A STUDY OF THE EFFECT OF CRACKS ON THE RESONANT FREQUENCY AND DYNAMIC MODULUS OF ELASTICITY OF INSULATING BRICK BY THE DYNAMIC TESTING METHOD

by

Lim Yock-Lio

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APPROVED:

Louis A. Pardue
Director of Graduate Studies

Dean of Engineering

APPROVED:

Head of Department

Supervisor

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TABLE OF CONTENTS

I

INTRODUCTION AND LITERATURE REVIEW

For a great many years, the sonic test has been recognized as a most outstanding non-destructive means for measuring Young's Modulus of Elasticity of various materials, and owing to its simplicity, accuracy and convenience in measuring, its usefulness has immensely progressed and expanded. Early in the year of 1935 the sonic testing method was first employed by Grime[1] in the determination of Young's Modulus of Elasticity for some building materials such as roofing tiles, brick, mortar and asbestos-cement sheeting. However, one of the most successful uses of this dynamic technique has been in the study of the deterioration and durability of concrete. In 1938 Powers[2] compared dynamically and statically determined values of Young's Modulus of Elasticity and indicated that they were in good agreement. Hornibrook[3], Thomson[4], Lewis and Batchelder[5] and many other investigators in their study of the deterioration of concrete stated that the sonic testing method is sensitive, accurate and suitable for the studies of this nature.

Late in 1939 the sonic method was used by Koenig[6] for the determination of the pitch and fatigue of some ceramic bodies by which correlation between composition, physical properties and the degree of firing were established. Forgue and Loomis[7] in 1941 showed that the sonic method gives more consistent results and requires less time than the stress-strain method.

Not until 1948 did Baab and Kraner[8] report that the sonic method of determining the dynamic modulus of elasticity was suitable to ascertain the degree of firing of brick. They also recommended that the test can be used by the manufacturer of refractories as a tool for control work and by consumers for the purpose of checking specifications.

E. R. Glabau[9] in 1951 reported that Pickett's[15] formula for the determination of dynamic modulus of elasticity is not only applicable to concrete but also to fire clay brick. Moreover, a positive correlation between the dynamic modulus of elasticity and modulus of rupture for the conventional fire clay refractory brick made by the stiff mud process was reported. However, because of the small number of samples, Glabau was unable to draw any definite conclusions regarding the relationship between the modulus of elasticity and the modulus of rupture. However, from an implication in the earlier work of Glabau, a more extensive study was made by A. J. Metzger, E. R. Glabau, and T. S. Smith[10] in 1953. They showed a positive correlation between dynamic modulus of elasticity and modulus of rupture for fire clay brick. They also reported that the dynamic modulus of elasticity can be used to predict the modulus of rupture for brick made by the same process and from the same area. About a year later Mayberry[11] also found a positive linear correlation between the dynamic modulus of elasticity and the modulus of rupture for both hard-burned and regular burned super duty fire brick.

In spite of the general good correlation between the dynamic modulus of elasticity and modulus of rupture it frequently happens that brick which would be expected to be quite strong, based on their elastic modulus, fail under rather small loads. This would indicate that one or more of the variables which influence the modulus of rupture do not exert the same influence on the modulus of elasticity. Or again, certain variables which cause a marked lowering of the rupture value might cause only a small lowering of the elasticity. Discontinuities in the structure of a brick would naturally be expected to lower the modulus of rupture. Would all discontinuities have the same effect on the modulus of elasticity? Perhaps the position or the orientation of the discontinuities would cause differences of opposite kinds in the rupture and elasticity values. The present investigation was undertaken to determine what effect, if any, the orientation and position of discontinuities, in the form of narrow slots, would have on the dynamic modulus of elasticity. Because of lack of time, no investigation of the effect of the discontinuities on the modulus of rupture was carried out.

II

METHOD OF PROCEDURE

1. Apparatus and Testing Procedure

The principal features of the dynamic testing apparatus used for this investigation can be seen from the picture in plate 1. The apparatus is similar to that designed and used by the Bureau of Standards on concrete specimens and has been described by Hornibrook[12]. It consists essentially of the following parts:

1. Crystal pick-up.
2. Driver Mechanism.
3. Vacuum tube voltmeter.
4. Variable frequency audio oscillator.
5. Oscillograph.

The general procedures of testing can be briefly described as follows:

The bar to be tested is supported at its nodal positions by two straight piano wires. The piano wire supports are adjustable to provide for different specimen lengths. For the flexural vibration of a free-free bar the nodal points have been found to be 0.224 times the length of the specimen from each end.[13] A movable loudspeaker with a light aluminum rod cemented to its voice coil is mounted on the channel of an iron frame. This device serves as a vibration exciter and is called the "driver." The point of excitation is located below the midpoint of the test bar. A Hewlett-Packard, model 200 AB, variable audio oscillator

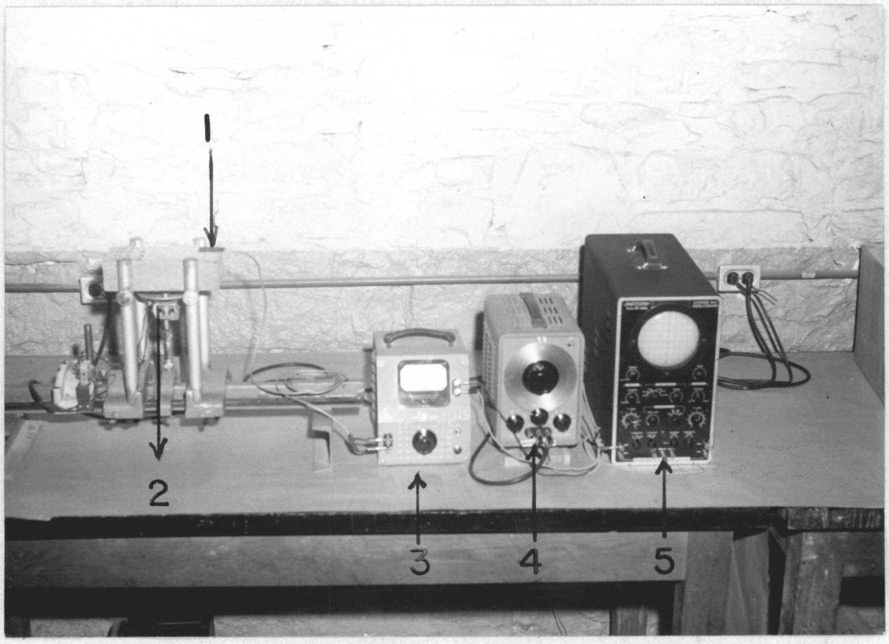


PLATE 1 Sonic Apparatus for Determining the Resonant Frequency of Specimen

1. Crystal pick-up
2. Driver
3. Vacuum tube voltmeter
4. Variable Audio Oscillator (Hawlett-Packard, Model 200AB)
5. Oscillograph

was provided for driving this excitation unit. When the frequency oscillator is in operation it vibrates the "driver" and the vibration is in turn transferred to the bar by the light aluminum rod. The vibration of the bar is transmitted to a crystal pick-up which is held by a rubber band on the upper surface at one end of the test specimen. The output of the pick-up is connected to the vertical plates of the oscillograph through a vacuum tube voltmeter, and the horizontal deflection plates of the oscillograph are connected directly to the oscillator. The function of the oscillograph is to compare on the screen of the cathode-ray tube the variation that occurs in an unknown quantity with that which occurs in a known quantity. In this case, the frequency of the vibrating bar and the frequency of the exciter constitute the unknown and the known quantities respectively. The frequency of the oscillator is varied and at the same time the screen of the oscillograph and reading of the vacuum tube voltmeter are observed. When the frequencies of the oscillator and the vibrating bar are equal an ellipse, a straight line or a circle depending upon the phase difference, will be seen on the screen of the cathode-ray oscillograph. These are the well known Lissajous figures. At the same instant a peak value or maximum deflection point of the vacuum tube voltmeter will occur. The resonant frequency is indicated by the dial of the oscillator.

A simple rule for determining the frequency ratio from a Lissajous pattern has been given by Rutter[14]. Imagine a horizontal line AB and a vertical line AC as shown in Figure 1.

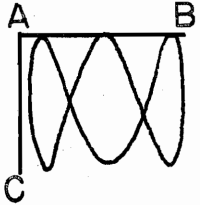


FIG. I

There are three loops tangent to AB and one loop tangent to AC. The ratio of the frequency applied to the vertical channel to the frequency applied to the horizontal channel is 3:1. Mathematically, it can be expressed as follows:

$$\frac{\text{Frequency applied to the vertical channel}}{\text{Frequency applied to the horizontal channel}} = \frac{\text{number of loops tangent to horizontal line}}{\text{number of loops tangent to vertical line}}$$

2. Experimental Detail

Forty commercial samples of 9" straight high temperature insulating brick were taken at random and marks were made for identification.

A steel rule 12 inches in length, graduated in hundreths of an inch was used for measuring the length, width and thickness of all the samples. To affirm the precision of reading the measurements, a magnifying glass was used in conjunction with the steel rule. Each measurement was recorded to the nearest 0.01 inch. A Toledo scale having a dial graduated in 0.01 pounds was used to determine the weight of the samples. The resonant frequency was determined as described previously for each specimen. From these data, namely, the dimensions, shapes, weights and fundamental frequencies, the dynamic modulus of elasticity was computed. A number of methods have been proposed for the calculation of the modulus of elasticity for bars vibrating in the fundamental mode of flexural vibration with free ends. The simplest and most accurate method for practical purposes is that of G. Pickett[15]. The equation is expressed as follows:

$$E = C\,W\,N^2$$

where :

E = Modulus of elasticity

W = Weight of specimen

N = A resonant frequency

C = A factor which depends upon the shape and size of specimen, the mode of vibration and Poisson's ratio

For the first mode of vibration of a rectangular prism the shape factor C can be expressed as:

$$C = 0.0024523 \left(\frac{l^3}{b\,t^3}\right) T_1$$

Where :

T_1 = correction factor depending upon Poissons ratio and on the ratio of the radius of gyration to the length of the specimen. Equations are also given for various values of Poisson's ratio (μ) in computing T_1.

For $\mu = 0$

$$T_1 = 1 + 79.02\left(\frac{r}{l}\right)^2 - \frac{1201\left(\frac{r}{l}\right)^4}{1 + 76.06\left(\frac{r}{l}\right)^2} - 125\left(\frac{r}{l}\right)^4$$

For $\mu = \frac{1}{6}$

$$T_1 = 1 + 81.79\left(\frac{r}{l}\right)^2 - \frac{1314\left(\frac{r}{l}\right)^4}{1 + 81.09\left(\frac{r}{l}\right)^2} - 125\left(\frac{r}{l}\right)^4$$

For $\mu = \frac{1}{3}$

$$T_1 = 1 + 86.12\left(\frac{r}{l}\right)^2 - \frac{1572\left(\frac{r}{l}\right)^4}{1 + 92.61\left(\frac{r}{l}\right)^2} - 125\left(\frac{r}{l}\right)^4$$

Where :

r = Radius of gyration

l = Length of specimen

b and t = Dimensions of cross section of prism, t being in direction of vibration

In order to shorten the calculation the value of the shape factor C can be obtained from the graph given by Pickett.

Based upon the dynamic modulus calculated, the samples were then classified into four groups, each group consisting of four specimens. The four specimens within each group were so selected that the range of the dynamic modulus of elasticity did not exceed a value of 2×10^4 p s i.

Each of the bricks was cut lengthwise into four equal parts using a silicon carbide wheel. Each group of specimens cut from a single brick was numbered from 1 through 4 starting consistently from the outside. Each of the 64 specimens thus obtained was numbered with its original brick number followed by a 1, 2, 3, or 4 depending upon the position from which it came. All specimens marked 1 and 4 were from the outer part of the original brick while those marked 2 and 3 were from the interior portion. In order to minimize the error of thickness measurements the cut faces of the specimens were ground on a silicon carbide plate. After smoothing, the specimens were measured, weighed and vibrated in the same manner as were the original brick. The modulus of elasticity was computed. Because the depth is the most important dimension in the calculation of the modulus of elasticity from the data six thickness measurements, using a micrometer caliper, were made on each specimen and averaged. Readings were taken at each corner and at the midpoint of each side. These readings were recorded to the nearest 0.001 inch.

To determine the effect of the location and orientation of discontinuity on the dynamic modulus of elasticity the following experiments were conducted:

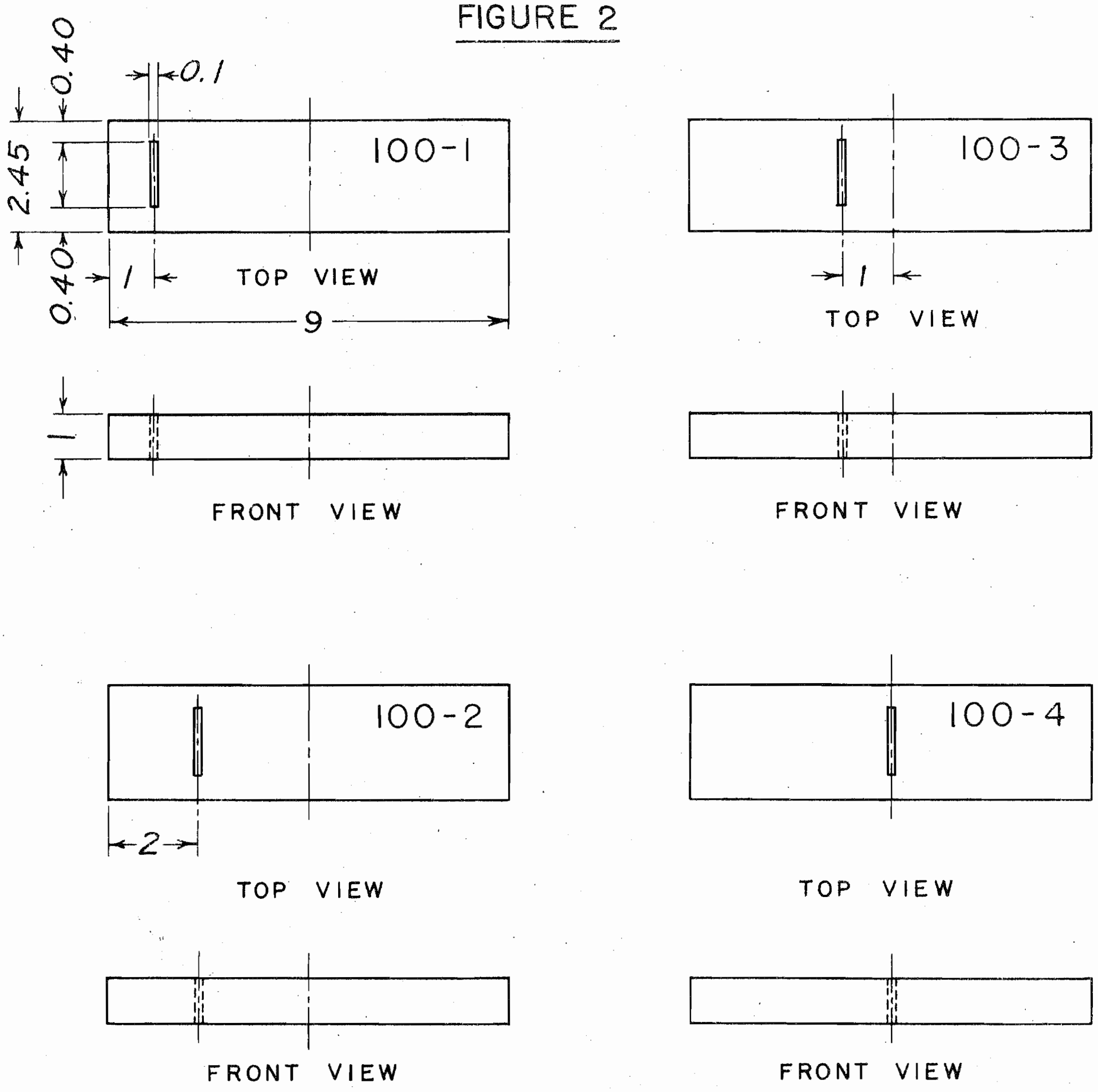
First, to each of the four small bars cut from the same 9" brick vertical slots normal to the major axis of the bars were made by means of a hack saw. Care was exercised to maintain the size and shape of the slots as consistent as possible throughout all the bars. See Figure 2.

Second, after the slots had been made, the weights and frequencies of the bars were redetermined, and the dynamic modulus of elasticity was computed. With all the necessary information obtained the change in frequency (ΔN), change in weight (ΔW), change in modulus of elasticity (ΔE), change in modulus of elasticity per unit change in weight $\frac{\Delta E}{\Delta W}$ and the percentage of change in modulus of elasticity $\frac{\Delta E}{E}$ were computed.

In order to investigate what was the real effect of orientation and location of discontinuities on the dynamic modulus of elasticity, an experiment on bars with slots parallel to their major axis was suggested. Because of the limited number of samples, an additional experiment was carried out to test the possibility of re-use of the previous test bars, that is, bars which already had slots normal to their major axis.

Following the same procedure as described before, a fifth group of samples consisting of eight small bars was obtained for the purpose of this experiment. The dimensions, frequencies, weights and dynamic

FIGURE 2



SCALE - 3" = 1'0"

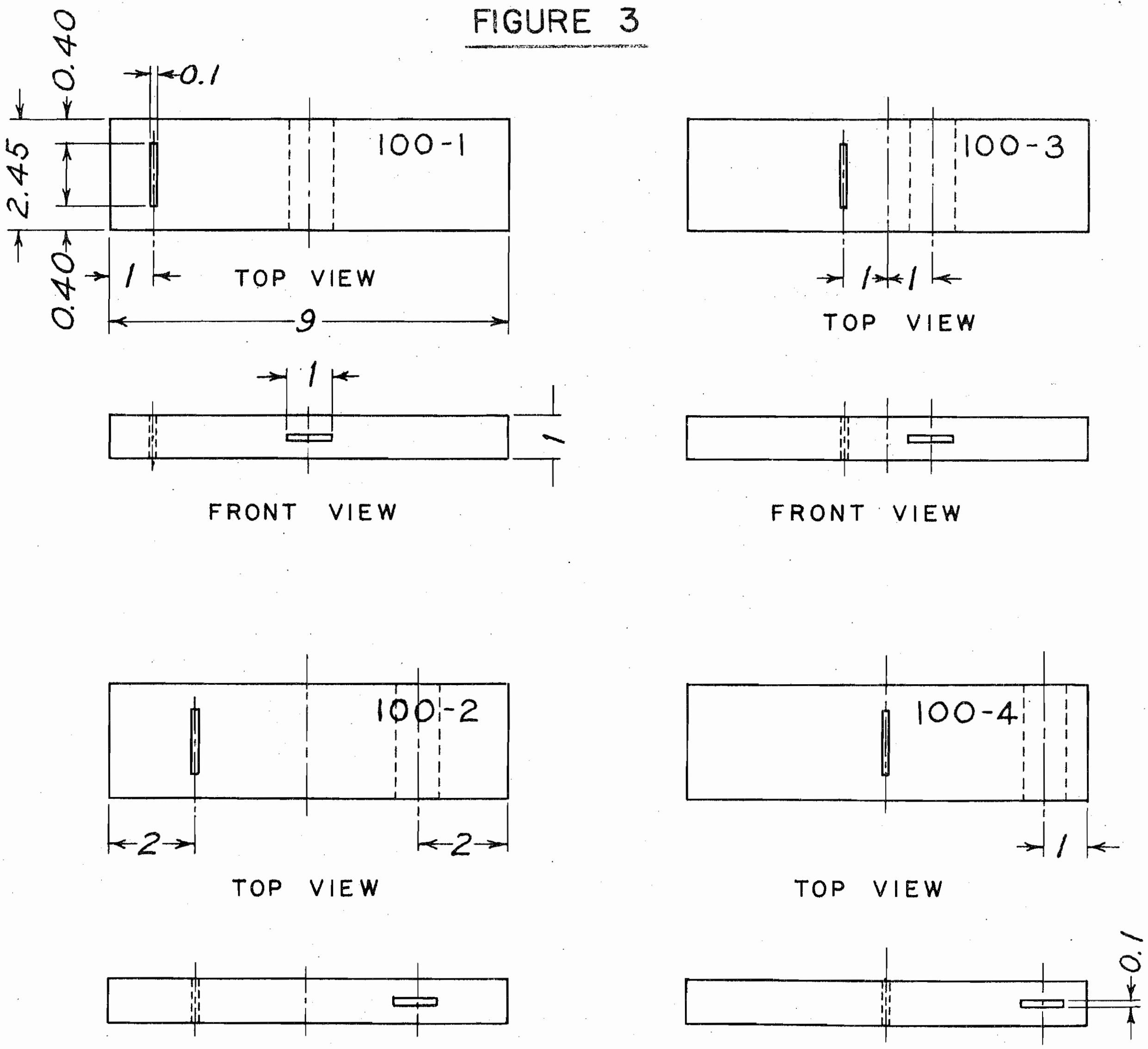
ALL DIMENSIONS IN INCHES

modulus of elasticity were determined as before. To each of the four bars cut from the same 9" brick slots were made parallel to the major axis. See Figure 3. Again, care was exercised to maintain the size and shape of the slots as consistent as possible throughout all the bars and the modulus of elasticity was redetermined. After the dynamic modulus had been obtained, additional slots normal to the major axis were made in the same bars and the modulus of elasticity was again determined.

Following the same pattern as shown in Figure 3, slots parallel to the major axis were made in the eight bars of Group I which already had slots normal to their major axis, and the modulus of elasticity was redetermined. The results of this experiment are recorded in Table I. The results of this experiment show the justification of re-use of the previous bars. This will be discussed in detail later. Hence, all the previous bars were used again and treated precisely in the same manner as Figure 3. The dynamic test was made.

In order to discover the effect of the thickness of a crack on the dynamic modulus of elasticity of the brick, eight small test bars were selected at random. Vertical slots were cut in each bar normal to the major axis. These slots were of the same width and thickness as those cut in all of the previous samples. After measuring and weighing, the bars were vibrated and the modulus of elasticity was calculated. After determining the modulus of elasticity the thickness of the slots was increased to twice the original thickness and again the bars were weighed and vibrated. The dynamic modulus of elasticity was again computed.

FIGURE 3



SCALE - 3" = 1'0"

ALL DIMENSIONS IN INCHES

Finally, each slot was once more enlarged; this time the thickness of the slot was increased to three times the original value. Once more the data were secured and the modulus of elasticity was calculated.

III

DATA AND RESULTS

TABLE I

Results for Changes in Resonant Frequency and Modulus of Elasticity Due to Introduced Discontinuity.

Sample	Horizontal Discontinuity Parallel to Major Axis		Vertical Discontinuity Normal to Major Axis	
	ΔN (CPS)	$\%\Delta E$	ΔN (CPS)	$\% \Delta E$
129-1	0	1.00	210	35.79
129-2	5	2.44	40	9.50
129-3	5	2.40	100	22.80
129-4	0	0.73	10	2.89
133-1	0	1.44	130	29.60
133-2	5	2.67	30	8.35
133-3	0	1.45	95	23.10
133-4	5	2.72	10	1.89

Sample	Vertical Discontinuity Normal to Major Axis		Horizontal Discontinuity Parallel to Major Axis	
	ΔN (CPS)	$\% \Delta E$	ΔN (CPS)	$\%\Delta E$
100-1	170	32.33	5	1.97
100-2	40	9.82	10	3.46
100-3	110	25.75	0	0.91
100-4	15	6.25	0	0.94
107-1	174	33.49	0	0.51
107-2	42	10.64	10	3.37
107-3	115	25.61	0	1.02
107-4	10	2.87	5	2.26

TABLE II

Data for Determination of Modulus of Elasticity of Nine-Inch Straight Insulating Brick.

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	Weight (lbs.)	N Cycle/sec.	E x 10^6 (Psi)
100	8.98	2.46	4.47	3.65	0.175	0.0391	3.633	2310	0.7580
101	8.97	2.45	4.47	3.66	0.176	0.0394	3.736	2230	0.7320
102	8.97	2.45	4.47	3.66	0.176	0.0394	3.777	2380	0.8429
103	8.97	2.45	4.47	3.66	0.176	0.0394	3.821	2400	0.8671
104	8.99	2.46	4.47	3.65	0.175	0.0391	3.745	2290	0.7679
105	8.96	2.45	4.46	3.65	0.176	0.0395	3.717	2310	0.7834
106	8.97	2.46	4.47	3.64	0.175	0.0391	3.776	2310	0.7878
107	8.97	2.46	4.47	3.63	0.175	0.0391	3.855	2240	0.7563
108	8.95	2.45	4.47	3.65	0.175	0.0391	3.712	2315	0.7778
109	8.95	2.46	4.47	3.64	0.174	0.0390	3.734	2320	0.7838
110	8.94	2.46	4.45	3.63	0.174	0.0391	3.573	2020	0.5700
111	8.95	2.46	4.46	3.64	0.174	0.0390	3.613	2060	0.5980
112	8.97	2.46	4.47	3.65	0.175	0.0391	3.953	2310	0.8247
113	8.97	2.46	4.47	3.65	0.175	0.0391	3.756	2270	0.7567

TABLE II Continued

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	Weight (lbs.)	N Cycle/sec.	E x 10^6 (Psi)
114	8.93	2.45	4.46	3.64	0.175	0.0392	3.577	1980	0.5497
115	8.93	2.45	4.46	3.64	0.175	0.0392	3.763	2225	0.7303
116	8.97	2.46	4.47	3.65	0.175	0.0391	3.776	2360	0.8223
117	8.97	2.46	4.46	3.65	0.175	0.0392	3.854	2290	0.7923
118	8.99	2.46	4.48	3.65	0.175	0.0391	3.867	2330	0.8208
119	8.96	2.46	4.46	3.66	0.175	0.0392	3.783	2320	0.7982
120	8.96	2.45	4.47	3.66	0.175	0.0391	3.566	1980	0.5466
121	8.97	2.46	4.47	3.64	0.175	0.0391	3.613	2370	0.7935
122	8.95	2.44	4.45	3.66	0.176	0.0395	3.703	2020	0.5968
123	8.97	2.46	4.46	3.64	0.175	0.0392	3.796	2370	0.8358
124	8.94	2.47	4.47	3.62	0.173	0.0387	3.687	2225	0.7064
125	8.94	2.48	4.46	3.60	0.170	0.0381	3.805	2190	0.6953
126	8.94	2.47	4.47	3.62	0.172	0.0385	3.706	2170	0.6718
127	8.93	2.46	4.45	3.63	0.174	0.0391	3.687	1960	0.5538
128	8.93	2.46	4.47	3.63	0.174	0.0389	3.714	2055	0.6101

TABLE II Continued

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	Weight (lbs.)	N Cycle/sec.	E x 10^6 (Psi)
129	8.94	2.47	4.46	3.62	0.173	0.0388	3.881	2400	0.8674
130	8.94	2.46	4.45	3.63	0.175	0.0393	3.607	2040	0.5899
131	8.96	2.47	4.46	3.62	0.172	0.0386	3.723	2190	0.6892
132	8.95	2.48	4.47	3.60	0.171	0.0382	3.853	2490	0.9126
133	8.96	2.46	4.47	3.62	0.173	0.0387	3.717	2260	0.7347
134	8.96	2.46	4.47	3.64	0.176	0.0394	3.758	2050	0.6222
135	8.94	2.47	4.47	3.62	0.173	0.0387	3.920	2220	0.7476
136	8.94	2.47	4.47	3.62	0.173	0.0387	3.726	2090	0.6298
137	8.95	2.47	4.47	3.62	0.172	0.0385	3.828	2400	0.8489
138	8.96	2.48	4.46	3.61	0.171	0.0382	3.938	2170	0.7084
139	8.95	2.47	4.46	3.62	0.172	0.0385	3.843	2440	0.8808

TABLE III

Classification of Brick According to Modulus of Elasticity.

	$\frac{L}{D}$	C b	C	Weight (lbs)	N (Cycle/sec.)	$E \times 10^6$ (Psi)
GROUP I						
100	3.65	0.175	0.0391	3.633	2310	0.7580
107	3.63	0.175	0.0391	3.855	2240	0.7563
113	3.65	0.175	0.0391	3.756	2270	0.7567
115	3.64	0.175	0.0392	3.763	2225	0.7303
GROUP II						
105	3.66	0.176	0.0395	3.717	2310	0.7834
109	3.64	0.174	0.0390	3.734	2320	0.7838
117	3.64	0.175	0.0392	3.854	2290	0.7923
119	3.66	0.175	0.0392	3.783	2320	0.7982
GROUP III						
112	3.65	0.175	0.0391	3.953	2310	0.8247
116	3.64	0.175	0.0391	3.776	2360	0.8223
118	3.65	0.175	0.0391	3.867	2330	0.8208
123	3.65	0.175	0.0392	3.796	2370	0.8358
GROUP IV						
110	3.63	0.174	0.0391	3.573	2020	0.5700
114	3.64	0.175	0.0392	3.577	1980	0.5497
120	3.66	0.175	0.0391	3.566	1980	0.5466
127	3.63	0.174	0.0391	3.687	1960	0.5538

TABLE IV

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group I - Table III

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	W_1 (lbs.)	N_1 (Cycle/sec.)	$E_1 \times 10^6$ (Psi)
100-1	8.97	0.919	2.46	9.76	2.46	1.000	0.723	900	0.5856
100-2	8.97	0.908	2.46	9.88	2.53	1.028	0.712	840	0.5165
100-3	8.97	0.852	2.46	10.53	3.05	1.240	0.673	810	0.5475
100-4	8.97	0.995	2.46	9.02	1.94	0.789	0.763	970	0.5664
107-1	8.97	0.942	2.46	9.52	2.29	0.931	0.790	990	0.7208
107-2	8.97	0.940	2.46	9.54	2.30	0.935	0.787	842	0.5217
107-3	8.96	0.972	2.45	9.22	2.19	0.894	0.800	860	0.5290
107-4	8.97	0.983	2.46	9.12	2.01	0.817	0.814	859	0.6116
113-1	8.97	0.720	2.45	12.46	4.82	1.967	0.601	750	0.6283
113-2	8.97	0.740	2.45	12.12	4.50	1.837	0.609	682	0.6650
113-3	8.97	0.720	2.45	12.46	4.82	1.967	0.608	681	0.5203
113-4	8.97	0.710	2.45	12.63	5.02	2.050	0.608	710	0.5546
115-1	8.97	0.854	2.47	10.50	3.00	1.214	0.690	820	0.5632
115-2	8.96	0.825	2.47	10.86	3.31	1.340	0.668	725	0.4705
115-3	8.96	0.852	2.47	10.52	3.02	1.223	0.699	755	0.4873
115-4	8.96	0.873	2.47	10.26	2.82	1.142	0.721	825	0.5604

TABLE V

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group II - Table III.

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	W_1 (lbs.)	N_1 (Cycle/sec.)	$E_1 x 10^6$ (Psi)
105-1	8.96	0.854	2.45	10.49	3.10	1.265	0.686	879	0.6705
105-2	8.97	0.893	2.45	10.45	2.98	1.216	0.721	840	0.6186
105-3	8.96	0.885	2.45	10.12	2.71	1.106	0.707	810	0.5130
105-4	8.97	0.856	2.44	10.47	2.99	1.225	0.701	880	0.6650
109-1	8.98	0.796	2.46	11.28	3.65	1.484	0.639	795	0.5993
109-2	8.97	0.861	2.46	10.42	3.10	1.260	0.700	310	0.5787
109-3	8.96	0.886	2.45	10.11	2.64	1.077	0.716	860	0.5703
109-4	8.96	0.864	2.46	10.37	2.90	1.179	0.701	900	0.6694
117-1	8.98	0.926	2.46	9.70	2.40	0.976	0.758	980	0.7105
117-2	8.98	0.886	2.47	10.14	2.72	1.101	0.732	795	0.5094
117-3	8.98	0.936	2.47	9.59	2.40	0.972	0.791	860	0.5686
117-4	8.98	0.913	2.47	9.84	2.50	1.012	0.777	975	0.7475
119-1	8.95	0.928	2.46	9.64	2.38	0.967	0.776	980	0.7206
119-2	8.96	0.835	2.46	10.73	3.17	1.289	0.688	785	0.5465
119-3	8.96	0.920	2.46	9.74	2.42	0.984	0.755	860	0.5494
119-4	8.96	0.868	2.46	10.32	2.88	1.171	0.702	875	0.6294

TABLE VI

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group III - Table III.

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	W_1 (lbs.)	N_1 (Cycle/sec)	$E_1 \times 10^6$ (Psi)
112-1	8.97	0.748	2.46	11.99	4.39	1.784	0.618	760	0.6368
112-2	8.97	0.756	2.46	11.86	4.25	1.728	0.633	710	0.5514
112-3	8.97	0.727	2.46	12.34	4.72	1.919	0.629	675	0.5499
112-4	8.97	0.743	2.46	12.07	4.41	1.793	0.661	730	0.6316
116-1	8.97	0.774	2.46	11.59	3.92	1.593	0.633	765	0.5901
116-2	8.97	0.763	2.46	11.76	4.11	1.671	0.615	710	0.5180
116-3	8.97	0.786	2.46	11.41	3.80	1.545	0.637	735	0.5317
116-4	8.97	0.805	2.46	11.14	3.52	1.431	0.661	840	0.6674
118-1	8.99	0.760	2.46	11.83	4.22	1.715	0.632	800	0.6937
118-2	8.99	0.824	2.46	10.91	3.35	1.362	0.680	745	0.5140
118-3	8.99	0.747	2.46	12.03	4.41	1.793	0.620	682	0.5170
118-4	8.99	0.813	2.46	11.08	3.42	1.390	0.689	855	0.7001
123-1	8.98	0.907	2.46	9.90	2.55	1.036	0.735	985	0.7388
123-2	8.98	0.868	2.46	10.34	2.71	1.102	0.710	810	0.5133
123-3	8.98	0.918	2.46	9.78	2.48	1.008	0.750	850	0.5462
123-4	8.98	0.934	2.46	9.61	2.41	0.980	0.763	985	0.7255

TABLE VII

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group IV - Table III.

Sample	Length (in.)	Depth (in.)	Width (in.)	$\frac{L}{D}$	C b	C	W_1 (lbs)	N_1 (Cycle/sec)	$E_1 \times 10^6$ (Psi)
110-1	8.95	0.861	2.47	10.39	2.91	1.178	0.666	750	0.4413
110-2	8.94	0.820	2.45	10.90	3.35	1.367	0.638	650	0.3685
110-3	8.94	0.838	2.46	10.66	3.11	1.264	0.651	680	0.3805
110-4	8.93	0.825	2.46	10.82	3.30	1.341	0.631	705	0.4206
114-1	8.96	0.837	2.46	10.70	3.19	1.296	0.648	725	0.4414
114-2	8.95	0.808	2.45	11.08	3.50	1.428	0.619	605	0.3235
114-3	8.95	0.862	2.45	10.38	2.90	1.184	0.667	660	0.3440
114-4	8.95	0.848	2.45	10.55	3.08	1.257	0.650	680	0.3778
120-1	8.95	0.817	2.46	10.95	3.40	1.382	0.624	720	0.4470
120-2	8.95	0.821	2.44	10.90	3.35	1.373	0.631	630	0.3438
120-3	8.96	0.832	2.46	10.77	3.22	1.309	0.642	650	0.3550
120-4	8.95	0.806	2.44	11.10	3.51	1.438	0.614	700	0.4326
127-1	8.95	0.797	2.45	11.23	3.62	1.477	0.641	690	0.4508
127-2	8.93	0.838	2.45	10.66	3.16	1.290	0.661	655	0.3658
127-3	8.94	0.816	2.45	10.96	3.41	1.392	0.655	630	0.3619
127-4	8.93	0.780	2.46	11.44	3.89	1.581	0.620	670	0.4400

TABLE VIII*

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group I - Table III. Vertical Discontinuity Normal to Major Axis.

Sample	G	W_2 (lbs)	N_2 Cycle/sec	$E_2 x10^6$ (Psi)	N_1-N_2 Cycle/sec	$(E_1-E_2)x10^6$ (Psi)	W_1-W_2 (lbs)	$\frac{E_1-E_2}{W_1-W_2}x10^6$	$\frac{E_1-E_2}{E1}$ (Per cent)
100-1	1.000	0.717	875	0.5490	15	0.0366	0.006	6.10	6.25
100-2	1.028	0.708	800	0.4658	40	0.0507	0.004	12.68	9.82
100-3	1.240	0.669	700	0.4065	110	0.1410	0.004	35.25	25.75
100-4	0.789	0.759	800	0.3833	170	0.1831	0.004	45.78	32.33
107-1	0.931	0.783	980	0.7001	10	0.0207	0.007	2.96	2.87
107-2	0.935	0.779	800	0.4662	42	0.0555	0.008	6.94	10.64
107-3	0.894	0.793	745	0.3935	115	0.1355	0.007	19.36	25.61
107-4	0.817	0.808	785	0.4068	174	0.2048	0.006	34.13	33.49
113-1	0.608	0.601	700	0.6037	10	0.0246	0.007	3.51	3.92
113-2	0.601	0.595	700	0.5735	50	0.0915	0.006	15.25	13.76
113-3	0.609	0.601	594	0.3895	88	0.1308	0.007	18.68	25.14
113-4	0.608	0.602	565	0.3780	116	0.1766	0.006	29.43	31.84
115-1	0.690	0.685	810	0.5456	10	0.0176	0.005	3.52	3.12
115-2	0.668	0.651	690	0.4153	35	0.0552	0.017	3.25	11.73
115-3	0.699	0.692	655	0.3631	100	0.1242	0.007	17.74	25.49
115-4	0.721	0.716	700	0.4006	125	0.1598	0.005	31.96	28.52

* - Subscript 2 in heading refers to data obtained from sample having vertical discontinuity normal to major axis only.

TABLE IX

Data for Determination of Modulus of Elasticity of 1 x 2½" x 9" Samples Cut from Group II - Table III. Vertical Discontinuity Normal to Major Axis.

Sample	G	W_2 (lbs.)	N_2 Cycle/sec.	$E_2 \times 10^6$ (Psi)	N_1-N_2 Cycle/sec.	$E_1-E_2 \times 10^6$ (Psi)	W_1-W_2 (lbs.)	$\frac{E_1-E_2}{W_1-W_2} \times 10^6$	$\frac{E_1-E_2}{E_3}$ (percent)
105-1	1.265	0.681	875	0.6595	4	0.0110	0.005	2.20	1.64
105-2	1.216	0.715	785	0.5358	55	0.0828	0.006	13.80	13.38
105-3	1.106	0.700	700	0.3794	110	0.1336	0.007	19.08	26.04
105-4	1.225	0.698	705	0.4249	175	0.2401	0.003	80.03	36.10
109-1	1.484	0.632	795	0.5928	0	0.0065	0.007	0.92	1.08
109-2	1.260	0.695	775	0.5258	30	0.0499	0.005	9.98	8.62
109-3	1.077	0.711	730	0.4040	130	0.1663	0.005	33.26	29.16
109-4	1.179	0.689	720	0.4211	180	0.2483	0.012	20.69	37.09
117-1	0.976	0.755	980	0.7077	0	0.0028	0.003	0.93	0.39
117-2	1.101	0.728	755	0.4569	40	0.0525	0.004	13.12	10.31
117-3	0.972	0.787	760	0.4418	100	0.1268	0.004	31.70	22.30
117-4	1.012	0.772	775	0.4692	205	0.2783	0.005	55.66	37.23
119-1	0.967	0.771	980	0.7160	0	0.0046	0.005	0.92	0.64
119-2	1.289	0.683	760	0.5085	25	0.0380	0.005	7.60	6.95
119-3	0.984	0.749	735	0.3981	125	0.1513	0.006	25.22	27.54
119-4	1.171	0.699	720	0.4243	155	0.2051	0.003	68.37	32.59

TABLE X

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group III - Table III Vertical Discontinuity Normal to Major Axis.

Sample	C	W_2 (lbs.)	N_2 Cycle/sec.	$E_2 \times 10^6$ (Psi)	N_1-N_2 Cycle/sec.	$E_1-E_2 \times 10^6$ (Psi)	W_1-W_2 (lbs.)	$\frac{E_1-E_2}{W_1-W_2 \times 10^6}$	$\frac{E_1-E_2}{E_1}$ (Percent)
112-1	1.784	0.612	755	0.6223	5	0.0145	0.006	6.92	2.28
112-2	1.728	0.630	680	0.5034	30	0.0480	0.003	16.00	8.87
112-3	1.919	0.625	585	0.4104	90	0.1395	0.004	34.88	25.37
112-4	1.793	0.658	615	0.4462	115	0.1854	0.003	61.80	29.35
116-1	1.593	0.631	760	0.5806	5	0.0095	0.002	4.75	1.61
116-2	1.671	0.612	700	0.5011	10	0.0169	0.003	5.63	3.26
116-3	1.545	0.632	650	0.4125	85	0.1192	0.005	23.84	22.42
116-4	1.431	0.659	700	0.4621	140	0.2053	0.004	51.32	30.76
118-1	1.715	0.630	800	0.6915	0	0.0022	0.002	1.10	0.32
118-2	1.362	0.675	715	0.4700	30	0.0440	0.005	8.80	8.56
118-3	1.793	0.617	605	0.4049	77	0.1121	0.003	37.37	21.68
118-4	1.390	0.684	720	0.4929	135	0.2072	0.005	41.44	29.57
123-1	1.036	0.731	985	0.7348	0	0.0040	0.004	1.00	0.54
123-2	1.102	0.708	780	0.4753	30	0.0380	0.002	19.00	7.40
123-3	1.008	0.748	760	0.4355	90	0.1107	0.002	55.35	20.27
123-4	0.980	0.759	785	0.4584	200	0.2671	0.004	66.78	36.82

TABLE XI

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group IV - Table III Vertical Discontinuity Normal to Major Axis.

Sample	C	W_2 (lbs.)	N_2 Cycle/sec.	$E_2 x 10^6$ (Psi)	$N_1 - N_2$ Cycle/sec.	$E_1 - E_2 x 10^6$ (Psi)	$W_1 - W_2$ (lbs.)	$\frac{E_1 - E_2}{W_1 - W_2 x 10^6}$	$\frac{E_1 - E_2}{E_1}$ (Percent)
110-1	1.178	0.662	745	0.4328	5	0.0085	0.004	2.12	1.93
110-2	1.367	0.632	620	0.3321	30	0.0364	0.006	6.07	9.87
110-3	1.264	0.648	580	0.2755	100	0.1050	0.003	35.00	27.60
110-4	1.341	0.628	580	0.2828	125	0.1378	0.003	32.76	32.76
114-1	1.296	0.642	720	0.4313	5	0.0101	0.006	1.68	2.29
114-2	1.428	0.613	585	0.2996	20	0.0239	0.006	3.98	7.39
114-3	1.184	0.661	565	0.2498	95	0.0942	0.006	15.70	27.38
114-4	1.257	0.645	565	0.2588	115	0.1190	0.005	23.80	31.50
120-1	1.382	0.627	710	0.4368	10	0.0102	0.003	3.40	2.28
120-2	1.373	0.629	610	0.3214	20	0.0224	0.002	11.20	6.52
120-3	1.309	0.639	578	0.2794	72	0.0756	0.003	25.20	21.30
120-4	1.438	0.610	578	0.2926	122	0.1400	0.004	35.00	32.36
127-1	1.477	0.638	690	0.4368	0	0.0140	0.003	4.66	3.10
127-2	1.290	0.658	625	0.3316	30	0.0342	0.003	11.40	9.35
127-3	1.392	0.650	560	0.2837	70	0.0782	0.005	15.64	21.61
127-4	1.581	0.615	555	0.2995	115	0.1405	0.005	28.10	31.93

TABLE XII*

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group I - Table III. Horizontal Discontinuity Parallel to Major Axis.

Sample	C	W_3 (lbs.)	N_3 Cycle/sec.	$E_3 x 10^6$ (Psi)	N_2-N_3 Cycle/sec.	$(E_2-E_3) x 10^6$ (Psi)	W_2-W_3 (lbs.)	$\frac{E_2-E_3}{W_2-W_3} x 10^6$	$\frac{E_2-E_3}{E_2}$ (Percent)
100-1	1.000	0.711	870	0.5382	5	0.0108	0.006	1.80	1.97
100-2	1.028	0.701	790	0.4497	10	0.0161	0.007	2.30	3.46
100-3	1.240	0.663	700	0.4028	0	0.0037	0.006	0.53	0.91
100-4	0.789	0.752	800	0.3797	0	0.0036	0.007	0.51	0.94
107-1	0.931	0.779	980	0.6965	0	0.0036	0.004	0.90	0.51
107-2	0.935	0.772	790	0.4505	10	0.0157	0.007	2.24	3.37
107-3	0.894	0.785	745	0.3895	0	0.0040	0.008	0.50	1.02
107-4	0.817	0.800	780	0.3976	5	0.0092	0.008	1.15	2.26
113-1	2.050	0.590	700	0.5926	0	0.0071	0.010	0.64	1.18
113-2	1.967	0.598	695	0.5681	5	0.0054	0.003	1.80	0.94
113-3	1.837	0.596	590	0.3811	4	0.0084	0.005	1.68	2.16
113-4	1.967	0.595	560	0.3670	5	0.0110	0.007	1.57	3.00
115-1	1.214	0.680	810	0.5416	0	0.0040	0.005	0.80	0.73
115-2	1.340	0.654	680	0.4052	10	0.0101	0.003	3.37	2.43
115-3	1.223	0.638	645	0.3500	10	0.0131	0.004	3.28	3.60
115-4	1.142	0.709	700	0.3967	0	0.0039	0.007	0.56	0.97

* - Subscript 3 in heading refers to data obtained from samples having both normal and parallel discontinuities to major axis.

TABLE XIII

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group II - Table III. Horizontal Discontinuity Parallel to Major Axis.

Sample	C	W_3 (lbs.)	N_3 Cycle/sec.	$E_3 x 10^6$ (Psi)	N_2-N_3 Cycle/sec.	$(E_2-E_3) x 10^6$ (Psi)	W_2-W_3 (lbs.)	$\frac{E_2-E_3}{W_2-W_3} x 10^6$	$\frac{E_2-E_3}{E_2}$ (Per cent)
105-1	1.265	0.676	870	0.6472	5	0.0123	0.005	2.44	1.86
105-2	1.216	0.710	780	0.5253	5	0.0105	0.005	3.10	1.96
105-3	1.106	0.692	700	0.3750	0	0.0044	0.008	0.55	1.16
105-4	1.225	0.691	710	0.4267	-5	-0.0018	0.007	-0.26	-0.42
109-1	1.484	0.628	780	0.5670	15	0.0258	0.004	6.45	4.35
109-2	1.260	0.690	770	0.5155	5	0.0104	0.005	2.08	1.98
109-3	1.077	0.705	725	0.3991	5	0.0049	0.006	0.82	1.21
109-4	1.179	0.683	720	0.4174	0	0.0037	0.006	0.62	0.88
117-1	0.976	0.750	980	0.7030	0	0.0047	0.005	0.67	0.66
117-2	1.101	0.721	755	0.4525	0	0.0044	0.007	0.63	0.96
117-3	0.972	0.781	760	0.4384	0	0.0034	0.006	5.67	7.69
117-4	1.012	0.767	770	0.4602	5	0.0090	0.005	1.80	1.92
119-1	0.967	0.766	980	0.7114	0	0.0046	0.005	0.64	0.92
119-2	1.289	0.679	740	0.4792	20	0.0293	0.004	7.32	5.76
119-3	0.984	0.743	740	0.4004	-5	-0.383	0.006	-0.38	-0.58
119-4	1.171	0.693	720	0.4207	0	0.600	0.006	0.60	0.85

TABLE XIV

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group III - Table III. Horizontal Discontinuity Parallel to Major Axis.

Sample	C	W_3 (lbs.)	N_3 Cycle/sec.	$E_3 x 10^6$ (Psi)	N_2-N_3 Cycle/sec.	$(E_2-E_3) x 10^6$ (Psi)	W_2-W_3 (lbs.)	$\frac{E_2-E_3}{W_2-W_3} x 10^6$	$\frac{E_2-E_3}{E_2}$ (Per cent)
112-1	1.784	0.606	745	0.6162	10	0.0061	0.006	1.02	0.98
112-2	1.728	0.621	680	0.4962	0	0.0072	0.009	0.80	1.45
112-3	1.919	0.619	585	0.4065	0	0.0039	0.006	0.65	0.95
112-4	1.793	0.650	610	0.4336	5	0.0126	0.008	1.58	0.29
116-1	1.593	0.622	760	0.5723	0	0.0083	0.009	0.92	1.14
116-2	1.671	0.606	685	0.4751	15	0.0260	0.006	4.33	5.47
116-3	1.545	0.628	645	0.4036	5	0.0089	0.004	2.22	2.20
116-4	1.431	0.657	700	0.4614	0	0.0007	0.002	0.35	0.15
118-1	1.715	0.622	790	0.6657	10	0.0258	0.008	3.22	3.88
118-2	1.362	0.669	705	0.4529	10	0.0171	0.006	2.85	3.78
118-3	1.793	0.608	600	0.3924	5	0.0125	0.009	1.39	3.18
118-4	1.390	0.680	715	0.4832	5	0.0097	0.004	2.42	1.97
123-1	1.036	0.722	985	0.7257	0	0.0091	0.009	1.01	1.25
123-2	1.102	0.701	770	0.4580	10	0.0173	0.007	2.47	3.78
123-3	1.008	0.738	740	0.4074	20	0.0281	0.010	2.81	6.90
123-4	0.980	0.750	780	0.4472	5	0.0112	0.009	1.24	2.44

TABLE XV

Data for Determination of Modulus of Elasticity of 1" x 2½" x 9" Samples Cut from Group IV - Table III. Horizontal Discontinuity Parallel to Major Axis.

Sample	C	W_3 (lbs.)	N_3 Cycle/sec.	$E_3 \times 10^6$ (Psi)	N_2-N_3 Cycle/sec.	$(E_2-E_3) \times 10^6$ (Psi)	W_2-W_3 (lbs)	$\frac{E_2-E_3}{W_2-W_3} \times 10^6$	$\frac{E_2-E_3}{E_2}$ (Percent)
110-1	1.178	0.657	745	0.4295	0	0.0033	0.005	0.66	0.76
110-2	1.367	0.628	620	0.3230	0	0.0218	0.004	5.45	6.32
110-3	1.264	0.640	580	0.2721	0	0.0034	0.008	0.42	1.23
110-4	1.341	0.620	580	0.2796	0	0.0032	0.008	0.40	1.13
114-1	1.296	0.632	720	0.4246	0	0.0067	0.010	0.67	1.55
114-2	1.428	0.607	585	0.2966	0	0.0030	0.006	0.50	1.00
114-3	1.184	0.654	565	0.2472	0	0.0026	0.007	0.37	1.04
114-4	1.257	0.638	565	0.2560	0	0.0028	0.007	0.40	1.08
120-1	1.382	0.623	710	0.4340	0	0.0028	0.004	0.70	0.64
120-2	1.373	0.620	600	0.3064	10	0.0150	0.009	1.67	4.67
120-3	1.309	0.631	575	0.2731	3	0.0063	0.008	0.79	2.25
120-4	1.438	0.601	575	0.2857	3	0.0069	0.009	0.77	2.36
127-1	3.62	0.631	680	0.4309	10	0.0059	0.007	0.84	1.35
127-2	3.16	0.651	615	0.3176	10	0.0140	0.007	2.00	4.22
127-3	3.41	0.642	555	0.2752	5	0.0085	0.008	1.06	2.99
127-4	3.89	0.609	550	0.2966	10	0.0029	0.006	0.48	0.97

TABLE XVI

Average Value for Change in Resonant Frequency and Modulus of Elasticity Due to Presence of Discontinuities.

Average Value of ΔF^* (Normal)				
Position of Cut	Group I	Group II	Group III	Group IV
1	11.25	1.00	2.50	5.00
2	41.75	36.25	25.00	25.00
3	103.25	116.25	85.50	84.25
4	146.25	178.75	147.50	119.25

Average Value of ΔF^* (Parallel)				
Position of Cut	Group I	Group II	Group III	Group IV
1	1.25	5	5	2.5
2	7.50	7.5	8.75	5.0
3	3.50	0	7.50	2.0
4	2.25	0	3.75	3.25

Average Value of $\% \Delta E^*$ (Normal)				
Position of Cut	Group I	Group II	Group III	Group IV
1	4.04	0.94	1.19	2.40
2	11.49	10.82	7.02	8.24
3	25.50	26.26	22.44	24.47
4	31.54	35.75	31.62	32.14

Average Value of $\% \Delta E^*$ (Parallel)				
Position of Cut	Group I	Group II	Group III	Group IV
1	1.10	1.94	1.81	1.08
2	2.55	2.66	3.62	4.05
3	1.92	2.37	3.30	1.88
4	1.79	0.80	1.21	1.38

* - Average of all specimen from same position of cut in original nine-inch brick, e.g. 100-1, 107-1, 115-1 for Group I; 105-1, 109-1, 117-1, 119-1 for Group II, etc.

TABLE XVII

Effect of Varying Width of Vertical Cut on Modulus of Elasticity for Samples Having Cut Normal to Major Axis.

Sample	$E \times 10^6$ (No Cut)	$E \times 10^6$ (Cut 0.1" Wide)	$E \times 10^6$ (Cut 0.2" Wide)	$E \times 10^6$ (Cut 0.3" Wide)
125-1	0.6430	0.6160	0.6044	0.5997
125-2	0.4533	0.4390	0.4310	0.4205
125-3	0.4759	0.3583	0.3568	0.3487
125-4	0.6700	0.4383	0.4288	0.4139
138-1	0.6903	0.6803	0.6758	0.6644
138-2	0.4873	0.4615	0.4526	0.4479
138-3	0.5195	0.4055	0.4022	0.3706
138-4	0.6896	0.4301	0.4276	0.4113

IV

RESULTS AND DISCUSSION

It was mentioned in the procedure that in the interests of conserving samples both vertical and horizontal cuts were made in each specimen. To justify this doubling up it was necessary to determine whether there was an interaction between the two cuts or whether their effects were simply additive. If the latter proved to be true, it would be satisfactory to reuse all of the bars having vertical cuts normal to the major axis for the horizontal cuts parallel to the major axis. If, however, there was an interaction between the two cuts present in the same specimen, it would be necessary to make the horizontal cuts in an entirely new set of bars. As described in the procedure, sixteen bars were selected and their moduli of elasticity were determined. In eight of these bars vertical slots were cut normal to the major axis. In the other eight, horizontal slots were cut parallel to the major axis. It should be pointed out that the bars in which the vertical slots were introduced were from Group I of the regular test design. After the data obtained was recorded, a second cut was made in each of the sixteen specimens, a horizontal cut in those containing a vertical cut and vice versa. From Table I it may be seen that the horizontal slots cause no change in the modulus of elasticity which might not be accounted for by experimental error. The vertical slots on the other hand cause marked changes in the modulus of elasticity, increasing in amount as the slot approaches the center of the bar.

Bars containing both horizontal and vertical slots reflect a modulus of elasticity change due to the vertical slots alone. In all of these bars the value of the modulus of elasticity varies with position in the same manner, and are of the same magnitude, as those in the bars having vertical slots only. From this it seems safe to conclude that any effects due to the presence of both horizontal and vertical slots are simply additive and in most cases are due to the contribution of the vertical slots alone. As a result of this experiment all subsequent tests were made using one bar to secure data for both types of cut.

Very early in the course of the investigation it was recognized that the dynamic modulus of elasticity decreases as the size of the specimen decreases. This is in good agreement with the recent work done by R. H. Lester[17]. Moreover, it was also of interest to note that in all the tests made the dynamic modulus of elasticity obtained for bars cut from the exterior portions of a brick is significantly greater than that for bars cut from the interior. One possible reason for this difference in the dynamic modulus of elasticity is the heat treatment of the brick. During the process of firing the brick, owing to the temperature gradient present, the exterior was exposed to a higher temperature than was the interior of the brick. Consequently, differences in certain physical properties and structure of the two different parts of the brick are likely to be expected. Close examination of the specimens show that the exterior portion of the brick is considerably harder than the interior portion. From the classical mechanical point of view, the modulus of elasticity is a unique property of a given material. However, the results of this experiment show that the modulus of elasticity varies with the size and the structure of the brick, even though they are made of the same materials and produced by the same process. This contradiction indicates that the modulus of elasticity is a complex function of many variables such as pores, cracks and other inhomogeneities and is statistically reproducible as long as the average of these variable factors are reproduced in the same order.[18]

Since previous studies have indicated a high degree of correlation between the modulus of rupture and the dynamic modulus of elasticity, it is of interest to compare the effects of certain inhomogeneities in a brick on both of these values. It would appear obvious that the softer portion of the brick used in this study, namely the interior, will have a lower strength than the exterior or harder portion. Assuming good linear positive correlation, this would be reflected in the moduli of elasticity of the two portions.

John Tucker, Jr.[19] in his theory of failure of structural members stated that when one of the internal fibers of the structure is relatively weak, it may fail before an outer fiber, even though the stress is greater at the outside, and that the failure of one such fiber will result in the failure of the beam. This is known as the "weakest link theory." Hence, if this brick is subjected to tensile stress, the inner portion being weaker than the outer portion of the brick, it will fail before the outer portion and cause the failure of the brick. For instance, some given materials show the same value of dynamic modulus of elasticity, but because of their differences in interior resistance, thus they show different values of modulus of rupture.

Griffith[20] in his "flaw theory" of rupture of solid bodies assumes that in general the presence of inhomogeneities of some kind (cracks, for example) in a test-piece prevent the applied exterior load from acting homogeneously over the full extent of a test piece, thus causing premature rupture as a consequence of stress-concentration effect. It is to be expected that "flaws", represented by cracks, in a refractory brick would in general weaken it, resulting in a lowering of the modulus of rupture. It is not so obvious, however, what the effect of these cracks would be on the dynamic modulus of elasticity. Obviously, if the dynamic modulus of elasticity is lowered by cracks in the brick, then the rupture-elasticity relationship will remain. If, on the other hand, these cracks produce no effect on the modulus of elasticity, then we are led astray in relying on this property to estimate the modulus of rupture. It should be noted that we are not interested here in making corrections in the elasticity equations for discontinuities, even though such correction will show the proper correlation between rupture and elasticity. Since the cracks in a brick, particularly in the interior, are not known to exist at the time of the test, one would proceed in the usual manner to collect his data. By deliberately introducing "cracks" in the brick used in this study and observing the resulting change in the dynamic modulus of elasticity, it is believed that a partial answer to the failure of some brick to follow the usual correlation pattern may be had. Tables VIII, IX, X and XI show that the modulus of elasticity of bars having vertical slots decreases as the slot location approaches the center of the bar. Figure 4 shows the relationship between the location of the slot and the percentage change in the dynamic modulus of elasticity.

The change in the dynamic modulus of elasticity with the location of horizontal cracks parallel to the major axis of the bars is shown in TablesXII, XIII, XIV and XV. The results thus obtained show that the drop off in the modulus of elasticity is independent of the location of the crack. Moreover, the change in the resonant frequency is only of the order of a few cycles per second.

The effect of the orientation and location of cracks on the average percentage of change in the dynamic modulus of elasticity is best illustrated by Figure 5. The comparisons can also be seen from Table XVI.

An interpretation of these results in terms of discontinuities in standard nine-inch straight fire brick would indicate that cracks in a plane normal to the major axis would give sharply reduced modulus of elasticity values. Also the location of such cracks with respect to the length of the brick would influence the amount of the lowering of the elastic modulus. From the mechanics of a beam it would be inferred that cracks in a plane perpendicular to the major axis would materially reduce the strength of the brick in flexure. Furthermore, the closer the crack to the center of the brick the greater will be the reduction in the breaking strength in the center-point loading method of the standard modulus of rupture test. Thus we see that for vertical cracks perpendicular to the major axis the modulus of elasticity will correlate with modulus of rupture. Cracks in a horizontal plane parallel to the major

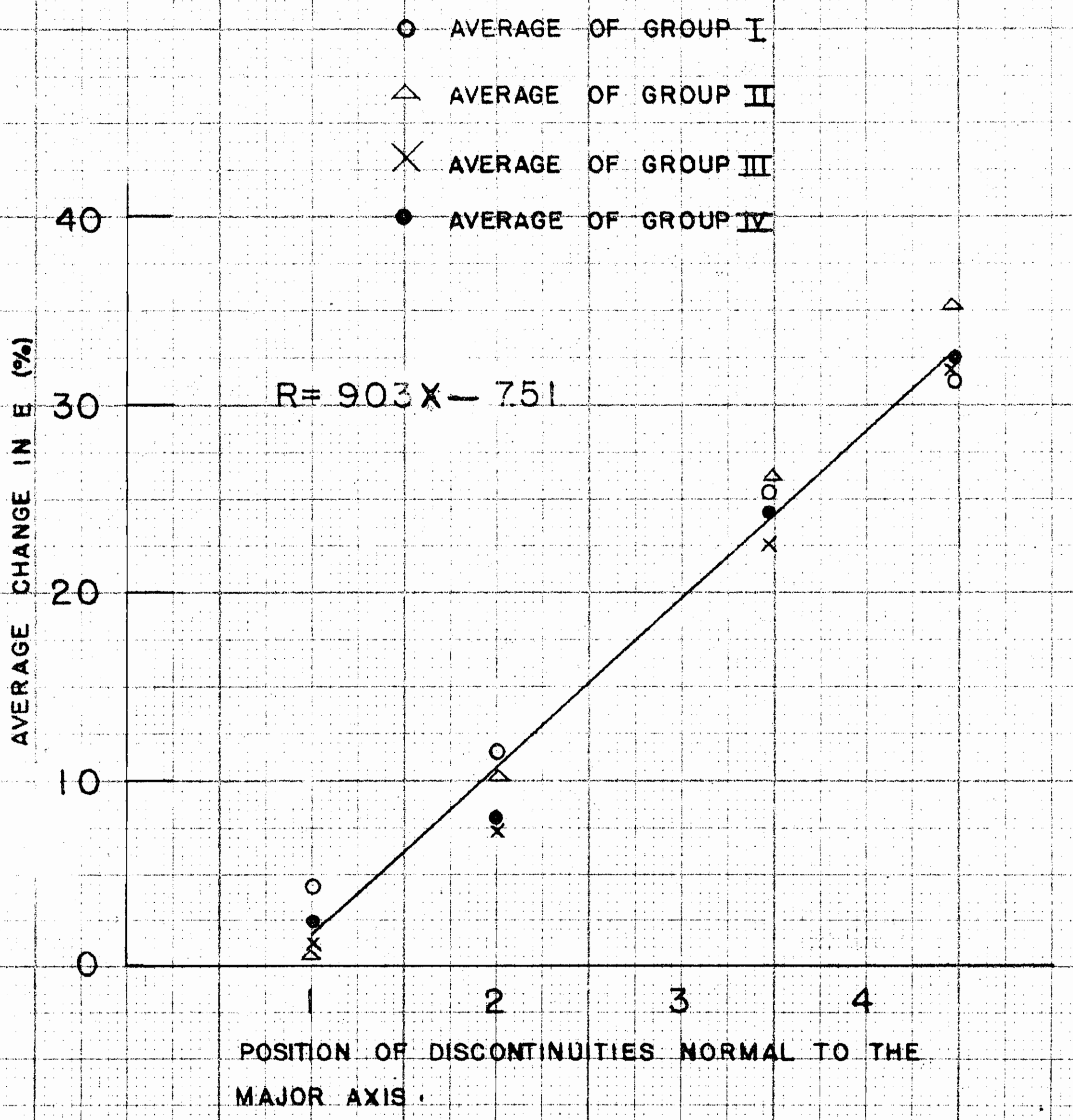

○ AVERAGE OF GROUP I
△ AVERAGE OF GROUP II
× AVERAGE OF GROUP III
● AVERAGE OF GROUP IV
R= 9.03X — 7.51
AVERAGE CHANGE IN E (%)
0
10
20
30
40
1
2
3
4
POSITION OF DISCONTINUITIES NORMAL TO THE MAJOR AXIS.

FIGURE 4

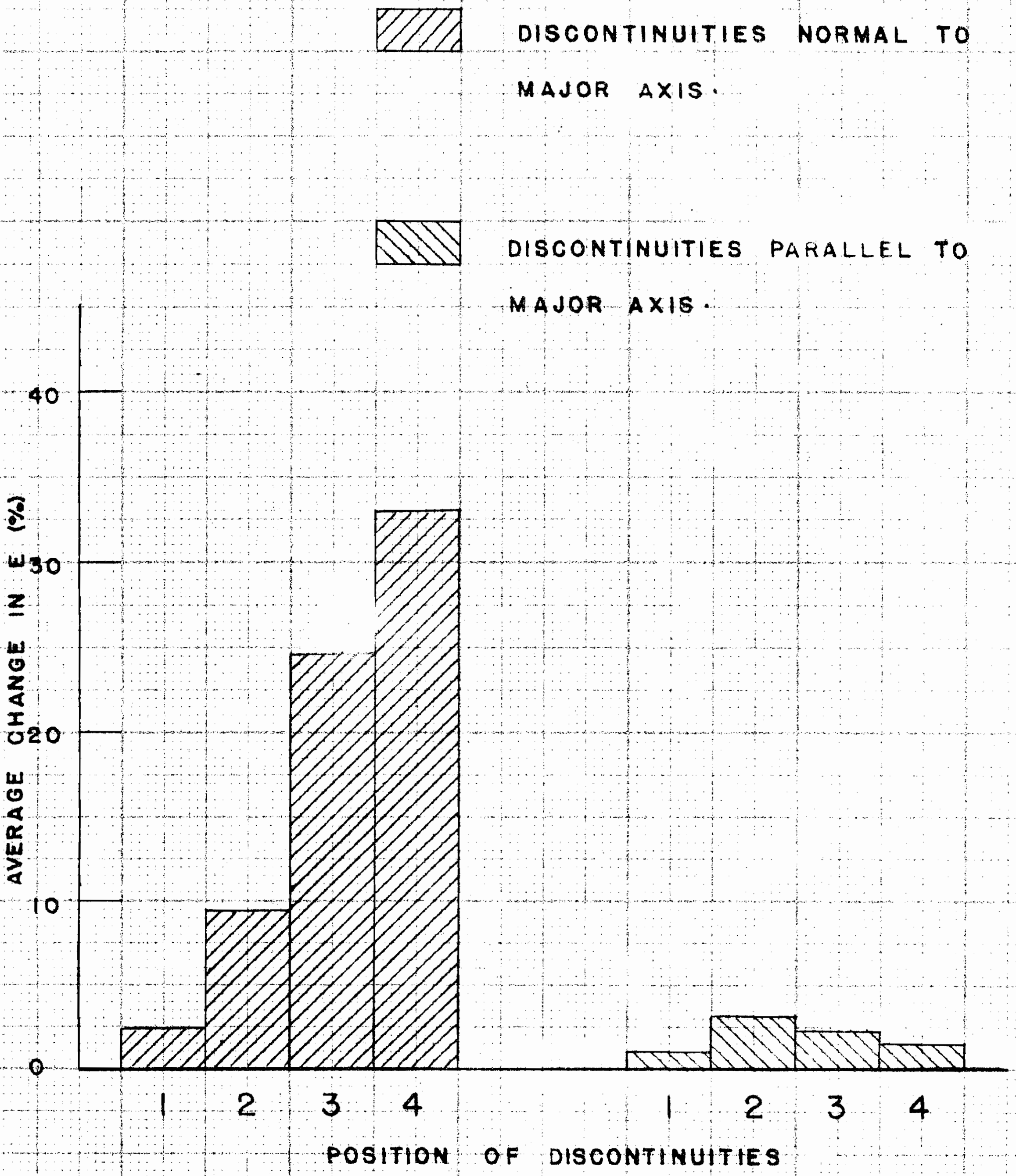

DISCONTINUITIES NORMAL TO MAJOR AXIS.
DISCONTINUITIES PARALLEL TO MAJOR AXIS.
AVERAGE CHANGE IN E (%)
40
30
20
10
0
1
2
3
4
1
2
3
4
POSITION OF DISCONTINUITIES

FIGURE 5

axis show little or no change in the dynamic modulus of elasticity regardless of their location with respect to the length of the brick. Inasmuch as such cracks would also lead to a reduction in the modulus of rupture value, it appears that the dynamic modulus of elasticity will not be well correlated with rupture in the case of brick having such defects.

The type of crack which occurs in shippable brick is usually undetectable by ordinary methods of examination. For fear that the thickness of the simulated cracks in the study might have been the real reason for the marked drop in the modulus of elasticity, a test was conducted to see whether or not the thickness of vertical cracks was a factor. Table XVII gives the results of the test. From these results it can be seen that there is no change in the resonant frequency with increase in thickness of the slot. There would be a slight lowering of the calculated modulus of elasticity because of the reduction in weight of the specimens. This test indicates that the first break in the continuity of the major axis is responsible for the sharp lowering of the resonant frequency and hence the modulus of elasticity. It would be expected, therefore, that even cracks of the size which would normally occur in a fire brick would evidence themselves by a reduction in the modulus of elasticity.

V

CONCLUSIONS

From the results of the experimental work carried out in this investigation the following results seem justified:

1. Vertical cracks normal to the major axis of a standard fire brick will cause a lowering of the dynamic modulus of elasticity calculated.
2. Horizontal cracks parallel to the major axis of a standard fire brick will not cause a change in the dynamic modulus of elasticity calculated.
3. Vertical cracks normal to the major axis of a standard fire brick will cause a lowering of the resonant frequency of flexural vibration.
4. Horizontal cracks parallel to the major axis of a standard fire brick will not cause a change in the resonant frequency of flexural vibration.
5. Vertical cracks normal to the major axis of a standard fire brick will cause a lowering of the dynamic modulus of elasticity calculated proportional to their distance from the end of the brick. The maximum lowering occurs at the center.
6. Vertical cracks normal to the major axis of a standard fire brick will cause a lowering of the resonant frequency of flexural vibration proportional to their distance from the end of the brick. The maximum reduction in the resonant frequency occurs at the center.

VI

SAMPLE CALCULATION

The dynamic modulus of elasticity was calculated from the following formula developed by Pickett as follows:

$$E = CWN^2$$

Where:

E = Modulus of elasticity (psi)

W = Weight of specimen (Pounds)

N = A resonant frequency (cycles per sec.)

C = A factor which depends upon the shape and size of specimen, the mode of vibration and Poisson's ratio (sec^2 per sq. in.)

Example:

Calculate the modulus of elasticity for sample 100-4 with the following data:

L = 8.97 (in.)

D = 0.919 (in.)

b = 2.46 (in.)

W = 0.763 (lbs.)

N = 970 (cycles per sec.)

Solution:

$$\frac{L}{D} = \frac{8.97}{0.919}$$

$$\frac{L}{D} = 9.02$$

Cb = 1.94 (From Pickett's Figure 3 when $\mu = \frac{1}{6}$.)

$$C = \frac{1.94}{2.46}$$

$$C = 0.789 \quad (\text{sec}^2 \text{ per sq. in.})$$

$$E = CWN^2$$

$$= 0.789 \times 0.763 \times \overline{970}^2$$

$$E = 0.5664 \times 10^6 \quad (\text{psi})$$

VII

APPENDIX

Calculation of Regression of Change in Modulus of Elasticity in Position for Bars with Vertical Slots.

Average Percentage of Change in Modulus of Elasticity

(% Δ E) and Position of Crack Normal to Major Axis

X	Y
1	4.04
1	0.94
1	1.19
1	2.40
2	11.49
2	10.82
2	7.02
2	8.24
3.48	25.50
3.48	26.26
3.48	22.44
3.48	24.47
4.48	31.54
4.48	35.75
4.48	31.62
4.48	32.14

X = Position of Vertical Slots Normal to Major Axis (in.)

Y = Percentage of Change in Modulus of Elasticity (% Δ E)

1. Regression Calculation

$Xi = 43.84$

$Yi = 275.86$

$XiYi = 1014.19$

$Xi^2 = 148.72$

$Yi^2 = 7,138.46$

$\bar{X} = 2.74$

$\bar{Y} = 17.24$

$$b = \frac{\Sigma XiYi - \sum \frac{Xi \quad Yi}{N}}{\Sigma Xi^2 - \sum \frac{(\ Xi)^2}{N}}$$

$$b = \frac{1014.19 - \frac{43.84 \times 275.86}{16}}{148.723 - \frac{(43.84)^2}{16}} = \frac{1014.19 - 755.85}{148.72 - 120.12}$$

$$b = \frac{258.34}{28.60} = 9.033$$

$$\overline{Y}x = \overline{Y} - b (X - \overline{X})$$

$$\overline{Y}x = 17.24 - 9.03 (X - 2.74)$$

$$\overline{Y}x = 17.24 - 9.03X - 24.75$$

$$\overline{Y}x = 9.03X - 7.51$$

2. Test for linearity of Regression:

$$\frac{Ty^2}{N} = \frac{(275.86)^2}{16} = 4756.17$$

$$\sum \frac{Tyi^2}{Ni} = \frac{(8.57)^2}{4} - \frac{(37.57)^2}{4} - \frac{(98.67)^2}{4} - \frac{(131.05)^2}{4}$$

$$\Sigma\ \Sigma\ Yij^2 = \frac{7098.71}{7138.46}$$

$$\Sigma \Sigma\ Yij^2 - \frac{Ty^2}{N} = 7138.46 - 4756.17$$

$$= 2382.29$$

$$\sum (\frac{Tyi^2}{Ni}) - \frac{Ty^2}{N} = 7098.71 - 4756.17$$

$$= 2342.54$$

$$b^2 \left[\Sigma \Sigma X^2_{ij} - \frac{\Sigma\ \Sigma (Xij)^2}{N} \right] = \overline{9.03}^2 (148.72 - 120.12)$$

$$= \overline{9.03}^2 (28.601) = 2332.15$$

	S.S.	D.F.	M.S.
Within Groups	39.75	12	3.31
Regression	2332.15	1	
About Regression	10.39	2	5.19
Total	2342.54	3	

$$F = \frac{5.19}{3.31} = 1.56$$

$$F0.95\ (2,12) = 3.89$$

Comparing F = 1.56 with F0.95 (2,12) = 3.98 we declared not significant. Therefore, we accept the hypothesis that the <u>regression is a straight line</u>.

3. Test for Independence:

1. Hypothesis: B = 0
2. 0.05% level of significance
3. $t = \dfrac{(b-0)\ Sx\ \sqrt{N-1}}{Sy.x}$
4. $$Sx^2 = \frac{120.12}{16-1} = 8.008$$

$$Sy^2 = \frac{7138.46 - \dfrac{(275.86)^2}{16}}{15} = 158.82$$

$$Sy.x^2 = \frac{16-1}{16-2}\left[\ 158.82 - \overline{9.03}^2\ (8.008)\ \right]$$

$$= 100.2$$

$$t = \frac{(9.03 - 0)\ (2.839)\ (3.872)}{10} = 9.91$$

$$t = 9.91$$

5. Degree of freedom = $16-2 = 14$

6. Critical region: $-2.14 > t > 2.14$

7. Here $t = 9.91$ which is larger than 2.14 and so we have sufficient reason to say, at the 5% level of significance, that Y is dependent on X.

VIII

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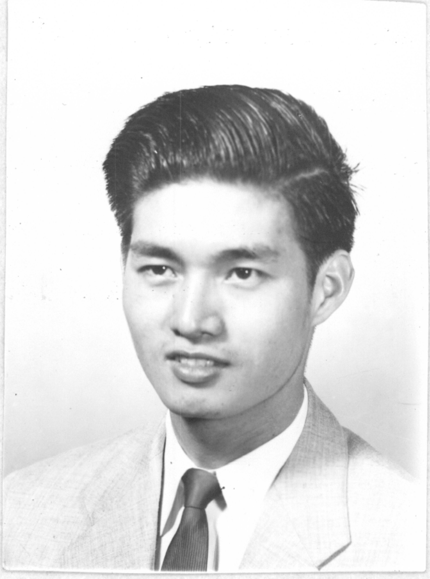
Chapter 5.

IX

VITA

Lim Yock Lio was born in Fookien, China on September 13, 1931. After completing his high school work at Mapua Institute of Technology, Manila, Philippines in 1951, he registered at the University of Santo Tomas, College of Engineering, where he was granted a Bachelor of Science degree in Chemical Engineering in April 1955. The following fall he enrolled at Virginia Polytechnic Institute as a candidate for the Master of Science degree in Ceramic Engineering.



Lim, Y.L.