ASSESSING THE RELATIONSHIP BETWEEN RECURRING AND NONRECURRING TRAFFIC CONGESTION

Mahathi B. Kuchi

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APPROVED

Donald R. Drew
Chair, Thesis Committee

Richard D. Walker

Antonio A. Trani

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Blacksburg, VA 24061
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Assessing the Relationship Between Recurring and Nonrecurring Traffic Congestion

by

Mahathi B. Kuchi

Committee Chair: Dr. Donald R. Drew

Civil Engineering

ABSTRACT

This discussion develops an approach for using fractal geometry and diffusion limited aggregation to describe highway traffic flow. The formulation is platoon based and is most applicable for describing uninterrupted-flow facilities. The model explains empirical models in terms of fractal dimensions. The concepts of change in length of a platoon and a discrete spacing unit are described for the first time in this paper. Boundary values of various fractal dimensions are calculated for different HCM freeway LOS designations. A state of flow equation established in the model represents both microscopic and macroscopic aspects of a traffic stream. Using the same traffic flow model, recurring and nonrecurring types of congestion were quantified. A congestion evaluation index was developed to address the two types of congestion which can also be used as an performance-measure in monitoring a Congestion Management System (CMS). A few ideas were suggested for quantifying indirect benefits of CMS and furthering the present research trend.
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Congestion, an everyday experience of an urban-life liability, can be very frustrating until one sees it from the view point of a transportation professional. After taking one educated glance at statistics, the feeling of frustration turns to surprise resulting from comprehending reality. The reality being congestion, expressed in terms of money in congestion costs, statistics expressing congestion in terms of energy by fuel-consumption, and statistics in person-hours of delay. A delineation of this reality in terms of the three precious elements of life, time, money and energy, causes awareness and concern in addition to the initial feeling of frustration. That insipid feeling lasts until one becomes cognizant of the efforts of the transportation professionals in taming the congestion. The efforts achieved through pilot programs, conferences and symposiums. The efforts through quantifying and analyzing congestion and taking remedial measures to minimize congestion and its consequent effects. The efforts through traffic operations, policy implementations and transportation planning. The single most important effort directed towards managing traffic congestion was in delineating Intermodal Surface Transportation
Efficiency Act (ISTEA) in 1991, which lead to the unravelling of an important era in controlling traffic congestion with the introduction of Congestion Management Systems (CMS).

Congestion Management Systems (CMS) is a regulation issued by the Secretary of Transportation in compliance with the directive of section 1034 of ISTE. The modified definition of CMS quoted from the Federal Register / Vol. 58 / No. 229 / 63463 [D.1] is:

**Initial Definition**

A system to monitor and analyze the magnitude of congestion on the multimodal transportation system and to plan and implement actions, appropriate to the scope of the problem, that reduce congestion and enhance the performance of the transportation system to the level desired

**Modified Addition**

Include the identification of alternative strategies to alleviate congestion and to enhance the mobility of persons and goods

In the wake of issued directives and proposed rules by the department of transportation, there are increased efforts by organizations and institutions like DOT's and MPO's in researching the methods of congestion measurement. There was little or no research effort catered towards the same in the past by the same institutions. The cynosure falls on identifying and quantifying the basic types of congestion and establishing performance measures to monitor the implemented CMS in the respective regions of concern. From previous research, the two major areas or types of congestion are recurring and
nonrecurring congestion. The proposed rule for management systems mentioned in the Federal Register Vol. 57, No. 107 regarding recurring and nonrecurring congestion states:

Recurring and Nonrecurring

Typically the planning process has dealt mainly with recurring congestion. However, an effective congestion management system will need to address both congestion that occurs regularly at the same locations and congestion due to isolated incidents.

And the modified version of the rule in Vol. 58, No. 229 states:

Modification

The phrase "recurring and nonrecurring" has been removed in response to concerns on how areas with existing or potential "nonrecurring" congestion would be identified. However, since "nonrecurring" congestion due to incidents often accounts for most of the congestion in many areas, such congestion is still be addressed by the CMS and strategies for dealing with "nonrecurring" congestion (e.g., incident management) are included in the list of strategies to be considered in § 500.507(c).

Objectives

The preceding discussion clearly purports the importance of the research on recurring and nonrecurring congestion, which is the reason for the delineating this project titled "Assessing the Relationship between Recurring and Nonrecurring Congestion." The required three objectives for this project are to:
(1) Establish initial relationship between recurring and nonrecurring types of congestion,

(2) Finalize the relationship between recurring and nonrecurring types of congestion, and

(3) Quantify indirect benefits of a Congestion Management System (CMS).

With the aid of the concepts behind fractal geometry, a new mathematical field, a traffic flow model will be developed to define a state of congestion equation, which will then be used to quantify recurring and nonrecurring types of congestion. The two types of congestion are addressed by developing an evaluation index relating the delay due to recurring and nonrecurring congestion. The third objective of the project concentrates mostly on air quality and land use. The indirect benefit due to air quality will be quantified by using the developed traffic state-of-flow equation and relating speed input in MOBILE 5.1 to the congested speeds defined by flow equation, during incidents. Similarly, land use characteristic of llnl-level travel time can be calculated by incorporating the modified flow equation.
Literature reviews usually intend to show the past research in the minor field of interest with which the document is associated with. The present theory falls in the areas of traffic flow theory and quantifying congestion. Having a huge repertoire of past research, this chapter mentions relevant and important works only. The traffic model developed is primarily akin to stream- and car-following models and to that reason, the majority of interest is emphasised towards those two areas of interest only. And the congestion measurement is principally delay-based and therefore the attentioned is focussed on mentioning delay measures. A few congestion indices are also mentioned due to the bearing of the same upon the developed theory.

2.1 TRAFFIC STREAM MODELS

Traffic stream models are generally classified into Speed-Concentration, Flow-Concentration, Speed-Flow and Travel time models. And car-following models are classified into linear and non-linear types. A brief summary of the above mentioned
models will be discussed in the following sections which are extracted primarily from ref.[C.1].

2.1.1 Speed-Concentration Models

The immediate observation one observes from being in a traffic is the relation between speed and concentration characteristics of the traffic. It seems quite obvious to a driver that the speeds are decreasing due to increasing number of cars and the contrary can't be true due to traffic laws. And so it didn't escape the scrutiny of the earlier researchers that speed and concentration are related and efforts into knowing how lead to formulation of various speed-concentration models of which a important few are presented next.

**Greenshields' Model**

Greenshields proposed a linear relationship between speed and concentration which is expressed in the form:

\[ u = u_f (1 - k / k_j) \]  \hspace{1cm} (2.1)

where \( u_f \) is the free-flow speed and \( k_j \) is the jam density. This model is simple to use and has good correlation with observed data. Also the application of this relation depends on the circumstances and practical value of the real-time situations.

**Greenberg's Model**

Greenberg used fluid state analysis to propose a speed-concentration relationship which can be expressed as:
\[ u = u_m \ln \left( \frac{k_j}{k} \right) \]  \hspace{1cm} (2.2)

where \( u_m \) is the speed at maximum flow. There is a good correlation between this model and field data for the congested flows, but at low concentrations it breaks down.

\textit{Underwood's Model}

Underwood developed an exponential model for low concentrations expressed as:

\[ u = u_f e^{-k/m} \]  \hspace{1cm} (2.3)

where \( k_m \) is the concentration at maximum flow. Also \( k_m = k_j / e \) and for Greenberg's model \( u_m = u_f / e \).

\textbf{Generalized Single-Regime Speed-Concentration Models}

The following are the generalized single-regime models developed after the speed-concentration models.

\textit{Pipes-Munjal Models}

Pipes and Munjal have proposed a family of models of the form:
\[ u = u_f (1 - k / k_j)^n \]  

where \( n \) is a real number greater than zero. When \( n = 1 \) the above relationship reduces to Greenshields model.

\textit{Drew Models}

Drew has proposed a family of models of the form:

\[ \frac{du}{dk} = u_m k^{(n-1) / 2} \]  

where \( m \) is a real number. When \( n = -1 \) the above relationship reduces to Greenberg's model.

\textit{Car-Following Models}

Car-following models fall in the single-regime speed-concentration class and are presented at the end of the chapter after flow models.

\textit{Bell-Shaped Curve Model}

Drake, Scofer and May have proposed a bell-shaped curve intrinsic in their speed-concentration model represented by:

\[ u = u_f e^{-\left(k_1 k_x k_x^2\right)} \]  

\[ (2.6) \]
where all the notations are as described earlier.

**Multiregime Speed-Concentration Models**

*Edie’s Model*

Edie made a composite regime model by using Greenberg's and Underwood's model to represent traffic flow at both low and high concentrations. The results are presented using normalized axes for speed and concentration.

*Underwood’s Two-Regime Model*

Underwood modified his earlier speed-concentration model at congested flows to represent two regimes during high concentrations.

*Dick’s Model*

With the assumption of a fixed upper speed limit in combination with Greenberg's model, Dick presented his model with a logarithmic concentration axis.

2.1.2 Flow-Concentration Models

Along with the speed-flow investigations, traffic engineers were also interested in the headway characteristics of traffic. Those two interests were unified to evolve the flow-
concentration model first proposed by Lighthill and Whitham. Haight has termed the flow-concentration as "the basic diagram of traffic". Certain important features of a flow-concentration (q-k) are summarily presented below:

(1) In the absence of concentration there can be no flow, thus the curve must pass through the origin. Furthermore, if space mean speed is taken as the ratio q/K, the slope with which the curve leaves the origin is the free-flow speed. (Which is the maximum slope of the speed.)

(2) It is an observable fact that it is possible to have high concentrations with no low where the leader of a stream has stopped and followers have thus been forced to stop. This effect may be seen in queues at traffic signals. Under certain situations it can also be seen on freeways, and although it occurs in many other situations, the two cited examples are best known. Thus, the curve has a point representing maximum (jam) concentration with zero flow.

(3) In as much as there are observable flows at intermediate concentrations, there must be one or more points of maximum flow between the two zero points.

(4) It is not necessary for the q-k curve to be continuous.

Parabolic Flow-Concentration Model

Greenshields' linear speed-concentration is adopted to arrive at a parabolic model as shown:
\[ q = k u = k u_f (1 - k / k_j) \]  \hspace{1cm} (2.7)

Now differentiating the equation with respect to concentration and setting \( dq / dk = 0 \) to obtain conditions for maximum flow, and defining \( q_m, k_m, \) and \( u_m \) viz. maximum flow, concentration and speed as:

\[
\begin{align*}
  k_m &= k_j / 2 \\
  u_m &= u_f / 2 \\
  q_m &= u_f k_j / 4 = u_m k_j / 4
\end{align*}
\]

**Logarithmic Flow-Congestion Model**

Greenberg's speed concentration model is used to arrive at the logarithmic flow-concentration model as:

\[ q = k u = k u_m \ln \left( \frac{k_j}{k} \right) \]  \hspace{1cm} (2.8)

The above equation is differentiated to obtain conditions for maximum flow:

\[
\begin{align*}
  k_m &= k_j / e \\
  u_m &= u_m \\
  q_m &= u_m k_j / e
\end{align*}
\]

For Underwood's model,
\[ q = k u_f \exp\left(-k/k_m\right) \tag{2.9} \]

where \( c_m = k_m u_f / e \), \( u_m = u_f / e \) and \( k_m = k_m \) (a parameter).

**Discontinuous Flow-Concentration Models**

Edie indicated that the traffic behaves differently at high and low concentrations and proposed two different flow-concentration curves which are disconnected. The flow-concentration relationships he arrived at were:

\[ q = 90 u \ln\left(\frac{46}{u}\right) \quad \text{for non-congested flow} \]
\[ q = 14.5 k \ln\left(\frac{250}{k}\right) \quad \text{for congested flow} \]

*Some flow-concentration models describe three-lane flow of traffic as against the usual one-lane flow representations. Usually, flow-concentration models are used as applications for capacity studies and freeway-flow control.*

**2.1.3 Speed-Flow Models**

Speed-flow models can be determined from speed-concentration models. The apex of a speed-flow curve will be at free-flow speed and zero flow. Regardless of the shape of the speed-concentration curve, the speed-flow curve will have one point at the origin and one point on the speed axis at the maximum value of speed. The rest of the curve is dependent upon the speed-concentration model. Greenshields' speed-concentration model results in a
parabolic speed-flow curve. In experimental speed-flow curves determined from a test track, roadway radius played an important role. Similarly in the British Road Research Laboratory speed-flow model, road width was important. The Highway Capacity Manual defines levels of service based on the speed-flow curve, developed by May[C.2].

2.1.4 Travel Time Models

There has been very little research concentrating on travel time-flow relationships. Notable work was performed by Haase, Wardrop, Rothrock and Keefer, Guerin, Smeed, Weinberg et al., and, Greenberg and Crowley. Some of these are presented below.

Haase's Model

Haase proposed freeway a travel time model having the form:

\[ t_i = n_i \left[ \frac{1}{q_1} - \frac{1}{q_0} \right] + \frac{d}{u} + n_i \left[ \frac{1}{q_2} - \frac{1}{q_1} \right] \]  \hspace{1cm} (2.10)

where \( t_i \) = total trip for the \( i \)th car,
\( n_i \) = the \( i \)th car to arrive on the on-ramp queue,
\( q_0 \) = the average arrival rate at the on-ramp queue,
\( q_1 \) = the average departure rate from the on-ramp queue,
\( q_2 \) = the average departure rate from the off-ramp queue,
\( u \) = effective steady-state velocity of the \( N \) cars on the freeway,
\( d \) = distance traveled on the freeway.
**Smeed's Model**

Smeed discussed a model on an area-wide basis taking into account the fact that drivers delay starting time for the trips to minimize travel time and is expressed as:

\[
T = \frac{t}{2} + \frac{(7.409/10^6)\sqrt{A}}{(1-n/33.1tfA)^{1/3}}
\]  

(2.11)

where  
T = average journey time measured from the time the first vehicle  
\( t \) = period over which entries to the CBD are spread,  
\( n \) = number of vehicles entering CBD during period \( t \),  
\( A \) = area of CBD in square feet, and,  
\( f \) = fraction of the CBD area devoted to streets.

**Wardrop's Model**

Wardrop postulated two principles concerning travel time by regular users over several alternate routes. In such situations regular users will distribute themselves over the various routes so that:

1. The journey times on all routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route, and

2. The average journey time is a minimum.
U. S. Department of Transportation's Planning Model

The Department of Transportation's planning model uses travel time as the function of the volume expressed as:

\[ T = T_{\text{min}} \left(1 + 0.15 \left( \frac{q}{q_{\text{m}}} \right)^4 \right) \]  \hspace{1cm} (2.12)

where \( T \) = the travel time for a particular flow \( q \),
\( q_{\text{m}} \) = the maximum flow or capacity of the facility,
\( T_{\text{min}} \) = the minimum travel time.

2.2 CAR-FOLLOWING MODELS

In essence, traffic is a continuous, dynamic manifestation of an individual driver's response to traffic stimuli as perceived by the senses. This idea is the basis for the theory behind car-following models and in general they are in the form of stimulus-response equation. The response is the reaction of a driver to the motion of the vehicle immediately preceding him in the traffic stream. The response of successive drivers in the traffic stream is to accelerate or decelerate in proportion to the magnitude of the stimulus at time \( t \) and is begun after a time lag \( T \). The basic equation is in the form:

\[ \text{Response (} t + T \text{) } = \text{ Sensitivity} \times \text{Stimulus (} t \text{)} \]  \hspace{1cm} (2.13)

Different theories and tests on car-following lead to different forms of the above equation (2.12), a few which are presented below.
**Linear Car-Following \{Two-Car\} Model**

The basic equation of the linear car-following model based on two-car response-stimulus dynamics is expressed as:

\[
a_{n+1}(t + T) = \frac{[u_n(t) - u_{n+1}(t) + 1]}{T}
\]  \hspace{1cm} (2.14)

where \(u_n(t)\) = velocity of \(n^{th}\) vehicle at time \(t\),

\(u_{n+1}(t)\) = velocity of the \((n+1)^{th}\) vehicle at time \(t\), and,

\(a_{n+1}(t + T)\) = acceleration of the \((n+1)^{th}\) vehicle after time lag \(T\).

which can be generalized to arrive at a response-stimulus form expressed as:

\[
a_{n+1}(t + T) = \alpha [u_n(t) - u_{n+1}(t)]
\]  \hspace{1cm} (2.14)

where \(\alpha\) = Stimulus, such that, \(\alpha \cdot T = C\) (a parameter).

**Non-Linear Car-Following \{Two-Car\} Model**

The assumption that the stimulus is constant in the linear car-following theory has been modified by Gazis, Herman and Potts to arrive at a realistic non-linear car-following model, in which they have proposed that the sensitivity be inversely proportional to the spacing so that
\[ a_{n+1}(t+T) = \{ \alpha_0 \left/ \left[ x_n(t) - x_{n+1}(t) \right] \right\} \cdot [ u_n(t) - u_{n+1}(t) ] \]  \hspace{1cm} (2.15)

where \( \{ \alpha_0 \left/ \left[ x_n(t) - x_{n+1}(t) \right] \right\} \) is a measure of the sensitivity and the units of \( \alpha_0 \) are distance/time, and rest of the nomenclature being the same, \( x_n(t) \) = position of the \( n \) th vehicle at time 't', and \( x_{n+1}(t) \) = position of the \( n+1 \) th vehicle at time 't'.

Car-Following Models have been extended to three- and five-car formulations and also have been applied to check stability of traffic stream, and to check acceleration noise. The most interesting application investigated by Gazis and et al. is the relationship between the car-following models and traffic stream models which is demonstrated as follows:

For Linear Car-Following Model >>>

(1) Acceleration of the \( n+1 \) st vehicle from equation (2.14) is

\[ a_{n+1}(t+T) = \alpha \left[ u_n(t) - u_{n+1}(t) \right] \]

(2) Integrating the equation to obtain the velocity of the \( n+1 \) st vehicle (the velocity of the traffic stream), we get

\[ u_{n+1}(t+T) = \alpha \left[ x_n(t) - x_{n+1}(t) \right] + C_0 \]
Since under the steady state the velocity at time (t+T) is the same as the velocity at time (t), the lag time T can be disposed of such that

\[ u_{n+1} = \alpha [x_n - x_{n+1}] + C_0 = \alpha s + C_0 \]

where \([x_n - x_{n+1}]\) is the average spacing between the vehicles, \(s (= 1/k)\). The expression \(s\alpha\)

(3) When velocity \(U = 0\), Spacing = \(s_j\) = jam spacing = \(1/k_j = (\text{jam density})^{-1}\). Therefore, \(s\alpha\)

(4) Substituting step 3 in step 2 and by expressing 'u' in terms of 'k', we get:

\[ u = \alpha [1/k - 1/k_j] \]

(5) Steady state form can be achieved by using the relationship \(q = uk\) as:

\[ q = k \alpha [1/k - 1/k_j] = \alpha [1 - k/k_j] \]

which is Greenshield's flow-concentration model derived from his postulated speed-concentration model.

(6) For \(K = 0\), the proportionality constant \(\_ = q_m\) the maximum flow does not concurs with

For Non-Linear Car-Following Model >>>

The non-linear car-following theory, the same procedure as described in the preceding section is followed to arrive at a traffic stream model, but in this case the end product will be Greenberg's model after step no. 5. Solving for the proportionality constant yields \(\alpha_0 = u_m\), which is theoretically correct. Although the speed cannot be infinity, it proved by
simple logic that the non-linear car-following theory is realistic. This statement has a bearing upon the developed model (refer chapter 5).

**Generalized Expression For Car-Following Models**

The generalized expression for car-following models first proposed by Gazis, Herman, and Rothery provided the missing link between microscopic and macroscopic traffic models and is expressed as:

\[
a_{n+1}(t+T) = \alpha \cdot \frac{u_{n+1}(t+T)}{x_n(t)-x_{n+1}(t)} [u_n(t)-u_{n+1}(t)]
\]  

(2.16)

where 'l' and 'm' are constants and rest of the nomenclature is the same as those in previous sections. With different permutations of values of 'l' and 'm' it is possible to arrive at different equations of state viz. traffic stream models, as demonstrated by May and Keller.

May and Keller developed with an interesting steady state flow equation derived from generalized car-following model. Using data from the Eisenhower Expressway at Chicago, May and Keller derived non-integer values for "l (2.8)" and "m (0.8)" to arrive at an empirical traffic stream model[c.6] expressed as:

\[
q = k_u f [1 - (k/k_j)^{1.8}]^5
\]

(2.17)

The importance of the above equation concerning the present study will be discussed in the subsequent chapters.
2.3 REVIEW OF CONGESTION METHODOLOGIES

The limited efforts in the past in quantifying congestion resulted in foundation for concepts which are employed as building bricks in new methodologies. The single most notable effort in quantifying congestion was by Jeffrey Lindley [D.2]. Other distinctive methodologies, most being based on Lindley's method, are by Michael Azer [D.8], Wayne Cottrell, Tim Lomax [D.4], and Adolph May's team from University of California, Berkeley [D.7]. CALTRANS distinguished itself one more time in leading research efforts in quantifying congestion by developing few detector-based and algorithm-based methodologies, mentioned in reference [D.7]. This section lists few of the congestion methodologies, most of which are delay-based models owing to its simplicity and popularity. Also it is prudent to mention here that in the past there were other efforts in quantifying congestion based on travel time, with empirical relationships and using congestion indices. Congestion index is the second most popular method in quantifying congestion, towards which a sub-section is allotted in this review.

JEFFREY LINDLEY'S MODEL

Lindley's method was the harbinger in quantifying congestion which gave an impetus to a field of research efforts. The reason for the popularity of Lindley's model lies in its simplicity and the use of its plausible characteristics. The model is based on calculating delay due to congested vehicle miles of travel and on calculating fuel economy for the calculated delay, as:
\[ \text{DELAY} = (\text{IDEAL} - \text{ACTUAL}) \times \text{PCT} \times \text{AADT} \times 260 \]  

Where
- Delay = annual delay due to recurring congestion,
- Ideal = ideal section travel time per vehicle (average speed = 55mph),
- Actual = actual section travel time (< ideal average speed),
- PCT = percentage of daily traffic under congested conditions,
- AADT = annual average daily traffic, and
- 260 = days per year when recurring congestion exists.

The fuel economy is calculated based on the average travel speed as:

\[ \text{MPG} = 8.8 + 0.25 \times \text{AVGSPD} \]  

Where
- MPG = average fuel economy (miles per gallon), and
- AVGSPD = average travel speed (miles per hour).

Lindley also developed general incident trees with incident frequencies for different types and locations of incidents, which were obtained from a study on low cost freeway incident management techniques. The formula for calculating congested vehicle miles of travel by Lindley was given as:

\[ \text{CVMT} = \text{PCT} \times \text{AADT} \times \text{LENGTH} \times 260 \]  

Where
- CVMT = total annual congested vehicle miles of travel,
- PCT = percentage of daily traffic under congested conditions.
Length = expanded section length, and rest being same as above.

MICHAEL AZER'S MODEL

Michael Azer's model is similar to Lindley's model in some respects, but delay calculation is based on different levels of service for a period of 24 hrs. Cost is calculated for recurring congestion based on percent of trips as:

\[
\text{Annual Delay} = \text{Daily Delay @ Levels of Service D, E and F} \\
\text{Delay} = \text{* Number of Days (261)}
\]

(2.21)

\[
\text{Daily Delay @ LOS} = \left( \frac{1}{\text{actual speed}} - \frac{1}{\text{ideal speed}} \right) \times L \times V
\]

(2.22)

Where
- Ideal speed = 54 mph for L.O.S C,
- Actual speed = actual average travel speed at this L.O.S,
- L = length of the section in miles, and
- V = Volume of traffic operating at this L.O.S.

\[
\text{Weighted Average Cost} = \text{Hourly Cost for Each Trip Purpose} \\
\text{Cost} = \text{* % of Trips / 100}
\]

(2.23)
Working Papers

There are a few research efforts in progress which are also trying to quantify recurring and nonrecurring congestion. The two most noteworthy among them is Timothy Lomax of TexDOT's effort for the TRB numbered NCHRP 7-13, and the research team from University of California, Berkeley under professor Adolph May's guidance.

Lomax's NCHRP 7-13 uses sampling and license plate matching to quantify congestion on an annual basis. It is too early to comment on these efforts before studying the final report.

U.C. Berkeley's efforts in collaboration with CALTRANS delivered a draft of the report which intelligently utilized Lindley's model and a few of detector based models to come out with a combination methodology which looks very promising.

A few other computer and detector based models developed by different districts in California like TASAS model, Traffic model(FREQ), Tach runs, detector-based models sound interesting, but they are seldom cost-effective.

CONGESTION INDICES

Congestion Indices are best viewed as performance measures though it is primarily intended as a quantifying measure. In the past, all the efforts in this field were geared towards deriving meaningful results for a semi-intuitive term called congestion. Listed below are the three effective indices developed by Lindley, Lomax and Cottrell:
Lindley's Index

Congestion Severity Index (CSI) = Total Freeway Delay / Freeway VMT \hspace{1cm} (2.24)

Lomax's Roadway Congestion Index

\[ \frac{[\text{Fwy. DVMT/Mi.} \times \text{Fwy. DVMT}] + [\text{Prin. Art. DVMT/Mi.} \times \text{Prin. Art. DVMT}]}{RCI} \]

\[ \frac{[13,000 \times \text{Fwy. DVMT}] + [5,000 \times \text{Prin. Art. DVMT}]}{2.25} \]

Cottrell's Lane-Mile Duration Index

\[ \text{LMDI}_f = \sum_{i=1}^{m} \left( \text{Congested Lane Miles}_i \times \text{Congestion Duration}_i \right) \hspace{1cm} (2.26) \]

The reason for placing emphasis on the flow model by May and Keller mentioned in the previous section and the congestion indices listed above, is that these models have an important bearing upon the developed methodology in this report by the author. This will become clearer in the subsequent chapters.
B. B. Mandelbrot lead the quest for understanding nature by introducing a new field of mathematics, by appellation Fractal Geometry[B.1]. The underlying principles of fractal geometry demand an open outlook and extended imagination from an individual. Originally intended as an answer to geometrically inexplicable shapes, fractal geometry was later applied to chaotic systems in science and nature. This gave birth to Chaos Theory and subsequently to chaotically dynamical systems.

The world of fractals began when Mandelbrot added a new dimension to dimensions. His hypothesis states that there exists dimensions of fractional values lying in between the first and the second dimensions, the second and the third dimensions and so on. This was not readily accepted by the mathematical world. Mandelbrot developed on the principles over the past twenty or so years to arrive at a monograph entitled "The Fractal Geometry of Nature."[B.1] The following sections are derived from the same monograph[B.1] which will explain fractal concepts and the relevance of the subject in this discussion.

3.1 HAUSDORFF-BESICOVITCH DIMENSION

25
The idea of fractal dimension introduced by Mandelbrot is based on the definition given by Hausdroff in 1919 and later developed by Besicovitch in 1937. This definition coincides with the definition of the 'Capacity of a Geometrical Figure' given by Kolmogorov in 1958.

### 3.1.1 Evolution

Cantor Point Set originated thinking in different dimensions during the late 19th century. Lebesgue disagreed with Cantor's hypothesis, but Caratheodory furthered Cantor Set in 1914 which was implemented in Hausdorff 1919.

A classical method for evaluating the area of a planar shape 'S' begins by approximating it to a collection of very small squares and by adding these squares' sides raised to the power \( D=2 \). Caratheodory extended this approach by replacing the squares with discs, in the process, deliberately neglecting the concept of standard Euclidean shapes of known dimension imbedded in a known \( \mathbb{R}^d \). The same in three dimensions will be a fortiori covered by balls of radius 'p'. When 'S' is really a surface, the contents of the area can be obtained by adding the individual areas of the discs. Therefore, generalizing the concept, it is expressed as:

\[
    h(p) = r(d) \cdot p^d
\]

where

\[
    r(d) = [ r(1/2) ]^d / r(1 + d / 2)
\]

\{ which is the contents of sphere of unit radius, and \}

\[ d = \text{euclidean dimension (integer)} \]

Hausdroff improved on Caratheodory with constraints as:

26
\[ \lim_{p \to 0} \left[ \inf_{p \leq p} h(p_m) \right] \] (3.2)

where \( \inf = \text{infimum} \), and
\[ p_m = \text{radius of balls covering the space (p)} \]

This limit might be either finite and positive, or infinite, or zero. The function \( h(p) \) may be called \textit{intrinsic for 'S'} and denoted by \( h_s(p) \) and if positive and finite, it may be called \textit{fractal measure of 'S'}. For standard shapes in Euclid, the intrinsic test function is always of the form shown in equation (3.1) with 'd' replaced by 'D(an integer value)'. Extending the same to statistically self-similar random fractals, it becomes more complicated for the intrinsic function and the \( h \)-measure of the form of equation (3.1) vanishes according to Hausdroff. Besicovitch developed the \( h \)-measure further, by applying it to non-integer values of 'd' and concluded that if 'd' is not an integer, 'S' is not a standard shape. He also proved that for every set 'S' there exists a real value 'D' such that the \( d \)-measure is infinite for \( d < D \) and vanishes for \( d > D \). \textit{This D is called the Hausdroff-Besicovitch dimension of S.}

\subsection*{3.2 EFFECTIVE DIMENSION}

The \( d \)-measure in the previous discussion named 'Topological Dimension' and denoted by \( D_T \), satisfies the Szpiroaj inequality,\(^1\)

\[ D \geq D_T \] (3.3)

\(^1\)In Hurewicz & Wallman 1941
$D_T$ is always an integer and $D$ need not be an integer in an Euclidean span $\mathbb{R}^E$. This invoked an intuitive notion called 'Effective Dimension', the value of which is allowed to be a fraction.

Effective dimension concerns the relation between mathematical sets and natural objects. Imagine a ball of thread, depending upon the resolution of the observation, the dimension varies from zero(0) through three(3). It indicates that size is scale dependent which when stated in simple terms can be said as a case of relativity. It is another challenge to talk about the complex theory of relativity postulated by Einstein. Consider now that the ball of thread can be geometrically analyzed by using fractals!

### 3.3 Fractal Dimension

**Def.** A fractal is by definition a set for which the Hausdroff-Besicovitch dimension strictly exceed

Fractals are self-similar figures by geometry and a phenomena by natural events which express the same kind of behavior or structure at any scale or phase in question. At different scales, they have different dimensions which are allowed to be fractions. These dimensions are usually fractional and therefore termed fractal dimension.

The general expression for defining a fractal dimension mathematically is:

$$D = \lim_{h \to 0} \left[ \frac{\log N(h)}{\log (1/h)} \right] \quad (3.4)$$

where $D =$ value of fractal dimension,

$h =$ ever decreasing measuring unit (variable),
\[ N = \text{number of segments at } n^{\text{th}} \text{ phase, and} \]
\[ N(h) = \text{a function of } h \text{ at } n^{\text{th}} \text{ phase.} \]

The above expression can be easily explained by illustrating the Cantor Set (refer to chapter no. 4). Kolmogorov in 1958 defined a similar expression for the capacity of a geometrical figure.

### 3.4 Fractal Geometrics

The properties of fractal dimensions vary as that of Euclidean and are parallel in the nature of their relationships.

For a circle of radius \( R \), the perimeter is given by \( 2\pi R \) and the area is given by \( \pi R^2 \) and it follows from that:

\[
\text{length(perimeter)} = 2\sqrt{\pi \text{area}}
\]

And similarly for squares:

\[
\text{length} = 4\sqrt{\text{area}}
\]

Generalizing the above concept, each family of standard planar shapes that are geometrically similar and have different linear extents, the ratio \( \text{length} / (\text{area})^{1/2} \) is a number entirely determined by the common shape.
In space (Euclidean $E = 3$), dimensions give evaluations of the linear extent, and the ratio of any two of them is a shape parameter independent of units of measurement (which lead to an important technique - dimensional analysis - Birkhoff, 1960).

3.4.1 FRACTAL LENGTH-AREA RELATION

For any fractal shape for which $D > 1$, the generalized ratio is:

\[(G\text{-length})^{1/D} / (G\text{-area})^{1/2}\]  

(3.5)

where $G = \text{yard stick length}$,

$G\text{-area} = \text{area measured in units of } G^2$, and

$D = \text{fractal dimension}$.

Equation (3.5) mentioned ratio is same for different geometrically-similar shapes.

3.4.2 FRACTAL AREA-VOLUME RELATION

The fractal area-volume relation is given by:

\[(G\text{-area})^{1/D} \sim (G\text{-volume})^{1/3}\]  

(3.6)
and the nomenclature is similar to the length-area relation and the ratio is same for all fractal natures.

3.4.3 FRACTAL DIAMETER-NUMBER RELATION

This relation is derived from Kocak law which initially formulated a wrong exponent, later corrected by Mandelbrot. Therefore, the diameter-number relation for a group of irregular shapes with fractal nature in them is:

\[ \text{Nr}(\lambda > \lambda_\text{-}) = F_{-D} \]  \hspace{1cm} (3.7)

where \( \text{Nr}(\lambda > \lambda_\text{-}) \) = is fashioned upon the probability notation and gives the distribution of an irregular shape of radius \( \lambda_\text{-} \), consisting of fractally similar shapes of radius \( \lambda_\text{-} \),

\[ F \] = a constant, and

\[ D \] = fractal dimension.

Though the concept of fractal geometry is used in developing a traffic flow model, the model is not in accordance with all the requirements of fractal concepts. Therefore, the model is a semi-application of fractals, developed to suit the needs of traffic flow.
The foundation for the present study originated from two dimensions. The first being that traffic at volumes less than the congestion threshold tend to travel in groups called platoons, and the second being that the discrete nature of traffic as reflected by car-following theories. This gives a more realistic picture of traffic flow. The proposed methodology ties in the above two aspects to represent both the microscopic and macroscopic aspects of traffic in one traffic stream model.

As mentioned in chapter (2), the macroscopic properties of traffic are reflected well by the different traffic stream models. This methodology also proposes a new traffic stream model which also accounts for the discrete nature of traffic with the help of fractal geometry. Chapter (2) discussed the various traffic stream models at length. In the following text, the discussion will involve the relevant parts of fractals as they are applied to the developed traffic stream model.

4.1 CANTOR POINT-SET

Cantor is one of the founders of set theory. In 1870 he proposed a theory of point-set which one could call the oldest fractal. This Cantor point-set can be built up by repeating
a simple principle indefinitely. The principle is simple, but visualizing the same in terms of fractals requires a bit of imagination.

Starting with a line of unit length, step one is to divide the line into three parts and to leave out the middle third, not the endpoints. Step two is to do a similar operation on the remaining two segments and the result will be four line-segments of equal lengths and three unequal gaps in between. The next steps follow the same operations to result in line-segments indefinitely. Eventually there will be discrete points with an unequal distribution of gaps. This is called Cantor set or cantor fractal[8.3] it can be illustrated graphically as shown below:

```
                      h    N
          _____________
        ____________  1/3    2
          ____________
                  1/9    4
          ____________
        ____________  1/81    8
```

Analyzing the Cantor fractal gives us the basic definition of fractal dimension. Since the original length of the line is 1, after step one, there will be 2 (or 2^1) segments of length 1/3 (or 3^-1). After step two, there will be 4 (or 2^2) segments of length 1/9 (or 3^-2) each, and the process continues indefinitely. After the n-th step, the number of segments N=2^n and the length of each segment will be h = 3^-n. The total length of the remaining line segments at n-th step will be N(h) = (2/3)^n. The notation N(h) is in functional form since in other cases 'h' will be in an exponential form, but in this case, the
original geometric dimension (Euclidean) is 1 for the line and $N(h)$ is simply a product of $N$ and $h$.

Now, imagining the Cantor set at $n$th step covered as sparingly as possible with line segments of length '$h$' and the smallest number of line segments needed for this be '$N(h)$', then according to Mandelbrot's, the definition of fractal definition is given by:

$$D = \lim_{h \to 0} \frac{\log N(h)}{\log (1/h)}$$

(4.1)

This can be simplified by disregarding the limit as:

$$D = \frac{\log N(h)}{\log(1/h)}$$

(4.2)

therefore

$$N(h) = (1/h)^D$$

(4.3)

The above expression states that length is scale-dependent and dimension(fractal)-dependent.

Lewis Fry Richardson, an English meteorologist, noticed from his attempts to measure a coast line that the results depended heavily on the scale of the map being used[B.3]. The coastline gets longer and longer when the scale descends down and it approaches a limiting value but never converges. Since the measure is usually a line of dimension(Euclidean) of 1, in the equation(4.1), '$h$' will be '1' and the equation becomes:
\[ N = (1/h)^D \]  

(4.4)

where 'N' is the number of times the measure 'h' is to be multiplied to get the total length 'S' as:

\[ S = N \cdot h = (1/h)^{D-1} \]  

(4.5)

Equation (4.5) can also be obtained from equation (4.3) by substituting 'N.h' for N(h).

### 4.2 DELINEATION

To delineate the concept of application of fractals to traffic flow becomes simple after taking one look at the Cantor point-set. It is difficult to miss the all too obvious resemblance between the Cantor set and the traffic stream. If it is possible to enumerate the broken fragments using the principle behind the fractals, then it is also possible to do the same thing to the traffic stream which resembles the Cantor set. That idea lead to incorporating the realistic aspects of car-following models through fractals along with the proposed traffic stream model in the methodology. How well the idea works can be enunciated from validation in chapter(6).

### 4.3 PLATOON PHENOMENA
In a few simple words, highway traffic flow is a complex phenomena. To represent traffic flow mathematically has been more than a challenge to traffic engineers. After considerable research in developing a traffic flow theory, there are still many gray areas contributing to an incomplete picture. Though the studies have advanced substantially enough to provide pragmatic solutions to traffic operation problems, the big picture is still missing, particularly when it comes to predicting congestion. "Traffic" either at low volume or at congested volume is traffic again, but only at a different state of flow. State of flow is a descriptive of certain key physical characteristics of the traffic stream at the instantaneous time and instantaneous volume. The qualifying physical characteristics flow are volume, speed, concentration, headway, spacing and the like. To represent such states of flow is no easy task and that is the reason why traffic engineers treat traffic flow as states of distinct entities for practical purposes. This methodology attempts to achieve that big picture with the help of tools like fractal geometry, galaxy-galaxy cluster aggregation phenomena and previous traffic research in addition to this primary objective.

During low volume through to a threshold volume, it has been observed that the traffic stream follows a pattern of flow called 'platoon behavior' [C.1] on the highways. Platoons are aggregation of cars formed over a section of highway, moving at a near constant speed which is nearly equal to the average speed of the platoon.[C.1] But, for the purpose of this methodology, the definition of the platoon will be altered in later sections to include more realistic aspects of traffic flow. In this methodology, the initiation is at a platoon description and the aspects of platoon aggregation and disaggregation follows later with a mathematical representation. Novel features of this methodology are, one, alteration of platoon definition to the needs of the methodology and two, the use of a new field of mathematics known as 'fractal geometry' to arrive at a more accurate description of traffic stream. And in the later parts of the methodology, a directive towards achieving a Unified
Traffic Flow Theory will be indicated by the suggestion of certain research ideas which can be used to further this study.

Due to steady influx of traffic onto the highway, the behavioral variations of an already existing platoon to the new traffic influx influences the future traffic flow characteristics. The incoming flux onto a platoon or to put simply, the new stream of vehicles either from a ramp or from a following platoon cause the change in length of a platoon which in turn influences traffic stream characteristics. A similar event happens at the front of the platoon when there is a discharge flux which again changes the length of the platoon. The flux is dependent on length and concentration of the platoons, preceding and succeeding and the platoon under consideration. Highway geometrics and different visual inputs contribute to driver psychology and the response to the same, in essence, dictates the traffic stream characteristics to a considerable extent. This is the logic behind the concept of length(and/or concentration)-dependent changes in platoons and are termed platoon aggregation and/or disaggregation relative to the presence of influx or discharge flux, respectively. And the same logic will be employed in formulating the present methodology, which is the principle behind the phenomena of aggregation in galaxy-galaxy clusters as discussed in the next chapter.
Ball and Witten[A.2] efforts in the Diffusion limited Aggregation (DLA) process provided an excellent base for this model in formulating the macroscopic component of traffic flow. Mandelbrot's fractal geometry supports the microscopic component to complete the circle. The reader will need a level of proficiency in mathematics to fully understand the model. Because fractal geometry involves pure mathematics and requires extensive explanation, certain mathematical expressions are quoted from a reference without explanation. The scope of this report restricts extensive explanations.

Idealizing phenomena is intrinsic in quantifying physical processes. Certain assumptions must be made before idealizing. Such assumptions pertinent to the present model are presented below.

The assumptions are:

(1) Traffic flow is uniform (though it is not realistic, the correction factor for the same will be introduced in the final form),

(2) The definition of a platoon is assumed as defined in the following sections,
(3) The concentrations of the platoons are taken as average values (dimension $D$-dependent),

(4) The effective length is the lane with the longest stream of cars for a platoon,

(5) The flux, either influx or discharge flux, is assumed to behave like platoons, and

(6) The only assumption which is common to the real dimension and the fractal dimension is that the process of aggregation is diffusion-limited and is abbreviated as DLA-process.

Some key elements are to be identified and defined for comprehending the various aspects of the methodology. Some new terms used in the model are defined below.

**PLATOON**

A platoon of traffic is defined as an aggregation of automobiles progressing on the highway at a speed called the average speed of the platoon. This is nearly equal to individual auto speeds, depending upon the volume and concentration. The least number of vehicles to be present to qualify as a platoon can be 'three' lengthwise.

**PLATOON FLOW RATE**

The flow rate of a platoon is similar to the flow rate (during the congested flow, the flow rate of a platoon will become the actual flow rate), defined as the change in number of vehicles of a platoon over time[A.2]. It is given by:
(3) The concentrations of the platoons are taken as average values (dimension 'D'-dependent),

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\[
d\left(\frac{N_p}{dt}\right) = k_{p+1} \cdot L_p^{E-2} \quad (5.1)
\]

where
- \(N_p\) = number of vehicles in the platoon under study,
- \(k_{p+1}\) = concentration of the following or incoming platoon,\(^1\)
- \(L_p\) = length of the platoon under study, and
- \(E\) = the Euclidean dimension of the platoon under study.

It is important to state that the exponent \((d-2)\) for \(L_p\) is not the true representation for the present analogy but, for this phase, it is being accepted temporarily to represent the flow rate.

**PLATOON DENSITY**

Platoon(or local) density is the number of vehicles present per unit length of the platoon under study and is represented as :\([A.2]\)

\[
d\left(\frac{N_p}{L_p}\right) = L_p^{D-1} \quad \text{from eq.(3.1)} \quad (5.2)
\]

where
- \(D\) = fractal dimension, and the rest are same as before.

\(^1\)The subscripts \(p, p+1\) and \(p-1\) denote the platoon under study, the \(0\) preceding the platoon under study, respectively.
CONTACT FLUX

The flux of vehicles onto an existing platoon is related in a simple way to its length. [A.2] Imagine the flow of the incoming vehicles (conc. k) in the absence of the existing platoon and, supposing each vehicle moves unit motion in unit time(unit motion). After a time t the density of the unit motion(all vehicles) is simply kt and the average number of contacts with the existing platoon is Nkt. The change in flux is related to the number as

\[ \frac{dC}{dt} = \frac{dN_p}{dt} \]  \hspace{1cm} (5.3)

where \( C = \) number of contacts, and rest are in usual notation.

DISCRETE SPACING UNIT (DSU)

The 1985 Highway Capacity Manual describes a methodology for the calculation of different levels of service based on Mean Service Flow(MSF), Peak-Hour Factor(PHF) and other correction factors. Here, there is a term called Passenger Car Equivalent(PCE) which converts all classes of vehicles with a length above 18ft into multiples of one base unit viz. one PCE of 18ft. In the ensuing discussion on fractal bounds, there is one such similar measure like PCE, termed Discrete Spacing Unit(DSU). The difference between PCE and DSU is the measure, which in the case of DSU is 3ft. This measure is applicable to both vehicle length and vehicle spacing. Lengths measuring greater than one DSU are converted to its multiples and are applied to the formulations subsequently.
THE METHODOLOGY

The scenario for this methodology includes a section of highway with distinct platoons with average densities and speeds and follow all the assumptions as stated above.

Change in average length of the platoon under consideration is depends upon the concentration of the flux on to the platoon which is limited by the length or capacity of the section, as: [A.2]

\[
\frac{d L_p}{dt} \quad (< C_s) \quad \propto \quad k_{p+1} \quad \quad (5.4)
\]

where \( L_p \) = length of the platoon under consideration,
\( C_s \) = (limited by) capacity of the section, and
\( k_{p+1} \) = concentration of the influx (treated as a platoon).

Change in length of the platoon over time is related to flow rate and platoon density as:[A.2]

\[
\frac{d L_p}{dt} = \frac{(d N_p / dt)}{(d N_p / d L_p)} \quad (5.5)
\]

Substituting equations (5.1), (5.2) and (5.4) in equation (5.5), we obtain

\[
\frac{d L_p}{dt} \propto k_{p+1} \quad L_p^{E-1-D} \quad (5.6)
\]

where \( E = \) euclidean dimension, and
D = fractal dimension.

Equation (5.6) represents the state equation for a platoon of length $L_p$ at a time $t$ and it is dependent on the concentration of the influx, or more generally the following platoon, and the fractal dimension of the platoon under consideration.

CASE 1
PLATOON FOLLOWING...for $t = p$ and $p+1$

From the definition of the contact flux, the average number of contacts on to a platoon under study is $N_{p+1} K_{p+1} t$. Since the aggregation (or unit motion) within a length grows as the exponent of its fractal dimension, the number of contacts per trajectory or lane grows as $L_p^{D+D_1-E}$ [A.2]. The number 'C' of first contacts on to the platoon is the total number divided by the number per lane and given as:

$$C \propto \frac{(N_{p+1} K_{p+1} t)}{(L_p^{D+D_1-E})}$$
$$\propto N_{p+1} K_{p+1} t \cdot L_p^{E-D-D_1}$$
$$\propto K_{p+1} t \cdot L_p^{E-D_1} \quad (5.7)$$

(since $N_{p+1} \propto L_p^D$ [A.2])

Every new contact changes the rate of flux and consequently the change in number of vehicles, therefore we arrive at equation (5.3) viz.:

$$\frac{dC}{dt} = \frac{dN_p}{dt} \quad (5.3)$$
By substituting equations (5.2), (5.3) and (after differentiating) \(t^1\) prime of (5.7), in equation (5.7) we obtain:

\[
\frac{dL_p}{dt} \propto K_{p+1} t \cdot L_p^{E-D_D1+1}
\]  

(5.8)

where \(D_1\) = fractal dimension of the following platoon, and rest of the notations are same as mentioned earlier.

**CASE 2**

**PLATOON FOLLOWING...for \(t = p\) and \(p-1\)**

The density of a platoon and subsequently the length decreases proportionally to the exponent of their fractal and euclidean dimensions, as shown below:

\[
K_p \left( \propto \frac{dN_p}{dL_p} \right) \propto L_p^{-(E-D)}
\]  

(5.9)

All notations being the same, in this case, there will be discharge flux from the front of the platoon to the preceding platoon (\(p-1\)) and all other terms are similar with a change in subscripts. The change in flux \(\frac{dC}{dt}\) is replaced by the change in discharge flux \(\frac{dC_d}{dt}\) and is given by \{follows from the previous case...for more detailed explanation refer [A.2]!:

\[
\frac{dC_d}{dt} \propto k_{p-1} t \cdot L_p^{D_2\cdot E}
\]  

(5.10)
and the change length of the platoon under study due to the discharge flux which is dependent on the the concentration of the platoon is:

\[
\frac{dL_p}{dt} \propto K_{p-1} t \cdot L_p^{D-D_2} \quad (5.11)
\]

where \( D_2 \) = fractal dimension of the preceding platoon,

'\( p-1 \)' subscript represents corresponding terms for preceding platoon,

and, the rest of the notations are same.

**INTEGRATION OF CASE STUDIES**

The effective rate of change of length of a platoon is the difference between the rates of change due to aggregation and discharge fluxes. Therefore, the integration of the two cases illustrating the change in length of a platoon proceeds as:

\[
\{ \text{EFFECTIVE RATE OF CHANGE IN} \} = \{ \text{RATE OF CHANGE IN LENGTH DUE} \} - \{ \text{RATE OF CHANGE TO AGGREGATION} \} - \{ \text{RATE OF CHANGE TO DISCHARGE} \} 
\]

\[
\frac{dL_p}{dt} = \{dL_p/\text{dt}\} \text{ of (case 1)} - \{dL_p/\text{dt}\} \text{ (case 2)} \quad (5.12)
\]
Substituting equations (5.8), (5.11) in (5.12), we obtain:

\[
\frac{dL_p}{dt} = \beta \left[ K_{p+1} \cdot L_p^{E-D_1+1} - K_{p-1} \cdot L_p^{D-D_2} \right] \quad (5.13)
\]

where all notations being the same as described earlier, the proportionality factor \( \beta \) will be defined in the later phases of this methodology. Equation (5.13) has a limiting factor viz. capacity or length of the section under study and can be integrated over time. The equation (5.13) is the skeleton-form for this methodology which needs minor variations and parameter calibrations to arrive at the final form.

**CALIBRATION OF FRACTAL DIMENSIONS**

From the discussion on Cantor point-set, it is evident that the fractal dimension of traffic flow is occupancy-dependent. The same dimensions are being used to calibrate fractal limits for different levels of service.

The fractal dimension can be calculated using the basic form derived as equation (4.2) expressed:

\[
D = \frac{\log N(h)}{\log(1/h)}
\]

(4.2)

The above expression, when applied to traffic stream, the logarithmic numerator of the right hand side, \( N(h) \) is the snap-shot lane occupancy. Simply put, the number of vehicles
in the section at a given instant. The denominator part is nothing but the discrete spacing unit (DSU) expressed as a fraction of a mile.

The 1985 HCM defines various parameter limits for the different levels of service and these same are used to calibrate the dimensional limits for LOS A to F. The calibration is performed by taking the values of density-limits defined for levels of service in the HCM and are substituted in equation (4.2) as multiples of the basic measure chosen viz. one DSU of length 3 feet and, 'h' is expressed as 3 over 5280 in miles. The results of the substitutions are presented in a tabular form numbered(5.1):

<table>
<thead>
<tr>
<th>LEVEL OF SERVICE</th>
<th>MAXIMUM DENSITY (pc/mi/ln)</th>
<th>Actual number of vehicles (N)</th>
<th>Converted number...N(h)</th>
<th>FRACTAL (dimensional) LIMIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12</td>
<td>12</td>
<td>72</td>
<td>0.57</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>20</td>
<td>120</td>
<td>0.64</td>
</tr>
<tr>
<td>C</td>
<td>30</td>
<td>30</td>
<td>180</td>
<td>0.69</td>
</tr>
<tr>
<td>D</td>
<td>42</td>
<td>42</td>
<td>252</td>
<td>0.74</td>
</tr>
<tr>
<td>E</td>
<td>67</td>
<td>67</td>
<td>402</td>
<td>0.80</td>
</tr>
<tr>
<td>F</td>
<td>'&gt;'</td>
<td>'&gt;'</td>
<td>'&gt;'</td>
<td>'&gt;'</td>
</tr>
</tbody>
</table>

5.1 Limiting values of fractal dimensions for different levels of service

**FRACTAL DIMENSION OF TRAFFIC**

The calculation of the snap-shot dimension for the traffic in terms of its measurable characteristics is necessary for the purpose of simplicity in the formula. But, controlling the degree of simplicity is limited by accuracy factor. The onus for traffic stream models
is on the frugality of the data measurement and on the wealth of information derived by using the model. This model tries to achieve that same balance between frugality and data requirement.

As described in the previous section, the logarithmic numerator part of the right hand side of the equation (4.2) viz. \( N(h) \) can be expressed in terms of traffic characteristics to evaluate the instantaneous traffic dimension which, when incorporated into the model gives the state of the traffic stream at the instant of measurement. The instantaneous values of certain traffic characteristics can be easily measured on the field. The way they are related to the equation (4.2) is:

\[
N(h) = N = K * L = \frac{1}{n} \sum_{i=1}^{1} \frac{1}{k_i} L_p = \frac{1}{n} \sum_{i=1}^{1} \frac{1}{h_i u_i} \tag{5.14}
\]

Where

- \( N \) = Number of vehicles,
- \( K \) = Concentration of the stream,
- \( L_s \) = Length of the stream,
- \( n \) = Number of vehicles for the sub-stream,
- \( h_i \) = instantaneous headway (not to be confused with DSU..."1/h"),
- \( k_i \) = instantaneous concentration or sub-stream concentration,
- \( u_i \) = instantaneous speed, and
- \( S_i \) = instantaneous spacing.

When equation (5.14) is substituted in equation (4.2) the fractal dimension of the traffic dimension can be calculated as:

48
\[ D = \frac{\log[N(h)]}{\log[1/h]} = \frac{\log[\frac{1}{n} \sum_{i=1}^{1} \frac{1}{h_i \cdot u_i}]}{\log(1/r)} \]  

(5.15)

where the DSU has been renamed as "r" instead of "h" to avoid the confusion with headway, and the term "D" is the fractal dimension rest of the notation being the same as mentioned earlier.

The equation (5.15) representing the fractal dimension is semi-recursive, meaning, it uses samples of traffic characteristics to define the numerator part of the formulation, this in turn accounts for the discrete nature of traffic by its definition of fractal dimension. Further, it will be incorporated into the generalized stream model to represent the state of traffic. That is the reason it is termed as semi-recursive owing to representative ability between microscopic and macroscopic aspects of traffic flow.

**CHANGE IN LENGTH**

When the equation (5.14) is substituted in the equation (5.15), the resulting equation will be the basic formulation of the methodology which will still be in the form of the equation (5.13) and the exponential parameters for different dimensions of platoons will be qualified with equation (5.15), for the sake of simplicity. That takes care of the right hand side of the formulation and this section concentrates on the left hand side of the equation (5.13).
Reverting back to equation (5.5), the change in length of the platoon is expressed in terms of change in number of vehicles over time (which is flow) and the change in number of vehicles over change in length of the platoon (which is density) as:

\[
\frac{d L_p}{dt} = \left(\frac{d N_p}{dt}\right) / \left(\frac{d N_p}{d L_p}\right) \quad (5.5)
\]

where \(\left(\frac{d N_p}{dt}\right)\) = platoon flow \(q_p\), and
\(\left(\frac{d N_p}{d L_p}\right)\) = platoon density \(k_p\)

from the definitions of the same as shown in equations (5.1) and (5.2).

And so

\[
\left(\frac{d N_p}{dt}\right) / \left(\frac{d N_p}{d L_p}\right) = q_p / k_p = u_p \quad (5.16)
\]

Equation (5.16) shows the relationships between flow, concentration and speed characteristics of a platoon with the change in length of the platoon.

**FINAL FORMULATION**

The final form of the traffic stream model can be arrived at by substituting equations (5.16) and (5.15) in equation (5.13) as:

\[
q_p = \beta \left\{ k_p \left[ K_{p+1} \cdot L_p \cdot E - D_D1 + 1 - K_{p-1} \cdot L_p \cdot D_D2 \right] \right\} \quad (5.17a)
\]

50
or it can be expressed in terms of speed of the platoon as:

\[ u_p = \beta \left[ K_{p+1} \cdot L_p \cdot E^{-D-D_1+1} - K_{p-1} \cdot L_p \cdot E^{-D-D_2} \right] \] (5.17b)

Generalizing the concept by visualizing the traffic stream made of a series of sub-flows and changing the subscripts of 'p' to 'i' for the instantaneous characteristics of sub-streams or sub-flows, the generalized representation of the developed traffic stream model is expressed as:

\[ q_i = \beta \left[ k_i \cdot \{ K_{i+1} \cdot (L_i \cdot E^{-D_i-D_{i+1}+1}) - K_{i-1} \cdot (L_i \cdot D_i-D_i-1) \} \right] \] (5.18)

where

- \( q_i \) = flow for \( i \) th sub-flow (not to be confused with the concept of instantaneous flow)
- \( L_i \) = length of the \( i \) th sub-flow,
- \( k_i \) = concentration of the \( i \) th sub-flow and is the corresponding dimension,
- \( K_{i+1} \) = concentration of the \((i+1)\)th sub-flow, or, the influx density on the \( i \) th flow
- \( K_{i-1} \) = concentration of the \((i-1)\)th sub-flow, or, the discharge flux density from the \( i \) th flow and is the corresponding dimension,
- \( E \) = Euclidean (spatial) dimension for the traffic stream depending on the analysis
- \( \beta \) = a proportionality factor (discussed later).
Equation (5.18) can also be expressed to obtain speed as:

\[
u_i = \beta \cdot \{ K_{i+1} \cdot (L_i \cdot E-D_i-D_{i+1}+1) - K_{i-1} \cdot (L_i \cdot D_i-D_{i-1}) \} \quad (5.18a)
\]

where \(u_i\) being the space mean speed of the \(i\)th flow and rest have the same notations as equation (5.18).

**Special Case 1**

**Absolute Congestion**

Taking the equation (5.18a), and applying boundary conditions results in interesting insights into traffic dimensions. Taking the case of absolute congestion, the speed \(u_{jam}\) = 0 and the stream has a near uniform concentration of \(k_{jam}\). Therefore, the terms of equation (5.18a) becomes:

\[
u_i = 0 \quad \text{and} \quad k_{p+1} = k_{p-1} = k_{jam}
\]

Substituting these conditions in the equation (5.18a), we obtain:

\[
\beta \cdot \{ K_{jam} \cdot (L_i \cdot E-D_i-D_{i+1}+1) - K_{jam} \cdot (L_i \cdot D_i-D_{i-1}) \} \approx 0 \quad (5.19)
\]

which results in:

\[
L_i \cdot E-D_i-D_{i+1} +1 \quad - \quad L_i \cdot D_i-D_{i-1} \quad (5.20)
\]

Now equating the exponents results as:
\[ \frac{E - D_i - D_{i+1} + 1}{D_i - D_{i-1}} = (5.21) \]

But in accordance with the principles of fractal geometry, (after taking into account the assumptions of the methodology, the following holds true, ideistically):

\[ D_i = D_{i+1} = D_{i-1} = D_{\text{jam}} \quad (5.22) \]

Substituting equation (5.22) in (5.21) and after simplification, we obtain the condition:

\[ D_{\text{jam}} = \frac{(E + 1)}{2} \quad (5.23) \]

which is the dimensional bound on traffic at jam concentration.

*Calculation of \( D_{\text{jam}} \)*

Imagine a section of length one mile being occupied with vehicles of average length 18 ft., with minimum spacing of one DSU(3 ft.), which is an idealistic scenario, and if there are 'n' vehicles, then there will be 'n-1' gaps, therefore,

\[ N \times \text{Average length} + (N-1) \times \text{Gap length} = 5280 \text{ ft} \]

\[ N \times \text{PCE} + (N-1) \times \text{DSU} = 5280' \]

\[ N \times 18 + (N-1) \times 3 = 5280' \]
the solution of which obtains the value of $N = 251$ (after rounding off the value), though the number of gaps will be counted more than the actual number by taking the average length of the vehicle. There can be a small deviation while calculating dimensions and traffic stream characteristics.

The total gap length is $(N-1) \times 3 = 753$ ft, and, the total vehicle length for the section will equal to $N \times 18 = 4518$ ft, for which a term has been coined by name "Knievel length"\textsuperscript{2} honoring the feat performed by Evel Knievel.

To calculate the fractal dimension from the equation (4.2), the term $N(h)$ is calculated as $N \times \text{PCE/DSU}$ which is $251 \times 6 = 1506$. Whereas, the term 'h' is calculated as the fractional value of the total initial segment length and so it is $3/5280$.

Therefore, the snap-shot dimension of the traffic at jam concentration can be calculated from the equation (4.2) as:

$$D_{jam} = \log\left(\frac{N(h)}{\frac{1}{h}}\right) = \frac{\log(1506)}{\log(\frac{1}{3/5280})} = 0.97$$

(5.24)

Substituting this result in equation (5.23), the actual(euclidean) dimension of a traffic stream can be calculated as:

$$D_{jam}(= 0.97) = (E + 1) / 2$$

this yields

\textsuperscript{2}Courtesy to a colleague who requested anonymity status
\[ E = (0.97 \times 2) - 1 = 0.94 \] (5.25)

which is the actual snap-shot dimension (although only theoretically) of the traffic stream, instead of one (1), for analysis purposes. The same can be extended to real traffic in three dimensions, but analysis of characteristics becomes difficult. Fitting the data from the Eisenhower Freeway in Chicago, May and Keller derived a traffic stream equation from non-linear car-following generalized model as:

\[ q = k u_f \left[ 1 - \left( \frac{k}{k_j} \right)^{1.8} \right]^5 \] (2.17)

May and Keller arrived at this equation by using the limiting values in the speed-density matrix. They didn't realize the importance of the exponents and dimensions for traffic flow representation. They chanced upon the value of real fractal dimension for traffic flow.

**Special Case 2**

*Congestion Threshold*

Usually, the level of service 'D' is considered congested and from table (5.1) which has fractal bounds 0.69 to 0.74. This is the state at which traffic transitions into one continuous stream of flow where, idealistically, has uniform concentration, speed and fractal dimension. Therefore, the parameters will become:

\[ k_p = k_{p+1} = k_{p-1} = k_{cong} \quad \text{and} \quad D = D_1 = D_2 = D_{cong} \]

substituting these in equation (5.18a):
\[ u_{\text{cong}} = \beta \cdot k_{\text{cong}} \cdot \{ L(1.94 - 2D_{\text{cong}}) - 1 \} \]  \hspace{1cm} (5.26)

since \( E = 0.94 \) and if \( D_{\text{jam}} \) is substituted for \( (E+1)/2 \) in equation (5.27) then:

\[ u_{\text{cong}} = \beta \cdot k_{\text{cong}} \cdot \{ L^2 (D_{\text{jam}} - D_{\text{cong}}) - 1 \} \]  \hspace{1cm} (5.27)

Here, the subscript 'cong' indicates parameters at and after the congestion threshold, and the parameter \( D_{\text{cong}} \) is the threshold dimension at which there is a transition from platoon flow to collective, continuous traffic flow at which the flow will have a concentration of \( k_{\text{cong}} \) travelling at a speed \( u_{\text{cong}} \). In the equation (5.27), the term \( D_{\text{jam}} = 0.97 \) and the rest of the parameters is site- and intensity of traffic dependent.

*Calculation of \( D_{\text{cong}} \)*

The 1985 HCM states that level of service 'D' borders on the unstable flow and therefore the fractal bound for LOS D will be taken as the value for \( D_{\text{cong}} \). Whence the value of \( D_{\text{cong}} \) is:

\[ D_{\text{cong}} = 0.74 \]  \hspace{1cm} (5.28)

Substituting this result in equation (5.27),

\[ u_{\text{cong}} = \beta \cdot k_{\text{cong}} \cdot \{ L^{0.46} - 1 \} \]  \hspace{1cm} (5.29)
is the state equation at congestion threshold.

Modifying Assumptions

This is good juncture to expound on the assumptions mentioned earlier and modify them if necessary. Recounting all the assumptions mentioned at the beginning of this chapter,

(1) Traffic flow is uniform (though it is not realistic, the correction factor for the same will be introduced in the final form),

(2) The definition of a platoon is assumed as defined in the following sections,

(3) The concentrations of the platoons are taken as average values (dimension 'D'-dependent),

(4) The effective length is the lane with the longest stream of cars for a platoon,

(5) The flux, either influx or discharge flux, is assumed to behave like platoons, and

(6) The only assumption which is common to the real dimension and the fractal dimension is that the process of aggregation is diffusion-limited and is abbreviated as DLA-process.

Assumption (1) states that the traffic is considered uniform, and just this assumption alone makes the methodology deviate from reality. But factors like 'β' which are needed in almost all stream-models to make them concur with real-time data, is introduced for the present methodology. Exploring the correction factor 'β' can be achieved by borrowing an important tool from Physics, namely, 'dimensional analysis.'
After substituting all the units for different traffic characteristics in the equation (5.18a), the units of the term \( \beta \) can be calculated as:

\[
miles/hour = \beta \left( \text{vehicles/mile} \ast \text{mile constant} \ - \ \text{vehicles/mile} \ast \text{miles constant} \right)
\]

which is simplified as:

\[
\beta = \frac{(\text{mile} \ast \text{mile} \ / \ \text{vehicles} \ast \text{hour} \ast \text{mile constant})}{(\text{mile} / \ \text{vehicles} \ast \text{hour})}
\]

(5.30)

since, mile raised to a constant will be a fraction of the same unit, it results in obtaining the equation (5.30).

Equation (5.30) qualifying \( \beta \) can be represented as the ratio of speed units and number of vehicles, and therefore the speed units belong to the sum of a set of instantaneous speeds and is divided by \( N \), the number of the vehicles and so \( \beta \) is nothing but 'spot speed' for the sample for which measurements are taken to calculate the corresponding fractal dimensions. Therefore:

\[
\beta = u_{\text{spot}} = 1/N \cdot \left( \_u_i \right)
\]

(5.31)

Assumption (2) states that a platoon is defined in the previous sections as:
A platoon of traffic is defined as an aggregation of automobiles progressing on the highway at a speed called the average speed of the platoon which is nearly equal to individual auto speeds, and, the least number to be present to qualify as a platoon can be 'three' lengthwise.

There were no specifications for the length of a platoon to qualify as a platoon, and the notion has always been semi-intuitive, in traffic theories. However, as an unwritten rule, the low density traffic was never included under platoon category. It is true that platoons merge into a single almost-uniform stream of flow above a threshold concentration (as discussed in special case 2, previous section). If the platoon definition is altered such that even two vehicles can be called a platoon, that sums up the traffic flow into only two broad categories to be accounted for. That is exactly the logic behind the altered definition of the platoon for this methodology and the developed equations can be applied to both states of flow.

Assumption (3) states that the concentrations used in the formulation are average values and as described in the earlier sections, the accuracy of the formulation can be achieved by accounting for discrete nature of traffic with the help of fractal dimension. It is also evident from chapter(3) that concentration is fractal dimension(D-) dependent.

Assumption (4) states a logical statement that the effective length of a platoon is the longest stream of vehicles. If the analysis is being performed over all the lanes, it is more accurate when done lane by lane.

Assumption (5) states that influx and discharge flux behave like platoons which in essence is restating assumption (2).
Assumption (6) states that the process of diffusion of traffic streams follows the concept of diffusion-limited aggregation (DLA) process as in galaxy-galaxy clusters, which is the basic assumption on which this model stands. Here, it is interesting to note that there is a difference between theoretical and real-time flow. If all drivers follow at minimum headway, idealizing traffic flow, then the flow is easy to predict. But when there is a deviation from theory, owing to driver psychology, vehicle and highway characteristics, there is a loss in spatial characteristics in between vehicles which cannot be used by other vehicles. In other words, that lost spacing distance is limited by diffusion. Other than the minimum spacing required, the spacing lost due to the diffusion limitation is called "shadow" (actually adopted from the analogous situation in galaxy-aggregation).[A.1]

There are concepts in fractal geometry which measure the intensity of the number of gaps to measure. This concept is known as "lacunarity of a fractal" which gives many interesting insights into gap distribution and colloids. However, these concepts are too complex to be included in the present discussion.

RECURRING CONGESTION

Recurring congestion occurring on freeways is due primarily to high volumes during peak-periods. Though the capacity reduction due to bad design and geometry of the section are included in the recurring type of congestion, there is a consensus that recurring congestion is associated with only the imbalance between volume and capacity which occurs during peak-hour only. Design and geometry aspects can be easily dealt with operational solutions.

As mentioned chapter (2), estimating delay is the most viable technique to quantify recurring congestion. Time being the important aspect of human life, the measured delay
due to increased volumes expresses the severity of the recurring type of congestion. Recounting the delay measures mentioned in chapter (2), delay can be calculated as:

A method for quantifying recurring congestion by delay-measure, partly influenced by the above mentioned delay measures, can be developed as:

\[
\text{Delay for the critical stream of flow (DS)} = (S/u_{cong} - S/u_f) \ast C \ast T_0
\]  

(5.32)

Where

- \(DS\) = Delay for the critical stream during peak hour,
- \(S\) = Section length,
- \(u_{cong}\) = Speed of the observed flow at saturation as given below,
- \(u_f\) = Free flow speed,
- \(C\) = Capacity of the section, and,
- \(T_0\) = Duration of flow data collection.

Where from equation (5.27),

\[
u_{cong} = \beta \cdot k_{cong} \cdot \{ L^2 (D_{jam} - D_{cong}) - 1 \}
\]  

(5.27)

and
\[ C = 2000 \times N \times W \times T \times D \times G \]  \hspace{1cm} (5.33)

Where

- \( N \) = Number of lanes,
- \( W \) = Width and lateral clearance factor,
- \( T \) = Truck presence factor,
- \( D \) = Driver Population factor, and
- \( G \) = Grade factor.

**Critical Stream**  
As mentioned in the equation (5.32), the delay-measure is for a critical-stream which is chosen as the critical stream or sub-flow in the critical hour. The concept is similar to the design hour (though not the same), and the choice of the critical hour can be as mentioned in appendix (A) or it can be based on engineering judgement.

Now proceeding to equation (5.32), the method is extended to an annual basis of measurement as follows:

\[ \text{Average} \mid \text{Hourly} \mid \text{Delay} \mid (\text{AHD}) \mid \]

\[ = \sum [ (HV_i / q_i) \times DS ] \]  \hspace{1cm} (5.34)
Where \( q_i \) = equivalent hourly flow rate for the critical stream, and 
\( HV_i \) = Hourly volume for which the flow data is taken.

Here AHD can be taken as the average for multiple critical sub-flows as suggested in appendix(A). The Average Hourly Delay can be extended to daily delay as:

{\[
\text{Average} \quad \begin{array}{c}
\text{Daily} \\
\text{Delay} \\
(ADD)
\end{array} \\
\begin{array}{c}
\sum \left[ \frac{\text{AADT}}{HV_i} \right] \cdot \text{AHD}
\end{array}
\]}

(5.35)

Where \( \text{AADT} \) = Annual Average Daily Traffic, and,

*Similar to equation (5.34), the above relation can be taken for critical

Total recurring delay per annum is calculated by the following equation:

{\[
\text{Total} \quad \begin{array}{c}
\text{Recurring} \\
\text{Delay per} \\
\text{Annum}
\end{array} \\
\text{TRDA} \quad = \quad ADD \cdot 260
\]}

(5.36)

Where the number 260 is for the number of working days. Concurrent to the trend of the method and similar to Lindley’s index[D.2], total delay per vehicle-mile of travel using the value of \( AVMT \) is calculated as:
Average |  
Annual |  
Recurring | AARDV = TRDA / AVMT (5.37)  
Delay per |  
Vehicle-mile |  
of Travel |  

Where \( AVMT \) = Annual Vehicle Miles of Travel  
*All other notations being the same.

Validity  
The validity of the model is dependent on the choice of the critical stream chosen and statistically speaking, the validity also depends on the number of streams, number of segments, choice of days and choice of months selected. Based on standard deviation and coefficient of variation, the samples can be chosen to incorporate in the above described model. Equations (5.34) and (5.35) have summation symbols to allow for multiple streams chosen as described in the appendix [A].

NONRECURRING CONGESTION

Nonrecurring congestion on highways is due primarily to unpredictable events which cause temporary capacity reduction. Accidents, construction, stalls, flats, spills and special events fall in the category of special events. The delay due to incidents is the primary factor adding to the annual congestion costs. Jeffrey Lindley[D.2] arrived at figures which indicate incident congestion to be more than 60% of the total annual congestion.
Therefore, the same concept of quantifying by delay is also developed for nonrecurring type of congestion as follows:

\[
\text{Delay per Incident} = (q_{ii} - q_{cong}) \cdot T_d \cdot T_o \quad \text{for } q_{ii} > q_{cong} \quad (5.38) \quad (\text{DI})
\]

Where
\[
q_{cong} = \text{flow at saturation (defined as the flow over v/c of 0.77)},
\]
\[
q_{ii} = \text{equivalent hourly flow rate for the } i \text{th sub-flow during } T_o,
\]
\[
T_d = \text{duration of the incident, and}
\]
\[
T_o = \text{duration of flow data collection.}
\]

\[
\text{Average Delay per Incident} = \frac{(\sum DI)}{NI} \quad (5.39)
\]

Where \( NI = \text{Number of incidents per year} \)

\[
\text{Total Nonrecurring Delay per Annum} = \sum DI \quad (5.40)
\]
Where \((\frac{TNDA}{NI}) \neq ADI\)

Annual average  
Nonrecurring  
Delay per  
Vehicle-mile   \[ = \frac{TNDA}{AVMT} \quad (5.41) \]
of Travel  
(AANDV)  

Where \(AVMT = \text{Total Vehicle Miles of Travel per year}\)

**CONGESTION MEASUREMENT**

It follows from above sections that total delay caused due to the recurring and nonrecurring types of congestion is the sum of their individual delays which can be expressed as:

Total  
Congested  \[ = TRDA + TNDA \quad (5.42) \]
Delay  
(TCD)  

subsequently,
The above equation is the Delay Measure of Congestion in vehicle-hours / vehicle-miles.

**Index for evaluating Congestion**

Delay-measure is an important measure of congestion. The picture is incomplete if a methodology terminates after quantifying congestion. There must be an established measure for evaluating the change in delay pertaining to the both recurring and nonrecurring types of congestion. The most logical direction points to relating the recurring and nonrecurring congestion with the aid of an evaluation index. Such an index can be used as analysis-, evaluation- and performance-measure for section, area and/or as a policy tool for monitoring the Congestion Management Systems. The developed index is based on annual delay and variation of delay-measures with respect to each other and with respect to the total delay. The index is a ratio of two delay ratios and the underlying principle is simply stated as:

*Congestion Evaluation Index (CEI) is a relativity index which captures the effect of severity of congestion by measuring the corresponding change in Accident Delay due to the relative change in the recurring delay for a section, an area, or a region.*
Therefore the index is ratio of two ratios, namely, Accident index (AI) and Recurring Delay Index (RDI), which are expressed as:

\[
\text{CEI} = \frac{\text{AI}}{\text{RDI}} \quad (5.44)
\]

Where

\[
\text{Ad} = \frac{\text{Accident Delay (AD)}}{\text{Total Incident Delay (TNDA)}} \quad (5.45)
\]

and

\[
\text{RDI} = \frac{\text{Recurring Delay (TRDA)}}{\text{Congested Delay (TCD)}} \quad (5.46)
\]

Equation (5.44) can also expressed differently by using equation (5.42) as:
\begin{equation}
\text{AD} \ast (\text{TRDA} + \text{TNDA}) = \frac{\text{CEI}}{(\text{TRDA} \ast \text{TNDA})}
\end{equation} (5.44a)

With appropriate data set, the variation of the delay-measures with respect to each other will give interesting insights into the relationship between recurring and nonrecurring types of congestion. As mentioned above, it can also be useful in evaluating and monitoring congestion and can be used as a performance measure by the federal government to monitor the progress of the implementation of developed congestion management systems in various State Departments of Transportation, Metropolitan Planning Organisations and other local transportation jurisdictions.
The implementation of a Congestion Management System in any regional jurisdiction results in causing few changes and benefits for which the onus is on the positive, since that is actual objective of implementing a management system. Benefits can be direct or indirect depending on the causality routing. This chapter tries to achieve the third objective of the project, which is quantifying the indirect benefits of the congestion management system, may be only conceptually.

The first three direct benefits from congestion reduction (as mentioned in chapter.1) which comes to mind are time, energy and money. When it comes to indirect benefits, depending on different perspectives, the list of benefits is unending. Considering from the pertinent perspectives of transportation, the concise list of benefits which take a positive turn are:

- Safety
- Air Quality
- Land Use
- Economic Growth
- Environment (other)
Water Quality

Noise

Architectural facades

Discussing about the different benefits listed above, how and how much, will fulfill the purpose of this chapter.

Intuition is one acute sense a human being possesses, which lead to the birth of science. Every one who drives to work every working day knows intuitively the effects of congestion and the importance of safety. It is also easy to perceive the simple causality between congestion and safety. But, there wasn't any theories formulated on paper supporting the same intuitive notion. One such concept was delineated in the course of this project is presented in appendix(A).

One of the primary concerns for both transportation and environmental fields is air quality. So far, air quality has been an indirect effect of congestion and owing to the increasing efforts by individuals and organization belonging to various fields, the status of air quality is fast becoming a direct concern. After all it is a matter of perception of the perspective. Air quality relation to the CMS and quantifying the same will be discussed in the next section.

Contemplating the genesis of Land-Use and Transportation is synonymous with the classic case of Chicken and Egg. Whatever the case, transportation and land-use derive mutual gains. Congestion definitely has a derogatory effect on land-use and consequently, land-use is an indirect benefit of a congestion management system. In the following sections, attempts to quantify the same will be presented.
Indicative of previous discussions on other benefits like time, money and energy, economic growth has positive direction by implementing a congestion management system. But, the direct benefits and land-use (indirect) are causal to economic growth, here comes again the futile argument of means-to-an-end or end-to-a-means. Whence it is difficult to approach from a transportation view-point though it is possible from an economist's view-point.

Other environmental concerns are to do with global warming and greenhouse effects, which rightfully falls in the category of air quality concerns, though it is not difficult to quantify derived benefits from the same. Environmentalists do it all the time while protesting on the same issues to the government.

Another environmental related concern is the quality of water, which is again similar to the previous concern and falls possibly in the same category but, the people from the wetlands division of federal highway might raise a hue and a cry.

Noise is one problem which has been neglected in the past, but, really gaining importance lately. Owing to less-important-concern status, noise is categorized under an indirect concern but from the view-point of author of this report, it should be included in the direct concerns to congestion. There are simple mathematical models already existing which express noise impacts.

Pollution is of different forms, a sizable amount of which is due industries and vehicle-emissions. Constructions of architectural and historical significance take a battering due to the same pollution. This is surely an important concern and it is categorized under indirect aspects of congestion. With a simple positive-negative indication in relation to vehicle miles of travel, one can represent the quantitative aspects of this part of the
indirect benefits. The similar procedure can be adopted to quantify(graphically) other benefits like noise, water quality, other environmental concerns and even economic growth too.

The next two sections proposes procedures to quantify indirect benefits resulting from implementing a CMS, like air quality and land-use, the two important concerns to the field of transportation.

*Quantifying Air Quality Benefits*

The twentieth century environmental concerns amongst the transportation community brought about many changes in issues and policies. Air quality is one of such similar issues which originated in response to the generation of excessive traffic. It is not fair to include air quality in the category of indirect impacts because air quality is a direct result of urban transportation. But, the causality is really not between transportation and air quality. Instead, from the view-point of an environmental person, human actions like generating traffic is indirectly related to air quality. That is the reason why air quality falls in the category of indirect impacts though it does seem otherwise to the field of transportation.

Quantifying an intuitive notion like air quality benefits from implementing a congestion management system (CMS) is not an easy task. Presently, air quality measurement is being performed by estimating emissions from the vehicles. Based on different criteria in relation to automobiles, EPA's MOBILE software calculates the emission factors to evaluate air quality. Before proposing any ideas on how to quantify air quality benefits, there is a need to know the theory[D.14] behind MOBILE, which is presented next.
The simple expression to calculate emissions on an area wide basis is:

\[ \text{Total Emissions} = \text{VMT} \times \text{CEF} \] (6.1)

Where \( \text{VMT} = \text{Total vehicle miles of travel for the area under consideration}, \)
\( \text{CEF} = \text{Composite Emission Factor calculated from MOBILE}. \)

From this juncture, MOBILE takes over, and calculates:

\[ \text{EF}_{i,j,k} = \sum_{m=1}^{n} \{ \text{VMT}_m \times (\text{BER}_{j,k,m} \times \text{CF}_{j,k,m}) \} \] (6.2)

Where \( \text{EF}_{i,j,k} = \text{fleet-average emission factor for calendar year } i, \text{ pollutant } j, \text{ and} \)
\( \text{VMT}_m = \text{fractional VMT attributed to model year } m \text{ (the sum of VMT}_m \)
\( \text{BER}_{j,k,m} = \text{Base emission rate for pollutant } j, \text{ process } k, \text{ and model } m, \)

and,
\( \text{CF}_{j,k,m} = \text{correction factor(s) (e.g., temperature, speed) for pollutant } j, \)

The individual components are calculated as:

\[ \text{VMT}_m = \frac{\text{REG}_m \times \text{MILES}_m}{\sum_{m=1}^{n} \{ \text{REG}_m \times \text{MILES}_m \}} \] (6.3)

Where \( \text{MILES}_m = \text{annual mileage accumulation for each model year } m, \)
\( \text{REG}_m = \text{registration fraction for the model year } m, \text{ and} \)
\( n = \text{total number of model years in fleet}. \)
\[ \text{BER} = 0.206 \times \text{cold start mode} + \\
0.521 \times \text{stabilized mode} + \\
0.273 \times \text{hot start mode} \]  

(6.4)

and the correction factors,

\[ \text{CF}_{j,k,m} = \text{TCF} \times \text{OMCF} \times \text{SCF} \]  

(6.6)

Where \text{TCF} = \text{Temperature correction factor} \text{ since the test temperature was 68}

\text{OMCF} = \text{Operating mode correction factor viz. Catalyst Hot-start,}

Catalyst Cold-start and Non-catalyst Cold-start with default values

\text{SCF} = \text{Speed correction factor against the test run at 19.6 mph for 7.5}

miles.

In equation (6.6), speed correction factor is the term of interest. The SCF was initially calibrated for driving cycle of 7.5 miles with an average speed of 19.6 mph. MOBILE actually calculates factors over that driving cycle. The emission factors are not for the speed of 19.6 mph, but, it is an approximation of the average rate of emissions over a trip with an average speed of 19.6 mph. MOBILE has three speed correction regimes to account for different speed inputs.

This is the ideal position to tie in the developed traffic stream model. There will be special data efforts if the following methodology is chosen to be adapted for quantifying indirect benefits.

Recalling equation (5.18a),
\[ u_i = \beta \cdot \{ K_{i+1} \cdot (L_i \cdot E-D_i-D_{i+1}+1) - K_{i-1} \cdot (L_i \cdot D_i-D_{i-1}) \} \]  

This equation measures average speed of a traffic stream for a given length and concentration. Average speed is the characteristic of a traffic stream which is used in MOBILE. The difference between default (test) and actual values are corrected for, by choosing a speed regime. The third speed regime is average travel speeds between 48 to 60 miles per hour.

**IDEA**

Corresponding VMT which is considered congested can be evaluated as mentioned in the previous chapter which will be substituted in equation (6.3) as an altered input to MOBILE. The output will be emission factor for CVMT alone. The changes can be monitored to arrive at benefits.

This proposed method of quantifying air quality benefits is presently an innovative idea alone, which can be developed upon to arrive at more meaningful results. The scope of the paper restricts expressing more than what has been proposed already, but in future, interested researchers may follow up on this idea, for that matter, all the ideas expressed in this report, to formulate better models.
Quantifying the air quality benefits resulting from implementing a congestion management system (CMS) can be performed by using the above mentioned method. For quantifying any indirect benefit, causality is important. The model tries achieve the same effort of establishing causality and then calculating the positive change to arrive at the benefit gained in the air quality due to the implemented CMS.

**Quantifying Land Use Benefits**

Transportation has a very dynamic interaction with Land use. Economy and Population has positive causality with Land use. The reason that dynamic causality is continuously changing, it is difficult quantify the same. As proposed in the case of air quality, it is possible to propose a conceptual methodology to measure the positive change in Land use with the help of traffic flow model developed in the previous chapter. The theory\[D.15\] behind land use described mathematically will assume the following form.

General interaction equation between transportation and land use is:

\[ T_{od} = k \cdot f(L_o) \cdot g(L_d) / Z(t_{od}) \]  \hspace{1cm} (6.9)

Where 
- \( T_{od} \) = traffic flow between two zones,
- \( L_o \) = Land use for the origin zone,
- \( L_d \) = Land use for the destination zone,
- \( t_{od} \) = travel time between origin and destination,
- \( k \) = traffic generation constant, and,
- \( f,g, & Z = \) functions.
After assuming the functions 'f' and 'g' as linear, and 'Z' as an exponential function of 't', then equation (6.9) becomes:

\[ T_{od} = k \cdot L_o \cdot L_d / t_{od}^n \]  

(6.10)

Where, \( n \) = exponent, and the rest of the nomenclature is same as before.

Equation (6.10) assumed the form of the famed gravity model. The transport impedance factor was originally formulated to account for congestion at the link level. As mentioned W.R.Blunden's work[D.15] on land use, the impedance equation can be formulated as:

\[ t_{od} = \sum_l A(l,m,n) \cdot t_l \quad (n=od) \]  

(6.11)

Where \( A(l,m,n) \) = complete Transport and Land use three dimensional matrix,

\( t_l \) = travel time on a link level.

Next part of the model includes congestion factors on the link levels which is tedious. The area wide planning restricts using the developed flow model in the present form to quantify indirect benefits of land use in relation to a CMS. The positive change in land use can be evaluated with the help of travel time measurement. If the changes in any of the indirect impact is negative, then it is not considered a benefit as a result asking for federal intervention. Therefore, by establishing causality it is possible to quantify indirect benefits of a CMS.
CONCLUSION

The methodology conceived and developed in the previous chapters are skeleton models for new applications in the transportation field. Data availability is the key in calibrating the model. But, despite a year's efforts and request, obtained data sets didn't meet the requirements of the model. Specific data collection for the present study has been ruled out because of the high costs involved and restrictions of the scope of the project.

Validation The traffic flow model was calibrated with simulated data (single stream data set) and a 97% accuracy was observed. Since this observation is not sufficient to calibrate the whole model, mentioning the same was discretionary and hence was omitted. Data sets for the congestion part followed the same itinerary and it was not possible without calibrating the flow model.

Concluding from the developed methodology, the potential model can be a working one after calibration. As mentioned earlier in the previous chapters, traffic flow model is a good representation of both microscopic and macroscopic characteristics. The application of fractals being a novel idea has opened up a new field intrinsic in the theory. Congestion methodology was influenced by Jeffrey Lindley's model and it incorporates traffic flow concepts in quantifying recurring and nonrecurring types of congestion. The most
important outcome of this project is the development of an index, Congestion Evaluation Index (CEI), which addresses both recurring and nonrecurring types of congestion and it also can be used as a performance measure in monitoring a Congestion Management System (CMS). Regarding indirect benefits, few ideas were mentioned in quantifying benefits of air quality and land use as result of implementing CMS.

**Suggestions**  Author of the report recommends use of the index CEI as a performance measure. It is suggested that researchers who were interested in the approach should pursue the model further or if they prefer, should use new fractal concepts such as tremas, wheys, lacunarity, succolarity, Sirpinski's carpet and some other interesting concepts as applicable to traffic and transportation. Attention is being drawn to the concept on gap distribution in Mandelbrot's treatise on fractal geometry(B.1).

**Future Research**  In the process of walking the designated path, the author is planning to publish few technical papers in future introducing and developing on the application of fractal geometry to the field of transportation. The research will take it's own course, because,

"*Nature never did betray the heart that loved it.*"
Appendix A

This appendix tries to describe a suitable sampling procedure for the developed methodology. The first choice is HPMS sampling procedure which is established and in use since a long time. The second choice can be as described in the reference [D.17]. This reference describes the size and choice of data sampling by percentage of sites, size, choice, type and nature of the sampling. It was originally proposed for establishing congestion patterns in the country which aptly fits into this methodology.
BIBLIOGRAPHY

A  Space Clusters and Colloids


B  Fractals


**C Traffic Flow**


**D Congestion Related**


OBJECTIVE
To secure a position as a Transportation Engineer and/or Planner

EDUCATION
Master of Science in Civil Engineering (Transportation) August 1994
Virginia Polytechnic Institute & State University, Blacksburg, Virginia
Coursework included traffic characteristics, land development, areawide planning, airport & highway
design and transit planning
Projects & Papers:
➢ Fractals and Traffic flow - a new concept (this report based paper pending selection for 1995 TRB conference )
➢ Evaluation of Transportation Systems Management(TSM) Strategies
➢ Superpower and the Classic : A Systems Approach (a systems engineering application)
➢ Taxi 2000 : A PRT feasibility study for Northern Virginia(NOVA)
➢ Design of Raleigh-Durham airport using REDIM (with simulated data)
Bachelor of Science in Civil Engineering 1987-91
Nagarjuna University, Guntur, India
Project: Stress/Strain analyses of cantilever structures using FEM method

EXPERIENCE
Research Fellow, Federal Highway Administration, U.S. DOT, Washington D.C., August 1993 to Present
Project: Assessing the Relationship between Recurring and Nonrecurring Congestion
• Designed and developed a congestion index as a performance measure for a Congestion Management System(CMS)
• Developed a traffic model with the aid of a new mathematical field called fractal geometry
• Established a relationship between recurring and nonrecurring congestion after quantifying them with the help of
delay-measure and then relating them by developing an index
• Proposed the developed index to federal highway to be adopted as a performance measure in monitoring a CMS
• Contributed as a team player to Congestion Management Systems Technical Working Group(CMS-TWG) on
system development and policy aspects
• Presently preparing a technical paper for 1995 TRB conference based on the report

Engineer Intern, VGT-DOT, Guntur, India, Summer 1990
Recorded, compiled and analyzed highway speed and headway data for the Congestion Alleviation Program(CAP)
which aided in operational decisions for congestion management

COMPUTER COGNIZANCE
Computers: Familiar with IBM 3090 Mainframe, IBM PC’s, Apple Macintosh, IBM RISC 6000
Languages: Fortran & Pascal
Software: Sufficient proficiency in QRS II, Script GML, DYNAMO, REDIM, MINUTP, MOBILE,

AFFILIATIONS
Student affiliate of ASCE, TRB, ITE

INTERESTS / AWARDS
Academic interests are in the fields of Traffic Flow, Congestion management, IVHS, Air quality, Mathematics and
Physics; Other personal interests are in Writing, Poetry, Cartooning, Quizzing, Scout & Social Service, Tennis, Soccer
and Martial arts
Awarded National Merit Scholarship 1985 & 88-90, Commander of Scout Camporee 1983, Served on the editorial board of
The Collegiate Times - Virginia Tech newspaper, and, numerous provincial level awards in Quizzing, Writing and Art