use and work with four-bar mechanisms. This is a useful first step, however watching a linkage oscillate or rotate does not provide the same impact as feeling the varying resistance of the mechanism. Presentation of complex synthesis theory after
introduction and use of such a tool will yield far greater results in terms of student understanding. Furthermore, the tool will stimulate the student’s innate “need to know” how to synthesize such a mechanism.

A planar four-bar linkage has been synthesized, analyzed, and fabricated which allows the student to feel the effect of a nonlinear mechanism rather than simply observe its motion. The linkage receives its input from flexion and extension of the wrist. The output is elevation and return of a weight stack.

The true effect and advantage of a linkage designed with a prescribed nonlinearity is seen when compared with a constant mechanical advantage exercise method. Although free weights are of constant magnitude, the force component perpendicular to the rotating limb is variable. The failing of free weight methods is that the variations is neither controllable nor guaranteed to be appropriate to the exercise (Darden, 1982). Prescribed nonlinear resistance allows provides more work output per repetition than free weights. It is important to note that the mechanism does not assist the user, but merely provides a guide to motion. The variation in resistance provided by the mechanism is prescribed to match the physiological capabilities of the forearm muscles.
A free weight is, by its very nature, unable to work with the body to maximize work. The psychological impact of the mechanism is realized when a user is able to load the mechanism with more weight than could be lifted with a free weight. This dramatically increases work done during each repetition of the exercise; and, therefore, efficiency.

The focus of this work is not on the linkage synthesis method itself. Extensive work has been done in the development of closed-form synthesis methods. Rather, the focus is on the use of this technology in a new way. Most previous work with this type of mechanism has focused on the development of analysis and synthesis techniques rather than on applications. This work has taken a broad view and pulled together advanced linkage synthesis and physiology to develop the tool described herein.

1.3 Physiology of the Forearm and Wrist

The synthesized mechanism is designed to work the muscles of the forearm through flexion and extension of the wrist. Anatomically, flexion (or bending) is defined as motion of a limb or digit which decreases the angle between body parts, whereas extension (or straightening) increases that angle (Damon, 1966). This definition requires the specification of a zero deflection position. This plane
is defined as the midfrontal plane of the body. The frontal plane is defined as any vertical plane which is at right angles to the midsagittal plane. The midsagittal plane is the vertical plane which divides the body into right and left halves (Kroemer, et al., 1990).

Figure 1 illustrates the motion associated with both flexion and extension of the wrist. The maximum strength of a muscle or muscle group is limited by the mechanical advantage with which it functions. The mechanical advantage of human muscles varies with limb position. It is this variation of maximum strength with limb position that results in the nonlinear strength curve.

There are four muscles involved in flexion and extension of the wrist; two each for flexion and extension. Flexion requires contraction of the flexor carpi radialis and flexor carpi ulnaris muscles. Extension requires contraction of the extensor carpi radialis brevis and extensor carpi ulnaris muscles (Thompson, 1973).

It is necessary to define several physiological terms which will be used extensively throughout this work. For consistency and to conform to convention, the definitions and data in the following paragraphs are summarized from Woodson (1981) and Tricker (1967).
Strength is defined as the maximum force that muscles can exert in a single, voluntary, isometric effort. Work, as expected, is a measure of dynamic effort and is expressed as the product of force and distance. A distinction is made between mechanical work, as defined above, and muscular work. Muscles can exert force, and hence expend energy, without causing motion. In this case, the mechanical work is zero since no motion has occurred. The muscular work, however, is nonzero and depends on the magnitude and duration of the effort. Throughout this thesis, use of the term work refers to mechanical work. Any references to muscular work will be explicitly stated as such.

![Figure 1: Flexion and Extension of the Wrist](image)

From: Thompson, C. W., *Manual of Structural Kinesiology*, 7th Ed., C. V. Mosby Co., St. Louis, 1973, Fig. 5-3, p. 40 and Fig. 5-4, p. 41. Used by Permission.
The effects, if any, of direction influence, acceleration, overall body position, and limb position on strength must be investigated while developing a linkage of this type. In this case, acceleration refers to acceleration of the entire body rather than of the mechanism during use. Test results presented by Woodson (1981) show that acceleration up to five times that of gravity does not affect strength. In addition, this mechanism is intended for use as a tabletop demonstration unit. For this reason, it is not expected that acceleration will be a factor in design.

Direction influence refers to the variation in strength which may be associated with the direction and/or absence of motion. The two dynamic motions are defined as *concentric* in flexion and *eccentric* in extension. The static strength measure is defined as a *hold*. Current research is aimed at determining the importance of direction influence. For this work, strength data was measured in flexion at several static positions. A dynamic measurement device is unavailable at this time. Therefore, the strength curve used in this work approximates the concentric strength curve. Concentric strength has been shown to be lesser in magnitude than both eccentric and hold strength in some tests, and is therefore the limiting factor in an exercise designed for both flexion and extension.

Body position is a general concern with regard to user comfort, and of specific concern regarding the position of the thumb. Thumb position has a direct effect
on the range of motion of the wrist and on the muscles used for flexion and extension. The range of motion is maximized when the fist is closed in extension and open in flexion (Brunnstrom, 1972). Use of this mechanism will require that the fist be closed at all times. This will have the effect of limiting the range of motion of the wrist in extension. Likewise, a closed fist will eliminate variability in muscle actuation due to thumb position. Use of this mechanism will not cause undue stress or discomfort to the user which could artificially affect the range of motion.

Limb position is a very important design consideration. As mentioned above, the variation of strength is largely dependent on the relative position of the moving limbs. In addition, the strength profile of a muscle group depends on the overall position of the limb relative to the rest of the body. For this mechanism, consistency in orientation is critical, rather than the specific posture chosen. Consistency must be achieved between the position used during strength curve generation and during mechanism use. The mechanism has been designed to allow a high degree of repeatability in forearm and wrist positioning.

In order to develop a mechanism of this type, it is important to accurately represent the full range of motion of the wrist joint. Damon (1966) and Woodson
(1981) present identical results for wrist articulation. The average forced motion angular range is 189 degrees. This is broken down into 90 degrees in flexion and 99 degrees in extension. The standard deviation of the data presented by Woodson is 12 degrees in flexion and 13 degrees in extension. In these two works, the wrist is defined as having zero angular deflection when the hand is aligned with the forearm. Note that this data is based on forced motion, that is, motion obtained by restraining the hand and forcibly rotating the forearm about the wrist. Forced motion testing establishes extreme limits of motion.

Free motion testing establishes a maximum range of motion based on rotation of the hand about the wrist without restraint of any body part. Damon (1966) presents data which places the free range of motion at 165 degrees. Damon also defines a comfortable range of motion as that obtained without strain of any relevant muscles. The comfortable range provides a lower bound for wrist articulation. Damon's data suggests an average comfortable range of motion of 80 degrees.

Other data has been presented (An, et al., 1991) which places the range of motion at 138.4 and 137.3 degrees. These two measurements were taken by hand and electronic goniometers, or three-axis hand mounted potentiometers. Standard deviations for An’s data are 36 degrees. This variation is in excess of Woodson’s
test results by approximately 14 degrees. Person to person variation of this magnitude is reasonable. To assure that a significant majority of users would not be restricted by the mechanism, the design range of motion was chosen to be 140 degrees. This minimizes the problems associated with human variability and agrees well with the published data.

While Damon’s data suggests that comfortable use will require less than 100 degrees of rotation, the typical user will more likely exercise through 120 degrees of rotation. Since the mechanism is designed to allow a maximum of 140 degrees of rotation, many users will “lose” 20 degrees of rotation. If not constrained by the mechanism, the lost rotation would typically consist of the final 10 degrees in both flexion and extension. For the purposes of fabrication, however, the mechanism will have a fixed starting position which requires the user to rotate through the full 70 degrees of extension. This will have the effect of lumping the lost motion at the end of the flexion stroke.

Finally, possible variations in strength profiles based on sex and/or age were investigated. None of the data suggested any significant statistical variation in ranges of motion between men and women, although both elderly persons and those with hand or wrist injuries typically have reduced ranges of motion.
1.4 Literature Review

1.4.1 Introduction to the Literature Review

The literature review began with early uses of force or torque generating mechanisms. Research continued with a review of published synthesis and analysis techniques. A search for similar applications was conducted. References for physiological data were thoroughly investigated. Finally, an in-depth review of current theories on interactive learning techniques and tools was conducted.

1.4.2 Mechanism Designs and Applications

Techniques for force synthesis have, in the past, been linked to position synthesis through specified position and velocity at a set of precision points. Recent work presents closed-form force synthesis techniques which are independent of position and velocity restraints. Force and torque generating mechanisms have been in use for hundreds of years.

The literature is replete with research which focuses specifically on specific examples of force generating mechanisms. Such mechanisms have been in
existence for hundreds of years. A common example, given in Hartenberg et al. (1964), is the use of a mechanism in 1586 to relocate the $3.21 \times 10^4$ N ($722 \times 10^3$ lb.) Vatican City obelisk to its present day position. In such applications, mechanisms may reduce the load to raise such a massive object to a fraction of its weight.

In general, the literature does not provide information regarding the development of mechanisms specifically for classroom instruction. The published work which relates to applications of torque or force generating mechanisms could be used as examples in such an environment, however students would again be left with a merely theoretical treatment of a topic which demands multisensory input to develop a thorough understanding. In addition, every existing mechanism of this sort has been designed using numerical methods, rather than the closed-form techniques used herein. These methods are, of course, important, however the closed-form synthesis techniques must not be ignored.

Two examples of technology transfer from the design stages to the exercise industry were located. Bokelberg and Gilmore (1990) and Soper (1995) present designs of mechanisms which match nonlinear strength curves. Soper employs a
closed-form synthesis technique, while Bokelberg and Gilmore rely on numerical methods to achieve solutions.

1.4.3 Analytical Methods

Soper, Scardina, Tidwell, Reinholtz, and LoPresti (1995) present a closed-form method for four-precision-position synthesis of force generating mechanisms. The linkage synthesized herein utilizes the technique presented in this paper. Huang and Roth (1992) have developed a method for position/force analysis of closed-loop linkages. A portion of this paper is concerned with planar four-bar linkages. General references for linkage design include Sandor and Erdman (1984), Tao (1967), and Mabie and Reinholtz (1987).

Bokelberg and Gilmore (1990) present a variable mechanical advantage design methodology for a rehabilitation machine. The variable torque output is designed to match the strength curve obtained from flexion of the elbow. This methodology uses the variable mechanical advantage of the human arm in conjunction with a linear spring to provide the correct resistance. The analysis technique utilizes a dyad solution in much the same way as the method contained herein. The mechanism developed by Bokelberg and Gilmore is designed using four precision points, leaving one infinity of solutions. A
drawback of this work is a reliance on numerical methods to derive only one solution set at a time. The closed-form method presented by Soper, et al. allows for multiple solutions to be viewed and rated against one another to determine the optimum data set. For the purposes of this work, an optimum data set is one which provides the best match of resistance to strength while meeting all other design and kinematic constraints.

Harmening (1974) used a four-precision-point technique to develop a mechanism for static mass balancing. As with Bokelberg and Gilmore, Harmening's synthesis technique requires the use of a spring in conjunction with a four-bar linkage to develop the required nonlinear output torque.

Mitchiner and Mabie (1975) present a catalog of curves which may be used to approximate link ratios to obtain a known torque ratio for four bar linkages for a specified range of input angles. The range of output angles which correspond to those input angles may be found through the data presented by Brown and Mabie (1969). This work is useful in that it provides guidelines by which an open-ended design problem may be narrowed by eliminating classes of linkages which will not produce the desired torque ratios. Curves are given for various link ratios with the ground link set equal to unity. This allows for scaling in
much the same way as will be conducted in the linkage synthesis presented below.

Tidwell, et al. (1995) has developed a closed-form analytical method for synthesis of wrapping cams. These cams find application in much the same manner as linkages with resistance torque prescribed to a strength curve. The solution method uses the condition of contact and conjugate geometry to develop a function which describes the cam profile.

1.4.4 Physiological Data

Thompson (1973) provides comprehensive data regarding the internal structures of the forearm. Biomechanical data and conventions regarding physiological terms are provided and clarified in this work.

Anatomical data for the wrist and forearm is presented in Woodson (1981). This work presents tabular data for ranges of motion as well as extensive definitions for physiological terms. In addition, Woodson presents qualitative data regarding the effects of various conditions on both strength and range of motion.
Damon (1966) presents corroborating data for the range of motion of the wrist. In addition, several design constraints are suggested from a human factors viewpoint which enhance the utility of the completed mechanism. Tricker (1967) calls attention to the fact that several human muscular motions act under mechanical disadvantage, that is, the limb moves through a much shorter distance than the load which requires proportionally greater forces.

An, et al. (1991), Johnson (1991), Brunnstrom (1972), Guyton (date), and Kirby and Roberts (1985) provide detailed background information on general biomechanics. Information regarding the psychological perception of work is taken from Borg (1977).

1.4.5 Interactive Teaching Methods and Tools

Review of the literature in the field of teaching research has provided ample evidence that this type of tool will greatly aid understanding in the classroom. A great deal of current and recent research is available for review. The main focus of most work is on methods of improving student understanding and cognition in a lecture environment.
A theme stressed in almost every source is the importance of interaction and feedback as an aid in rapid and complete cognition. Data regarding the effectiveness of interactive teaching methods was provided by Bösser (1994) and Anderson (1983). Although Bösser is directly concerned with the development of interactive computer tools for education, he makes several points which are relevant to this investigation. As will be the case in general, Bösser states that presentation of information in conjunction with direct manipulation provides for a much higher potential retention rate than information presented on its own. Bösser also stresses that any interactive tool must have fidelity to the concept being taught based on the student’s, not the instructor’s, point of view. This importance of this is also stressed by Mayo and Gilliard, whose work is discussed below.

Mayo and Gilliard (1979) describe a theory of how humans choose to learn. The rationality principle (page 214-215) states that humans choose the course of action with the highest utility and lowest relative costs. Bösser asserts that, to many students, the utility of learning is judged to be low relative to the costs, therefore learning is not pursued. The success of a teaching tool should be judged based on its impact on the student’s perceived utility of learning.
In a study of effective teaching methods in a university environment, Guskey (1988) points out that active students are far more likely to achieve full understanding of concepts than passive students. The difference between the two types of students depends to a large degree on the presentation given by the professor. The use of demonstration pieces has been shown to promote learning at a more rapid pace than presentation of information without tangible demonstrations. Coupled with the use of these demonstrations is the chance for the students to get immediate feedback regarding the object lesson. Guskey states that during a lecture attention should focus on the most powerful analogies, examples, illustrations, explanations, and demonstrations. No single tool provides more effective teaching and learning. Rather, the combination of feedback tools and information presented in a manner which requires active student participation provides the key. Guskey states that the learner must be involved both mentally and physically in a two-way interactive process, otherwise the student may receive nothing more than entertainment from a spartan presentation of facts and theory.

Another benefit of this type of teaching method is presented by Mayo and Gilliard (1979), whose work was mentioned above. Hands-on tools allow the student to get data and analyze it in an order which is logical and consistent to the learner. The order in which a student may be best prepared to learn is
usually different from the order in which a teacher, who is thoroughly familiar with the material, would instinctively present information. This work also mentions how learners who make an, “overt response during the learning experience,” (page 42) will transfer the information to permanent storage more efficiently than those who do not. Mayo emphasizes that this effect is dramatically increased when the information being presented is novel and/or difficult to the student. Experience with advanced kinematics courses demonstrates that most students find this type of material both novel and difficult at first, which increases the utility of this tool.

Williams (1983) presents a detailed analysis of the human brain and the methods by which learning is accomplished. This study examines the effect of the dramatically different ways in which each hemisphere of the brain processes information. The orderly, linear, and analytical left hemisphere is best suited to a rule-based theory of learning. Williams likens rule-based learning to expert computer systems. For a given situation, a certain series of steps must be taken to develop the solution to the problem. Standard lecture formats speak almost exclusively to the left hemisphere, ignoring the right hemisphere’s capacity to see patterns and develop spatial relationships.
Williams emphasizes the importance of direct experience in balancing the presentation of information between the hemispheres. Through direct experience, students achieve a more holistic view of the problem and are able to apply all five senses to cataloging and analyzing the data, rather than only listening to a step-by-step linear presentation of theory which satisfies the left hemisphere. Use of the tactile and kinesthetic senses allows for more information to be input to the student than if only the auditory and visual senses were used. Williams likens the student to a television set which may receive information on several channels, which correspond to the senses. The likelihood is that one or more of these channels will provide clearer reception to the information presented. Learning techniques based only on lectures limit the student to receiving on two channels, neither of which may provide the strongest reception.

The importance of time constraints in the classroom was mentioned above and is also addressed by Williams. Conventional wisdom states that teaching with direct experience requires a great deal more time than textbook lecture teaching. This philosophy is challenged by Williams who argues that direct experience teaching allows students to achieve an understanding on their own terms. This enhances learning by preventing failures associated with past learning methods. In short, the student does not have to backtrack or catch up to others, thereby
placing experience and lecture teaching on an equal time footing. Williams also stresses the importance of, "...materials that can be manipulated by students..." (Williams, 1983) as a part of the direct experience teaching method. The tangible items which are left to the students to assimilate (in the right hemisphere) complement the theory which is presented in lecture (for the left hemisphere).

Williams states that use of tactile/kinesthetic senses by employing direct experience teaching is a self-reinforcing method which often provides instruction and understanding of a fundamental nature which is retained far longer that information obtained through other senses.

Linn (1988) promotes the theory that science education in this country is in need of reform. The reforms outlined by Linn are based on psychological research into the manner in which people learn and include the importance of such direct experience techniques as have been discussed above. The emphasis of the reformed system would be on, "thinking tools" (page 155) to aid cognition and instruction in technological subjects. Linn assesses the potential of interactive teaching methods as considerable.

literature review did not reveal a single published work which described the effects of active student involvement as detrimental. In fact, every source indicates that interactive learning provides far better rates of cognition, retention, and understanding than standard lecture-based education.

1.4.6 Conclusions

From the literature review, several conclusions can be drawn regarding the utility and necessity of this work. First and foremost, this tool allows students to achieve a thorough understanding of prescribed mechanical advantage mechanisms. The research which has been conducted in the field of education strongly supports the introduction of interactive tools in highly technological subject areas to aid in cognition and retention. No references were found regarding existing tools of this type. This mechanism will find immediate use in both academia and industry as a demonstration model for force synthesis of linkages. Industrial use in the exercise field should begin to alter the industry's mindset regarding the efficacy of mechanisms which match physiological strength profiles. The need for which is illustrated by the fact that professional bodybuilders have been reticent to embrace innovations of this sort (Darden, 1982).
Chapter 2  

Background and Theoretical Development

2.1  Background and Definition of Terms

Muscular strength in humans has historically been determined by measuring the magnitude of a constant weight propelled through a specified range of motion. Tracking an increase in weight has been taken as both an indication and accurate measure of increasing strength. Qualitative and quantitative strength measurements, therefore, rely directly on the magnitude of weight and do not take into account any measurement of the work done during exercise. In addition, constant resistance methods do not account or allow for the variation in strength associated with the direction of motion, as described in Chapter 1. These methods will be shown to be inefficient and inaccurate.

Constant resistance methods of measurement ignore the fact that strength is a function of limb position. That is, as a muscle progresses through its range of
motion, the mechanical advantage of the limb varies as does the maximum resistance the muscle is capable of overcoming. Typically, a muscle is weakest at the extreme positions in its range of motion and strongest near its rest position where the mechanical advantage is the greatest (Thompson, 1973).

When using a free weight, the work done is limited by the maximum weight a person can lift at the weakest point in the range of motion. It is important to consider the effect of rotating the free weight in the gravity field to accurately depict the benefits of this device. The synthesis methodology assumes that the input force remains perpendicular to the input link at all times. To be consistent, the component of the free weight perpendicular to the hand must be considered when describing the constant resistance work. In this case, the component of force normal to the input limb varies with the cosine of the input angle, $\beta$. The input angle varies from 70° to zero, and then back to 70° degrees. Therefore, the magnitude of the resistance normal to the input varies from 34.2% of the weight, to 100% and then back to 34.2%. Note that 100% resistance is still defined by the weakest point in the strength curve.

Development of a linkage with a prescribed nonlinearity obviously requires a set of data to which the output of the mechanism must conform. In the case of a linkage designed to match the strength of a muscle or muscle group, this data is
termed the strength curve. A typical strength curve for extension of the calf is depicted in Fig. 2. This represents a data set taken from one person. Measurements are taken at incremental positions, as indicated by the data points. This curve is used, rather than that of the forearm, to provide a basis for comparisons made in a subsequent section. This curve is taken from the Nautilus force synthesis software database and includes the discrete data points and a polynomial curve fit for the data. Strength is shown as a percentage of maximum. Beta, the input angle, is given in degrees.

The strength curve graphically depicts muscle strength relative to limb position. Limb position is typically measured in degrees or inches (depending on the type of action) from a convenient reference position. Strength may likewise be measured in any convenient force units.

In this work as well as Soper (1995) and Bokelberg and Gilmore (1990), the strength axis is nondimensional. This is accomplished by normalizing the strength data by the maximum weight. As a result, the graph depicts strength as a percentage of its maximum value. Normalization removes the disparity in maximum strength evidenced from person to person.
Normalization also allows the designer to observe that the shape of strength curves for a given type of motion does not vary a great deal from person to person. That is, although the magnitude and precise position of each data point will vary widely from person to person, after normalization, the curves will have very similar shapes. This allows for generalization and development of one linkage which will feel appropriate to a wide range of users. This subjective, rather than qualitative, assessment of the linkage performance is unavoidable.

![Graph](image)

**Figure 2: Strength Curve for Extension of the Calf**

The strength curve is a measure of a person’s ability to apply force to an output link as a function of position. The *resistance curve*, is the resisting force the output member actually exerts on the user. Ideally, the strength curve and the resistance curve should match to maximize the efficiency of the exercise.
The strength curve is best conceptualized as a chart of muscular capability. For this work, the efficiency of an exercise repetition will be judged by the amount of work done by the muscles. This method necessarily neglects the various forms of explosive exercise and variations which do not include the full range of motion. Integration of strength multiplied by distance between the upper and lower ranges of motion yields the maximum theoretical work which could be done by a muscle or muscle group, per repetition. This is, obviously, a measure of the area under the strength curve, which is an important part of the synthesis theory presented below. Keep in mind that only the force applied perpendicular to the input limb is considered when calculating work.

As mentioned above, free weights inherently limit the efficiency of the exercise. The plot in Fig. 3 qualitatively depicts the difference between maximum theoretical work and work done using a free weight. In Fig. 3, the area under the strength curve represents the maximum theoretical work the muscle group is capable of performing across the range of motion. The area under the constant resistance curve demonstrates qualitatively the increase in efficiency which profiled resistances are able to achieve. Obviously, there is a vast muscular potential which free weights fail to exploit. Note that the magnitude of the free weight is governed by the minimum strength, regardless of location.
Figure 3: Comparison of Theoretical and Constant Resistance Work

It is obvious from this graph that the utility of free-weight training is limited by the inherent weakness of muscles at the extremes of their ranges of motion. Since free weights maintain a constant resistance during exercise, a great deal of the potential value of a workout is lost. A mechanism which maximizes the work of a muscle will be an excellent tool for both classroom education and exercise trainers. A person who can lift no more than 178 N (40 lb.) with a free weight would typically load the mechanism with 335 N (75 lb.) for the forearm exercise. The student will feel the mechanism continually vary the resistance in accordance with strength. This discussion is general in nature and has not, as yet, specifically addressed the forearm strength curve. The general presentation
will continue for the development of the synthesis theory and turn to specifics in the next chapter.

Figure 4 is the standard synthesis model. The model includes offset links, \( l_{in} \) and \( l_{w} \), fixed at angles of \( \theta_{in} \) and \( \theta_{w} \), respectively. The ground, input, coupler, and output links are given by \( l_{i} \) where \( i \) is 1, 2, 3, and 4 respectively. Likewise, the angles links one through four make with the horizontal are represented by \( \Theta_{i} \). The input angle is given by \( \beta \). The output angle is given by \( \Theta_{4} \). The input force is \( F_{in} \). All angles are measured dextrally.

![Figure 4: Standard Four-Bar Synthesis Model](image)

The synthesis methodology detailed below incorporates several assumptions, as described by Soper, et al. (1995). First is that all links are considered massless.
and that the effects of acceleration (i.e., dynamic forces) will be neglected. Justification is provided by requiring that the link mass be negligible relative to the mass of the weight stack. Also, proper exercise technique, as defined in several standard texts and in Nautilus guidelines, requires the weight stack to travel smoothly through the entire range of motion with no rapid starts or stops. It is acknowledged that some very new theories have been introduced which present test data supporting explosive exercise or other variations on this type of exercise. These theories present strength profiles which vary from those for concentric motion tested statically, and, as such, are outside the scope of this work.

The second assumption is that the input force will at all times remain perpendicular to the input arm. This assumption is justified by the same argument which allows dynamic loading to be neglected. That is, proper use of the mechanism will dictate that the user consistently remain in the proper orientation. The mechanism has been designed to allow high repeatability of positioning and for proper application of force from that position. The third assumption is that energy is neither stored nor dissipated in the system. Justification of this assumption requires the exclusion of springs and dampers from the design. In addition, friction must be negligible. In this mechanism, no
energy storing devices are used and joint friction is minimized through the use of Teflon bearings at the joints.

2.2 Synthesis Methodology

The theoretical work described below was presented by Soper (1995). Synthesis begins by inverting the prescribed mechanical advantage problem to give an input/output velocity relationship. This input is then converted to a function generation problem through integration. The first step is to consider the linkage in terms of virtual work. The process will proceed then to four-precision-point Burmester analysis. It is important to note that this solution method is exact at only those four points.

One of the strengths of this design approach is the capability to scale and rotate the solution. The input, output, and coupler links are scaled relative to the ground link. \( L_s \), the scaled link length, is defined in Eq. 2.2.1.

\[
\text{Eq. 2.2.1} \quad L_s = \frac{l_s}{l_1} \quad \text{where} \quad L_1 = 1
\]
The solution obtained by this method does not restrict the orientation of the mechanism in the global coordinate system. That is, any rotation, $\Theta_i$, may be assigned to the ground link provided all other links are given an identical baseline rotation.

Keeping the assumptions listed above in mind, Eq. 2.2.2 gives a general form of the virtual work equation for a static system. Recall that all virtual displacements must correspond to the motion of the mechanism, be it linear, $y$, or angular, $\Theta$. The virtual work equation which is specific to this synthesis problem is given in Section 2.3. This equation is now applied to the specific case at hand.

Eq. 2.2.2

$$\sum F_{in} \cdot \delta y + \sum F_{out} \cdot \delta s_{out} = 0$$

Equation 2.2.2 may be modified to represent power equilibrium by dividing through by $\delta t$ to form a velocity expression. Taking the limit as $\delta t$ goes to zero yields Eq. 2.2.3 (Mitchiner and Mabie, 1975).

Eq. 2.2.3

$$F_{in} \cdot v_{in} = F_{out} \cdot \omega_{out}$$
In general, mechanical advantage is defined as the ratio of output to input force. Equation 2.2.3 may be rearranged to give the mechanical advantage of this system as in Eq. 2.2.4, where \( \mu \) is defined as the mechanical advantage.

Eq. 2.2.4

\[
\mu = \frac{F_{\text{out}}}{F_{\text{in}}} = \frac{v_{\text{in}}}{v_{\text{out}}}
\]

As the mechanism to be synthesized has, at all points, a prescribed mechanical advantage, Eq. 2.2.3 suggests that the problem may be represented in terms of prescribed velocity and force ratios. The analysis method will use the velocity ratio condition along with the inverse of the mechanical advantage, \( S^{-1} \), to convert this problem to one which may be solved by a standard technique, such as function generation. Note that \( S \) is nothing more than the inverse of \( \mu \), which is solely a function of the input angle, \( \beta \). These relationships are summarized in Eq. 2.2.5.

Eq. 2.2.5

\[
S = \frac{T_{\text{in}}}{W} = \frac{1}{\mu}
\]
Integrating $S$ across the range of motion yields the area under the strength curve, $A_{sc}$, as shown in Eq. 2.2.6. This represents the normalized maximum work which could theoretically be done by the muscle group in question.

$$A_{sc} = \int_{\beta_o}^{\beta} \frac{F_I \cdot I_{in}}{W} d\beta$$

2.3 Transformation to Function Generation

Equation 2.2.2 is a general form of the virtual work equation. The virtual work expression for the linkage described above can be written as in Eq. 2.3.1. In this case, the work done by an input torque acting through a virtual angular displacement, $d\beta$, is equal to the work done by the resistance weight moving through a virtual linear displacement, $dy$.

$$T_{in} \cdot d\beta = W \cdot dy$$

The velocity ratio expressed in Eq. 2.2.4 could be obtained at this point by dividing each side of the expression by $dt$. However, in order to use standard position-based synthesis techniques, the solution is obtained by integrating each side as shown in Eq. 2.3.2.
Eq. 2.3.2  
\[ \int_{\beta}^{\gamma} T_{in} d\beta = \int_{y_0}^{y} Wdy \]

Noting that the input torque is nothing more than the input force times the input link length, \( F \cdot l_{in} \), a substitution may be made for \( T_{in} \). The constants \( l_{in} \) and \( W \) may then be brought outside the integral, as shown in Eq. 2.3.3.

Eq. 2.3.3  
\[ l_{in} \cdot \int_{\beta}^{\gamma} F d\beta = W \cdot \int_{y_0}^{y} dy \]

Dividing each side of Eq. 2.3.3 through by \( W \) yields:

Eq. 2.3.4  
\[ l_{in} \int_{\beta}^{\gamma} \frac{F}{W} d\beta = \int_{y_0}^{y} dy \]

The ratio of input force to weight, \( F/W \), is nothing more than \( S \), the resistance curve. The integral of the force ratio has likewise been defined above as \( A_{\alpha'} \). The solution of the right hand side of Eq. 2.3.4 is trivial and is given in Eq. 2.3.5.
Eq. 2.3.5 \[ \int_{y_o}^{y} dy = y - y_o \]

A displacement in the vertical direction for a weight attached to a rotating link is described in Eq. 2.3.6.

Eq. 2.3.6 \[ y = l_w \sin \Phi \quad \text{and} \quad y_o = l_w \sin \Phi_o \]

The area under the strength curve, \( A_{sc} \), may be substituted for the integral of the force ratio on the left hand side of Eq. 2.3.4. The right hand side of Eq. 2.3.4 may be replaced by Eq. 2.3.5. The result of these substitutions is shown in Eq. 2.3.7,

Eq. 2.3.7 \[ l_{in} A_{sc} = y - y_o \]

Equations 2.3.6 may be substituted for the right hand side of Eq. 2.3.7. Solving the resulting expression for \( \Phi \) yields Eq. 2.3.8.

Eq. 2.3.8 \[ \Phi = \sin^{-1} \left[ \frac{l_{in}}{l_w} A_{sc} + \sin \Phi_o \right] \]
At this point, a functional relationship between $\Phi$ and $\beta$ has been established. In any linkage synthesis problem, a maximum number of precision points may be established. For every design parameter specified, the number of available precision points is reduced by one. For the case of four-precision-point synthesis, the designer is free to choose the reference angle $\Phi_o$ and the lengths of the input and weight offset links.

The area under the strength curve is inherently a function of the input angle beta only. For any rotation of the input link, the corresponding rotation of the output link can be determined by integrating to find the area under the strength curve up to that point. The original prescribed mechanical advantage design task has now been rendered in standard function generation format.

It is important to note that the work obtained through the linkage cannot exceed the work done by moving the weight stack through its maximum range of motion. In terms of this synthesis model, the maximum extent of motion is constrained to be $l_w$, the length of the offset weight arm. The full extent is reached only when $\Phi$ is equal to 90 degrees. This is true regardless of the starting position of the weight link. Substitution of $\Phi$ and $l_w$ will yield a very
useful result. Equation 2.3.8 may be rewritten after making these substitutions and simplification as:

\[ l_w = \frac{l_{in} A_{sc}}{1 - \sin \Phi_o} \]

Eq. 2.3.9

Equation 2.3.9 establishes a lower bound on the length of the offset weight arm, and is true for the case of \( \Phi = \Phi_{\text{max}} = 90 \) degrees. Generality is restored to the expression by the use of an inequality as shown in Eq. 2.3.10.

\[ l_w \geq \frac{l_{in} A_{sc}}{1 - \sin \Phi_o} \]

Eq. 2.3.10

2.4 Inversion and Transformation to Body Guidance

The aim of this portion of the synthesis is to use standard loop-closure methods to synthesize the linkage. The linkage is not solved directly as a function generation problem, however. Instead, this approach requires a transformation to body guidance synthesis. This allows graphical display and evaluation of the synthesis results (Soper, 1995).
If the mechanism is inverted by fixing the output link and allowing the ground and input links to rotate, the problem becomes one of body guidance for the input link. Figure 5 represents the linkage in inverted space. Refer to Fig. 4 for the relevant synthesis variables.

Figure 5: Inverted Linkage Model

The body guidance solution is obtained by considering the mechanism at two general precision points, position 1 and position \( n \), as illustrated in Fig. 6. Position 1 is determined by the vectors \( Z \) and \( M \), which represent the input and coupler links, respectively. Position \( n \) is determined by a rotation of \( Z \) and \( M \). At position \( n \), \( Z \) has been rotated through an angle \( \alpha_s \) and \( M \) has been rotated.
through an angle $\Psi_n$. The rotation of $Z$ is measured from a reference position on the body.

Link rotations are expressed mathematically in complex notation. This is due to the intuitive convenience of representing the linkage in polar coordinates.

Figure 6: Inverted Dyad Model
Therefore, the rotated input link becomes $Z \cdot e^{\alpha \cdot \gamma_*}$ and the coupler link becomes $M \cdot e^{\gamma \cdot \gamma_*}$. It is important to note that the input angle $\beta$ is specified at each of these precision points. Finally, the vector connecting position 1 with position $n$ has been designated $\delta_n$.

At this point it is possible to write a loop-closure equation for this linkage in much the same manner as for standard position, velocity, and acceleration analysis of four-bar linkages. The loop-closure equation for the dyad shown in Fig. 6 is given in Eq. 2.4.1.

\[
\text{Eq. 2.4.1} \quad \delta_n + Z \cdot e^{\alpha \cdot \gamma_*} + M \cdot e^{\gamma \cdot \gamma_*} = Z + M
\]

Solving Eq. 2.4.1 for $\delta_n$ and grouping terms yields:

\[
\text{Eq. 2.4.2} \quad \delta_n = Z - Z \cdot e^{\alpha \cdot \gamma_*} + M - M \cdot e^{\gamma \cdot \gamma_*}
\]

\[
\delta_n = Z \cdot (1 - e^{\alpha \cdot \gamma_*}) + M \cdot (1 - e^{\gamma \cdot \gamma_*})
\]

For four precision points, $n$ takes the values 2, 3, and 4 as measured from a reference position (position 1). Equation 2.4.3 gives this complex linear system.
of equations in matrix form. To complete the synthesis, it is necessary to obtain simultaneous solutions for both \( \mathbf{Z} \) and \( \mathbf{M} \) (Sandor and Erdman, 1988). Given three equations and two unknowns, it is obvious that one of the equations must be linearly dependent on the other two. This equation will give rise to the compatibility equation, as presented below.

\[
\begin{bmatrix}
1 - e^{\alpha_2} & 1 - e^{\alpha_3} \\
1 - e^{\alpha_3} & 1 - e^{\alpha_4} \\
1 - e^{\alpha_4} & 1 - e^{\alpha_1}
\end{bmatrix}
\begin{bmatrix}
\mathbf{Z} \\
\mathbf{M}
\end{bmatrix}
= 
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
\delta_4
\end{bmatrix}
\]

Eq. 2.4.3

Note that the second column of the 3 x 2 matrix on the left side of the equation and the column vector on the right side contain information relating to input conditions. The first column of the 3 x 2 matrix carries rotational data. To obtain a solution for this system of equations it is first necessary to regroup the data as in Eq. 2.4.4.

\[
\begin{bmatrix}
1 - e^{\alpha_2} & 1 - e^{\alpha_3} \\
1 - e^{\alpha_3} & 1 - e^{\alpha_4} \\
1 - e^{\alpha_4} & 1 - e^{\alpha_1}
\end{bmatrix}
\begin{bmatrix}
\mathbf{Z} \\
\mathbf{M}
\end{bmatrix}
= 
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
\delta_4
\end{bmatrix}
\]

Eq. 2.4.4

\[
\begin{bmatrix}
1 - e^{\alpha_2} & 1 - e^{\alpha_3} & \delta_2 \\
1 - e^{\alpha_3} & 1 - e^{\alpha_4} & \delta_3 \\
1 - e^{\alpha_4} & 1 - e^{\alpha_1} & \delta_4
\end{bmatrix}
\begin{bmatrix}
\mathbf{Z} \\
\mathbf{M}
\end{bmatrix}
= 0
\]

\[
= 
\begin{bmatrix}
\delta_2 \\
\delta_3 \\
\delta_4
\end{bmatrix}
\]

\[
= 
\begin{bmatrix}
1 \\
-1
\end{bmatrix}
\]
In order for this equation to be true, the rank of the augmented matrix must be 2 (Erdman and Sandor, 1988). The rank, \( r \), of a matrix is defined as the size of the largest nonzero \( r \times r \) minor. This is another way of stating that the augmented matrix must be singular, and that its determinant must be zero. Previously it was mentioned that a compatibility equation would be derived from the augmented matrix. This is accomplished by expanding the augmented matrix about the center column (Erdman and Sandor, 1988). The center column is chosen since it contains the unknown angles. The compatibility equation is given in Eq. 2.4.5.

\[
\text{Eq. 2.4.5} \quad D_1 + D_2 e^{i\gamma_2} + D_3 e^{i\gamma_3} + D_4 e^{i\gamma_4} = 0
\]

In Eq. 2.4.5, the \( D_n \) are the cofactors obtained by expanding the singular 3x3 matrix about the center column. Equations 2.4.6 give the expanded form of the cofactors (Erdman and Sandor, 1988).

\[
\text{Eq. 2.4.6} \quad D_2 = \begin{vmatrix} 1 - e^{i\alpha_3} & \delta_3 \\ 1 - e^{i\alpha_4} & \delta_4 \end{vmatrix} \quad D_3 = \begin{vmatrix} 1 - e^{i\alpha_2} & \delta_2 \\ 1 - e^{i\alpha_4} & \delta_4 \end{vmatrix} \quad D_4 = \begin{vmatrix} 1 - e^{i\alpha_2} & \delta_2 \\ 1 - e^{i\alpha_3} & \delta_3 \end{vmatrix}
\]

where \( D_1 = -D_2 - D_3 - D_4 \)
The compatibility equation given in Eq. 2.4.5 consists of known coefficients given by the $D_n$ and unknown rotations in the exponentials, which is very similar to standard loop-closure equations. The difference is that, in this case, the coefficients are complex. Erdman and Sandor (1988) suggest a graphical solution procedure from this point, however a different method will be used. Noting that Eq. 2.4.5 consists of both real and imaginary terms the $D_n$ may be separated into real and imaginary components, as shown in Eq. 2.4.7.

Eq. 2.4.7  
\[ |D_1|e^{i\arg(D_1)} + |D_2|e^{i(\Psi_2 + \arg(D_2))} + |D_3|e^{i(\Psi_3 + \arg(D_3))} + |D_4|e^{i(\Psi_4 + \arg(D_4))} = 0 \]

In this case, all of the real numbers are grouped into the coefficients and all imaginary terms are represented by exponential terms. Introducing the variable $\Omega_n$ to simplify the exponents and $\partial_n$ to represent the coefficients yields:

Eq. 2.4.8  
\[ \partial_1 e^{i\Omega_1} + \partial_2 e^{i\Omega_2} + \partial_3 e^{i\Omega_3} + \partial_4 e^{i\Omega_4} = 0 \]

The equation has now been reduced to the standard loop-closure form. Four position synthesis allows for one infinity of solutions to be developed with the free choice of two variables. It is now possible to solve for $Z$ and $M$. These vectors establish a Burmester point pair, which is one set of ground pivot
locations which satisfy the body guidance problem. The magnitude of the vectors define the respective link lengths.

2.5 Dyad Solution in Real Space

In the current form of the solution, vectors $\mathbf{Z}$ and $\mathbf{M}$ are defined in inverted (or body guidance) space. The utility of the solution is increased by reinversion of the solution to real, or function generation, space. This is accomplished by a rotation of equal magnitude and opposite direction. For a general vector $\mathbf{X}$, such a rotation may be represented as in Eq. 2.5.1.

\[
\text{Eq. 2.5.1} \quad \mathbf{X}'_{n_{FG}} = -\mathbf{X}_{n_{BG}} \cdot e^{\Phi_n}
\]

The prime indicates the vector which has been reinverted. The subscripts $FG$ and $BG$ refer to Function Generation and Body Guidance space and $\Phi$ is the angle of rotation. The reinversion is accomplished by:

\[
\text{Eq. 2.5.2} \quad \mathbf{Z}'_n = -\mathbf{Z}_n \cdot e^{i\alpha_n} \quad \text{and} \quad \mathbf{M}'_n = -\mathbf{M}_n \cdot e^{i\gamma_n}
\]
The entire linkage may now be described in terms of $Z'$ and $M'$. These vectors locate the endpoints of the input and coupler links. Since one ground pivot is known, and the synthesis is predicated on a unit ground link length, these vectors define the mechanism geometrically. Recall that the link lengths may be scaled by any factor. The input link is given by $Z'$ and the coupler link by $Z' + M'$.

The efficiency of this process is greatly enhanced by the development of software for the Burmester synthesis process. The software has been developed by Michael Scardina and R. Randall Soper in conjunction with Nautilus. This software includes a module which allows for force analysis as well as linkage parameter development. Synthesis is performed by fine tuning the computer model to best match the resistance curve to the strength curve. This helps to minimize or eliminate costly and time consuming iterations of physical prototyping.

Manipulation of the computer model involves manipulation of the precision points, the offset link lengths, and the initial angle of the offset weight arm. A complete discussion of the methods used to optimize the fit of the linkage resistance to the strength curve is included in Section 3.4.
2.6 Linkage Defects

Mechanisms of this type, i.e., single degree of freedom planar four-bar linkages, are subject to two main types of defect, either one of which may render the linkage useless. These are commonly termed circuit and branch defects. For the purposes of this work, the definitions applied by Chase and Mirth (1990) will be adopted. Additional mechanical problems may be associated with dead spots in the range of motion. This problem and its solution are discussed in Section 3.5.

Circuit defect is most critical since it renders a mechanism totally unacceptable in all circumstances. One circuit of a mechanism consists of all the possible orientations of the links which may be achieved without disassembly. That is, a circuit is made up of one of the branches of a mechanism. When a linkage undergoes circuit defect, the geometry does not allow assembly throughout the full range of motion. The dividing point between different branches of a circuit is termed the stationary point. Hunt (1978) defines stationary configurations as those positions where the derivative of the angle of the output link with respect to the input link is infinite.

Branch defect, while less serious, still demands attention due to the potential for the linkage to suffer drivability problems as it passes through the stationary
configurations (Mabie and Reinholtz, 1987). Branch defect can also occur under the effect of gravity, should a linkage become unloaded.

A prototype linkage was used to confirm the absence of defects from this linkage. In the case of the linkage synthesized for this work, the range of motion is mechanically limited to the necessary 140 degrees of rotation. This prevents branch defect by placing the stationary configuration outside the range of motion of the linkage. Branch defect due to unloading was taken into consideration during design and is also prevented from occurring.

2.7 Transmission Angle

The transmission angle of a four-bar linkage is formed by the coupler and output links. This angle directly affects the torque at the output of the linkage. As the transmission angle varies, either increasing or decreasing, from 90 degrees the torque at the output decreases. In synthesis cases where maximum torque transmission is a critical design criterion, tight control of the transmission angle is important. In these cases, maintenance of the transmission angle within a small range of values centered on 90 degrees may be required. When synthesizing for prescribed mechanical advantage, however, this is not the case.
Prescribing the mechanical advantage, or resistance torque, of a linkage effectively removes the designers' ability to specify transmission angles. For example, when a very low mechanical advantage is required, and hence low resistance torque, the transmission angle may very well be far from 90 degrees to limit the force transmitted. This does not absolve the designer from considering and minimizing joint friction problems which may be due to transmission angles far from 90 degrees however. A further discussion of the transmission angle and its relevance to the prescribed mechanical advantage linkage problem is provided in Section 3.5.
Chapter 3       Design, Fabrication, and Testing

3.1   Conceptual Design

Two main requirements of the mechanism were identified during conceptual
design. Development of a linkage which clearly demonstrates the properties of
variable mechanical advantage mechanisms in an interactive manner was given
first priority. The second priority was to make the device portable. This, in
turn, required the device to be of minimum overall size and weight while still
producing the required range of motion.

Given the cooperative research currently underway between this institution and
Nautilus, the utility of an exercise machine in this application was immediately
obvious. The requirement for active participation on the part of the student, not
only as an observer but as the prime mover in the demonstration, assures a high
educational impact. To further heighten the effectiveness of the demonstration,
this linkage may be used in side-by-side comparison tests with constant mechanical advantage methods, such as free weights. In this way, a qualitative measure of the capabilities of the linkage is possible through direct comparison.

Such comparisons would be compromised if the linkage did not propel the weight stack through a displacement. The vertical displacement of the weight stack is more critical than path, since work is independent of the path taken. This condition is met during design by proper orientation of the mechanism to allow similar displacements.

Having selected the form of the mechanism, it was necessary to select a single muscle group to provide the input to the linkage. Selection was based on the nonlinearity of the strength curve. A direct relationship exists between the nonlinearity of the strength curve and the difference in resistance between constant and variable mechanical advantage methods. As strength curves become more nonlinear, this difference in resistance is felt as increased physical effort on the part of the user. The strength curves for several types of arm, wrist, and leg motion were examined in order to select the optimal muscle group and associated motion. The relevant strength curves were taken from Nautilus data and from the force synthesis software database.
Selection of the muscle group was not solely based on the nonlinearity of the strength curve, however. As mentioned above, it is important to minimize the size of the mechanism for classroom use and transportation. Size considerations include physical dimensions, range of motion, and maximal weight required by a typical user. The strength curve of the wrist in flexion and extension provides the best mix of nonlinear characteristics and minimal size.

From the data shown in Fig. 7, it can be seen that forearm muscle strength varies by as much as 42%. This is an acceptable degree of nonlinearity to effectively demonstrate the properties of this mechanism. Other motions do show higher degrees of nonlinearity, however these are not compatible with the size limitations placed on the mechanism. Flexion and extension of the muscles of the forearm propels the wrist through a maximum range of approximately 180 degrees. The typical user, however, will operate through only 140 degrees of rotation. The short length of the hand and the relatively lower strength of the muscles in question minimizes the size of the required linkage both in physical dimensions and weight required. The strength is especially low relative to the strength of the calf. A mechanism designed to exercise the muscles of the forearm provided the best combination of both size and nonlinearity characteristics.
Figure 7: Strength Curve for Flexion and Extension of the Human Wrist

A linkage which takes input from the gripping motion of the hand was ruled out because the strength curve was too flat to clearly illustrate the advantage of the linkage. Specifically, the strength variation is less than half of that for flexion/extension of the wrist. With regards to the size requirement, this exercise would function well. Typically, the grip motion is weaker than the flexion/extension motion, therefore this exercise is slightly better in this regard. Unfortunately, the benefits of minimal size do not offset the problems associated with a nearly linear strength curve.

A linkage which takes input from flexion/extension of the calf was also considered. A benefit of this exercise is the extreme nonlinearity of the strength curve. In some tests, the calf shows a strength variation of approximately 70%
across the range of motion, as shown in Fig. 2. If maximum nonlinearity had been the only design constraint, this exercise would have been chosen; however, the increase in utility is mitigated by the required increase in size.

3.2 Data Generation

Once the form of the mechanism was chosen, it was necessary to obtain the strength curve. Strength curves are obtained by measuring the force developed by a muscle or muscle group throughout a specified range of motion. Experimentally, this data may be obtained using a load cell in conjunction with appropriate measuring devices and data collection hardware and software.

The output of the load cell is directed through an analog-to-digital conversion unit into a data collection program. The experiment itself is conducted by taking data while a test subject applies a maximal force to the load cell. The test apparatus is arranged to duplicate the motion for which the strength curve is desired. The subject attempts to maintain a constant force at each stage of data taking. The stages are determined by the extent of the range of motion. Typically measurements are taken at a set number of angular or linear intervals, depending on the motion required. The stages are incremented by, for example,
addition of links to a chain incrementing the motion by a known amount to the subsequent data collection point.

In the future, this methodology could be improved by the advent of real-time dynamic force measuring devices. This will eliminate any error due to representing dynamic strength with a strength curve measured at static positions.

In addition to this approximation, the accuracy of this method is limited by the ability of the subject to maintain maximal force at each step in the measuring process. The signal noise associated with the human subject generally exceeds the error due to the limitations of the measuring equipment. The negative effect of this variation is mitigated by taking several test runs with adequate sampling at each point and averaging the data before performing a curve fit.

The strength curve used in this work was developed in this manner by the engineering department at Nautilus. Only one reference, Roberson (1985), was identified in the literature which includes strength curves for comparison. This work, a manual of structural kinesiology, is notable for its inclusion of this data where no other kinesiology or physiology texts do so. Published data is generally focused on relationships between muscle length and limb orientation,
rather than on overall strength. This data does provide qualitative correlations between muscle length and overall strength. It appears that muscle strength varies with muscle length and limb position in much the same manner. Guyton (1981), Kroemer, Kroemer, and Kroemer-Elbert (1990), and Clark (1984) present data to support this generalization.

3.3 Wrist Flexion/Extension Strength Curve

Selection and/or experimental generation of a strength curve forms the basis for the linkage synthesis process. A difficulty associated with this is the inherently qualitative nature of strength curves as measurement devices. The variations associated with measurement described above and normal variability from person to person prevent strength curves from representing absolute data. Strength curves should be taken as indicators of trends, not as precise numerical solutions. In order to mitigate the effect of this uncertainty it is necessary to carefully investigate the data used to assure its accuracy.

These investigations include verification of range of motion and, to the extent possible, the shape and magnitude of the strength curve itself. Section 1.3 describes the supporting evidence which confirms the accuracy of the strength
curve chosen. Figure 7 depicts the curve used during synthesis to represent the variation of strength with hand position for flexion and extension of the wrist.

Note that the range of motion is defined with zero degrees at the horizontal position where the hand is in the midfrontal plane of the body, as defined in Section 1.3. The mechanism is designed to rotate through 140 degrees, measured dextrally. The discrete data points of Fig. 7 are determined by data averaged from eight complete and independent strength tests. Each strength test consisted of taking data at the reference position (70 degrees below the horizontal) and at increments of 20 degrees thereafter to 70 degrees above the horizontal.

The raw data is normalized to the maximum strength to represent strength as a percentage of the maximum value. This allows generalization of strength curves among various users. It is worthwhile to note that for a given exercise, the overall shape of a strength curve varies little from user to user. This generalization is based on the similarity in muscle length variation, as a percentage of maximum, from user to user which provides a similar mechanical advantage variation (Guyton, 1981).

A third order polynomial fit was then used to obtain the smooth data curve. Table 1 lists the discrete data points used for the curve fit. Note that these values
are averaged from eight independent tests performed at Nautilus and normalized to the maximum strength. As can be seen from the table, the strength of the wrist varies by as much as 42% over its range of motion.

Table 1: Averaged and Normalized Strength Curve Data

<table>
<thead>
<tr>
<th>Position (degrees)</th>
<th>Strength (percentage of maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-70</td>
<td>58</td>
</tr>
<tr>
<td>-50</td>
<td>65</td>
</tr>
<tr>
<td>-30</td>
<td>92</td>
</tr>
<tr>
<td>-10</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>95</td>
</tr>
<tr>
<td>30</td>
<td>80</td>
</tr>
<tr>
<td>50</td>
<td>69</td>
</tr>
<tr>
<td>70</td>
<td>60</td>
</tr>
</tbody>
</table>

As indicated in Section 3.1, maximum educational impact will be derived from a mechanism using muscles which exhibit extremely nonlinear strength curves. Although a quantitative degree of nonlinearity was not specified, the variation inherent in this motion is acceptable.

3.4 Synthesis/Analysis

Foremost among the analytical tools used in this work is the force synthesis software under development at Virginia Polytechnic Institute and State
University by Michael Scardina and R. Randall Soper (Soper, 1995). This package allows rapid analysis and computer modeling of proposed linkages in order to determine acceptability prior to fabrication.

This stand-alone software package employs a user-friendly interface to accept the strength curve data from the designer. The analysis begins with the four precision points chosen by the software. The designer is responsible for specification of the input and output offset link lengths and the initial angle of the offset weight link. In addition, the software allows for multiple solutions to be viewed for one set of link parameters. The designer specifies the initial offset weight link rotation, $\Phi_o$, a step size, and the number of solution sets to be generated. The synthesis results are mapped as sets of Burmester point pairs.

Once the initial analysis is complete, the software provides a map of the Burmester point pairs. The software performs an analysis as presented in Chapter 2, albeit much more efficiently than can be accomplished by hand calculations. A force analysis is performed to ascertain the acceptability of the linkage. This analysis allows for visual comparison of the resistance curve provided by the linkage and the original strength curve. Relationships between input and output angles are established using a module for analysis and animation of the linkage through its range of motion.
The initial Burmester analysis will yield a set of linkage parameters. The initial force analysis allows comparison of the resistance provided by a linkage of those parameters to the strength curve. The synthesized linkage may or may not fit the design constraints. In most cases, several iterations of the synthesis process will be required to achieve an optimal solution, as defined previously.

During these iterations, the designer has control over the location of the four precision points, offset link parameters and initial output offset link rotation. This method of synthesis for four precision points with a selection of two parameters allows one infinity (i.e., a one-parameter locus) of solutions to be viewed.

Maintenance of one infinity of solutions requires that one of the original free parameters remain free. Since three parameters (the two offset lengths and $\Phi_n$) are chosen by the designer, it may be unclear how four precision points remain, instead of three. Any confusion which may arise is due to the selection of $\Phi_n$ as a parameter. In fact, this angle is not a true free choice but is defined relative to the first precision point. That is, $\Phi_n$ is merely the position of the offset weight link at $\beta_1$ and is, therefore, not a free parameter.
Varying the location of precision points on the strength curve provides the maximum return in terms of achieving an acceptable linkage in the minimum time. The designer must use experience and a knowledge of the specific strength curve in question to choose appropriate precision points. Experience has shown that users are very sensitive to linkage performance in the vicinity of inflections in the strength curve. Other methods for grouping and spacing precision points are discussed below.

Although the designer also has choice of offset link lengths, Eq. 2.3.9 provides a constraint which limits the effectiveness of this approach. Since the maximum work can not exceed the work done by simply moving the weight from its lowest to highest position, these link characteristics are constrained to vary in relation to one another. Manipulating the length of these links is more useful when adjusting overall linkage geometry than in obtaining a close fit between the strength curve and the resistance curve. That leaves variation of the precision point locations as the main design tool, as noted above.

A common scheme for selection of points in a range of data is Chebyshev spacing. This method was developed to minimize the structural error associated with a linkage. Structural error is defined as the difference between the desired function and the actual function generated by the linkage (Sandor and Erdman,
1988). Since this method is only strictly applicable to symmetric functions and other special cases, there is no guarantee that Chebyshev spacing will generate an acceptable linkage. In fact, for an analysis of this type, it is likely that these precision points will not generate an acceptable linkage. Chebyshev spacing often places the precision points in such a manner that the linkage generated will suffer from circuit defect.

Other methods for spacing precision points are available, including Freudenstein respacing and the Rose-Sandor technique. These are similar methods which reduce overall error by decreasing the distance between adjacent precision points (Erdman and Sandor, 1988). A rigorous mathematical treatment is neither necessary nor practical when choosing spacing for the Burmester analysis. The speed of the software allows for many scenarios to be viewed in much less time than one iteration would take by hand.

It is possible to use the intent of these other methods without working out precise positions merely by dividing the precision points into two sets and grouping them together. This method was chosen with the precision points, $\beta_1$ through $\beta_4$ set at -69, -64, 50, and 52 degrees in a range from -70 to 70 degrees with zero defined when the hand is in the midfrontal plane of the body.
The precision points must be reassigned to emphasize the most critical points of the strength curve. As mentioned above, these areas are typically near inflections in the strength curve. In the case of this strength curve, a concave down parabolic arc, there are no inflection points but rather a peak and subsequent decrease in strength. This peak occurs between -30 and 30 degrees of rotation, as shown in Fig. 7. It was necessary to develop a linkage with a mechanical advantage that gradually increased to a maximum in this range before decreasing through the remaining motion. As mentioned above, it is important to maintain a precision point at or near the extents of the range of motion to ensure that the linkage will assemble at all required positions, i.e., prevent circuit defect. Outside of these constraints, the designer is free to manipulate the variables to more accurately fit different regions of the strength curve.

The synthesis process may continue indefinitely, however extensive refinements typically yield diminishing returns. The synthesis process results in a linkage with a resistance curve which approximates the strength curve of the forearm. How close the two curves are required to be in order to identify an acceptable linkage is a matter of judgment. For this work, the judgment to halt computer modeling was reached when the model proved insensitive to precision point alteration and met the general curve fit parameters outlined in Section 4.3.
In the case of the final design, variation of the grouping by as much as 25 degrees does not result in significant resistance curve variation. Synthesis was concluded with this set of linkage parameters. Note that this synthesis method ensures that all precision points are met, but does not ensure that all of the points lie on the same branch of the mechanism (Soper, 1995). Since circuit defect is always unacceptable, it is necessary to check for this condition. This may be accomplished by a computer animation and/or by construction of a prototype mechanism.

3.5 Fabrication and Testing

The fabrication process began with construction of a wooden prototype mechanism. This was used as a low cost method to identify flaws such as dead points in the range of motion or branch or circuit defects not identified through computer modeling. Once a successful prototype was completed, mechanical drawings for shop fabrication of the mechanism were prepared and the design was submitted for construction. The mechanical drawings were prepared using AutoCAD Release 12 and shop facilities were provided by Nautilus. The finished design uses two identical linkages in parallel to increase rigidity without altering the mechanisms' kinematic properties.
Although normalization of strength data is useful in the synthesis and analysis process, the mechanism will be used by persons of widely varied strengths. Variation in maximum resistance may be achieved either by altering the weight or by altering the mechanical advantage. Since this mechanism is intended for use as an instructional aid, it was decided to design the mechanism as a plate loading machine. That is, the maximum resistance is varied by physically altering the load on the mechanism. This provides a much greater psychological impact on the user by forcing the recognition that the linkage actually takes the constant resistance of the weight stack and varies the resistance at the input.

The completed linkage is constructed of 6061 aluminum with steel pivots and uses Teflon bearings to minimize friction at the joints. The mechanism was designed for a maximum load of 445 N (100 lb.). Design for higher loads is not necessary based on typical user strengths and realistic expectations regarding weights used for demonstration. Mechanical drawings for the linkage components are contained in the Appendix. Design stress calculations are included below.

As intended, the input link operates through a range of 140 degrees. The range is, in fact, limited to 140 degrees by mechanical stops in order to assure that the linkage does not operate in a region where it could be susceptible to branch
defect. This condition may be predicted by Grashof's law and also identified physically on a prototype.

Grashof's law determines the type of motion a four-bar linkage will undergo, be it crank/rocker, double crank, or double rocker. The prediction is made based on geometric relationships between the lengths of two pairs of links (Mabie and Reinholtz, 1987). If the sum of the longest and shortest link lengths is equal to the sum of the middle two link lengths, as for this mechanism, the resulting mechanism may take on any of the forms listed above. In this case, the mechanism is a double rocker.

Linkages which conform to this case of Grashof's law may operate in a region where the links become collinear. In this position, it is possible for the mechanism to change branch and/or direction of rotation. In some applications the inertia of the links, or of a supplied flywheel, is sufficient to propel them through this region. Exercise mechanisms, however, are designed to be used at speeds far too low for inertia to be a factor. Since prevention of branch changes is desirable, this mechanism is prevented from operation beyond its design range of motion by mechanical stops.
A second concern which arises in all four-bar linkages, and especially linkages which are of this Grashof type, is the range of the transmission angle. The theoretical and physical implications of the transmission angle are provided in Section 2.7. Analytical determination of the range of transmission angle for this mechanism is provided in Section 4.2.

The following stress analyses refer to components of the completed mechanism. For reference, the reader should consult Fig. 9. This linkage was designed to allow for a maximum load of 445 N (100 lb.), or 223 N (50 lb.) per weight pin. This restriction allows for ease of portability without artificially limiting users to less than maximal resistance. Typically the mechanism will be loaded well below the maximum design weight. Analysis of the mechanism was conducted to assure that the strength of the components is sufficient to withstand all foreseeable loading. Analysis was conducted for static and fatigue failures only. Dynamic analysis of this mechanism is not necessary since proper use demands that acceleration loading be negligible.

Static stress analysis is performed for shear at the ground pivots, coupler pivots, and handle pivots. Fatigue analysis of the bolted connections is not necessary since the members are not subjected to reversed stress. The coupler pivots,
however, are subjected to repeated shear loading and are analyzed for fatigue life.

Analysis of ground pivot forces was conducted to assure design adequacy. There are four ground pivots in the mechanism, each with a nominal diameter of 0.953 cm (0.375 in.) at the shear location. For ductile materials, failure is most accurately predicted by the Distortion Energy theory. This approach defines failure at the onset of yield. This theory gives the shear strength of ductile materials as 57.7 percent of the tensile yield strength (Shigley and Mitchell, 1993).

In some designs, it is necessary to correct the yield strength for shear across the threaded portion of bolted connections. This mechanism is designed such that all shear loads are transferred across the shoulder of the connecting bolts. Therefore no further modifications of the shear strength are necessary.

Design specifications require that the ground pivots be bolts of at least SAE grade 1 (ASTM A307) quality. The average yield strength for bolts of this quality is 248 MPa (36,000 psi) (Shigley and Mitchell, 1993). Using this figure and correcting for shear yields an allowable shear stress, $\sigma_{ult}$, of 143 MPa (20,800 psi) per bolt. The stress area, $A$, of each bolt is $2.39 \times 10^4$ m$^2$ (0.0037 in$^2$) for the
coarse thread bolts used in this mechanism (Marks' Standard Handbook for Mechanical Engineers, 1993). This data is used in Eq. 3.5.1 to determine the maximum allowable load, \( P_{all} \), which can then be compared to the actual loading to calculate the factor of safety.

Eq. 3.5.1 \[
\sigma_{all} = \frac{P_{all}}{A} \quad P_{all} = \sigma_{all} \cdot A
\]

Inserting the known values for allowable stress and bolt cross-sectional area yields Eq. 3.5.2.

Eq. 3.5.2 \[
P_{all} = 143 \text{ MPa} \cdot 2.39 \times 10^{-6} \text{ m}^2 = 342 \text{ N} \quad (76.8 \text{ lb.})
\]

Assuming that all four bolts share the direct shear evenly the applied load is 25 percent of 445 N, or approximately 112 N. The factor of safety, \( n \), at maximum design load is given by Eq. 3.5.3.

Eq. 3.5.3 \[
n = \frac{P_{all}}{P} = n = \frac{342 \text{ N}}{112 \text{ N}} = 3.05
\]

71
The shafting specified is grade 303 stainless steel and has a yield strength of 241 MPa (35,000 psi) (Shigley and Mitchell, 1993). Taking 57.7 percent of the yield strength as the allowable shear strength gives 139 MPa (20,200 psi). The cross-sectional area of each shaft is $50.3 \times 10^{-4} \text{ m}^2$ (0.0779 in$^2$). Equation 3.5.1 may be used to determine the allowable load on each shaft, as shown in Eq. 3.5.4.

Eq. 3.5.4 \[ P_{alt} = \sigma_{alt} \cdot A = 139 \text{ MPa} \cdot 50.3 \times 10^{-4} \text{ m}^2 = 6990 \text{ N} \quad (1570 \text{ lb.}) \]

The maximum coupler force, determined from Fig. 8, is seven times the applied load. The vertical axis is normalized to the maximum coupler link force, just as the strength curve data is normalized to maximum strength. This gives a maximum coupler force of 1560 N (350 lb.) for each side of the linkage. Figure 8 is taken directly from the force synthesis analysis module. Assuming that each of the two shafts used as coupler pivots takes half of the load, the factor of safety is given by Eq. 3.5.5.

Eq. 3.5.5 \[ n = \frac{6990 \text{ N}}{780 \text{ N}} = 8.96 \]
Figure 8: Coupler Link Force vs. Input Angle

The handle pivot bolts are identical to the ground pivot bolts, however there are two, rather than four. Obviously, this location will be more susceptible to failure than the ground pivots. Equation 3.5.6 gives the factor of safety for shear at the handle pivots.

Eq. 3.5.6  \[ P = \frac{445 \text{ N}}{2} = 223 \text{ N} \quad n = \frac{P_{\text{all}}}{P} = \frac{342 \text{ N}}{223 \text{ N}} = 1.53 \]

Bolted connections should be designed for a static factor of safety of at least 1.5 (Shigley and Mitchell, 1993). As the results of Eqs. 3.5.3, 3.5.5, and 3.5.6 establish, this mechanism is adequate for the design loads under static
conditions. It is now necessary to calculate the fatigue life of the mechanism at the design load.

Fatigue analysis of the shafts is necessary to determine the expected life of the mechanism. The predicted load history of this mechanism is best described as short periods of concentrated use separated by periods of inactivity. The total number of cycles applied to the mechanism is difficult to predict, therefore the analysis will be performed for infinite life (more than $10^6$ cycles).

Fatigue will occur in the coupler pivot shafts and in the handle pivot screws. The fatigue analysis will examine the shafting and the screws separately. The cyclic loads will be represented by mean and alternating stress components. The mean stress, $\tau_m$, is simply the average of the maximum and minimum stress. The alternating stress, $\tau_a$, is equal to one-half of the difference between the maximum and minimum stress. These terms are used with the modified Goodman failure criteria to determine the factor of safety for infinite life. To complete the analysis, it is necessary to determine the endurance strength of the component in question.

The endurance strength will be calculated using the method described in Shigley and Mitchell (1993). This method is actually developed for cases of completely
reversed bending or fluctuating tensile loading, however the Distortion energy theory allows for correction to shear strength. To correct for the reduced strength of metals in shear, the shear endurance strength will be taken as 57.7 percent of the standard corrected endurance strength, $S_e$.

Equation 3.5.7 gives the standard formula for calculation of endurance strength. For infinite life, $S_e$ is typically 35 percent of the ultimate tensile strength of the material. For 303 stainless steel, this is equivalent to 217 MPa (31,500 psi) (Shigley and Mitchell 1993). The dimensionless modifying factors, $k_a$ through $k_p$, are used to account for the effects of surface finish, size, reliability, temperature, stress concentration, and other problems on endurance strength. These other problems include load time dependency, corrosion, residual stresses, fretting corrosion, and other factors.

Eq. 3.5.7
$$S_e = k_a \cdot k_b \cdot k_c \cdot k_d \cdot k_e \cdot k_f \cdot S_e.$$  

Shigley and Mitchell (1993) give the surface finish factor for a ground shaft as 0.900. For a reliability of 95 percent, $k_i$ is 0.868. The eight millimeter diameter shaft does not require size correction. There will be no high temperature operating regimes and there are no significant stress concentrations since the
retaining rings specified do not require a groove. For these reasons, the size temperature, and stress concentration factors are set equal to unity.

The miscellaneous effects factor, \( k_p \) is meant to be a reminder to the designer that many other factors affect endurance strength. In this application, only cyclic frequency and frettage corrosion need be addressed. There are no residual stresses due to shot peening or cold working in the shafting. The mechanism will not be used in a chemically or environmentally corrosive environment. Finally, the shafting is neither plated nor case hardened which eliminates other potential stress concentration effects.

Loading which is time or frequency dependent can lessen the fatigue life of a material. Since this mechanism is designed to provide variable resistance, the loading is obviously time dependent. It is only to be determined if the frequency of this loading will affect the fatigue strength, or if the loading should be considered as imparting fluctuating stresses to the mechanism.

In general, fatigue failure is independent of frequency except in a chemically corrosive or high temperature environment. The intended uses of this mechanism do not allow for operation in either of these conditions. Therefore,
the varying load will be accounted for by determination of alternating and mean stress components.

Fretting corrosion, due to contact stresses at the handle pivots may reduce the fatigue life of the handle screws. Fretting may occur due to the metal to metal contact at these joints. To account for fretting, the miscellaneous effects factor will be set to 0.900.

Having established all reduction factors, substitutions may be made into Eq. 3.5.7 yielding:

\[
S_r = 0.900 \cdot 0.868 \cdot 0.900 \cdot 217 \text{ MPa} = 153 \text{ MPa}
\]

This gives the endurance strength for reversed bending. The endurance strength for shear loading is given by taking 57.7 percent of \( S_r \) (Shigley and Mitchell, 1993). This gives the final value for the shear endurance strength, \( S_{wr} \) as 88.3 MPa.

To complete a modified Goodman analysis, it is necessary to determine the maximum, minimum, mean, and alternating shear stresses for the shafting. These calculations are provided in Eqs. 3.5.9 through 3.5.12.
Eq. 3.5.9  \[
\tau_{\text{max}} = \frac{1560 \text{ N}}{50.3 \times 10^{-6} \text{ m}^2} = 31.0 \times 10^6 \text{ Pa (4500 psi)}
\]

Eq. 3.5.10  \[
\tau_{\text{min}} = \frac{223 \text{ N}}{50.3 \times 10^{-6} \text{ m}^2} = 4.43 \times 10^6 \text{ Pa (643 psi)}
\]

Eq. 3.5.11  \[
\tau_m = \frac{\tau_{\text{max}} + \tau_{\text{min}}}{2} = 17.7 \times 10^6 \text{ Pa (2570 psi)}
\]

Eq. 3.5.12  \[
\tau_a = \frac{\tau_{\text{max}} - \tau_{\text{min}}}{2} = 13.3 \times 10^6 \text{ Pa (1930 psi)}
\]

In Eq. 3.5.10, the minimum stress, as given in Fig. 8, is overstated, to be conservative. The Goodman criteria, modified for shear loading, is given in Eq. 3.5.13. The mean and alternating stress values are given in Eqs. 3.5.11 and 3.5.12. The primed shear endurance strength is 35 percent of the ultimate tensile strength. In reversed bending, the corresponding term is the unmodified ultimate tensile strength, therefore it is felt that this calculation is conservative. The corrected shear endurance strength is given above. Since the factor of safety is in excess of 1.00, this design is suitable for infinite life.
Eq. 3.5.13 \[
\frac{\tau_m}{S_{se}} + \frac{\tau_a}{S_{se}} = \frac{1}{n} \quad n = 4.31
\]

Fatigue analysis of the handle pivots is performed in an identical manner. The mean and alternating shear stresses are 73.7 and 19.6 MPa, respectively. The Marin reduction factors are 0.900 for surface finish, 0.975 for size, 0.868 for reliability, and 0.900 for miscellaneous effects. There are no temperature or stress concentration effects. The corrected endurance strength for reversed bending is 99.4 MPa. Correction of this value for fatigue in shear gives \( S_e \) as 57.4 MPa. The factor of safety for these screws is 1.18.

Although this factor of safety is very close to one, consideration must be given to the conservative nature of the calculations. Note that the calculations are made for infinite life at maximum design load. This condition will not occur in practice. In addition, the reduction factors have been applied generously to mitigate the uncertainty associated with certain parameters.

Finally, all structural members are pinned at their end points and as such are not capable of supporting a moment. The margins of safety associated with this mechanism assure confidence in a long life cycle. Overload of the mechanism
beyond its structural capabilities is unlikely due to the intentionally restricted
dimension of the weight pins.
Chapter 4  Results

4.1  Introduction

During conceptual design, it was decided that an acceptable mechanism would meet two specific criteria. The linkage synthesized clearly surpasses the minimum requirements for acceptability. Most significantly, the linkage demonstrates the properties of prescribed mechanical advantage mechanisms in a manner requiring active student participation. Further, the mechanism is portable for ease of use in the classroom and industry. Evidence supporting the need for and utility of interactive tools such as this is presented in Section 1.4.4.

This chapter presents data which indicates that the linkage synthesized meets the needs described above. That is, this mechanism will function in a manner which allows students to achieve a more thorough and deeper understanding of prescribed mechanical advantage linkages. A kinematic analysis is provided
which focuses on assessing the degree of fit between the strength and resistance curves. This assessment includes both qualitative and quantitative methods for evaluating curve fit.

4.2 Physical Description

The completed four-bar linkage is depicted in Fig. 9, with the arm rest omitted for clarity. During use, the arm rest is screwed to the base plate to provide a stable and adjustable platform. The user places the forearm on the rest with the palm facing up to grip the input handle. In this position, the wrist is extended 70 degrees below the midfrontal plane of the body. The forearm is secured to the arm rest with a velcro strap to prevent motion relative to the ground links. The wrist joint is aligned as closely as possible with the input link ground pivot. Adjustment for variation in user hand size is made by sliding the arm rest in the slots provided. The rest is secured to the base plate from below by four screws.

As can be seen in Fig. 9, the mechanism consists of two symmetric four-bar linkages. This is necessary to provide a balanced, double-supported load, but is not necessary from a kinematic viewpoint. The intent of this symmetric design is to increase the overall rigidity of the mechanism. In this application, the input force cannot be applied in the plane of a single linkage. A single-sided linkage
would be forced to operate under significant bending loads. These bending loads would be due to the input force applied at the end of the cantilevered input handle. It is possible to design for a single-sided mechanism, however, it is more feasible to develop a mechanism such as this for demonstration purposes. It is also possible to develop a mechanism for dual wrist exercise.

![Figure 9: Isometric View of the Assembled Mechanism](image)

1 - Input Link
2 - Coupler Link
3 - Output Link
4 - Ground Link
5 - Input Handle
6 - Base Plate

A single sided linkage would develop unacceptably high frictional loads at the joints due to this eccentric loading. High frictional loads will prevent smooth
operation. As smooth, low-friction operation is an assumption made during synthesis, this characteristic is critical to a successful design. By using two linkages which are kinematically identical, the bending loads are balanced and the mechanism will operate more smoothly throughout its entire range of motion.

4.3 Kinematic Description

A kinematic skeleton diagram of the linkage in its starting position is provided in Fig. 10. The ground link, L1, is not shown for clarity. Note that this figure is not drawn to scale. Dimensioned drawings of the linkage components are contained in the Appendix.

The ground link is the stationary body which connects and locates the grounded pivots. The offset input link is rigidly attached to the input link at a fixed angle of 166 degrees. The input handle is attached to this offset link. The offset weight link is rigidly attached to the output link at a fixed angle of 140 degrees. The weight stack is attached to this offset link. Open circles indicate coupler joints, while closed boxes represent ground joints. This mechanism contains only revolute joints and operates with one degree of freedom.
Each mechanism component is listed in Table 2 with the applicable linear or angular dimension. Linear dimensions (i.e., link lengths) are taken from pivot center to pivot center. Offset angle dimensions are taken between lines connecting the pivot centers.

It is important to keep the provision for scaling (Section 2.2) in mind when designing the linkage for fabrication. The synthesis software returns a set of linkage parameters based on a ground link length of unity. Theoretically, any
scale factor may be applied without affecting the kinematic properties of the mechanism. In practice, however, radical scale factors should be avoided.

Very high scale factors are a concern with regard to weight and portability. The dynamic (i.e., moving) weight of the linkage should be minimized. This is in accordance with the assumption of negligible weight mass. In addition, the overall size should be minimized to assure easy portability. Very low scale factors present problems with fabrication and user comfort. A scale factor of one was chosen for this mechanism.

Table 2: Summary of Linkage Dimensions

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground Link</td>
<td>2.54 cm (1.00 in.)</td>
</tr>
<tr>
<td>Input Link</td>
<td>17.3 cm (6.80 in.)</td>
</tr>
<tr>
<td>Coupler Link</td>
<td>4.42 cm (1.74 in.)</td>
</tr>
<tr>
<td>Output Link</td>
<td>15.4 cm (6.06 in.)</td>
</tr>
<tr>
<td>Input Offset Link</td>
<td>8.89 cm (3.50 in.)</td>
</tr>
<tr>
<td>Output Offset Link</td>
<td>10.4 cm (4.10 in.)</td>
</tr>
<tr>
<td>( \Theta_{in} )</td>
<td>166 deg.</td>
</tr>
<tr>
<td>( \Theta_{w} )</td>
<td>140 deg.</td>
</tr>
</tbody>
</table>

A general discussion of the transmission angle and its significance in the design of four-bar linkages is contained in Section 2.7. A transmission angle analysis specific to this mechanism is now presented. Figure 11 depicts a general four-
bar linkage model with the transmission angle indicated by $\tau$. Strictly speaking, mechanisms have not one, but a range of transmission angles the extents of which are determined by geometric constraints.

![Figure 11: Transmission Angle in a General Linkage Model](image)

This magnitude of this angle determines the percentage of coupler link force which is converted to torque at the output. In many synthesis cases, one design goal is to maximize the torque at the output. This condition requires that the range of transmission angles be maintained close to 90 degrees. In the present case, the output torque is prescribed to vary from a minimum to a maximum and then to decrease again. This necessitates transmission angles which might be considered outside the acceptable range for torque transmitting linkages.
Figure 12 gives the details required for a geometric determination of the transmission angle, $\tau$, and for geometric position analysis. Position analysis allows calculation of the output angle, $\Theta_4$, for any given input angle. This method is adapted for a crossed linkage from the method presented in Mabie and Reinholtz (1987). There are other position analysis methods such as vector or complex number methods which could work equally as well for this mechanism.

![Geometric Transmission Angle Analysis Model](image)

**Figure 12: Geometric Transmission Angle Analysis Model**

Equations 4.2.1 and 4.2.2 are derived by applying the law of cosines to triangles $Z, l_y, l_4$ and $Z, l_y, l_t$. These relationships provide the two governing equations
needed to determine the transmission angle and to perform position analysis for the linkage.

Eq. 4.3.1 \[ Z^2 = L_1^2 + L_2^2 - 2L_3L_4 \cos(180 - \Theta_2) \]

Eq. 4.3.2 \[ Z^2 = L_3^2 + L_4^2 - 2L_3L_4 \cos(\tau) \]

Obviously, the right hand sides of Eqs. 4.2.1 and 4.2.2 are equivalent. These equations may be rearranged and simplified to yield:

Eq. 4.3.3 \[ \tau = \cos^{-1} \left[ \frac{Z^2 - L_3^2 - L_4^2}{-2L_3L_4} \right] \]

This equation gives the transmission angle in terms of the input angle, which is embedded in Z, and the physical dimensions of the linkage only. From any specified input angle it is possible to determine the transmission angle. For a double rocker mechanism, such as the solution linkage described above, it is only necessary to determine the extremes of the transmission angle range. These calculations result in a transmission angle of 176 degrees for the initial input angle of six degrees and 79.8 degrees for the final angle of 146 degrees. During
this analysis it is important to consider the double-valued nature of inverse cosines to assure that the proper branch of the linkage is being considered.

The magnitude of the transmission angle makes it necessary to address the problem of high friction at the joints. A high value of the transmission angle, (one that deviates significantly from 90 degrees) such as exists at or near stationary points, will directly lead to high joint friction under load. This condition exists for approximately the first 30% of the input range. In this region the load is low, but steadily increasing.

To alleviate any binding or operability problems this might cause, the linkage has been designed with reinforced Teflon bearings at the coupler joints. The coefficient of static friction for Teflon in contact with steel is 0.04 (Marks’, 1987). This effectively eliminates any problems which might otherwise occur due to the magnitude of the transmission angle. The reinforced Teflon bearings are also light weight, require no maintenance, and have a high life expectancy. The use of ball or roller bearings would also lower friction, but would result in higher cost and weight. In addition, the applied loads and expected cycles during the life of the mechanism are far below the level needed to justify this type of bearing.
A second concern with boundary lubrication bearings involves unit pressure and velocity concerns. General ratings do not exist, however Shigley and Mitchell (1993) provide the PV test as a measure of acceptability. This rating is determined by multiplying the maximum unit pressure, \( P \), and the velocity of the bearing. The units for this term may be kept in terms of pressure (psi) times velocity (feet per minute). The maximum PV rating for plain Teflon bearings is 1000, note that these calculations do not apply at elevated temperatures.

The maximum load of 1560 N (375 lb.) is shared by two bearings, therefore the unit pressure (at the smallest bearing location) is approximately 700 psi. The velocity of the shaft in the bearing is difficult to define with certainty. however for a user who completes the full range of motion in two seconds, the rotational speed of the shaft will be approximately 1.0 feet per minute. This gives a PV value of 700, which is below the maximum.

The rotational speed is based on the shaft revolving through 40% of its circumference during the two second exercise. This calculation, although extrapolated from several assumptions, should provide an acceptable description of the forces acting on the bearing. If it proves necessary after use, reinforced Teflon may be substituted for plain Teflon at negligible expense.
The kinematic review provided in this section is followed by a description of the qualitative characteristics of an acceptable strength/resistance curve fit. This is augmented by development of a quantitative method for measure of curve fit.

4.4 Qualitative Curve Fit Analysis

Figure 13 shows a comparison of the forearm strength curve used in this work and the resistance curve provided by the linkage. Section 1.3 includes a detailed discussion of wrist flexibility and expected range of motion. The linkage was designed to allow a maximum input rotation of 140 degrees although most users consider 120 degrees to be comfortable during exercise. Therefore, a typical input range will be 20 degrees less than the maximum allowable rotation. Fit of the two curves will be judged over this range, which corresponds to input angles of -70 to 50 degrees.

Since the linkage requires the user to begin with 70 degrees of forearm extension, users who do not achieve the maximum range of motion will lose the final 20 degrees of flexion. In Fig. 13, this region begins at 50 degrees and continues to the end of the range of motion. The typical users will therefore work in the range of -70 to 50 degrees. In this region, the resistance curve of the mechanism provides a close match to the strength curve of the forearm.
As mentioned in several preceding sections of this work, the area under the strength curve represents the maximum theoretical work which the muscles in question are capable of performing. Figure 3 demonstrates the limitations of free weights with respect to maximizing work. Figure 13 is a comparison of the strength curve and the resistance curve of the mechanism. The resistance curve must be adjusted down the strength axis during use until the resistance is tangent to the strength curve at the maximum resistance. This is accomplished by adjusting weight to fit each user. By comparing Fig. 3 with Fig. 13, the increase in work done due to the linkage is immediately evident.

Fig. 13: Strength and Linkage Resistance Curves

Figure 13: Comparison of Strength and Resistance Curves
A set of criteria for gauging fit, applicable to all synthesis problems of this sort, has been established. Whether these criteria are met by a mechanism is solely the decision of the designer. Obviously, it is not necessary to check mechanisms which are subject to kinematic defects, such as described in Chase and Mirth (1990). The main criteria for an acceptable fit is that the resistance curve match inflections of the strength curve in both position and direction. Also important is a close match in the position and magnitude of local maxima and minima.

Matching changes of inflection, in either direction, is the most important and easiest to identify of the two criteria. Experience has shown that users notice and comment on mechanism feel based on its performance at and in the vicinity of inflections in the strength curve. It is the responsibility of the designer to ensure that changes in strength curve inflection are matched by the resistance curve in both position and direction.

Regarding position, the linear or angular tolerance on inflection point location is impossible to define for all synthesis cases. In general, the degree of variation should be limited in accordance with the overall range of motion and the proximity of other inflection changes. A mechanism with a range of motion of 180 degrees could allow more variability in inflection changes than one with
only 75 degrees of motion. Likewise, a strength curve with two inflections widely spaced could allow more variability in position than one with closely spaced inflections. Obviously, no error is allowable with regard to the direction of the inflection. If range of motion data is obtained from sources which provide statistical data, the standard deviation may be used to guide decisions regarding acceptable positional variation.

A second criteria involves the maxima and minima of the curves. These should match in position and in magnitude. Position refers not to a single point at a minimum or maximum, but includes the region surrounding such points. The extent of this region is up to the designer and will vary depending on the characteristics of the strength curve in question. Tolerance on positional errors may be evaluated as described above. Variations in magnitude should be much more restricted.

Variation above the strength curve is less acceptable than below. A resistance curve which falls below the strength curve diminishes the efficiency of the mechanism. Variation above the strength curve may result in a mechanism which prevents some users from achieving the full range of motion. For such a person, the efficiency of the mechanism is reduced to zero in certain regions.
Note that variation of the resistance curve above the polynomial fit for the strength curve may still be acceptable if not in excess of 100 percent strength.

Variation in location and magnitude of minima is also important, although such differences will not typically render the mechanism useless. It is important to consider that resistance in excess of strength near minima will typically require improper lifting technique on the part of the user. This may be motion of the forearm and body relative to the ground links or rapid accelerations applied to the input in order to progress through the difficult region.

The criteria above are concerned with the fit at discrete points in a continuous system, specifically at extrema and inflection points. It is important that the overall shape of the resistance curve matches the strength curve. This demands that the designer ascertain the overall fit of the curves in addition to the specific points mentioned above.

4.5 Quantitative Curve Fit Analysis

A standard mathematical measure of curve fit would be a useful analysis tool. Such a standard has, to this point, not been established. During this work, a measure of fit was adopted and a set of criteria established for determining the
acceptability of a resistance curve. The method is suitable for implementation in a computer synthesis and analysis package, such as the Nautilus force synthesis program. Reliance on the skill of the designer remains an integral part of the analysis process, however.

A purely mathematical comparison method is not a desirable analysis tool. Maintenance of a skilled engineer in the design loop provides flexibility in the design process. The designer is also responsible for ascertaining the overall acceptability of the resistance curve. This refers to visual inspection of the resistance curve for overall fit.

One measure of fit is achieved by determining the loss in area between the resistance curve shown in Fig. 13 and the same curve once it is adjusted down to allow a full range of motion. That the weight applied must be reduced until the maxima of the resistance curve is just tangent to the strength curve. The loss of efficiency is measured by the difference in area between these two curves. The departure from maximum theoretical work may also be measured. The difference in area between the strength curve and the adjusted resistance curve will provide this measure.
Note that this is not a measure of error, as such. As mentioned previously, strength curves are taken as indicators of standards which are broadly applicable to many users. Strength curves are neither absolute nor definitive. For this reason it is inappropriate to discuss the error of a resistance curve with respect to a given strength curve.

Determining the difference between the two curves first requires functional expressions which may be treated mathematically. The appropriate expressions are subtracted from each other and the result integrated over the range of motion. It is not necessary to square the result, as it will at all time be positive if the strength curve, $S$, is subtracted from the resistance curve, $R$. While this method does not allow for a completely automated fit check, this calculation provides the optimal gauge of precision when combined with engineering oversight.

To begin, an expression for the strength curve is necessary. The synthesis software provides this relationship by performing a polynomial curve fit for the strength curve data. The polynomial is calculated by the synthesis software to double precision (15 digits) and is given in Eq. 4.5.1 to four significant figures.

Eq. 4.5.1 \[ S = 0.9440 - (7.604 \times 10^{-2})\beta - (0.2623)\beta^2 + (6.353 \times 10^{-2})\beta^3 \]
For comparison, it is necessary to develop a functional relationship for the resistance curve of the forearm. The software does not provide a functional relationship, therefore it is necessary to return to basic principles. As in Chapter 2, the analysis begins with a virtual work expression for the linkage as given in Eq. 4.5.2.

Eq. 4.5.2 \[ F_{l_{in}} \cdot \delta \beta = W \cdot \delta y \]

The virtual displacement indicated by \( \delta y \) is a virtual displacement of the weight stack in the vertical direction. At this point, dividing through by \( \delta t \) yields a velocity relationship between the input and output links. Noting that the velocity of the weight stack depends only on the length of the offset weight arm and the angle of inclination \( \Phi \) yields Eq. 4.5.3. The angle \( \Phi \) is the difference between the output angle and the angle of the offset weight link.

Eq. 4.5.3 \[ F_{l_{in}} \cdot \dot{\Theta}_2 - W \cdot I_w \cdot \dot{\Theta}_4 \sin \Phi = 0 \]

This may be further reduced to give the resistance of the mechanism as shown in Eq. 4.5.4.
Eq. 4.5.4  \[ R = \left( \frac{l_w}{l_n} \right) \cdot \left( \frac{\Theta_4}{\Theta_2} \right) \sin \Phi \]

For an exact fit, an ideal case which does not occur in practice, the velocity ratio in Eq. 4.5.4 will reduce to the expression for the strength curve. For fits which are not perfect, it is necessary to perform a position and velocity analysis for the linkage to determine the velocity ratio. Position analysis may be performed geometrically, as described above, however a loop-closure method will be employed here. The work which follows is adapted from Mabie and Reinholtz (1987) for the case of a crossed linkage.

Figure 14 depicts the loop-closure model used in this analysis. All angles are measured in a positive right-hand sense. The ground link, \( l_r \), is shown in the horizontal position, rather than rotated by 270 degrees as in the actual mechanism. This does not affect the calculations, since all angles are measured from ground as a reference.
Figure 14: Loop-closure Model

Note that this figure is not drawn to scale. The loop-closure equation is given in Eq. 4.5.5. Complex notation is used to simplify the expression.

Eq. 4.5.5  
\[ l_1 e^{i\Theta_1} + l_2 e^{i\Theta_2} + l_3 e^{i\Theta_3} - l_4 e^{i\Theta_4} = 0 \]

Equation 4.5.5 may be broken up into real and imaginary parts, using the Euler identity. In addition, the exponential in the first term may be eliminated, since \( \Theta_1 \) is the reference angle. These modifications are shown in Eqs. 4.5.6.
Eq. 4.5.6  \hspace{1cm} \text{Re: } \quad l_1 + l_2 \cos(\Theta_2) + l_3 \cos(\Theta_3) - l_4 \cos(\Theta_4) = 0

\text{Im: } \quad l_2 \sin(\Theta_2) + l_3 \sin(\Theta_3) - l_4 \sin(\Theta_4) = 0

In these equations, the link lengths and the input angle are known, leaving the coupler and output angles as unknowns. The two transcendental equations may be solved for the two unknowns and velocity analysis may begin. The solution is performed as in Mabie and Reinholtz (1987).

To begin velocity analysis, it is necessary to take the first derivatives of Eqs. 4.5.6. The results, after rearranging and grouping terms, are given in Eqs. 4.5.7.

Eq. 4.5.7  \hspace{1cm} [l_4 \sin(\Theta_4)]\dot{\Theta}_4 - [l_3 \sin(\Theta_3)]\dot{\Theta}_3 = [l_2 \sin(\Theta_2)]\dot{\Theta}_2

[l_4 \cos(\Theta_4)]\dot{\Theta}_4 - [l_3 \cos(\Theta_3)]\dot{\Theta}_3 = [l_2 \cos(\Theta_2)]\dot{\Theta}_2

The coefficients of each velocity term consist of known quantities, since the velocity analysis has already been performed to determine the coupler and output angles. It appears that the angular velocity of the input link is required, however this term will drop out after subsequent calculations. Equation 4.5.7 gives two expressions for two unknowns, the coupler and output link angular
velocities. These expressions may be simplified to yield the following equation for the angular velocity of the output link:

\[
\dot{\Theta}_4 = \frac{[\cos(\Theta_2) \cdot \sin(\Theta_3) - \cos(\Theta_3) \cdot \sin(\Theta_2)] \cdot (l_2 \dot{\Theta}_2)}{[\cos(\Theta_4) \cdot \sin(\Theta_3) - \cos(\Theta_3) \cdot \sin(\Theta_4)] \cdot (l_4)}
\]

The velocity ratio in Eq. 4.5.4 may now be given in terms of the geometry of the linkage only. As mentioned above, the angular acceleration of the input link, which depends on the user, drops out of the equation at this point. The functional relationship for \( R \), the resistance curve, is given in Eq. 4.5.9.

\[
R = \left( \frac{l_w}{l_{in}} \right) \cdot \left( \frac{\cos(\Theta_2) \cdot \sin(\Theta_3) - \cos(\Theta_3) \cdot \sin(\Theta_2)}{\cos(\Theta_4) \cdot \sin(\Theta_3) - \cos(\Theta_3) \cdot \sin(\Theta_4)} \right) \cdot \left( \frac{l_2}{l_4} \right) \sin \Phi
\]

This may be simplified to give the unadjusted resistance curve as:

\[
R = \left( \frac{l_w}{l_{in}} \right) \cdot \left( \frac{\sin(\Theta_3 - \Theta_2)}{\sin(\Theta_3 - \Theta_4)} \right) \cdot \left( \frac{l_2}{l_4} \right) \sin \Phi
\]
Equation 4.5.10 gives an expression for the resistance of the linkage as a function of the input angle, $\beta$, only. This is a form which may be used for difference analysis.

The difference function, $E$, for this linkage will be defined as in Eq. 4.5.11.

\[
E = \int_{\beta_0}^{\beta} (S - R) \, d\beta
\]

The expressions developed in this section provide the information necessary to determine a numerical value for the curve fit. Solution of the integral in Eq. 4.5.11 may be best solved numerically, through the use of Newton-Raphson numerical analysis, or a related technique. Note that, the solution will require that the system be treated at discrete points. However, implementation into a software package will eliminate errors by performing the calculations for a large number of data points.

Use of this method for a variety of mechanisms will establish a knowledge base for curve fits. The designer will be able to rely on the information contained in that database to judge the quality of the curve fit.
Several other methods exist which could be used to determine the suitability of the linkage, however none provide as accurate a picture. For example, a sum of the squares method could be used at each data point. This method could, in general, be very misleading as to the accuracy of a given resistance curve since the curves are continuous and the testing method is discrete. Again, it falls to the designer to ascertain the potential for aliasing and determine an appropriate method for measuring fit.

4.6 Discussion of Difference Analysis

This section provides a review of curve fit as applies to this mechanism. In addition, a discussion of the mechanism with respect to original design constraints, objectives, and assumptions is included. Overall, a good fit has been achieved. The design objectives have been met, the constraints followed, and no assumptions have been violated.

There are two local minima and one maximum in the strength curve used for this work, see Fig. 7. The first 15 degrees of rotation are referred to as the range of the minima. The initial minima matches both position and magnitude extremely closely. A positional error of up to 15 degrees is acceptable, however no tolerance is necessary in this location. This number was determined by
considering the angular displacement associated with rotating the input link through the thickness of the hand. As shown in Fig. 13, the positional and magnitude variation is below the tolerance of the plotting device.

The maximum strength range of the mechanism lags the strength curve in flexion by approximately twenty degrees. The magnitude of the resistance is in excess of the polynomial fit, but does not exceed 100 percent strength. By adjusting the resistance down the strength axis, it is possible to maximize the efficiency of the exercise, without restricting use. A variation of twenty degrees is just greater than the angle subtended by the hand in the plane of the mechanism. This variation is within acceptable limits given the range of motion and the absence of other maxima.

The least precise fit occurs in the range of the second minimum. While the curves do cross, the slope of the resistance curve exceeds that of the strength curve. This translates to a more rapid change in mechanical advantage than the strength curve dictates. Although it may seem counterintuitive, this is a desirable feature in a resistance curve. Many users will have a maximum range of motion of 120 degrees; however, others will be able to achieve the full 140 degrees. The mechanism must not artificially limit the range of motion by providing a resistance which is too high for the user. A resistance curve which
drops off more rapidly than is necessary at the end of the range of motion ensures that this will not occur.

Overall, the resistance curve meets the criteria set in Section 4.4 very well. For comparisons of this type it is useful to consider the strength curve as a range of data rather than a specific value. In this way resistance curves are not required to directly match the strength curve at all points, but to follow the trend established by the strength curve.

The linkage achieves the main design goal by varying mechanical advantage to provide resistance as prescribed by the strength curve. This, among other requirements, is necessary to maximize the teaching potential of the linkage. The highly nonlinear nature of the flexion/extension strength curve allows even inexperienced users to readily feel the effect of the linkage.

The linkage is small enough to allow easy transport and use in a classroom or seminar setting. The exercise chosen requires the use of a minimum of weight plates to achieve the full benefit of the design.

The linkage also performs well in that the links function as a guide to motion and do not serve as an aid to lifting. This mechanism provides its benefit from
the exploiting the portion of muscular ability left untouched by constant resistance methods. This is not to say that the *internal* link loads will not be significant. In many regimes, link forces will exceed the loading, as evidenced by Fig 8. Also, use of the linkage demonstrates that the action of the linkage is to provide a varying torque resistance at the input, and that the variation is not random, but may be designed to match any set of data.

The requirement that the motion of the weight stack on the mechanism match that of a free weight has been met. This was accomplished by rotation of the mechanism in the global coordinate system to a position which allowed the mechanism to match vertical displacement of the weight stack. This allows for easy comparison of variable and constant resistance methods.

Figure 15 is a plot of $\Theta_4$, the output link angle, versus $\beta$, the input angle. To make a valid comparison between the variable and constant mechanical advantage methods, it is necessary for the weight stack to move through a similar vertical displacement. This condition was set during conceptual design and has been met by the final design. The plot is taken directly from the linkage force synthesis program used to synthesize this mechanism.
During use, the input pivot begins its motion slightly below the weight pivot. As the linkage is rotated, the weight pivot catches up to the input pivot near the middle of the stroke. At the extreme end of the range of motion, the weight pivot has progressed to slightly past the input pivot.

The assumptions made at the outset of the analysis have been maintained in the final design. The most critical of these assumptions is that the links could be treated as massless, i.e., that the effects of acceleration are negligible. The total mass of the dynamic portion of the mechanism (not including the weight stacks)
is approximately 28 N (6.5 lb.). This figure is valid for a linkage constructed of 6061 aluminum, as specified. A typical resistance load will be on the order of 290 N (65 lb.). The links will, therefore, typically have only ten percent of the mass of the resistance.

Further reduction of the dynamic mass of the linkage is impractical. Since the maximum design load is 444 N (100 lb.), the physical size of the linkage is required. Optimization of the design could be accomplished by development of the linkage as a solid model. A stress analysis/weight reduction module could optimize the design for minimum weight.

A further assumption was that the input force would remain perpendicular to the input throughout the range of motion. Inclusion of an input handle which freely rotates about its own axis allows the wrist to rotate as needed to minimize radial loading on the input link.

The virtual work equation was simplified for this analysis by neglecting friction and assuming that neither energy storing or dissipating devices would be used in the design. No springs or dampers of any sort have been used in the design. Friction is minimized through the use of Teflon bearings and washers at the pivots. Special attention must be paid to joint friction where the transmission
angle of the mechanism deviates greatly from 90 degrees. In these positions, the bearings minimize static friction and should eliminate binding in the mechanism. In addition, only revolute joints are used, which exhibit far less frictional resistance that higher order pairs such as cam mechanisms.
Chapter 5  Future Work

5.1  Background

As indicated, this work is aimed specifically at increasing understanding of prescribed mechanical advantage mechanisms. This class of mechanisms represents only one aspect of the field of kinematics and mechanism design. As the teaching methods described in Chapter 1 become more and more common in universities, the need for tools of this type will become more profound, not only in kinematics, but in all fields. Limiting the discussion to technological applications still allows for a great deal of development in many aspects of engineering and science.

With this in mind, preliminary synthesis of a second interactive teaching tool has been completed. This section will begin by presenting the work already completed and conclude with a discussion of the potential for the preliminary
work already discussed. Specifically, a first-generation cam mechanism has been synthesized. This cam matches the strength curve of the forearm in flexion and extension of the wrist, just as the linkage does. Cam design is often perceived by students to be even more esoteric than linkage design, especially when the classical graphical synthesis methods are considered.

Closed-form synthesis theory which incorporates conjugate geometry and computer modeling is beginning to be introduced in kinematics coursework. A mechanism such as this will give students a better opportunity to understand and retain advanced cam synthesis theory. Demonstration of the cam and linkage in a side-by-side comparison will illustrate that both cams and linkages are viable options for many synthesis problems.

As described in Section 1.4, lecture-based demonstration of this point using only mathematical theory is not as likely to result in deep understanding of technological subjects. The interactive nature of both demonstration pieces will promote student cognition of the options available when synthesizing a mechanism for prescribed nonlinearity. This is obviously and enhancement to a student’s problem solving abilities.
Force generating cam mechanisms also have significant industrial applications. Use of the cam mechanism in a seminar or sales environment will provide validation of the variable mechanical advantage design concept. The direct result of this convincing demonstration is an increase in acceptance and confidence in the design methodology and resulting exercise equipment.

5.2 Cam Synthesis and Results

Cam synthesis has been accomplished through use of the software developed at Virginia Tech by Tidwell, Bandukwala, Dhande, Reinholtz, and Webb. The synthesis technique is closed-form and has been implemented as a Mathcad model. The brief description of the theoretical basis for this work which follows is taken from Tidwell, et al. (1994).

The Mathcad model uses conjugate geometry for closed-form wrapping cam synthesis. The initial step is to develop a loop-closure equation for the cam, flexible link (typically a chain or belt), and the circular sprocket. The loop consists of four vectors. The first locates the centers of the cam and sprocket. The second locates the point of contact on the cam from the center of the cam. The third connects the points of contact on the cam and sprocket. The final vector connects the point of contact on the sprocket to the center of the sprocket.
The closure equation may be solved for the vector, $P$, which locates the point of contact on the cam.

For all rotations of the cam, $\eta$, the vector $P$ must follow the surface of the cam. Therefore the vector $dP / d\eta$ must at all times point along the tangent line at the point of contact. Since the tangent and the normal to the surface are orthogonal, the dot product of their vectors must equal zero. By taking the dot product of the loop-closure equation with the normal to the surface, an equation for the length of the flexible link is derived. In a follower cam, this length would be analogous to the width of the follower face.

Substitution of geometric relationships and the torque function at the cam transform this equation into one which describes the vector from the center of the cam to the point of contact. The torque function depends on the weight and the moment arm, which is a function of cam rotation. At this point an expression for the cam surface has been established. This equation is a function of the derivative of torque with respect to cam rotation, which is given by the strength curve. The designer is responsible for selection of the weight, center distance, and sprocket radius. The strength curve shown in Fig. 7 was used for this synthesis.
The mechanism synthesized takes user input from rotation of a circular sprocket which drives the cam through a flexible connection. The weight stack is suspended from a circular sprocket which is fixed to the cam. Figure 16 is a conceptual design of the completed mechanism. The cam profile shown in this figure is not specific to this synthesis problem, but represents a general cam. Specific data is provided below. The outlines of both the circular sprocket and the cam represent the pitch lines of these components.

![Diagram of Conceptual Cam Mechanism for Wrist Flexion/Extension](image)

**Figure 16: Conceptual Cam Mechanism for Wrist Flexion/Extension**
There are several parameters which must be defined during cam synthesis. Most important are the center distance, the sprocket sizes, the maximum weight displacement, and whether the cam uses a crossed chain or a straight chain.

For this model, a center distance of 30.5 cm (12 in.) was specified. This is in keeping with the desire to maximize the portability of the mechanism. The mechanism is fairly insensitive to this parameter after a point. The minimum depends on cam and sprocket diameter. For a given sprocket and cam, any flexible link length above a certain minimum will not cause a large change in the associated angles, thereby minimizing the effect of the increase. The weight stack was specified to move no more than 56 cm (22 in.) vertically. This is well within the range of acceptable motion for a tabletop demonstration unit.

Standard ANSI Type 40 chain was specified in the model. This chain has the advantages of being standard in Nautilus equipment and well in excess of the strength requirements for this application. The minimum tensile strength of ANSI Type 40 chain is 13,920 N (3,130 lb.), thereby providing an adequate margin of safety (Shigley and Mitchell, 1993).

The input sprocket was specified to have 60 teeth. The pitch of ANSI type 40 chain is 1.27 cm (0.50 in.), therefore the radius of the sprocket is calculated to be
12.1 cm (4.78 in.). The output sprocket, which is attached to the cam, is specified to have 56 teeth, for a radius of 11.6 cm (4.56 in.).

Given these parameters, Figure 17 shows a first-generation cam profile which will match the strength curve for the forearm. This figure is taken directly from the synthesis software discussed above. Obviously, this profile is very radical and would present difficulties during fabrication and use. The profile is included in this work since it is the end product of the first stage of modeling.

![Cam Surface Plot - Polar Coordinates](image)

**Figure 17: Cam Surface Profile - Polar Coordinates**
To reach this point, the initial conceptual model of the mechanism was first evaluated and found to be unacceptable. Originally, the mechanism was intended to locate the cam mechanism at the input, to increase educational impact. The weight would be suspended from a circular sprocket. However, computer modeling proved that this configuration is not feasible.

All synthesis attempts for this configuration resulted in a cam which required backtracking, or retrograde motion, to achieve the required resistance. That is, all cams synthesized for this configuration had multiple radii for single angular positions. This defect occurred over a range of approximately 30 degrees of rotation.

Obviously, this profile may not be fabricated. It is postulated that the problem is due to the rapid change in mechanical advantage of the forearm muscles. In order to match the strength curve, the cam profile is required to double-back on itself. At present, an analytical method for predicting and/or proving this concept does not exist. This leads directly to the suggestion for additional future work in this area.

At this point, the configuration was reevaluated. The decision was made to synthesize a mechanism as shown in Fig. 16. Significant manipulation of the
design parameters was necessary to achieve the profile shown above, which, although far from optimal, does provide a starting point for future work.

Complex notation and polar coordinates lend themselves extremely well to cam synthesis. Such representations are less useful for fabrication purposes. Figure 18 is a plot of the cam surface in rectangular coordinates. The vertical, $Y$, position is the imaginary component of the complex surface vector.

Figure 18: Cam Surface Profile - Rectangular Coordinates
Comparison of the resistance and strength curves is not necessary for cam mechanisms. The closed-form synthesis technique results in a cam profile which will provide the required resistance. The precision of the resistance curve is limited by the accuracy of the fabrication technique. Fabrication may be improved by calculation of a larger data set for surface positions.

The utility of such a mechanism is obvious. In additional side-by-side comparisons with a linkage-based device students will be able to further identify the physical result of design for prescribed mechanical advantage. Use of these devices is easily coupled with topics covered in mechanisms and kinematics courses. These topics include mechanism inversion, loop-closure analysis, classical and closed-form synthesis techniques, conjugate geometry, and Burmester theory.

The existence of both a higher and lower pair mechanism designed for the same resistance profile has another educational benefit. Beyond comparison of the mechanisms to their constant advantage counterparts, the mechanisms may be compared to one another. Topics such as the frictional properties, manufacturing requirements, and cost analysis of both higher and lower pair mechanisms may be pursued.
Since the human body is not constrained to motion in only one plane, future work could include a three-dimensional demonstration mechanism (Bokelberg and Gilmore, 1990). A spatial mechanism, and the required synthesis and analysis techniques, could be presented in advanced kinematics courses. The industrial value of such a mechanism would be found in the further increase in efficiency coupled with more accurate modeling of human motion. The extent of the efficiency increase depends to a large extent on the exercise undertaken. The greater the deviation from planar motion, the more effective a spatial mechanism will be.
Chapter 6  Conclusions and Recommendations

This work presents detailed documentation of the synthesis, analysis, and fabrication of a prescribed nonlinear mechanical advantage demonstrator. The focus of this work has been to develop a tool for use in education and industry to illustrate the properties of this class of mechanism. Educational uses will focus on classroom instruction. In industry, the mechanism may find use as a proof-of-concept demonstrator during sales or as an instructional aid during seminars.

Closed-form techniques greatly facilitate the synthesis of mechanisms such as this. Use of these synthesis techniques as well as an appreciation for cutting edge educational theories and practices has led to the development of a tool for use in classrooms of the near future. Application of this tool in education and industry will enhance the transfer of knowledge and understanding to the student. Interactive learning aids such as this, when coupled with synthesis theory such as detailed in Chapter 2, result in more thorough cognition of highly technological subjects.
It is recommended that this tool be used as part of an overall plan to encourage a change in engineering education. To continually improve the standards of education, a shift from standard lecture/dictation format to a more interactive and hands-on environment is required. This environment will not necessarily be unable to meet syllabus requirements. In fact, interactive tools may save time by allowing students to tailor their learning pace to their comprehension rate.

Often, students rely on course-end review sessions to gain a firm grasp on the theory. If the misunderstood theory is used by the instructor as foundation for later, more advanced work, as is often the case, any chances of complete understanding have been sabotaged. The time lost due to this lack of understanding is difficult to estimate, but when the number of students who may suffer from this condition is considered, the potential waste becomes significant. Interactive tools allow students to achieve understanding the first time through the material, thereby preventing wasted time, as described above.

In education, the result of these changes will be the production of students more fully equipped to enter industry. Any and all steps which institutions of higher learning can take to create graduates of this caliber should be exploited. Use of interactive tools such as this, in concert with the necessary theoretical background, will generate engineers more capable of creative thinking and on-
the-spot problem solving. This improvement is directly due to the students’ hands-on experience which leads to more complete cognition and retention.

In industry, this mechanism represents a chance for exercise professionals to advance the level of technology associated with their field. Acceptance of variable resistance mechanisms will result in new industry standards for efficiency in exercise and muscular development. The linkage synthesized herein will demonstrate the merit of this type of mechanism.

Nautilus has begun to stress linkages to improve customer satisfaction. Successful designs feel more smooth and are provide no “spongy” feeling. Deflection in the mechanism is often described as the most detrimental of all defects. Chains and belts, even when Kevlar reinforced, will stretch and alter the feel and performance of the mechanism. This is not to say that cams will be eliminated from the marketplace, most rotational motions, such as pronation, are too well suited to cam mechanisms.

It is naive to think that such changes can be brought about in a school year or even during the entire course of a career. Incremental changes are being made, however. Effort must continue to be directed toward the discovery and implementation of more effective teaching methods. To be effective, true
interactive education must become a reality. Moreover, this change in the fundamental paradigm of education must be embraced by post-secondary educational institutions in general. The effectiveness of the method is enhanced by its thorough integration into a students’ entire college career.

The day of true interactive learning, where the use of tools such as the one presented herein is approaching. That advent of that day is should be encouraged, promoted, and implemented by the educational community in a forthright and aggressive manner. Coupling this change in the theory of post-secondary education with interactive teaching tools will better serve today’s student and tomorrow’s engineer.
References


Sherman, T., 1984, Proven Strategies for Successful Learning, Charles E. Merrill Publishing Co., Columbus, Missouri.


Tao, D. C., 1967, Fundamentals of Applied Kinematics, Addison-Wesley, Reading, Massachusetts


Appendix
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Bill of Materials 138

Notes:

The assembly drawing shows only the left side of the mechanism for clarity. In addition, the coupler/output joint is not shown. The right side of the mechanism is a mirror image of the left side. The coupler/output joint is identical to the coupler/input joint shown.

The item numbers shown on the assembly drawing refer to the Bill of Materials on page 147.

The mechanism requires two each of the input, coupler, output, and ground links. Only one base plate is required.
Input Link
Base Plate

- 2X Ø.281 THRU BY 2.50 LONG SLOT
- √ Ø.507 X 82°
- 6X .25-20 UNC-2B .75 DEEP

Dimensions:
- 2X .893
- 2X 2.50
- 2X 2.50
- 2X 2.625
- 2X 1.00
- 2X .50
- 7.15
- 8.50
- 6X .375
Ground Plate
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Vita

Keith Garguilo was born on June 19, 1970 and has no idea why this fact must be included here. Since then Mr. Garguilo has spent entirely too much time in school. This is mostly due to the fact that he kept leaving to get work experience. These important years have included stretches at Rensselaer Polytechnic Institute, Pratt and Whitney, Vermont Yankee Nuclear Power Corporation, and, obviously, Virginia Tech.

And that's all he wrote.