CHAPTER IV
ANALYTICAL PREDICTIONS AND COMPARISONS

4.1 INTRODUCTION

Three analytical procedures to predict the modal properties of long span deck floor systems are studied in this chapter. The predicted results are compared to the results obtained from the in-situ tests studied in Chapter 2 and from the laboratory footbridge studied in Chapter 3, to determine their accuracy. First, the natural frequencies of the tested floors are predicted with finite element models (FEM) developed using a commercial finite element software. Second, the floor frequencies and accelerations are evaluated with the provisions given in the AISC Design Guide 11 *Floor Vibrations Due to Human Activity* (Murray et al, 1997). Third, the natural frequency and response acceleration of the tested floors are predicted using the criteria of the SCI publication "Design of Floors for Vibration: A New Approach" (Smith et al, 2007). The results of the analytical predictions and the comparisons with the measured natural frequencies and accelerations are discussed in the following sections.

4.2 FINITE ELEMENT MODELS AND COMPARISONS

Finite element models were created for the thirteen bays tested in Chapter 2 and the laboratory footbridge studied in Chapter 3. SAP2000 version 11.0.8 (Computers and Structures Inc., 2007) was used to create the finite element models of the floors.

In this section the modeling procedure used to conduct the finite element analyses of the floors is presented first. The finite element model results for the floors studied in Chapter 2 are presented next. The results obtained from the finite element models of the laboratory footbridge are presented and studied at the end of this section.

4.2.1 Modeling of the Floors

Elements. Two types of elements were used to create the finite element models of the floor structures. Thin shell elements were used to model the slab. Frame elements were used to model the supporting beams and girders of the floors. These two types of elements have been used in the past to create finite element models of floor structures with accurate results (Sladki, 1999; Perry, 2003).

Materials. Two types of materials were used to create the models. The first is named STEEL, and is predefined in SAP2000. This material was used to model the steel membres of the floor structures. The second is a user defined material named VIBCON. This material has the properties of the concrete used to construct the slab, with two modifications. VIBCON has orthotropic properties to account for the difference in stiffness present in a composite slab in each bending direction. Consider, for example, the concrete-steel deck composite section shown in Figure 4.1. The moment of inertia of the slab about the *x*-axis is the transformed moment of inertia of the section shown in Figure 4.1(a), which includes the steel deck. In the other direction, the moment of inertia is equal to the transformed moment of inertia of the concrete rectangle shown in Figure 4.1(b).

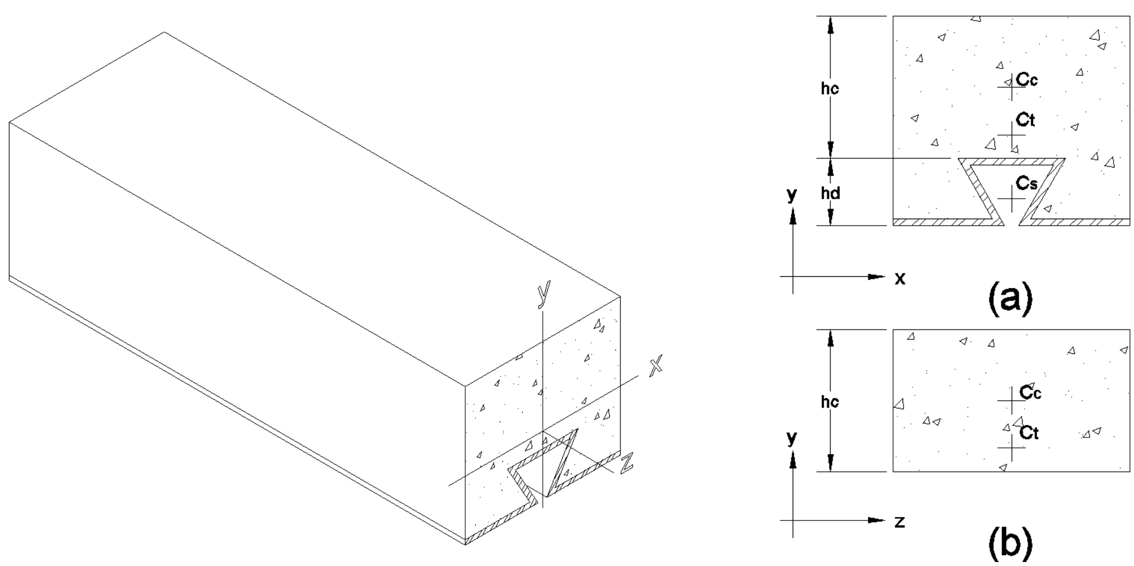


Figure 4.1 Cross-Sections of a Composite Slab

To account for this difference, VIBCON has two moduli of elasticity, E. In one direction, E is equal to the dynamic modulus of elasticity of the concrete; that is 1.35 times the static modulus of elasticity (Murray et at, 1997). In the other direction, E is given by:

$$E_x = \frac{(1.35 \cdot E_c) \cdot I_x}{I_r} \tag{4.1}$$

where

E_c = static modulus of elasticity of the concrete

I_x = transformed moment of inertia about the *x*-axis

I_c = transformed moment of inertia of the concrete rectangular section

The other modification is the weight and mass of the material. SAP2000 calculates the natural frequencies of a structure considering only the mass of the slab. The live or dead loads acting on the floor are not included in the modal analysis. In VIBCON, the loads are included by modifying the specific weight of the material, W. The following formula was used to calculate the specific weight of a floor including the loads:

$$W = \frac{\frac{W_c}{A_s} + w_{deck} + w_{dead} + w_{live}}{h_c} \tag{4.2}$$

where

W = specific weight of the material

W_c = weight of the slab without the deck

A_s = area of the slab

w_{deck} = deck weight

w_{dead} = distributed dead load

w_{live} = distributed live load

h_c = height of the concrete (see Figure 4.1)

Area Sections. Area sections with thin shell elements and with a thickness equal to the concrete height, h_c, were used to model the slabs. The material used to define the area sections was VIBCON. Thus, the slab is defined as a uniform rectangular section with orthotropic properties and with a specific weight that considers the applied loads. This model was used to define the area sections in all the floors modeled.

Restraints and Constraints. The slab ends were modeled as fully fixed when supported by CMU, steel stud, or masonry walls. Free support conditions were considered in the other two sides of the floors.

4.2.2 Finite Element Models of the Building Floors

The finite element models for the thirteen floors tested in-situ are presented in this section. One bay, Bay 5 of the Concord and Cumberland building was selected to show all the steps involved in the creation of a finite element model.

Slab Cross Section and Deck Geometry : The slab cross section and the deck geometry are shown in Figure 4.2.

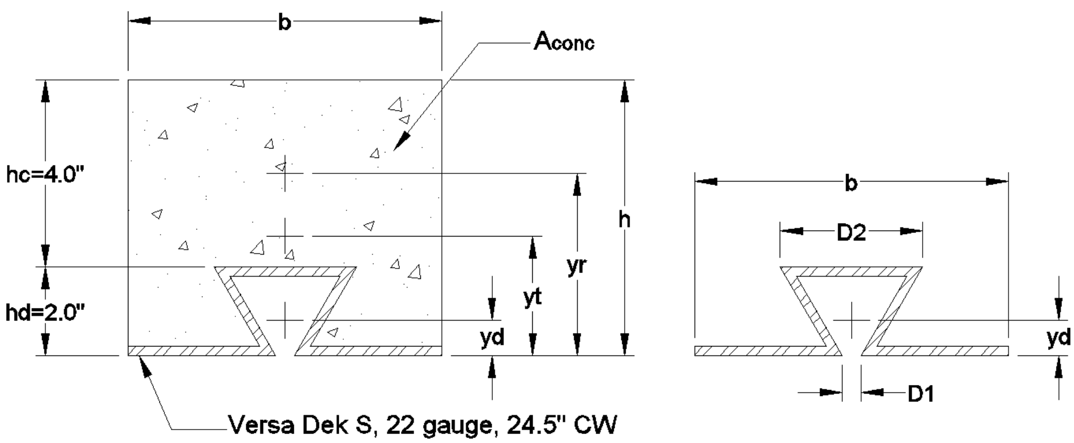


Figure 4.2 Bay 5, Slab Cross Section and Deck Geometry

Steel Deck Properties: The steel deck used in this slab is the Versa Dek S, 22 gauge, 24.5" CW (Metal Dek Group®). The following properties were obtained from the deck manufacturer:

Versa Dek S, 22 gauge, 24.5" CW

Area, A_d	$= 0.6518\ in^2$
Distance to Neutral Axis, y_d	= 0.6695 in.
Moment of Inertia, I_d	$= 0.4175\ in^4$
Weight, w_{deck}	= 2.22 psf
D1	= 0.5 in.
D2	= 1.5 in.

b = 6 1/8 in.

Nominal Concrete Properties:

Normal Weight Concrete

$f'_c = 3.0$ ksi

$w = 150$ pcf

$E_c = w^{1.5}\sqrt{f'_c} = 150^{1.5}\sqrt{3.0} = 3{,}182$ ksi

Modular Ratio, n:

$$n = \frac{E_s}{1.35 \cdot E_c} = \frac{29{,}000}{1.35 \cdot 3{,}182} = 6.75$$

Transformed Moment of Inertia, I_t: To calculate the transformed moment of inertia, I_t, the section shown in Figure 4.2 is split into four parts: one concrete rectangle, two concrete trapezoids, and the steel deck.

- Trapezoids:

$$b_1 = \frac{b - D1}{2n} = \frac{6.125 - 0.5}{2 \cdot 6.75} = 0.417 \text{ in.}$$

$$b_2 = \frac{b - D2}{2n} = \frac{6.125 - 1.5}{2 \cdot 6.75} = 0.343 \text{ in.}$$

$$y_{trap} = h_d - \frac{h_d(2b_1 + b_2)}{3(b_1 + b_2)} = 2.0 - \frac{2.0(2 \cdot 0.417 + 0.343)}{3(0.417 + 0.343)} = 0.967 \text{ in.}$$

$$A_{trap} = \frac{h_d(b_1 + b_2)}{2} = \frac{2.0(0.417 + 0.343)}{2} = 0.759 \text{ in}^2$$

$$I_{trap} = \frac{h_d^3(b_1^2 + 4b_1b_2 + b_2^2)}{36(b_1 + b_2)} = \frac{2.0^3(0.417^2 + 4 \cdot 0.417 \cdot 0.343 + 0.343^2)}{36(0.417 + 0.343)} = 0.252 \text{ in}^4$$

- Rectangle:

$$y_r = \frac{h - h_d}{2} + h_d = \frac{6.0 - 2.0}{2} + 2.0 = 4.0 \text{ in.}$$

$$A_r = (h - h_d)\frac{b}{n} = (6.0 - 2.0)\frac{6.125}{6.75} = 3.629 \text{ in}^2$$

$$I_r = \frac{b(h - h_d)^3}{12n} = \frac{6.125(6.0 - 2.0)^3}{12 \cdot 6.75} = 4.839 \text{ in}^2$$

- Total Section:

$$A = A_d + 2A_{trap} + A_r = 0.6815 + 2 \cdot 0.759 + 3.629 = 5.48 \text{ in}^2$$

$$y = \frac{A_d y_d + 2A_{trap} y_{trap} + A_r y_r}{A} = \frac{0.6815 \cdot 0.6695 + 2 \cdot 0.759 \cdot 0.967 + 3.629 \cdot 4.0}{5.48} = 2.958 \text{ in.}$$

$$\begin{aligned} I_x &= \left[I_d + A_d (y - y_s)^2\right] + 2\left[I_{trap} + A_{trap}(y - y_{trap})^2\right] + \left[I_r + A_r \cdot (y - y_r)^2\right] \\ &= \left[0.4175 + 0.6815(2.958 - 0.6695)^2\right] + 2\left[0.252 + 0.759(2.958 - 0.967)^2\right] + \\ &\quad \left[4.839 + 3.629 \cdot (2.958 - 4.0)^2\right] \\ &= 17.26 \frac{\text{in}^4}{6\,1/8 \text{ in.}} = 33.81 \frac{\text{in}^4}{\text{ft}} \end{aligned}$$

Dynamic Modulus of Elasticity About the *x*-axis, E_x: The moment of inertia of the concrete rectangular section about the principal *x*-axis is:

$$I_{t-x} = \left[I_r + A_r \cdot (y - y_r)^2\right] = \left[4.839 + 3.629 \cdot (2.958 - 4.0)^2\right] = 8.782 \text{ in}^4$$

$$E_x = \frac{(1.35 \cdot Ec) \cdot I_x}{I_{t-x}} = \frac{(1.35 \cdot 3{,}182) \cdot 33.81}{8.782} = 8{,}440 \text{ ksi}$$

Effective Weight and Mass:

- Total Weight, W_c:

$$A_{conc} = 34.5 \frac{\text{in}^2}{6\,1/8 \text{ in.}}$$

$$\text{Width} = 30 \text{ ft } 5\,1/2 \text{ in.} = 365.5 \text{ in.}$$

$$\text{Length} = 21 \text{ ft } 8\,3/8 \text{ in.} = 260.38 \text{ in.}$$

$$W_c = (A_{conc} \cdot \text{Width}) \cdot \text{Length} \cdot w = (34.5 \cdot 365.5) \cdot 260.38 \cdot 150 = 46.5 \text{ kips}$$

- Effective Weight, W:

$$W = \frac{\frac{W_c}{A_s} + w_{deck} + w_{dead} + w_{live}}{h_c} = \frac{\frac{46.5}{260.38 \cdot 365.5} + \frac{2.22 \cdot 10^{-3}}{12^2}}{4} = 1.26 \cdot 10^{-4} \text{ kips/in}^3$$

- Effective Mass, M:

$$M = \frac{W}{386} = \frac{1.26 \cdot 10^{-4}}{386} = 3.26 \cdot 10^{-7} \frac{\text{kips-sec}^2/\text{in}}{\text{in}^3}$$

The finite element model created with the material VIBCON with properties E_c, E_x, W, and M is shown in Figure 4.3. The frequency calculated by SAP2000 for this model is 12.14 Hz; the corresponding mode shape is shown in Figure 4.4.

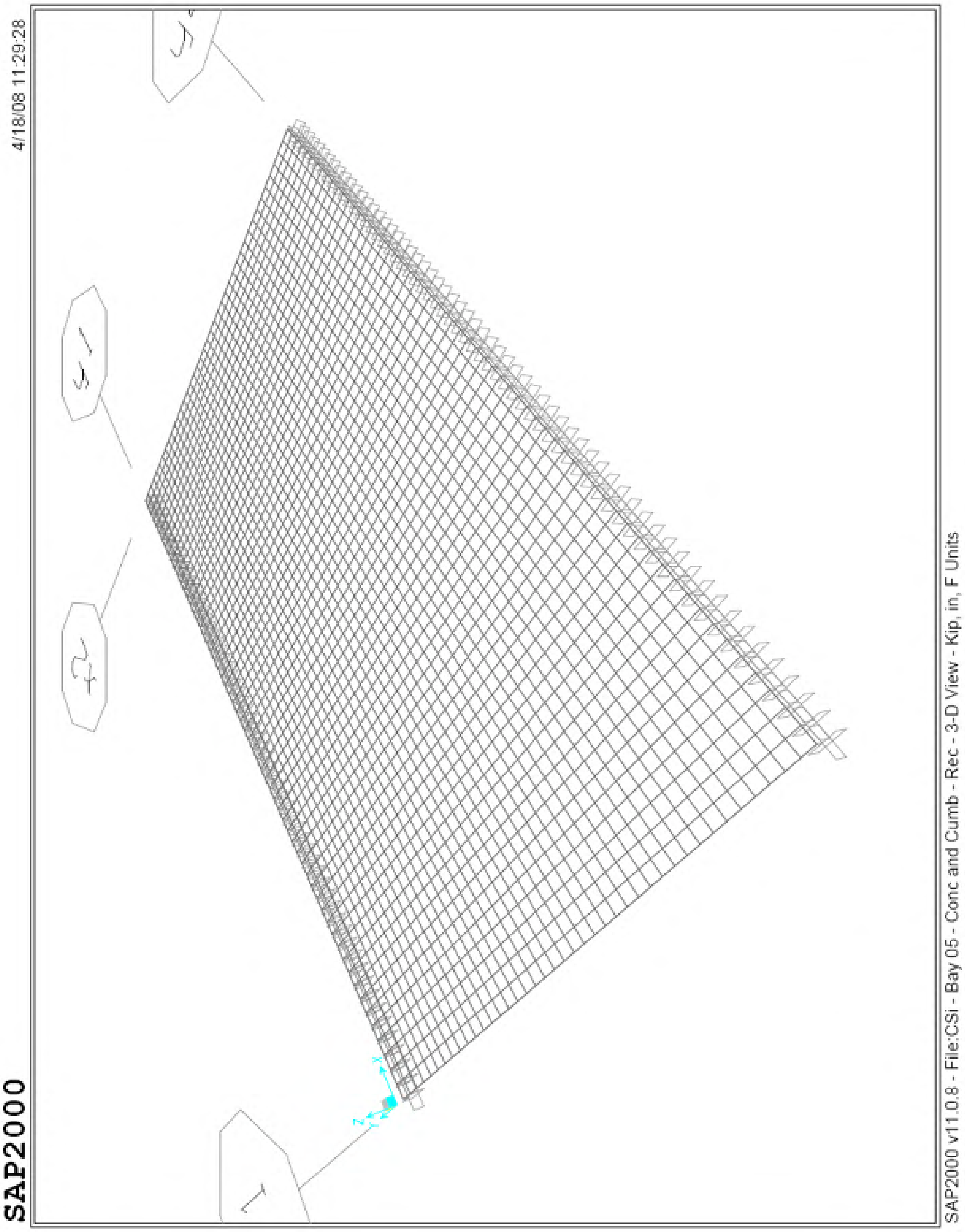


Figure 4.3 Definition of the Finite Element Model for Bay 5

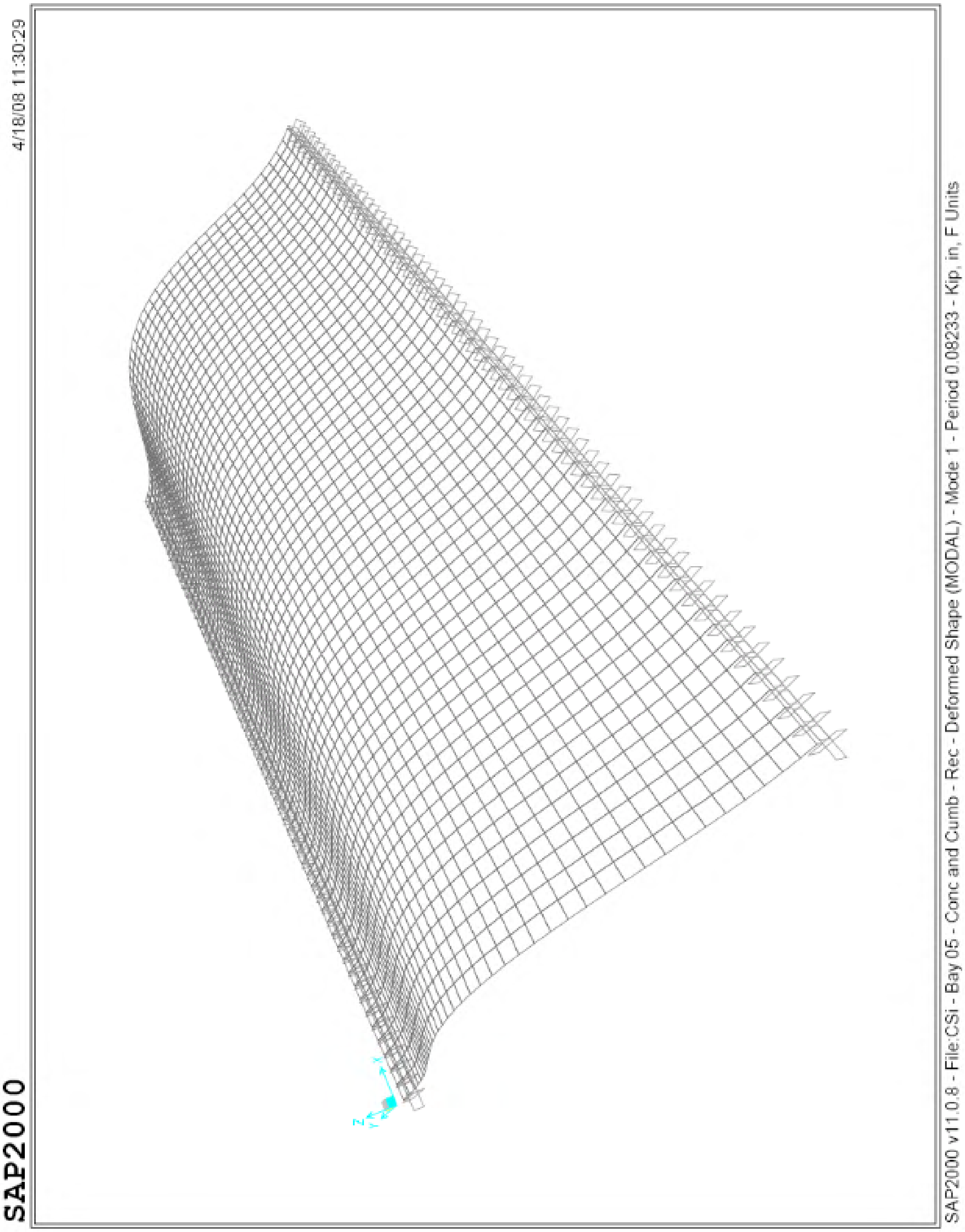


Figure 4.4 First Mode Shape for Bay 5, $f_n = 12.14$ Hz

The process developed to create the finite element model for Bay 5 was the same process used to model the other bays. Table 4.1 summarizes the VIBCON material properties, the predicted frequencies obtained from the finite element models, and the floor frequencies measured in the in-situ tests.

Table 4.1 Summary of Results Obtained from the Finite Element Models

Bay Number	I_r (in^4/pitch)	I_t (in^4/pitch)	E_x (ksi)	W (kips/in^3)	M (W/g) (kips-sec^2/in^3*in)	FEA Predicted f_n (Hz)	Measured f_n (Hz)
1	10.20	20.04	9741	1.269E-04	3.288E-07	11.50	13.30
2	10.20	20.04	9741	1.269E-04	3.288E-07	15.04	16.80
3	9.98	19.62	9753	1.261E-04	3.267E-07	27.70	26.00
4	9.98	19.62	9753	1.261E-04	3.267E-07	10.85	11.80
5	8.78	17.26	8440	1.261E-04	3.267E-07	12.14	13.50
6	9.98	19.44	9665	1.072E-04	2.776E-07	14.27	14.50
7	9.84	19.94	9554	1.448E-04	3.751E-07	27.17	27.80
8	10.20	20.04	9741	1.269E-04	3.288E-07	12.38	11.50
9	10.20	20.04	9741	1.269E-04	3.288E-07	13.09	20.50
10	10.20	20.04	9741	1.269E-04	3.288E-07	9.33	13.50
11	8.78	17.26	8440	1.261E-04	3.267E-07	12.95	13.00
12	8.78	17.26	8440	1.261E-04	3.267E-07	12.95	10.50
13	8.78	17.26	8440	1.261E-04	3.267E-07	12.95	10.75

As shown in Table 4.1, the results obtained from the finite elements models are close to the frequencies measured in the in-situ tests. The predicted and measured frequencies in Bays 9 and 10 are different because these bays had a partition wall between the supports that stiffens the floor. Since the bays were modeled as bare surfaces, the FEM has a lower frequency. For most cases, the frequencies predicted by the finite element analyses are lower than the measured frequencies. A possible reason is that nominal values of the concrete properties were used for the analytical predictions. The modulus of elasticity of the concrete, E_c, is calculated using the nominal value of the concrete compressive strength, f'_c. Generally, the actual concrete compressive strength is higher than the nominal strength. Correspondingly, the actual concrete modulus of elasticity is higher than that calculated with the nominal properties. As a result, the actual floors should be stiffer than the finite element models, and the actual floor frequencies should be higher than those predicted by the analytical models.

4.2.3 Finite Element Models of the Laboratory Footbridge

Finite elements models for the three construction stages of the laboratory footbridge studied in Chapter 3 were created. The modeling procedure was the same as used to model the in-situ floors. The slab was modeled as an orthotropic shell with a different dynamic modulus of elasticity in each bending direction. The walls were modeled as isotropic shells. The dynamic modulus of elasticity for the walls and the slab were evaluated based on the results obtained from the experimental tests (Chapter 3). The properties of the VIBCON material used to model the composite slab and the properties of the CMU walls are:

VIBCON material:

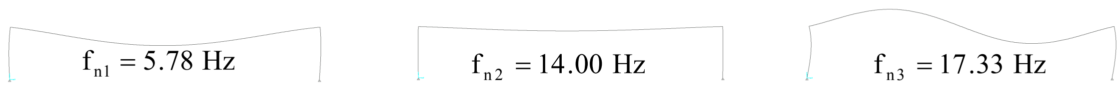
$$f'c = 1.45 \text{ ksi}$$
$$W = 1.187\times10^{-4}\ {}^{\text{lbs}}\!/_{\text{in}^3}$$
$$M = 3.075\times10^{-4}\ {}^{\text{lbs}-\text{sec}^2}\!/_{\text{in}^3\cdot\text{in.}}$$
$$E_c = 4{,}741 \text{ ksi}$$
$$E_x = 20{,}208 \text{ ksi}$$

CMU wall:

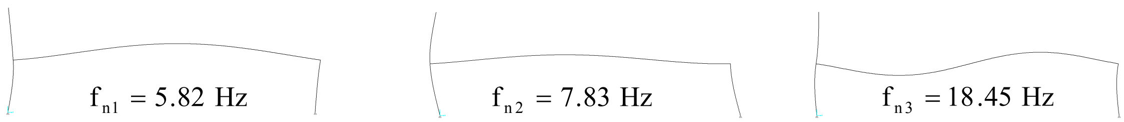
$$f'c = 3.0 \text{ ksi}$$
$$W = 6.829\times10^{-5}\ {}^{\text{lbs}}\!/_{\text{in}^3}$$
$$M = 1.767\times10^{-7}\ {}^{\text{lbs}-\text{sec}^2}\!/_{\text{in}^3\cdot\text{in.}}$$
$$E_c = 1{,}150 \text{ ksi}$$

All the degrees of freedom at the connection between the composite slab and the CMU walls were constrained to represent a rigid connection.

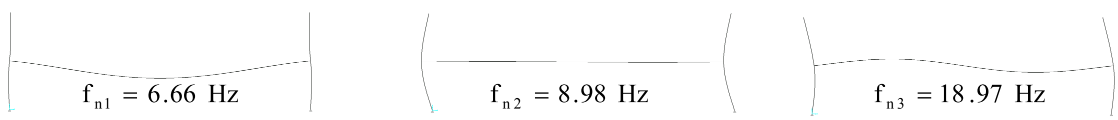
Figure 4.5 shows the mode shapes and the associated frequencies obtained from the finite element analysis for the three construction stages. Figure 4.6 shows the mode shapes and frequencies obtained from the experimental tests conducted with the impulse hammer. The mode shapes presented in Figure 4.6 were determined by measuring the structure accelerations in several points along the footbridge centerline as described in Section 3.2.2.1. As shown in the figures, except for the second vibration mode of Stage 1, the finite element models are an accurate representation of the actual structure behavior. The frequencies and mode shapes predicted by the model are close to the frequencies and mode shapes measured in the structure.



FEM Stage 1

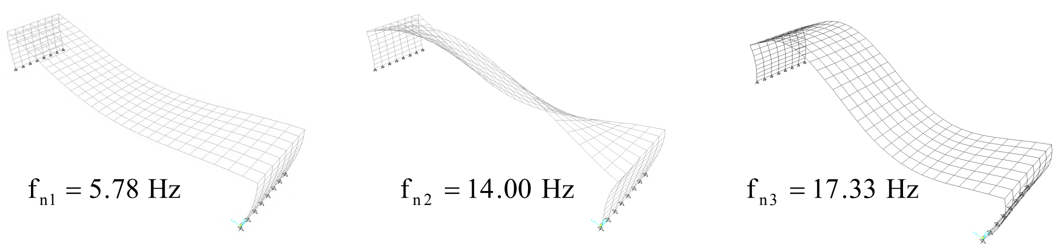


FEM Stage 2

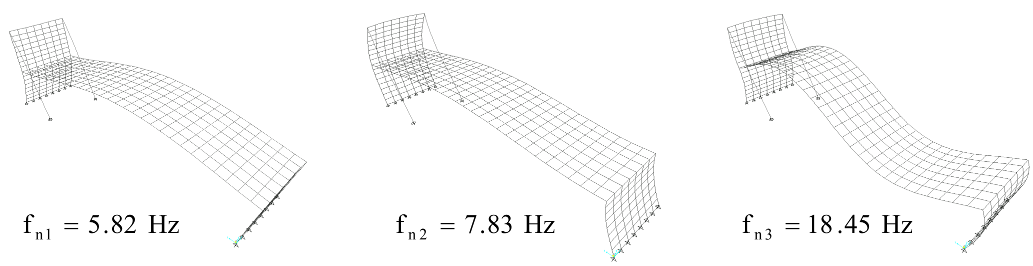


FEM Stage 3

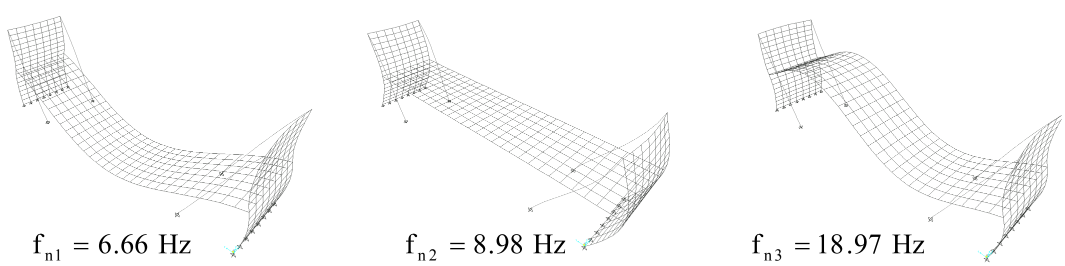
a) Two Dimensions



FEM Stage 1



FEM Stage 2



FEM Stage 3

b) Three Dimensions

Figure 4.5 Laboratory Footbridge Frequencies and Mode Shapes Determined from the FEM

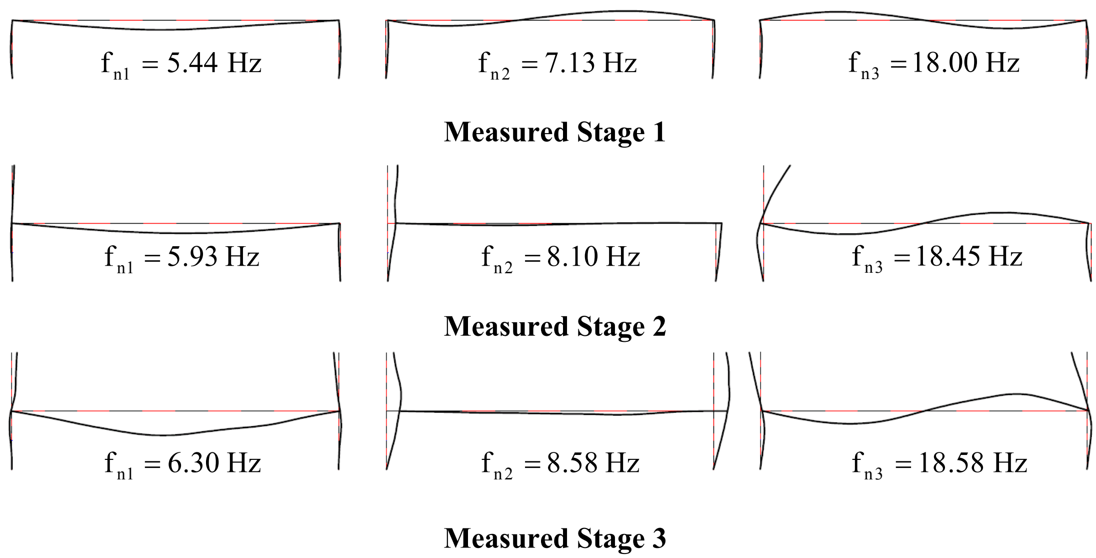
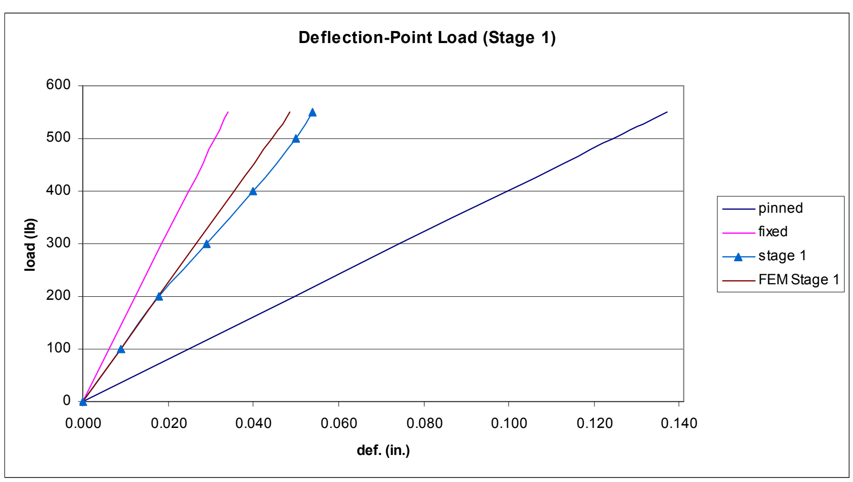


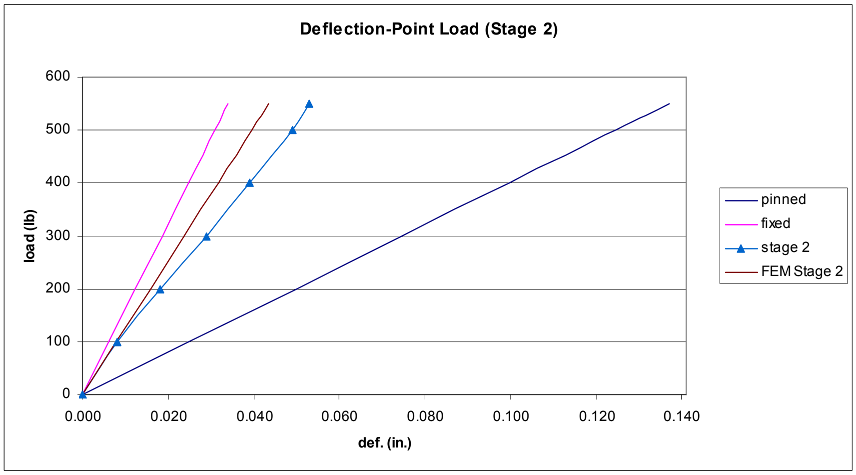
Figure 4.6 Laboratory Footbridge Frequencies and Mode Shapes Determined from EMA

Figure 4.7 shows the force-displacement plots for the static tests presented in Chapter 3, including the finite element predictions. In the plots, the leftmost and rightmost lines represent the force-displacement relationship for a beam with fixed ends and a simply supported beam, respectively. The interior response curves are the FEM prediction and the measured response. Plots are presented for the three construction stages, with the point and the distributed loads. As shown in the figures, the FEM prediction is a reasonable representation of the actual floor behavior. The response of the FEM approaches the response of the ideal beam with fully fixed ends.

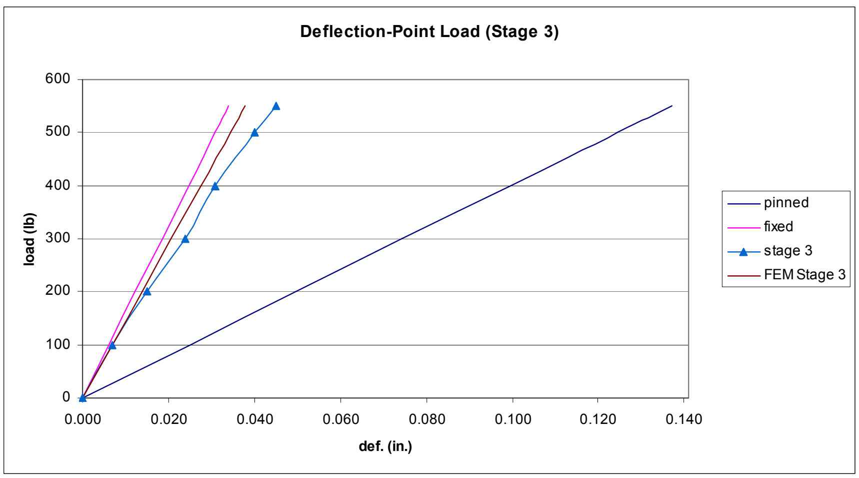


a) Point Load, Stage 1

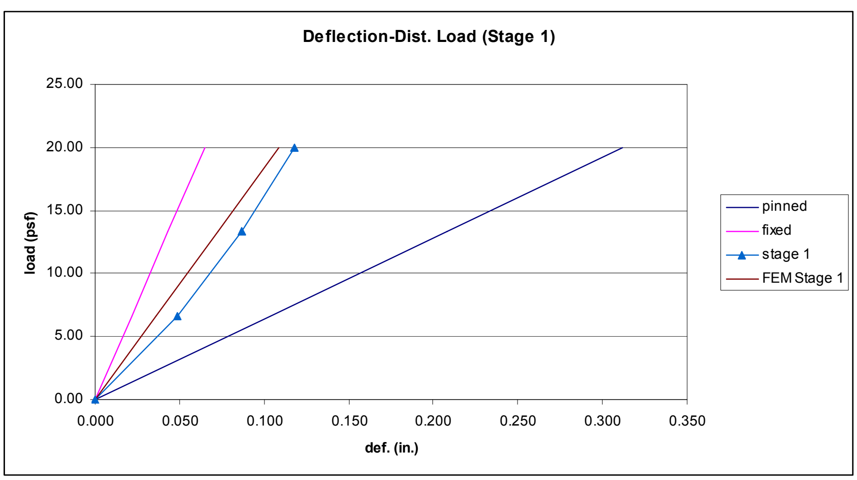
Figure 4.7 Measured and Predicted Force-Displacement Relationships for Static Tests



b) Point Load, Stage 2

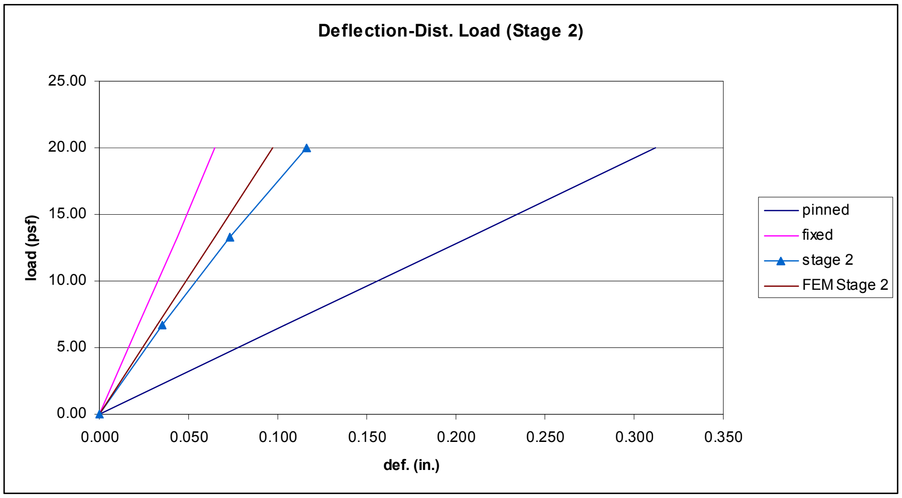


c) Point Load, Stage 3

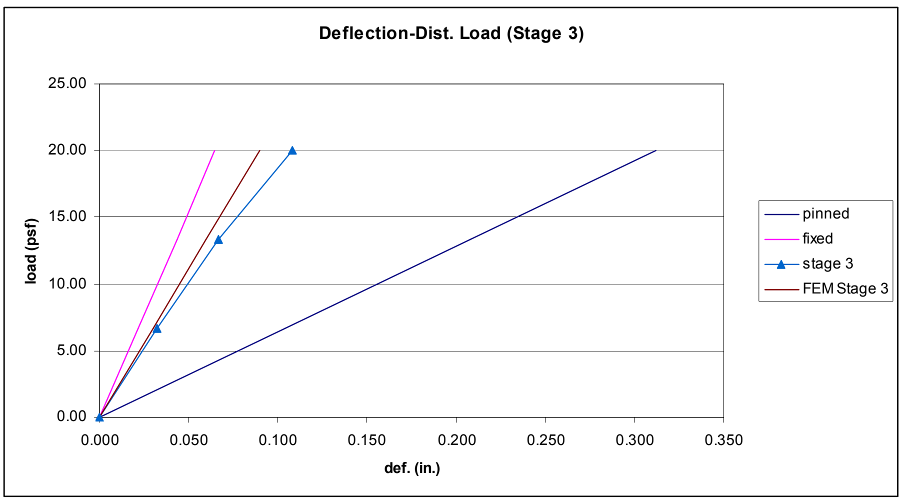


d) Distributed Load, Stage 1

Figure 4.7 Measured and Predicted Force-Displacement Relationships for Static Tests, Continued



e) Distributed Load, Stage 2



f) Distributed Load, Stage 3

Figure 4.7 Measured and Predicted Force-Displacement Relationships for Static Tests, Continued

The first vibration mode is usually the most important mode in floor vibration analysis. The fundamental natural frequencies of the footbridge determined with the impulse hammer, the shaker, and the finite element prediction are presented in Table 4.2. As shown in the table, the FEM predictions are a good approximation of the frequencies measured in the experimental tests.

Table 4.2 Comparison of Fundamental Natural Frequencies

Stage	Fundamental Natural Frequency		
	Impulse Hammer Excitation (Hz)	Shaker Excitation (Hz)	FEM (Hz)
1	5.44	5.73	5.78
2	5.93	5.89	5.82
3	6.30	6.14	6.66

4.3 AISC DG 11 PREDICTIONS, COMPARISONS AND MODIFICATIONS

4.3.1 Frequency and Acceleration Predictions for the Building Floors

Frequency and acceleration predictions for the building floors analyzed in Chapter 2 are presented in this section. The provisions of the AISC Design Guide 11 (Murray et al, 1997) are used for this purpose. The complete procedure used to compute the natural frequency and the peak acceleration for Bay 5 of the Concord and Cumberland building is presented, followed by a summary of these properties for the remaining buildings.

For a steel framed floor, the Design Guide 11 procedure computes the modal properties of beams and girders independently, and then combines them to find the vibration properties of the entire floor. To adapt this procedure for the study of long span deck floor systems, the composite slab is divided in equivalent beams. Each beam has a width equal to the pitch of the steel deck. For Bay 5, the beam width is b = 6.125 in., as shown in Figure 4.8. The number of equivalent beams is equal to the floor width divided by the beam width. For Bay 5, shown in Figure 4.8, the floor the width is 30 ft 5-1/2 in. (365.5 in.). The number of equivalent beams is 365.5 in. divided by 6.125 in. or 60.

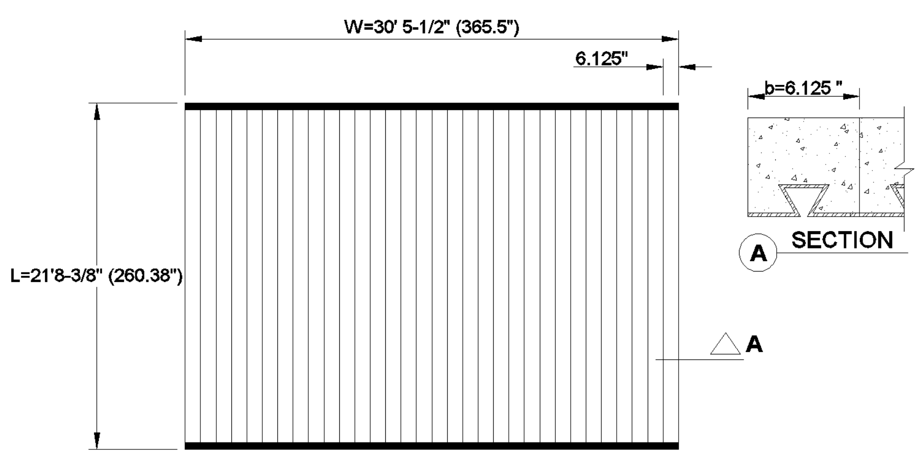


Figure 4.8 Bay 5 Plan View, Equivalent Beam Concept

In steel framed floors the beams are supported by girders. For some long span deck systems, the equivalent beams are supported by walls. The walls are assumed to be rigid girders. Thus, the vibration properties of the floor only depend on the equivalent beam modal properties.

4.3.2 Analyses of Bay 5, Concord and Cumberland

In this section, Bay 5 is analyzed according to the Design Guide 11 procedure. First, the floor natural frequency is calculated and compared to the frequency measured in the experimental tests. A modification of the Design Guide 11 method to calculate the natural frequency is then proposed. Second, the peak acceleration is computed with the Design Guide 11 provisions and compared to the measured values.

4.3.2.1 Natural Frequency

The floor natural frequency is calculated with Equation 1.4:

$$f_n = 0.18\sqrt{\frac{g}{\Delta}} \tag{1.4}$$

where,

g = acceleration of gravity

Δ = midspan deflection

$$= \frac{5 \cdot w_j \cdot L^4}{384 \cdot E \cdot I_t}$$

From Section 4.2.2, the weight of the concrete in the slab, W_c, the weight of the deck, w_d, and the transformed moment of inertia, I_t, are:

$$W_c = 46.5 \text{ kips}$$

$$w_d = 2.22 \text{ psf}$$

$$I_t = 17.26 \frac{\text{in}^4}{6\,1/8 \text{ in.}} = 17.26 \frac{\text{in}^4}{\text{eq. beam}}$$

The load due to the slab self weight, w_c, is:

$$w_c = \frac{46.5}{30.46 \cdot 21.70} = 70.35 \text{ psf}$$

$$w_j = (70.35 + 2.22)\frac{6.125}{12} = 37.04 \text{ plf}$$

The deflection and natural frequency are then:

$$\Delta_j = \frac{5 \cdot w_j \cdot L^4}{384E \cdot I} = \frac{5 \cdot 37.04 \cdot 21.70^4 \cdot 1{,}728}{384 \cdot 29{,}000 \cdot 1000 \cdot 17.26} = 0.369 \text{ in.}$$

$$f_n = 0.18\sqrt{\frac{386}{0.369}} = 5.82 \text{ Hz}$$

The frequency measured in the in-situ tests was 13.50 Hz. As shown in the calculations above, the predicted frequency with the Design Guide 11 assumption of pinned ends is 5.82 Hz, which is considerably below the measured frequency, 13.50 Hz, and the FEM predicted frequency, 12.14 Hz. If fixed ends are considered, the deflection is:

$$\Delta_j = \frac{w_j \cdot L^4}{384E \cdot I} = \frac{37.04 \cdot 21.70^4 \cdot 1{,}728}{384 \cdot 29{,}000 \cdot 1000 \cdot 17.26} = 0.0738 \text{ in.}$$

and the frequency is:

$$f_n = 0.18\sqrt{\frac{386}{0.0738}} = 13.01 \text{ Hz}$$

which is very close to the measured frequency.

4.3.2.2 Peak Acceleration

In the Design Guide 11, the peak acceleration is calculated with the following equation:

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta \cdot W} \qquad (4.3)$$

where,

$\frac{a_p}{g}$ = predicted peak acceleration

P_o = a constant force representing the excitation

f_n = fundamental natural frequency

β = modal damping ratio

W = effective weight

Effective weight: The transformed moment of inertia of the equivalent beam per unit of width, D_j, and of the concrete above the deck, D_s, per unit of width is:

$$D_j = \frac{I_t}{S} = \frac{17.26}{6.125} \cdot 12 = 33.81 \frac{in^4}{ft}$$

$$D_s = \frac{d_e^{\ 3}}{n} = \frac{(4+1)^3}{6.75} = 18.52 \frac{in^4}{ft}$$

The beam effective width, B_j, is given by:

$$B_j = C_j \left({D_s}/{D_j} \right)^{0.25} L_j = 2.0 \cdot \left({18.52}/{33.81} \right)^{0.25} 260.38 = 448 \text{ in.} > 2/3 \cdot 365.5 = 244 \text{ in.} \Rightarrow B_j = 244 \text{ in.}$$

The supported weight per unit area, w, is:

$$w = \frac{46.5}{365.5 \cdot 260.38} + \frac{2.22}{12^2 \cdot 1{,}000} = 5.04 \times 10^{-4} \frac{kips}{in^2}$$

The effective panel weight, W_j, is then:

$$W_j = w \cdot B_j \cdot L_j = 5.04 \times 10^{-4} \cdot 244 \cdot 260.38 = 32.02 \text{ kips}$$

At the time of testing, the Concord and Cumberland building was in construction and the slab was bare (see Figure 2.7). For this condition, the Design Guide 11 recommends a modal damping ratio of 0.01. The predicted peak acceleration, using the fixed end frequency is then:

$$\frac{a_p}{g} = \frac{P_o e^{-0.35 f_n}}{\beta \cdot W} = \frac{65 \cdot e^{-0.35 \cdot 13.02}}{0.01 \cdot 32{,}020} \cdot 100 = 0.21 \,\%g$$

4.3.3 Comparison of Measurements and Predictions

The procedure to predict the floor natural frequency and the peak acceleration was repeated for the rest of floors studied in Chapter 2. The predicted natural frequencies are presented in Table 4.3, and the predicted peak accelerations in Table 4.4. For comparison purposes, the measured floor natural frequencies and accelerations are also presented in Table 4.3 and 4.4, respectively. The measured values are highlighted in gray.

As shown in the tables, the natural frequency predictions are reasonable when the frequencies are calculated assuming fixed conditions. The predicted peak accelerations, however, are much smaller than the actual accelerations. Hence, it is concluded that the Design Guide 11 procedure to calculate the peak acceleration is not applicable for high frequency floors, $f_n > 10$ Hz.

Table 4.3 Predicted vs. Measured Natural Frequency

Building/Mock-up	Bay Number	n	Ir (in^4/pitch)	It (in^4/b)	Δj-pinned (in.)	Δj-fixed (in.)	Δg-pinned (in.)	fn-pinned (Hz)	fn-fixed (Hz)	Measured fn (Hz)
Hampton Inn (Norfolk, VA)	1	5.85	10.20	20.04	0.384	0.077	0.000	5.71	12.76	13.30
	2	5.85	10.20	20.04	0.384	0.077	0.000	5.71	12.76	16.80
	3	5.85	9.98	19.62	0.069	0.014	0.000	13.50	30.19	26.00
Caribe Cove (Kissimmee, FL)	4	5.85	9.98	19.62	0.447	0.089	0.025	5.15	10.45	11.80
Concord and Cumb. (Charleston, SC)	5	6.75	8.78	17.26	0.369	0.074	0.000	5.82	13.01	13.50
Royal Reef (North Caicos, BWI)	6	5.85	9.98	26.10	0.311	0.062	0.000	6.34	14.17	14.50
	7	5.85	9.84	19.94	0.056	0.011	0.000	14.99	33.51	27.80
Seybold Flats (Tampa, FL)	8	5.85	10.20	20.04	0.338	0.068	0.002	6.06	13.40	11.50
	9	5.85	10.20	20.04	0.535	0.107	0.000	4.84	10.81	20.50
	10	5.85	10.20	20.04	0.114	0.023	0.027	9.44	15.92	13.50
Regency (Sunset Beach, NC)	11	6.75	8.78	17.26	0.237	0.047	0.000	7.27	16.25	13.00
	12	6.75	8.78	17.26	0.237	0.047	0.000	7.27	16.25	10.50
	13	6.75	8.78	17.26	0.237	0.047	0.000	7.27	16.25	10.75

Table 4.4 Predicted vs. Measured Peak Acceleration

Building/Mock-up	Bay Number	Ds (in^4/ft)	Dj (in^4/ft)	Bj (in.)	Bg (in.)	Wj (kips)	Wg (kips)	W (kips)	β	ap/g (%g)	Measured Peak Acc. (%g)
Hampton Inn (Norfolk, VA)	1	21.37	39.26	162	0	22.21	0.0	22.21	0.01	0.34	1.41
	2	21.37	39.26	162	0	22.21	0.0	22.21	0.01	0.34	1.32
	3	21.37	38.44	213	0	19.00	0.0	19.00	0.01	0.00	***
Caribe Cove (Kissimmee, FL)	4	21.37	38.44	110	188	15.65	19.0	16.38	0.01	1.02	1.05
Concord and Cumb. (Charleston, SC)	5	18.52	33.82	244	0	32.02	0.0	32.02	0.01	0.21	0.60
Royal Reef (North Caicos, BWI)	6	32.50	51.13	256	0	35.65	0.0	35.65	0.01	0.13	0.84
	7	15.58	39.07	235	0	19.93	0.0	19.93	0.01	0.00	0.94
Seybold Flats (Tampa, FL)	8	21.37	39.26	141	0	18.83	0.0	18.29	0.01	0.33	1.73
	9	21.37	39.26	141	0	21.11	0.0	21.11	0.01	0.70	***
	10	21.37	39.26	234	134	23.73	25.0	24.42	0.01	0.10	0.38
Regency (Sunset Beach, NC)	11	18.52	33.82	152	0	17.87	0.0	17.87	0.01	0.12	2.57
	12	18.52	33.82	152	0	17.87	0.0	17.87	0.01	0.12	0.78
	13	18.52	33.82	152	0	17.87	0.0	17.87	0.01	0.12	0.84

4.3.4 Frequency Prediction for the Laboratory Footbridge

The laboratory footbridge natural frequency was computed following the same procedure used to calculate the predicted natural frequency of the in-situ floors. The calculations required to find the predicted frequency are the following:

From Section 4.2.2, the area of concrete per unit of width, A_{conc}, is:

$$A_{conc} = 34.5 \frac{in^4}{6\frac{1}{8}\,in.} = 67.59 \frac{in^4}{ft}$$

The weight per unit area of the Deep-Dek 4.5, gauge 16, w_d, is 5.17 psf. Then, the total supported weight per unit area, w, is:

$$w = 134 \frac{lbs}{ft^3} \times \frac{1\ ft^2}{12^2\ in^2} \times 67.59\ in^2 + 5.17 \frac{lbs}{ft^2} \times 1\ ft = 68.07 \frac{plf}{ft}$$

The transformed moment of inertia, I_t, is:

$$I_t = 46.77 \frac{in^4}{ft}$$

The deflection and the natural frequency are then:

$$\Delta = \frac{5 \cdot 68.07 \cdot 30^4 \cdot 1{,}728}{384 \cdot 29{,}000 \cdot 1{,}000 \cdot 46.77} = 0.915\ in.$$

$$f_n = 0.18\sqrt{\frac{386}{0.915}} = 3.68\ Hz$$

If fixed end conditions are assumed, the deflection and the natural frequency are:

$$\Delta = \frac{68.07 \cdot 30^4 \cdot 1{,}728}{384 \cdot 29{,}000 \cdot 1{,}000 \cdot 46.77} = 0.183\ in.$$

$$f_n = 0.18\sqrt{\frac{386}{0.183}} = 8.22\ Hz$$

The measured natural frequencies for the three construction stages of the laboratory footbridge are 5.44 Hz, 5.93 Hz, and 6.30 Hz. These frequencies correspond to experiments conducted with the impulse hammer. For the experiments conducted with the shaker, the floor frequencies are 5.73 Hz, 5.89 Hz, and 6.14 Hz. The measured frequencies are closer to the value predicted assuming fixed ends than to the frequency computed assuming pinned ends, especially for Stage 3.

4.4 SCI DESIGN GUIDE PREDICTIONS AND COMPARISONS

4.4.1 Frequency and Acceleration Predictions for the Building Floors

The predicted frequencies and accelerations for the floors tested in-situ according to the SCI Design Guide (Smith et al, 2007) are presented in this section. Complete calculations are presented for Bay 5 of the Concord and Cumberland building, followed by a summary of the frequencies and accelerations determined for the rest of tested buildings.

The SCI design guide was developed and calibrated for the analysis of steel framed floors. In this design guide, the members in the direction parallel to the steel deck ribs are defined as primary beams. The elements that support the slab in the perpendicular direction are the secondary beams. To adapt the procedure for the analysis of LSDFS, it is assumed that there are no primary beams, and that the secondary beams are the supporting walls, as shown in Figure 4.9.

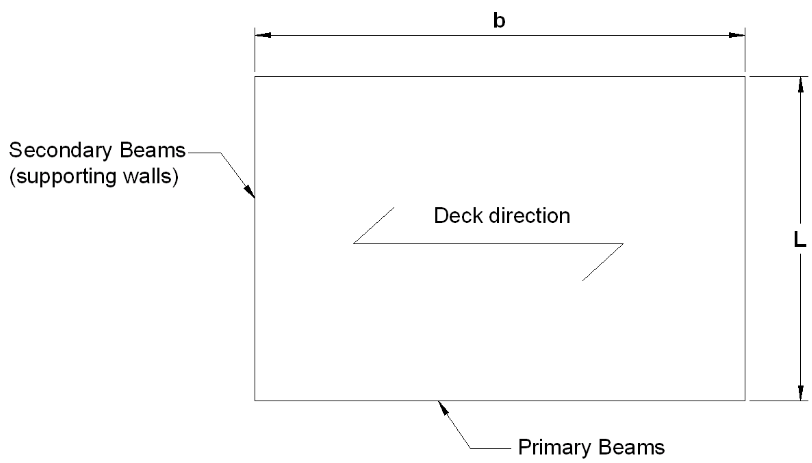


Figure 4.9 SCI Design Guide Nomenclature

For the calculation of the natural frequency, the SCI design guide gives a formula for floors without filler or intermediate secondary beams, as the shown in Figure 4.9. The floor frequency is computed with Equation 1.4 with the deflection, δ , given by:

$$\delta = \frac{w \cdot b}{384 \cdot E}\left(\frac{5L^4}{I_b} + \frac{b^3}{I_s}\right) \tag{4.4}$$

where

δ	= deflection at midspan
w	= distributed load per unit area
b	= slab length
E	= steel elastic modulus
L	= secondary beam length
I_b	= effective moment of inertia of the secondary beam
I_s	= slab moment of inertia per unit of length

Expanding Equation 4.4 results in:

$$\delta = \frac{5 \cdot w \cdot b \cdot L^4}{384 \cdot E \cdot I_b} + \frac{w \cdot b^4}{384 \cdot E \cdot I_s} \tag{4.5}$$

In the equation above, the first term is the deflection of the secondary beam assuming pinned ends, and the second term is the deflection of the slab assuming fixed ends. For LSDFS supported by walls, the first term is zero. The slab deflection is the same deflection computed with the Design Guide 11 procedure assuming fixed ends. Therefore, the floor natural frequencies predicted using the SCI design guide procedure are the same as the frequencies predicted using the modified Design Guide 11 method.

The SCI design guide provides two equations to predict the root mean square (rms) accelerations, depending on the floor type. For low frequency floors, $f_n \leq 10\ \text{Hz}$, the acceleration is given by:

$$a_{w,rms} = \mu_e \mu_r \frac{0.1 \cdot Q}{2\sqrt{2} M \zeta} W \cdot \rho \tag{4.6}$$

where

a_{rms}	= rms floor acceleration
μ_e	= mode shape factor at the point of excitation
μ_r	= mode shape factor at the point of response
Q	= weight of a person, normally taken as 746 N
M	= effective modal mass
ζ	= damping ratio

W = weighting factor for human perception of vibrations

ρ = resonance build-up factor

The mode shape factors, μ_e and μ_r, scale the floor response depending on the point where the excitation is applied and the response measured, respectively. For a LSDFS, the mode shape is approximately a half sinusoidal curve, as shown in Figure 4.10. In the figure, the excitation and the response are measured at two different locations. The mode shape excitation and response factors will be equal to 1.0 if the excitation and the response are applied and measured at midspan.

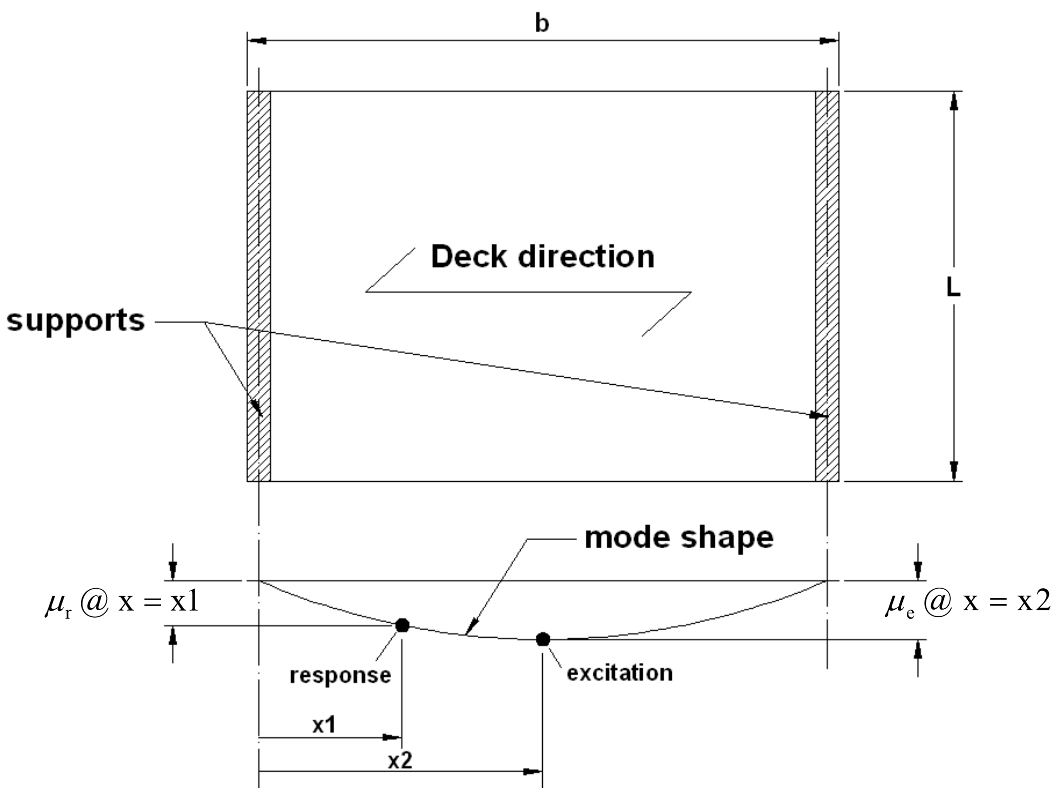


Figure 4.10 Excitation and Response Mode Shape Factors

The effective modal mass, M, is calculated with the following equation:

$$M = m \cdot S \cdot L_{eff} \tag{4.7}$$

where

m = floor mass per unit area

S = effective floor width

L_{eff} = effective floor length

The effective floor width and the effective floor length, S and L_{eff}, define the area of the floor that experiences motion due to the external excitation. For LSDFS analysis, it is assumed that the entire area of the floor is excited. Thus, S is equal to the floor width, L, and L_{eff} is equal to the floor length, b.

The weighting factor, W, scales the floor response depending on the value of the floor frequency. For a frequency range between 5.0 and 16.0 Hz, W is equal to 1.0; for frequencies outside of this range, W is less than 1.0.

The SCI design guide provides the following equations to compute the weighting factor:

$$
\begin{aligned}
&W = 0.4 && \text{for } 1\text{ Hz} < f < 2\text{ Hz} \\
&W = \frac{f}{5} && \text{for } 2\text{ Hz} \le f < 5\text{ Hz} \\
&W = 1.0 && \text{for } 5\text{ Hz} \le f \le 16\text{ Hz} \\
&W = \frac{16}{f} && \text{for } f > 16\text{ Hz}
\end{aligned}
\tag{4.8}
$$

The response build-up factor, ρ, scales the floor response based on the length of the walking path. This factor reduces the predicted response if the walking path is too short and insufficient to excite the floor and reach steady state response. The response build-up factor is given by:

$$\rho = \left(1 - e^{\frac{2\pi\varsigma L_p}{v}}\right) \tag{4.9}$$

where

ς = damping ratio

L_p = length of the walking path

v = velocity of walking

For high frequency floors, the response acceleration is given by:

$$a_{w.rms} = 2\pi\mu_e\mu_r \frac{185}{M \cdot f_n^{\,0.3}} \frac{Q}{700} \frac{1}{\sqrt{2}} W \tag{4.10}$$

with M in kg and Q taken as 746 N.

4.4.2 Example Calculations for Bay 5, Concord and Cumberland

The rms acceleration predicted with the SCI design guide provisions for Bay 5 are presented in this section and compared to the measured acceleration. The SCI predicted frequency is the same as the Design Guide 11 frequency, 13.01 Hz.

Bay 5 is a high frequency floor. Therefore, the predicted rms response acceleration is given by Equation 4.10. Assuming that the point of excitation and the point of response are both at the center of the bay, $\mu_e = \mu_r = 1$. With

$$M = \frac{47.96}{386} = 0.124\,\frac{\text{kips}-\text{sec}^2}{\text{in}} = 21{,}746\ \text{kg}$$

and since $5\ \text{Hz} \le f_n = 13.01\ \text{Hz} \le 16\ \text{Hz}$, W = 1.0

The rms acceleration is then:

$$a_{w.rms} = 2\pi \cdot 1 \cdot 1 \cdot \frac{185}{21{,}746 \cdot 13.01^{0.3}}\frac{746}{700}\frac{1}{\sqrt{2}}1.0 = 0.0187\,\frac{\text{m}}{\text{s}^2} = 0.19\ \%\text{g}$$

The rms response accelerations for the other bays were calculated using the above procedure. Table 4.5 summarizes de calculations. The measured values are highlighted in gray. As shown in the table, the rms accelerations predicted with the SCI design guide are reasonably comparable to the measured values.

Table 4.5 Predicted rms Response Acceleration According to the SCI Design Guide

Building/Mock-up	Bay Number	M (kg)	W	$a_{w,rms}$ (%g)	Measured $a_{w,rms}$ (%g)
Hampton Inn (Norfolk, VA)	1	15232	1.00	0.27	0.37
	2	15232	1.00	0.27	0.37
	3	12948	0.53	0.13	***
Caribe Cove (Kissimmee, FL)	4	10667	1.00	0.40	0.35
Concord and Cumberland (Charleston, SC)	5	21746	1.00	0.19	0.23
Royal Reef (North Caicos, BWI)	6	24522	1.00	0.16	0.31
	7	13645	0.48	0.11	0.23
Seybold Flats (Tampa, FL)	8	12914	1.00	0.32	0.41
	9	14479	1.00	0.30	***
	10	16279	1.00	0.24	0.12
Regency (Sunset Beach, NC)	11	12179	1.00	0.32	0.68
	12	12179	1.00	0.32	0.23
	13	12179	1.00	0.32	0.21

CHAPTER V
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 RESEARCH SUMMARY

The purpose of this research was to address the vibration properties of long span deck floor systems. Experimental and analytical studies were conducted to assess the dynamic characteristics of this type of floor system. First, tests in thirteen in-situ bays were carried out to determine the natural frequency and the walking acceleration response of the floors. The results of the experiments demonstrated that the vibration properties of these floors are different to those of conventional beam and girder steel framed floors. The measurements show that the natural frequency of a LSDFS supported by CMU walls, steel stud walls, concrete beams, and masonry walls is considerably higher than the natural frequency of conventional steel framed floors.

As an attempt to understand the behavior of LSDFS supported by walls, a laboratory footbridge was constructed and tested with three different support conditions. The purpose of the experiment was to determine the influence of the support condition change in the modal properties of the structure. Static tests with point and distributed loadings were conducted to measure the midspan deflection of the floor. The measured deflections were compared to the theoretical deflections of a simply supported beam and a fixed-end beam, to determine the degree of fixity present in the slab-CMU wall connection. Dynamic tests were also conducted to measure the floor natural frequency and to determine the vibration mode shapes for the three construction stages.

Finally, the floors tested in-situ and the laboratory footbridge were studied using three analytical procedures. The program SAP2000 was used to create finite element models of the floors. The floors were also evaluated with the provisions given in the AISC Design Guide 11 *Floor Vibrations due to Human Activity* and the SCI publication *Design of Floors for Vibration: A New Approach.* The frequencies predicted by the finite element models and the two design guides were compared to the results obtained from the in-situ tests and from the laboratory footbridge, to determine their accuracy. The accelerations measured in the experimental test were also compared to those predicted by the design guides.

5.2 CONCLUSIONS

The results of the study show that the natural frequency of long span deck floor systems supported on walls is best predicted when fixed-end conditions are assumed. The tests conducted in the laboratory footbridge with the static and dynamic loadings support this conclusion. The floor natural frequencies obtained from finite element analyses, modeling the supports as fully restrained, are also close to the measured frequencies.

A modified version of the Design Guide 11 procedure to compute the floor natural frequency provides results that are consistent with the measured frequencies. If the deflection used in the frequency calculations is computed assuming fixed ends rather than pinned ends, the resulting natural frequencies are close to the measured values.

In the SCI design guide, if the deflection equation used to calculate the floor frequency, is modified to account for wall supports, a fixed-fixed condition results. The frequencies obtained with the SCI guide method assuming rigid supports are the same as those obtained using the modified Design Guide 11 procedure.

The frequencies measured in the in-situ floors are above 10 Hz. Therefore, the basic procedure in Design Guide 11 did not accurately predict walking accelerations because this procedure was specifically calibrated for low frequency floors. The predicted peak accelerations are much lower than the accelerations measured in the tests. The Design Guide 11 procedure to calculate the modal mass of a LSDFS needs to be modified to obtain more accurate peak acceleration predictions. The rms accelerations predicted using the SCI design guide reasonably match the measured rms accelerations if the modal mass is taken as the entire mass of the bay.

5.3 RECOMMENDATIONS

It is recommended that the natural frequency of LSDFS be determined assuming fixed support conditions. It is also recommended that when calculating the rms acceleration response with the SCI design guide procedure for high frequency floors, the modal mass be taken equal to the mass of the entire bay.

5.4 RECOMMENDATIONS FOR FUTURE RESEARCH

The analyses of the building floors presented in this study were limited to the measurement of the natural frequency and the acceleration at one location. It is recommended that experimental modal analyses be done to determine mode shapes and several natural frequencies on building floors. This will permit increase the data base of tested LSDFS and also provide data to improve and calibrate the analysis using finite element models. The results of experimental tests also might help to address the modal mass question.

The experiments and results presented in this study were limited to the analysis of LSDFS supported on walls. It is recommended to conduct research of LSDFS supported by steel beams and girders to study the modal characteristics of this type of floor system.

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