Development of a Cantilever Beam, Capacitive Sensing Skin Friction Gage and Supporting Instrumentation for Measurements

by

Istvan Horvath

Thesis submitted to the Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of Master of Science in Mechanical Engineering
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Abstract

A cantilever beam type, capacitive sensing, skin friction gage has been developed. A prototype along with supporting electronics has been constructed. The cantilever beam gage is a change of area variable capacitive transducer. It is designed to measure the wall shear stress in a short duration, supersonic flow. The supporting electronics consists of an electrical oscillator for frequency modulation, and a frequency demodulator. The change in capacitance due to the shear stress in the flow modulates the output signal of the oscillator, which is then demodulated to extract a voltage signal which corresponds to the change in capacitance of the gage. The gage and the electronics were constructed from simple, inexpensive components for the purpose of proving the concept of a capacitive sensing transducer. Static calibrations have been completed and statistical analysis has been done to test the performance of the gage. A 0.12 mV response due to the expected 98.1 g m/s² force input of the skin friction of the Mach 2.9 design flow, over the 0.49 in² (316.1 mm) area of the gage's sensing head, was measured as the average output of the skin friction gage
instrumented with stainless steel strips.
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Chapter 1

INTRODUCTION

1.1 Rationale For The Research

Current research and development of hypersonic aircraft has created the need for accurate time-resolved skin-friction measurements. This thesis describes the development, theory of operation, and testing of a prototype cantilever beam type, capacitance skin friction gage.

Aircraft such as the National Aerospace Plane (NASP plane) will experience supersonic internal airflow within its Supersonic Combustion RAMJET (SCRAMJET) propulsion system while the plane is traveling at hypersonic speeds. The shape of the combustion nozzle inside the SCRAMJET determines, among other things, the amount of wall shear stress of the air flow, or skin friction. The skin
friction forces in a supersonic combustor nozzle are relatively large. An accurate skin friction gage is necessary to measure these forces to be able to determine combustor efficiency, since this is where the significant losses are. Also, the skin friction forces limit the available thrust of a SCRAMJET. An accurate skin friction gage can be used not only in the design of a efficient nozzles, injectors, etc., which allows SCRAMJETS to function but it can be used to fine tune a SCRAMJET nozzle as well.

During actual SCRAMJET operation, the skin friction gage will have to withstand and make accurate measurements in supersonic flows having temperatures up to 7500°C (4200 K), gas velocities of approximately Mach 2.5, as well as static pressures of 5 to 15 psia (34.5 to 103 kPa). The gage will need to have a response time of 3 to 9 ms.

With the support of the National Aeronautics Space Administration (NASA), Virginia Polytechnic Institute and State University (VPI&SU) is currently conducting research to develop a floating element and cantilever beam type resistance gage, and a cantilever capacitance gage for the measurement of skin friction. Also, a shock tunnel for the testing of the skin friction gages is being developed.

The resistance skin friction gages use a differential amplifier to measure the voltage output signal from a wheatstone bridge. The capacitance gage described here uses a frequency oscillator and demodulator to measure the change in capacitance of the gage. Wheatstone bridge output signals have a greater
temperature dependence and decreased sensitivity compared to signals using frequency modulation.

The capacitance approach was taken because of the low temperature dependence of capacitors and their high frequency a.c. electronic capability. The high frequency electronics is highly sensitive to small changes in capacitance which is beneficial in constructing a sensitive transducer. The cantilever beam configuration is used due to the cantilever’s lack of response to axial loading and its high frequency design possibilities.

The design, construction and static calibration of a cantilever beam, capacitance skin friction gage instrumented with 0.0625 in. (1.59 mm) wide stainless steel capacitor plates has been completed. In addition, copper etching with photolithography has been demonstrated to construct a more precise capacitor plate pattern on the gage surfaces that will exhibit a greater change in area when the gage is deflected. This procedure will allow the construction of smaller skin friction gages in the future. Static calibration results along with their statistical analysis are presented. The thrust of this thesis is to present the development, theory of operation, and testing of the cantilever capacitance gage and its instrumentation.
1.2 Thesis Overview

Chapter one presents the rationale for the research efforts to develop skin friction gages. The Background section is used to describe some of the origins of skin friction measurement techniques. Then this section goes into explaining more current methods of skin friction measurements as well as some test apparatuses for simulating SCRAMJET environments.

Chapter two introduces other current research projects at VPI&SU dealing with the development of skin friction gages. Namely, the cantilever beam capacitance gage, the floating element and cantilever beam resistance gages are described.

Chapter three deals with the cantilever beam design development for the capacitance gage. It also covers the design of the capacitance plate patterns and the photolithography process used to construct the gage. Chapter four presents the development of the cantilever capacitance gage. Principle of operation including electrical capacitance theory is described. Also, the instrumentation used to make measurements is discussed, which includes a description of the electronic oscillator and frequency demodulator.

Chapter five presents the results of the cantilever capacitance gage calibration and testing procedures. Chapter six contains conclusions and future recommendations for the further advancement of the capacitive skin friction gage research.
1.3 Background

The measurement of wall shear stress, or skin friction, has been a challenge of interest in fluid dynamics for many years. At first, the study of skin friction mostly addressed the area of fluid resistance acting against the motion of ships and airplanes. In these early studies made around the 1930s, skin friction experiments were conducted in water tanks and low speed wind tunnels. More modern studies used concepts of similitude and tested theoretical model setups (i.e., flat plates, etc.) in high speed air tunnels.

Early Skin Friction Measurements

Froude, among the first scientists to experimentally study surface-friction, developed laws of model testing. Some of his work is presented in his report written in 1872 [1]. Froude’s method assumed that the frictional component of the total resistance of ships can be determined independently and subtracted and that the remainder resistance can be determined by model experiments. Froude’s experiments consisted of towing thin wood planks through still water. He then plotted the results and fit a line through the mean of the data. Using this technique, it was found that at low speeds 85 percent of the resistance of ships is due to a frictional component [2].

In similar manners, other experimenters (including: Gebers,
Baker, Zahn, Gibbons, and others) repeated Froude’s experiments and tried to curve fit the data in order to deduce mathematical relations to describe different experimental situations. The experiments of all the different scientists included variation in speed and length, but they lacked variation in kinematic viscosity. In addition, their efforts were uncoordinated and many redundant experiments were performed and none of them standardized. Upon Schoenherr’s review of the assortment of experimental data, he discovered that the experimenter’s results spanned a wide range, indicating a high mean experimental error. Due to this lack of coordination among earlier researchers it was necessary to supplement their data with new tests.

Numerous scientists made studies similar to Froude’s, some of whom used the U.S. Experimental Model Basin (U.S.E.M.B.). The U.S.E.M.B hardware consisted of a thirty five foot long water tank, a pulley system which used a silk cord to transmit the force of a suspended weight on the horizontally oriented plank, and a strip recorder on which the distance traveled by the plank was recorded in time. A falling weight was suspended by the cord whose opposite end was attached to the plank. The plank would accelerate due to the force of the falling weight until the water resistance would equal the force of the weight. The plank would travel at constant velocity which would be apparent from the strip recorder trace, and the water resistance could be determined. The sensitivity of the U.S.E.M.B. apparatus is greater than one ten-thousandths of a pound with a maximum capacity of approximately one half pound [2].

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A report made by the (Great Britain) Aeronautical Research Committee, for the year 1931-32, indicates that studies of air drag on airplane surfaces and water drag on seaplane hulls were made by the employment of wind tunnels and water tanks, respectively. Techniques for measuring drag in The Compressed Air wind tunnels were not very precise or repeatable with discrepancies among different test results of up to fifteen percent.

Numerous flow visualization techniques were used to detect turbulence, knowing that if turbulence is decreased, the total resistance of an aircraft is also decreased. An ultra-microscope was used to observe the motion of particles in an appropriately illuminated moving fluid. Photographic techniques using heated wires and electric sparks to mark fluid flow were very effective techniques for visualizing flows. In dense fluid medium, such as water, oil drops can be used to mark flow and make visual observations.

At the time of the report, 1932, standardized pitot static tubes were widely used to get an idea of the velocity fields in a flow. In the ultra microscope technique, particles deviating from the main streamline of the flow give some estimation of the turbulence present in the flow. Observing photographs with the aid of a measuring microscope, the amount of unsteady flow around models towed through water can be determined with a fair degree of accuracy by calculating the circulation around a wing tip. In addition to these techniques, hot-wire thermocouple instruments were also used to measure the distribution of temperature in the
wake downstream of the hot-wire. From the temperature distribution it was determined whether the air flow was laminar or turbulent [3].

Modern Skin Friction Measurements

Modern flow measurements, namely skin friction measurement techniques can be classified in two categories: indirect and direct. Indirect techniques include hot film or hot wire, Preston tube, Stanton tube, and pressure gradient and momentum balance. Direct methods consist of floating element, cantilever beam, and torsional beam configurations. Unlike indirect methods, direct methods do not require a flow field assumption. Direct methods measure the force, tangentially applied to the wall, directly – as implied by the name.

Indirect Skin Friction Measuring Techniques

Indirect skin friction measurement techniques incorporate the use of physical analogies which relate key parameters of velocity, thermal, or concentration boundary layers [4]. The Reynolds analogy is one such relation from which the skin friction coefficient can be inferred from heat transfer information. This analogy can be used with hot-wire or hot-film gages, and is given by the relation,
\[
\frac{C_f}{2} = St, \quad St = \frac{h}{\rho V C_p}
\]

where \(C_f\) is the skin friction coefficient, and \(St\) is the Stanton number, \(h\) is the convection coefficient, \(\rho\) is the mass density, \(V\) is the velocity and \(C_p\) is the specific heat at constant pressure.

Another common correlation used with indirect methods is the relation between the wall shear stress and the velocity profile of a flow. As with all indirect methods, the velocity distribution is assumed to be universal and the distribution must adhere to Prandtl's law of the wall

\[
u^* = \frac{u}{u^*} = F\left(\frac{\nu u^*}{\nu}\right), \quad u^* = \left(\frac{\nu}{\nu}\right)^{1/2}
\]

where the quantity \(u^*\) is the friction velocity [5]. Prandtl's law of the wall describes the geometric structure of the turbulent flow boundary layer. The law of the wall is an underlying principle among some indirect methods and all methods depend on it.

The law of the wall states that the velocity profile of a turbulent boundary layer consists of four regions: linear \((y^+ \approx 5)\), buffer \((y^+ \approx 5 \text{ to } y^+ \approx 45)\), logarithmic \((y^+ \approx 45 \text{ to } y/\delta \approx 0.2)\), and wake \((y/\delta \approx 0.2 \text{ to } \delta)\) regions, where \(y=\delta\) is the edge of the outer turbulent layer. The wake region occupies 80 percent of the boundary layer thickness. The logarithmic region is dependent on the Reynolds number, increasing for increasing Reynolds numbers.
The log region contains eddies and has higher shear stress than the rest of the boundary layer. This adjusts quickly to flow conditions such as pressure gradients and is a more reliable indicator of wall flow conditions than outer flow regions (Haritonidis). Indirect methods should not extend beyond the linear region, $y^+ = 3$, if accuracies of $\pm 1\%$ are to be obtained [6]. More information about Prandtl's law of the wall can be found in references [6, 7, 8, and 9].

The principle of operation of the pressure gradient technique is based on the fact that the pressure drop over a given length of duct is directly balanced by the integrated shear stress over the same length [10]. This is shown by the relation,

$$\Delta p \bar{A} = \int_0^L \overline{\tau_x} \, ds$$

where $\Delta p$ bar is the average pressure drop over the length $L$, $\bar{A}$ is the cross sectional area, $S$ is the area of the surface over which the stress acts, and $\overline{\tau_x}$ bar is the average shear stress over length $L$. The limitations of this method are that the duct dimensions need to be constant over the length used for measurement, the length needs to be long enough to make an accurate pressure drop measurement, and that the flow is fully developed so all streamline gradients except pressure are zero. The mean pressure gradient determines the mean wall shear, therefore the pressure gradient technique is not satisfactory for obtaining time resolved skin
friction measurement [11]. This method, due to its limitations, is most useful for calibration purposes [10].

If velocity profiles and momentum thickness gradients for two-dimensional (2-D) and three-dimensional (3-D) flows could be determined easily and precisely enough, von Karman’s momentum theorem would be an ideal tool for determining the wall shear stresses in a laminar or turbulent flow. Von Karman’s theorem relates momentum thickness to the mean wall shear stress by the relation,

$$\frac{\bar{r}_x}{\rho U_0^2} = \frac{d\theta}{dx} + (H+2) \frac{\theta}{U_0} \frac{dU_0}{dx}$$

where $U_0$ is the boundary layer edge velocity, $\theta$ is the momentum thickness, $x$ is the streamwise direction, and $H$ is the shape factor. However, since measuring velocity profiles accurately is time consuming and it is an understatement to say that measuring the momentum gradient is difficult, this technique is not the most practical approach to measuring wall shear stresses. References [7, and 10] discuss the momentum thickness gradient technique more extensively.

Figure 1.3.1 shows a cross sectional schematic of a surface hot-film gage. The technique using this gage is based on analogies such as Reynolds analogy between local skin friction and heat transfer. It assumes that the heat transfer is proportional to the power input to the thin film. The film forms one arm of a
Figure 1.3.1: Schematic of hot-film skin-friction gage. Heat loss of the hot-film to the flow puts the bridge out of balance which creates a voltage that is amplified as the output $U_B$. 

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constant-temperature anemometer bridge. An electric current is passed through the film to maintain a constant temperature in the presence of convective losses to the flow. The voltage, across the bridge, applied to the film correlates to the shear stress acting on the film. The calibration equation which gives the relation of the shear stress to the square bridge voltage, $U_B^2$, of the bridge in which the hot-film is an element is,

$$\tau^{1/3} = AU_B^2 + B$$

where $A$ and $B$ are constants that can be obtained from calibration in steady flow at different free-stream velocities [12]. In using hot-film techniques, care must be taken in selecting wall and gage materials with proper thermal properties to reduce the effect of heat loss to the surrounding wall [11]. Further discussion of hot-film methods can be found in references [7, 10, 13-14, 15, 16-17, and 18].

The Preston tube shown in Figure 1.3.2, is another type of instrument which uses a flow field analogy from which the skin friction can be inferred. The Preston tube is very practical for laboratory experimentation and the apparatus required is easy to construct.

The Preston tube is simple and easy to use. The diameter of a Preston tube is usually large enough to reach into the log region of a velocity profile (described above). The Preston tube measures dynamic pressure which corresponds to local velocities in the flow.
**Figure 1.3.2:** Schematic diagram of Preston tube. Flow of the logarithmic portion of the velocity profile enters the Preston tube at the inlet on the left, and dynamic pressure is measured at the outlet on the right.
Using calibration curves, a correlation between the measured dynamic pressures and skin friction can be determined. The calibration curves for standard setups are described by,

$$
\tau_p^+ = \left( \frac{U_d^2}{2} \right)^2, \quad q^+ = \frac{1}{2} \left( \frac{U}{U_r} \frac{d}{2} \right)^2
$$

where the dimensionless shear stress $\tau_p^+$ and the dimensionless pressure $q^+$ can be determined from boundary layer variables. For standard cases (turbulent pipe and flat plate flows) the calibration curves can be calculated from the law of the wall. For different cases whose law of the wall deviate from the standard case, the calibration curves have to be modified according to the appropriate boundary layer laws. In severe pressure gradients, favorable and adverse, the Preston tube has been found to overestimate the skin friction [8]. Also, the output of Preston tube measurements increases with increasing turbulence intensities [6]. References [6, 8, 10, 14, 19, and 20] present previous work with Preston tubes in greater detail.

The Stanton tube is a local measuring device, due to its smaller size, which responds faster to shear stress fluctuations. The Stanton tube usually operates within the linear region of the velocity profile which adjusts most rapidly to changes in wall conditions, therefore this is the most suitable portion of the boundary layer to infer wall conditions [10]. Like the Preston tube, a Stanton tube also measures dynamic pressures, and
applicable calibration equations depend on the value of the Reynolds number,

\[ Re_r = \frac{\tau_x \rho H^2}{\mu^2} \]

which depends on the most critical dimension of the Stanton tube, \( H \) (Fig 1.3.3). Because Stanton tubes can sense instantaneous changes in dynamic pressure, the gage is sensitive to wall pressure fluctuations, and therefore can be more difficult to use than Preston tubes [10]. Another drawback to surface tube measurements such as the Stanton tube is that the instrument has to be properly aligned with the flow direction. In turbulent flows, this can be a problem since a small error in alignment can lead to a difference on the order of 30% in the resulting skin friction coefficient [16]. Skin friction measurement studies using Stanton tubes have also been done by [20].

**Direct Skin Friction Measuring Techniques**

Direct force sensing skin friction gages are more practical than their indirect counterparts in that they directly measure the tangential force exerted on a surface by the flow in consideration. In this way direct methods do not rely on any information or assumption about the flow field in question. For instance, in practical applications, a direct gage would be the first choice of anyone in need of a skin friction measurement since the output
Figure 1.3.3: Schematic diagram of the Stanton tube. Flow of linear velocity profile enters the Stanton tube inlet (on the left) of height h; dynamic pressure is measured at the outlet (hole on bottom).
gives a true, numerical, calibrated measurement. With a direct gage, there is no need to refer to cumbersome and approximate calibration curves as with indirect gages.

There are two types of direct measurement skin friction gages – displacement and feedback gages. The corresponding deflection under the applied force is measured when using a displacement type gage. With a feedback instrument, the force required to maintain the sensor element at its null-position is correlated to the applied force.

Three general geometric configurations of direct measuring skin friction gages exist: a floating element, a cantilever beam, and a torsional beam. A schematic of each of these types of gages is shown, respectively, in Figures 1.3.4 – 1.3.6. The thin film floating element type configuration consists of an active area which is attached to the gage surface via tabs which contain designed stress concentration. When the gage is in operation, this active element is flush with the surface which is tangential to the flow under study (wall of the tunnel). Because of the designed stress concentration, the tabs will experience more strain than the rest of the gage components. This makes an ideal location for strain measuring devices.

A floating element gage does not have to be of a thin film variety. There are also types which involve micro-machined grooves as the separation between the gage body and the active area. In this case the groove is the area of stress concentration. Floating components can also consist of blocks assembled between gage body
Figure 1.3.4: Schematic diagram of floating element gage configurations: a) thin film; b) mass floating element; c) surface floating element
Figure 1.3.5: Schematic diagram of cantilever gage.
Figure 1.3.6: Schematic diagram of torsion beam gage.
halves. In this case the area of stress concentration used to
detect displacement of the floating element is the gap between the
floating head and the gage body. More information about floating
element techniques is available in references [7, 10, 21, and 22].

The main disadvantage of floating element designs is that they
are sensitive to normal pressure forces in the flow. Wall
pressures can be an order of magnitude greater than the wall stress
generated by skin friction [22]. Pressures in the flow will
deflect the floating element in a direction normal to the wall of
the flow. This deflection can be large and dominate the amplitude
of the measurement, giving erroneous and invalid data. This
necessitates the choice and calibration of instrumentation that is
not sensitive to floating element motion in a direction other than
that of the skin friction force.

Torsional beam configurations are beam designs that are
pivoted at half lengths, or other locations, of the beam. The
deflection of the end of the beam flush with the flow is sensed at
the opposite end of the beam, through the pivot. Again various
deflection sensing instrumentations can be employed with torsional
beams as well. Torsional beams can be designed to be of the
category known as self-nulling. A self-nulling gage only senses
the component of a force that is of interest. For example, the
active component of a torsional beam gage, the beam, is only
sensitive to moments applied to the mounting pin. Therefore,
torsional beams are not excited by the lateral force on the gage
body exerted by the shock, on the flow tunnel, of the sudden flow.
Detailed coverage of null balance and/or torsional beam type instruments can be found in references [5, 6, 9, 10, 23, and 24].

Cantilever beam designs also have an area flush with the wall of the tunnel, but its active element is a beam which is normal to the flow surface. The beam deflects laterally as the flow exerts a skin friction force on the sensing area. As with the floating element design, cantilever beam deflection can be measured with numerous instrumentation techniques including strain, capacitance, and optical methods. Unlike floating element gages, cantilever designs are less sensitive to normal forces exerted on it.

Cantilever designs are of greater mass than an equivalently sized floating element gage. Therefore the natural frequency of a cantilever gage is lower than that of a floating element design. The natural frequency can be increased by increasing the stiffness of the beam. Increasing the stiffness however, lowers the sensitivity of the gage. This decreased sensitivity necessitates instrumentation that is highly sensitive to small displacements. Capacitance techniques are adaptable to this application. Due to the high frequency electronic capabilities of capacitors, capacitance instrumentation prove to be very sensitive to small inputs and provide clean outputs: [19, 21, and 25].

Strain gages are popularly used instrumentation to detect deflections, however, it would also be possible to sense the small deflections by capacitance, optical (such as laser interferometry) [26, 27, 28-29, and 30], or any other means appropriate for the task of measuring displacement. Strain gages undergo a change in
resistance when a strain is applied by the deflection of the gage mechanism. This change in resistance can be measured with a Wheatstone resistance bridge. The overlapping area or the separation distance of capacitance plates can be varied by the deflection of gages instrumented with capacitive sensors. In this way, a capacitance is varied by the displacement of the gage mechanism. A change in capacitance can be measured with an impedance bridge or a more sensitive approach of using a tuned electronic oscillator such as is done in this research.

Laser interferometry is a sensitive and precise optical method. This technique interferometrically senses the time rate of thinning of an oil film on a polished surface subjected to aerodynamic shear [28-29]. Lubrication theory is then used to relate the rate of thinning to the applied shear stress at the oil-air interface [30].

Skin friction has been measured in many types of flow fields: laminar, turbulent, adiabatic, compressible, etc. Also, it has been of interest in flows over flat plates, in pipes, in ducts, etc. The purpose of this research is to measure the skin friction of shock waves similar to those that exist in a SCRAMJET. To develop a shock wave, shock tubes seem to be a practical tool. Shock tubes suddenly release pressurized gas which join to form a shock wave as the gas propagates down the length of the shock tube. Cal Tech went one step further and built a high enthalpy shock impulse facility which also simulates the temperature and pressure of a SCRAMJET environment. The CalTech tunnel can produce flows
with enthalpies up to 13,000 BTU/lb (6200 kJ/kg) and speeds of 24,600 ft/sec (7500 m/s) [31]. NASA AMES' combustor tunnel has flow of total pressure of 275-400 psia (1.9 - 2.8 MPa) and total temperature of 5800-8400 R (3200 - 4700 K) with enthalpies of 1700-3300 BTU/lb (810 - 1600 kJ/kg) [25]. Additional shock tube references: [32].

**Current Research In Skin Friction**

Currently there are three skin friction gages in development at VPI & SU. The design of all three gages are aimed at ultimately operating in supersonic combustion flows. The gage configurations consist of a resistance floating element, and a cantilever beam with resistance and capacitance instrumentation.

Lattimer and Diller are developing a floating element gage that uses strain gage instrumentation. The most recent version of the floating element style gage is being researched at VPI & SU. A schematic of the current design is shown in Figure 1.3.7.

This gage employs the use of four strain gages mounted underneath the floating element to give directional sensitivity as well as the skin friction force measurement. The strain gages are mounted under the floating element to insulate the semiconductor strain gages from the high temperature gradient of the supersonic flow. This is necessary because the semiconductor material of the strain gages is very sensitive to temperature changes. The effected resistance changes due to temperature can dominate
Figure 1.3.7: Schematic diagram of floating element resistance skin friction gage.
measurement signals.

Ball bearings are used to limit the motion of the floating element in the normal direction, so as to make the gage less sensitive to normal forces acting on it. This limits the motion of the floating element in the lateral direction (tangent to the flow) somewhat, but by making the whole mechanism stiffer it counterbalances the effect of the high mass on the natural frequency. By placing the strain gages across the gap between the floating element and the gage body, a high amount of strain is sensed. This would tend to balance the effects of decreased displacement due to the high stiffness.

DeTurris, Chadwick, and Schetz have developed a water cooled cantilever beam resistance skin friction gage [25]. This gage uses strain gage instrumentation to measure skin friction forces. Also, this gage is designed to perform in high enthalpy flow conditions. Tests have been completed in NASA AMES' SCRAMJET combustor tunnel. Currently, Novean and Schetz are working on improvements for the gage.

A schematic of the cantilever beam resistance skin friction gage developed by Chadwick and Schetz is shown in Figure 1.3.8. As can be seen from the figure, this gage is water cooled to keep temperature changes, local to the strain gages, down to a minimum. The lip around the head was designed to cancel effects of misalignment of the gage with the wall of the tunnel, as explained in the study completed by J. Allen [23]. The beam is made of a stiff quartz material. The time response of the gage in the flows
Figure 1.3.8: Schematic diagram of cantilever beam resistance skin friction gage.
under study is 5 kHz. This gage has performed in combustor model tests with a maximum deviation from the mean of 8% [25].

This thesis presents research done to develop a cantilever beam type skin friction gage instrumented with capacitance sensing electronics designed to be tested in short duration, supersonic turbulent flow. The cantilever beam design is advantageous due to its sensitivity to tangential forces and low response to normal loads applied to its sensing head. Capacitance sensing electronics are responsive to small changes in capacitance. Frequency modulation is used to provide adequate resolution to measure the fluctuating skin friction forces in turbulent supersonic flow.
A cantilever beam type skin friction gage instrumented with capacitive sensing electronics has been designed to be operated in Mach 2.9 short duration (4 ms start-up, 10 ms run time) air flow. The cantilever beam capacitance skin friction gage is designed to be a non-obtrusive direct shear force measuring transducer with high sensitivity to tangential loading and minimum response due to normal forces and temperature effects. In order to instrument the prototype gage with a conductive surface, strips of stainless steel were attached to the Plexiglas surfaces. These stainless steel strips were used as the capacitance plates. Sputtering and photolithography was also demonstrated to transfer a precision capacitance plate pattern of copper onto the Plexiglas pieces. Although the sputtered capacitor plate pattern was not made to work on a gage because it was difficult to attach lead wires to the fine copper pattern, this is the preferred method of constructing a plate pattern.

In comparison to floating element gages, cantilever beam gage
designs prove to be less sensitive to input forces in the normal direction. In addition, capacitive elements are affected less by temperature than resistive semiconductor strain gages. The temperature coefficient of capacitors are three orders of magnitude less than temperature coefficients of semiconductor strain gages. Also, high frequency, 100 MHz, electronics are utilized to measure the change in capacitance of the gage, to enable adequate resolution to measure the fluctuating shear forces in supersonic, steady and impulse flows. The estimated frequency of the 4.0 ms start-up time of the Mach 2.9 short duration flow intended for testing is 250 Hz. To ensure a gage response of relatively low error a gage with a calculated 2.5 kHz resonant frequency is necessary.

2.1 Cantilever Beam Design

The design of the cantilever beam was one of the most critical considerations in the development of this gage. The other was the actual sensor - the capacitor plate patterns. Maximum deflection of the beam and maximum change in overlapping area of the capacitor plate pattern under the resulting displacement were the main design goals. Also, the beam had to be made adjustable so the separation distance of the capacitor plates could be fine tuned. A secondary concern with the design of this version of the gage was the natural frequency of vibration of the beam because this gage essentially serves for concept testing purposes. A schematic of the cantilever
gage design is shown in Figure 2.1.1.

A detectable deflection of the beam, in the design flow, was the primary criterion that had to be met by the design of the mechanics of the cantilever beam. A pair of capacitance plates whose overlapping area changes as the beam deflects serve as the electronic sensing element of the gage. Therefore, the beam's deflection has to be sufficient to provide a change in capacitance that is measurable. Also, the beam had to be of a material that is an electrical and thermal insulator - plexiglass was used to meet these requirements. Final design parameters of the cantilever gage allow it to detect a tangential shear force in the design flow by deflecting $5.2 \times 10^{-4}$ in. ($1.32 \times 10^{-2}$ mm) at the end of the beam.

The skin friction gage was designed to sense a minimum shear stress of 285 N/m$^2$, 0.0414 lb/in$^2$, which is estimated to exist in the shock tube facility of VPI&SU's Mechanical Engineering Department that is to be used for gage testing. The sensing head of the beam has to be big enough to transmit enough force to deflect the beam effectively. A radius of 0.395 in. (10.03 mm) gives the head a surface area of 0.490 in$^2$ (316 mm$^2$). With the expected shear stress, this head area results in a force of 0.0203 lb (0.0903 N) exerted on the end of the beam. Since the change in overlapping area of the capacitor plates is determined by the amount of deflection of the beam, the length of the beam had to be chosen such that it deflects sufficiently under the given force to change the area of the capacitor plates by a predetermined amount.
Figure 2.1.1: Schematic diagram of capacitance cantilever gage.
Under basic static deflection theory, the elastic deflection of the end of a cantilever beam is given by the relation, 

$$\delta_{\text{max}} = \frac{PL^3}{3EI}$$

where P is the applied force, L is the length of the beam, E is the modulus of elasticity, and I is the moment of inertia. Using this equation a length of 1.333 in. (33.86 mm) was calculated for the beam. The head area and beam length under the given shear stress determine the lateral deflection of the gage head, 5.2 X 10^-4 in. (1.32 X 10^-2 mm). The size of the sensing head also determines the available area to which the capacitance surface can be attached. An area of 0.46 in^2 (297 mm^2) is provided by the size of the head. The critical dimensions of the cantilever gage are shown in the schematic diagram in Figure 2.1.2.

The cantilever beam is attached to a mounting base that is of much greater mass than the beam to ensure that the resonant frequency of the beam would be decoupled from that of the base. In this way the mass of the beam is known to be not affected by the lower resonant frequency of the base. Had the base and the beam been of more similar masses, the lower natural frequency of the base would have lowered the effective natural frequency of the beam as the beam would have tried to vibrate the base as well.

The calculated equivalent mass and equivalent stiffness of the plexiglass beam are 1.777 X 10^-6 lb s^2/in (3.112 X 10^-4 kg) and 6.8595 lb/in (1.20 N/mm), respectively, to make the natural
Figure 2.1.2: Critical dimensions of capacitance cantilever skin friction gage.
frequency of the plexiglass capacitance gage approximately 313 Hz. This is one order of magnitude lower than the estimated 1 kHz start-up frequency of the Mach 2.9 short duration turbulent flow to be used for testing. This low natural frequency is due to the fact that this gage was oversized for conceptual testing purposes. Given the sensitivity of the capacitive electronics, the gage geometry can be easily changed to increase the natural frequency to 5.9 kHz while still obtaining deflections large enough to yield measurable outputs. Higher natural frequencies can be obtained if a material with a higher modulus of elasticity is used along with a redesigned geometry of the beam.

When designing any precision instrument, regardless of the accuracy of modern machining processes, the critical dimensions of the instrument (i.e., separation distance of the capacitor plates) must be made adjustable. This adjustment is necessary so any stochastic error due to machining or extraneous effect of environmental deterioration can be compensated for. The beam is fixed to the plexiglass base and made adjustable by attaching this base to the gage body with adjusting screws threaded into the base. The sensing head of the gage discussed here can be adjusted by adjusting the base of the beam with the screws on the bottom of the gage body (Fig 2.1.1). These adjusting screws are located in 120 degree radial increments to allow full adjustment of the head in three dimensions. The heads of the adjusting screws are held stationary by individual securing clamps on the bottom of the gage body. These clamps are needed since there are no threads in the
gage body for the adjusting screws. The mating threads for the adjusting screws are in the base of the beam. As the adjusting screws are turned, their heads are held stationary by the securing clamps and motion of the sensing head relative to the gage body results.

Plexiglas was used as the material for the beam. Plexiglas is flexible enough to deflect adequately under small forces. Plexiglas also has the thermal and electrical insulating properties desired for the gage. In the thermal sense, Plexiglas is a good insulator (1.4 W/m/K) which is preferred to keep the influence of temperature effects on the capacitor performance to a minimum. Plexiglas is an electrical insulator as well. This is desirable because the capacitance plates are conductors and require an insulator for mounting. Also, the electrical insulation provided by the Plexiglas surrounding the capacitance plates shields the plates from any stray capacitance or electric field effects. The choice of Plexiglas simplified the design of the gage in more than one way by providing insulation electrically and thermally.

A cantilever type gage can be designed to have a high resonant frequency. High resonant frequency instruments have excellent time-resolution capabilities for accurate measurement of rapidly changing quantities such as shear stresses in high speed turbulent flows. Cantilevered beams are insensitive to normal pressure forces present in supersonic flows. The gage designed here has a natural frequency of 313 Hz and has low sensitivity to forces applied in the normal direction.
2.2 Capacitor Plate Design

Change of area capacitors are ideal for measuring longitudinal displacement measurements. Capacitive transducers have been used for the measurement of extremely small displacements down to the order of molecular dimensions ($10^{-9}$ cm, $4 \times 10^{-10}$ in). The cross section of a serrated capacitive transducer such as the one used in the gage presented in this thesis is shown in Figure 2.2.1. As reported by Lion, a sensitivity of 1 pF/0.0001 in. (1 pF/0.00254 mm) has been obtained with this type of transducer [33]. A capacitive transducer has the principal advantage that the physical mechanism involved in its action does not depend on the physical property of any plate material [33].

The purpose of this research is to test the concept of a capacitive measuring device and to build a working model of a capacitance gage. Therefore, this gage was designed to be unidirectional and measure a force along one axis only, to simplify the instrumentation of the gage. The capacitance of a capacitor is linearly proportional to the overlapping area of its plates. The capacitance plates of this gage are located on the underside of the head and on the top of the crown (Fig 2.2.2).

In this locale, the plates face each other. In addition, the capacitor is insulated by the layer of plexiglass between the capacitance plates and the flow.

The conductive surfaces were laid out in parallel lines,
Figure 2.2.1: Schematic diagram of capacitance surface pattern showing the serrated cross-section of the overlapping surfaces.
Figure 2.2.2: Capacitor plate location on gage.
0.0625 in. (1.59 mm) wide, across the diameter of the underside of the head and on top of the parallel surface just underneath the head (the crown). The conductive lines are separated by nonconductive spaces which are also 0.0625 in. (1.59 mm) wide. This enabled the measurement of force in one direction, and provided for a simple pattern to superimpose on the surfaces. The strips are staggered by half width, 0.03125 in. (0.794 mm), on the opposing faces (Fig. 2.2.1). This serrated arrangement is done so that a deflection in one direction increases the overlapping area of the capacitor thereby increasing the capacitance. A deflection in the opposite direction will decrease the overlapping area thereby causing the capacitance of the gage to decrease. Stainless steel was chosen as the capacitance plate material because of the ease of attaching it to the Plexiglas surfaces and its good conductivity.

Multiple strips were used to increase the effect of the deflection of the beam on the overlapping area of the capacitor. A lateral deflection in the beam of $5.2 \times 10^{-4}$ in. ($1.32 \times 10^{-2}$ mm) is expected in the design flow. The size of the stainless steel strips enabled 7 conductance lines to be positioned across the diameter of the sensing head of the gage. Since the width of the lines are greater than the lateral deflection of the head, increasing the number of lines increases the change in overlapping area of the capacitor surface. For example, if only one line (or one pair of plates) were used, the effective change in area would only be the deflection times the length of the overlapping surface.
Because 7 pairs of capacitance lines exist, the change in area is 7 times the deflection times the length of the overlapping surface. This multiplies the change of the overlapping area of the capacitance lines thereby increasing the sensitivity of the gage.

In the design calculations of the area of overlap of the surfaces, the varying length across the face of the circular head had to be taken into consideration. This was easily done by the aid of a computer program. The calculation of the area of one line is explained in this paragraph. The rest of the area was done by repeating the same procedure in a program loop. First, a coordinate system origin was chosen at the center of one of the circles. In this way, the length of a rectangle (line) running across the circular surface can be determined by calculating the points of intersection with the perimeter of the circle. The distance formula is then used to calculate the length. The second circle was assumed to be the active one. Its center was offset from the first circle by 0.03125 in. (0.794 mm) to account for the half-line-width overlap that exists in the null position of the gage. To calculate the overlapping areas, the length of each individual rectangle was multiplied by the amount of offset, 0.03125 in. (0.794 mm), of the circles. This gave the amount of overlap in the null position for one pair of lines. Upon deflection of the beam, this offset was either decreased, for deflection in one direction, or increased, for deflection in the other direction. In this manner the capacitance value was decreased in one direction and increased in the other,
respectively. The calculated area of overlap in the null position is 0.0854 in\(^2\) (55.1 mm\(^2\)). The minimum and maximum areas of overlap resulting from the expected deflection in one direction or the other is 0.0840 in\(^2\) (54.2 mm\(^2\)) and 0.0869 in\(^2\) (56.1 mm\(^2\)).

2.3 Photolithography and Etching

To demonstrate the precision with which a capacitor plate pattern can be attached to a nonconducting surface, a photolithographic technique was used to transfer the capacitor plate pattern onto a layer of copper sputtered on the surface of the Plexiglas. Initially, the Plexiglas surfaces had to be cleaned very well to ensure that the sputtered copper coating would adhere sufficiently. A thin copper layer was then sputtered onto the Plexiglas substrates. Next, photolithography was used to transfer and etch the desired patterns, created on AUTOCAD at a scale of 20:1, onto the copper covered surfaces. First, the patterns were cut out of red vinyl sheets. Once the background was peeled off the 20:1 vinyl pattern to leave a positive of the original pattern, a photoreduction technique was used to reduce the images to actual size. This provided for a positive black and white mask to be used in the photolithographic process during which the copper is exposed to UV light along with the use of a positive photoresist. The copper masked from the UV light was developed and removed with an etchant. The final step in the instrumentation of the gage was to wire the capacitance surfaces to the oscillator electronics. This
was done by attaching gold leads to the plates. Silver paint was then used to attach the gold leads to the copper wires with which the capacitor plates were connected to the electronics.

Before the Plexiglas surfaces could be sputtered, they had to be cleansed of all surface contaminants. A solution of approximately 5 drops of Microclean and 150 ml of deionized water was used in an ultrasound cleaning device to clean the lucite pieces. Ultrasound had to be used because the alternative cleaning method would have employed a strong acid. Solvents, however, such as acetone, attack the Plexiglas surface. This results in cracks in the Plexiglas where thermal stresses were built up from the machining process.

Once the Plexiglas is cleaned, the surfaces can be sputtered with copper so the image can be etched onto the copper. Sputtering is a method by which a surface is covered by a layer of metal of thickness on the order of $10^{-5}$ in. ($2.54 \times 10^{-4}$ mm), by bombarding it with electrons of a chosen metal. For the capacitor gage being developed here, copper was the metal chosen to be sputtered onto the surface of Plexiglas. Copper was chosen because of its high electrical conductivity and its ease of sputtering and etching.

In order to instrument the cantilever beam capacitance gage effectively, a pattern (Section 2.2) had to be superimposed on the capacitance surfaces of the gage. This pattern was made of a conductive material so an electrical charge could be applied to it in order to form a capacitor. A photolithographic technique was used to create a pattern of conductive material on the plexiglass
surface. Photolithography is the process of forming images in photopolymers and then transferring these images into the substrate or to other materials. The result is a pattern, as in Figure 2.2.1, of copper of thickness on the order of $10^{-5}$ in. ($10^{-4}$ mm) on top of the plexiglass surface [34].

Creating two patterns to transfer onto the copper covered surfaces for the etching process was the first thing done for the photolithographic process. The lines of the pattern, discussed in Section 2.2, were created on an AUTOCAD technical drafting software system. The pattern drawn on AUTOCAD was drafted at a scale of 20:1. This was done so the pattern transferred to a vinyl sheet is large enough to work with by hand. Once on AUTOCAD, the pattern was cut into a red vinyl sheet with a plotter for the purpose of creating a mask with the photoreduction procedure. A diamond cutter was used in place of a plotter pen to cut into the vinyl at a depth equal to the thickness of the vinyl to leave the clear acetate backing intact. The vinyl background of the pattern was then peeled off of the acetate substrate to create a positive of the original pattern.

After the positive of the 20X pattern was transferred onto the vinyl, the image was reduced to actual size. The reduction was done using a photoreduction method. The hardware used in the photoreduction setup consists of a lighted board and a camera on a frame. The camera is mounted on a bed so its distance from the board can be adjusted. In this way, with the use of different lenses, various reduction ratios can be achieved. The vinyl
pattern is clipped onto the lighted screen. The camera is adjusted on the sled at an appropriate distance from the screen to obtain a reduction factor of 20 of the image on the screen. This technique enabled the reduction of the 20X blown up pattern to actual size. The vinyl (pattern) is a red color while the substrate (background) is clear. This results in a black image with a clear background when a black and white photosensitive film is used in the photoreduction procedure. The photograph of the positive vinyl pattern then becomes a black and white positive of the original AUTOCAD drawing.

The black and white pattern is used as a mask to expose the sputtered copper surfaces to ultra violet (UV) light. This process was performed using a positive photoresist chemical to transfer the image of the positive of the pattern to the copper surface.

The photoresist was applied to the copper surface using a spinning technique. With this technique the plexiglass is spun on a turntable at 3000 RPM after the photoresist chemical is applied. This gives a thin, even coating of photoresist on the copper surface. After baking the photoresist, the positive mask is placed on the copper and the unmasked surface is exposed to UV light. With a positive mask, the background of the pattern lets the substrate material be exposed to the UV light.

A mercury light was used to provide a light of short wavelength, near-UV, light for exposing the part through the pattern. A 365-436 nm wavelength high-intensity light is used with an exposure time of approximately 10 s. Short wavelength light
sources are preferred since the potential resolution of the exposed image becomes smaller (better) as the wavelength decreases [34]. The photoresist covered copper that is exposed to the light undergoes a photochemical reaction that can be developed. In this way, the exposed part of the substrate (copper), the part that is the background on the mask pattern, can be removed with an etchant solution. The etchant solution used was 5 % ferric chloride. It was found that this solution does not attack the plexiglass. Since a positive photoresist was used, a positive of the original image of copper was left behind after the etching process.

After the capacitance surface pattern was etched into the copper, 0.001 in. (0.0254 mm) diameter gold leads were used to connect the capacitor plates to the oscillator circuit. Silver paint was used to attach the gold leads to conventional copper wiring used to connect the gage to the oscillator circuit. The lead to the head was attached at the center of the head, by pinching it between the tip of the beam and the bevelled hole in the head. A 0.001 in. (0.0254 mm) diameter lead was chosen so the thickness of the wire would not throw the head off center. Gold was chosen as the material for the leads because it is soft. Its pliability would allow it to interfere less with the alignment of the head on the beam.

The pattern on the crown was wired with thin material, but the diameter of the lead wire used here is not as critical because the ring is larger than the head and would not be misaligned as much due to the wire thickness. Also, the ring is not an active part of
the gage, therefore its misalignment is not as crucial. The conductance surface on the crown was wired by pinching the gold lead between the crown and the ring layers. The lead used to wire the conductance surface on the crown was run through the plexiglass through a hole through the thickness of the crown (Fig. 2.2.2). The lead was then run through the gage body. Both the crown and the head lead wires were attached to contact pads on the inside of the gage body. Copper wire was then used to run the leads outside the gage body where they can be hooked up to the oscillator.

The contact pads were attached to the gage body with silver paint. Silver paint could not be directly applied to the plexiglass material because the cementing agent in it would attack the plexiglass surface. Scotch tape was first put on the plexiglass where the contact pads were to be placed, to protect the plexiglass from the silver paint. The contact pads were then glued in place on the scotch tape surface with the silver paint. The gold and copper lead wires were also glued to the contact pads with silver paint to ensure good electrical contact.

Although it was found difficult to attach lead wires to the copper pattern and this copper etched pattern was not used, it was shown that photolithography allows the accurate transfer of precision images to substrate materials. The capacitor in this gage requires a tight tolerance for the gap between the capacitor plates. The sputtering process enabled the application of a precise, even thickness of copper onto the plexiglas. The photolithography and copper etching procedures used for this gage
provides the ability to fabricate a pair of capacitor plates of high resolution. It was possible to transfer a pattern of 177 parallel conducting lines across the diameter of the sensing head. Calculations show that compared to the 7 lines attached to the sensing head by hand, the pattern created by photolithography increases the variable capacitor’s change in overlapping area by a factor of 170. This needs to be verified by further tests.
Chapter 3

USING CAPACITANCE ELECTRONICS TO MEASURE WALL SHEAR STRESS

It has been successful thus far to measure the amount of deflection with strain gages attached to the sides of the beam [25]. The gage discussed here uses a high frequency, capacitance electronics approach to measure the deflection of its sensing head. A high frequency measuring system is necessary to be able to measure the rapid fluctuations in skin friction of supersonic flow. High frequency, $10^8$ Hz, electronics is also favorable considering that it is 7 orders of magnitude greater in frequency than commonly troublesome 60 Hz noise. Due to this difference in operating frequency, noise contamination of the signal is minimized. Capacitive instrumentation is preferred due to its low temperature dependence, high sensitivity and high frequency electronics capabilities.

The cantilever skin friction gage presented here employs a variable capacitor transducer to make measurements. The variable capacitor follows basic electrical capacitance theory in its principle of operation. A capacitor is an a.c. device, that acts as an open circuit to d.c. voltage, and as a frequency dependent impedance to a.c. voltage. It has frequency dependent reactance
properties, with a.c. voltage, which deem it an oscillatory component in certain circuit configurations. The change in capacitance of the gage is used to modulate the frequency output of an electronic oscillator. After transmission, this signal is demodulated to measure the change in capacitance of the gage. This chapter covers the fundamentals of capacitor, electronic oscillator, and frequency demodulator theory.

3.1 Electrical Capacitance Theory

The main, active, electrical component of the capacitance skin friction gage is a change of area, variable capacitor. A capacitor is an electrical device capable of storing electrical energy [35]. A schematic diagram of a simple capacitor is shown in Fig. 3.1.1-(a). Part (b) of this diagram shows how more complicated capacitors can have multiple layers of capacitor plates, and dielectrics. The performance characteristics of a capacitor dictate how much energy a capacitor can store and how long it can store this energy.

Capacitance in Farads is given by the relation,

\[ C = \frac{q}{V} \]

where \( q \) is the charge in Coulombs, and \( V \) is the electrical potential in volts.

The capacitance not only depends on the area, \( A \), available for
Figure 3.1.1: Schematic diagram of capacitor configurations.

- Simple capacitor
- Capacitor with multiple dielectric and plate layers.
the charge to accumulate on, it is also inversely proportional to
the distance, d, of separation of the plates and directly
proportional to the permittivity, ε, of the dielectric material.
This is shown by the relation,

\[ C = \frac{\varepsilon A}{d} \]

The permittivity of a dielectric material is related to the
permittivity of free space, \( \varepsilon_0 \), by a constant, k, known as the
dielectric coefficient. This is shown by,

\[ \varepsilon = k \varepsilon_0 \]

Figure 3.1.2-(a) shows the technical symbol for an ideal
capacitor. In addition to having capacitance effects on a circuit,
a capacitor’s leads and plates have inductance effects. A
capacitor also introduces resistance into a circuit in two ways:
leakage of electrons across the dielectric (parallel leakage
resistance) and the electrical resistance of the leads and plates
themselves (series effective resistance). These real effects of a
capacitor are shown in an equivalent circuit in Fig. 3.1.2-(b).
Due to these real, non-ideal, effects of a capacitor, energy
absorption is an existent factor in its operation, i.e. energy is
dissipated during the charging and discharging of a capacitor [35].

Other considerations that need to be made when using
capacitors for an electrical application are: power factor,
Figure 3.1.2: Technical symbol for a) capacitor and b) capacitive equivalent network showing: L inductance, RS series effective resistance, C capacitance, RP parallel leakage resistance.
temperature and humidity effects, and dielectric and plate material choices. The power factor of a capacitor is the total power losses involved in the charging process. In the case of the skin friction gage capacitor discussed here, the losses of the charging process is not a consideration. The goal of this research is to make an operational gage which is charged while in operation and discharged afterwards. This is unlike capacitors in other types of circuits which are charged and discharged continuously during operation.

Temperature and humidity are two ambient conditions that affect the operational characteristics of a capacitor the most. Most capacitors' capacitance values increase as the ambient temperature increases, but some can be designed to behave in an opposite manner. The temperature coefficient of capacitors, the amount of capacitance decrease with temperature increase, are within a range of 33 to 750 \( \times 10^{-6}/^\circ\text{C} \). Temperature coefficients of capacitors are three orders of magnitude less than temperature coefficients of semiconductor strain gages. The semiconductor strain gages used with resistance instrumentation have a range of temperature coefficients of resistance change of 6 to 100 \( \times 10^{-3}/^\circ\text{C} \).

Humidity tends to lower the dielectric strength by making insulators such as paper or air be less resistant to electrical flow. Dielectric strength determines the voltage that can be applied without breaking down the dielectric. The break down of a dielectric means that the capacitor gets shorted out and loses its capacity to store energy. The choice of the materials for the
dielectric and the capacitor plates is a critical step in the
design of a capacitor. This step is of primary importance if a
capacitor is to have the necessary performance factors and
electrical properties, discussed above, for the intended
application.

Temperature effects on the capacitor gage have been reduced by
using a thermal insulator, plexiglass, for its construction. The
plexiglass sensing head separates and thereby insulates the
capacitance surfaces from the flow. This is necessary since there
is a high temperature gradient in the flow. The gage body is also
made of plexiglass to insulate the capacitance plates from the flow
tunnel wall.

One of the most useful properties of a capacitor is that it
performs differently in direct current than in alternating current
(ac) voltage applications. A capacitor behaves like an open
circuit in direct current applications. When alternating current
is applied to a capacitor, its reactance or impedance varies with
the frequency of the applied voltage. The reactance of a capacitor
depends on the frequency of the applied alternating current and the
value of capacitance. The capacitor reactance is given by,

\[ X_C = \frac{1}{I\omega C} \]

where i is the unit vector in the positive imaginary axis
direction, \( \omega \) is the frequency in radians per second, and C is the
value of capacitance in Farads. This variable reactance trait
gives the capacitor resonant qualities which can be employed rather effectively. An inductor in parallel with a capacitor forms a resonant circuit that can be used to construct an electric oscillator. An electric oscillator can be used to measure the values of a capacitor by measuring the change in resonant frequency due to the change in capacitance. An oscillator of this type has been used in this research to measure the change in capacitance of the capacitor skin friction gage, as described in Section 3.2.

A capacitor is very adaptable to precise measurement applications. Since a capacitor is an ac component, it can be used with high frequency electric signals to form an instrument that can make time-resolved measurements with high resolution. Capacitors are less sensitive to temperatures than other instrumentation alternatives such as strain gages. Capacitors are capable of steadier signals than resistance gages due to the relative temperature insensitivity. The output signal of the capacitor gage exhibits little drift compared to the output signal of a resistance strain gage.

3.2 Electronic Oscillator

Oscillators are used in applications such as: radar systems; guidance, tracking and communications for missiles and spacecraft; and time and frequency metrology.

An oscillator is used in this research to measure the change in capacity of the capacitance gage. An oscillator can be used to
do this by incorporating the variable capacitor of the gage in the oscillator circuit. As the capacitor changes in capacity, the resonant frequency of the oscillator is modulated. The frequency of a frequency modulated (FM) signal depends on the magnitude of the measured variable. FM is the alternative signal carrier system to amplitude modulation (AM). In amplitude modulated signals, the amplitude of a sinusoidal signal depends on the magnitude of the measured variable. FM systems have the following advantages over AM systems:

a) The external interference affects signal amplitude more than it affects signal frequency; FM is therefore inherently more resistant to interference than AM.

b) By counting pulses over a fixed time interval, a frequency signal can easily be converted to digital form [36].

An electronic oscillator produces a continuous sinusoidal electric signal of either constant or varying amplitude and frequency characteristics. Three fundamental components of the oscillator used in instrumenting this gage are a transistor, a capacitor, and an inductor. A series or parallel combination of an inductor and a capacitor inherently forms a resonant circuit due to the components' frequency dependent reactances and principle of operation. The capacitive reactance is given by,

\[ X_C = \frac{1}{j\omega C} \]
where $X_C$ is the reactance or resistance to current flow of the capacitor, $\omega$ is the frequency of oscillation of the current in rad/s, and $C$ is the capacitance value in Farads. The inductive reactance is expressed as,

$$X_L = i\omega L$$

where $\omega$ is the frequency in rad/s, and $L$ is the value of inductance in Henries. At resonance, the capacitive and inductive reactances are equal. A capacitor discharges through an inductor which causes a change of current in the inductor. The current in the inductor induces a voltage which recharges the capacitor. As this cycle repeats itself with the aid of an amplifier, to overcome the internal resistances of the circuit, a sinusoidal signal is generated. The capacitor's and inductor's frequency dependent reactances define the frequency of oscillation according to the expression,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

where $\omega_r$ is the resonant frequency, $L$ is the value of inductance, and $C$ is the value of capacitance.

An oscillator maintains a sinusoidal signal by using a positive or regenerative feedback loop. Unity or greater gain must exist around a regenerative feedback loop in order for an oscillator to maintain oscillation. A schematic of a basic
feedback amplifier is given in Figure 3.2.1. If the gain of the amplifier is $A$ and that of the feedback circuit is $B$, the transfer function of the circuit is given by,

$$A_F = \frac{V_o}{V_i} = \frac{A}{1+AB}$$

where $A_F$ is the gain of the feedback network. With nonresistive feedback networks, the feedback gain, $A_F$, is infinite allowing the circuit to maintain a constant output with no external input which is the definition of an oscillator circuit. This can be seen by considering that $V_{in} = V_o = V_f = 0$, initially before any input voltage is applied to the circuit (Fig. 3.2.1). An initial voltage, $V_{in}$, is applied to start the oscillation. The output of the circuit, $V_o$, becomes $A(V_{in} - V_f) = AV_{in}$, making the input to the feedback network nonzero. The feedback input becomes $AV_{in}$. Now, the feedback output becomes $ABV_{in}$. With the external input, $V_{in}$, removed, the input to the amplifier is $-ABV_{in}$. Under the condition that $AB=-1$, the input to the amplifier will remain $-ABV_{in} = -(1)V_{in} = V_{in}$. This shows that the output of the amplifier is still $AV_{in}$ after the external input to the circuit is made zero. The condition that requires the feedback loop gain, $-AB$, to equal 1 is known as the Barkhausen criterion. This criterion is what makes a feedback network oscillate [37].

A commercial FM microphone transmitter circuit was used in this research to carry out the oscillator and transmitter functions. A drawing of the oscillator and transmitter circuit is
Figure 3.2.1: Schematic of feedback amplifier circuit.
shown in Figure 3.2.2. This circuit transmits a signal with a carrier of 88 MHz to 108 MHz frequency. Frequency modulation is used to impose the information to be transmitted on the carrier signal. A frequency demodulator is then used to separate the information from the carrier signal in order to extract the information.

Again referring to Figure 3.2.2, the oscillator and transmitter components and operation are described. The oscillator is made up of transistor Q2, capacitors C4, C5, C7, and inductor L1. Inductor L1 is variable to allow the selection of a favorable carrier frequency. Resistor R7 is used to forward bias the transistor base-emitter junction such that it amplifies the current across the collector and emitter in order to provide a regenerative feedback for the oscillator circuit. The three capacitors and the inductor determine the frequency of oscillation of the carrier signal according to the relation,

$$\omega_r = \frac{1}{\sqrt{LC}}$$

where C is the series combination of C7 and C5 in parallel with C4. C in the previous equation is therefore given as,

$$C = C4 + \frac{C5 + C7}{C5C7}$$
Figure 3.2.2: Oscillator and transmitter circuit. The circuit of transistor Q2 is the oscillator, circuit of transistor Q3 is the transmitter antenna.
Modulation of the carrier frequency is accomplished by varying one of the capacitors in the oscillator. Since the value of the gage's capacitance plates, 9.0 pF to 12.2 pF, is close to the value of C5, 33 pF, C5 was replaced by the gage's leads. To retain the same oscillator performance, the gage's capacitor in parallel with a 25 pF capacitor was used to replace the 33 pF capacitor, C5, in the oscillator circuit. A change of capacitance of the gage consequently modulates the carrier frequency in this configuration.

The output of the oscillator is coupled to the input of the transmitter via capacitor C6. This capacitor serves as an a.c. coupling. Its relatively low capacitance value, 10 pF, allows it to have a higher impedance against low frequencies. This is shown by,

$$X_C = \frac{1}{\omega C}$$

High frequencies, however, are coupled to the output transistor since C6 offers less impedance at high frequencies. In this manner, any d.c. component of the output signal from the oscillator is filtered out and a pure a.c. signal, without a d.c. offset, is transmitted.

The output of an oscillator with a varying component, such as the variable capacitive transducer of this gage, is an FM signal. The FM signal can be broken down into two components - the carrier and information. The carrier frequency is used to carry the reference for the FM signal. The information component is a bit
more complex. The information in an FM signal is represented by a change in frequency of the carrier. The modulation of the carrier frequency represents amplitude information. The rate of change of the modulation of the carrier frequency represents the frequency of the information signal.

Electronic oscillators produce output frequency signals that are very steady and can be monitored accurately. Oscillators can be designed such that external effects like temperature changes and power source frequencies do not affect the output signal. High frequency, 10 kHz to 100 MHz, LC oscillators exhibit excellent frequency stability and are highly sensitive to small capacitance changes. Oscillators produce frequency modulated output signals which are favorable for processing and digital conversion operations.

3.3 Frequency Demodulator

A basic radio frequency, 90 MHz to 110 MHz, receiver and demodulator is used to demodulate the signal generated by the variable capacitance gage, and the oscillator circuit described in Sections 3.1 and 3.2. A radio frequency receiver consists of detector, filter and amplifier stages with which it demodulates modulated input signals. A circuit diagram of the frequency demodulator used is shown in Figure 3.3.1.

The detector stage is made up of transistor TR1, inductor L1, and capacitors C2, C3, C4, and C5. The filter stage consists

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Figure 3.3.1: Frequency demodulator circuit showing: tank circuit as L1, C4, C3; r-f amplifier as circuit of TR1; bandpass filter circuit as circuit consisting of RFC, C8, R1, C7.
of components R1, C6, C7, and a radio frequency choke (RFC). The amplifier section is comprised of the remainder of the receiver circuit, everything above of what is not mentioned in the detector and filter stages, in Figure 3.3.1.

A schematic block diagram of the principle of operation of the demodulator is given in Figure 3.3.2. As can be seen in the block diagram, the frequency modulated input signal is initially passed through a tuned tank circuit. The input signal is first encountered by capacitor C1. It is part of the antenna circuit which is tuned to the frequency of the carrier that is received with the variable capacitor, C4, in the tank circuit. Capacitor C5 bypasses the transistor amplifier TR1 and applies the input signal at the tank circuit. The tank essentially blocks the radio frequency (r-f) carrier component of the input signal and allows deviations from this frequency to pass to the amplifier formed by transistor TR1. These deviations consist of the information signal which was used to modulate the carrier. In this manner, the information component of the input signal is extracted from the r-f carrier. Finally, the audio frequency (a-f) information signal is amplified, filtered, and then applied to the output a-f amplifiers.

The tuned circuit in the demodulator is made up of a tank circuit which consists of inductor L1 in parallel with capacitors C3 and C4. Capacitor C4 is variable and is used to tune the tank and antenna circuits to the desired carrier frequency by adjusting the tank’s resonant frequency to equal that of the carrier frequency. The tank circuit has a high impedance to any signal at
Figure 3.3.2: Block diagram of operation of demodulator.
its resonant frequency, or the carrier frequency. At resonance, the impedance of the tank is given by,

\[ Z_r = QX_L = \frac{X_L^2}{R} \]

where \( X_L \) is the inductive reactance, \( R \) is the inductive resistance, and \( Q \) is the ratio of the inductance reactance to the resistance [38]. Also, at resonance, the reactance of the capacitors, \( X_C \), equals the reactance of the inductor, \( X_L \),

\[ X_C = \frac{1}{i \omega C} \]

\[ X_L = i \omega L \]

\[ \frac{1}{i \omega C} = i \omega L \]

where \( i \) is the imaginary axis unit vector, \( \omega \) is the signal frequency in radians per second, \( C \) is the value of capacitance equal to \( C_3 + C_4 \), and \( L \) is the value of inductance \( L_1 \).

The tank acts as a detector by having a high impedance to the carrier frequency signal it is tuned to and a lower impedance to modulations of the carrier which are caused by the information signal. A parallel tank circuit functions in this way because the current through the tank is smallest at the frequency for which the

69
inductive and capacitive reactances are equal. The frequency this condition occurs at is known as the resonant frequency. At the resonant frequency, the current through the inductor is canceled by the equivalent, $180^\circ$ out-of-phase current through the capacitor, so that the current is determined by the resistance $R$ of the inductor coil. At frequencies below the resonance the current through $L1$ is larger than that through $C$, because the reactance of $L1$ is smaller and that of $C$ higher at low frequencies. There is only partial cancellation of the two reactive currents and the current through $L1$ is higher than through $R$ alone. At frequencies above resonance the situation is reversed and more current flows through $C$ than through $L1$ so the current through the tank is again increased relative to the current through $R$ alone [39].

In this manner, the carrier signal is blocked, and the information is allowed to pass through the tank. The information signal at the output of the tank is then fed back via capacitor $C6$ and the 10 K resistor to the emitter and the base of the BJT, common emitter transistor TR1, respectively. This feedback signal forward biases the base-emitter junction of the transistor such that it amplifies the information component of the input signal. This selective impedance of the tank causes the transistor amplifier to amplify only the information signal.

The filter stage performs the function of smoothing the amplified output signal of the tank circuit. The filter also uses a.c. coupling to remove lower frequency and d.c. components from the signal. The filter used here can be classified as a choke
input filter, RFC, combined with a capacitor input filter, C6, R1, C7. The first element in the filter is the RFC. It smooths out the ripples in the current by opposing changes in the current. The amplified tank output consists of cycles of increasing and decreasing current. During the portion of the signal in which the current increases, the strength of the magnetic field about the inductor increases. During decreasing portions of the signal, the magnetic field about the inductor collapses. This action of the inductor opposes changes in the current, thereby having a smoothing effect. The output of the inductor has an a.c. as well as a d.c. component as shown in Figure 3.3.3.

The capacitor input portion of the filter is a low pass filter used to filter out the higher frequency components of the detector stage output. During the time that the transistor output has a positive value, energy is stored in the capacitor. During the cycle that the tank output is less than zero, the capacitor discharges through the resistor in series with it. The output of a capacitor discharged through a resistor in this way decreases exponential during the discharge portion (Fig. 3.3.3). The second capacitor in parallel with the first is another section of the filter which further reduces the percent ripple content of the signal. The RC time constant of this stage causes the capacitor input filter section’s rate of discharge to be such that it attenuates frequencies above approximately 6 kHz. Frequencies higher than these are filtered out. The capacitors also act as open circuits to the d.c. component in the signal, thereby removing
Filter action of a capacitor and an inductor. (a) Current or voltage output of a full-wave rectifier. (b) Output from a capacitor. (c) Output from an inductor.

Figure 3.3.3: Filter action of a capacitor and an inductor.
any d.c. drifts or low frequency disturbances which may be present. The output of the filter stage is an audio frequency signal which is the input to the a-f amplifier stage.

The resulting signal carries information which corresponds to the original FM signal which was input into the demodulator receiver circuit. The voltage (amplitude) of the output signal corresponds to the change in frequency of the FM signal, and the frequency of the output signal corresponds to the rate of change of frequency of the original FM signal.

3.4 Application of Frequency Modulation and Demodulation

Electronics to Capacitance Measurements

The skin friction measurement gage developed in this research changes in capacitance when a tangential (wall shear) force is applied to it. This change in capacitance occurs in a variable capacitor which is part of the total capacitance in an oscillator circuit. The output of the oscillator circuit is an a.c. signal whose frequency corresponds to the change in capacitance. This signal is used to modulate a carrier frequency of 88-108 MHz and then transmitted. Next, the signal is demodulated with a standard FM receiver. The demodulated output of the frequency demodulator is an ac voltage that is proportional to the change in capacitance of the capacitance sensing skin friction gage. The voltage amplitude and frequency correspond to the change in frequency and the rate of change of frequency of the FM signal, respectively.
The output of the FM receiver can be viewed with a frequency counter, to measure the frequency change in the oscillator and thereby the change in capacitance. The d.c. component of the signal was measured within the circuit before it is filtered out to allow static calibrations of the skin friction gage. Also, since the variable capacitor gage changes the resonant frequency of the oscillator, a phase angle change in the oscillator’s output exists. This phase angle corresponding to the change in capacitance of the gage was measured to provide a reference for comparison of the calibration data obtained with the demodulator circuit. The tangential force of the shear stress in a flow can then be deduced, based on the change in capacitance, using a calibration curve.

An FM microphone transmitter system and an FM radio receiver are used to measure the change in capacitance of the skin friction gage. The microphone transmitter has an output with a carrier frequency within the range of an FM radio, 88-108 MHz. The microphone transmitter unit has a built in oscillator that is used to monitor the frequency (voice) input into the microphone. The microphone is removed from the circuit leaving an open circuit in its place, in order to utilize the oscillator stage alone. The variable capacitor gage is then used to replace a capacitor within the oscillator so the change in capacitance of the variable capacitor modulates the resonant frequency of the oscillator. The modulated resonant frequency of the oscillator is the transmitted output frequency.

This microphone oscillator was employed by the skin friction
gage by installing the variable capacitor in place of one of the
capacitors, capacitor C5, in the transmitter's local oscillator
(Fig 3.2.2). The change in the capacitance of the variable
capacitor of the skin friction gage was measured by measuring the
change in the resonant frequency of the oscillator due to the
variable capacitor. In this way, the use of a manufactured
oscillator was employed because the problems of stray capacitance,
inductance, and noise effects, which would have presented
themselves had an attempt been made to build an oscillator in the
lab, were eliminated.

Since the demodulated output signal of the FM receiver is only
affected by transient or dynamic input to the capacitance gage, a
static calibration procedure is of no use with the electronics
developed here. The demodulator circuit's output corresponds only
to transient inputs, hence d.c. offsets are not contained in the
output and a static load cannot be measured, however the
demodulator output is perfectly suitable for monitoring the
transient response of the gage due to dynamic input. D.c.
components of the demodulator signal are contained within the
electronics before filtering and a.c. coupling occur. A d.c.
component due to static loads can be measured across resistor R1
within the demodulator electronics (Fig. 3.3.1). A dynamic
 calibration is required to interpret the transient input of the
flow. A dynamic calibration would also show the dissimilarities in
the way the gage responds to static vs. transient excitation. In
order to perform a dynamic calibration, a harmonic shaker can be
employed as the input medium. A force transducer and an accelerometer can be used to measure the input and the output, respectively, of the gage.

High frequency electrical components, such as the modulator and demodulator used here at 100 MHz, are very sensitive to stray capacitance and inductance effects. Stray inductance effects are caused by external magnetic fields which can distort the output of the electronics. External electric fields can cause stray capacitance effects. Both can be eliminated or reduced by proper shielding and grounding of the shield. In an effort to curb stray effects a ferromagnetic metal shield is used to block external fields from the electronics. The metal shield is grounded to provide a low resistance path to one common ground thereby reducing the possibility of ground loops.

A frequency demodulator is an excellent device for monitoring the output of the oscillator used in this research. Any version of today's quartz or transistorized FM receiver can be used to demodulate the oscillator output. Modern FM receivers will give clean undistorted output signals.
Chapter 4

TESTING OF PROTOTYPE

In order to verify that the performance characteristics of the finished gage are as intended by the design, a complete testing of the gage is necessary. These tests should include: frequency response testing of the electronics and the beam mechanics, static and dynamic calibration of the gage, and supersonic and shock tunnel testing of the final gage. Transfer functions of the electronics and gage mechanics are useful in interpreting calibration and test results. A complete calibration should be done to test the output of the gage for inputs well below and above the expected input due to the flow. The final test, to prove the functionality of the gage, that is necessary is supersonic and shock tunnel testing of the gage. A static calibration of a prototype of the capacitive sensing cantilever beam skin friction gage was completed in this research.

A prototype of the cantilever beam skin friction gage was instrumented with strips of stainless steel tape to form the variable capacitor of the gage. The capacitance surfaces on the plates consisted of 0.0625 in. (1.59 mm) wide stainless steel strips placed 0.0625 in. (1.59 mm) apart. A piece of 0.001 in. (0.0254 mm) thick mylar was placed between the two capacitor plates and was used as the dielectric. The calculated minimum and maximum
capacity of the model capacitor plate setup is 0 pF and 38.4 pF, respectively. The measured minimum and maximum capacity of the plates, corresponding to minimum and maximum overlap of the stainless steel strips (respectively), is 9.0 pF and 12.2 pF.

This discrepancy is due to the non-ideal performance of the gage's capacitor. When the capacitance was measured, the plates with the mylar dielectric in between them were simply laid on top of each other without any normal force pressing the plates together. Since the mylar wasn't perfectly smooth, the dielectric didn't completely fill the gap between the plates and the separation of the plates was actually more than 0.001 in. (0.0254 mm) - which is the thickness of the mylar used in the calculation which yields a maximum of 38.4 pF. Upon further investigation it was found that with an approximate 0.005 in. (0.127 mm) thick layer of air between the plates along with the 0.001 in. (0.0254 mm) mylar creates a composite dielectric which makes the maximum capacitance 20.6 pF. This is consistent with the measured maximum capacitance since a gap of air, of at least 0.005 in. (0.127 mm) thick, was likely to exist due to the uneven mylar dielectric. Further measurement error can be reasoned from the fact that the plates weren't perfectly parallel due to the basic construction of the gage plates during measurement. The discrepancy between the calculated and measured minimum capacitance values can be attributed to edge effects. Because the distance between the plates is very small, the plates don't have to be overlapping to build up a charge. As shown in Figure 2.0.1, the edges of the
conducting strips are close enough (on the same order as the widths of the strips) for a charge to accumulate on the surfaces when the theoretical overlapping area of the plates is zero.

During operation of the oscillator, the frequency output was observed to be very sensitive to displacement inputs applied to the capacitor plates. A static calibration was performed on the gage being developed in this research. A static load was exerted on the gage by suspending a known weight from a string attached to the gage's sensing head. The frequency modulation due to static loading is represented in the demodulator circuit as a d.c. component of the output signal of the detector. This d.c. change in output voltage is measured across resistor R1 in the demodulator circuit (Fig 3.3.1). An a.c. as well as the strongest d.c. component of the detector output is present across these nodes. A static calibration is allowed in this manner by measuring the d.c. offset in the detector output which corresponds to the change in capacitance of the gage.

A static calibration has been conducted on the prototype of the capacitance skin friction gage. The static calibration procedure consisted of applying a known weight in a tangential direction to the gage's sensing head. The output of the gage was then measured as described above.

Known weights of values: 58.86, 68.67, 98.1, 147.15, 196.2 g m/s², were applied to the transducer during the calibration procedures. Input weights spanning the expected shear force of 98.1 g m/s² due to the design flow were selected to test for
linearity of the transducer's response about this range of input forces. 58.86 and 68.67 g m/s² weights were chosen to test the resolution of the gage's sensitivity to small input changes. The gage has an average output of 0.12 mV change in potential to the expected 98.1 g m/s² force input. A difference of 0.04 mV output was observed due to the inputs of 58.86 and 68.67 g m/s² forces.

To verify the performance of the frequency modulation electronics developed here, a calibration was performed using a phase angle comparator to measure the change in phase of a reference signal due to the complex impedance of the skin friction gage's capacitor. A correlation coefficient squared was calculated between the calibration data obtained with the frequency modulation electronics and that obtained with the phase angle comparator. This indicates how well the data obtained with the electronics developed here corresponds with the phase angle measurements.

Finally, a series of supersonic and shock tunnel tests are suggested. Supersonic tunnel tests will show how the gage responds to sustained turbulent flow. This will show the gage's sensitivity to temperature effects, drift characteristics, and other idiosyncrasies. Shock tunnel tests will show the transient response characteristics of the gage by how well it responds to the shock wave that is blown past it. The values of the gage output will indicate how accurately it measures skin friction forces by comparing it to theoretically calculated skin friction forces of the flow. Turning the gage around 180 degrees in both flows should
indicate that the gage is measuring true skin friction by yielding outputs of opposite magnitudes.
Chapter 5

RESULTS AND DISCUSSION

Due to its high sensitivity to tangential forces and low sensitivity to normal forces and temperature effects, the cantilever beam capacitance skin friction gage appears to be a more favorable approach than the other gage and electronics combinations currently under study. Calibration results show a sensitivity of 0.02 mV per g m/s² with no amplification. A 0.12 mV response due to the expected 98.1 g m/s² (0.0221 lb) force input of the skin friction of the Mach 2.9 flow, over the 0.49 in² (316 mm²) area of the sensing head, was measured as the average output of the gage instrumented with the stainless steel strips. An output of 0.05 mV results when a 196.2 g m/s² (0.0441 lb) weight is applied to the head in a normal direction.

Table I. shows three calibration set results. Calibration No. 1 is the average of five calibrations performed by loading the gage in the streamwise, zero degree direction, Calibration No. 2 was performed by loading the gage in the streamwise 180 degree direction, and Calibration No. 3 was done in the cross-stream 90 degree direction. Calibration No. 1 shows the repeatability of the calibration results. Calibration No. 2 shows the gage calibration results due to tangential loading in the 180 degree direction.
relative to Calibration No. 1. This data set shows the gage’s sensitivity to directionality and that tangential force was indeed measured since the negative of the output due to loading in the zero degree direction was obtained. Calibration No. 3 shows the output of the gage due to loading in the cross-stream 90 degree direction relative to the zero degree calibration. This data set shows the expected low output compared to the stream-wise directions, because of the lack of area change due to deflection in the 90 degree direction due to the gage’s capacitor pattern layout.

Table II. shows the results of the calibration performed by recording changes in the phase angle of a reference sinusoidal signal passed through the variable capacitor of the gage. This procedure was performed to provide a reference to compare to the output of the electronics presented here. This data set indicates that the calibration curves obtained with the electronics presented here are reasonable. Correlation calculations show that a correlation coefficient squared of 0.986 exists between the calibration outputs using phase angle measurements and measurements with the demodulator electronics. The correlation coefficient squared shows that 98.6 % of the two output data sets, using the demodulator electronics and the phase comparator techniques, correspond to one another.
### Table I.: Calibration results.

<table>
<thead>
<tr>
<th>CALIBRATION</th>
<th>WEIGHT [g m/s²]</th>
<th>OUTPUT [mV d.c.]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>No. 1</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streamwise</td>
<td>58.86</td>
<td>0.04</td>
</tr>
<tr>
<td>0 Degree</td>
<td>68.67</td>
<td>0.08</td>
</tr>
<tr>
<td>Direction</td>
<td>98.1</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>147.15</td>
<td>0.22</td>
</tr>
<tr>
<td></td>
<td>196.2</td>
<td>0.33</td>
</tr>
<tr>
<td><strong>No. 2</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Streamwise</td>
<td>58.86</td>
<td>-0.04</td>
</tr>
<tr>
<td>180 Degree</td>
<td>68.67</td>
<td>-0.08</td>
</tr>
<tr>
<td>Direction</td>
<td>98.1</td>
<td>-0.16</td>
</tr>
<tr>
<td></td>
<td>147.15</td>
<td>-0.22</td>
</tr>
<tr>
<td></td>
<td>196.2</td>
<td>-0.28</td>
</tr>
<tr>
<td><strong>No. 3</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cross-Stream</td>
<td>58.86</td>
<td>0.03</td>
</tr>
<tr>
<td>90 Degree</td>
<td>68.67</td>
<td>0.04</td>
</tr>
<tr>
<td>Direction</td>
<td>98.1</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>147.15</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>196.2</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Table II.: Results of phase angle measurement calibration.

<table>
<thead>
<tr>
<th>CALIBRATION</th>
<th>WEIGHT [g m/s²]</th>
<th>PHASE ANGLE [deg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 4</td>
<td>58.86</td>
<td>0.2</td>
</tr>
<tr>
<td>Streamwise</td>
<td>68.67</td>
<td>0.2</td>
</tr>
<tr>
<td>0 Degree</td>
<td>98.1</td>
<td>0.25</td>
</tr>
<tr>
<td>Direction</td>
<td>147.15</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>196.20</td>
<td>0.45</td>
</tr>
</tbody>
</table>

Table III. shows the results of the statistical analysis of the streamwise, zero degree calibration results presented previously. The average, variance, and 95% confidence bounds are given for the average of five samples of calibrations in the streamwise zero degree direction and for the calibration results for one of the five calibrations in the zero degree direction. The output obtained in the calibration procedure are random variables assumed to have a normal distribution about their mean. Since a small number of samples were taken, five samples for the average results and three for the single calibration result presented, a t-distribution was used in the statistical calculations. The variance of a random variable indicates the variability of the observations about the mean. The sample variance is given by,
\[ S^2 = \frac{1}{n-1} \sum (x_i - \overline{x})^2 \]

where \( n \) is the sample size, \( x_i \) is the \( i \)th observation, and \( \overline{x} \) bar is the sample average. For small sample sizes, approximately < 30 observations, the values of \( S^2 \) fluctuate considerably from sample to sample [40]. Due to the nature of the electronics used in this research, the sample variance was expected to fluctuate even more because of the bandpass of the demodulator circuit. With each calibration, the portion of the bandpass that was used was random and unknown. The magnitude and sensitivity of the output signal depended on the slope of the bandpass at which the demodulator was operated. Care was taken to use approximately the same portion of the band, however, it was an unknown and caused fluctuations of \( S^2 \) in addition to the fluctuations inherent because of the small sample size. This can be seen in Table III which shows the variance of the average of the calibrations to be about one to two orders of magnitude greater than the variance of the single calibration. This means that the gage's output is accurate if it is recalibrated with every use.
Table III.: Statistical results of calibrations.

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Weight</th>
<th>Sample Average $\bar{x}$</th>
<th>Sample Variance $s^2$</th>
<th>95% Confidence Interval For Population Mean $\mu_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[gm/s^2]</td>
<td>[mV]</td>
<td>[mV^2]</td>
<td></td>
</tr>
<tr>
<td>Average of 5 Streamwise</td>
<td>58.86</td>
<td>0.0398</td>
<td>2.17 e-4</td>
<td>0.031 $\leq \mu_x \leq$ 0.067</td>
</tr>
<tr>
<td>0 Degree Direction</td>
<td>68.67</td>
<td>0.0765</td>
<td>4.25 e-4</td>
<td>0.052 $\leq \mu_x \leq$ 0.086</td>
</tr>
<tr>
<td>Calibrations</td>
<td>98.1</td>
<td>0.121</td>
<td>1.18 e-3</td>
<td>0.115 $\leq \mu_x \leq$ 0.141</td>
</tr>
<tr>
<td></td>
<td>147.15</td>
<td>0.225</td>
<td>1.07 e-3</td>
<td>0.212 $\leq \mu_x \leq$ 0.242</td>
</tr>
<tr>
<td></td>
<td>196.20</td>
<td>0.325</td>
<td>2.35 e-3</td>
<td>0.302 $\leq \mu_x \leq$ 0.349</td>
</tr>
<tr>
<td>Results For One Calibration: Streamwise, 0 Degree Direction</td>
<td>58.86</td>
<td>0.030</td>
<td>1.33 e-6</td>
<td>0.008 $\leq \mu_x \leq$ 0.114</td>
</tr>
<tr>
<td></td>
<td>68.67</td>
<td>0.090</td>
<td>8.00 e-4</td>
<td>0.029 $\leq \mu_x \leq$ 0.126</td>
</tr>
<tr>
<td></td>
<td>98.1</td>
<td>0.1467</td>
<td>3.33 e-5</td>
<td>0.088 $\leq \mu_x \leq$ 0.164</td>
</tr>
<tr>
<td></td>
<td>147.15</td>
<td>0.220</td>
<td>3.1 e-3</td>
<td>0.164 $\leq \mu_x \leq$ 0.250</td>
</tr>
<tr>
<td></td>
<td>196.20</td>
<td>0.273</td>
<td>4.33 e-4</td>
<td>0.219 $\leq \mu_x \leq$ 0.357</td>
</tr>
</tbody>
</table>

Table III. also shows the confidence bounds of the mean of the calibration output results. Since three observations were made for each calibration point and this was repeated for five calibration procedures, the mean estimate in the data set presented for the five calibrations is a random variable. As other random variables,
an interval can be estimated which includes the parameter being estimated with a known degree of certainty. This interval associated with the certainty that it contains an estimated variable is known as a confidence interval and is given by,

\[
\left\{ \bar{x} - \frac{st_{n;\alpha/2}}{\sqrt{N}} \leq \mu_x < \bar{x} + \frac{st_{n;\alpha/2}}{\sqrt{N}} \right\}, \quad n=N-1
\]

where, \( \mu_x \) is the true mean, \( \bar{x} \) is the estimate of the mean obtained from the estimation equation that was calculated using linear regression, \( st_{n;\alpha/2} \) is the t-value for \( (1-\alpha)100\% \) confidence with \( n=N-1 \) degrees of freedom, \( s \) is the square root of the sample variance, and \( N \) is the sample size. Statistical analysis results are shown for both the average of the five zero degree calibrations as well as a single zero degree calibration to point out that the variance is greater for the average of the calibrations than any one of the individual calibrations. Because of this fact, the gage with the electronics presented here will have to be calibrated before each experiment due to the uncertainty of the operating point within the passband of the demodulator.

A regression analysis was done on the calibration data to obtain an estimation equation of the gage output. The line estimate of calibration No. 1, Table I., is given by,

\[
[mV] = 0.01974 [gm/s^2] - 0.06941
\]

where \( gm/s^2 \) is the force input in mN, and mV is the estimated
output in millivolts. The square of the correlation coefficient for the input and output data is 0.997. This indicates that 99.7% of the output is correlated to the input. A linear regression analysis was also completed on the calibration data points of an individual calibration. The estimation equation for a single calibration is expressed as,

\[ mV = 0.01621 \ [gm/s^2] - 0.03614 \]

where the variables are the same as in the previous equation.

To investigate whether the calibration lines pass through the origin, confidence intervals for the regression line constants were calculated. These statistical calculations performed at the 0.05 level of significance, \( \gamma \), are presented in Table IV. The results show that for the calibration data set containing the average of five calibrations, the y-axis intercept, \( \alpha \), in the regression line \( \mu_{y|x} = \alpha + \beta x \), does not pass through the origin. However, for the calibration data set for one of the individual calibrations, the y-axis intercept does pass through the origin, since the confidence bounds for the y-intercept constant spans the value zero. This is attributed to the fact that the operating point on the bandpass of the frequency demodulator is random and unknown. This causes a bias in the output of a group of individual calibrations. Little bias exists in the data set of an individual calibration.
Table IV: Statistical results of estimation equation.

<table>
<thead>
<tr>
<th>Statistical Calculation</th>
<th>Average of 5 Calibrations</th>
<th>Individual Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimation Equation</td>
<td>mV=0.01974 [g m/s^2]</td>
<td>mV=0.01621 [g m/s^2]</td>
</tr>
<tr>
<td></td>
<td>- 0.06941</td>
<td>- 0.03614</td>
</tr>
<tr>
<td>95% Confidence Interval For Slope</td>
<td>0.017 ≤ β ≤ 0.022</td>
<td>0.0093 ≤ β ≤ 0.023</td>
</tr>
<tr>
<td>95% Confidence Interval For Y-Intercept</td>
<td>-0.100 ≤ α ≤ -0.039</td>
<td>-0.124 ≤ α ≤ 0.052</td>
</tr>
</tbody>
</table>

The confidence bounds for the regression line constants were calculated using the statistical relations as follows. A \((1-\xi)100\%\) confidence interval for the parameter \(\beta\) in the regression line \(\mu_{Y|x}=\alpha+\beta x\) is,

\[
b - \frac{t_{\xi/2} s}{\sqrt{S_{xx}}} < \beta < b + \frac{t_{\xi/2} s}{\sqrt{S_{xx}}}
\]

where \(t_{\xi/2}\) is a value of the t-distribution with \(n-2\) degrees of freedom. A \((1-\xi)100\%\) confidence interval for the parameter \(\alpha\) in the regression line \(\mu_{Y|x}=\alpha+\beta x\) is,
\[ a - \frac{t_{\gamma/2} s\sqrt{\sum x_i^2}}{\sqrt{nS_{xx}}} < a < a + \frac{t_{\gamma/2} s\sqrt{\sum x_i^2}}{\sqrt{nS_{xx}}} \]

where \( t_{\gamma/2} \) is a value of the t-distribution with \( n-2 \) degrees of freedom. In the above two statistical relations, \( s \) is the variance of the data points, \( S_{xx} \) is the value of the autospectral density function, and \( n \) is the sample size of the data set.

Statically loading the gage with a 196.2 g m/s² force in a direction normal to the plane of the surface of the sensing head yielded an output of 0.05 mV which is less than half of that in the cross-stream, 90 degree direction. In fact, for the two lowest weights used, 58.86 and 68.67 g m/s², the output was insignificant.

The static pressure in the design flow is approximately 4 psia (27.6 kPa absolute). This results in a negative pressure of 10 psi (68.9 kPa) on the gage's sensing head. The amount of force exerted on the 0.49 in² (316 mm²) gage head is 5 lb (22.24 N) in a direction away from the wall. This force is two orders of magnitude greater than the 196.2 g m/s² force applied during static calibration testing. With this in mind it can be seen that the output of the gage during testing in the shock tube may be governed by the normal wall pressure input. This high output is due to the present construction of the cantilever beam. Due to the limitations of the photolithographic techniques employed in this research, the sensing head of the cantilever beam is attached to the cantilever beam by a screw. This type of attachment decreases the stiffness of the construction. Increasing the stiffness of the sensing head and
beam attachment will decrease the sensitivity of the gage to normal loading. Also, oil into the cavity between the gage body and the cantilever beam/sensing head components will probably be essential to balance the pressure on either side of the sensing head so no force is exerted on the head in a normal direction due to the static pressure of the flow.

Figures 5.0.1 through 5.0.4 show the plots of the calibration curves. Figure 5.0.1 contains the calibration points obtained in the five repeated streamwise zero degree calibrations along with the fitted calibration line using the average of the five data sets. Figure 5.0.2 shows the calibration points obtained by measuring the relative phase change of a reference signal due to the influence of the gage. Figure 5.0.3 shows the points obtained in the streamwise 180 degree direction indicating the negative response of the gage. Figure 5.0.4 contains the data points obtained by loading the gage in the cross-stream 90 degree direction indicating the relatively low response compared to the streamwise outputs.
Figure 5.0.1: Graph of the 5 sets of calibration points recorded. The gage was loaded tangentially in the streamwise zero degree direction. The line, which was calculated using a linear regression procedure, represents the estimated equation of the gage output.
Figure 5.0.2: Graph of the calibration points obtained by measuring the relative phase change of a reference signal due to the influence of the complex impedance of the gage.
Figure 5.0.3: Graph of calibration points obtained by loading the gage in the streamwise 180 degree direction.
Figure 5.0.4: Graph of calibrations points obtained by loading the gage in the cross-stream 90 degree direction.
To test if the regression model is correct, that $\mu_{Y|x}$ is related to $x$ linearly in the parameters, an analysis of variance was conducted. This consisted of a test for lack of fit of the linear regression model to the data where the error sum of squares is divided into two parts: the amount due to the variation between the value of $x$ and a component that is normally called the lack of fit contribution. The first component reflects mere random variation or pure experimental error, while the second component is a measure of the systematic variation brought about by higher order terms.

If the $\mu_{Y|x}$ fall on a straight line, there is no lack-of-fit when a linear model is assumed so that the sample variation around the regression line is pure error resulting from the variation that occurs among repeated observations. The hypothesis that the regression is linear in $x$ vs. nonlinear in $x$ was tested at the 0.05 level of significance. The data set consists of five samples of five average data points each which yields a critical $f$-value of 3.10 with degrees of freedom equal to 5 and 25. The analysis of variance for the regression model is shown in Table V.
Table V.: Results of analysis of variance for regression model.

<table>
<thead>
<tr>
<th>SOURCE OF VARIATION</th>
<th>SUM OF SQUARES</th>
<th>DEGREE OF FREEDOM</th>
<th>MEAN SQUARE</th>
<th>COMPUTED f</th>
</tr>
</thead>
<tbody>
<tr>
<td>REGRESSION</td>
<td>0.2674</td>
<td>1</td>
<td>0.2674</td>
<td>262.2</td>
</tr>
<tr>
<td>ERROR</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) LACK OF FIT</td>
<td>0.02127</td>
<td>23</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.0008697</td>
<td>3</td>
<td>0.0002899</td>
<td>0.2842</td>
</tr>
<tr>
<td>b) PURE ERROR</td>
<td>0.0204</td>
<td>20</td>
<td>0.00102</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>0.2886</td>
<td>24</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analysis of variance results indicate a significant variation accounted for by the linear model ( $f=262.2 \gg f_c=3.10$ ), and an insignificant amount of variation due to lack of fit ( $f=0.2842 < f_c=3.10$ ). Based on these calculations, the data do not seem to suggest the need to consider terms higher than first order in the model.
Chapter 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

The purpose of this research was to develop a skin friction gage instrumented with capacitance sensing electronics and its supporting electronic systems. The gage had to be operational and insensitive to normal forces exerted on it. The following conclusions can be drawn from the research completed.

- A prototype of a capacitive transducer constructed with 0.0625 in. wide stainless steel capacitor plates was successfully constructed and tested performing static calibrations. This suggests that future skin friction gage research would be a worthwhile effort.

- The sensitivity of the gage to normal loading was not found to be low enough to be acceptable. Further design consideration of making the attachment of the sensing head to the cantilever beam more rigid is necessary. Also, during shock tunnel testing it is suggested to inject oil in the cavity of the gage between the gage body and the sensing head/cantilever beam to balance the pressure on either side of the sensing head.
- Frequency modulation and demodulation electronics were successfully utilized to support the capacitive skin friction gage. The electronics presented here exhibits low temperature dependence and insignificant drift characteristics.

- The results of the calibrations suggest that accurate measurements can be made with the gage and electronics developed here at a relatively low cost. Significant improvements in gage sensitivity and output signal distortion can be expected at a reasonable cost.

6.2 Recommendations

Based upon what was learned about the cantilevered beam capacitance skin friction gage through conducting the research presented in this thesis, the following recommendations are made for future advancements in this research, concerning the mechanics of the gage and the development of the electronics:

- The gage should be made smaller to obtain higher natural frequency characteristics. The prototype gage presented here has a natural frequency of 313 Hz. As pointed out in the thesis, with minor geometric modifications a natural frequency of 5.9 kHz can be obtained while still maintaining an acceptable sensitivity.

- Make modifications to the gage mechanism. In specific, the
diameter of the base of the beam relative to the beam needs to be larger to allow greater and more accurate adjustment of the sensing head. The small pattern created by photolithography could be used if the separation distance of the capacitor plates is adjusted more precisely.

- The attachment of the gage’s sensing head to the cantilever beam needs to be made more rigid to reduce the gage’s output to normal forces. Oil should also be injected into the cavity between the sensing head and the cantilever beam to balance the pressure on either side of the sensing head.

- The lead wires connected to the capacitor plate pattern need to be soldered or attached with a screw to ensure proper contact after assembly of the gage. As is, good contact is made between the lead wires and the pattern, however, the assembly would be much better if the contact of the lead wires weren’t a consideration.

- A more precise transmission oscillator and demodulator would be a step in obtaining more precise measurements. The oscillator and demodulator used in this research are adequate yet very basic and output signal distortion and measurement inaccuracies can be eliminated with higher precision electronics.

- To obtain an accurate, time-resolved trace of the response of the gage to supersonic flow, the electronics needs to be interfaced
with data acquisition systems using any of the available switched-capacitor interface chips.

- If it is necessary to move toward other measurement principles to obtain better accuracy, phase comparator electronics are believed to be a worthwhile method.

- Frequency response tests should be conducted including: transfer function measurements of both the electronics and the gage mechanism, and dynamic calibration.

- The gage should be tested in a supersonic wind tunnel with a run time of 10 to 30 seconds. This will show the response of the gage under sustained flow conditions.

- Shock tunnel tests should also be completed. Tests in an impulse facility will show the gage's transient response characteristics.
REFERENCES


23. Allen, J. M., "an Improved Sensing Element for Skin-Friction


Vita

The author of this thesis received his Bachelor of Science in Mechanical Engineering from Virginia Polytechnic Institute and State University in August of 1991. The writing of this thesis concludes a Master of Science in Mechanical Engineering program started in August of 1991. The Motor Company served as a great deal of motivation during the author's college career.

Istvan Horvath

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