

Diameter and Height Increment and Mortality Functions for  
Loblolly Pine Trees in Thinned and Unthinned  
Plantations

by

Michael C. Smith

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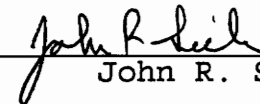
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Forestry

Approved:

  
Harold E. Burkhardt, Chairman

  
Richard G. Oderwald

  
John R. Seiler

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Harold E. Burkhart, Chairman

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(ABSTRACT)

Although there remains some controversy as to the benefits of individual tree growth and yield prediction models over stand level models, individual tree models still have wide acceptance. A generalized thinning response variable which can account for the intensity of thinning, as well as the age of the stand at the time of thinning and the time elapsed since thinning, was applied to two existing models for loblolly pine (Pinus taeda) in cutover site-prepared plantations. A site index equation for predicting mean total height of dominant and codominant trees and a diameter increment model were developed to incorporate the thinning response variable. New fits of height increment and mortality functions to the available data were also completed. Separate mortality functions were fit to data from unthinned and thinned stands.

The base models for this analysis were from the individual tree growth simulation model PTAEDA2. Evaluations

for predictive ability of these models were done in a reduced version of the growth simulator which was modified to accept external data. The mean total height model had improved predictive ability over the original PTAEDA2 model for this variable. The diameter increment model produced no significant improvement in simulation comparisons. Fitting the two mortality functions to the multiple observation data resulted in reduced predictive ability of the simulator compared to the original mortality model from PTAEDA2 which was fit to data from unthinned stands only.

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## INTRODUCTION

Much work has been done recently with individual tree growth and yield models. Although there remains some controversy as to the benefits of individual tree prediction over stand level models, individual tree models still have wide acceptance.

In the individual tree growth simulation model PTAEDA2 (Burkhart et al. 1987), response to thinning was implicit in changes in each tree's competitive status. Amateis et al. (1989) developed separate diameter increment models for unthinned and thinned stands. A more efficient approach is to have one prediction model which can function well in either unthinned stands or stands thinned to a wide range of different intensities.

Recent work in tree and stand response to thinning practices has focused on developing a generalized thinning response variable which can account for the intensity of thinning, as well as the age of the stand at the time of thinning and the time elapsed since thinning (Short and Burkhart, 1992; Liu et al. in press). These recently-published analyses were focused on describing crown growth. A thinning response variable which would explain diameter growth and tree mortality across time under different intensities of thinning has not been developed previously.

In this paper, a thinning response variable developed by Liu et al. was applied to a site index equation for predicting mean total height and to a DBH (diameter at breast height) increment model. The base models for this work were those developed in PTAEDA2. The evaluations for predictive ability of these models were done in a reduced version of the growth simulator which was modified to accept external data.

### **The Data**

The data used in this analysis are from a thinning study using permanent plots established by the Loblolly Pine Growth and Yield Research Cooperative. The thinning study contains 186 permanent locations established from 1980 to 1982 in plantations on cutover, site-prepared lands spread across twelve southeastern states. This area includes most of the natural range of loblolly pine. At each location, three similar plots were established, and individual trees were tagged and measured. Plots had to be similar in site index, stocking, and basal area. Diameter, total height, and height to live crown were measured on volunteer, as well as planted loblolly pines, and competing hardwoods. Stem maps were generated for each plot at the time of plot establishment.

One plot at each location was randomly selected as a control, one was thinned to about 70 percent residual basal area (light thin), and the third plot was thinned to

approximately 50 percent (heavy thin) of the starting basal area. The thinning treatments were primarily from below, with an occasional row removed for access. An effort was made to avoid confounding the thinning with release from competing hardwoods by only removing those natural pines and hardwoods that would normally be cleared in an operational thinning (Burkhart et al. 1987).

There have been three remeasurements at three-year intervals since plot establishment. At each remeasurement, mortality was recorded, as well as diameter at breast height, total height, and height to the base of the live crown. A fourth remeasurement is under way at this time. In this remeasurement, one half of the plots are being thinned a second time for future analysis of stand response to multiple thinnings.

Only the interior trees in the Coop dataset were used for model analysis and development in this project. Interior trees are those for which all competitors within a 10 BAF (basal area factor) angle gauge sweep are within the measured research plot. Thus, only these trees could have valid competition indices calculated for them based on actually measured competitors. Table 1 provides a detailed description of the data used.

Table 1. Mean stand and tree values for interior trees from the Coop thinning study dataset, by thinning treatment at plot establishment and the third remeasurement.

Variables	Plot Establishment			Third Remeasurement		
	Un-Thinned	Light Thinned	Heavy Thinned	Un-Thinned	Light Thinned	Heavy Thinned
<b>Stand</b>						
MTH	41.14	41.33	41.40	56.23	56.60	56.56
VOLUME	36.00	106.55	81.08	35.94	153.30	107.69
AGE	15.17	15.17	15.18	24.17	24.17	24.18
I	1.00	0.73	0.59	-----	-----	-----
<b>Tree</b>						
DBH	4.61	5.61	5.85	5.71	7.28	7.76
HT	32.64	36.10	36.78	46.35	51.33	51.73
CR	0.44	0.48	0.50	0.31	0.37	0.40
CI	1.26	0.64	0.46	1.57	0.95	0.74

All values are for planted loblolly pines only.

MTH = mean total height of dominant/codominant trees in each plot in feet, VOLUME = cubic feet per plot, outside bark, I = intensity of thinning = basal area after thinning / basal area before thinning, DBH = diameter at breast height in inches, HT = total height in feet, CR = crown ratio, CI = competition index.

## PREVIOUS WORK

Individual tree growth simulation models have gained much acceptance in recent years. Prior models were largely based on stand level predictions or distributional analysis (Daniels and Burkhart, 1975). Newnham (1964) is credited with the first stand model based on individual tree simulation (Daniels and Burkhart, 1975). He put forth a diameter increment model for Douglas-fir plantations in which individual tree diameter growth was estimated as open-grown growth modified by a measure of competition.

Burkhart et al. (1987) developed an individual tree growth simulation model for loblolly pine. In this distance-dependent model, called PTAEDA2, diameter and height increment are predicted on an annual basis, then added to the current diameter and height values to be carried forward to the next growing season. Mortality is predicted annually, and the value of the competition index is calculated for each tree dependent on current tree size and the growth or loss of competitors.

PTAEDA2 is the revised version of PTAEDA, originally developed by Daniels and Burkhart (1975). The most notable improvements were an option to consider hardwood competition

in the growth simulation, improved estimates of crown ratio, and improved versatility for the user in output volume determination. Also, the growth models were refitted to new data, and height increment prediction was improved by a new site index equation used to determine the potential height increment.

The core of any growth simulation program is the individual prediction models. The height increment model in PTAEDA2 predicted height growth as the potential height increment modified by a function of crown ratio and competition index. The potential height increment is calculated within PTEADA2 from a site index equation developed by Amateis and Burkhart (1985), and is based on stand age, site index, and the average height of dominant and codominant trees. The height increment model with its modifier is shown below.

$$HIN=PHIN*[\beta_1+\beta_2CR^{\beta_3}*\exp(-\beta_4CI-\beta_5CR)]$$

where: HIN = predicted height increment  
PHIN = potential height increment, computed as  
the first difference in height from a



site index equation

CR = crown ratio

CI = competition index

The above model has some good biological properties. Crown ratio expresses the tree's photosynthetic potential. It is in the model twice to describe its positive relationship to potential height increment, and its negative relationship to it when the tree approaches an open-grown condition.

As with the height increment model, the predicted diameter increment is the potential diameter increment modified by a function of the tree's crown ratio and competition index. The potential diameter increment was developed and fitted in the first version of PTEADA and was based on data gathered from 81 open-grown loblolly pines. It uses the current height increment of each tree to predict the tree's potential diameter increment. The diameter increment model is shown below.

$$DIN = PDIN * [\beta_1 CR^{\beta_2} * \exp(-\beta_3 CI)]$$

where:      DIN = predicted diameter increment

PDIN = potential diameter increment, computed  
from an equation fitted to data from  
open-grown trees  
other variables are as described previously

Potential diameter increment model:

$$PDIN=0.286583*HIN+0.209472$$

where: HIN = observed current height increment

Mortality prediction is also based on each trees competitive stress and vigor expressed in its competition index and crown ratio. The mortality equation determines a value between 0 and 1, which is then compared to a uniform random variate. The tree is considered dead if the calculated value is less than the uniform random variate. This survival equation is:

$$PLIVE=\beta_1 CR^{\beta_2} * \exp(-\beta_3 CI^{\beta_4})$$

where: PLIVE = probability that a tree remains alive

other variables are as described previously

The competition index used in PTAEDA2 was applied by Daniels and Burkhart in the first version of this program. Developed by Hegyi (1974), this compact measure of competition has worked well in several applications. The index is a summation of the ratios of the diameters of each competitor to the subject tree, divided by their separation distance. Competitors were determined annually in a 10 BAF angle gauge sweep. The competition index is calculated as:

$$CI_i = \sum_{j=1}^n (D_j/D_i) / DIST_{ij}$$

where:  $CI_i$  = competition for the  $i^{th}$  subject tree  
 $D_i$  = diameter breast height of the  $i^{th}$  subject tree  
 $D_j$  = diameter breast height of the  $j^{th}$  competitor  
DIST = distance between the  $i^{th}$  tree and the  $j^{th}$  competitor  
 $n$  = number of competitors within a 10 BAF sweep

Final estimates for the models were fit to data from the unthinned control plots over the first three year remeasurement period of the Coop thinning study dataset. When thinned stands in the same study area were used as a semi-independent validation data set for the individual models, the researchers found only a slight underprediction.

In these models, response to thinning is incorporated via the competition index. Elapsed time since thinning is not explicitly included, but rather is implicit in the annual change in the trees crown ratio and in its competition index due to the growth of its competitors.

In 1989, Amateis, Burkhart, and Walsh developed separate distance-independent diameter increment equations for thinned and unthinned stands. These models were of the same form as the diameter increment model applied in PTAEDA2, but were designed for use where stand spatial data were not available. The potential diameter increment used in these models was identical to that used in PTAEDA2.

In place of an individual tree competition index, a spatially insensitive analog to it was used. The ratio of stand mean quadratic DBH to each tree's DBH, subtracted from one, was used to describe the competitive stress of each tree.

The model developed for application in unthinned stands was of the form shown below.

$$DIN=PDIN*[\beta_1 CR^{\beta_2} * \exp(-\beta_3(1-D_q/D))]$$

where:  $D_q$  = mean quadratic DBH of the stand  
D = diameter breast height  
other variables are as described previously

A separate model was developed for thinned stands with two variables added to the exponential portion. The stand age at thinning was added, and the thinning intensity in the stand was described by a ratio of stand basal area after thinning to the basal area before thinning. In the below model, both  $B_4$  and  $B_5$  must be negative to ensure the proper response of the model.

$$DIN=PDIN*[\beta_1 CR^{\beta_2} * \exp(-\beta_3(1-D_q/D) + \beta_4 A_t + \beta_5 (B_a/B_b))]$$

where:  $A_t$  = stand age at thinning  
 $B_a$  = stand basal area after thinning

$B_0$  = stand basal area before thinning

other variables are as described previously

Amateis et al. (1989) also fit models to describe diameter growth in thinned and unthinned stands when the basal area of competing hardwoods is considered. For each of these four conditions, a distance-independent survival equation was fitted. The survival equations were of the same form as the PTAEDA2 model.

The models for unthinned and thinned conditions were fitted to the data from unthinned and thinned plots respectively from the same Coop data set used for PTAEDA2. The researchers used data from the first remeasurement period, and all of the second remeasurement data that was available at that time. Analysis of the model predictions showed a slight increasing underprediction over time. Deviations of .02" at the first remeasurement and .06" at the second one were seen for both the thinned and unthinned models.

What was lacking at this point was a thinning response variable which would allow individual tree diameter increment prediction across a wide range of conditions; i.e., from unthinned to heavily thinned stands. Also, an important characteristic that is missing from the thinned stand model

above is a term to describe the elapsed time since thinning. This factor is necessary to give a more biologically valid shape to the growth curve in response to thinning.

Working with crown height increment models, Short and Burkhart (1992) developed a thinning response variable which describes thinning intensity and age at thinning. This thinning response variable enables prediction of crown recession in thinned and unthinned stands using the same model. In this work, a distance-dependent, a distance-independent, and a stand level model were developed. Also, there was an in depth analysis of various model forms and associated biases. Below is the multiplicative thinning response variable developed.

$$T = \left( \frac{BA_a}{BA_b} \right)^{\left( \frac{TA}{A} \right)}$$

where:       $BA_a$  = stand basal area after thinning  
               $BA_b$  = stand basal area before thinning  
               $TA$  = stand age at thinning  
               $A$  = present stand age  
other variables are as described previously

T will equal 1 in unthinned stands since  $BA_a$  and  $BA_b$  are equal. With the above variable, heavier thinning results in a smaller T and thus less crown recession. The negative impact of thinning on crown height increment increases as age at thinning increases. Also note that the variable T is at its minimum immediately after thinning, then slowly returns to 1 as time since thinning increases. The final form of the distance-dependent, individual tree model for predicting crown height increment was:

$$HLCIN = \beta_0 T^{\beta_5} HT^{\beta_1} * \exp(\beta_2 CR^{0.5} + \beta_3 CI + \beta_4 A)$$

where: HLCIN = crown height increment

HT = total tree height

other variables are as described previously

When the distance-independent model was fit without T to the unthinned data, analysis in thinned stands showed an overprediction that increased with thinning intensity. The increment model with T showed an overprediction in the unthinned stands and a trend towards underprediction as thinning intensity was increased. Raising the thinning variable to a power greatly improved the fit and reduced these



biases.

The one failing of this thinning variable is that it is monotonically decreasing across time. The thinning response variable does not accurately describe the actual relationship between thinning and elapsed time since thinning. The variable predicts a maximum response to thinning at the time of thinning, with response declining thereafter. In reality, a trees response is more curvilinear, reaching its maximum at some point in time after thinning has occurred and then declining.

Liu et al. (in press) improved upon the thinning response variable from Short and Burkhart by incorporating time elapsed since thinning as an actual measure, rather than as a ratio. This variable gives a more biologically natural shape to the thinning response function. In fitting an allometric model for predicting crown ratio (CR), they developed a thinning response variable (TRV) that accounts for the age at thinning, the thinning intensity, and the elapsed time since thinning. This thinning variable is:

$$T = \left( \frac{BA_a}{BA_b} \right)^{\frac{r[-(A_g - A_t)^2 + K(A_g - A_t)]}{A_s^2}}$$

where:       $r$  = rate parameter  
               $K$  = duration parameter  
               $A_s$  = current stand age  
               $A_t$  = stand age at thinning  
              other variables are as described previously

The "rate" parameter,  $r$ , is unitless. It helps to describe the shape of the response curve primarily by its relationship to  $K$ . The closer the estimates for  $r$  and  $K$  are to each other, the more intense the level of modification becomes. Parameter  $K$ 's units are years, and it determines the duration of the thinning response, as well as the year of maximum response which is dependent on the relationship between  $K$  and  $A_t$ .

This TRV is designed to neutralize when the elapsed time since thinning equals the duration parameter, thus the value of  $T$  equals 1 and no modification occurs. The intensity of modification is also controlled by the relationship between the age at thinning and the elapsed time since thinning. As age at thinning is increased, less modification is produced. The base ratio, the intensity of thinning, also has control of the level of modification. Less intense thinning results in less intense modification.

The final form of the allometric CR model was:

$$CR=1-[T*\exp(-(\beta_0+\beta_1/A_s)*D/H)]$$

where: H = total tree height

other variables are as described previously

The above allometric model, using the new thinning variable, was compared to the increment model developed by Short and Burkhart for predicting crown recession. When the originally published thinning response variable was retained in the increment model, they found that an allometric determination of crown ratio, rather than an increment approach in predicting crown height, produced less biased results.

To test the general applicability of the new TRV, the researchers refit the increment model developed by Short and Burkhart, replacing the original TRV with the newly developed one. They found little difference between the increment and the allometric approach to predicting crown development in thinned stands. Thus, they concluded that model selection

could be based on model efficiency and application concerns, rather than predictive ability.

The above analysis demonstrated that the new TRV could be applied to a variety of prediction applications in which the expected response to a particular treatment would be null initially, increase to some maximum over time, then gradually return to the null condition. Since the TRV is unitless, it is applicable to a variety of silvicultural treatments.

Imprecise estimates of parameters can result from the temporal correlation inherent in repeated observations over several remeasurement periods. In Liu et al., the effect of this correlation was examined. The researchers used two different methods in splitting the data for fitting and validation. One dataset was a random split of the entire dataset, and a second dataset was generated by randomly selecting one remeasurement observation from each tree, thereby eliminating the temporal correlation. After fitting each of the models they examined to each dataset, they found "no substantial differences" in bias or precision.

Thus, their final recommendation was to use all the data in fitting any final models to ensure the most stable parameter estimates. This was qualified in their conclusions

by stating that additional remeasurement periods or shorter intervals may have a much greater impact on model fitting.

A similar analysis, aimed at assessing the effects of unmeasured time-dependent variables was conducted by Avila and Burkhart (1992) in their development of distance-dependent and distance-independent mortality functions for loblolly pine. The models were fit to one-half of a random split dataset and validated on the other half, and they were fit to the first remeasurement period and validated on the second. They found "no practical differences" in validation statistics between the two data splitting methods.

The distance-dependent logistic model developed for predicting survival of individual trees was of the form:

$$PLIVE = \frac{1}{1 + \exp[-(\beta_1 + \beta_2 CR + \beta_3 HH - \beta_4 CI)]}$$

where: HH = ratio of total height to average height of dominant and codominant trees  
other variables are as described previously

Analysis of this model in thinned stands showed that no improvement in model performance was achieved by including variables to describe intensity of thinning, age at thinning, or time elapsed since thinning. When this model was fit to data from unthinned stands and compared to the survival equations published by Burkhart et al. (1987) and Amateis et al. (1989), only modest improvement was found for the above equation.

## Analysis

All model fitting for this thesis was done using the statistical analysis package (SAS) developed by SAS Institute Incorporated, Cary, NC. All non-linear regressions were done with the automated regression method DUD.

Since I was using the multiple observation Coop dataset, temporal correlation was a concern. However, based on the results of Liu et al. (in press) in their analysis of temporal correlation in this dataset, I proceeded as though there were no temporal correlation problem.

Another consideration was the validity of dividing three-year interval observations by three and using those values to fit one-year interval models. It is a common procedure to use what is known as the "linear averaging method", which assumes linear growth over each three-year period, to obtain annual data from multi-year data. McDill and Amateis (1993) compared four methods for fitting difference equations when the desired time interval is not the same as the interval at which the data used to fit the equation were collected. They found that the linear averaging method worked best for the younger ages, but it underpredicted for the older ages. Because the linear averaging method has generally produced

satisfactory results and because it is the simplest method to apply, it was used in these analyses.

Analyses of model performance were also based on these observed values divided by three in order to validate the models on a one-year basis. Thus, given the three-year measurement interval of the data, predictions could only be made and analyzed for the first, fourth, and seventh growing seasons following plot establishment.

Note that throughout this discussion, residual or residuals are defined as observed minus predicted values.

The first step was to determine if a thinning response variable was warranted in the diameter increment, height increment, and mortality functions. If no significant differences in model parameters across thinning regimes existed, then the increased complexity of an additional variable to describe thinning response would not be necessary.

For each of the three models under examination, separate parameters were fit for the no thin, light thin, and heavy thin data, and again across all of the data. The sums of squares for error from these fits were used in a simple F-test as described by Swindel (1970) to determine if there were



significant differences between the parameters fit to different thinning intensities.

For example, when testing the diameter increment (DIN) model, which has three parameters, the null and alternative hypotheses of this F-test were as follows:

$$H_0: B_{11} = B_{12} = B_{13} \quad \text{and}$$

$$B_{21} = B_{22} = B_{23} \quad \text{and}$$

$$B_{31} = B_{32} = B_{33}$$

Ha: at least one of the above three conditions is false

where  $B_{11}$ ,  $B_{12}$ , and  $B_{13}$  represents the first parameter in the no thin, light thin, and heavy thin fits, respectively;  $B_{21}$  through  $B_{23}$  symbolizes the second parameter in each of the three models, etc. In this test, the full model is the model in which separate parameters have been fit for each thinning regime, thus totalling nine parameters for the DIN model. The reduced model is the three parameter model fit to the entire dataset. The form of the F statistic is:

$$F = \frac{(SS_r - SS_f) / (df_r - df_f)}{SS_f / df_f}$$

where:  $SS_r$  = sum of squares for error of the reduced model  
 $SS_f$  = total of the sums of squares for error of each  
of the separate fit models  
 $df_r$  = degrees of freedom of the reduced model  
 $df_f$  = sum of the degrees of freedom of the separate  
fit models

Within the simulator, mean total height is calculated first. This in turn is used to determine a potential height increment which is used in the height increment determination. The height increment result is the basis of the potential diameter increment calculation, which is modified in the DIN model. Considering this flow in the simulator, the HIN model was the first focus.

### Height Increment Model

Prediction of height increment (HIN) is accomplished by the modification of the potential height increment with a function of crown ratio and competition index. The potential height increment (PHIN) is based on the difference between the observed mean total heights of dominant and codominant trees in each plot at two consecutive remeasurement periods. The HIN model from PTAEDA2 is restated here:

$$HIN=PHIN*[\beta_1+\beta_2CR^{\beta_3}*\exp(-\beta_4CI-\beta_5CR)]$$

Fitting the HIN model to the multi-year data proved to have several problems. Attempts were first made at separate fits of one-year increment models for each thinning regime. These were necessary for the F-test for significant differences. The data from unthinned plots fit well, with only  $B_1$  being insignificant (indicated by its asymptotic confidence interval including zero).

It was very difficult to obtain a non-linear regression convergence with data from either the light thinned or heavy thinned plots. Several sets of starting parameters were tried. When convergence was achieved, the estimates for  $B_1$  and  $B_2$  had no standard errors in each of the thinned data fits, and SAS reported "the Jacobian is singular". This indicated that all information in these two parameters was completely explained by the other three parameters in the model. Estimates of this type will hereafter be called singular estimates. The estimates for  $B_3$ ,  $B_4$ , and  $B_5$  were all insignificant. No set of starting parameters could improve upon this result.

It was decided to attempt three-year prediction fits in the hope of getting valid convergence with the data from the thinned plots. Convergence was more successful using three-year increments but again all estimates for the fit to heavy thin data were insignificant, and only  $B_2$  was significant in the fit to light thinned data. Correlation matrix analysis identified part of the cause of these problems. In both of the datasets from thinned plots, correlation was greater than 0.98 between  $B_3$ ,  $B_4$ , and  $B_5$ . In the data from heavy thin plots, correlation between  $B_1$  and  $B_2$  exceeded 0.99, and was greater than 0.94 for the fit to data from light thinned plots. Correlations were low (less than 0.6) for  $B_1$  or  $B_2$  to  $B_3$ ,  $B_4$ , or  $B_5$ , even in the fit to all of the data.

Originally, this model was fit to data from unthinned plots only for PTAEDA2. In that fit, correlations were similarly high, but not nearly as high as the current fit to unthinned data. The original fit was across only one three-year increment period of unthinned plot data. In that fit, over 10,000 observations were used by translating the plot so that border trees would have competitors on the opposite plot edge. However, only 2,500 observations were used in this analysis, due to only interior trees being used in the current fits. These are expanded to 5,700 observations by including second and third remeasurement data.

To test whether temporal correlation was affecting the fits, non-temporally correlated datasets were generated by randomly selecting one of the three remeasurement period observations for each tree. This reduced the total available data by two-thirds. Fits to data from unthinned, light, and heavy thinned plots were done for one-year and three-year increments.

Using non-temporally correlated data gave no reduction in the correlations between  $B_3$ ,  $B_4$ , and  $B_5$  for any of the thinning regimes or increment levels. The correlations between these estimates and  $B_1$  and  $B_2$  either greatly increased or decreased depending on the thinning intensity in the three-year increment fits. These correlations could not be compared with temporally correlated one-year increment fits because these values were not available for the singular estimates for  $B_1$  and  $B_2$ .

The HIN fits were not improved using the non-temporally correlated data. All estimates in the fits to data from thinned plots were again insignificant, and the estimate for  $B_2$  was again singular in both the three-year and one-year increment fits to heavy thinned data. The standard errors for each estimate increased dramatically, particularly for  $B_1$  and  $B_2$ . The fit statistics, such as they are, are presented in

table 2 for the non-temporally and temporally correlated fits, for both the one and three-year fits. The significance of the estimates is also given.

The unthinned plot data were randomly split and the one-year HIN model was fit and validated. The  $S_{y,x}$  for this fit was 0.6604, an improvement over both of the other fits to unthinned plot data. Validation residuals were similar to the fit residuals, and indicated no particular bias.

When high correlations exist between parameters, it usually indicates that one or more of those parameters should be fixed at a particular value, rather than being fitted through iterations. This was tried with  $B_3$  and  $B_5$ . When  $B_3$  was set to 0.12, the model failed to converge with several sets of starting parameters. Fixing  $B_5$  at the approximate value of previous fits generated better results.

The  $B_5$  parameter was set to 0.5 for unthinned, 0.6 for light thinned, and 0.3 for heavy thinned plot data. The fits were improved for all three thinning regimes, for both one-year and three-year increments over previous fits using five parameters. The improvements were not significant, but some estimates which were insignificant became significant and no singular estimates occurred. Future analysis may prove this

to be a better design for the height increment model. The fit statistics and status for these fits for one and three years are given in table 3, and can be compared with the table 2 fits to temporally correlated data.

To go one step further, an attempt was made to fit the HIN model with  $B_5$  set and to incorporate the thinning response variable developed in Liu et al. (in press) multiplied against CI in the exponent. Convergence was achieved, but the estimates for R and K were found to be insignificant. The parameter K was estimated at 725 which was obviously not valid.

Other variations of this model were tried. The estimate for  $B_1$  was always insignificant. Fitting the model without this parameter allowed more stable estimates for the other parameters, but there was no improvement in the residual analysis and no practical gain was seen from this design. When the model was fit without CR or its parameter ( $B_5$ ) in the exponent of the model, it resulted in the worst fit of any of the forms tried.

Spreadsheet analysis was done on the models fitted to data from light and heavy thinned plots to determine the impact of each of the variables CR, CI, and PHIN on the HIN

Table 2. Fit statistics and significance of estimates for height increment models fit to temporally and non-temporally correlated data.

Model	SSE	MSE	Sy.x	Df	Significance
1	2536.9304	0.4520	0.6723	5613	B1 insignificant
2	5526.2522	0.4271	0.6535	12940	All insignificant
3	3695.1913	0.4171	0.6458	8859	All insignificant
4	22858.1896	4.0724	2.0180	5613	B1 insignificant
5	49982.4027	3.8632	1.9655	12938	only B2 significant
6	33480.1545	3.7801	1.9442	8857	All insignificant
7	863.2771	0.4614	0.6793	1871	B1, B3, B5 insig.
8	1813.7477	0.4280	0.6542	4238	All insignificant
9	1284.8736	0.4379	0.6617	2934	All insignificant
10	7769.4939	4.1526	2.0378	1871	B1, B3, B5 insig.
11	16323.7290	3.8518	1.9626	4238	All insignificant
12	11563.8625	3.9413	1.9853	2934	All insignificant

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = fitting degrees of freedom.

**Models:**

- 1 1 Year unthinned data fit to temporally correlated data
- 2 1 Year light thinned data fit to temporally correlated data
- 3 1 Year heavy thinned data fit to temporally correlated data
- 4 3 Year unthinned data fit to temporally correlated data
- 5 3 Year light thinned data fit to temporally correlated data
- 6 3 Year heavy thinned data fit to temporally correlated data
- 7 1 Year unthinned data fit to non-temporally correlated data
- 8 1 Year light thinned data fit to non-temporally correlated data
- 9 1 Year heavy thinned data fit to non-temporally correlated data
- 10 3 Year unthinned data fit to non-temporally correlated data
- 11 3 Year light thinned data fit to non-temporally correlated data
- 12 3 Year heavy thinned data fit to non-temporally correlated data



Table 3. Fit statistics and significance of estimates for height increment models fit with parameter B5 set to 0.5 for unthinned data, 0.6 for light thinned data, and 0.3 for heavy thinned data.

Model	SSE	MSE	Sy.x	Df	Significance
1	2533.7036	0.4513	0.6718	5614	all significant
2	5507.3832	0.4257	0.6525	12936	B1 insignificant
3	3692.3467	0.4168	0.6456	8858	only B3 significant
4	22803.3317	4.0619	2.0154	5614	all significant
5	49567.3491	3.8317	1.9575	12936	B1 insignificant
6	33231.1141	3.7515	1.9369	8858	only B3 significant

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = fitting degrees of freedom.

Models:

- 1 1 Year unthinned data fit with B5 set to 0.5
- 2 1 Year light thinned data fit with B5 set to 0.6
- 3 1 Year heavy thinned data fit with B5 set to 0.3
- 4 3 Year unthinned data fit with B5 set to 0.5
- 5 3 Year light thinned data fit with B5 set to 0.6
- 6 3 Year heavy thinned data fit with B5 set to 0.3

prediction. Each of these variables was varied across its observed range while holding the other two variables constant at their means, and the HIN prediction was analyzed.

It was found that HIN changed by less than a foot across the entire range of either competition index or crown ratio. The HIN prediction was about equal to PHIN across the range of potential heights (about 5 feet). This indicated that CR and CI seemed to have very little power in the modification of PHIN, thus the value of PHIN largely controls the HIN prediction. This condition was more prominent for the estimates fit to data from heavy thin plots. The lack of impact of changes in CR or CI as thinning intensity is increased was another possible explanation for the difficulty in obtaining valid fits to the thinned data.

There now appeared to be two possible explanations for the difficulty in fitting the height increment model to data from thinned plots. One was that there was no measurable variation in height increment response between different intensities of thinning, a possibility supported by much previous research. This lack of variation, coupled with the inherent error in field measurements of height could easily explain the fitting difficulties.

The second possibility was that the response is already incorporated in the model in the observed potential height increments. This second possibility would explain why it becomes increasingly difficult to obtain valid estimates for the parameters associated with CR and CI as the intensity of thinning is increased. Modification of PHIN in the HIN model seems to become insignificant as thinning intensity is increased. This was indicated by the spreadsheet analysis.

When the observed PHIN's were plotted, the second possibility became very likely. Refer to figure 1 and note the inversion of potential growth between intensities as elapsed time since thinning increases. These plots show that thinning the stand slows the decrease in height growth with increasing age, at least on a mean analysis level.

During analysis for this thesis, fourth remeasurement data for 44 of the 186 plots became available. The three remeasurement dataset is labeled REM3. When the data from this fourth remeasurement (REM4) was included in the unthinned plot fit of the HIN model, results similar to the three remeasurement fit to REM3 data were obtained. However, when only three remeasurements from the REM4 dataset were used, fit statistics were similar but all estimates were insignificant except  $B_4$ .

### Means of Potential Height Increments Observed values by MTH Subtraction

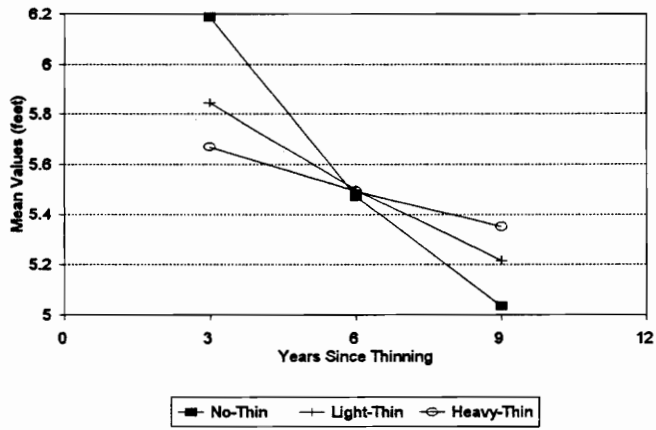


Figure 1. Means of potential height increments from the observed data.

### Means of Potential Height Increments Observed Values Using REM4 Data

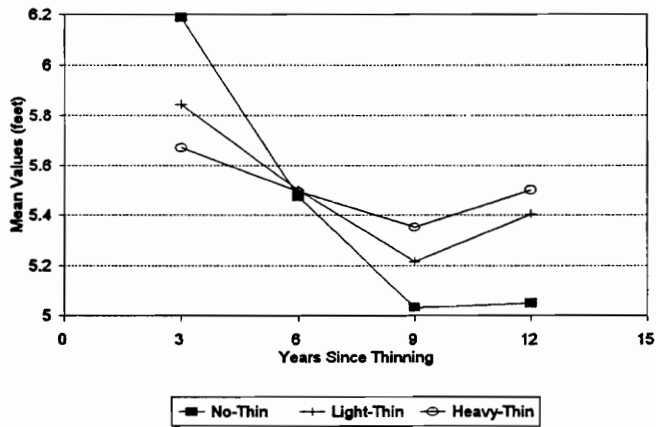


Figure 2. Means of potential height increments including the observed fourth remeasurement data.

When PHIN plots were generated to include the available fourth remeasurement data, the PHIN's of all three thinning intensities rose sharply between nine and twelve years (see figure 2). Since these plots are essentially current annual increment (CAI) plots and loblolly pine reaches its peak CAI at four to five years of age, PHIN should not increase again.

A comparison of the mean site indices between the 44 fourth remeasurement plots and the total of 186 plots found the mean values to be only two feet higher on the REM4 plots, but the minimum site indices were about eight feet higher on these plots. It is unknown if this could account for the differences seen between the two PHIN plots, but it is one possible explanation.

Thus, based on this analysis and the HIN fitting problems with three remeasurements from REM4 data, it was decided that only three remeasurements of REM3 data should be used in fitting the HIN model to avoid confounding the results, and that it be fit to unthinned data only, as was done originally for PTAEDA2. The final estimates from this unthinned data fit are presented in table 4.

To maintain consistency between the different models in this project, all of the models were fit to only three

Table 4. Final estimates for the fitted models.

	MTH Model	HIN Model	DIN Model	PLIVE Unthinned	PLIVE Thinned
B1	-0.016003	-0.547476	0.789332	1.057593	1.012884
B2	-3.020419	2.420062	0.773562	0.062350	0.021765
B3		0.207179	0.703278	0.006169	0.000962
B4		0.171205		2.518345	4.763163
B5		0.381199			
R	-0.689758		8.866352		
K	10.773257		5.027573		
Sy.x	1.7898	0.6723	0.0901	0.1330	0.0759

Sy.x = standard error of the estimate.

remeasurements using the REM3 dataset. The 44 plots with REM4 data were used in certain validation analyses described later.

The goal then became to develop a model incorporating a thinning response variable (TRV) to describe the response of mean total height of dominant and codominant trees to thinning intensity. This model would then provide the predicted PHIN's in a height increment model fitted to unthinned plot data only. The modeled PHIN would then, in effect, become the modifier to the HIN prediction, rather than the modified portion of the model. In this way, thinning response would be incorporated into the HIN prediction.

#### Mean Dominant and Codominant Height Model

Examination of the PHIN plots indicated that the model should be conditioned such that the mean dominant/codominant height at all thinning intensities would be equal directly after thinning and at six years after thinning. To test this conditioning, the data were equally divided into four groups defined by the plot ages at thinning. The groups were 8 - 12 years, 12 - 14 years, 14 - 18 years, and 18 - 25 years of age at the time of thinning. Figure 3 shows plots of observed PHIN's in each of these four groups.

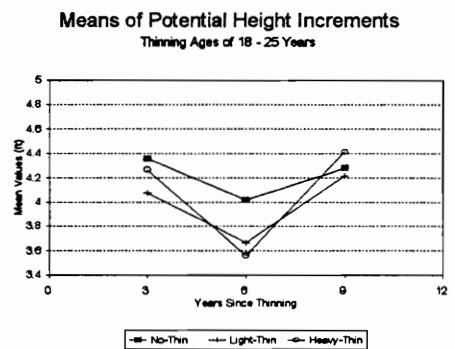
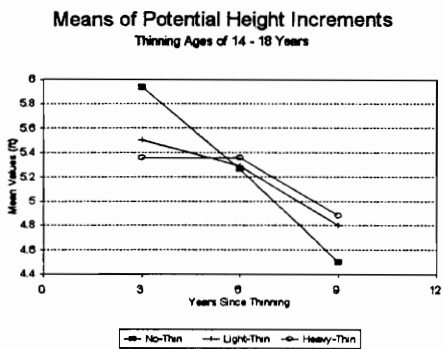
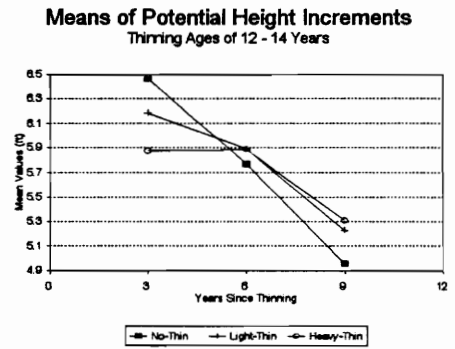
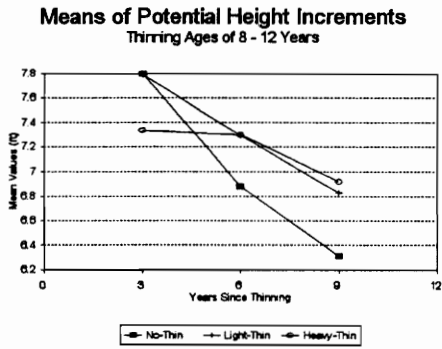


Figure 3. Plots of the means of observed potential height increments for four ranges of plot ages at thinning. Each plot represents 25% of the total number of research plots.

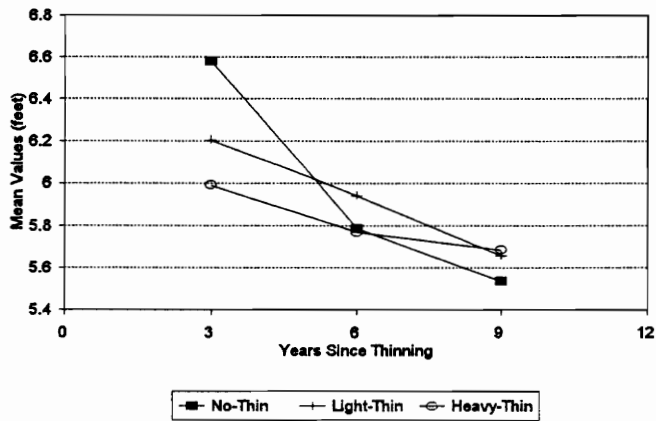


Each PHIN plot shows a variation in mean dominant/codominant height between thinning intensities over time, indicating the need for a model to describe these variations. However, they show very different patterns for the youngest and oldest plots than was indicated by the plots of all the data. Thus, the idea of conditioning the model was discarded.

Another consideration was that the definition of mean total height (MTH) was at the core of the observed variation. In the United States, MTH is defined as the mean total height of the dominant and codominant trees on a plot. Identification of these trees is always subjective, depending on the cruiser's skill and experience. If variation in MTH between thinning intensities proved to be absent under a different definition of MTH, then this could provide an alternate path of analysis for the thinning response modeling. Two other definitions for mean total height were examined. MTH defined as the mean total height of the tallest 100 trees per hectare (40 trees per acre), as is common practice in many European countries, and the mean total height of only the dominant trees on each plot.

The plots of observed PHIN's under each of these definitions shown in figure 4, demonstrate a similar variation in MTH between thinning intensities over time as was observed

### Observed PHIN's by MTH Subtraction MTH Defined by the Tallest 40 TPA



### Observed PHIN's by MTH Subtraction MTH Defined by Dominant Trees Only

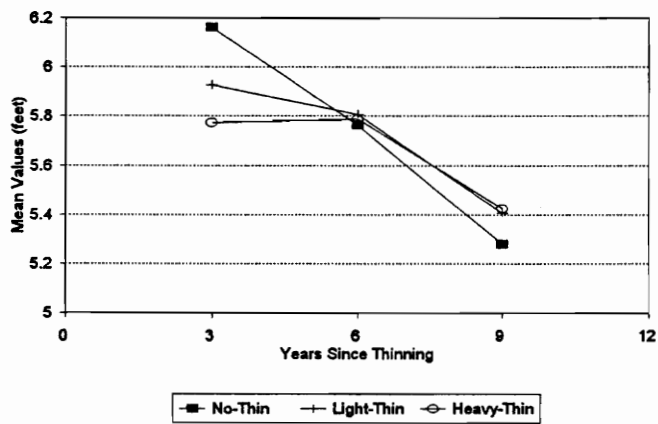


Figure 4. Observed means of potential height increments calculated by subtraction of consecutive mean total heights under two definitions of MTH.

under the traditional definition. The variations of course differed between definitions, but they still existed, thus this path of analysis was abandoned.

Development of a model to describe the variations between thinning intensities in mean dominant/codominant height proceeded under the traditional definition of MTH described above. This model is hereafter referred to as the MTH model. The candidate models for predicting MTH were variations on site index equations. The general form of the model is

$$MTH = f(\text{age}) * T$$

where  $f(\text{age})$  is a site index equation to describe the age/height relationship, and  $T$  is a thinning response variable.

The thinning response variable applied in all following analyses of the MTH model was that developed by Liu et al. for their crown ratio model. This previously described TRV is restated below.

$$T = \left( \frac{BA_a}{BA_b} \right) \frac{r[-(A_g - A_r)^2 + K(A_g - A_r)]}{A_g^2}$$

The first step was to select potential site index equations for the MTH model. One consideration in model selection was the number of parameters involved. Plots of the mean total heights over time indicated very little variation between thinning intensities. Thus, the number of parameters to be estimated would have to be limited to avoid difficulty in fitting.

Several site index equations were examined (Clutter et al., 1983, Amateis and Burkhart, 1985). A paper by Cao (1993) provided the best source of candidate models. In this paper, the author fit different transformations of several site index models to determine the best form of each equation in terms of fit statistics. The author labeled these different model forms as Method A, Method B, etc., and they will hereafter be referred to in the same manner. The models examined were from Schumacher (1939), Bailey and Clutter (1974), and Amateis and Burkhart (1985). The five models selected are shown below.

Schumacher, Method A:

$$\ln(MTH) = [\beta_1 + \beta_2/age] * T$$

Schumacher, Method C:

$$MTH2 = [MTH1 * \exp(\beta_1 * (1/age2 - 1/age1))] * T$$

Bailey and Clutter, Method C:

$$MTH2 = [\exp(\beta_1 + (\ln(MTH1) - \beta_1)) * (age2/age1)^{\beta_2}] * T$$

Amateis and Burkhart, Method B:

$$\ln(MTH2) = [\ln(MTH1) * (age1/age2)^{\beta_1} * \exp(\beta_2 * (1/age2 - 1/age1))] * T$$

Amateis and Burkhart, Method C:

$$MTH2 = [\exp(\ln(MTH1) * (age1/age2)^{\beta_1} * \exp(\beta_2 * (1/age2 - 1/age1)))] * T$$

where: MTH1 is the mean total height at the start of a prediction period

MTH2 is the mean total height at the end of a

prediction period

age1 is the plot age at the start of a prediction period

age2 is the plot age at the end of a prediction period

Each of these models was fitted to the entire dataset. The fits were compared on the basis of residuals (observed - predicted) and on the plots of the predicted PHIN's calculated by subtraction. The Schumacher, method A had an entirely different pattern of residuals when plotted compared to the other four models. This could have been due to this model being strictly anamorphic. It was decided that a more polymorphic model was needed for this application. Thus, the non-differential Schumacher, method A was eliminated.

There was little variation in fit statistics between the four differential models (see table 5). All estimates were significant with the exception of  $B_1$  in the Amateis and Burkhardt, method B fit. Since the Amateis and Burkhardt, method B could not be directly compared with the other models, the selection criteria became the residual and PHIN plots. All four residual plots showed a change to an underprediction from three to six years with the Schumacher, method C being the worst of these (see figure 5). From six to nine years,

variation in mean fit residuals between thinning intensities increased for all four models. The Bailey and Clutter, method C model had the best residuals, varying less than 0.2 from zero. The Schumacher, method C model had far worse residuals in this third remeasurement period than the other three models.

The PHIN plots for each of the four models were similar to one another, and were similar to the observed data for the first six years after thinning. These plots are shown in figure 6. From the second to the third remeasurement (6 to 9 years), the predicted PHIN's for the thinned plots increased in all cases. As was discussed earlier, PHIN values should not increase again. The Schumacher, method C was again the worst of these plots.

To determine if this third remeasurement variation was resulting from the TRV, spreadsheet plots of the fitted TRV's for each model were generated for each of four ages at thinning (8, 12, 18, and 25). The plots shown in figure 7 for a thinning age of 8 demonstrate that this TRV has all of the required characteristics. Modification of the MTH prediction decreases as age at thinning increases (indicated by the other age at thinning plots not displayed), and increases with the intensity of thinning. However, these plots all continue to

rise after the duration parameter K is reached and show no indication of returning to 1 even 18 years after thinning. After the TRV exceeded 1, it was hoped that it would return to 1 at some point, indicating that the effect of thinning was eventually negated.

It is quite possible that this continuous increase in the TRV above 1 after K is reached is responsible for the increasing PHIN's during the third remeasurement. If so, there were only two choices, redesign the TRV to return to 1 after a certain period has elapsed, or simply truncate the TRV at 1 in application and analyze the MTH model performance.

The plots of the TRV's (figure 7) revealed that once again the Schumacher, method C was the most extreme case. Its rise above 1 far exceeded the other three models. Considering this, and the poor residual and PHIN plots for this model, the Schumacher, method C was dropped from further analysis.

A vital requirement of any prediction model lies in the path invariance property. A predictive model should be able to generate the same result regardless of the length of the prediction period. That is, performing two five year predictions, using the first result as input to the second, should generate the same result as one ten year prediction.



Table 5. Fit statistics and estimates for K and R in the thinning response variable for the mean total height model analysis.

Model	SSE	MSE	Sy.x	K	R
1	4905.0562	3.1605	1.7778	6.08	0.98
2	4418.2868	2.8487	1.6878	6.48	0.23
3	2.2555	0.0015	0.0381	6.67	0.57
4	5023.6650	3.2390	1.7997	6.35	0.76
5	4472.6445	2.8837	1.6981	4.71	0.42
6	2.2446	0.0014	0.0380	9.63	-0.64
7	2.2450	0.0014	0.0380	10.12	-0.61
8	4968.3817	3.2033	1.7898	10.77	-0.69
9	4959.4939	3.1976	1.7882	8.62	-0.87
10	1.0561	0.0014	0.0371	8.58	-0.86
11	2290.2976	2.9978	1.7314	7.64	-1.06

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate. R and K are estimated parameters in the TRV's.

**Models:**

- 1 Schumacher, Method C with 1 TRV
- 2 Bailey and Clutter, Method C with 1 TRV
- 3 Amateis and Burkhart, Method B with 1 TRV
- 4 Amateis and Burkhart, Method C with 1 TRV
- 5 Bailey and Clutter, Method C with 2 TRV's
- 6 Amateis and Burkhart, Method B with 2 TRV's
- 7 Amateis and Burkhart, Method B with 4 TRV's
- 8 Amateis and Burkhart, Method C with 2 TRV's
- 9 Amateis and Burkhart, Method C with 4 TRV's
- 10 Split data analysis of Amateis and Burkhart, Method B with 2 TRV's
- 11 Split data analysis of Amateis and Burkhart, Method C with 2 TRV's

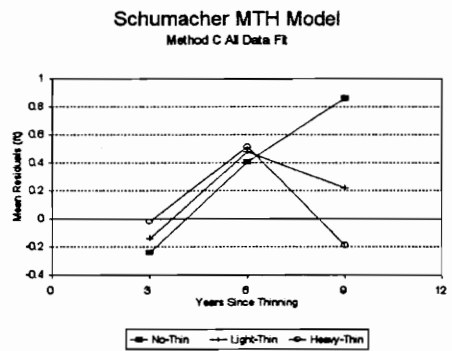
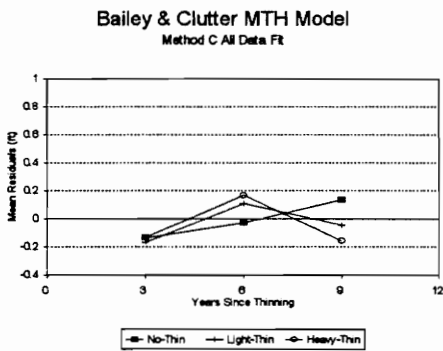
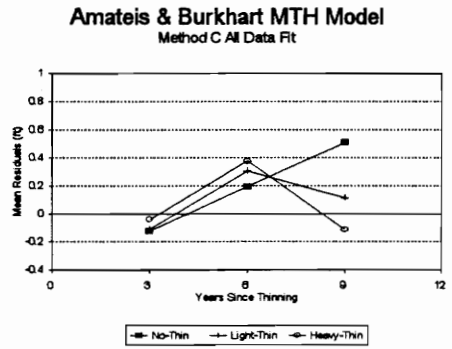
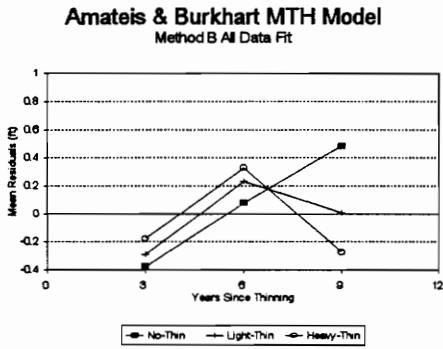


Figure 5. Mean fit residuals for four mean total height models fit to all available data.

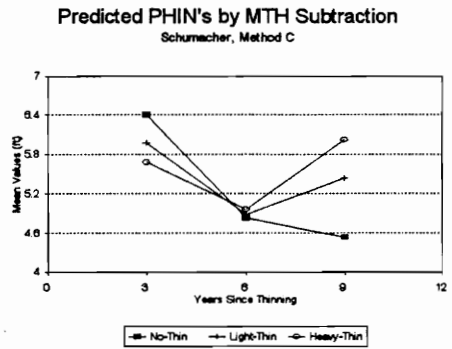
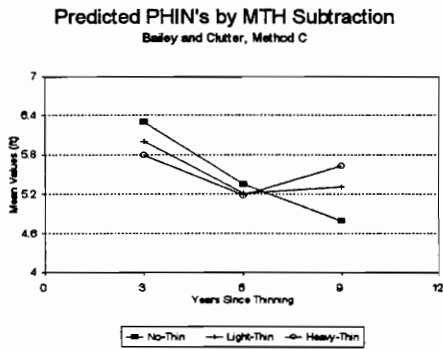
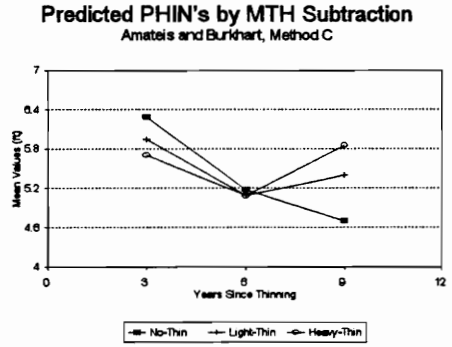
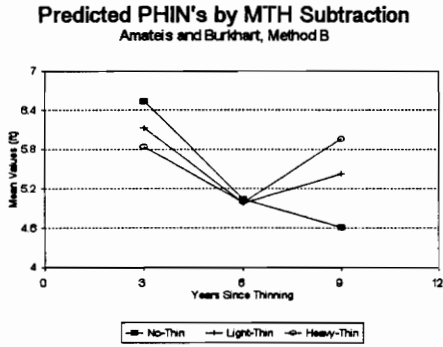


Figure 6. Means of predicted potential height increments from four mean total height models.

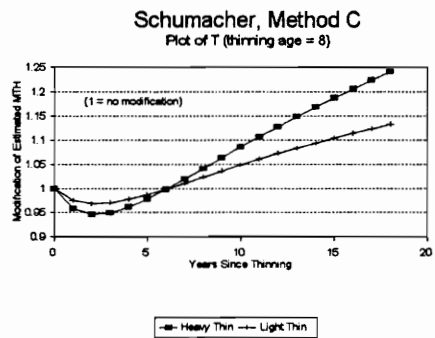
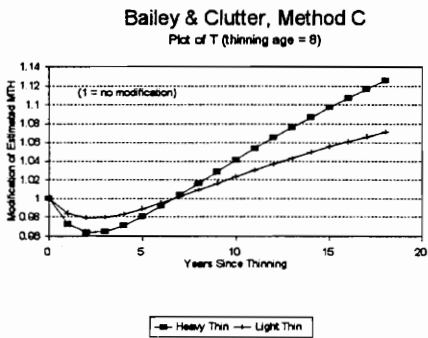
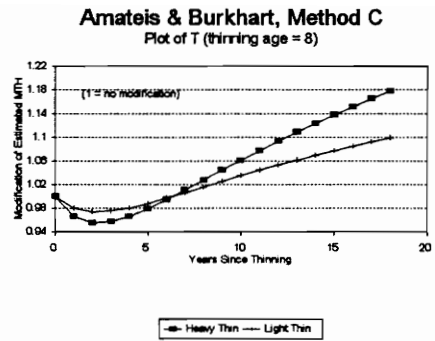
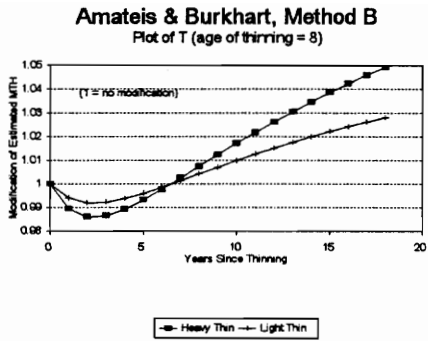


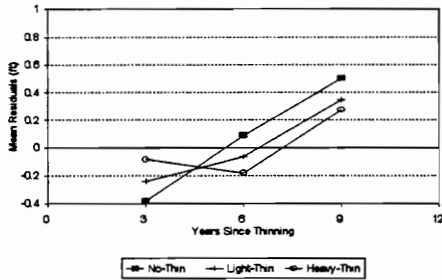
Figure 7. Plots of the fitted thinning response variable for four mean total height models. Residual basal areas are 0.5 for the heavy thinned data and 0.67 for the light thinned data.

This property was tested on each of the three remaining differential models. The tests revealed that all three models were path variant in their present form. Two five year predictions did not yield exactly the same results as one ten year prediction. It was identified that this was the result of multiplying the entire model by the TRV. Since these are differential models, each age factor must be multiplied by its own corresponding TRV to maintain path invariance.

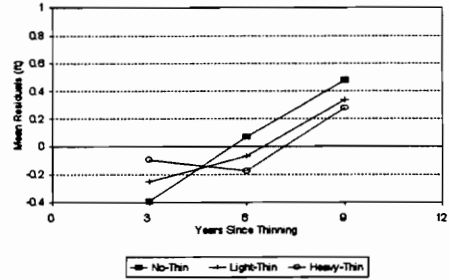
The three models were refit with each age factor multiplied by a TRV based on the same age as that factor (age1 or age2), and fitting the same parameters to both. In the Amateis and Burkhart models, a ratio of ages and a difference of ages are used. Since it was uncertain whether the ratio ages would require separate TRV's as well as the difference ages, these two models were fit both ways, using two TRV's and using four TRV's.

The fit statistics for these five models are also shown in table 5 with the corresponding estimates for K and R. The estimate for  $B_1$  was again insignificant in each Amateis and Burkhart, method B fit. Note that using multiple TRV's rather than multiplying the entire model by the TRV resulted in improved statistics in every case except the Bailey and Clutter model. In figure 8, the residual plots for the Bailey

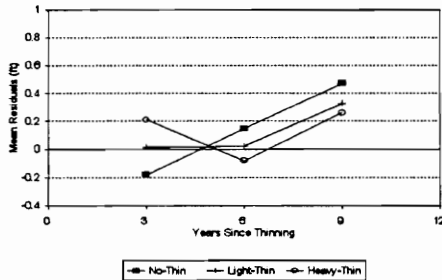
**Amateis & Burkhart MTH Model with 2 T's**  
Method B Fit Residuals



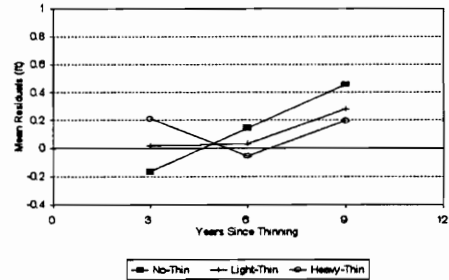
**Amateis & Burkhart MTH Model with 4 T's**  
Method B Fit Residuals



**Amateis & Burkhart MTH Model with 2 T's**  
Method C Fit Residuals



**Amateis & Burkhart MTH Model with 4 T's**  
Method C Fit Residuals



**Bailey & Clutter MTH Model with 2 T's**  
Method C Fit Residuals

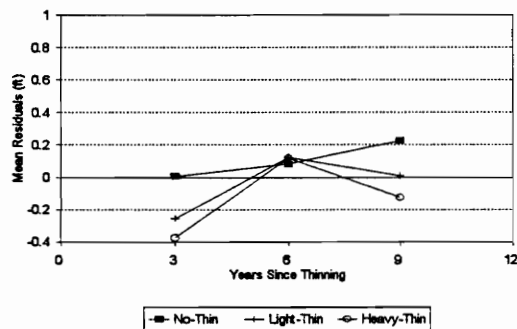


Figure 8. Plots of mean fit residuals from five mean total height models fit to all the data using multiple thinning response variables.

and Clutter model were markedly different from those of the other four models. This is possibly attributable to the significantly smaller value of K for this fit, compared to the other models. The Bailey and Clutter fit was the only one in which the estimated value of K became smaller.

All of the previously examined plots of mean PHIN's indicate that stand thinning has an impact on height growth through at least nine years. Thus, it was expected that the duration parameter would be estimated somewhere in the range of nine to 12 years. This was the case for the four variations of the Amateis and Burkhart model. The fact that k became smaller in the Bailey and Clutter model than its value in the single TRV fit makes this model questionable.

These five fitted models were then analyzed for path invariance. Only the Amateis and Burkhart models using TRV's multiplied against the difference ages were path invariant. The three models in which the ratio ages were multiplied failed the test. This eliminated the Bailey and Clutter, method C model from further analysis and reduced the selection process to two models. These two are shown below.

Amateis and Burkhart, Method B:

$$\ln(MTH2) = \ln(MTH1) * (age1/age2)^{\beta_1} * \exp(\beta_2 * (TRV2/age2 - TRV1/age1$$

Amateis and Burkhart, Method C:

$$MTH2 = \exp(\ln(MTH1) * (age1/age2)^{\beta_1} * \exp(\beta_2 * (TRV2/age2 - TRV1/age1$$

where: TRV1 is the previously described TRV with age1 in the denominator of the exponent, and elaps = age1 - age at thinning.

TRV2 is the previously described TRV with age2 in the denominator of the exponent, and elaps = age2 - age at thinning.

Again, fit statistics could not be used for model comparison, thus selection was based on residual and PHIN plots. The method C residual plot showed considerably less bias than the method B plot (see figure 8). There was little difference in the mean predicted PHIN plots between the two models shown in figure 9. The method B plots had a slightly steeper decline over time than the method C model, but it was not known if this was significant for comparison. Both plots



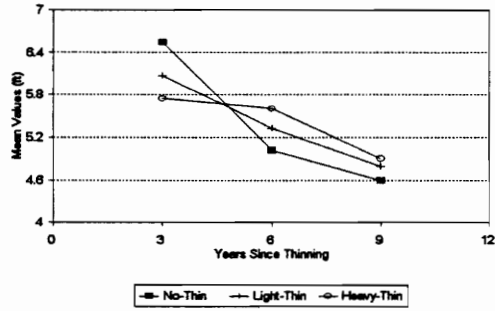
had declining mean PHIN's through nine years after thinning, which was the desired condition.

The data were randomly split and the two models were fit to one half of the data and validated on the other half. Figure 10 compares the residuals from these two validations. From these plots, it is difficult to determine which model is less biased, but the method C plot appears slightly better. Once again, fit statistics for these fits can not be directly compared. Both had somewhat better  $S_{y.x}$ 's than their full data counterparts. These statistics are included in table 5.

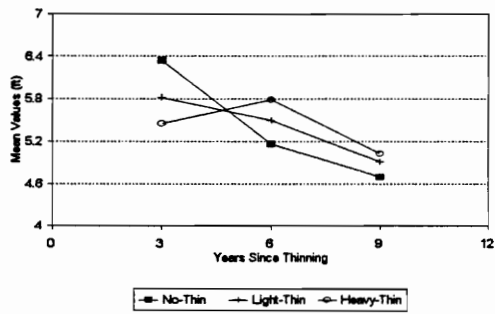
The only other factor to consider was the value of the duration parameter for each fit.  $K$  was 9.63 for method B and 10.77 for method C. Either value seems reasonable, but it was thought that a larger  $K$  was more appropriate for this application. Based on this, the better residual plots, and the fact that  $B_1$  was insignificant in the method B fits, the Amateis and Burkhardt, method C model with two TRV's was selected.

Figure 9 also contains a plot of predicted PHIN's from the MTH model fit without TRV's. Fit statistics were considerably worse for this fit, and as can be seen, PHIN prediction is greatly compromised without the TRV's

**Predicted PHIN's by MTH Subtraction**  
Amateis and Burkhart, Method B



**Predicted PHIN's by MTH Subtraction**  
Amateis and Burkhart, Method C



**Predicted PHIN's by MTH Subtraction**  
Amateis & Burkhart, Method C, No TRV's

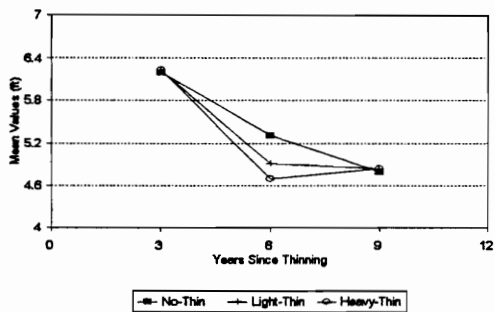
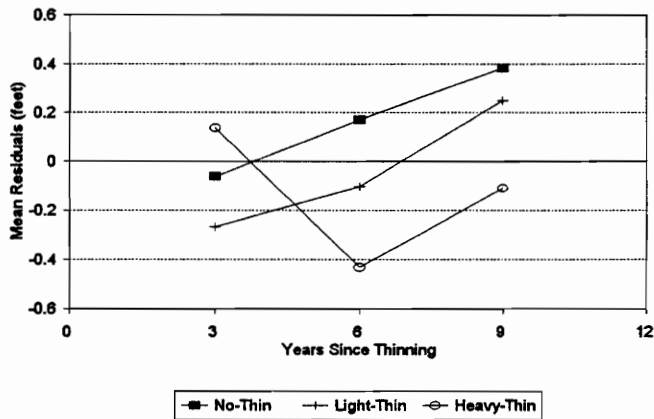


Figure 9. Predicted PHIN's for the final two mean total height models, and for the MTH model fit without a thinning response variable.

**Amateis & Burkhart MTH Model with 2 T's**  
 Method B Split-Data Validation



**Amateis & Burkhart MTH Model with 2 T's**  
 Method C Split-Data Validation

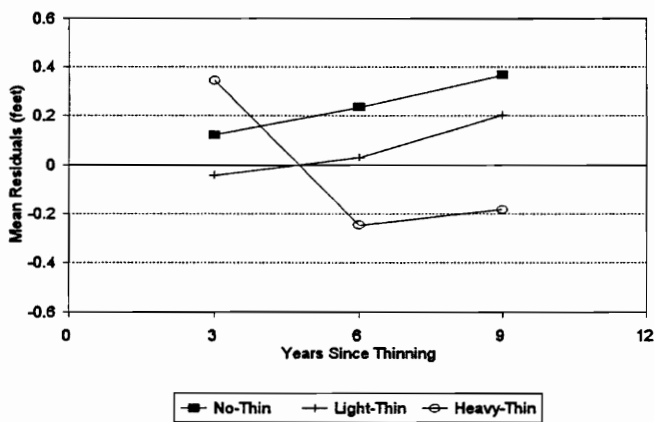


Figure 10. Validation residuals from split-data fit/validation analysis on the final mean total height model choices.

The final estimates are presented in table 4. These estimates were used to generate site index curves for site indices of 50, 60, and 70. Three sets of curves were generated for three ages at thinning, 12, 25 and 30 years. Curves for unthinned stands and stands thinned by fifty percent have been superimposed. They are displayed in figures 11, 12, and 13.

Note particularly in the curves for an age at thinning of 12 that the curves for unthinned stands initially exceed the thinned stand projected growth, but later fall below the thinned stands. Also note a greater separation between the growth of thinned and unthinned stands for higher site indices at a plot age of thirty-five years. Finally, mean total height is equal to the correct site index at the base age of 25. These curves indicate that the desired relationship between thinned and unthinned stands has been correctly modeled.

# Site Index Curves for the MTH Model

Age at Thinning = 12, Intensity = 0.5

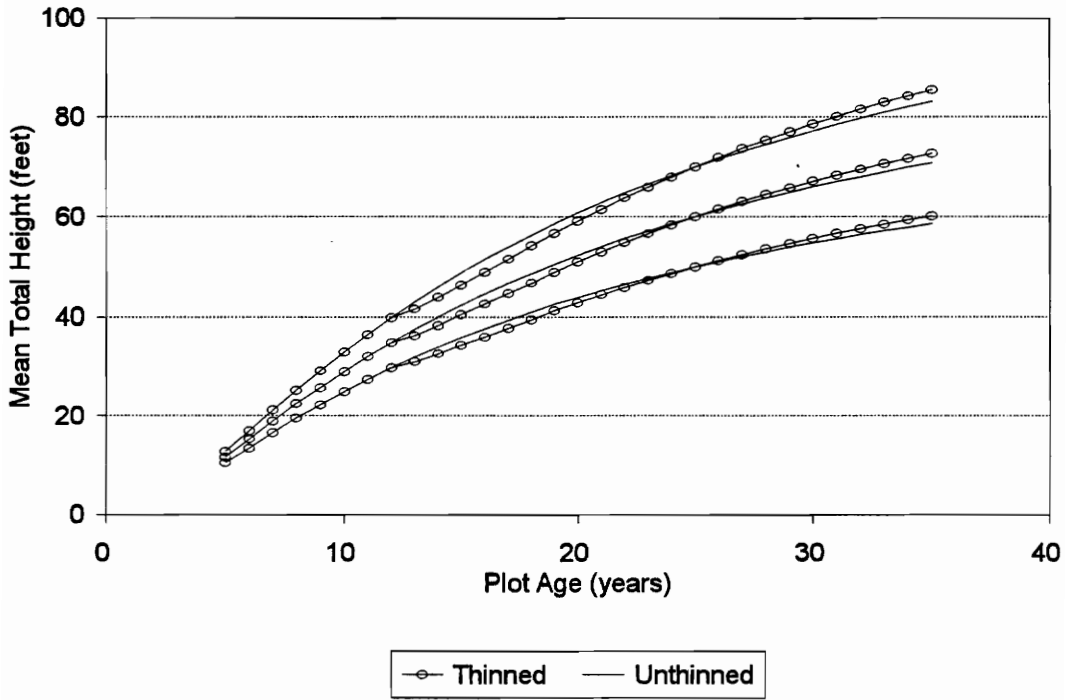


Figure 11. Site index curves for an age at thinning of 12.

# Sit Index Curves for the MTH Model

Age at Thinning = 25, Intensity = 0.5

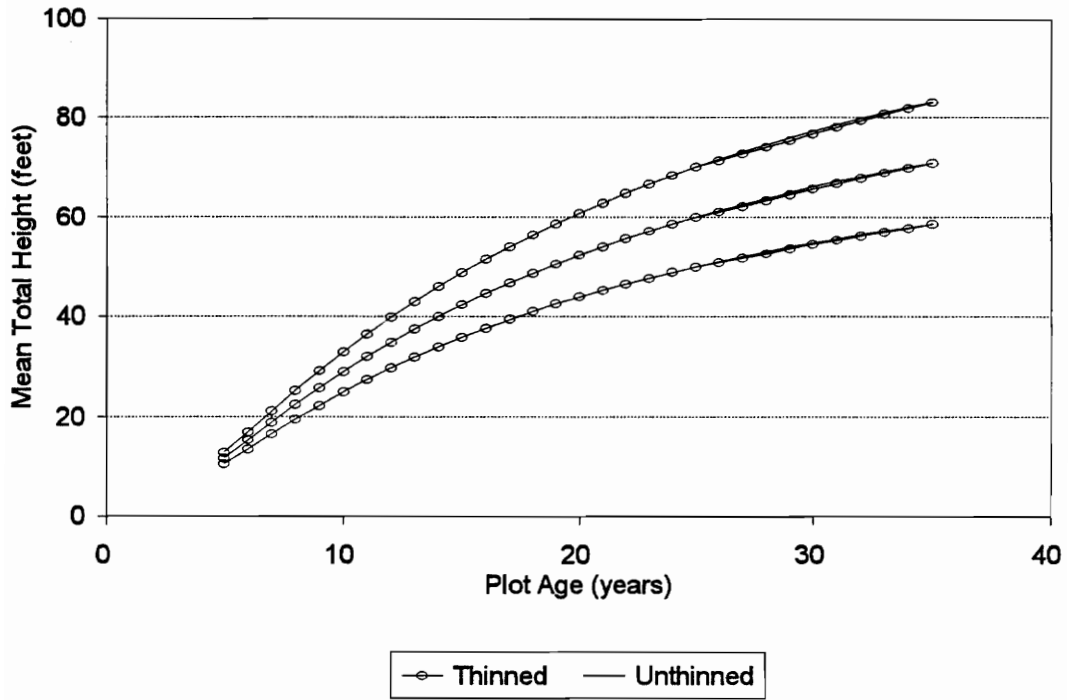


Figure 12. Site index curves for an age at thinning of 25.

# Site Index Curves for the MTH Model

Age at Thinning = 30, Intensity = 0.5

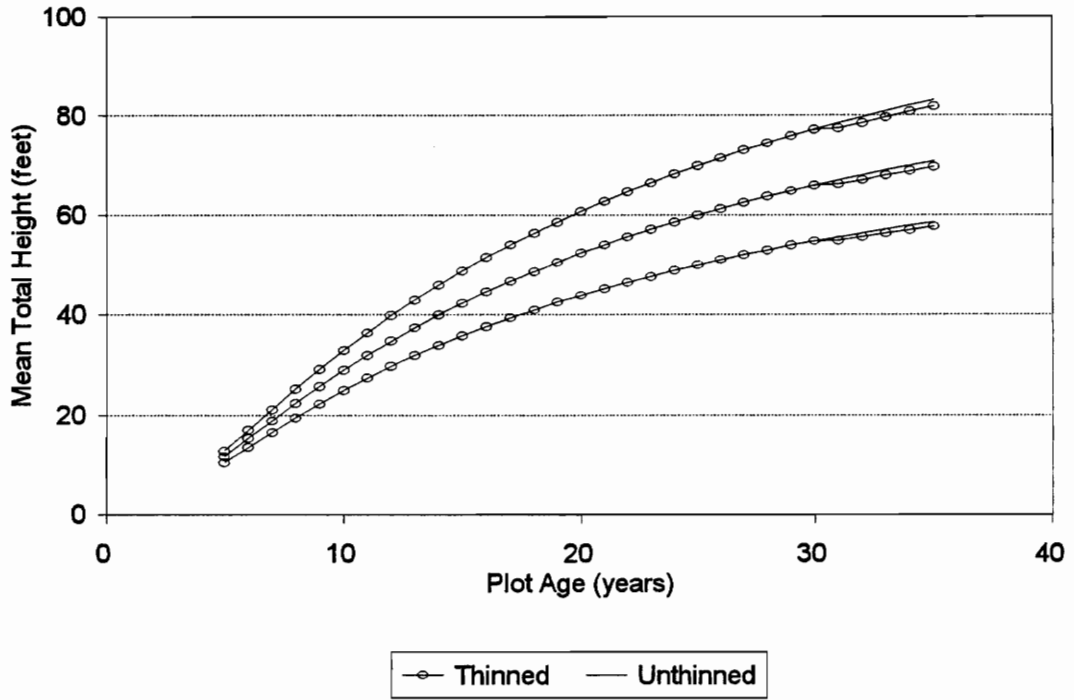


Figure 13. Site index curves for an age at thinning of 30.

## Diameter Increment Model

Tree diameter is commonly known to have a large response to changes in stand density. Therefore, thinning should have the strongest impact on growth simulation through changes in tree diameter and the resulting changes in competition index, which impacts the other models. A thinning response variable which describes these changes should be of most value in the diameter increment model.

The predicted diameter increment is the potential diameter increment modified by a function of the tree's crown ratio and competition index. For all model fitting, PDIN was derived from the previously described model (Daniels and Burkhardt, 1975) based on open grown trees and using observed height increments. During application in the growth simulator, PDIN is derived by subtraction of the previous mean total height from the current one. The diameter increment model from PTAEDA2 is restated here.

$$DIN=PDIN*[\beta_1 CR^{\beta_2} * \exp(-\beta_3 CI)]$$



Separate fits of the DIN model across all of the data in each thinning regime and across all of the data were first done for calculation of the F statistic. The statistics from these four fits are shown in table 6. Analysis of these fit residuals showed a large underprediction (0.01 to 0.025) at the first remeasurement which increased with thinning intensity. Residuals were less than 0.005 at the second and third remeasurement.

This analysis also showed the residuals from the fit to light thin data to be nearly identical to the all data fit residuals. This was not surprising since the light thinned data comprises about fifty percent of all the available data, but it indicates that a model fit to all of the data would underpredict DIN in heavily thinned stands and overpredict it in unthinned stands.

The first fits of the DIN model incorporating a TRV were of the form shown below using the previously described TRV developed by Liu et al.

$$DIN = PDIN * T * [\beta_1 CR^{\beta_2} * \exp(-\beta_3 CI)]$$

where: T = the Liu et al. thinning response variable

The separate fits of this model were improved in all cases over the fits without a TRV, with the exception of the fit to the unthinned data of course, since the TRV's value is 1 in unthinned stands (see table 6). The mean residuals were improved at the first remeasurement, however, they were made considerably worse at the third remeasurement.

Although mean residuals were improved at the first remeasurement by multiplying the DIN model by the TRV, it is the performance of the model over time that is the major concern. Stand thinning results in sudden changes in the calculated competition index which can sometimes be quite large. Large levels of mortality can also result in sudden changes in CI in subsequent remeasurements.

Natural systems such as trees take time to respond to changes in competition. The DIN model assumes an instant response to these changes and since one-year predictions are being modeled, this discrepancy could result in erroneous predictions. These changes in CI may not be severe in an annual simulator, but when using three-year data, they can possibly have a large impact on the fitted estimates.

Table 6. Fit statistics for diameter increment models fit to all of the data.

Model	SSE	MSE	Sy.x	Df
1	28.1075	0.005006	0.0708	5615
2	104.4083	0.008069	0.0898	12937
3	90.5932	0.010226	0.1011	8859
4	224.8853	0.008202	0.0906	27417
5	28.1075	0.005006	0.0708	5615
6	100.6405	0.007780	0.0882	12935
7	85.4583	0.009649	0.0982	8857
8	216.1075	0.007883	0.0888	27415
9	28.1075	0.005006	0.0708	5615
10	102.0641	0.007891	0.0888	12935
11	87.9961	0.009935	0.0997	8857
12	220.7055	0.008051	0.0897	27415

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = degrees of freedom.

**Models:**

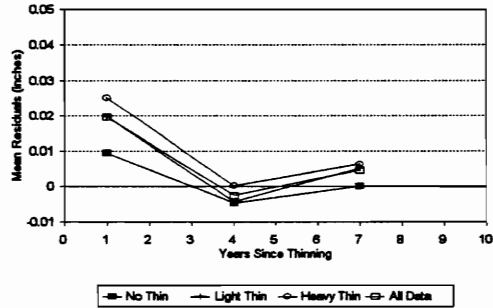
- 1 Fit to unthinned data with no TRV
- 2 Fit to light thinned data with no TRV
- 3 Fit to heavy thinned data with no TRV
- 4 Fit to all the data with no TRV
- 5 Fit to unthinned data with TRV multiplied against model
- 6 Fit to light thinned data with TRV multiplied against model
- 7 Fit to heavy thinned data with TRV multiplied against model
- 8 Fit to all the data with TRV multiplied against model
- 9 Fit to unthinned data with TRV multiplied against CI
- 10 Fit to light thinned data with TRV multiplied against CI
- 11 Fit to heavy thinned data with TRV multiplied against CI
- 12 Fit to all the data with TRV multiplied against CI

A model form which allows this change in CI to gradually impact the prediction would be more consistent with the natural system. One way to accomplish this is to multiply the TRV times CI directly in the exponent of the DIN model, rather than multiplying it times the entire model. The following model formulation was fitted to each thinning regime to parallel the analysis with the TRV multiplied against the entire model.

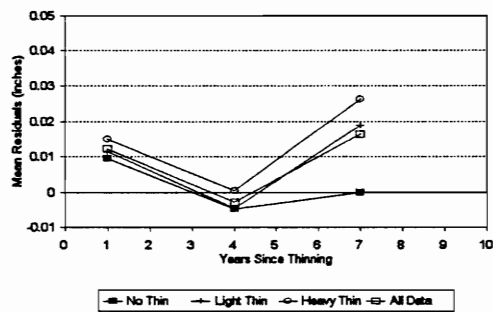
$$DIN = PDIN * [\beta_1 CR^{\beta_2} * \exp(-\beta_3 CI * T)]$$

The statistics for these model fits are also displayed in table 6 for comparison to the previous TRV fits and to the fits without a TRV. The table shows that using the TRV in the exponential position results in poorer fits for all regimes than multiplying the TRV against the entire model. These fits are however still better than the fits without a TRV. The fit residual plots also confirm the poorer fits with the TRV in the exponent. Mean fit residuals were slightly worse in all three remeasurements. Figure 14 compares the fit residual plots for the three analyses described above.

**Separate Fit DBH Increment Models**  
No TRV Used



**Separate Fit DIN Models with a TRV**  
TRV Multiplied Against the Entire Model



**Separate Fit DIN Models with a TRV**  
TRV Multiplied Against CI in Exponent

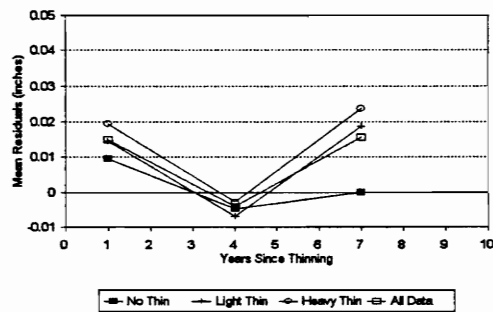


Figure 14. Fit residuals from separate fits of the diameter increment model to each thinning regime with and without a TRV.

The idea of modifying competition index rather than the entire model seemed sound, even though preliminary analysis was dissatisfying. It was decided to try some other forms of thinning response variables.

It was believed that in order for an individual tree's response to thinning be accurately modeled, an expression of the change in the individual tree's competitive status must be included. Note that the previously discussed TRV's are stand level variables. Their base is a ratio of after thinning basal area to basal area before thinning, thus the TRV's value is the same for every tree in each particular stand. A tree level TRV was developed which is based on a ratio of each trees competition index after to competition index before thinning, rather than the residual basal area ratio previously used. This TRV design is shown below.

$$T = \left( \frac{CIA}{CIB} \right)^{\frac{r[-(A_g - A_t)^2 + K(A_g - A_t)]}{A_g^2}}$$

where: CIA = the trees competition index after thinning  
at plot establishment

CIB = the trees competition index calculated for

before thinning competitive status  
other parameters and variables are as described  
previously for the tree level TRV

The model was fit as before with the new TRV multiplied against the entire model and multiplied against CI in the exponent. The TRV developed in Short and Burkhart (1992) was also analyzed in this fashion using the basal area ratio based and CI ratio based TRV's. Table 7 provides a comparison of these six models which include a TRV. For simplicity, the fits to unthinned data have not been included in this table, since they are not altered by different approaches to including a TRV.

As can be seen in this table when compared to table 6, not one of these TRV designs improved upon the Liu TRV multiplied against the entire model. The same is true for the original Liu TRV multiplied against CI, with the exception of the design using the Liu TRV with a CI ratio base multiplied against the entire model (models 2 and 3 in table 7 ). The fit to light thinned data did not perform as well as the basal area ratio based TRV in the exponent, but the fits to heavy thinned data and to all the data are somewhat improved when compared to models 11 and 12 in table 6.

Table 7. Fit statistics for diameter increment models fit to all of the data, using various thinning response variables.

Model	SSE	MSE	Sy.x	Df
1	102.9050	0.007956	0.0892	12935
2	87.8937	0.009924	0.0996	8857
3	219.7937	0.008017	0.0895	27415
4	103.0570	0.007967	0.0893	12935
5	88.9278	0.010040	0.1002	8857
6	221.6138	0.008084	0.0899	27415
7	110.1898	0.008517	0.0923	12937
8	100.9302	0.011393	0.1067	8859
9	264.9213	0.009663	0.0983	27417
10	103.6169	0.008009	0.0895	12937
11	89.3093	0.010081	0.1004	8859
12	224.9926	0.008206	0.0906	27417
13	137.5653	0.010633	0.1031	12937
14	145.2180	0.026392	0.1625	8859
15	338.2935	0.012339	0.1111	27417
16	104.7965	0.008101	0.0900	12937
17	90.6628	0.010234	0.1012	8859
18	225.9657	0.008242	0.0908	27417

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = degrees of freedom.

Models:

- 1 Fit to light thinned data using a CI ratio TRV multiplied against model
- 2 Fit to heavy thinned data using a CI ratio TRV multiplied against model
- 3 Fit to all the data using a CI ratio TRV multiplied against model
- 4 Fit to light thinned data using a CI ratio TRV multiplied against CI
- 5 Fit to heavy thinned data using a CI ratio TRV multiplied against CI
- 6 Fit to all the data using a CI ratio TRV multiplied against CI
- 7 Light thinned fit with A. Short B/A ratio TRV multiplied against model
- 8 Heavy thinned fit with A. Short B/A ratio TRV multiplied against model
- 9 All data fit with A. Short B/A ratio TRV multiplied against model
- 10 Light thinned fit with A. Short B/A ratio TRV multiplied against CI
- 11 Heavy thinned fit with A. Short B/A ratio TRV multiplied against CI
- 12 All data fit with A. Short B/A ratio TRV multiplied against CI
- 13 Light thinned fit with A. Short CI ratio TRV multiplied against model
- 14 Heavy thinned fit with A. Short CI ratio TRV multiplied against model
- 15 All data fit with A. Short CI ratio TRV multiplied against model
- 16 Light thinned fit with A. Short CI ratio TRV multiplied against CI
- 17 Heavy thinned fit with A. Short CI ratio TRV multiplied against CI
- 18 All data fit with A. Short CI ratio TRV multiplied against CI



The Short and Burkhart TRV with a CI ratio base performed extremely poorly compared to the other designs, particularly when multiplied against the entire model. Although fitting of models utilizing the Short and Burkhart TRV is easier due to fewer parameters, it seems obvious that the exponent of this TRV is not descriptive enough to utilize the information in a CI ratio base. Residual plots similarly indicated poorer predictive ability for the TRV based on a ratio of CI values.

It should be noted that in all of the above described fits, all parameter estimates were significant. The DIN models did not have any of the fitting problems found with the HIN model, probably because there is much more variation in the diameter data, compared to the height data, and much less inherent measurement error.

Based on these results and to remain consistent with the other models in the simulator, it was decided to limit further analysis to the Liu TRV using the basal area ratio base. This TRV has been shown in the MTH analysis above and in LIU et al. to have better characteristics for describing biological systems than previous variables. Thus, analysis was focused on the configuration of the TRV in the model.

The data were randomly split, using one-half as the fitting dataset and one-half for validation. Each of the two model forms, the TRV multiplied against the entire model, hereafter called the multiplicative model, and multiplying CI in the exponent by the TRV, called the exponential model, were fit and validated. These split-data fit statistics are shown in table 8. In these fits, as before, the multiplicative model had slightly better statistics than the exponential model. However, in the residual plots, validation was much better for the exponential model (see figure 15).

The two models were again fit to all of the data, then validation residuals were generated for each thinning regime using the fitted estimates. These fit statistics are also shown in table 8. Figure 16 shows the validation of these two models on each thinning intensity. Although this validation is not on an independent dataset, it demonstrates how well or poorly the thinning response variable is performing. The exponential model again had slightly worse fit statistics, as earlier analysis found, but the validation shows a much tighter relationship between intensities. There was very little bias compared to the multiplicative model.

Table 8. Fit statistics and estimates for K and R in the thinning response variable for diameter increment models with different elaps/age configurations in the thinning response variable.

Model	SSE	MSE	Sy.x	K	R
1	105.6967	0.007776	0.0882	-4.45	7.47
2	107.6455	0.007920	0.0890	9.01	7.69
3	216.1075	0.007883	0.0888	-4.59	7.51
4	220.7055	0.008051	0.0897	9.16	7.66
5	110.1552	0.008104	0.0900	-0.28	9.57
6	108.6441	0.007993	0.0894	8.38	5.01
7	219.3568	0.008001	0.0894	-6.20	4.96
8	222.7511	0.008125	0.0901	8.87	5.03
9	216.0319	0.007880	0.0888	-6.07	7.46
10	218.7138	0.007978	0.0893	-8.91	5.00

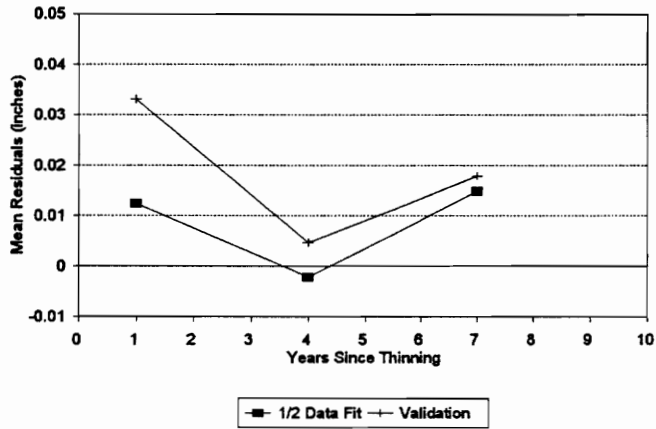
SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate. R and K are estimated parameters in the TRV's.

Models:

- 1 Split-data fit with TRV multiplied against entire model
- 2 Split-data fit with TRV multiplied against CI
- 3 All data fit with TRV multiplied against entire model
- 4 All data fit with TRV multiplied against CI
- 5 Split-data fit with TRV multiplied against entire model (1 year elaps)
- 6 Split-data fit with TRV multiplied against CI (1 year elaps)
- 7 All data fit with TRV multiplied against entire model (1 year elaps)
- 8 All data fit with TRV multiplied against CI (1 year elaps)
- 9 All data fit using two TRV's
- 10 All data fit using two TRV's (1 year elaps)

### DIN Model with Multiplicative TRV

Split-Data Fit/Validation



### DIN Model with Exponential TRV

Split-Data Fit/Validation

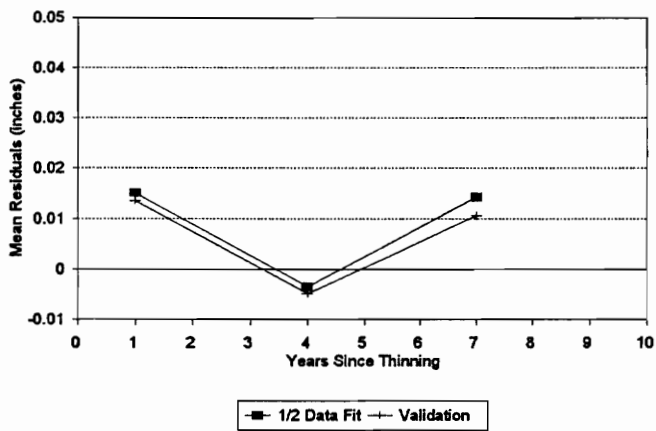
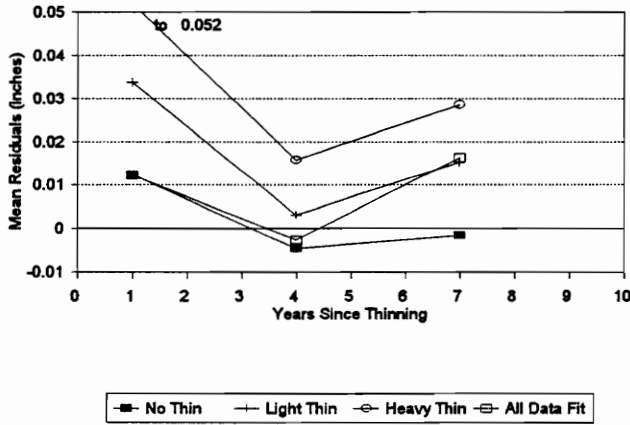


Figure 15. Residuals from split-data fit/validation analysis on the diameter increment models using a three-year elaps/age configuration.

### DIN Model with Multiplicative TRV All Data Fit with Validation on Regimes



### DIN Model with Exponential TRV All Data Fit and Validation on Regimes

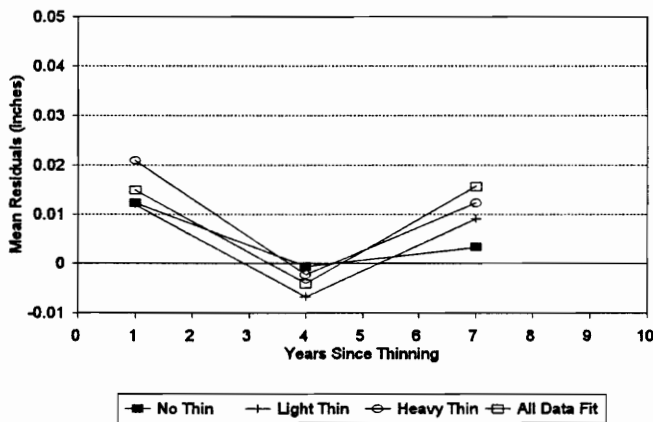


Figure 16. Fit residuals from diameter increment models fit to all of the data using a three-year elaps/age configuration in the thinning response variable, with validation on each thinning regime.

At this time, the structure of the variables in the TRV came into question. Up to this point in the analysis, the elapsed time (age minus age at thinning), termed elaps, used in the TRV was 3 years, 6 years, and 9 years. This was also true of the age factor in the denominator of the exponent of the TRV. Since these are one-year models, it seems logical that the elapsed time the TRV is predicting across should also be one year.

Split-data analyses were again run on the two models. This time, the values for elaps used in the TRV were 1, 4, and 7. Also, the age factor in the denominator was set to age at thinning plus 1, plus 4, and plus 7. This age factor must be the same as the age used to calculate elaps. In this configuration, the exponential model performed slightly better than the multiplicative model; however, there was very little difference in the validation residuals for the two fits (see figure 17). The exponential model was better at the second remeasurement and the multiplicative was better at the third.

Each model was refit to all of the data using the new elaps/age configuration. The fit statistics for this and the split-data fits are also in table 8 for comparison to the three-year elaps fits. Once again, the multiplicative model had a higher quality fit. But again, in the validation plots

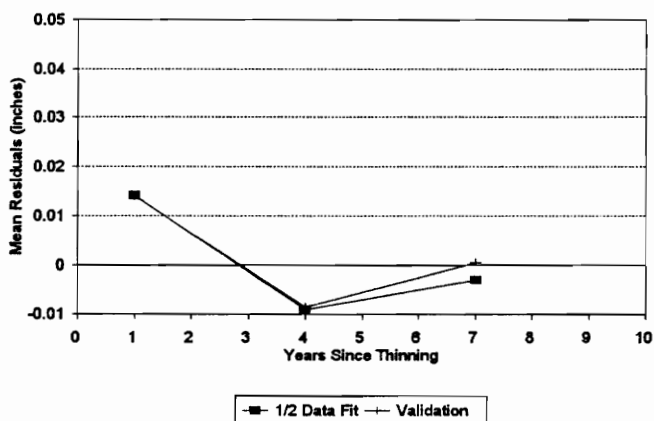
of these estimates on each regime shown in figure 18, the exponential model performed better than the multiplicative design. Included in figure 18 is a validation of estimates fit without a TRV for comparison to the TRV plots.

In every case using the one-year elaps/age configuration, fit statistics were slightly worse than the similar fit with the original configuration. It is difficult to explain why the fit statistics were worsened by using the intuitively correct one-year configuration. However, the change resulted in a substantial improvement in the validation plots for the multiplicative model, when figures 16 and 18 are compared. The exponential model shows little change in this comparison.

Another factor which seemed to indicate that the exponential model design was superior were the estimates for K. In all of the fits, both three-year and one-year, in which the TRV was multiplied directly against CI, this estimate was always between 8.8 and 9.2 (see table 8). Even the split-data fits fell in this range. This estimate for the multiplicative model was very variable, and thus appears unstable.

To ensure the correct model choice, the six previously described model forms with variations of the TRV were refit using the one-year elaps/age configuration. Not one of these

### DIN Model with Multiplicative TRV Split-Data Fit/Validation



### DIN Model with Exponential TRV Split-Data Fit/Validation

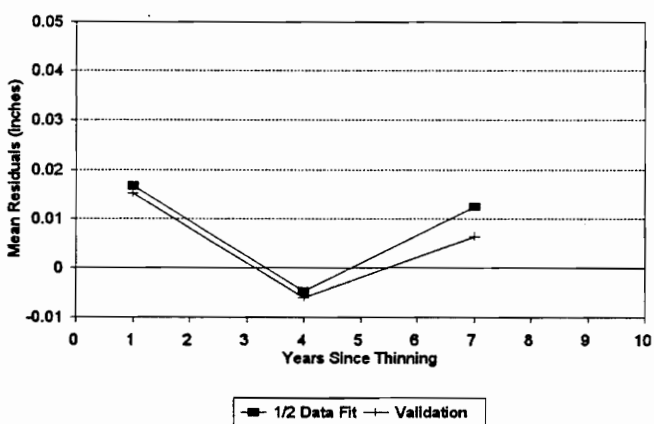
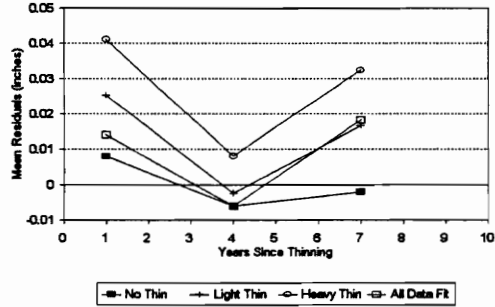


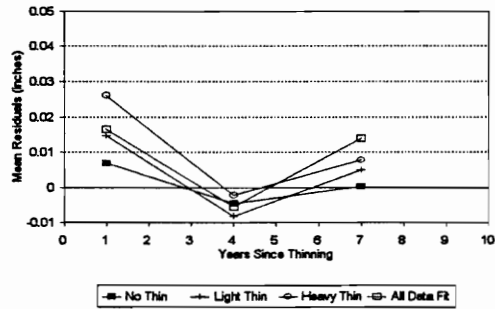
Figure 17. Residuals from split-data fit/validation analysis on the diameter increment models using a one-year elaps/age configuration in the thinning response variable.



**DIN Model with Multiplicative TRV**  
All Data Fit with Validation on Regimes



**DIN Model with Exponential TRV**  
All Data Fit with Validation on Regimes



**DIN Model with No TRV**  
All Data Fit with Validation on Regimes

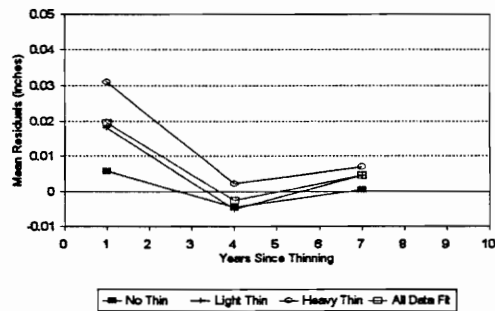


Figure 18. Fit residuals from diameter increment models fit to all of the data using a one-year elaps/age configuration in the thinning response variable.

models generated fit statistics which were better than the three-year elaps/age configuration fits previously discussed.

Two more fits were done to analyze the performance of the DIN model incorporating two TRV's, one multiplied against the entire model and one multiplied against CI. This fit was done for each of the two elaps/age configurations. These statistics are the last two entries in table 8. Using two TRV's resulted in slightly improved fit statistics, but a validation of these estimates on each regime demonstrated the worst performance of any of the models examined.

The exponential model had slightly worse fit statistics than the multiplicative model, but appeared to perform considerably better. The idea of modifying CI directly still seemed sound. Using a one-year elaps/age configuration also generated poorer statistics but better performance. This configuration still seemed intuitively correct. Based on these results and the goal of sound, long-term prediction stability, the DIN model form in which the TRV is multiplied directly against CI in the exponent using a one-year elaps/age configuration was selected as the best choice. The final estimates are given in table 4.

Spreadsheet plots were generated to examine the TRV behavior in this fitted model. Plots were generated for ages at thinning of 8 and 25. These are shown in figure 19. This TRV behaves very similar to the TRV in the MTH model. After  $K$  is exceeded, the response variable increases without bound for 20 years (the range of the graph). The bottom plot is on an expanded scale to more closely display the thinning response during the first 8 years after thinning for a thinning age of 8.

The plots indicated that this TRV would probably have to be truncated at 1 in application similar to the TRV in the MTH model. This would be determined through simulator analysis. Since the time period between thinning and  $K$  years is the most critical for modification of the diameter increment prediction in thinned stands, truncation after  $K$  is exceeded should not have a large impact on the prediction.

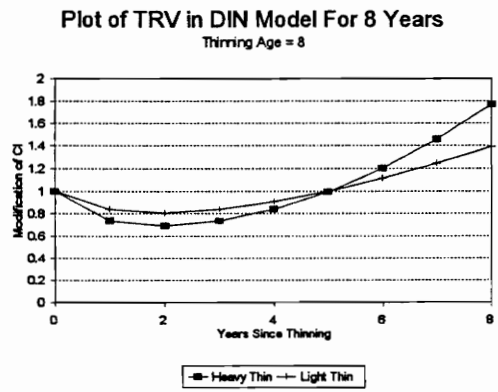
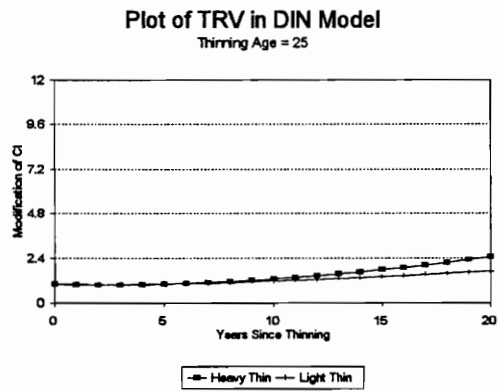
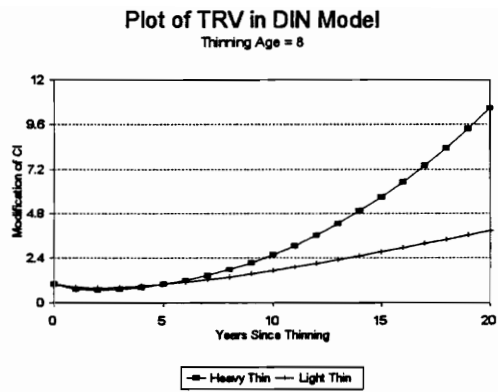


Figure 19. Behavior of the thinning response variable in the diameter increment model for two ages at thinning.

## Mortality Function

The mortality function estimates values between 0 and about 1. The high end of this range is fixed by the  $B_1$  estimate, which in past applications has been near 1.0. The resulting values are then compared to a uniform random variate. The tree is considered dead if the calculated value is less than the uniform random variate. This function is restated here:

$$PLIVE = \beta_1 CR^{\beta_2} * \exp(-\beta_3 CI^{\beta_4})$$

where: PLIVE = probability that a tree remains alive  
other variables are as described previously

Several changes had to be made to the dataset prior to fitting and analysis for this model. Several of the plots have been damaged by severe storms and southern pine beetle infestation since plot establishment. Many of these were completely destroyed. An analysis of the percent mortality in each plot at each remeasurement was used to determine which observations were suspect. To avoid confounding the model fits with large levels of catastrophic mortality, all plots

which had been completely destroyed since the previous remeasurement have had all observations for that and all subsequent remeasurements deleted.

Also, since this is a one-year prediction model, only mortality which occurred during the first year of each remeasurement period was retained. All observations in which the estimated year of death was the second or third year of a remeasurement period were deleted. These combined deletions resulted in about a 15% reduction in the total amount of data available for fitting.

The first fits of this model to each thinning regime were completed for calculation of the F statistic. The fit statistics are shown in table 9. The estimate for  $B_3$  was insignificant in the fit to light thinned data. Both estimates associated with competition index ( $B_3$  and  $B_4$ ) were insignificant in the fit to heavy thinned data. The fit to all of the data converged in four iterations and all estimates were significant. The thinned data fit results seemed to indicate that CI becomes less important in predicting mortality as thinning intensity increases.

The calculated F statistic for significant differences between thinning intensities was 24.7. This indicates that

mortality prediction could benefit from either separate models for different thinning intensities or a single model incorporating some form of thinning response variable. Previous research has shown that there are significant differences in mortality response between unthinned and thinned stands (Avila and Burkhart, 1992). There remained a question as to how much difference in response there was between different intensities in thinned stands. If these differences are not significant, then a thinning response variable would be of no value to the model.

The model was fit to all of the thinned plot data. The estimate for  $B_3$  was insignificant in this fit. The Swindel F-test was applied to the two fits to thinned plot data, treating them as the full model, while the reduced model was that fit to all thinned plot data. This F statistic was 5.36, much lower than the statistic involving unthinned plot data. This F value indicated a much smaller variation in parameter estimates than between the three thinning regimes.

This F statistic does not provide a lot of power for rejecting the null hypothesis, considering the large number of degrees of freedom. It could not be said for certain that there are significant differences in the parameter estimates. Considering the other controls on thinning intensity response

in the reduced simulator, particularly the MTH and DIN models, an additional thinning response variable in the PLIVE function could possibly overcompensate for what appears to be only slight variations in mortality response between thinning intensities.

The data were randomly split into fitting and validation datasets. Models were fit to data from unthinned and thinned plots, and to all of the data. Standard errors of the estimate were better for the fits to all the data and to unthinned plots than for their all data counterparts (see table 9). However, the split-data fit to thinned plot data required many more iterations than when all of the thinned plot data was used, and the estimates for  $B_3$  and  $B_4$  were both highly insignificant. Apparently, reducing the amount of data by one-half greatly impacted the fit to thinned plot data.

The plots of fit and validation residuals for these split-data fits indicated fairly good performance of the fitted estimates on semi-independent datasets for the fits on thinned plot data and on all the data. However, the validation of the split-data fit to unthinned plot data was not as good. These three plots are displayed in figure 20.



Table 9. Fit statistics and significance of estimates for the mortality function.

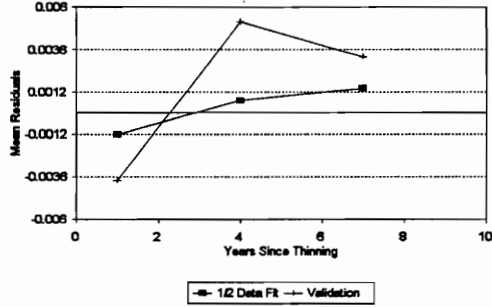
Model	SSE	MSE	Sy.x	Df	Significance
1	103.7195	0.017700	0.1330	5860	All significant
2	70.1972	0.005340	0.0731	13146	B3 insignificant
3	57.2638	0.006365	0.0798	8997	B3 & B4 insignificant
4	127.5845	0.005761	0.0759	22147	B3 insignificant
5	232.8119	0.008311	0.0912	28011	All significant
6	51.2612	0.017406	0.1319	2945	All significant
7	71.2356	0.006443	0.0803	11056	B3 & B4 insignificant
8	114.5197	0.008174	0.0904	14011	All significant

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = fitting degrees of freedom.

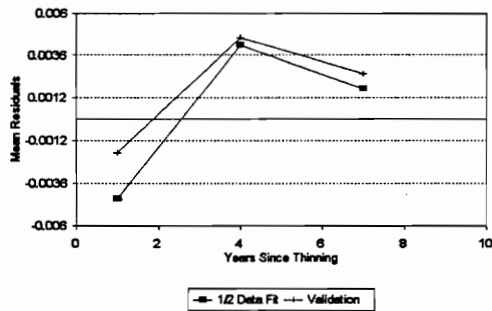
Models:

- 1 Fit to unthinned data
- 2 Fit to light thinned data
- 3 Fit to heavy thinned data
- 4 Fit to all thinned data
- 5 Fit to all the data
- 6 Split-data fit/validation on unthinned data
- 7 Split-data fit/validation on thinned data
- 8 Split-data fit/validation on all the data

**Split-Data Fit of Mortality Function**  
Fit and Validation on Unthinned Data



**Split-Data Fit of Mortality Function**  
Fit and Validation on Thinned Data



**Split-Data Fit of Mortality Function**  
Fit and Validation on All the Data

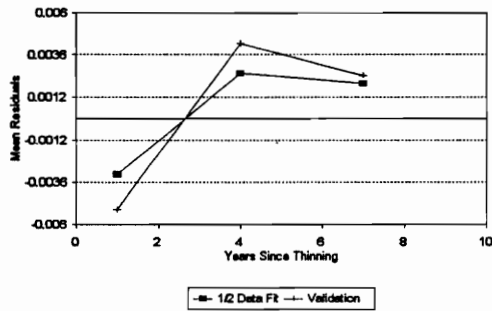


Figure 20. Residuals from split-data fit/validation analysis of the mortality function on unthinned, thinned, and all of the data.

It was decided to fit two PLIVE models, one to data from unthinned plots and one to data from thinned plots, then analyze their performance in the reduced simulator before making a decision about incorporating a TRV. All of the data was used in these two final fits. These estimates are presented in table 4.

It was discovered that the estimate for  $B_1$ ,  $B_2$ , and  $B_3$  varied very little between the different intensity fits. However,  $B_4$  changed greatly. Its value was 2.5 for unthinned plot data and 5.75 for the fit to light thinned data. Correlations between the parameters were examined. Correlations were low in most cases, but  $B_3$  and  $B_4$  had correlations between one another of greater than 0.97 in all three thinning regimes.

To determine if the fits could be improved by stabilizing the value of  $B_4$ , each intensity was refit with  $B_4$  fixed. Three values for  $B_4$ , 2.5, 3.0, and 3.5 were tried for each of the three thinning intensities. There was a very slight improvement in the fit to data from unthinned plots when  $B_4$  was set at 2.5 (see table 10). For the six fits on the thinned plot data, while the sums of squares for error improved, the MSE was increased in all cases. Improvement was seen, however, in the significance of the estimates. All

estimates were significant in these fits except  $B_3$  in the fits to data from heavy thinned plots. This model form was also fit across all thinned plot data.

Isolated simulations were run on these models to determine if differences in performance existed between a model fit to all thinned plot data and models fit to each thinning intensity. Of course, separate models for each thinning intensity are not practical, but the analysis showed whether a TRV for the PLIVE model was needed and should be pursued. These results are described later.

Table 10. Fit statistics and status of estimates for mortality functions with parameter B4 fixed at 2.5, 3.0, and 3.5.

Model	SSE	MSE	Sy.x	Df	Status
1	103.7193	0.017690	0.1330	5861	All significant
2	103.8819	0.017724	0.1331	5861	All significant
3	104.2647	0.017790	0.1334	5861	All significant
4	70.5804	0.005369	0.0733	13147	All significant
5	70.4424	0.005358	0.0732	13147	All significant
6	70.3334	0.005350	0.0731	13147	All significant
7	57.2645	0.006364	0.0798	8998	B3 insignificant
8	57.2633	0.006364	0.0798	8998	B3 insignificant
9	57.2633	0.006364	0.0798	8998	B3 insignificant
10	127.9965	0.005779	0.0760	22148	All significant
11	127.8440	0.005772	0.0760	22148	All significant
12	127.7199	0.005767	0.0759	22148	All significant

SSE = sum of squares for error, MSE = mean square error, Sy.x = standard error of the estimate, Df = fitting degrees of freedom.

Models:

- 1 Unthinned data fit with B4 set to 2.5
- 2 Unthinned data fit with B4 set to 3.0
- 3 Unthinned data fit with B4 set to 3.5
- 4 Light thinned data fit with B4 set to 2.5
- 5 Light thinned data fit with B4 set to 3.0
- 6 Light thinned data fit with B4 set to 3.5
- 7 Heavy thinned data fit with B4 set to 2.5
- 8 Heavy thinned data fit with B4 set to 3.0
- 9 Heavy thinned data fit with B4 set to 3.5
- 10 Thinned data fit with B4 set to 2.5
- 11 Thinned data fit with B4 set to 3.0
- 12 Thinned data fit with B4 set to 3.5

## The Reduced Simulator

The ultimate test of individual model performance is in a growth simulator. The simulator used for this analysis was a reduced version of PTAEDA2. The majority of the subroutines in PTAEDA2 are for user interface or initializing the juvenile stand. These include the INPUT, PLANT, JUV, and OUTPUT subroutines, as well as the management subroutines THIN and FERT. If external data from existing stands is used, then none of these are required for PTAEDA2 to perform its basic function of simulating stand development. The only subroutines that were retained from PTAEDA2 were the initialization, competition, and growth subroutines (INIT1, INIT2, COMP1, and GROW2). Two new subroutines were written, SI and STANDARD.

Site index is a user input to PTAEDA2, and this value for each stand is not available in the Coop thinning study data. In the subroutine SI, the site index of each stand was calculated from the stand age and the average height of dominant and codominant trees in the stand. Since there are four observations of dominant height in each stand, the subroutine was designed to select the dominant height associated with the stand age which is closest to the site index base age of twenty-five years.

The competition subroutine in PTAEDA2 was written to process rectangular plots, i.e. the total number of trees must have a whole number square root. This condition is inherent in the plots generated in the juvenile subroutine in that simulator, but it is not the case for the natural plots used as input in the current analysis. Thus, the subroutine STANDARD generates dummy trees up to the necessary count, and adds these to the existing trees in the stand. These trees are entered as dead, so their presence has no impact on competition determination or growth prediction of the existing trees.

The reduced simulator was designed to generate outputs of predicted growth after three, six, and nine years of simulation. These results were then uploaded to SAS and compared to the associated observed data at the first, second, and third remeasurements. The analysis of model performance was based on plots of mean differences between the observed and predicted values of height, mean total height, and diameter at breast height. Differences equaled observed minus predicted values for this analysis.

Comparison of mortality was done by determining the total number of surviving trees for each plot at each remeasurement period of the observed data and subtracting the total number

of live trees in each plot at each three-year prediction output. All of the plots and remeasurements deleted from the observed data for the fitting of the PLIVE equations were also deleted from the observed data used for the comparison.

There was some indecision as to whether the mortality analysis should be done on the basis of dead trees or surviving trees. Since the numbers are much larger for living trees, it was decided to base the analysis on the surviving trees to avoid any erroneous results from small dead tree counts.

A variety of simulations were run incorporating the newly developed models. Some of the simulations involved all of the models, while others isolated a particular model while retaining the other models in their original PTAEDA2 form. The goal was to draw specific conclusions as to the performance of each model separately and together as a unit.

Each simulation discussed is actually a block of three simulations. For each simulator configuration, separate simulations were run for each thinning intensity, unthinned, light and heavy thinned. The comparisons in SAS were also done in these blocks of three. This allowed mean difference analysis by thinning intensity as well as by remeasurement



period. It also kept the amount of analysis data at manageable levels.

Isolated simulations were done on the MTH model fitted with and without the new TRV's. These were done to determine if the TRV's benefitted the prediction or in fact, were not needed. In these simulations, and in all simulations which incorporated the new MTH model, the site index model in the subroutine SI was also of the new model fit and form, with or without the TRV's.

Similar, isolated simulations were run on the DIN model fitted with and without its TRV. Separate simulations were also used to determine the validity of truncating the TRV in this model at less than or equal to 1. These comparisons were also done for the TRV's in the MTH model.

Isolated simulations were run for the PLIVE fits in which  $B_4$  was set to a particular value. These two simulations compared the performance of three separate models for predicting mortality to the original design of two models, one for unthinned plots and one for thinned plots.

No isolated simulations were completed for the new fit of the HIN model since the performance of this model in thinned

stands is dependent on the thinning response incorporated into it via the new MTH model. Thus, analysis of the new HIN fit was only accomplished during simulations involving all of the new models.

One simulation was run in which all of the original PTAEDA2 models were retained. This simulation provided a baseline for comparison. Comparisons were visually drawn between each new model simulation and this baseline in order to determine if improvement was realized by fitting the models to multiple remeasurement data, or by the incorporation of thinning response variables.

The ultimate goal of growth simulation is some form of volume prediction. A total volume determination by plot was done in the comparison phase in SAS for a simulator configuration isolating the new MTH model. A similar calculation was completed for a simulation incorporating all of the new models. The results of these two determinations were visually compared. The question here was whether the new MTH model, which directly impacts the HIN and DIN predictions in the simulator, does a better job of predicting total volume working alone than all of the new models working as a unit. As discussed in the PLIVE analysis, it is possible that multiple TRV's and model changes may overcompensate for the

response to thinning in a stand.

It would have been highly desirable to analyze volume and mortality on a per acre basis. However, this was not possible because only interior trees were analyzed in this study. The size of each plot is determined by the maximum X and Y coordinates of the trees on that plot. Since only interior trees were examined, the analysis plots were smaller than the original research plots. The original plot areas were the only values available for calculating per-acre volume and mortality, thus the expansion of the comparison results to a per acre basis would not have been valid.

Crown ratio is an important component of the HIN, DIN, and PLIVE models. Crown ratio is calculated within the simulator from an equation by Dyer and Burkhart (1986). To determine if simulator performance could be improved, the crown ratio model incorporating the general thinning response variable developed in Liu et al. (in press) was included in duplications of some of the above described simulations.

Lastly, a simulation was completed on the forty-four plots which now have data available for the fourth remeasurement. These simulations were run only on the simulator configuration with all new models.

A complete listing of the source code for the reduced simulator is provided in appendix A.

## Results and Conclusions

### Results

The F-tests for significant differences in parameter estimates between thinning intensities yielded varied results. The F value for the DIN model was 36.5. This value clearly indicates that the predictive ability of the DIN model should benefit from a thinning response variable that can describe these differences. The F-tests for the mortality function were discussed in the analysis of that model. The three-year thinned data fits of the HIN model with several insignificant estimates were the only ones available for calculation of the F statistic. This brings into question the validity of the F-test result for this model.

The F-test for the HIN model resulted in a negative value (-3.03). This occurred because the reduced model had a lower sum of squares for error than the full model. Difficulties with separate fits to the light thinned and heavy thinned data resulted in such poor statistics that the full model had a higher sum of squares for error than the reduced model.

The sum of squares for error for the reduced model must be greater than the full model because fewer parameters are

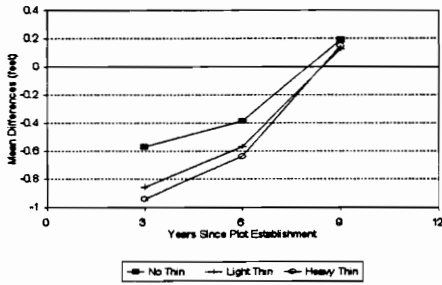
being estimated. These results invalidated the F-test for the HIN model, and became one more factor in the decision to fit the model to data from unthinned plots without a TRV.

The first simulation was performed on the original unaltered PTAEDA2 models. This provided the previously discussed baseline for comparison in subsequent analyses. Plots of mean differences for the comparison of observed to predicted values for the MTH, HIN, DIN, and PLIVE models are shown in figure 21. The plots for the HIN, DIN, and PLIVE models all indicate a trend towards an increasing underprediction over time. The original MTH model starts with an overprediction at the first remeasurement, then shows a trend similar to the other models that results in a large reduction in mean differences by the third remeasurement.

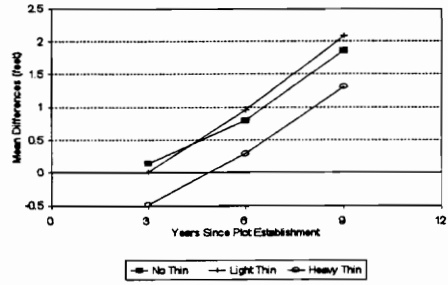
Several preliminary isolated simulations were completed for the MTH and DIN models to determine the validity of truncating the TRV's in these models. These were necessary in order to obtain the best simulator configuration for subsequent analyses.

Simulation of the DIN model without its TRV truncated to 1 resulted in an increase in the trend towards underprediction over time. The change was not great, but this indicated that

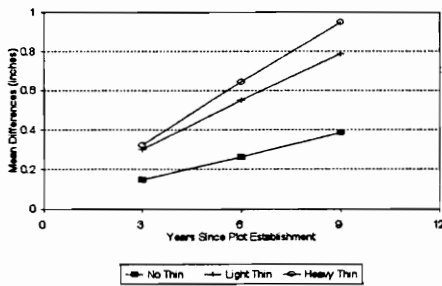
**Simulation Performance of MTH Model**  
 Simulator with Original PTAEDA2 Models



**Simulation Performance of HIN Model**  
 Simulator with Original PTAEDA2 Models



**Simulation Performance of DIN Model**  
 Simulator with Original PTAEDA2 Models



**Simulation Performance of PLIVE Model**  
 Simulator with Original PTAEDA2 Models

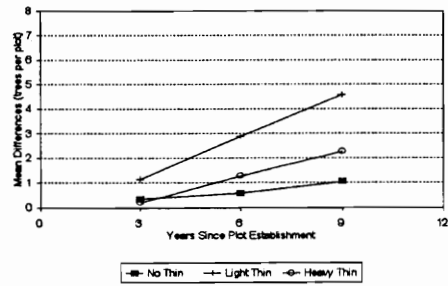


Figure 21. Simulation comparison for the four original PTAEDA2 models. Comparison is observed values minus simulator predicted values at the first, second, and third remeasurements.

the unbounded increase in the TRV after K was exceeded was detrimental to the prediction. Thus, in application, the thinning response variable for the DIN model should be truncated to less than or equal to 1.

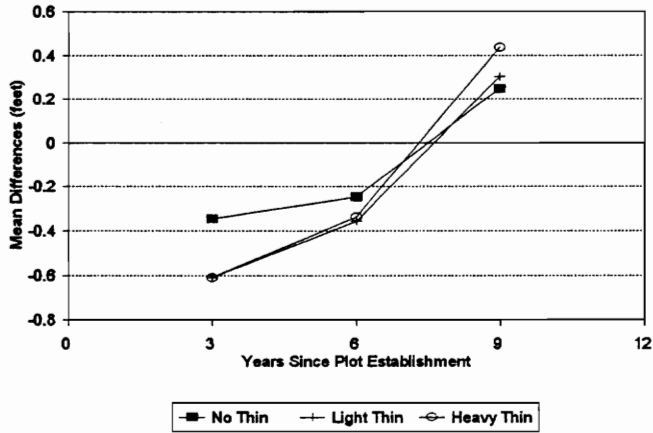
There was little difference in the performance of the MTH model with or without truncation of TRV1. This TRV is fixed at a constant value for each plot, which may be less than or greater than 1, depending on the age at thinning of the plot. Thus, truncation of this TRV offers no practical gain and would probably be detrimental to the model.

Simulation without the varying TRV (TRV2) truncated in the MTH model resulted in a large change in the comparison. Figure 22 compares these two simulations. The vertical scale of these two plots has been shifted, but the range is identical. The comparison for unthinned stands is unchanged. However, the comparisons for thinned stands are shifted to an underprediction, and they are much more separated without TRV truncation, indicating increased bias in the prediction across thinning intensities.

The shift resulted in an improvement for the light thinned data comparison in the first three years of prediction. However, when this analysis was repeated with the



### Simulation Performance of MTH Model TRV2 Truncated To 1



### Simulation Performance of MTH Model TRV1 and TRV2 Not Truncated To 1

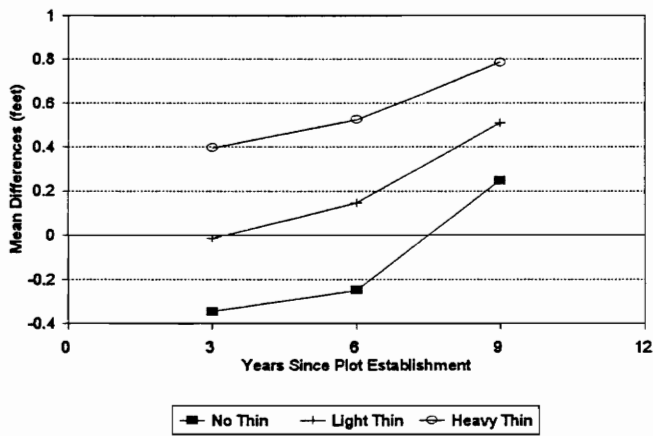


Figure 22. Simulation comparison on the mean total height model with and without TRV2 truncated to less than or equal to 1.

new HIN model in the simulator, the HIN prediction became considerably more biased (about one-half foot) towards underprediction. Thus, considering the progression of the growth simulator from MTH to HIN to DIN, and the apparent increase in bias in the MTH comparison without truncation, TRV2 was truncated in all subsequent simulations.

It was expected that since diameter growth is highly sensitive to changes in stand density, a large improvement would be realized by incorporating a thinning response variable. Simulation analysis proved otherwise. The isolated simulations of the diameter increment model fitted with and without a TRV showed virtually no difference in the performance of the simulator. In addition, this model fit to three remeasurement periods and using a TRV performed slightly more poorly (0.05 to 0.1 inches) than the original PTAEDA2 DIN model which was fit to only one remeasurement period and to unthinned data only.

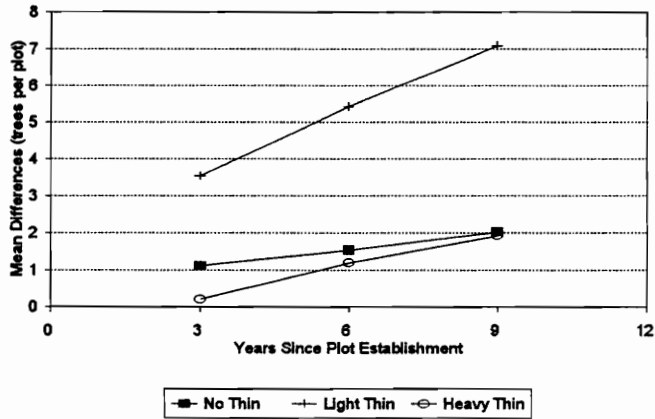
The new MTH model, the "front end" of the simulator, was run with and without its TRV's. The site index equation in the subroutine SI was also fitted and run with and without TRV's coincident with the above two simulations. This analysis showed a slight improvement in the comparison plots when the TRV's were included. The decrease in overprediction

was 0.05 feet at the first remeasurement for all thinning intensities. This simulator configuration also performed better than with the original PTEADA2 MTH model, with or without TRV's included in the new model (about 0.2 feet at the first remeasurement for all thinning intensities).

The fits of the PLIVE model in which  $B_4$  was fixed at a particular value were used in a pair of simulations to determine if separate models for unthinned and thinned stands were sufficient for mortality prediction. The alternative is the development of a single function which includes a TRV for prediction of mortality across all intensities of thinning. One simulation used two mortality functions, one for unthinned data and another for thinned data. The second simulation used three functions, one for each thinning intensity.

The comparison of mean differences in surviving tree counts on a per plot basis showed that using separate models produced a slight improvement in the mortality prediction for light thinned plots (less than 0.5 trees per plot). The prediction across heavy thinned plots however, suffered a one tree loss in the mean difference analysis at all three remeasurements (see figure 23). The comparison for unthinned plots was unchanged because the same model was used in both simulations.

### Simulation Performance of PLIVE Model Separate Fits for Each Thinning Regime



### Simulation Performance of PLIVE Model Separate Fits to Unthinned & Thinned

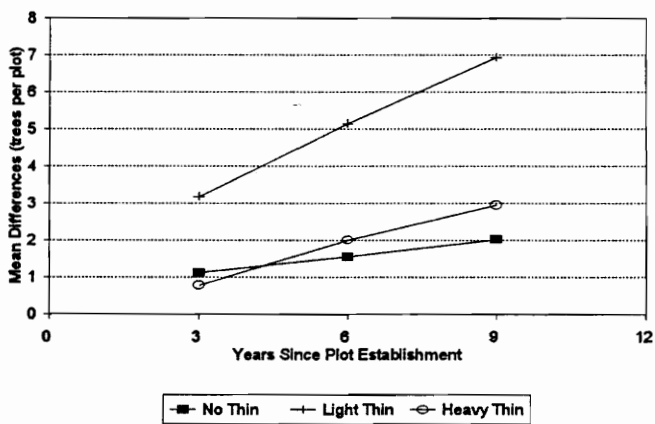
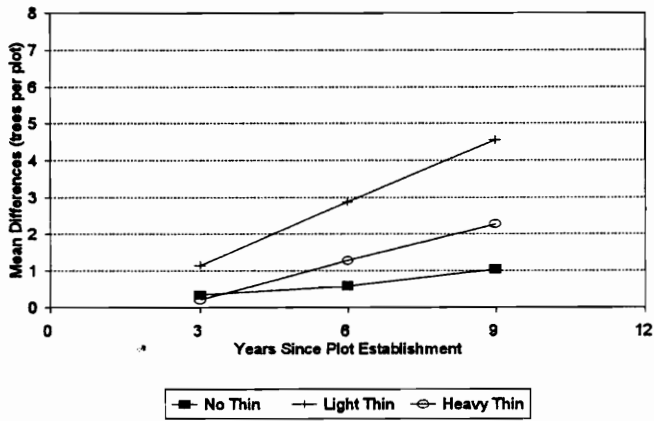


Figure 23. Simulation comparison of the mortality function performance using separate models for each thinning intensity or models fit to the unthinned and thinned data only. The parameter B4 is set in these model fits.

### Simulation Performance of PLIVE Model Simulator with Original PTAEDA2 Models



### Simulation Performance of PLIVE Model Growth Simulator with All New Models

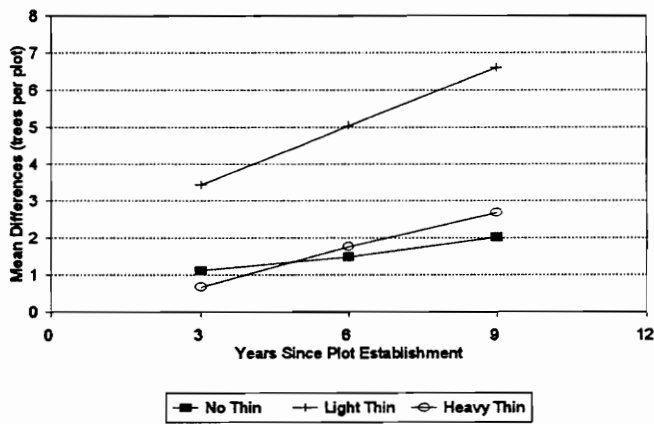


Figure 24. Simulation comparison between the original PTAEDA2 mortality function and the final models fit to unthinned and to thinned data.

The results of the above PLIVE analysis provided evidence that the light thinned plots have a significant impact on a model fit across all thinning intensities. The analysis also showed that a thinning response variable for this model should be investigated. However, the results do not clearly indicate an obvious need for a TRV, the goal of this path of analysis.

Several simulations using all of the new models were completed. In these simulations, the PLIVE models were the final fit models previously described with four estimated parameters. A loss in predictive accuracy (about one-half tree per plot) was realized for the mortality configuration using two PLIVE models over the original PTAEDA2 model fit to one remeasurement of unthinned plot data. This loss was quite considerable (better than two trees per plot) for the comparison across light thinned plots (see figure 24).

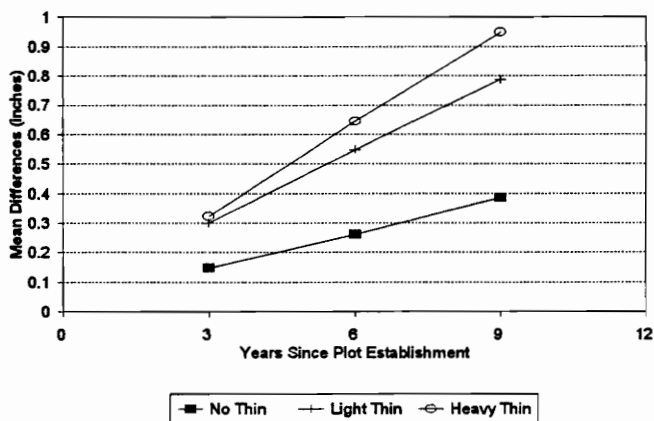
It is quite possible that the strong bias exhibited by the mortality comparison across light thinned plots would not be so apparent if mortality prediction were analyzed on a per acre basis. The original research plots that were lightly thinned are on average twice the size of the unthinned plots, thus they include many more trees resulting in larger prediction variations. The plots which were heavily thinned are similar in size to the light thinned plots, but more trees

were removed from these plots. Examining all thinning intensities on a per acre basis could neutralize this discrepancy in plot size and provide a more balanced comparison.

The comparison of the DIN performance between a simulation with all new models and one with the original PTEADA2 model had varied results. Figure 25 shows a very slight loss in performance of the new model on data from light thinned plots and a slight loss on unthinned plots (about 0.05 inches). Performance improved slightly for the heavy thinned plot comparison, with the total bias after nine years decreased from 0.95 inches with the original model to 0.92 inches for the new model with a TRV.

The MTH model had measurable improvement in prediction over the original PTAEDA2 model. The overprediction at the early remeasurements was noticeably reduced, while the third remeasurement was unaffected. The reduction in bias was about 1.8 feet at the first remeasurement for all three thinning intensities using the new model. This comparison is displayed in figure 26. Thus, including the TRV's in the MTH model improved the early prediction ability without impacting the nine year prediction.

### Simulation Performance of DIN Model Simulator with Original PTEADA2 Models



### Simulation Performance of New DIN Model Growth Simulator with All New Models

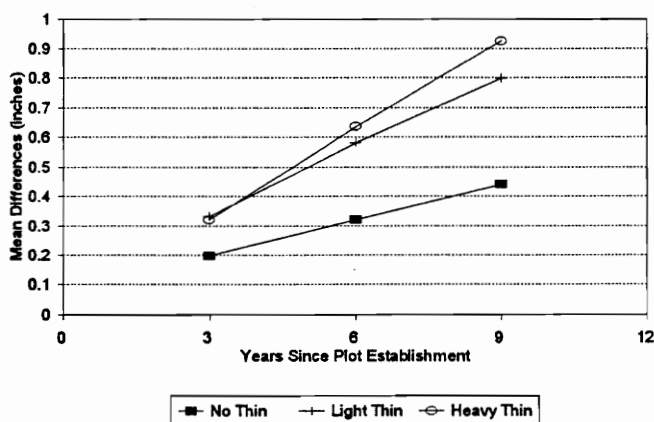
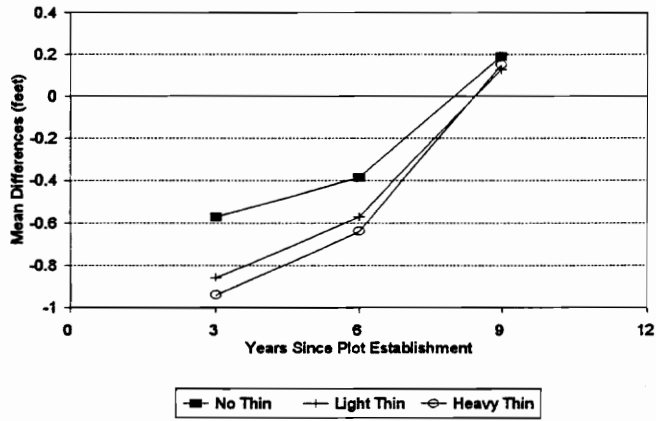


Figure 25. Simulation comparison between the original PTAEDA2 diameter increment model and the new model with a thinning response variable.



### Simulation Performance of MTH Model Simulator with Original PTAEDA2 Models



### Simulation Performance of New MTH Model Growth Simulator with All New Models

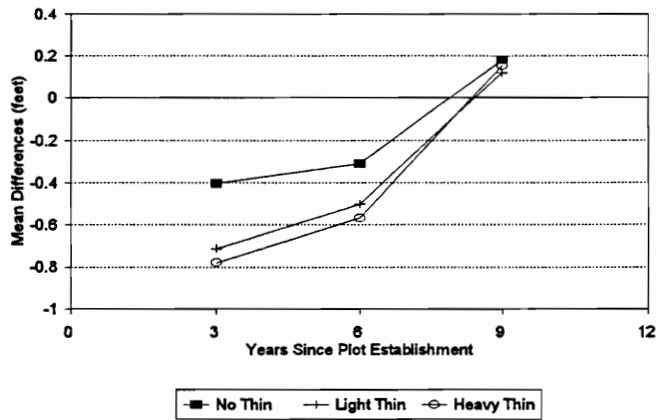


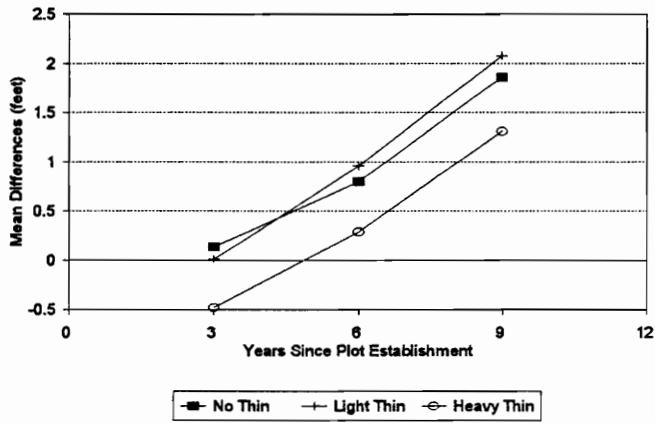
Figure 26. Simulation comparison between the original PTAEDA2 mean total height model and the new model with two thinning response variables.

This improvement, however, was not reflected in the HIN prediction. Mean difference comparison of this model's prediction with the original PTAEDA2 model performance shows no change in the bias relationship between thinning intensities, but the entire pattern of comparison results was shifted upwards to a greater underprediction. Only prediction for heavy thinned plots at three years after thinning could be said to have improved (see figure 27).

Total volume outside bark was determined on a per plot basis for four different simulations; the original PTAEDA2 simulator configuration, a simulation with the new MTH model isolated, a simulation with all new models, and one in which the original crown ratio model was replaced by the CR model including a TRV developed by Liu et al.

The comparison of the simulation with the new MTH model isolated to that in which all models were new found the former to be superior in predictive efficiency. The new MTH model working with the other original PTAEDA2 models had better mean differences (5 to 10 cubic feet per plot) for unthinned and light thinned plots. There was a very little change in volume prediction for heavy thinned plots. This comparison is shown in figure 28.

### Simulation Performance of HIN Model Simulator with Original PTEADA2 Models



### Simulation Performance of New HIN Model Growth Simulator with All New Models

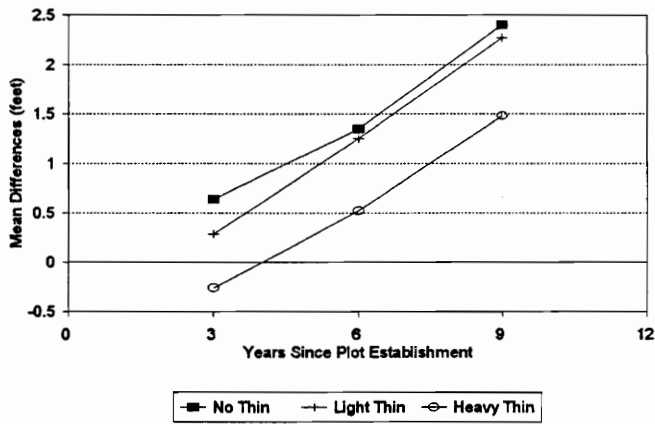
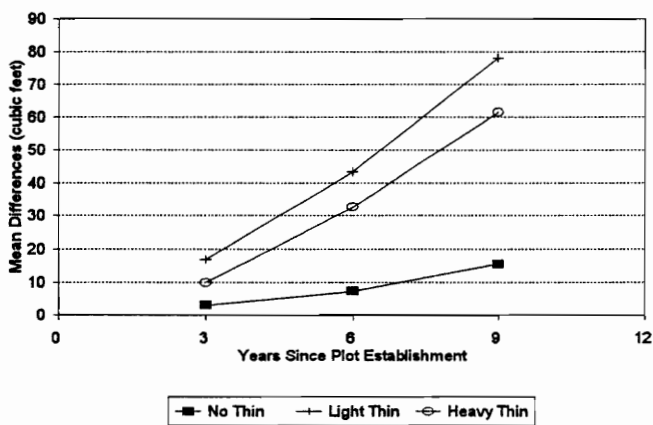


Figure 27. Simulation comparison between the original PTAEDA2 height increment model and the new model fit to unthinned data.

### Volume Prediction in Reduced Simulator New MTH Model, Other Models Original



### Volume Prediction in Reduced Simulator Growth Simulator with All New Models

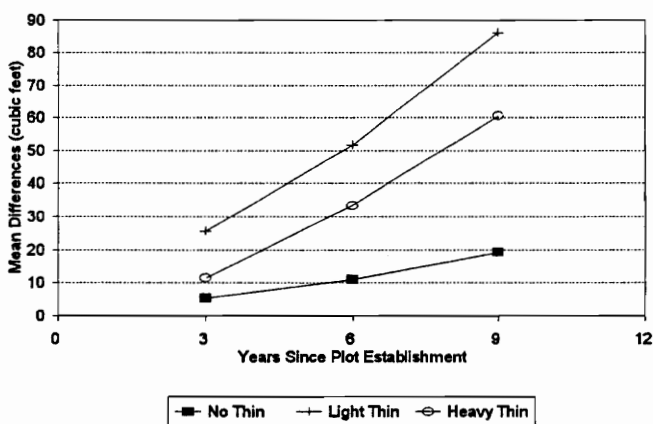


Figure 28. Simulation comparison of volume prediction performance between a simulator configuration with only the original mean total height model replaced by the new MTH model and one with all new models.

The above result reinforces the theory discussed in the analysis of the PLIVE model that multiple model changes with several TRV's can overcompensate in the explanation of the response to thinning, and thus result in a poorer prediction. When this isolated MTH simulation was compared to the original PTAEDA2 model for predicting MTH, virtually no difference was found. That is, no improvement was realized in volume prediction by including a thinning response variable in the model for predicting mean total height of dominant and codominant trees.

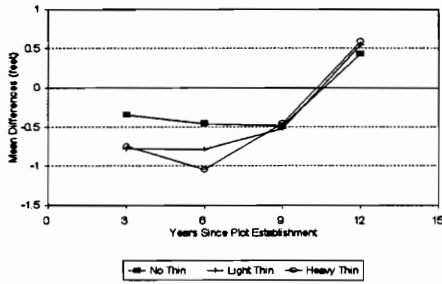
The simulation in which all new models were used was repeated with the original CR model replaced by the CR model including the TRV developed by Liu et al. All of the models which utilize CR in their prediction were slightly improved by incorporating the new crown ratio model in the simulation. The HIN, DIN, and PLIVE models all performed slightly better. The MTH model does not use CR in its prediction and this model showed no change with the new CR model compared to the original one. Volume prediction also showed a slight improvement with the new CR model. All of the above improvements seen were for thinned plots. The new CR model seemed to have no impact on the unthinned data simulations.

The forty-four plots with fourth remeasurement data available were used in a pair of simulations including all of the new models. One simulation utilized the original PTAEDA2 CR model while a second used the CR model developed by Liu et al. The mean difference comparisons for the MTH, HIN, and DIN models, as well as volume determination are shown in figure 29. Mortality was not analyzed for the REM4 simulations.

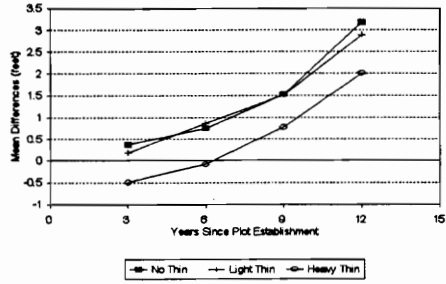
The HIN and DIN model comparisons across four remeasurements demonstrated the same increasing underprediction over time found in the analyses of simulations on three remeasurements; however, no sudden increases were seen in these trends. The MTH model performed quite differently in the fourth three-year prediction period than in the first three periods. As was indicated by the original analysis of PHIN's in this fourth period (figure 2), there was a large change in bias from the third to the fourth remeasurement (about 1 foot). MTH prediction on these forty-four plots changed from a consistent overprediction to an underprediction at the fourth remeasurement.

There was also a change in volume prediction during this fourth period. The increasing bias virtually stalled for the comparison across light thinned plots and actually reduced for the heavy thinned plots. Incorporating the new CR model in

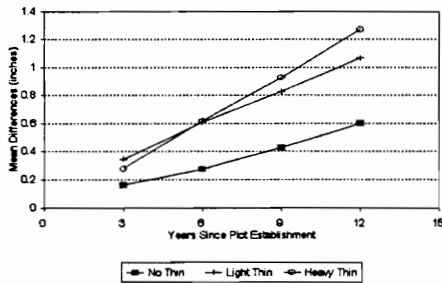
**Simulation Performance of New MTH Model**  
Growth Simulator with All New Models



**Simulation Performance of New HIN Model**  
Growth Simulator with All New Models



**Simulation Performance of New DIN Model**  
Growth Simulator with All New Models



**Volume Prediction in Reduced Simulator**  
Growth Simulator with All New Models

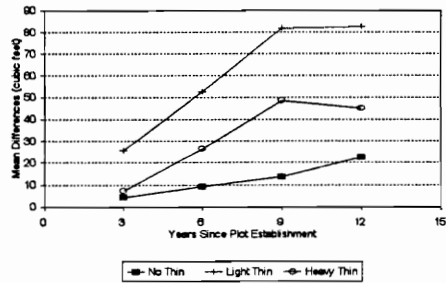


Figure 29. Simulation comparison for three of the new models and for volume determination across forty-four plots with fourth remeasurement data available.

the simulation resulted in a very slight improvement in all cases with the exception of the MTH model which was unchanged.

### Conclusions

The mean total height prediction was improved using a model which included a thinning response variable over the original PTAEDA2 design of this model. This improvement, however, did not have a positive impact on the height increment prediction. The HIN model performance was deteriorated somewhat for all thinning intensities. It is apparent that although thinning response was incorporated into the HIN prediction, it did not produce the desired reduction in bias. Further analysis of the relationship between these two models is recommended.

There was little variation between any of the analyses of the DIN model performance. This was true of the isolated analyses as well as simulations in which all models were new. No model or simulator configuration changes seemed to have any great impact on the DIN prediction, except to deteriorate it from the original PTAEDA2 model performance.

Mortality prediction was deteriorated by the refits of the PLIVE function. A significant difference does exist



between mortality in unthinned and thinned stands; however, fitting separate models to these two conditions did not improve the prediction. The F statistic comparing estimates in fits to light thinned and heavy thinned data was 5.36, and differences were seen in simulation comparisons between using separate models and one model for thinned stand prediction. Thus, there is sufficient evidence to suggest that a thinning response variable may benefit the mortality prediction and it is recommended that a model incorporating a TRV be investigated.

Total volume prediction on a per plot basis was not changed by including a TRV in the MTH model. Total volume prediction was deteriorated by changing several models in the simulator. Actual volume prediction should be analyzed on a per acre basis to determine if performance can be improved using the new MTH model with its TRV's.

The analysis indicated that mean total height prediction can be improved by including a TRV in a mean total height model. Thus, it is recommended that future versions of the PTAEDA simulator incorporate this model change.

Crown ratio is a basic component of the HIN, DIN, and PLIVE models. Including the CR model with a TRV developed by

Liu et al. improved each of these models over the original PTAEDA2 CR model in simulation comparisons. Volume prediction also benefitted from this new model. Any new versions of the PTAEDA growth simulator should include this new model.

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## Appendix A. Source Code For the Reduced Growth Simulator

```
C
C
C THIS PROGRAM DOES NINE YEARS OF PREDICTION, USING EXTERNAL DATA
C OBSERVED ON 186 SITES IN THE COASTAL PLAINS AND PIEDMONT AREAS OF THE
C SOUTHEAST. IT GENERATES AN OUTPUT TO AN ASCII DATASET EVERY THREE YEARS
C FOR EACH PLOT. THE PROGRAM USES EXCERPTS FROM THE GROWTH SIMULATION
C MODEL PTAEDA2. THE PURPOSE OF THIS PROGRAM IS TO DETERMINE THE VALIDITY
C OF THE HEIGHT AND DIAMETER INCREMENT MODELS IN PTAEDA2, AS WELL AS THE
C MORTALITY FUNCTION, BY COMPARING THE PREDICTED OUTPUT TO THE OBSERVED
C DATA OVER THE NINE YEAR PERIOD. THE PROGRAM ALSO HAS MODELS WHICH CAN
C BE SELECTED THAT ARE MODIFIED TO INCLUDE A THINNING RESPONSE VARIABLE
C DESIGNED TO IMPROVE PREDICTION.
C
C VARIABLES:
C
C ODEAD = 1 CODE FOR TREE STATUS OF DEAD
C OALIVE = 2 CODE FOR TREE STATUS OF ALIVE
C
C COMMON BLOCK 1
C
C X, Y, D, H, CL, CIP, LMORT, KMORT, TAG, LVIGR(MTREES) =
C ARRAYS OF THESE VARIABLES FROM 1 TO MTREES.
C ACRES = PLOT SIZE (from PLOT DATA)
C ID = COMPANY/LOCATION CODE
C PLOT = 1, 2, OR 3. INDICATES NO, LIGHT, OR HEAVY THINNED
C PERCENT = % OF TOTAL BASAL AREA IF QFERT TRUE (from INIT1)
C HD = AVERAGE HEIGHT OF DOMINANT AND CODOMINANT TREES (from PLOT DATA)
C IX = A RANDOM NUMBER (from INIT1)
C N = NROWS*NROWS (from STANDARD)
C K = PRESENT STAND AGE & CURRENT GROWING SEASON (from PLOT DATA)
C M = TREE COUNT FOR EACH PLOT (from PLOT DATA)
C V = 3, 6, OR 9 YEARS OF PREDICTED GROWTH, OR REMEASUREMENT PERIOD
C KTHIN = PLOT AGE AT THINNING (from MAIN PROGRAM)
C LTHIN = INDICATES THINNED (= 1) OR UNTHINNED (= 0)
C NROWS = # OF PLANTED ROWS AND TREES/ROW (from INIT1 or STANDARD)
C RESBA = RESIDUAL BASAL AREA FACTOR (I IN SAS, from PLOT DATA)
C K3, K6, & K9 = AGES AT 1ST, 2ND, AND 3RD REMEASUREMENTS (from PLOT DATA)
C HD3, HD6, & HD9 = HD'S AT 1ST, 2ND, AND 3RD REMEASUREMENTS (PLOT DATA)
C
C COMMON BLOCK 2
C
C SITE = SITE INDEX (from SI)
C PX = DISTANCE RATIO BETWEEN TREES (from INIT1)
C PY = DISTANCE RATIO BETWEEN ROWS (from INIT1)
C PLOTX = NROWS*PX (from INIT2)
C PLOTY = NROWS*PY (from INIT2)
C VARX = % VARIANCE BETWEEN TREES (from INIT1)
C VARY = % VARIANCE BETWEEN TREES (from INIT1)
C NYEARS = # OF GROWING SEASONS TO SIMULATE (from MAIN PROGRAM)
C
C MAIN PROGRAM
C
IMPLICIT INTEGER*4 (I-N)
INTEGER*4 IX, ID, PLOT, TAG
INTEGER*4 ODEAD, OALIVE
INTEGER*4 K3, K6, K9, Z, Q, V
REAL HD3, HD6, HD9
CHARACTER*80 FILEN
PARAMETER (MROWS = 20, MTREES = MROWS*MROWS)
```

```

PARAMETER (ODEAD = 1, OALIVE = 2)
PARAMETER (ZERO = 0.0)
COMMON /BLOK1/ X(MTREES), Y(MTREES), D(MTREES), H(MTREES), CL(MTREES),
1 CIP(MTREES), LMORT(MTREES), KMORT(MTREES), TAG(MTREES), ACRES, ID,
2 PLOT, PERCNT, HD, IX, N, K, M, V, NROWS, RESBA, K3, K6, K9, HD3, HD6, HD9,
3 LVIGR(MTREES), LTHIN, KTHIN
COMMON /BLOK2/ SITE, PX, PY, PLOTX, PLOTY, VARX, VARY, NYEARS
C
C CALL HEADER
C
C GET THE NAME FOR THE OUTPUT FILE.
C
10 WRITE(*, 20)
20 FORMAT(5(/), 31X, 'ASCII Output File' , //,
1 ' Please enter the file name (or CON or PRN): ')
READ(*, 30) FILEN
30 FORMAT(A80)
IF (FILEN.EQ.' ') GO TO 10
C
C RUN ENTIRE PROGRAM ON FIRST ID, THEN LOOP TO START, RESET, AND RUN
C PROGRAM ON EACH SUCCESSIVE ID UNTIL LAST ID IS REACHED.
C
OPEN(1, FILE = FILEN, STATUS = 'NEW')
OPEN(2, FILE = 'PLOT2.DAT', STATUS = 'OLD')
OPEN(3, FILE = 'TREE2.DAT', STATUS = 'OLD')
Q = 1
DO 180 Z = 1, 185
CALL INIT1
CALL INIT2
C
C READ IN PLOT DATA FOR THIS ID.
C
READ (2, 110) ID, PLOT, K, HD, ACRES, RESBA, K3, K6, K9, HD3, HD6, HD9, M
110 FORMAT(3I4, F6.1, F8.4, F9.5, 3I4, 3F6.1, I4)
KTHIN = K
CALL SI
C
C READ IN TREE DATA FOR THIS ID.
C
DO 130 I = 1, M
READ (3, 120, END = 140) ID, PLOT, TAG(I), X(I), Y(I), CL(I), D(I), H(I),
1 LMORT(I)
120 FORMAT(3I4, 3F4.0, F5.1, F4.0, I4)
C
C ADD CIB(I) AND F9.5-IF RUNNING PLOTS 101 TO 1109 TO TREE INPUT.
C
C CONVERT FROM VIGR CODE TO LMORT CODE. VIGR CODES ARE: 1 = ALIVE,
C 2 = DEAD, AND 3 = THINNED. VIGR VALUES ARE READ IN AS LMORT.
C
IF (LMORT(I).EQ.1) THEN
LMORT(I) = OALIVE
ELSE
LMORT(I) = ODEAD
ENDIF
N = N + 1
130 CONTINUE
140 CALL STANDARD
C
C ENABLE CALL DATALIST TO DISPLAY A LIST OF THE INITIAL DATA.
C
C CALL DATALIST

```

```

    KBEGIN = K + 1
C
C CHANGE LENGTH OF GROWTH SIMULATION TO MATCH ACTUAL PLOT SURVIVAL.
C
    IF (HD3.EQ.0.0.AND.HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
        WRITE (*,145)
145 FORMAT(/,1X,'PLOT DESTROYED PRIOR TO FIRST REMEASUREMENT.')
        Q = Q + 1
        GO TO 165
    ELSEIF (HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
        NYEARS = K + 3
    ELSEIF (HD9.EQ.0.0) THEN
        NYEARS = K + 6
    ELSE
        NYEARS = K + 9
    ENDIF
C
C COMPUTE COMPETITION INDEX AND GROW TREES.
C
    DO 160 K = KBEGIN, NYEARS
        WRITE (*,150) K
150 FORMAT(/,20X,'Computing Growth for Growing Season',I4)
        CALL COMP1
        CALL GROW2
C
C GET OUTPUTS FOR 3, 6, AND 9 YEARS OF GROWTH. OUTPUTS ARE ONLY GENERATED
C IF GROWTH WAS ACTUALLY DONE FOR THAT REMEASUREMENT PERIOD.
C
    IF (K.EQ.KBEGIN + 2) THEN
        V = 3
        CALL TREES
    ELSEIF (K.EQ.KBEGIN + 5) THEN
        V = 6
        CALL TREES
    ELSEIF (K.EQ.KBEGIN + 8) THEN
        V = 9
        CALL TREES
    ELSE
        GO TO 160
    ENDIF
160 CONTINUE
    K = NYEARS
    Q = Q + 1
165 IF (Q.EQ.186) GO TO 180
    WRITE (*,170) Q
170 FORMAT(/,24X,'Growing Plot',I4,' of 185 Plots')
180 CONTINUE
    CLOSE(1)
    CLOSE(2)
    CLOSE(3)
    STOP 'END OF PROGRAM'
    END
C
C SUBROUTINE HEADER
C
C Write program heading
C
    WRITE (*,20)
20 FORMAT(10(/),1X,78('-',/,' |',T79,'|',/,' |',5X,4(8('*'),2X),
1 7('*'),3X,8('*'),2X,6('*'),T79,'|',/,' |',5X,'** '
2 2('*'),5X),'** ** **',8X,'** ** ** **',8X,'**',

```

```

3 T79,'|',/,,'|',5X,8(''),2(5X,''),6(''),2X,6(''),
4 2(4X,''),2X,8(''),3X,6(''),T79,'|',/,,'|',5X,'',11X,
5 ' * * * * *',8X,2(' * * * * *'),' * *',T79,'|',/,,'|',
6 5X,'',11X,' * * * * *',8(''),2X,7(''),
7 ' * * * * *',8(''),T79,'|',/,,'|',T79,'|')
WRITE(*,30)
30 FORMAT(' |',23X,'COPYRIGHT 1987  VERSION 1.0',T79,'|',/,,'|',
1 7X,'Simulation of Individual Tree Growth and Stand ',
2 'Development in',T79,'|',/,,'|',9X,'Loblolly Pine Plantations ',
3 'on Cutover, Site-Prepared Areas',T79,'|',/,,'|',T79,'|',/,
4 ' |',5X,'By K. D. Farrar, R. L. Amateis, H. E. Burkhart, ',
5 'and R. F. Daniels',T79,'|',/,,'|',18X,
6 'School of Forestry and Wildlife Resources',T79,'|',/,,'|',13X,
7 'Virginia Polytechnic Institute and State University',T79,'|',/,
8 ' |',25X,'Blacksburg, Virginia 24061',T79,'|',/,,'|',T79,'|',/,
9 ' |',21X,'Modified by Michael C. Smith, 1993',T79,'|',/,,'|',
* T79,'|',/,,'|',1X,78('-'))
CALL CONT
RETURN
END

C
SUBROUTINE INIT1
C
C This routine initializes common variables to initial default values
C
IMPLICIT  INTEGER*4 (I-N)
INTEGER*4  IX,ID,PLOT,TAG
INTEGER*4  K3,K6,K9
REAL      HD3,HD6,HD9
PARAMETER  (MROWS = 20,MTREES = MROWS*MROWS)
COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
3 LVIGR(MTREES),LTHIN,KTHIN
COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS

C
PERCNT = 0.048
IX = 68767
NYEARS = 35
NROWS = 15
SITE = 60.0
PX = 1.0
PY = 1.0
VARX = 10.0
VARY = 10.0
END

C
SUBROUTINE INIT2
C
C Subroutine INIT2 initializes the individual tree data to zero
C before each simulation begins.
C
IMPLICIT  INTEGER*4 (I-N)
INTEGER*4  IX,ID,PLOT,TAG
INTEGER*4  ODEAD,OALIVE
INTEGER*4  K3,K6,K9
REAL      HD3,HD6,HD9
PARAMETER  (MROWS = 20,MTREES = MROWS*MROWS)
PARAMETER  (ODEAD = 1,OALIVE = 2)
PARAMETER  (ZERO = 0.0)
COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),

```



```

1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
3 LVIGR(MTREES),LTHIN,KTHIN
COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS

```

C

```

DO 10 I=1,MTREES
  X(I) = ZERO
  Y(I) = ZERO
  D(I) = ZERO
  H(I) = ZERO
  CL(I) = ZERO
  CIP(I) = ZERO
  TAG(I) = ZERO
  LVIGR(I) = 1
  LMORT(I) = OALIVE

```

10 KMORT(I) = -1

```

  N = 0
  M = 0
  K = 0
  K3 = 0
  K6 = 0
  K9 = 0
  HD = ZERO
  HD3 = ZERO
  HD6 = ZERO
  HD9 = ZERO
  RESBA = ZERO
  ACRES = ZERO
  PLOTX = ZERO
  PLOTY = ZERO
  NROX = 0
  NROY = 0
  KTHIN = 0
  LTHIN = 0
  RETURN
END

```

C

SUBROUTINE SI

C

C THIS ROUTINE DETERMINES WHICH REMEASUREMENT PERIOD IS CLOSEST TO THE  
C SI BASE AGE OF 25, THEN USES THE DOMINANT HEIGHT FROM THAT REMEASUREMENT  
C PERIOD TO DETERMINE THE SITE INDEX. PLOT ESTABLISHMENT K AND HD ARE  
C RETAINED IN THE MAIN PROGRAM.

C

```

IMPLICIT INTEGER*4 (I-N)
INTEGER*4 IX,ID,PLOT,TAG
INTEGER*4 ODEAD,OALIVE
INTEGER*4 K3,K6,K9
REAL HD3,HD6,HD9
PARAMETER (MROWS=20,MTREES=MROWS*MROWS)
PARAMETER (ODEAD=1,OALIVE=2)
PARAMETER (ZERO=0.0)
COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
3 LVIGR(MTREES),LTHIN,KTHIN
COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS

```

C

```

A = ABS(25 - K)
B = ABS(25 - K3)
C = ABS(25 - K6)

```

```

E = ABS(25 - K9)
IF (HD3.EQ.0.0.AND.HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
  TEMP = A
ELSEIF (HD6.EQ.0.0.AND.HD9.EQ.0.0) THEN
  TEMP = MIN(A,B)
ELSEIF (HD9.EQ.0.0) THEN
  TEMP = MIN(A,B,C)
ELSE
  TEMP = MIN(A,B,C,E)
ENDIF
IF (TEMP.EQ.A) THEN
  L = K
  DH = HD
ELSEIF (TEMP.EQ.B) THEN
  L = K3
  DH = HD3
ELSEIF (TEMP.EQ.C) THEN
  L = K6
  DH = HD6
ELSE
  L = K9
  DH = HD9
ENDIF
C
C Equation (Site Index) developed by Michael Smith
C from Amateis and Burkhart (1985)
C Fit without TRV's
C
C SITE = (1.0/ALOG(DH))*((1.0/REAL(L))/(1.0/25))**(-0.03754117)*
C 1 EXP(-2.61807656*(1.0/REAL(L)-1.0/25))
C
C Equation (Site Index) developed by Michael Smith
C from Amateis and Burkhart (1985)
C K = AGE2 (prediction age), KT = elapsed time since thinning
C
KT = K-KTHIN
KT1 = 25-KTHIN
TRV1 = RESBA**(-0.68975758*(-(KT1**2) + 10.77325694*KT1)/(25**2))
TRV2 = RESBA**(-0.68975758*(-(KT**2) + 10.77325694*KT)/(K**2))
IF (TRV2.GT.1.0) THEN
  TRV2 = 1.0
ELSE
  TRV2 = TRV2
ENDIF
SITE = (1.0/ALOG(DH))*((1.0/REAL(L))/(1.0/25))**(-0.01600294)*
1 EXP(-3.02041884*(TRV2/REAL(L)-TRV1/25))
C
C Equation (Site Index) from Amateis and Burkhart (unpublished)
C
C SITE = (1.0/ALOG(DH))*((1.0/REAL(L))/(1.0/25.0))**(-0.02205)*
C 1 EXP(-2.83285*(1.0/REAL(L)-1.0/25.0))
SITE = 1.0/SITE
SITE = EXP(SITE)
C WRITE (*,100) SITE,DH,L
C 100 FORMAT(/,1X,'SITE INDEX = ',F6.1,', FROM HD = ',F6.1,
C 1 ', AND K = ',I4)
RETURN
END
C
SUBROUTINE STANDARD
C

```

C This routine standardizes a plot to rectangular for input by adding  
 C dead trees to make NROWS a whole number.

C  
 IMPLICIT INTEGER\*4 (I-N)  
 INTEGER\*4 IX,ID,PLOT,TAG,DUMMIES  
 INTEGER\*4 ODEAD,OALIVE  
 INTEGER\*4 K3,K6,K9  
 REAL HD3,HD6,HD9  
 PARAMETER (MROWS = 20,MTREES = MROWS\*MROWS)  
 PARAMETER (ODEAD = 1,OALIVE = 2)  
 PARAMETER (ZERO = 0.0)  
 COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),  
 1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,  
 2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,  
 3 LVIGR(MTREES),LTHIN,KTHIN

C  
 N2 = N  
 ROWS = ANINT(SQRT(REAL(N)) + 0.499999)  
 NROWS = INT(ROWS)  
 N = INT(ROWS)\*INT(ROWS)  
 DUMMIES = N - N2  
 IF (DUMMIES.EQ.0) RETURN  
 DO 20 I = N2 + 1,N  
 X(I) = 0.0  
 Y(I) = 0.0  
 D(I) = 0.0  
 H(I) = 0.0  
 CL(I) = 0.0  
 LMORT(I) = ODEAD  
 KMORT(I) = K  
 20 CONTINUE  
 RETURN  
 END

C  
 C SUBROUTINE DATALIST

C  
 IMPLICIT INTEGER\*4 (I-N)  
 INTEGER\*4 IX,ID,PLOT,TAG  
 INTEGER\*4 ODEAD,OALIVE  
 INTEGER\*4 K3,K6,K9,Z,Q  
 REAL HD3,HD6,HD9  
 CHARACTER\*80 FILEN  
 PARAMETER (MROWS = 20,MTREES = MROWS\*MROWS)  
 PARAMETER (ODEAD = 1,OALIVE = 2)  
 PARAMETER (ZERO = 0.0)  
 COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),  
 1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,  
 2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,  
 3 LVIGR(MTREES),LTHIN,KTHIN

C  
 C THIS SUBROUTINE HAS NOT BEEN UPDATED FOR CHANGES IN THE INPUT DATASET

C  
 CALL COMP1  
 WRITE(\*,10)  
 10 FORMAT(//1X,' ID PLOT X Y K CL D H HD LMORT  
 1 ACRES SITE CIP',/,1X,73('-'))  
 DO 30 I = 1,N  
 WRITE (\*,20) ID,PLOT,X(I),Y(I),K,CL(I),D(I),  
 1 H(I),HD,LMORT(I),ACRES,SITE,CIP(I)  
 20 FORMAT(1X,2I4,2F6.1,I4,F6.1,F5.1,2F6.1,I4,F8.4,F6.1,F8.4)  
 30 CONTINUE

```

RETURN
END
C
SUBROUTINE COMP1
C
C Subroutine COMP1 calculates a modified Hegyi competition index
C on all live trees in a stand. Competitors are found by sampling
C neighbors based on their size and distance away by essentially
C taking a point sample at each subject tree with a BAF-10 prism.
C
IMPLICIT INTEGER*4 (I-N)
INTEGER*4 IX,ID,PLOT,TAG
INTEGER*4 ODEAD,OALIVE
INTEGER*4 K3,K6,K9
REAL HD3,HD6,HD9
PARAMETER (MROWS = 20,MTREES = MROWS*MROWS)
PARAMETER (ODEAD = 1,OALIVE = 2)
PARAMETER (PLOTX = 2.75)
DIMENSION JDIS(9),MID(MTREES),IDIS(4),DIST(9)
COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
3 LVIGR(MTREES),LTHIN,KTHIN
COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
DATA JDIS/1,9,8,7,6,5,4,3,2/
C
C Initialize
C
DO 5 I = 1,MTREES
5 CIP(I) = 0.0
IDIS(1) = 1
C
C Find internal trees
C
DMAX = 0.0
DO 10 I = 1,N
10 DMAX = AMAX1(DMAX,D(I))
D2 = PLOTX*DMAX
DISMAX = D2-PX/2.0
DISMAY = D2-PY/2.0
DMX = PLOTX-DISMAX
DMY = PLOTY-DISMAY
DO 20 I = 1,N
MID(I) = 2
20 IF (X(I).GT.DISMAX.AND.X(I).LT.DMX.AND.
1 Y(I).GT.DISMAY.AND.Y(I).LT.DMY) MID(I) = 1
C
C Calculate competition index
C
DO 130 I = 1,N-1
IF (LMORT(I).NE.OALIVE) GO TO 130
DO 120 J = I+1,N
IF (LMORT(J).NE.OALIVE) GO TO 120
INTIOR = MID(I) + MID(J)
XDIST = X(J)-X(I)
YDIST = Y(J)-Y(I)
DIST(1) = SQRT(XDIST*XDIST + YDIST*YDIST)
IF (INTIOR.LT.3) GO TO 100
IF (XDIST.LT.0.0) GO TO 30
DIST(5) = SQRT(((XDIST-PLOTX)**2) + (YDIST*YDIST))
IDIS(2) = 5

```

```

GO TO 40
30  DIST(6) = SQRT(((XDIST + PLOTX)**2) + (YDIST*YDIST))
   IDIS(2) = 6
40  IF (YDIST.GE.0.0) GO TO 50
   DIST(3) = SQRT((XDIST*XDIST) + ((YDIST + PLOTY)**2))
   IDIS(3) = 3
   ICODE = IDIS(2) + IDIS(3) - 7
   GO TO (60,70,100,100,100,80,90),ICODE
50  DIST(8) = SQRT((XDIST*XDIST) + ((YDIST - PLOTY)**2))
   IDIS(3) = 8
   ICODE = IDIS(2) + IDIS(3) - 7
   GO TO (60,70,100,100,100,80,90),ICODE
60  DIST(2) = SQRT(((XDIST - PLOTX)**2) + ((YDIST + PLOTY)**2))
   IDIS(4) = 2
   GO TO 100
70  DIST(4) = SQRT(((XDIST + PLOTX)**2) + ((YDIST + PLOTY)**2))
   IDIS(4) = 4
   GO TO 100
80  DIST(7) = SQRT(((XDIST - PLOTX)**2) + ((YDIST - PLOTY)**2))
   IDIS(4) = 7
   GO TO 100
90  DIST(9) = SQRT(((XDIST + PLOTX)**2) + ((YDIST - PLOTY)**2))
   IDIS(4) = 9
100  RJI = D(J)/D(I)
   RIJ = 1.0/RJI
   DO 110 L=1,4
   LC = IDIS(L)
   LCC = JDIS(LC)
   IF (DIST(LC).LT.D(J)*PLOTX) CIP(I) = CIP(I) + RJI/DIST(LC)
   IF (DIST(LC).LT.D(I)*PLOTX) CIP(J) = CIP(J) + RIJ/DIST(LC)
   IF (INTIOR.LE.3) GO TO 120
110  CONTINUE
120  CONTINUE
130  CONTINUE
   RETURN
   END

```

```

C
SUBROUTINE GROW2
C

```

```

C Subroutine GROW2 does the annual growth of the individual trees
C in the stand. This routine does NOT take into consideration the
C amount of hardwood competition in terms of percent of total basal
C area - it assumes a 4.8% basal area.
C

```

```

C
IMPLICIT INTEGER*4 (I-N)
INTEGER*4 IX,ID,PLOT,TAG
INTEGER*4 ODEAD,OALIVE
INTEGER*4 K3,K6,K9
INTEGER*4 KT,KT1,K1
REAL HD3,HD6,HD9,TRV1,TRV2,TRVD
PARAMETER (MROWS=20,MTREES=MROWS*MROWS)
PARAMETER (ODEAD=1,OALIVE=2)
COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
3 LVIGR(MTREES),LTHIN,KTHIN
COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C

```

```

C
C Initialize tree counter
C

```

```

NTSIM = 0

```

```

C
C Compute potential height increment for all trees and begin
C individual tree growth
C
C Equation (Site Index) developed by Michael Smith
C from Amateis and Burkhart (1985)
C Fit without TRV's
C
C   POTH = ALOG(SITE)*((1.0/K)/(1.0/25))**(-0.03754117)*
C   1 EXP(-2.61807656*(1.0/K-1.0/25))
C
C Equation (Site Index) developed by Michael Smith
C from Amateis and Burkhart (1985)
C   K = AGE2 (prediction age), KT = ELAPSED TIME SINCE THINNING
C
C   KT = K-KTHIN
C   KT1 = 25-KTHIN
C   TRV1 = RESBA**(-0.68975758*(-(KT1**2) + 10.77325694*KT1)/(25**2))
C   TRV2 = RESBA**(-0.68975758*(-(KT**2) + 10.77325694*KT)/(K**2))
C   IF (TRV2.GT.1.0) THEN
C     TRV2 = 1.0
C   ELSE
C     TRV2 = TRV2
C   ENDIF
C   POTH = ALOG(SITE)*((1.0/K)/(1.0/25))**(-0.01600294)*
C   1 EXP(-3.02041884*(TRV2/K-TRV1/25))
C
C Equation (Site Index) from Amateis and Burkhart (unpublished)
C
C   POTH = ALOG(SITE)*((1.0/K)/(1.0/25.0))**(-0.02205)*
C   1 EXP(-2.83285*(1.0/K-1.0/25.0))
C   POTH = EXP(POTH)
C   PHIN = POTH-HD
C
C   DO 10 I=1,N
C     IF (LMORT(I).NE.OALIVE) GO TO 10
C     CR = CL(I)/H(I)
C
C Determine tree mortality
C
C New fits of PLIVE to each thinning regime with B4 set. Model
C selection is controlled by the plot number.
C
C   IF (PLOT.EQ.3) THEN
C     LTHIN = 2
C     PLIVE = 1.01194722*CR**0.02292409*EXP(-0.00097984*CIP(I)**
C     1 3.0)
C   ELSEIF (PLOT.EQ.2) THEN
C     LTHIN = 1
C     PLIVE = 1.00885339*CR**0.01394739*EXP(-0.00384231*CIP(I)**
C     1 3.5)
C   ELSE
C     PLIVE = 1.05756047*CR**0.06222136*EXP(-0.00631486*CIP(I)**
C     1 2.5)
C   ENDIF
C
C New fit of PLIVE to unthinned and thinned data. If thinned data
C is input (plots 2 or 3), LTHIN = 1 and the first model is used.
C
C   IF (PLOT.EQ.2.OR.PLOT.EQ.3) THEN
C     LTHIN = 1

```

```

C PLIVE = 1.01002349*CR**0.01862881*EXP(-0.00028157*CIP(I)**
C 1 5.75017158)
C ELSE
C PLIVE = 1.05439170*CR**0.05821756*EXP(-0.00627880*CIP(I)**
C 1 2.49931569)
C ENDIF
C
C New fit of PLIVE to unthinned and thinned data. If thinned data
C is input (plots 2 or 3), LTHIN = 1 and the first model is used.
C (THESE ARE THE NEW FITS INCORPORATING ALL OF THOSE DELETES)
C
C IF (PLOT.EQ.2.OR.PLOT.EQ.3) THEN
C LTHIN = 1
C PLIVE = 1.01288380*CR**0.02176457*EXP(-0.00096182*CIP(I)**
C 1 4.76316297)
C ELSE
C PLIVE = 1.05759255*CR**0.06234989*EXP(-0.00616873*CIP(I)**
C 1 2.51834543)
C ENDIF
C Originally fit PLIVE model from PTAEDA2.
C
C PLIVE = 1.02797295*CR**0.03789773*EXP(-0.00230209*CIP(I)**
C 1 2.65206263)
C IF (U(IX).GE.PLIVE) THEN
C NLIVE = NLIVE-1
C LMORT(I) = ODEAD
C KMORT(I) = K
C GO TO 10
C ENDIF
C
C Compute Height and Diameter increment on all trees
C
C R = STNORM(IX)
C
C New fit of HIN model to REM3 data:
C
C HRED = -0.54747576 + 2.42006199*CR**0.20717950*EXP(-0.17120539*
C 1 CIP(I)-0.38119877*CR)
C
C Originally fit HIN model from PTAEDA2:
C
C HRED = 0.26324665 + 2.11118696*CR**0.56188187*EXP(-0.26375086*
C 1 CIP(I)-1.03076126*CR)
C HIN = AMAX1(PHIN*HRED,0.0)
C
C HINMAX is not actually used
C HINMAX = 0.72785206*PHIN + 0.88373520
C HIN = AMIN1(HIN,HINMAX)
C
C NTSIM = NTSIM + 1
C PDIN = 0.28658336*HIN + 0.2094718
C HIN = AMAX1((HIN + R*0.672290115),0.0)
C
C New fit of DIN model to REM3 data without a TRV:
C
C DRED = 0.81124405*CR**0.79631061*
C 1 EXP(-0.71247612*CIP(I))
C
C New fit of DIN model to REM3 data with the TRV:
C
C TRVD = RESBA**(8.86635219*(-(KT**2) + 5.02757251*KT)/(K**2))

```

```

IF (TRVD.GT.1.0) THEN
  TRVD = 1.0
ELSE
  TRVD = TRVD
ENDIF
DRED = 0.78933211*CR**0.77356231*
1 EXP(-0.70327755*CIP(I)*TRVD)
C
C Originally fit DIN model from PTAEDA2:
C
C   DRED = 0.72511188*CR**0.98014576*
C 1 EXP(-0.37397613*CIP(I))
  DIN = AMAX1((PDIN*DRED + R*0.089724578),0.0)
  D(I) = D(I) + DIN
  H(I) = H(I) + HIN
10 CONTINUE
  HD = POTH
C
C Determine crown length:
C Crown ratio equation by Dyer and Burkhart, 1986
C
  DO 20 I=1,N
    IF (LMORT(I).NE.OALIVE) GO TO 20
    CR = 1.0-EXP((-1.35243-37.02600/K)*D(I)/H(I))
C
C Crown ratio equation by Liu, et al., 1994
C Estimates are metric, thus D and H are converted within.
C
C   TRVC = RESBA**((0.13130*(-(KT**2) + 67.042*KT)/(K**2))
C   CR = 1.0-TRVC*EXP((-0.14780-4.7233/K)*(2.54*D(I))/(0.3048*H(I)))
  CL(I) = AMIN1(AMAX1(0.0,(H(I)*CR)),H(I))
20 CONTINUE
  WRITE(*,30) NTSIM
30 FORMAT(I33,' Trees Simulated')
  RETURN
  END
C
  SUBROUTINE TREES
C
C This subroutine TREES outputs a file containing the individual
C tree data in ASCII format.
C
  IMPLICIT INTEGER*4 (I-N)
  INTEGER*4 IX,I4,ID,PLOT,TAG
  INTEGER*4 ODEAD,OALIVE
  INTEGER*4 K3,K6,K9,V
  REAL HD3,HD6,HD9
  PARAMETER (MROWS=20,MTREES=MROWS*MROWS)
  PARAMETER (ODEAD=1,OALIVE=2)
  COMMON /BLOK1/ X(MTREES),Y(MTREES),D(MTREES),H(MTREES),CL(MTREES),
  1 CIP(MTREES),LMORT(MTREES),KMORT(MTREES),TAG(MTREES),ACRES,ID,
  2 PLOT,PERCNT,HD,IX,N,K,M,V,NROWS,RESBA,K3,K6,K9,HD3,HD6,HD9,
  3 LVIGR(MTREES),LTHIN,KTHIN
  COMMON /BLOK2/ SITE,PX,PY,PLOTX,PLOTY,VARX,VARY,NYEARS
C
  WRITE (*,10)
10 FORMAT(/)
C WRITE(1,20)
C 20 FORMAT(1X,'ID',2X,'PLOT',1X,'TAG',3X,'X',5X,'Y',4X,'DBH',2X,
C 1 'Height',3X,'CL',5X,'CIP',2X,'DETHVIGR',2X,'HD',4X,'V',1X,'AGE')
  DO 30 I=1,N

```



C  
C CONVERT FROM LMORT CODE TO VIGR CODE FOR USE WITH COOP DATASET.

C  
IF (TAG(I).EQ.0) RETURN  
IF (LMORT(I).EQ.OALIVE) THEN  
LVIGR(I) = 1  
ELSE  
LVIGR(I) = 2  
ENDIF  
WRITE (1,40) ID,PLOT,TAG(I),X(I),Y(I),D(I),H(I),SITE,CIP(I),  
1 KMORT(I),LVIGR(I),HD,V,K  
30 CONTINUE  
40 FORMAT(3I4,2F6.1,F6.2,F7.2,F7.1,F8.4,2I4,F6.1,2I4)  
RETURN  
END

C  
C SUBROUTINE CONT

C  
C This subroutine asks the user if he/she wishes to continue  
C running PTAEDA2. If not, the program is halted.

C  
IMPLICIT INTEGER\*4(I-N)  
CHARACTER\*1 ANS

C  
WRITE(\*,10)  
10 FORMAT(/,' Do you wish to continue? (YES or NO): '  
CALL GETYN(ANS)  
IF (ANS.EQ.'N') THEN  
WRITE(\*,20)  
20 FORMAT(25(/))  
STOP 'Program Terminated By User'  
ENDIF  
RETURN  
END

C  
C SUBROUTINE GETYN(ANS)

C  
C This routine reads in character input for YES/NO questions.

C  
CHARACTER\*1 ANS,TEMP

C  
C Read in string

C  
10 READ(\*,20) TEMP  
20 FORMAT(A1)

C  
C Convert to upper case

C  
LETTER = ICHAR(TEMP)  
IF ((LETTER.GE.97).AND.(LETTER.LE.122)) LETTER = LETTER-32  
TEMP = CHAR(LETTER)

C  
C Check to be sure value is "Y" or "N"

C  
IF ((TEMP.EQ.'Y').OR.(TEMP.EQ.'N')) GO TO 40  
WRITE(\*,30) CHAR(7)  
30 FORMAT(1X,A1,'Please enter YES or NO: '  
GO TO 10

C  
C End: wrap-up

```

C
40 ANS = TEMP
RETURN
END
C
FUNCTION GAMMA(XX)
C
C Based on program from Scientific Subroutine Package, IBM
C
IMPLICIT INTEGER*4 (I-N)
IF (XX.LE.10.0) GO TO 20
10 GX = 0.0
GO TO 100
20 X = XX
ERR = 1.0E-6
GX = 1.0
IF (X-2.0) 50,50,40
30 IF (X.LE.2.0) GO TO 90
40 X = X-1.0
GX = GX*X
GO TO 30
50 IF (X-1.0) 60,100,90
C
C See if X is near negative integer or zero
C
60 IF (X.GT.ERR) GO TO 80
Y = FLOAT(INT(X))-X
IF (ABS(Y).LE.ERR) GO TO 10
C
C X not near a negative integer or zero
C
70 IF (X.GT.1.0) GO TO 90
80 GX = GX/X
X = X + 1.0
GO TO 70
90 Y = X-1.0
GY = 1.0+Y*(-0.5771017+Y*(0.9858540+Y*(-0.8764218+Y*(0.8328212+
1 Y*(-0.5684729+Y*(0.2548205+Y*(-0.05149930))))))
GX = GX*GY
100 GAMMA = GX
RETURN
END
C
FUNCTION STNORM(IX)
C
C Generates a standard Normal random variate
C Based on the routine "GAUSS" in "Scientific Subroutine Package";
C IBM; 1968; page 77. The algorithm is based on "Numerical Methods
C for Scientists and Engineers"; R. W. Hamming; McGraw-Hill Pub.;
C 1962; pages 34 and 389.
C Assumes a standard deviation (S) of 1.0 and a mean (AM) of 0.0
C
IMPLICIT INTEGER*4 (I-N)
INTEGER*4 IX
PARAMETER (S = 1.0,AM = 0.0)
C
A = 0.0
DO 10 I = 1,12
10 A = A + U(IX)
STNORM = (A-6.0)*S + AM
RETURN

```

```

END
C
FUNCTION TRIANG(IX)
C
C This routine TRIANG returns a triangular distributed random
C number from -1.0 to 1.0.
C
INTEGER*4 IX
REAL X,Y
C
X = U(IX)
IF (X.GT.0.5) THEN
Y = 1.0-SQRT(0.5*(1.0-X))
ELSE
Y = SQRT(0.5*X)
ENDIF
C This converts the range from (0,1) to (-1,1)
TRIANG = (2.0*Y)-1.0
RETURN
END
C
FUNCTION U(ISEED)
C
C From: 'Simulation, Statistical Foundations and Methodology'
C 'Mathematics In Science and Engineering', Vol. 92
C by G. Arthur Mihram, 1972, Academic Press, NY.
C pages 44-57.
C
C M = 2**b where b = number of bits in integer
C A = M-3 (actually A = (M-((4*I)-1)), I = 1,2,...)
C C = (M/2)-1
C
INTEGER*4 ISEED
REAL*8 A,C,M
PARAMETER (M=2147483648.DO,A=2147483645.DO,C=1073741823.DO)
C
ISEED = IDINT(DMOD(((A*DBLE(ISEED))+C),M))
U = DBLE(ISEED)/M
RETURN
END

```

## VITA

Michael C. Smith was born July 3, 1955 in Prince George's County, Maryland. Raised in Minnesota, he served six years in the U.S. Navy after high school. After his discharge, he worked as an electronics technician until 1988, when he enrolled in the forestry program at the University of Minnesota. He graduated with a Bachelor of Science degree in Forestry in 1992, and accepted a graduate assistantship in forest biometrics at the College of Forestry and Wildlife Resources at Virginia Polytechnic Institute and State University.

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Michael C. Smith