An Improved Method for Estimating the Moderator Temperature Coefficient of Pressurized Water Reactors Using Multivariate Autoregression

by

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(ABSTRACT)

An improved method for evaluating the moderator temperature coefficient (MTC) of reactivity in pressurized water reactors based on a newly established technique is described. Previous work has correlated stochastic fluctuations in the reactor core-exit temperature with similar fluctuations in the in-core neutron flux. The frequency response function between these two stochastic signals has been shown to be proportional to the moderator temperature coefficient. Though this method has been shown to agree well with reactor core design and measured values of the MTC, the method requires over three hours of data processed via the fast Fourier transform, and has certain limitations which suggest that the data analysis be done another way. In the present work, the multivariate autoregressive analysis method is utilized to process under ten minutes of the plant data used previously. Nearly identical results were obtained using only 5 % of the data used in the prior analysis.

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INTRODUCTION

1.1 Prelude

Safety has been and will continue to be of primary concern in the operation of nuclear reactors. United States utilities using nuclear energy must adhere to the many safety regulations set by the Nuclear Regulatory Commission (NRC) to maintain their operating licenses. Among these regulations is the requirement for the periodic evaluation of reactor safety parameters. If the reactor is found to be operating outside the technical specifications set for the plant, the reactor must be shut down, costing the utility nearly one million dollars for each day of reactor down time. In the interest of safe economical production of electrical power, new methods are being developed to evaluate reactor safety parameters.

This research is an attempt to improve a newly established method for evaluating one reactor safety parameter, the moderator temperature coefficient of reactivity (MTC) in pressurized water reactors (PWR). Several previous papers [1,2,3,4] have described a new method for determining the MTC for pressurized water reactors by correlating stochastic fluctuations in reactor core-exit coolant temperature with similar fluctuations in the in-core neutron flux. These earlier results were obtained from over three hours of plant data analyzed in the frequency domain via the fast Fourier transform (FFT). Though results with this method have been shown to be better or at least as reliable as current MTC measurement techniques [1], improvements were sought in order to reduce the amount of data required. Instead of using the frequency domain analysis of the FFT, the autoregressive (AR) time series method was selected to model the data as a time series.

The AR approach to time series analysis has been established as a viable method for

data analysis when small data sets are available. Since noise is included in the AR model of the time series, smaller data sets can give good results. Another advantage of the time series approach to signal processing is high spectral resolution. Since some problems have occurred over the three hour data collection span, it was proposed [1] that similar results could be obtained using much less data and an autoregressive approach.

This thesis describes the AR analysis of computer generated and actual measured nuclear power plant data. The theoretical basis for the AR approach to time series analysis, and the formulation of the harmonic mean algorithm used in this work will be presented. Results through use of an optimized AR algorithm for this application will then be used to display the final results for ten minutes of power plant data taken at several times during the fuel cycle of one reactor.

1.2 Moderator Temperature Coefficient in PWR's

The moderator temperature coefficient in pressurized water reactors , denoted α_c, is defined as the change in reactivity ρ per degree change in reactor coolant temperature T_c [5]:

$$\alpha_c = \frac{d\rho}{dT_c} = MTC.$$

To understand how the MTC works in a PWR, the fission reaction sequence must be explained. For a U^{235} atom, the fissionable fuel atom in a PWR fuel pin, to undergo fission, it must be bombarded with a free neutron with a certain energy level. The fission process produces two or more fission fragments, energy, and several free neutrons. The neutrons produced by the reaction are traveling too fast thus having too high an energy level for fission and must be thermalized (slowed) to energy levels suitable for nuclear fission if a reactor is to have a sustainable reaction process. The free neutrons collide with other atoms in the fuel pin, slowing very little until, they pass through the fuel pin cladding and collide with the moderator. The moderator in a PWR is light water or ordinary H_2O. In PWR's

this water serves a dual purpose as neutron moderator and coolant. The hydrogen atoms in the moderator are particularly effective in slowing fast neutrons because the masses of the two particles are similar [5]. These collisions cause the fast neutrons to lose energy to levels suitable for fission where they may be utilized for more fission reactions.

The moderator temperature plays an important role in how the MTC regulates reactor power level. As the moderator absorbs heat from the fission reaction, its density decreases. The water molecules become less closely packed and provide less efficient fast neutron moderation, and thus fewer thermal neutrons are available to continue the fission reaction. With fewer fission reactions occuring, the coolant temperature will decrease, and the moderator once again becomes a more efficient neutron moderator and the stable fission reaction can continue [6].

The MTC describes the relation between neutron population (reactivity) and coolant temperature in the process described above. Reactivity is defined as the fractional change in neutron population per neutron generation. For a stable reactor, the MTC must act as a negative feedback element in the reactor control sequence. An increase in coolant temperature must cause a decrease in reactivity. Typically the MTC is a small negative number at the beginning of the reactor fuel cycle and increases in magnitude as the core ages as fuel burnup is compensated for by decreasing the concentration of the soluble neutron absorber. This increase in magnitude is of concern because of a hypothetical accident scenario. In the event of a main steam line rupture, the reactor core would be flooded with cold emergency cooling water. With a large negative MTC, an enormous increase in power would follow, possibly beyond the capability of the neutron absorbing control rods to control. To address this possible accident scenario, The Nuclear Regulatory Commission has set limits on the MTC magnitude [1].

Twice each fuel cycle, the utility must perform a measurement of the MTC to ensure that the reactor is operating within the technical specifications set for that plant. If the MTC is found to be out of specification, the utility must perform MTC evaluations every two weeks until the MTC is within allowable specification, or else reduce power [1].

1.3 MTC Measurement: Current Method

Two utilities cooperating throughout this research effort, Duke Power and Virginia Power, currently use the boron-dilution method for MTC determination. This process involves opening the steam throttle to extract more heat from the coolant and thus lowering the coolant temperature. To offset the resulting increase in reactivity, soluble boron, a neutron absorber, is added to the coolant. Over several hours, the core temperature is lowered several degrees and the amount of boron added to the coolant is monitored. The amount of negative reactivity added by the boron to maintain criticality and the core average temperature drop over the time span quantify the MTC [1] . This method has several inherent disadvantages [1,7]:

1. Lowering the average temperature of the core changes the axial power distribution in the core, leading to fission product poison transients which must be monitored for about 24 hours.

2. The longer the test takes to execute, the larger the variance of the results due to induced transients.

3. The boron concentration is small near the end of the fuel cycle, and is difficult to accurately measure. Also, processing the large volume of high purity water needed for the test is costly.

4. Varying the reactor power level disrupts normal reactor plant operations and is costly in terms of reduced generator output.

1.4 MTC Measurement: Stochastic Signal Analysis

The boron-dilution method uses macroscopic changes in reactor coolant temperature and reactivity over several hours. The analysis of stochastic fluctuations relies on microscopic

rather than macroscopic changes in nuclear reactor power signals. Though the coolant temperature and neutron flux signals are macroscopically constant when the reactor power level is constant, microscopic changes occur about the mean signal level. Random fluctuations in coolant flow rate and coolant inlet temperature can initiate similar fluctuations in the neutron flux level and the core-exit temperature. These small fluctuations contain information about the system response of the power plant, and the proper analysis of these signals can reveal this useful information. The frequency response function for the reactor has been shown to change with core-burnup and has also been shown to be proportional to the MTC [1]. This relationship will again be shown to exist using the AR approach to signal analysis in a later chapter.

1.5 Project Background

In May, 1988, work began on a joint effort between Virginia Polytechnic Institute and State University (VPI) and Virginia Power to develop a technique for determining the moderator temperature coefficient of reactivity in PWR's. Another nuclear utility, Duke Power, became involved later by supplying VPI with data taken at their McGuire Nuclear Station. The Electric Power Research Institute (EPRI) has also funded this project in an effort to make United States electric utilities aware of the benefits of reactor noise analysis.

Virginia Power's North Anna Power Station was having some trouble at that time meeting the NRC's specifications for the MTC. Since the MTC measurement method then used involved a reactor transient, was costly to perform and was of questionable accuracy, a passive technique was sought. This method was to be based on the analysis of random fluctuations in the core-exit thermocouple signals and the in-core neutron flux detectors. By correlating similar fluctuations, an indication of the MTC could be obtained [4].

To perform the analysis, data from the same fuel assembly is taken for the core-exit thermocouple and the in-core neutron flux detectors. The thermocouple is located about twelve inches above the fuel assembly, and the neutron detector can be positioned axially

within the fuel assembly. The raw signals are first stripped of the DC portion of the signal by using a DC-offset amplifier. The resulting signals are then low-pass filtered at 15 Hz and gained before either being recorded via FM tape recorder or digitized at 45 samples per second via a VPI-developed data acquisition system.

The VPI data acquisition system samples the data and simultaneously performs a fast Fourier transform of the signals. Over three hours (500 23-second data blocks) of data are taken and the averaged FFT results are saved for analysis. The averaged power spectral densities (PSD) and cross-power spectral densities (CPSD) are used to compute a frequency response function (FRF) for the reactor system. This FRF is then averaged over the frequency band of highest signal coherence (0.1 to 0.5 Hz). The average value of the magnitude of the FRF is then used to predict the MTC by normalizing the value to a known MTC value [1]. The results from this analysis trend well with the design prediction for the MTC and the measured MTC values.

One problem associated with this method is the long data acquisition time. On several occasions thermocouple signal drift has occurred and the DC-offset amplifier has had to be adjusted by a technician, rendering the system dependent on supervision. A method requiring less data acquisition time would increase the liklihood of a drift-free signal and possibly the elimination of the need for supervision of the signal level altogether.

Another stage of the research described above involved the development of a FORTRAN code that simulates the transient temperature profile and neutron flux of a PWR fuel cell [8]. This model uses finite-differenced heat transfer equations and numerical solution of the neutron kinetics equations to simulate the time record of a PWR fuel cell. Input parameters to the model include core-inlet temperature perturbations, coolant velocity perturbations, and MTC. Results from this model were used to study the dominant noise sources in PWR's and were compared with actual plant data.

For this reasearch, results from this transient temperature PWR model was used to apply the AR method of time series analysis to the resulting perfectly correlated data. This study allowed the AR method to be refined for use on the actual plant data. The selection

of AR model order and data record length is system and application dependent, and had to be studied.

Chapter 2

LITERATURE REVIEW

2.1 MTC Estimates Using Noise Analysis

Thie [9] was the first to suggest the use of coolant temperature and neutron flux perturbations to estimate the MTC. In his model, the RMS values of the core-exit coolant temperature and ex-core neutron flux were measured in the low-frequency band of 0.1 to 0.5 Hz. The ratio of the RMS flux value and the RMS temperature perturbation was expected to be proportional to α_c, the MTC,

$$\frac{RMS[\delta\phi/\phi_o]}{RMS[\delta T_c]} = G_o(f)\alpha_c,$$

where G_o is the local transfer function. Türkcan [10] reported success in using this technique for the Borselle reactor, and further research was done by Pór [11] who reported that the method is biased.

Shieh et. al [13] described a computer model of the loss-of- fluid-test (LOFT) reactor to study the effect of changing the sign of the moderator temperature coefficient on the low-frequency phase behavior between neutron flux and core-exit temperature. Results from this study showed that as frequency approaches 0 Hz, the phase angle extrapolates to -180 degrees for negative MTC and 0 degrees for positive MTC when the noise perturbations result from coolant flow fluctuations, core-inlet temperature perturbations, or random heat transfer [13]. These analytical results were compared with experimental data from the LOFT reactor and the dominant noise source was found to be coolant velocity fluctuations.

Aguilar and Pór [12] applied a new formula for the estimation of the MTC based upon earlier work by Thie [9] and Pór [11]. After noting the stationarity of the temperature PSD

with time and the increasing nature of the neutron flux PSD, a correlation relating MTC to boron concentration was reported.

The method on which the present research effort is based was developed by Herr and Thomas [14]. After recognizing that the temperature PSD was stable with time and the neutron flux PSD increased with time, they proposed to estimate the MTC by utilizing the frequency response function between the temperature and neutron flux. The FRF is averaged over the frequency band of 0.1 to 0.5 Hz, and this average value is then normalized to a known MTC value to estimate the MTC over the remainder of the fuel cycle. Their method was verified with data from Virginia Power's North Anna Power Station and Duke Power's McGuire Nuclear Station. Results were found to agree quite well with core design predictions and measured MTC values.

2.2 AR Analysis in Nuclear Reactors

One of the first to report the successful application of the multivariate autoregressive (MAR) technique in the nuclear field was Fukunishi (1977) [15]. In his study of a 460-MW boiling water reactor (BWR), the neutron flux PSD was examined for noise contributions from up to six sources. The MAR method was shown to appropriately model the complex dynamics associated with the multiple-input, single-output system. The result of the analysis showed that the fluctuation in the neutron flux of the BWR was principally due to the inherent noise of the neutron flux and was not strongly correlated with other reactor process signals such as core flow rate and reactor vessel pressure [15].

Many researchers have chosen the MAR method for time series analysis of nuclear reactor processes since 1977 with many different purposes. Upadhyaya et. al [16] made the MAR method available to those not yet familiar with the process by publishing an algorithm for MAR noise analysis. Noteworthy are the efforts of Oguma and Türkcan [17] for their introduction of the signal transmission path (STP) analysis method for the identification of noise sources and the detection of feedback in multivariate systems. Oguma has also

contributed to the field through his presentation of the extended partial coherence analysis (EPCH) method that allows the determination of multivariate signal commonality whether or not feedback exists [18].

All the applications of the AR method for the detection of nuclear reactor dynamics seen in the literature have determined the AR parameters from the Yule-Walker equations for multivariable systems. This method utilizes estimates of the autocorrelation and cross-correlation functions obtained from twelve to thirty minutes of data. In the univariate case, Marple [19] has shown this technique to be biased. The Burg algorithm is said to avoid this bias by working entirely within the data set [20]. Better spectral resolution is also a characteristic of the Burg algorithm [19].

Though the Burg algorithm does not directly generalize to the multivarite case [20], multivariate versions of Burg's algorithm have been developed independently by Nuttall [22] and Strand [21], and also by Morf et. al [23]. Both methods have been seen to give similar results in this research effort and similar results have also been shown by Marple as well [19].

Though it is not yet clear whether these multivariate versions of Burg's algorithm produce results superior to those of the Yule-Walker equations [20], the trend seems to be towards these methods and away from the Yule-Walker equations. For the analysis involved with this research, a FORTRAN program given by Marple [19] was used to determine the AR parameters and the AR spectral properties. The program allows either the selection of the Nuttall-Strand [19] method or the method of Morf et al [19].

Chapter 3

TIMES SERIES ANALYSIS

3.1 Formulation of the Yule-Walker Equations

The autoregressive (AR) approach to time series analysis is based on the premise that a member of a set of equally spaced observations in time can be modelled as a function of previous observations. That is, given a stationary time series with zero mean, a value of that series at some time t, X_t, can be represented by a parametric expression involving X_{t-1}, X_{t-2}, etc [20]:

$$X_t = \sum_{i=1}^{p} A_i X_{t-i} + e_t, \tag{3.1}$$

where $A_1, A_2, \ldots, A_p$ are the parameters sought and e_t is an error term assumed to represent white noise. The number of parameters or AR coefficients p is the order of the autoregression.

Fitting Eq. 3.1 to a time series of N observations under the constraint of minimizing the square of the error terms results in $N-p$ equations that can be solved for the p autoregressive coefficients. An $AR(p)$ system of equations in matrix form is represented as

$$\begin{bmatrix} X_t \\ X_{t+1} \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} X_{t-1} & X_{t-2} & \ldots & X_{t-p} \\ X_t & X_{t-1} & \ldots & X_{t-p+1} \\ \vdots & \vdots & \ddots & \vdots \\ X_{N-1} & X_{N-2} & \ldots & X_{N-p} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \end{bmatrix} \tag{3.2}$$

This system is in the form $\{B\} = [X]\{A\}$ and can be represented by an equivalent $p \times p$ system by performing the following matrix operation:

$$X^T B = [X^T X]\{A\}. \tag{3.3}$$

The resulting system is then

$$\begin{bmatrix} \sum_{t=p+1}^{N} X_t X_{t-1} \\ \sum_{t=p+1}^{N} X_t X_{t-2} \\ \vdots \\ \sum_{t=p+1}^{N} X_t X_{t-p} \end{bmatrix} = \begin{bmatrix} \sum_{t=p+1}^{N} X_{t-1} X_{t-1} & \cdots & \sum_{t=p+1}^{N} X_t X_{t-p} \\ \sum_{t=p+1}^{N} X_t X_{t-1} & \cdots & \sum_{t=p+1}^{N} X_t X_{t-p+1} \\ \vdots & \ddots & \vdots \\ \sum_{t=p+1}^{N} X_t X_{t-p} & \cdots & \sum_{t=p+1}^{N} X_{t-p} X_{t-p} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \end{bmatrix} \tag{3.4}$$

Another equation can be added to this system by multiplying Eq. 3.1 by X_t and taking the expectation value over the data set [19]:

$$E[X_t X_t] = E[X_t \sum_{i=1}^{p} A_i X_{t-i}] + E[e_t(\sum_{i=1}^{p} A_i X_{t-i} + e_t)] \tag{3.5}$$

Simplifying Eq. 3.5 yields

$$X_t X_t = \sum_{i=1}^{p} A_i X_t X_{t-i} + \rho, \tag{3.6}$$

where ρ is the variance associated with the white noise of the process.

These $p + 1$ equations are usually written in terms of the autocorrelation function for lags 0 to p [20],

$$R(p) = \sum_{t=1}^{N-p} X_t X_{t+p}. \tag{3.7}$$

The system then becomes:

$$\begin{bmatrix} R(0) & R(1) & \cdots & R(p) \\ R(1) & R(0) & \cdots & R(p-1) \\ \vdots & \vdots & \ddots & \vdots \\ R(p) & R(p-1) & \cdots & R(0) \end{bmatrix} \begin{bmatrix} 1 \\ A_1 \\ \vdots \\ A_p \end{bmatrix} = \begin{bmatrix} \rho \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.8}$$

This system forms the Yule-Walker equations for an autoregression of order p, and can be solved for the AR coefficients and the noise variance ρ [19].

The Yule-Walker equations can be solved using Guassian elimination, but this would require a number of operations proportional to p^3 and storage requirements proportional to p^2. [19] Another algorithm originally written by Levinson [1947] for the linear prediction of

a discrete-time process, and later proposed by Durbin [1960] for the fitting of an autoregressive model [19] is much more efficient. The Levinson-Durbin algorithm requires operations proportional to p^2 and memory storage of $2p$. [19]. A further advantage of this solution method is that it is recursive in nature; that is, in solving for the $AR(p)$ coefficients, the coefficients for the $AR(1), AR(2), \ldots AR(p-1)$ are also obtained. Having the coefficients available for p autoregressions allow a comparison of the various parameters for the evaluation of the optimal model order. Using a non-recursive solution method would make this evaluation much more costly in terms of computational time.

3.2 Recursive Univariate Autoregression

The process for fitting a univariate autoregression utilizing the Levinson-Durbin recursion begins by setting the noise variance ρ equal to the autocorrelation of lag zero [20],

$$\rho(0) = R(0). \tag{3.9}$$

The first AR coefficient is then obtained as the ratio of the autocorrelation for lags one and zero,

$$A_1(1) = \frac{R(1)}{R(0)}. \tag{3.10}$$

The estimate for the noise variance is then updated for the second AR coefficient estimation:

$$\rho(1) = [1 - (A_1(1))^2]\rho(0). \tag{3.11}$$

The step for the estimation of the p^{th} AR coefficient is then

$$A_p(p) = \frac{[R(p) - \sum_{i=1}^{p-1} A_i(p-1)R(p-i)]}{\rho(p-1)}. \tag{3.12}$$

This is called the partial autocorrelation coefficient [20] or reflection coefficient of order p because it represents the normalized correlation between X_t and X_{t-k} with correlation of $X_{t-1}, \ldots, X_{t-k+1}$ removed. [19] The general step for the p^{th} noise variance is then [20],

$$\rho(p) = [1 - (A_p(p))^2]\rho(p-1). \tag{3.13}$$

With the p^{th} AR coefficients now available, the coefficients of order $1,\ldots,(p-1)$ can be updated according to

$$A_i(p) = A_i(p-1) - A_p(p)A_{p-i}(p-1), \qquad i = 1,\ldots,p-1. \tag{3.14}$$

This process was further specialized by Burg [1967] to give better results. Burg sought to minimize the error terms or residuals of both forward prediction autoregression and backward autoregression [20]. The method utilizes the residuals based on a forward autoregression of order p, $e_t^f(p)$, which can be represented as

$$e_t^f(p) = X_t - \sum_{i=1}^{p} A_i X_{t-i}, \qquad t = p+1,\ldots,N. \tag{3.15}$$

If the coefficients from the $(p-1)$ autoregression are available from a previous recursion, substitution from Eq. 3.14 into Eq. 3.15 gives

$$e_t^f(p) = X_t - \sum_{i=1}^{p-1}[A_i(p-1) - A_p(p)A_{p-i}(p-1)]X_{t-i} - A_p(p)X_{t-p}. \tag{3.16}$$

This equation contains only one unknown $A_p(p)$, and can be simplified by recognizing the residual of the backward autoregression of order $(p-1)$ contained in the equation. Defining this backward residual as,

$$e_t^b(p-1) = X_t - \sum_{i=1}^{p-1} A_i X_{t+i}, \qquad t = 1,\ldots,N-p-1, \tag{3.17}$$

and substituting this into Eq. 3.16 then gives,

$$e_t^f(p) = e_t^f(p-1) - A_p(p)e_{t-p}^b(p-1), \qquad t = p+1,\ldots,N. \tag{3.18}$$

A similar process for the backward residual of order p yields,

$$e_t^b(p) = e_t^b(p-1) - A_p e_{t+p}^f(p-1), \qquad t = 1,\ldots,N-p. \tag{3.19}$$

Minimizing the squared residuals for forward and backward autoregressions, $A_p(p)$ can be estimated: [20]

$$A_p(p) = \frac{2\sum_{t=p+1}^{N} e_t^f(p-1)e_t^b(p-1)}{\{\sum_{t=1}^{N-p}[e_t^b(p-1)]^2 + \sum_{t=p+1}^{N}[e_t^f(p-1)]^2\}}. \tag{3.20}$$

Having obtained the coefficient $A_p(p)$ and revised the lower order coefficients, the recursion can be continued with the next AR order until the autoregression of the desired order is complete.

3.3 Multivariate Autoregression

The multivariate autoregression is simply a matrix application of the univariate autoregression. Let X represent a time series of $m \times N$ observations [19],

$$X = \{X_1, X_2, \ldots X_t, \ldots X_N\}, \tag{3.21}$$

where each X_t is a $m \times 1$ column vector. Writing out the autoregression for one of the $m \times 1$ vectors gives,

$$X_t = \sum_{i=1}^{p} A_i X_{t-i} + e_t^f, \tag{3.22}$$

resulting in $m \times m$ autoregressive equations. Fitting Eq. 3.22 to the entire data set results in $m \times m \times (N - p)$ equations. Equation 3.22 can be rewritten as a block vector inner product,

$$\mathbf{e}_t^f = \underline{\mathbf{a}}\mathbf{X}_t, \tag{3.23}$$

where $\underline{\mathbf{a}}$ is the block row vector of AR coeficients,

$$\underline{\mathbf{a}} = (\mathbf{I} \quad \mathbf{A_1} \quad \ldots \quad \mathbf{A_p}). \tag{3.24}$$

I is an $m \times m$ identity matrix and each A_i is also an $m \times m$ matrix of AR coefficients. The block column vector of data values is,

$$\underline{\mathbf{X_t}} = \begin{pmatrix} \mathbf{X_t} \\ \vdots \\ \mathbf{X_{N-p}} \end{pmatrix}. \tag{3.25}$$

Each X_t is an $m \times 1$ data column vector.

Equation 3.23 can be written in terms of the matrix autocorrelation by multiplying by $\underline{\mathbf{X_t}}^T$. Defining $\underline{\mathbf{R}}(p)$ as the block matrix of $(p+1) \times (p+1)$ autocorrelation values, $R_{xx}(k)$ for lag k as [19],

$$\underline{\mathbf{R}}(p) = \begin{pmatrix} R_{xx}(0) & R_{xx}(1) & \dots & R_{xx}(p) \\ R_{xx}(-1) & R_{xx}(0) & \dots & R_{xx}(p-1) \\ \vdots & \vdots & \ddots & \vdots \\ R_{xx}(-p) & R_{xx}(-p+1) & \dots & R_{xx}(0) \end{pmatrix}, \tag{3.26}$$

and simplifying the result gives the Yule-Walker equations for a multivariable system [19],

$$\underline{\mathbf{aR}}(p) = \begin{pmatrix} \mathbf{P^f} & \mathbf{0} & \dots & \mathbf{0} \end{pmatrix}, \tag{3.27}$$

where $\mathbf{P}^f$ is the forward prediction matrix of $m \times m$ noise variances. This system of block matrix equations can be solved recursively for the AR coefficients using the multivariable version of the Levinson-Durbin algorithm as originally proposed by Whittle [24] and later by Robinson [25].

3.4 Autoregressive Spectral Estimation

Once the time series is modelled by the autoregressive parameters, the power spectral density function can be obtained. Taking the Z-transform of Eq. 3.1 gives

$$X(z)Z^0 = \sum_{k=1}^{p} A_k X(z) Z^{-k} + e(z). \tag{3.28}$$

where $Z = exp(2\pi fT)$ and T is the sampling interval. The above equation can be rewritten as:

$$X(z) = \frac{e(z)}{1 - \sum_{k=1}^{p} A_k Z^{-k}} \tag{3.29}$$

Multiplying Eq. 3.29 by the complex conjugate yields

$$X(z)X(z)^* = \frac{e(z)e(z)^*}{\mid A(z) \mid^2}, \tag{3.30}$$

where $A(z) = 1 - \sum_{k=1}^{p} A_k e^{-2\pi f k T}$ and T is the sampling interval. Equation 3.30 can now be rewritten by substituting for the noise variance ρ and the PSD of the time series, $P_{xx}(f)$, and by scaling by the sampling interval T, which gives [19]

$$P_{xx}(f) = \frac{T\rho}{\mid A(f) \mid^2}. \tag{3.31}$$

Equation 3.31 is the formula for the univariate AR power spectral density function and can be used to express the time series in the frequency domain.

The multivariate AR power spectral density function is a matrix of $m \times m$ values representing the spectral properties for a given frequency [19],

$$\mathbf{P_{ar}} = T[\mathbf{A}(f)]^{-1}\mathbf{P}^f[\mathbf{A}(f)]^{-H}, \tag{3.32}$$

where $\mathbf{A}(f) = \mathbf{I} + \sum_{k=1}^{p} \mathbf{A}_k e^{-2\pi f k T}$ and the superscript $-H$ denotes the hermitian transpose operation. From Eq. 3.32, the auto-spectra and the cross-spectrum for the time series can be obtained to express the time series in the frequency domain and to establish a causal relationship between the time series.

3.5 Comparison of AR and FFT Spectral Estimation

If similar results are expected from two methods of data analysis, one would expect some similarity among those two methods. Though the autoregressive method of time series analysis may appear to be quite different from the Fourier transform technique, similarities exist. In a Fourier transform of N data points, the frequency content of the data is modelled by N/2 sinusoids. These sinusoids have frequencies that are harmonic of one sinusoid, so the frequency content of the data is displayed by the N/2 evenly spaced spectral lines. In the AR process, the number of sinusoids to model the data is chosen by choosing the model order. An $AR(2)$ gives one sinusoid, and an $AR(4)$ gives two sinusoids. If the AR model of N data points is expanded to a model order of N, the result would be a model utilzing N/2 sinusoids to model the data. In the statistical sense and in information content, the $AR(N)$ and the FFT converge to the same result.

Chapter 4

AR ANALYSIS APPLIED TO MODEL-GENERATED DATA

4.1 Model Order Selection

The AR method was first applied to data generated by the computer model of a PWR [8]. Since little was known about applying the AR method to nuclear reactor data, this introductory approach to the signal processing was used to develop some knowledge about the proper model order and the data record length needed for accurate AR results. The inherent advantage to using the computer-generated data lies in the fact that the reactor core-exit temperature and the neutron flux data are perfectly correlated, having a signal coherence of unity. This allows the use of very little data, which is a further advantage since the generation of twenty-five seconds of data requires approximately 20 minutes of processor time on the IBM 3090 mainframe computer.

To investigate the effects of different model orders on the power spectral densities, the results from several AR model orders were compared to the results obtained from a Fourier transform of the same data. For the Fourier transform results, four averages of 5-second data records (1024 pts) were used. To reduce spectral leakage, a Hanning window was applied to the time series. The 'optimal' model order was chosen based on the results of Akaike's Information Criterion (AIC) [16] for the selection of proper AR order. This criterion considers the matrix of noise variances, the number of points in the model, and the AR order in the estimation of the proper model order.

Figure 4.1 and Fig. 4.2 show the results of AR analysis using model orders 40, 45, and 50 with a Hanning window applied. From these figures it is evident that the AR models

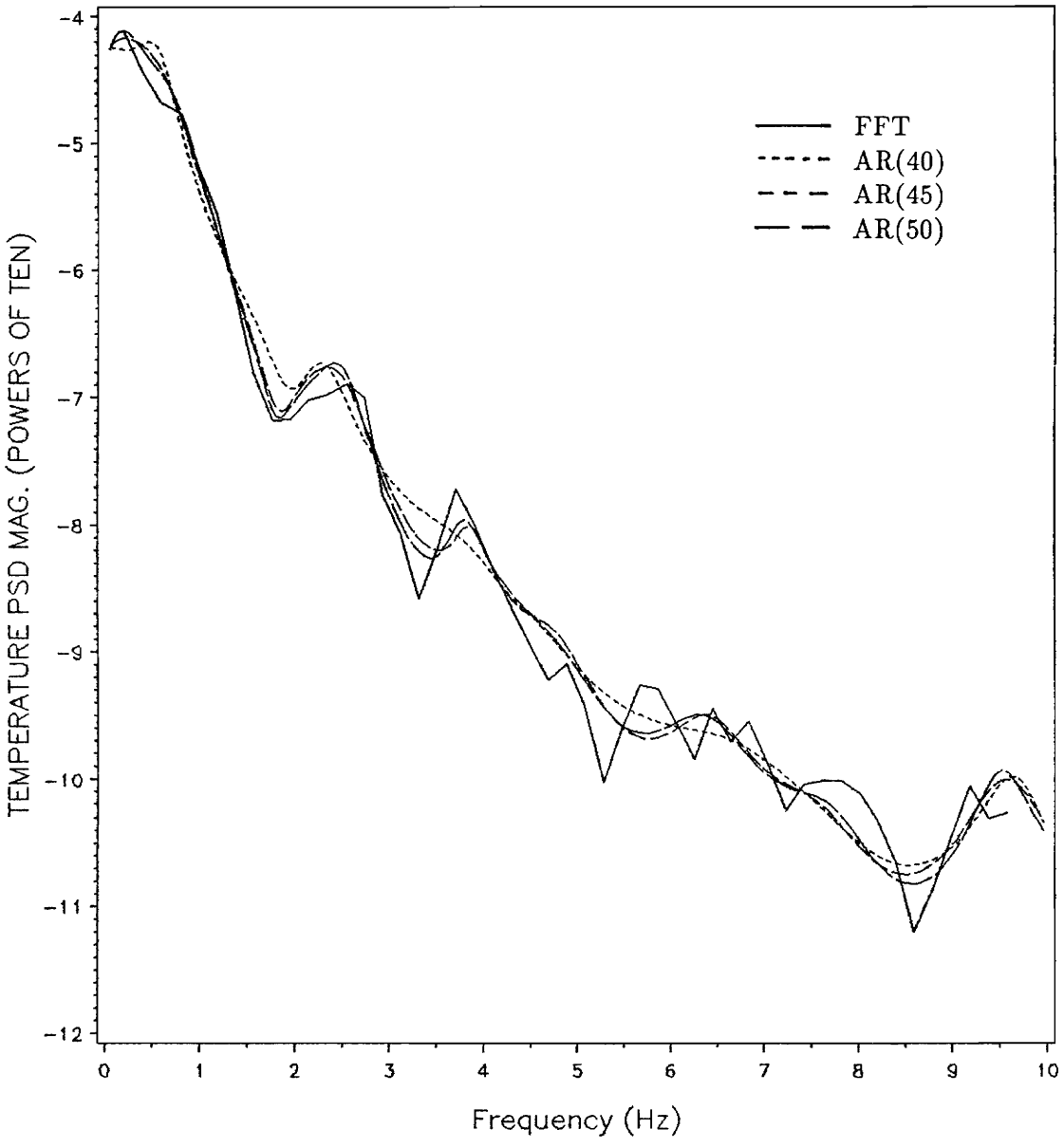

Figure 4.1: Temperature Fluctuation PSD for AR Model Orders 40,45, and 50 Compared with FFT with a Hanning Window Applied

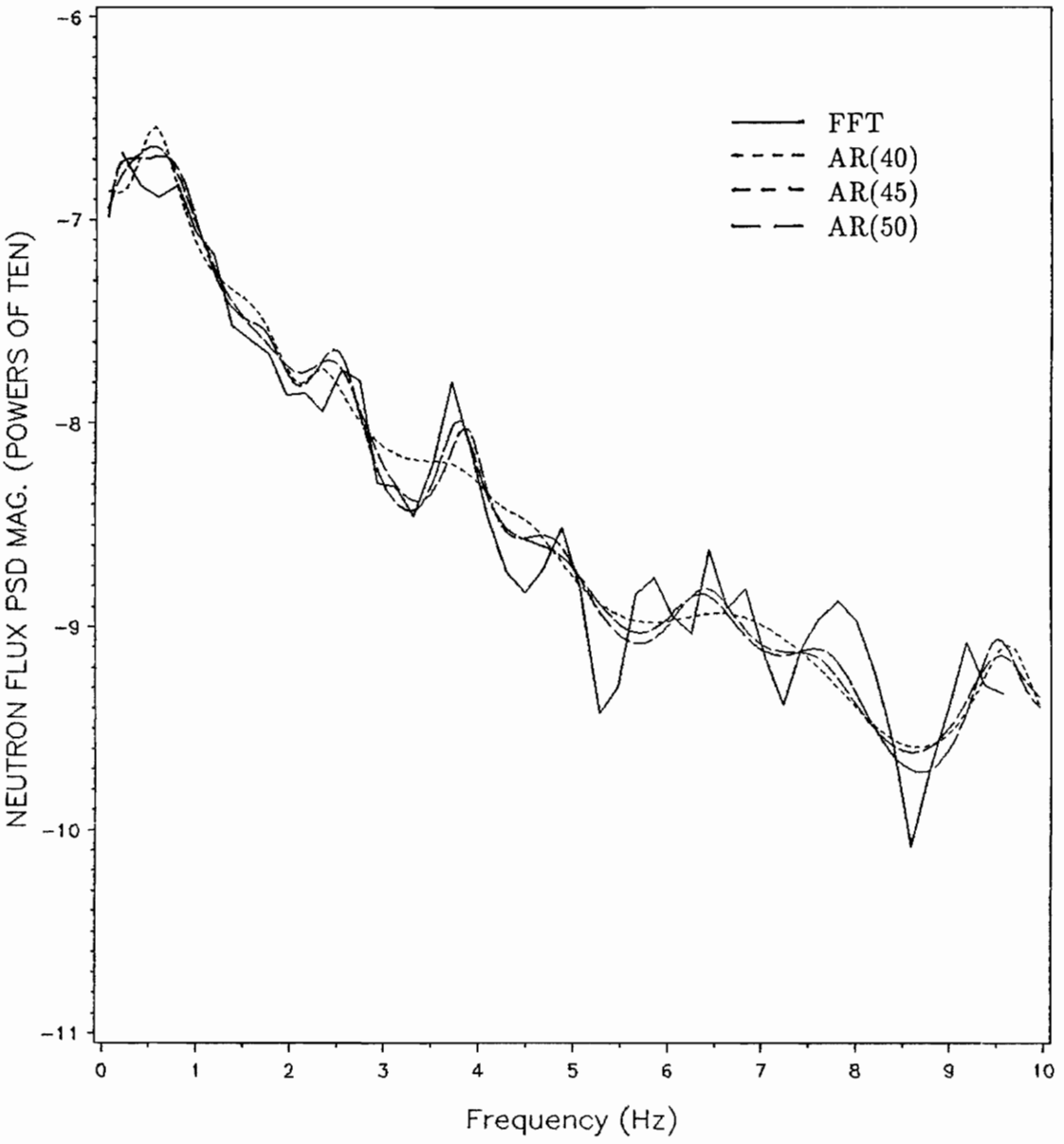

Figure 4.2: Neutron Flux Perturbation PSD for AR Model Orders 40,45 and 50 Compared with FFT with a Hanning Window Applied

estimate both the PSD magnitude and spectral peaks comparable to the FFT. These curves also show that the model order chosen to be the best, order 50, is indeed the best of those shown, and the PSD's from the lower model orders are converging to those of the proper model order. These results are significant because the AIC is verified as a viable criterion for model order selection, and the AR results do not appear too sensitive to model order.

4.2 Data Record Length Selection

To determine how large a time record is needed to produce accurate AR results, the effect of varying data record length was also studied. All of the data (twenty seconds) was first used to establish a basis for comparison with shorter data lengths. The model order was fixed at 50 to eliminate the effect of varying model order in this portion of the analysis. Three different data record lengths were used for comparison with the 20 second standard (15, 10, & 5 seconds).

The effects of the different record lengths on the temperature fluctuation PSD and the neutron flux PSD are shown in Figs. 4.3 and 4.4. There seems to be some decline in accuracy in the PSD's when compared to the 20 second standard, though this is only apparent in the 5 second data length results. Though this analysis seems to be a bit inconclusive as to how long a data record is needed for accurate AR results, ten seconds of data appears to be a minimum. These results were used as a guide in preparing for the analysis of the actual power plant data.

4.3 Estimating the MTC

The final test for the multivariate AR analysis involves estimating the MTC from the analytical data. Since the MTC is a known input to the computer model, the accuracy of the predicted MTC using AR analysis can be verified.

The MTC is predicted by using the frequency response function (FRF) between the core-exit temperature and the neutron flux. Figure 4.5 shows three FRF's for a model

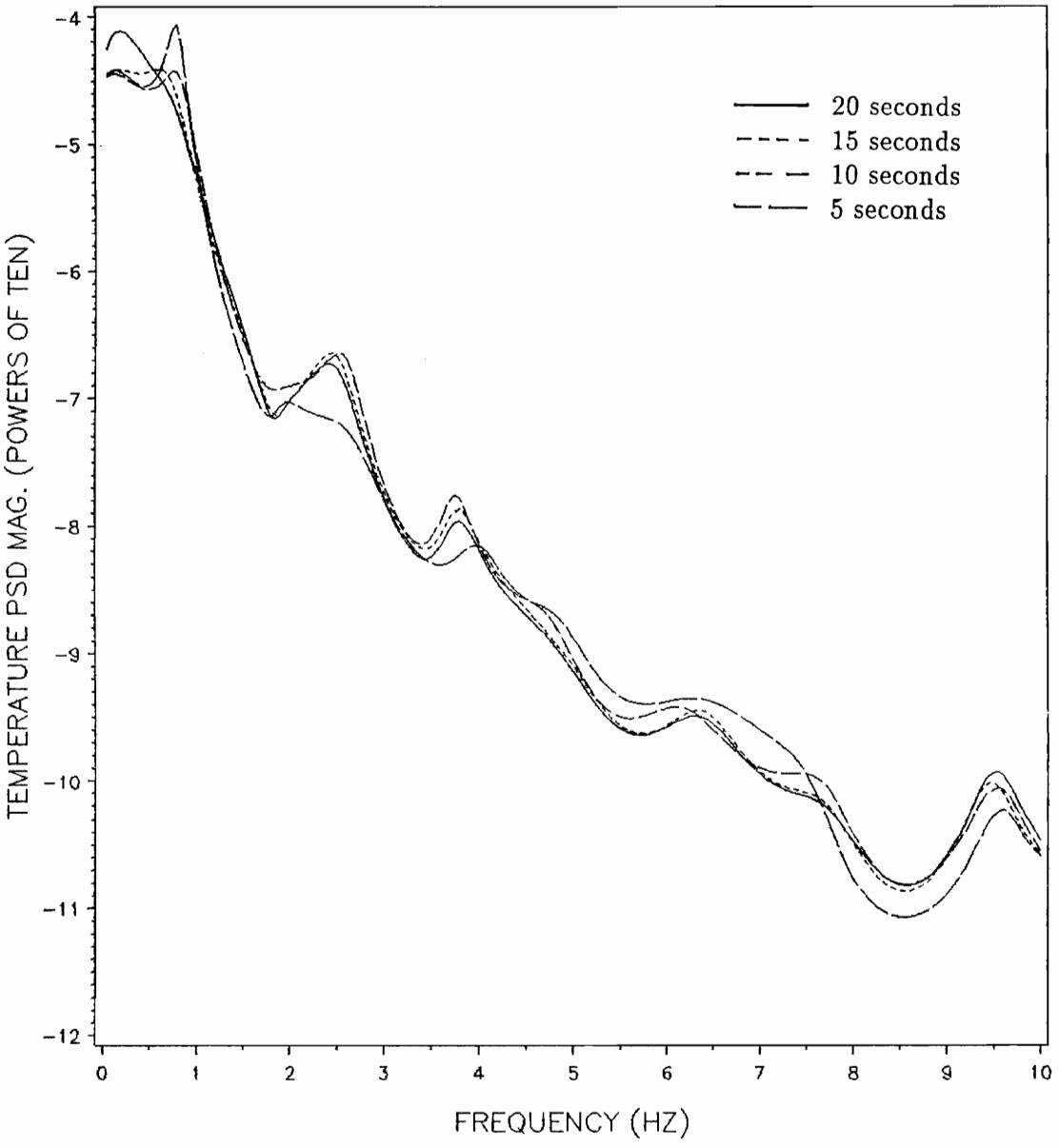

Figure 4.3: Temperature Fluctuation PSD for 5, 10, 15, and 20 Seconds of Data for AR Model Order 50

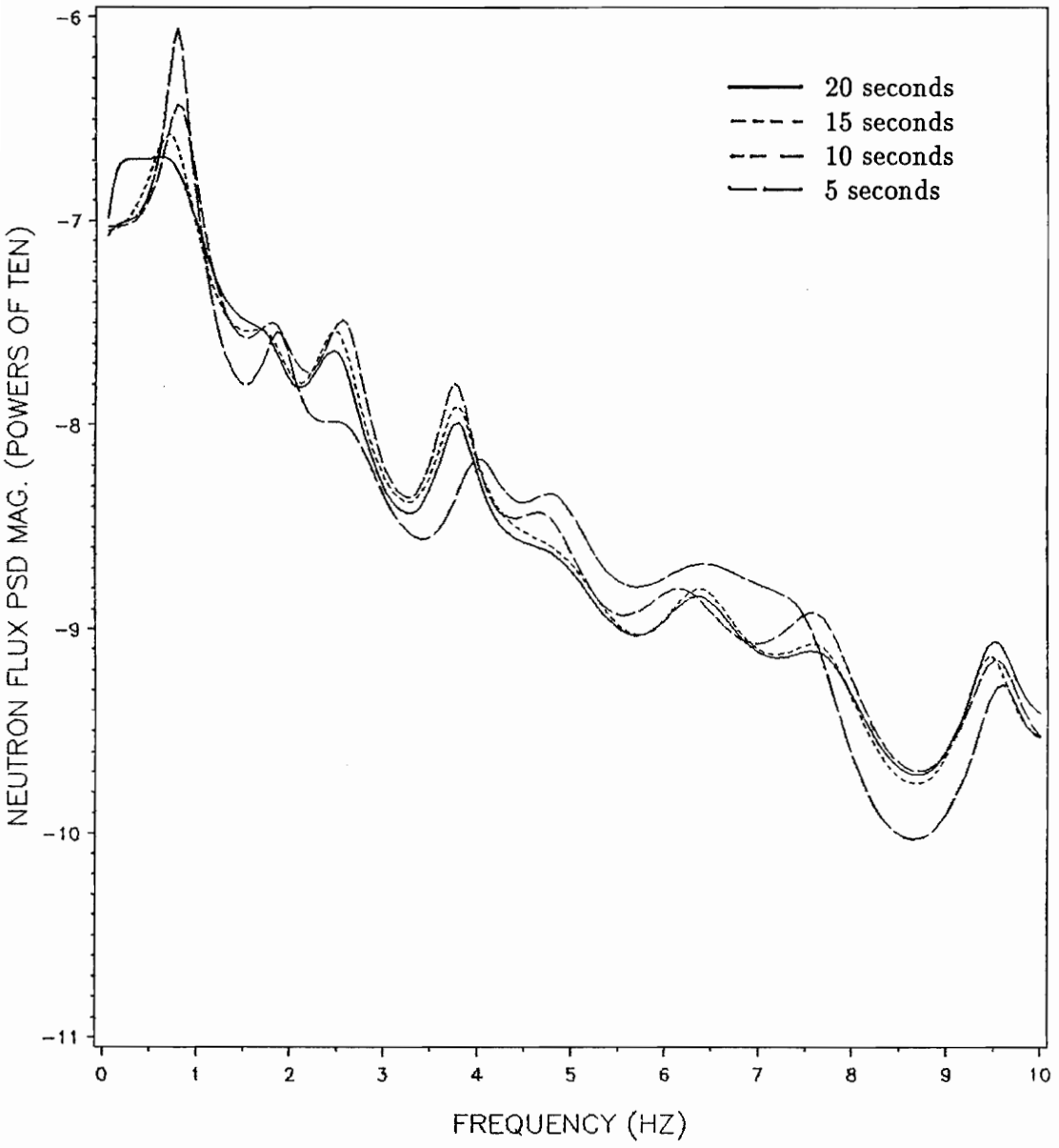

Figure 4.4: Neutron Flux Perturbation PSD for 5, 10, 15, and 20 Seconds of Data for AR Model Order 50

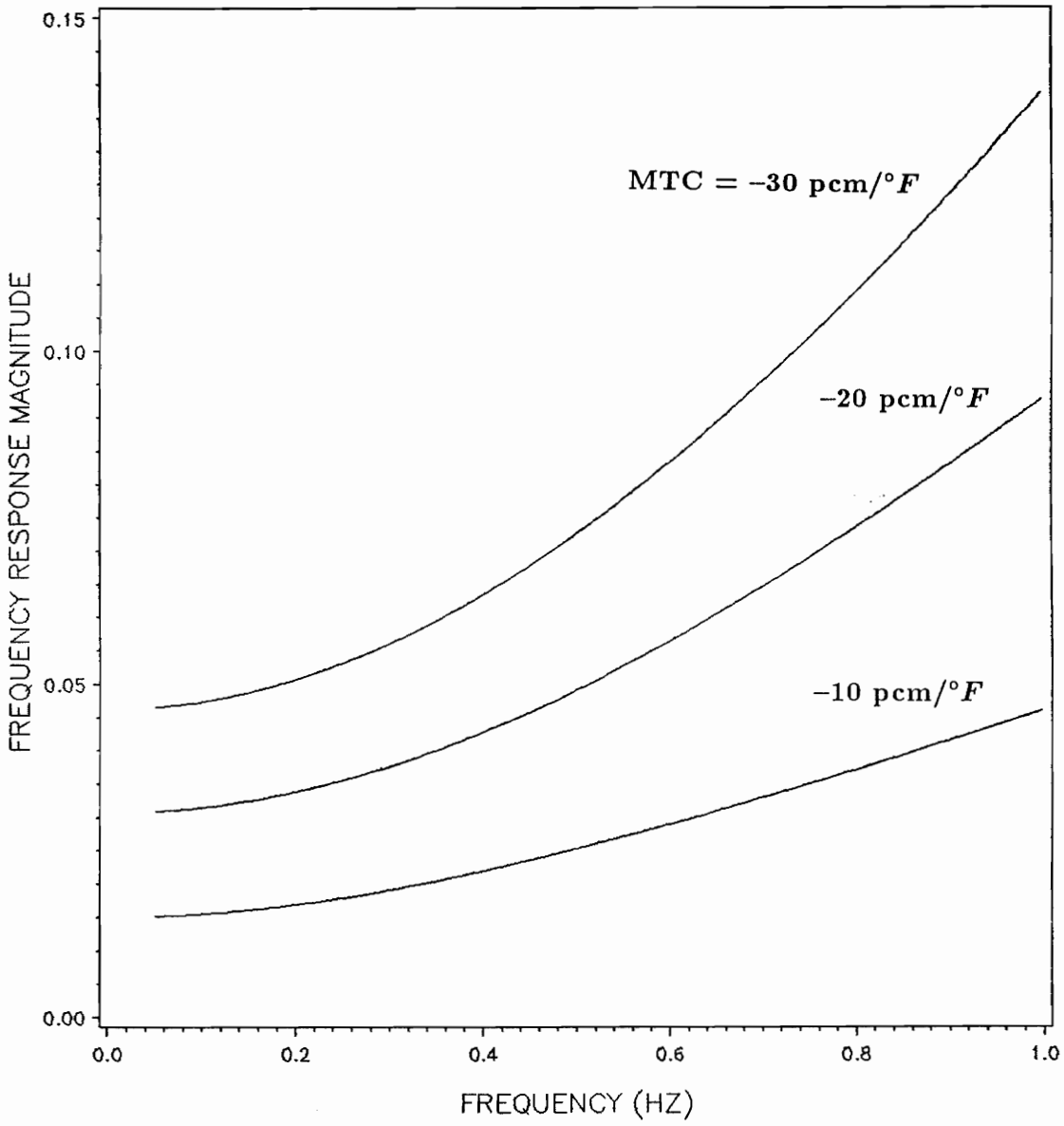

Figure 4.5: Frequency Response Function for MTC of -10, -20, and -30 pcm/°F for AR Model 50

MTC of -10, -20, and -30 pcm/°F. To establish a calibration factor relating the magnitude of the FRF to the MTC, an average value of the FRF between 0.1 to 0.5 Hz is calculated for the response curve for -10 pcm/°F. This calibration factor is then used to predict the MTC of the remaining two cases based on the average magnitude of the FRF for those cases. By using the known FRF magnitude as a calibration factor, the unknown MTC is estimated. This method estimates the MTC as -19.79 and -29.51 pcm/ °F for the -20 and -30 pcm/ °F curves respectively. A similar calculation by Wood [8] using the FFT resulted in nearly identical values of -19.76 and -29.53 pcm/°F for the model MTC. This result further establishes the AR method as a viable alternative to FFT spectral estimation.

Chapter 5

ANALYSIS OF MEASURED DATA

5.1 Data Acquisition

The data used in this research effort was obtained in cooperation with Duke Power. As a utility active in reactor noise analysis, Duke Power has an entire fuel cycle of data for its McGuire Nuclear Station available on analog tape. Approximately ten minutes of data were copied from the Duke tape at each of the eight data points during the fuel cycle. Data for an ex-core thermocouple and an in-core neutron detector in the same fuel assembly was then available for analysis.

The data is transferred from the dubbed analog tape to digital format using the data collection system developed by Herr [1] for an earlier stage of this research project. This system consists of an IBM PC equipped with an analog-to-digital converter with a triggering circuit for synchronous sampling of the available data channels. An anti-aliasing filter with a 15 Hz low-pass capability is used prior to digitization. For more specifics of the data aquisition system the reader is referred to Herr [1]. Consistent with the prior work of Herr, the data is sampled at 45 samples per second and then written to disk. Use of the direct memory access controller of the IBM PC allows the data to be simultaneously sampled and written to disk so that contiguous time series are available.

Before analyzing the thermocouple and neutron flux data, the digital data has to be converted from electrical units to more useful units. The fluctuating neutron flux signal, $\delta\phi$, is normalized to the steady-state DC flux voltage to make the resulting neutron signal representative of $\frac{\delta\phi}{\phi_0}$, and the thermocouple voltage is converted to degrees F. The data files are then ready for analysis.

5.2 Data Analysis

Typically, the multi-channel Yule-Walker equations have been used for the analysis of nuclear reactor dynamics, and much success has been reported in the literature. This research has not continued that tradition for several reasons. Most prevalent of those reasons is the lack of availability of an an algorithm to efficiently solve the multi-channel Yule-Walker equations. Without such an algorithm readily avaliable, one would have had to be written, which is a sizable task in itself. Other reasons for using another analysis method are the inherent bias of the method and the greater spectral resolution of other AR algorithms [19].

An algorithm generalizing Burg's method was used for the AR analysis. Subroutines for the AR analysis and the AR spectral properties were available in the text by Marple [19]. Having these subroutines available made the AR analysis relatively easy. A main FORTRAN program had to be written to utilize the subroutines.

This research differs further from the work of others by using averaged AR results. When the multi-channel Yule-Walker equations and other AR algorithms are solved, typically one estimate of the spectral properties is available. With the available algorithm, a 25,600 point model would have to be used. Instead, the the avaliable measured data is broken up into several smaller data sets for individual AR analysis, giving several estimates of the AR spectral properties. Averaging these results then give the estimate for the entire data set. For stationary data, the results are statistically the same.

5.3 Computer Program Description

The computer program written to do the AR analysis is set up to do the entire analysis for the ten minutes of plant data. Input to the program are the number of points per data set and the model order for the autoregression. The program reads in a data set and does the analysis for the AR parameters in a recursive fashion. If the model order is chosen to be p, the AR parameters for the orders 2 to p are available during the program run. Each

set of AR parameters is used in the evaluation of a model order selection criterion, and the best model order for that data set is saved for comparison with the best model orders of the remainder of the data sets. For each of the data sets, the AR power spectral densities and cross-power spectral densities are written to disk for later analysis. The average model order is also saved for comparison with the selected model order. If the selected model order is largely different from that given by the program, the program should be run again with a model order closer to that produced by the program. This program takes over 2 minutes of processor time on the IBM 3090 mainframe computer or about 1 hour on a 80286 IBM PC to produce results for a model order of 30. Another program is utilized to average the AR spectral properties. The computer program code listings are included in Appendix A for reference.

5.4 Data Record Length

Since long-term signal drift had been observed as a possible characteristic of the reactor data, questions concerning the proper stationarity of the data arose. Weakly stationary data is an assumption in both the FFT and in the AR methods of signal analysis, and non-stationary data could give biased or possibly incorrect results. Dividing the data into smaller sets could be an asset since with smaller data sets, the likelihood of data stationarity is increased.

How to subdivide the data for best results became the next concern. Longer data sets could possibly give better low-frequency models but would provide fewer sets to average. Shorter data sets could provide more data to average, but could compromise the accuracy of the AR model. The first and perhaps the most logical choice for data record length involves the lowest frequency considered important in the analysis. A set of data should contain the lowest frequency content of the signal considered important. The reciprocal of the frequency should give the data record length in seconds. For this project, the 0.1 Hz lower limit on frequency suggested a ten second data length, and 50 11-second data sets

(512 pts) were first used. While this produced acceptable results, it was observed that the low-frequency coherence, or measure of signal commonality, increased with longer data set length. To test for better results, 25 23-second (1024 pts) data sets were included in the next analysis with a small, but significant improvement in results. The similarity of the results between the two record length cases indicate that there is indeed a tradeoff for model accuracy and number of data sets for averaging.

5.5 Model Order Selection

The selection of the proper AR model order is also an important part of the analysis. As an aid in the selection of the proper model order the Akaike Information Criterion (AIC) [16] is used. This criterion uses the matrix of noise variances along with the number of points and the model order to assign a value for the AIC. For a multi-channel AR model the form of the AIC is [16]:

$$AIC_p = N log \mid P \mid +2m^2p, \tag{5.1}$$

where N is the number of points, P is the noise variance matrix, m is the number of data channels, and p is the order of the autoregression. The model order that gives the smallest value of the AIC is the best model order.

When the AIC is used to determine the 'optimal' model order for an autoregression, values relating the quality of the autoregression to particular models are obtained. Figure 5.1 shows this relationship for one data set for model orders 2 through 50. The criterion shows that there is a rather broad range of acceptable model orders for this data, since the AIC value changes little from 30 to 50. With such a broad range to consider, model orders from 30 to 50 were used in experimental analysis of the McGuire data. Though good results were also obtained with the higher model orders, model order 30 gave the best results. A further advantage of using model order 30 is a significant reduction in the computer run time for the autogregression.

Though the AIC does not clearly identify the best model order for an autoregressive

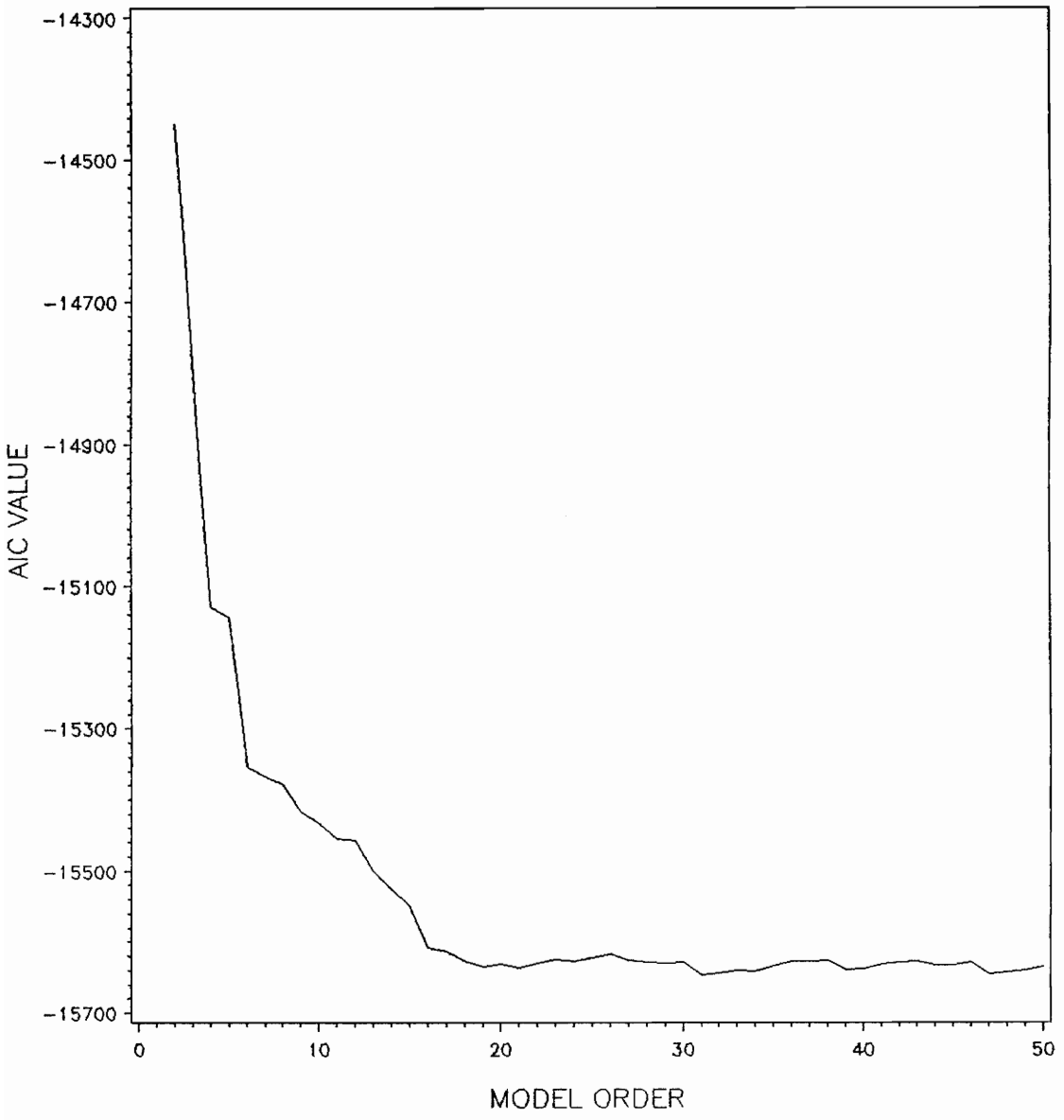

Figure 5.1: Akaike's Information Criterion for Optimal AR Order Selection

model, it does indicate a range of acceptable model orders. Very similar results can be expected from AR models whose AIC values are similar. In the above analysis, one would expect the results of an $AR(30)$ to be similar to the results of an $AR(45)$. Were another analysis to be done, an AIC value of -14500 would indicate a poor model, and an AIC value of -15600 would indicate an acceptable model.

5.6 Experimental Results

The power spectral densities and cross-power spectral densities for the 25 data sets were averaged in the frequency domain to produce average PSD's and CPSD's for the ten minutes of data. Figure 5.2 shows the average temperature perturbation PSD, and Fig. 5.3 shows the average neutron flux PSD. Figure 5.4 is the phase angle between the temperature and neutron flux obtained from the CPSD. Each of these figures give the results of nearly one year of plant data for the McGuire Unit 2 reactor overlayed to show trends with time.

The temperature fluctuations in the frequency range of interest (0.1 to 1.0 Hz) show little change with reactor core burn up. This indicates that the nature of temperature fluctuations essentially do not vary with time, and suggests that the dominant noise sources such as coolant flow velocity and core-inlet temperature driving core-exit temperature fluctuations are independent of the effects of fuel depletion.

The magnitude of the in-core neutron flux PSD shows a different trend, increasing with core burnup. This monotonic increase in neutron flux PSD magnitude is significant since it suggests an indication of changing reactor response to the driving noise sources.

The phase angle between neutron flux and core-exit temperature fluctuations also remains constant with fuel depletion. In the frequency band of highest coherence, a linear trend between phase and frequency can be seen, approaching -180 degrees at zero frequency. This trend indicates a negative MTC and further supports the idea that correlating neutron flux fluctuations with temperature fluctuations can provide an indication of the MTC.

Figure 5.4 shows the amount of coherence or signal commonality between the temper-

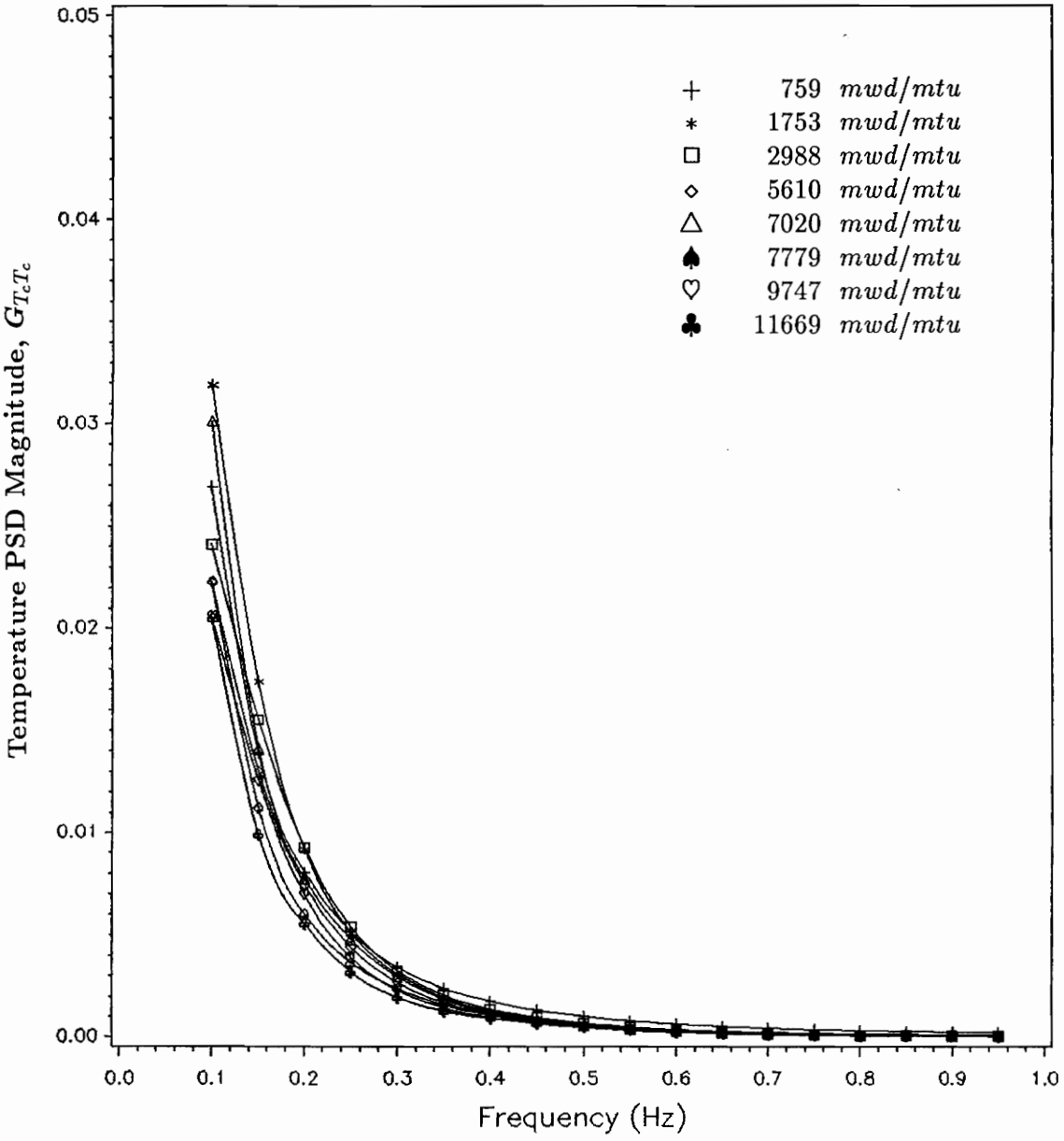

Figure 5.2: Core-exit Temperature Fluctuation PSD's for McGuire Unit 2, Cycle 5 for AR Model 30

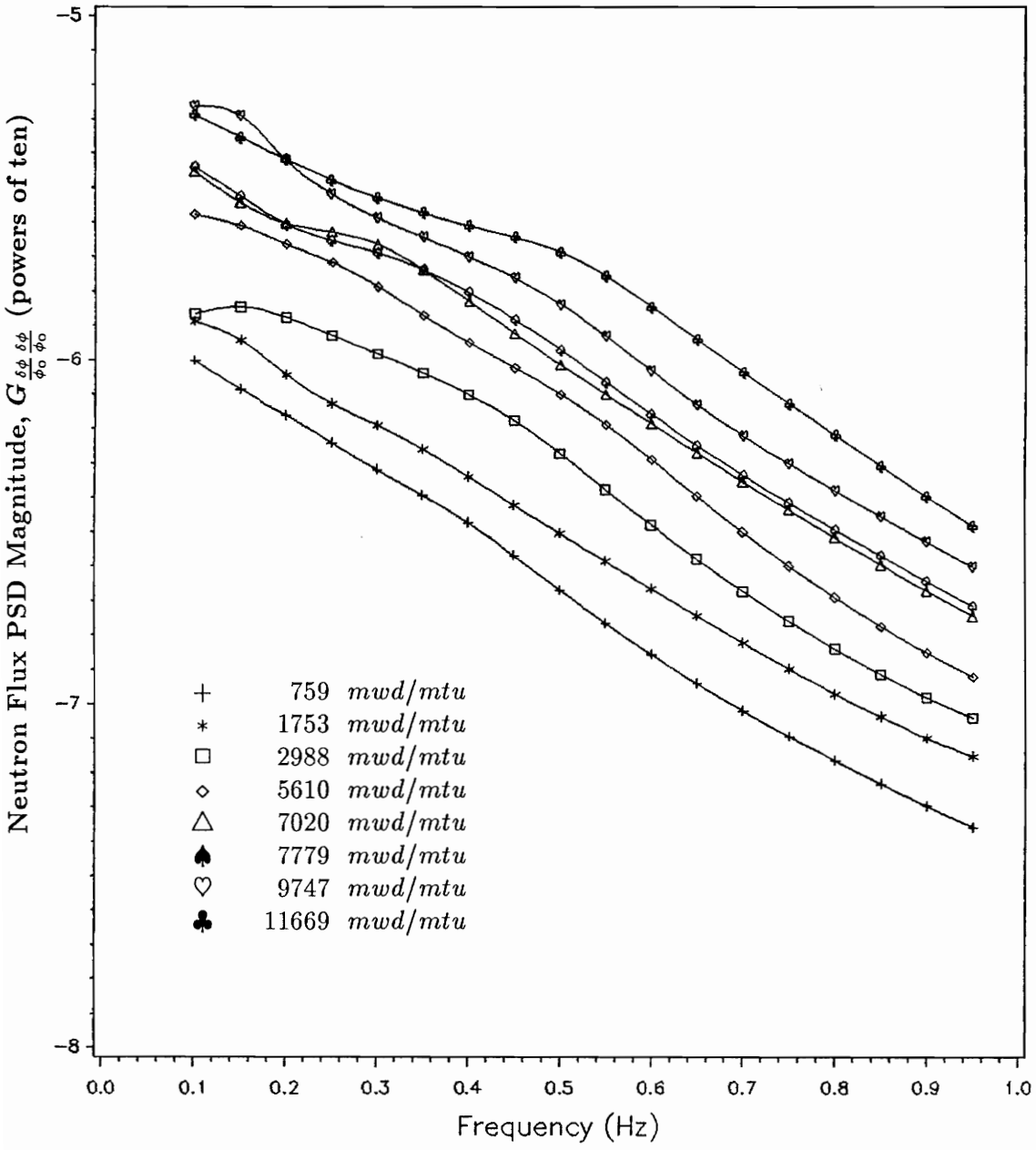

Figure 5.3: In-core Neutron Flux detector Perturbation PSD's for McGuire Unit 2, Cycle 5 for AR Model 30

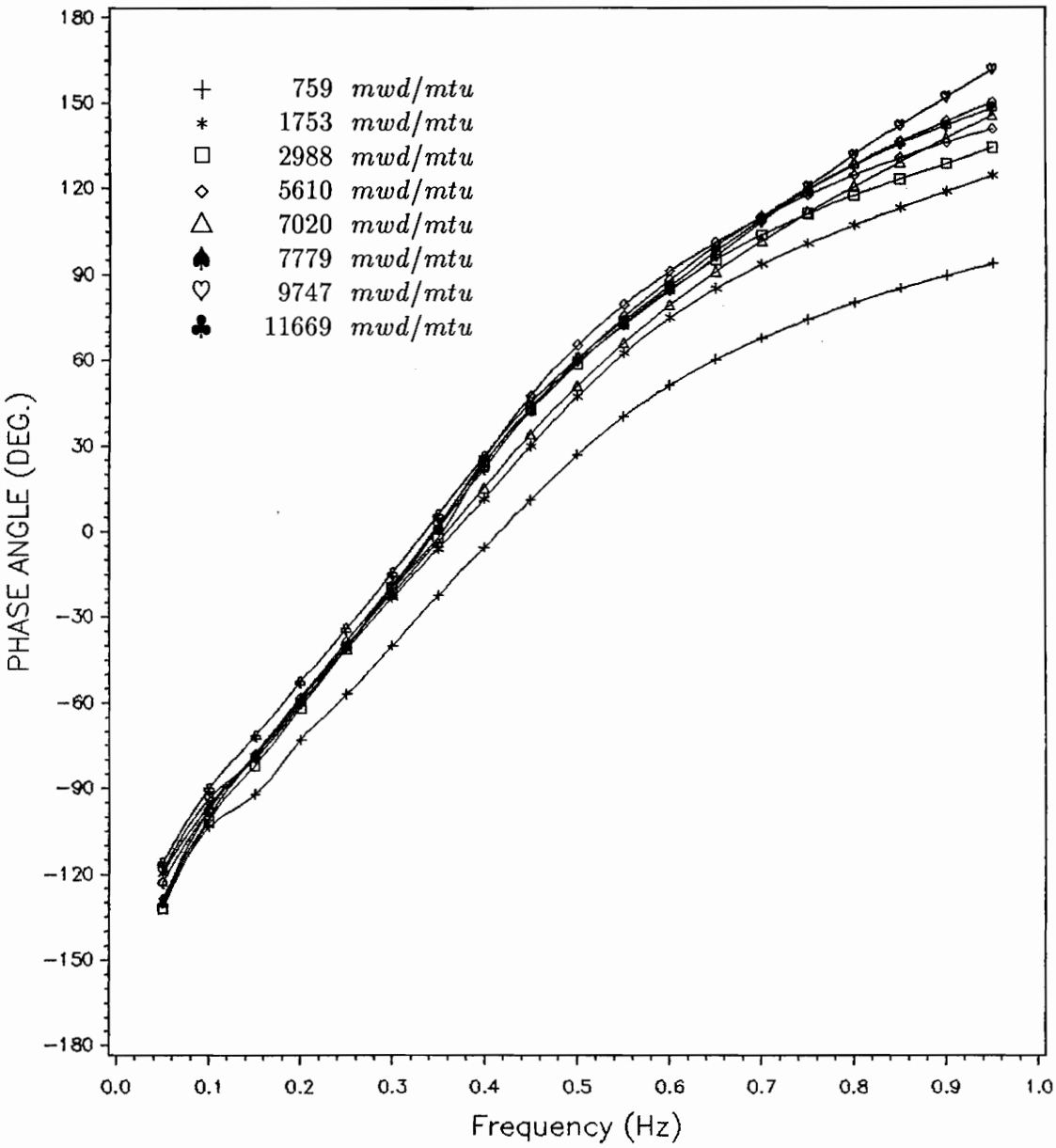

Figure 5.4: Phase Angle Between Core-exit Thermocouple and In-core Neutron Flux detector for McGuire Unit 2, Cycle 5 for AR Model 30

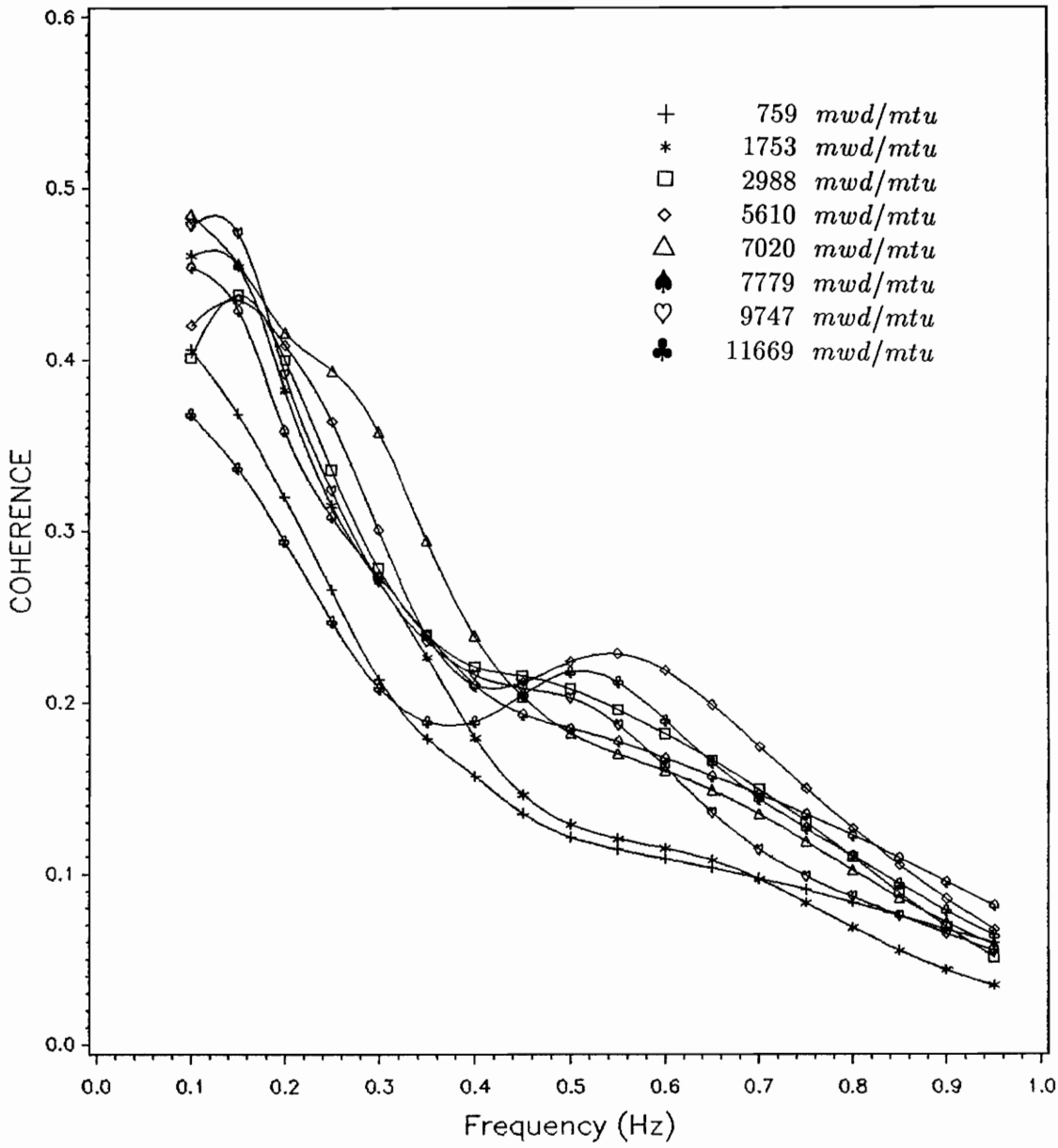

Figure 5.5: Coherence Between Core-exit Thermocuple and In-core Neutron Flux Detector

ature and neutron flux. The coherence is highest in the frequency band of 0.1 to 0.5 Hz. There is no apparent trend for coherence with fuel depletion.

5.7 Frequency Response Estimation

To establish a causal relationship between core-exit thermocouple and in-core neutron flux signals a system model must be assumed. The temperature perturbations, the input, must be related to the neutron flux, the output, by means of a transfer function in the frequency domain. Since the system input, the temperature fluctuations, remains fairly constant with fuel depletion, and the output, the neutron flux, increases with core burnup, one would expect the frequency response function to increase with burnup as well. Knowing that the MTC magnitude also increases with fuel depletion, the frequency response function should be able to indicate the MTC. This system response function is obtained using the auto spectra for the two signals and the cross spectrum between the two signals.

If the noise sources for the signals are not available for measurement, as was the case in this analysis, a frequency response estimator must be used to estimate the transfer function, the FRF. Bendat [26] proposed the use of the H_1 FRF estimator when the system noise is present only in the output. The H_1 FRF estimator is the ratio of the CPSD beween the two signals and the PSD of the input signal,

$$H_1 = \frac{G_{xy}}{G_{xx}}. \tag{5.2}$$

Mitchell [27] reported that this estimator will predict a response that is biased low from the true response because of the noise in the input signal. While this bias may be acceptable in cases where there is negligible noise contribution to the input signal, thermocouples are noisy transducers so a better FRF estimator should be used.

Another FRF estimator proposed by Bendat [26] is the H_2 estimator and is based on a system model that assumes noise in the input signal only. The H_2 FRF estimator is given

as the ratio of the ouput signal PSD and the cross power spectrum,

$$H_2 = \frac{G_{yy}}{G_{yx}}. \tag{5.3}$$

This method of response estimation is biased high [27] and when used with the H_1 estimator suggests the use of an average FRF estimator. The H_3 estimator as originally proposed by Mitchell [27] to better approximate the true frequency response function when noise is present in both the signals is simply the mean of the H_1 and the H_2 FRF estimators. To illustrate how the H_3 response may better approximate the true response, Fig. 5.5 shows the three estimators for data taken on October 8, 1988 at the McGuire nuclear facility. In the region of high signal coherence (0.1 to 0.5 Hz) the curves have similar values, but as the coherence decreases, the H_1 and H_2 curves diverge, with the H_3 curve representing the less biased frequency response.

The results from the analysis of the McGuire data show the desired trend of increasing FRF magnitude with reactor core burnup. Figure 5.6 shows the results using the H_1 FRF estimator. A trend of separation between the various curves exists though some curves do cross. The H_1 results proved to be the poorest estimation, as perhaps expected.

A clearer trend was obtained using the H_2 estimator as can be seen in Fig. 5.7. The expected increase in FRF with burnup is seen at the frequencies below 0.5 Hz. Since less than three weeks separate the data taken on January 18, 1989 and on February 6, 1989, the responses are similar and the curves cross at some frequencies.

The best results were obtained using the H_3 frequency response estimator, since problems with the H_1 and H_2 estimators are not seen with the H_3 method. Figure 5.8 shows the results of this analysis. Notice how the H_3 estimator shows the clearest trend of separation among curves and maintains the increase in FRF below 0.5 Hz. Having established the desired trend of increase in FRF magnitude with fuel depletion, it is now desired to correlate this increase with the increase in MTC magnitude.

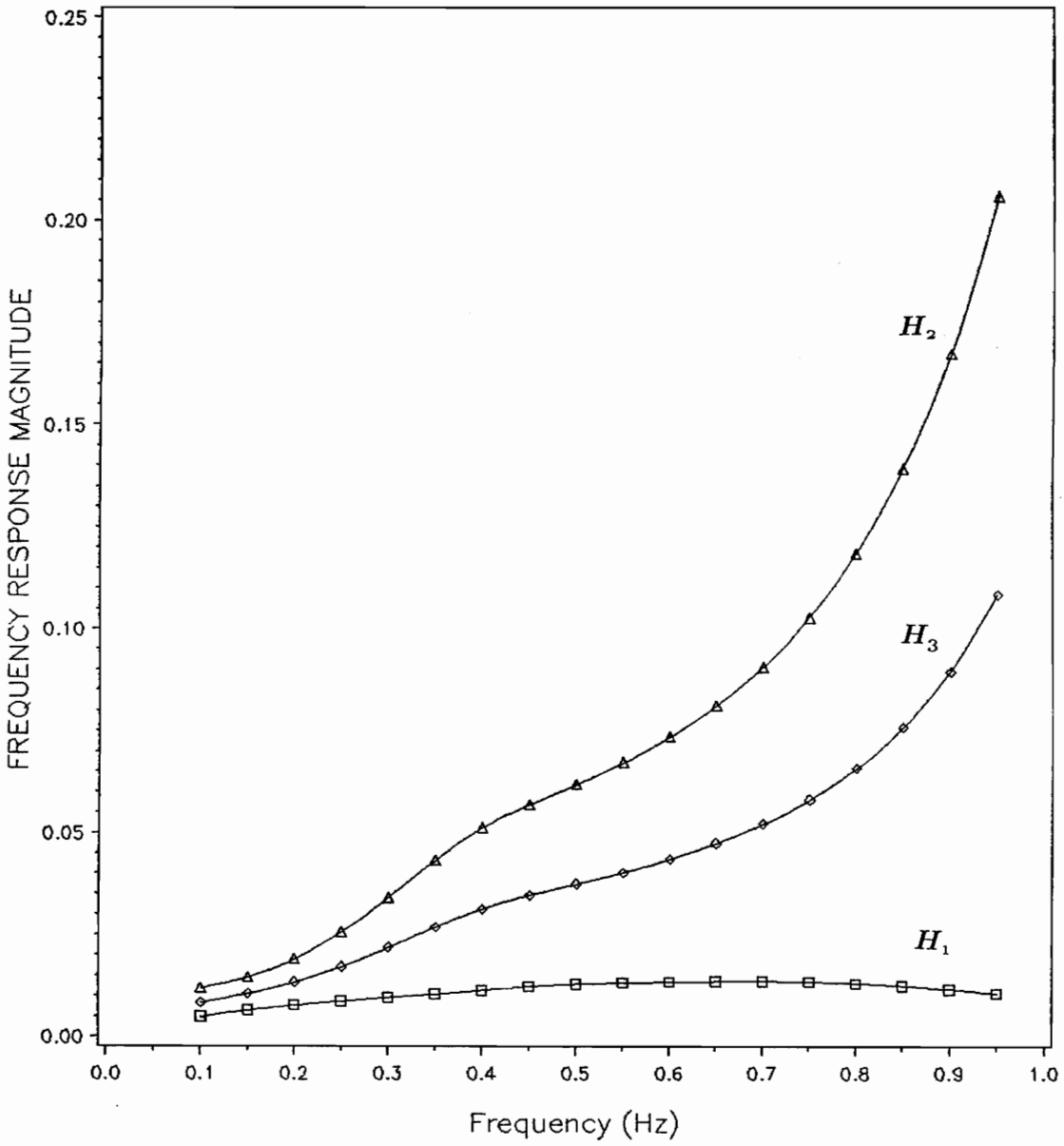

Figure 5.6: Comparison of H_1,H_2, & H_3 FRF Estimators for Data Taken on October 8, 1988

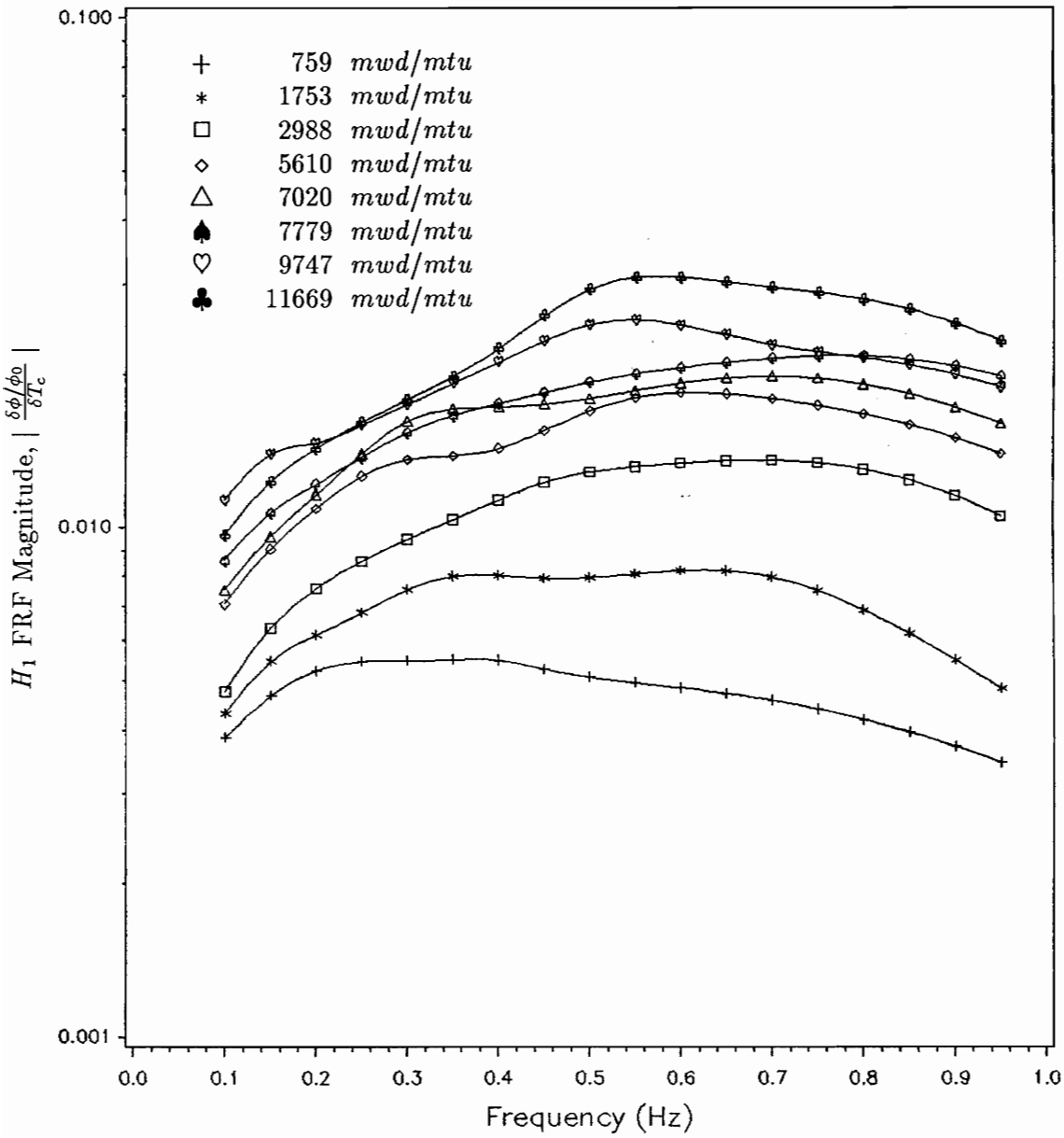

Figure 5.7: H_1 FRF Estimator Results for McGuire Unit 2, Cycle 5 for AR Model 30

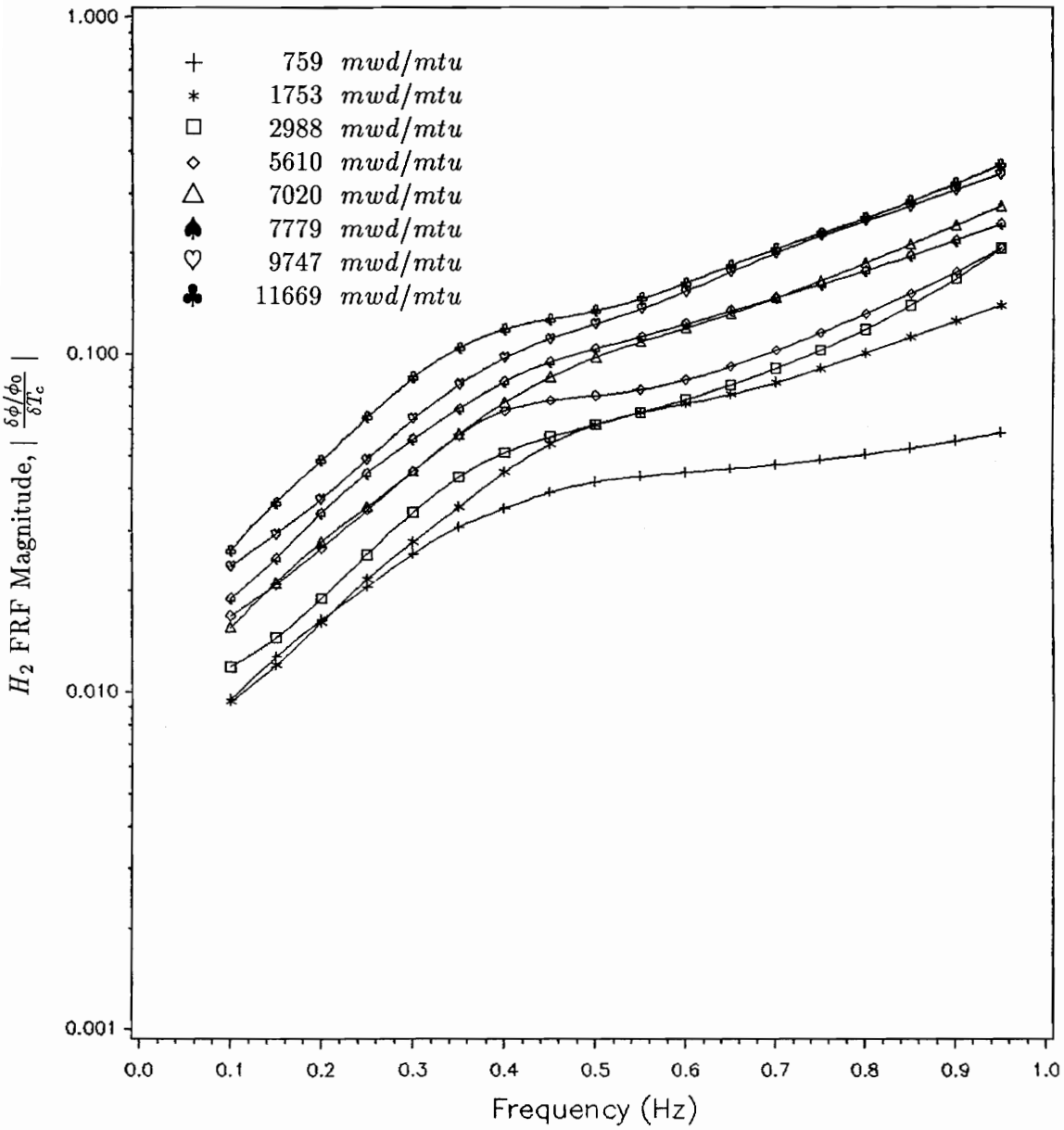

Figure 5.8: H_2 FRF Estimator Results for McGuire Unit 2, Cycle 5 for AR Model 30

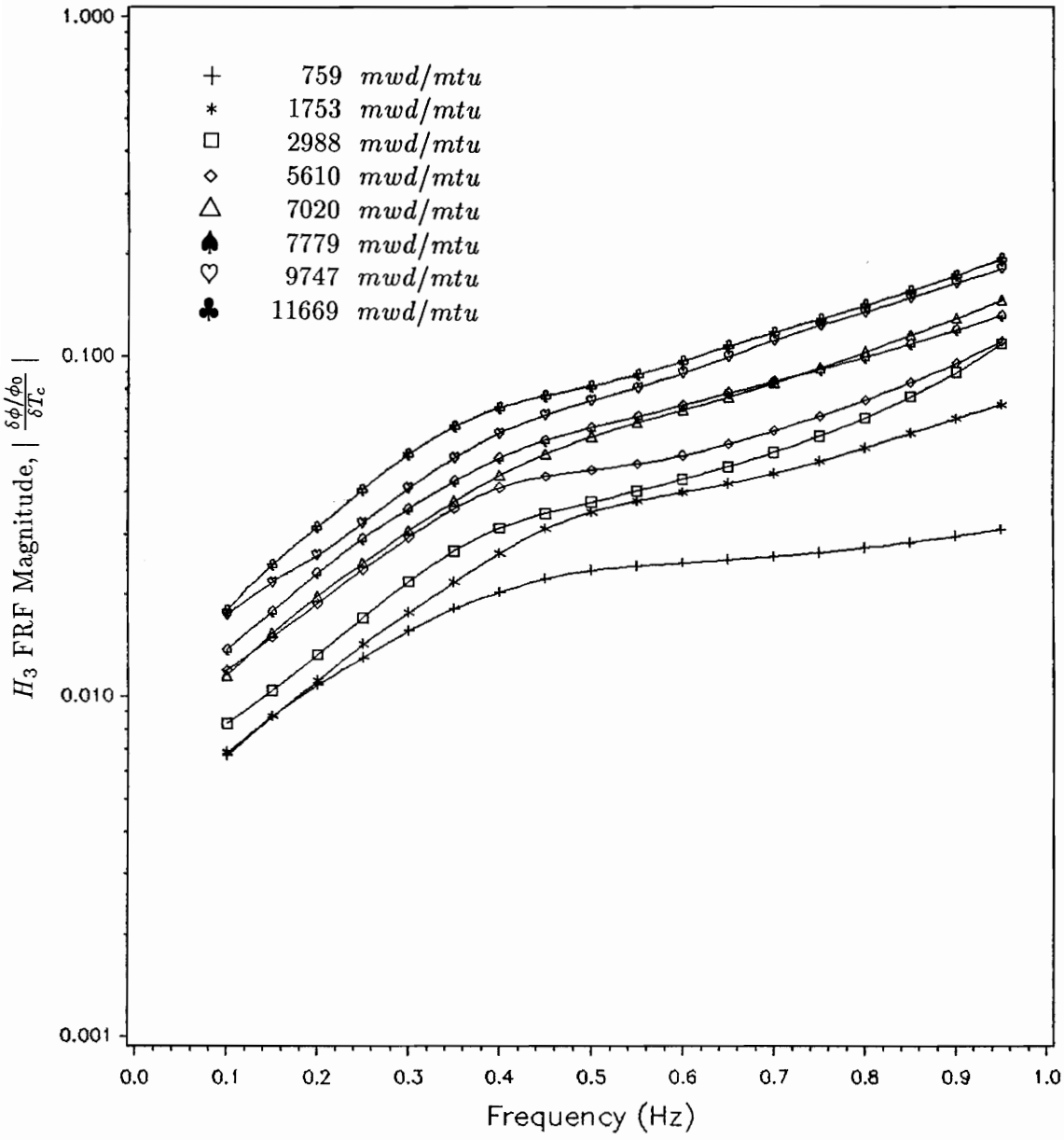

Figure 5.9: H_3 FRF Estimator Results for McGuire Unit 2, Cycle 5 for AR Model 30

5.8 MTC Prediction

The magnitude of the frequency response function between the core-exit temperature fluctuations and the in-core neutron flux fluctuations has been shown to increase monotonically with fuel depletion in the McGuire Unit 2 fuel cycle. Since the MTC is also related to the fuel depletion during the cycle, the FRF should be able to provide a measure of the MTC. A direct calculation of the MTC from the FRF was attempted earlier without success [1], so this research effort will not attempt to make this correlation. Instead, this project attempts to repeat the successful MTC prediction of performed by Herr [1] using a normalization method. Both core design values of the MTC and those values predicted by Herr will be used to judge the success of the autoregressive results.

The FFT results for MTC estimation were obtained by normalizing an average value of the FRF magnitude to a known MTC value. The FRF was averaged over the frequency band of 0.1 to 0.5 Hz where the signal coherence is highest to produce an average value of the FRF for each point in the fuel cycle. This FRF value was then ratioed to the known MTC value to predict the MTC for that point in the fuel cycle.

Results from this analysis agree quite well with the earlier results obtained by the FFT analysis. Figure 5.9 depicts the predicted MTC values obtained by the AR method and those obtained using the FFT. Only two core design values were available for the McGuire reactor and the core design MTC curve has been appoximated by a straight line between those two points. The AR results trend well with both the core design and the predicted MTC values of the FFT method. These results suggest that the method is quite powerful when it is considered that nearly identical results were obtained using only five percent of the data used in the FFT analysis.

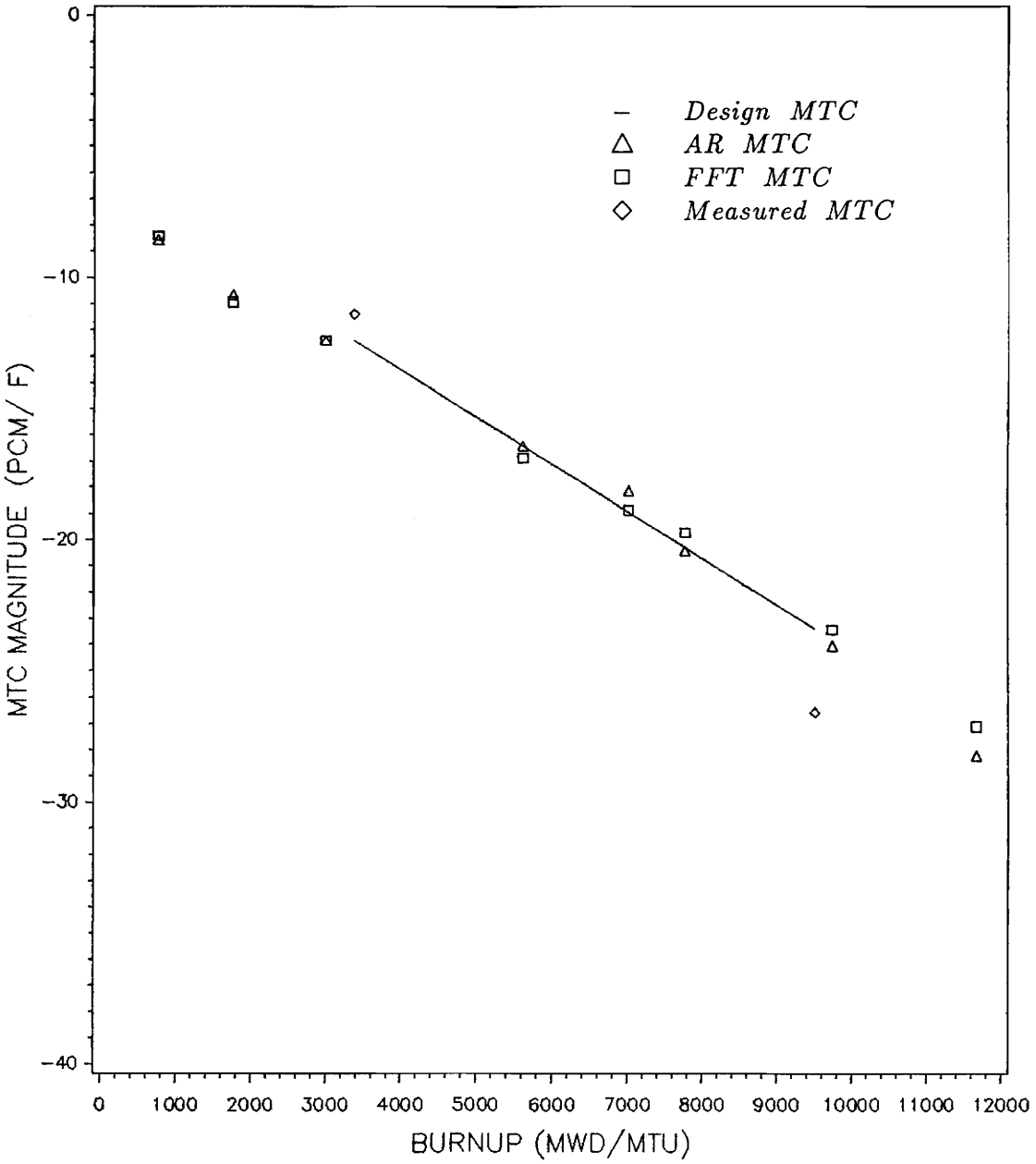

Figure 5.10: Averaged H_3 FRF Estimator Results Corrected to the Core Design MTC for McGuire Unit 2, Cycle 5 for AR Model 30 and FFT

Chapter 6

CONCLUSIONS, LIMITATIONS, & RECOMMENDATIONS

6.1 Conclusions

The correlation of stochastic fluctuations in the reactor core-exit temperature and the in-core neutron flux has again been shown to be capable of providing a measure of the MTC in pressurized water reactors. The time invariant nature of the temperature perturbation PSD, and the increasing nature of the neutron flux PSD has suggested the use of the frequency response function between the core-exit temperature and the neutron flux as a measure of the reactor's changing response to fuel depletion and the MTC. By normalizing an average value of the FRF magnitude to a known calibration factor, the magnitude of the MTC can be obtained. The sign of the MTC is indicated by the phase angle between the two fluctuating signals. The use of the multivariate autoregressive technique for signal analysis using only 5 % of the data has been shown to give nearly identical results when compared with the Fourier transform method.

Since so much less data is required for this analysis method, the likelihood of long-term thermocouple signal drift is greatly reduced. The AR approach to the signal processing also has the advantage of having the time series available for multiple analyses instead of only having the spectral properties saved as in the case of the on-line FFT method. If there is a DC component in the recorded data, an undesirable characteristic, the AR analysis allows for the subtraction of that component to give a zero-mean time series. This adjustment is not possible with the other method of data analysis, and may lead to better results.

6.2 Limitations

Although the MAR method has advantages over the FFT method, it is not without limitation. The question of statistical confidence is difficult to answer because of the nature of the AR process. Since the AR process models out noise in the signals, the statistical confidence of AR results is difficult to quantify. With this limitation, there is some uncertainty in the statistical accuracy of the obtained results.

A further disadvantage in the analysis used for the estimation of the MTC is the assumption of the stationarity of the data. To attempt to verify data stationarity, each 23 second data set was checked for having a zero mean and a constant variance, tests that indicate weakly stationary data. A disappointing result was that the data is not stationary in that time span. There is a low-frequency signal drift present in the data that diminishes the statistical confidence that various data sets are indeed from the same process. Figure 6.1 shows this low-frequency drift tendency. As a further check, a "run test" as described by Bendat [26] was performed on the data to check for the presence of underlying trends. The data also fails this test. The variance of the 25 data sets is much better behaved as can be seen in Fig. 6.2. A long-term signal drift was addressed as a limitation of the FFT analysis [1], and it was thought that this effect would disappear since the AR process uses so much less data. Though there does not appear to be any such drift in the data used for the AR analysis, the data is still not stationary within the time span considered for the analysis of each data set. Since both the AR and the FFT methods have used the same data set length, the apparent non-stationarity of the data is a weakness of both techniques.

Though there is a low-frequency drift that causes the data to be nonstationary with respect to the data set mean, the higher frequency fluctuations could indeed be stationary with time. Signal fluctuations in the frequency range of interest (0.1 to 0.5 Hz) could be stable and thus stationary, indicating that the data analysis method and averaging technique is a valid way to do the signal processing. Time does not allow for the development and testing of this theory.

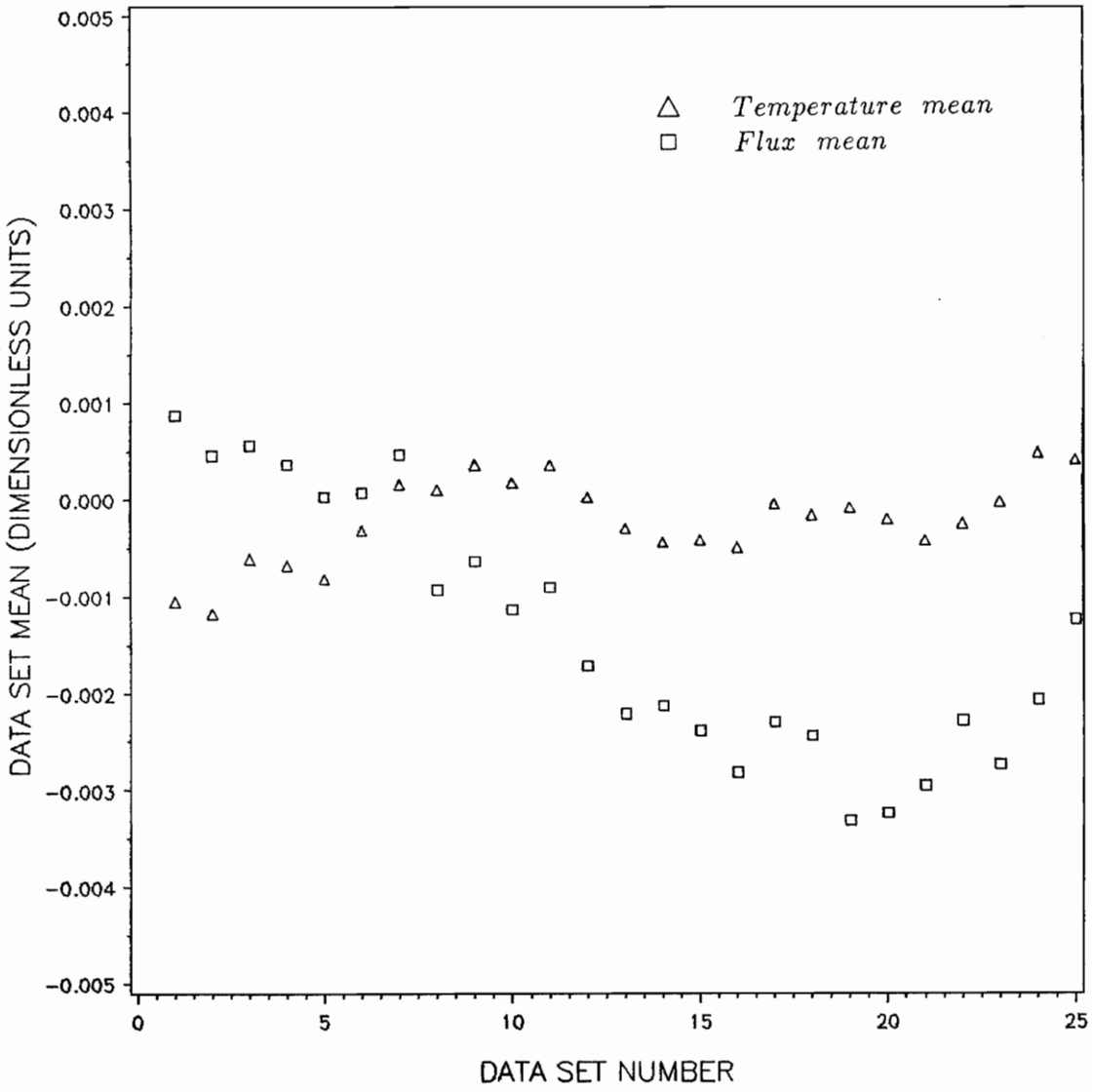

Figure 6.1: Variation of Temperature and Neutron Flux Data Set Mean

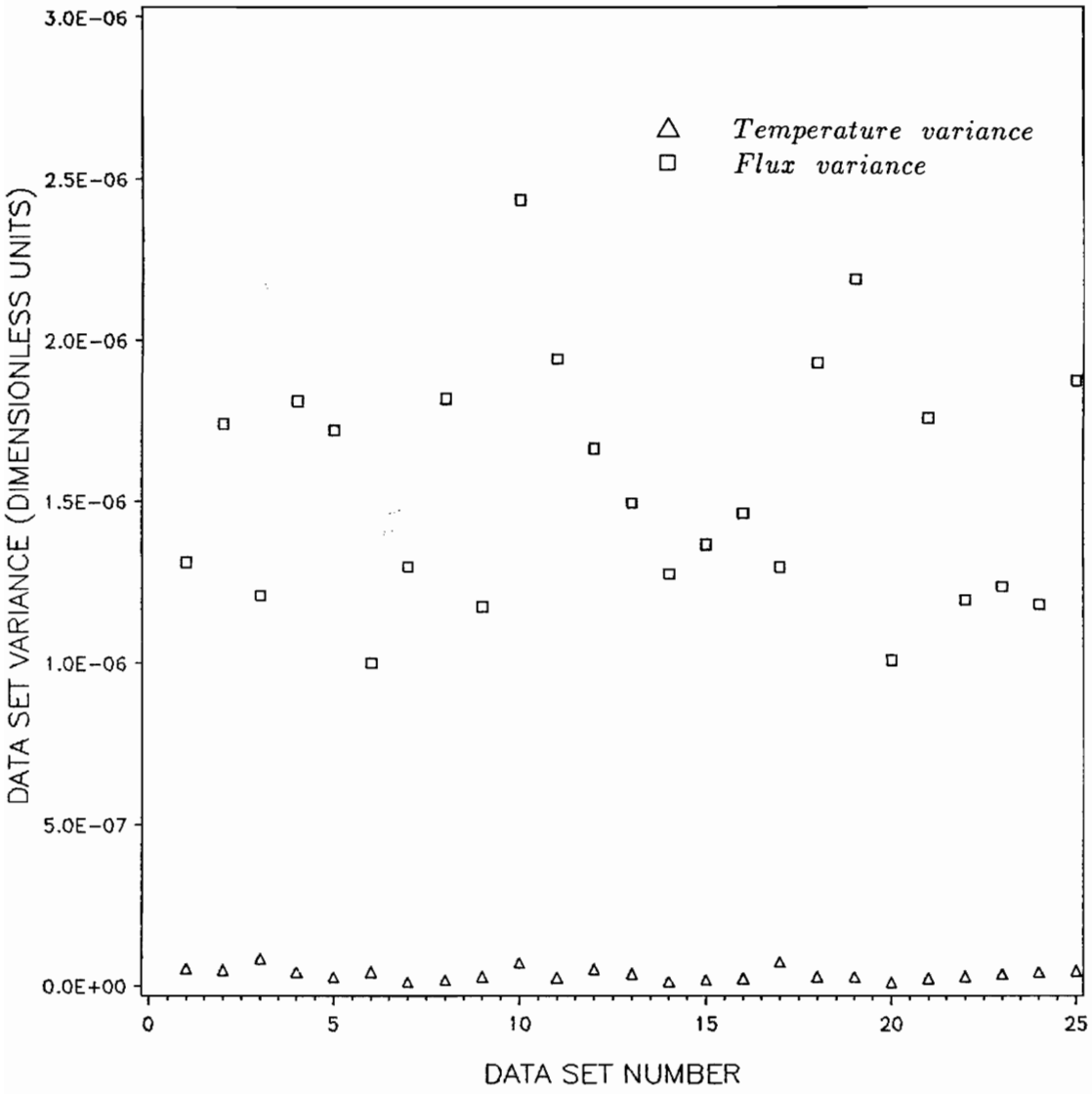

Figure 6.2: Comparison of Temperature and Neutron Flux Variance

6.3 Recommendations

Despite the apparent lack of proper stationarity in the data, correlating stochastic fluctuations in the core-exit temperature with similar fluctuations in the neutron flux has been shown to be a valid passive technique for determining the MTC in pressurized water reactors. It suggested that further research be done to establish whether the data is stationary with respect to the frequency range of interest to prove the merits of the analysis. More research should also be done to establish a theoretical and an experimental basis for the determination of an absolute calibration factor for a given reactor so that the ultimate superiority of this method can be more firmly established. It is also recommended that nuclear utilities begin to practice using this method to gather more experience. This experience could be of great importance if the NRC is to consider making MTC estimation by noise analysis an acceptable alternative to current test methods.

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Appendix A

COMPUTER PROGRAMS

```
C*****************************************************************
C                                                                *
C PROGRAM: MAR-FRF              ALLEN W. CLEM  10-29-90          *
C                               MECHANICAL ENGINEERING DEPT      *
C                               VIRGINIA POLYTECHNIC INSTITUTE   *
C                               & STATE UNIVERSITY               *
C                                                                *
C REVISED BY:                                                    *
C LATEST REVISION DATE:                                          *
C                                                                *
C PROGRAM DESCRIPTION:                                           *
C                                                                *
C     THIS PROGRAM IS SET UP TO DO A TWO CHANNEL AUTO-           *
C REGRESSIVE ANALYSIS FOR A GIVEN TIME SERIES. THE PROGRAM       *
C UTILIZES SUBROUTINES GIVEN BY S. L. MARPLE, IN "DIGITAL        *
C SPECTRAL ANALYSIS WITH APPLICATIONS" TO COMPUTE THE AR         *
C PARAMETERS AND TO COMPUTE THE AR PSD AND CPSD. THIS PROGRAM    *
C REQUIRES THE INPUT OF A TWO CHANNEL TIME SERIES REPRESENTING   *
C THE CORE-EXIT TEMPERATURE AND THE IN-CORE NEUTRON FLUX FOR     *
C A PRESSURIZED WATER REACTOR. THE TIME SERIES IS BROKEN UP      *
C INTO SMALLER DATA SETS FOR INDIVIDUAL AR ANALYSIS. THE         *
C SPECTRAL PROPERTIES OF EACH DATA SET ARE WRITTEN TO A DATA     *
C FILE WHERE THEY CAN BE AVERAGED TO PRODUCE AN ENSEMBLE         *
C AVERAGE FOR THE ENTIRE TIME SERIES.                            *
C                                                                *
C                                                                *
C VARIABLE DESCRIPTION:                                          *
C                                                                *
C AICAVE        -AVERAGE MODEL ORDER FOR ENTIRE TIME SERIES      *
C A             -COMPLEX ARRAY OF FORWARD LINEAR PREDICTION      *
C                AUTOREGRESSIVE COEFFICIENTS                     *
C B             -COMPLEX ARRAY OF BACKWARD LINEAR PREDICTION     *
```

```
C                AUTOREGRESSIVE COEFFICEINTS                          *
C COH           -ORDINARY COHERENCE FUNCTION VALUE                    *
C DT            -TIME INTERVAL BETWEEN DATA POINTS                    *
C F             -FRACTION OF SAMPLING FREQUENCY SENT TO MCARPSD*
C FINPSD        -FINAL PSD ARRAY                                      *
C FLUX          -NEUTRON FLUX DATA INPUT FROM DATA FILE               *
C FMAX          -MAXIMUM FREQUENCY DESIRED FOR SPECTRAL EST.          *
C H1            -H1 FREQUENCY RESPONSE FUNCTION ESTIMATOR             *
C H2            -H2 FREQUENCY RESPONSE FUNCTION ESTIMATOR             *
C H3            -AVERAGE BETWEEN H1 AND H2 FRF ESTIMATORS             *
C IP            -AR ORDER CHOSEN FOR ENTIRE ANALYSIS                  *
C ISTAT         -ERROR FLAG (SEE SUBROUTINES MCAR,MCARPSD)            *
C MAXPTS        -MAXIMUM NUMBER OF POINTS PER DATA SET                *
C MAXORDER      -MAXIMUM ORDER OF AUTOREGRESSION                      *
C MEANX         -MEAN VALUE FOR INDIVDUAL TEMPERATURE DATA SET *
C MEANY         -MEAN VALUE FOR INDIVIDUAL FLUX DATA SET              *
C METHOD        -DETERMINES WHICH METHOD THE PROGRAM USES TO DO*
C                THE AR ANALYSIS:                                     *
C                   1--VIEIRA-MORF ALGORITHM                          *
C                   2--NUTTALL-STRAND ALGORITHM                       *
C NAVE          -NUMBER OF INDIVIDUAL DATA SETS FOR THE ANALYSI*
C NF            -NUMBER OF FREQUENCY COMPONENTS WITHIN THE            *
C                FREQUENCY BAND                                       *
C NUMCHS        -NUMBER OF CHANNELS IN THE TIME SERIES                *
C PB            -COMPLEX BACKWARD LINEAR PREDICTION ERROR             *
C                COVARIANCE ARRAY                                     *
C PF            -COMPLEX FORWARD LINEAR PREDICTION ERROR              *
C                COVARIANCE ARRAY                                     *
C PSD           -PSD VALUES RETURNED FROM SUBROUTINE FOR GIVEN *
C                FREQUENCY                                            *
C TEMP          -CORE-EXIT TEMPERATURE DATA INPUT FROM FILE           *
C X             -COMPLEX ARRAY OF TIME SERIES                         *
C XSUM          -SUM OF TEMPERATURE VALUES FOR EACH DATA SET          *
C YSUM          -SUM OF FLUX VALUES FOR EACH DATA SET                 *
C                                                                     *
C                                                                     *
C DATA FILE DESCRIPTION:                                              *
C                                                                     *
C INPUT FILE:    MNS106 DAT                                           *
C OUTPUT FILES:  PSDDATA OUT                                          *
C                FRFDATA OUT                                          *
```

```
C                                                                      *
C SUBROUTINES CALLED:                                                  *
C     MCAR                                                             *
C     MCARPSD                                                          *
C     FRF                                                              *
C                                                                      *
C**********************************************************************

      PARAMETER (IP = 30)
      PARAMETER (NAVE = 25)
      PARAMETER (N = 1024)
      PARAMETER (NF = 20)
      PARAMETER (METHOD = 2)
      PARAMETER (NUMCHS = 2)
      PARAMETER (MAXPTS = 1024)
      PARAMETER (MAXORDER = 52)


      COMPLEX*16 X(NUMCHS,MAXPTS),PF(NUMCHS,NUMCHS),PB(NUMCHS,NUMCHS)
     $ ,A(NUMCHS,NUMCHS,MAXORDER),B(NUMCHS,NUMCHS,MAXORDER),
     $ PSD(NUMCHS,NUMCHS),
     $ FINPSD(NUMCHS,NUMCHS,NF)
      REAL*8 TEMP(MAXPTS),FLUX(MAXPTS),XSUM,YSUM,
     $ MEANX,MEANY,H1(NF),F,COH(NF),DUMMY
     $   , H2(NF),H3(NF),DT,FMAX

      FMAX = 1.0D0
      DT = 0.0222D0

C     OPEN (UNIT=8,FILE = 'MNS106.DAT',STATUS = 'OLD')
C     OPEN (UNIT=10,FILE = 'FRFDATA',STATUS = 'NEW')
C     OPEN (UNIT=9,FILE = 'PSDDATA.OUT',STATUS = 'NEW')

C PRINT FILE HEADINGS
      WRITE (9,9)
 9    FORMAT (2X,'FREQ',3X,'TEMP PSD',5X,'FLUX PSD', 4X,
     $ 'CPSD (REAL)',2X,'CPSD (IMAG)',2X,'COHERENCE')

      WRITE (10,10)
 10   FORMAT (2X,'FREQ',2X,'H1 FRF MAG',3X,'H2 FRF MAG',
     $  3X,'H3 FRF MAG',3X,'COHERENCE')
```

```
C*****************************************************************
C THIS SECTION INPUTS THE TEMPERATURE AND NEUTRON FLUX DATA     *
C AND SUBTRACTS THE MEAN FROM THE DATA TO ENSURE ZERO-MEAN      *
C TIME SERIES                                                   *
C*****************************************************************

      DO 1 JJ = 1,NAVE
        XSUM = 0.0
        YSUM = 0.0

        DO 2 I = 1,N
          READ (8,*) TEMP(I),FLUX(I)
          XSUM = TEMP(I) + XSUM
          YSUM = FLUX(I) + YSUM
  2     CONTINUE

        MEANX = XSUM/DBLE(N)
        MEANY = YSUM/DBLE(N)

      DO 3 I = 1,N
        TEMP(I) = TEMP(I) - MEANX
        FLUX(I) = FLUX(I) - MEANY
        X(1,I) = DCMPLX(TEMP(I),0.D0)
        X(2,I) = DCMPLX(FLUX(I),0.D0)
 3    CONTINUE

C*****************************************************************
C THIS SECTION CALLS THE SUBROUTINE "MCAR" TO DO THE MULTIVARIATE *
C AUTOREGRESSIVE ANALYSIS AND RETURNS THE AR COEFFICIENTS, THE    *
C NOISE VARIANCES, AND THE BEST MODEL ORDER FOR THE DATA SET      *
C*****************************************************************
      CALL MCAR(METHOD,N,IP,X,PF,A,PB,B,ISTAT,NUMCHS,MAXORDER,MAXPTS,
     $ IPBEST)
      AICSUM = AICSUM + DBLE(IPBEST)/DBLE(NAVE)
C     WRITE (*,*) ISTAT

C*****************************************************************
C THIS SECTION CALLS THE SUBROUTINE "MCARPSD" TO DO THE MULTI     *
C CHANNEL SPECTRAL ESTIMATION AND RETURNS THE APSD AND THE CPSD   *
C FUNCTIONS FOR A GIVEN FREQUENCY                                 *
```

```
C*****************************************************************

      DO 4 K = 1,NF
        F = DBLE(K)/DBLE(NF)*DT*FMAX
        CALL MCARPSD (IP,F,PF,A,PSD,ISTAT,NUMCHS,MAXORDER,MAXPTS)

        DO 4 J = 1,NUMCHS
        DO 4 I = 1,NUMCHS
          FINPSD(I,J,K)  = PSD(I,J)
 4      CONTINUE

C*****************************************************************
C THIS SECTION CALLS THE SUBROUTINE "FRF" TO COMPUTE THE         *
C FREQUENCY RESPONSE FUNCTION AND RETURNS THE H1,H2 AND H3 FRF   *
C FREQUENCY RESPONSE FUNCTION ESTIMATORS AS WELL AS THE COHERENCE*
C*****************************************************************

      CALL FRF (FINPSD,H1,H2,H3,NF,NUMCHS,COH)

C*****************************************************************
C THIS SECTION PRINTS THE OUTPUT TO DATA FILES                    *
C*****************************************************************

      DO 5 I = 1, NF
        F = DBLE(I)/DBLE(NF)*FMAX

C       PRINT FREQ   TAPSD  NAPSD  CPSD COHERENCE
          WRITE (9,7) F,CDABS(FINPSD(1,1,I))*DT,
     $    CDABS(FINPSD(2,2,I))*DT, (FINPSD(1,2,I))*DT, COH(I)

C UNCOMMENT THE NEXT LINES TO GET FRF OUTPUT
C
C       PRINT FREQ  H1 H2 H3  COH
C         WRITE (10,6) F, H1(I), H2(I), H3(I), COH(I)

  7       FORMAT (1X,F5.2,5E13.5)
  6       FORMAT (1X,F5.2,4E14.5 )
  5   CONTINUE

 1    CONTINUE
```

```
      WRITE (9,8) AICSUM
 8    FORMAT (1X,'BEST AVERAGE ORDER IS ',F4.0)

      STOP
      END

      SUBROUTINE MCAR (METHOD,N,IP,X,PF,A,PB,B,ISTAT,NUMCHS,
     $ MAXORDER, MAXPTS,IPBEST)

C  COMPUTES THE MULTICHANNEL AUTOREGRESSIVEMATRIX COEFFICIENTS USING
C  EITHER THE VIEIRA-MORF ALGORITHM (A GENERALIZATION OF THE SINGLE-
C  CHANNEL GEOMETRIC METHOD) OR THE NUTTALL-STRAND ALGORITHM ( A GENER-
C  IZATION OF THE SINGLE CHANNEL HARMONIC METHOD).
C
C  INPUT PARAMETERS:
C
C     METHOD - INTEGER VARIABLE; IF 1 THEN THE VIEIRA-MORF ALGORITHM
C              IS SELECTED; IF 2 THE NUTTALL-STRAND ALGORITHM IS USED
C     N      - NUMBER OF SAMPLE POINTS PER CHANNEL (INTEGER)
C     IP     - ORDER OF AUTOREGRESSIVE MODEL (INTEGER)
C     X      - COMPLEX ARRAY OF NUMCHS CHANNEL DATA SAMPLES
C
C  OUTPUT PARAMETERS:
C
C     PF     - COMPLEX NUMCHS X NUMCHS FORWARD LINEAR PREDICTION ERROR
C              COVARIANCE
C     A      - COMPLEX ARRAY OF FORWARD LINEAR PREDICTION MATRIX
C              COEFFICIENTS
C     PB     - COMPLEX BACKWARD LINEAR PREDICTION ERROR COVARIANCE ARRAY
C     B      - COMPLEX ARRAY OF BACKWARD LINEAR PREDICTION MATRIX
C              COEFFICIENTS
C     ISTAT  - INTEGER STATUS INDICATOR AT TIME OF SUBROUTINE EXIT
C              0 - NORMAL EXIT WITH NO ERRORS DETECTED
C              1 - PROCESSING OF MORE THAN TWO CHANNELS REQUESTED
C                  OF SUBROUTINES BILINEAR, INVERT, SQROOT
C              2 - SINGULAR MATRIX ENCOUNTERED BY SUBROUTINE SQROOT
C              3 - IP EXCEEDS THE MAXIMUM ORDER PERMITTED; REDIMENSION
C                  APPROPRIATE SUBROUTINE ARRAYS
C
C  NOTES: ARRAY DIMENSIONS CONTROLLED IN THE PARAMETER STATEMENTS BELOW
C
```

```
C     PARAMETER NUMCHS = 2
C     PARAMETER MAXORDER = 52
C     PARAMETER MAXPTS = 1024

      COMPLEX*16 X(NUMCHS,MAXPTS),PF(NUMCHS,NUMCHS),PB(NUMCHS,NUMCHS)
      COMPLEX*16 A(NUMCHS,NUMCHS,MAXORDER),B(NUMCHS,NUMCHS,MAXORDER)
      COMPLEX*16 PFHAT(2,2),PBHAT(2,2),S
      COMPLEX*16 PFBHAT(2,2),RHO(2,2),COL1(2)
     $ ,EF(2,1024),EB(2,1024),TEMP1(2,2)
     $ ,TEMP2(2,2),TEMP3(2,2),COL2(2)
     $ ,TEMP4(2,2),TEMP5(2,2),COL3(2)
      REAL*8 AICVAL,DETPF
C
C  ** INITIALIZATION **
C
      AICMIN = 1.0D10

      ISTAT = 0
      IF (IP.LE.MAXORDER) GO TO 5
      ISTAT = 3
      RETURN
  5   CALL ZERO (NUMCHS,NUMCHS,PF)
      DO 10 K = 1,N
        CALL ASSIGN (NUMCHS,1,X(1,K),EF(1,K))
        CALL ASSIGN (NUMCHS,1,X(1,K),EB(1,K))
        CALL OUTPROD (NUMCHS,X(1,K),X(1,K),TEMP1)
  10    CALL ADD (NUMCHS,NUMCHS,PF,TEMP1,PF)
      S = DCMPLX(1.D0/DBLE(N),0.D0)
      CALL SCALE (NUMCHS,NUMCHS,S,PF,PF)
      CALL ASSIGN (NUMCHS,NUMCHS,PF,PB)
C
      M = 0
C
C  ** MAIN LOOP **
C
 100  CONTINUE
C
C  UPDATE THE ESTIMATED ERROR COVARIANCES
C
      CALL ZERO (NUMCHS,NUMCHS,PFHAT)
      CALL ZERO (NUMCHS,NUMCHS,PBHAT)
```

```
      CALL ZERO (NUMCHS,NUMCHS,PFBHAT)

      DO 20 K = M+2,N
        CALL ASSIGN (NUMCHS,1,EF(1,K),COL1)
        CALL ASSIGN (NUMCHS,1,EB(1,K-1),COL2)
        CALL OUTPROD (NUMCHS,COL1,COL1,TEMP1)
        CALL ADD (NUMCHS,NUMCHS,PFHAT,TEMP1,PFHAT)
        CALL OUTPROD (NUMCHS,COL2,COL2,TEMP1)
        CALL ADD (NUMCHS,NUMCHS,PBHAT,TEMP1,PBHAT)
        CALL OUTPROD (NUMCHS,COL1,COL2,TEMP1)
 20     CALL ADD (NUMCHS,NUMCHS,PFBHAT,TEMP1,PFBHAT)
C
      M = M + 1
      IF (METHOD.EQ.2) GO TO 30
C
C  COMPUTE ESTIMATED NORMALIZED PARTIAL CORRELTAION MATRIX
C        EQ 15.88  (VIEIRA-MORF ALGORITHM ONLY)
C
      CALL SQROOT (NUMCHS,PFHAT,TEMP1,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL INVERT (NUMCHS,TEMP1,TEMP3,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL SQROOT(NUMCHS,PBHAT,TEMP1,ISTAT)
      CALL INVERT (NUMCHS,TEMP1,TEMP2,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL XPOSE (NUMCHS,TEMP2,TEMP4)
      CALL MULT (NUMCHS,NUMCHS,TEMP3,PFBHAT,TEMP1)
      CALL MULT (NUMCHS,NUMCHS,TEMP1,TEMP4,RHO)
C
C  UPDATE FORWARD AND BACKWARD REFLECTION COEFFICIENTS
C     EQS(15.82),(15.83)(VIEIRA-MORF ALGORITHM ONLY)
C
      CALL SQROOT (NUMCHS,PF,TEMP3,ISTAT)
      CALL SQROOT (NUMCHS,PB,TEMP4,ISTAT)
      CALL MULT (NUMCHS,NUMCHS,TEMP3,RHO,TEMP1)
      CALL XPOSE (NUMCHS,RHO,TEMP5)
      CALL MULT (NUMCHS,NUMCHS,TEMP4,TEMP5,TEMP2)
      CALL INVERT (NUMCHS,TEMP4,TEMP5,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL MULT (NUMCHS,NUMCHS,TEMP1,TEMP5,A(1,1,M))
      CALL INVERT (NUMCHS,TEMP3,TEMP5,ISTAT)
```

```
      IF (ISTAT.NE.0) RETURN
      CALL MULT (NUMCHS,NUMCHS,TEMP2,TEMP5,B(1,1,M))
      GO TO 35

C
C COMPUTE ESTIMATED PARTIAL CORRELATION MATRIX EQ. (15.98)
C         NUTTALL-STRAND ALGORITHM ONLY
C
 30   CALL INVERT (NUMCHS,PF,TEMP1,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL MULT (NUMCHS,NUMCHS,PFHAT,TEMP1,TEMP2)
      CALL INVERT (NUMCHS,PB,TEMP3,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL MULT (NUMCHS,NUMCHS,TEMP3,PBHAT,TEMP4)
      S = (2.D0,0.D0)
      CALL SCALE (NUMCHS,NUMCHS,S,PFBHAT,TEMP5)
      CALL BILINEAR (NUMCHS,TEMP2,TEMP4,TEMP5,RHO,ISTAT)
      IF (ISTAT.NE.0) RETURN

C
C  UPDATE THE FORWARD AND BACKWARD REFLECTION COEFFICIENTS
C  EQS. (15.73),(15.74),(15.78) (NUTTAL-STRAND ALGORITHM ONLY)
C
      CALL MULT ( NUMCHS,NUMCHS,RHO,TEMP3,A(1,1,M))
      CALL XPOSE (NUMCHS,RHO,TEMP2)
      CALL MULT (NUMCHS,NUMCHS,TEMP2,TEMP1,B(1,1,M))
 35   S = (-1.D0,0.D0)
      CALL SCALE (NUMCHS,NUMCHS,S,A(1,1,M),A(1,1,M))
      CALL SCALE (NUMCHS,NUMCHS,S,B(1,1,M),B(1,1,M))
C
C  UPDATE FORWARD AND BACKWARD ERROR COVARIANCES- EQS (15.75),(15,76)
C
      CALL MULT (NUMCHS,NUMCHS,A(1,1,M),B(1,1,M),TEMP1)
      CALL MULT (NUMCHS,NUMCHS,TEMP1,PF,TEMP2)
      CALL SUB (NUMCHS,NUMCHS,PF,TEMP2,PF)
      CALL MULT (NUMCHS,NUMCHS,B(1,1,M),A(1,1,M),TEMP1)
      CALL MULT (NUMCHS,NUMCHS,TEMP1,PB,TEMP2)
      CALL SUB (NUMCHS,NUMCHS,PB,TEMP2,PB)
C
C UPDATE FORWARD AND BACKWARD PREDICTOR COEFFIFIENTS - (15.84),(15.85)
C
```

```
      IF (M.EQ.1) GO TO 50
      DO 40 K = 1,M-1
        MK = M-K
        CALL ASSIGN (NUMCHS,NUMCHS,A(1,1,K),TEMP1)
        CALL ASSIGN (NUMCHS,NUMCHS,B(1,1,MK),TEMP2)
        CALL MULT (NUMCHS,NUMCHS,A(1,1,M),TEMP2,TEMP3)
        CALL ADD (NUMCHS,NUMCHS,TEMP1,TEMP3,A(1,1,K))
        CALL MULT (NUMCHS,NUMCHS,B(1,1,M),TEMP1,TEMP3)
 40     CALL ADD (NUMCHS,NUMCHS,TEMP2,TEMP3,B(1,1,MK))

C CALC AIC ORDER PREDICTION
      DETPF = DREAL(PF(1,1)*PF(2,2)-PF(1,2)*PF(2,1))
      AICVAL = DBLE(N)*DLOG(DETPF)+8.0D0*DBLE(M)
        IF (AICVAL.LT.AICMIN) THEN

            AICMIN = AICVAL
            IPBEST = M
        ELSE
        ENDIF


C
C CHECK FOR PROPER ORDER
C
 50   IF (M.EQ.IP) RETURN
C
C CALCULATE THE RESIDUALS
C
      DO 60 K = N,M+1,-1
        CALL ASSIGN (NUMCHS,1,EF(1,K),COL1)
        CALL ASSIGN (NUMCHS,1,EB(1,K-1),COL2)
        CALL MULT (NUMCHS,1,A(1,1,M),COL2,COL3)
        CALL ADD (NUMCHS,1,COL1,COL3,EF(1,K))
        CALL MULT (NUMCHS,1,B(1,1,M),COL1,COL3)
 60     CALL ADD (NUMCHS,1,COL2,COL3,EB(1,K))
      GO TO 100
      END
C
      SUBROUTINE MCARPSD (IP,F,P,A,PSD,ISTAT,NUMCHS,MAXORDER,
     $ MAXPTS)
C
```

```
C  THIS SUBROUTINE COMPUTES THE MULTICHANNEL POWER SPECTRAL DENSITY
C  MATRIX AT A GIVEN FREQUENCY F
C
C  INPUT PARAMETERS:
C
C     IP    - ORDER OF AUTOREGRESSIVE MODEL (INTEGER)
C     F     - FRACTION OF SAMPLING FREQUENCY (RANGES FROM -0.5 TO 0.5)
C     P     - COMPLEX ARRAY OF DRIVING NOISE COVARIANCE
C     A     - COMPLEX ARRAYS OF AUTOREGRESSIVE MATRIX COEFFICIENTS
C
C  OUTPUT PARAMETERS:
C
C     PSD   - COMPLEX ARRAYS OF POWER SPECTRAL DENSITY VALUES
C     ISTAT- INTEGER STATUS INDICATOR AT TIME OF SUBROUTINE EXIT
C            0 FOR NORMAL EXIT
C            1 IF NONSINGULAR MATRIX ENCOUNTERED BY INVERT
C
C  NOTES : ARRAY DIMENSIONS ARE CONTROLLED BY THE PARAMETER STATEMENTS
C          BELOW. SUPPORTING SUBROUTINES ARE PROVIDED BELOW.
C
C     PARAMETER NUMCHS = 2
C     PARAMETER MAXORDER = 52

      COMPLEX*16 P(NUMCHS,NUMCHS),A(NUMCHS,NUMCHS,MAXORDER)
      COMPLEX*16 PSD (NUMCHS,NUMCHS),R(2,2),S(2,2)
      COMPLEX*16 T(2,2),TEMP
      REAL*8 F,ARG

      ISTAT = 0
      TWOPI = 2.D0*DACOS(-1.D0)
      ARG = -TWOPI*F
      CALL IDENT (NUMCHS,T)
      DO 10 K = 1,IP
        TEMP = CDEXP(DCMPLX(0.D0,ARG*DBLE(K)))
C
        CALL SCALE (NUMCHS,NUMCHS,TEMP,A(1,1,K),R)
 10     CALL ADD (NUMCHS,NUMCHS,T,R,T)
      CALL INVERT (NUMCHS,T,R,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL XPOSE (NUMCHS,R,S)
      CALL MULT (NUMCHS,NUMCHS,R,P,T)
```

```
      CALL MULT (NUMCHS,NUMCHS,T,S,PSD)
      RETURN
      END
C
C
      SUBROUTINE FRF ( PSD, H1,H2,H3,NF, NUMCHS,COH)

C COMPUTES THE H1, H2, H3 FRF ESITIMATOR
C MAGNITUDE FROM THE INPUT AUTO AND CROSS PSD'S

C INPUT PARAMETERS:
C     PSD     - COMPLEX MATRIX OF POWER SPECTRAL DENSITY
C               VALUES
C     NF      - NUMBER OF FREQUENCIES REQUIRED FOR FRF
C     NUMCHS  - NUMBER OF DATA CHANNELS

C OUTPUT PARAMETERS:
C     H1      - H1 ESTIMATOR FREQUENCY RESPONSE FUNCTION
C               MAGNITUDE
C     H2      - H2 ESTIMATOR FREQUENCY RESPONSE FUNCTION
C     H3      - H3 AVERAGE OF H1 AND H2 FREQ RESP FUNCTION
C     COH     - ORDINARY COHERENCE FUNCTION ESTIMATE
C DEFINE AND DIMENSION VARIABLES
      COMPLEX*16 PSD(NUMCHS,NUMCHS,NF)
      REAL*8 H1(NF), COH(NF),H2(NF),H3(NF)

C COMPUTE FRF  AND COHERENCE
      DO 10 I = 1,NF
        H1(I) = CDABS(PSD(2,1,I)/PSD(1,1,I))
        H2(I) = CDABS(PSD(2,2,I)/PSD(1,2,I))
        H3(I) = (H1(I) + H2(I))/2.0D0
        COH(I) = CDABS(PSD(1,2,I))**2/(PSD(1,1,I)*PSD(2,2,I))
  10  CONTINUE
      RETURN
      END

C *** SUPPORTING SUBROUTINES ***
C
      SUBROUTINE ADD (M,N,A,B,C)
C ADDS M X N MATRICES A AND B WITH RESULT IN C
      COMPLEX*16 A(M,N),B(M,N),C(M,N)
```

```
      DO 10 J = 1,N
      DO 10 I = 1,M
 10     C(I,J) = A(I,J) + B(I,J)
      RETURN
      END
C
      SUBROUTINE ASSIGN (M,N,A,B)
C  ASSIGNS CONTENTS OF M X N MATRIX A TO MATRIX B
      COMPLEX*16 A(M,N),B(M,N)
      DO 10 J = 1,N
      DO 10 I = 1,M
 10     B(I,J) = A(I,J)
      RETURN
      END
C
      SUBROUTINE BILINEAR (M,A,B,C,X,ISTAT)
C SOLVES THE BILINEAR EQUATION AX + XB = C FOR M = 1,2 ONLY
      COMPLEX*16 A(M,M),B(M,M),C(M,M),X(M,M),TEMP(2,2)
      REAL*8 C1,C2
      IF (M.EQ.1) GO TO 10
      IF (M.EQ.2) GO TO 20
      ISTAT = 1
      RETURN
 10   X(1,1) = C(1,1)/(A(1,1)+B(1,1))
      RETURN
 20   C1 = DREAL(A(1,1)+A(2,2)+B(1,1)+B(2,2))
      C2 =DREAL(A(1,1)*A(2,2)-A(1,2)*A(2,1)-B(1,1)*B(2,2)+B(1,2)*B(2,1))
      CALL MULT (M,M,C,B,TEMP)
      X(1,1) = A(2,2)
      X(1,2) =-A(1,2)
      X(2,1) = -A(2,1)
      X(2,2) = A(1,1)
      CALL MULT (M,M,X,C,A)
      CALL ADD (M,M,TEMP,A,TEMP)
      B(1,1) = C1*B(1,1)+C2
      B(2,2) = C1*B(2,2)+C2
      B(1,2) = C1*B(1,2)
      B(2,1) = C1*B(2,1)
      CALL INVERT ( M,B,A,ISTAT)
      IF (ISTAT.NE.0) RETURN
      CALL MULT (M,M,TEMP,A,X)
```

```
      RETURN
      END
C
      SUBROUTINE IDENT (M,A)
C  RETURNS IDENTITY MATRIX IN A
      COMPLEX*16 A(M,M)
      DO 10 J = 1,M
      DO 10 I = 1,M
         A(I,J) = (0.D0,0.D0)
         IF (I.EQ.J) A(I,J) = (1.D0,0.D0)
 10   CONTINUE
      RETURN
      END
C
      SUBROUTINE INVERT (M,A,B,ISTAT)
C  INVERTS MATRIX A WITH RESULT IN MATRIX B FOR M=1,2 CASES ONLY
      COMPLEX*16 A(M,M),B(M,M),DET,T
      IF (M.EQ.1) GO TO 10
      IF (M.EQ.2) GO TO 20
      ISTAT = 1
      RETURN
 10   B(1,1) = 1.D0/A(1,1)
      RETURN
 20   DET = A(1,1)*A(2,2)-A(1,2)*A(2,1)
      IF (CDABS(DET).NE.0.) GO TO 30
      ISTAT = 2
      RETURN
 30   T = 1.D0/DET
      B(1,1) = A(2,2)*T
      B(1,2) =-A(1,2)*T
      B(2,1) =-A(2,1)*T
      B(2,2) = A(1,1)*T
      RETURN
      END
C
      SUBROUTINE MULT (M,N,A,B,C)
C  MULTIPLIES M X M MATRIX A WITH M X N MATRIX B
C  WITH RESULT IN M X N MATRIX C
      COMPLEX*16 A(M,M),B(M,N),C(M,N),S
      DO 10 J = 1,N
      DO 10 I = 1,M
```

```
        S = (0.D0,0.D0)
        DO 20 K = 1,M
 20       S = S + A(I,K)*B(K,J)
 10     C(I,J) = S
      RETURN
      END
C
      SUBROUTINE OUTPROD (M,X,Y,A)
C  COMPUTES OUTER PRODUCT OF VECTORS X AND Y WITH
C  RESULT IN MATRIX A
      COMPLEX*16 X(M),Y(M),A(M,M)
      DO 10 J = 1,M
      DO 10 I = 1,M
 10     A(I,J) = X(I)*DCONJG(Y(J))
      RETURN
      END
C
      SUBROUTINE SCALE (M,N,S,A,B)
C  SCALES A M X N MATRIX A BY A SCALAR S WITH RESULT IN MATRIX B
      COMPLEX*16 S,A(M,N),B(M,N)
      DO 10 J = 1,N
      DO 10 I = 1,M
 10     B(I,J) = S*A(I,J)
      RETURN
      END
C
      SUBROUTINE SQROOT (M,A,B,ISTAT)
C  COMPUTES LOWER TRIANGULAR "SQUARE ROOT" MATRIX OF HERMITIAN
C  MATRIX A WITH RESULT IN MATRIX B FOR M = 1,2 CASES ONLY
      COMPLEX*16 A(M,M),B(M,M)
      REAL*8 C,X
      IF  (M.LE.2) GO TO 10
      ISTAT = 1
      RETURN

 10   X = DREAL(A(1,1))
      C =DSQRT(X)
      B (1,1) = DCMPLX(C,0.D0)
      IF (M.EQ.1) RETURN
      B(2,1) = A(2,1)/C
      B(1,2) = 0.D0
```

```
      C = DREAL (A(2,2)-A(1,2)*A(2,1)/A(1,1))
      B(2,2) = DCMPLX(DSQRT(C),0.D0)
      RETURN
      END
C
      SUBROUTINE SUB (M,N,A,B,C)
C  SUBTRACTS M X N MATRIX B FROM MATRIX A WITH RESULT IN MATRIX C
      COMPLEX*16 A(M,N),B(M,N),C(M,N)
      DO 10 J = 1,N
      DO 10 I = 1,M
 10     C(I,J) = A(I,J)-B(I,J)
      RETURN
      END
C
      SUBROUTINE XPOSE (M,A,B)
C  COMPLEX CONJUGATE TRANSPOSE OF MATRIX A PLACED IN MATRIX B
      COMPLEX*16 A(M,M),B(M,M)
      DO 10 J = 1,M
      DO 10 I = 1,M
 10     B(I,J) = DCONJG(A(J,I))
      RETURN
      END
C
      SUBROUTINE ZERO (M,N,A)
C  PLACES ZEROS IN M X N MATRIX A
      COMPLEX*16 A(M,N)
      DO 10 J = 1,N
      DO 10 I = 1,M
 10     A(I,J) = (0.D0,0.D0)
      RETURN
      END
\end{singlespace}
```

```
C*************************************************************
C PROGRAM: FRFAVE              ALLEN W. CLEM       10-30-90  *
C                              MECHANICAL ENGINEERING DEPT   *
C                              VIRGINIA POLYTECHNIC INSTITUTE*
C                              & STATE UNIVERSITY            *
C                                                            *
C LATEST REVISION:                                           *
C REVISED BY:                                                *
C                                                            *
C PROGRAM DESCRIPTION:                                       *
C                                                            *
C THIS PROGRAM USES THE OUTPUT FILE FROM MAR-FRF TO OBTAIN AN *
C ENSEMBLE AVERAGE FOR THE CORE-EXIT TEMPERATURE PSD, THE    *
C IN-CORE NEUTRON FLUX PSD, AND THE CPSD BETWEEN THE TWO. THE *
C PSD'S AND THE CPSD ARE READ IN AND AVERAGED IN THE FREQUENCY*
C DOMAIN TO PRODUCE AVERAGED RESULTS FOR THE PSD'S, THE CPSD, *
C AND THE FREQUENCY RESPONSE FUNCTIONS. THE FRF IS THEN      *
C AVERAGED OVER THE FREQUENCY BAND OF HIGHEST COHERENCE      *
C (0.1 TO 0.5 HZ) TO GIVE AN AVERAGE VALUE OF THE FRF.       *
C                                                            *
C                                                            *
C VARIABLE DESCRIPTION:                                      *
C                                                            *
C COH       - AVERAGE COHERENCE VALUE FOR A GIVEN FREQUENCY  *
C F         - FREQUENCY (HZ)                                 *
C GXX       - TEMPERATURE PERTURBATION PSD VALUE READ IN FROM *
C             FILE                                           *
C GXXAVE    - AVERAGE TEMPERATURE PSD VALUE FOR GIVEN FREQ   *
C GXY       - CPSD VALUE                                     *
C GXYAVE    - AVERAGE CPSD VALUE FOR A GIVEN FREQUENCY       *
C GXYIMAG   - IMAGINARY PART OF CPSD READ IN FROM FILE       *
C GXYREAL   - REAL PART OF CPSD READ IN FROM FILE            *
C GYY       - NEUTRON FLUX PERTURBATION PSD VALUE READ IN    *
C             FROM FILE                                      *
C GYYAVE    - AVERAGE FLUX PSD VALUE FOR A GIVEN FREQUENCY   *
C H1        - H1 FRF ESTIMATOR FOR A GIVEN FREQUENCY         *
C H1AVE     - AVERAGE H1 FRF MAGNITUDE OVER FREQUENCY BAND   *
C H2        - H2 FRF ESTIMATOR FOR A GIVEN FREQUENCY         *
C H2AVE     - AVERAGE H2 FRF MAGNITUDE OVER FREQUENCY BAND   *
C H3        - H3 FRF ESTIMATOR FOR A GIVEN FREQUENCY         *
C H3AVE     - AVERAGE H3 FRF MAGNITUDE OVER FREQUENCY BAND   *
```

```
C ISTART    - BEGINNING FREQUENCY FOR FRF AVERAGE              *
C IEND      - ENDING FREQUENCY FOR FRF AVERAGE                 *
C NAVE      - NUMBER OF AVERAGES TO BE USED IN ANALYSIS        *
C NF        - NUMBER IF FREQUENCIES PER AVERAGE                *
C PHASE     - PHASE ANGLE FOR GIVEN FREQUENCY                  *
C                                                              *
C DATA FILE DESCRIPTION:                                       *
C                                                              *
C INPUT FILE:   PSDDATA OUT                                    *
C OUTPUT FILE   FRFAVE OUT                                     *
C                                                              *
C***************************************************************

      PARAMETER (NAVE = 50)
      PARAMETER (NF = 20)
      PARAMETER (ISTART = 2)
      PARAMETER (IEND = 10)

      REAL*8 GXX(NAVE,NF),GYY(NAVE,NF),H1(NF),H2(NF),H3(NF)
      REAL*8 GXXAVE(NF),GYYAVE(NF),F(NF),GXYREAL,GXYIMAG
      REAL*8 COH(NF),PHASE
      COMPLEX*16 GXY(NAVE,NF), GXYAVE(NF)

C     OPEN (UNIT = 1,FILE = 'PSDDATA.OUT',STATUS = 'OLD')
C     OPEN (UNIT = 2,FILE = 'FRFAVE.OUT',STATUS = 'NEW')
C     OPEN (UNIT = 3,FILE = 'PSDAVE.OUT',STATUS = 'NEW')

      WRITE (3,10)
 10   FORMAT (1X,'FREQ',5X,'TEMP PSD', 5X,'FLUX PSD',13X,'CPSD'
     $ ,13X,'PHASE')

      WRITE (2,11)
 11   FORMAT (1X,'FREQ',9X,'H1',11X,'H2',11X,'H3',13X,'COH')
      WRITE (*,1)
  1   FORMAT (1X,'ENTER NUMBER OF AVERAGES ')
      READ(*,*) N

C INPUT PSD, CPSD DATA FROM FILE
      DO 2 I = 1,N
        DO 3 J = 1,NF
          READ(1,*) F(J),GXX(I,J),GYY(I,J),GXYREAL,GXYIMAG
```

```
            GXY(I,J) = CMPLX(GXYREAL,GXYIMAG)
3         CONTINUE
2       CONTINUE

C AVERAGE PSD, CPSD DATA
        DO 5 J = 1,NF
          GXXAVE(J) = 0.0D0
          GYYAVE(J) = 0.0D0
          GXYAVE(J) = 0.0D0
          DO 6 I = 1,N

            GXXAVE(J) = GXX(I,J) + GXXAVE(J)
            GYYAVE(J) = GYY(I,J) + GYYAVE(J)
            GXYAVE(J) = GXY(I,J) + GXYAVE(J)
  6       CONTINUE

        GXXAVE(J) = GXXAVE(J)/DBLE(N)
        GYYAVE(J) = GYYAVE(J)/DBLE(N)
        GXYAVE(J) = GXYAVE(J)/DBLE(N)

        H1(J) = CDABS(GXYAVE(J)/GXXAVE(J))
        H2(J) = CDABS(GYYAVE(J)/GXYAVE(J))
        H3(J) = (H1(J) + H2(J))/2.0D0
        COH(J) = (CDABS(GXYAVE(J)))**2/(GXXAVE(J)*GYYAVE(J))
        WRITE (2,7) F(J),H1(J),H2(J),H3(J),COH(J)
        PHASE = DATAN(DIMAG(GXYAVE(J))/DREAL(GXYAVE(J)))*57.29578D0

C PRINT OUT AVERAGE PSD, CPSD, PHASE
        WRITE (3,33) F(J),GXXAVE(J),GYYAVE(J),GXYAVE(J),PHASE
 33      FORMAT (1X,F5.2,4E13.5,F7.2)

  7     FORMAT(1X,F5.2,4E14.5)

  5     CONTINUE

        H1AVE = 0.0
        H2AVE = 0.0
        H3AVE = 0.0

        DO 8 J = ISTART, IEND
          H1AVE = H1AVE + H1(J)
```

```
        H2AVE = H2AVE + H2(J)
 8      H3AVE = H3AVE + H3(J)

      H1AVE = H1AVE/DBLE(IEND-ISTART)
      H2AVE = H2AVE/DBLE(IEND-ISTART)
      H3AVE = H3AVE/DBLE(IEND-ISTART)

      WRITE (2,9) H1AVE,H2AVE,H3AVE
 9    FORMAT(1X,3E14.5)

      STOP
      END
```

VITA

The author was born on October 19, 1966 in Winchester, Virginia. He has spent the majority of his years living in rural Gordonsville, Virginia.

Allen entered the Virginia Tech College of Engineering in the fall of 1984, and received his B. S. degree in mechanical engineering in May, 1989. Allen decided to continue his engineering education through graduate level studies in mechanical engineering at VPI to become more knowledgeable about his chosen field. He received his Master's degree in December, 1990.