REDUCTION OF ENERGY LOSSES:

A KEY TO

IMPROVED ROCK DRILLING?

by

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LIST OF SYMBOLS

a	Real part of λ.
A_c	Cross-sectional area of column. (m^2)
A_j	Real part of a complex number; $j = x, u, H(x-x_o)$, $L-x_o$ or L.
A_s	Surface area of cylinder. (m^2)
b	Imaginary part of λ.
B_j	Imaginary part of a complex number. (see "A_j")
D_H	Diameter of hole. (m)
D_i	Inside diameter of drill column. (m)
D_o	Outside diameter of drill column. (m)
E	Young's modulus of column material. (N/m^2)
F_D	Drag force due to friction. (N)
F_{D_i}	Viscous drag force on the inside of the column. (N)
F_{D_o}	Viscous drag force on the outside of the column. (N)
F_o	Amplitude of forcing function. (N)
F_1	$\frac{-F_o}{A_c E}$ (Dimensionless).
g	Acceleration of gravity. (m/s^2)
G_L	Real part of a complex number.
H_L	Imaginary part of an imaginary number.
$H(x-x_o)$	Heaviside "unit" function. $H(x-x_o) = \begin{cases} 0 & x < x_o \\ 1 & x \geq x_o \end{cases}$
i	$\sqrt{-1}$
k	$\sqrt{\frac{\omega\rho}{2\mu}}$

L	Length of column. (m)
P	Internal force in rod. (N)
P_D	Viscous power losses. (W)
r	Radial coordinate.
s	Laplace transform variable.
S_{L-x_o}	Real part of a complex number.
T	Period of vibration. [section 5.2] (s^{-1})
T	Torque [section 5.3] (N.m)
$u(x,t)$	Axial displacement of a point on the column. (m)
$u_c(x,t)$	Complex displacement. (m)
$U(x)$	Real displacement amplitude. (m)
$U_c(x)$	Complex displacement amplitude. (m)
$V_c(x)$	Complex velocity. (m/s)
V_o	Amplitude of fluid velocity. (m/s)
$v(x,r,t)$	Fluid velocity. (m/s)
x	Coordinate direction along the axis of column.
y	Radial coordinate; $y = 0$ on the surface of the cylinder.

Greek Symbols

β_1	Coefficient of inertial term in equation of motion of longitudinal vibration.
β_2	Coefficient of damping term in equation of motion of longitudinal vibration.
γ	Weight density of column material. (N/m^3)
ε	Strain (unit elongation of column).

θ	Argument of a complex number.
λ	Complex coefficient, a + ib. [section 5.2]
μ	Absolute viscosity of drilling fluid. (Pa.s)
v_θ	Tangential component of velocity. (M/s)
ρ	Mass density of fluid. (kg/m^3)
τ	Shear stress. [section 5.2] (Pa)
$\tau_{r\theta}$	Shear stress in cylindrical coordinates. (Pa)
$\phi(x)$	Phase angle. (rad)
ω	Frequency. (rad/s)

1. INTRODUCTION

Improved rock drilling methods will soon become more important in meeting the basic world-energy needs, if the development of alternate sources such as solar and nuclear energy fails or encounters extensive delays. The need for drilling systems superior to conventional rotary has been recognized for a long time. Serious attempts to develop improved drilling methods in the 50's showed some promise but were discontinued. A survey of attempts to develop rotary-vibratory methods revealed that a reason for the discontinuation of the major effort was low-power output. Viscous shear loss is a possible cause of low-power output. This is an analysis of the viscous shear losses into the drilling fluid. A brief explanation of basic concepts in rock drilling follows this introduction. Then the literature on attempts to develop improved drilling systems is discussed. The power-loss analysis follows the literature review. The power loss results are discussed next.

In Appendix C, the reasons for the failure of previous attempts to improve rock drilling are discussed. Following the survey of "failures" are suggestions for immediate research directions. These are derived by synthesizing lessons that could be learned from "failures". Special emphasis is placed on design approaches that become obvious after a recognition of the energy waste that is prevalent in previous designs.

2. FUNDAMENTALS OF ROCK DRILLING

The concepts needed for an adequate understanding of section 5 (analysis of losses due to viscous drag) will be reviewed here. A glossary explaining some secondary concepts is included in Appendix A. Further details are available in standard texts in petroleum technology [1,2]*.

Rotary Drilling†

Two parts of the rotary drilling system will be considered in detail in this study: the drill string or drill column and the drill bit. The drill string consists of the drill pipe, the upper section of the drill string, and the drill collar. The drill string consists of coupled lengths of pipes usually about 9.1 m (30 ft) long. Drill pipes considered in this study have outside diameters of 0.1407 m (5.54 in), and inside diameters of 0.1186 m (4.67 in). The drill collar consists of coupled sections of thick-walled tubing. The bit is forced into the rock by the weight of the drill collar and the thrust at the top of the column. The whole drill string is rotated at the top of the hole. The rock cuttings are removed by circulating a drilling fluid. Usually the drill fluid flows down the central passage and up the annular space between the drill string and the sides of hole. The drilling fluid may be air, water, or various drilling "muds".

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* Numbers in braces refer to references listed at the end of this thesis.

† Adapted from Simon [6].

The term "drill column" will be used primarily in this study. "Conventional rotary drilling" will be used to refer to the system described above.

Percussive Drilling*

Rock is fragmented by repetitive impaction. A pneumatic rock drill consists of a piston which gives a series of impacts to the drill rod, and a rotating mechanism to re-index the bit so that a fresh rock surface is presented for each blow. The rotation is not intended to fracture rock.

In a down-the-hole percussion drill, the piston strikes the drill bit directly. Rotation of the drill column provides indexing.

Rotary-Percussive and Rotary-Vibratory Drilling†

In the literature, rotary percussive drilling machines are regarded as tools in which a section of round steel shafting 0.0762 m to 0.1016 m (3 or 4 ins) in diameter and about 3.048 m (10 ft) in length has been reciprocated vertically inside of the drill column just above the bit and has acted as a hammer delivering blows downward on top of the bit. The hammer is actuated by the flow of drilling mud through the tool.

In rotary-vibratory drilling, the bit is vibrated and also rotated. The rotary drilling process is analogous to the rotary-vibratory

* Adapted from McGregor [2].

† Adapted from DRI [7], Simon [6], McCray [1].

drilling process. In rotary drilling, the mechanical power input is developed by the rotation of the bit. The torque reaction to rotation results from the sinking of the teeth of the bit into the rock, their penetration being caused by the static thrust load. In rotary vibratory the penetration is produced by a "dynamic" load and sometimes a static thrust load.

The rotary-vibratory tool that DRI attempted to develop in the 50's is generally referred to as a "vibratory" drill. In this thesis, the term "rotary-vibratory" is used generally to avoid confusion. "Vibratory" is used only in quotations and in the literature survey. Whenever the term "vibratory" is intended to mean a process that does not include rotation, the term "pure vibratory" is used.

Rotary-vibratory drilling generally refers to appreciably continuous contact between the vibrating tool and the rock. As the amplitude of vibration is increased, the bit may begin to lose contact with the rock for a fraction of the cycle of vibration. This occurs at lower amplitudes of vibration for the lower static loads or for the harder rocks. At still greater amplitudes of vibrations, the action of the bit may become that of a series of impacts each lasting for a small fraction of the cycle of vibration. The mechanical power input going to the rock may then be generated primarily by the vibratory motion instead of by the rotation. This result, strictly speaking, is "percussive drilling" instead of vibratory drilling.

These precise definitions were used by DRI only. Most other researchers used "percussive" and "rotary-percussive" rather loosely.

The DRI rotary-vibratory drilling machine driven by a magnetostrictive transducer was frequently referred to as "rotary-percussive" in the literature.

Probably the most precise definitions of these terms were given by Prof. A. M. Freudenthal in his DRI report [7]. Briefly, "rotary-vibratory" refers to drilling action during which the drill bit remains in continuous contact with the rock. Strictly speaking, "rotary-percussive" drilling refers to drilling action in which the drill bit periodically loses contact with the rock. Conventional "rotary-percussive" drilling refers to drilling action in which no steady force or torque is exerted on the drilling tool, but the disruption of the rock is produced by a succession of shocks alone. Rotary-vibratory and rotary-percussive, strictly speaking, involve a steady force in the drill bit produced by an oscillating force.

Viscous Drag Losses

A concept that will also be used extensively in this study is the "viscous losses into the drilling fluid". An explanation adapted from DRI [23], will be useful.

As the drill column vibrates longitudinally or rotates, a layer of fluid in contact with the walls follows the motion of the column exactly. Fluid a short distance from the walls is undisturbed. This difference in motion gives rise to a viscous wave in the liquid and to reaction forces on the column.

3. LITERATURE REVIEW

This section is a review of relevant literature on attempts to develop improved drilling methods. A more detailed study of these attempts is necessary to glean insight into productive future research in this area. This will be considered in Section 6.

Most of the work done on the development of improved drills was done between 1946 and 1958. A large number of these attempts were sponsored by the petroleum industry. Hence a very limited amount of useful information is available since much of the work is proprietary in nature.

Ledgerwood [3], and Gray and Young [4] reviewed all crucial developments up to 1973. Pfleider and Lacabanne [5] and Simon [6] reviewed developments in rotary-percussive and rotary-vibratory* drilling and provided more of technical information. A review of some of the most significant developments in these combination drilling systems will be useful here.

Several papers reported the results of feasibility studies on rotary-percussive and rotary-vibratory drilling [3,5,6,10]. In these studies, tests were run with laboratory and preliminary field models.

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* The terms "rotary-percussive" and "rotary-vibratory" were explained in Section 2. "Rotary-percussive" has been used loosely in the literature to refer to rotary-percussive and rotary-vibratory systems. In most cases the meanings intended were not clear from the context. In these cases, this author will adopt the terminology used in these papers. In cases where the context indicates the meaning intended, the less confusing terms "rotary-percussive" and "rotary-vibratory" will be used. DRI's vibratory drilling is rotary-vibratory drilling.

Their performances were compared with conventional rotary drilling. In all these tests, rotary-percussive drilling and rotary-vibratory drilling were reported to be superior to conventional rotary drilling in drilling rates.

For example, Drilling Research, Incorporated (DRI) reported that laboratory and preliminary field models of a magnetostrictive-vibration drilling system* had two to three times the drilling rate of conventional rotary [6,7]. DRI (1948-1957) constitutes the most extensive attempt of the industry to develop improved drilling systems [1]. DRI did not complete the development of any method. This effort was discontinued between 1957 and 1959 because of "lack of funds to overcome transmission-line problems and the inability of the down-hole vibrator to develop sufficient power output into the load represented by the rock" [3].

In the concluding report [8,9] it was estimated that rotary-vibratory drilling would have to "result in at least tripling the rate of penetration achievable by optimum rotary drilling techniques in medium to medium-hard formation" in order to be economically competitive with rotary drilling. Based on these criteria, Simon [8,9] made the tentative conclusion that "vibratory drilling in rock of medium hardness may be just about economically competitive with conventional rotary drilling". This conclusion was based on a cost-of-drilling analysis [11] involving drilling rates, capital equipment costs, and operating costs of the DRI magnetostriction-vibration drilling system.

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* The reader is reminded that explanations of this term and other rock-drilling concepts are provided in the glossary in Appendix A. Please refer to Reference [1] and [2] for further details.

Other research efforts had similar results with DRI's. Papers on these efforts do not provide much useful information. All these attempts showed very promising results but were discontinued before a commercial tool was developed. Borg-Warner's development of the eccentric-weight percussion (or sonic) drilling system [10] apparently showed some promise. It had attractive drilling rates and had a full range of cutting possibilities. However, it was discontinued in 1958 "when additional funds to prevent mechanical failure of down-hole components could not be justified" [3].

Pfleider and Lacabanne [5] claim that rotary-percussion drilling rates in the United States of America varied from 5 to 25 times those of conventional rotary drilling. Similar developments were also reported in Europe.

The Russians [5] reported drilling rates 3 to 20 times those attainable by conventional rotary drilling systems with "similar thrusts and rotational speeds". Raynal [12] reported that Moscow Drilling Institute was working on a "vibro-drill" in 1958. In laboratory tests, this hydraulic vibrator had drilling rates twice those of conventional rotary.

Most of these attempts were discontinued as a result of mechanical and electrical problems [3]. This author knows about only three developments that were not discontinued. Research on the Russian vibro-drill was still in progress at the end of 1959 [12]. Gulf Oil Corp. had reported [3] that Continental-Emsco Co., their licensee, was making final tests on the Gulf Oil Corp. percussor (a liquid-actuated tool) before offering it commercially. Ledgerwood [3] claimed that the only commercial rotary-percussion tool available in 1960 was the tool "initiated by Ross Bassinger, refined by Pan American Petroleum Corp. and offered commercially by Mission Mfg. Co. The tool is air-actuated; unfortunately, this actuation severely restricts its general utility because industry's inability to cope with water influx in air drilling currently limits the use of air to 3 percent or less of hard-rock footage".

McGregor [2], Mission Manufacturing Co. [13], and Schumacher [14] present more current information on the status of this development. The Mission Hammerdril and Megadril are currently offered commercially by TRW Mission Manufacturing Co. [13]. The Hammerdril and Megadril are "pneumatically operated, bottom-hole drills" that "combine the percussive action of cable tool drilling with the rotary action of rotary drilling. They can be used on standard rotary rig with the necessary air-compressor capacity. They are used for fast and economical drilling of medium-hard to hard formations in mining, quarry, construction, water well, oil well and geophysical work".* Details are not given on the performance of these drilling machines. Mission Manufacturing Co. claims that "fast penetration" results from the fact that the Hammerdril and Megadril are down-hole machines. These machines are rotary-percussive since they "combine the percussive action of cable tool drilling with the rotary action of rotary drilling". They look like the Bassinger rotary-percussion drilling machine. One wonders whether the performance of the Hammerdril and Megadril is comparable to the Bassinger machine [15,16]. For example, the Bassinger machine had "penetration rates of 300 to 1000

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* The reader is reminded that [13] is an operation and maintenance manual, and is probably used for sales promotion.

percent improvement over conventional means" in brittle shales and cherty formations (hard rock formations) [15].

McGregor [2] provides less information than Mission [13]. A brief explanation of the operation of the Hammerdril is given. McGregor considers the Hammerdril with other British and American down-hole machines.

The preceeding section was an attempt to establish a view of the status of the only commercial rotary-percussion tool* developed as a result of attempts made in the 50's.

It is very likely that there is no rotary-vibratory drilling machine available currently. A Manager of Research and Development, Reed Drilling equipment, claimed [14] that he did "not know of any vibratory assisted drilling rig available today".

This author does not have precise information on the current status of rotary-vibratory and rotary-percussive drilling systems. However, it is clear that they are not used extensively. Writing about the rotary method, Gray [4] claims that "86 percent of the rigs are now drilling by this method to make 92 percent of the hole". Therefore, less than 14 percent of the rigs were drilling by the rotary-percussive method and other methods in 1973.

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* The ROTARY-PERCUSSION Equipment Division of Reed Tool Company manufactures the "BIG RED", Reed RPT-76 Hammer (patent pending). It is an "air-operated percussion hammer", and looks very similar to the Bassinger machine. It is not clear whether this machine is percussive or rotary-percussive.

Another company apparently associated with "Percussion-Rotary" drilling is Widder Corp., Mamaroneck, New York [Thomas Register, 1975]. No information has been received from Widder Corp. on this tool.

The final outcome of many of the attempts to develop improved rock-drilling methods in the 50's are not known. Ledgerwood [3] listed the major attempts that were discontinued before 1960. Very vague reasons were given for discontinuance. Only DRI gave useful information on the reasons for the discontinuation of their developments.

Very little technical information on these attempts is available. However, a survey of literature on rock drilling [2,3,4] shows that extensive attempts have been made to apply pre-1960 technology to develop improved drilling systems. Ledgerwood [3] states that various means of actuating vibration have been tried. In addition, a broad range of frequencies have been tried; experimental work has been done for the frequencies between 6 Hz and 300 Hz. DRI even considered 300 to 1000 Hz [8]. Vibration amplitudes ranging from 0.0 016 m to 0.0 889 m (1/16 in to 3 1/2 in) have been tried experimentally [3]. Power outputs of the vibration mechanisms ranging from 14 912 W to 59 648 W (20 to 80 hp) have been obtained [3]. However, there is an unanswered question as to where the output power goes: into the rock or into other nonproductive loss mechanisms.

In addition to theoretical and laboratory feasibility studies and tests on field models of rotary-percussive drilling systems, there has been considerable research done in rotary, percussive, and turbo-drilling [1,2,3,6,17,18] that are probably applicable to combination systems (rotary-vibratory and rotary-percussive). In-depth study has been done on drilling rates, for example [4]. Theoretical and experimental analysis of the effects of various factors on drilling rate has been done and attempts have been made to use these analyses to optimize drilling rates.

An overview of the literature on development of improved drilling methods reveals some common trends. One of these trends is the significance of power to researchers in this field. Dr. Maurer's remark [19] is a typical illustration of this trend. He asserts that "the most economical way to drill rock is not necessarily the method which can remove the rock most efficiently". Continuing he writes: "We can afford to have a method which requires a lot more energy if it allows more power to be transmitted (to the rock)*. ... Don't rule out exotic drills just because they take a lot of energy. The cost of power is a small percentage of drilling costs. ... The cost of the power† is insignificant in petroleum drilling and is quite negligible in most mining drilling". This attitude underlies most major design decisions available in the literature.

Perhaps this might explain why very little work has been done in published literature on power losses in drilling systems. Some researchers [21,22] mentioned these losses. For example, Hartman [20] mentioned the "oft-repeated estimate that penetration processes are less than one percent efficient". When rotary, percussion, or a combination method is used, limited amounts of power are transmitted [6,19]. High power input has not improved the power output. Maurer [19] states that the power output of large oil well rigs, employing 0.2 m (8-in) rotary

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* Inserted by the author.

† Dr. Maurer is referring to the power used in large drilling rigs. More will be written about this attitude in section 6.

bits, is about 14.9 kW (20 hp), when the input power is 745.6 kW to 1491.2 kW (1000 to 2000 hp); some mining rotary bits are limited to power outputs of about 0.745 kW or 1.49 kW (1 or 2 hp). In these cases, over 98% of the power input is lost. Some portion of the power is lost into the power generator and into the drilling fluid. Other sources of power loss are joint friction, structural losses, and frictional losses on the sides of the hole.

For example, power losses in rotary drilling includes "increased frictional losses on the sides of the hole and the increased viscous losses into the drilling fluid as the drill string gets longer"[6]. Simon [6] claims that about "five to ten times more rotary power must be supplied at the top of a 15,000 foot (4572 m)* hole than is required to turn the bit at the bottom".

Percussive drills lose most of their imparted power in rod vibration [18]. Writing about vibratory drilling, Singh [22] conjectured that losses into the drilling fluid would only be large for very long columns.

There are probably other references to rotary or rotary-vibratory power losses in the literature†. Probably, the only published analysis of viscous losses into the drilling fluid was done by B. A. Wise for DRI in 1952 [23]. Wise was interested in estimating down-hole losses. The viscous drag losses of the transducer column were estimated in his

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* Inserted by the author.

† Simon [21] is another reference to power losses in vibratory drilling systems.

analysis. Apparently, Wise assumed that the drill string was vibration-isolated from the transducer (the vibration-generating element). Hence, losses into the drilling fluid occurred only at the transducer. However, this is conjecture on the author's part since Wise did not explain the limitation of his analysis to the transducer section. Wise's computations showed that the value of the power loss was small in the frequency range of 100 to 1000 Hz, drilling fluid densities of 10^3 to 2×10^3 kg/m^3 and viscosities of 0.02 to 0.1 Pa·s. It was concluded that the viscous drag losses on the transducer were negligible compared to other losses.

However, DRI tried to develop "vibration-isolation systems to prevent excessive losses into the drill string and into the drilling fluid" [6]. The first model did not work and DRI became inactive before a second model could be tested. In their final report [24], DRI observed that "operation at vibration frequencies in the range of the high hundreds to low thousands of cycles per second would result in the need for solving several problems in regard to filtering, drilling fluid losses, and transmission of electrical power". DRI also recommended that these problems be considered if operation at high frequencies was indicated.

Many papers report research done on various characteristics of drilling fluids. Some studies [25] on the effect of drilling fluids on drilling rate are relevant to this study. However, most of these papers report phenomena at the interface of the bit and the rock; effects of the drilling fluid head and circulation rate on the drilling rate were studied. For example, these studies show that drilling rates decrease with increase in fluid viscosity and fluid head. Investigators [4] speculated on the reason why some properties of the drilling fluid caused a decrease in the drilling rate.

This section had considered the major features of attempts to improve rock drilling that are relevant to a study of drilling systems involving both rotary and vibratory or percussive motion. The methods considered were those methods involving the mechanical attack of rock. Most of the attempts to improve rock drilling sought to improve these tools by increasing their penetration rates. References to power losses in the literature, and many of the design approaches used in attempts to improve rock drilling, indicate that the problem posed by power losses is recognized. Probably, this problem has not been considered seriously because power costs were regarded as insignificant*. This is especially true when the drilling is being carried out by an energy resource company such as a petroleum company.

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* See Appendix C for further details.

4. PURPOSE OF STUDY

The primary purpose of this study was to estimate the viscous drag losses in the entire drilling column when the column is driven with an imposed driving force at some arbitrary position along its length. This problem is a practical one in the cases where the installed vibration isolation has failed between the drill and drill column or where none has been installed. Failure of such vibration isolation equipment has been reported [7]. Wise [23] estimated only down-hole losses, apparently assuming that the drill column was effectively isolated from the vibration. Part of the theoretical basis for making decisions on whether or not to vibration-isolate the drill column from the vibrator will be provided in this study.

There is a basic difference in approach between this study and DRI's. It can be deduced from information gleaned from various sections of DRI's Collected Report [8,26] that DRI's decision to include a vibration-isolation system was not based on any theoretical analysis. One would expect a column vibrating in a viscous fluid and striking a rock surface to dissipate energy into the fluid and to be susceptible to fatigue failure. Probably DRI based their decision on vibration isolation on such an expectation. Effective vibration isolation would solve these problems.

However, vibration isolation would also introduce many problems. Vibration isolators designed into the down-hole system will complicate other elements of the machine, will provide additional potential for unreliability, and will have to carry large thrust loads and may have to accommodate large deflections. DRI considered the isolation of vibration as the most important design consideration for a down-hole machine because of its influence on the remaining problems of the design [27]. Apparently, DRI did not base their decision to introduce vibration-isolation in the drill column on theoretical or experimental analysis [7]. The importance of vibration isolation in a down-hole machine and undesirable features associated with vibration isolation creates a need for such an analysis. Some theoretical and experimental analysis is necessary to decide on the advisability of the introduction of a vibration isolation system in a drill column.

Wise's study was based on a vibratory system or any other source that generates prescribed displacement (motion) inputs. Force inputs will be used in this study. By modifying the form of the equation of motion and the boundary conditions, a similar analysis could be applied to systems driven by displacement inputs. Therefore, this study will have broader application; it can be applied directly to systems driven by means such as rotating and reciprocating unbalance. Eccentric-weight machines and electrodynamic shakers are examples of such sources.

This study also involves some other details not included in Wise's. For example, inertia due to the fluid that remains attached to the surface of the drill column is taken into account in computing the losses. This can become significant for deep holes.

This study is applicable to more systems than Wise's. It provides a theoretical basis for deciding on whether or not to introduce vibration isolation in the design. Finally, the results of this analysis are not limited to any frequency range because of the continuous nature of the model. Therefore, these results could be used in estimating losses in the frequency range including the "high hundred to low thousands of cycles per second" as recommended in DRI's final report [24].

The results of this analysis, combined with an analysis of past "failures" in attempts to develop improved drills will provide a basis for considering design directions in this field.

5. THEORETICAL ANALYSIS OF VISCOUS POWER LOSSES

5.1 GENERAL REMARKS ON ANALYSIS

This section considers losses resulting from the motion of the drill column of a rotary-vibratory drill in the drilling fluid. This analysis was based on simple models. The motion of a rotary-vibratory drill column is complex and nonlinear. However, these simple models are adequate for a preliminary analysis.

The rotational motion and longitudinal motion of a rotary-vibratory drill column are probably coupled. In this analysis, it is assumed that they can be uncoupled and considered separately.

The actual system consists of a long pipe that simultaneously vibrates longitudinally and rotates steadily in a hole. A large portion of the pipe is immersed in the drilling fluid. The pipe strikes the bottom of the hole periodically.

5.2 LONGITUDINAL VIBRATION OF DRILL COLUMN

Viscous losses will be computed for 2 cases. The first case, model A, represents a drill system whose entire column vibrates. In the second case, only the down-hole portion of the column vibrates. This case, model B, represents a drill column that is perfectly vibration-isolated from the vibrating element. Only the portion of the drill column beneath the vibrator is free to vibrate.

The first part of this section is the derivation of the equation of motion applicable to both models A and B. A general form of the steady-state solution is derived and has terms that will be evaluated by applying appropriate boundary conditions.

5.2.1 Derivation of Equation of Motion

Assumptions

A drill column in a deep well has the relative dimensions of a length of thread [29]. It is long and slender. The drilling fluid is pumped down into the hole via the drill pipe and is pumped out of the hole through the annulus between the drill pipe and the hole. This means that the drill column vibrates in a fluid that had some initial velocity distribution.

The drill bit is attached to the lower end of the column. A concentric passage in the drill bit provides restricted communication between the descending and ascending columns of fluid.

The bit-end of the column acts like a plunger, generating a turbulent velocity distribution at the bottom of the hole. This turbulence is neglected. The drilling fluid is non-Newtonian [29]. Usually, a thin laminar boundary layer exists close to the walls while the remainder of the fluid is in turbulent motion [29].

Tests with strain gages at the bit shank indicated that DRI's rotary-vibratory drill column vibrated sinusoidally near the bit [30]. In addition to the motion generated by the driving force, some motion is due to the impact blows struck upon the rock.

The drill column is loaded axially by the driving force, the reaction force from the rock, and by thrust loads at the top of the column.

In a deep well the acceleration of gravity would vary along the length of the column. Large pressure gradients due to the drilling fluid head exist in deep wells. Temperatures of 422 K (300 F) exist at depths of over 4267 m (14,000 ft) [1].

The drill column cross section is not uniform. The drill collar has a large outside diameter and smaller inside diameter than that of the rest of the column. The drill bit is another non-uniformity on the drill column.

The preceding was an outline of some main features of a rotary-vibratory drill column. To carry out a simple analysis of the longitudinal vibration of a drill column many of these features will be simplified.

Major assumptions used in analysing the longitudinal vibration of a continuous column in a fluid are:

I. During vibration, the cross sections of the column remain plane and the particles in these cross sections execute motion only in the axial direction. Chen [28] states that the lateral displacements of the particles can be neglected without substantial errors in the case of "thin" rods whose cross-sectional dimensions are small in comparison with the wavelengths of longitudinal motion. The term "long column" will be used in place of "thin rod" because it is a better description of the vibrating drill string which meets the criteria for thin rods.

II. The column vibrates in a fluid that is at rest initially. The extreme boundary of the fluid is at rest at all times.

III. End effects are not considered; only the section of the column away from the ends is considered.

IV. The fluid is Newtonian; fluid flow close to the vibrating column is laminar.

V. The section of the column under consideration in this analysis is always immersed in the fluid.

VI. The motion of the column is sinusoidal; any motion due to impact blows struck upon the rock is not taken into account.

VII. The column behaves elastically.

VIII. The acceleration of gravity is constant over the length of column considered.

IX. Pressure gradient due to the drilling fluid head is not considered.

X. Isothermal conditions are assumed.

XI. The column has uniform cross section.

XII. The column is loaded only by the driving force; the thrust load at the top of the column is neglected. The drill column does not buckle under given axial loads.

Assumptions VIII, IX, XI will not be used in the derivation of the equation of motion. They will be used solving for the steady-state motion.

A drill column is a long column. It meets the requirement of assumption I. That is, the cross-sectional dimension, D_o is small compared with the wavelength of longitudinal motion, λ where

$$\lambda = \frac{c}{f}$$

c is the velocity of sound in steel and f is the vibration frequency. Therefore a drill column is a long column when

$$D_o << \frac{c}{f}$$

Therefore, a drill column with D_o = 0.1407 m (5.54 in) is "long" when

$$f << \frac{c}{D_o} \quad .$$

Using this relation and realizing that c = 5050 m/s for a steel bar [41], one finds that for frequencies less than 35 892 rad/s, the drill column can be considered as "long".

Assumptions IV, V, VI, and VII are good approximations of a real case. The other assumptions are not.

The Derivation

The equation of motion is derived by carrying out a force balance on an element of a solid circular cylinder. Since a drill column is a pipe, the drag force on the inside surface of the pipe is included.

The forces acting on a cylindrical element are given in Fig. 1:

The inertia force of an element of column $= - \frac{\gamma A_c}{g} dx \frac{\partial^2 u(x,t)}{\partial t^2}$

$$\text{where } A_c = \frac{\pi(D_o^2 - D_i^2)}{4}$$

The tensile force across any section, $P = A_c E \varepsilon$

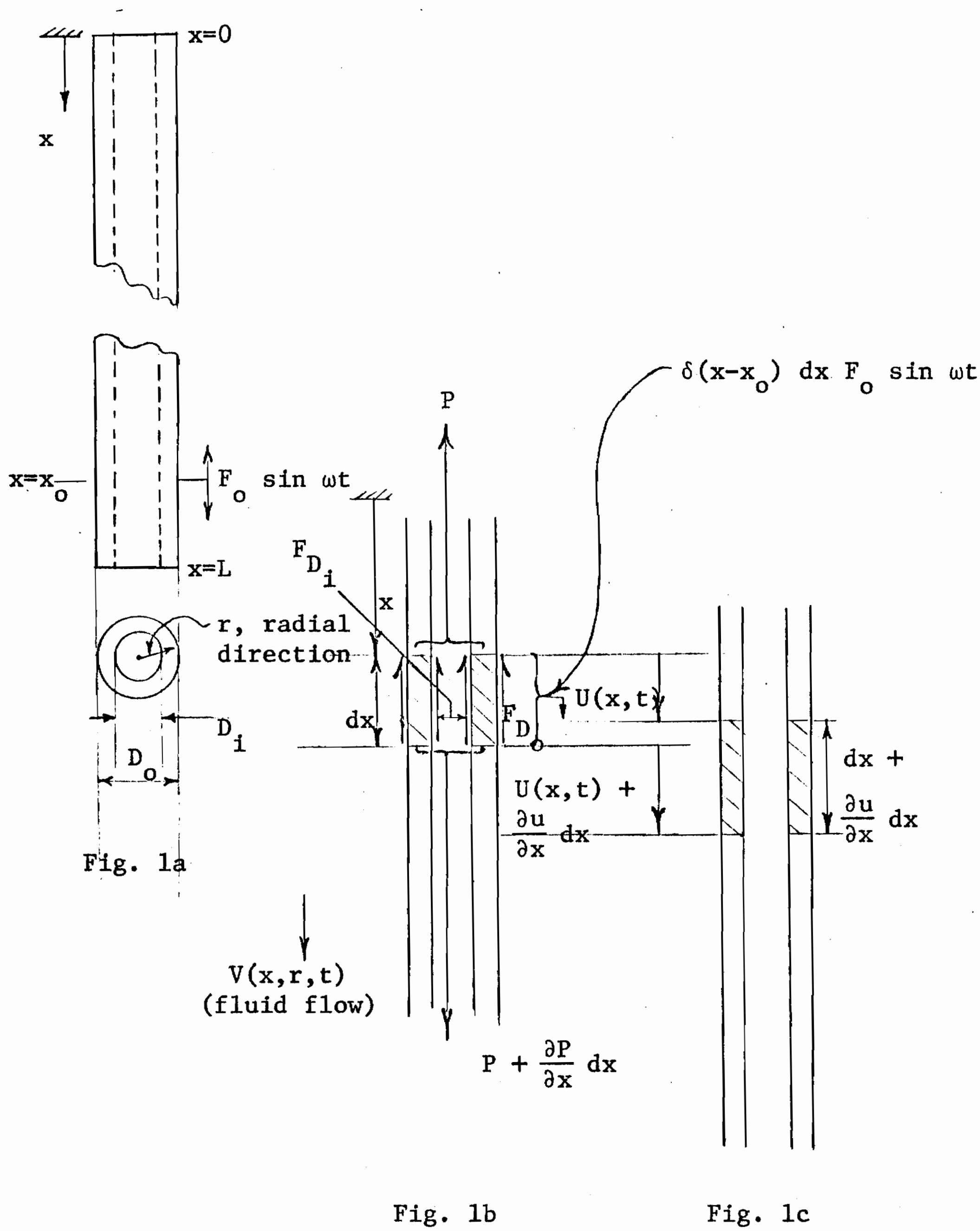

Fig. 1. Column element in longitudinal vibration.

$$\text{or } \frac{P}{A_c} = E \frac{\partial u}{\partial x}$$

$$\text{Thus } P = A_c E \frac{\partial u}{\partial x}$$

$$\text{and } \frac{dP}{dx} = A_c E \frac{\partial^2 u}{\partial x^2} \quad .$$

The damping force on the inside or outside surfaces of the element is

$$F_D = \tau A_s \tag{1a}$$

Using the reference directions in Fig. 2a, the frictional shearing stress τ is given by

$$\tau = \mu \frac{\partial v}{\partial r}$$

where $\frac{\partial v}{\partial r}$ is the shear rate in the fluid. As one will find later, the coordinates used in the analysis of the fluid-drill column interface are attached to the moving element (Fig. 2b). In this case $\frac{\partial v}{\partial r}$ is negative and the revised shearing stress equation is given by

$$\tau = -\mu \frac{\partial v}{\partial r} \tag{1b}$$

Substituting equation 1b in equation 1a yields

$$F_{D_o} = -\mu \left[\frac{\partial v(x,r,t)}{\partial r}\right]_{r = \frac{D_o}{2}} \pi D_o dx$$

where $A_s = \pi D_o dx$, $\mu \left[\frac{\partial v(x,r,t)}{\partial r}\right]$ is the shear rate on the surface of the column, and F_{D_o} is the drag force on the outside surface of the drill column. In the same way, the drag force on the inner surface of the

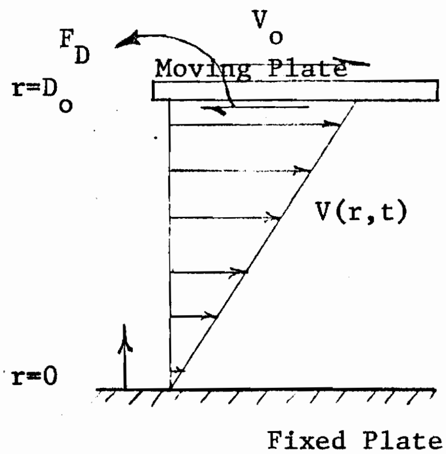

Fig. 2a. Fixed reference plane

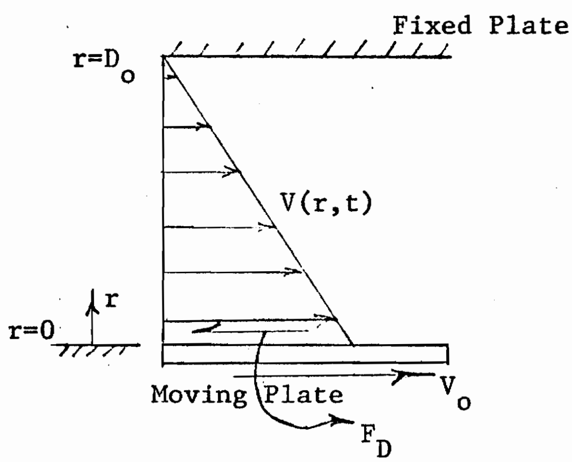

Fig. 2b. Moving reference plane

Fig. 2. Velocity distribution in a viscous fluid between two parallel plates.

column is given by

$$F_{D_i} = -\mu \left[\frac{\partial v(x,r,t)}{\partial r} \right]_{r = \frac{D_i}{2}} \pi D_i dx$$

The external driving force = $\delta(x-x_o)\ dx\ F_o \sin \omega t$

where $\delta(x-x_o)$, the dirac delta function, is given by [36]

$\delta(x-x_o) = 0$ for $x \neq x_o$.

$\int_o^\infty \delta(x-x_o) dx_o = 1$

The amplitude of the external driving force is the area under the force-position curve; that is $F_o dx$. Summing all the forces acting on the element yields

$$\frac{\gamma A_c}{g} dx \frac{\partial^2 u(x,t)}{\partial t^2} = -P + P + \frac{dP}{dx} dx + \pi D_o dx\ \mu \left[\frac{\partial v(x,r,t)}{\partial r} \right]_{r = \frac{D_o}{2}}$$

$$+ \pi D_i dx\ \mu \left[\frac{\partial v(x,r,t)}{\partial r} \right]_{r = \frac{D_i}{2}}$$

$$+ \delta(x-x_o)\ dx\ F_o \sin \omega t$$

Replacing $\frac{dP}{dx}$ by $A_c E \frac{\partial^2 u}{\partial x^2}$ and dividing through by dx, yields

$$-A_c E \frac{\partial^2 u}{\partial x^2} + \frac{\gamma A_c}{g} \frac{\partial^2 u}{\partial t^2} - \pi D_o \mu \left[\frac{\partial v(x,r,t)}{\partial r} \right]_{r = \frac{D_o}{2}}$$

$$- \pi D_i \mu \left[\frac{\partial v(x,r,t)}{\partial r} \right]_{r = \frac{D_i}{2}} = \delta(x-x_o)\ F_o \sin \omega t \qquad (2)$$

The third and fourth terms in equation 2 are evaluated using Schlichting's solution for an oscillating flat plate on a viscous fluid* [31].

Figure 3 shows a flat plate when it is given a harmonic motion with velocity amplitude V_o in a fluid whose extreme boundary is at rest. The resulting fluid velocity distribution is given by [31]:

$$v(y,t) = V_o e^{-ky} \cos[\omega t - ky] \tag{3}$$

$$\text{where } k = \sqrt{\frac{\omega\rho}{2\mu}} \tag{4}$$

Note that at the surface of the oscillating member, y = o, that fluid has a velocity equal to the member velocity, $V_o \cos wt$. To evaluate equation 3 for use in equation 2 properly, y must be replaced by some function of r. Assume that the plate is wrapped into a cylindrical column. The y direction becomes the radial direction and

$$r = y + \frac{D}{2} \tag{5}$$

In order to apply Schlichting's solution, equation 5 is substituted into equation 4. Moreover, to accommodate the velocity distribution along the length of the cylinder, the x-direction, let

$$V_o = V_c(x) \tag{6}$$

where the subscript c indicates that a complex velocity coefficient is allowed. This is done to accommodate a velocity response that has a

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* See Langlois [32] for a detailed derivation.

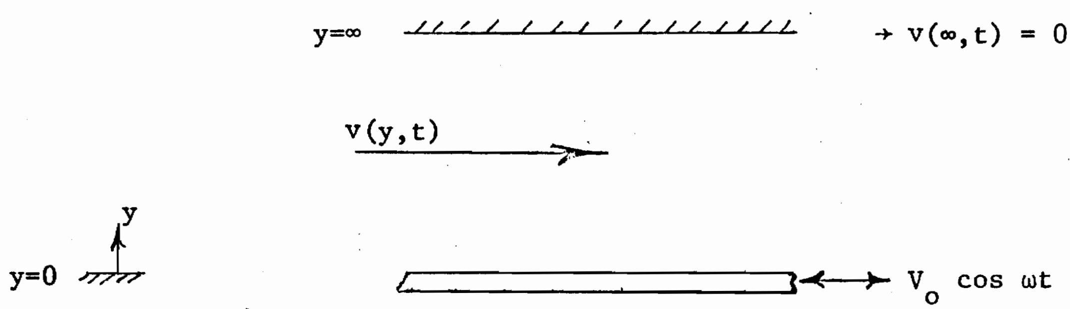


Fig. 3. Flow near an oscillating plate.

phase lag with respect to the driving function. Various phase lags are expected along the length of the drill column. Equation 3 can now be re-written as

$$v(x,r,t) = V_c(x)\ e^{-k(r - \frac{D}{2})} \cos [\omega t - k(r - \frac{D}{2})] \tag{7}$$

The shear rate at the surface of the cylinder must be evaluated. The fluid shear rate, in general, is given by equation 8 by taking the partial derivative of equation 7 with respect to r. Thus

$$\frac{\partial v(x,r,t)}{\partial r} = k\ V_c(x)\ e^{-k(r - \frac{D}{2})} \sin [\omega t - k(r - \frac{D}{2})]$$

$$- k\ V_c(x)\ e^{-k(r - \frac{D}{2})} \cos [\omega t - k(r - \frac{D}{2})] \tag{8}$$

Evaluating at the surface where $r = \frac{D_o}{2}$, one gets

$$\frac{\partial v(x,\frac{D_o}{2},t)}{\partial r} = k\ V_c(x) \sin \omega t - k\ V_c(x) \cos \omega t \tag{9a}$$

$$\frac{\partial v(x,\frac{D_i}{2},t)}{\partial r} = k\ V_c(x) \sin \omega t - k\ V_c(x) \cos \omega t \tag{9b}$$

Equation 9 will be simplified further before being substituted into equation 2. Assuming that the fluid in contact with the column does not slip

$$v(x,\frac{D_o}{2},t) = v(x,\frac{D_i}{2},t) = \frac{\partial u(x,t)}{\partial t} \tag{10}$$

where u is the elemental column displacement shown in Fig. 1c. From equations 7 and 10 one finds

$$v(x,\frac{D}{2},t) = V_c(x) \cos \omega t = \frac{\partial u(x,t)}{\partial t} \tag{11a}$$

where D stands for D_o and D_i. Differentiating equation 11a with respect to time and rearranging, yields

$$\bar{V}_c(x) \sin \omega t = -\frac{1}{\omega} \frac{\partial^2 u(x,t)}{\partial t^2} \tag{11b}$$

Substituting equations 11a and 11b into equation 9, one finds

$$\left[\frac{\partial v(x,r,t)}{\partial r}\right]_{r=\frac{D}{2}} = \frac{-k}{\omega} \frac{\partial^2 u(x,t)}{\partial t^2} - k \frac{\partial u(x,t)}{\partial t} \tag{12}$$

Observe that u and its derivatives were assumed not to be functions of r; u and its derivatives do not vary in the radial direction since the column is "long".

Substituting equation 12 into equation 2 yields equation 13

$$-A_c E \frac{\partial^2 u}{\partial x^2} + \frac{\gamma A_c}{g} \frac{\partial^2 u}{\partial t^2} - \pi \mu (D_o + D_i)\left(\frac{-k}{\omega} \frac{\partial^2 u}{\partial t^2} - k \frac{\partial u}{\partial t}\right)$$

$$= \delta(x-x_o) F_o \sin \omega t \tag{13}$$

Collecting terms and simplifying:

$$\frac{\partial^2 u}{\partial x^2} + \frac{1}{A_c E}\left[\frac{-\pi \mu k(D_o + D_i)}{\omega} - \frac{\gamma A_c}{g}\right] \frac{\partial^2 u}{\partial t^2} - \frac{\pi \mu k(D_o + D_i)}{A_c E} \frac{\partial u}{\partial t}$$

$$= -\frac{F_o}{A_c E} \delta(x-x_o) \sin \omega t \tag{14}$$

For a hollow cylinder,

$$A_c = \frac{\pi(D_o^2 - D_i^2)}{4} \quad .$$

Equation 14 becomes

$$\frac{\partial^2 u}{\partial x^2} + \left[\frac{-4\,\mu\,k}{E\omega(D_o - D_i)} - \frac{\gamma}{gE}\right]\frac{\partial^2 u}{\partial t^2} - \frac{4\,\mu\,k}{(D_o - D_i)}\frac{\partial u}{\partial t} =$$

$$-\frac{F_o}{A_c E}\sin\omega t \qquad \delta(x - x_o) \tag{15}$$

In general form

$$\frac{\partial^2 u}{\partial x^2} - \beta_1\frac{\partial^2 u}{\partial t^2} - \beta_2\frac{\partial u}{\partial t} = \delta(x - x_o)\,F_1 \sin\omega t \tag{16a}$$

where

$$F_1 = \frac{-F_o}{A_c E} \tag{16b}$$

$$u = u(x,t) \qquad t \geq 0 \tag{16c}$$

$$\beta_1 = \frac{\gamma}{gE} + \frac{4\,\mu\,k}{(D_o - D_i)\,E\omega} \tag{16d}$$

$$\beta_2 = \frac{4\,\mu\,k}{(D_o - D_i)\,E} \tag{16e}$$

$$A_c = \frac{\pi(D_o^2 - D_i^2)}{4} \tag{16f}$$

$$k = \sqrt{\frac{\omega\rho}{2\mu}} \tag{16g}$$

Equation 16 describes the damped longitudinal vibration equation of motion of a long, hollow rod.

5.2.2 Steady-State Solution of Equation of Motion

A general form of solution applicable to various kinds of boundary conditions will be derived. Then boundary conditions appropriate for various representations of a vibrating drill column are used in evaluating this solution.

The steady state of a dynamic system is the state in which the dependent variables describing the system behavior are either invariable with time or are periodic functions in time [33]. That is, the complementary solution has been neglected or has been allowed to decay to zero. This is valid since only the steady-state power loss is sought in the analysis.

General Form of Solution

Following the procedure outlined by Thompson [34,35,36], one replaces sin ωt in equation 16 by $e^{i\omega t}$ and u by $u_c(x,t)$, where the subscript c indicates that a complex displacement amplitude is allowed. After completing the symbolic solution, one disregards the real part of the harmonic solution. This procedure retains only the response caused by the sin ωt driving force. Thus

$$\frac{\partial^2 u_c}{\partial x^2} - \beta_1 \frac{\partial^2 u_c}{\partial t^2} - \beta^2 \frac{\partial u_c}{\partial t} = \delta(x-x_o)\, F_1\, e^{i\omega t} \qquad (17)$$

For the steady state solution

$$u_c(x,t) = U_c(x)\, e^{i\omega t} \qquad (18)$$

$$= U(x)\, e^{-i\phi(x)}\, e^{i\omega t} \qquad (19)$$

$U_c(x)$ is the complex amplitude of the steady-state solution. Substituting equation 18 into equation 17 and dividing through by $e^{i\omega t}$, one obtains

$$\frac{d^2\, U_c(x)}{dx^2} + \beta_1\omega^2\, U_c(x) - i\, \beta_2\omega\, U_c(x) = \delta(x-x_o)\, F_1\, . \tag{20}$$

$$\frac{d^2U_c(x)}{dx^2} + \lambda^2\, U_c(x) = \delta(x-x_o)\, F_1 \tag{21}$$

$$\text{where } \lambda^2 = \beta_1\omega^2 - i\, \beta_2\omega \tag{22}$$

Laplace transforming equation 21 with respect to x, yields

$$s^2\, \overline{U}_c(s) - s\, U_c(o) - U_c'(o) + \lambda^2\, \overline{U}_c(s) = e^{-x_o s}\, F_1$$

$$\text{where } U_c'(o) = \left[\frac{d\, U_c(x)}{dx}\right]_{x\, =\, o}$$

$\overline{U}_c(s)$ is the Laplace transform of $U_c(x)$. Rearranging yields equation 23

$$\overline{U}_c(s) = \frac{e^{-x_o s}}{s^2+\lambda^2}\, F_1 + \frac{s}{s^2+\lambda^2}\, U_c(o) + \frac{1}{s^2+\lambda^2}\, U_c'(o) \tag{23}$$

where $U_c(o)$ and $U_c'(o)$ are boundary conditions that are given or are determinable using given boundary conditions. The inverse Laplace Transform of equation 23 is

$$U_c(x) = \frac{F_1}{\lambda}\, \sin\,[\lambda(x-x_o)]\, H(x-x_o) +$$

$$U_c(o)\ \cos \lambda x + U_c'(o)\ \frac{\sin \lambda x}{\lambda} \tag{24a}$$

where $H(x-x_o)$, the Heaviside "unit function", is defined by

$$H(x-x_o) = \begin{bmatrix} 0 & x < x_o \\ 1 & x \geq x_o \end{bmatrix}$$

Therefore

$$u_c(x,t) = \left[F_1\ \frac{\sin[\lambda(x-x_o)]}{\lambda}\ H(x-x_o) + U_c(o)\ \cos \lambda x \right.$$

$$\left. + U_c'(o)\ \frac{\sin \lambda x}{\lambda} \right] e^{i\omega t} \tag{24b}$$

However, to make equation 24b useful λ must be evaluated. Thus, one must find the root of the complex number given in equation 22.

$$\lambda^2 = \beta_1 \omega^2 - i\ \beta_2 \omega$$

The complex number, $\lambda = (\beta_1 \omega^2 - i\ \beta_2 \omega)^{1/2}$, has 2 roots [37] given by equation 28.

$$\lambda_{1,2} = [\beta_1{}^2 \omega^4 + \beta_2{}^2 \omega^2]^{1/4}\ \{\cos\ [\tfrac{\theta+2m\pi}{2}] +$$

$$i\ \sin\ [\tfrac{\theta+2m\pi}{2}]\} \tag{28}$$

$$\text{where } \theta = \tan^{-1}\left[\frac{-\beta_2}{\beta_1 \omega}\right] \text{ and } m = 0,\ 1. \tag{29}$$

$$\lambda_1 = -\lambda_2\ . \tag{30}$$

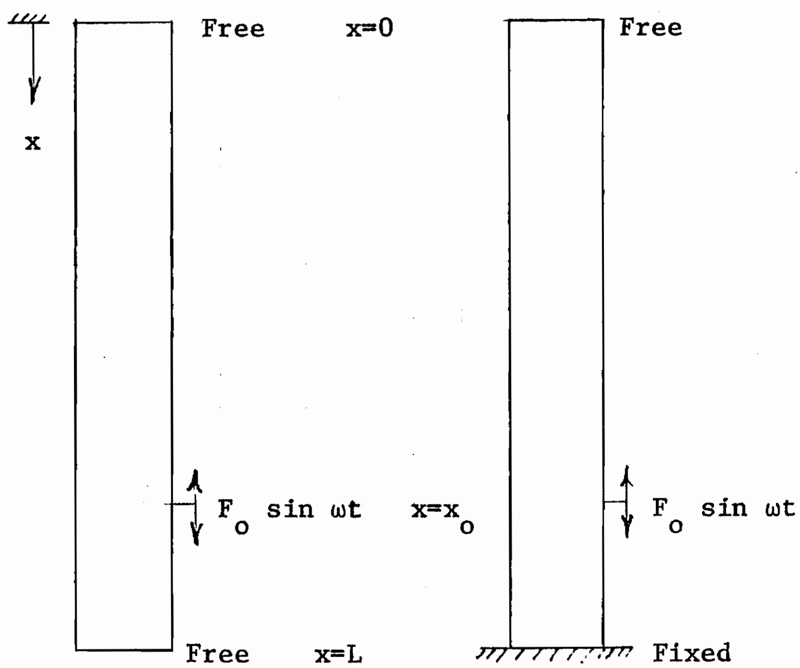
From equation 28, λ has the form

$$\lambda_{1,2} = a + ib \tag{31}$$

To complete the solution of equation 24b one must evaluate the boundary conditions.

Boundary Conditions for Longitudinal Vibration

Two sets of boundary conditions are used to represent the fact that the lower end of the drill column is periodically in and out of contact with the rock during each cycle of vibration. Model A represents vibration of the entire drill column. The first set of boundary conditions for model A, the "first limit on power loss", represents the free-free state of the vibrating column. In this state, the column has lost contact with the rock; its lower end is free and its upper end is also free as far as vibration is concerned [8]. The other set of boundary conditions, the "second limit on power loss" represents a fixed-free column. The "second limit" applies to the "on-times", the period of contact. These two sets of boundary conditions for model A are illustrated in Fig. 4a and 4b. It is important to realize that these two sets represent an upper and a lower limit under the assumptions given in section 5.2.1. That is, a rotary-vibratory drill column that meets all the requirements given in section 5.2.1 has a higher level of power dissipation than one of the limits but a lower level than the other limit. If it does not meet all the requirements, then these boundary conditions do not necessarily represent two limiting cases. For example, one of the limits is not necessarily an upper limit for a rotary-vibratory drill string vibrating in a non-Newtonian



Model A: "FIRST LIMIT"

Fig. 4a

Model A: "SECOND LIMIT"

Fig. 4b

Fig. 4: "Full-Length" Longitudinal Vibration.

flow. More will be written about this point in section 5.2.5.

The boundary conditions for model A are also used in characterizing model B. Model B represents the longitudinal vibration of only the down-hole section of the drill column, that section beneath the driving force. In model A, the length $L-x_o$ represents the "down-hole" section of the drill column. To apply the results of the analysis of model A to model B, assign a very small value to x_o. This gives a useful approximation to a vibration-isolated down-hole machine.

Mathematical representations of these sets of boundary conditions are given below.

Most of these relations are available in texts on engineering mathematics [38,39] or strength of materials texts [40]. The relation for an internal force in the column, P, is

$$P = AE\varepsilon$$

$$= AE \frac{\partial u}{\partial x}(x,t)$$

When applied to the free ends, x = L and x = o in Fig. 4a, yields

$$P = AE \frac{\partial u}{\partial x}(o,t) = AE \frac{\partial u}{\partial x}(L,t) = 0 \quad , \tag{32}$$

since a free end is free from load. From equation 32, the boundary conditions for the column in Fig. 4a is

$$\frac{\partial u}{\partial x}(o,t) = \frac{\partial u}{\partial x}(L,t) = 0 \tag{32a,b}$$

For the fixed end x = L in Fig. 4b

$$u(L,t) = 0 \tag{34}$$

The results of this section are summarized below for reference. The boundary conditions for the free-free representation (first limit of power loss) are:

$$\frac{\partial u}{\partial x}(o,t) = 0 \tag{33a}$$

$$\frac{\partial u}{\partial x}(L,t) = 0 \tag{33b}$$

The boundary conditions for the fixed-free representation (second limit of power loss) are:

$$\frac{\partial u}{\partial x}(o,t) = 0 \tag{33a}$$

$$u(L,t) = 0 \tag{34}$$

Solutions for Various Representations of the Vibrating Column

The solution for the free-free column (model A - "First Limit") is derived by applying the boundary conditions, equations 33a and 33b, to the general form of solution given in equation 24. From equation 18,

$$\frac{\partial u}{\partial x}(x,t) = U_c'(x)\, e^{i\omega t} \tag{35}$$

where the prime indicates a partial derivative with respect to x. Equations 33a, 33b, and 35 imply that

$$U_c'(o) = 0 \tag{36a}$$

$$U_c'(L) = 0 \tag{36b}$$

Substituting equation 36a into equation 24a yields

$$U_c(x) = \frac{F_1}{\lambda} \sin [\lambda(x-x_o)] \, H(x-x_o) + U_c(o) \cos \lambda x$$

where $H(x-x_o)$, the Heaviside "unit function" is defined by

$$H(x-x_o) = \begin{bmatrix} 0 & x < x_o \\ 1 & x \geq x_o \end{bmatrix} \tag{37}$$

Taking the derivative of $U_c(x)$ gives

$$U_c'(x) = F_1 \cos [\lambda(x-x_o)] \, H(x-x_o) +$$

$$\frac{F_1}{\lambda} \sin [\lambda(x-x_o)] \, \delta(x-x_o) - \lambda \, U_c(o) \sin \lambda x \tag{38}$$

The fact that the derivative of the unit step function is equal to the dirac delta function [42], is used in determining equation 38. Substituting equation 36b into equation 38 and using definitions of $H(x-x_o)$ and $\delta(x-x_o)$ one finds that

$$U_c'(L) = 0 = F_1 \cos [\lambda(L-x_o)] - \lambda \, U_c(o) \sin \lambda L$$

Thus

$$U_c(o) = \frac{F_1 \cos\,[\lambda(L-x_o)]}{\lambda \sin \lambda L} \tag{39}$$

Substituting equation 39 into equation 24a yields

$$U_c(x) = \frac{F_1}{\lambda} \sin\,[\lambda(x-x_o)]\; H(x-x_o) + \frac{F_1 \cos\,[\lambda(L-x_o)]}{\lambda \sin\lambda L} \cos \lambda x \tag{40}$$

where $\lambda^2 = \beta_1\omega^2 - i\beta_2\omega$.

From equation 24b

$$u_c(x,t) = \left[\frac{\sin\,[\lambda(x-x_o)]}{\lambda} H(x-x_o) + \frac{\cos\,[\lambda(L-x_o)]}{\lambda \sin \lambda L} \cos \lambda x\right] F_1\, e^{i\omega t} \tag{41}$$

The steady-state solution is the imaginary part of equation 41 as seen in the time domain. The imaginary parts of λ are carrying phase information coming about because of the spatial coordinate, x. Thus

$$u_{ss}(x,t) = \mathrm{Im}[U_c(x)\; e^{i\omega t}] = \mathrm{Im}[U(x)\; e^{-i\phi(x)} e^{i\omega t}]$$

$$= U(x) \sin\,[\omega t - \phi(x)] \tag{42}$$

where

$$U_c(x) = U(x)\; e^{-i\phi(x)}$$

Equation 42 follows from writing $U_c(x)$ as

$$U_c(x) = A_1(x) - iB_1(x)$$

where A_1 and B_1 are the real and imaginary parts respectively. Expressing $U_c(x)$ in its polar form

$$U_c(x) = U(x)\, e^{-i\phi(x)}$$

$$\text{where } U(x) = \sqrt{A_1^2(x) + B_1^2(x)}$$

and

$$\phi(x) = \text{arc tan}\left[\frac{B_1(x)}{A_1(x)}\right] .$$

To extract the steady-state solution, equation 41 is reduced to a complex number. To simplify this reduction the following notation is used.

$$\sin\,[\lambda(x-x_o)]\; H(x-x_o) = A_{H(x-x_o)} + i\, B_{H(x-x_o)} \tag{43a}$$

$$\cos\,[\lambda(L-x_o)] = A_{L-x_o} + i\, B_{L-x_o} \tag{43b}$$

$$\sin \lambda L = A_L + i\, B_L \tag{43c}$$

$$\cos \lambda x = A_x + i\, B_x \tag{43d}$$

To evaluate, for example, $\cos \lambda x$, where $\lambda = a + ib$, one uses equation 44 as given by Kreyszig [43].

$$\cos \lambda x = \cos ax \cosh bx - i \sin ax \sinh bx \tag{44}$$

A comparison of equations 44 and 43d yields

$$A_x = \cos ax \cosh bx \tag{45a}$$

$$B_x \;=\; -\sin ax \sinh bx \tag{45b}$$

Similarly

$$A_{H(x-x_o)} \;=\; \sin\,[a(x-x_o)]\cosh\,[b(x-x_o)]\,H(x-x_o) \tag{45c}$$

$$B_{H(x-x_o)} \;=\; \cos\,[a(x-x_o)]\sinh\,[b(x-x_o)]\,H(x-x_o) \tag{45d}$$

$$A_{L-x_o} \;=\; \cos\,[a(L-x_o)]\cosh\,[b(L-x_o)] \tag{45e}$$

$$B_{L-x_o} \;=\; -\sin\,[a(L-x_o)]\sinh\,[b(L-x_o)] \tag{45f}$$

$$A_L \;=\; \sin aL \cosh bL \tag{45g}$$

$$B_L \;=\; \cos aL \sinh bL \tag{45h}$$

For convenience the notation given in equations 43a-d and defined in equations 45a-h will be used. Substituting equations 43a-d and $\lambda = a + ib$ into equation 41

$$u_c(x,t) \;=\; \left[\frac{A_{H(x-x_o)} + i\,B_{H(x-x_o)}}{a+ib} + \right.$$

$$\left.\frac{(A_{L-x_o} + i\,B_{L-x_o})(A_x + i\,B_x)}{(a+ib)(A_L + i\,B_L)}\right] F_1\,e^{i\omega t} \tag{46}$$

Rearranging yields

$$u_c(x,t) \;=\; \frac{1}{(a+ib)(A_L + i\,B_L)}\left[(A_{H(x-x_o)} + i\,B_{H(x-x_o)})\right.$$

$$\left.(A_L + i\,B_L) + (A_{L-x_o} + i\,B_{L-x_o})(A_x + i\,B_x)\right] F_1\,e^{i\omega t} \tag{47}$$

$$= \frac{1}{(a\ A_L - b\ B_L) + i(a\ B_L + b\ A_L)} \Big[(A_{H(x-x_o)}\ A_L - B_{H(x-x_o)}\ B_L$$

$$+ A_{L-x_o}\ A_x - B_{L-x_o} B_x) + i(A_{H(x-x_o)}\ B_L + B_{H(x-x_o)}\ A_L + A_{L-x_o}$$

$$B_x + B_{L-x_o}\ A_x) \Big] F_1\ e^{i\omega t}$$

Next, multiplication of the numerator and denominator by the complex conjugate of the denominator yields

$$= \frac{1}{(a\ A_L + b\ B_L)^2 + (a\ B_L - b\ A_L)^2} \Big[[(A_{H(x-x_o)}\ A_L - B_{H(x-x_o)}\ B_L$$

$$+ A_{L-x_o}\ A_x - B_{L-x_o}\ B_x)(a\ A_L - b\ B_L) + (A_{H(x-x_o)}\ B_L + B_{H(x-x_o)}$$

$$A_L + A_{L-x_o}\ B_x + B_{L-x_o}\ A_x)(a\ B_L + b\ A_L)] - i\ [(A_{H(x-x_o)}\ A_L$$

$$- B_{H(x-x_o)}\ B_L + A_{L-x_o}\ A_x - B_{L-x_o}\ B_x)(a\ B_L + b\ A_L) - (A_{H(x-x_o)}$$

$$B_L + B_{H(x-x_o)}\ A_L + A_{L-x_o}\ B_x + B_{L-x_o}\ A_x)(a\ A_L - b\ B_L)] \Big] F_1 e^{i\omega t} \quad (48)$$

Thus the solution is of the form

$$u_c(x,t) = [A_u(x) - i\ B_u(x)]\ F_1\ e^{i\omega t} \quad (49)$$

where

$$A_u(x) = \frac{1}{(a\ A_L - b\ B_L)^2 + (a\ B_L + b\ A_L)^2} \Big[(A_{H(x-x_o)}\ A_L$$

$$- B_{H(x-x_o)} B_L + A_{L-x_o} A_x - B_{L-x_o} B_x)(a A_L - b B_L)$$

$$+ (A_{H(x-x_o)} B_L + B_{H(x-x_o)} A_L + A_{L-x_o} B_x + B_{L-x_o} A_x)$$

$$(a B_L + b A_L)\Big] \qquad (50)$$

and where

$$B_u(x) = \frac{1}{(a A_L - b B_L)^2 + (a B_L + b A_L)^2} \Big[(A_{H(x-x_o)} A_L - B_{H(x-x_o)}$$

$$B_L + A_{L-x_o} A_x - B_{L-x_o} B_x)(a B_L + b A_L) - (A_{H(x-x_o)} B_L$$

$$+ B_{H(x-x_o)} A_L + A_{L-x_o} B_x + B_{L-x_o} A_x)(a A_L - b B_L)\Big] \qquad (51)$$

Writing equation 49 in polar form yields

$$u_c(x,t) = \sqrt{A_u^{\,2}(x) + B_u^{\,2}(x)}\; F_1\, e^{-i\phi(x)}\, e^{i\omega t} \qquad (52)$$

$$\text{where } \phi(x) = \text{arc tan}\left[\frac{B_u(x)}{A_u(x)}\right] \qquad (53)$$

Invoking equation 42 one finds that

$$u_{ss}(x,t) = U(x) \sin\,[\omega t - \phi(x)]$$

$$u_{ss}(x,t) = \sqrt{A_u^{\,2}(x) + B_u^{\,2}(x)}\; F_1 \sin\,[\omega t - \phi(x)] \qquad (54)$$

where A_u, B_u and $\phi(x)$ are given by equations 50, 51 and 53, and where, in turn, terms in these equations are defined by equations 43 and 45.

The Solution for the Fixed-Free Column (Model A)

This solution can be derived in the same way as the free-free column. It is simpler to use the general form of solution, equation 24b, and to evaluate for $U_c(o)$ and $U_c'(o)$ using the boundary conditions. This approach will be used here. Repeating equation 24a for convenience

$$U_c(x) = \frac{F_1}{\lambda} \sin [\lambda(x-x_o)] \, H(x-x_o) +$$

$$U_c(o) \cos \lambda x + U_c'(o) \frac{\sin \lambda x}{\lambda} \tag{24a}$$

From boundary conditions given in equations 33a and 34 one finds that

$$\frac{d\, U_c(o)}{dx} = 0 \tag{55a}$$

$$U_c(L) = 0 \tag{55b}$$

Substituting equation 55a into equation 24b yields

$$U_c(x) = \frac{F_1}{\lambda} \sin [\lambda(x-x_o)] \, H(x-x_o) + U_c(o) \cos \lambda x \tag{56}$$

Substituting equation 55b into equation 56 yields

$$U_c(L) = \frac{F_1}{\lambda} \sin [\lambda(L-x_o)] + U_c(o) \cos \lambda L = 0 \tag{57}$$

from which one can find $U_c(o)$

$$U_c(o) = - \frac{F_1 \sin [\lambda(L-x_o)]}{\lambda \cos \lambda L} \tag{58}$$

Substituting equation 58 into 56 yields

$$U_c(x) = \left[\frac{\sin\,[\lambda(x-x_o)]\; H(x-x_o)}{\lambda} - \frac{\sin\,[\lambda(L-x_o)]}{\lambda\,\cos\,\lambda L}\;\cos\,\lambda x \right] F_1 \tag{59}$$

Applying the same notation defined in equations 43a-d along with

$$\sin\,[\lambda(L-x_o)] = S_{L-x_o} + i\,T_{L-x_o} \tag{60a}$$

and

$$\cos\,\lambda L = G_L + i\,H_L \tag{60b}$$

where

$$S_{L-x_o} = \sin\,[a(L-x_o)]\,\cosh\,[b(L-x_o)] \tag{61a}$$

$$T_{L-x_o} = \cos\,[a(L-x_o)\,\sinh\,[b(L-x_o)] \tag{61b}$$

$$G_L = \cos\,aL \quad \cosh\,bL \tag{61c}$$

$$H_L = -\sin\,aL \quad \sinh\,bL \tag{61d}$$

Applying the same method used in the free-free case, the following relations are obtained:

$$u_c(x,t) = U_c(x)\,e^{i\omega t}$$

$$= \sqrt{A_u^{\,2}(x) + B_u^{\,2}(x)}\;\; F_1\, e^{-i\phi(x)}\, e^{i\omega t} \tag{62}$$

where

$$A_u(x) = \frac{1}{(a\,G_L - b\,H_L)^2 + (a\,H_L + b\,G_L)^2} \Big[(A_{H(x-x_o)}\,G_L - B_{H(x-x_o)}$$

$$H_L - S_{L-x_o}\,A_x + T_{L-x_o}\,B_x)(a\,G_L - b\,H_L) + (A_{H(x-x_o)}\,H_L$$

$$+ B_{H(x-x_o)}\,G_L - S_{L-x_o}\,B_x - T_{L-x_o}\,A_x)(a\,H_L + b\,G_L) \Big] \qquad (63)$$

and where

$$B_u(x) = \frac{1}{(a\,G_L - b\,H_L)^2 + (a\,H_L + b\,G_L)^2} \Big[(A_{H(x-x_o)}\,G_L - B_{H(x-x_o)}$$

$$H_L - S_{L-x_o}\,A_x + T_{L-x_o}\,B_x)(a\,H_L + b\,G_L) - (A_{H(x-x_o)}\,H_L$$

$$+ B_{H(x-x_o)}\,G_L - S_{L-x_o}\,B_x - T_{L-x_o}\,A_x)(a\,G_L - b\,H_L) \Big] \qquad (64)$$

Using equation 29 yields

$$u_{ss}(x,t) = U_c(x) \sin [\omega t - \phi(x)] \qquad (65)$$

$$\text{where } U_c(x) = \sqrt{A_u^{\,2}(x) + B_u^{\,2}(x)} \quad F_1$$

and

$$\phi = \text{arc tan} \left[\frac{B_u(x)}{A_u(x)} \right]$$

The Solution for "Down-Hole" Longitudinal Vibration (Model B)

Results of the analysis of model A can be applied to model B by assigning a very small value to x_o. For the free-free case, set x_o equal to a small number in equation 54. For the fixed-free case, assign x_o a small value in equation 65.

5.2.3 Steady-State Viscous Power Losses

The following development follows the lead of Kinsler and Frey [41]. The power supplied to the system is equal to the total work done by the driving force per complete vibration. At steady state, the power being supplied by the driving force is equal to that being dissipated by the viscous force. Therefore, the energy dissipated into the drilling fluid per cycle at steady state is given by

$$\text{Work done per cycle} = \int_o^T F(t) \frac{\partial u_{ss}(x_o, t)}{\partial t} dt \tag{66}$$

where the driving function, F(t) is applied at $x = x_o$. This force gives rise to a steady-state displacement $u_{ss}(x_o, t)$ at $x=x_o$, with a period of T. Thus power dissipated, P_D, is

$$P_D = \frac{1}{T} \int_o^T F(t) \frac{\partial u_{ss}(x_o, t)}{\partial t} dt \tag{67}$$

where

$$u_{ss}(x,t) = U(x) \sin [\omega t - \phi(x)] \tag{68}$$

That is,

$$P_D = \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} F_o \sin \omega t \left[\omega\, U(x_o) \cos [\omega t - \phi(x_o)] \right] dt \tag{69}$$

Rearranging into a convenient form

$$P_D = \frac{\omega^2\, U(x_o)\, F_o}{2\pi} \int_0^{\frac{2\pi}{\omega}} \sin \omega t \cos [\omega t - \phi(x_o)]\, dt \tag{70}$$

After integration, one finds

$$P_D = \frac{\omega\, U(x_o)\, F_o \sin \phi(x_o)}{2} \tag{71}$$

Equation 71 can be simplified further by applying the following relations:

$$U(x_o) = F_1 \sqrt{A_u^{\,2}(x_o) + B_u^{\,2}(x_o)} \tag{72}$$

and $$\phi(x_o) = \arctan \left[\frac{B_u(x_o)}{A_u(x_o)} \right] \tag{73}$$

which are applicable to full-length longitudinal vibration (Model A) and to down-hole longitudinal vibration (Model B). From equation 73,

$$\sin \phi(x_o) = \frac{B_u(x_o)}{\sqrt{A_u^{\,2}(x_o) + B_u^{\,2}(x_o)}} \tag{74}$$

Substituting expressions for sin $\phi(x_o)$ and $U(x_o)$ into equation 71 gives

$$P_D = \frac{\omega\ B_u(x_o)\ F_o\ F_1}{2} \tag{75}$$

Substituting $F_1 = -\frac{F_o}{A_c E}$ into equation 75 yields

$$P_D = -\frac{\omega\ B_u(x_o)\ F_o^{\ 2}}{2\ A_c E} \tag{76}$$

At steady state, power dissipated into the drilling fluid is given by equation 76 when the drill column is driven by a harmonic driving function F_o sin ωt applied at $x=x_o$.

Numerical Estimates of Power Losses

P_D, the viscous losses into the drilling fluid, is given by equation 76. Numerical estimates of P_D for various values of ω are shown in Fig. 5 and Fig. 6 for model A, and in Fig. 7 for model B.

For the free-free case and the fixed-free case the vibratory viscous power losses are considerably small for L = 152.4 m (500 ft), except at the resonance peaks. Numerical estimates of P_D for various values of L show the same trend. P_D increases as μ increases.

For model A, the power losses away from the peaks in the free-free case are much higher than those of the fixed-free case for frequencies less than approximately 450 rad/s. Away from the resonance peaks, the fixed-free case had power losses of the order of 7.456 W (10^{-2} hp) while the free-free case had losses ranging from 74.56 to

74.560 W (0.1 hp to 10 hp). Close to resonance peaks the free-free case had much higher losses than the fixed-free case for frequencies less than approximately 450 rad/sec. The free-free and fixed-free representations seem to interchange roles for driving frequencies greater than about 450 rad/s.

The plot for model B has a smaller number of peaks. These resonance frequencies are much higher than those of model A. Peak viscous power losses of model B are larger than those of model A when the same driving force is used. Away from the peaks, model B has smaller power losses than model A.

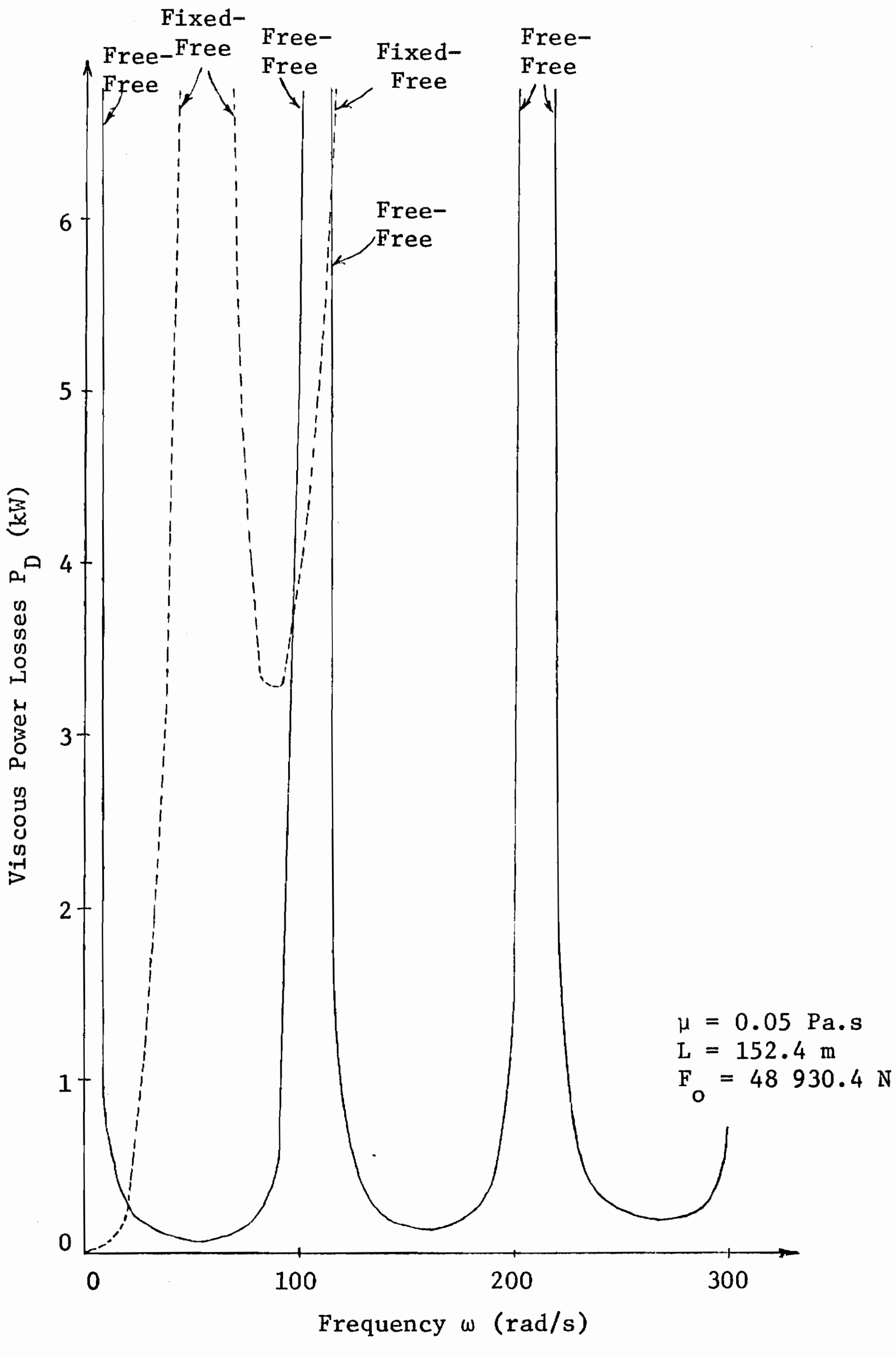


Fig. 5. Viscous Vibratory Power Losses for ω less than 300 rad/s (Model A).

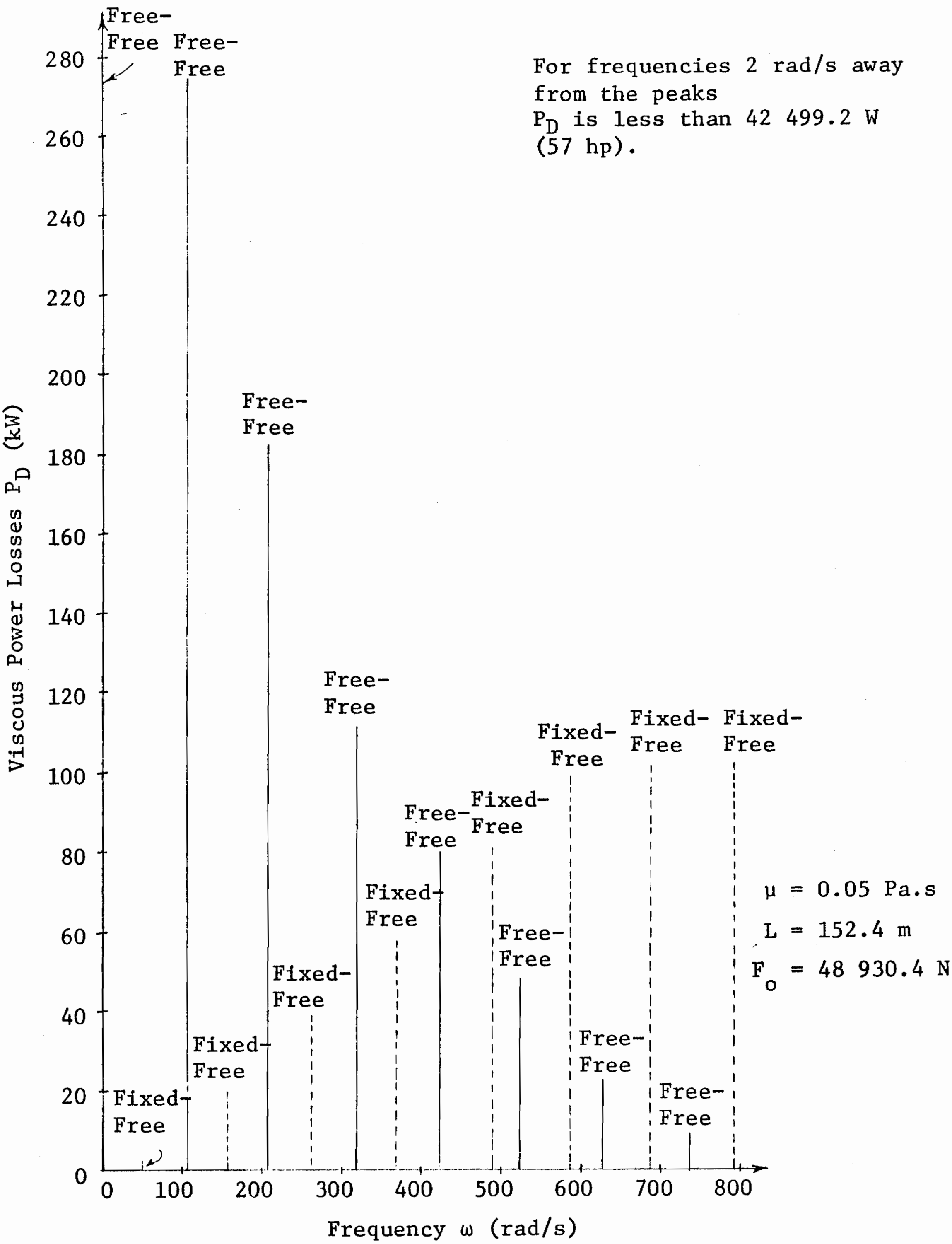


Fig. 6. Viscous Vibratory Power Losses for ω less than 800 rad/s (Model A) (only resonant peaks shown).

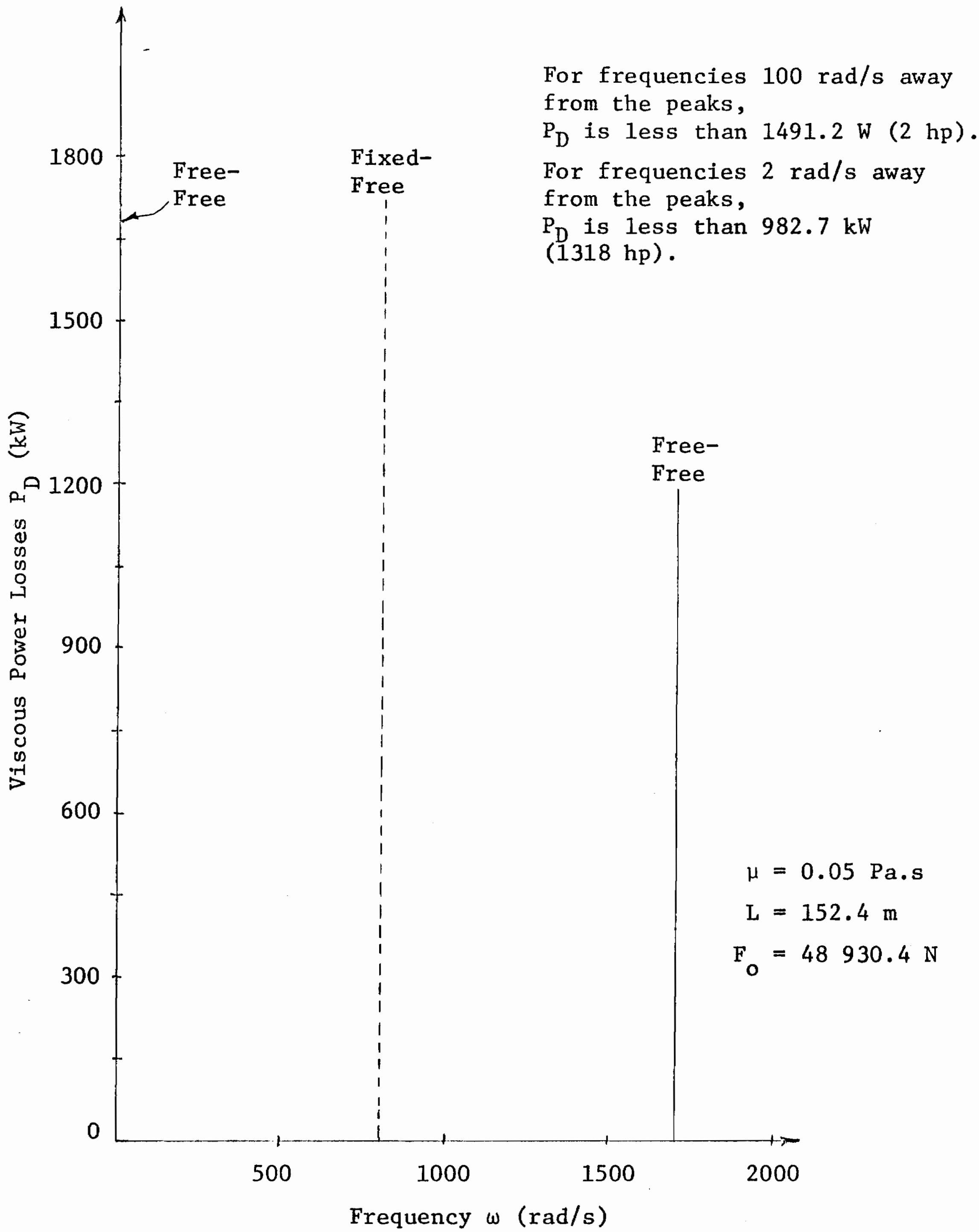


Fig. 7. Viscous Vibratory Power Losses for ω less than 1800 rad/s (Model B) (only resonant peaks shown)

5.2.4 Discussion of Results

Before drawing conclusions from the results of this analysis, it is necessary to establish the relationship between the actual system and the model. This relationship will be specified in terms of the conditions under which the model approximates the actual system.

It is also necessary to determine the limitations of the analytical tools used. In this analysis the Laplace transform method and the complex-algebra method for steady-state solutions have been used. Some remarks need to be made about their limitations. Finally the results of the analysis of damped longitudinal vibration done in section 5.2 will be compared with those of the undamped longitudinal vibration of a rod [36].

This discussion is especially important in any analysis where simple models have been used to describe complex systems.

It will be useful to outline the over-all "strategy" employed in this analysis. This outline should help to place this discussion in a correct perspective. First, the system was described in terms of a simple model. A mathematical relation was derived for this model and a solution was sought. This analysis gives a solution and conditions under which this solution is applicable. The final outcome of the analysis is a solution which is applicable to the original system under the conditions when the model approximates the original system, and under the conditions when the mathematical analysis is valid. Subject to these two sets of conditions, the system behavior and the solution will be approximately the same.

The following discussion should provide two sets of conditions. From these constraints, one could determine when the analytical solution will approximate the system response very closely.

Checks on the Analytical Solution of Model A (Full-Length Vibration)

Before applying the analytical solutions formulated from the models to the actual system, it will be necessary to establish the validity of the mathematical tools applied. Two primary analytical methods were used: the complex algebra method of deriving steady-state solutions and Laplace transforms.

The Laplace Transform method is applicable in this problem. The primary variable involved in this analysis was the displacement. The displacement of a column under the conditions specified in this analysis is of the exponential order and is piece-wise continuous. Therefore, the Laplace transforms exist and the application of this method is valid [35].

The complex algebra method for determining steady-state solutions is based on the fact that the steady-state solution of a system excited by a harmonic force is also harmonic, with the same natural frequency [36]. The assumed form of solution given in equation 19 was deduced from the fact that the impressed force leads the resulting displacement by the phase angle ϕ. The complex algebra method is used extensively in the solution of problems similar to the damped longitudinal vibration of a column as the following discussion will show.

A very similar problem to the damped longitudinal vibration of a column is the damped vibration of a string. Rayleigh [44] solved the free damped vibration of a string by the complex algebra method. The solution looked very similar to the solution for model A. However, the correspondence between these solutions is not clear since the end conditions for a column and a string are different. Snowdon [45] had solutions for the longitudinal vibration of internally damped rods in terms of driving point impedances. The expressions for these impedances were very similar to terms in the steady-state solution, but no simple comparison can be made between these results. Chen [46] solved for the undamped vibration of a string driven by a concentrated force. Chen's solution before the boundary conditions U(o) and U'(o) were evaluated, was almost exactly the same as equation 24a (the general form of solution). The only difference was the complex constant λ in equation 24a which was replaced by a real constant ω/c in Chen's. Several solutions of the undamped longitudinal vibration of rods with various boundary conditions were located [28,48,49,50]. These solutions are usually not in forms that are directly comparable.

The preceeding paragraph had some examples of very closely related problems solved by either Laplace transforms or complex algebra. Chen [47] states that "all methods developed for the string can be borrowed to solve problems for the rod" since "the differential equation of a longitudinally vibrating rod is identical to that of a vibrating string". Therefore, one can claim that the methods employed in this analysis, the complex algebra method and Laplace transform method, are valid since they are valid for the vibrating string problem.

As a further check, the results of this analysis will be compared with those of simpler systems. At low frequencies, the column vibrates almost like a rigid body. This rigid body motion is expected to approximate simple rigid-body motion. To determine what is a "low" frequency one must find the natural frequencies of an elastic rod. One-tenth of the first internal natural frequency is regarded as a low frequency.

In Thomson's formulation of the longitudinal vibration of free-free rod [50], the natural frequencies are given by

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{Eg}{\gamma}}$$

where $n = 0, 1, 2, \ldots, \infty$

This expression will be used in finding the natural frequencies. Using $E = 0.2068 \times 10^{12}$ N/m^2 (30×10^6 psi), $g = 9.8067$ m/s^2 (386 in/s^2), $L = 152.4$ m (500 ft) and $\gamma = 76\ 820$ N/m^3 (0.283 lbf/in^3)

$$\omega_1 = 105.93 \text{ rad/s.}$$

Now, a simple rigid body has the equation of motion

$$m\ddot{x} = F \cos \omega t$$

Using complex algebra, X, the displacement amplitude is given by

$$X = \frac{F}{-\omega^2 m} \qquad (77)$$

F is 49 033 N (11,000 lbf) and m, the mass of the column is

$$\frac{\gamma\pi(D_o^2 - D_i^2) L}{4 g}$$

For D_o = 0.1407 m (5.54 in), D_i = 0.1186 m (4.67 in) and L = 152.4 m (500 ft), γ = 76 820 N/m^3 (0.283 lbf/in^3), m = 52 097 kg. At $\omega = \frac{1}{10}$ ω_1 = 10 rad/s, (a low frequency relative to the natural frequency)

$$X = \frac{F}{-\omega^2 m}$$

$$= 0.091 \text{ m } (3.58 \text{ in}).$$

One would expect $U(x_o)$ computed from the results in section 5.2.2 to be of the order of 0.091 m when ω is 10 rad/s. The displacement for the free-free column at ω = 10 rad/sec is 0.085 m as computed from equation 54. This is close to the estimate made from the rigid body motion. The first damped natural frequency calculated from the results of this analysis is 105.2 rad/s, while the undamped, closed-form, natural frequency estimated is 105.93 rad/s. The second damped natural frequency found here is 211 rad/s while the undamped natural frequency is $2\omega_1$ or 211.86 rad/s. These results check adequately since damping will yield a peak response below the undamped natural frequency. For μ = 0, (no damping) the peak response frequencies were shifted up to 106 rad/s, and 212 rad/s as would be expected. They are very close to the undamped natural frequency estimated above. Increased resolution on the computer's frequency steps would allow greater accuracy in the determination of the eigenvalues. Undoubtedly they will coincide with those given by Thomson [50].

Timoshenko [49] gives the natural frequencies for a fixed-free rod as

$$\omega_n = \frac{n\pi}{2L}\sqrt{\frac{Eg}{\gamma}} \quad , n = 1, 3, 5, \ldots, \quad \infty$$

Using the same values as in the case of the free-free rod,

$$\omega_1 = 52.9 \text{ rad/s} \quad \text{and} \quad \omega_2 = 3\omega_1 \text{ or } 158.7 \text{ rad/s.}$$

The damped analysis used here gives maximum response peaks as 52.4 rad/s and 157.9 rad/s. For μ=0, this analysis gives

$$\omega_1 = 53 \text{ rad/s} \quad \text{and} \quad \omega_2 = 159 \text{ rad/s.}$$

All these results check well.

Results derived from simple theories are consistent with these results. One could claim that this analysis is valid.

From the results given in the preceding section it is clear that some viscous losses are present in the system. The power losses for the lower limit are between about 0.746 W to 74.56 W (0.001 hp to 0.1 hp) for frequencies between 0.1 rad/sec to 350 rad/sec, excluding the peaks. It is not clear whether the losses given by the free-free case represent an upper limit. These losses range from 74.56 W to 7 456 W (0.1 hp to 10 hp) away from the resonance points. It is possible that substantial losses exist. But further analysis will be necessary to confirm this. Losses at the resonant peaks will be discussed later as they have particular relevance to the vibratory drilling problems and deserve separate attention.

The Models Versus the Drill Column

The validity of the solution to the mathematical model has been discussed. The conditions under which the model approximates the system and the extent to which it can approximate the model need to be considered before drawing conclusions. Many of the assumptions have been discussed in section 5.1 and 5.2.

Assumptions on Fluid Flow Pattern and Properties

Many assumptions were made about fluid properties. Some of them were mentioned in section 5.2. Most of these assumptions were conservative, that is, they would give lower estimates of losses than the actual amount of losses present. Some remarks will be made about some of the assumptions not discussed already.

The flow rate of the drilling fluid in a hole ensures that the flow pattern is turbulent [1]. In section 5.2, some remarks were made about McCray's [1] conjecture regarding the presence of a thin laminar boundary layer close to the drill column. However, McCray's remark was made under a discussion of rotary drilling. So, it is very probable that the flow around a rotary-vibratory drill column is turbulent. Some other factors tend to aid turbulence.

Eccentrically-situated drill columns also aid turbulence. If the drill column is not well-centered in the hole, it tends to make the flow more disorderly and perhaps to aid turbulence. The lateral vibration of the column aids turbulence too.

It could be stated with some certainty that most of the assumptions on fluid properties were conservative as far as viscous losses were concerned. What is needed at this point is to determine how the underestimation of losses due to assumptions about fluid properties would compare with the over-estimation due to other assumptions.

The assumption regarding the flow of the fluid down through the inner pipe of the drill column and up through the annular space needs some consideration. It was assumed that the fluid in the inside pipe and in the outer annulus were stationary. This assumption might not make much difference for rotational effects, but it could have a large effect on the longitudinal vibratory effects. In any case, the fluid flows through two constrictions in the annulus and the inside pipe. One "constriction" is at the drill bit and the other is the drill collar. These non-uniformities will certainly make the fluid pattern more turbulent. It seems as if most aspects of the drilling process tend towards making the flow pattern more and more turbulent. The only property that tends to make the flow less turbulent is its non-Newtonian behavior of the drilling fluid. There is much more to be said about these assumptions. But this will all amount to stating that most of the assumptions made about fluid flow pattern and properties would give an underestimate of the losses.

Application of the Flat Plate Theory to Cylindrical Geometry

There is no experimental or theoretical justification for applying Schlichting's solution for the oscillating flat plate to cylindrical geometry. This approach is used widely. Some examples of its use are in Streeter [52], and Spotts [53]. Further remarks on this point are included in Appendix B. It will be desirable to formulate a solution without making this assumption or to check this assumption analytically. But all evidence indicates that this is a good assumption.

The Models

The boundary condition representations were identified as "first limit" and "second limit". These representations constitute an upper and a lower limit as far as viscous losses are concerned and under all the assumptions made in section 5. The results of analysis based on these representations should give good estimates of the range in which these losses fall. To narrow this range of values, some further analysis of the assumptions is needed.

These representations serve not only to bracket the viscous losses but they describe the two phases of operation of a rotary-vibratory machine. The "first limit", the fixed-free case, represents the "on-times", while the "second limit" represents the remainder of the cycle. By combining these representations, one could get a better estimate of the losses. It is not worthwhile explaining in detail how these representations could be used to compute an average power loss given the "on-time" because these over-all models are very simplified. Hence, the value of carrying out extensive calculations based on them is doubtful.

When more sophisticated models are developed, the following effects should be considered: the bit-rock interaction, the static loading at the top of the column, the nature of the driving function, moving reference frame, the steady advance of the drill column while drilling, and impedance of the ends of the drilling column.

Based on some simple examination of these assumptions and comparison with the actual system, one could say that the free-free case provides a high estimate of losses.

Viscous Losses in Resonant Vibratory Drilling Systems

Figure 5 and Fig. 6 show that the free-free case of model A had losses of about 296 451 W (397.6 hp) at the first internal mode peak response while the fixed-free case has power losses of about 3 802.6 W (5.1 hp) at first mode. It should be noted that the object of adding vibratory effects to the rotary drilling process is to induce high dynamic loads. One wishes to run the drilling system at or very near the damped resonant peaks. It is clear that large amounts of power will be required to be delivered to the down-hole vibratory rig in order to maintain vibratory amplitude. This may be the reason DRI failed to get adequate vibratory effects in their drilling effort. The DRI magnetostrictive transducer was designed to operate at resonance. Thus the analysis applies. However, DRI designed vibration isolation between their vibratory transducer and the drilling column. This isolation never worked. Thus the DRI transducer was attempting to drive the whole system in a resonant mode. This must have demanded high driving powers. The DRI transducer was to deliver on the order of 29 824 W (40 hp) in the down-hole position. This is well below the first mode power possibly required of 296 451 W (397.6 hp). This may be one of the contributing causes of failure of this venture. Figure 7 shows the power loss in a vibration-isolated down-hole machine (model B). The power lost by model B is greater than that of model A when the driving force is the same. However, one would expect that a smaller driving force will be required for model B to obtain the same power output into the rock. Thus lower power losses would be expected when the drilling column is vibration-isolated since power loss is proportional to the square of the force amplitude. This is mere conjecture on the part of the author. The models used in this analysis do not account for the power output into the rock. A model that includes a terminal impedance at the lower end of the column will be required for an accurate study of the effect of vibration isolation on viscous power losses.

One should note that DRI's magnetostriction-vibration drilling system was not driven by a force input as was assumed in this analysis. DRI's machine was essentially driven by a displacement input. Therefore, the conclusions made here about DRI's machine are tentative. Further analysis will be required to make precise conclusions.

5.3 STEADY ROTATION OF DRILL COLUMN

Viscous shear losses into the drilling fluid due to rotation will be estimated using the exact solution of the Navier-Stokes equation for flow between two concentric rotating cylinders [31,51]. In this case, the outer "cylinder", the sides of the hole, is at rest. The inner cylinder, the outside surface of the drill pipe, has a steady rotational speed, ω_r.

Only losses into the annulus between the drill pipe and the hole are considered. Losses into the fluid in the drill pipe will be very small relative to annular losses since this fluid will come to a steady state rotation speed equal to that of the drill column.

It is also assumed that the fluid is at rest at its walls of the hole. Fluid behavior is assumed to be Newtonian and laminar. The drill column is assumed to be rigid; torque is constant along its length. Finally, the flow is assumed to be peripheral; only the tangential component of the velocity, v_θ, is considered. The tangential component of the velocity, v_θ, is given by [39]

$$v_\theta = \frac{1}{r_2^{\,2} - r_1^{\,2}} \left[(\omega_2 r_2^{\,2} - \omega_1 r_1^{\,2})\, r - \frac{r_1^{\,2}\, r_2^{\,2}}{r} (\omega_2 - \omega_1) \right] \tag{78}$$

where r_1, r_2, ω_1 and ω_2 are given in Fig. 8. Applying

$$\tau_{r\theta} = \mu \left[\frac{d}{dr} \left(\frac{v_\theta}{r} \right) \right] \quad , \tag{79}$$

yields a shearing stress

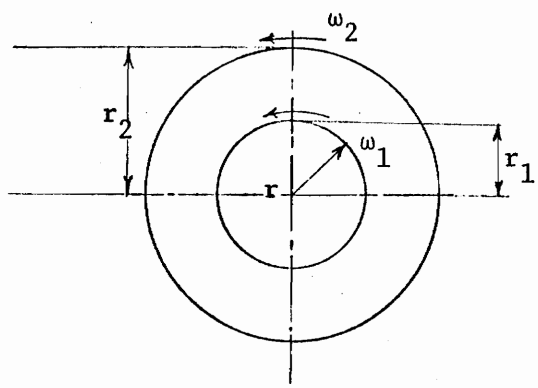


Fig. 8. Flow between two concentric cylinders.

$$\tau_{r\theta} = \frac{2\,\mu}{r_2^2 - r_1^2}\left[\frac{r_1^2\; r_2^2}{r^2}\right](\omega_2 - \omega_1) \tag{80}$$

The shearing stress at the walls of the inner cylinder is obtained by letting $r = r_1$. Thus,

$$\tau_{r\theta}\Big]_{r=r_1} = \frac{2\,\mu}{r_2^2 - r_1^2}\; r_2^2\; (\omega_2 - \omega_1) \tag{81}$$

Letting $\omega_2 = 0$, since the outside "cylinder" is stationary, yields

$$\tau_{r\theta} = -\frac{2\,\mu\, r_2^2\, \omega_1}{r_2^2 - r_1^2} \tag{82}$$

The sign in the preceding equation indicates that fluid motion, which gives rise to the shearing stress at the walls of the cylinder, opposes the motion of the cylinder. The torque transmitted into the fluid

$$T = \frac{2\,\mu\, r_2^2\, \omega_1}{r_2^2 - r_1^2}\; 2\,\pi r_1\, L \cdot r_1 \tag{83}$$

$$= \frac{4\,\mu\pi\, r_1^2\, r_2^2\, \omega_1\, L}{r_2^2 - r_1^2} \tag{84}$$

where L is the length of the column. This expression applies to a case where $\tau_{r\theta}$ is invariant along the column.

Replacing ω_1 by ω_r, r_1 by $\frac{D_o}{2}$, r_2 by $\frac{D_H}{2}$ and substituting T in the equation for power:

$$\text{Power} = T\,\omega_r$$

$$= \frac{\pi\mu\, L(\omega_r\, D_H\, D_o)^2}{D_H^{\,2} - D_o^{\,2}} \tag{85}$$

Equation 85 gives, in watts, the power dissipated into the drilling fluid due to steady rotation. A sample calculation is made for a column rotating at 10.47 rad/s (100 rpm), in a quiescent fluid. If μ = 0.05 Pa.s (0.5 poise), D_H = 0.262 m (10.5 in), D_o = 0.1407 m (5.54 in), a column 152 m (500 ft) long dissipates 73.03 W($\approx$0.1 hp) into the drilling fluid as a result of its steady rotation. Since the power lost is directly proportional to the drill column, one can easily in a mile deep hole expect a loss in excess or 730.3 W ($\approx$1 hp). Moreover, drill column-hole rubbing could only increase this power loss.

Discussion

More numerical results are not given because the relationship between the power dissipated into the fluid and primary parameters is evident from equation 85.

The power dissipated into a fluid by a rigid column, rotating steadily in a Newtonian fluid at rest, is proportional to the square of the angular speed of rotation, to the length of the column, and to the viscosity of the fluid. This assumes that the angular velocity, the fluid properties and annular dimensions ensure laminar flow. Under these conditions, power loss increases as the diameter of the column increases.

An analysis of the assumptions indicates that the power losses derived from equation 85 will be lower than power losses by an actual rotating drill column. An outline of this analysis follows.

The characteristics of the fluid in which a rotary-vibratory drill column is immersed were discussed in some detail in sections 5.1 and 5.2. Assumptions made about these characteristics in estimating rotational losses were similar to assumptions made in estimating longitudinal losses. The effects of these assumptions on the results of these analysis are similar.

Assumptions made about fluid properties would generally give a lower power loss than in an actual system. Fluid flow will probably be turbulent especially in section of the column with low viscosity due to high temperature. Fluid surrounding the upper section would have a higher viscosity than the fluid at the bottom because it has a lower temperature. Higher viscosity and turbulence would give rise to higher power dissipation than was estimated above. The effect of fluid characteristics at the lower section of a deep hole would be more difficult to estimate. Higher temperatures lower the viscosity; higher pressures would tend to increase the viscosity. The total effects of all these properties on the power dissipated is not clear.

It was assumed in this analysis that the fluid was at rest. However, fluid in the annulus between the drilling column flows orthogonal to the calculations of power dissipation based on helical flows may be more accurate.

Rotational losses into the fluid in the drill column were neglected in the longitudinal vibrational energy loss analysis. Drill operators try to maintain a constant rotational speed at the bottom of the hole as various strata are traversed. As the depth of the hole increases, the torque required to maintain the same rotational speed increases. Simon [6] claims that five to ten times more rotary power must be supplied at the top of a 4572 m (15,000 ft) hole than is required to turn the bit at the bottom. Since the rotational speed is kept approximately constant, one could conjecture that a major part of the increased power input goes into frictional losses on the sides of the hole and viscous drag from the drilling fluid.

Two important factors were not taken into account in the preceding analysis -- static and dynamic thrusts. It is not clear how these would affect the estimates of the losses. Static loads of 133 440 to 155 680 N (30,000 to 35,000 pounds) are common loads on the top of 0.1715 m (6 3/4 inch) bits. The effect of not including these factors in the analysis is not clear.

The primary aim of this analysis was to estimate the power dissipated into the drilling fluid as a result of the rotation of the entire drill column in the drilling fluid. A second important purpose of this analysis is related to the analysis of the longitudinal vibration of the drill column. In the analysis in section 5, it has been assumed that the rotational and the vibratory effects could be uncoupled. This is probably a crude assumption. These effects are not coupled together in a simple manner. The steady rotation of the drill column has some effect on the fluid velocity distribution generated by longitudinal vibration. It has a direct effect on the longitudinal vibration of the column. Steady rotation would have little effect on the longitudinal vibration of the column if the bit-bock interface is smooth. However, the rock surface is full of inhomogeneities. These inhomogeneities give rise to severe bouncing of the drill column [8]. McCray [1] states that the rotary "drill string naturally vibrates while rotating rolling cutter-type bits on the bottom, and it is impossible to rotate such bits on bottom without causing drill-string vibrations". The tendency of the drill column to deflect laterally under static loads gives rise to the third way in which steady rotation causes longitudinal vibration. The drill column in a deep well has the relative dimensions of a length of thread. Under the static load and its own weight, the drill column bends laterally. The steady rotation of the column in this configuration, in addition to the irregular rubbing of the column against the sides of the hole, results in longitudinal vibration.

The preceding are three ways in which the rotation of the column has a direct effect on the longitudinal vibration. The rotation of the column has also effects on the velocity distribution of the fluid generated by the longitudinal vibration. In analysing the velocity distribution resulting from longitudinal vibration, it was assumed that fluid flow had no rotational components. For rotary-vibratory drill columns, the rotational components are considerable as indicated by equation 78. This is the second purpose of this analysis of rotational effects. It is clear that the assumption that rotational effects could be estimated independent of longitudinal-vibrational effects is incorrect. However, this analysis gives one an idea of the magnitude of the effects that are present. Numerical estimates based on accurate analysis of rotational effects would provide a basis for trying a more complex analysis of rotary-vibratory phenomena. If rotational effects are small, then it will be relatively accurate to uncouple these effects or even to neglect rotational effects completely. This analysis indicates that rotational effects (tangential velocity and rotational power losses) are present. This indicates that uncoupling rotational and vibratory effects might be suspect.

CONCLUSIONS

This analysis indicates that rotational losses into the drilling fluid might be moderate. The assumptions related to fluid properties would result in estimates of losses that would be lower than the actual losses.

The expression for power losses derived from a simple model indicates that the most important parameters that determine rotational losses are the rotational speed, length of column immersed in the fluid, diameter of the column, and the fluid viscosity. Reduction of fluid viscosity, area of moving surfaces, and rotational speeds would reduce viscous losses.

Equation 78 could provide some useful estimates of the magnitude of the tangential velocity present in the fluid surrounding a rotary-vibratory drill column. If this velocity component is not small compared to the vibratory velocities, then the uncoupling of rotational and vibrational effects should be reconsidered.

RECOMMENDATIONS

Improved Rotational Loss Analysis

If an accurate estimate of the losses is required the following factors deserve consideration:

I. Experimental methods of determining rotational losses should be considered. The rotating-cylinder viscometer could be used as a starting point for designing the experimental equipment. Since the energy dissipation is essentially a conversion of mechanical energy into heat energy, the power dissipated could be estimated through the measured temperature increase in the drilling fluid.

II. It might be useful to eliminate some of the simplifying assumptions about fluid properties, or to deduce the kinds of effects these assumptions would make on the estimated losses.

III. It will also be useful to design inexpensive ways of estimating viscous losses directly from field data. This will give more accurate estimates of the losses in actual systems.

Measures to Reduce Rotational Losses

Most of the obvious measures have been tried already: down-hole rotation, use of thrust bearings to prevent rotation of upper section of the column, and the use of double-channel drill columns. The underlying principle for most of these attempts can be deduced from equation 85. These measures were probably intended to minimize the fluid viscosity, speed of rotation, surface area of moving surfaces, torque required to turn the bit at the bottom, and to increase the free annular space.

There is a need to identify sources of viscous losses and to estimate the levels in down-hole machines. Some factors that should be considered are indirect viscous losses induced by rotation.

Direct and indirect losses due to rotation of down-hole machines should be estimated. The indirect losses are losses due to the longitudinal vibrations induced by the rotation. Systems that should be considered are hydraulic-driven rotary vibratory systems such as systems combining turbo- and vibratory action.

5.4 GENERAL DISCUSSION ON ANALYSIS, CONCLUSIONS, AND RECOMMENDATIONS

Coupled Rotational and Vibratory Effects

There is still some need to examine the relationship between the rotational and vibratory effects in a combination system. There is need to formulate a less approximate representation for the system. For example, it will be useful to consider representing a rotary-vibratory system vibrating in a drilling fluid in terms of a pair of partial differential equations defining the fluid velocity and column displacement. It will consist of the Navier-Stokes equation in cylindrical coordinates and equation of motion of a cylindrical column.

After formulating these equations, then various assumptions will be applied to them, until the problem is solvable. This approach can enable the analyst to find the conditions under which the rotational effects can be uncoupled from the longitudinal vibration.

This approach may be a solution to the problem of having to make unrealistic assumptions, especially when one cannot establish the relationship between the assumed behavior and the actual system behavior.

Perhaps the use of experimental models or numerical methods might be desirable in the longitudinal vibration analysis.

Reduction of Viscous Losses and Maximum Power Transfer

It is important to realize that reduction of losses does not necessarily result in increased power output into the rock. A well-known linear theory states that the power output to the load is maximum when the load is adjusted so that the dissipation of energy into the load and in the generator are the same [9]. This indicates that the power output is not necessarily maximum when the power dissipated in sources other than the generator are reduced.

A vibratory drill system has many sources of power dissipation. These include viscous losses into the drilling fluid, internal losses in the vibrator, useful work done in the rock and frictional losses on the sides of the hole. Reduction of viscous losses does not necessarily result in higher power output to the rock.

This problem will not be discussed in detail here, but it is important to realize that reduction of power losses is not necessarily advantageous. But it will make more power available should the impedance matching be appropriate for rock acceptance.

Reduction in power losses could lead to a lower mechanical power output. For example, elimination of the drill column of a rotary drilling machine would reduce rotational viscous losses. The drill column provides the means for transmitting torque to the rock. But the weight of the drill column, especially the weight of the drill collar, in addition with the static load at the top of the column makes it possible for the rock to accept the mechanical power. The weights of these components maintains good contact between the bit and the rock, thus increasing the torque transmitted to the rock. The elimination of this column would reduce losses but it will also lead to reduced power output to the rock. However, if a substitute hydraulic thrustor system is provided, power output could be maintained with less power input.

There is a need to determine the conditions under which the power dissipated into the load is equal to the power dissipated in the vibration generator (impedance matching). However, the load in this case includes many components. These include the rock reaction and the fluid inertial damping effects. The rock reaction includes stiffness and structural damping effects. It would be useful, in deciding about best design alternatives, to develop a model for studying the relationship between the reduction of viscous losses and maximum power transmission. to the rock. This model would be one where only one of many power outputs was to be maximized by adjusting the other power outputs (impedances).

5.5 CONCLUSIONS AND RECOMMENDATIONS

I. The analysis of rotational effects indicates that viscous power effects due to rotation could be significant. Since most of the assumptions were conservative, a less approximate model will be helpful. However, it seems that these losses are generally recognized in the literature. An analysis based on a specific drilling system might be more valuable than another broad treatment.

II. Estimates of the viscous losses due to longitudinal vibration and steady rotation indicate substantial losses.

Improved Models

The following features should be included in the next analysis:

(i) Simultaneous rotation and longitudinal vibration of the column should be considered. It should be possible to derive from this formulation conditions under which it is accurate to study rotational and vibratory effects separately.

(ii) Dynamic rock properties need to be modelled [54,45]. The rock reaction may be introduced as a boundary condition describing an elastic support [56]. The general form of this boundary condition is $au + b\frac{\partial u}{\partial x} = c$ where c is a constant. It might also be useful to include the static load at the top of the column.

(iii) Inclusion of more realistic fluid properties in the model is needed. Turbulence should be considered.

(iv) Numerical impedance methods for multi-degree of freedom systems could be used.

(v) An experimental model could be used in verifying the results of this analysis.

(vi) The forcing function was represented as a concentrated harmonic force. Solutions involving displacement forcing functions will be more relevant to the DRI drilling system.

(vii) A solution of the problem of longitudinal vibrations taking into account lateral displacements [55].

(viii) A study of multiple output impedance matching and the effects of the reduction of non-useful power losses on the output into the rock.

Measures for Reducing Losses

This analysis indicates the dependence of viscous losses on various factors such as fluid viscosity and density, area of moving surfaces, operating frequencies, and cross sectional dimensions of the drill column. Its dependence on the rate of fluid flow can be deduced. The following measures, deduced from this dependence, could be used in reducing rotational and vibratory losses:

(i) Reduction of fluid viscosity; this can be done using less additives in the drilling fluid, by using gases as drilling fluids, or by using double-channel drill pipes.

(ii) Consider low operating frequencies to reduce shear velocities.

(iii) Consider the possibility of avoiding operation right on the resonance peaks as power loss is maximum here. However, off resonance operation would significantly lower vibratory output to the rock.

(iv) Consider vibration isolation of the drill column from longitudinal vibration.

(v) Consider use of down-hole rotary-vibratory machines.

(vi) Develop methods to control or utilize rotationally induced longitudinal vibration.

(vii) Reduce the areas of moving surfaces: consider the use of compact vibrators, the elimination of the drill column, or the use of a flexible drill column.

Many of these measures and many others have been tried. Usually, these measures reduce losses but give rise to other problems. The loss analysis indicates that power losses involved might be large enough to compensate for the problems associated with these measures.

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APPENDIX A. A GLOSSARY OF ROCK-DRILLING CONCEPTS

(McCray [1], McGregor [2], DRI [7], Ledgerwood [3], were used in compiling these explanations.)

CABLE DRILLING: The drill bit is suspended by a steel rope and is repeatedly raised and dropped to the hole bottom. Penetration depends on gravity, and therefore, holes must be in a vertically downward direction. No fluid is circulated in the hole.

ECCENTRIC-WEIGHT DRILL: See Sonic Drill.

MAGNETOSTRICTION-VIBRATION DRILLING: This is a rotary-vibratory drilling method investigated by DRI in the 50's. The operation of this method is based on the magnetostrictive property exhibited by such materials as nickel, iron, and many alloys of these materials. Magnetostriction is the change of dimensions of a material caused by a change in the magnetic field surrounding the material. The vibratory component of this drilling machine was a magnetostrictive transducer, an electromechanical transducer. An alternating current flowing into the transducer was used to induce vibrations in the bit. The bit was rotated and mud was pumped down through the drill string and bit in the usual manner.

MAGNETOSTRICTIVE TRANSDUCER: See Magnetostriction-Vibration Drilling.

ON-TIME: The portion of the cycle of vibration during which the bit is in contact with the rock. It is determined by the relative magnitudes of the static and dynamic forces.

ROLLING-CUTTER TYPE BIT: The most widely used bit. They are designed for soft, medium, and hard rock. Rows of teeth are cut into rolling members.

SONIC DRILL: This tool uses a series of unbalanced rotating members to set up unbalanced vertical forces. Mud is pumped down the drill string and through an axial-flow mud turbine in the bottom-hole apparatus. The mud turbine drives the eccentric weights which are attached to a long section of drill collars. The motion of the drill collar is transmitted directly to the drill bit. The sonic drill is rotary-vibratory.

APPENDIX B. FURTHER DISCUSSION OF ASSUMPTIONS

Application of Vibrating Plate Theory to a Vibrating Cylinder

Flat plate theory has been applied to cylindrical geometry in the theory of hydrodynamic lubrication. Design methods based on this theory have been acceptable. This is another justification for applying the vibrating plate theory to a vibrating cylinder.

Petroff's equation was derived in this way. It was assumed that the equation for shearing stress in a fluid over which a flat plate is moving could easily be adapted to the cylindrical or journal bearing, provided that the speed and the viscosity are high and the load is very light so that the journal is in a central position in the bearing. Thus, the plate in Fig. B-1 is assumed to be wrapped into the cylindrical shaft in Fig. B-2. The thickness h becomes the radial clearance c, or the difference between the radius of the bearing and the radius of the shaft [53]. The results of applying these assumptions is Petroff's equation for the "hypothetical case of zero load and centrally located journal".

These assumptions seem reasonable. In the preceeding example, flow in a straight line is transformed into circular flow. In the analysis in section 5.2.1, the flat plate is wrapped into a cylinder, but the straight line flow still remained straight line.

It is very likely that this assumption is appropriate. Should more accurate analysis be required, it will be necessary to justify the validity of applying planar theory to axially symmetric

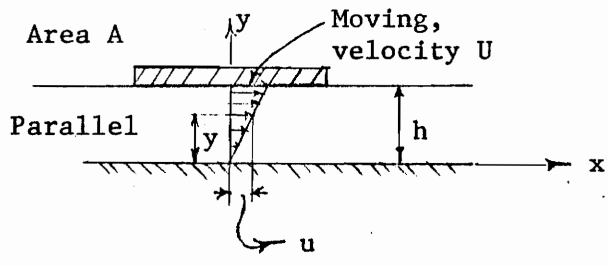

Fig. B-1

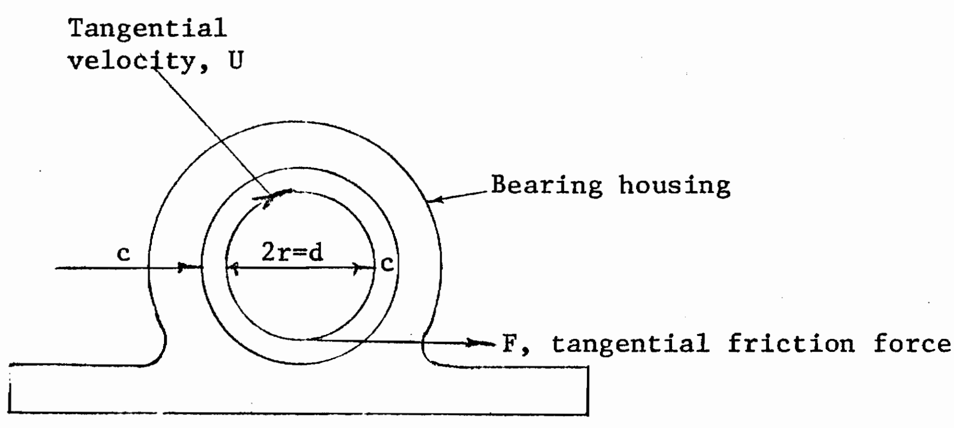

Fig. B-2

Fig. B. Application of flat plate theory to cylindrical geometry (adapted from Spotts [53]).

geometry or the conditions under which transformation is valid.

APPENDIX C. IMPROVED ROCK DRILLING

The preceding sections have considered the problem of viscous losses into the drilling fluid. This section will consider the overall problem of developing an improved drilling system. Some general trends in previous attempts to improve rock drilling will be discussed and, based on this discussion, recommendations will be made on possible future research directions.

Rotary-Vibratory Drilling and the Limitations of Major Drilling Systems*

Researchers initiated attempts to develop improved drilling systems as soon as they recognized the limitations of the conventional rotary drilling method. The conventional rotary drilling system has excellent performance in soft rock. But its performance in hard rocks or in holes deeper than about 4,572 m (15,000 ft.) is very poor. Another major problem of the conventional rotary method is the instability of the drill column [7]. There is also the need for materials that can withstand very large torques; drill column fatigue failure is a serious problem. Conventional rotary drill systems lose much power as a result of rotating the entire drill column.

The turbo-drilling method was an answer to the problem of rotating the entire drill column. The turbo-drill makes more efficient use of power [57]. There is no frictional loss resulting from pressure of

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* Further details on the limitations of major drilling systems are given in Simon [6, 11], McCray [1], and Uren [57].

rotating-drill pipe against the walls of the hole. Lighter drill pipes can be used. The turbo-drilling equipment is less expensive since equipment such as the costly rotary table are eliminated. Uren [57] claims that turbo-drilling equipment makes straighter holes. In turbo-drilling, the bit is rotated at 10 to 16.7 Hz by means of a turbine driven by the flow of the drilling fluid [6]. The load is applied through a thrust bearing so that only the bit is rotated at the bottom of the hole.

One of the major problems in turbo-drilling is the sealing of bearings from the high pressure abrasive drilling fluid [6]. It has some of the conventional rotary drilling problems; the thrust load is limited by practical considerations [6]. Its performance in deep wells is not good. In Russia, where turbo-drills are used in 80 percent of current drilling [6], turbo-drills had poor performance beyond 3 048 m to 4 876.8 m (10,000 to 16,000 ft.) [12]. Turbo-drilling machines operate at higher power levels.

Turbo-drilling was one method for overcoming the problems created by the need to turn the entire drill column in a deep hole. Many other forms of down-hole machines were also tried for the same purpose. All of them had various kinds of limitations.

Down-hole rotary percussive and rotary vibratory machines were more economical than conventional rotary under some conditions. They had faster penetration rates in hard rock and less bit wear under some conditions [5]. Pure percussive drilling systems had limitations as well. Percussive action is a discontinuous process and most of the power developed in this machine is lost in rod vibration [5]. However, percussion drills can drill hard rocks with reasonable wear [5].

The limitations of one drilling system is usually the advantage of another system. This is one of the common features of attempts to develop improved drilling methods. The conventional rotary method drills soft rocks very economically. It has drill column problems and performs poorly in hard rocks. The percussive drills have good performance in hard rock formations. They have no problems with column instability. Unlike rotary drilling, percussive drilling is a discontinuous process.

The same trend could be observed in the attempts to develop improved rotary-percussive and rotary vibratory tools. The Borg-Warner development of Bodine's sonic drill was discontinued as a result of mechanical failure of certain down-hole components. One of the major reasons for the discontinuation of DRI's development of the magnetostrictive-vibration drill was low power output into the rock. The sonic drill operated at 60 Hz, while the magnetostrictive drill operated at 300 Hz.

There is insufficient information on most of the attempts to develop improved drilling methods but it seems as if the advantages of one system constitutes the major problems of another. All of the systems perform well only under certain conditions. This has been regarded as the major problem in developing a method that is superior to the conventional rotary drilling method. But it is likely that there is a great potential in this problem. The combination systems were answers to the limitations of major drilling systems, but they have limitations, too.

For example, the rotary percussive and rotary-vibratory methods have very good performance in hard rocks. One could conjecture that these methods would have better performance than others in deep holes. All rocks are hard in deep holes; soft rocks are harder to drill in deep holes because they become ductile under high pressures while hard rocks are more brittle under these conditions. In addition, most other methods depend on static loads at the top of the hole. Static loads are useful in rotary-vibratory drilling, but rotary-vibratory tools could depend on dynamic loads in deep holes when it becomes impractical to use static loads. Simon [9] states that the "primary interest in oil-well drilling is apparently in the economical drilling of rocks of medium hardness". It is possible that future primary interest in rock drilling will be in rapid drilling of hard rocks in deep holes. The U. S. Energy Outlook [58] claimed that "drilling will be carried out in increasingly deeper formations". In addition to deeper holes, a greater number of shallow exploratory holes would be needed. Currently, 70 drillings are needed for discovery of a yielding oil well [59]. Fuels become increasingly hard to obtain because more easily accessible supplies have been depleted and the only resort is less accessible supplies. In 1945, 25 new-field wildcat wells were required per profitable discovery. In 1961, 70 wells were required per discovery. This indicates the need for deeper wells as well as for rapid drilling. Hubbert [60] writes that 60 percent of the world's present production of energy for industrial purposes, and 67 percent of the U. S.'s is obtained from petroleum and natural gas. Therefore, the decline in annual supplies of these fuels, primarily as a result of the difficulty in making discoveries, poses a problem of immediate concern. The need for deep-hole drilling and for rapid drilling provide good reasons for considering rotary vibratory and rotary-percussive drilling. These methods have probably the best potential for good performance in deep wells. Feasibility studies on rotary-percussive and rotary-vibratory drilling showed that these methods yielded higher drilling rates than conventional rotary method.

These studies indicated that these combination drilling methods consumed more power than standard rotary methods. But it is possible that these drilling methods could be made economically competitive with conventional rotary drilling methods. If this does not happen, these methods could still be used in drilling where power costs are not a main concern. For example, Russia needs to drill quickly regardless of cost [12]. Evidence in various publications on the energy problem [58,59,60] indicates that it might also become necessary for the United States of America, for example, to drill rapidly regardless of cost. Probably the cost of power in deep-hole drilling will not be a primary concern. Therefore, rotary-percussive and rotary-vibratory drilling have a potential in future deep-hole and rapid drilling.

Energy Waste

A general trend in the development of major drilling systems and in attempts to improve rock drilling has been related to power considerations. This is the main thrust of this study.

Developers, just like most people, had very little concern for the finite energy supplies on this planet. Researchers in the United States of America were more concerned about this problem than the Russians [12], but reports on various attempts [3,7] show inadequate concern for the power used. Perhaps one could justify considering frequencies ranging from the "high hundreds to the low thousands of cycles per second" [8]. Perhaps one could justify saying that "we can afford to have a method which requires a lot more energy if it allows more power to be transmitted.......Don't rule out exotic drills just because they take a lot of energy" [19]. But it seems as if most major design decisions are based on this kind of attitude. DRI's reports illustrate this. DRI considered most forms of the major drilling systems. For example, DRI considered hydraulic vibratory motors during a feasibility study. Writing about this study [7], DRI claims that "Hydraulic vibratory motors, operated by the drilling fluid, have been tried with moderate success. The inherent disadvantage of the hydraulic motor arises from the nature of the drilling fluid: (1) high viscosity limits the efficiency, (2) abrasive action results in excessive wear, and (3) solid particles suspended in the fluid tend to cause jamming and failure in service". The conclusion derived from the preceeding claim and other results was: "These problems can be eliminated by using an electrically operated solenoid motor". Perhaps this was the best decision to make under those circumstances. However, DRI chose to use magnetostrictive transducers later and for the following four years spent considerable time "bonding", "laminating", and "etching" various fixtures in this equipment. This illustrates what might be one of the reasons why this effort and many others failed. Whatever criteria were employed in making decisions led DRI to the use of largely electrical systems that were very susceptible to failures and posed insulation problems. These criteria led to the use of monstrous equipment that included several high-power generators. The magnetostrictive transducer, for example, was 9.1 m (30 ft.) long and had a cross-sectional area of 0.021 m^2 (35 $in.^2$). The use of more compact and simpler equipment would have yielded less power losses into various sources and more reliability. Possibly, these decisions could be justified in terms of the available technology then.

Power costs are not significant in most drilling, but design decisions that seek to conserve power would probably lead to economical tools.

Miscellaneous Observations

Many attempts to improve rock drilling were expensive. There seemed to be a general tendency to employ expensive field tests and designing. Expenditures on the four rotary-percussive and rotary vibratory tools probably approach $10 million [3].

Many of these efforts were not cooperating. This probably led to duplication of efforts. Many developers who might have succeeded in producing commercial tools, became inactive because of lack of funds to continue. Ever present scarcity of useful information on tool development is a continuation of this trend.

A "New" Model

Considering present drilling needs [49,50], one could justifiably choose rotary-percussive or rotary-vibratory as a tool with a good potential. Another attempt should be made to develop a commercial combination tool. A good starting point for such a development would be a study of past efforts. There is a great deal to learn from the apparent failures of these attempts. Limitations of various approaches are evident from these attempts. One could also glean information on major advantages and problems in these approaches.

One could deduce from information on past efforts that an attractive model that could be tried would consist of vibratory action superimposed on a turbo-drilling machine. This would combine all the advantages of down-hole drilling offered by turbo-drilling with all the advantages of vibratory drilling. Some of the advantages of turbo-drilling includes its use of the drilling fluid in generating the input power. The drilling fluid is used primarily in conveying rock particles from the hole. Turbo-drilling machines are also used in straight hole drilling. They do not lose power into the drill fluid as a result of the rotation of the entire drilling column and they can develop much higher rotary speeds and input power than conventional methods. Pure vibratory drilling has good performance in hard rock and has a fast drilling rate. However, turbo- and vibratory-drilling use higher power levels than conventional tools. Turbo-drilling machines have several other disadvantages. Although turbo-drilling tools can develop fast rates of rotation and greater horsepower, the extent to which the increased rotary speeds and horsepower are used in an economical manner probably depends largely on the nature of the formation being drilled, along with other factors. McCray [1] also includes two other turbo-drilling problems. In deep-drilling operations, drilling rates tend to be less important to the total feet drilled per bit. If the faster drilling rates of turbo-drilling result in increased bit wear, then turbo-drilling might not be economical. "For those cases in which the volume rate of circulation of drilling fluid must be increased for the sole purpose of developing power at the bottom of the hole, the additional flow-friction pressure losses occurring in the circulating system make the hydraulic transmission of power to the bottom of a deep hole inefficient as compared to mechanically rotating a string of pipe in the hole." Pure vibratory drilling has very problems; most of them have not been studied definitively. For example, pure vibratory drilling machines lose power into the drilling fluid; the analysis in section 5.2 indicates that these losses may be large. Bit wear in pure vibratory drilling has not been studied conclusively. In spite of these problems, turbo- and vibratory drilling have good potential. In deep holes, turbo-drilling machines have drilling rates of one and one-half to five times those of conventional rotary drilling [6]. Laboratory and preliminary field models of rotary-vibratory tools had drilling rates of two to three times those obtained with rotary drilling [6]. Turbo-vibratory drilling should be useful when drilling has to be done at any cost. It might also be possible to optimize these methods to such an extent that they become economically competitive with conventional methods.

Optimization

Perhaps the greatest need in developing improved drilling methods is optimization. Most methods that have been tried worked. Many of them could not compete with conventional rotary economically. Some, of course, had various kinds of mechanical and electrical failures. But a development like DRI's needed optimization more than any other approach, to produce a commercial tool.

Various studies [4] indicate that most attempts to improve drilling processes by modifying significant parameters have differential effects. For example, it has been shown that the drilling fluid head and viscosity tend to reduce drilling rates. The preceding loss analysis shows the dependence of the power loss into the drilling fluid on the fluid viscosity. However, the drilling fluid has several important functions in drilling. In addition to serving as a means of conveying rock particles from the hole, it prevents some rock formations from caving in, it cools the drill bit, and it serves various other functions in several types of formations. Low-density drilling fluids and the use of gases have been tried and have differential effects as well. There are several similar examples. This indicates that an optimization study would be useful. Useful results may not result from modification of one or two variables, but an optimization study involving many of these variables might make a difference. Some optimization work has been done on profitable combination of parameters during drilling operations [4]. Perhaps what is needed more is an optimization study on the drilling hardware in combination with drilling fluid parameters.

For example, a turbine-driven eccentric weight rotary-vibratory machine seems to have a good potential for development. Before Borg-Warner discontinued its development, this machine must have been more expensive than conventional rotary machines. This means that if it had been developed, it would have had applications where the initial cost and power costs were not a problem. It would have been employed for hard rock drilling. This machine would have had limited application, but it has a potential for more extensive use. Attempts to optimize the performance and cost of this tool will be helpful in making use of this potential. In addition to considering the optimal size and reliability of various components, a feasibility study would consider the possibility of running this machine in 5 modes: pure rotary, pure percussive, pure vibratory, rotary-percussive, and rotary-vibratory. All the main drilling systems have limitations, as has been mentioned. Each system drills very well in certain rock formations. Very often, many different kinds of rock formations are available in one hole. This means that a drilling machine does not perform well throughout a drilling operation. A machine that is capable of operating in most of the five modes, mentioned above, will be economical. Such a machine could switch to its rotary mode in shallow soft rock formations, vibrate in very hard rock formations, and employ the percussive mode when there is danger of not drilling straight.

Perhaps this suggests a need for variable frequency operation, automatic, and remote control. The choice of drill bits will also be pertinent.

These parameters and many others, and their effects on the drilling system performance could be the subject of an optimization study. A model of a drilling system would also be useful in this study. Computer-aided design could be used. Some important design criteria that could be included are reliability, costs, and drilling rate.

A Lesson from Past "Failures"

A point that has been alluded to, needs to be emphasized. A possible cause of the failure of all attempts to develop improved drilling methods is the lack of cooperation and a failure to learn from past mistakes. Most of the approaches tried in the 1950's had already been tried before the beginning of the 20th century [3]. Information on these attempts was probably scarce, as it currently is. Therefore, subsequent developers failed to draw from these experiences either because they refused to or because information was withheld from them. DRI tried to draw on these resources and at the same time did a great deal of fundamental work. Less than ten years later, DRI "became inactive because of lack of funds........". A year later Borg-Warner discontinued its development of Bodine Sonic Drill "when additional expenditure........ could not be justified". The American Percussion Tool Co. [3], which was developing a solenoid-hammer type percussive machine, became "unable to secure the financial backing........". The scarcity of information in this field encourages speculation. One, however, cannot help conjecturing about some of the trends in these developments. It is probably evident that the development of drilling methods is not usually short-term. Any attempt to develop an improved tool that involves "fundamental" work should expect to produce a commercial tool in one or two decades. The only organizations that are involved in such long-term programs are government agencies.

Therefore, attempts to develop improved tools should draw very heavily on past experiences. For example, this author thinks that the most attractive way to start another development is a brief feasibility study of those approaches that have been worked on most extensively. These include the Borg-Warner development of the sonic drill and DRI's magnetostriction-vibration. This could also include combination systems involving systems that have been studied. For example, a feasibility study on a hydraulic-driven rotary-vibratory system should first consider the turbo-drill as a possible hydraulic component. The Russians have done much work on this system and have solved major problems such as the sealing of bearings from the high-pressure, abrasive drilling fluid [6]. The hydraulic rotary component should be a turbodrill while the hydraulic-driven vibrator should be Bodine's turbine-driven eccentric weight machine. This approach is less likely than more basic research or the "flash-of-genius approach" to produce phenomenal improvements. But it has the best chance of yielding good results in a short time. Many components of the drilling methods which have been tried must have been improved considerably. This makes this approach attractive.

In 1960, Ledgerwood [3] felt that the primary need in the field of rock drilling was basic research. Ledgerwood based his opinion on pre-1960 research. This author, after reviewing some pre-1960 literature on rock drilling, feels that tool development is needed now more than basic research. Considerable work has been done. There is a need to synthesize the results of past "failures" and successes. Meld this with the basic research since the 60's and considerable insight into the optimum paths may result. Basic research is important, but tool development is more timely. Sufficient basic research has been done and it is time for development.

Recent energy problems have shown the urgent need for improved rock drills. Easily accessible energy resources are being used up rapidly. It is becoming increasingly necessary to drill many explatory holes to make a discovery and then to drill very deep holes in order to tap the resource. An alternate fuel source such as geothermal energy also involves drilling very deep holes. Even ERDA has interest in drilling large, very deep holes in order to store atomic reactor wastes. These atomic reactors are yet another source for power now and in the future. Rapid tool development is needed more than basic research.

Summary of Discussion and New Design Directions

I. Consider initiating the development of a rotary-vibratory or rotary-percussive drilling machine.

II. A good starting point is a rotary-vibratory drilling machine consisting of Bodine's eccentric weight machine and the turbo-drilling machine.

III. Consider an optimization study of this rotary-vibratory machine.

IV. A machine which can operate either as a rotary-vibratory, rotary, or pure vibratory machine with variable frequencies seems promising.

V. A computer model of a drilling system will be useful in an optimization study; computer-aided design will also be helpful in designing economical tools.

VI. There is a need for a change of some primary design criteria. Power losses are not primary economic factors in rock drilling, but would yield economical tools if used as primary design criteria.

VII. There is an immediate need for tool development, not basic research.

VITA

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REDUCTION OF ENERGY LOSSES:

A KEY TO

IMPROVED ROCK DRILLING?

by

Donatus Chukwubueze Ohanehi

(ABSTRACT)

The viscous power losses of a rotary-vibratory drilling system are estimated in this study. Rotational and pure vibratory losses are considered separately. This analysis indicates that considerable power is dissipated into the drilling fluid. Pure vibratory power losses of over 3 728 W (5 hp) are expected at frequencies equal to the first mode maximum response point for a drill column that is 152.4 m (500 ft) long. This is a frequency slightly less than the first undamped natural frequency of the drill column.

For the same column, viscous rotational power losses of over 74.56 W (0.1 hp) are expected at rotational speeds of 10.47 rad/s (100 rpm). Rotational and vibratory losses increase with increases in the length of the column, fluid viscosity and operating frequency.