DEMONSTRATION OF ACTIVE STRUCTURAL ACOUSTIC CONTROL OF CYLINDERS

by

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Active control is applied to reduce noise emission from a vibrating elastic cylinder by exerting forces on the cylinder that cancel the noise-generating vibration. This technique is called Active Structural Acoustic Control (ASAC) (Fuller, 1987). Sensors are implemented using piezoelectric film, and actuators are implemented using piezoceramic material. Both analog and digital noise cancellation control algorithms are used to reduce the noise emission from the cylinder. Two cylinder boundary conditions are taken as case studies. The first boundary condition is the open cylinder case. The second boundary condition is where the cylinder has an end plate bolted to each end.

Actuator placement and the sensor design are done by first obtaining the natural frequencies and mode shapes of the cylinder using both analytical and experimental methods. Modal sensors developed and tested in previous work (Lee, 1989) are applied.

After preliminary control experiments with analog feedback loop show that control can be done with the sensors and the actuators, digital signal processing hardware programmed with the filtered-x Least-Mean-Square adaptive control algorithm is used to
control the vibration of the cylinder. The excitation is single-tone on-resonance. Acoustic testing demonstrates that ASAC reduces the sound pressure level generated by the vibrating cylinder by up to 29 dB in the reverberant field. Vibration measurement reveals that the reduction in sound emission from the cylinder is a result of reduction in vibration. The adaptive controller reduces the vibration level by up to 68 dB.
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Nomenclature

Roman letters:

$A_s$  sensor area

$a(\theta,t)$  acceleration as a function of angle, $\theta$, and time, $t$

$b$  width of PVDF sensor

$C$  capacitance

$C_a$  actuator capacitance

$C_{ij}$  interface circuit capacitive impedance

$C_{ij}$  modal amplitude

$C_s$  sensor capacitance

$c$  speed of sound in air

$c$  piezoelectric material stiffness matrix [6x6]

$D$  electric displacement (charge density) vector [3x1]

$d$  strain / electric field constant matrix, or
electric charge density / stress constant matrix [3x6]

$d_k$  desired system response at time step $k$

$E$  modulus of elasticity

$E$  electric field vector [3x1]

$E[.]$  expected value operator

$e$  piezoelectric stress/charge constant matrix [6x3]

$e$  PVDF sensor signal
F \quad \text{polarization pattern of PVDF film sensor}

F_c \quad \text{actuator in-plane force amplitude}

f \quad \text{mode shape function}

f \quad \text{actuator in-plane force}

f \quad \text{actuator in-plane force}

f_{sch} \quad \text{Schroeder cutoff frequency}

g \quad \text{PVDF sensor output due to PZT actuator excitation}

h \quad \text{cylinder shell thickness}

h_s \quad \text{sensor film thickness}

I \quad \text{sensor current in Laplace domain}

I \quad \text{current}

i \quad \text{circumferential modal index}

i \quad \text{sensor current}

j \quad \text{axial modal index or modal index of beam}

j \quad \sqrt{-1}

k_T \quad \text{torsional spring constant}

L \quad \text{length of cylinder or beam}

L \quad \text{inductance}

M_c \quad \text{actuator bending moment amplitude}

m_x \quad \text{actuator bending moment}

P \quad \text{PZT actuator power}

Q \quad \text{sensor charge in Laplace domain}

q \quad \text{sensor charge}

R \quad \text{cylinder midplane radius}

R_o \quad \text{cylinder outside radius}
$R_i$  interface circuit resistive impedance
$R_s$  PVDF sensor resistance
$S$  sensor strain vector [6x1]
$s$  Laplace variable
$s$  compliance matrix = $1 / \text{Young's Modulus}$ [6x6]
$T$  sensor stress vector [6x1]
$T_c$  transfer function between amplifier input and PVDF sensor output
$T_c$  estimated transfer function between controller output and PVDF sensor output
$T_1$  torsional spring parameter = $k T L / (E I)$
$T_{60}$  chamber reverberation time
t  time
$u$  axial displacement of cylinder shell element
$u_s$  unit step function
$V$  sensor output voltage in Laplace domain
$V$  voltage
$V$  reverberation chamber volume
$v$  circumferential displacement of cylinder shell element
$v$  sensor output voltage
$W$  linear combiner weight vector = $[w_0 \; w_1]^T$
$W$  cylinder mode shape function
$w$  radial displacement of cylinder shell element
$x$  reference signal = $[x_k \; x_{k-1}]^T$
$x$  estimated reference signal = $[x_k \; x_{k-1}]^T$
$x$  distance from end of cylinder or beam
Y  Young’s modulus of elasticity
z  distance from neutral plane of cylinder shell
z_r distance from neutral plane of base structure to mid plane of sensor layer

Greek letters:
\( \gamma_j \) a constant evaluated for j\textsuperscript{th} mode shape of beam
\( \delta \) the Kronecker delta function
\( \varepsilon \) strain vector [6x1]
\( \varepsilon \) dielectric permittivity constant [3x1]
\( \varepsilon_k \) (adaptive control variable) error signal at time step k
\( \lambda_j \) j\textsuperscript{th} eigenvalue of beam
\( \lambda_{ij} \) mode (i,j) eigenvalue of cylinder
\( \mu \) density
\( \mu \) convergence parameter
\( \mu_{10} \) width scale factor of PVDF modal sensor
\( \nu \) Poisson’s ratio
\( \phi_j \) j\textsuperscript{th} axial or beam mode shape function
\( \Psi \) phase angle between current and voltage
\( \theta \) position angle
\( \theta_0 \) position angle of an antinode
\( \xi \) nondimensional distance from end of beam
\( \xi \) mean square error
\( \omega_{ij} \) angular natural frequency of cylinder
Subscripts:

s  pertaining to PVDF sensor film
i  pertaining to interface circuit
i  cylinder circumferential mode index
j  cylinder axial mode index or beam mode index
k  time step
x  axial direction of cylinder
θ  circumferential direction of cylinder
1.1 Motivation

Noise emission from vibrating elastic structures creates problems in many applications such as machinery, home appliances, aerospace structures, and underwater structures. The problems created by noise ranges from minor irritation to serious hazards. Particularly, in military applications noise emission from a structure can impair ability to evade detection. Reducing the noise emission from vibrating structures is therefore an important subject.

Control of noise emission from cylinders is important because many aerospace and underwater structures have cylindrical shell components. For this reason, this thesis studies acoustic control of a cylinder. There are many methods to control the acoustic emission, however, the particular application of the cylinder imposes several restrictions which narrow the choice of methods. The need for the active acoustic control technique used in this research work is described below.

Reduction of noise emission from a structure is usually done by passive acoustic control, i.e., by relying on absorption and reflection of the sound waves, or by modifying the structure (Wilson, 1989). However, the structure studied in this thesis, which is a simplified model of an underwater vehicle component, does not allow passive acoustic control for the following reasons. The addition of material for passive noise damping
increases the mass of the structure—an effect which is undesirable in many cases. Modification or redesign of the structure for acoustic control may not be feasible, or may be too costly. Designing an efficient passive damping system may also be difficult because, as in most practical cases, predicting the resulting noise reduction is difficult. Passive noise control techniques are often not effective in low frequency range. Moreover, the application of the structure requires a very low noise level which may not be achieved by passive acoustic control alone. The above reasons lead to what is called Active Structural Acoustic Control (ASAC), which requires the generation of secondary sound field which interferes destructively with the noise to be controlled.

ASAC generally requires the use of sensors which sense the noise, and control actuators which drive the structure with forces that oppose the noise-generating vibration. Several types of sound and vibration transducers can be used as feedback sensors and as actuators in active noise control. However, for the cylinder controlled in this project the choice is limited because the application does not allow sensors other than structure-borne sensors. The cylinder models an outer part of an underwater vehicle. Piezoelectric film instead of a microphone is used as a sensor; and piezoelectric ceramic material is used to form actuators. The reasons for choosing piezoelectric sensors and actuators are as follows.

Several features of piezoelectric film can be exploited to make the film superior to other structure-born vibration sensors such as accelerometers or strain gages. The film is a distributed sensor that extracts spatial vibration information and can be tailored into such shapes that it is sensitive only to selected vibration modes. Being lightweight, thin and very compliant, the film does not load the structure. Furthermore, the film has good dynamic characteristics, such as very high bandwidth, and ability to be adjusted to respond to strain or strain rate depending on the electronic interface circuit. Film made
of Polyvinylidene Fluoride (PVDF) can be used as sensors because the material has a considerably higher piezoelectric charge density/strain constant than other polymers (Murayama et al., 1976).

Piezoelectric material used as actuators have several advantages. They are lightweight and they occupy negligible space compared to other types of actuators such as shakers, and they have a broad band response compared to other actuators. Also, the fact that they are distributed actuators can be exploited to increase their control performance (Cudney, 1989). The use of one type of such crystal, Lead Zirconate Titanate (PZT), has been proven to be successful in controlling sound radiation from a vibrating structure (Fuller et al., 1989a).

With the sensor and actuator material selected, the next step in the design of a scheme to control the cylinder is to design the sensors and actuators, and to determine their locations on the cylinder. For the sensor and actuator design and placement, the natural frequencies and the mode shapes of the cylinder are required. The existing analytical work on the vibration characteristics of cylinders has been done with either an open-end condition or a boundary condition where the ends of the cylinder are closed with flexible membranes which do not have any resistance to the bending of the shell. However, the cylinder model studied in this present work has a stiff plate bolted onto each end. Finding the natural frequencies and mode shapes of the cylinder with this boundary condition is therefore explored as a part of this thesis.

In addition to the characterization of the cylinder and the study of the sensors and actuators, selection of the control scheme is an important part of the work. The nature of the control problem and the dynamics of the structure to be controlled are important in
the selection of a control strategy to be used in acoustic control. The vibration characteristics of the cylinder under study, like the dynamics of shells in general, is difficult to model analytically. An adaptive control algorithm is a very good control strategy for controlling the vibration of the cylinder, because this type of control algorithm requires no specific knowledge of the dynamics of the structure (Clark and Fuller, 1990).

Success has been achieved in previous research work on noise control of cylinders (Fuller, 1987; Abler and Silcox, 1987; Fuller and Jones, 1987; Lester and Fuller, 1990; Lefebvre, 1991, etc.). However, the previous work concentrated mainly on the noise field inside the cylinders; the disturbance excitation source was typically a noise source outside the cylinder; and microphones were used as error sensors. The present work attempts to reduce the noise emitted to the outside of cylinder when the cylinder is mechanically excited by a vibrating motor or shaker, and to do this using structure-borne sensors instead of microphones.

1.2 Objective and Approach

The objective of this research work is to demonstrate reduction of noise from a cylinder by selectively controlling the structural vibration using piezoelectric sensors and actuators. The noise is a result of a single tone excitation from a disturbance shaker. It should be noted that this work is not intended to be a treatise in ASAC. The engineering contribution of this work is mainly in three other research areas: 1) characterization of cylinders, 2) distributed sensing, and 3) demonstration of acoustic reduction through vibration control.
The approach is as follows. First, the vibration characteristics of the cylinder are studied by obtaining the cylinder’s natural frequencies and mode shapes both analytically and experimentally. Then, acoustic testing is performed to determine the effective sound radiating modes and to determine which frequencies are to be chosen for the control experiments. To select the frequencies, acoustic directivity patterns generated by a disturbance shaker excitation are compared with acoustic directivity patterns generated by the PZT actuators. If at a certain frequency the PZT actuators generate the same acoustic directivity patterns as the shaker, then at that frequency the PZT actuators can be used to cancel the acoustic radiation caused by the shaker excitation.

In the next step, the piezoelectric film sensors and piezoceramic actuators are studied experimentally to investigate the performance of the film as a sensor and to determine if the piezoceramic actuators have enough authority to counteract disturbance vibrations in the cylinder.

The final step of the approach is to perform experiments to demonstrate ASAC on the cylinder. Two control strategies are implemented in the experiments. The first control strategy uses analog components. This control strategy is chosen because its implementation is simple. The main purpose of this control strategy is to show experimentally that vibration control can indeed be achieved by using the piezoelectric film sensor and the piezoceramic actuators together as part of the control system. The second control strategy utilizes microcomputer-based digital signal processing (DSP) hardware programmed with an adaptive control algorithm to control the acoustic emission of the cylinder. This control strategy is chosen because previous work has proven that it gives very good results in ASAC. Experiments are done with both control strategies. The reduction in total sound pressure level generated by the cylinder is measured in a
reverberation chamber.

1.3 Overview

The presentation of the research is outlined as follows. Chapter 2 discusses previous work done on dynamic modelling of cylinders, control of cylinder vibration and acoustic radiation, and piezoelectric sensors and actuators. The description of the cylinder and its instrumentation is given in Chapter 3. In that chapter the analysis and experiments done to obtain the natural frequencies, mode shapes and acoustic emission of the cylinder are presented. Chapter 4 explains the basic principles of piezoelectric sensors and actuators, and presents results of several proof-of-concept experiments on the sensors and actuators. Control experiments on the cylinder are discussed in Chapter 5. First, this chapter describes a preliminary vibration control experiment using the piezoelectric sensors and actuators with an analog control scheme. Then, the adaptive control algorithm and its implementation with digital signal processing hardware is described. Chapter 6 presents results of the experiments using analog feedback control and an adaptive control scheme with the piezoelectric sensors and actuators. The last chapter gives conclusions and proposes future work to be done in the next phase of the research.
Chapter 2
Previous Work

Previous research done in the past has studied the important elements presented in this thesis, namely: cylinder vibration analysis, piezoelectric sensors and actuators, active vibration control, and active structural acoustic control. Some relevant previous work is reviewed here. The review begins with a historical background of active acoustic control. Then previous research work relevant to the research discussed in this thesis are discussed.

2.1 Concept of Active Acoustic Control

Secondary sound sources provided the first means of reducing noise inside cylinders. The idea of making silence by adding in a secondary, anti-phase wave field to cancel an existing one appeared in 1934 (Elliot et al., 1990). Unfortunately, the electronic technology at that time was not advanced enough for general experimental verification. It was not until 1953 that a real advancement in active noise control took place. At that time the description of an "electronic sound absorber" was published (Olson and May, 1953). Experimental work was done on this concept and published in 1956 (Olson, 1956), and a book was published in 1957 (Olson, 1957). The first practical application of active noise control was demonstrated in an effort to reduce noise generated by a transformer (Connover, 1955 and 1956).

Later analytical work showed that transformer noise within a certain angle from the
horizontal plane can be reduced by the use of additional sound sources (Onada and Kido, 1968). The development of computer technology made possible the implementation a control scheme that adapts the velocities of the additional sources by periodic perturbation. Adaptive control techniques were developed further by Widrow, who in 1975 implemented adaptive signal processing to reconstruct signals corrupted by additive noise (Widrow et al., 1975). His applications included cancellation of periodic interference in speech signals and cancellation of broad-band interference in the side lobes of an antenna array. In 1984, active control was used to reduce noise generated by a turbulent flame, achieving a peak reduction of 17 dB, and a 10 dB reduction over the band of 300 - 700 Hz (Dines, 1984). The active control scheme was adaptive control with the light emission from the flame as reference signal. The error signal was given by microphones in the far field. Other work in active noise control done in the 1980’s were reported in a paper by Stevens and Ahuja (Stevens and Ahuja, 1990).

The application of active control in marine structures was shown in 1983 (Chaplin, 1983). Chaplin’s concepts included the silencing of noise in ships caused by the engines and the propellers. The noise reduction improved passenger comfort. Chaplin also achieved a reduction of exhaust noise by means of a "synchronized waveform synthesizer" in which one vibration cycle was used to predict the following cycles so that the anti-phase version could be synthesized well before it was required.

2.2 Active Structural Acoustic Control

In one of the early active acoustic control experiments, loudspeakers were used inside an aircraft cabin to generate a control sound field to reduce the interior noise (Swinbanks, 1973). But later a new concept emerged, which was reducing sound radiation from a
vibrating structure by controlling the structural vibration. In 1987, an experiment was conducted which was the original effort to control acoustic level inside cylinders by controlling the structural vibration (Fuller, 1988). In this experiment, Fuller used a shaker as control actuator. The influence of actuator locations for controlling the noise field in cylinders was also studied in 1987 (Fuller and Jones, 1987b).

2.3 Piezoelectric Transducers for Active Vibration Control and Active Structural Acoustic Control

The use of point force sources such as shakers as described above can cause undesirable spillover into structural vibration modes because point sources are spectrally white in a spatial sense. This spillover problem degrades control performance (Dimitriadis and Fuller, 1989). Moreover, for many applications shakers as control actuators are too heavy, occupy too much space, and require complicated mounting. A very good solution to these problems was found in piezoelectric actuators. Being distributed, piezoelectric actuators can reduce spillover. They also have desirable physical characteristics: they are lightweight, compact, inexpensive, and have a large bandwidth.

Before their use to control acoustic emission, piezoelectric crystals had been used to control vibration of cylinders (Swigert and Forward, 1981). Swigert and Forward’s theoretical and experimental work involved an "electronic damper", which was a system of electro-mechanical transducers as elements of electronic feedback loops controlling the mechanical vibration of an end-supported mast. The structure used was a hollow fiberglass cylinder called an omni-antenna mast. The electronic damping was applied by using piezoceramic sensing transducers mounted directly to the structure to detect dynamic strains, amplifying and phase-shifting the signal, and reintroducing the signals
to piezoceramic actuators elsewhere on the structure. Amplitude reduction greater than 30 dB in two vibration modes was achieved.

A method for optimizing the design of piezoelectric actuators for vibration control purposes was proposed in 1990. This method minimizes control effort (Jia, 1990). In the same year a comprehensive set of experiments using PZT actuators to excite simply supported plates was done (Fleming, 1990). These experiments produced plate vibrations whose modal compositions agreed very well with analytical prediction developed previously (Fuller and Dimitriadis, 1989).

The original sensors for active acoustic control, still the most commonly used to date, are error microphones. However, microphones impose certain limitations in use, especially when active acoustic control is to be done on moving structures such as vehicles. Structure borne sensors are likely to be the only choice in this case. Fuller and Jones used an array of accelerometers as sensors when controlling acoustic field in a cylinder (Fuller and Jones, 1987a). An array of accelerometers instead of a single accelerometer was necessary to obtain spatial information of the vibration. Spatial information can also be obtained in real time using distributed sensors (Lee, 1987). Distributed sensors obtain spatial information from the structure in real time like an array of point sensors. However, a distributed sensor needs only one signal channel as opposed to several channels needed for an array of point sensors. The reduced number of channels is obviously an advantage from the signal processing and hardware points of view.

A very good distributed sensor can be made of Polyvinylidene Fluoride (PVDF) film because it is light, compact, durable, compliant, and easy to apply. A particular advantage of using this polymer film as a sensor lies in the fact that the film can be cut in such
shapes that it gives a spatial filtering effect (Lee, 1987). This type of distributed sensors are capable of acting as "modal filters", being sensitive only to selected modes depending on the shapes of the sensors. Lee’s experiments with PVDF modal sensors on a cantilever beam demonstrated the validity of his theory. Since the publication of his papers (for example, Lee and Moon, 1989, Lee and Moon, 1990), many researchers have investigates the concept of modal sensors. For example, Collins et al. used PVDF film as modal sensors for controlling a space manipulator link (Collins et al., 1990). Experimental investigation of the influence of different excitation on the response of the modal sensors has also been studied (Sumali and Cudney, 1990). PVDF modal sensors have also been applied successfully in ASAC (Clark and Fuller, 1990). To date, modal sensors are made of commercially available PVDF film.

The fact that the piezoelectric polarization of such PVDF film is constant throughout the film makes it impossible to create in practice a two-dimensional modal sensor. However, analysis and experiments have shown that it is possible to use a one-dimensional modal sensor on a two-dimensional structure such as a plate (Zhou et al., 1990). The modal sensor produces a modal filtering effect in one dimension. Therefore, instead of sensing only one mode, the sensor senses a family of modes having a common modal index. It is this concept which is used to design a sensor to be used for the active acoustic control application studied in this thesis.

Active structural acoustic control has been done on beams and plates, using adaptive controllers in many cases. Significant reduction of sound radiation from a vibrating plate has been achieved using this technique together with piezoelectric transducers (Clark and Fuller, 1990). Application of this technique to control sound transmission through plates in the reverberant field has also been successful (Zhou and Cudney, 1991.).
2.4 Active Structural Acoustic Control of Cylinders

Vibration analysis of cylinders have resulted in methods to obtain natural frequencies and mode shapes for several boundary conditions. Much research has been done to reduce noise generated by vibration of elastic cylinders (Abler and Silcox, 1987). This research area has been an important research topic mainly because of efforts to reduce interior noise in aircraft fuselages (Bullmore, 1986, Fuller and Jones, 1987, and Lester and Fuller, 1990). Recently, piezoceramic actuators have been used with an adaptive controller to reduce sound pressure level inside a cylinder (Lefebvre, 1991). The cylinder was a model of an aircraft fuselage; the noise source was a loudspeaker outside the cylinder; and the sensors were error microphones inside the cylinder.

2.5 Summary

The previous work mentioned above have given useful results in the study of the elements of the research discussed in this thesis, namely, the study of cylinder vibration and acoustics, application of adaptive control, theory of piezoelectric sensors and their application, and the application of piezoelectric actuators for vibration and acoustic control.
Chapter 3
Characterization of the Cylinder

The acoustic radiation from the cylinder can be controlled by selectively controlling the vibration. To sense the vibration properly and apply forces on the cylinder to cancel the vibration effectively, actuators and sensors have to be placed on the cylinder using the knowledge of the mode shapes of the cylinder. The shape of the sensor is also determined by the mode shape to be controlled. Thus, knowledge of the natural frequencies and mode shapes of the cylinder is necessary.

It is also important to determine in the beginning of the research the frequency to be controlled, because as mentioned earlier the objective of the research is to control single tone acoustic emission from the cylinder. A reasonable choice is the frequency corresponding to an effective acoustic radiating mode, because at this frequency the vibration signals have a good signal-to-noise ratio. Experiments are done to determine this frequency.

To obtain the natural frequencies, mode shapes and acoustic emission of the cylinder, the following approach is taken. First, Flugge shell theory is applied in conjunction with a Rayleigh-Ritz technique to predict the natural frequencies and mode shapes of the open cylindrical shell without end-caps. In the method used, this boundary condition in the axial direction corresponds to the free-free beam boundary condition. This method is given in the literature (Blevins, 1979). A vibration experiment on the open cylinder gives the actual natural frequencies and mode shapes. The agreement between analytically
predicted natural frequencies and experimentally obtained resonant frequencies shows that the mathematical model is accurate and applicable, at least, to the open cylinder.

The above model is then used to analyze the more complex case where the cylinder is closed with end-caps. In this case, the model is modified to accommodate the boundary condition at the joints between the shell and the end caps. This non-classical boundary condition is difficult to model. An analytical model is created to approximate the boundary condition. This model requires a torsional spring parameter which needed to be obtained experimentally. Therefore, one experimentally obtained resonant frequency and the corresponding mode shape are used to determine the torsional spring parameter to be used in the analytical model. The analytical model is then used to predict other natural frequencies and mode shapes. All experimental resonant frequencies and analytical natural frequencies are compared to determine the accuracy the mathematical model.

After the mode shapes are determined, an acoustic test is done to determine which frequencies and mode shapes are effective acoustic radiators. These will then be used to demonstrate active control of single frequency noise emissions. The acoustic test is done in an anechoic chamber so that the directivity pattern of the sound emission can be obtained. The directivity pattern has to be obtained for the following reason. The way the PZT actuators exert forces on the cylinder is different from the way the shaker exerts forces on the cylinder. If at a certain frequency the sound directivity pattern generated by the cylinder under PZT actuator excitation can match the sound directivity pattern generated by the cylinder under the shaker excitation, then it can be expected that the PZT actuator is capable of cancelling the sound field generated by the shaker excitation at that frequency.
3.1 Description of the Instrumented Cylinder

The cylinder used in the experiments consisted of an aluminum cylindrical shell and two end-caps fastened onto the shell with bolts. The dimensions and mechanical properties of the cylinder are listed in Table 3.1. Figure 3.1 shows the detailed dimensions of the cylinder, and Appendix A contains photographs of the cylinder. The two end-caps were made with unequal thicknesses in an attempt to model the underwater cylindrical part according to the original blue print drawing of the part.

Figure 3.2 shows the positions of piezoelectric sensors and actuators on the cylinder. The numbered grid provides a coordinate frame to be used when referring to positions on the cylinder. Point (0,6) is the drive location whenever a shaker is used as exciter. Four PVDF film sensors and eleven PZT actuators are bonded on the outside of the cylinder shell. On the inside, one PVDF film sensor is bonded in the axial direction.

The piezoelectric film sensors are shaped according to the modal sensor theory which will be described in section 4.3. This sensor is attached to the cylinder with an adhesive layer. The piezoceramic actuators are bonded on surfaces on the cylinder which were milled flat because the PZT actuators cannot be bent without breaking. These actuators, when subjected to an electric field, expand or contract, thereby causing strain in the cylinder surface. These actuators will be discussed further in section 4.4.
Table 3.1 Dimensions and Mechanical Properties of Cylinder

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside diameter</td>
<td>0.254 m</td>
</tr>
<tr>
<td>Shell length</td>
<td>0.4064 m</td>
</tr>
<tr>
<td>Shell thickness</td>
<td>0.00635 m</td>
</tr>
<tr>
<td>Density</td>
<td>2700 kg/m³</td>
</tr>
<tr>
<td>Young's modulus</td>
<td>64 GPa</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>
Figure 3.1 Dimensions of the Cylinder (All Dimensions Are in mm).
Figure 3.2 Positions of Piezoelectric Sensors and Actuators on the Cylinder
3.2 Open Cylinder Natural Frequencies and Mode Shapes

3.2.1 Analytical Natural Frequencies and Mode Shapes

The curvature of the cylindrical shell couples extensional and flexural deformations. In the literature, the linear differential equations describing the deformations of a shell have no general agreement because of differences in assumptions and in neglecting small terms. The analysis part in this chapter is done using methods given in the literature (Blevins, 1979). In the analysis, the following assumptions are made:

1. The thickness of the shell is constant.
2. The wall thickness is less than 10% of the radius.
3. The shell material is linear, elastic, homogeneous, and isotropic.
4. The deformation of the shell is small compared to the radius.
5. Rotary inertia and shear deformation are negligible.

Flugge shell theory gives the following differential equations which relate strains to deflections (Blevins, 1979). At any position indicated by the coordinates \( (x, \theta, z) \), the strains \( \varepsilon \) are,

\[
\varepsilon_x = \frac{\partial u}{\partial x} - z \frac{\partial^2 w}{\partial x^2}, \tag{3.1.a}
\]

\[
\varepsilon_\theta = \frac{1}{R} \frac{\partial v}{\partial \theta} - \frac{z}{R(R + z)} \frac{\partial^2 w}{\partial \theta^2} + \frac{w}{R + z}, \tag{3.1.b}
\]

\[
\varepsilon_x = \frac{1}{R + z} \frac{\partial u}{\partial \theta} + \frac{R + z}{R} \frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x \partial \theta} \left( \frac{z}{R} + \frac{z}{R + z} \right), \tag{3.1.c}
\]
\[ \epsilon_{xx} = \epsilon_{yy} = \epsilon_{zz} = 0, \]  

(3.1.d)

where \( u, v, w \) are deflections in the axial (x) direction, circumferential (\( \theta \)) direction, and radial (w) direction as indicated in Fig. 3.3; \( z \) is the distance between the point \((x, \theta, z)\) and the mid-plane of the shell, and \( R \) is the mid-plane radius of the cylindrical shell.

To obtain the natural frequencies of a vibrating cylindrical shell, Eqs. 3.1.a through 3.1.d are used in conjunction with the Rayleigh-Ritz technique with the assumption that the deformation of the freely vibrating shell takes the form of

\[ w(\xi, t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \phi_j(\xi) \cos i\theta \cos \omega_t, \]  

(3.2)

where \( \xi = x / L; x \) is distance from one end of the cylindrical shell and \( L \) is the length of the shell, \( \theta \) is position angle, and \( \omega \) is the natural frequency. The modal indices \( i \) and \( j \) have the following physical meaning: vibration mode \((i,j)\) has \( i \) complete circumferential waves and \( j \) axial half-waves (Fig. 3.4). The circumferential mode shape is always sinusoidal in polar coordinates. In this model the axial mode shape, \( \phi_j \), is the same as the \( j^{th} \) mode shape of a beam with boundary conditions corresponding to the end conditions of the shell of the cylinder. For the open cylinder, \( \phi_j \) takes the form of free-free beam mode shape function

\[ \phi_j = \cosh \lambda_j \xi + \cos \lambda_j \xi - \gamma_j (\sinh \lambda_j \xi + \sin \lambda_j \xi), \]  

(3.3.a)

where \( \lambda_j \) is the \( j^{th} \) eigenvalue, and

\[ \gamma_j = \frac{\cosh \lambda_j - \cos \lambda_j}{\sinh \lambda_j - \sin \lambda_j}. \]  

(3.3.b)
Figure 3.3 Deflections of Cylinder Wall. (From Blevins, Formulas for Natural Frequency and Mode Shape, Krieger, Florida, 1979, p 292.)
Figure 3.4 Circumferential and Axial Nodal Lines of the Cylinder. (From Blevins, Formulas for Natural Frequency and Mode Shapes, Krieger, Florida, 1979, p. 297.)
The axial part of the mode shape function, such as the one in Eq. 3.3.a, will be referred to as the beam mode shape function in the rest of this chapter.

The natural frequency associated with mode \((i,j)\) is,

\[
\omega_{ij} = \frac{\lambda_{ij}}{R_0 \mu (1-\nu)/E^{1/2}}, \tag{3.4}
\]

where \(R\) is the mid-plane radius, \(\mu, \nu\) and \(E\) are the density, Poisson's ratio and Young's modulus, respectively. The eigenvalue \(\lambda_{ij}\) is the smallest positive root of the characteristic polynomial given by

\[
\alpha_2 \lambda_{ij}^6 - (a_{12} + \alpha_2 a_{22} + \alpha_2 a_{33}) \lambda_{ij}^4 - \\
(a_{12}^2 + a_{13}^2 + \alpha_2 a_{22}^2 - \alpha_2 a_{22} a_{33} - a_{11} a_{33} - a_{11} a_{22} \lambda_{ij}^2) \lambda_{ij}^2 + \\
a_{12} a_{33} + a_{23} a_{11} + a_{23} a_{22} - a_{11} a_{22} a_{33} - 2 a_{12} a_{22} a_{13} = 0 \tag{3.5}
\]

where

\[
a_{12} = -\nu i \beta_j \alpha_1 - (1-\nu) i \beta_j \frac{\alpha_2}{2}, \tag{3.6}
\]

\[
a_{13} = -\nu \beta_j \alpha_1 + k \beta_j [-\beta_j^2 + (1-\nu)i^2 \alpha_2/2], \tag{3.7}
\]

\[
a_{22} = i^2 + (1+3k)(1-\nu) \beta_j^2 \alpha_2/2, \tag{3.8}
\]

\[
a_{23} = i + k i \beta_j^2 [\nu \alpha_1 + 3(1-\nu) \alpha_2/2], \tag{3.9}
\]

and

\[
a_{33} = 1 + k [\beta_j^4 + (i^2-1)^2 + 2\nu i^2 \beta_j^2 \alpha_1 + 2(1-\nu)i^2 \beta_j^2 \alpha_2]. \tag{3.10}
\]

Parameters \(\alpha_1\) and \(\alpha_2\) are obtained from the appropriate beam mode shape function, \(\phi\), by the following integration,

\[
\alpha_1 = -\int_0^1 \frac{d^2 \phi(\xi)}{\phi^2(\xi)} \phi(\xi) \, d\xi, \tag{3.11}
\]
\[ \alpha_2 = \int_0^1 \left[ \frac{d \phi(\xi)}{d \xi} \right]^2 d\xi . \]  \quad (3.12)

Parameter \( k \) is a geometric property of the cylinder cross section:

\[ k = h^2 j(12R^2), \]  \quad (3.13)

where \( h \) is the thickness and \( R \) is the mid-plane radius of the cylindrical shell.

Parameter \( \beta_j \) is a product of the beam eigenvalue and the cylinder aspect ratio, i.e.,

\[ \beta_j = \lambda_j R/L. \]  \quad (3.14)

The mode shape function of mode \((i,j)\) is obtained from Eq. 3.2 and Eq. 3.3, which give

\[ W_{ij}(\theta, \xi) = \cos(i \theta) \phi_j \]
\[ = \cos(i \theta) \left[ \cosh \lambda_j \xi + \cos \lambda_j \xi - \gamma_j (\sinh \lambda_j \xi + \sin \lambda_j \xi) \right] , \]  \quad (3.15)

where \( i \) and \( j \) are the circumferential and axial mode indices, \( \theta \) is position angle, \( \xi \) is a non-dimensional axial distance \( x/L \), and \( \lambda_j \) and \( \gamma_j \) are as in Eq. 3.3.

In addition to the mode shapes where the axial variations are like those of a beam, two other types of mode shapes are present in the open cylinder. For these two types, straight axial lines along the cylinder shell remain straight, consequently, the axial mode index \( j \) is zero. These modes do not have any axial nodes (Fig. 3.4 does not show these modes). For these modes, the open cylinder undergoes only flexural deformation and no extensional deformation. As a result, the calculation of the natural frequencies and mode shapes are very simple.
The first of the two types are called the Rayleigh modes, in which there is no axial variation in the deflection. Straight lines along the shell therefore remain straight and parallel to each other. The radial deformation of the cylinder in these modes can be expressed as

$$w_{i0}(\theta, t) = A \cos i\theta \cos \omega t,$$  \hspace{1cm} (3.16)

where $A$ is a constant. The natural frequency of Rayleigh mode $(i,0)$ (commonly abbreviated as Rayleigh mode $i$) is

$$\omega_{i0} = \frac{i(i^2 - 1)}{R\sqrt{i^2 + 1}} \sqrt{\frac{\kappa^2}{12R^2} \frac{E}{\mu(1 - \nu^2)}}.$$  \hspace{1cm} (3.17)

where $R$ and $h$ are the mid-plane radius and wall thickness of the cylinder, $E$, $\rho$, and $\nu$ are the modulus of elasticity, density, and Poisson's ratio of the cylinder, respectively. The fact that the natural frequencies of the Rayleigh modes do not depend on the length of the cylinder can be explained as follows. Because in the axial direction there is no variation in deflection, there is no interaction between adjacent cross-sections. Therefore, the whole cylinder vibrates in Rayleigh motion as if it were made of independent circular rings. A cross-section can be made in any axial position of the cylinder, and the radial deflection of one section will still be independent of the remaining section. Thus, the axial dimension of the section does not affect the radial deflection.

For the second type of the two inextensional modes, called the Love modes, straight lines along the shell also remain straight, but not parallel. For any given radial position on the cylinder the deflection at one end of the cylinder is opposite to the deflection at the other end. For the $i^{th}$ Love mode, the radial deflection of the cylindrical shell is,
The natural frequency is

$$\omega_{i_0} = \frac{i(i^2 - 1)}{R \sqrt{i^2 + 1}} \frac{h}{R \sqrt{i^2 + 1}} \sqrt{\frac{E}{\mu (1 - \nu^2)}} \left(1 + \frac{24(1 - \nu)R^2}{i^2 L^2} \right) - \frac{12 R^2}{i^2(i^2 + 1)L^2} . \quad (3.19)$$

where \( L \) is the length of the cylinder.

Application of equations 2 through 18 results in analytical natural frequencies as listed in Table 3.2. Appendix B contains the listing of a Matlab program written to predict these frequencies using the above procedures. The program also generates analytical mode shape functions.

It is interesting to see that for each particular axial modal index \( j > 0 \), the natural frequencies do not always increase with increasing circumferential modal index \( i \). For example, mode \((2,1)\) natural frequency is higher than mode \((4,1)\) natural frequency.

### 3.2.2 Experimental Resonant Frequencies and Mode Shapes

Figure 3.5.a shows the experiment setup used to obtain the natural frequencies and mode shapes of the open cylinder. A shaker was used to excite the cylinder. Burst random tests were performed on the cylinder. As indicated in Fig. 3.5.b, accelerometer number 1 measured the vibration on points along a line on the cylinder, and accelerometer number 2 measured the vibration on points around the cylinder in a constant axial position.
Table 3.2 Open Cylinder Analytical Natural Frequencies

<table>
<thead>
<tr>
<th>$i$</th>
<th>Rayleigh Freq (Hz)</th>
<th>Love Freq (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>261</td>
<td>307</td>
</tr>
<tr>
<td>3</td>
<td>737</td>
<td>798</td>
</tr>
<tr>
<td>4</td>
<td>1413</td>
<td>1480</td>
</tr>
<tr>
<td>5</td>
<td>2285</td>
<td>2356</td>
</tr>
<tr>
<td>6</td>
<td>3353</td>
<td>3425</td>
</tr>
<tr>
<td>7</td>
<td>4615</td>
<td>4687</td>
</tr>
<tr>
<td>8</td>
<td>6071</td>
<td>6144</td>
</tr>
</tbody>
</table>

a. Rayleigh and Love Modes, $j = 0$

<table>
<thead>
<tr>
<th>$i \backslash j$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tr>
<td>1</td>
<td>4209</td>
<td>5666</td>
<td>6063</td>
<td>6362</td>
</tr>
<tr>
<td>2</td>
<td>2270</td>
<td>4118</td>
<td>5165</td>
<td>5839</td>
</tr>
<tr>
<td>3</td>
<td>1586</td>
<td>3005</td>
<td>4269</td>
<td>5267</td>
</tr>
<tr>
<td>4</td>
<td>1860</td>
<td>2758</td>
<td>3877</td>
<td>4981</td>
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</tr>
<tr>
<td>7</td>
<td>4943</td>
<td>5409</td>
<td>6075</td>
<td>6942</td>
</tr>
</tbody>
</table>

b. Mode $(i,j)$ Natural Frequencies
Figure 3.5  Experiment to Obtain Natural Frequencies and Mode Shapes of the Cylinder,  
a. Hardware Schematic,  
b. Positions of Accelerometer.
Tektronix 2630 Fourier Analyzer computed the frequency response functions between the shaker force and the accelerations. Figure 3.6 shows a frequency response function obtained with point (0,6) as the drive point and point (0,5) as the response point.

The mode shapes were obtained by the following method. The imaginary parts of the FRF's at resonant frequencies in different positions along and around the cylinder were used to render the circumferential and axial mode shape plots at those frequencies. A cubic spline technique was used to interpolate experimental data points and plot smooth curves. Note that because the curves obtained by this technique pass through all experimentally-obtained data points, this technique relies heavily on the experimentally-obtained transfer function values and does not attempt to minimize errors.

The experiment revealed seventeen mode shapes of the open cylinder and their corresponding frequencies. Table 3.3 shows that the experimental natural frequencies agree closely with the analytical prediction. The experimental mode shapes corresponding to those frequencies are shown in Appendix C. As mentioned earlier, the knowledge of the mode shapes is essential. At a later stage of the research work, acoustic tests were done to determine which frequency was to be chosen as the frequency to be controlled; then the actuator placement and the sensor design would be done according to the mode shape corresponding to that frequency.

Two points are worth noting regarding the accuracy of the analytical model. For each particular axial mode index \( j > 0 \), the agreement between the analytical model and the experimental results improves with increasing circumferential mode number \( i \). This result is as predicted in the literature (Blevins, 1979). The second point is that this agreement is very good when \( j = 1 \). In this case the differences between analytical and experimental
Figure 3.6 Typical FRF between Acceleration and Force, Open Cylinder
Table 3.3 Analytical and Experimental Natural Frequencies of the Open Cylinder

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical frequency (Hz)</th>
<th>Experimental frequency (Hz)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rayleigh 2</td>
<td>261</td>
<td>263</td>
<td>0.77</td>
</tr>
<tr>
<td>Rayleigh 3</td>
<td>737</td>
<td>747</td>
<td>1.36</td>
</tr>
<tr>
<td>Love 3</td>
<td>798</td>
<td>803</td>
<td>0.63</td>
</tr>
<tr>
<td>Rayleigh 4</td>
<td>1413</td>
<td>1425</td>
<td>0.85</td>
</tr>
<tr>
<td>Rayleigh 5</td>
<td>2285</td>
<td>2250</td>
<td>-1.53</td>
</tr>
<tr>
<td>Love 5</td>
<td>2356</td>
<td>2310</td>
<td>-1.95</td>
</tr>
<tr>
<td>Rayleigh 6</td>
<td>3353</td>
<td>3350</td>
<td>-0.09</td>
</tr>
<tr>
<td>3, 1</td>
<td>1586</td>
<td>1570</td>
<td>-1.01</td>
</tr>
<tr>
<td>4, 1</td>
<td>1860</td>
<td>1850</td>
<td>-0.54</td>
</tr>
<tr>
<td>5, 1</td>
<td>2637</td>
<td>2630</td>
<td>-0.27</td>
</tr>
<tr>
<td>6, 1</td>
<td>3684</td>
<td>3680</td>
<td>-0.11</td>
</tr>
<tr>
<td>2, 2</td>
<td>4118</td>
<td>3825</td>
<td>-7.12</td>
</tr>
<tr>
<td>3, 2</td>
<td>3005</td>
<td>2880</td>
<td>-4.16</td>
</tr>
<tr>
<td>4, 3</td>
<td>3877</td>
<td>3735</td>
<td>-3.66</td>
</tr>
<tr>
<td>5, 3</td>
<td>4139</td>
<td>4030</td>
<td>-2.64</td>
</tr>
<tr>
<td>6, 3</td>
<td>4927</td>
<td>4825</td>
<td>-2.07</td>
</tr>
<tr>
<td>5, 4</td>
<td>5182</td>
<td>4895</td>
<td>-5.54</td>
</tr>
</tbody>
</table>
natural frequencies are considerably less than 2%. But for higher axial mode number \( j > 1 \), the agreement decreases. The worst agreement is for mode (2,2), where the difference is almost 8%. It can be concluded that in general the mathematical model predicts the natural frequencies with sufficient accuracy. However, it should be noted that this model may need refining for axial mode number \( j \) greater than 1.

The experiment obtains almost all modes in the 0 - 5 kHz frequency range. Several Love modes are not excited because the shaker point of action lies on an antinodal line of these modes.

3.3 Closed Cylinder Natural Frequencies and Mode Shapes

3.3.1 Analytical Natural Frequencies and Mode Shapes

The closed cylinder has end plates attached to the shell. The connection between the shell and the end plates is shown in Fig. 3.7.a. Each end plate is bolted to the shell with thirty-six 1/8" steel bolts. Additionally, each end plate has a raised face 1 mm thick inserted into the shell. This boundary condition is difficult to model accurately. An assumption that both end caps are rigid (the thicknesses of the end caps are 25.4 mm and 19 mm; the thickness of the shell is 6.35 mm) and that the boundary conditions at both ends are equal simplifies the modeling. It is well known that clamped-clamped boundary condition is not likely to be achieved in real physical systems, especially at high frequencies. On the other hand, if the fastening bolts were perfectly flexible and the shell rotation were not constrained at the joints with the caps, the boundary condition would be pinned-pinned. The fact that the fastening bolts give some resistance to bending of the shell at the joints suggests that the boundary condition be modelled with a pin connection.
Figure 3.7  Connection between Cylinder Shell and End Plate
a. Physical Description,  
b. Pin and Torsion Spring Model,  
c. Analytical Mode Shape in Axial Direction, Compared to Pinned-pinned and Clamped-clamped Beam.
and a torsional spring at each end, as shown in Fig. 3.7.b. A spring constant is used to model the bending stiffness at the shell-to-cap joints.

Equations 3.4 through 3.14 remain valid for this boundary condition; however, the right hand side of Eq. 3.3 has to be replaced by the jth axial mode shape function of a beam with a pin and a torsional spring at each end, which is given by (Gorman, 1975),

\[
\phi_j = \sin \lambda_j \xi - \sinh \lambda_j \xi + \gamma_j [\cos \lambda_j \xi - \cosh \lambda_j \xi - (2 \lambda_j / T_1) \sinh \lambda_j \xi],
\]

(3.20)

where

\[
\gamma_j = \frac{\sinh \lambda_j - \sin \lambda_j}{\cos \lambda_j - \cosh \lambda_j - (2/T_1) \sinh \lambda_j}.
\]

(3.21)

Here \( T_1 = k_T L/EI \) is a torsional spring parameter defined for a beam of length \( L \), section modulus \( EI \), with a pin plus torsional spring of stiffness \( k_T \). A pinned joint has a \( T_1 \) value of 0. At the other extreme, a clamped joint has a \( T_1 \) value of infinity. In the case of the cylinder with end plates, \( T_1 \) will be determined by using experimentally obtained natural frequencies. Once \( T_1 \) is obtained, the eigenvalue \( \lambda_j \) is found by solving the characteristic equation

\[
T_1^2 + 2T_1 \frac{\sin \lambda_j \cosh \lambda_j - \cos \lambda_j \sinh \lambda_j}{1 - \cos \lambda_j \cosh \lambda_j} + \frac{2 \lambda_j^2 \sin \lambda_j \sinh \lambda_j}{1 - \cos \lambda_j \cosh \lambda_j} = 0.
\]

(3.22)

Then the natural frequencies of the closed cylinder are computed using Eqs 3.4 through 3.14.

For comparison, the natural frequencies of the cylinder with pinned-pinned and clamped-
clamped boundary conditions are also computed. For the pinned-pinned boundary conditions, the \( j^{th} \) eigenvalue is simply

\[
\lambda_j = j \pi .
\]  

(3.23)

For the clamped-clamped boundary conditions, the eigenvalues are found by solving

\[
\cos \lambda_j \cosh \lambda_j = 1 .
\]  

(3.24)

The computed natural frequencies of the cylinder with pinned-pinned and clamped-clamped boundary conditions are given in Appendix B. This appendix also presents natural frequencies of the closed cylinder modelled with pin-and-torsional-spring boundary conditions with various \( T_1 \) values between 1 and 16. Note that for any given mode the computed natural frequency of the closed cylinder with pin-and-torsional-spring boundary conditions is always between the natural frequencies of the cylinder with pinned-pinned boundary condition and the natural frequency of the cylinder with clamped-clamped boundary condition. These results of the frequency calculation agree with intuitive sense that the joint rotational stiffness is between zero (pinned joint case) and infinity (clamped joint case).

Equations 3.21 and 3.20 are incorporated in the Matlab program in Appendix B, so that the program also generates the mode shape functions of the closed cylinder. For example, this program is used to compute the beam mode shape function for mode (i,1). This beam mode shape function is the axial part of the cylinder mode shape function. Figure 3.7.c shows a comparison of the beam first mode shapes for \( T_1 = 0 \), \( T_1 = \infty \) (clamped-clamped boundary condition), and \( T_1 = 4 \) (pin plus torsion spring boundary condition).
This example shows that the predicted mode shape function is between that of a pinned-pinned cylinder and that of a clamped-clamped cylinder.

### 3.3.2 Experimental Resonant Frequencies and Mode Shapes

The experiment setup to determine the mode shapes of the closed cylinder was similar to that for the open cylinder. A shaker was used as exciter and a force transducer was used to measure the shaker force. Two accelerometers were used in the same positions as on the open-cylinder experiment. Burst random excitation was used in 1 Hz to 3 kHz and 3 kHz to 5 kHz ranges. Frequency-response-function (FRF) curves for the closed cylinder were obtained with point (0,6) as the drive location and point (12,6) as the response location. Approximate mode shapes were obtained by the same procedure used in obtaining the approximate mode shapes of the open cylinder. Six mode shapes were obtained. Comparison between the calculated natural frequencies and the first four experimentally obtained resonant frequencies of the closed cylinder leads to a conclusion that the cylinder has a $T_1$ value of approximately 4. Substituting this experimental stiffness ratio into Eq. 3.22 makes it possible to solve for the eigenvalue $\lambda_j$. The natural frequencies are obtained by using Eq. 3.5 and Eq. 3.4. The mode shapes of the cylinder are then calculated by Eq. 3.2 and Eq. 3.20.

Among the six experimental natural frequencies, only mode (1,1) frequencies were close to those predicted by analysis. As predicted in the previous section which deals with the open cylinder, the disagreement between the analytically predicted natural frequencies and the experimental resonant frequencies shows that the approximation of the axial mode shapes by equivalent beam mode shapes may need refining for high axial modal indices ($j > 1$). Although the bending stiffness of the end supports has been modeled with
reasonable accuracy by introducing the torsional spring constraints, another significant boundary condition has not been taken into account. This boundary condition is that the ends of the cylinder shell are not totally free to move in the axial direction. The axial restraints to the movement of the ends of the cylinder comes from the inertia of the end-plates. Unlike the case of beams where axial constraints may be neglected without resulting in significant errors, axial constraints in shells are important in determining the eigenvalues, and taking the mass into account in modeling the cylinder will increase the accuracy for higher axial model indices.

Table 3.4 shows comparison between frequencies obtained by calculation with $T_1 = 4$ and frequencies obtained by experiments using a shaker as an exciter. For the mode shapes whose frequencies are predicted with close accuracy, the experimental mode shapes are also close to the analytically-predicted ones. Figure 3.8 shows the variation of accelerance amplitude in the circumferential and axial direction when the cylinder is excited at 1550 Hz. This experimental result shows that the mode corresponding to this frequency is mode (4,1). The analytical prediction for this mode shape is drawn (with the cylinder unwrapped for clarity) also in Fig. 3.8. Other mode shapes obtained experimentally are shown in Appendix C.

3.4 Closed Cylinder Acoustic Radiation

An acoustic test was done in an anechoic chamber to determine the acoustic radiation pattern of the closed cylinder. Figure 3.9 shows the setup for the acoustic experiment. The cylinder was first excited with a shaker working at point (0,6) (see Fig. 3.2), and then with PZT actuator No. 6, which is diametrically opposite from the shaker point of action. The resulting sound pressure was sensed by 12 microphones in a semicircular array, 670
Table 3.4 Analytical and Experimental Natural Frequencies of the Closed Cylinder

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical frequency (Hz)</th>
<th>Experimental frequency (Hz)</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>1019</td>
<td>1010</td>
<td>-0.88</td>
</tr>
<tr>
<td>2.1</td>
<td>1121</td>
<td>1050</td>
<td>-6.33</td>
</tr>
<tr>
<td>4.1</td>
<td>1526</td>
<td>1545</td>
<td>1.24</td>
</tr>
<tr>
<td>5.1</td>
<td>2358</td>
<td>2400</td>
<td>1.78</td>
</tr>
</tbody>
</table>
Figure 3.8 Mode Shapes of the Closed Cylinder
a. Analytical (with the Cylinder Unwrapped)
b. Experimental in Axial Direction
c. Experimental in Circumferential Direction.
Figure 3.9 Acoustic Experiments in the Anechoic Chamber
mm from the cylinder shell. At this distance the microphones may be assumed to be in the far field, since the distance is about two times greater than the length of the cylinder. The microphones in the array were evenly spaced at 16.4°. (Note that there was no microphone number 7.) The whole array was moved axially step by step to collect data in positions 0, 2, 4, 6, 8, 10, 12 along the cylinder (see Fig. 3.9). In addition to those positions, positions -2 and 14, which were beyond the ends of the cylinder, were also used to collect data.

When the excitation is given by the shaker, the acoustic FRF is the ratio of the sound pressure to the shaker force. When the excitation is given by the PZT actuator, the acoustic FRF is the ratio of sound pressure to the PZT voltage. Appendix D contains plots of the variation in the acoustic FRF around the cylinder in axial position 6 for both the shaker excitation and PZT actuator excitation. Because the microphone array used in the experiment was semicircular and symmetry is assumed, the lower half of the plots are obtained by mirroring the upper parts. At several frequencies, the sound pressure field generated by the PZT actuator was somewhat different from the sound pressure field generated by the shaker. The two results are different because the mode shapes are distorted when the cylinder was excited by the PZT actuator with the idle shaker attached to the cylinder. In this case the shaker acts as an idle mass, spring and damper which alter the structural characteristics of the cylinder.

Despite the loading effect of the idle shaker, at 1550 Hz, which is found to be a very effective acoustic radiating frequency, the PZT actuator generates almost the same acoustic pattern as the shaker (see Fig. 3.10). The vibration mode at this frequency is mode (4,1). A previous test to determine the mode shape also shows very little distortion of the mode shape due to the idle shaker. Moreover, mode (4,1) was the most easily
Figure 3.10 Directivity Patterns of Acoustic Radiation from the Closed Cylinder at 1550 Hz
(a) Response to Shaker Excitation
(b) Response to PZT Actuator Excitation
identifiable mode shape among all mode shapes of the closed cylinder because the experimen
tal mode shape was found to be very close to the theoretical mode shape. Thus, 1550 Hz is a good frequency to use in subsequent experiments involving single tone "on resonance" excitation.
Chapter 4
Piezoelectric Sensors and Actuators

As already mentioned in the objective statement of this research, the cylinder application requires that the sensors and actuators be bonded on or embedded in the cylinder, and piezoelectric materials are used as structure borne sensors and actuators. This chapter studies the piezoelectric sensors and actuators.

A piezoelectric polymer polyvinylidene fluoride (PVDF) material is used as sensors, and a piezoelectric crystal lead zirconate titanate (PZT) material is used as actuators. The reasons for choosing those materials will be given in the beginning of this chapter. Then the constitutive relations which govern the behavior of piezoelectric materials are presented to explain the principle of operation of the piezoelectric sensors and actuators and to introduce the constants which describe piezoelectric properties.

Active acoustic control of the cylinder is done by sensing the vibration of the cylinder and exerting forces on the cylinder that reduce the vibration. Sensing is a very important process in any type of control. The sensor characteristics, i.e. the relation between the mechanical input to the sensor and the electrical output that the sensor deliver, must therefore be obtained. In the second section of this chapter, piezoelectric constitutive relations and theory of elasticity are used to derive the relation between strain in the cylinder and electric charge developed in a PVDF sensor. Then electric circuit theory is used to determine the relation between the charge and output voltage from the sensor. The discussion includes the theory of modal sensors, a special class of sensors with spatial filtering capability. Spatial filtering is a feature that is useful in feedback control because
it can reduce observation spillover. After theoretical relations are derived for the PVDF sensors, experiments are done to verify the analytical results. These experiments verify the sensor strain-voltage relation, give an idea of the output levels and the general behavior of the sensors, and prove the modal filtering capability of modal sensors.

The characteristics of the actuators must also be studied to ensure that the actuators are capable of exerting forces on the cylinder which cancel the cylinder vibration. For this purpose, the actuators are studied in the later part of this chapter. The discussion on the actuators explains how the electric input to the actuators produces mechanical strains in the cylinder. To investigate the authority of the actuators, an experiment is done to compare the vibration response of the cylinder due to bending forces produced by the actuators and due to the force given by the disturbance shaker.

4.1 Advantages of the Piezoelectric Transducers

4.1.1 PVDF Film Sensors

PVDF film is the chosen sensor for the cylinder. This material has several interesting features. The film can easily be implemented as a distributed sensor. Film made of PVDF has several excellent physical properties. PVDF is very flexible and can be bonded to the curved cylinder wall very easily. Furthermore, sensors can be made up of several layers of the film for increased sensitivity. PVDF is available commercially as thin sheets which are easy to cut into shapes, a feature which makes piezo polymers better sensors than piezo ceramics because ceramics are very brittle, difficult to bond to the cylinder, and difficult to cut into desired shapes. Among polymers, PVDF has by far the highest piezoelectric strain constant (or charge density per unit stress).
Sensors made of PVDF film are lightweight, the density being about one fourth of that of PZT, and very compliant, with the Young's modulus about 33% of that of PZT. Therefore, the sensors are not likely to impede the mechanical motion to be sensed and do not put considerable load on the vibrating structures. Transducers made of PVDF film occupy almost no space because the film is thin. Yet, they are tough and durable. PVDF is resistant to most chemicals. Examples of application of PVDF as protective coating against harsh environment include chemical vat liners, cable insulation and molded pipes (Collins et al., 1990). The piezoelectric effect of PVDF film is extremely stable. It endures time, temperature (up to about 120°C), and mechanical shock (up to several hundred g's). Also, it is a high bandwidth device, having a typical range of sensing from about 0.1 Hz to 10^7 Hz. Another advantage of PVDF film sensors is that they do not require complicated electronic interfaces, unlike strain gages which require special conditioning circuits. The interface circuit impedance requirement will be discussed in section 4.3.

4.1.2 PZT Actuators

The cylinder control application studied in this research project requires that the actuators be structure borne. Piezoelectric actuators are obviously better than other types of structure-born actuators such as proof mass actuators, because the latter are bulky and heavy. Point actuators like shakers are also spectrally 'white' in spatial sense, leading to the problem of spillover. Piezoelectric actuators have the advantage of being distributed actuators. Previous research work has shown that PZT actuators can approach the "perfect distributed vibration actuator" more closely than distributed point actuators, thus reducing the problem of spillover (Dimitriadis and Fuller, 1989). Piezoelectric actuators take negligible space on the structure. They are also light and can be attached to the structure
without other mounting parts. They are wide bandwidth devices compared to other actuators such as electromechanical shakers, and they have good control authority on the structure (as will be shown experimentally in this chapter). Being small and very lightweight, piezoceramic actuators do not substantially alter the dynamic characteristics of the structure by adding mass loading. Once mounted on the structure, they are rugged because of lack of moving parts. PZT, the piezoelectric crystal used in this research, is commercially available as actuators in a ready-to-use form of a thin piezoelectric layer sandwiched between two metal electrodes. The actuators can be cut into desired dimensions and thus are very versatile in many applications. Attachment to the structure is easily done with a thin layer of strong adhesive. Once attached properly to the structure, the actuators need virtually no maintenance. Compared with other structural actuators, PZT actuators are very inexpensive.

4.2 Piezoelectric Constants

The piezoelectric sensors work by generating electric charge when they are subjected to mechanical strain (a phenomenon called the direct piezoelectric effect). The piezoelectric actuators work by creating mechanical strain when subjected to electric field (a phenomenon called the converse piezoelectric effect). The fundamental mathematical relations among electrical and mechanical quantities are reviewed below. The constants described in this section apply to both sensors and actuators.

In a non-piezoelectric material, Hooke's law relate stress and strain only through the Young's modulus; and the dielectric equation relates electrical displacement to electrical field only through a dielectric constant. In a piezoelectric material the mechanical and electrical fields are coupled, resulting in the following constitutive equations (Holland and
\[ S = s^E T + d^T E \]  
\[ D = d T + e^T E \]

where,

\[ S = 6 \times 1 \text{ strain vector,} \]
\[ T = 6 \times 1 \text{ stress vector,} \]
\[ s^E = 6 \times 6 \text{ compliance matrix,} \]
\[ d = 3 \times 6 \text{ stress/electric displacement constant matrix} \]
\[ E = 3 \times 1 \text{ electric field vector,} \]
\[ D = 3 \times 1 \text{ electric displacement vector,} \]
\[ e^T = 3 \times 3 \text{ permittivity constant matrix,} \]

The superscripts indicate which variables are held constant in the definition. For example, the compliance \( s^E \) is defined at constant electric field and the dielectric constant \( e^T \) is defined at constant stress. Table 4.1 gives definitions of the elastic, dielectric and piezoelectric constant matrices.

Another electrical property required in calculation of voltages in piezoelectric materials is capacitance. Calculation of this property is required in experiments described later in this chapter. For a sensor, the capacitance is

\[ C_s = \varepsilon_s \frac{A_s}{h_s}, \]

where \( \varepsilon_s \) = permittivity of the material, \( A_s \) = sensor electrode area, and \( h_s \) = thickness.
Table 4.1 Definitions of Piezoelectric Constant Matrices

<table>
<thead>
<tr>
<th>Constant</th>
<th>Definition</th>
<th>Unit</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_{mn})</td>
<td>(\frac{\partial S_m}{\partial E_n}</td>
<td>_{T=\text{const}})</td>
<td>m/V</td>
</tr>
<tr>
<td></td>
<td>(\frac{\partial D_m}{\partial T_n}</td>
<td>_{E=\text{const}})</td>
<td>C/N</td>
</tr>
<tr>
<td>(e_{mn})</td>
<td>(\frac{\partial T_m}{\partial E_n}</td>
<td>_{S=\text{const}})</td>
<td>N/(Vm)</td>
</tr>
<tr>
<td></td>
<td>(\frac{\partial D_m}{\partial S_n}</td>
<td>_{E=\text{const}})</td>
<td>C/m²</td>
</tr>
<tr>
<td>(g_{mn})</td>
<td>(\frac{\partial E_m}{\partial T_n}</td>
<td>_{D=\text{const}})</td>
<td>Vm/N</td>
</tr>
<tr>
<td></td>
<td>(\frac{\partial S_m}{\partial D_n}</td>
<td>_{T=\text{const}})</td>
<td>m²/C</td>
</tr>
<tr>
<td>(h_{mn})</td>
<td>(\frac{\partial E_m}{\partial S_n}</td>
<td>_{D=\text{const}})</td>
<td>V/m</td>
</tr>
<tr>
<td></td>
<td>(\frac{\partial T_m}{\partial D_n}</td>
<td>_{S=\text{const}})</td>
<td>N/C</td>
</tr>
</tbody>
</table>
The capacitance of piezoelectric actuator can be calculated using the same formula with the actuator area substituted for $A_r$.

### 4.3 PVDF Film Sensors

Kynar PVDF film made by Atochem is used as sensors on the cylinder. The thickness of the PVDF layer is 52 μm. Table 4.2 gives the physical properties of this type of film sensor. The film is bonded to the cylinder with a double-sided adhesive tape layer to provide a uniform thin layer of adhesive. The input-output relations, i.e., the relation between the deflection of the cylinder and the output signal generated by the PVDF sensors, is derived in the following subsection. The next subsection discusses the modal filtering effect, a feature of distributed PVDF film sensors which is useful in feedback control.

#### 4.3.1 Relation between Strain and Voltage

Because PVDF sensors are used to provide feedback in controlling the cylinder vibration, the knowledge of the relation between the output of the sensor and the deflection of the cylinder has to be derived. One important question is whether the sensor output is proportional to strain, strain rate, or other variables. The objective of this subsection is to show how the cylinder vibration produces electric signals in the PVDF film sensors and to derive a mathematical model to relate the voltage generated by the sensors to the vibration magnitudes of the structure. Most of the equations given in this section are special forms of equations based on theories given in a well known reference (Lee, 1989), with modifications for the cylinder case. The theory of elasticity is used to derive the strains developed in the PVDF film sensor. These strains are then used to derive voltage
Table 4.2 Properties of Kynar Piezoelectric PVDF Film

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Piezo Strain Constants</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$d_{31}$</td>
<td></td>
<td>$23(10)^{12}$</td>
<td>$(C/m^2)/(N/m^3)$</td>
</tr>
<tr>
<td>$d_{32}$</td>
<td></td>
<td>$3(10)^{12}$</td>
<td>$(C/m^2)/(N/m^3)$</td>
</tr>
<tr>
<td>$d_{33}$</td>
<td></td>
<td>$-33(10)^{12}$</td>
<td>$(C/m^2)/(N/m^3)$</td>
</tr>
<tr>
<td><strong>Young’s Modulus</strong></td>
<td>$Y$</td>
<td>$2(10)^9$</td>
<td>$N/m^2$</td>
</tr>
<tr>
<td><strong>Permittivity</strong></td>
<td>$\varepsilon$</td>
<td>$106(10)^{-12}$</td>
<td>$F/m$</td>
</tr>
<tr>
<td><strong>Capacitance</strong></td>
<td>$C_s$</td>
<td>$379(10)^{-12}$</td>
<td>$F/cm^2$</td>
</tr>
<tr>
<td><strong>Volume Resistivity</strong></td>
<td>$\rho_v$</td>
<td>$1.5(10)^{13}$</td>
<td>$\Omega m$</td>
</tr>
<tr>
<td><strong>Max. Operating Field</strong></td>
<td>$E_0$</td>
<td>$30$</td>
<td>$V/\mu m$</td>
</tr>
<tr>
<td><strong>Density</strong></td>
<td>$\rho$</td>
<td>$1.78(10)^3$</td>
<td>$kg/m^3$</td>
</tr>
</tbody>
</table>
signal developed by the sensor.

The orientation of the sensor with respect to the cylinder and the coordinate system used in the sensor study have to be established first. The direction of the poling field and the direction of stretching during manufacture determines the coordinate axes to be used when referring to the constants which describe the material properties. For Kynar PVDF film, poling is always done in the thickness direction, also called the 3-direction. Because of the position of the electrodes, electric charge can be collected from the film only in the 3-direction. The direction of stretching is called the 1-direction. The 2-direction is perpendicular to both 1- and 3-directions.

Figure 4.1 shows the layers in the sensors and the orientation of the PVDF film. The PVDF drawing direction is aligned with the cylinder circumferential direction in order to make the sensor more sensitive to circumferential bending than to axial bending. The reason for choosing the above sensor orientation is related to the final control objective, which is to control mode (4,1) as mentioned in the previous chapter. With the type of PVDF film commercially available currently, only one-dimensional modal sensors are feasible (Zhou, 1991). To control the selected mode (4,1), the one-dimensional modal sensor can be made sensitive to mode 4 in the circumferential direction (modes (4,j)), or to mode 1 in the axial direction (modes (i,1)). Appendix B shows that for the cylinder, within the frequency range of interest (0-5000 Hz) there are more (i,1) modes than (4,j) modes. A sensor which is sensitive to mode (4,j) filters out more irrelevant modes than a sensor which is sensitive to mode (i,1). Therefore, modal filtering is better achieved with a sensor sensitive to modes (4,j) than with a sensor sensitive to modes (i,1). Another reason for using a mode (4,j) sensor instead of a mode (i,1) sensor is that the former undergoes a higher strain level than the latter when the cylinder vibrates in mode (4,1).
Figure 4.1 Layers and Orientation of PVDF Film Sensor on the Cylinder
The strain levels can be visualized intuitively by considering the mode shape. Figure 3.8 shows that the mode shape has four full sinusoidal waves in the circumferential direction and only one half wave in the axial direction. Moreover, the peak-to-peak variation in the circumferential direction is twice as high as the peak-to-peak variation in the axial direction. One drawback in using a circumferential mode 4 sensor is that the sensor is more sensitive to error in shape than lower mode sensors. However, the error can be avoided by precision in cutting the film and in positioning the sensor on the cylinder.

The coordinate axes used in the equations given in the discussion of PVDF sensors can be seen in Fig. 4.2. The PVDF film drawing direction and cylinder circumferential direction is called the x-direction. Because the film sensor is bonded around the cylinder, the range of x is 0 ≤ x < 2π R₀, where R₀ is the outside radius of the cylinder. The cylinder radial direction, which is also the film thickness direction, is called the z-direction. The y-direction is perpendicular to both the x- and z-directions. PVDF film deformations in the x, y, and z directions are u, v, and w, respectively.

An assumption made in the following derivations is that the cylinder vibration is highly dominated by flexural modes. This assumption is justified because the modal participations of the purely extensional (breathing) modes are very low due to the very high frequencies of the extensional modes (Blevins, 1979). With this assumption, strains induced by bending can be expressed in terms of radial deformation, w. With z₁ equal to the distance between cylinder neutral plane and film mid-plane (see Fig. 4.2), the strains S₁, S₂ and S₆ are

\[
\begin{align*}
S_1 &= -\frac{\partial^2 w}{\partial x^2} \\
S_2 &= \frac{\partial^2 w}{\partial y^2} \\
S_6 &= -2\frac{\partial^2 w}{\partial x \partial y}
\end{align*}
\]
Figure 4.2 Strain in PVDF Film Sensor Due to Bending in the x-y Plane
These strains produce electric displacement $D$. Among the three components of the electric displacements, only $D_3$ is of interest because the 3-direction is the only direction in which a pair of electrodes can collect charge from the PVDF layer. Furthermore, when the film is used as a sensor, there is no applied electric field in the film and the charge developed by the PVDF layer is collected in the electrodes. Therefore $E = 0$. In this case, the fourth equation in Table 4.1 describes the electric displacement as

$$D = e S .$$  \hspace{1cm} (4.5)

To obtain the charge developed in the PVDF film, Gauss' law

$$q(t) = \oint_S D \cdot d\sigma$$  \hspace{1cm} (4.6)

is used. From the last three equations, the charge is expressed as

$$q(t) = -z_f \oint_S F(x,y) \left[ e_{31} \frac{\partial^2 w}{\partial x^2} + e_{32} \frac{\partial^2 w}{\partial y^2} + e_{36} \frac{\partial^2 w}{\partial x \partial y} \right] d\sigma ,$$  \hspace{1cm} (4.7)

where the polarization function $F(x,y)$ is equal to 1 if at point $(x,y)$ the PVDF poling direction is the positive $z$ direction. $F(x,y)$ is equal to -1 if at that point the poling direction is the negative $z$ direction, and 0 if point $(x,y)$ is not covered by the PVDF sensor.

The above equation uses the charge density/strain constants $e$. However, technical references, particularly the Kynar reference manual, give the charge/stress constant $d$ instead of $e$ as a piezoelectric property. To express Eq. 4.7 in terms of the charge/stress constant matrix $d$, the relation

$$e = d c^e ,$$  \hspace{1cm} (4.8)

where $c^e$ is the compliance matrix (the inverse of the matrix of stiffness moduli), is used.
Equation 4.7 expresses charge as a function of normal strain in the x-direction, normal strain in the y-direction, and shear strain in the x-y plane. However, it can be shown that for Kynar PVDF film used on the cylinder the charge is generated mainly by strain in the x-direction because the charge density/strain constant $e_{31}$ is much greater than $e_{32}$ and $e_{36}$. Therefore, the charge calculation is approximated by considering the strain only in the circumferential direction. Figure 4.2 shows the strain in the sensor at a circumferential position $\theta$, where $\theta$ is expressed in a curved linear coordinate $x = R_o \theta$, and $R_o$ is the outside radius of the cylinder. In this case, charge developed in the sensor is

$$q = -e_{31} z_f \int_0^{2\pi R_o} F(x) \frac{\partial^2 w(x)}{\partial x^2} dx,$$  

(4.9)

where $F(x)$ represent the variation of sensor profile in the circumferential direction.

The above equation expresses the charge developed in the PVDF as a function of strain. However, because the electrical signal commonly used in signal processing is voltage instead of charge, the relation between voltage and charge has to be obtained. The output voltage of the piezoelectric sensor is determined not only by the charge, but also by the sensor capacitance and the input impedance of the circuit to which the sensor is connected. Figure 4.3.a shows the circuit model of a PVDF film sensor as a piezoelectric charge generator and a capacitor. Because the internal resistance of the film is very high (the resistivity is $1.5 \times 10^{12} \ \Omega$-m), it can be ignored, as shown in Fig. 4.3.b. Figure 4.3.c shows the complete circuit model when the sensor is connected to an interface circuit having an input resistance $R_i$. The dashed-line rectangles indicate the Thevenin circuit equivalents of the PVDF sensor and the interface circuit. Furthermore, the capacitive part of the input impedance, $C_i$, is usually very small compared to the resistive part $R_i$. Therefore, the input impedance of the interface circuit is assumed to be the same as the
Figure 4.3 Circuit Model of a PVDF Film Sensor
a. Complete Model of the PVDF Sensor
b. Film Resistance Neglected
c. PVDF Sensor with Interface Circuit.
(From Pennwalt, Inc., *Kynar Piezo Film Technical Manual*, Valley Forge, 1987, p. 27)
input resistance. The output voltage $V$ can be calculated by considering the fact that current is the time rate of change of charge, and applying Kirchhoff current law to split the current generated by the piezoelectric charge generator into the current going to the capacitor and the current going to the resistor. The output voltage is obtained by calculating the voltage across the capacitor, i.e.,

$$v = \frac{1}{C_s} \int \left[ \frac{dq}{dt} - \frac{v}{R_i} \right] dt . \quad (4.10)$$

where $C_s$ is the capacitance of the PVDF sensor, $q$ is the charge, and $R_i$ is the input resistance of the interface circuit. The capacitive part of the interface input impedance has been neglected because it is usually several order of magnitudes smaller than the capacitance of the PVDF sensor film. In the Laplace domain with zero initial conditions, the above equation can be expressed as the transfer function from the electric charge to the output voltage, i.e.,

$$\frac{V(s)}{Q(s)} = \frac{1}{C_s} \frac{s}{s + \frac{1}{R_i C_s}} . \quad (4.11)$$

From the above equation, it is clear that the transfer function between charge and voltage is a function of $R_i$, the input resistance of the interface circuit connected to the PVDF sensor. When $R_i$ approaches zero, the output voltage approaches zero as well. For very high interface input resistance, the output voltage is

$$V(t) = \frac{q(t)}{C_s} . \quad (4.12)$$

The above equation shows that the output voltage is proportional to the charge. Equation 4.9 shows that $q(t)$, the charge generated in the PVDF film, is proportional to strain.
Therefore, it is clear that when a PVDF sensor film is connected to an interface circuit with a very high input resistance, the output voltage generated is directly proportional to the strain in the sensor.

Operational amplifier circuits that provide very high input impedances for the PVDF film sensor can be used with the PVDF film sensor to provide strain measurement. High quality signal processing instruments usually have input impedances in the order of tens of megohms. Therefore, connecting the PVDF film sensors directly to such instruments provides sufficiently high impedance for the PVDF film sensors. However, many other instruments such as A/D boards from a wide variety of manufacturers have input impedances less than 100 kΩ. If they are used with a PVDF sensor, a buffer circuit must be used to ensure that strain is being measured.

The voltage-charge relation can be shown numerically and graphically. The mode 4 PVDF sensor, which has a capacitance $C_s$ of $24 \times 10^{-9}$ F is used as an example. The resistance is neglected since it is very high. According to Eq. 4.11, the transfer function between the output voltage and charge depends on the input impedance, $R_i$, of the interface circuit. Figure 4.4 shows the magnitude of the transfer functions for several input impedances. The voltage/charge transfer function has a high pass characteristic. The transfer function for $R_i = 1$ MΩ is highlighted as a high impedance example. For this input impedance value, the transfer function is practically zero for frequencies lower than 10 Hz, and constant for frequencies higher than about 1 kHz. On the other hand, for a low input impedance such as 100 Ω, the transfer function is mainly zero for frequencies below 1 kHz, and does not have a constant value unless the frequency is higher than about 1 MHz.

An equation similar to Eq. 4.11 can be derived to relate the PVDF output current and
Figure 4.4 Effect of Interface Input Impedance on the Transfer Functions Between Charge and Voltage of a PVDF Sensor
strain rate. With that equation, it can be shown that with a very low input impedance the PVDF sensor output is proportional to strain rate. For the mode 4 sensor, the region between $R_i = 100 \, \Omega$ and $R_i = 100 \, k\Omega$ is a transition region where the sensor gives neither strain nor strain rate measurement. Operating a PVDF film sensor with an interface circuit in such a transition region will give a signal which lags strain rate by an angle other than 90 degrees for sinusoidal strains. Eq. 4.11 shows that the input resistance transition region depends on frequency as well as the sensor’s properties. Previous research work done on beams shows that, for a particular PVDF film sensor operating with a particular interface circuit in the input impedance transition region, a higher frequency tends to shift the measurement towards strain measurement (Zhou et al., 1991).

4.3.2 Modal Sensor

The sensor used in most of the vibration control experiments discussed later in this thesis is a special type of distributed sensor called the modal sensor. The features of this sensor and the basis for the design procedure is discussed in this sub-section.

Equation 4.7 can be exploited to create a type of vibration sensor whose signals contain such spatial information that the sensor is sensitive only to a selected vibration mode. This sensor feature is useful in structural vibration control (Lee, 1989). Previous research work has shown that creating a PVDF modal sensor which is sensitive to only one specific mode of a two-dimensional structure is not possible except in a few cases. Even in those cases, creating such a two-dimensional modal sensor is difficult because it requires that the polarization of the PVDF be varied over the area (Zhou, 1991). The commercially available piezo film cannot be used to make a two-dimensional modal sensor because the polarization is constant throughout the film. Despite the limitation
mentioned above, a one-dimensional modal sensor which is sensitive to a family of modes can be created using a theory described below. A one-dimensional modal filter bonded on the cylinder in the circumferential direction is selectively sensitive to a particular circumferential modal index i, but it is not capable of making distinction among signals from different axial modes. Furthermore, the piezoelectric charge density/strain constant e_{32} is at least one order of magnitude smaller than e_{31}, so that a sensor oriented with the drawing direction in the circumferential (x) direction of the cylinder is much less sensitive to bending in the axial planes. Thus, application of one-dimensional modal sensors on two-dimensional structures is feasible. Experiments have shown successful application of one-dimensional modal sensors to a two-dimensional structure (Zhou et al., 1991). The equations derived in this section mainly extend the equations developed for one-dimensional modal sensors by Lee to a two-dimensional structure, particularly the cylinder.

The radial deflection of the cylinder can be expressed as

\[ w(x,y,t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} A_{ij} \phi_j(y) \cos \left( \frac{i \pi x}{R} \right) \cos(\omega_{ij} t) \, , \]  

(4.13)

where \( \phi_j(y) \) is the \( j \)th axial mode shape function and \( \cos(\pi x/R) \) is the \( i \)th circumferential mode shape function with \( x \) as the circumferential distance from a reference point, \( A_{ij} \), and \( \omega_{ij} \) are the modal participation and the natural frequency of mode \((i,j)\). The one-dimensional sensor is only capable of sensing deflection in the circumferential direction. To this sensor, the spatial variation of the deflection of the cylinder is a function of \( x \) only, i.e.,

where \( C_{ij} \) is a constant representing the contribution of vibration mode \((i,j)\). Furthermore,
\[ w(x,t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} C_{ij} \cos \left( \frac{i\pi x}{R_o} \right) \cos \left( \omega_{ij} t \right) . \] (4.14)

because the sensor is assumed to be sensitive primarily to strain in the x direction, Eq. 4.7 reduces to

\[ q(t) = -z f e_{31} \int_0^{2\pi R_o} \frac{\partial^2 w}{\partial x^2} dx . \] (4.15)

where the new variable \( b(x) \) expresses the width of the PVDF sensor film as a function of \( x \). Combining the last two equations results in,

\[ q(t) = z_f e_{31} \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} C_{ij} (i/R_o)^2 \int_0^{2\pi R_o} F(x) \cos \left( \frac{i\pi x}{R_o} \right) dx \cos(\omega_{ij} t) . \] (4.16)

To create a modal sensor, the last equation is used to exploit the orthogonality of the second derivatives of the normal mode functions with respect to \( x \). In general, this orthogonality condition for two mode shape functions \( f_n \) and \( f_i \) of a one dimensional structure is,

\[ \int_0^L \frac{d^2 f_n(x)}{dx^2} \frac{d^2 f_i(x)}{dx^2} dx = \delta_{ni} \frac{\lambda_n^4}{L^3} , \] (4.17)

where \( L \) is the whole length of the one dimensional structure, \( \delta_{ni} \) is the delta Kronecker function, and \( \lambda_n \) is the \( n^{th} \) eigenvalue.

If \( f_i \) is taken as the circumferential mode shape of the cylinder, (the one-dimensional sensor is placed on the cylinder in the circumferential direction), then \( L \) must be the same as the circumference of the cylinder, and the orthogonality principle is expressed simply...
as

\[ \int_{0}^{2\pi R_o} \cos \left( \frac{n x}{R_o} \right) \cos \left( \frac{i x}{R_o} \right) \, dx = \delta_{ni} \pi R_o. \] (4.18)

where \( \delta_{ni} \) is the Kronecker delta function, and \( R_o \) is the outside radius of the cylinder. Equations 4.16 and 4.18 show that if the width of the sensor film is varied along the circumference of the cylinder according to,

\[ b(x) = \mu_n \cos(i_0 \frac{x}{R_o}), \] (4.19)

where \( \mu_n \) is the amplitude of the width of the sensor, then the charge equation for such a sensor can be obtained by substituting \( b(x) \) in Eq. 4.19 for \( F(x) \) in Eq. 4.16 and applying Eq. 4.18 to the result. These substitutions result in the expression for the sensor charge

\[ q(t) = z_t e_{31} \mu_n \pi \frac{i_0^2}{R_o} \sum_{j=0}^{\infty} C_{i_0 j} \cos \left( \omega_{i_0 j} t \right), \] (4.20)

where \( z_t \) is the distance between the cylinder neutral plane and the sensor mid-plane, \( e_{31} \) is the charge density/strain constant, \( \mu_n \) is the width amplitude of the sensor, \( R_o \) is the outside diameter of the cylinder, \( C_{i_0 j} \) and \( \omega_{i_0 j} \) are the modal participation and natural frequency of mode \((i_0,j)\). Notice that orthogonality of mode shapes (Eq. 4.18) leaves only \( C_{i_0 j} \) as non-vanishing terms in the summation over the index \( i \). This result show that the sensor will be sensitive only to vibration in the \( i_0^{th} \) circumferential modes. That is, the sensor will act as a modal filter, insensitive to vibration modes other than modes \((i_0,j)\), where \( j \) is any axial mode index and \( i_0 \) is the circumferential mode index to which the sensor is designed to be sensitive.
The analysis done above on modal sensors is verified by an experiment which will be described in the next section. In another experiment, Eqs. 4.20 and 4.12 are used as a basis for predicting the output voltage produced by the sensor when subjected to a vibration with known strains.

4.3.3 Experiments with PVDF Film Sensors

The first purposes of these experiments is to predict the output voltage of the sensor to determine the feasibility of using the sensor in a control system. The second purpose is to study the modal filtering effect of PVDF modal sensors. As will be shown later in the chapter dealing with vibration control experiments, modal filtering effects increase the stability of the feedback control system when an analog feedback control is applied. The third purpose of the experiments is to study the effect of different sensor geometry to determine the best sensor for the control problem.

The experiments were done on the cylinder without the end caps. The open cylinder is a good structure for investigating the film sensors because it can act as a one-dimensional structure, i.e., for Rayleigh modes the cylinder's radial deflection is just like the deflection of a simply-supported beam. The similarity between the Rayleigh mode shape and the simply supported beam mode shape can be observed by considering Eq. 3.16. The \( i^{th} \) Rayleigh mode shape is

\[
W_{i,0} = \cos (i \theta),
\]

where \( \theta \) is the position angle measured from an antinode (e.g. from the shaker point of action). If the cylinder is unwrapped, or the polar coordinate is transformed into a linear coordinate with
then the expression of the Rayleigh mode shape becomes

\[ W_{i,0} = \sin (i \pi \xi), \]  

(4.23)

which is the \(i\)th mode shape function of a simply supported beam with \(\xi\) as a non-dimensional linear coordinate.

Figure 3.2 in the previous chapter shows the positions of five PVDF sensor films on the outer surface and one sensor film on the inner surface of the cylinder shell. The PVDF layers in all of the sensors were 52 \(\mu\)m thick. Five sensors were experimentally studied:

1. Rectangular (also referred to as "plain") sensor
2. Mode 4 sensor
3. Incomplete mode 2 sensor
4. Axial sensor
5. Double-layered sensor.

The sensors were tested under burst random excitation from a shaker. The shaker worked at point (0,6) in radial direction, except in the testing of the axial sensor, where the shaker was attached onto the end of the open cylinder at point (6,0) and worked in the axial direction. The sensors and their responses are described below.

**Rectangular PVDF Film:** The first sensor is a rectangular strip, 25.4 mm wide, attached with the center line lying between point (18,10) and point (6,10). This position was selected to enable comparison with the mode 4 PVDF sensor, because this position covers a part of the cylinder between two circumferential nodes (see Fig. 3.2). A frequency
response function (FRF) taken between the sensor voltage and the force generated by a
shaker working at point (0,6) is shown in Fig. 4.5. For comparison, another FRF obtained
with an accelerometer on top of the center of the PVDF sensor is overlaid on the plot.

Two things can be learned from this experiment. First, the output voltage level of the
PVDF is in the order of millivolts for every Newton of shaker force. Second, the FRF
peaks of the PVDF stays in the same order of magnitude across the frequency range of
study. This is in contrast with the accelerance, which generally increases with frequency.
The rectangular sensor film senses much fewer modes than the accelerometer. This shows
that the rectangular PVDF sensor has some modal filtering effect because of its position.
Among the modes sensed by the accelerometer but not by the PVDF sensor are Modes
(3,1) (1570 Hz), and mode (5,1) (2630 Hz).

Mode 4 Sensor Film: As shown in Chapter 3, the circumferential mode shapes of the
cylinder are sinusoidal. A modal sensor to sense mode (4,j) vibration is thus shaped like
four full periods of sinusoid around the cylinder. It has been shown experimentally
(Sumali and Cudney, 1990) that metal-coated polyvinylidene fluoride (PVDF) film shaped
into sinusoids acts as good modal sensors on simply-supported beams. The mode 4 sensor
consists of eight identical parts, with a 25.4 mm maximum width, attached around the
cylinder with alternating polarities. An FRF between the sensor voltage and the force
generated by a shaker working at point (0,6) is shown in Fig. 4.6. Comparison between
this FRF to the FRF given by the rectangular sensor film demonstrates that the mode 4
sensor film is very sensitive to mode (4,j), and that it filters out other modes. When Fig.
4.6 is redrawn on a linear scale (see Fig. 4.7), it reveals more clearly that the three modes
which are sensed much more strongly than other modes are Rayleigh mode 4 (1420 Hz),
mode (4,1) (1845 Hz) and mode (4,2) (3690 Hz). Mode (4,3) is not sensed strongly
Figure 4.5 FRF between Plain PVDF Sensor and Shaker Force, Compared with FRF between Accelerometer and Shaker Force.
Figure 4.6 FRF between Mode 4 Sensor Voltage and Shaker Force.
Figure 4.7 FRF between Mode 4 Sensor Voltage and Shaker Force, Drawn on a Linear Scale to Show Modal Filtering Effect
because the PVDF sensor happens to be very close to the nodal line of this mode. This result demonstrates the modal filtering effect of the sensor.

**Sensor voltage level**: Because the mode 4 PVDF film sensor is to be used as a feedback sensor in controlling the cylinder vibration, better knowledge of the sensor voltage level is needed. A test was done in time domain to investigate the voltage generating capability of the PVDF film sensor and to verify Eq. 4.12, (relation between voltage and charge) and Eq. 4.9, (relation between charge and deflection). For this purpose, the open cylinder was excited with a shaker with a sinusoidal excitation at 1420 Hz, the natural frequency of Rayleigh mode 4 vibration. The acceleration at point (0,5), an antinode, was measured. The sensor was connected directly to the frequency analyzer. Equations 4.20 and 4.12 can be combined to obtain the relation between the voltage developed by the sensor and acceleration at an antinode. At this Rayleigh mode 4 natural frequency, it can be assumed that the contribution of other modes are negligible.

First, the deflection of the cylinder is obtained from the accelerometer output by using Eq. 4.14. Under the assumption of single mode vibration mentioned above, the only non-zero term is the one with the circumferential mode index $i = 4$ and axial mode index $j = 0$. The frequency $\omega_0$ is the natural frequency of Rayleigh mode 4 (1420 Hz). The constant $C_{ij}$, which is the modal amplitude for the Rayleigh mode 4 is determined by using

$$a(\theta_0,t) = -\omega^2 C_{40} \sin (\omega_0 t)$$

(4.24)

where $a(\theta_0,t)$ is the measured acceleration at the antinode. With this antinode acceleration, the deflection of the cylinder is

where $\theta$ is the position angle. Next, the effect of the interface circuit input impedance on the sensor output voltage is investigated to determine the voltage-deflection relationship
for the PVDF film modal sensor. The input impedance of the Tektronix 2630 Fourier Analyzer is 100 kΩ parallel with 200 pF (Tektronix, Inc., 1991). The capacitance of the PVDF modal sensor could be computed using Eq. 4.3. With the permittivity and thickness data given in the Kynar manual, this equation results in the sensor capacitance, $C_s$, of 21 nF. However, measurement with a capacitance meter gave a $C_s$ of 24 nF. The difference between the measured and the theoretically predicted capacitances might be due to the fact that the dielectric constant given in the Kynar manual is only accurate at the specified frequency, given as 10 kHz in the specification. Also, the thickness of the PVDF film might differ slightly from the specified value. The measured capacitance value $C_s$ is used in Eq. 4.12. The interface input impedance dependency of the mode 4 PVDF film sensor used in this experiment at the frequency of 1420 Hz can be found in Fig. 4.4. The plots show that if the input resistance is less than 100 Ω the sensor voltage is zero because the interface circuit practically shorts the electrodes, letting the current flow freely between them. For the low input resistances, the sensor signal is in-phase with strain rate. Current or 'charge' amplifiers work in this region for this sensor. For an input resistance of 100 kΩ or higher, the sensor gives about 5 volts per milli Coulomb of charge developed, and the sensor signal is proportional to strain. Thus, at this input resistance region the sensor measures strain.

Because the Tektronix Fourier analyzer used in this experiment has an input resistance of about 100 kΩ, the plots in Fig. 4.4 state that connecting the PVDF mode 4 film sensor to the Tektronix Fourier analyzer makes the sensor output proportional to strain. To verify this theory by experiment, first it is necessary to verify the charge and voltage equations
developed in the previous section. In this experiment the cylinder vibration is assumed
to consist only of mode (4,0), only one term of the summation in Eq. 4.20 is non-zero.
To calculate the charge developed in the film, the integration in Eq. 4.16 is performed to
obtain the constant $C_{40}$ in Eq. 4.20. The charge obtained is substituted into Eq. 4.12 to
obtain an equation which predicts the complex voltage generated by the sensor based on
the knowledge of acceleration on an antinode. The resulting equation relates voltage
directly to the measured acceleration, \textit{i.e.},

$$V = z_r e_{31} \mu_{40} \pi \frac{i^2 a(\theta_0, t)}{\omega R_0} \frac{j R_i}{1 + j \omega R_i (C_f + C_i)}.$$  \quad (4.26)

The numerical values of the variables in the last equation are,

- $z_r$ = distance between sensor mid-plane and cylinder shell mid-plane,
  (assumed to be $1/2 \times$ cylinder shell thickness $= 3.175 \times 10^{-3}$ m),
- $e_{31} = 7.35 \times 10^{-2}$ C/m$^2$ (from Eq. 4.8 with electric displacement/charge
  constants, $d_{31}$ and $d_{32}$ obtained from Kynar manual),
- $\mu_{40}$ = width amplitude of the mode 4 sensor $= 25.4 \times 10^{-3}$ m,
- $i$ = circumferential mode index $= 4$,
- $R_o$ = outside radius of cylinder $= 0.127$ m,
- $\omega$ = 1440 Hz = 8922 rad/s,
- $a(\theta_0, t) = 77.50 \sin(8922 t)$ m/s$^2$, (from measurement),
- $R_i$ = input resistance of Tektronix 2630 Fourier analyzer $= 100$ k\,\Omega,
- $C_s$ = sensor capacitance $= 24 \times 10^{-9}$ F,
- $C_i$ = input capacitance of Tektronix 2630 Fourier analyzer $= 200 \times 10^{-12}$
  F,
- $j = \sqrt{-1}$. 
With the measured acceleration, the above formula gives a sensor output voltage amplitude of 94.28 mV at an angle of 4.1 degrees leading the strain or charge. The measurement result was a 104.6 mV output voltage amplitude with a phase about the same as that of deflection. Thus, there is an 11 percent difference between the predicted and the calculated sensor voltage magnitudes.

**Incomplete Mode 2 Sensor Film:** The third sensor film consists of sinusoidally-shaped parts (33.5 mm at the widest) designed to sense circumferential mode 2 (see Fig. 3.2). However, only two out of four sinusoidal parts are used. The purpose of testing this incomplete sensor is to see if a piezoelectric film sensor can act as a modal sensor when its shape agree only piecewise with the second derivative of the structure’s mode shape with respect to distance. The FRF given by this sensor film on a linear scale in Fig. 4.8 shows that this sensor does sense Rayleigh mode 2 (263 Hz) much more strongly than other modes. However, this sensor is not very successful in filtering out Rayleigh mode 3 and it also fails to sense modes (2,n), where n > 0.

The sensor’s failure to sense other modes (2,1) and failure to filter out Rayleigh 3 mode show that a shaped sensor which does not cover the entire circumference of the cylinder does not work well as a circumferential mode sensor. The failure to filter out the modes other than the intended ones is due to the fact that the space interval covered by the sensor is not the interval which causes the film width function b(x) to be orthogonal with the second derivative of the mode shape function, φ(x), with respect to position. This experiment shows that for the incomplete modal sensor, the limits of integration in Eq. 4.18 do not cause the function to be zero for the modes other than the intended one.

**Axial Sensor Film:** The fourth sensor is a rectangular 25.4 mm wide PVDF strip attached
Figure 4.8 FRF between the Incomplete Mode 2 Sensor Voltage and Shaker Force, on a Linear Scale.
along the inside of the cylinder shell. The center line of the sensor lies between point (6,0) and point (6,12). The shaker was attached onto the end of the cylinder at point (6,0) so that the shaker force was in the cylinder axial direction. The excitation was random in 0-10 kHz range. For comparison, an accelerometer was placed at the end of the cylinder shell at point (6,12).

For the FRF between the force and the axial PVDF sensor (Fig.4.9.a), the prominent peaks in the 0 - 5 kHz range are identifiable as: 262 Hz (Rayleigh 2 mode), 750 Hz (Rayleigh 3 mode), 803 Hz (Love 3 mode), 1570 Hz (mode (3,1)), 2250 Hz (Rayleigh 5 mode), and 4030 Hz (mode (5,3)).

The FRF between the force and the acceleration (Fig.4.9.b) has so few distinctive peaks that it is difficult to determine the modes sensed by the accelerometer. One possible mode of axial vibration is the "accordion" mode, where the cylinder shell contracts and expands only in the axial direction. In this case, the natural frequency is that of a rod, which can be calculated by (Thomson, 1988)

\[ \omega = \frac{n \pi}{L} \sqrt{\frac{E}{\mu}}, \]  

(4.27)

where \( L \) is the length of the cylinder, \( E \) is the modulus of elasticity, \( \mu \) is the density, and \( n \) is the mode number. According to this formula, the axial natural frequencies are 5990 Hz and its integer multiples. Only the first of these axial natural frequencies is within the test frequency range. A peak at this frequency is found on both the FRF obtained with the PVDF sensor and the FRF obtained with the accelerometer. However, without \textit{a priori} knowledge of the frequency, it would be impossible to tell from the experiment that this frequency is an axial mode frequency without further testing.
Figure 4.9 Comparison between Axial PVDF Sensor and Accelerometer
a. FRF between Axial Sensor Voltage and Axial Force
b. FRF between Axial Acceleration and Axial Force.
A conclusion drawn from this experiment is that neither the accelerometer nor the axial PVDF film sensor is good for the experimental determination of the axial natural frequencies of the cylinder. The difficulties in detecting the axial modes are: first, the axial natural frequencies are very high, and second, the cylinder vibration is dominated by flexural modes even when the excitation is in the axial direction.

**Two-layer Sensor Film:** The fifth sensor is shaped like one of the sinusoidal parts which form the mode 4 film sensor (see Fig. 3.2); however, it is made of two PVDF film layers joined together with opposing polarities (see Fig. 4.10). The positive sides of the PVDF layers are connected together to the positive lead. The negative sides are connected together to the negative lead and are grounded. The purpose of this arrangement is to provide shielding to isolate and to ground disturbances.

The response of the two-layer sensor to burst random excitation from PZT actuator no. 6 is shown in Fig. 4.11. As a comparison, in that figure is also shown the response of the sinusoidal part of mode 4 sensor film whose position is symmetric with the two layer sensor with respect to the cylinder mid-plane. The response of the two-layer sensor at the most prominent resonant frequencies (Rayleigh 2, 265 Hz; Rayleigh 3, 747 Hz, Rayleigh 4, 1430 Hz; Rayleigh 5, 2360 Hz) is 6 dB higher than the response of the one-layer sensor. Moreover, the two layer sensor appears to give a cleaner FRF than the one layer sensor, as seen by comparing the two plots in Fig. 4.11.
Figure 4.10 A Two-Layer PVDF Sensor on the Cylinder
Figure 4.11 FRF between Two-Layer PVDF Sensor Voltage and PZT Actuator Voltage (Solid Line), Compared with FRF between One-Layer PVDF Sensor Voltage and PZT Actuator Voltage (Dashed Line).
4.4 PZT Actuators

4.4.1 Principles of Operation

The PZT actuators are bonded to the outer surface of the cylinder wall. When voltage (and hence electric field) is applied to the electrodes of an actuator, the actuator works by imparting strain to the cylinder. Because the actuator is bonded to one side of the cylinder wall, this strain effectively exerts a bending moment and an extensional force on the cylinder wall (see Fig. 4.12).

Several models have been developed for the action of piezoelectric actuators on flat plates. These models can be adapted for use in the cylinder case. In the case where the actuator is bonded to only one surface of the cylinder wall, the net action of the actuator can be expressed as a line moment along the edges of the actuator plus an in-plane force also from the edges of the actuator. The moment loading provided by the line moment distribution can be expressed as (Lefebvre, 1991),

\[
    f_n(x, \theta) = \frac{\partial m_\theta}{\partial x} + \frac{1}{R_0} \frac{\partial m_x}{\partial \theta}, \tag{4.28}
\]

where

\[
    m_x = M_c [u_s(x - x_1) - u_s(x - x_2)] [\delta (R_0 \theta - R_0 \theta_2) - \delta (R_0 \theta - R_0 \theta_1)], \tag{4.29}
\]

and

\[
    m_\theta = M_c [u_s(R_0 \theta - R_0 \theta_1) - u_s(R_0 \theta - R_0 \theta_2)] [\delta (x - x_2) - \delta (x - x_1)]. \tag{4.30}
\]

The function \( u_s(x) \) is the unit step function and \( \delta(x) \) is the Dirac delta function. The piezoelectric complex bending amplitude, \( M_c \), is a function of the geometric
Figure 4.12 Bending Moments and In-Plane Forces Exerted by a PZT Actuator on the Cylinder Wall
a. Stretching Force on the Surface
b. Equivalent Force and Moment Model.
characteristics and piezoelectric constants of the actuator. The parameters \( x_1 \) and \( x_2 \) are the positions of the ends of the actuator in the axial direction, and \( \theta_1 \) and \( \theta_2 \) are the positions of the ends of the actuator in the circumferential direction. \( R_o \) is the outside radius of the cylinder.

Similarly, the in-plane force loading provided by the actuator can be expressed as

\[
f_x = F_c \left[ u_x(R_0 \theta - R_0 \theta_1) - u_x(R_0 \theta - R_0 \theta_2) \right] \left[ \delta(x-x_2) - \delta(x-x_1) \right], \tag{4.31}
\]

\[
f_\theta = F_c \left[ u_\theta(x-x_1) - u_\theta(x-x_2) \right] \left[ \delta(R_0 \theta - R_0 \theta_2) - \delta(R_0 \theta - R_0 \theta_1) \right]. \tag{4.32}
\]

\( F_c \), the piezoelectric actuator stretching amplitude, is a function of the geometric characteristics and piezoelectric constants of the actuator. The calculation of \( M_c \) and \( F_c \) is complicated, especially for shells such as the cylinder. This topic has been a subject of much research and is considered outside the scope of this thesis.

### 4.4.2 Experiments

Experiments were done to determine the authority of the PZT actuators. Figure 3.2 shows the locations of the eleven piezoceramic actuators on the cylinder. The actuators were bonded to the cylinder during fabrication, before exact knowledge of the proper locations was obtained and before the mode to be controlled was known. The locations of the actuators were on antinodes of the prediction of the modes which seemed likely to be controlled. All of the actuators are made of Lead Zirconate Titanate (PZT), 0.19 mm thick, 37.5 mm long and 30 mm wide. These actuators have nickel electrode layers. Each of the actuators is bonded with M-bond 200 adhesive to a rectangular flat-milled surface.
on the cylinder (see Fig. 4.13). There are actuators on the antinodes of circumferential modes 2, 3, 4 and 6, and also on the antinodes of axial modes 1 and 3 of the closed cylinder.

In the experiments, it is necessary to drive the PZT actuators with voltages ranging from 0 V to about 120 V (rms). A transformer with an output/input ratio of 18.7 is used to generate this voltage. PZT actuators are almost purely capacitive loads, so that the power factor is very poor. Because of this poor power factor, the current needed to drive the actuators is high -- sometimes too high for the power amplifier to supply. To minimize current requirement, an inductor is used in parallel with the PZT actuator. The current can be expressed as (Bobrow, 1985)

$$I_{(\text{rms})} = \frac{P}{V_{(\text{rms})} \cos(\psi)},$$  \hspace{1cm} (4.33)

where $P$ is the average power supplied to the PZT actuator, $V$ is the voltage, and $\psi$ is the power factor, which is expressed as,

$$\psi = \tan^{-1} \left[ \frac{\omega L - \frac{1}{\omega C}}{R} \right],$$  \hspace{1cm} (4.34)

where $\omega$ is the frequency, $C$ is the capacitance of the PZT actuator, and $L$ is the inductance. The required inductance to minimize the current is calculated by maximizing the power factor, i.e.,

$$L = \frac{1}{(\omega^2 C)}. \hspace{1cm} (4.35)$$

With $C = 90 \text{ nF}$, $\omega = 2 \pi \times 1550$ (1550 Hz is the frequency of mode (4,1) vibration of the closed cylinder), $L$ was calculated to be 117 mH. A 150 mH inductor was used, also in all other experiments involving the PZT actuators, although the inductance was
Figure 4.13 Bonding of a PZT Actuator on the Cylinder Wall
calculated only for 1550 Hz. To supply the resistive part of the load, a 37 kΩ resistor was also used in parallel with the PZT actuator and the inductor. This resistance value was determined by assuming a maximum voltage of 140 V and considering the 0.5 W power rating of the resistor.

The test was done with a swept sine excitation, first with the shaker, and later with PZT actuator No. 6, whose position was diametrically opposite the shaker point of action. An accelerometer measured the cylinder response at point (12,6). The test frequency range was from 250 to 320 Hz. Such a narrow frequency range and swept sine excitation were intended to obtain a good resolution around the resonant peak. The peak of the FRF is at 263 Hz -- Rayleigh mode 2 frequency. This mode was chosen for the actuator test because, being the lowest-index mode, this mode has the longest structural wavelength around the cylinder. The long structural wavelength helps reduce the effect of the inaccuracies of the positions of the PZT actuator with respect to the shaker point of action. Due to fabrication error, PZT actuator No. 6 was not exactly diametrically opposite from the shaker point of action. The position error of the actuator was about 4 degrees in the radial direction. This position error is about 18 % of the distance between a node and an antinode of Rayleigh mode 4, but only about 9 % of that distance for Rayleigh mode 2.

The shaker excitation resulted in a frequency response function in Fig. 4.14.a. The excitation with PZT No. 6 resulted in a frequency response function in Fig. 4.14.b. The acceleration/force FRF has a peak of about -8 dB (0 dB = 1 g/N). The acceleration/PZT volt FRF has a peak of about -25 dB (0 dB = 1 g/V). Comparison between the two FRF indicates that at this frequency applying a force of 1 N to the shaker point of action is equivalent to applying about 7.1 V to PZT actuator no. 6.
Figure 4.14 Comparison between PZT Actuator and Shaker
a. FRF between Acceleration and Shaker Force
b. FRF between Acceleration and PZT Voltage
Chapter 5
Control Experiment Design and Setup

The analysis and experiments discussed in the previous chapters show that PVDF film can function well as modal sensors and the PZT patches have sufficient authority to cancel the vibration induced by the shaker. This chapter describes how experiments were set up and performed to utilize the sensors and the actuators together with a controller to reduce the vibration of the cylinder, with the final objective of reducing the acoustic emission from the cylinder. The purposes of the experiments will be explained first, followed by the approach taken to achieve those purposes. Then the selection of the adaptive digital control algorithm is discussed, followed by the explanation of how the algorithm works. After this discussion on the software, the hardware and experiment setup are described. This chapter is concluded with a summary listing the important details of all the experiments and a brief explanation of each experiment. The results of these experiments will be given in the next chapter.

5.1 Objectives and Approach

The final objective of the experiments described in this chapter was to minimize the sound pressure level (SPL) generated by the cylinder by controlling the vibration with an adaptive control algorithm implemented on a digital signal processing (DSP) board. Another objective was to study different combinations of actuators and the effects of actuator locations to the control results. Comparison between structure-borne piezoelectric transducers and a microphone as sensors is also made.
The approach was as follows. First, preliminary control experiments were done on the open cylinder with a simple analog closed loop to prove that the sensors and actuators were indeed capable of operating together to cancel the vibration caused by a shaker. (The experiments will be referred to as analog control experiments in this chapter.) The open cylinder was used because it has a one-dimensional mode shape, which simplifies analysis. The cylinder was excited with a single-tone harmonic excitation, and the resulting sound pressure levels before and after control were monitored in a reverberation chamber. In these experiments several variables were measured, such as the voltage level of the error signal, acceleration at a point, and the control voltage applied to the PZT actuators. Knowledge of this voltage is important because the maximum allowable actuator voltage is one of the constraints limiting the performance of the control. In these experiments the effect of the location and number of actuators was also studied by using different combinations of actuators. Experiments were first done with the PVDF mode 4 film as a sensor. Then a PZT patch was also used as sensor in place of the PVDF mode 4 sensor, so that the performances of the two types of sensors could be compared.

In the rest of the control experiments, a digital controller programmed with the filtered-x version of the Least-Means-Squares (LMS) adaptive control algorithm was used to control the vibration of the cylinder. Most of the experiments were done on the closed cylinder because the ultimate research objective was to control acoustic emission from the cylinder with this boundary condition. However, experiments were also done on the open cylinder to compare the results of the LMS control with the results of the analog control, which was done on the open cylinder. In these adaptive control experiments, the effect of the location and numbers of actuators was studied. Finally, another experiment was done to compare the performances of three different types of sensors: plain PVDF film sensor, PVDF film modal sensor, and an error microphone.
5.2 Control Algorithm

The selection of a control scheme is a very important element in the research work, and a study of the control strategy is necessary before implementing control. Furthermore, knowledge of the control mechanism is important in the analysis of the results of the control experiments. Therefore, the digital control algorithm used in the experiments is studied in this section.

The control algorithm used in the control experiments was the filtered-x version of the Least-Mean-Square (LMS) algorithm. The reason for selecting this algorithm for the control experiments was that the system-identification part of the algorithm is capable of identifying the plant by computing two filter weights which represent the changes in magnitude and phase between the plant input and the plant output. This feature was particularly useful in the control of the cylinder because the dynamics of the cylinder is complex and difficult to model. Characterization of the cylinder vibration in Chapter 3 is useful in identifying the vibration modes of the cylinder. However, determination of the transfer function of the plant requires a higher level of analysis, which can be bypassed by the filtered-x LMS algorithm. Moreover, this algorithm has been chosen and proven to be effective by many researchers in the field of active structural acoustic control.

The following discussion on the Least-Mean-Square (LMS) adaptive control algorithm and its filtered-x version is based on the literature well-known in the field of adaptive control (Widrow et al. 1975, Widrow and Stearns, 1985). Figure 5.1 shows the components of the control system. The objective of the control algorithm is to minimize the expected value (mean) of the square of the PVDF sensor error signal.
Figure 5.1 Components of the Adaptive Control System
where $E[.]$ denotes the expected value, and $e_k$ is the PVDF sensor signal. The subscript $k$ indicates time step. The following paragraphs explain how the LMS algorithm processes a reference signal to produce an output which drives the PZT actuators to cancel the effect of the shaker excitation on the cylinder. Figure 5.2 shows a more detailed description of the control system for the discussion.

If noise is neglected, the PVDF sensor signal can be considered to be a superposition of the PVDF output $d_k$ due to the shaker excitation alone, and the PVDF output $g_k$ due to the PZT actuators excitation alone, or

$$e_k = d_k + g_k.$$  \hfill (5.2)

The controller output, denoted by $u_k$, is produced from a reference signal $x$ taken from the signal generator which drives the shaker. The controller processes the reference signal with a two-weight finite impulse response (FIR) filter. Two adaptive weight coefficients $w_0$ and $w_1$ are used to multiply the reference signal $x_k$ and $x_{k-1}$. The controller output is the sum of the resulting weighted signals, i.e.,

$$u_k = w_{0k} x_k + w_{1k} x_{k-1}.$$  \hfill (5.3)

Two weights are sufficient because for this pure sinusoidal excitation the controller only scales the magnitude and determines the phase shift from the reference signal to produce the control signal. The purpose of the LMS algorithm is to adjust the weights $w_0$ and $w_1$, 

$$\xi = E[e_k^2].$$  \hfill (5.1)
Figure 5.2 The Filtered-\( x \) Adaptive Control Algorithm
so that $u_k$ drives the PZT actuator in such a way that the resulting $g_k$ cancels $d_k$, thus minimizing $\xi$. If the transfer function from the controller output to the PVDF output is denoted by $T_c$, the PVDF output due to $u_k$ alone is

$$g_k = T_c w_0 x_k + T_c w_1 x_{k-1}.$$  \hspace{1cm} (5.4)$$

Substituting Eq. 5.4 into Eq. 5.2 results in the total PVDF output

$$e_k = [d_k + T_c w_0 x_k + T_c w_1 x_{k-1}]$$  \hspace{1cm} (5.5)$$

The last equation and Eq. 5.1 show that the expected value of the square of the error, $\xi$, is a quadratic function of the weights $w_0$ and $w_1$. Therefore the error curve has a single minimum. Adaptive control algorithms work by adjusting the values of $w_0$ and $w_1$ towards the values which result in this minimum. Weight adjustment can be done at each time step by adding to each weight a term which is a scaled negative of the gradient of $\xi$ with respect to the weight. This weight adjustment process can be expressed as

$$w_{i(k+1)} = w_{i(k)} - \mu \left( \frac{\partial \xi}{\partial w_i} \right)_{(k)}; \ i = 0, 1.$$  \hspace{1cm} (5.6)$$

The convergence parameter $\mu$ is used to determine how fast the weights are adjusted, and also determines the stability of the control system. In the experiments described below, this parameter is set by the user through a user interface program. The gradients can be calculated by using Eq. 5.1 and Eq. 5.5, which results in,

$$\frac{\partial \xi}{\partial w_i} = 2 E [ e_k T_c x_{k-i} ]; \ i = 0, 1.$$  \hspace{1cm} (5.7)$$
However, instead of using the above gradient values, the LMS algorithm uses estimates for those values, which are obtained by approximating the expected (mean) values of the gradient with the present value of the gradients, i.e.,

\[ \frac{\partial \xi}{\partial w_i} = 2 e_k T_c x_{k-1} ; \ i = 0, 1. \] (5.8)

The above gradient approximation makes the LMS algorithm much simpler than most of other steepest-descent adaptive control algorithms. However, Eq. 5.8 still cannot be used to obtain the gradient estimates, and at this point the weight coefficient update of Eq. 5.6 still can not be executed. The reason is that \( T_c \), the transfer function from the controller output to the PVDF sensor output, is not known. This problem is solved by simulating the transfer function \( T_c \) with a filter having a transfer function \( \hat{T}_c \). Since the plant is linear and time-invariant and the excitation is harmonic, the transfer function \( T_c \) can be represented by a two-weight finite impulse response (FIR) filter. These weights are obtained by a system identification program which obtains the transfer function between the PVDF output \( g_k \) and the controller output \( u_k \) before control is applied. The transfer function is used to filter the original reference signal \( x_k \). The resulting new reference signal is

\[ \hat{x}_k = \hat{T}_c x_k . \] (5.9)

Now Eq. 5.8 can be further approximated by substituting \( \hat{x}_k \) for \( T_c x_k \), which results in

\[ \frac{\partial \xi}{\partial w_i} = 2 e_k \hat{x}_{k-i} ; \ i = 0, 1. \] (5.10)

The gradient estimate from the last equations are then substituted into Eq. 5.6. Thus, the
This technique is called the filtered-x algorithm because the reference signal to be multiplied by the weights $w_0$ and $w_1$ are the output of the FIR filter instead of the original reference signal $x$. Equation 5.11 shows that the filtered-x LMS algorithm can be implemented in a practical system without complicated computational operations such as squaring, averaging, or differentiation.

The Filtered-x LMS algorithm and the system identification ($T_c$ estimation) routine are programmed on a Spectrum TMS320C30 digital signal processing board residing in a 80386 host computer. The programs were created by Jeff Vipperman, a graduate research assistant in the Vibrations and Acoustic Laboratory, Mechanical Engineering Department at Virginia Polytechnic Institute and State University (Vipperman et al., 1991).

### 5.3 Experiment Setup

#### 5.3.1 The Reverberation Chamber

As shown by experiments described in Chapter 3, the acoustic emission from the cylinder is directional. Vibration control on the cylinder may change the directivity pattern of the acoustic emission. Therefore minimizing the sound pressure level in a particular direction may result in an increase of sound pressure level in other directions. To demonstrate acoustic control, a set of directions can be selected and then the SPL's in those directions before and after control are measured. A simpler approach to demonstrate the SPL
reduction is to observe a measure of the total acoustic emission from the cylinder in a reverberation chamber. In the diffuse field in this chamber, the contributions of all rays that radiate from the cylinder can be measured by a microphone at a single point.

A set of experiments were done on the cylinder in a reverberation chamber. The positions of the cylinder, as well as the dimensions of the reverberation chamber, is as shown in Fig. 5.3. A Bruel & Kjaer type 4134 microphone was used to monitor the sound pressure level generated by the cylinder. The microphone was placed in an arbitrarily chosen position, marked as position 1 and position 2 in Fig. 5.3. To determine whether or not microphone locations are important in this reverberation chamber, the reverberation time $T_{60}$ was measured in five randomly chosen points in the room, the values were averaged, and then the Schroeder cutoff frequency was calculated by

$$f_{sch} = \left(\frac{c^3}{4 \ln 10}\right)^{1/2} \left(\frac{T_{60}}{V}\right)^{1/2}, \quad (5.12)$$

where

- $c = \text{speed of sound in air} = 340 \text{ m/s},$
- $T_{60} = \text{reverberation time (averaged from measurements at five points)} = \frac{(3.98 + 3.99 + 3.97 + 3.85 + 4.042)}{5} = 3.96 \text{ s},$
- $V = \text{volume of chamber} = 45.9 \text{ m}^3,$

which gave a Schroeder cutoff frequency of 607 Hz. This calculation result is in agreement with a previous calculation done in a research work utilizing the reverberation chamber (Zhou, 1991). Since the test frequencies of 1420 Hz (Rayleigh mode 4 frequency) and 1515 Hz (Mode (4,1) frequency) were well above the Schroeder frequency, it was predicted that the acoustic field in the room anywhere reasonably far from the cylinder was diffuse. This prediction was proven experimentally by exciting the
Height of Chamber
= 4.216 m

Figure 5.3 Position of Cylinder in Reverberation Chamber
cylinder at 1420 Hz and finding that the sound pressure level (SPL) at twelve randomly chosen points in the room further than 1 m from the cylinder did not vary considerably. With the excited cylinder generating an SPL of about 95 dB, the spatial variation of SPL was found to be ± 4 dB, even lower than the time variation at any chosen point. To test whether the microphone was in the reverberant field, the microphone was moved from position 1 towards the cylinder so that the distance to the cylinder was halved. No increase in SPL was observed as a result. Therefore, the microphone position was in the reverberant field. The same test was done and the same result was observed in position 2. Thus, both microphone positions are in the reverberant field.

For the LMS control experiments, the microphone in position 1 was directed towards the cylinder; in position 2 the microphone was directed away from the cylinder, facing a wall at an angle. The uncontrolled SPL's caused by the closed cylinder vibration at 1515 Hz for those two positions were found to be almost identical, as shown in Fig. 5.4.

5.3.2 Analog Control Experiment Setup

The preliminary (analog) control experiments were done as shown in Fig. 5.5. Only the open cylinder was used. Error signals from PVDF or PZT sensors were input to a Frequency Devices 9002-LP02 analog filter. This filter had a low pass 8-pole Bessel characteristic. In addition to filtering out high frequency noise from the sensors, this filter provided a high input impedance for the piezoelectric sensors. With the input impedance of 1 MΩ, this filter made the sensors measure strain according to Fig. 4.4. This filter also provided an adjustable gain. A high gain in the feedback loop was necessary because the signal level from the PVDF film sensors was only in the order of tens of millivolts. In some cases as will be shown in the next chapter, the control voltage needed to drive the
Figure 5.4 Uncontrolled SPL in Two Microphone Positions
Figure 5.5 Analog Control Experiment Setup
PZT actuators with sufficient authority might exceed 100 V (rms). An HP 6842A power supply amplifier provided power and gain to the loop. To further elevate the voltage level in the loop, a transformer with a turn ratio of 18.7 was used.

Because the PZT actuators are basically a capacitive load, an AC driving voltage may generate very high currents which may overload the amplifier. To alleviate this high current problem, a 37.5 KΩ resistor was used in parallel with the PZT actuators to provide resistive load. Built from several resistors, this resistor was also designed to serve as a voltage divider which scaled down the voltage going to the Fourier analyzer by a factor of 1/10. (Signals going to the analyzer must be within ± 10 V.)

When PVDF mode 4 film sensor was used, the controlled frequency was 1420 Hz, which is the resonant frequency of Rayleigh mode 4 vibration. Therefore, the actuators used were those which were on the antinodal line of circumferential mode 4. Experiments were done using groups of actuators no (1, 2), (1, 2, 7, 8), and (7, 8, 11). All of those actuators are on antinodal lines of circumferential mode 4. An accelerometer was attached at point (6,5). This point is on an antinodal line of Rayleigh mode 4 vibration.

When a PZT patch was used as a sensor, experiments were also done at 263 Hz (Rayleigh mode 2 frequency) as well as at 1420 Hz. Control at 263 Hz was possible with the PZT patch as a sensor because the PZT patch, unlike the PVDF modal filter, was sensitive to Rayleigh mode 2 frequency. As will be explained in the next chapter, the control performance was limited by stability problem, i.e., undesired modes being excited and magnified in the loop. At 263 Hz, this problem could be overcome to a great extent by the use of the analog filter. Because 263 Hz was the open cylinder's lowest natural frequency, setting the filter cutoff frequency just above 263 Hz would remove all other
vibration modes from the feedback loop. Therefore, tests were done at this frequency to investigate the authority of the PZT actuators. The shaker excitation was set to a high force, and a high voltage was applied to the PZT actuators.

5.3.3 Adaptive Control Experiment Setup

The setup for the adaptive digital control experiments is shown in Fig. 5.6. For comparison with the analog control experiments, an experiment was done first on the open cylinder. Later, the cylinder was closed using two end caps and most of the adaptive digital control experiments were done on this closed cylinder. The excitation frequency used on the experiments was 1515 Hz. The system components were the same as those used in the analog control experiments, except that an adaptive digital controller was used in the feedback loop. The adaptive digital controller was a Texas Instruments TMS320C30 digital signal processing (DSP) board. A PC/386 was used as a host computer for downloading programs to the DSP board, and as an interface with the user. The controller received feedback signals from the filter, which in turn obtained the error signal from one of the three error sensors: plain PVDF film sensor, mode 4 PVDF film sensor, or error microphone. The reference signal for the controller was obtained directly from the excitation signal generator.

PZT actuator groups used in these experiments were groups of actuators number (1, 2), (1, 2, 6), (6), (6, 7, 8), (7, 8), (7, 8, 11), (11), and (1, 2, 6, 7, 8, 11). All of these actuators are on antinodal lines of circumferential mode 4. The error microphone was first used in position 1 and then moved to position 2 (see Fig. 5.3) to investigate the effect of microphone locations on the control results.
Figure 5.6 Adaptive Control Experiment Setup
The final and most important set of experiments was done with the LMS adaptive control on the closed cylinder. In these experiments, the excitation frequency was 1515 Hz, the frequency of mode (4,1) vibration, which was chosen to be the vibration mode to control based on the preliminary acoustic testing discussed in Chapter 3. Several different combinations of PZT actuators were used and compared. Also, the use of three types of sensors was investigated: plain PVDF film sensor, mode 4 PVDF film sensor, and a microphone.

5.4 Summary of Experiments Performed

After the experiment setup and the hardware connections are described in the previous section, this section explains how the experiments were conducted and what variables were studied in each experiment. Table 5.1 gives a summary of all control experiments done on the cylinder. The purposes and the important details of the experiments are summarized below. The experiment numbers are given on the left column of Table 5.1.

Experiments No. 1, 2, and 3:

One purpose of these experiments was to investigate the relation between reduction in vibration level and reduction in acoustic radiation. Because the controlled mode was an efficient acoustic radiator, it was expected that reduction in vibration level would result in reduction in SPL. Another purpose was to determine the resulting reduction of the signals in the control system, e.g., to determine the sensor and actuator voltage levels.

Experiments No. 4 and 5:
Table 5.1 Summary of Control Experiments Performed on Cylinder

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<tbody>
<tr>
<td>1</td>
<td>Sensor Signal, Acceleration, SPL, Voltage to Actuator</td>
<td>Analog</td>
<td>Open</td>
<td>Mode 4</td>
<td>1420</td>
<td>1, 2</td>
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<td>Open</td>
<td>Mode 4</td>
<td>1420</td>
<td>7, 8</td>
</tr>
<tr>
<td>3</td>
<td>Sensor Signal, Acceleration, SPL, Voltage to Actuator</td>
<td>Analog</td>
<td>Open</td>
<td>Mode 4</td>
<td>1420</td>
<td>7, 8, 11</td>
</tr>
<tr>
<td>4</td>
<td>SPL</td>
<td>Analog</td>
<td>Open</td>
<td>Mode 4</td>
<td>1420</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>SPL</td>
<td>Analog</td>
<td>Open</td>
<td>Mode 4</td>
<td>1420</td>
<td>1, 2</td>
</tr>
<tr>
<td>6</td>
<td>SPL</td>
<td>LMS</td>
<td>Open</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2, 6</td>
</tr>
<tr>
<td>7</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>6, 7, 8</td>
</tr>
<tr>
<td>8</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>7, 8, 11</td>
</tr>
<tr>
<td>9</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2, 6, 7, 8, 11</td>
</tr>
<tr>
<td>10</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>11</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>12</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>13</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>14</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>15</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Mode 4</td>
<td>1515</td>
<td>1, 2</td>
</tr>
<tr>
<td>16</td>
<td>SPL</td>
<td>LMS</td>
<td>Closed</td>
<td>Plain</td>
<td>Microphone</td>
<td>1515</td>
</tr>
</tbody>
</table>
A purpose of these experiments was to study the possibility of using a PZT patch as a sensor. PZT has a piezoelectric strain constant which is about 5 times greater than PVDF. Therefore, it was expected that the PZT patch would give a higher output voltage than a PVDF sensor of comparable size under the same strain. However, the PZT patch lacks the distributed feature which the PVDF sensor has. Problems arising from using a PZT patch as a sensor (instability in some cases) were discovered by these experiments.

Another purpose of these experiments was to determine the limits of the actuator authority. The PZT patches are capable of operating at relatively high voltages -- the manufacturer's suggested maximum voltage is 150 V rms. In experiment No. 5, a feedback control loop was closed and the shaker force was increased to a level at which the resulting PZT actuator control voltage approaches the manufacturer's suggested maximum.

Additionally, this test would also demonstrate that the PZT actuator, which is very fragile by itself, has enough authority to excite stiff structures. The PZT chip has a dynamic tensile strength of $2.1 \times 10^7$ Pa, (compare to, for instance, $0.262 \times 10^9$ Pa of aluminum). This low strength, brittleness, and the thinness of the chips make them very fragile. However, once the PZT chips are bonded on the surface of the metal structure, they can impart considerable forces on the structure. Demonstrating the mechanical strength of the PZT actuator could also be done without a control loop, i.e., by applying a high voltage to the actuator without the cylinder being excited by a shaker. However, this method may not demonstrate the authority of the actuator in the real control problem, since the resulting vibration obtained in the cylinder by this method is not the same as the resulting vibration obtained by exciting the cylinder with the shaker and controlling the vibration with a PZT actuator. Moreover, merely applying a high voltage to the PZT actuator may
result in an excessive vibration level which endangers the integrity of the actuator. Therefore, applying a high control voltage to the PZT actuator in a control loop is a better method to test the actuator.

**Experiments No. 6 and 7:**

The purpose of these experiments was to compare the performances of the analog feedback control and the LMS (digital) control.

**Experiments No. 8 through 15:**

These experiments were done to compare the performances of different actuator combinations. Because the digital controller had only one output channel, basically the actuators work as one actuator distributed in different positions on the structure. Previous research work on optimal actuator placement had resulted in such actuator placements (Jia, 1990). For example, an optimal actuator placement for controlling mode 3 vibration of a simply supported beam was achieved by placing three actuators on the beam, each on an antinode of the mode shape. In the experiments on the cylinder, all of the PZT actuators used were located on antinodal lines of the controlled mode (Rayleigh mode 4 for the open cylinder, and mode (4,1) for the closed cylinder).

PZT actuators No. 3, No. 4, No. 5, No. 9 and No. 10 were not used, because their positions were not near the optimal positions for controlling mode (4,1) vibrations. Those actuators were extra actuators bonded on the cylinder because at the time of fabrication and instrumentation of the cylinder the mode shapes to be controlled had not been known and the actuator placement was done based on the prediction of the most likely mode
shapes to be controlled.

Expériments No. 16, 10, and 17:

The purpose of these experiments was to compare the performances of three different sensors: a plain PVDF sensor, the mode 4 PVDF sensor, and a microphone. Since the final control objective was to reduce SPL, it was expected that the microphone is the best sensor. However, because the motivation of the whole research work discussed here included the investigation of the use of structure-borne sensors in place of a microphone, the performance of PVDF film sensors was compared with the performance of the microphone.

The results of the experiments described in this chapter will be given in the next chapter.
Chapter 6
Results of Control Experiments

This chapter presents results of the control experiments described in the previous chapter. Graphs presented in this chapter present auto spectra from the Tektronix 2630 Fourier Analyzer. The reference value for the measured sound pressure level (SPL) is 20 μPa. For other variables, the reference values are stated in the graphs. Reductions in vibration levels were measured by measuring the acceleration at an antinode. The experiment numbers in the following discussion refer to the numbers in the first column of Table 5.1 in the previous chapter.

6.1 Analog Control Experiments

6.1.1 Analog Control with PVDF Mode 4 Sensor and Various PZT Actuators (Experiments No. 1, 2 and 3)

The analog control experiments implemented a strain feedback loop and were performed as described in sub section 5.3.2 in the previous chapter. In experiments No. 1, 2 and 3, PVDF mode 4 sensor was used, and several combinations of PZT patches were used as actuators. The cylinder was excited by the shaker at 1420 Hz (Rayleigh mode 4) to produce an uncontrolled sound pressure level of 88 dB. Then the analog feedback loop was closed by activating the feedback amplifier. The cutoff frequency of the analog filter was changed gradually to vary the phase angle of the signal fed back to the PZT actuator. Changes in vibration levels were monitored. When a phase angle was found which gave
a maximum SPL reduction, the feedback gain was increased manually up to the verge of instability. The result was reductions in vibration sensor signal, acceleration, and SPL. This technique is based on vibration cancellation by the PZT actuator, and is based on the ASAC technique invented by Fuller (1987).

Table 6.1 shows the reductions achieved with mode 4 PVDF film sensor and various actuators. The acceleration was measured at point (12,5), which is on an antinodal line of Rayleigh 4 mode. The autospectra of those variables for experiment No. 1 are shown in Figure 6.1. From the plots, it is clear that the reduction of vibration levels produces the reduction of acoustic emission.

In the above experiments, the performance of the analog control was limited by instability caused by spillover. When the cylinder is excited by the shaker with a single frequency on-resonance harmonic excitation at 1420 Hz, Rayleigh mode 4 vibration dominates the response, but other modes are also excited (to a much smaller extent). The mode 4 sensor helps prevent the other modes from being sensed (see Fig. 4.7). However, in a two-dimensional structure like the cylinder, the one-dimensional modal sensor also admits (4,j) modes other than the controlled Rayleigh mode 4. Also, the imperfection of the modal sensor and distortion in the cylinder's mode shapes allow other modes to be sensed. Therefore, observation spillover cannot be completely eliminated. When Rayleigh mode 4 vibration is suppressed, the new vibration modes may be of significant magnitudes relative to the Rayleigh mode 4 vibration. Feeding back vibration signals from some of these modes cause instability at lower gains than the gain that would cause instability when feeding back a vibration signal from mode 4 alone. To prevent this instability problem in the experiments, the manual increment of the feedback loop gain was done slowly and carefully and the cylinder acceleration was constantly monitored in frequency
Table 6.1 Results of Analog Control on Open Cylinder.

Excitation Frequency: 1420 Hz
Sensor: PVDF mode 4 film
Microphone Position: Position 1
Uncontrolled SPL: 88 dB
Background Noise: 50 dB

<table>
<thead>
<tr>
<th>Actuators PZT No.</th>
<th>Sensor Signal Reduction (dB, ref 22 mV)</th>
<th>Accel. Reduction (dB, ref 1.5 g)</th>
<th>SPL Reduction (dB)</th>
<th>PZT Actuator Voltage (V rms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,2</td>
<td>13</td>
<td>13</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>7,8</td>
<td>12</td>
<td>11</td>
<td>13</td>
<td>10</td>
</tr>
<tr>
<td>7,8,11</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>8</td>
</tr>
</tbody>
</table>
Figure 6.1 Open Cylinder with and without Control. Autospectra of
a. SPL,
b. Acceleration at an Antinode
c. Sensor Output Voltage.
domain on a linear scale. As soon as a peak other than the 1420 Hz peak appeared on the real time acceleration auto spectrum plot, the loop gain was reduced. Thus, spillover caused instability which limited the control performance.

When PZT No. 7, 8, and 11 are used as actuators, the analog feedback control can achieve reductions of about 16 dB in acceleration, 15 dB in PVDF sensor signal, and 23 dB in sound pressure level.

An important conclusion can be drawn regarding the effect of numbers of actuators on control performance. Using PZT actuators No. 7, 8, and 11 resulted in higher reductions than using PZT actuators No. 7 and 8 alone, even when the voltage applied to the two actuators was slightly higher than the voltage applied to the three actuators (see Table 6.1). This result indicates that using more actuators may result in a greater vibration reduction in general. The effects of numbers of actuators and their positions will be further studied later in this chapter.

The actuator voltage measurements show that for the disturbance level introduced on this particular test, actuator voltage limitation is not an important constraint in the performance of the feedback control, because the actuator control voltage is only about 10% of the manufacturer's suggested maximum voltage (see Table 6.1).

6.1.2 Analog Control with PZT Patches as Sensors and Actuators (Experiments No. 4 and 5)

Two experiments were done using a PZT patch as a sensor and other PZT patches as actuators. One objective of the experiments was to compare the performances of PZT
patches and shaped PVDF film as sensors. These experiments also gave an indication of the range of the PZT actuator voltage.

In the first of the two experiments, the excitation was 1420 Hz (Rayleigh mode 4 frequency). Table 6.2 shows the reduction in sensor signal, acceleration and sound pressure level in this experiment. In Table 6.1, it was shown that a Rayleigh mode 4 vibration with an antinode acceleration of 1.48 g produced a PVDF modal sensor output voltage amplitude of 22.3 mV. On the other hand, it is shown in Table 6.2 that a Rayleigh mode 4 vibration with an antinode acceleration of 2.4 g produced a PZT sensor output voltage of 118 mV. Normalizing these two sensor voltages with respect to the measured acceleration show that the PZT sensor is more sensitive than the PVDF mode 4 sensor. However, because the PZT patches do not carry spatial information that produces the modal filtering effect, many modes other than the one desired to be fed back to the actuators are sensed. The feedback signal at some frequencies were amplified in the feedback loop. This observation spillover created instability problems which limited the reductions to levels lower than those achieved with the modal sensor. In the experiment using PZT patch No. 6 as a sensor and PZT patches No. 1 and No. 2 as actuators, the SPL reduction achieved was 12.4 dB. The previous experiment using the PVDF mode 4 sensor and the same actuators achieved an SPL reduction of 13.3 dB. In both cases the feedback gains were the maximum possible gains obtained by increasing the power amplifier gains gradually up to the verge of instability.

The second experiment with PZT patch No. 6 as a sensor and PZT patches No. 1 and 2 as actuators (experiment No. 5) was done to determine the actuator authority and vibration excitation level which can be controlled by using PZT actuators. In this experiment, the excitation frequency of 263 Hz (Rayleigh mode 2 vibration) was chosen. The fact that this
Table 6.2 Results of Using a PZT Sensor with Analog Control on the Open Cylinder.

Sensor: PZT No. 6
Actuator: PZT No. 1, 2
Microphone Position: 1
Background Noise: 50 dB

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Error Signal</th>
<th>Acceleration</th>
<th>SPL</th>
<th>Actuator Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>no control (mV)</td>
<td>no control (dB)</td>
<td>no control (g)</td>
<td>no control (dB)</td>
</tr>
<tr>
<td>1420</td>
<td>118</td>
<td>9.6</td>
<td>2.4</td>
<td>12.5</td>
</tr>
<tr>
<td>263</td>
<td>87</td>
<td>21.0</td>
<td>4.5</td>
<td>23.0</td>
</tr>
</tbody>
</table>
frequency is the lowest natural frequency of the open cylinder was exploited to conduct an experiment with a high control actuator voltage, intended to give some insight of the range of voltage which might be required to control the vibration of the cylinder. With the filter cutoff frequency set just above the Rayleigh mode 2 frequency (263 Hz), feedback signals from all modes sensed by the PZT sensor other than Rayleigh 2 mode were attenuated by the filter in the feedback loop. Therefore, the low cutoff frequency made it possible to increase the feedback gain to a high magnitude before instability occurred. The excitation produced an antinode acceleration of 4.5 g (0-to-peak). Although the uncontrolled SPL in this experiment was only 79.4 dB, which was low in comparison to the SPL in the experiment with Rayleigh mode 4 vibration discussed in the previous paragraph (91.3 dB), the excitation force in this experiment was actually much higher than the excitation force in the previous experiment. Calculation using the FRF shown in Fig. 3.6 shows that the force necessary to produce this level of acceleration was

\[ \frac{4.5 \text{ g}}{0.8384 \text{ g/N}} = 5.37 \text{ N,} \]  \hspace{1cm} \text{(6.1)}

or 8.6 times higher than the previous (Rayleigh mode 4) experiment where the force was only,

\[ \frac{2.4 \text{ g}}{3.84 \text{ g/N}} = 0.625 \text{ N.} \]  \hspace{1cm} \text{(6.2)}

Therefore, it was expected that the control actuator voltage required to control the Rayleigh mode 2 vibration in this experiment would be much higher than the control voltage in the Rayleigh mode 4 experiment.

This control scheme with the PZT sensor and an analog filter resulted in a sensor signal reduction of 21 dB. The actuator control voltage in this experiment was 128 V, as
compared to 9 V in the Rayleigh mode 4 experiment with 0.625 N excitation force and the same sensor and actuators. The manufacturer's suggested maximum voltage is 150 V. The important thing learned here is that although the SPL generated by the shaker does not appear to be very high, the force given by the shaker may result in a high actuator voltage requirements. Thus, it is important to always monitor the actuator voltage when running control experiments with PZT actuators.

6.2 Adaptive Control versus Analog Control

The results of the adaptive digital control experiments done on both the open cylinder and closed cylinder are presented below.

6.2.1 Sampling Problem and Measurement of SPL Reduction

With the analog control scheme discussed above, the acoustic field in the reverberation chamber was generated by the cylinder vibration almost totally at the excitation frequency because the contribution of vibrations at other frequencies was insignificant (see Fig. 6.1). Therefore, measurement of the SPL reduction by reading the value of the sound autospectrum at the excitation (peak) frequency can be justified. On the other hand, with the digital adaptive control scheme the SPL reduction cannot be measured by the reduction of the SPL autospectrum value at the excitation frequency alone. The reason for this inequality was that the SPL controlled by the digital controller was not dominated by a single tone acoustic emission. The controller excited new harmonic vibrations in the cylinder, which can be explained as follows.

The output of the digital signal processing board is in the form of steps, which can be
expressed as a result of a sample and a hold operation. If a 1420 Hz harmonic signal is sampled and held at 6000 Hz, then the resulting signal is as shown in time domain in Fig. 6.2.a. The frequency domain representation of this signal can be obtained as follows. In frequency domain, the output of the sampling operation done on a signal \( F(s) \) at a sampling rate of \( \omega_s \) is (Astrom and Wittenmark, 1990)

\[
F^*(s) = \omega_s \sum_{n=-\infty}^{\infty} F(s + jn \omega_s),
\]

(6.3)

where \( j = \sqrt{-1} \). The result of sampling a 1420 Hz harmonic signal at a rate of 6000 samples/second is shown in frequency domain in Fig. 6.2.b. In the 0-5 kHz frequency range, the spectrum of this result has two non-zero spectral lines instead of only one at 1420 Hz. The second frequency content, which is generated by the sampling operation, is at the frequency of

\[
f_{\text{extra}} = f_{\text{sampling}} - f.
\]

(6.4)

Therefore, the sampling operation induced a harmonic at 4580 Hz. The hold operation has a transfer function

\[
\frac{F_{\text{output}}}{F_{\text{input}}} = \frac{1}{s} \left( 1 - e^{-2\pi k} \right),
\]

(6.5)

which is illustrated in Fig. 6.2.c. Thus, the frequency domain representation of the sampled-and-held harmonic signal in Fig. 6.2.a is shown in Fig. 6.2.d. This result shows that if the controller generates an output that would be a single-frequency harmonic signal in a continuous-time system, the sampling operation will distort that output signal so that the resulting signal has another content at a higher frequency. This analysis explains why
Figure 6.2 Results of Sampling and Hold
a. Time Signal
b. Result of Sampling
c. Transfer Function of Holding Process
d. Spectrum of Sampled and Held Signal.
a new peak in the FRF appears in the experiment results.

These sampling-induced vibrations could have been easily suppressed by filtering the controller output with an analog low pass filter. Even without filtering, the bandwidth limitation of the power amplifier and the dynamics of the plant suppress the high frequency contents of the controller output to some extent. In the experiments, however, the first of the controller-generated harmonics was not sufficiently suppressed. As a result, the controlled cylinder vibrates at both the excitation frequency and the controller-generated frequency. This is the reason why the reduction in acoustic emission must be measured by measuring the broad band SPL instead of comparing the value of the SPL autospectrum at the excitation frequency. To measure the SPL, a B&K type 2231 portable SPL meter was used with a B&K type 4155 microphone. This microphone was attached to the first microphone which was connected to the Fourier Analyzer and which was used earlier to monitor the SPL in the analog control experiments. The time weighting on the SPL meter was set to 'slow' (1 minute averaging time) to average out time variations, and the frequency weighting was linear with a range of 10 Hz to 20 kHz.

6.2.2 Adaptive versus Analog Control on the Open Cylinder (Experiments No. 6 and 7)

Table 6.3 and Fig. 6.3 compare the result of analog control with the result of digital adaptive control on the open cylinder. Compared to the analog control, the adaptive control results in a much greater reduction in signal (32 dB compared to 15 dB). The adaptive control also reduces the SPL at the excitation frequency more dramatically than the analog control (34 dB compared to 23 dB). The excitation frequency was 1420 Hz, and the sampling rate was 6000 Hz, so that the controller excited a new peak at 4580 Hz.
Table 6.3  Analog versus Adaptive Control on the Open Cylinder

Excitation Frequency: 1420 Hz  
Sensor: PVDF mode 4 film  
Actuators: PZT No. 1, 2  
Uncontrolled SPL: 94 dB  
Background Noise : 53 dB

<table>
<thead>
<tr>
<th>Controller</th>
<th>Error Signal Reduction (dB)</th>
<th>SPL Reduction (dB)</th>
<th>1420 Hz</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analog</td>
<td>15</td>
<td>23</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>Adaptive</td>
<td>32</td>
<td>34</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.3 Analog versus LMS Control: SPL of Open Cylinder
(see Eq. 6.4). However, the SPL at this frequency is more than 30 dB lower than the uncontrolled SPL because the bandwidth limitation of the power amplifier and the dynamics of the cylinder suppressed the high frequency contents of the controller output. Therefore, the overall SPL reduction resulting from the adaptive control was still greater than the overall SPL reduction resulting from the analog control.

6.3 Effects of Actuator Positions

All the experiments described above were done on the open cylinder. The rest of this chapter presents results of the adaptive control experiments on the closed cylinder.

In this section, the effects of using different combinations of actuators are discussed. The actuators used in the control experiments were only the ones located on antinodal lines of mode (4,j). The positions of those actuators are shown in Fig. 6.4. The uncontrolled SPL was 95 dB, and the background noise was 50 dB. The sampling frequency was 5000 Hz. Table 6.4 lists the results of the control experiments.

6.3.1 PZT Actuator No. 6 and PZT Actuator No. 11 (Experiments No. 8 and 9)

With mode 4 PVDF film as a sensor, the adaptive control system reduced PVDF sensor signal almost three orders of magnitude. The resulting reduction in SPL was about one order of magnitude. Control with PZT patch No. 6 as an actuator resulted in a 54 dB reduction in PVDF sensor signal (see Fig. 6.5), a 25 dB reduction in SPL autospectrum at 1515 Hz, and a 21 dB reduction in total SPL (see Table 6.4). The sampling process excited a vibration at 3485 Hz. However, the SPL autospectrum value at 3485 Hz was 33 dB lower than the FRF peak of the uncontrolled SPL. The adaptive control reduced the
Figure 6.4 Positions of PZT Actuators Used in Control Experiments
Table 6.4 Results of Adaptive Control with Various PZT Actuator Combinations

Sensor: PVDF Mode 4
Excitation Frequency: 1515 Hz
Microphone Position: Pos 1
Uncontrolled SPL: 95 dB
Background Noise: 50 dB

<table>
<thead>
<tr>
<th>Actuator, PZT No.</th>
<th>Error Signal Reduction (dB)</th>
<th>SPL Reduction (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1515 Hz</td>
</tr>
<tr>
<td>6</td>
<td>54</td>
<td>25</td>
</tr>
<tr>
<td>11</td>
<td>58</td>
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</tr>
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<td>1,2</td>
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<td>39</td>
</tr>
<tr>
<td>6,7,8</td>
<td>62</td>
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</tr>
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<tr>
<td>1,2,6,7,8,11</td>
<td>61</td>
<td>36</td>
</tr>
</tbody>
</table>
Figure 6.5 Closed Cylinder SPL with and without Control.
Sensor: Mode 4 PVDF Film, Actuator: PZT No. 6.
total SPL from 95 dB to 74 dB.

The FRF peaks at 3030 Hz and 4545 Hz indicated harmonics of the excitation frequency. These harmonics were also excited in the uncontrolled system (see Fig. 5.4 in Chapter 5). The SPL autospectrum value at 3030 Hz was about 25 dB lower than the SPL autospectrum value at 1515 Hz, and the SPL autospectrum value at 4090 Hz is even much lower. Therefore, the harmonics did not give a significant contribution to the total SPL.

Like PZT patch No. 6, PZT patch No. 11 was also located on an antinode of mode (4,1) (see Fig. 6.4). Because of symmetry of the mode shape, PZT actuator No. 11 was expected to exert line moments on the cylinder in the same way as PZT No. 6. The resulting vibration controlled with PZT actuator No. 11 should also be similar to the controlled vibration when PZT patch No. 6 was used. The experiments show that the controlled SPL with PZT patch No. 11 was similar to the controlled SPL with PZT No. 6 (compare Fig. 6.5 to Fig. 6.6). PZT No. 11 had a higher authority and thus gave a better reduction in vibration and SPL autospectrum at 1515 Hz. However, PZT No. 11 also induced more vibration at 3485 Hz. The resulting total SPL reduction by PZT patch No. 11 was 20 dB, about the same as the resulting total SPL reduction by PZT patch No. 6. In both cases, the control voltage applied to the PZT actuator was about 13 V. The difference in actuator authority might be due to differences in bonding, such as bonding layer thickness.

6.3.2 PZT Actuators No. 1 and 2 (Experiment No. 10)

PZT actuators No.1 and 2 were designed to be almost collocated with the disturbance source, therefore they were located very close to the shaker point of action (see Fig. 6.4).
Figure 6.6 Closed Cylinder SPL with and without Control.
Sensor: Mode 4 PVDF Film, Actuator: PZT No. 11.
With regard to mode shapes, this pair of actuators is almost equivalent to PZT No. 6 and PZT No. 11 because it is located at the antinodes of mode (4,1). The line moments exerted on the cylinder by PZT No. 1 and 2 in the axial direction were expected to be greater than the line moments given by PZT No. 6 or No. 11 alone in the same direction. The experiment result (see Fig. 6.7) shows that the signal reduction achieved by using this pair of PZT is at least 10 dB greater than the sensor signal reduction achieved by using PZT No. 6 or PZT No. 11 alone (see Fig. 6.5 and Fig. 6.6). However, the SPL reduction is only slightly greater than the SPL reduction achieved by using PZT No. 11.

The above comparison between the performance of PZT No. 6 or 11 and the performance of PZT No. 1 and 2 is actually complicated by two factors which were overlooked when the experiments were conducted, namely, the power amplifier gain and the controller sampling period. In experiments 8 through 15, an attempt was made to maintain an equal power amplifier gain by marking the gain adjustment dial on the amplifier. This method later appears to be erratic, because the actual gain of the power amplifier is determined not only by the dial setting, but also by the number of PZT actuators loading the amplifier. Experiments done later (after the control experiments discussed in this chapter) showed that the power amplifier output at any particular gain setting actually dropped with the number of actuators loading the amplifier. Therefore, the actuator voltage in the experiment using PZT No. 1 and 2 together were lower than the actuator voltage in the experiment using PZT No. 6 or PZT No. 11 separately.

The other factor which affected the result was the controller sampling rate. The experiments using PZT 6 or 11 (experiments No. 8 and 9) were done with a sampling rate of 5000 samples/second. However, the experiment using PZT 1 and 2 (experiment No. 10) was done with a sampling rate of 6000 samples/second. (Chronologically, the latter
Figure 6.7 Closed Cylinder SPL with and without Control. Sensor: Mode 4 PVDF Film, Actuators: PZT No. 1 and 2.
experiment was done in a sequence with experiments No. 16 and 17 to compare the performance of different sensors.) This difference in sampling rate caused a difference in the frequencies of the harmonics generated by sampling. Experiments No. 8 and 9 resulted in a harmonic at 3485 Hz, and experiment No. 11 resulted in a harmonic at 4485 Hz. The different harmonics gave different contributions to the total SPL, although the contributions of the high frequency harmonics were not very significant.

The two factors mentioned above show that the experiment results may not be a good basis for making comparison between the controlled SPL in the three cases. However, qualitative comparison can still be made with reasonable confidence.

6.3.3 PZT Actuators No. 7 and 8 (Experiment No. 11)

PZT actuators No. 7 and 8 are not on the antinodes of mode (4,1). Fig. 6.8 shows that when these actuators were used, the excited vibration at 3485 Hz was rather high (80 dB), so that although the SPL reduction at 1515 Hz was high (58 dB), the total SPL reduction was only 14 dB. Comparison between Fig. 6.8 and Fig. 6.7 shows that PZT actuators No. 1 and 2 give greater reductions than PZT No. 7 and 8. PZT actuators No. 1 and 2 are very close to the antinode of Mode (4,1), while PZT actuators No. 7 and 8 are not (see Fig. 6.4). These results corroborate the theory that better actuator authority is achieved when the actuator is closer to the antinodes of the controlled mode.

6.3.4 PZT Actuators No. 1, 2, and 6 (Experiment No. 12)

When PZT No. 6 was used in addition to PZT No. 1 and 2, the SPL autospectrum reduction at 1515 Hz was improved (see Table 6.4 and Fig. 6.9). However, both the
Figure 6.8  Closed Cylinder SPL with and without Control. 
Sensor: Mode 4 PVDF Film, Actuators: PZT No. 7 and 8.
Figure 6.9 Closed Cylinder SPL with and without Control. 
Sensor: Mode 4 PVDF Film, Actuators: PZT No. 1, 2, 6.
sensor signal reduction and the overall SPL reduction were not as good as the reduction achieved with PZT No. 1 and 2 only. In this case adding PZT No. 6 did not improve the control performance. However, because of the difference in power amplifier gains (as explained above), no firm conclusion from this experiment can be drawn regarding the effect of the number of actuators on control performance.

6.3.5 PZT Actuators No. 6, 7, and 8 (Experiment No. 13)

PZT actuators No. 7 and No. 8 are in one axial line with PZT actuator No. 6, and this axial line is a line where antinodes of mode (4,1) are located. The combination of these three actuators is a good configuration to cancel mode (i,1) vibrations. For this reason, the three PZT patches were used in-phase. Figure 6.10 shows the controlled SPL. The resulting reduction in sensor signal was 62 dB, which was about the same as the sensor signal reduction achieved by using PZT No. 1, 2, and 6 together. The sensor signal reduction was also better than that given by PZT patch No. 6 alone (54 dB), although both the SPL autospectrum reduction at 1515 Hz and the total SPL reduction were about the same as those given by PZT patch No. 6 alone (see Table 6.4). However, vibration at 3485 Hz was strongly excited. The controlled total SPL reduction was 20 dB.

6.3.6 PZT Actuators No. 7, 8, and 11 (Experiment No. 14)

PZT actuators no 7, 8 and 11 together resulted in an SPL reduction of 28 dB (see Fig. 6.11) and Table 6.4, which is greater than the reduction achieved with the two other three-actuator combinations.

6.3.7 PZT Actuators No. 1, 2, 6, 7, 8, 11 (Experiment No. 15)
Figure 6.10  Closed Cylinder SPL with and without Control. Sensor: Mode 4 PVDF Film, Actuators: PZT No. 6, 7, 8.
Figure 6.11  Closed Cylinder SPL with and without Control.
Sensor: Mode 4 PVDF Film, Actuators: PZT No. 7, 8, 11.
Combining all available PZT actuators used previously (PZT patches No. 1, 2, 6, 7, 8, 11) resulted in the greatest total SPL reduction (29 dB) (see Fig. 6.12). The reduction in vibration level is also high (see Table 6.4). This result suggests that increasing the number of actuators results in greater reductions both in vibration and in acoustic emission.

6.3.8 Conclusion on Actuator Locations

From the experiments with the various actuators described in this section, the following conclusions can be drawn.

Best Actuator Location: PZT No. 6 and No. 11 are located at antinodes of mode (4,1), and the sensor signal reduction and SPL reduction resulting from using each of these actuators were better than the reductions given by PZT No. 7 and 8 together. PZT No. 7 and 8 are not located at antinodes. PZT No. 1 and 2, used together, gave greater reductions than PZT No. 7 and 8 used together. Therefore, it can be concluded that the best actuator locations are at antinodes. This result may be compared to a result from previous analytical work on beams (Jia, 1991), which shows that the optimum actuator locations for minimum control effort are at the antinodes of the beam.

Number of Actuators: Results from experiments No. 1 through 3 (see Table 6.1) show that using more actuators results in better reductions in sensor signal or SPL.

6.4 Comparison among Three Sensors (Experiments No. 16, 10, and 17)

To compare three types of sensors, experiments were done on the closed cylinder with
Figure 6.12 Closed Cylinder SPL with and without Control.
Sensor: Mode 4 PVDF Film, Actuators: PZT No. 1, 2, 6, 7, 8, 11.
the adaptive controller. Figure 6.13 and Table 6.5 show the controlled SPL with plain PVDF film sensor, mode 4 PVDF film sensor, and a microphone. The microphone gave the greatest SPL reduction (26 dB). This result was as expected, since the microphone measured the SPL directly. The use of the PVDF sensors only controls the vibration of the cylinder, and reduction in SPL may not have a simple relation with the reduction in vibration. Despite this shortcoming, the use of the mode 4 PVDF modal sensor resulted in an SPL reduction of 24 dB, which was comparable to the reduction achieved with the microphone. The plain PVDF film sensor only gave a 14 dB reduction in total SPL.

6.5 Summary

The following paragraphs summarize the research work done for this report. Experiments have been done on a cylinder which was a simplified scale model of an underwater structural part to demonstrate that noise emission from the cylinder could be actively controlled. The cylinder was instrumented with structure-borne piezoelectric sensors and actuators. Active acoustic control was successfully done on the cylinder by controlling the cylinder vibration with digital signal processing hardware programmed with an adaptive control algorithm, shaped PVDF film used as a sensor, and PZT patches used as actuators. The excitation was on-resonance single frequency forces applied with a shaker.

Prior to the experiments mentioned above, analytical study and preliminary experiments were done to characterize the elements of the control system. In particular, the cylinder, the sensor and the actuators were studied. First, the natural frequencies and mode shapes of the cylinder were analytically predicted for both the open cylinder case and the closed cylinder case. Experiment results agreed with the analytical results. A preliminary acoustic testing was done to determine which mode was to be used in later experiments to
Figure 6.13 Closed Cylinder SPL with Three Different Sensors: Plain PVDF Film, Mode 4 PVDF Film, and Microphone.
Table 6.5 Results of Adaptive Control with Three Different Sensors.

Excitation Frequency: 1515 Hz  
Microphone Position: Pos 1  
Actuators: PZT No. 1, 2  
Uncontrolled SPL: 95 dB  
Background Noise : 50 dB

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Error Signal Reduction (dB)</th>
<th>SPL Reduction (dB)</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1515 Hz</td>
<td>Overall</td>
<td></td>
</tr>
<tr>
<td>PVDF mode 4</td>
<td>68</td>
<td>33</td>
<td>24</td>
<td></td>
</tr>
<tr>
<td>plain PVDF</td>
<td>42</td>
<td>14</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>microphone</td>
<td>26</td>
<td>36</td>
<td>26</td>
<td></td>
</tr>
</tbody>
</table>
demonstrate active control. From the results of these experiments, mode \((4,1)\) was chosen to be the mode to be controlled.

The next analytical and experimental study was concerned with piezoelectric sensors and actuators. A piezoelectric sensor theory developed by Lee was used as a basis for developing the relation between strain and sensor output voltages. Experiments were done to investigate the applicability of modal sensors to the cylinder vibration control problem. Analysis and experiments proved that one-dimensional modal sensors can be used on two-dimensional structures such as a cylinder. Based on Lee’s theory, equations were developed to predict PVDF sensor output voltage from the knowledge of the structural mode shape and acceleration at an antinode. The output voltage also depended on the input impedance of the electronic interface circuit connected to the PVDF sensor. An experiment done on the open cylinder with a modal sensor resulted in a sensor output voltage which agreed with the output voltage predicted by the developed equations. Analysis and experiments done to investigate the use of PVDF film as a structure borne distributed sensor have given significant results which may also be useful for future projects.

Experiments also proved that PZT patches bonded onto the cylinder wall had enough authority to cancel the vibration caused by the shaker excitation. After the PVDF sensors and the PZT actuators were tested on the cylinder, a preliminary vibration control experiment was done with a simple strain feedback loop. This experiment showed that the sensors and the actuators were able to work together in controlling the vibration of the cylinder.

To achieve the final goal of the research project, acoustic control experiments were done
on the cylinder in a reverberation chamber. The filtered-x version of the Least Mean Square (LMS) adaptive control algorithm was used with a TMS320C30 digital signal processing board to make the DSP board a controller for controlling the acoustic emission from the cylinder. PZT patches on the vibration antinodes were used as actuators. Three sensors were used and compared: the plain PVDF film, the shaped PVDF film, and an error microphone. Comparison showed that the best control performance resulted from using a microphone as a sensor. However, the use of mode 4 PVDF sensor gave a control performance which was comparable to the use of the microphone.
Chapter 7
Conclusions and Recommendations

7.1 Conclusions

Experiments have been done to demonstrate that sound radiation from a vibrating cylinder can be reduced by controlling the structural vibration using piezoelectric sensors and actuators. This research work demonstrates the potential of Active Structural Acoustic Control (ASAC). The work involved characterization of the cylinder, study of the sensors and actuators, and implementation of the control strategies. From the analysis and experiments, several conclusions can be drawn.

The method given in the literature (Blevins, 1979) predicts natural frequencies and mode shapes of the cylinder accurately for the open cylinder case. For seventeen modes whose frequencies are lower than 5 kHz, the difference between experimentally obtained resonant frequencies and analytically predicted natural frequencies is generally less than 5%.

For the closed cylinder case, the method can be combined with formulas given by Gorman (1975), which calculate the eigenvalues of a beam with a pin and torsional spring boundary condition. The resulting method predicts the natural frequencies accurately for low-index modes. For the first four modes, the difference between the predicted natural frequencies and the experimentally obtained resonant frequencies is less than 7%. However, for higher axial modal indices this technique needs refining.
Analysis and experiments have shown that PVDF modal sensors are applicable to cylinders. When applied on the cylinder, PVDF modal sensor exhibits modal filtering effect by sensing a selected family of modes. This effect is useful in reducing observation spillover.

Analysis has been done to express the PVDF sensor output voltage as a function of the cylinder vibration amplitudes and mode shapes. An experiment verified the relation.

Comparison between the effect of a shaker force and the effect of PZT actuation on the cylinder shows that applying a point force of 1 N at the shaker point of action is equivalent to applying 7.08 V to the PZT actuator No. 6, whose position is diametrically opposite the shaker point of action.

At 1420 Hz (Rayleigh mode 4 frequency), the analog feedback control with PVDF mode 4 sensor resulted in a 15 dB SPL reduction, 13 dB acceleration reduction, and 13 dB reduction in error signal. With a PZT patch as a sensor, a 17.5 dB SPL reduction can be achieved, provided that the excitation frequency is the cylinder's lowest natural frequency (Rayleigh mode 2 frequency, 263 Hz) and a that low pass filter is used in the feedback loop. The low pass filter is necessary to stabilize the control because, lacking modal filtering capability, PZT patches do not alleviate observation spillover problems.

The best actuator locations are at the antinodes of the mode to be controlled. If all actuators are located at these positions, increasing the number of actuators results in an increase in actuator authority.

The microphone was the best sensor as far as the total SPL reduction was concerned. This
is because the microphone measures the SPL directly, unlike the PVDF sensors which actually measure vibrations. The PVDF modal sensor gave better reductions than the rectangular PVDF sensor.

Adaptive control produced a much greater reductions in SPL and error signal than analog control. With the adaptive control, an SPL reduction of 36 dB at the autospectrum peak and 29 dB overall were achieved. The SPL reduction achieved by both analog and adaptive control was a result of reduction in vibration levels. However, a greater reduction in vibration does not always result in a greater reduction in SPL.

7.2 Recommendations

The main objective of this thesis was to demonstrate experimentally that active acoustic control of the cylinder could be done through vibration control with PVDF film as a sensor and PZT patches as actuators. Beyond this objective, much research is yet to be done to achieve better knowledge of the elements of the control system, and finally to integrate the knowledge to achieve an ultimate objective of controlling acoustic emission from cylinders.

Possible research work as a continuation of the research project discussed in this thesis is as follows. First, the combined dynamics of the cylinder, the shaker and the actuator is determined using available mathematical models and modal analysis. This study involves modeling the coupling of the PZT actuator forces into the cylinder vibration. The result includes a general expression of the response at any point of the cylinder to the disturbance excitation, and the response of the cylinder to the actuator voltage. Then the coupling of the structural vibration to the acoustic field is studied. The result will be
analytical expressions of sound pressure level as a function of shaker force and actuator voltage. A more advanced study involves fluid loading because for this research project ultimately the medium of interest is water, whose density imposes considerable loading on the structure. In this case, the structural vibration couples with the acoustic field.

Another future research area is the design of structure-borne sensors. The objective is to design sensors which measure structural vibration but whose output is proportional to acoustic pressure levels. Thus, these sensors can replace microphones as acoustic sensors. One design approach is the wavenumber sensor approach, which has been proposed and pursued by several researchers (e.g., Clark and Fuller, 1991). This approach is geared towards tailoring distributed vibration sensors in such a way that the sensors are sensitive to certain wavenumbers. The theory can be combined with the structural-acoustic model of the cylinder to design structure-borne sensors.

When the dynamic models of the cylinder vibration and acoustic field are available, and after structure-borne sensors which can replace microphones are designed, the ultimate research work will integrate the results by closing the control loop. Considerable work can be done in the future for the development of different control strategies such as state space control, classical feedback control, and adaptive control. Comparison of various control strategies both analytically and experimentally will be very useful in identifying the advantages and disadvantages of the control strategies under various conditions. Ultimately, acoustic control will be able to minimize acoustic pressure level generated in water by the cylinder under any general vibration excitation, using structure-borne sensors and actuators.
References


Dimitriadis, E. K., and C. R. Fuller, 1989. "Investigation on Active Control of Sound


Appendix A
Photographs of the Cylinder
Figure A.1 Photograph of the Closed Cylinder, Showing the Side with PZT No. 1, PZT No. 2, and Shaker Point of Action.
Figure A.2 Photograph of the Closed Cylinder, Showing the Side with PZT No. 6, 7, and 8
Appendix B
Analytical Natural Frequencies of the Cylinder

The natural frequencies and mode shapes of the closed cylinder are computed by a method described in Section 3.3. The equivalent beam boundary condition of the closed cylinder is modeled with a pin and a torsion spring at each end. The natural frequencies of the cylinder are computed and tabulated for several values of torsion spring constant, $T_1$. The actual torsional spring constant for the closed cylinder is determined by experimentally obtaining a mode shape and the corresponding natural frequency, and then finding the appropriate table which has the matching natural frequency for the mode.

The computed natural frequencies of mode $(i,j)$ for several torsion spring constant are tabulated below. The computation is done with a Matlab program listed after the tables.

B.1 Tables of Natural Frequencies

Pinned-pinned Boundary Condition ($T_1 = 0$):

<table>
<thead>
<tr>
<th>$i \downarrow, j \rightarrow$</th>
<th>1</th>
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<th>3</th>
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<tr>
<td>3</td>
<td>928</td>
<td>1675</td>
<td>2560</td>
<td>3308</td>
</tr>
<tr>
<td>4</td>
<td>1489</td>
<td>1874</td>
<td>2520</td>
<td>3308</td>
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<tr>
<td>5</td>
<td>2338</td>
<td>2570</td>
<td>3028</td>
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Clamped-clamped Boundary Condition \((T_1 \to \infty)\)

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<td>5616</td>
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<td>3170</td>
<td>4383</td>
<td>5300</td>
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<td>3</td>
<td>1284</td>
<td>2407</td>
<td>3570</td>
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<td>4</td>
<td>1651</td>
<td>2361</td>
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Pin and Torsion Spring Boundary Condition

\(T_1 = 1:\)

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\(T_1 = 2:\)

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<td>2400</td>
<td>3224</td>
<td>3224</td>
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</table>
B.2 Matlab Program

Two main programs were used in this thesis. The first program computes natural frequencies of cylinders with classical boundary conditions. This program was used to compute the natural frequencies of the open cylinder. The second program computes natural frequencies of the closed cylinder with pin plus torsional spring boundary conditions. Only the second program is listed in this appendix.

Main Program

```matlab
fprintf('NATURAL FREQUENCIES OF CIRCULAR CYLINDER\n');
fprintf('from Blevins, Formulas for Natural Frequency and Mode Shape\n');
fprintf('Hartono Sumali July 1990\n');

outfil = input('enter filename to write to, without ext''n :','s');
outfil = [outfil,'.m'];

%highest mode index
maxind = 6;

% Dimensions of Cylinder, SI units
R = 0.123825
L = 0.4064
h = 0.00635
mu = 2700
E = 64e9
nu = 0.3

k = h ^2/(12*R^2);

%specify torsion spring constants
for T1 = 1:1:16,
  fprintf('T1 = %.2g \n',T1)
  for indj = 1:maxind,
    lambguess = indj * pi;
    lamb(indj) = fsolve('blambda',lambguess,zeros(16,1),[T1,1]);
  fprintf('lambda(%d) = %.2g \n',indj, lamb(indj));
```

\[
\text{beta}(\text{indj}) = \lambda(\text{indj}) \cdot \frac{R}{L};
\]

integrate product of mode shape function and its second derivative
integrand is written in bdalp1.m

\[
\text{alpha1(}\text{indj}) = -1 \cdot \text{bquad8('bdalp1',T1,lambda(indj),0,1)};
\]

integrate square of first derivative of mode shape function
integrand is written in bdalp2.m

\[
\text{alpha2(}\text{indj}) = \text{bquad8('bdalp2',T1,lambda(indj),0,1)};
\]

%fprintf\('alpha1(%2.0f)= %12.6g ', indj, alpha1(indj)\);
%fprintf\('alpha2(%2.0f)= %12.6g\n', indj, alpha2(indj)\);
end

for indii = 1:maxind+3,
indii = indii - 1;
for indji = 2:maxind+1,
indj = indji - 1;

\[
\text{all} = \text{beta(indj)}^{1/2} + 0.5 \cdot (1 + k) \cdot (1 - \nu) \cdot \text{indi}^{1/2} \cdot \text{alpha2(indj)};
\]

\[
\text{a12} = -1 \cdot \nu \cdot \text{indi} \cdot \text{beta(indj)} \cdot \text{alpha1(indj)};
\]

\[
\text{a13} = -1 \cdot \nu \cdot \text{beta(indj)} \cdot \text{alpha1(indj)};
\]

\[
\text{a22} = \text{indi}^{2} + 0.5 \cdot (1 + 3 \cdot k) \cdot (1 - \nu) \cdot \text{beta(indj)}^{1/2} \cdot \text{alpha2(indj)};
\]

\[
\text{a23} = \text{indi} + \text{beta(indj)}^{2} \cdot (\nu \cdot \text{alpha1(indj)} + 1.5 \cdot (1 - \nu) \cdot \text{alpha2(indj)});
\]

\[
\text{a33} = \text{beta(indj)}^{4} + (\text{indi}^{2} - 1)^{2} + 2 \cdot \nu \cdot \text{indi}^{2} \cdot \text{beta(indj)}^{2} \cdot \text{alpha1(indj)}^{2} \cdot \text{alpha2(indj)};
\]

\[
\text{c0} = \text{a12} \cdot \text{a12} \cdot \text{a33} + \text{a23} \cdot \text{a23} \cdot \text{a11} + \text{a13} \cdot \text{a13} \cdot \text{a22} - \text{a11} \cdot \text{a22} \cdot \text{a33} - 2 \cdot \text{a12} \cdot \text{a23} \cdot \text{a13};
\]

\[
\text{c6} = \text{alpha2(indj)};
\]

\[
\text{c4} = -1 \cdot (\text{a11} + \text{alpha2(indj)} \cdot \text{a22} + \text{alpha2(indj)} \cdot \text{a33});
\]

\[
\text{c2} = \text{a12} \cdot \text{a12} + \text{a13} \cdot \text{a13} + \text{alpha2(indj)} \cdot \text{a23} \cdot \text{a23};
\]

\[
\text{coeff} = \begin{bmatrix} \text{c6} & 0 & \text{c4} & 0 & \text{c2} & 0 & \text{c0} \end{bmatrix};
\]

solve characteristic equation using matlab routine roots;
lambda = min(abs(lambda(vec));
omega(indii,indji) = lambda / ( \( R \cdot \sqrt{\mu \cdot (1-\nu \cdot \nu) / E} \));
fn = omega(indii,indji) / (2*pi);
fprintf('freq \ %g \ %g = \ %6.0f \ \n', indi, indj, fn);
apc(indii,indji) = (a33 - lambda^2)*(a22 - lambda^2) - a23*a23;
apc(indii,indji) = apc(indii,indji) / (a12*a23 - a22*a13);
bpc(indii,indji) = a12*(a33 - lambda^2) - a13*a23;
bpc(indii,indji) = bpc(indii,indji) / (a22*a13 - a23*a12);

end % {of indji}
end % {of indii}

fprintf(outfil,'T1 = %12.6g \n', T1);
for indj=1:maxind,
    fprintf(outfil,'lambda(%2.0f)= %9.6g; ',indj,lamb(indj));
    fprintf(outfil,'alpha1(%2.0f)= %9.6g; ', indj, alphal(indj));
    fprintf(outfil,'alpha2(%2.0f)= %9.6g; \n', indj, alphal2(indj));
end:
fprintf(outfil,\n Natural frequencies : \n');
fprintf(outfil,' j = 1 2 3 4 5 6 \n');
for indii = 1:maxind+3, 
    indi = indii-1;
    fprintf(outfil,'\n', %2.0f', indi)
    for indji = 2:maxind+1, %indj = indji-1;
        fn = omega(indii,indji) / (2*pi);
        fprintf(outfil,'%7.0f ',fn)
    end
    fprintf(outfil,\n');
end
fprintf(outfil,\)n');
end %of T1 loop

Subroutine to compute beam mode shape function

% bphi.m

function y = bphi(T1, alpha, jj, x);

lambguess = (jj - 0.4)*pi;
lambd = fsolve('blambda', lambguess, zeros(16,1), [T1, alpha])

gam = (sinh(lambd) - sin(lambd)) / ... 
    (cos(lambd) - cosh(lambd) - 2*lambd*sinh(lambd)/T1);
xx = lambd*x;

y = sin(xx) - sinh(xx) + gam*(cos(xx) - cosh(xx) - 2*lambd*sinh(xx)/T1);

Subroutine to Integrate mode Shape Functions and Their Derivatives
function y = bdalp2(T1,lambda,x);
gam = (sinh(lambda)-sia(lambda)) / ...
    (cos(lambda)-cosh(lambda)-2*lambda*sh(lambda)/T1);
xx = lambda^x;

drdxx = cos(xx)-cosh(xx)-gam*(sin(xx)+sh(xx)+2*lambda*cosh(xx)/T1);
y = drdxx.^2;
Appendix C
Experimental Mode Shapes of the Cylinder

The approximate mode shapes of the cylinder were obtained experimentally as described in Section 3.2. In this appendix, each of the approximate mode shapes of the cylinder is shown in two parts. The first part is the circumferential variation (shown as polar plots), and the second part is the axial variation.
Figure C.1  Open Cylinder Rayleigh Mode 2 (263 Hz)

Figure C.2  Open Cylinder Rayleigh Mode 3 (747 Hz)

Figure C.3  Open Cylinder Love Mode 3 (803 Hz)
Figure C.4  Open Cylinder Rayleigh Mode 4 (1425 Hz)

Figure C.5  Open Cylinder Love Mode 5 (2310 Hz)

Figure C.6  Open Cylinder Mode (3,1) (1570 Hz)
Figure C.7 Open Cylinder Mode (4,1) (1850 Hz)

Figure C.8 Open Cylinder Mode (5,1) (2630 Hz)

Figure C.9 Open Cylinder Mode (6,1) (3680 Hz)
Figure C.10  Open Cylinder Mode (2,2) (3825 Hz)

Figure C.11  Open Cylinder Mode (3,2) (2880 Hz)

Figure C.12  Open Cylinder Mode (4,3) (3735 Hz)
Figure C.13  Open Cylinder Mode (5,3) (4030 Hz)

Figure C.14  Open Cylinder Mode (6,3) (4825 Hz)
Figure C.15 Closed Cylinder Mode (3,1) (1010 Hz)

Figure C.16 Closed Cylinder Mode (2,1) (1050 Hz)

Figure C.17 Closed Cylinder Mode (4,1) (1545 Hz)
Figure C.18  Closed Cylinder Mode (5,1) (2400 Hz)
Appendix D
Variations of Acoustic FRF Around the Closed Cylinder

The magnitude ratio of SPL around the cylinder to shaker excitation force or PZT excitation voltage was obtained experimentally as described in section 3.4. A typical acoustic FRF obtained with a microphone is shown in Fig. D.1. Figures D.2, D.3, and D.4 show the variation of FRF with circumferential position at three resonant frequencies.
Figure D.1  
a. FRF between SPL and Shaker Force  
b. FRF between SPL and PZT Voltage.  
Closed Cylinder, Microphone No. 1, Position 6.
Figure D.2 Acoustic Directivity of Closed Cylinder at 1050 Hz, Measured with Semicircular Microphone Array in Axial Position 6.

a. Response from Shaker Excitation
b. Response from PZT No. 6 Excitation.
Figure D.3  Acoustic Directivity of Closed Cylinder at 1550 Hz, Measured with Semicircular Microphone Array in Axial Position 6.
a. Response from Shaker Excitation
b. Response from PZT No. 6 Excitation.
Figure D.4 Acoustic Directivity of Closed Cylinder at 2375 Hz, Measured with Semicircular Microphone Array in Axial Position 6.
a. Response from Shaker Excitation
b. Response from PZT No. 6 Excitation.
Hartono Sumali, also known as Anton, was born on January 4, 1964 in Jakarta, Indonesia. After graduating from a Jesuit High School in that city in 1982, he studied mechanical engineering at an Institute of Technology in Bandung and graduated in 1987. Upon graduation, he worked as an instrumentation engineer for McDermott Indonesia, a company that builds offshore oil platform. He came to the United States in August 1989, and enrolled in the Master of Science program in the Mechanical Engineering Department at Virginia Polytechnic Institute and State University. With Prof. Cudney, he wrote and co-authored three papers on modal sensing and vibration control. In 1992, he began working towards a PhD degree in the same department.

Hartono Sumali