NONLINEAR ANALYSIS OF PLANE FRAMES

by

Joel Clinton Bradshaw, III

Thesis submitted to the Graduate Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Civil Engineering

APPROVED:

Siegfried Holzer

S. M. Holzer, Chairman

R. D. Walker

R. D. Walker, Department Head

Arnold E. Somers, Jr.

A. E. Somers

May, 1975
Blacksburg, Virginia

ACKNOWLEDGEMENTS

My appreciation is extended to Dr. Siegfried M. Holzer for his suggestions throughout the work and for his editing of the report. I also acknowledge Dr. A. E. Somers, Dr. M. A. Plamondon, and R. G. Galloway for their suggestions and comments on writing the manuscript. The efforts of Dr. Richard D. Walker, which resulted in extending the time to complete this thesis, are greatly appreciated. Special thanks are for Kamol Wongkoltoot who gave me encouragement and ideas during my work on the computer programs which are significant to the thesis. Technical support was given by the Air Force Weapons Laboratory at Kirtland Air Force Base, New Mexico.

TABLE OF CONTENTS

LIST OF FIGURES

Figure Number		Page

LIST OF TABLES

CHAPTER I

INTRODUCTION

Structural elements subjected to axial and flexural loads must be treated as beam-column elements. When axial and transverse loads are acting simultaneously and large displacements occur, a proper analysis must consider the change in magnitude and direction of the applied forces caused by the interaction between axial and transverse components of displacement. The analysis can no longer consider the external forces as acting independently of each other. Several studies have been made investigating the effects of large displacements in beam-columns [2,5,6,7,9,13,15].

The purpose of this investigation is to test a finite element model of a beam-column with geometrical and material nonlinearities. The geometrical nonlinearities result from inclusion of a nonlinear strain-displacement relation and from the formulation of the equilibrium conditions based upon the deformed geometry. Material nonlinearities are considered to result from nonlinearly elastic stress-strain relationships.

The reliability of the proposed finite element model is investigated by addressing three classes of problems. 1. The first investigation considers the totally linear element which is compared with other exact solutions. 2. The model is used to investigate problems consisting of geometrical nonlinearities, and solutions are compared with analyses in the literature that involve both analytical and numerical solutions. 3. The ability of the model to address material nonlinearity is presented to show the capability of the model in analysing more complex systems which have nonlinearly elastic material properties.

The investigation is made in the following manner. First, the mathematical models are presented in two forms: 1. the finite element model of a beam-column, and 2. the system model which is made up of the discrete element models. The analysis process describes how the mathematical models are used to analyse a structural plane frame. An algorithm is developed that is incorporated into computer programs to test the reliability of the finite element model. Several demonstration problems are solved with the computer programs to demonstrate the capability of the beam-column model.

The proposed finite element model of a beam-column may be used in algorithms for the analysis of structural systems that may undergo large displacements and that may become inelastic. Since the element reference axis is not restricted to any position within the element, the model can be used for reinforced concrete structures where the material properties produce a variation in the location of the neutral axis. The model is ideal for the energy minimization method of analysis since it is a function only of the generalized coordinates and the element dimensions. The model is currently used in the SINGER computer program which is described in the report by Holzer, et al. [8].

CHAPTER II

MATHEMATICAL MODELS

A plane frame is represented by an assemblage of one-dimensional finite elements rigidly connected at the external nodes. The nodal displacements form the unknowns in the analysis process. The state of the discrete mathematical model of the structure is described by the total potential energy.

Finite Element Model

The finite element model is based on the fundamental assumptions of the classical beam-column: Plane sections remain plane and normal to the deformed reference axis. Normal strains and rotations are infinitesimally small. The constitutive law at any point of the element is defined by the constitutive law of the material. The effect of shear deformations is negligible.

The state of each element is described by its strain energy, which is defined by the distortion components $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$, $\bar{u}_4$ of the element; the components $\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ represent the relative external nodal displacements, and the component $\bar{u}_4$ represents the internal nodal displacement (Figure 2). The modeling process of the finite element is illustrated in Figure 1: u and v define the deformed state of the reference axis (Figure 2); $\bar{u}$ is the four dimensional element distortion vector; x and y are the element coordinates (Figure 2); ε and σ denote

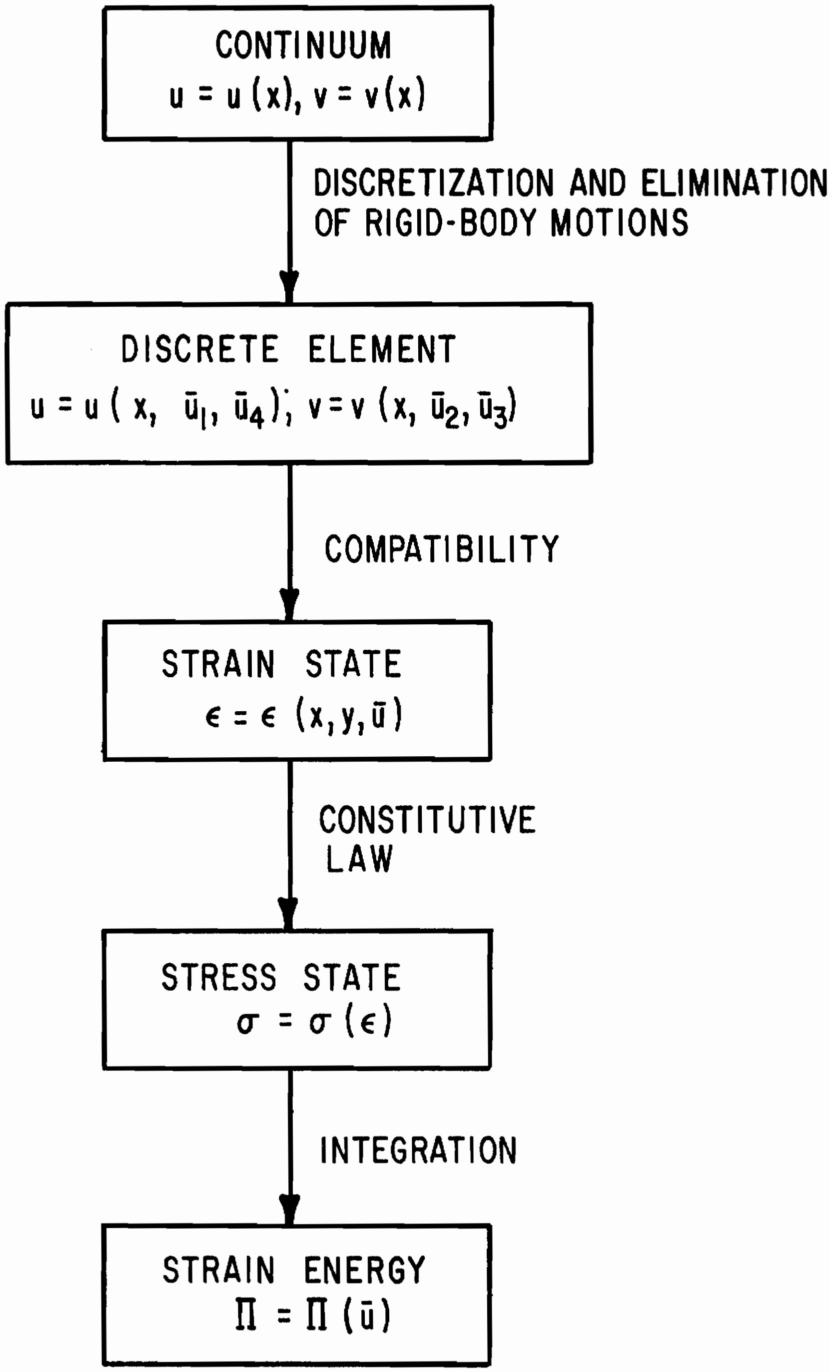


FIG. 1 FINITE ELEMENT MODELING PROCESS

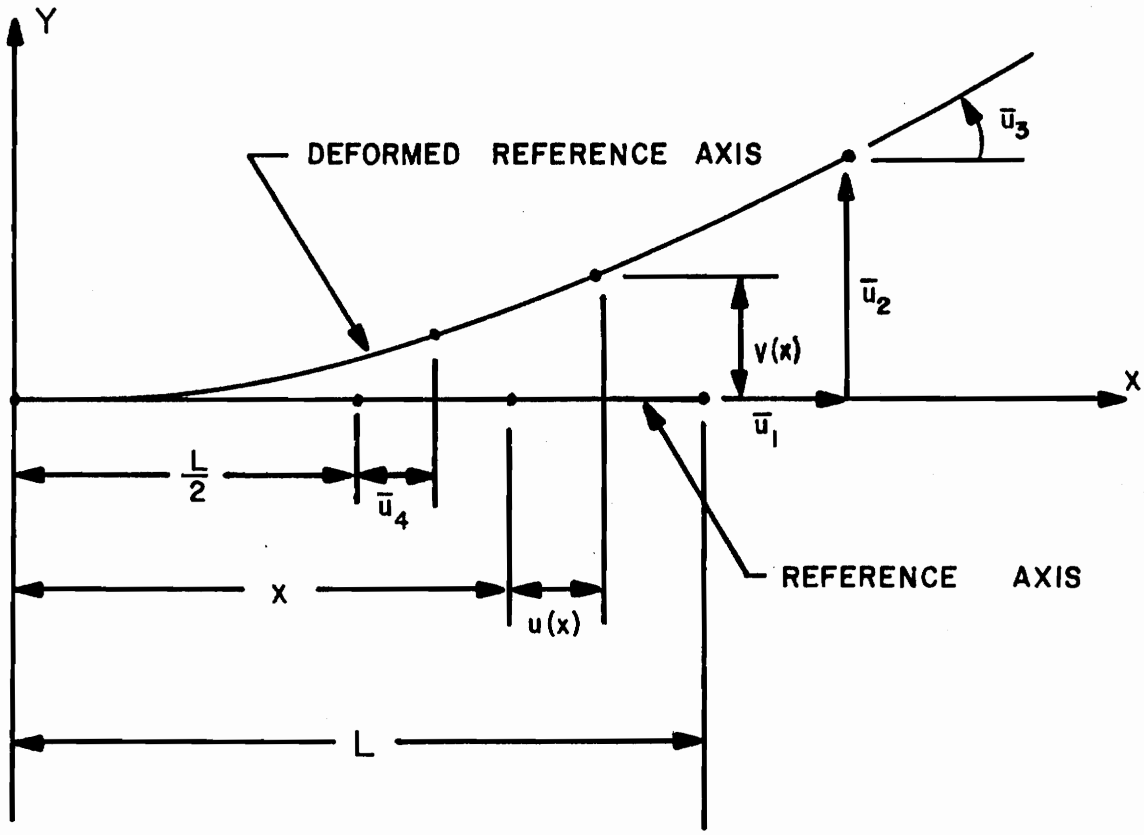


FIG. 2 ELEMENT REFERENCE AXIS

the strain and stress at a point of the element; and Π represents the strain energy of the element.

A special feature of the finite element model is that the reference axis can be located arbitrarily in the longitudinal plane of symmetry of the element (Figure 3). This permits modeling of assemblages composed of elements with various depths without causing eccentricities at the joints (Figure 4). Moreover, it allows modeling of beam-columns with linearly varying neutral axes. The only restriction on the location of the reference axis is that it must be parallel to a longitudinal edge of the beam-column element.

The configuration of the deformed reference axis is defined as

$$u(\xi) = \phi_1(\xi)\bar{u}_1 + \phi_4(\xi)\bar{u}_4 \tag{1}$$

$$v(\xi) = \phi_2(\xi)\bar{u}_2 + \phi_3(\xi)\bar{u}_3 \tag{2}$$

where

$$\phi_1(\xi) = 2\xi^2 - \xi \tag{3}$$

$$\phi_2(\xi) = -2\xi^3 + 3\xi^2 \tag{4}$$

$$\phi_3(\xi) = L(\xi^3 - \xi^2) \tag{5}$$

$$\phi_4(\xi) = 4(-\xi^2 + \xi) \tag{6}$$

and

$$\xi = x/L \ . \tag{7}$$

Linear variation in the normal strain along the reference axis is accomplished via the internal distortion component $\bar{u}_4$ [8].

The deformations at a point are defined by the strain-displacement

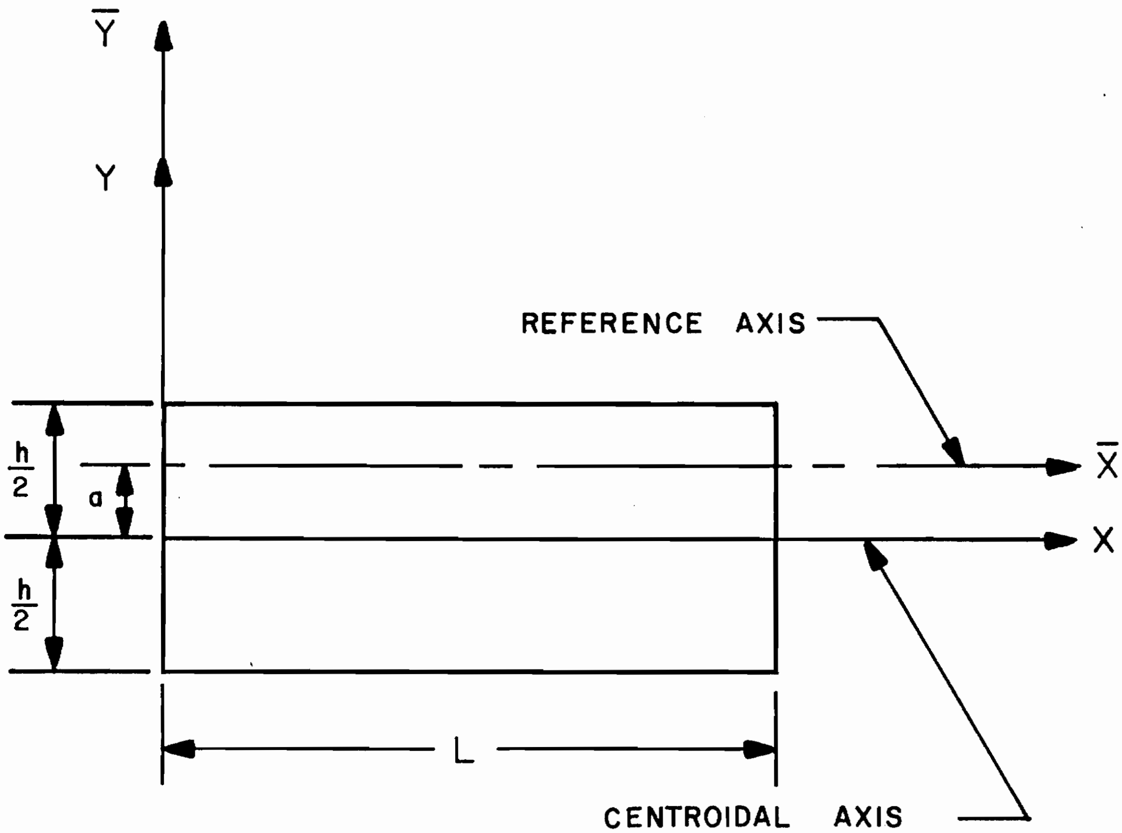


FIG. 3 TRANSLATED REFERENCE AXIS

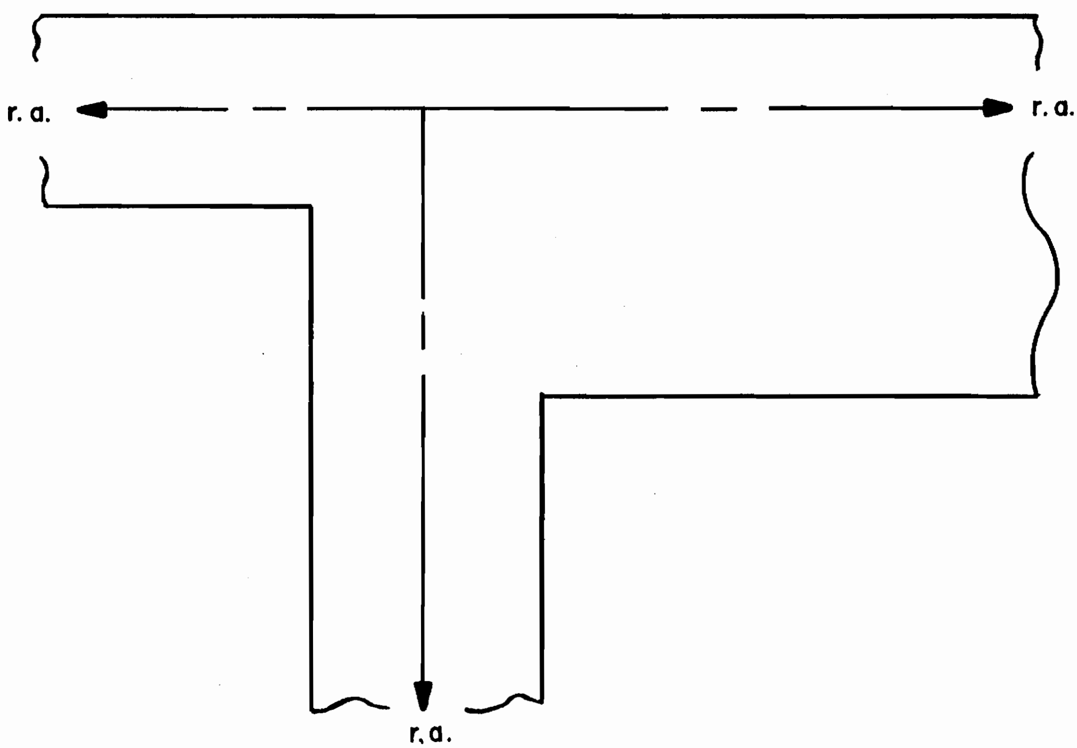


FIG. 4 ELEMENTS OF VARIOUS DEPTHS

relation

$$\varepsilon(x,y) = \frac{du}{dx} + \frac{1}{2}\left(\frac{dv}{dx}\right)^2 - y\,\frac{d^2v}{dx^2} \tag{8}$$

where $\varepsilon(x,y)$ is the normal strain at any point (x,y). The first term on the right-hand side of equation 8 represents the normal strain caused by axial deformation of the reference axis, the second term accounts for the coupling of axial and flexural distortions, and the last term represents the elementary bending strain.

Equation 8 is valid only if the strains and rotations are small compared to unity. This limitation is not very restrictive since the strains of most structural materials are infinitesimal even at fracture and the rotations can be kept small through the subdivision of the element.

With the aid of equations 1 and 2, equation 8 can be expressed in discretized form

$$\varepsilon(\xi,\eta) = \phi_1' \frac{\bar{u}_1}{L} + \phi_4' \frac{\bar{u}_4}{L} + \frac{1}{2}\left(\phi_2' \frac{\bar{u}_2}{L} + \frac{\phi_3'}{L}\bar{u}_3\right)^2$$

$$- \eta\left(\phi_2'' \frac{\bar{u}_2}{L} + \frac{\phi_3''}{L}\bar{u}_3\right) \tag{9}$$

where

$$\phi_1' = 4\xi - 1 \tag{10}$$

$$\phi_4' = 4(-2\xi + 1) \tag{11}$$

$$\phi_2' = 6(-\xi^2 + \xi) \tag{12}$$

$$\phi_3' = L(3\xi^2 - 2\xi) \tag{13}$$

$$\phi_2'' = 6(-2\xi + 1) \tag{14}$$

$$\phi_3'' = L(6\xi - 2) \tag{15}$$

and

$$\eta = y/L \ . \tag{16}$$

The strain energy is defined by the volume integral

$$\Pi = \int_V \Pi^* \, dV \tag{17}$$

where the strain-energy density is

$$\Pi^* = \int_0^{\varepsilon} \sigma \, d\varepsilon \tag{18}$$

For linearly elastic materials (Figure 5a), equation 18 assumes the form

$$\Pi^* = \frac{E\varepsilon^2}{2} \tag{19}$$

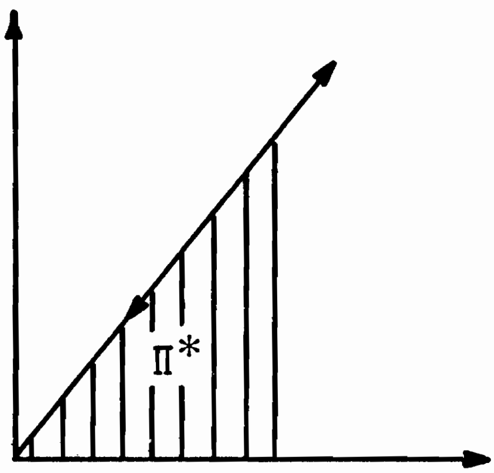
where E is the modulus of elasticity. On the basis of equations 9 - 15, the strain energy can be expressed in terms of the element distortion components

$$\Pi = \frac{AE}{6L}(7\bar{u}_1^2 - 16\,\bar{u}_1\bar{u}_4 + 16\bar{u}_4^2) + \frac{aAE}{L^2}(-4\bar{u}_1\bar{u}_2 + 3L\bar{u}_1\bar{u}_3$$

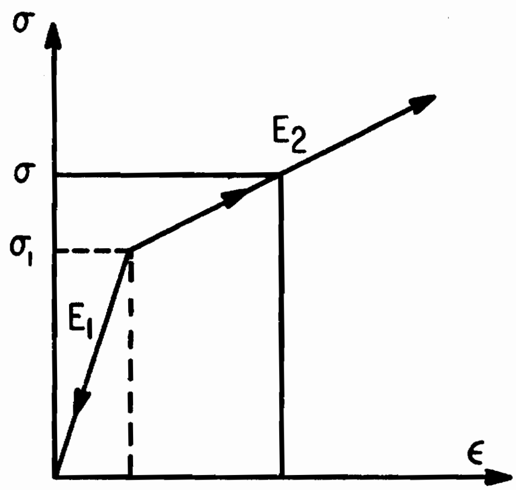
$$+ 8\bar{u}_4\bar{u}_2 - 4L\bar{u}_4\bar{u}_3) + \frac{IE}{2L^3}(12\bar{u}_2^2 - 12L\bar{u}_2\bar{u}_3 + 4L^2\bar{u}_3^2)$$

$$+ \frac{AE}{30L^2}(18\bar{u}_1\bar{u}_2^2 + 3L\bar{u}_1\bar{u}_2\bar{u}_3 + 4L^2\bar{u}_1\bar{u}_3^2 - 12L\bar{u}_4\bar{u}_2\bar{u}_3 - 4L^2\bar{u}_4\bar{u}_3^2)$$

$$+ \frac{aAE}{6}(\bar{u}_3^3) + \frac{AE}{280L^3}(L^3\bar{u}_2\bar{u}_3^3 + 18L^2\bar{u}_3^2\bar{u}_2^2 - 36L\bar{u}_3\bar{u}_2^3$$



(a) LINEARLY ELASTIC MATERIAL



(b) NON-LINEARLY ELASTIC MATERIAL

FIG. 5 STRAIN ENERGY DENSITY

$$+ 72\bar{u}_2^4 + 2L^4\bar{u}_3^4) \qquad (20)$$

where L is the length of the element, I is the gross moment of inertia, and A is the cross-sectional area of the element.

For nonlinearly elastic material, which is described by a bilinear stress-strain relation (Figure 5b), the strain energy must be evaluated numerically. The particular numerical integration scheme used is the Gauss-Legendre quadrature method which establishes integration points at unequal intervals to best approximate the integral. The method and integral tables are presented by Scheid [12]. This particular quadrature takes the form

$$\int_y \int_x f(x,y)\ dxdy = \sum_{j=1}^{m} \sum_{i=1}^{n} A_i B_j f(x_i, y_j) \qquad (21)$$

where A_i and B_j are the weighting coefficients and n and m are the number of Gauss points (integration points) in the x and y directions, respectively. The Gauss points are defined by the coordinates x_i, y_j which can be found in Gauss-Legendre integral tables [12]. A 2-point by 4-point quadrature example is illustrated in Figure 6. Equation 21 represents the general form of energy computation. It is assumed that the strain-energy density does not vary across the width of the element.

The algorithm that determines the strain energy is based on equation 21. The strain, ε, is determined from equation 9 for prescribed element distortion components at each Gauss point of the element. On the basis of equation 18 and Figure 5b, the strain-energy density is determined as follows: If $0 \leq \varepsilon \leq \varepsilon_1$,

$$\Pi^* = \frac{1}{2} E_1 \varepsilon^2 \qquad (22)$$

FIG. 6 GAUSS POINTS FOR NUMERICAL INTEGRATION

and if $\varepsilon > \varepsilon_1$,

$$\Pi^* = \frac{1}{2} E_1 \varepsilon_1^2 + E_1 \varepsilon_1 (\varepsilon - \varepsilon_1) + \frac{1}{2} E_2 (\varepsilon - \varepsilon_1)^2 \tag{23}$$

where ε_1, ε_2, E_1, and E_2 are determined from the predefined stress-strain curve (Figure 5b). Presentation of equation 17 in the form of equation 21 yields the strain energy of the element

$$\Pi = b \sum_{j=1}^{m} \sum_{i=1}^{n} A_i B_j \Pi^*(x_i, y_j) \ . \tag{24}$$

System Model

The state of the system model, the mathematical model of the structure, is described by the total potential energy

$$V = \Pi + \Omega \tag{25}$$

which is defined in terms of the generalized coordinates, q_i, and the generalized external forces, Q_i. The generalized coordinates consist of the displacements of the nodes at which the elements are interconnected and the internal nodal displacements of the elements. The potential energy of the external forces is

$$\Omega = - \sum_{i=1}^{n} Q_i q_i \tag{26}$$

where n denotes the number of generalized coordinates. The strain energy of the system is defined by

$$\Pi = \sum_{i=1}^{m} \Pi_i \tag{27}$$

where Π_i is the strain energy stored in element i, and m represents the number of elements comprising the assemblage. Π_i is defined in terms of the element distortion components, which are determined by the following relations [8]:

$$\bar{u}_1 = \bar{c}\ \Delta X_1^* + \bar{s}\ \Delta X_2^* - b_1 \tag{28}$$

$$\bar{u}_2 = -\bar{s}\ \Delta X_1^* + \bar{c}\ \Delta X_2^* - b_2 \tag{29}$$

$$\bar{u}_3 = U_{j3} - U_{i3} \tag{30}$$

$\bar{u}_1$, $\bar{u}_2$, $\bar{u}_3$ represent the relative end-displacements of an element going from node i to node j (Figure 7). The internal nodal displacement of the element, $\bar{u}_4$, is specified directly in the analysis process. In equations 28 - 30,

$$\bar{c} = \cos\gamma \tag{31}$$

$$\bar{s} = \sin\gamma \tag{32}$$

and

$$\gamma = \alpha + U_{i3} \tag{33}$$

where U_{i3} is the rotation of node i, and the angle α locates the initial axis of the element relative to the global 1-axis (Figure 7);

$$\Delta X_k^* = \Delta X_k + \Delta U_k\ , \quad k=1,2 \tag{34}$$

where ΔX_k is the projection of the initial element axis onto the global k-axis (Figure 7), and ΔU_k is the relative nodal deflection in the direction of the global k-axis, i. e.,

$$\Delta U_k = U_{jk} - U_{ik} \tag{35}$$

$$b_1 = c\Delta X_1 + s\Delta X_2 \tag{36}$$

$$b_2 = -s\Delta X_1 + c\Delta X_2 \tag{37}$$

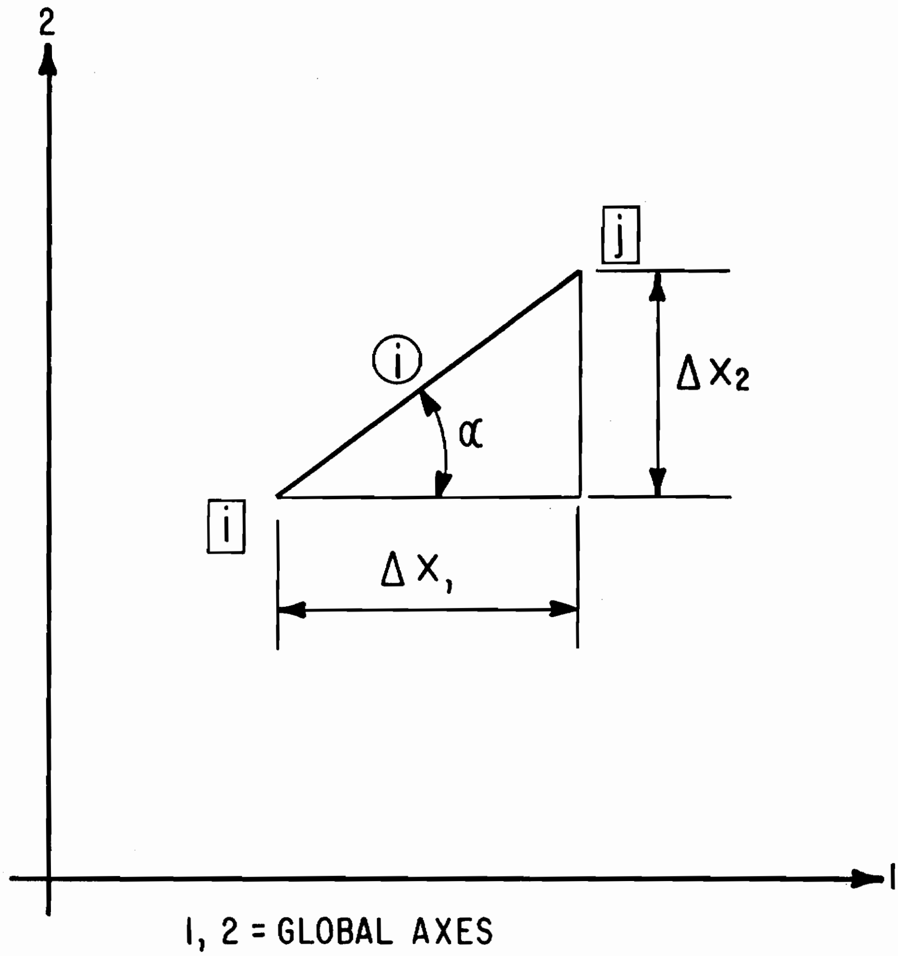


FIG. 7 ELEMENT GEOMETRY

where

$$c = \cos \alpha \tag{38}$$

$$s = \sin \alpha \tag{39}$$

and finally, U_{j3} is the rotation of node j.

There is no intrinsic limitation on the magnitude of the generalized coordinates. However, the relative displacements of nodes linked by elements must meet the small deformation requirements of the element.

CHAPTER III

ANALYSIS PROCESS

The objective in the analysis is to obtain the generalized coordinates of the system model corresponding to prescribed nodal forces, the generalized external forces. This is accomplished via minimization of the total potential energy of the system. Since the total potential energy assumes a relative minimum at a stable equilibrium state [10], the solution process converges to the stable equilibrium state corresponding to the prescribed nodal forces. The Davidon-Fletcher-Powell minimization algorithm [3,4,6] is used in this analysis.

In principle, the solution process is not affected by the geometric nonlinearities of the system model; the strain energy of the system is a nonlinear function of the generalized coordinates in any case. However, proofs for the convergence of minimization algorithms are restricted to quadratic object functions, which correspond to the potential functions of linear system models. For nonlinear models, the total potential energy approaches the quadratic form only in the vicinity of the desired equilibrium state (consider a Taylor series expansion). Consequently, the solution process for nonlinear models is susceptible to numerical instability; i. e., it can converge to nonstationary configurations. To minimize this possibility, the search for the desired equilibrium state should be initiated in its vicinity. This can be accomplished by seeking the solution to a prescribed load level via a series of sufficiently small load increments. Accordingly, the equilibrium states corresponding to the beginning and end of a load increment can be made arbitrarily close.

The Algorithm

The computer programs for the analysis of linearly elastic and nonlinearly elastic systems are presented in Appendices A and B, respectively. The program in Appendix A has the strain energy of each element and the corresponding gradients expressed in explicit form. Consequently, only linearly elastic systems can be analysed with this program. The program in Appendix B computes the strain energy of each element and the corresponding gradients via the Gaussian quadrature method and is based on nonlinearly elastic materials defined in Figure 5b.

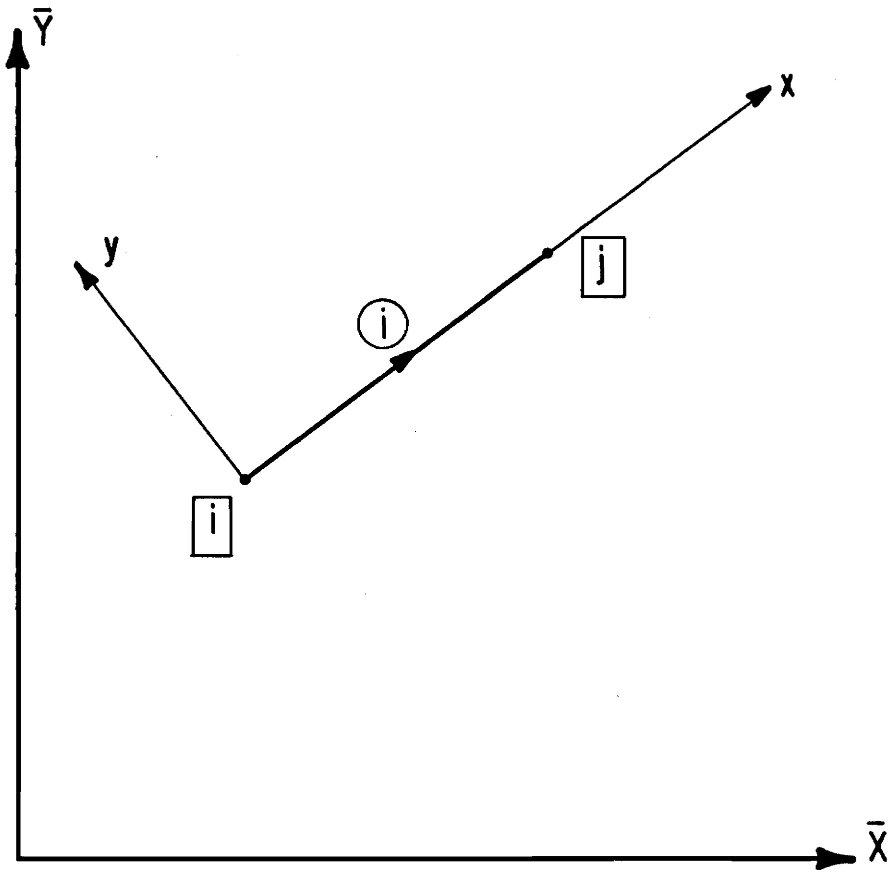
The following algorithm forms the basis of the computer program in Appendix A:

1. Specify the global coordinates of the nodes at which the elements are interconnected, and give the status of each joint displacement (free or constrained).

2. Specify the nodes at which each element is incident and the origin of the element (e. g., element i in Figure 8 goes from node i to node j).

3. Input the element properties with reference to the local element coordinate system.

4. Prescribe the nodal forces relative to the global frame of reference.



$\bar{X}$, $\bar{Y}$ = GLOBAL COORDINATES

x, y = LOCAL COORDINATES

FIG. 8 FRAMES OF REFERENCE

5. Begin the function minimization process for each loading configuration by making an initial guess on the generalized coordinates of the system.

6. The following steps must be performed for each iteration of the minimization process.

a. Determine the potential energy of the external forces on the basis of equation 26.

b. Compute the element distortion components on the basis of equations 28 - 30.

c. Calculate the strain energy of each element on the basis of equation 20.

d. Determine the strain energy of the system on the basis of equation 27.

e. Compute the gradient components of the element strain energy relative to the element distortion components:

$$\frac{\partial \Pi_i}{\partial \bar{u}_k}, \qquad k = 1,2,3,4 \tag{40}$$

f. Calculate the partial derivatives of the element distortion components with respect to the generalized coordinates (see Table 1).

g. Combine the results of steps e and f by the chain rule to determine the contribution of element i to the j-th gradient component of the total potential energy:

$$\frac{\partial \Pi_i}{\partial q_j} = \sum_{k=1}^{4} \frac{\partial \Pi_i}{\partial \bar{u}_k} \frac{\partial \bar{u}_k}{\partial q_j} \tag{41}$$

TABLE 1. - PARTIAL DERIVATIVES OF ELEMENT DEFORMATION COMPONENTS WITH RESPECT TO GLOBAL DISPLACEMENTS

	$\frac{\partial \bar{u}_1}{\partial(\ )}$	$\frac{\partial \bar{u}_2}{\partial(\ )}$	$\frac{\partial \bar{u}_3}{\partial(\ )}$	$\frac{\partial \bar{u}_4}{\partial(\ )}$
U_{i1}	$-\bar{c}$	$+\bar{s}$	0	0
U_{i2}	$-\bar{s}$	$-\bar{c}$	0	0
U_{i3}	$-\bar{s}\ \Delta X_1^* + \bar{c}\ \Delta X_2^*$	$-\bar{c}\ \Delta X_1^* - \bar{s}\ \Delta X_2^*$	-1	0
U_{j1}	$+\bar{c}$	$-\bar{s}$	0	0
U_{j2}	$+\bar{s}$	$+\bar{c}$	0	0
U_{j3}	0	0	$+1$	0
$\bar{u}_4$	0	0	0	$+1$

$\bar{c} = \cos(\alpha + U_{i3})$, $\bar{s} = \sin\ (\alpha + U_{i3})$

Compute the contribution of the strain energy of the system to the j-th component of the gradient of the total potential energy:

$$\frac{\partial \Pi}{\partial q_j} = \sum_{i=1}^{m} \frac{\partial \Pi_i}{\partial q_j} \tag{42}$$

h. Compute the total potential energy of the system on the basis of equation 25.

i. Compute the components of the gradient of the total potential energy:

$$\frac{\partial V}{\partial q_j} = \frac{\partial \Pi}{\partial q_j} - Q_j \ , \quad j = 1,2,. \ . \ .,n \tag{43}$$

7. Step 6 is repeated until a relative minimum of the total potential energy has been attained.

This algorithm is presented in flowchart form in Figure 9. Step 6 of the algorithm is modified for the computer program in Appendix B in that the computation of the element strain energies and the corresponding gradient components is conducted numerically.

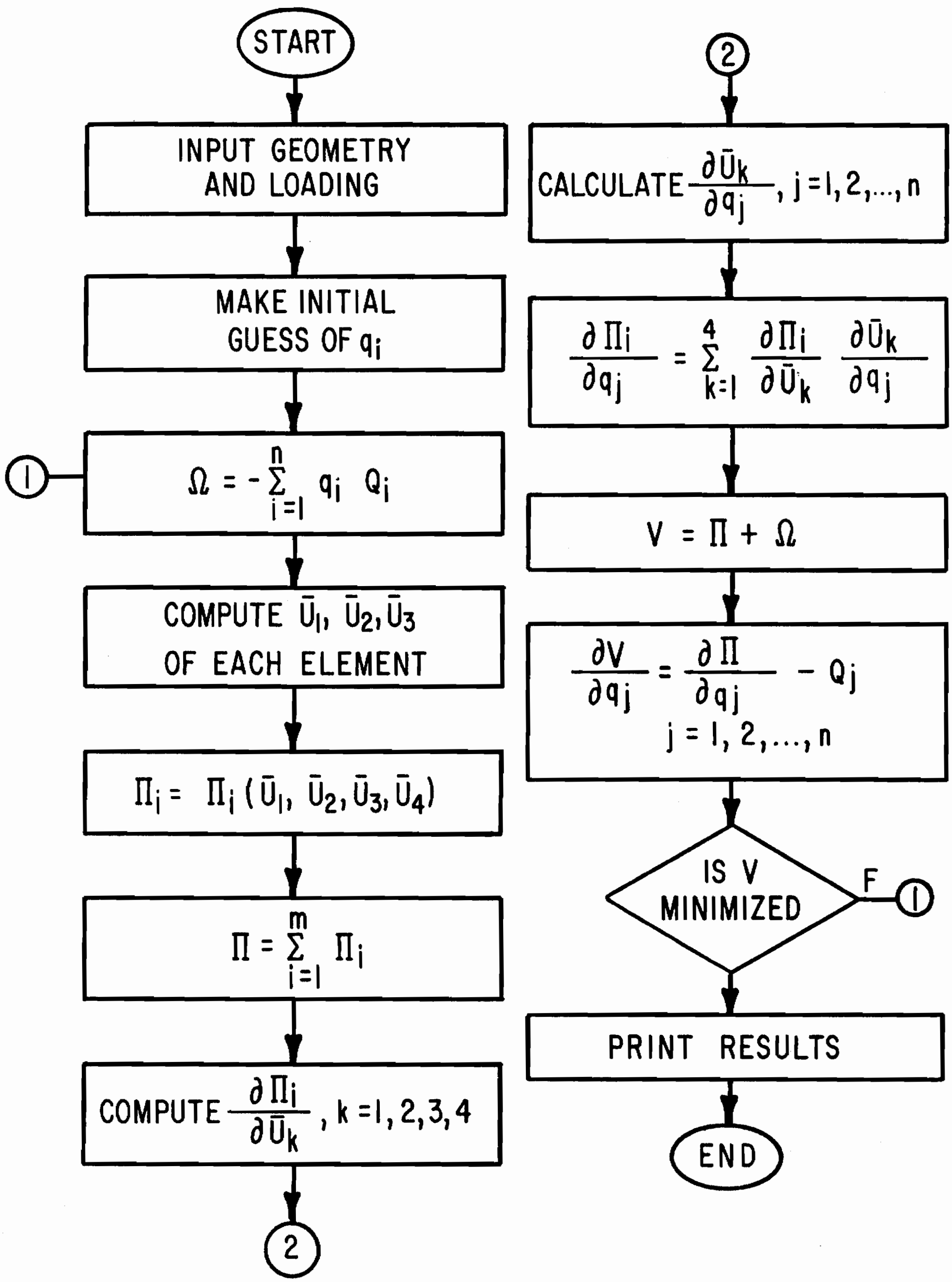


FIG. 9 THE SOLUTION PROCESS

CHAPTER IV

DEMONSTRATION PROBLEMS

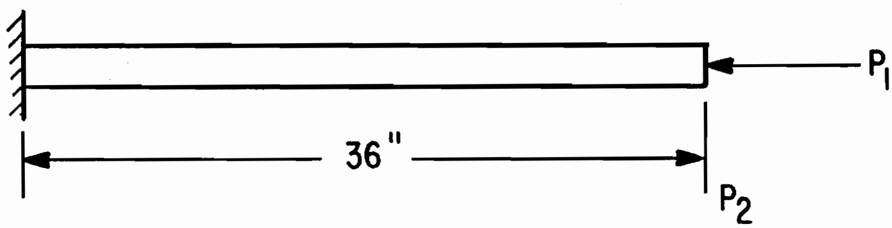
Three categories of problems have been investigated to show the reliability of the proposed finite element model. These are: 1. an elastic, geometrically linear beam-column, 2. an elastic, geometrically nonlinear beam-column, and 3. a nonlinearly elastic, geometrically linear beam-column. These specific problems have been solved by methods independent of the energy approach to provide a basis for determining the reliability of the model. The geometry of the structural element used for all test cases is a cantilever beam with the dimensions given in Figures 10a and 10b.

Elastic, Geometrically Linear Beam-Column

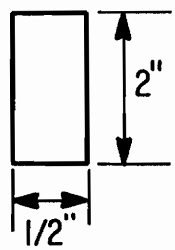
The first problem investigated was a geometrically linear model incorporating the assumption that there is no interaction between axial and flexural distortions. With this assumption, the nonlinear term in the strain-displacement relation can be eliminated to yield

$$\varepsilon(x,y) = \frac{du}{dx} - y\,\frac{d^2v}{dx^2} \tag{44}$$

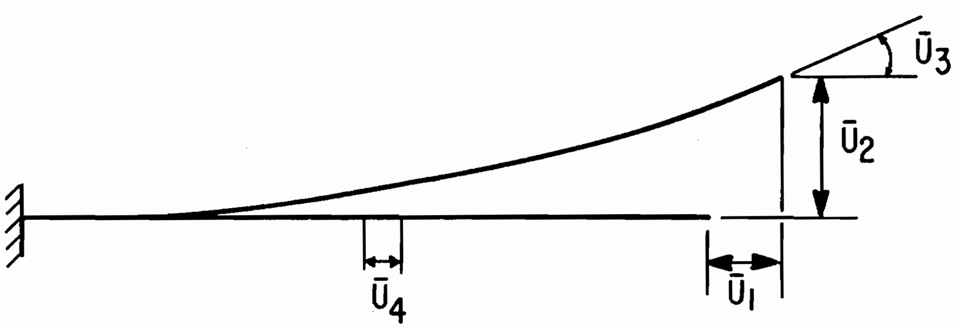
The first term on the right side of equation 44 represents the uniform normal strain induced by axial deformations of the reference axis, and the last term describes the elementary bending strain. The material of the element is assumed to be linearly elastic which results in the following expression for the strain energy:



a. LONGITUDINAL SECTION



b. CROSS SECTION



c. DISPLACEMENT OF THE REFERENCE AXIS

FIG. 10 SAMPLE BEAM-COLUMN ELEMENT

$$\Pi = \frac{E}{2} \int_V \varepsilon^2(x,y)\ dV \tag{45}$$

Having established the form of the internal energy of the element and knowing the external forces, the total potential energy can be determined which is then minimized to obtain the unknown displacements. As a check on these results, the flexural displacements, $\bar{u}_2$ and $\bar{u}_3$, can be determined by the moment-area method [11] since it is applicable to this case. The axial displacements are determined by the relation, PL/AE, where P, L, A, and E are axial load, length of the element, cross-sectional area, and Young's modulus, respectively.

The particular test problem used is described in Figure 10. The cantilever has an axial load, p_1, of -9,000 pounds and a transverse load, p_2, of 166.6667 pounds. The elastic modulus is 29,000,000 psi. Displacements obtained by the moment-area method and direct calculation are contained in Table 2.

For an elastic, geometrically linear model, the strain energy equation may be integrated directly and minimized for the unknown displacements,

$$\Pi = \frac{AE}{6L}\,(7\bar{u}_1^2 - 16\bar{u}_1\bar{u}_4 + 16\bar{u}_4^2) + \frac{IE}{2L^3}\,(12\bar{u}_2^2 - 12L\bar{u}_2\bar{u}_3 + 4L^2\bar{u}_3^2) \tag{46}$$

Adding this equation to the external work expression, $p_1\bar{u}_1 + p_2\bar{u}_2$, and determining the gradients, the work function is minimized to obtain the displacements. The minimization is performed via the computer program in Appendix A but with the nonlinear terms removed from the strain energy function and from the gradients. The displacement

TABLE 2. - DISPLACEMENT RESULTS FOR A LINEAR, ELASTIC BEAM-COLUMN

Displacement Vector[a]	Exact Solution in inches	Model Displacement in inches
$\bar{u}_1$	-.0111724	-.0111724
$\bar{u}_2$	.2681379	.2681380
$\bar{u}_3$	.0111724	.0111724
$\bar{u}_4$	-.0055862	-.0055862

[a] The displacement vector is described in Figure 10c.

results are listed in Table 2, and the axial displacements at the tip of the element are shown in Figure 11.

In a case where the reference axis does not coincide with the centroidal axis of the element, the strain energy equation takes the following form when integrated over the height:

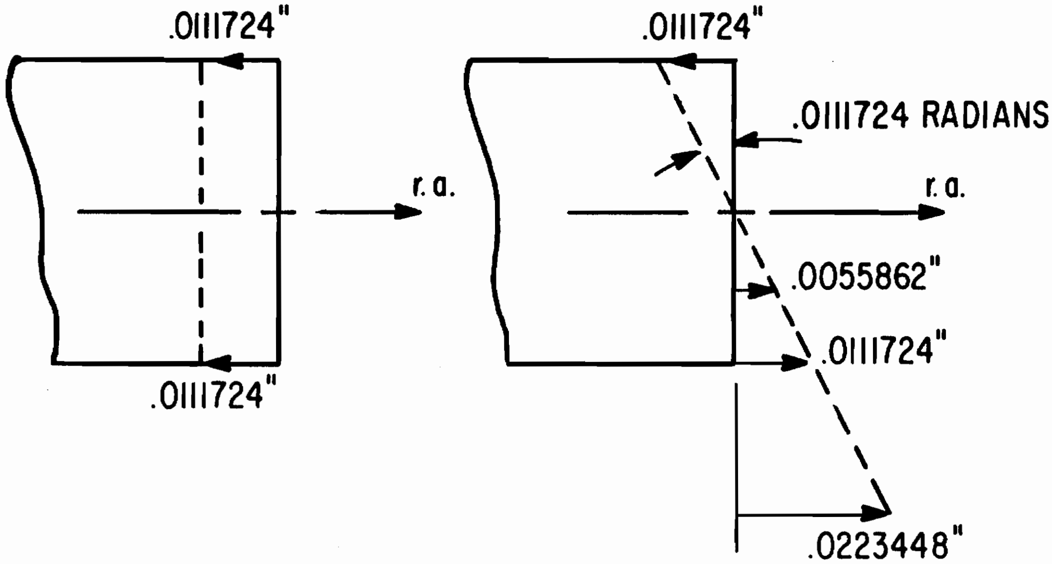
$$\Pi = \frac{bEL}{2}\int_0^1 [h(u')^2 - 2hau'v'' + (\frac{h^3}{12} + ha^2)(v'')^2] \, d\xi \tag{47}$$

where a is the distance from the reference axis to the centroidal axis.

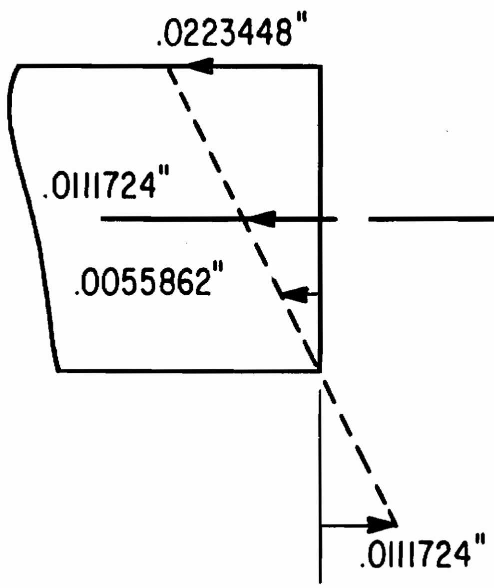
Regardless of the location of the reference axis, as in Figure 3, the displacement results are as expected. The flexural displacements remain constant wherever the reference axis is shifted, and the axial displacements vary linearly with respect to the location of the reference axis. The resulting displacements with the reference axis at several locations are listed in Table 3 which are identical to the displacement vectors in Figure 11b.

Elastic, Geometrically Nonlinear Beam-Column

This test problem was investigated to evaluate the capability of the model to analyse structures which undergo large displacements. Numerous studies have been made on the elastic, geometrically nonlinear beam-column [1,9,15]. This is also known as the "elastica" theory of thin rods. The computer program in Appendix A is used to obtain the displacement results for comparison with the classical approaches.



a. AXIAL DISPLACEMENT DUE TO COMPRESSION AND ROTATION AT THE TIP OF AN ELEMENT



b. COMBINED AXIAL DISPLACEMENT

FIG. 11 AXIAL DISPLACEMENT AT THE TIP OF A CANTILEVER BEAM

TABLE 3. - DISPLACEMENT RESULTS FOR A LINEAR, ELASTIC BEAM-COLUMN WITH A TRANSLATED REFERENCE AXIS

Location, a[a] in inches	Displacements[b] in inches			
	$\bar{u}_1$	$\bar{u}_2$	$\bar{u}_3$	$\bar{u}_4$
0.0	-.0111724	.2681380	.0111724	-.0055862
-0.5	-.0055862	.2681380	.0111724	-.0013966
-1.0	.0	.2681380	.0111724	.0027931
-2.0	.0111724	.2681380	.0111724	.0111724
1.0	-.0223448	.2681380	.0111724	-.0139656

[a] The location of the reference axis is taken from the centroidal axis of the element in the direction of the y-axis.

[b] The displacement vector is described in Figure 10c.

Eccentrically Loaded Beam-Column

These specific examples were chosen to calculate the nonlinear response of the eccentrically loaded column of Figure 12a and a column with axial load and applied moment on the free end in Figure 12b. The column in Figure 12a has an eccentricity, e, which is equivalent to the ratio M/P of the column in Figure 12b. These loading configurations give identical displacement results when used in the computer program in Appendix A.

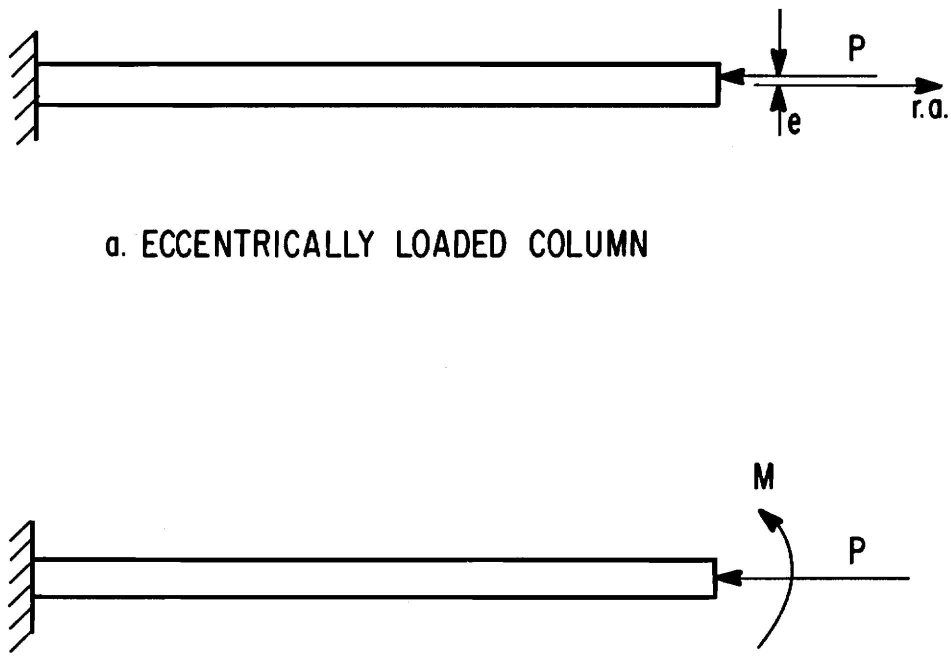
Since the beams attain large displacement configurations, they are subdivided into several elements so that each element has small relative rotations. The loadings are incrementally applied, and each displacement configuration estimate is given by the solution to the previous load step. These loadings are nondimensionalized by the Euler load

$$P_E = \frac{\pi^2 EI}{4L^2} \tag{48}$$

and the displacements are normalized by the length of the beam-column.

Vertical displacements of the free end of the beam-column are listed in Table 4 along with results obtained from the equation for the deflection curve of a simply supported beam with axial load and couples on the ends derived by Timoshenko and Gere [15]. The equation for center-line displacement of the simply supported beam is altered to give results for vertical tip displacement of a cantilever beam,

$$\Delta = \frac{ML^2}{EI}\frac{(1 - \cos(u))}{u^2\cos(u)} \tag{49}$$



a. ECCENTRICALLY LOADED COLUMN

b. COLUMN WITH MOMENT AND AXIAL LOAD

FIG. 12 BEAM-COLUMNS WITH EQUIVALENT ECCENTRICITIES

TABLE 4. - COMPARISON OF VERTICAL TIP DISPLACEMENTS OF A CANTILEVER BEAM WITH AN AXIAL LOAD AND APPLIED MOMENT ON THE FREE END

P/P_E	Nondimensional Displacement (Δ/L)	
	Timoshenko[a]	Model[b]
0.1	.0013747	.0013746
0.3	.0053336	.0053322
0.5	.0125218	.0125134
0.7	.0294062	.0293406
0.8	.0505792	.0502640
0.9	.1141934	.1094751

[a] From Timoshenko and Gere [15].

[b] A beam with a mesh of nine elements.

where

$$u = L\sqrt{\frac{P}{EI}} \tag{50}$$

and P and M are defined in Figure 12.

Equation 49 is limited to loads less than P_E since secant(u) approaches infinity as P approaches P_E. As P is increased, M is also increased by the same percentage to maintain a ratio e/L of .01. The results from a problem with only four elements are almost the same as those with nine elements. The displacement results obtained from a column of nine elements are identical to the results of a column with twelve elements. The function minimization time is increased as the mesh size is increased. The refinement from four elements to nine elements is unnecessary for practical problems due to the little change in accuracy.

Analysing the beam-column near the Euler load and in the post buckled state requires making small load steps in the region of the Euler load. This region is approximately 90% to 110% of the Euler load. If large increments in load are made, the solution process will fail to converge because it relies on the previous displacement results as the current estimate of the displaced configuration, and in the region of the Euler load there is a rapid change in the geometry of the structure.

As a comparison of the deflected shape of the beam-column up to 200% of the Euler load, Figures 13 and 14 show results obtained by Huddleston [9]. Huddleston employs a numerical technique in obtaining the deflected shape. He describes the behavior of the thin beam-column

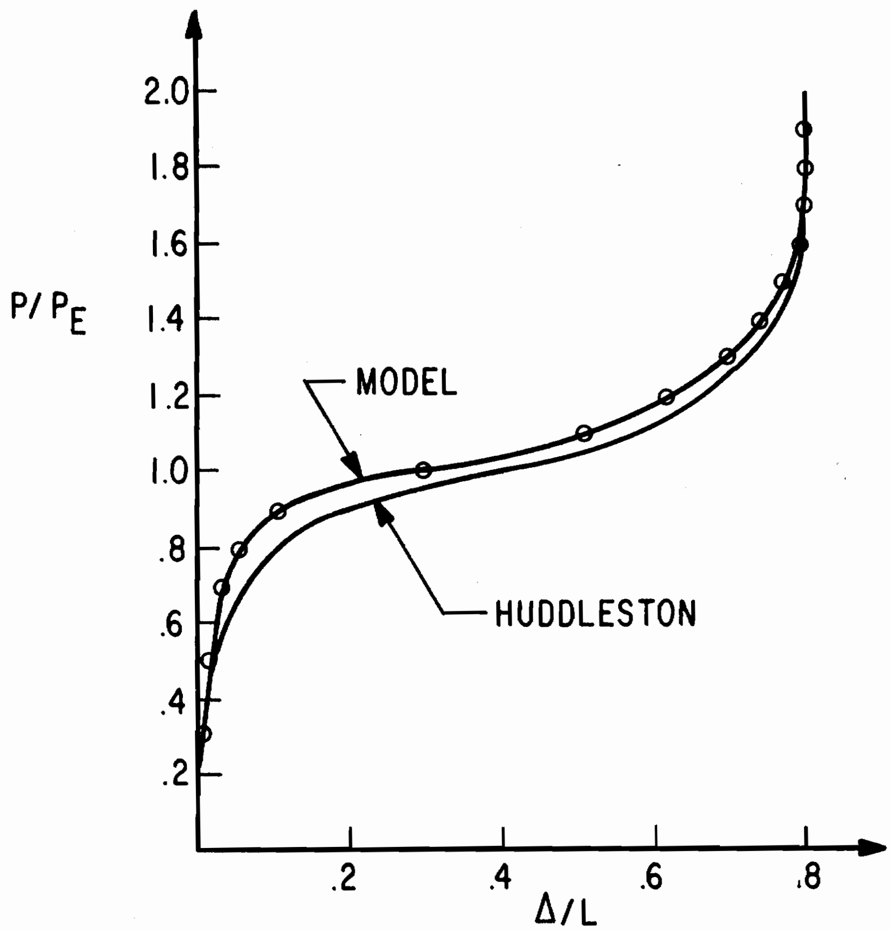


FIG. 13 FOUR-ELEMENT BEAM-COLUMN, ECCENTRICALLY LOADED - TRANSVERSE TIP DISPLACEMENT

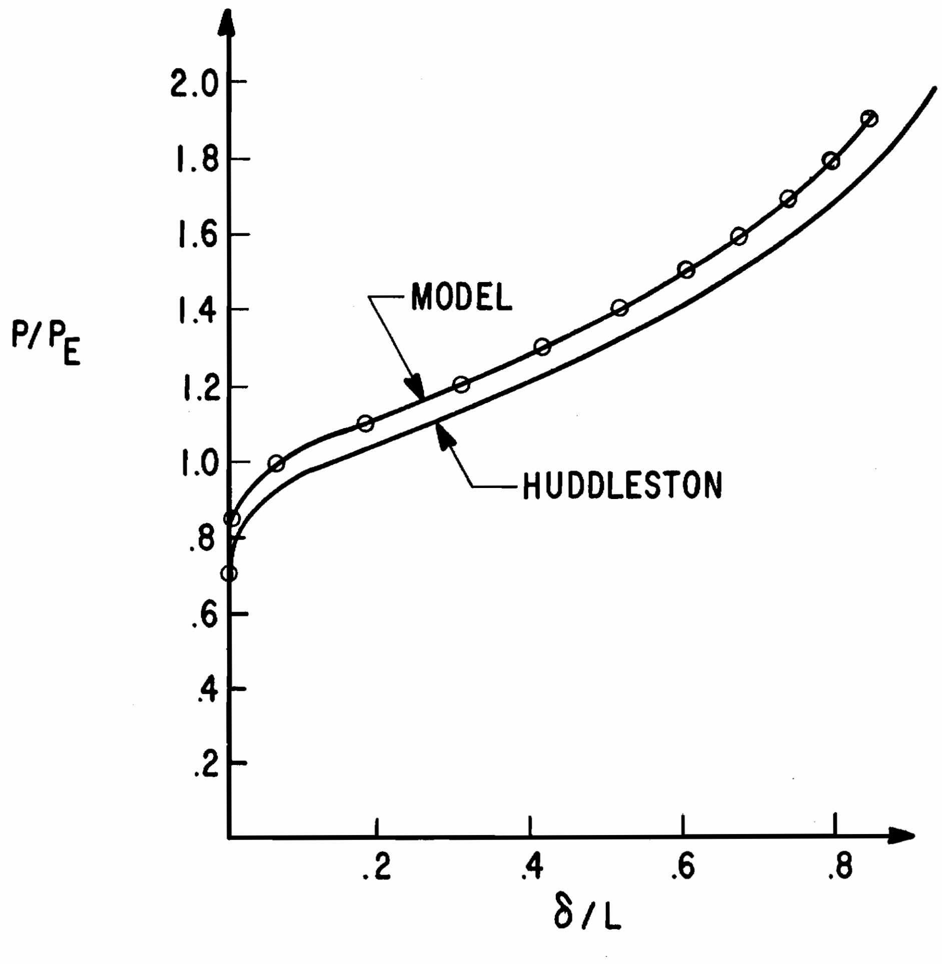


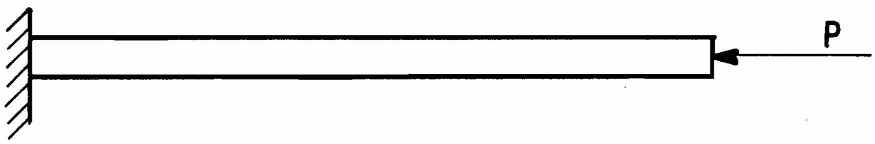
FIG. 14 FOUR-ELEMENT BEAM-COLUMN WITH MOMENT AND AXIAL LOAD - AXIAL TIP DISPLACEMENT

as three simultaneous, first-order, nonlinear differential equations. The method used to solve the two-point boundary-value problem is one in which initial conditions are assumed, integration is performed by a predictor-corrector method, and terminal conditions are computed. An iterative process is used to adjust the initial conditions until the boundary conditions at the terminal point are attained. This solution process does not consider the effect of axial strain.

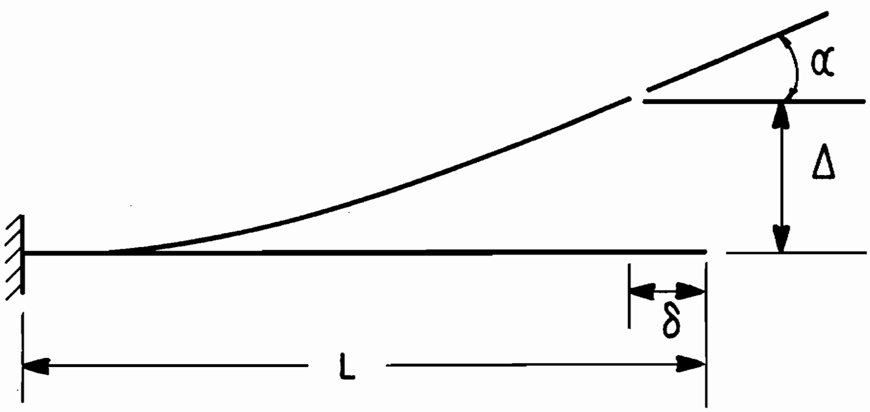
Concentrically Loaded Beam-Column

This test problem is presented to show the capability of the model to predict the post buckling path of a perfect column (Figure 15a). The beam-column in Figure 15a is loaded on the reference axis which coincides with the centroidal axis. The displacement of the reference axis is shown in Figure 15b.

The analysis is performed by giving an initial displacement configuration at the buckling load. The loading is started at the Euler load and incremented by small amounts up to approximately 10% above the Euler load because of the rapid change in geometry in that region. Figures 16, 17, and 18 give the displacements as a comparison with results obtained from a solution via elliptic integrals [15]. The column used for the model results is divided into four elements. The vertical tip displacement results for a nine element column are given in Figure 19.



a. CONCENTRICALLY LOADED COLUMN



b. DEFORMATION OF THE REFERENCE AXIS

FIG. 15 POST BUCKLING OF A PERFECT COLUMN

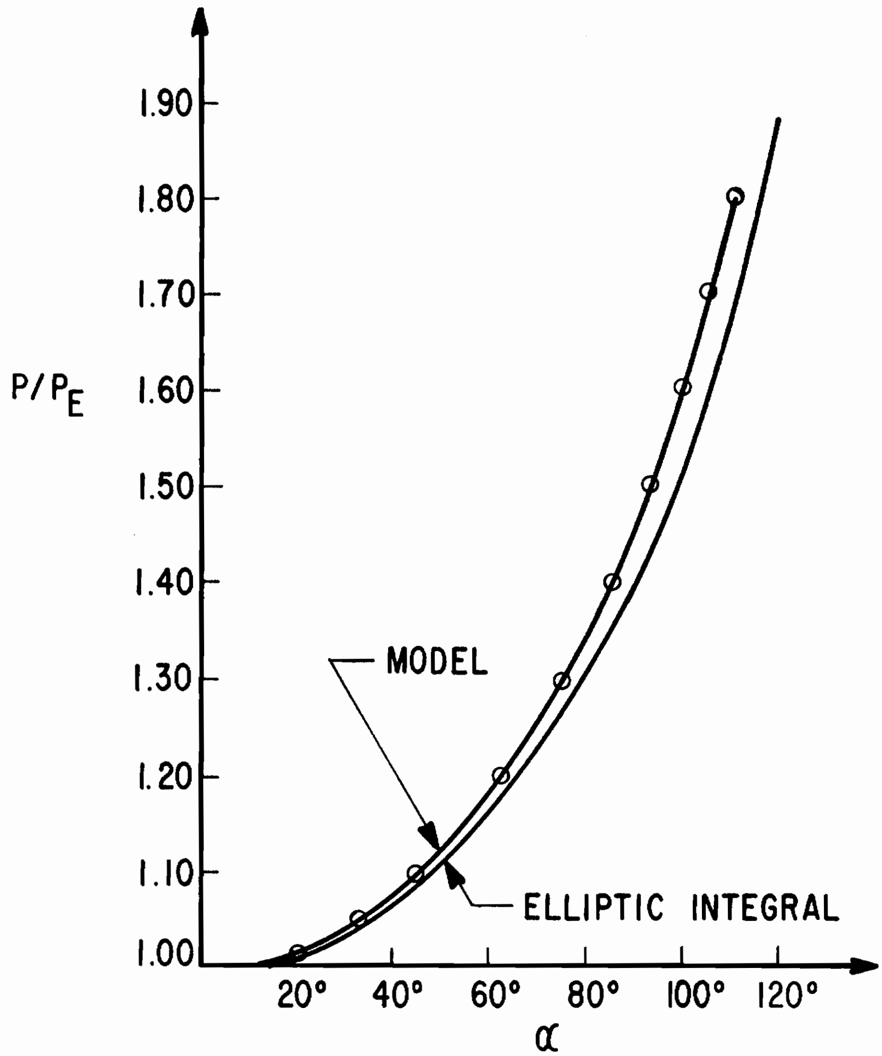


FIG. 16 FOUR-ELEMENT BEAM-COLUMN, CONCENTRICALLY LOADED - TIP ROTATION

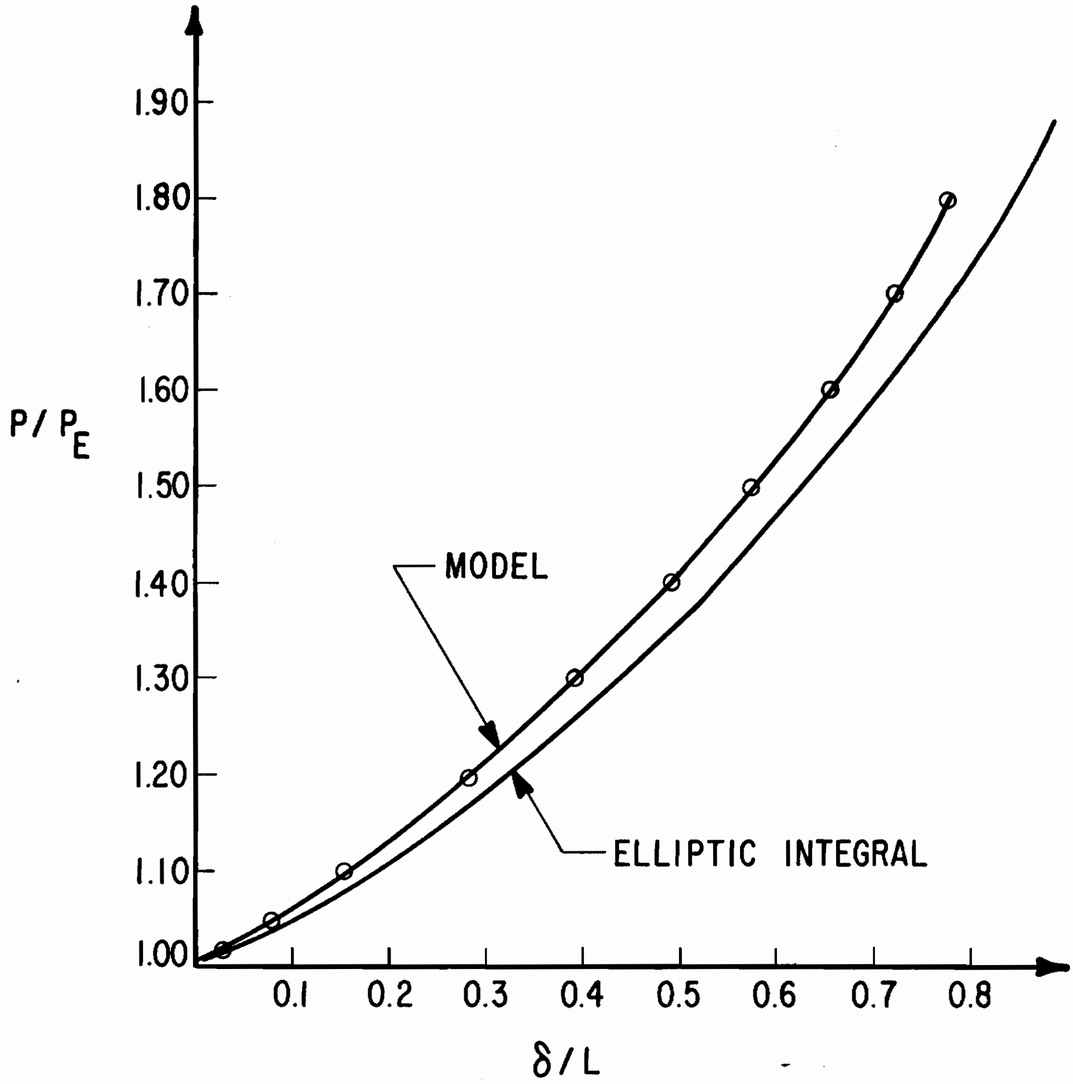


FIG. 17 FOUR-ELEMENT BEAM-COLUMN, CONCENTRICALLY LOADED - AXIAL TIP DISPLACEMENT

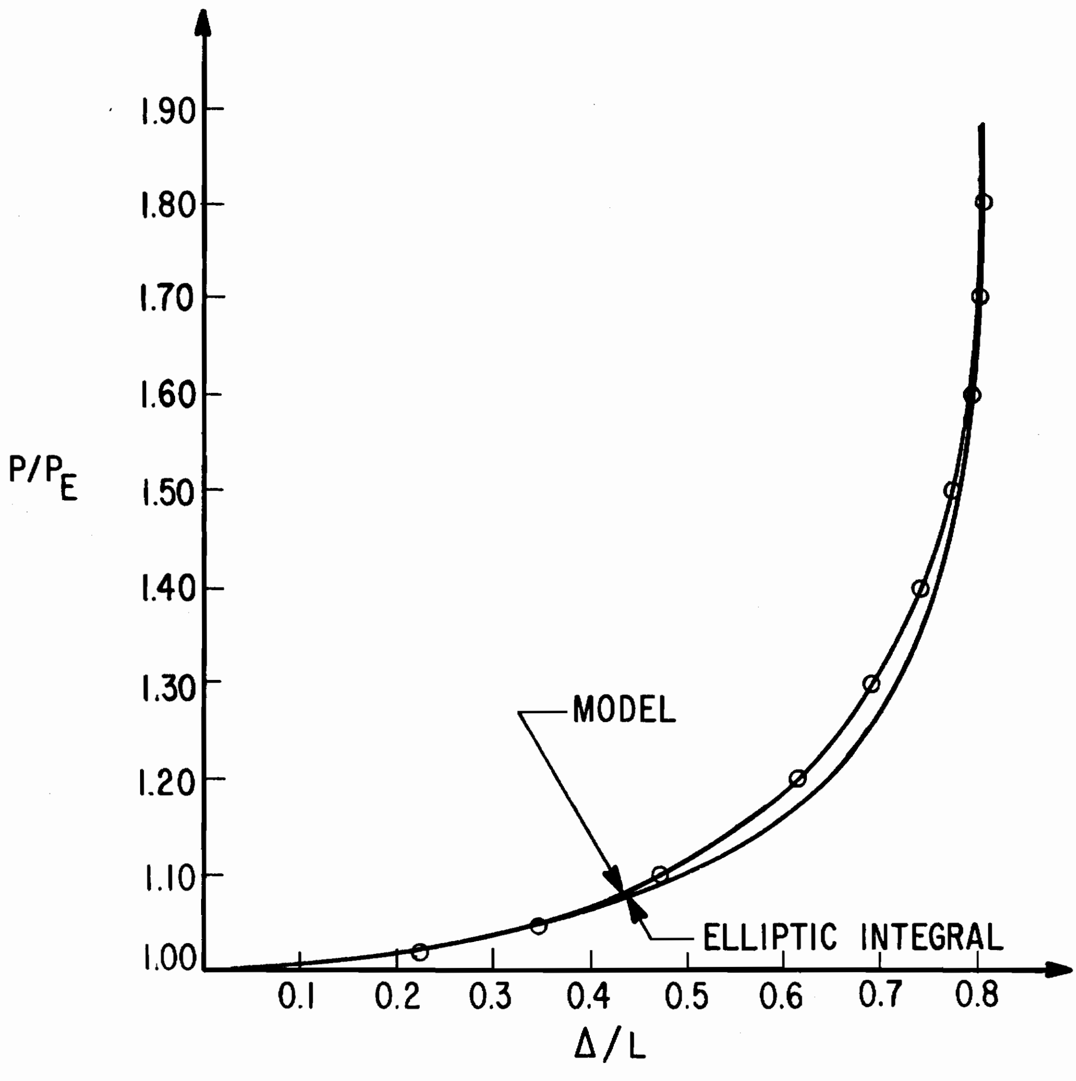


FIG. 18 FOUR-ELEMENT BEAM-COLUMN, CONCENTRICALLY LOADED - TRANSVERSE TIP DISPLACEMENT

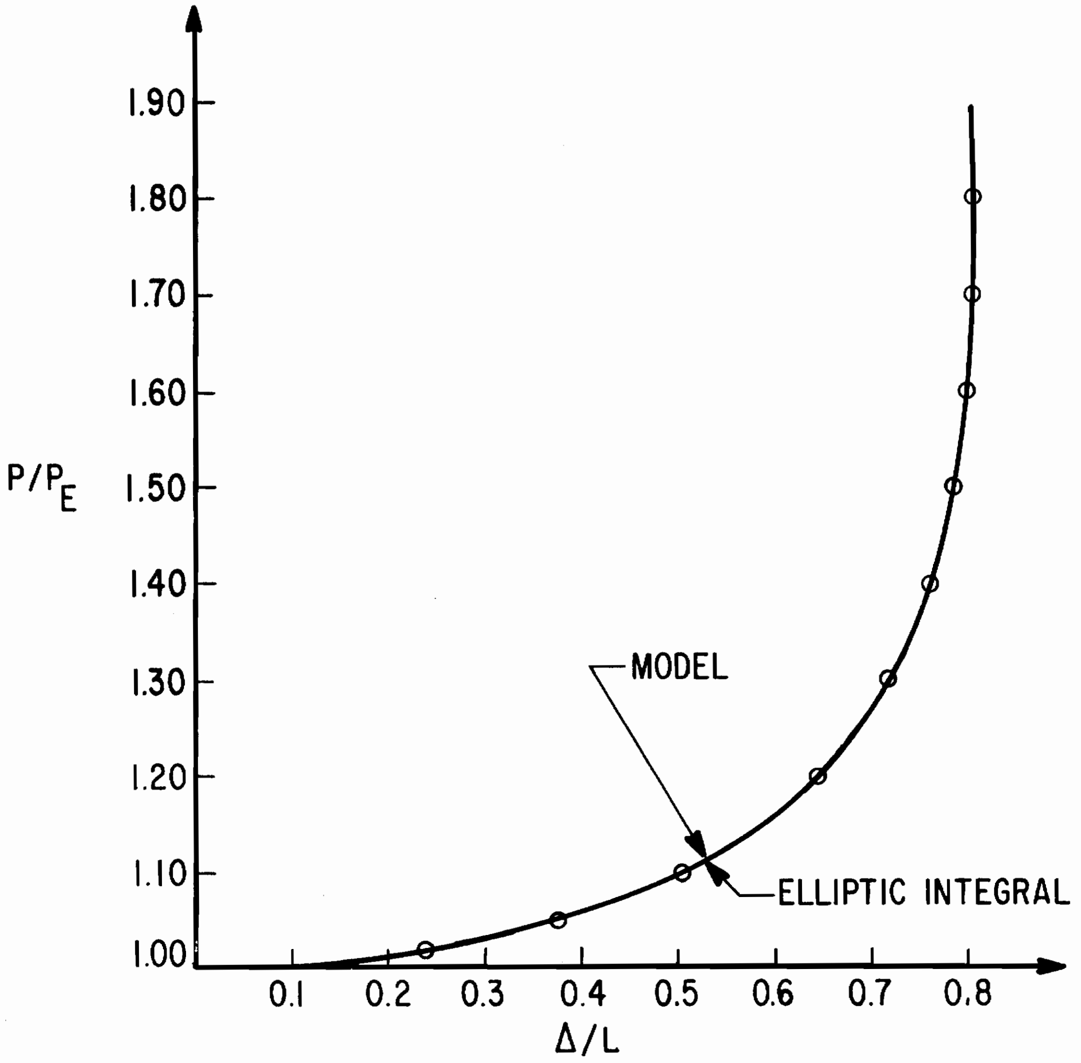


FIG. 19 NINE ELEMENT BEAM-COLUMN, CONCENTRICALLY LOADED - TRANSVERSE TIP DISPLACEMENT

Beam-Column with a Vertical Tip Loading

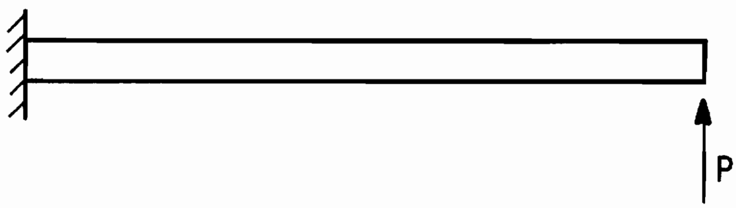
A beam with a transverse loading on the free end is shown in Figure 20a and the corresponding displacement of the reference axis is given in Figure 20b. This example illustrates the ability of the model to predict the nonlinear equilibrium path of a beam-column with only a flexural loading.

The displacement results obtained from the computer program as a comparison with a solution via elliptic integrals presented by Bisshopp and Drucker [1] are given in Figure 21. The beam in this example is composed of nine elements.

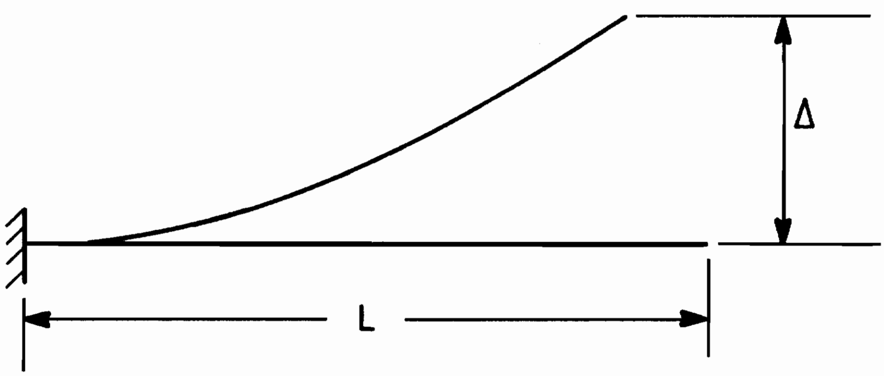
Nonlinearly Elastic, Geometrically Linear Beam-Column

This test problem is used to illustrate the capability of the model to analyse a beam-column that does not possess material linearity. The computer program in Appendix B is used to obtain the displacement and strain state results. The stress-strain curve of Figure 22b gives the material properties that are used in the program for this example.

The beam in Figure 22a is analysed for the strain state at four sections as shown. The results from the computer program are given in Tables 5 and 6. These examples use a beam made of four elements. Each element has four integration points along the length of the reference axis and four integration points across the height of the element. The beam used in Table 7 is a four element beam with a six by six quadrature mesh and has a load of 450 pounds. In Table 8, an eight element beam with a four by four quadrature is analysed.



a. BEAM WITH TRANSVERSE TIP LOADING



b. DISPLACEMENT OF THE REFERENCE AXIS

FIG. 20 CANTILEVER BEAM WITH VERTICAL LOAD ON THE FREE END

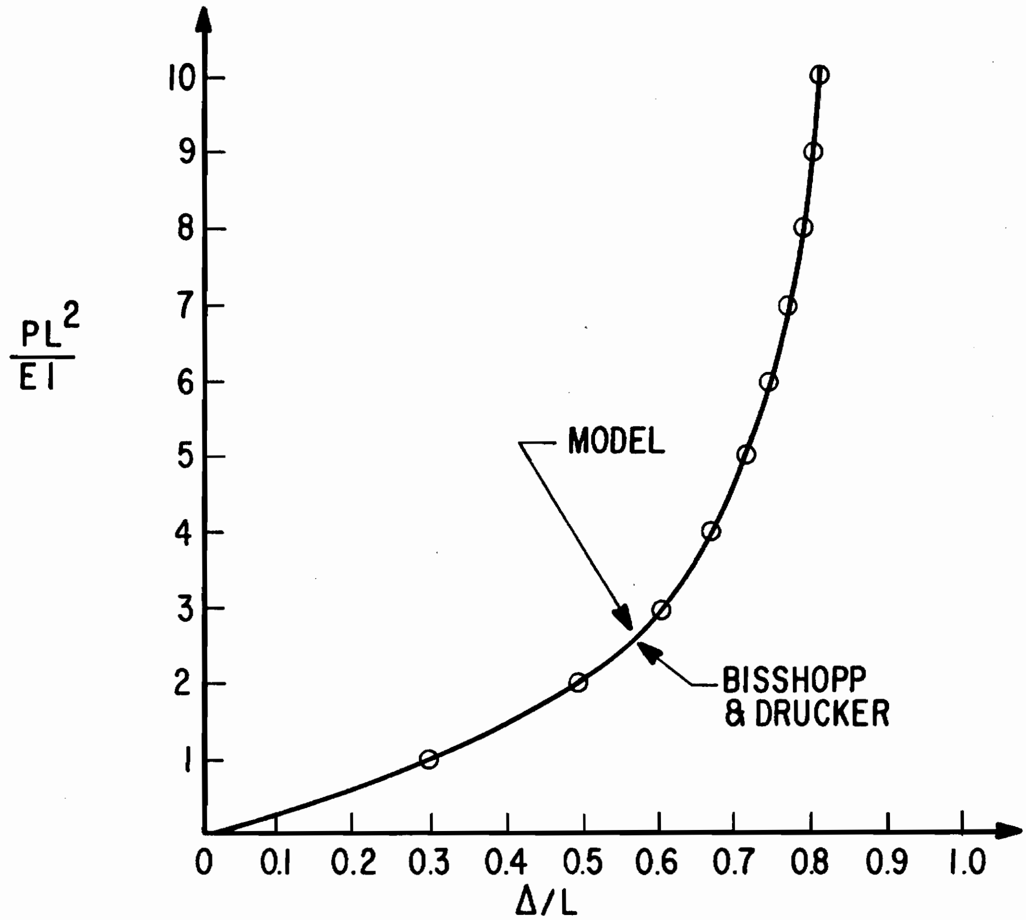
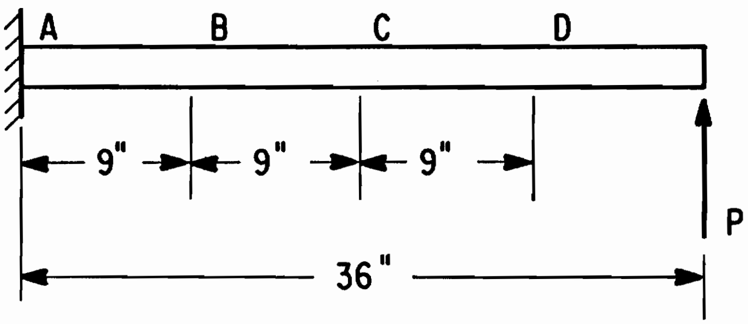
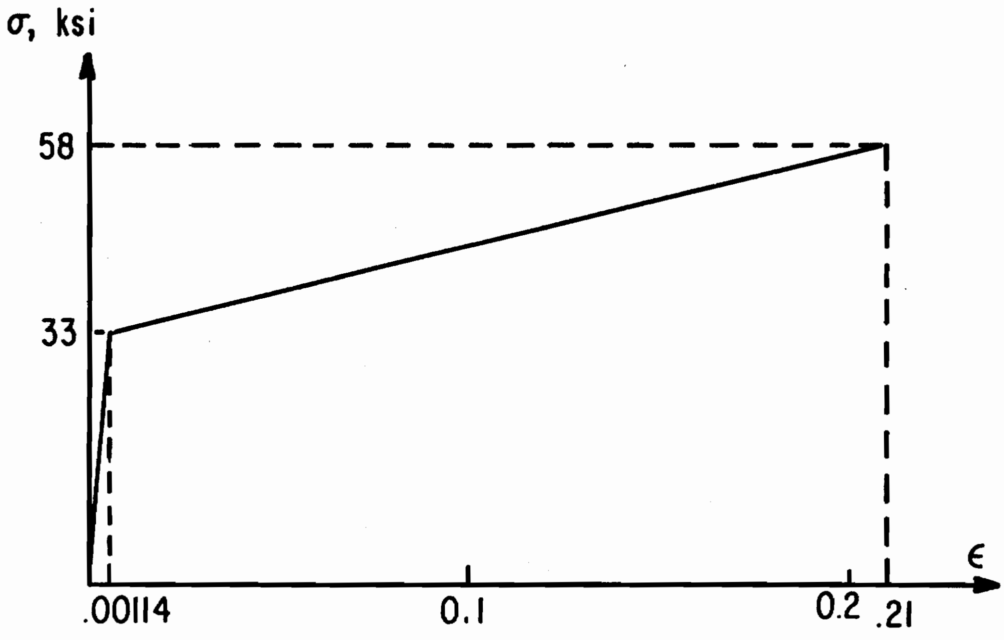


FIG. 21 COMPARISON OF VERTICAL DISPLACEMENT OF FREE END FROM BISSHOPP AND DRUCKER WITH RESULTS FROM FINITE ELEMENT MODEL - NINE ELEMENTS



a. SECTIONS FOR ANALYSIS



b. STRESS-STRAIN DIAGRAM

FIG. 22 A NONLINEARLY ELASTIC BEAM

TABLE 5. - COMPARISON OF STRAINS AT THE TOP OF A CANTILEVER BEAM WITH A VERTICAL TIP LOAD OF 400 POUNDS

Section[a]	Strain	
	Gurfinkel & Robinson	Model
A	-.0018293	-.0018170
B	-.0011193	-.0011135
C	-.0007462	-.0007493
D	-.0003731	-.0003725

TABLE 6. - COMPARISON OF STRAINS AT THE TOP OF A CANTILEVER BEAM WITH A VERTICAL TIP LOAD OF 450 POUNDS

Section[a]	Strain	
	Gurfinkel & Robinson	Model
A	-.0040630	-.0027973
B	-.0012807	-.0012516
C	-.0008395	-.0008363
D	-.0004197	-.0004188

[a] See Figure 22a.

TABLE 7. - COMPARISON OF STRAINS AT THE TOP OF A CANTILEVER BEAM WITH A SIX BY SIX QUADRATURE AND A VERTICAL TIP LOAD OF 450 POUNDS

Section[a]	Strain	
	Gurfinkel & Robinson	Model
A	-.0040630	-.0029061
B	-.0012807	-.0012550
C	-.0008395	-.0008363
D	-.0004197	-.0004188

TABLE 8. - COMPARISON OF STRAINS AT THE TOP OF A CANTILEVER BEAM WITH AN EIGHT ELEMENT MESH AND A VERTICAL TIP LOAD OF 450 POUNDS

Section[a]	Strain	
	Gurfinkel & Robinson	Model
A	-.0040630	-.0028607
B	-.0012807	-.0012562
C	-.0008395	-.0008377
D	-.0004197	-.0004187

[a] See Figure 22a.

The method used for comparing the nonlinearly elastic strain state results is one presented by Gurfinkel and Robinson [6]. This is a numerical analysis in which the strain state is assumed, and an iterative process is used until the strain state is adjusted to correspond to the given loading at the section being analysed. The computer program in Appendix C is based on this procedure. There are two major disadvantages with this method of analysis. The loading at the section being analysed must be known, and the initial guess of the strain state must be close to the actual strain state.

CHAPTER V

DISCUSSION OF RESULTS AND CONCLUSION

The test problems presented in Chapter IV show the reliability of the finite element model under several conditions. The first of these is the elastic, geometrically linear beam-column. The displacement results obtained are identical to those obtained by direct solution. When the reference axis is shifted to different locations within the element and even outside the element, the flexural displacements remain constant, and the axial displacements vary linearly with respect to the reference axis. This problem indicates that the internal node gives a high degree of performance since the strain state is predicted accurately regardless of the location of the reference axis. From this test it can be seen that the model gives good predictions for an elastic, geometrically linear beam-column.

Several examples of an elastic, geometrically nonlinear beam-column are examined. The first of these is a cantilever beam with an eccentrically applied axial load. A comparison with a solution obtained by a method presented by Timoshenko and Gere [15] for loads up to approximately 90% of the Euler load give almost identical displacement results. When the same problem is compared with results from the Huddleston curves, there seems to be a discrepancy. Within a range of 60% below and 60% above the Euler load, the displacement results obtained from the four element model are smaller in magnitude than those of Huddleston's. A test problem with nine elements give displacement results that are closer to Huddleston's results, but only by a small amount. An increase from nine elements to twelve causes no change in the magnitude of the displacements, which illustrates that further refinement of the element mesh is unnecessary. Huddleston's solution assumes axial incompressibility while the finite element model considers the effects of axial strain. A cantilever beam with a concentrically applied axial load is compared with results obtained by Timoshenko and Gere [15]. The loading for this case is started at the Euler load to obtain results for the post buckled state of the classical perfect column. Displacements obtained from a four element beam-column are close to the results from Timoshenko and Gere, but when a nine element column is used, the results from the two methods are identical. As an alternate example for large geometry change is a cantilever with a vertical load on the free end. Displacement results obtained with a nine element beam are identical to those results obtained by Bisshopp and Drucker [1] via an elliptic integral solution.

The results obtained from the nonlinearly elastic, geometrically linear beam-column are almost identical to the results obtained by the method of Gurfinkel and Robinson [6] within the linear range of the stress-strain curve. When the displacements become large, the Gurfinkel and Robinson routine tends to give higher estimates of the strain state of the section being analysed as compared to the results via the finite element model. Changing the number of integration points seems to have a stronger effect on the results than refining the mesh.

The finite element model presented proves to be a reliable model for use in an algorithm using an energy method of analysis. The elastic, geometrically nonlinear beam-column test problem shows that the model is good for structural elements which undergo large displacements that involve the coupling of axial and flexural distortions. The nonlinearly elastic test problem shows that the model can be used in an algorithm that incorporates inelastic material.

Further studies should be performed on this finite element model. Refinements on the optimum mesh size used for a structural member should be performed, and with this, some type of a mesh generator should be developed which could be based on an even distribution of the energy within the member being analysed. Test problems with more than one internal node could be investigated to determine if this would reduce the necessity of refining the mesh to get improved displacement results. Additional studies need to be performed on the nonlinearly elastic case so that more general stress-strain diagrams can be used in the analysis. Also, as a more complete study of the model, tests on an inelastic, geometrically nonlinear beam-column should be conducted.

REFERENCES

1. Bisshopp, K. E., and Drucker, D. C., "Large Deflections of Cantilever Beams," Quarterly of Applied Math, Vol. 3, 1945, pp. 272-275.

2. Bleich, Friedrich, Buckling Strength of Metal Structures, McGraw-Hill Book Company, Inc., New York, 1952, pp. 1-9.

3. Fletcher, R., and Powell, M. J. D., "A Rapidly Convergent Descent Method for Minimization," Computer Journal, Vol. 6, Iss. 2, 1963, pp. 163-168.

4. Fox, Richard L., Optimization Methods for Engineering Design, Addison-Wesley Publishing Company, Reading, Massachusetts, 1971, pp. 60-111.

5. Ghali, A., and Neville, A. M., Structural Analysis, Intext Educational Publishers, Scranton, Pennsylvania, 1972, pp. 356-393.

6. Gurfinkel, G., and Robinson, A., "Determination of Strain Distribution and Curvature in a Reinforced Concrete Section Subjected to Bending Moment and Longitudinal Load," Journal ACI, Vol. 64, No. 7, July 1967, pp. 398-403.

7. Hoff, Nicholas John, The Analysis of Structures, John Wiley and Sons, Inc., New York, 1956, pp. 151-173.

8. Holzer, S. M., et al., "SINGER: A Computer Code for General Analysis of Two Dimensional Reinforced Concrete Structures," Vol. 1, Technical Report, Kirtland AFB, N. Mex., September 1974.

9. Huddleston, John V., "A Numerical Technique for Elastica Problems," Journal of the Engineering Mechanics Division, ASCE, Vol. 94, EM5, October 1968, pp. 1159-1165.

10. Langhaar, Henry L., Energy Methods in Applied Mechanics, John Wiley and Sons, Inc., New York, 1962.

11. Laursen, Harold I., Structural Analysis, McGraw-Hill Book Company, New York, 1969, pp. 125-138.

12. Scheid, Francis, "Numerical Analysis," Schaum's Outline Series, McGraw-Hill Book Company, New York, 1968, pp. 125-149.

13. Sechler, Ernest E., Elasticity in Engineering, John Wiley and Sons, Inc., New York, 1952, pp. 359-381.

14. Stewart, G. W., "A Modification of Davidon's Minimization Method to Accept Difference Approximations of Derivatives," Journal of the Association for Computing Machinery, Vol. 14, No. 1, January 1967, pp. 72-83.

15. Timoshenko, Stephen P., and Gere, James M., Theory of Elastic Stability, McGraw-Hill Book Company, New York, 1961.

APPENDIX A

```
CELIN  1C  10   0
C
C     MAIN PROGRAM TO ANALYSE STRUCTURES THAT MAY BECOME GEOMETRICALLY
C     NONLINEAR.  THE ELEMENT MATERIAL MUST BE LINEARLY ELASTIC SINCE
C     THE STRAIN ENERGY OF EACH ELEMENT IS INTEGRATED EXPLICITLY.
C
C     WRITTEN IN AMERICAN STANDARD FORTRAN BY J. C. BRADSHAW, III.
C
C     CARD 1 :  TITLE CARD.  FORMAT(20A4)
C
C     CARD 2 :  CONTROL CARD.  FORMAT(E10.0,6I5,4E10.0)
C     E=YOUNG'S MODULUS.
C     NJ=NUMBER OF JOINTS TO BE INPUT.
C     NM=NUMBER OF ELEMENTS TO BE INPUT.
C     NF=NUMBER OF FORCES TO READ IN THE FORCING FUNCTION.
C     NINF=OPTION NOT TO READ FORCES IN THE FORCING FUNCTION, BUT
C          TO INPUT FORCES THROUGH THE LOAD INCREMENT FEATURE. (GT.0)
C          OPTION TO READ IN INITIAL DISPLACEMENTS. (LT.0)
C     LIMIT=LIMIT ON THE NUMBER OF MINIMIZATIONS.
C     NPLOT=OPTION TO PLOT LOAD VERSUS DEFLECTION.   (GT.0)
C     EPS=DESIRED ACCURACY OR ROUND-OFF OF GRADIENTS.
C     EST=AN ESTIMATE OF THE FUNCTION VALUE.
C     FACT=FACTOR TO NORMALIZE LOADS FOR PLOTTING.
C     DFAC=FACTOR TO NORMALIZE DISPLACEMENTS FOR OUTPUT.
C
C     CARDS 3 :  JOINT COORDINATES AND STATUS.
C                FORMAT(I5,5X,2D10.0,2X,3A1)
C     I=JOINT NUMBER.
C     X=X OR 1 DIRECTION COORDINATE.
C     Y=Y OR 2 DIRECTION COORDINATE.
C     IR(1)="R" IF RESTRAINED IN THE 1 DIRECTION.
C     IR(2)="R" IF RESTRAINED IN THE 2 DIRECTION.
```

```
C     IR(3)="R" IF RESTRAINED IN THE 3 DIRECTION.
C
C     CARDS 4 :  ELEMENT CARDS.  FORMAT(3I5,5X,3D10.0)
C     M=ELEMENT NUMBER.
C     IP=MEMBER START.
C     IQ=MEMBER END.
C     B=WIDTH OF THE ELEMENT.
C     H=HEIGHT OF THE ELEMENT.
C     A=DISTANCE OF REFERENCE AXIS FROM CENTER OF ELEMENT.
C
C     CARDS 5 :  JOINT FORCING FUNCTION.  FORMAT(8D10.0)
C
C     CARDS 6 :  JOINT LOADS.  FORMAT(2I5,2D10.0)
C     IJOI=JOINT TO BE LOADED.
C     IDIR=DIRECTION OF LOAD (1,2, OR 3).
C     DS=LOAD MULTIPLICATION CONSTANT.
C     ADD=LOAD INCREMENT VALUE.
C
C     JOINT LOADS MUST BE ENDED WITH A BLANK CARD.
C
C     CARDS 7 :  INITIAL DISPLACEMENTS. FORMAT(8D10.0)
C
C     ENTIRE DATA DECK MUST BE ENDED WITH TWO BLANK CARDS.
C
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MAINEK/ U(80),G(80)
      COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM
      COMMON /PLOTS/ YD(3,21,40),P(41),NF
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      DIMENSION TITLE(20), IR(3,21), HN(3480), FOR(40)
      DATA IRES/1HR/,FOR/40*0.E0/
   20 CONTINUE
```

```
      READ 160, TITLE                                                   ELIN 640
      READ 170, E,NJ,NM,NF,NINF,LIMIT,NPLOT,EPS,EST,FACT,DFAC           ELIN 650
      IF (E.LE.0.E0) GO TO 140                                          ELIN 660
      IF (FACT.EQ.0.E0) FACT = 1.E0                                     ELIN 670
      IF (DFAC.EQ.0.E0) DFAC = 1.E0                                     ELIN 680
      IF (LIMIT.EQ.0) LIMIT=40                                          ELIN 690
      IF (EPS.EQ.0.E0) EPS=1.E-7                                        ELIN 700
      PRINT 150                                                         ELIN 710
      PRINT 220, TITLE                                                  ELIN 720
      READ 180, (I,X(I),Y(I),IR(1,I),IR(2,I),IR(3,I),K=1,NJ)            ELIN 730
      PRINT 230                                                         ELIN 740
C                                                                       ELIN 750
C     DETERMINE THE JOINT STATUS, AND INDEX FOR ASSEMBLY.               ELIN 760
C                                                                       ELIN 770
      DO 50 J=1,NJ                                                      ELIN 780
      DO 40 I=1,3                                                       ELIN 790
      IJ(I,J)=0                                                         ELIN 800
      XDJ(I,J)=0.E0                                                     ELIN 810
      IF (IR(I,J).EQ.IRES) GO TO 30                                     ELIN 820
      NDF=NDF+1                                                         ELIN 830
      IJ(I,J)=NDF                                                       ELIN 840
30    CONTINUE                                                          ELIN 850
C     INITIALIZE THE FORCE MATRIX.                                      ELIN 860
      DO 40 K=1,NF                                                      ELIN 870
      F(I,J,K)=0.E0                                                     ELIN 880
40    CONTINUE                                                          ELIN 890
      PRINT 240, J,X(J),Y(J),IJ(1,J),IJ(2,J),IJ(3,J)                    ELIN 900
50    CONTINUE                                                          ELIN 910
      N=NDF+NM                                                          ELIN 920
      NH=N*(N+7)/2                                                      ELIN 930
      PRINT 250                                                         ELIN 940
      DO 60 K=1,NM                                                      ELIN 950
```

```
      READ 190, M,IP(M),IQ(M),B,H,A
      XL=(X(IQ(M))-X(IP(M)))**2+(Y(IQ(M))-Y(IP(M)))**2
      XL=SQRT(XL)
      AX=B*H
      ZI=B*H**3/12.E0+AX*A**2
      PRINT 260, M,IP(M),IQ(M),B,H,A,XL,AX,ZI
C
C     ESTABLISH COEFFICIENTS OF CONSTANT MEMBER PROPERTIES.
C
      C(1,M)=AX*E/6./XL
      C(2,M)=A*AX*E/XL**2
      C(3,M)=ZI*E/2./XL**3
      C(4,M)=AX*E/30./XL**2
      C(5,M)=A*AX*E/6.E0
      C(6,M)=AX*E/280./XL**3
      XLEN(M)=XL
60    CONTINUE
      IF (DFAC.NE.1.E0) PRINT 365, DFAC
      IF (NINF.GT.0) GO TO 70
      READ 200, (FOR(I),I=1,NF)
70    CONTINUE
      READ (5,210) IJOI,IDIR,DS,ADD
      IF (IJOI.EQ.0) GO TO 90
      IF (DS.EQ.0.E0) DS=1.E0
      AD=ADD
      DO 80 ILOAD=1,NF
      F(IDIR,IJOI,ILOAD)=(FOR(ILOAD)+AD)*DS
      AD=AD+ADD
80    CONTINUE
      GO TO 70
90    CONTINUE
      DO 100 I=1,N
```

```
      U(I)=0.E0                                                         ELIN1280
  100 CONTINUE                                                          ELIN1290
      IF(NINF.LT.0) READ 200, (U(I),I=1,N)                              ELIN1300
      DO 130 IL=1,NF                                                    ELIN1310
      NMIN=-1                                                           ELIN1320
      ILOAD=IL                                                          ELIN1330
C                                                                       ELIN1340
C     PRINT OUT THE JOINT LOAD MATRIX.                                  ELIN1350
C                                                                       ELIN1360
      PRINT 370                                                         ELIN1370
      PRINT 345, ILOAD                                                  ELIN1380
      PRINT 350                                                         ELIN1390
      DO 110 J=1,NJ                                                     ELIN1400
      PRINT 340, J,(F(I,J,ILOAD),I=1,3)                                 ELIN1410
  110 CONTINUE                                                          ELIN1420
C                                                                       ELIN1430
C     CALL THE MINIMIZATION ROUTINE FOR EACH LOAD STEP.                 ELIN1440
C                                                                       ELIN1450
      CALL DFMFP (NH,N,FUN,EST,EPS,LIMIT,IER,HN)                        ELIN1460
C                                                                       ELIN1470
C     PRINT OUT THE RESULTS FROM THE MINIMIZATION PROCESS.              ELIN1480
C                                                                       ELIN1490
      IF (IER.EQ.0) PRINT 270                                           ELIN1500
      IF (IER.EQ.1) PRINT 280                                           ELIN1510
      IF (IER.EQ.-1) PRINT 290                                          ELIN1520
      IF (IER.EQ.2) PRINT 300                                           ELIN1530
      PRINT 310, KOUNT,NMIN                                             ELIN1540
      PRINT 320, FUN,(G(I),I=1,N)                                       ELIN1550
      PRINT 330                                                         ELIN1560
      DO 120 J=1,NJ                                                     ELIN1570
      DO 115 I=1,2                                                      ELIN1580
  115 YD(I,J,ILOAD)=XDJ(I,J)/DFAC                                       ELIN1590
```

```
      YD(3,J,ILCAD) = XDJ(3,J)
      PRINT 340, J,(YD(I,J,ILCAD),I=1,3)
  120 CCNTINUE
      PRINT 342
      DC 124 M=1,NM
      PRINT 343, M,U(NDF+M)
  124 CCNTINUE
      PX = ABS(F(1,NJ,ILOAD))
      P(ILOAD) = PX/FACT
  130 CCNTINUE
      P(NF+1) = 0.E0
      IF (NPLOT.GT.0) CALL YDEF
      GO TO 20
  140 CCNTINUE
      PRINT 150
      PRINT 360
      STOP
C
  150 FCRMAT (1H1)
  160 FCRMAT (20A4)
  170 FCRMAT (E10.0,6I5,4E10.0)
  180 FCRMAT (I5,5X,2E10.0,2X,3A1,45X)
  190 FCRMAT (3I5,5X,3E10.0,30X)
  200 FCRMAT (8E10.0)
  210 FCRMAT (2I5,2E10.0,50X)
  220 FCRMAT (1H0,T10,20A4//)
  230 FCRMAT (1H0,T20,28HJCINT CCORDINATES AND STATUS//T7,5HJOINT,9X,1HX
     1,12X,1HY,9X,2HDX,3X,2HDY,3X,2HDZ//)
  240 FORMAT (1H ,T9,I2,6X,2(F8.3,5X),3(I3,2X)/)
  250 FORMAT (1H0,T29,18HELEMENT PROPERTIES//T6,7HELEMENT,4X,5HSTART,4X,
     13HEND,3X,5HWIDTH,6X,6HHEIGHT,9X,1HA,7X,6HLENGTH,7X,4HAREA,6X,7HINE
     2RTIA//)
```

```
260    FORMAT (1H ,T9,I2,7X,I3,4X,I3,6(4X,F8.4)/)                           ELIN1920
270    FORMAT (1H0,T10,24HCONVERGENCE WAS OBTAINED/)                         ELIN1930
280    FORMAT (1H0,T10,34HNO CONVERGENCE IN LIMIT ITERATIONS/)               ELIN1940
290    FORMAT (1H0,T10,30HERRORS IN GRADIENT CALCULATION/)                   ELIN1950
300    FORMAT (1H0,T10,33HLINEAR SEARCH TECHNIQUE INDICATES/T10,41HIT IS ELIN1960
      1LIKELY THAT THERE EXISTS NO MINIMUM/)                                 ELIN1970
310    FORMAT (1H0,T10,I4,29H MINIMIZATIONS HAVE BEEN MADE/T10,I4,27H FUNELIN1980
      1CTION SUBROUTINE CALLS.////)                                          ELIN1990
320    FORMAT (1H0,T10,17HFUNCTION VALUE = ,E16.8,//T10,11HGRADIENTS :// ELIN2000
      1  20(T10,6(E16.8,2X)/))                                               ELIN2010
330    FORMAT (1H0,T24,19HJOINT DISPLACEMENTS//T7,5HJOINT,12X,2HDX,19X,2HELIN2020
      1DY,19X,2HDZ//)                                                        ELIN2030
340    FORMAT (1H ,T9,I2,6X,3(E16.7,5X)/)                                    ELIN2040
342    FORMAT(1H0,T17,26HINTERNAL NODE DISPLACEMENT//T18,7HELEMENT,12X,      ELIN2050
      1        2HU4//)                                                       ELIN2060
343    FORMAT(1H ,T21,I2,7X,E16.8/)                                          ELIN2070
345    FORMAT(1H ,T10,10HLOAD STEP ,I2/)                                     ELIN2080
350    FORMAT (1H0,T28,11HJOINT LOADS//T7,5HJOINT,13X,1HX,20X,1HY,20X,1HZELIN2090
      1//)                                                                   ELIN2100
360    FORMAT (1H0,T10,46H=====  NORMAL COMPLETION OF THE PROGRAM  =====/ELIN2110
      1/)                                                                    ELIN2120
365    FORMAT (1H0,T6,50HDISPLACEMENTS HAVE BEEN NORMALIZED BY A FACTOR OELIN2130
      1F ,F16.8///)                                                          ELIN2140
370    FORMAT (1H0,60(1H-)//)                                                ELIN2150
       END                                                                   ELIN2160
CFUNC  10  10  -0                                                                   0
       SUBROUTINE FUNCT (N,FUN)                                              FUNC  10
       COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ      FUNC  20
       COMMON /MAINBK/ U(80),G(80)                                           FUNC  30
       COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM            FUNC  40
       COMMON /TAB/ ILOAD,NMIN,KOUNT                                         FUNC  50
       COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3          FUNC  60
```

```
      NMIN=NMIN+1
      FUN=0.E0
      DO 10 J=1,NJ
      DO 10 I=1,3
      IF (IJ(I,J).EQ.0) GO TO 10
C
C     PUT VALUES INTO THE JOINT DISPLACEMENT MATRIX FROM THE
C     GENERALIZED COORDINATES MATRIX.
C
      XDJ(I,J)=U(IJ(I,J))
C
C     CALCULATE THE PART OF EACH GRADIENT WHICH IS DERIVED FROM
C     THE EXTERNAL WORK.
C
      G(IJ(I,J))=-F(I,J,ILOAD)
10    CONTINUE
      DO 20 M=1,NM
      U4=U(NDF+M)
      CALL DEFO (M)
      CALL STRN (M)
      CALL GRDT (M)
      CALL GRAD (M)
C
C     ACCUMULATE THE STRAIN ENERGY FROM EACH ELEMENT.
C
      FUN=FUN+STRAIN(M)
20    CONTINUE
C
C     COMPUTE THE TOTAL EXTERNAL WORK.
C
      CALL POTE (OMEG)
C
```

```
C     CALCULATE THE TOTAL WORK OF THE SYSTEM.
C
      FUN=FUN+OMEG
      RETURN
      END
CDEFO  10  10  -0
      SUBROUTINE DEFO (M)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      I=IP(M)
      J=IQ(M)
      DX1=(X(J)-X(I))
      DX2=(Y(J)-Y(I))
      DU1=(XDJ(1,J)-XDJ(1,I))
      DU2=(XDJ(2,J)-XDJ(2,I))
      UI3=XDJ(3,I)
      DU3=XDJ(3,J)-UI3
C
C     GEOMETRICALLY NONLINEAR: UNLIMITED RIGID-BODY MOTIONS,
C     BEAM-COLUMN DISTORTIONS.
C     ROTATIONAL TRANSFORMATION PARAMETERS: GLOBAL TO DEFORMED JOINT I.
C
      SI3= SIN(UI3)
      CI3= COS(UI3)
      SI32= SIN(UI3/2.E0)**2
C
C     DISTORTIONS OF MEMBER M IN JOINT-I COORDINATES (TRANSLATIONAL).
C
      D1=-2.E0*SI32*DX1+SI3*(DX2+DU2)+CI3*DU1
      D2=-SI3*(DX1+DU1)-2.E0*SI32*DX2+CI3*DU2
C
```

```
C     ROTATIONAL TRANSFORMATION PARAMETERS: GLOBAL TO LOCAL (UNDEFORMED
C     MEMBER) CONVENTION FOR MEMBER AXES: 1-AXIS FROM JOINT I TO J,
C     3-AXIS WITH SAME SENSE AS 3-GLOBAL, 2-AXIS TO FORM A
C     RIGHT-HANDED TRIAD.
C
      C3=DX1/XLEN(M)
      S3=DX2/XLEN(M)
C
C     ROTATIONAL TRANSFORMATION: GLOBAL TO INITIAL LOCAL.
C
      U1=C3*D1+S3*D2
      U2=-S3*D1+C3*D2
      U3=DU3
      RETURN
      END
CSTRN   10  10  -0
      SUBROUTINE STRN (M)
      REAL L
      COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      L=XLEN(M)
      STRAIN(M)=C(1,M)*(7.*U1**2-16.*U1*U4+16.*U4**2)+C(2,M)*(-4.*U1*U2+
     13.*L*U1*U3+8.*U4*U2-4.*L*U4*U3)+C(3,M)*(12.*U2**2-12.*L*U2*U3+4.*L
     2**2*U3**2)+C(4,M)*(18.*U1*U2**2+3.*L*U1*U2*U3+4.*L**2*U1*U3**2-12.
     3*L*U4*U2*U3-4.*L**2*U4*U3**2)+C(5,M)*U3**3+C(6,M)*(U2*U3**3*L**3+1
     48.*U3**2*U2**2*L**2-36.*U3*U2**3*L+72.*U2**4+2.*U3**4*L**4)
      RETURN
      END
CGRDT   10  10  -0
      SUBROUTINE GRDT (M)
      REAL L
      COMMON /DERIV/ DU(4)
```

```
      COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      L=XLEN(M)
      DU(1)=C(1,M)*(14.*U1-16.*U4)+C(2,M)*(-4.*U2+3.*L*U3)+C(4,M)*(18.*U
     12**2+3.*L*U2*U3+4.*L**2*U3**2)
      DU(4)=C(1,M)*(-16.*U1+32.*U4)+C(2,M)*(8.*U2-4.*L*U3)+C(4,M)*(-12.*
     1L*U2*U3-4.*L**2*U3**2)
      DU(2)=C(2,M)*(-4.*U1+8.*U4)+C(3,M)*(24.*U2-12.*L*U3)+C(4,M)*(36.*U
     11*U2+3.*L*U1*U3-12.*L*U4*U3)+C(6,M)*(U3**3*L**3+36.*U3**2*U2*L**2-
     2108.*U3*U2**2*L+288.*U2**3)
      DU(3)=C(2,M)*(3.*L*U1-4.*L*U4)+C(3,M)*(-12.*L*U2+8.*L**2*U3)+C(4,M
     1)*(3.*L*U1*U2+8.*L**2*U1*U3-12.*L*U4*U2-8.*L**2*U4*U3)+C(5,M)*3.*U
     23**2+C(6,M)*(3.*U2*U3**2*L**3+36.*U3*U2**2*L**2-36.*U2**3*L+8.*U3*
     1)*(3.*L*U1*U2+8.*L**2*U1*U3-12.*L*U4*U2-8.*L**2*U4*U3)+C(5,M)*3.*U
     23**2+C(6,M)*(3.*U2*U3**2*L**3+36.*U3*U2**2*L**2-36.*U2**3*L+8.*U3*
     3*3*L**4)
      RETURN
      END
CGRAD  10  10  -C
      SUBROUTINE GRAD (M)
      COMMON /DERIV/ DU(4)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MAINBK/ U(80),G(80)
      COMMON /MEMB/ C(6,20),STRAIN(20),XLEN(20),IP(20),IQ(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      I=IP(M)
      J = IQ(M)
      ALPHA = ACOS (DX1/XLEN(M))
      GAMMA = UI3 + ALPHA
      CB =  COS(GAMMA)
      SB =  SIN(GAMMA)
      IF(IJ(1,I).NE.0) G(IJ(1,I))=G(IJ(1,I))+(-CB*DU(1)+SB*DU(2))
```

```
      IF(IJ(1,J).NE.0) G(IJ(1,J))=G(IJ(1,J))+( CB*DU(1)-SB*DU(2))
      IF(IJ(2,I).NE.0) G(IJ(2,I))=G(IJ(2,I))+(-SB*DU(1)-CB*DU(2))
      IF(IJ(2,J).NE.0) G(IJ(2,J))=G(IJ(2,J))+( SB*DU(1)+CB*DU(2))
      IF(IJ(3,I).NE.0) G(IJ(3,I))=G(IJ(3,I)) - DU(3)
     1  +(-SB*(DU1+DX1)+CB*(DU2+DX2))*DU(1)+(-CB*(DU1+DX1)-SB*(DU2+DX2))
     2 *DU(2)
      IF(IJ(3,J).NE.0) G(IJ(3,J)) = G(IJ(3,J)) + DU(3)
      G(NDF+M)=DU(4)
      RETURN
      END
CPOTE  10  10  -0
      SUBROUTINE POTE (OMEG)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      OMEG=0.0E0
      DO 10 J=1,NJ
      DO 10 I=1,3
      OMEG=OMEG-XDJ(I,J)*F(I,J,ILOAD)
10    CONTINUE
      RETURN
      END
CFMFP  10  10  -0
      SUBROUTINE DFMFP (NH,N,F,EST,EPS,LIMIT,IER,H)
      DIMENSION H(NH)
      COMMON /MAINBK/ U(80),G(80)
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      CALL FUNCT (N,F)
      IER=0
      KOUNT=0
      N2=N+N
      N3=N2+N
      N31=N3+1
```

```
10    K=N31
      DO 40 J=1,N
      H(K)=1.E0
      NJ=N-J
      IF (NJ) 50,50,20
20    DO 30 L=1,NJ
      KL=K+L
30    H(KL)=0.E0
40    K=KL+1
50    KOUNT=KOUNT+1
      OLDF=F
      DO 90 J=1,N
      K=N+J
      H(K)=G(J)
      K=K+N
      H(K)=U(J)
      K=J+N3
      T=0.E0
      DO 80 L=1,N
      T=T-G(L)*H(K)
      IF (L-J) 60,70,70
60    K=K+N-L
      GO TO 80
70    K=K+1
80    CONTINUE
90    H(J)=T
      DY=0.E0
      HNRM=0.E0
      GNRM=0.E0
      DO 100 J=1,N
      HNRM=HNRM+ABS(H(J))
      GNRM=GNRM+ABS(G(J))
```

```
100   DY=DY+H(J)*G(J)
      IF (DY) 110,540,540
110   IF (HNRM/GNRM-EPS) 540,540,120
120   FY=F
      ALFA=2.E0*(EST-F)/DY
      AMBDA=1.E0
      IF (ALFA) 150,150,130
130   IF (ALFA-AMBDA) 140,150,150
140   AMBDA=ALFA
150   ALFA=0.E0
160   FX=FY
      DX=DY
      DO 170 I=1,N
170   U(I)=U(I)+AMBDA*H(I)
      CALL FUNCT (N,F)
      FY=F
      DY=0.E0
      DO 180 I=1,N
180   DY=DY+G(I)*H(I)
      IF (DY) 190,390,220
190   IF (FY-FX) 200,220,220
200   AMBDA=AMBDA+ALFA
      ALFA=AMBDA
      IF (HNRM*AMBDA-1.E10) 160,160,210
210   IER=2
      RETURN
220   T=0.E0
230   IF (AMBDA) 240,390,240
240   Z=3.E0*(FX-FY)/AMBDA+DX+DY
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))
      DALFA=Z/ALFA
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA
```

```
      IF (DALFA) 540,250,250
250   W=ALFA* SQRT(DALFA)
      ALFA=DY-DX+W+W
      IF (ALFA) 260,270,260
260   ALFA=(DY-Z+W)/ALFA
      GO TO 280
270   ALFA=(Z+DY-W)/(Z+DX+Z+DY)
280   ALFA=ALFA*AMBDA
      DO 290 I=1,N
290   U(I)=U(I)+(T-ALFA)*H(I)
      CALL FUNCT (N,F)
      IF (F-FX) 300,300,310
300   IF (F-FY) 390,390,310
310   DALFA=0.E0
      DO 320 I=1,N
320   DALFA=DALFA+G(I)*H(I)
      IF (DALFA) 330,360,360
330   IF (F-FX) 350,340,360
340   IF (DX-DALFA) 350,390,350
350   FX=F
      DX=DALFA
      T=ALFA
      AMBDA=ALFA
      GO TO 230
360   IF (FY-F) 380,370,380
370   IF (DY-DALFA) 380,390,380
380   FY=F
      DY=DALFA
      AMBDA=AMBDA-ALFA
      GO TO 220
390   IF (OLDF-F+EPS) 540,400,400
400   DO 410 J=1,N
```

```
      K=N+J                                                             FMFP1070
      H(K)=G(J)-H(K)                                                    FMFP1080
      K=N+K                                                             FMFP1090
  410 H(K)=U(J)-H(K)                                                    FMFP1100
      IER=0                                                             FMFP1110
      IF (KOUNT-N) 450,420,420                                          FMFP1120
  420 T=0.E0                                                            FMFP1130
      Z=0.E0                                                            FMFP1140
      DO 430 J=1,N                                                      FMFP1150
      K=N+J                                                             FMFP1160
      W=H(K)                                                            FMFP1170
      K=K+N                                                             FMFP1180
      T=T+ABS(H(K))                                                     FMFP1190
  430 Z=Z+W*H(K)                                                        FMFP1200
      IF (HNRM-EPS) 440,440,450                                         FMFP1210
  440 IF (T-EPS) 590,590,450                                            FMFP1220
  450 IF (KOUNT-LIMIT) 460,530,530                                      FMFP1230
  460 ALFA=0.E0                                                         FMFP1240
      DO 500 J=1,N                                                      FMFP1250
      K=J+N3                                                            FMFP1260
      W=0.E0                                                            FMFP1270
      DO 490 L=1,N                                                      FMFP1280
      KL=N+L                                                            FMFP1290
      W=W+H(KL)*H(K)                                                    FMFP1300
      IF (L-J) 470,480,480                                              FMFP1310
  470 K=K+N-L                                                           FMFP1320
      GO TO 490                                                         FMFP1330
  480 K=K+1                                                             FMFP1340
  490 CONTINUE                                                          FMFP1350
      K=N+J                                                             FMFP1360
      ALFA=ALFA+W*H(K)                                                  FMFP1370
  500 H(J)=W                                                            FMFP1380
```

```
      IF (Z*ALFA) 510,10,510                                             FMFP1390
510   K=N31                                                              FMFP1400
      DO 520 L=1,N                                                       FMFP1410
      KL=N2+L                                                            FMFP1420
      DO 520 J=L,N                                                       FMFP1430
      NJ=N2+J                                                            FMFP1440
      H(K)=H(K)+H(KL)*(H(NJ)/Z)-H(L)*(H(J)/ALFA)                         FMFP1450
520   K=K+1                                                              FMFP1460
      GO TO 50                                                           FMFP1470
530   IER=1                                                              FMFP1480
      RETURN                                                             FMFP1490
540   DO 550 J=1,N                                                       FMFP1500
      K=N2+J                                                             FMFP1510
550   U(J)=H(K)                                                          FMFP1520
      CALL FUNCT (N,F)                                                   FMFP1530
      IF (GNRM-EPS) 580,580,560                                          FMFP1540
560   IF (IER) 590,570,570                                               FMFP1550
570   IER=-1                                                             FMFP1560
      GO TO 10                                                           FMFP1570
580   IER=0                                                              FMFP1580
590   RETURN                                                             FMFP1590
      END                                                                FMFP1600
CYDEF  10  10  -0                                                               0
      SUBROUTINE YDEF                                                    YDEF  10
C     DUMMY ROUTINE FOR PLOTTING.                                        YDEF  20
      RETURN                                                             YDEF  30
      END                                                                YDEF  40
CBLOC  10  10  -0                                                               0
      BLOCK DATA                                                         BLOC  10
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ   BLOC  20
      DATA NDF/0/,IJ/63*0/                                               BLOC  30
      END                                                                BLOC  40
```

APPENDIX B

```
CNLIN  10  10   0
C
C     MAIN PROGRAM TO ANALYSE STRUCTURES THAT MAY BECOME GEOMETRICALLY
C     NONLINEAR.  THE ELEMENT MATERIAL MAY BE NONLINEARLY ELASTIC
C     SINCE THE STRAIN ENERGY OF EACH ELEMENT IS INTEGRATED NUMERICALLY.
C
C     WRITTEN IN AMERICAN STANDARD FORTRAN BY J. C. BRADSHAW, III.
C
C     CARD 1 :  TITLE CARD.  FORMAT(20A4)
C
C     CARD 2 :  CONTROL CARD.  FORMAT  (10X,6I5,4E10.0)
C     NJ=NUMBER OF JOINTS TO BE INPUT.
C     NM=NUMBER OF ELEMENTS TO BE INPUT.
C     NF=NUMBER OF FORCES TO READ IN THE FORCING FUNCTION.
C     NINF=OPTION NOT TO READ FORCES IN THE FORCING FUNCTION, BUT
C          TO INPUT FORCES THROUGH THE LOAD INCREMENT FEATURE. (NE.0)
C     LIMIT=LIMIT ON THE NUMBER OF MINIMIZATIONS.
C     NPLOT=OPTION TO PLOT LOAD VERSUS DEFLECTION.   (GT.0)
C     EPS=DESIRED ACCURACY OR ROUND-OFF OF GRADIENTS.
C     EST=AN ESTIMATE OF THE FUNCTION VALUE.
C     FACT=FACTOR TO NORMALIZE LOADS FOR PLOTTING.
C     DFAC=FACTOR TO NORMALIZE DISPLACEMENTS FOR OUTPUT.
C
C     CARD 3 : MATERIAL PROPERTIES.  FORMAT(4D10.0)
C     STN,STRS=PAIRS OF STRAIN AND CORRESPONDING STRESS.
C
C     CARDS 4 :  JOINT COORDINATES AND STATUS.
C                FORMAT(I5,5X,2E10.0,2X,3A1)
C     I=JOINT NUMBER.
C     X=X OR 1 DIRECTION COORDINATE.
C     Y=Y OR 2 DIRECTION COORDINATE.
C     IR(1)="R" IF RESTRAINED IN THE 1 DIRECTION.
```

```
C       IR(2)="R" IF RESTRAINED IN THE 2 DIRECTION.
C       IR(3)="R" IF RESTRAINED IN THE 3 DIRECTION.
C
C       CARDS 5 :  ELEMENT CARDS.  FORMAT(3I5,5X,3D10.0,2I5)
C       M=ELEMENT NUMBER.
C       IP=MEMBER START.
C       IQ=MEMBER END.
C       B=WIDTH OF THE ELEMENT.
C       H=HEIGHT OF THE ELEMENT.
C       A=DISTANCE OF REFERENCE AXIS FROM CENTER OF ELEMENT.
C       NK=NO. OF GAUSS POINTS ALONG THE LENGTH OF THE ELEMENT.
C       MK=NO. OF GAUSS POINTS ACROSS THE HEIGHT OF THE ELEMENT.
C
C       CARDS 6 :  JOINT FORCING FUNCTION.  FORMAT(8D10.0)
C
C       CARDS 7 :  JOINT LOADS.  FORMAT(2I5,2D10.0)
C       IJOI=JOINT TO BE LOADED.
C       IDIR=DIRECTION OF LOAD (1,2, OR 3).
C       DS=LOAD MULTIPLICATION CONSTANT.
C       ADD=LOAD INCREMENT VALUE.
C
C       JOINT LOADS MUST BE ENDED WITH A BLANK CARD.
C
C       ENTIRE DATA DECK MUST BE ENDED WITH TWO BLANK CARDS.
C
        COMMON /CONBK/STN(2),STRS(2),E(2)
        COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
        COMMON /MAINBK/ U(80),G(80)
        COMMON /MEMB/ STRAIN(20),XLEN(20),B(20),A(20),H(20),IP(20),IQ(20),
       1              NKM(20),MKM(20),NM
        COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
        COMMON /PLOTS/ YD(3,21,40),P(41),NF
```

```
      COMMON /SAVBK/ ETOP1,ETOP2,EBOT1,EBOT2
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      DIMENSION TITLE(20), IR(3,21), HN(3480), FOR(40)
      DATA IRES/1HR/,FOR/40*0.E0/
 20   CONTINUE
      READ 160, TITLE
      READ 170,    NJ,NM,NF,NINF,LIMIT,NPLOT,EPS,EST,FACT,DFAC
      IF (NJ.LE.0) GO TO 140
      IF (FACT.EQ.0.E0) FACT = 1.E0
      IF (DFAC.EQ.0.E0) DFAC = 1.E0
      IF (LIMIT.EQ.0) LIMIT=40
      IF (EPS.EQ.0.E0) EPS=1.E-7
      PRINT 150
      PRINT 220, TITLE
      READ 175, (STN(I),STRS(I),I=1,2)
      E(1) = STRS(1)/STN(1)
      E(2) = (STRS(2)-STRS(1))/(STN(2)-STN(1))
      PRINT 225,STN,STRS
      READ 180, (I,X(I),Y(I),IR(1,I),IR(2,I),IR(3,I),K=1,NJ)
      PRINT 370
      PRINT 230
C
C     DETERMINE THE JOINT STATUS, AND INDEX FOR ASSEMBLY.
C
      DO 50 J=1,NJ
      DO 40 I=1,3
      IJ(I,J)=0
      XCJ(I,J)=0.E0
      IF (IR(I,J).EQ.IRES) GO TO 30
      NDF=NDF+1
      IJ(I,J)=NDF
 30   CONTINUE
```

```
C      INITIALIZE THE FORCE MATRIX.                                          NLIN 960
       DO 40 K=1,NF                                                          NLIN 970
       F(I,J,K)=0.E0                                                         NLIN 980
       F(I,J,K)=0.E0                                                         NLIN 980
40     CONTINUE                                                              NLIN 990
       PRINT 240, J,X(J),Y(J),IJ(1,J),IJ(2,J),IJ(3,J)                        NLIN1000
50     CONTINUE                                                              NLIN1010
       N=NDF+NM                                                              NLIN1020
       NH=N*(N+7)/2                                                          NLIN1030
       PRINT 370                                                             NLIN1040
       PRINT 250                                                             NLIN1050
       DO 60 K=1,NM                                                          NLIN1060
       READ 190, M,IP(M),IQ(M),B(M),H(M),A(M),NKM(M),MKM(M)                  NLIN1070
       XL=(X(IQ(M))-X(IP(M)))**2+(Y(IQ(M))-Y(IP(M)))**2                      NLIN1080
       XL=SQRT(XL)                                                           NLIN1090
       AX = B(M)*H(M)                                                        NLIN1100
       ZI = B(M)*H(M)**3/12.E0 + AX*A(M)**2                                  NLIN1110
       PRINT 260, M,IP(M),IQ(M),B(M),H(M),A(M),XL,AX,ZI                      NLIN1120
C                                                                            NLIN1130
C      SET DEFAULT VALUES ON THE NUMBER OF GAUSS POINTS.                     NLIN1140
C                                                                            NLIN1150
       IF(NKM(M).EQ.0) NKM(M) = 4                                            NLIN1160
       IF(MKM(M).EQ.0) MKM(M) = 2                                            NLIN1170
       XLEN(M)=XL                                                            NLIN1180
60     CONTINUE                                                              NLIN1190
C                                                                            NLIN1200
C      PRINT OUT THE NUMBER OF GAUSS POINTS FOR EACH ELEMENT.                NLIN1210
C                                                                            NLIN1220
       PRINT 264                                                             NLIN1230
       DO 65 M=1,NM                                                          NLIN1240
       PRINT 266, M,NKM(M),MKM(M)                                            NLIN1250
65     CONTINUE                                                              NLIN1260
```

```
      PRINT 370
      IF (DFAC.NE.1.E0) PRINT 365, DFAC
      IF (NINF.NE.0) GO TO 70
      READ 200, (FOR(I),I=1,NF)
70    CONTINUE
      READ (5,210) IJOI,IDIR,DS,ADD
      IF (IJOI.EQ.0) GO TO 90
      IF (DS.EQ.0.E0) DS=1.E0
      AD=ADD
      DO 80 ILOAD=1,NF
      F(IDIR,IJOI,ILOAD)=(FOR(ILOAD)+AD)*DS
      AD=AD+ADD
80    CONTINUE
      GO TO 70
90    CONTINUE
      DO 100 I=1,N
      U(I)=0.E0
100   CONTINUE
      DO 130 IL=1,NF
      NMIN=-1
      ILOAD=IL
C
C     PRINT OUT THE JOINT LOAD MATRIX.
C
      PRINT 370
      PRINT 345, ILOAD
      PRINT 350
      DO 110 J=1,NJ
      PRINT 340, J,(F(I,J,ILOAD),I=1,3)
110   CONTINUE
C
C     CALL THE MINIMIZATION ROUTINE FOR EACH LOAD STEP.
```

```
C
      CALL DFMFP (NH,N,FUN,EST,EPS,LIMIT,IER,HN)
C
C     PRINT OUT THE RESULTS FROM THE MINIMIZATION PROCESS.
C
      IF (IER.EQ.0) PRINT 270
      IF (IER.EQ.1) PRINT 280
      IF (IER.EQ.-1) PRINT 290
      IF (IER.EQ.2) PRINT 300
      PRINT 310, KOUNT,NMIN
      PRINT 320, FUN,(G(I),I=1,N)
      PRINT 330
      DO 120 J=1,NJ
      DO 118 I=1,2
118   YD(I,J,ILOAD)=XDJ(I,J)/DFAC
      YD(3,J,ILOAD) = XDJ(3,J)
      PRINT 340, J,(YD(I,J,ILOAD),I=1,3)
120   CONTINUE
      PX = ABS(F(1,NJ,ILOAD))
      P(ILOAD) = PX/FACT
      P(NF+1) = 0.E0
      PRINT 342
      DO 124 M=1,NM
      PRINT 343, M,U(NDF+M)
124   CONTINUE
      PRINT 344
      DO 115 M=1,NM
      CALL DEFC(M)
      U4 = U(NDF+M)
      MNEG = -M
      CALL STNG(MNEG)
      PRINT 346, M,IP(M),ETOP1,EBOT1
```

```
      PRINT 346, M,IQ(M),ETOP2,EBOT2                                    NLIN1910
  115 CONTINUE                                                          NLIN1920
  130 CONTINUE                                                          NLIN1930
      IF (NPLOT.GT.0) CALL YCEF                                         NLIN1940
      GO TO 20                                                          NLIN1950
  140 CONTINUE                                                          NLIN1960
      PRINT 150                                                         NLIN1970
      PRINT 360                                                         NLIN1980
      STOP                                                              NLIN1990
C                                                                       NLIN2000
  150 FORMAT (1H1)                                                      NLIN2010
  160 FORMAT (20A4)                                                     NLIN2020
  170 FORMAT (  10X,6I5,4E10.0)                                         NLIN2030
  175 FORMAT(4E10.0,40X)                                                NLIN2040
  180 FORMAT (I5,5X,2E10.0,2X,3A1,45X)                                  NLIN2050
  190 FORMAT (3I5,5X,3E10.0,2I5,20X)                                    NLIN2060
  200 FORMAT (8E10.0)                                                   NLIN2070
  210 FORMAT (2I5,2E10.0,50X)                                           NLIN2080
  220 FORMAT (1H0,T10,20A4//)                                           NLIN2090
  225 FORMAT (1H0,T10,21HMATERIAL PROPERTIES ://T10,8HSTRAIN :,5X,      NLIN2100
     1       2(E16.8,2X)//T10,8HSTRESS :,5X,2(E16.8,2X)///)             NLIN2110
  230 FORMAT (1H0,T20,28HJOINT COORDINATES AND STATUS//T7,5HJOINT,9X,1HXNLIN2120
     1,12X,1HY,9X,2HDX,3X,2HDY,3X,2HDZ//)                               NLIN2130
  240 FORMAT (1H ,T9,I2,6X,2(F8.3,5X),3(I3,2X)/)                        NLIN2140
  250 FORMAT (1H0,T29,18HELEMENT PROPERTIES//T6,7HELEMENT,4X,5HSTART,4X,NLIN2150
     13HEND,3X,5HWIDTH,6X,6HHEIGHT,9X,1HA,7X,6HLENGTH,7X,4HAREA,6X,7HINENLIN2160
     2RTIA//)                                                           NLIN2170
  260 FORMAT (1H ,T9,I2,7X,I3,4X,I3,6(4X,F8.4)/)                        NLIN2180
  264 FORMAT (1H0,T10,24HNUMBER OF GAUSS POINTS ://T6,7HELEMENT,4X,     NLIN2190
     1       6HX-AXIS,4X,6HY-AXIS/)                                     NLIN2200
  266 FORMAT (1H0,T9,I2,8X,I2,8X,I2/)                                   NLIN2210
  270 FORMAT (1H0,T10,24HCONVERGENCE WAS OBTAINED/)                     NLIN2220
```

```
280   FORMAT (1H0,T10,34HNO CONVERGENCE IN LIMIT ITERATIONS/)
290   FORMAT (1H0,T10,30HERRORS IN GRADIENT CALCULATION/)
300   FORMAT (1H0,T10,33HLINEAR SEARCH TECHNIQUE INDICATES/T10,41HIT IS 
     1LIKELY THAT THERE EXISTS NO MINIMUM/)
310   FORMAT (1H0,T10,I4,29H MINIMIZATIONS HAVE BEEN MADE/T10,I4,27H FUN
     1CTION SUBROUTINE CALLS.///)
320   FORMAT (1H0,T10,17HFUNCTION VALUE = ,E16.8,//T10,11HGRADIENTS :// 
     1  20(T10,6(E16.8,2X)/))
330   FORMAT (1H0,T24,19HJOINT DISPLACEMENTS//T7,5HJOINT,12X,2HDX,19X,2H
     1DY,19X,2HDZ//)
340   FORMAT (1H ,T9,I2,6X,3(E16.7,5X)/)
342   FORMAT(1H0,T17,26HINTERNAL NODE DISPLACEMENT//T18,7HELEMENT,12X,
     1      2HU4//)
343   FORMAT(1H ,T21,I2,7X,E16.8/)
345   FORMAT(1H ,T10,10HLOAD STEP ,I2/)
350   FORMAT (1H0,T28,11HJOINT LOADS//T7,5HJOINT,13X,1HX,20X,1HY,20X,1HZ
     1//)
344   FORMAT(1H0/T20,20HELEMENT STRAIN STATE//T6,7HELEMENT,2X,5HJOINT,
     1      8X,10HTOP STRAIN,9X,13HBOTTOM STRAIN//)
346   FORMAT(1H ,T9,I2,6X,I2,2(5X,E16.8)/)
360   FORMAT (1H0,T10,46H=====  NORMAL COMPLETION OF THE PROGRAM  =====/
     1/)
365   FORMAT (1H0,T6,50HDISPLACEMENTS HAVE BEEN NORMALIZED BY A FACTOR O
     1F ,F16.8///)
370   FORMAT (1H0,60(1H-)//)
      END
CFUNC  10  10  -0
      SUBROUTINE FUNCT (N,FUN)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MAINEK/ U(80),G(80)
      COMMON /MEMB/ STRAIN(20),XLEN(20),B(20),A(20),H(20),IP(20),IQ(20),
     1              NKM(20),MKM(20),NM
```

```
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      NMIN=NMIN+1
      FUN=0.E0
      DO 10 J=1,NJ
      DO 10 I=1,3
      IF (IJ(I,J).EQ.0) GO TO 10
C
C     PUT VALUES INTO THE JOINT DISPLACEMENT MATRIX FROM THE
C     GENERALIZED COORDINATES MATRIX.
C
      XDJ(I,J)=U(IJ(I,J))
C
C     CALCULATE THE PART OF EACH GRADIENT WHICH IS DERIVED FROM
C     THE EXTERNAL WORK.
C
      G(IJ(I,J))=-F(I,J,ILOAD)
10    CONTINUE
      DO 20 M=1,NM
      U4=U(NDF+M)
      CALL DEFC (M)
      CALL STNG (M)
      CALL GRAD (M)
C
C     ACCUMULATE THE STRAIN ENERGY FROM EACH ELEMENT.
C
      FUN=FUN+STRAIN(M)
20    CONTINUE
C
C     COMPUTE THE TOTAL EXTERNAL WORK.
C
      CALL POTE (OMEG)
```

```
C
C     CALCULATE THE TOTAL WORK OF THE SYSTEM.
C
      FUN=FUN+OMEG
      RETURN
      END
CDEFO  10  10  -0
      SUBROUTINE DEFO (M)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MEMB/ STRAIN(20),XLEN(20),B(20),A(20),H(20),IP(20),IQ(20),
     1              NKM(20),MKM(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      I=IP(M)
      J=IQ(M)
      DX1=(X(J)-X(I))
      DX2=(Y(J)-Y(I))
      DU1=(XDJ(1,J)-XDJ(1,I))
      DU2=(XDJ(2,J)-XDJ(2,I))
      UI3=XDJ(3,I)
      DU3=XDJ(3,J)-UI3
C
C     GEOMETRICALLY NONLINEAR: UNLIMITED RIGID-BODY MOTIONS,
C     BEAM-COLUMN DISTORTIONS.
C     ROTATIONAL TRANSFORMATION PARAMETERS: GLOBAL TO DEFORMED JOINT I.
C
      SI3= SIN(UI3)
      CI3= COS(UI3)
      SI32= SIN(UI3/2.E0)**2
C
C     DISTORTIONS OF MEMBER M IN JOINT-I COORDINATES (TRANSLATIONAL).
C
      D1=-2.E0*SI32*DX1+SI3*(DX2+DU2)+CI3*DU1
```

```
      D2=-SI3*(DX1+DU1)-2.E0*SI32*DX2+CI3*DU2
C
C     ROTATIONAL TRANSFORMATION PARAMETERS: GLOBAL TO LOCAL (UNDEFORMED
C     MEMBER) CONVENTION FOR MEMBER AXES: 1-AXIS FROM JOINT I TO J,
C     3-AXIS WITH SAME SENSE AS 3-GLOBAL, 2-AXIS TO FORM A
C     RIGHT-HANDED TRIAD.
C
      C3=DX1/XLEN(M)
      S3=DX2/XLEN(M)
C
C     ROTATIONAL TRANSFORMATION: GLOBAL TO INITIAL LOCAL.
C
      U1=C3*D1+S3*D2
      U2=-S3*D1+C3*D2
      U3=DU3
      RETURN
      END
CSTNG  10  10   0
      SUBROUTINE STNG (M)
      REAL L
      COMMON /CONBK/STN(2),STRS(2),E(2)
      COMMON /DERIV/ DU(4)
      COMMON /GPTS/ AH(10),X(10),BH(10),T(10),NK,MK
      COMMON /MEMB/ STRAIN(20),XLEN(20),B(20),A(20),H(20),IP(20),IQ(20),
     1              NKM(20),MKM(20),NM
      COMMON /SAVBK/ ETOP1,ETOP2,EBOT1,EBOT2
      COMMON /SHAPE/ UP,VP,VPP,S,Y,P1P,P2P,P3P,P4P,P2PP,P3PP,L
      DX(PXP) = PXP/L
      DY(PXP,PXPP) = (VP*PXP -Y*PXPP/L)/L
      L = XLEN(IABS(M))
      IF(M.LT.0) GO TO 30
      STRAIN(M) = 0.E0
```

```
      NK = NKM(M)                                                       STNG 150
      MK = MKM(M)                                                       STNG 160
      CALL GAUSS                                                        STNG 170
      CONST = E(M)*L/2.E0*(H(M)/2.E0)                                   STNG 180
      DO 10 K=1,4                                                       STNG 190
      DU(K) = 0.E0                                                      STNG 200
10    CONTINUE                                                          STNG 210
C     GAUSSIAN QUADRATURE.                                              STNG 220
      DO 20 K=1,NK                                                      STNG 230
      DO 20 J=1,MK                                                      STNG 240
C     CHANGING THE ARGUMENT SO THAT THE LIMITS FOR GAUSS INTEGRATION    STNG 250
C     ARE SATISFIED ALONG THE AXIS.                                     STNG 260
      S=(X(K)+1)/2.                                                     STNG 270
C     CHANGING THE ARGUMENT SO THAT THE LIMITS FOR GAUSS INTEGRATION    STNG 280
C     ARE SATISFIED ACROSS THE HEIGHT OF THE ELEMENT.                   STNG 290
      Y = A(M) + T(J)*H(M)/2.E0                                         STNG 300
C     THE COMBINED WEIGHTING FACTORS AND THE CONSTANT VALUES.           STNG 310
      BHK = AH(K)*BH(J)*CONST                                           STNG 320
      AHK = BHK*E(1)/2.E0                                               STNG 330
C     THE STRAIN AT THE GAUSS POINT (X,Y) .                             STNG 340
      CALL SRAN(STRN)                                                   STNG 350
C     THE STRAIN ENERGY AT THE GAUSS POINT (X,Y) .                      STNG 360
      USTAR = BHK*(-E(1)*STN(1)**2/2.E0+E(1)*STN(1)* ABS(STRN)          STNG 370
     1        +E(2)/2.E0*( ABS(STRN)-STN(1))**2)                        STNG 380
      IF( ABS(STRN).LE.STN(1)) USTAR = AHK*STRN**2                      STNG 390
      STRAIN(M) = STRAIN(M) + USTAR                                     STNG 400
C     CALCULATE THE GRADIENTS.                                          STNG 410
      SGN = 0.E0                                                        STNG 420
      IF (STRN.NE.0.E0) SGN =  ABS(STRN)/STRN                           STNG 430
      VALUE = BHK*((E(1)-E(2))*STN(1)+E(2)* ABS(STRN))*SGN              STNG 440
      IF( ABS(STRN).LE.STN(1)) VALUE = 2.E0*AHK*STRN                    STNG 450
      DU(1) = DU(1) + VALUE*DX(P1P)                                     STNG 460
```

```
      DU(2) = DU(2) + VALUE*DY(P2P,P2PP)
      DU(3) = DU(3) + VALUE*DY(P3P,P3PP)
      DU(4) = DU(4) + VALUE*DX(P4P)
20    CONTINUE
      GO TO 40
30    CONTINUE
      M = IABS(M)
      S = 0.E0
      Y= A(M) + H(M)/2.E0
      CALL SRAN(ETOP1)
      Y = A(M) - H(M)/2.E0
      CALL SRAN(EBOT1)
      S = 1.E0
      CALL SRAN(EBOT2)
      Y = A(M) + H(M)/2.E0
      CALL SRAN(ETOP2)
40    CONTINUE
      RETURN
      END
CSRAN  10  10    0
      SUBROUTINE SRAN(STRN)
      REAL L
      COMMON /SHAPE/ UP,VP,VPP,S,Y,P1P,P2P,P3P,P4P,P2PP,P3PP,L
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
C     THE DERIVATIVES OF THE SHAPE FUNCTIONS :
      P1P=4.*S-1.
      P2P=6.*(-S**2+S)
      P3P=L*(3.*S**2-2.*S)
      P4P=4.*(-2.*S+1.)
      P2PP=6.*(-2.*S+1.)
      P3PP=L*(6.*S-2.)
C     THE COMPONENTS OF THE STRAIN FUNCTION :
```

```
      UP = (P1P*U1+P4P*U4)/L
      VP = (P2P*U2+P3P*U3)/L
      VPP = (P2PP*U2+P3PP*U3)/L**2
C     THE STRAIN IN THE ELEMENT AT THE POINT (S,Y).
      STRN = UP + .5E0*VP**2 - Y*VPP
      RETURN
      END
CGAUS  10  10   0
      SUBROUTINE GAUSS
      COMMON /GPTS/ AH(10),X(10),BH(10),T(10),NK,MK
      DIMENSION AA(10,7), AX(10,7)
      DATA AA/2*1.E0,8*0.E0,.88888889,2*.55555556,7*0.E0,2*.65214515,2*.
     134785485,6*0.E0,.56888889,2*.47862867,2*.23692689,5*0.E0,2*.467913
     293,2*.36076157,2*.17132449,4*0.E0,.41795918,2*.38183005,2*.2797053
     39,2*.12948497,3*0.E0,2*.36268378,2*.31370665,2*.22238103,2*.101228
     454,2*0.E0/
      DATA AX/.57735027,-.57735027,8*0.E0,0.E0,.77459667,-.77459667,7*0.
     1E0,.33998104,-.33998104,.86113631,-.86113631,6*0.E0,0.E0,.53846931
     2,-.53846931,.90617985,-.90617985,5*0.E0,.23861919,-.23861919,.6612
     30939,-.66120939,.93246951,-.93246951,4*0.E0,0.E0,.40584515,-.40584
     4515,.74153119,-.74153119,.94910791,-.94910791,3*0.E0,.18343464,-.1
     58343464,.52553241,-.52553241,.79666648,-.79666648,.96028986,-.9602
     68986,2*0.E0/
      DO 10 K=1,NK
      AH(K)=AA(K,NK-1)
      X(K)=AX(K,NK-1)
10    CONTINUE
      DO 20 K=1,MK
      BH(K)=AA(K,MK-1)
      T(K)=AX(K,MK-1)
20    CONTINUE
      RETURN
```

```
      END
CGRAD  10  10   0
      SUBROUTINE GRAD (M)
      COMMON /DERIV/ DU(4)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /MAINBK/ U(80),G(80)
      COMMON /MEMB/ STRAIN(20),XLEN(20),B(20),A(20),H(20),IP(20),IQ(20),
     1              NKM(20),MKM(20),NM
      COMMON /TRANS/ U1,U2,U3,U4,DX1,DX2,DU1,DU2,C3,S3,SI3,CI3,UI3
      I=IP(M)
      J = IQ(M)
      ALPHA =   ACOS(DX1/XLEN(M))
      GAMMA = UI3 + ALPHA
      CB =  COS(GAMMA)
      SB =  SIN(GAMMA)
      IF(IJ(1,I).NE.0) G(IJ(1,I))=G(IJ(1,I))+(-CB*DU(1)+SB*DU(2))
      IF(IJ(1,J).NE.0) G(IJ(1,J))=G(IJ(1,J))+( CB*DU(1)-SB*DU(2))
      IF(IJ(2,I).NE.0) G(IJ(2,I))=G(IJ(2,I))+(-SB*DU(1)-CB*DU(2))
      IF(IJ(2,J).NE.0) G(IJ(2,J))=G(IJ(2,J))+( SB*DU(1)+CB*DU(2))
      IF(IJ(3,I).NE.0) G(IJ(3,I))=G(IJ(3,I)) - DU(3)
     1  +(-SB*(DU1+DX1)+CB*(DU2+DX2))*DU(1)+(-CB*(DU1+DX1)-SB*(DU2+DX2))
     2 *DU(2)
      IF(IJ(3,J).NE.0) G(IJ(3,J)) = G(IJ(3,J)) + DU(3)
      G(NDF+M)=DU(4)
      RETURN
      END
CPOTE  10  10   0
      SUBROUTINE POTE (OMEG)
      COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      OMEG=0.0E0
      DO 10 J=1,NJ
```

```
      DO 10 I=1,3
      OMEG=OMEG-XDJ(I,J)*F(I,J,ILOAD)
10    CONTINUE
      RETURN
      END
DFMFP  10  10   0
      SUBROUTINE DFMFP (NH,N,F,EST,EPS,LIMIT,IER,H)
      DIMENSION H(NH)
      COMMON /MAINBK/ U(80),G(80)
      COMMON /TAB/ ILOAD,NMIN,KOUNT
      CALL FUNCT (N,F)
      IER=0
      KOUNT=0
      N2=N+N
      N3=N2+N
      N31=N3+1
10    K=N31
      DO 40 J=1,N
      H(K)=1.E0
      NJ=N-J
      IF (NJ) 50,50,20
20    DO 30 L=1,NJ
      KL=K+L
30    H(KL)=0.E0
40    K=KL+1
50    KOUNT=KOUNT+1
      OLDF=F
      DO 90 J=1,N
      K=N+J
      H(K)=G(J)
      K=K+N
      H(K)=U(J)
```

```
      K=J+N3
      T=0.E0
      DO 80 L=1,N
      T=T-G(L)*H(K)
      IF (L-J) 60,70,70
60    K=K+N-L
      GO TO 80
70    K=K+1
80    CONTINUE
90    H(J)=T
      DY=0.E0
      HNRM=0.E0
      GNRM=0.E0
      DO 100 J=1,N
      HNRM=HNRM+ABS(H(J))
      GNRM=GNRM+ABS(G(J))
100   DY=DY+H(J)*G(J)
      IF (DY) 110,540,540
110   IF (HNRM/GNRM-EPS) 540,540,120
120   FY=F
      ALFA=2.E0*(EST-F)/DY
      AMBDA=1.E0
      IF (ALFA) 150,150,130
130   IF (ALFA-AMBDA) 140,150,150
140   AMBDA=ALFA
150   ALFA=0.E0
160   FX=FY
      DX=DY
      DO 170 I=1,N
170   U(I)=U(I)+AMBDA*H(I)
      CALL FUNCT (N,F)
      FY=F
```

```
      DY=0.E0                                                           FMFP 590
      DO 180 I=1,N                                                      FMFP 600
  180 DY=DY+G(I)*H(I)                                                   FMFP 610
      IF (DY) 190,390,220                                               FMFP 620
  190 IF (FY-FX) 200,220,220                                            FMFP 630
  200 AMBDA=AMBDA+ALFA                                                  FMFP 640
      ALFA=AMBDA                                                        FMFP 650
      IF (HNRM*AMBDA-1.E10) 160,160,210                                 FMFP 660
  210 IER=2                                                             FMFP 670
      RETURN                                                            FMFP 680
  220 T=0.E0                                                            FMFP 690
  230 IF (AMBDA) 240,390,240                                            FMFP 700
  240 Z=3.E0*(FX-FY)/AMBDA+DX+DY                                        FMFP 710
      ALFA=AMAX1(ABS(Z),ABS(DX),ABS(DY))                                FMFP 720
      DALFA=Z/ALFA                                                      FMFP 730
      DALFA=DALFA*DALFA-DX/ALFA*DY/ALFA                                 FMFP 740
      IF (DALFA) 540,250,250                                            FMFP 750
  250 W=ALFA* SQRT(DALFA)                                               FMFP 760
      ALFA=DY-DX+W+W                                                    FMFP 770
      IF (ALFA) 260,270,260                                             FMFP 780
  260 ALFA=(DY-Z+W)/ALFA                                                FMFP 790
      GO TO 280                                                         FMFP 800
  270 ALFA=(Z+DY-W)/(Z+DX+Z+DY)                                         FMFP 810
  280 ALFA=ALFA*AMBDA                                                   FMFP 820
      DO 290 I=1,N                                                      FMFP 830
  290 U(I)=U(I)+(T-ALFA)*H(I)                                           FMFP 840
      CALL FUNCT (N,F)                                                  FMFP 850
      IF (F-FX) 300,300,310                                             FMFP 860
  300 IF (F-FY) 390,390,310                                             FMFP 870
  310 DALFA=0.E0                                                        FMFP 880
      DO 320 I=1,N                                                      FMFP 890
  320 DALFA=DALFA+G(I)*H(I)                                             FMFP 900
```

```
      IF (DALFA) 330,360,360                                            FMFP 910
  330 IF (F-FX) 350,340,360                                             FMFP 920
  340 IF (DX-DALFA) 350,390,350                                         FMFP 930
  350 FX=F                                                              FMFP 940
      DX=DALFA                                                          FMFP 950
      T=ALFA                                                            FMFP 960
      AMBDA=ALFA                                                        FMFP 970
      GO TO 230                                                         FMFP 980
  360 IF (FY-F) 380,370,380                                             FMFP 990
  370 IF (DY-DALFA) 380,390,380                                         FMFP1000
  380 FY=F                                                              FMFP1010
      DY=DALFA                                                          FMFP1020
      AMBDA=AMBDA-ALFA                                                  FMFP1030
      GO TO 220                                                         FMFP1040
  390 IF (OLDF-F+EPS) 540,400,400                                       FMFP1050
  400 DO 410 J=1,N                                                      FMFP1060
      K=N+J                                                             FMFP1070
      H(K)=G(J)-H(K)                                                    FMFP1080
      K=N+K                                                             FMFP1090
  410 H(K)=U(J)-H(K)                                                    FMFP1100
      IER=0                                                             FMFP1110
      IF (KOUNT-N) 450,420,420                                          FMFP1120
  420 T=0.E0                                                            FMFP1130
      Z=0.E0                                                            FMFP1140
      DO 430 J=1,N                                                      FMFP1150
      K=N+J                                                             FMFP1160
      W=H(K)                                                            FMFP1170
      K=K+N                                                             FMFP1180
      T=T+ABS(H(K))                                                     FMFP1190
  430 Z=Z+W*H(K)                                                        FMFP1200
      IF (HNRM-EPS) 440,440,450                                         FMFP1210
  440 IF (T-EPS) 590,590,450                                            FMFP1220
```

```
450    IF (KOUNT-LIMIT) 460,530,530                                FMFP1230
460    ALFA=0.E0                                                   FMFP1240
       DO 500 J=1,N                                                FMFP1250
       K=J+N3                                                      FMFP1260
       W=0.E0                                                      FMFP1270
       DO 490 L=1,N                                                FMFP1280
       KL=N+L                                                      FMFP1290
       W=W+H(KL)*H(K)                                              FMFP1300
       IF (L-J) 470,480,480                                        FMFP1310
470    K=K+N-L                                                     FMFP1320
       GO TO 490                                                   FMFP1330
480    K=K+1                                                       FMFP1340
490    CONTINUE                                                    FMFP1350
       K=N+J                                                       FMFP1360
       ALFA=ALFA+W*H(K)                                            FMFP1370
500    H(J)=W                                                      FMFP1380
       IF (Z*ALFA) 510,10,510                                      FMFP1390
510    K=N31                                                       FMFP1400
       DO 520 L=1,N                                                FMFP1410
       KL=N2+L                                                     FMFP1420
       DO 520 J=L,N                                                FMFP1430
       NJ=N2+J                                                     FMFP1440
       H(K)=H(K)+H(KL)*(H(NJ)/Z)-H(L)*(H(J)/ALFA)                  FMFP1450
520    K=K+1                                                       FMFP1460
       GO TO 50                                                    FMFP1470
530    IER=1                                                       FMFP1480
       RETURN                                                      FMFP1490
540    DO 550 J=1,N                                                FMFP1500
       K=N2+J                                                      FMFP1510
550    U(J)=H(K)                                                   FMFP1520
       CALL FUNCT (N,F)                                            FMFP1530
       IF (GNRM-EPS) 580,580,560                                   FMFP1540
```

```
560    IF (IER) 590,570,570                                              FMFP1550
570    IER=-1                                                            FMFP1560
       GO TO 10                                                          FMFP1570
580    IER=0                                                             FMFP1580
590    RETURN                                                            FMFP1590
       END                                                               FMFP1600
CYDEF   10  10   0                                                              0
C      DUMMY ROUTINE FOR PLOTTING.                                       YDEF  10
       SUBROUTINE YDEF                                                   YDEF  20
       RETURN                                                            YDEF  30
       END                                                               YDEF  40
CBLOK   10  10   0                                                              0
       BLOCK DATA                                                        BLOK  10
       COMMON /JOINTS/ X(21),Y(21),XDJ(3,21),F(3,21,40),IJ(3,21),NDF,NJ  BLOK  20
       DATA NDF/0/,IJ/63*0/                                              BLOK  30
       END                                                               BLOK  40
```

APPENDIX C

```
CSTAT  10  10   0
C
C      THIS PROGRAM IS WRITTEN IN FORTRAN EXTENDED FOR A CDC 6600,
C      VERSION 3.4.  IT MAY BE USED IN WATSIX OR WATFIV ON AN IBM 370
C      BY CHANGING THE END-OF-FILE (EOF) STATEMENTS TO THE FORM USED
C      WITH IBM COMPUTERS.
C
C      WRITTEN BY J. C. BRADSHAW, III
C
C      THIS PROGRAM IS BASED ON AN ALGORITHM PRESENTED BY GURFINKEL
C      AND ROBINSON AT THE UNIVERSITY OF ILLINOIS, *DETERMINATION OF
C      STRAIN DISTRIBUTION AND CURVATURE IN A REINFORCED CONCRETE
C      SECTION SUBJECTED TO BENDING MOMENT AND LONGITUDINAL LOAD*,
C      ACI JOURNAL, JULY 1967.  THE METHOD USED IS AN ITERATIVE PROCESS
C      BASED ON A TAYLOR EXPANSION.  A DISADVANTAGE TO THIS PROGRAM
C      IS THAT THE LOADING MUST BE KNOWN AT THE PARTICULAR SECTION
C      THE STRAIN STATE IS TO BE CALCULATED.
C
C      FOUR DATA CARDS ARE REQUIRED FOR THIS PROGRAM.
C
C      CARD 1 :  TITLE CARD.  FORMAT (20A4)
C
C      CARD 2 :  CONTROL CARD.  FORMAT (7E10.0,2I5)
C      PA=AXIAL LOAD ON THE FREE END OF THE ELEMENT.
C      Q=VERTICAL LOAD ON THE FREE END OF THE ELEMENT.
C      BMP=APPLIED MOMENT ON THE FREE END, COUNTER-CLOCKWISE DIRECTION.
C      XL=LENGTH OF THE ELEMENT.
C      BW=WIDTH OF THE ELEMENT.
C      C=DISTANCE FROM CENTROIDAL AXIS TO THE TOP OF THE ELEMENT.
C      EPS=DESIRED ACCURACY IN THE ITERATION PROCESS.
C      NL=NUMBER OF INCREMENTS TO REACH LOADING.  DEFAULTS TO 1.
C      MCYC=LIMIT ON THE NUMBER OF CYCLES.
```

```
C
C     CARDS 3,4 :    STRESS-STRAIN CURVE.  FORMAT (8E10.0)
C     STRAIN,STRESS=PAIRS OF STRAIN AND STRESS UP TO SIX POINTS,
C                    BUT NOT INCLUDING THE INITIAL ZERO POINT.
C
C     THE MAIN PROGRAM DOES THE TAYLOR EXPANSION AT SPECIFIC POINTS
C     OF THE ELEMENT.  THESE POINTS ARE DETERMINED BY THE DIMENSION X(*)
C     STATEMENT AND THE DATA X/------/,NLOAD/*/ STATEMENT.
C     THE PROGRAM COMPUTES THE BENDING MOMENTS AT EACH OF THESE POINTS.
C     AT A SECTION WITH BENDING MOMENT AND AXIAL LOAD, THE STRAIN
C     STATE IS FIRST COMPUTED FOR THE AXIAL LOAD.  A HALVING PROCESS
C     IS USED ON THE MOMENT TO APPROXIMATE THE STRAIN STATE AFTER
C     OBTAINING THE STATE WITH AXIAL LOAD ONLY.
C
C
      COMMON/MBK/ICYC,NSTAT
      COMMON/SBK/STRESS(7),STRAIN(7),C,H,BW,BMA,BSIGN
      COMMON/STBK/S(14),E(14),Y(14),BM(12),F(12),YB(12),K,KI
      COMMON /ZBK/ E4,PHI,E4D,PHID
      DIMENSION TITLE(20)
C
C     THE FOLLOWING TWO STATEMENTS MUST BE CHANGED IF MORE POINTS
C     ARE TO BE CONSIDERED.
      DIMENSION X(5)
      DATA X/0.E0,.25E0,.50E0,.75E0,1.E0/,NLOAD/5/
C
      DATA STRESS(1),STRAIN(1)/2*0.E0/,PHIR,E4R/2*1.E-6/
1     CONTINUE
      READ (5,390) TITLE
      IF (EOF(5)) 200,2
2     CONTINUE
      READ (5,400) PA,Q,BMP,XL,BW,C,EPS,NL,MCYC
```

```
      IF (EOF(5)) 200,3
3     CONTINUE
      READ (5,501) (STRAIN(I),STRESS(I),I=2,7)
      IF (EOF(5)) 200,4
4     CONTINUE
      IF (IABS(NL).EQ.0)  NL = 1
      IF(IABS(MCYC).EQ.0) MCYC = 100
      IF (ABS(EPS).EQ.0.E0) EPS = 1.E-7
      APA = PA/FLOAT(NL)
      AQ = Q/FLOAT(NL)
      ABMP = BMP/FLOAT(NL)
      H = 2.*C
      DO 100 L=1,NL
      PA = APA*FLOAT(L)
      Q = AQ*FLOAT(L)
      BMP = ABMP*FLOAT(L)
      PRINT 380, TITLE
      PRINT600,XL,BW,H,PA,Q,BMP,EPS
      PRINT603, STRAIN,STRESS
      DO 100 J=1,NLOAD
      NSTAT = 0
      BMR = -(1.E0-X(J))*XL*Q - BMP
      DIST = X(J)*XL
      PRINT601,PA,BMR,DIST
      NCYC = ICYC = 0
C     CHECK FOR NO LOADINGS.
      IF (ABS(PA).EQ.0.E0.AND.ABS(BMR).EQ.0.E0) GO TO 85
      IF (ABS(PA).NE.0.E0)  BSIGN = ABS(PA)/PA
      IF (ABS(BMR).NE.0.E0)  BSIGN = ABS(BMR)/BMR
      PHI = BSIGN*PHIR
      E4 = BSIGN*E4R
      BMA = 0.E0
```

```
      E4D = PHID = BSIGN*(E4*1.E-05 + 10.E-11)
C     CHECK FOR BENDING MOMENT ONLY.
      IF (ABS(PA).EQ.0.E0)  BMA = BMR
C  START CYCLING.
10    CONTINUE
      CALL CURVE(E4,PHI,PA,E3,PB,BMB)
      IF (NSTAT.EQ.1)  GO TO 100
      IF( ABS(PA-PB).LT.EPS.AND. ABS(BMA-BMB).LT.EPS.AND.ABS(BMR-BMB).
     1  LT.EPS)  GO TO 90
      IF( ABS(PA-PB).LT.EPS.AND. ABS(BMA-BMB).LT.EPS)
     1  CALL COMP (1,NCYC,BMR)
      IF (NCYC.GE.(MCYC/8))  CALL COMP(-1,NCYC,BMR)
      IF(ICYC.LT.MCYC) GO TO 105
      PRINT701,ICYC
701   FORMAT(1H0/T10,"THE ITERATION IS TERMINATED BECAUSE THE LIMIT OF "
     1 ,I4," CYCLES IS EXCEEDED"/////)
      GO TO 100
105   CONTINUE
      CALL CURVE(E4,PHI+PHID,PA,E3,PPHI,BMPHI)
      IF (NSTAT.EQ.1)  GO TO 100
      DPPHI = (PPHI-PB)/PHID
      DBMPHI = (BMPHI-BMB)/PHID
      CALL CURVE(E4+E4D,PHI,PA,E3,PE4,BME4)
      IF (NSTAT.EQ.1)  GO TO 100
      DPE4 = (PE4-PB)/E4D
      DBME4 = (BME4 - BMB)/E4D
      DET = DPPHI*DBME4-DPE4*DBMPHI
      IF (ABS(BMA).NE.0.E0) GO TO 60
C     COMPUTE VARIATIONS FOR AXIAL LOAD ONLY.
      PHID = PHI = 0.E0
      E4D = (PA-PB)/DPE4
      IF (DET.EQ.0.E0.AND.BMA.EQ.0.E0) GO TO 70
```

```
60      CCNTINUE                                                        STAT1280
        IF (DET.EQ.C.E0) GO TO 86                                       STAT1290
C       COMPUTE VARIATIONS FOR GENERAL LCADING.                         STAT1300
        PHID = (DBME4*(PA-PB) - DPE4*(BMA-BMB))/DET                     STAT1310
        E4D =(-DBMPHI*(PA-PB) + DPPHI*(BMA-BMB))/DET                    STAT1320
70      CCNTINUE                                                        STAT1330
        PHI = PHI + PHID                                                STAT1340
        E4 = E4 + E4D                                                   STAT1350
        ICYC = ICYC + 1                                                 STAT1360
        NCYC = NCYC + 1                                                 STAT1370
        GC TC 10                                                        STAT1380
85      CCNTINUE                                                        STAT1390
        PB = BMB = PHI = E4 = C.E0                                      STAT1400
        GO TO 95                                                        STAT1410
86      CCNTINUE                                                        STAT1420
        PRINT 640                                                       STAT1430
        GC TO 100                                                       STAT1440
90      CCNTINUE                                                        STAT1450
        PRINT103                                                        STAT1460
        PRINT101,(I,S(I),E(I),Y(I),I=1,K)                               STAT1470
        PRINT102                                                        STAT1480
        PRINT101,(I,F(I),BM(I),YB(I),I=1,KI)                            STAT1490
95      CCNTINUE                                                        STAT1500
        PRINT602,PB,EMB,PHI,E4,ICYC                                     STAT1510
100     CCNTINUE                                                        STAT1520
        GO TO 1                                                         STAT1530
200     CCNTINUE                                                        STAT1540
        PRINT 68C                                                       STAT1550
        STOP                                                            STAT1560
390     FCRMAT (20A4)                                                   STAT1570
400     FCRMAT (7E10.0,2I5)                                             STAT1580
501     FCRMAT(8E10.C)                                                  STAT1590
```

```
380   FORMAT (1H1,///T20,20A4//)
600   FORMAT(1H ///5(T10,1H*/),T10,52(1H*)/3(T10,1H*,50X,1H*/),
     1  T10,1H*,50X,1H*,10(1H-),3H> P/3(T10,1H*,50X,1H*/),T10,
     2  52(1H*)/T10,1H*,50X,1H&/4(T10,1H*,50X,1H'/),T63,1HQ/
     3 T10,20(1H-),5H>   X//
     4 T10,"LENGTH = "F10.4,"  INCHES   ","  WIDTH = ",F10.4,"  INCHES  "
     5 ,"  HEIGHT = ",F10.4,"  INCHES"//T10,"P = ",F10.4,"  KIPS  ",
     6  "  Q = ",F10.4,"  KIPS","       M = ",F10.4,"  IN-KIPS"//
     7  T10,"CONVERGENCE PRECISION = ",E16.8//,108(1H=))
101   FORMAT(1H ,(T11,I4,3(3X,E12.5)/))
102   FORMAT(1H0,T12,"I",9X,"F",14X,"BM",13X,"YB"//)
103   FORMAT(1H0/T13,"I", 8X,"S",14X,"E",14X,"Y"/)
601   FORMAT(1H///T10,"ACTUAL P = ",F10.4," KIPS",5X,"ACTUAL M = ",F10.4
     1 ," IN-KIPS"//T17,"X = ",F10.4," INCHES"/)
602   FORMAT(1H //T17,"P = ",F10.4
     1 ," KIPS",12X,"M = ",F10.4," IN-KIPS"//T15,"PHI = ",E14.7,12X,
     2 "E4 = ",E14.7//T10,"NUMBER OF CYCLES = ",I4//70(1H-)///)
603   FORMAT(1H0//T10,"THE STRAINS ARE : "//T10,7(E12.6,2X)//T10,
     1 "THE CORRESPONDING STRESSES ARE : "//T10,7(E12.6,2X)//108(1H=)/)
640   FORMAT (1H0,T5,68H***** THE DETERMINANT OF THE TAYLOR EXPANSION IS
     1 EQUAL TO ZERO *****///)
680   FORMAT (1H0//T15,29H===== NORMAL COMPLETION =====///////)
      END
CCOMP  10  10   0
      SUBROUTINE COMP (N,NCYC,BMR)
      COMMON/SBK/STRESS(7),STRAIN(7),C,H,BW,BMA,BSIGN
      COMMON /ZBK/ E4,PHI,E4D,PHID
      NCYC = 0
      IF (N.EQ.-1) GO TO 20
      BMA = BMA*2.E0
      IF (BMA.EQ.0.E0) BMA = BMR
      PHID = BSIGN*(PHI*1.E-05 + 10.E-11)
```

```
      RETURN
20    CONTINUE
      BMA = BMA/2.E0
      E4 = PHI = BSIGN*1.E-06
      E4D = PHID = BSIGN*(E4*1.E-05 + 10.E-11)
      RETURN
      END
CCURV  10  10   0
      SUBROUTINE CURVE(E4,PHI,PA,E3,P,BMT)
      COMMON/MBK/ICYC,NSTAT
      COMMON/SBK/STRESS(7),STRAIN(7),C,H,BW,BMA,BSIGN
      COMMON/STBK/S(14),E(14),Y(14),BM(12),F(12),YB(12),K,KI
      CG(A,B,D) = D*(2.*B+A)/3./(B+A)
      S1 = S2 = S3 = S4 = 1.E0
      IF(E4.LT.0.E0) S4 = -1.E0
      IF(PHI.EQ.0.E0.OR.BMA.EQ.0.E0) GO TO 80
      A = E4/PHI
      B = A - H
      E3 = PHI*B
      IF(E3.LT.0.E0) S3 = -1.E0
      IF(A.GT.H) S2 = -1.E0
      IF(A.LT.0.E0) S1 = -1.E0
C     FIND THE FORCE AND MOMENT RESULTANTS FROM THE N.A. TO E4.
      Y(1)=S(1)=E(1)=0.E0
      DO 20 I=2,7
      IF (ABS(STRAIN(I)).EQ.0.E0) GO TO 22
      II = I-1
C     LOCATE E4 ON THE STRESS-STRAIN DIAGRAM.
      IF (ABS(E4).LT.STRAIN(I)) GO TO 23
C     STORE STRAIN, STRESS, AND THE LOCATION FROM THE N.A.
      E(I) = S4*STRAIN(I)
      S(I) = S4*STRESS(I)
```

```
      Y(I) = E(I)/PHI
      GO TO 24
23    CONTINUE
      E(I) = E4
      S(I) = S4*(STRESS(II)+(STRESS(I)-STRESS(II))/(STRAIN(I)-
     1         STRAIN(II))* ABS(E4))
      Y(I) = A
24    CONTINUE
C     COMPUTE THE ARM TO EACH STRESS BLOCK.
      YB(II) = Y(II) + CG(S(II),S(I),(Y(I)-Y(II)))
C     COMPUTE THE FORCE AND THE MOMENT.
      F(II) = BW*(ABS(Y(I)-Y(II)))*(S(II)+(S(I)-S(II))/2.E0)
      BM(II) = YB(II)*F(II)
      IF(Y(I).EQ.A) GO TO 25
20    CONTINUE
21    CONTINUE
      PRINT 210, E4,PHI,ICYC
      NSTAT = 1
      RETURN
22    CONTINUE
      PRINT 250,I,STRAIN(I)
      GO TO 21
25    CONTINUE
C     FIND THE FORCE AND MOMENT RESULTANTS FROM THE N.A. TO E3.
      E(I+1)=S(I+1)=Y(I+1)=0.E0
      DO 40 J=2,7
      IF (ABS(STRAIN(J)).EQ.0.E0) GO TO 42
      K = I + J
      KK = K - 1
      KI = K - 2
C     LOCATE E3 ON THE STRESS-STRAIN DIAGRAM.
      IF (ABS(E3).LT.STRAIN(J)) GO TO 43
```

```
C      STORE STRAIN, STRESS, AND THE LOCATION FROM THE N.A.
       E(K) = S3*STRAIN(J)
       S(K) = S3*STRESS(J)
       Y(K) = E(K)/PHI
       GO TO 44
43     CONTINUE
       E(K) = E3
       S(K) = S3*(STRESS(J-1)+(STRESS(J)-STRESS(J-1))/
      1        (STRAIN(J)-STRAIN(J-1))* ABS(E3))
       Y(K) = B
44     CONTINUE
C      COMPUTE THE ARM TO EACH STRESS BLOCK.
       YB(KI) = Y(KK) + CG(S(KK),S(K),(Y(K)-Y(KK)))
C      COMPUTE THE FORCE AND THE MOMENT.
       F(KI) = BW*(ABS(Y(K)-Y(KK)))*(S(KK)+(S(K)-S(KK))/2.E0)
       BM(KI) = YB(KI)*F(KI)
       IF(Y(K).EQ.B) GO TO 45
40     CONTINUE
41     CONTINUE
       PRINT 220, E3,PHI,ICYC
       NSTAT = 1
       RETURN
42     CONTINUE
       PRINT 250,J,STRAIN(J)
       GO TO 41
45     CONTINUE
C      SUM THE AXIAL FORCE RESULTANTS AND THE BENDING RESULTANTS.
       P=BMT=0.E0
       DO 60 M=1,II
       P = P + F(M)*S1
       BMT = BMT + BM(M)*S1
60     CONTINUE
```

```
      DO 70 M=I,KI
      P = P + F(M)*S2
      BMT = BMT + BM(M)*S2
70    CONTINUE
      BMT = BMT + (A-C)*PA
      GO TO 90
C     FIND THE FORCE RESULTANT IF THERE IS NO MOMENT.
80    CONTINUE
      DO 120 I = 2,7
      IF (ABS(STRAIN(I)).EQ.0.E0) GO TO 122
      II = I - 1
C     LOCATE E4 ON THE STRESS-STRAIN DIAGRAM.
      IF(ABS(E4).LT.STRAIN(I)) GO TO 123
120   CONTINUE
121   CONTINUE
      PRINT 230, E4,ICYC
      NSTAT = 1
      RETURN
122   CONTINUE
      PRINT 250,I,STRAIN(I)
      GO TO 121
123   CONTINUE
      K = 1
      KI = 2
      E(1) = E(2) = E4
      S(1) = S(2) = S4*(STRESS(II)+(STRESS(I)-STRESS(II))/
     1              (STRAIN(I)-STRAIN(II))* ABS(E4))
      P = F(1) = BW*H*S(1)
      BM(1) = Y(1) = Y(2) = YB(1) = BMT = 0.E0
90    CONTINUE
      RETURN
210   FORMAT (1H0,T5,48H***** TOP STRAIN EXCEEDS STRAIN ON DIAGRAM *****
```

```
     1 /T5,11H***** E4 = ,E16.8,10H    PHI = ,E16.8/T5,6H***** ,I4,      CURV1210
     2 28H CYCLES HAVE BEEN MADE *****///)                               CURV1220
 220  FORMAT (1H0,T5,51H***** BOTTOM STRAIN EXCEEDS STRAIN ON DIAGRAM **CURV1230
     1***/T5,11H***** E3 = ,E16.8,10H    PHI = ,E16.8/T5,6H***** ,I4,    CURV1240
     2 28H CYCLES HAVE BEEN MADE *****///)                               CURV1250
 230  FORMAT (1H0,T5,50H***** AXIAL STRAIN EXCEEDS STRAIN ON DIAGRAM ***CURV1260
     1**/T5,11H***** E4 = ,E16.8/T5,6H***** ,I4,                         CURV1270
     2 28H CYCLES HAVE BEEN MADE *****///)                               CURV1280
 250  FORMAT (1H0,T5,13H***** STRAIN(,I1,4H) = ,E16.8,12H IN/IN *****//)CURV1290
      END                                                               CURV1300
```

VITA

Joel Clinton Bradshaw, III was born in Franklin, Virginia on August 11, 1951. He graduated from Windsor High School, Windsor, Virginia in 1969 and received a Bachelor of Science degree at Virginia Polytechnic Institute and State University in 1973. The degree of Master of Science in Civil Engineering was begun in March, 1973. He is currently working at the Air Force Weapons Laboratory, Kirtland Air Force Base, New Mexico as a Civil Engineering Project Officer.

Joel C. Bradshaw, III

Joel C. Bradshaw, III

NONLINEAR ANALYSIS OF PLANE FRAMES

by

Joel Clinton Bradshaw, III

(ABSTRACT)

An investigation was made to test a finite element model of a beam-column with geometrical and material nonlinearities. The geometrical nonlinearities result from inclusion of a nonlinear strain-displacement relation and from the formulation of the equilibrium conditions based upon the deformed geometry. Material nonlinearities are represented by nonlinearly elastic stress-strain relationships. A typical analysis proceeds by calculating the equilibrium states corresponding to specific load levels along the load-deflection path. These equilibrium states are obtained by minimization of the total potential energy of the system.

The reliability of the proposed finite element model was investigated on the basis of three classes of problems: 1. an elastic, geometrically linear beam-column, 2. an elastic, geometrically nonlinear beam-column, and 3. a nonlinearly elastic, geometrically linear beam-column. Each class of problems was solved by a method independent of the energy method to provide a basis for comparison. Computer programs were developed to compare the finite element model with the other methods.

It was found that the model is reliable for an elastic system ranging from the linear displacement state to the post-buckled state.