AN ECONOMIC OPTIMIZATION MODEL FOR CAPACITY EXPANSION DECISIONS

by

Carl W. Cormia

A Master's Thesis submitted to the faculty of the

Department of Industrial Engineering and Operations Research

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

in

Industrial Engineering and Operations Research

Approved:

Joel A. Nachlas, Chairman

Marvin H. Agee

Laurence J. Moore

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ABSTRACT

A model is developed to identify the optimal capital expansion decisions for producers of consumable products. The model features an optional selection of corporate level in a product tree hierarchy and a choice of several optimizing objective functions.

The model assumes that intermediate product demand is directly dependent on demand for consumable products. Intermediate product demands are derived from demand for consumable products using a product tree similar to a bill of materials. Restrictions exist on the productive capacity of all products in the product tree, and interdependences exist between producers of the various products. Likewise, the availability of labor limits production capacities for all products in the product tree. The capital available for the capacity expansions can either be capital equity or corporate debt.

The model identifies the expansion strategy which optimizes the chosen economic objective function. A case study is analyzed with linear programming software to determine the optimal expansion strategy for a tire manufacturer given a hypothetical market demand for automobiles.

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TABLE OF CONTENTS

LIST OF TABLES AND FIGURES

CHAPTER 1
INTRODUCTION

Planning expansions for a company's manufacturing operations has evolved into a very complex process of forecasting product demands, taking market surveys, evaluating options, and choosing the best strategy. Top managers expend large quantities of time evaluating alternatives and employ all the tools at their disposal to assist with such decisions. Even so, they have not always experienced a high success rate with their investment decisions. Many succeed, but many fail as well.

In an effort to eliminate or at least reduce losses, internal corporate planning departments analyze investments in terms of cash flows, interest deductions, and depreciation deductions. Yet the best operations plans fail if the manager fails to consider possible uncertainties in the market. Often investments are less profitable than originally planned because market interactions with other sectors of the economy influence product demand. To overcome the obstacles in an uncertain market, managers use many planning tools to track demand over time, search for seasonal fluctuations in the market, and so on. But any individual forecast might not agree with forecasts for the economy as a whole.

This research is concerned with developing a model whereby macro-economic forecasts for the economy can be applied to the micro-economic decision of a single company's capacity expansion strategy. Such a model avoids the pitfall of failing to consider macro-economic and micro-economic market interactions. Furthermore, a model merging the micro-economic and the macro-economic interactions would extend the literature on capacity expansion models.

1.1 STATEMENT OF THE PROBLEM

There are two basic problems with expansion models intended for use as planning tools.

The first problem is that the models only evaluate a single industry's expansion decisions. Expansion models have typically focused on finding the optimum expansion strategy for electric utilities. The models only incorporate materials consumed by the industry being studied. An example is the expansion models as applied to expansions in generating capacity for electric utilities. Such models generally only incorporate coal, oil, natural gas, and uranium availabilities into the decision strategy. Other economic factors such as private and industrial consumption of oil, the dependence on overseas oil and gas reserves, the fluctuation of industrial useage of electricity based on demands for products of heavy industries, etc., have generally not been considered in the expansion models.

Economists such as Leontief[1] have called for managers directing micro-economic planning--electricity demands for a geographical region, for instance--to consider macro-economic forecasts for consumable products in their forecasts of market demand. Such a synergism would hopefully eliminate the losses from trial-and-error type decisions while retaining the benefits of those investment decisions.

This combination of micro-economic and macro-economic forecasts and decisions leads to the second problem inherent in capacity expansion models. The second problem concerns the extensive data requirements necessary to implement the models. Production capacities, labor conusumption rates, and material requirements on a host of industrial sectors are needed in a sizeable expansion model. In addition, proprietary cost data is also required if the model is to find the strategy maximizing present worth (or whatever cost objective is desired) of more than one industry in the economy. One store of data for expansion models is the U.S. Bureau of Labor Statistics. The U.S. government has compiled great quantities of the required data for industries on the critical components list for militarily important industries. If the manager can obtain access to such data bases, then this second problem with capacity expansion models may be significantly reduced.

1.2 DESCRIPTION OF THE PROPOSED MODEL

The model is based upon a product tree representation of the U.S. industrial infrastructure developed by Frisch[2]. In the tree, raw materials are converted into basic material stocks such as aluminum and steel plate, copper wire, fabric, etc., by refining, smelting, and other conversion processes. Components such as engine blocks and aluminum siding are manufactured from the basic material stocks. Components are then assembled into assemblies which are, in turn, aggregated into products. The highest level in the product tree consists of systems such as automobiles, houses, and computers, where the systems are assembled from products. These six levels constitute what Frisch called the product tree.

The model developed here represents the entire conversion of raw materials into final systems which are then sold to individual, industrial, or governmental consumers. Figure 1.1 depicts this process. At each product level, consumer demand for systems during each production period causes a demand for products, assemblies, and components during the preceding time periods. These derived demands may then be used to compute production requirements for each time period. If, for example, productive capacity does not currently exist to meet production requirements, a shortage in product supply will occur. Since product demand exceeds current productive capacity, the industrial infrastructure will respond to the shortages by constructing new plants or increasing productive capacity at existing plants.

The representation of the product tree and the material, labor, and capital resources that are consumed in the production of systems for consumer purchase make up the model. The following six materials supply and production relationships summarize the model equations:

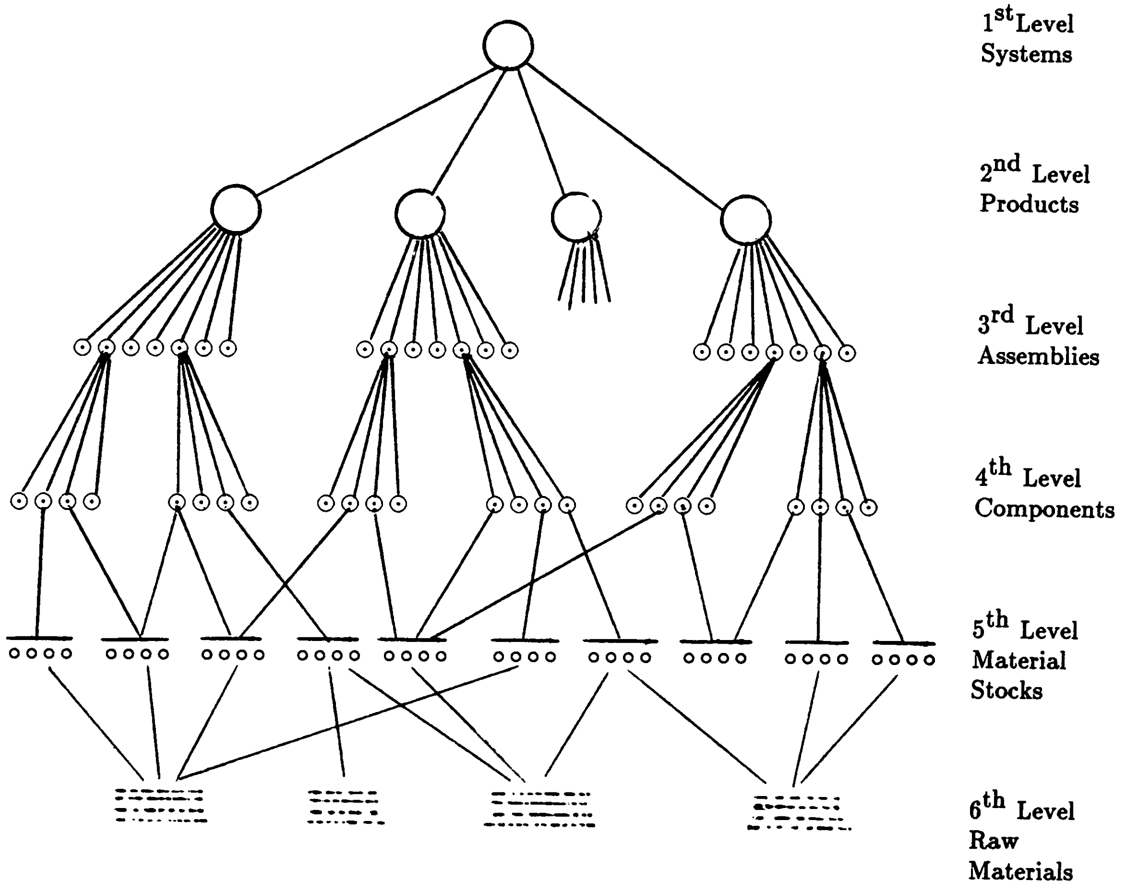


FIGURE 1.1 PRODUCT TREE AS PROPOSED BY THE MODEL

Optimize: The expansion strategy for an individual firm which maximizes net revenues

Subject to:

1) Cascading demands over time must be met at all production levels--may include material substitutions, initial inventories, and inventories carried over from one period to the next, but does not include backorders.
2) Production rates cannot exceed existing production capacity.
3) Production capacity may expand over time.
4) Production rates cannot exceed the current man-hours available to support production--may include substitutions of labor among the labor classes and among various production facilities.
5) A limited number of production expansions are allowed--may be relaxed to allow for unlimited number of expansions, may require a fixed number of expansions if desired.
6) Expansion costs cannot exceed capital availability for the expansion.

The most important feature of the model is the size and occurrence of the capacity expansions. The model formulation is flexible enough to model any individual firm in the product tree. The model is compatible with analytical tools to identify the optimal expansion strategy.

1.3 MODEL OBJECTIVE

The primary objective of this research is to develop the methodology of an analytical model that identifies the optimum capacity expansion decision and that provides a mechanism by which the optimal solution's sensitivity to model parameters such as the interest rate may be found. The proposed capacity expansion model can be applied to any company manufacturing items at any level in the product tree and used to compute optimal expansion strategies. Expansions of any size may be considered. The optimal strategy is then the strategy which maximizes the company's net revenues.

Eight assumptions are important to the model development. The assumptions are as follows:

1) A company's market share is constant over the study period.
2) Administrative costs are based on direct labor hours required to manufacture one unit of product.
3) Material substitutions are permitted at the basic raw material level, occasionally at the component level, but not at higher product levels.
4) No backordering is allowed.
5) All types of capitial assets are assumed to have a single depreciable life.
6) Straight-line depreciation is assumed to adequately approximate MACRS depreciation (or the method used by the company in question).
7) Growth in production capacity is assumed to occur in discrete events rather than in a continuous fashion (as would be the case if progress along a learning curve were included in the model).
8) Interest rates for equity capital and borrowed funds are assumed constant in the model.

Two possible objectives are to 1) maximize present worth of the company, and 2) maximize present worth while minimizing product inventories, subject to the six supply and production relationships. The model will then identify the expansion strategy to optimize the chosen objective function.

1.4 OVERVIEW OF THE RESEARCH

The remaining chapters of this paper contain a review of the literature pertinent to economic modeling of capacity expansion decisions, a discussion of the algebraic development of the model, a discussion of the model's computational implementation, a case study demonstrating the capabilities of the model, and final conclusions and recommendations on the model. Listings of the LINDO input and output files are included in Appendices 1 and 2.

CHAPTER 2
LITERATURE REVIEW

2.1 INTRODUCTION

Macro-economics is concerned with numerically estimating economic inter-relationships with various sectors of the economy.[2] The results from economic studies are used to study three specific economic issues: structural analysis (verification and validation of theories), economic forecasting, and policy evaluation.[2]

Structural analysis is used to evaluate competing economic theories on a quantitative basis. That is, proposed theories may be compared with other economic theories, evaluated, and verified. Forecasting is used to predict the behavior of certain variables by extrapolating expected future results from observed data. Forecasts of future economic conditions are then used as a motivation for pursuing specific economic actions. Policy evaluation is used to select between alternative economic actions, that is, to justify pursuing a given economic action.[2]

Given the three applications of econometric studies, a literature search was undertaken to find which aspects of econometric models are relevant to capacity expansion decisions.

2.2 DEMAND FORECAST AND PRODUCT TREE

After a brief examination of the product tree, it is clear that the most important variable in any capacity expansion is the market demand for materials and parts which the company of interest manufactures. The market demand for materials and parts originates from consumer demand for systems from companies manufacturing consumable items. The supply of consumable items is constrained by the acquisition of materials for production. Thus, any solutions to capacity expansion questions of an individual firm in the product tree must be a solution to some system of constraint equations which describe: 1) the availability of all materials and parts inputs for systems' production, and 2) the economic materials and parts demand for a given company in the product tree.

The work of Wassily Leontief in the area of input-output economics is very similar to the problem of interest, especially his development of input-output tables[4].

Leontief says that economic progress is achieved through a trial and error process similar to Charles Darwin's theory of natural selection. The trial-and-error process can be considered successful when, over the long run, net gains exceed net losses and the economy grows as a whole. This trial-and-error method is acceptable to a nation but, to an individual businessman, it is better to plan operations in advance to reduce or eliminate the risk of a project failing while retaining the expected gain.

To illustrate the type of economic interactions and risk reduction by operations planning, consider the following example.

> It takes coal to produce steel. It takes steel to produce railroad cars. But it also takes railroad cars to produce coal. Production targets for railroad cars must take into account not only how much steel will be needed to produce them and how much coal will be needed to produce the steel, but also how many railroad cars will be needed for the coal and steel needed to produce the railroad cars.

Leontief therefore has developed parts of the desired econometric model by recognizing the inter-relationships between the various levels in the product tree. He captures these interactions with his "magic" input-output tables. The tables are essentially huge systems of simultaneous equations. The effect of a change in output of one economic sector is transmitted to the rest of the economy. The tables allow production planners to compute the effect of a specific capacity expansion on the economy as a whole.

The work of Franz Frisch[1] in analyzing military surge capabilities is also very similar to the capacity expansion problem of interest. Frisch examines the organization of the industrial sector and various economic growth possibilities to model the satisfaction of military demand in wartime.

Frisch says that "The purpose of industrial mobilization is to increase the supply of war-essential military products over their peacetime supply rate. Hence, the thinking about industrial mobilization must start with the required products and proceed from there to the industrial processes of how such products are made." He says that all industrial growth is bounded by 1) growth through increased manpower, 2) growth through extension or multiplication of existing facilities, and 3) growth through construction of new facilities. Frisch then develops the industrial anatomy of his model.

He says that the value added at each product level consists of three elements. They are as follows: 1) cost of the labor needed to make the product, 2) the cost of materials used in the product, and 3) the cost of capital for the producer's investments in facilities. What the producer of systems considers "material" will be the combination of labor, materials, and capital value added for the product's producer. The decomposition of the product tree can be continued down through the raw materials level. An increase in manpower or facilities investment at any product level will result in an increase in the capacity for the given product. Frisch then describes industrial conditions for which instantaneous, or very rapid, growth can be obtained. Figure 2.1 depicts Frisch's breakdown of the manufacturing process. Frisch has developed parts of the desired econometric model by decomposing the manufacturing process into three elements--labor, materials, and facilities--by developing the six level structure to represent the process of manufacturing a system, and by developing the relationships between acquisition of systems and acquisition of material and parts inputs necessary for systems acquisition at the six product tree levels.

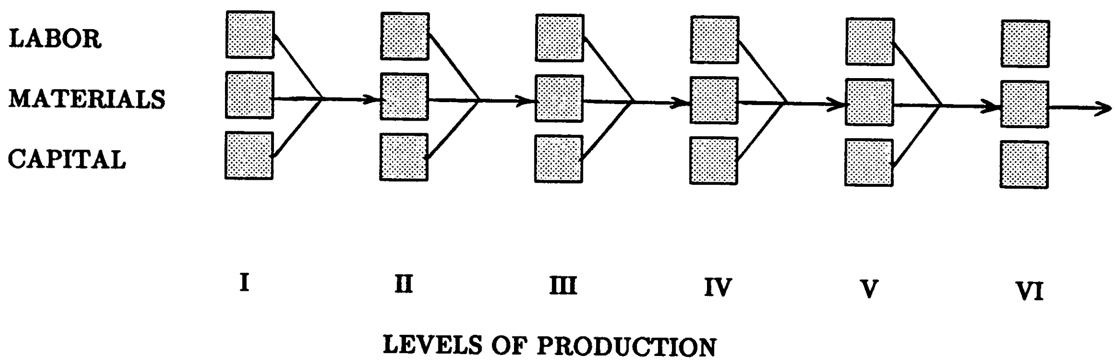
Frisch also details three possible means for expanding productive capacity. The three growth patterns are:

1. Growth through increased manpower.
2. Growth through additions (modernizations) to existing manufacturing facilities.
3. Growth through construction of new manufacturing facilities.

Each of the three growth patterns contribute to the proposed econometric model.

The work of J.A. Nachlas and Marvin Agee[2] also significantly contributes to the proposed econometric model. Nachlas and Agee investigated the vulnerability of United States military preparedness which results from excessive dependence on non-domestic production for critical parts and raw materials.

The systems purchased by the military can be viewed as an aggregation of products, assemblies, components, etc., which are required to manufacture the system. Over the last fifteen or twenty years, U.S. producers have grown increasingly dependent on foreign raw materials, components, assemblies, etc. At the initiation of hostilities with a foreign country, the U.S. might loose access to certain non-domestic markets, and U.S. military suppliers who rely on non-domestic producers in those markets will then be unable to meet the war-time product demands. The authors divide non-domestic producers up into three categories and associated a supply risk with each category. Domestic production represented a fourth production category. They then model U.S. production of military systems using Frisch's six product levels and a cascading set of demands for output at the various product levels.



The levels are defined as:

I = Raw Material
II = Basic Material Stock
III = Component
IV = Assembly
V = Product
VI = System

[6]Frisch, 1985.

FIGURE 2.1 BREAKDOWN OF MANUFACTURING PROCESS

Production at each level is constrained by the materials available for manufacture, capacity of the facilities, and labor requirements necessary to operate the facilities. Production equipment and spare parts production are also included in the model.

The proposed econometric model is very similar to the econometric model developed by Nachlas and Agee. The model has additional constraints on capital availability, and capital requirements for capacity expansion.

The work of Deckro, Spahr, and Herbert[4] in capital budgeting also contributes to the proposed econometric model. The authors model the loss between pursuit of various capital investment decisions by applying goal programming to a set of six decision variables. Net present value, payback period, dividend payments, investment risk, growth, and corporate borrowing constitute the set of decision variables. By using goal programming, the minimum loss from pursuing the set of sometimes conflicting decision variables is found.

2.3 UTILITY EXPANSIONS

Several authors have developed LP econometric models to identify optimum expansion of electrical generating capacities and are worthy of mention.

Anderson[5] develops an econometric model for optimizing expansions in electrical utility plants. He searches for the optimum plant production of electricity where current and future maximum generating capacity comprises the set of decision variables. Plant output is required to meet peak power demands during each time period (assuming electricity cannot be inventoried) and is constrained by the maximum plant operating capacity. Anderson includes additional locational and plant constraints in the model. The model minimizes the cost of operating current plants plus the cost of constructing new plants so as to meet present and future power demands.

Bienstock and Shapiro[6] also develop an econometric model for utility expansion decisions. They divide the model into two sets of variables--resource utilization variables (capacities) and energy variables (electricity generated by the plants and fuel consumed by the plants). The authors employ upper and lower bounds on the capacity expansion variables where the initial capacity is assumed to be known. A 0/1 integer variable forces the model to find the single capacity expansion decision (construction of a new plant) which minimizes the utility's investment costs. The model also solves for the optimal purchase quantities of material resources where contract requirements define lower and upper bounds for purchasing the material resources.

Modiano and Shapiro[7] develop an econometric model to optimize coal allocation to the U.S. energy sector. The authors allow for growth in the energy sector industries and minimize the costs of supply, conversion, and distribution of electricity. The sets of resource consumption variables are very similar to those of Bienstock and Shapiro. Modiano and Shapiro represent power demand with a piecewise demand function and minimize coal, oil, and nuclear resource utilization over a 20 year planning horizon.

2.4 CONTRIBUTION TO THE LITERATURE

The main contribution of this research to the literature on capacity expansion strategy is in the generality of the proposed model. All the expansion models described in the literature (except the model of Nachlas and Agee) integrate a single level of production into the problem formulation. The expansion models only describe capacity expansions in the electrical utilities.

Leontief detailed the interrelationship between the various economic sectors. That is, he described how companies at the micro-economic level often fail to consider the effects of macro-economic forecasts on economic decisions. Thus, he states that many investment decisions are entirely incorrect or much less profitable than originally expected.

Frisch described how a macro-economic model would rely on the product tree to derive demands at the various levels of the tree. Production expansions would then be based on forecasted product demand which is derived from demand for system level goods. The guess work inherent in forecasting product demand could thus be significantly reduced.

Nachlas and Agee used Frich's product tree to develop a model to capture shortages in military system demand. Their model is not intended to optimize a single company's capacity expansion decisions though. Rather, it minimizes the shortages in military system demand over time. Nachlas and Agee also do not include capital in the model since it is assumed that the government can procure all the needed funds to finance capacity expansions.

The proposed model extends the work of Leontief and Frisch to model capacity expansion of a single firm. The firm may exist at any level of the product tree, and the product of interest may be a material input for any number of final systems. The expansion strategy is then to choose the set of expansions which maximize the firm's net present worth. This generalization of the capacity expansion problem extends the electrical utility expansions into a single model for all sectors of the economy.

2.5 SUMMARY

Each of the works summarized contribute to the proposed model of capacity expansion strategy. The model is actually a synergism of the individual models. Each of the modeling techniques found in the literature are combined to construct a model for capacity expansion decisions which can be applied to any firm in the economy.

CHAPTER 3
MODEL DEVELOPMENT

3.1 INTRODUCTION

The algebraic form of the portrayal of the product tree is a modified input-output model. Equations representing the conversion of inputs to outputs at each level of the product tree are expressed in terms of the variables that represent the actual consumption of materials and the production processes required for the conversion. The equations are defined for each of the six product levels where the company of interest can occur at any given level. The algebraic representation of the production processes is intended to highlight the dependence of capacity expansion decisions on consumer purchases of end products.

The production of products can be viewed as having six levels of product conversion. The six levels are: the extraction of raw materials from the environment, the refining of the raw material into basic material stock, the manufacture of components from basic material stock, the manufacture of assemblies from components, the assembly of products from assemblies, and the assembly of systems from products[2].

The manufacture of outputs from inputs requires labor and capital resources as well as the basic material resources. Agee and Nachlas have developed an economic modeling strategy for capturing the manufacturing operations algebraically. This modeling strategy is used here with a few alterations of the initial assumptions.

3.2. MODEL ASSUMPTIONS

The model development is guided by eight assumptions. The first of the assumptions is that the individual corporations' market shares are constant over the study period. The focus of the model is on capacity expansion rather than capturing market share from competitors. A corporation often does realize an increase in market share upon completing a new facility. A portion of this increase in the market share results from corporate sales efforts. The remainder of the increase can be attributed to the market consuming the additional products.

The second assumption is that administrative costs are based on the direct labor hours required to manufacture the product. The overhead application rate is input as a parameter to the model.

The third assumption is that material substitutions may occur at the basic materials level and the component level but not for the higher levels of assembly, product, and system. This assumption reflects the ability of the manufacturer to substitute similar materials to avoid work stoppages which would otherwise result from material shortages.

The fourth assumption is that no backordering is allowed. In other words, unsatisfied product demand cannot be met during future time periods. This assumption reflects an infinite shortage cost for backordered sales, eliminating backordering for the product.

The fifth model assumption is that the capital assets purchased to expand production capacity all have the same depreciable life. That is, the property and buildings used for the expansion, all production equipment, and other support equipment are assumed to be depreciated over the same life. For certain types of expansions this assumption is not very appropriate. For other expansions, the assets will have approximately the same life. This assumption may later be relaxed once the optimal strategy is identified. Then, sensitivity analysis may be performed on the assumption to validate the solution, if the assumption appears to significantly affect the optimal decision strategy.

The sixth model assumption is that all capital assets will be depreciated by straight line depreciation. Straight line depreciation is to approximate the preferred accelerated depreciation methods because with straight line depreciation the percentage deduction allowed on an asset does not change over time. The depreciable life on investments is input as a model parameter. The capital equipment may be either a three, five, or a ten year property depending on the industry being studied.

The seventh assumption is that production capacity increases in a discrete fashion. That is, growth only occurs due to either increases in the labor force or as a result of the purchase of new production equipment (modernizations / new facilities). Note also that growth along the learning curve is not included in the model formulation. A possible extension to the model is to include learning in the model.

The eighth assumption is that interest rates obtained on capital equity and the interest rate obtained on corporate debt remain constant during the study period.

3.3. MODEL CONSTRUCTION

3.3.1 Notation

The model construction begins with the definition of material consumption relationships between the six levels of extraction, manufacture, and assembly. Recall the product tree where each system (end product) consists of the aggregation of inputs from the five lower levels of the tree. Let each of the six product levels be indexed using the following convention:

$H = \{h\}$ – raw materials (level 1)

$I = \{i\}$ – production material stocks (level 2)

$J = \{j\}$ – components (level 3)

$K = \{k\}$ – assemblies (level 4)

$L = \{L\}$ – products (level 5)

$M = \{m\}$ – systems (level 6)

The consumption of outputs from one product level by the next level are as follows:

$Z_{h,i}$ = the quantity of raw material h required to produce one unit of type i material stock,

$Z_{i,j}$ = the quantity of material stock i required to produce one type j component,

$Z_{j,k}$ = the number of component j required to produce one type k assembly,

$Z_{k,l}$ = the number of assembly k required to produce one type l product,

$Z_{l,m}$ = the number of product l required to produce one type m system. system.

The first subscript refers to the material input, the second subscript refers to the output product. It is assumed that consumption rates will remain constant over time. The above notation for material consumption is used for the remainder of this paper.

A second set of notation is required to define the labor consumption rates for manufacturing one unit of output at any product level. Let labor consumption be defined by:

$a_{p,r,q}$ = the required input of class q labor required to create one unit of output of product r at level p,

where there are four classes of labor q—skilled (q=1), unskilled (q=2), managerial (q=3), and technical (q=4), r is the specific product at level p to which the consumption rate applies. Note that the labor consumption rates are assumed to remain constant over time.

The third required notation set refers to the production capacity and production rates for the conversion processes (extracting, refining, assembling, etc.) Production rates are decision variables and are allowed to fluctuate over time. Define the production rate variables as:

$\pi_{p,r,t}$ = the production rate during time period t of output item r at level p.

The production rates found using the model will result from the derived product demands.

The production capacities have the same subscripts as the production rates and are decision variables in the model. The model solves for the optimal growth in production capacity over time so the capacity must be indexed for each time period. Define the production capacities for each product as:

$C_{p,r,t}$ = the production capacity making output r at product level p during time t.

Production capacity and production rates form the third set of variables. Initial production capacities are model parameters.

3.3.2 Constraint Sets

The capacity expansion problem is defined by seven constraint sets. Each set models one of the seven operational restrictions in the industrial base. The seven categories of constraint sets are:

1. Cascading product demands over time are derived from system demand.
2. Production must meet demand (no backorders allowed).
3. Production cannot exceed existing production capacity.
4. Plant capacity at any given time period equals original plant capacity plus plant expansions.
5. Production is restricted by the man-hours available to operate the plant.

6. A limited number of plant expansions are allowed. (Allows for minimum and maximum size of expansions--a policy variable.)
7. Plant expansions are restricted by capital resource availability.

Each constraint set is discussed separately in the next seven sections.

1) Deriving Product Demand

Product demand originates from consumers purchasing systems level products. Demand for systems can be defined by:

$d_{6,m,t}$ = the number of type m systems required at product tree level 6 during time interval t to meet consumer demand for systems.

Note that systems demands are not constant over time. Example: On a given month private and industrial consumers may purchase 65,000 automobiles and the next month only 45,000 automobiles. The demand parameters are therefore indexed over time. The demand for systems implies a demand on production for the other levels of the product tree. In particular, each copy of system m requires $Z_{l,m}$ of each product level good of type l. The total demand for product of type l at level 5 is then the sum over the set of systems which require $Z_{l,m}$ copies of product type l to make one copy of system m. Demand at level five is defined by:

$$d_{5,l,t} = \sum_{m} Z_{l,m} \, d_{6,m,t} \qquad \forall \; t,l \qquad (1)$$

Equation (1) would be correct if there were no time lags between when a manufacturing order is received and when the products are delivered to the customer. However, time lags do exist and so equation (1) must be altered to include time lags. Let the lag between levels l and m be defined as:

e_r = the number of time periods between when production of product type r begins and when the product is available for use at the next product level, where r = h, i, j, k, l to denote the product level of concern.

Note that both production and delivery lags are included in lag e_r. Given that product demands at each product tree level are derived from systems demands, and that the lag between each product tree level is e_r, it is possible to write the set of derived product demands. The derived demand equations are:

$$
\begin{aligned}
d_{5,l,t-e_l} &= \sum_m Z_{l,m}\, d_{6,m,t} \\
d_{4,k,t-e_l-e_k} &= \sum_l Z_{k,l}\, d_{5,l,t-e_l} \\
d_{3,j,t-e_l-e_k-e_j} &= \sum_k Z_{j,k}\, d_{4,k,t-e_l-e_k} + \text{substitutions} \\
d_{2,i,t-e_l-e_k-e_j-e_i} &= \sum_j Z_{i,j}\, d_{3,j,t-e_l-e_k-e_j} + \text{substitutions} \\
d_{1,h,t-e_l-e_k-e_j-e_i-e_h} &= \sum_i Z_{2,h,j}\, d_{2,i,t-e_l-e_k-e_j-e_i}
\end{aligned}
\tag{2}
$$

Note that the total time required to convert a raw material into a consumer product is then the sum of the time lags for all levels in the product tree.

Example of computing product demands: Demand for tanks creates a demand for tank engines, fuel systems, weapon systems, tank frames, air conditioners, and so forth. Demand for the input parts creates a demand for carburetors, armor plating, spark plugs, etc. Likewise, consumer demand for automobiles and replacement parts creates a demand for spark plugs. Demands for every item in the product tree are found in the same fashion. Figure 3.1 demonstrates how demand for any product can be derived from consumer demand for systems at product level 6. Figure 3.2 represents the time lag between the various levels in the product tree.

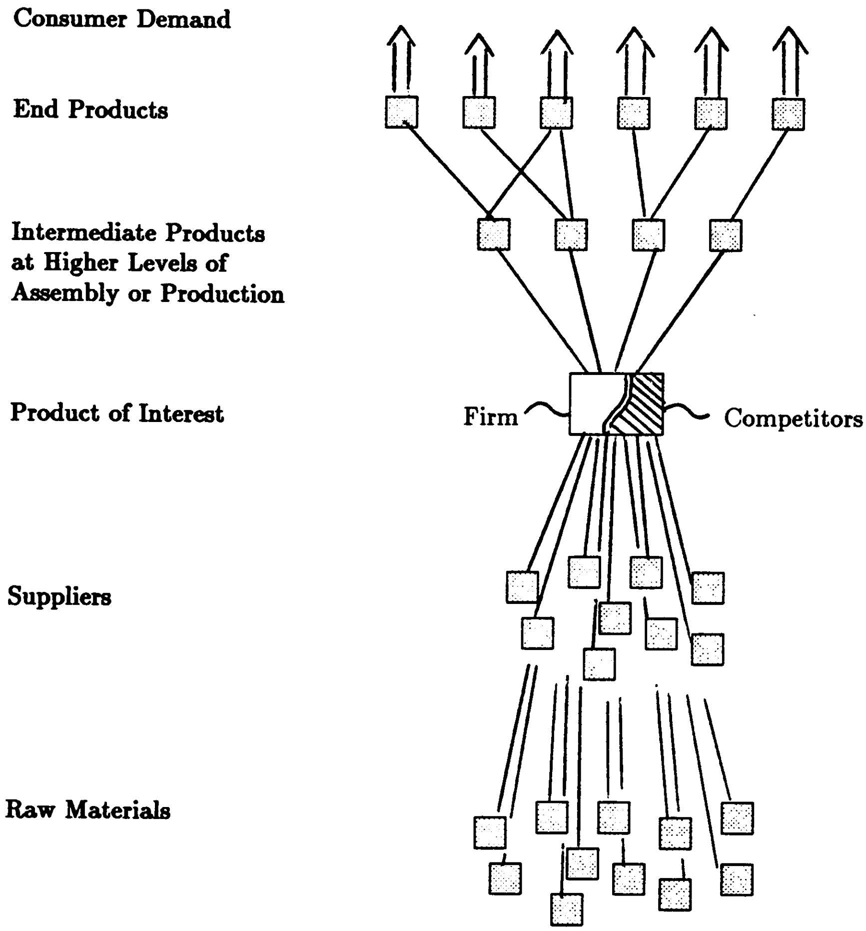


FIGURE 3.1 PRODUCT TREE APPLIED TO A SINGLE FIRM

PRODUCT TREE EXTENDED OVER TIME

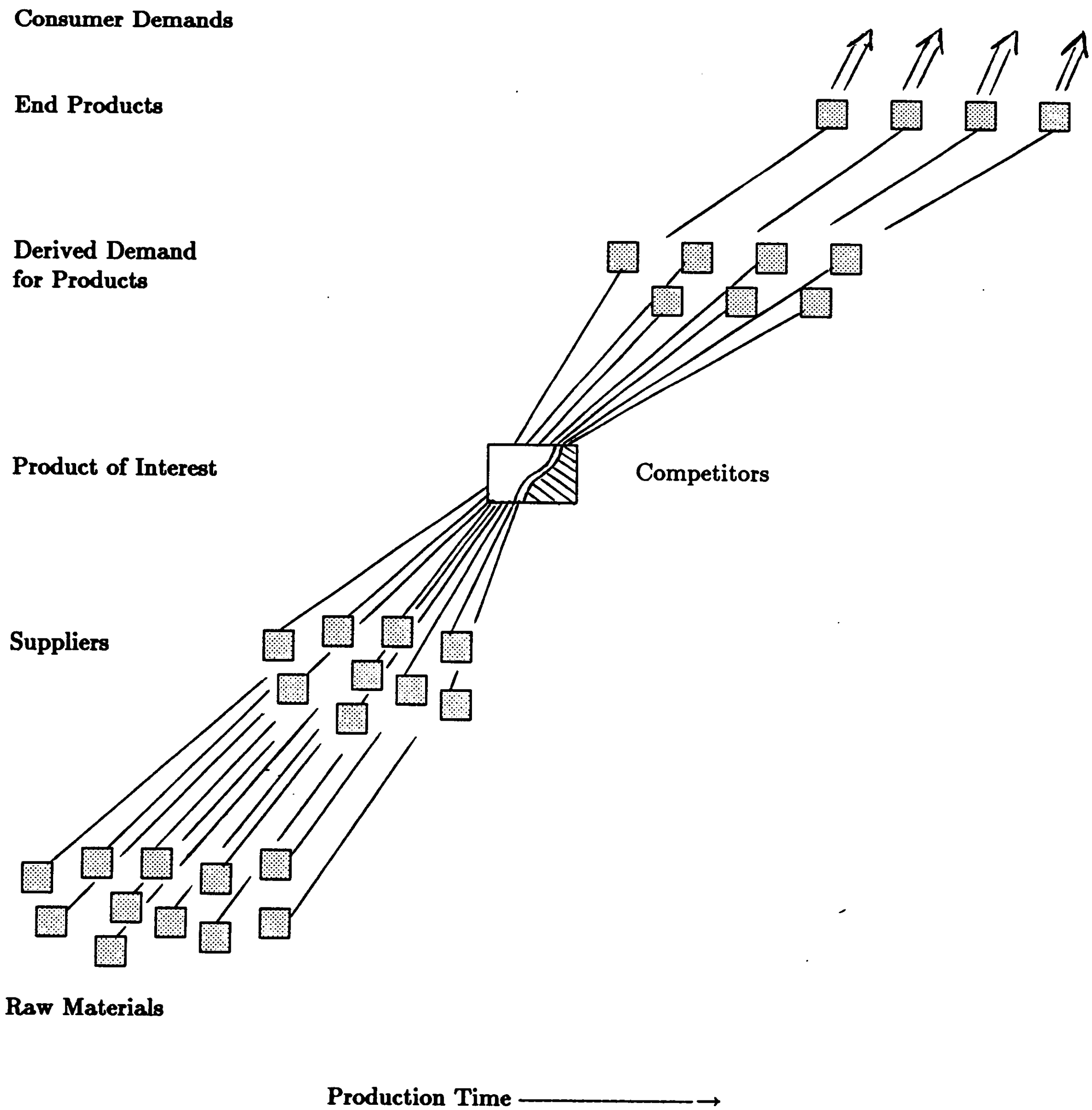


FIGURE 3.2 PRODUCT TREE EXTENDED OVER TIME

Two of the equations included in the above equation set contain additional quantities. The quantities are material substitutions at product levels two and three. Materials substitutions are considered as possible alternative sourcing strategies at the basic material stock and component levels. Define substitution variables by:

$s_{2,i_1,i_2,j}$ = the quantity substitution ratio for replacing material i_1 with material i_2 in the production of component j,

$s_{3,j_1,j_2,k}$ = the quantity substitution ratio for replacing component j_1 with component j_2 in the production of assembly k,

$\sigma_{2,i_1,i_2,j,t}$ = the decision variable indicating the extent to which the level two substitution (basic material stocks) will be used during period t,

$\sigma_{3,j_1,j_2,k,t}$ = the decision variable indicating the extent to which the level three substitution (components) will be used during period t,

The definitions for substitutions lead to the following forms for the component and assembly level demand equations:

$$\sum_k \left(Z_{j,k} \,(1-\sum_r \sigma_{3,j,r,k,t}) + \sum_r Z_{r,k}\, \sigma_{3,r,j,k,t}\, s_{3,r,j,k} \right) d_{4,k,t} = d_{3,j,t}$$
$$\sum_j \left(Z_{i,j} \,(1-\sum_r \sigma_{3,i,r,j,t}) + \sum_r Z_{r,j}\, \sigma_{3,r,i,j,t}\, s_{3,r,i,j} \right) d_{3,j,t} = d_{2,i,t} \qquad (3)$$

The derived demands for products d_{3jt} and d_{2it} are the sum of the standard product demands plus the uses for which the product is a substitute good minus those uses for which the product is substituted.

One further algebraic piece must be added to the derived demand equations to model capacity expansion of a single company--market share. Except in regulated monopolies and a few specific products (example: recently patented products), a company does not possess one hundred percent of the market for that particular product. Rather, several companies constitute the market's suppliers, and each holds a percent of the market. When applying the algebraic model to an individual company, the derived product demand must be multiplied by market share to determine the company's product demand. Let r^* be the company's product(s) at product level n. Define market share by:

$\mu_r =$ the estimated market share for product r^*,

where market share remains constant over time. The company's derived demand equation for product r^* would then be defined as:

$$\sum_r \mu_r \, Z_{r,r+1} \, d_{p+1,r+1} = d_{p,r^*,t} \qquad \forall \; p,r,t \qquad (4)$$

Note that the company of interest can exist at any level in the product tree and can make more than one product of interest.

A flaw in the definition of market share is that market share is not constant over time. New plants have lower unit cost (generally), more timely production, and higher product performance standards. Thus, a new facility may allow a company inroads into previously closed markets as well as increased sales in the current markets. If market share were a decision variable, the resulting derived demand equation would be non-linear in the derived demand and market share variables. Additionally, market share cannot be a decision variable because it is not subject to choice. That is, the model should not solve for the market share because at each time period market share is an unknown but fixed quantity. Instead, it is treated as an estimated value and is an input parameter. Market share is also assumed constant for the duration of the planning horizon. Example: If the firm manufactures spark plugs (assume spark plugs are a level 3 product), the demand equations for spark plugs would be:

$$d_{3,k,t-e_k-e_l-e_m} = \sum_k \mu_{4,k,l}\, Z_{4,k,l}\, d_{4,l,t-e_l-e_m}$$

where $\mu_{4,k,l}$ is the firm's estimated market share. All other demand equations would remain unchanged.

The demand equations are now complete. In section two below, the second set of constraint equations are developed.

2) Product Demand Satisfied Cannot Exceed Product Supply

Production must meet demand. That is, the accumulated production over every time period must equal or exceed the accumulated product demand over every time period. Note that product demand may be satisfied either by production during the time period or by inventories accumulated in prior periods. The set of equations representing per period production constraints on demand is defined by:

$$\pi_{p,r,t} + \text{inventories} \geq d_{p,r,t} \qquad \forall\ p,\ r,\ t \qquad (5)$$

and accumulated production greater than accumulated demand over time:

$$\sum_t \pi_{p,r,t} \geq \sum_t d_{p,r,t} \qquad \forall\ p,\ r,\ t \qquad (6)$$

where $\pi_{p,r,t}$ = the production rate during time period t of output item r at level p, and $d_{p,r,t}$ is the demand at product level p for product r at time t. The production rates should be determined by the model as a consequence of the derived demands. Equation (5) satisfies model assumption number four--that no backorders are allowed, and equation (6) requires production to meet demand throughout the planning horizon.

Note that the production constraint allows for the accumulation of inventory during periods of reduced demand. That is, the model allows the industrial base to build up product inventories over time in preparation for periods of higher product demand. Define the inventory variables by:

$V_{p,r,t-1}$ = the inventory level of product r, level p, at the end of the t-1th time period.

$V_{p,r,t}$ = the inventory level of product r, level p, at the end of the tth time period.

$\Delta_{p,r,t}$ = the safety stock required for product r, level p, at time t.

$V_{p,r,t-1}$ additional units are available during time t, plus the units manufactured during time t. All unsold units at time t, $V_{p,r,t}$, are then carried over to the time t+1. By including inventory variables in equations (6), the constraint equations on product demand become:

$$\pi_{p,r,t} + V_{p,r,t-1} \geq d_{p,r,t} + V_{p,r,t} \qquad \forall\ p,r,t \qquad (7)$$

where initial inventories $V_{p,r,0}$ are equal to current stockpiles of each product and are input as a model parameter. All other inventory variables are derived by the model. Safety stocks may be included in the model by setting simple lower bounds in the algorithmic solution to the model. The inclusion of safety stocks with simple lower bounds introduces the following restriction on inventory variables:

$$V_{p,r,t} \leq \Delta_{p,r,t} \qquad \forall\ p,r,t \qquad (8)$$

Note that the inventory term for safety stocks, $\Delta_{p,r,t}$, represents model parameters rather than decsion variables. If no safety stocks are desired in the model formulation, the equation set (8) should not be included in the model's set of constraint equations.

3) Production Capacity Constrains Production During Each Time Period

The production rate during any time period t cannot exceed the production capacity currently available for use. Define production capacity by:

$C_{p,r,t}$ = the production capacity available for making good r at product level p during period t.

$C_{p,r,0}$ = the production capacity available for making good r at product level p during the initial period t=0.

The initial capacity is known at time t=0. Production rates cannot exceed capacity at time t but can be expanded over time. The equations for capacity constraints become:

$$\pi_{p,r,t} \leq C_{p,r,0} + \sum_{t} \text{expansions} \qquad \forall\ p,r,t \qquad (9)$$

where the production rate at time t cannot exceed the original production capacity, plus the expansions in production capacity up to time t. Figure 3.3 depicts the breakdown of the manufacturing process into individual levels, starting from raw materials and ending with end product manufacture. The arrows represent derived demands; the boxes represent the production (manufacturing operations) at each level. Equations governing capacity expansion are developed in section 4.

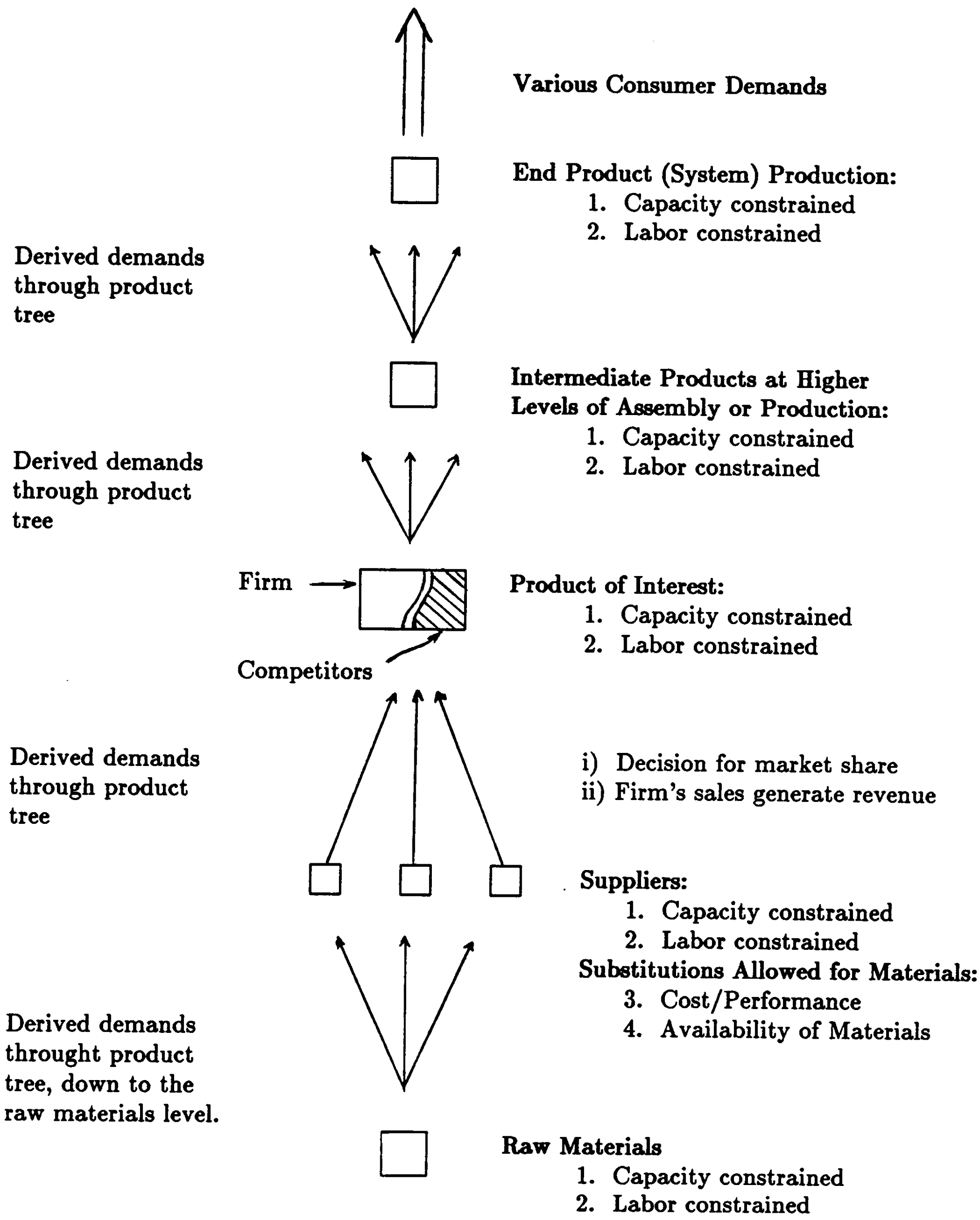


FIGURE 3.3 END PRODUCT BROKEN DOWN BY LEVEL

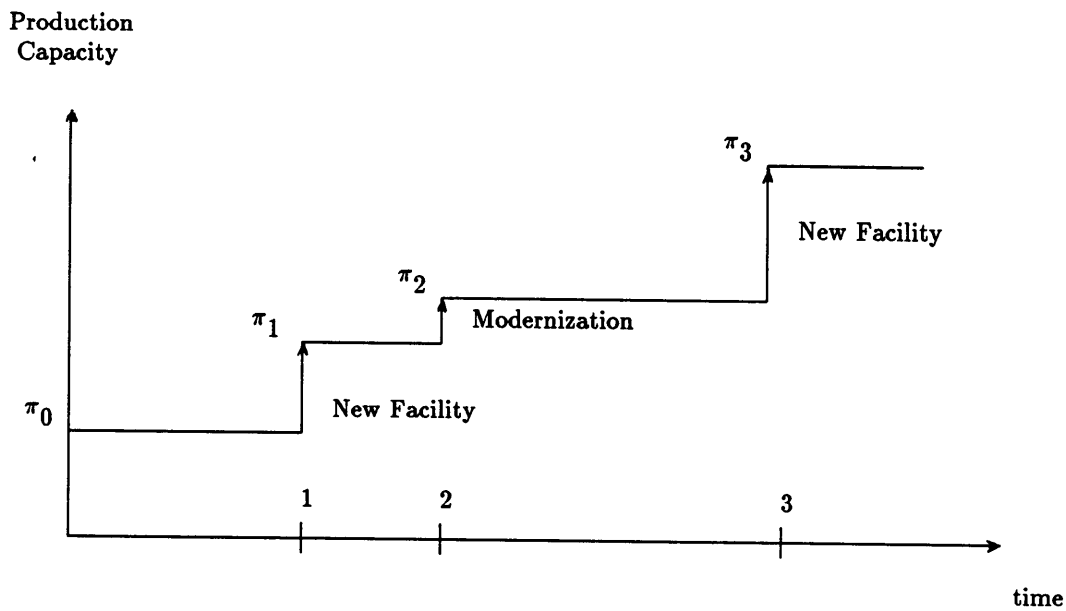
4) Production Capacity Equals Current Capacity Plus Future Capacity

In section 3, capacity at time t is defined as initial capacity plus capacity expansions in the interval $\tau \leq t$. Frisch described three methods to increase production capacity: through increased manpower, facilities modernizations, and construction of new facilities.[2] The labor constraints allow for productivity increases from increased manpower. Modernizations and new facilities construction are reflected in the following growth equation for production capacity:

$$C_{p,r,t} = C_{p,r,0} + \sum_{\tau \leq t+\theta} \delta_{p,r,\tau} \tag{10}$$

where $\delta_{p,r,t}$ is the additional capacity becoming available at time $t+\theta$. The index θ represents the time lag between when an expansion decision is made (construction begun) and when the facility is ready for production activity. The current capacity is then the sum of the original capacity and all capacity expansions made during the time $\tau \leq t+\theta$. Also note the discrete nature of the capacity expansions. Expansions occur at specific times. Two examples are as follows. As new equipment becomes operational, plant layouts are improved. A computer control system comes on-line. Capacity may increase due to progress along the learning curve, but the bulk of the capacity becomes available at the initiation of actual operation. Therefore, capacity expansions which result from learning effects are not treated in the model. Figure 3.4 gives an example of the discrete nature of capacity expansions, where two new facilities have been constructed and one plant modernization occurred in the interval $\tau \leq t+\theta$. The figure shows three capacity expansions over time, where the first and third expansions (π_1 and π_3) are new facilities, the second expansion (π_2) is a plant modernization, and π_0 is the original production capacity.

There also exist limitations on the size of any production expansion. Size limitations can be represented by upper and lower bounds on the expansion variable δ_{prt}. Expansions less



Expansions Over Time

FIGURE 3.4 CAPACITY EXPANDS IN DISCRETE INCREMENTS than size ϵ_1 can be considered operational decisions rather than strategic level decisions. Expansions greater than size ϵ_2 are considered unattractive due to the investment risks, corporate restrictions on project size, or other rationale. Let lower and upper bounds on thesize of the capital expansion δ_{prt} be defined as:

$\epsilon_{1,p,r}$ = lower bound on the capacity expansion of product r at level p during period t,

$\epsilon_{2,p,r}$ = upper bound on the capacity expansion of product r at level p during period t.

Define an 0/1 integer variable as follows:

$\rho_{p,r,t}$ = a 0/1 integer variable for all possible expansions related to p,r,t.

The equations bounding capacity expansions become:

$$\epsilon_{1,p,r} \ \rho_{p,r,t} \leq \delta_{p,r,t} \leq \epsilon_{2,p,r} \ \rho_{p,r,t} \qquad \forall p,r,t \qquad (11)$$

Note that the 0/1 variables bounding capacity expansions make the solution of the model more difficult. This set of constraints need only be implemented for the company of interest, and the model is then more easy to solve using integer programming methods.

When ρ is 1, the lower and upper bounds on the expansion will be ϵ_{1pr} and ϵ_{2pr}. When ρ is 0, the lower and upper bounds on the expansion will be zero and no expansion will occur at time t. Both ϵ_1 and ϵ_2 are input as parameters values for a given problem formulation.

5) Labor Requirements Constrain Production

The other important input to the model is the consumption of the labor resources. The labor inputs to each production process is defined as $a_{p,r,q}$. The labor resource can then be constrained in the same manner as the material resource. Define the labor constraint set as:

$$a_{p,r,q}\ \pi_{p,r,t} \leq A_{p,r,q,t} \qquad \forall p,r,t,q \qquad (12)$$

where the quantities of $A_{p,r,q,t}$ are defined as:

$A_{p,r,q,t}$ = the total quantity of class q labor available during period t for making output r at product level p,

$A_{p,r,q,0}$ = the initial quantity of class q labor available.

Production rates cannot exceed the amount of plant operation that can be supported by the labor force $A_{p,r,q,t}$. Figure 3.3 shows labor constraints at each stage (level) of the manufacturing process.

The actual labor resource allocation during any time period may differ from the initial labor pool value as a result of two circumstances. The first of the circumstances is growth in the labor force. The labor force can expand/contract over time as a result of employee training and retirement of older workers. Growth in the labor force is assumed to result from corporate training expenditures during period t. Define the increase in labor as:

$\kappa_{p,r,q,t}$ = the additional labor resource available at the beginning of period t.

The growth in the labor force can then be defined by:

$$A_{p,r,q,t} = A_{p,r,q,0} + \sum_{\tau \leq t} \kappa_{p,r,q,\tau} \quad + \text{substitutions} \qquad \forall p,r,t \qquad (13)$$

where the quantity of labor resource available to operate plants during time t is equal to the original labor resource plus the sum of all increases/decreases in the labor force up to the end of time t.

The second circumstance leading to an increase in the initial value of the labor pool is the substitution of one labor resource for another. Just as one material may be substituted for another, substitutions between the labor resources may increase the initial labor value. Examples of such substitutions are job switching, cross-job training programs, and salaried employees operating plants during wage disputes. Substitutions among labor resources are represented by letting:

$u_{p_1,r_1,q_1,p_2,r_2,q_2}$ = the quantity substitution ratio for using class q_2 labor that is normally assigned to the production of output r_2 at level p_2 in place of class q_1 labor for the production of output r_1 at level p_1, and

$\chi_{p_1,r_1,q_1,p_2,r_2,q_2,t}$ = the extent to which this substitution is used. More specifically, this is the proportion of the available labor resource that is so used during period t.

A special case for the variable χ is also used. When $p_1 = p_2$ and $r_1 = r_2$, χ represents the proportion of the labor resource that is "used as intended". Using these definitions, the equations for labor availability are described as follows:

$$\begin{aligned} A_{p_1,r_1,q_1,t} = \Big(A_{p_1,r_1,q_1,0} + \sum_{\tau \leq t} \kappa_{p_1,r_1,q_1,\tau}\Big) \chi_{p_1,r_1,q_1,p_2,r_2,q_2,t} \\ - \sum_{r_2} \chi_{p_1,r_1,q_1,p_2,r_2,q_2,t}\, u_{p_1,r_1,q_1,p_2,r_2,q_2} \\ + \sum_{r_2} \chi_{p_2,r_2,q_2,p_1,r_1,q_1,t}\, u_{p_2,r_2,q_2,p_1,r_1,q_1}. \end{aligned} \tag{14}$$

By defining the variable χ as the proportion of the labor available to support productive activity, the model allows for the labor substitutions to be used as needed to meet labor requirements. This approach imposes a need to constrain the values of the allocation variables, χ, to always sum to one. The constraints are:

$$\sum_{r_2} \sum_{q_2} \chi_{p_1,r_1,q_1,p_2,r_2,q_2,t} = 1.0 \qquad \forall\ t,r_1,q_1 \qquad (15)$$

so that the allocation of the labor resource to both usual and alternate tasks matches the available resource quantity.

Note that the substitution formulation accounts for two types of labor substitution. The first is the shift of people among labor classes of skilled, unskilled, technical, and managerial labor classes. The second is the shift of workers among various manufacturing facilities.

Finally, observe that equations (13) do not constrain labor allocation. They count it by allowing switching between labor classes and facilities. Equations (12) and (14) constitute the constraints on allocation. The net labor pool may expand (immigrants and high school graduates entering the labor pool) or contract (retirements and domestic casualties). These shifting employment patterns are captured by setting the sign of κ (expansion variable) as negative or positive, representing respectively a decrease or increase in the available labor resource.

6) Number of Plant Constructions and/or Modernizations Allowed

In the case of utility expansions and certain types of industrial expansions, only one capacity expansion (or some integer number of expansions) is desired during the planning horizon. Consider the utility industry. A single generating plant (a nuclear plant, for example) may be constructed to meet power needs for the next 20 years. The model should specify when the next plant should be constructed and what the plant's generating capacity should be. Therefore, the number of expansions may be constrained in the general algebraic model. This constraint may be eliminated if an unlimited number of expansions are allowed.

The number of expansions allowed can be limited to λ by using the 0/1 integer programming variable $\rho_{p,r,t}$. A value of 1 corresponds to an expansion decision at time t for product r at level p. The sum of the expansions over time can then constrained by a decision variable λ. Define:

$\lambda_{p,r}$ = the number of capacity expansions allowed for making product r at level p.

Two alternatives exist for restricting the number of capacity expansions--require a fixed number of expansions, and allow up to some maximum number of expansions during the planning horizon. Define the number of expansion allowed by:

$$\sum_{t=1}^{horizon} \rho_{p,r,t} \leq \lambda_{p,r} \qquad \forall\ p,r \qquad (16)$$

Note that by replacing the "$\leq$" with an "$=$" satisfies the first case (a fixed number of expansions during the horizon). This completes the equations governing capacity expansion. Equations governing capital availability are developed in section 7 below.

7) Constraints on Available Capital

Capital availability limits the size of the production expansions $\delta_{p,r,t}$. Before a corporation can expand production, the corporation must secure financing for the project. The corporation may invest yearly revenues to provide funds for expansions or secure a variety of outside financing through stock sales, bond issues, and other forms of debt. Capital availability in any time period is then the sum of capital equity and corporate borrowing.

Assume that a corporation can either undertake an expansion in a given period, repay previous borrowing, or invest the period's revenues. Then capital availability is the sum of net revenues, current borrowing, and previous lending, minus past borrowing and current lending.

The set of capacity expansion constraints need only be implemented for one firm at one level. That is, the set of capital constraints is only included for product r at level p which the company of interest manufactures.

Define the following variables:

$\zeta_{p,r}$ = the cost of a capital expansion for product r at level p,

in dollars per unit of capacity expansion,

$Q_{p,r,t}$ = the amount of funds available for product r at level p,

during time t (capital availability),

$Q_{p,r,0}$ = the initial budget for product r at level p at time 0.

The size of the expansion $\delta_{p,r,t}$ is constrained by the financing currently available for the project. By multiplying the cost of the expansion and the size of the expansion (in production units), the expansion can then be constrained by the availability of the capital resource. Define capital resource constraints as:

$$\zeta_{p,r}\, \delta_{p,r,t} \leq Q_{p,r,t} \qquad (17)$$

where the capital availability at time t is equal to 1) the corporation's net revenues at time t, plus 2) the previous time period's lending and borrowing, plus 3) the current period's lending and borrowing. In addition, define the following derived cash flow variables:

$F_{p,r,t}$ = The before-tax cash flow for product r at level p during time t. The cash flow is obtained by subtracting all wage, material and operating expenses from the period's total revenues which results in net corporate earnings. $F_{p,r,t}$ is in units of dollars per time period.

$N_{p,r,t}$ = The after-tax cash flow for product r at level p during time t. the after-tax cash flow is obtained by subtracting interest and depreciation deductions from the before-tax cash flow to compute taxable income, multiplying the taxable income by the income tax, and then subtracting income taxes from the net earnings. $N_{p,r,t}$ is in units of dollars per time period.

$L_{p,r,t}$ = amount of lending for product r at level p during time t in dollars per time period,

$B_{p,r,t}$ = amount of borrowing for product r at level p during time t in dollars per time period,

r_L = interest rate received when lending funds,

r_B = interest rate paid when borrowing funds,

N = planning horizon,

$MARR_{p,r}$ = Minimum acceptable rate of return on investment.

$I_{p,r}$ = the Corporate income tax for product r at level p.

Assuming no borrowing or lending prior to the current time, t = 0, the capital availability during time period t may be defined as:

$$Q_{p,r,0} = N_{p,r,0} + B_{p,r,0} - L_{p,r,0} \qquad \forall\ p,r,t=0 \qquad (18)$$

$$Q_{p,r,t} = N_{p,r,t} + B_{p,r,t} - B_{p,r,t-1} - L_{p,r,t} + (1 + r_L (1 - I_{p,r})) L_{p,r,t-1} \qquad \forall\ p,r,\ 0< t< N$$

$$Q_{p,r,N} = N_{p,r,N} - B_{p,r,N-1} - L_{p,r,t} + (1 + r_L(1 - I_{p,r})) L_{p,r,N-1} \qquad \forall\ p,r,t=N$$

The above equations allow for the corporation to either expand capacity at time t, invest the net revenues for future projects, or repay debts. The cash flow equation for time period t does not allow for any borrowing past the planning horizon. By not allowing any borrowing during the last time period the company is required to repay all debts by time t. The imposition of no borrowing during the last time period forces the model to find the discounted payback period at the MARR for all capital expansions. If borrowing is allowed during the last time period, the middle constraint has bounds $0 < t \leq N$, and the third cash flow constraint is eliminated. By allowing borrowing in the last period, capacity expansions in the later time periods of the planning horizon will be more likely when borrowing is allowed at time t than when no borrowing is allowed at time t.

Furthermore, the company may impose budget limitations on a given expansion due to the risks inherent in large plant constructions and/or modernizations. Define the maximum budget for any given expansion as:

$\phi_{p,r}$ = the maximum allowable capital budget for product r at level p, where ϕ is a parameter input in the model.

The equation governing the size of the capital expansion becomes:

$$0 \leq Q_{p,r,t} \leq \rho_{p,r,t}\, \phi_{p,r} \qquad \forall\ p,r,t \tag{19}$$

If the production expansion occurs during time t, the capital available during time t is $Q_{p,r,t}$, otherwise $Q_{p,r,t} = 0$. That is, if no expansion occurs at time t, revenues are either 1) invested or 2) debts are repaid during period t, with no expansion costs being incurred. Note that if the corporation does not restrict the project budget size, let $\phi_{p,r}$ take on a value BIG M, to ensure that capital must be re-invested or used to pay off corporate liabilities if no expansion occurs at time t. Figure 3.5 demonstrates the cash flows over a sample planning horizon, where three expansions (two new facilities, and one modernization) have been undertaken. The figures shows how capital costs are incurred θ_i time units before the expansion is completed and actually becomes operational. The figure also shows that capital costs are incurred at the initiation of the capacity expansions. Expansion costs will only be incurred during time periods where the 0/1 varialbe $\delta_{p,r,t}$ assumes a value of one.

<u>Computing the Revenues During Time t</u>:

The before-tax income in any given period is the sum of product sales, O&M costs, taxes, inventory holding costs, material and labor costs, and overhead costs (applied at a percentage of direct labor). Define the cost/revenue variables by:

$P_{p,r}$ = the price in dollars of the corporation's product r at level p,

$M_{p,r,t}$ = the operating and maintenance costs per production unit of product r at level p during time t,

$P_{p-1,r}$ = the price in dollars for all materials/intermediate products r at level p-1 used as inputs for product r at level p,

$D_{p-1,r,t}$ = the derived demand for input products used to make product r at level p during time t,

$HC_{p,r,t}$ = the inventory holding cost for unsold inventories of product r at level p at the end of time period t,

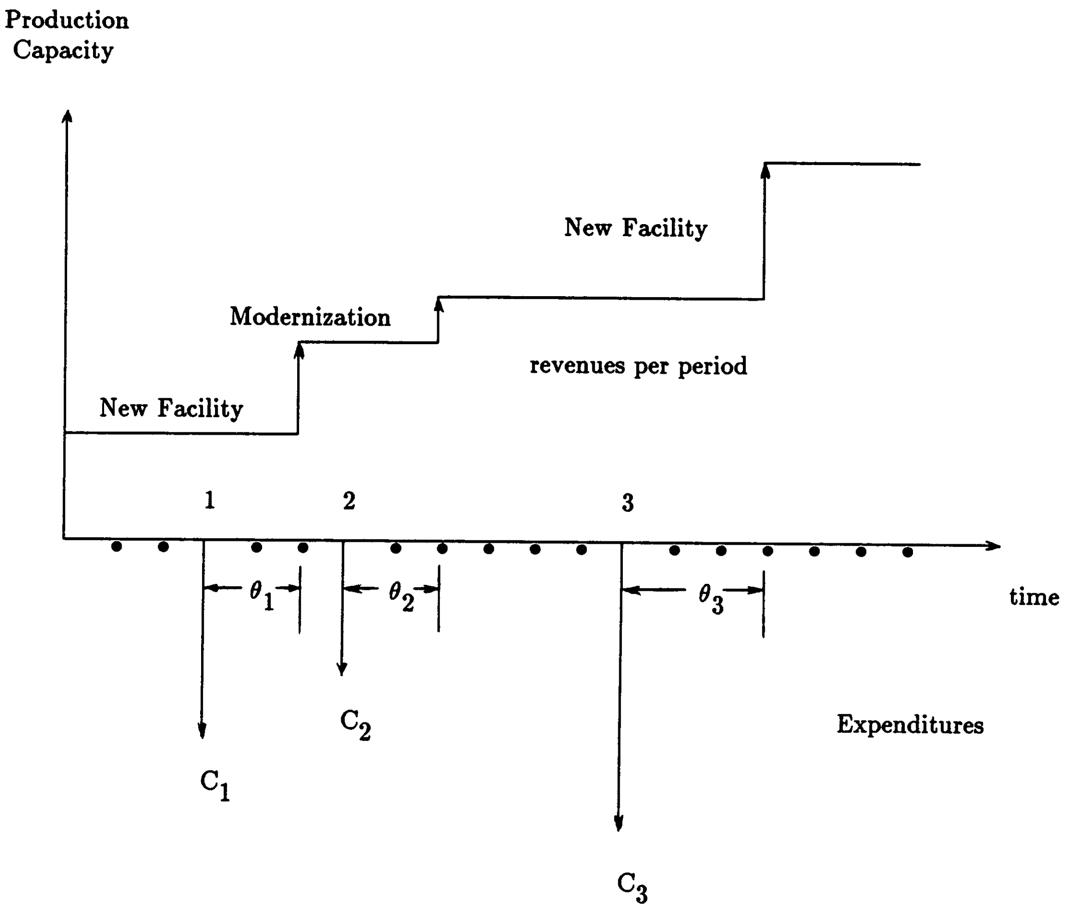


FIGURE 3.5 CASH FLOW FOR CAPITAL INVESTMENTS

$W_{p,r,q}$ = the direct wages paid to employees per hour for product r at level p in dollars per hour,

$O_{p,r}$ = the overhead application rate (salaried employees, administrative costs, all other indirect expenses) per direct labor dollar of product r at level p,

The equation for before-tax revenues for product r at level p during period t becomes:

$$F_{p,r,t} = P_{p,r}\, D_{p,r,t} - M_{p,r,t}\, \pi_{p,r,t} - \sum_{r} P_{p-1,r}\, D_{p-1,r,t} - HC_{p,r,t}\, V_{p,r,t} - \left(\sum_{q} W_{p,r,q}\, a_{p,r,\tau}\, \pi_{p,r,t}\right)(1 + O_{p,r}) \tag{20}$$

where $\pi_{p,r,t}$ is the production of product r at level p during time t, and $V_{p,r,t}$ is the number of product r at level p remaining in inventory at the end of period t. The cash flow generated from product sales each time period is then used to compute the after-tax cash flow of the firm.

The taxable income of the firm is found by subtracting interest payments and the depreciation incurred during the period from the net revenues during the period. The firm's net income is then given by revenues minus taxes, interest payments, and expenditures. Define the depreciation variables in the model as:

$\Psi_{p,r}$ = Depreciable life for capital investments at level p for product r in units of percent income deduction per time period t,

$\Gamma_{p,r}$ = Proportion of capital investment spent on depreciable items,

where the depreciable life is assumed constant for all investments. The equation for after-tax revenues for product r at level p during period t becomes:

$$N_{p,r,t} = F_{p,r,t} \quad - \left(F_{p,r,t} - \Gamma_{p,r} \frac{Q_{p,r,0}}{\Psi_{p,r}}\right) I_{p,r} \qquad t{=}0 \quad (21)$$

$$N_{p,r,t} = F_{p,r,t} \quad - \left(F_{p,r,t} - r_B\, B_{p,r,t\text{-}1} - \sum_{\substack{\tau=t\text{-}\Psi_{p,r} \\ \tau\geq 0}}^{t} \Gamma_{p,r} \frac{Q_{p,r,\tau}}{\Psi_{p,r}}\right) I_{p,r}$$

$$- r_B\, B_{p,r,t\text{-}1} \qquad t>0$$

where the company's after-tax income, $N_{p,r,t}$, is equal to before-tax income, $F_{p,r,t}$, minus corporate taxes, and also minus interest paid on corporate debt. Taxible income, shown inside the brackets, is equal to before-tax income minus depreciation deductions and interest payments (note that there are no interest deductions in the first equation since there is no borrowing previous to time 0). Taxible income is multiplied by $I_{p,r}$, the corporate tax rate, to deterimine the taxes paid by the corporation. The interest paid in any period is given by r_B * $B_{p,r,t\text{-}1}$, where r_B is the interest rate paid on corporate debt and $B_{p,r,t\text{-}1}$ is the amount of funds borrowed during the previous period. Note that the interest terms occuring inside the brackets in the second equation represent the tax deductions on interest payments, and the interest terms outside the brackets represent interest payments during the given time period. The depreciation deduction during any time period t is equal to the striaght-line depreciation deduction on expansion expenditures multiplied by the factor $\Gamma_{p,r}$, the proportion of the expenditure which is depreciable--that is, the proportion of the expenditure which was spent on machinery, buildings, etc. The summation in the second equation sums up depreciation deductions for previous capacity expansions. Depreciation deductions occur for each expenditure over the depreciable life of the investments, $\Psi_{p,r}$. The summation has lower limits of $t-\Psi_{p,r}$ and $t\geq 0$ and an upper limit of t, the current time. Example: If the depreciable life is 5 years, the units are in business quarters, and the current time is the first quarter in the fourth year of the planning horizon, the summation should sum from t=0 to t=13. If the current time is the fourth business quarter in the eight year of the planning horizon, the summation should sum from t=12 to t=32. The after-tax income is then used in the capital budgeting equations, as developed previously.

The next step in the analytical model development is to formulate LP objective functions for the various optimization schemes.

3.3.3 Formulation of the LP Objective Function

The final element of the model of the capacity expansion process is to define an appropriate objective function. Whatever the objective function, the motivation for the problem is to identify the best possible expansion scheme. Three objectives have been identified which will do this. The one that seems the most applicable to the capacity expansion problem is to maximize the present worth of the corporation. The present worth is found by discounting all future cash flows to the present, and the objective function is:

$$\text{maximize} \quad \sum_{t=0}^{N} (1 + \text{MARR}_{p,r})^{-t} \, (N_{p,r,t} - Q_{p,r,t}) \tag{22}$$

Note that the per period net worth is the sum of net revenue minus borrowing plus lending. The per period net worths are then discounted back to the present at the interest rate obtained for loaned funds because the rate for loaned funds is more reflective of the actual market rate of interest.

Two other possible objectives are to maximize revenues while minimizing product inventories and to maximize growth over the planning horizon.

3.4 SUMMARY OF THE PROPOSED MODEL

Given the choice of the objective function, the entire capacity expansion model may be written as:

MAXIMIZE: $$\sum_{t=0}^{N} (1 + MARR_{p,r})^{-t} (N_{p,r,t} - Q_{p,r,t})$$

SUBJECT TO:

Demand:

$$d_{5,l,t-e_l} = \sum_m Z_{l,m}\, d_{6,m,t}$$

$$d_{4,k,t-e_l-e_k} = \sum_l Z_{k,l}\, d_{5,l,t-e_l}$$

$$d_{3,j,t-e_l-e_k-e_j} = \sum_k \Big(Z_{j,k} (1-\sum_r \sigma_{3,j,r,k,t}) + \sum_r Z_{r,k}\, \sigma_{3,r,j,k,t}\, s_{3,r,j,k} \Big) d_{4,k,t-e_l-e_k}$$

$$d_{2,i,t-e_l-e_k-e_j-e_i} = \sum_j \Big(Z_{i,j} (1-\sum_r \sigma_{3,i,r,j,t}) + \sum_r Z_{r,j}\, \sigma_{3,r,i,j,t}\, s_{3,r,i,j} \Big) d_{3,j,t-e_l-e_k-e_j}$$

$$d_{1,h,t-e_l-e_k-e_j-e_i-e_h} = \sum_i Z_{2,h,j}\, d_{2,i,t-e_l-e_k-e_j-e_i}$$

Production Capacity:

$$\pi_{1,r,t} + V_{1,r,t-1} \geq d_{1,r,t} + V_{1,r,t} \qquad \forall\ r,t$$

$$\pi_{2,r,t} + V_{2,r,t-1} \geq d_{2,r,t} + V_{2,r,t} \qquad \forall\ r,t$$

$$\pi_{3,r,t} + V_{3,r,t-1} \geq d_{3,r,t} + V_{3,r,t} \qquad \forall\ r,t$$

$$\pi_{4,r,t} + V_{4,r,t-1} \geq d_{4,r,t} + V_{4,r,t} \qquad \forall\ r,t$$

$$\pi_{5,r,t} + V_{5,r,t-1} \geq d_{5,r,t} + V_{5,r,t} \qquad \forall\ r,t$$

$$\pi_{6,r,t} + V_{6,r,t-1} \geq d_{6,r,t} + V_{6,r,t} \qquad \forall\ r,t$$

$$V_{6,r,t} \leq \Delta_{6,r,t} \qquad \forall\ r,t$$

$$V_{5,r,t} \leq \Delta_{5,r,t} \qquad \forall\ r,t$$

$$V_{4,r,t} \leq \Delta_{4,r,t} \qquad \forall\ r,t$$

$$V_{3,r,t} \leq \Delta_{3,r,t} \qquad \forall\ r,t$$

$$V_{2,r,t} \leq \Delta_{2,r,t} \qquad \forall\ r,t$$

$$V_{1,r,t} \leq \Delta_{1,r,t} \qquad \forall\ r,t$$

$$\pi_{1,r,t} \leq C_{1,r,t} \qquad \forall\ r,t$$

$$\pi_{2,r,t} \leq C_{2,r,t} \qquad \forall\ r,t$$

$$\pi_{3,r,t} \leq C_{3,r,t} \qquad \forall\ r,t$$

$$\pi_{4,r,t} \leq C_{4,r,t} \qquad \forall\ r,t$$

$$\pi_{5,r,t} \leq C_{5,r,t} \qquad \forall\ r,t$$

$$\pi_{6,r,t} \leq C_{6,r,t} \qquad \forall\ r,t$$

$$C_{1,r,t} = C_{1,r,0} + \sum_{\tau \leq t+\theta} \delta_{6,r,\tau} \qquad \forall\ r,t$$

$$C_{2,r,t} = C_{2,r,0} + \sum_{\tau \leq t+\theta} \delta_{5,r,\tau} \qquad \forall\ r,t$$

$$C_{3,r,t} = C_{3,r,0} + \sum_{\tau \leq t+\theta} \delta_{4,r,\tau} \qquad \forall\ r,t$$

$$C_{4,r,t} = C_{4,r,0} + \sum_{\tau \leq t+\theta} \delta_{3,r,\tau} \qquad \forall\ r,t$$

$$C_{5,r,t} = C_{5,r,0} + \sum_{\tau \leq t+\theta} \delta_{2,r,\tau} \qquad \forall\ r,t$$

$$C_{6,r,t} = C_{6,r,0} + \sum_{\tau \leq t+\theta} \delta_{1,r,\tau} \qquad \forall\ r,t$$

$$\epsilon_{1,1,r}\ \rho_{1,r,t} \leq \delta_{1,r,t} \leq \epsilon_{2,1,r}\ \rho_{1,r,t} \qquad \forall\ r,t$$

$$\epsilon_{1,2,r}\ \rho_{2,r,t} \leq \delta_{2,r,t} \leq \epsilon_{2,2,r}\ \rho_{2,r,t} \qquad \forall\ r,t$$

$$\epsilon_{1,3,r}\ \rho_{3,r,t} \leq \delta_{3,r,t} \leq \epsilon_{2,3,r}\ \rho_{3,r,t} \qquad \forall\ r,t$$

$$\epsilon_{1,4,r}\ \rho_{4,r,t} \leq \delta_{4,r,t} \leq \epsilon_{2,4,r}\ \rho_{4,r,t} \qquad \forall\ r,t$$

$$\epsilon_{1,5,r}\ \rho_{5,r,t} \leq \delta_{5,r,t} \leq \epsilon_{2,5,r}\ \rho_{5,r,t} \qquad \forall\ r,t$$

$$\epsilon_{1,6,r}\ \rho_{6,r,t} \leq \delta_{6,r,t} \leq \epsilon_{2,6,r}\ \rho_{6,r,t} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{1,r,t} \leq \lambda_{1,r} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{2,r,t} \leq \lambda_{2,r} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{3,r,t} \leq \lambda_{3,r} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{4,r,t} \leq \lambda_{4,r} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{5,r,t} \leq \lambda_{5,r} \qquad \forall\ r,t$$

$$\sum_{t=1}^{N} \rho_{6,r,t} \leq \lambda_{6,r} \qquad \forall\ r,t$$

Labor:

$$a_{1,r,q}\ \pi_{1,r,t} \leq A_{1,r,q,t} \qquad \forall\ q,r,t$$

$$a_{2,r,q}\ \pi_{2,r,t} \leq A_{2,r,q,t} \qquad \forall\ q,r,t$$

$$a_{3,r,q}\ \pi_{3,r,t} \leq A_{3,r,q,t} \qquad \forall\ q,r,t$$

$$a_{4,r,q}\ \pi_{4,r,t} \leq A_{4,r,q,t} \qquad \forall\ q,r,t$$

$$a_{5,r,q}\ \pi_{5,r,t} \leq A_{5,r,q,t} \qquad \forall\ q,r,t$$

$$a_{6,r,q}\ \pi_{6,r,t} \leq A_{6,r,q,t} \qquad \forall\ q,r,t$$

$$A_{1,r,q,t} = A_{1,r,q,0} + \sum_{\tau \leq t} \kappa_{1,r,q,\tau} \qquad \forall\ q,r,t$$

$$A_{2,r,q,t} = A_{2,r,q,0} + \sum_{\tau \leq t} \kappa_{2,r,q,\tau} \qquad \forall\ q,r,t$$

$$A_{3,r,q,t} = A_{3,r,q,0} + \sum_{\tau \leq t} \kappa_{3,r,q,\tau} \qquad \forall\ q,r,t$$

$$A_{4,r,q,t} = A_{4,r,q,0} + \sum_{\tau \leq t} \kappa_{4,r,q,\tau} \qquad \forall\ q,r,t$$

$$A_{5,r,q,t} = A_{5,r,q,0} + \sum_{\tau \leq t} \kappa_{5,r,q,\tau} \qquad \forall\ q,r,t$$

$$A_{6,r,q,t} = A_{6,r,q,0} + \sum_{\tau \leq t} \kappa_{6,r,q,\tau} \qquad \forall\ q,r,t$$

Capital (Corporation of Interest):

$$0 \leq \zeta_{p,r}\, \delta_{p,r,t} \leq Q_{p,r,t}$$

$$Q_{p,r,0} = N_{p,r,0} + B_{p,r,0} - L_{p,r,0}$$

$$Q_{p,r,t} = N_{p,r,t} + B_{p,r,t} - (1 + r_B)\, B_{p,r,t-1} - L_{p,r,t} + (1 + r_L(1 - I_{p,r}))\, L_{p,r,t-1}$$

$$Q_{p,r,N} = N_{p,r,N} - (1 + r_B)\, B_{p,r,N-1} + (1 + r_L(1 - I_{p,r}))\, L_{p,r,N-1}$$

$$0 \leq Q_{p,r,t} \leq \rho_{p,r,t}\, \phi_{p,r}$$

$$F_{p,r,t} = P_{p,r}\, D_{p,r,t} - M_{p,r,t}\, \pi_{p,r,t} - \sum_{r} P_{p-1,r}\, D_{p-1,r,t} - HC_{p,r,t}\, V_{p,r,t} - \Big(\sum_{q} W_{p,r,q}\, a_{p,r,\tau}\, \pi_{p,r,t}\Big)(1 + O_{p,r})$$

$$N_{p,r,t} = F_{p,r,t} - \left(F_{p,r,t} - \Gamma_{p,r} \frac{Q_{p,r,0}}{\Psi_{p,r}}\right) I_{p,r} \qquad t=0$$

$$N_{p,r,t} = F_{p,r,t} - \left(F_{p,r,t} - r_B\, B_{p,r,t-1} - \sum_{\substack{\tau=t-\Psi_{p,r}\\ \tau\geq 0}}^{t} \Gamma_{p,r} \frac{Q_{p,r,\tau}}{\Psi_{p,r}}\right) I_{p,r} - r_B\, B_{p,r,t-1} \qquad t>0$$

The model is large, but software algorithms exist which will handle the model requirements. The LINDO[8] software will be used for the computational requirements of this paper.

CHAPTER 4
MODEL IMPLEMENTATION

4.1. INTRODUCTION

In Chapter 4, the data required by the model are discussed. In Section 4.2, the collection of data pertinent to the problem is described, in Section 4.3 possible simplifying assumptions and size reduction strategies are presented, and in Section 4.4 computer software and hardware limitations are discused. The limitations and data requirements are then summarized.

4.2. LISTING OF DATA REQUIREMENTS

The data require by the economic model are quite extensive. The model demonstrates the synergism of micro-economic data and macro-economic solutions to capacity expansion problems. That is, very detailed production data, labor data, and materials requirements for a system of products are used to answer such questions as, "When shall new manufacturing facilities be built?", and "How large should plant modernizations be?" As a result of this synergism, the micro-economic data requirements for each item in the product tree model are extensive. Since the model requires data on all systems that use the product of interest either directly or indirectly as material inputs, a large quantity of product tree data is required.

In addition to requiring a large quantity of data, the data is not necessarily easily accessible to corporate analysts. In fact, the data may not even exist. In order to obtain information on productive capacities, labor consumption rates, and labor availabilities, the analyst must secure the cooperation of the manufacturers to obtain data on the items in the product tree. If such data cannot be acquired, the analyst must estimate this data. The data inputs required for each item in the product tree are shown in Table 4.1.

Data such as the material consumption rates, production capacities, and labor

TABLE 4.1 INPUT DATA REQUIRED BY THE MODEL

General Data

- Length of Time Period
- Length of Model Run
- Increase in System Demand
- Systems Using the Product of Interest
- Product Trees for Systems

Data Required for Every Item in the Product Tree

- Production Units (ex. pounds/month, units/month)
- Material Consumption Rates
- Initial Inventories
- Time Lag between Production Levels
- Safety Stock Levels
- Current Productive Capacities
- Percent Expansion/Contraction of Productive Capacity over Time
- Labor Consumption Data:
  - Unskilled Labor Consumption Rate
  - Skilled Labor Consumption Rate
  - Managerial Labor Consumption Rate
  - Technical Labor Consumption Rate
- Labor Availabilities:
  - Availability of Unskilled Labor
  - Availability of Skilled Labor
  - Availability of Managerial Labor
  - Availability of Technical Labor
- Labor Substitutions:
  - Allowed Substitutions
  - Percent Utilization of Substitutions
  - Substitution Ratios

Required for System Level Items

- Consumer/Industrial Demand for Systems in Each Time Period

Required for Component and Assembly Level Items

- Allowed Material Substitutions
- Percent Utilization of Substitutions
- Substitution Ratios

consumptions can be obtained if the analyst is familiar with the organization which manufactures the product. Furthermore, the analyst must select the production units for each item. If the analyst has an idea of the production rates for the individual products, choosing the appropriate production units and lags between production levels will facilitate the selection of production units. Selection of the time units for the model depends primarily on the desired planning horizon--months or quarters are two likely choices. The time lags between production levels represent the time to manufacture products and ship the products to the next level of production (comparable to the time between placing an order and receiving the order). Data such as the demand for final systems will probably be forecast by someone other than the analyst and can be treated as model input parameters.

Some of the cost data, such as the cost of material inputs, is readily available to the analyst. The interest rates--on loaned as well as borrowed funds--should be fairly easy to obtain. The interest rates obtained on borrowed funds and on loaned funds represent the time value of money based on market interest rates. Specifically, bond market rates and the prime lending rate for blue-chip corporations are good initial estimates of the long and short term rates. Cost data for wages, overhead application rates, current operating expenses, current maintenance costs, and inventory holding costs should be available through sources within the company. The cost per unit of capacity expansion is more difficult to obtain. Estimates from the accounting and engineering departments should provide guidelines to estimate this cost. Since cost of expansion is an estimated cost and is subject to a high amount of variability, it should be varied over several model runs to provide sensitivity information on the effect the cost has on times and sizes of production expansions. Also of interest in the final analysis is the sensitivity of the optimal solution (maximum sales, PW, or growth) to the cost of expansion. Other input parameters are the minimum/maximum expansion size allowed and the number of allowed expansions. The required cost data for the product of interest are summarized in Tables 4.2 and 4.3.

Once the data needs for the model are defined, the data base for the model can be built by collecting the required data for each item in the product tree and the cost/expansion data for the company of interest. Building the data base for the model can be divided into seven steps.

The seven steps are as follows:

1) Select the product to be studied.
2) Identify all of the product markets (final systems) the firm supplies.
3) Develop the product trees for each of the final systems.
4) Gather material consumption, labor consumption, production capacity, and production expansion data on each product in the product tree.
5) Gather cost data for the firm of interest.
6) Identify substitute goods (if applicable).
7) Select/estimate model parameters such as market share and interest rates, maximum and minimum expansion sizes, and the number of expansions.

Once all the data are collected and stored electronically, the next step in model implementation is to reduce the size of the problem by focusing on a specific section of the product tree. If the problem is not restricted to a subset of the product tree the model grows too large to be easily solved. If desired, Each section of the product tree can be studied and the results synergized into a single expansion strategy. Strategies to reduce the size of the model are given in Section 4.3 (Model Simplification).

TABLE 4.2 COST DATA FOR THE PRODUCT OF INTEREST

Market Share
Product's Selling Price (dollars per production unit)
Cost of Materials (dollars per production unit)
Operating & Maintenance Cost (dollars/unit)
Holding Cost (dollars/unit)
MARR (Minimum Acceptable Rate of Return)
Wages:
Unskilled Labor
Skilled Labor
Managerial Labor
Technical Labor
Overhead Application Rate (percent of direct labor)
Corporate Income Tax
Maximum Budget size
Cost of Expansion Per Unit of Capacity Increase

TABLE 4.3 CAPACITY EXPANSION DATA

Minimum Capacity Expansion
Maximum Capacity Expansion
Maximum Number of Expansions
Payback Required (Y/N)

4.3 MODEL SIMPLIFICATIONS AND SIZE REDUCTION TECHNIQUES:

As seen in section 4.2, the data base for a given set of final systems which utilize an intermediate product as a material input can be of immense proportions. The problems in simply identifying the product trees are significant. Once the product trees are developed, collecting data for material resource utilization, production capacities, labor consumption rates, and labor availabilities from every industry included in the product tree is a sizeable task, providing firms are willing to share such data with the model analyst. Furthermore, the set of constraint equations for the complete model when examining a reasonable time horizon would contain hundreds of thousands of variables and constraint equations. Current mainframe LP languages accessible to most corporate planners cannot handle such a data set. The data set must therefore be compressed so as to reduce it to a tractable working size.

The size constraints of the model are not new to econometric models. The general model examines macro-econometric problems with micro-economic data and computations. The result of the combined macro-micro modeling approach is an often intractable set of constraint equations. One answer to solving econometric questions in such a fashion is to solve the micro-economic problem and then extrapolate the results to answer macro questions. That is, the model can optimize capacity expansion on a relatively small set of the product tree data base and then permit the analyst to optimize on larger data sets once the micro answers are known. An alternate plan is to solve the model using the Dantzig-Wolf type decomposition.[9]

If the simplified product tree is used in the problem, the second step in the model implementation is to make any simplifications necessary to reduce the dimensionality of the problem. Eight recommended strategies are as follows:

1) Compress the model vertically--that is, limit the number of levels in the product tree,

2) Compress the model horizontally--that is, limit the breadth of the product tree,
3) Approximate short time lags by assuming the lag is zero,
4) Limit selection of labor types,
5) Include only critical components in the data set,
6) Eliminate labor substitutions for all products except the item of interest,
7) Input production expansions as fixed values for all products except the item of interest--that is, only solve for the capacity expansions of the selected firm.
8) Input labor expansions as fixed values for all products except the item of interest--that is, only solve for the labor expansions of the selected firm.

The first two reduction methods limit the scope of the problem. The third method compresses the model size by reducing the model's start-up time. That is, the time between when products at level p are manufactured and when they are used as inputs of the final systems. The remaining strategies all explicitly reduce the number of constraints in the problem formulation. The reduction strategies are discussed in the next seven sections.

4.3.1 Compress Vertically

The first technique is to reduce the number of levels in the data set. The model allows for up to six levels in the product tree. If less than six levels are included, the model can be truncated to suit the data set. Example: If three product levels capture the derived market demand and production of automotive exhaust systems, the model can be reduced to the system, product, and assembly levels.

Another approach is to combine the product levels not used as direct inputs or outputs of the product of interest--both above and below the firm's product level. The model compresses most readily from the bottom up, since consumer demand for final systems is the driving factor in intermediate product and raw material demands. This reduction strategy is probably the best approach. Figures 4.1 and 4.2 demonstrate vertical compression of the product tree and the product tree resulting from the vertical compression.

4.3.2 Compress Horizontally

The model can be compressed horizontally by limiting the number of products at each level. Products could then be included only if they are used as a material input for the portion of the data set being studied. Non-functional products (such as white-walls on tires) also provide a significant size reduction if excluded from the problem formulation. Once the optimal capacity expansion strategy is known, the non-functional products may then be re-inserted into the formulation to verify that they do not affect the expansion decision. Figures 4.3 and 4.4 demonstrate horizontal compression of the product tree and the resulting product tree from the horizontal compression. Figure 4.5 shows the resulting product tree from the vertical and horizontal compressions (Figures 4.1 and 4.3).

If the number of products included in the product tree can be limited to 35 or fewer, the model can be applied to a longer planning horizon and may thus be used to obtain expansion strategy results for medium and long range corporate planning.

4.3.3 Approximate Small Time Lags

The third technique to reduce the model size is to approximate time lags where possible. The time lag refers to the amount of time needed for a product to be manufactured and made

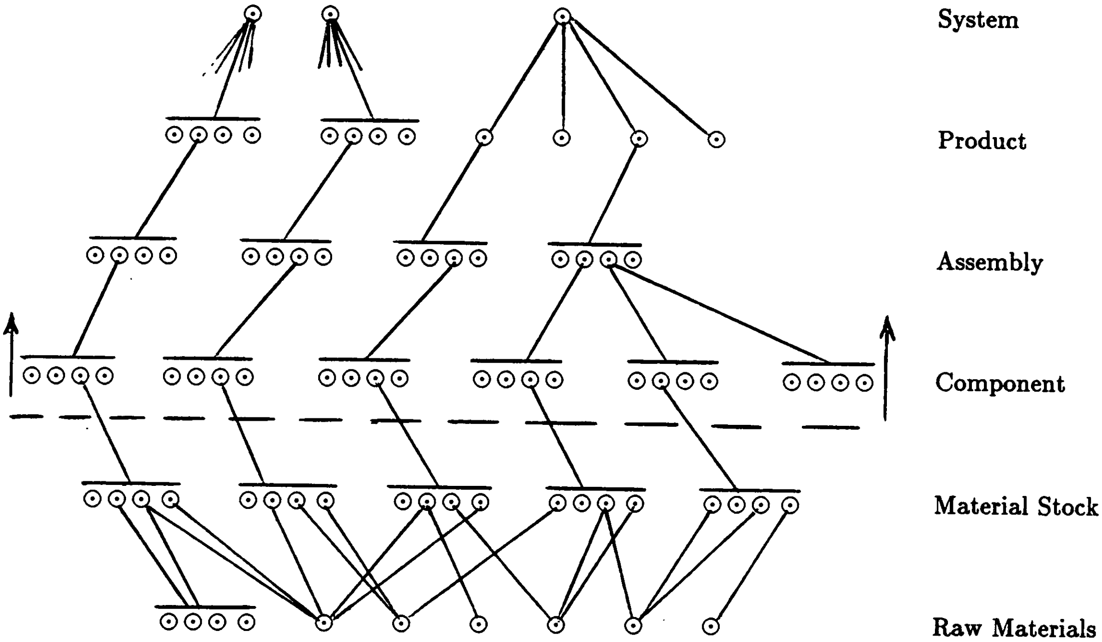


FIGURE 4.1 VERTICAL COMPRESSION OF PRODUCT TREE

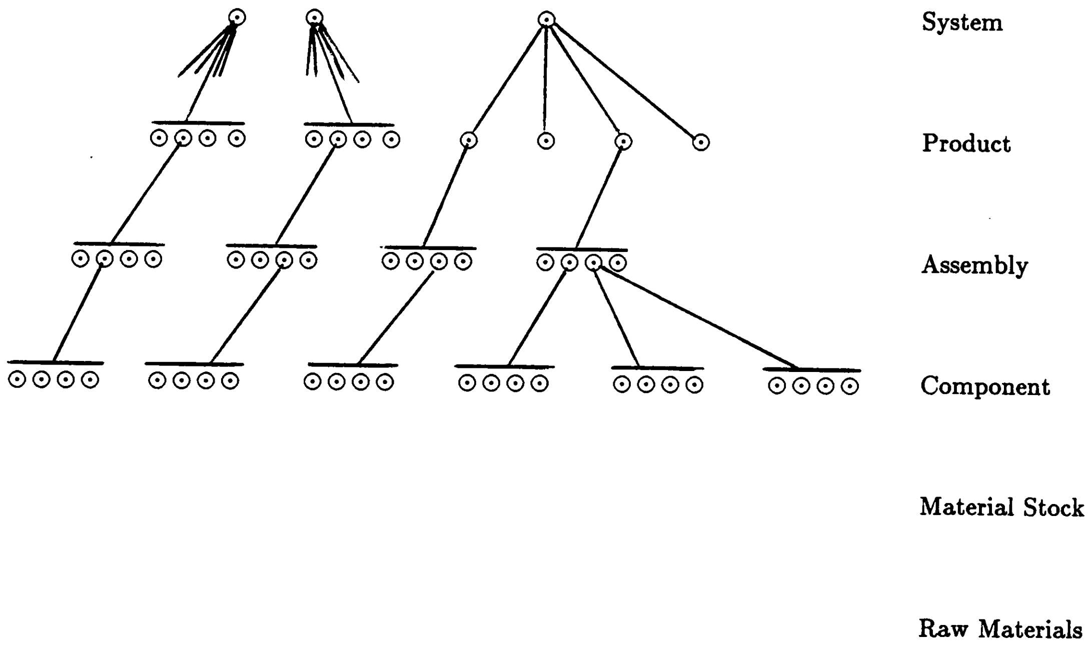


FIGURE 4.2 RESULTING PRODUCT TREE (VERTICAL COMPRESSION)

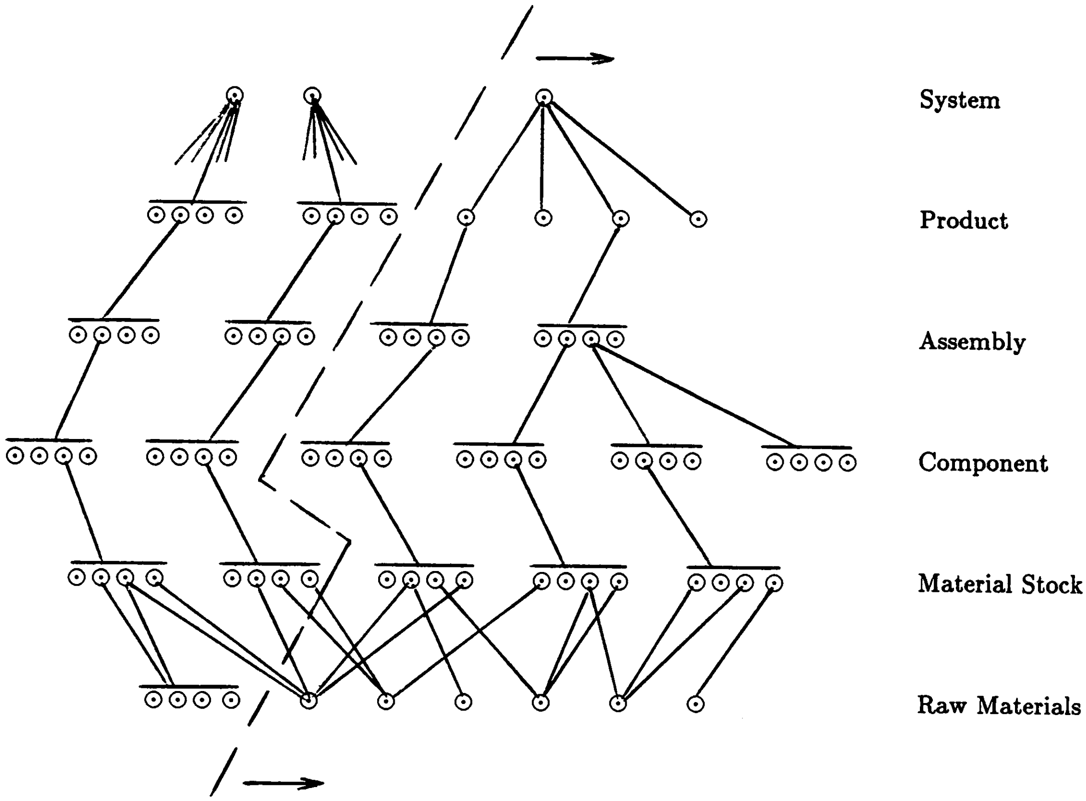


FIGURE 4.3 HORIZONTAL COMPRESSION OF PRODUCT TREE

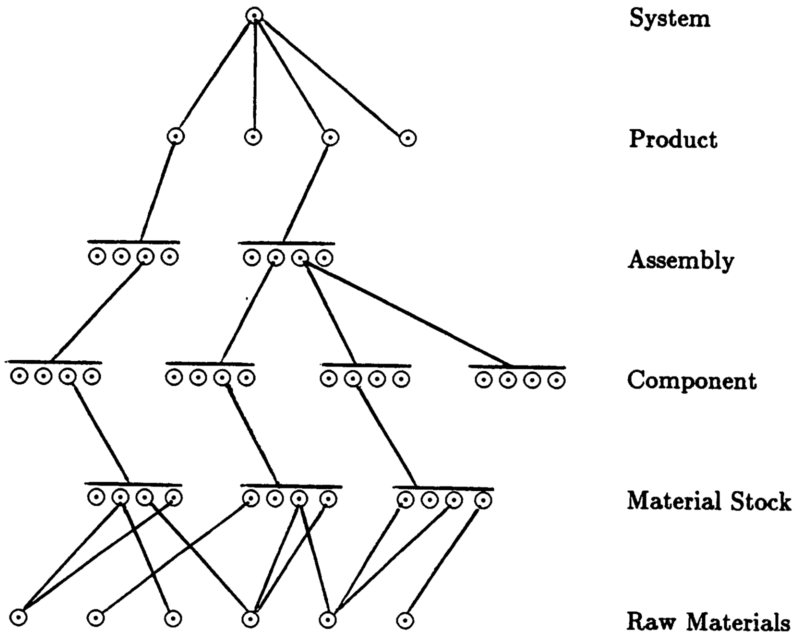


FIGURE 4.4 RESULTING PRODUCT TREE (HORIZONTAL COMPRESSION)

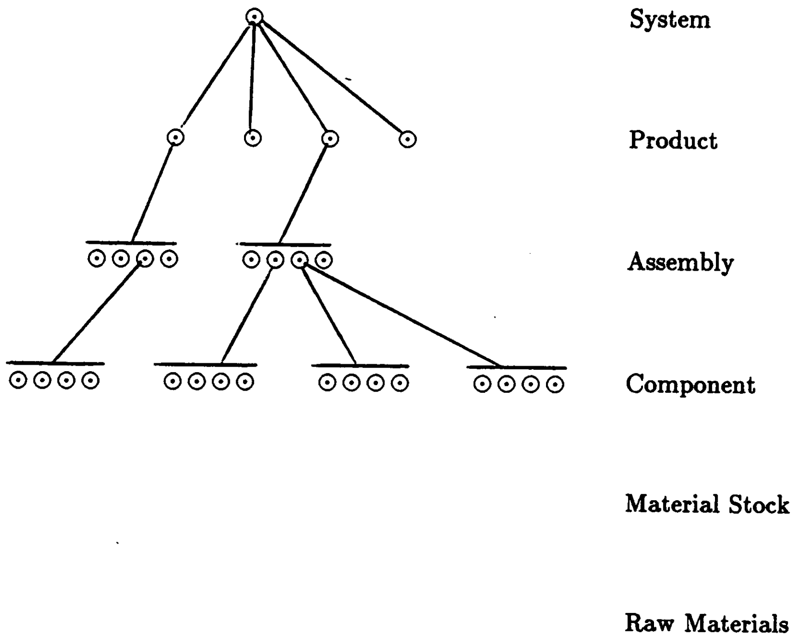


FIGURE 4.5 TOTAL REDUCTION IN MODEL SIZE

available to the next product level as a material input. For example: Given that the time period is in months, a time lag of one for mufflers implies that the mufflers (products) manufactured in February are not available for assembly into cars (systems) until March. In the case of longer lags (for instance, airplanes), the lags should be included as is.

Often small time lags for intermediate products can be rounded down to zero without seriously affecting the optimal capacity expansion strategy. By approximating small time lags with a lag of zero, the start-up time of the model can be reduced by four or five time periods. Start-up time refers to the time necessary for raw materials to be converted into final systems and is the sum of the largest time lag at each level over product levels 1 through 5, $(e_1 + \ldots + e_5)$.

Example: Given 50 products in the product tree, and a planning horizon of 40 months, if time lags between the component, assembly, and product levels could be approximated by 0, from a previous lag of 1 at each level, where the firm of interest was at the component level, the size reduction would be in the range of 100 to 200 constraint equations (3 x 50 x threefold reduction in constraints). Figures 4.6 and 4.7 show an example of the time needed to reach a steady state (time necessary for raw materials to be converted into systems) when 1) intermediate time lags are 1 time period for all product levels and 2) intermediate time lags are 0 time periods for all levels except the first and sixth product levels.

4.3.4 Selection of Labor Types

Limiting the number of labor types included in the data set results in a large reduction in the model size. Each product has labor allocation constraints for unskilled, skilled, managerial, and technical labor. For each labor type, there exists a two-fold reduction in the number of constraints for each labor type excluded from the constraint set at time t. One approach is to only include labor variables for the product of interest. A second approach is to include one

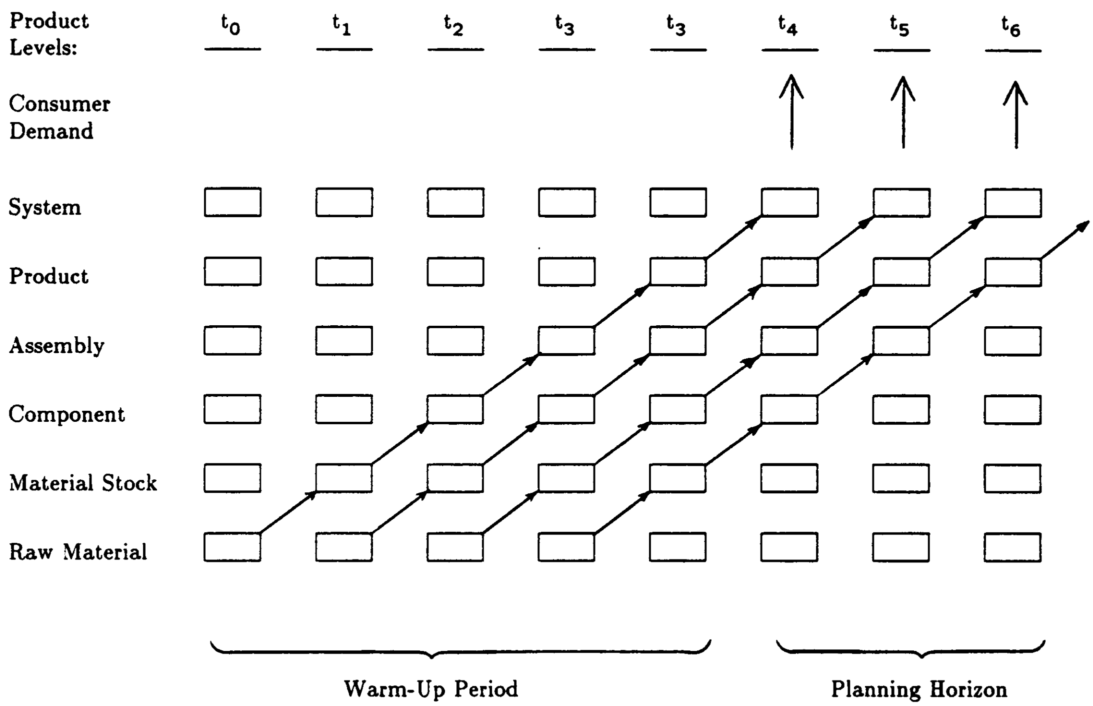


FIGURE 4.6 FLOW OF PRODUCTS

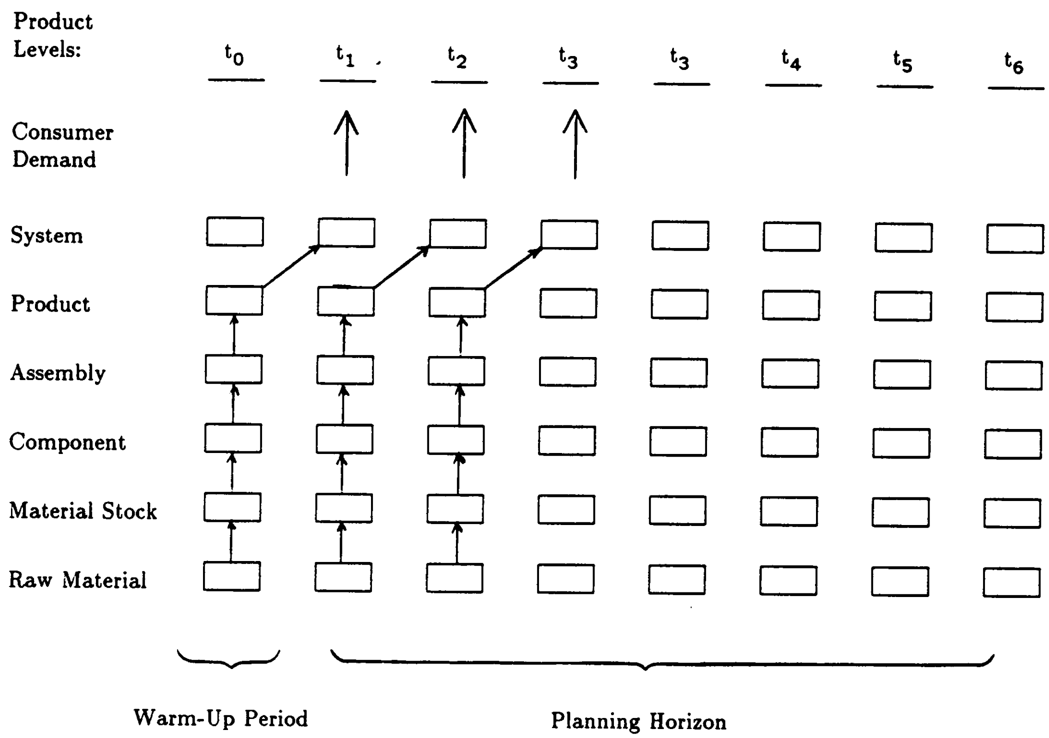


FIGURE 4.7 REVISED FLOW OF PRODUCTS

labor type in any given model run, and then to state the optimal capacity expansion in terms of each of the model runs. In this case, only one of the labor types, if any, should constrain the expansion decision.

4.3.5 Inclusion of Critical Components

Another technique to significantly reduce the model size is to include only those components critical for the system's proper functioning. Engineering judgement should be used when identifying critical components. That is, if there are 25 very important assembly level variables, but 250 moderately important variables in the model, a 250 variable reduction in the data set can be realized by including only the very important variables. By including only 1) functionally important products, and 2) products for which productive capacity is highly constrained, significant reductions are realized in the model size.

4.3.6 Labor Substitutions

Labor substitutions constitute the greatest opportunity to reduce the model size. By including labor substitutions only for the product of interest, the model size is reduced to workable levels.

4.3.7 Production Expansion Variables

Production expansion variables can be implemented in either of two ways: 1) the expansions may be entered as variables for all products, or 2) expansions for products other than the product of interest may be estimated and then entered as predetermined inputs to the model. For instance, if the automotive industry was expected to expand production at a rate of 0.02 % per month over the next 18 months, the production capacities could be increased at such a rate and treated as input parameters. The expansion variables for the product of interest (for example, tires) are input as decision variables and all other expansions are input as parameters. If the automotive industry expected several discrete jumps in the productive capacity (for example, the completion of new manufacturing facilities 15 months and 24 months from the current time), the estimated expansion values could be input in order to eliminate expansion variables (and thus constraint equations) from the set of model equations and decision variables.

If expansion variables are only included for the product of interest, simple upper bounds may be used for the other production variables. The simple upper bounds replace the production capacity constraints as well as the production constraints in the problem formulation.

4.3.8 Labor Expansion Variables

Labor expansion variables can be implemented in either of two ways: 1) the expansions may be entered as variables for all products, or 2) expansions for products other than the product of interest may be estimated and then entered as predetermined inputs to the model. The expansion variables for the product of interest (for example, tires) are input as decision variables and all other expansions are input as parameters.

If expansion variables are only included for the product of interest, simple upper bounds may be used for the other labor variables. The simple upper bounds replace the labor availability constraints as well as the labor constraints in the problem formulation.

4.4 COMPUTATIONAL BASIS OF THE MODEL

The model consists of multiple sets of linear constraints on an objective function. The preprocessor reads the data base and converts it into an interactive constraint format accepted by the LINDO software which is then used to solve the model.

The operation of the model is structured as follows:

1) Input data for resource allocations into LOTUS 123 spreadsheet files,
2) Preprocessor converts data base into constraint equations,
3) The LINDO LP software is used to import the constraint file, find the optimal solution, and divert output to an ASCII file, and
4) Post processor displays optimal solution.

The flow chart shown in Figure 4.8 gives a more detailed picture of the operation of the computer software. A random number generator is employed to generate all unknown but necessary data for the automobile example. Specifically, labor consumption, availability, and substitution data must be randomly generated in order to demonstrate the full capabilities of the model. Production capacities are hypothetical and also may be randomly generated on some positive interval. Cost data are also hypothetical but are input rather than randomly generated to assure validity of the data. Examples of cost data considerations are to ensure that the selling price is greater than material prices and to assure that the wages are correct relative to other wages (managerial time costing more than unskilled labor). Material consumption data is taken from General Motor Corporation's 1984 Shop Manual[10] covering passenger cars.

4.5 COMPUTATIONAL LIMITATIONS

Current mainframe software which will solve linear programs of up to 10,000 constraint equations exists. The LINDO software has a maximum size of 5,000 constraint equations, 15,000 variables, and 60,000 zero-one variables. LINDO allows for the use of simple upper and lower bounds on all variables. Thus, the inventory safety stock constraints may be treated as lower bounds rather than constraints. Likewise, if the labor and production size reduction

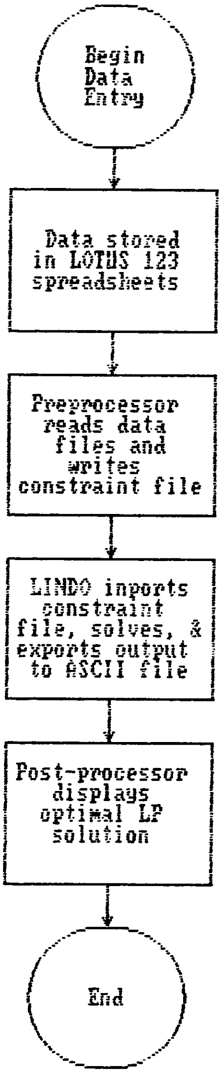


FIGURE 4.8 FLOWCHART OF COMPUTATIONAL SOFTWARE

techniques are implemented, simple upper bounds on the labor and production variables may replace the labor availability, labor consumption, production consumption, and production capacity-constraint sets.

Given even a relatively sparse product tree and months as the time units, if the desired planning horizon is greater than three or four years, the model exceeds the upper bound on LP constraints. The number of constraints in the model is given by:

$$\begin{aligned}\text{constraints} = \quad & t * (2q + 8) * (m + l + k + j + i + h) \\ & + \text{labor substitution equations} \\ & + (m + l + k + j + i + h).\end{aligned}$$

If labor and capital expansion constraints are only implemented for the company of interest, the number of constraint equations will be greatly reduced. The number of constraints would then be:

$$\begin{aligned}\text{constraints} = \quad & t * (2q + 2) * (m + l + k + j + i + h) \\ & + (5 + q) * t \\ & + \text{labor substitutions within company} + 1.\end{aligned}$$

Furthermore, if simple upper and lower bounds are used on production and labor constraints for all products except the product of interest, the number of constraints is then given by:

$$\begin{aligned}\text{constraints} = \quad & 2\,t * (m + l + k + j + i + h) \\ & + (5 + q) * t \\ & + \text{labor substitutions within company} + 1.\end{aligned}$$

The structure of the cascading product demands and the constraints on productive capacity remains the same when the labor constraints are only included for the company of interest. Once the optimal expansion strategy to this revised formulation is found, labor constraints for other manufacturers in the product tree may be added to the set of LP constraints using sensitivity analysis to verify that the optimal solution satisfies the original formulation. An example size of the set constraint equations generated by the model is given in Table 4.4.

Table 4.4 SIZE OF CONSTRAINT SET FOR THE MODEL

Type of Reduction Strategy Implemented	Size of Constraint Set
Full Model	32,100 Constraints
Labor & Production Expansions only at Product of Interest; Lower Bounds used for Inventory Safety Stock	20,181 Constraints
Labor & Production Expansions only at Product of Interest; Upper Bounds Used for Labor and Production Variables; Lower Bounds used for Inventory Safety Stock	4,181 Constraints

Number of:

Systems (m)	= 1
Products (l)	= 9
Assemblies (k)	= 20
Components (j)	= 38
Basic Materials (i)	= 26
Raw Materials (h)	= 6

Number of Time Periods = 20

Number of Labor Types = 4

4.6 MODEL APPLICATIONS

The most likely applications for the model are in planning medium and long range capacity expansions in the capital-intensive industrial sector of the economy. In an industrial environment, the model could be used to identify optimal times (minimum cost/maximum profit) for constructing plants, modernizing current plants, and expanding current plants. In military settings, the model could be used to identify the industrial sectors which would not be able to meet a surge in product demands as a result of military involvements with foreign countries. The model could be used to study any item in the product tree--from conversion of raw materials into basic material stocks to the assembly of final systems.

The model would be most effective if used by the staff in charge of long-term corporate planning. As an example of the model's application to corporate planning, the first case study examines capacity expansion decisions over a four year horizon. The model identifies the optimal months to expand and the size of the expansions. If desired, the analyst could select quarters or years as the unit of time in order to study a longer planning horizon.

4.7 SUMMARY

Chapter 4 highlighted the three keys to successful model implementation--1) definition of data requirements and gathering the data, 2) simplification of the problem by concentrating on a subset of the product tree, and 3) algorithmic analysis with current linear programming software. The two major limitations to the model implementation are the availability of required production and labor data for each item in the product tree and the polynomial increase in problem size as the number of items in the product tree increase.

Much of the required data already exists in government data bases in agencies such as the Bureau of Labor Statistics. Labor availabilities and consumption rates for all economic sectors are probably available in some form of electronic media.

The problem size may be reduced using any combination of the strategies presented in this chapter. Furthermore, the linear programming routines are well-defined and a large LP software package could either be purchased or programmed to take advantage of current mainframe computational capabilities (large meaning the ability to handle fifteen to twenty thousand constraint equations). By pursuing both model reduction strategies and the purchase/programming of a very large LP package, the model may be applied to a large variety of industrial and governmental capacity expansion problems.

CHAPTER 5
MODEL ANALYSIS

5.1 INTRODUCTION

In order to illustrate the empirical capabilities of the model, a case study was developed for modeling capacity expansion decisions and the corresponding data was input into the model. A description of the company of interest and the data base applicable to the company of interest are contained in Section 5.2. In Section 5.3 the data for the case study are given. The model output from maximizing the company's present worth is presented in Section 5.3. The optimal solution's sensitivity to model parameters such as the interest rate for borrowed funds, the MARR, planning horizon, and corporate income tax are presented in Section 5.4. The sample results generated here represent the type of output that the model can produce for any company which manufactures products at any level of the product tree.

5.2 DESCRIPTION OF CASE STUDY

The company being analyzed in the case study manufactures automotive tires. The product of interest is double-belted radial tires which are used in passenger cars. Figure 5.1 shows the double-belted tire being studied. The tire is assumed to be at the assembly level in the product tree and all the constituent materials of the tire at the component level. The product tree used in the model is the decomposition of a passenger automobile. A single system level product--the automobile--uses the tires manufactured by the company of interest. The level of detail needed to model each make of automobile and each type of tire goes beyond the scope of the case study. Rather, a single automobile class is assumed to represent all makes of automobiles using the tires and a single tire class is assumed to encompass all types of double-belted radial tires. That is, the production facilities for tire manufacture are

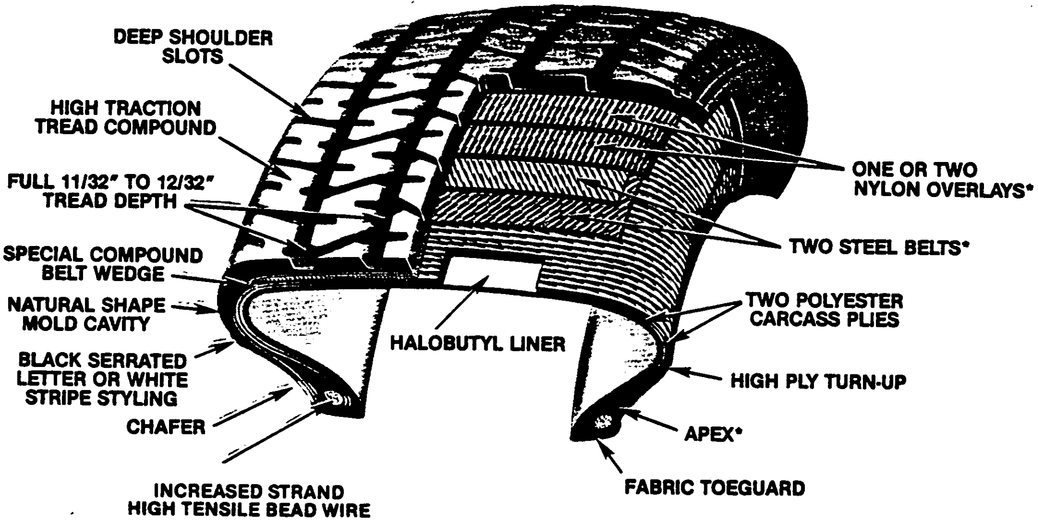


FIGURE 5.1 CUTAWAY DIAGRAM OF AUTOMOBILE TIRE

assumed to be able to manufacture any of the automotive tires which are used in passenger cars.

The capacity expansion strategy of a tire manufacturer is being studied over a 3 to 5 year planing horizon. The corporate planners would like to know 1) what the optimal capacity expansion strategy is for the company and 2) how is the sensitivity of the optimal strategy to fluctuations in the cost parameters used in the analysis.

The company of interest supplies tires to the automotive industry for use in passenger automobiles. The tires are also sold in bulk quantity to wholesale and retail tire distributors for sale as replacement tires for existing automobiles. The major markets for automotive and tires are:

- automotive industry: Ford Motor Corp. , Toyota, ...
- wholesale markets for replacement tires: distributors, wholesalers.

The tire manufacturer being considered only supplies a portion of the domestic and foreign tire market. Market share is estimated and input into the data base (In actuality, a company's market share is known and generally does not change significantly over time.) The optimal capacity expansion reflects this market share.

The economy is assumed to be growing at a rate equivalent to the GNP during each year. Demand for automobiles reflects this rate of growth. Derived demands for the constituent parts for the automobile therefore also increase at the assumed rate of the GNP. Seasonal fluctuations in demand for systems may be included in the model if desired but are not included in this case study. The demand for systems per year is estimated and used as the basis for all system demands during subsequent time periods.

5.3 DATA BASE FOR CASE STUDY

The first data requirement for the case study is to develop the product tree for the automobile. The automobile (system level) is composed of products such as wheels, chassis, engine, and fuel system. Each product can then in turn be broken down into the assemblies such as tires, wheel rims, and hubcaps which constitute the wheel.

Table 5.1 shows the system-product breakdown of the automobile. Examples of the wheel (product-to-assembly) decomposition is shown in Table 5.2. The assemblies are then decomposed into component parts; examples for the tire and wheel assemblies are shown in Table 5.3 and Table 5.4. The entire product tree can be listed in such a fashion.

The time units selected for the case study is quarterly production rates. One year is represented by four time periods. Quarters are used to allow for a five-year time horizon to be studied without generating too many constraint equations to be readily solved. Furthermore, the algorithmic literature on LINDO suggests a range of 0.001 to 100,000 on variables and RHS values. By selecting quarters as the time units, the production units are more suitable for the analysis of the case study.

Material consumption rates for each of the components, assemblies, and products which constitute the automobile are listed in the "Number Required" column of Tables 5.1 – 5.4 and the units are shown beside the number required. Labor consumption and production capacity data are not readily available and are simulated with random numbers in the case study. Table 5.5 lists the production capacity data that is used in the case study. Note that all production capacities are given in terms of quarterly production capacities. Other pertinent production data and general data are shown in Table 5.6.

Cost data is then gathered on employee wages, holding costs, material prices, the tire's selling price, etc. The cost data used in the model are purely hypothetical. Inflation is not considered in any of the cost data. That is, both material prices and the company's selling

TABLE 5.1 PARTS REQUIREMENTS FOR AUTOMOBILE

System†	Products††	Number Required
Automobile	Engine	1
	Fuel System	1
	Exhaust System	1
	Coolant System	1
	Electrical System	1
	Transmission (clutch if manual)	1
	Transmission Drive Assembly	1
	Propulsion Shaft	1
	Front Axle	1
	Rear Axle	1
	Air Conditioning System	1
	Drum Brakes (rear)	2
	Disc Brakes (front)	2
	Wheels	5*
	Chassis	1
	Springs	4
	Shock Absorbers	4
	Body	1
	Front Seats	2
	Rear Seat	1
	Heating and ventilation System	1
	Dashboard Assembly	1
	Front Windshield	1
	Rear Windshield	1
	Front Side Windows	2
	Rear Side Windows	2

†The products constituting each automotive system will be similar only for automobiles of the same class (example, economy class)--similar enough that the suppliers can manufacture the product for each automobile without large set-up times for the production run.

††Note: This is only a partial listing of the products which constitute an automobile

* Assuming the car comes equipped with a full-size spare tire.

TABLE 5.2 PARTS REQUIREMENTS FOR AUTOMOBILE WHEELS

Product	Assemblies	Number Required
Wheel	Wheel Rim Assembly	1
	Tire	1
	Aluminum Hubcap	1

Table 5.3 DECOMPOSITION OF TIRE

Assembly	Components	Number Required
Tire	Nylon Overlay	1
	Steel Belt	2
	Polyester Piles	2
	Apex	1
	Bead Bundle	2
	Fabric Toeguard	2
	Ply Turn-Up	1
	Belt Wedge	1
	Rubber Composite	1
	Carbon Black	1
	Halobutyl Liner	1

TABLE 5.4 DECOMPOSITION OF WHEEL RIM

Assembly	Components	Number Required
Wheel Rim	Inner Rim	1
	Outer Rim	1
	Air Valve	1
	Valve Cap	1
	Lug Bolt, 3/4 in	6
	Lug Nut, 3/4 in	6

TABLE 5.5 PRODUCTION CAPACITY DATA FOR CASE STUDY

Products	Production Capacity	Expected Growth
Automobiles	402	4 % per quarter
Wheels	2422	5 % per quarter
Hubcaps	2616	4 % per quarter
Wheel Rims	2290	4 % per quarter
Tires	504	4 % per quarter
Nylon Overlay	1420	5 % per quarter
Steel Belt	1420	6 % per quarter
Polyester Plies	1500	4 % per quarter
Apex	1400	6 % per quarter
Bead Bundle	1450	6 % per quarter
Fabric Toeguard	1400	7 % per quarter
Ply Turn-Up	1360	6 % per quarter
Belt Wedge	1510	5 % per quarter
Rubber Composite	1400	7 % per quarter
Carbon Black	1460	6 % per quarter
Halobutyl Liner	725	4 % per quarter

TABLE 5.6 INPUT DATA REQUIRED BY THE MODEL

General Data	
Time Period	Quarters
Length of Model Run	5 years
Systems Using the Product of Interest	Automobiles
Interest on Borrowed Funds	9 percent/year
Interest on Capital Equity	6 percent/year
Data Required for Every Item in the Product Tree	
Production Units (ex. pounds/month, units/month	1,000 / period
Initial Inventories	random on U(0,50)
Time Lag between Production Levels	0 time units
Safety Stock Levels	random on U(0,50)
Time Lag Between Time an Expenditure Occurs and Production Begins for Capacity Expansions	0 time units
Required for System Level Items	
Demand for Automobiles at Time 0	610 thousand
Increase in System Demand	3.8 % per year

price are assumed to remain constant over time. This is a relatively good assumption during periods of relatively low inflation (three to six percent) but might cause results in high inflation time periods to be affected adversely. The inclusion of inflation in the problem is left as a model extension. Table 5.7 lists the cost data required by the model and the hypothetical values assumed for the case study. In actual practice, the cost data needed by the model could be attained from the accounting department within the company. Table 5.8 shows data for capacity expansions of the tire manufacturer.

For the capacity expansion problem, the following strategy is used to reduce the product tree to a manageable size:

1. assume all automobiles can be classified by a single system variable,
2. only include wheels at the product level, and
3. decompose the wheel rim assembly, the tire, and the hubcap only to the component levels.

The revised product tree is much more tractable but still captures the essential market demand for tires. Figure 5.2 shows the product tree which is used in the case study. Note that the product tree could be expanded both vertically and horizontally if a greater level of detail is desired in the analysis.

Furthermore, production expansions at all levels except for the product of interest are approximated by assuming a fixed percent that the productive capacity expands during each quarter. That is, rather than solve specifically for the expansion decisions for each company in the product tree, the expansions of the entire substitutions are only implemented at the product of interest. Additionally, labor constraints are only implemented for the company of interest since their exclusion at other levels in the product tree does not alter the functionality of the model.

TABLE 5.7 COST DATA FOR TIRE MANUFACTURER

Data Requirement	Assumed Cost (in dollars)
Cost of Expansion Per Unit of Capacity Increase	240/unit
Maximum Budget size	45,000,000
Price of the Product (dollars/unit)	71.00/unit
Market Share	20 %
Cost of Materials (dollars/unit)	
Nylon Overlay	2.23/unit
Steel Belts	4.61/unit
Polyester Plies	1.96/unit
Ply Turn-Up	1.21/unit
Apex	0.83/unit
Fabric Toeguard	1.10/unit
Halobutyl Liner	2.90/unit
Bead Bundle	1.00/unit
Belt Wedge	0.90/unit
Rubber Polymer	4.23/unit
Carbon Black	3.12/unit
Operating & Maintenance Cost (dollars/unit)	0.25/unit
Holding Cost (dollars/unit)	3.00/month
Wages:	
Unskilled Labor	6.50/hr.
Skilled Labor	11.50/hr.
Managerial Labor	23.50/hr.
Technical Labor	21.25/hr.
Overhead Application Rate (percent of direct labor)	2.10 %
Corporate Income Tax	30.0 %

TABLE 5.8 CAPACITY EXPANSION DATA FOR TIRE MANUFACTURER

Minimum Capacity Expansion	1 unit/month
Maximum Capacity Expansion	500 units/month
Maximum Number of Expansions	unrestricted

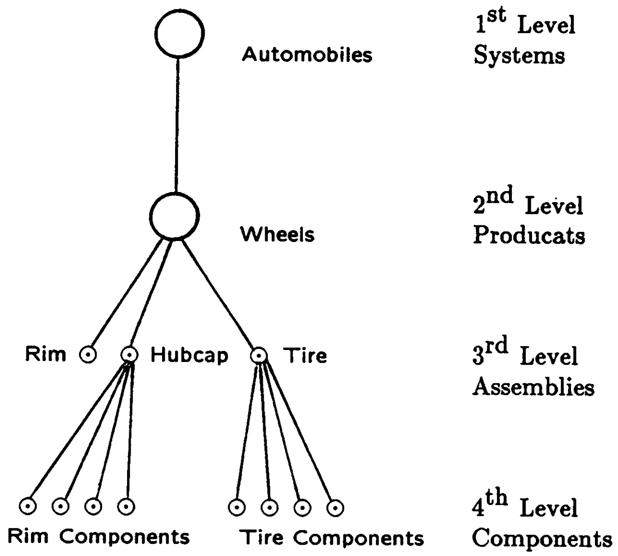


FIGURE 5.2 PRODUCT TREE USED IN THE CASE STUDY

5.4 DESCRIPTION OF CASE STUDY RESULTS

The model outputs a sizeable amount of information, as evidenced by Appendix 2. This output will be analyzed in this section. Furthermore, sensitivity analysis on interest rates, the increase in system demand (based on GNP), the corporate income tax, and planning horizon are presented in this section.

5.4.1 Optimum Expansion Times Generated

The primary information resulting from the model is the company's optimum expansion schedule which maximizes the company's present worth. The model output can also be used to verify that the solution satisfies the model parameters as specified in the problem formulation. For the case study, the present worth of the company was \$ 69,170 and five capacity expansions were specified in the optimal strategy. Table 5.9 shows the present worth (PW) of the company, the optimal expansion strategy, and shows how the optimal strategy satisfies the maximum budget size constraint, the maximum number of capacity expansion allowed in the strategy, and that production is less than the production capacity during each time period. The optimal expansion strategy for the tire manufacturer is then graphed in Figure 3.3 to show how the production capacity expands over time and to show the optimal expansion strategy versus the growth in the market demand for tires. The figure shows the capacity expansions when interest rates are 9 % for borrowed funds and 6 % obtainable on capital equity, a corporate tax rate of 30 %, and a planning horizon of 20 business quarters.

Here, capacity expansions closely match growth in the market demand for tires (adjusted to reflect the company's market share, in thousands) for the first four years. No expansions are undertaken over the last year due to the length of time needed to recover the costs of an expansion (capital expenditures are recovered in approximately 6-8 quarters). A lower bound of one production units is assumed for all capacity expansions. The right-hand side (RHS)

TABLE 5.9 OPTIMAL EXPANSION STRATEGY FOR TIRE MANUFACTURER

LP Objective Value (Present Worth) $ 69,170.50

Optimal Expansion Strategy: 5 Expansions

Time	Size of Expansions (in Thousands)	Expenditures for Each Eaxpansion (in dollars)
0	28.809 units	6,914,121
2	14.194 units	3,406,582
4	5.155 units	1,237,207
5	32.064 units	7,695,352
9	5.422 units	1,301,289

Total Expenditures = $ 20,554,550
(below budget limit of $ 45,000,000)

Tire Production Versus Production Capacity per Period

Time	Production (in Thousands)	Capacity (in Thousands)
0	613.8	613.8
1	601.4	613.8
2	628.0	628.0
3	626.5	628.0
4	633.2	633.2
5	665.2	665.2
6	644.9	665.2
7	653.6	665.2
8	659.6	665.2
9	649.3	665.2
10	670.6	670.6
11	670.6	670.6
12	670.6	670.6
13	670.6	670.6
14	670.6	670.6
15	670.6	670.6
16	670.6	670.6
17	670.6	670.6
18	670.6	670.6
19	670.6	670.6
20	670.6	670.6

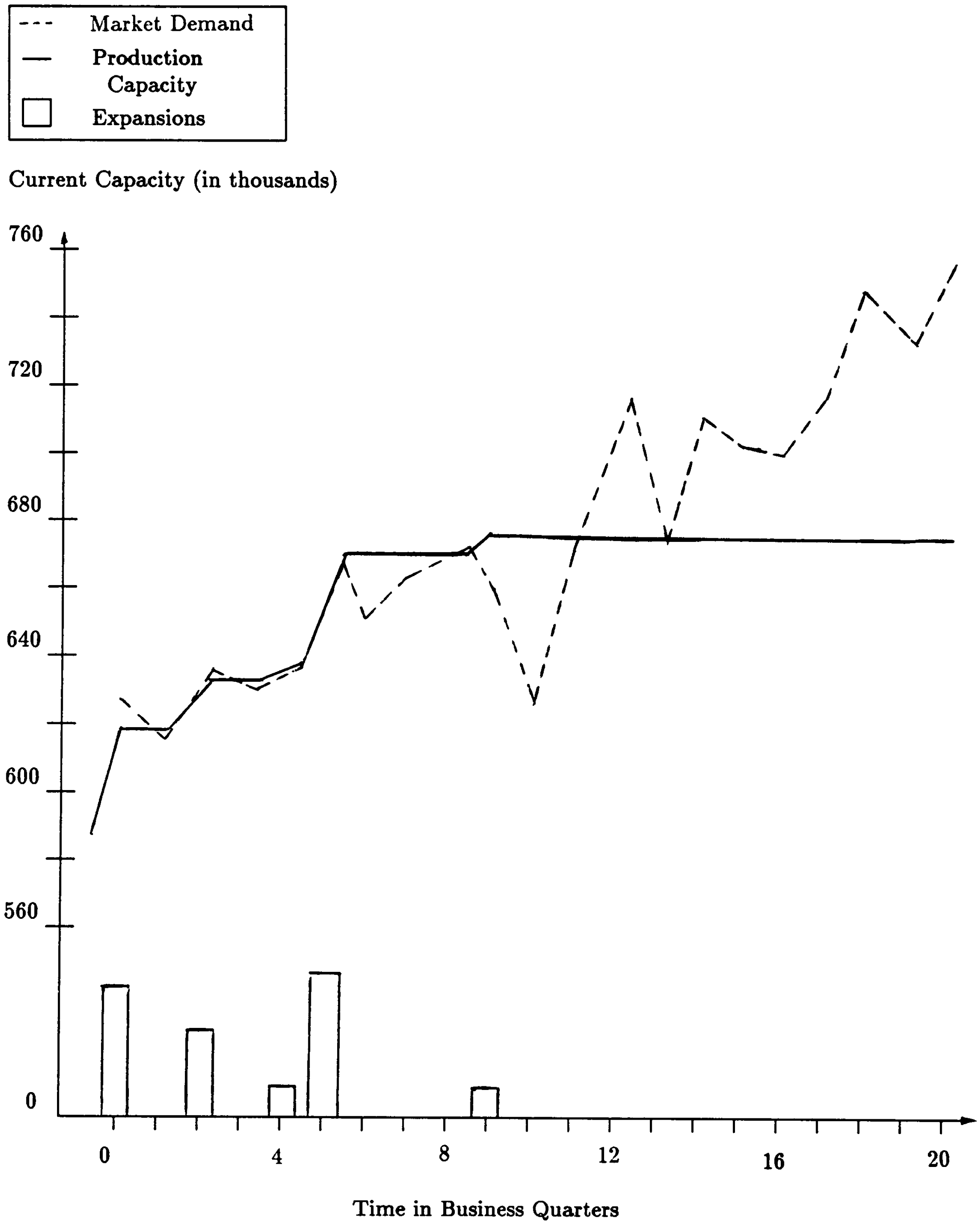


FIGURE 5.3 OPTIMAL STRATEGY: GROWTH IN CAPACITY VERSUS MARKET DEMAND FOR TIRES

constraint slack values and the dual prices for system level demand (automobiles) for the optimal strategy are given in Table 5.10. By examining the RHS slacks and dual prices, tire production meets tire demand during periods 0 through 16, but production does not meet demand during the last year of the planning horizon (periods 16 through 20). That is, a decrease of one unit in system demand causes a reduction in the objective function equivalent to the value of the dual price for that system variable.

The executive making an expansion decision could use the dual prices to estimate potential losses if the market estimates for automobiles were off by certain amounts. If, for instance, market estimates on automobile sales over the next two years (periods 0 through 8) were 10 percent higher, the loss in revenues could be found by multiplying forecasted demands by 10 percent and then multiplying the resulting figure by the dual prices over the next two years. Such a loss would be approximately $3,660 for the expansion strategy shown previously in Figure 5.3.

Table 5.11 shows the ranges of system demand for which the current LP basis is unchanged. Changes of the RHS values within the given range do not alter the current LP optimal solution, but the objective function will change in value depending on the dual prices of the RHS's. If, for instance, the RHS value for automobile demand at time 10 were to either be increased by 5.65 units or decreased by 6.4 units, the expansion strategy shown in Figure 5.3 would still be optimal but the company would either make more or less money, depending on whether the change was to increase or to decrease automobile demand. The manager could use the RHS ranges to test the optimal expansion strategies to fluctuations in the forecasted system demand to validate the solution for a given set of increases or decreases.

5.4.2 Sensitivity Analysis on Parameters

Sensitivity analysis on such variables as the interest rates, corporate tax rates, inventory

TABLE 5.10 DUAL PRICES FOR MARKET DEMAND CONSTRAINTS

Automobile Demand at	Sign	Slack or Surplus	Dual Prices
0	$\leq$	0	3.87
1	$\leq$	0	5.86
2	$\leq$	0	4.80
3	$\leq$	0	5.75
4	$\leq$	0	5.19
5	$\leq$	0	3.91
6	$\leq$	0	5.58
7	$\leq$	0	5.52
8	$\leq$	0	5.47
9	$\leq$	0	5.41
10	$\leq$	0	3.86
11	$\leq$	0	2.30
12	$\leq$	0	0.76
13	$\leq$	0	1.51
14	$\leq$	38.84	0.00
15	$\leq$	32.09	0.00
16	$\leq$	27.48	0.00
17	$\leq$	38.72	0.00
18	$\leq$	82.20	0.00
19	$\leq$	85.58	0.00
20	$\leq$	68.47	0.00

TABLE 5.11 RIGHT-HAND RANGES FOR WHICH THE BASIS IS UNCHANGED

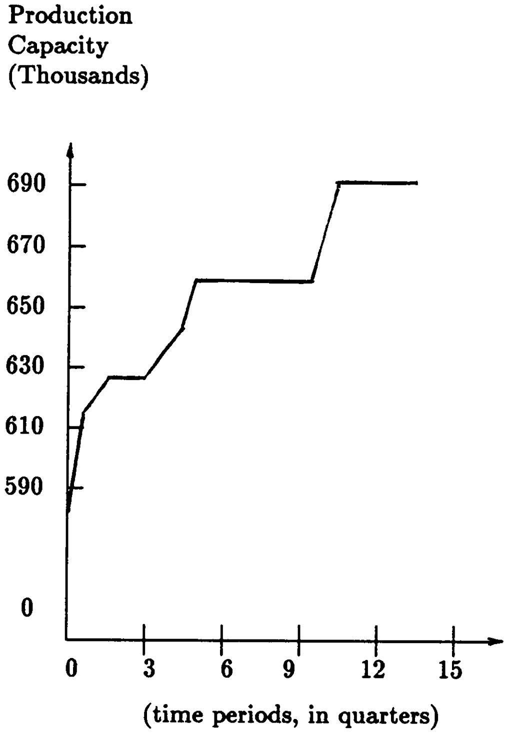
Automobile Demand at	Current RHS	Allowable Increase	Allowable Decrease
0	610	13.87	9.04
1	600	12.37	384.40
2	628	5.15	1.47
3	626	1.47	625.47
4	633	32.06	5.15
5	665	5.42	5.65
6	644	20.35	643.81
7	653	12.05	652.11
8	659	5.65	658.51
9	649	21.37	648.22
10	622	58.26	16.26
11	671	58.26	16.26
12	718	29.43	38.84
13	664	6.38	38.84
14	715	INFINITY	38.84
16	698	INFINITY	32.09
15	702	INFINITY	27.48
17	709	INFINITY	38.72
18	752	INFINITY	82.20
19	756	INFINITY	85.58
20	739	INFINITY	68.46

holding costs, the MARR, the length of the planning horizon, and other model parameters may also be performed with the model. Such sensitivity analysis requires the model to be re-run with the altered variable coefficients and objective function cost coefficients. The model results may then be compared with the original optimal solution to find the affects of the changes. In the case study, interest rates, the corporate tax rate, the planning horizon, and inventory holding cost were varied to demonstrate the model's sensitivity to these input parameters.

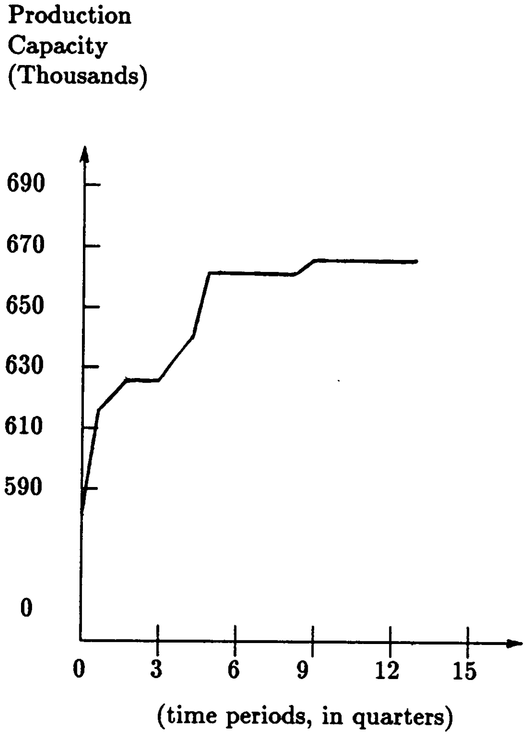
First, the corporate tax rate was varried from 30 percent to 40 percent taxation on net income minus possible deductions (interest and depreciation). It was expected that an increase in income tax would cause expansions to occur earlier in time. Figure 5.4 demonstrates the change in the optimal capacity expansion strategy when the tax rate is varied. The optimal strategy remained the same over the first 8 business quarters but an expansion during the 10^{th} quarter under a 30 percent tax rate was moved forward to the 9^{th} quarter under a 40 percent corporate tax. In revised expansion strategy, the expansion had a smaller magnitude (5 units rather than 30 units) due to a smaller expected profit from the expansion. The results did confirm expectations for changes in the tax rate.

Figure 5.5 demonstrates the changes in the expansion strategy when interest is varied. Interest was increased from 9 percent on borrowed funds and 6 percent on equity to 12 percent on borrowed funds and 11.5 percent on equity. The optimal expansion strategy for the revised interest rates was to move the bulk of an expansion originally made during time 5 to time period 4. An expansion made during the 9^{th} period with the original tax rates was eliminated under the higher interest rates. It was expected that when the interest rates for borrowed funds and capital equity are relatively closer together, expansions will occur sooner than when the difference between the rates is relatively greater. The optimal strategies for the two sets of interest rates demonstrates such a change in the expansion strategy.

Figure 5.6 demonstrates the sensitivity of the optimal expansion strategy to changes in



Original: Tax Rate = 30 %

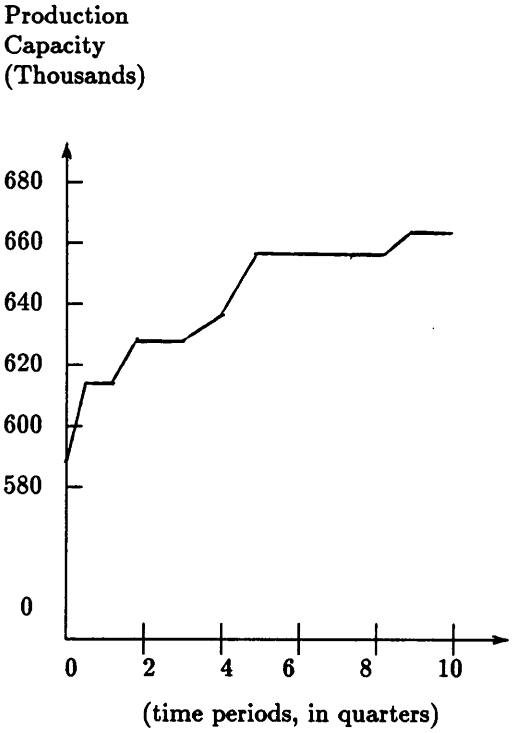


New: Tax Rate = 40 %

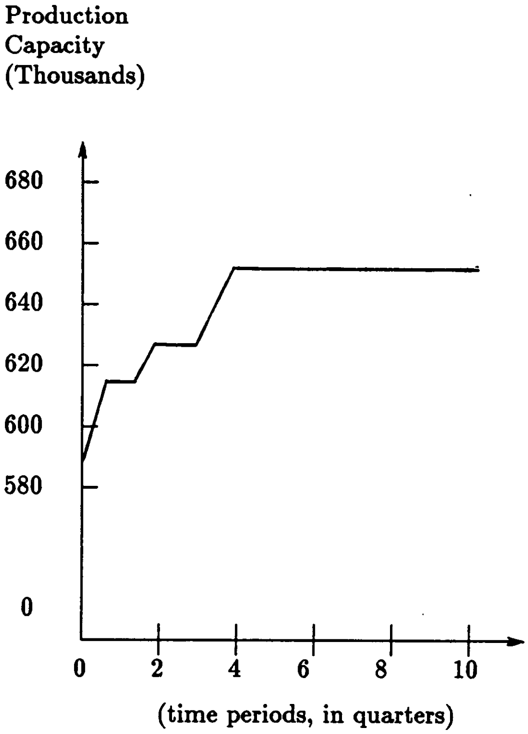
Time Period	Size of Expansion (in Thousands)
0	28.80
2	14.19
4	10.80
5	20.76
9	0
10	30.50

Time Period	Size of Expansion (in Thousands)
0	28.80
2	14.19
4	10.80
5	20.76
9	5.73
10	0

FIGURE 5.4 GROWTH IN CAPACITY--TAX VARIED



Original Rates: Debt = 9 %
Equity = 6 %

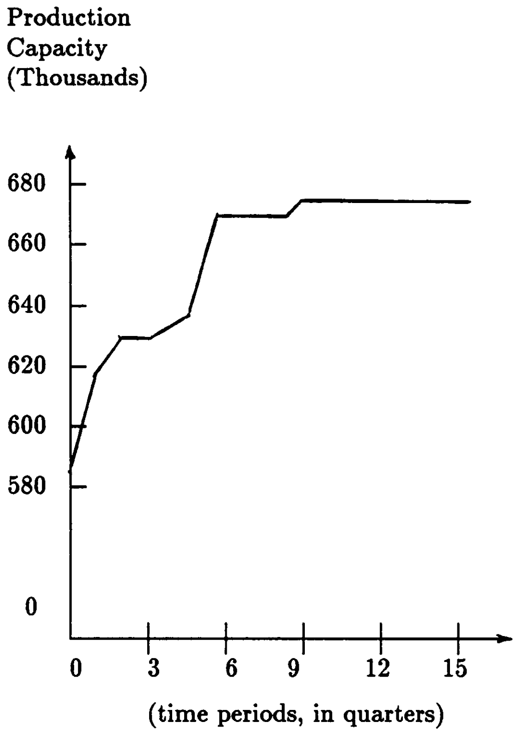


New Rates: Debt = 12 %
Equity = 11.5 %

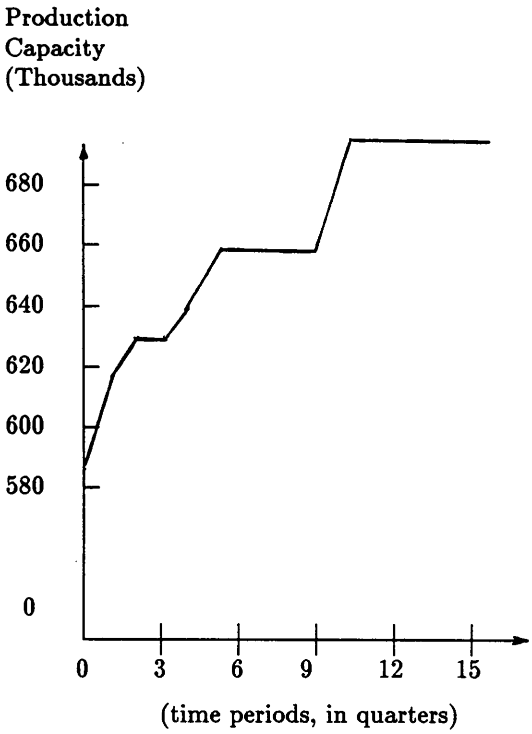
Time Period	Size of Expansions (in Thousands)
0	28.80
2	14.19
4	5.15
5	20.76
9	5.72
10	0

Time Period	Size of Expansions (in Thousands)
0	28.80
2	12.72
4	22.57
5	0
9	0
10	0

FIGURE 5.5 GROWTH IN CAPACITY--INTEREST VARIED



Orignal: Horizon = 20 Periods



New: Horizon = 21 Periods

Time Period	Size of Expansion (in Thousands)
0	28.80
2	14.19
4	5.15
5	32.06
9	5.42
10	0

Time Period	Size of Expansion (in Thousands)
0	28.80
2	14.19
4	10.80
5	20.76
9	0
10	30.50

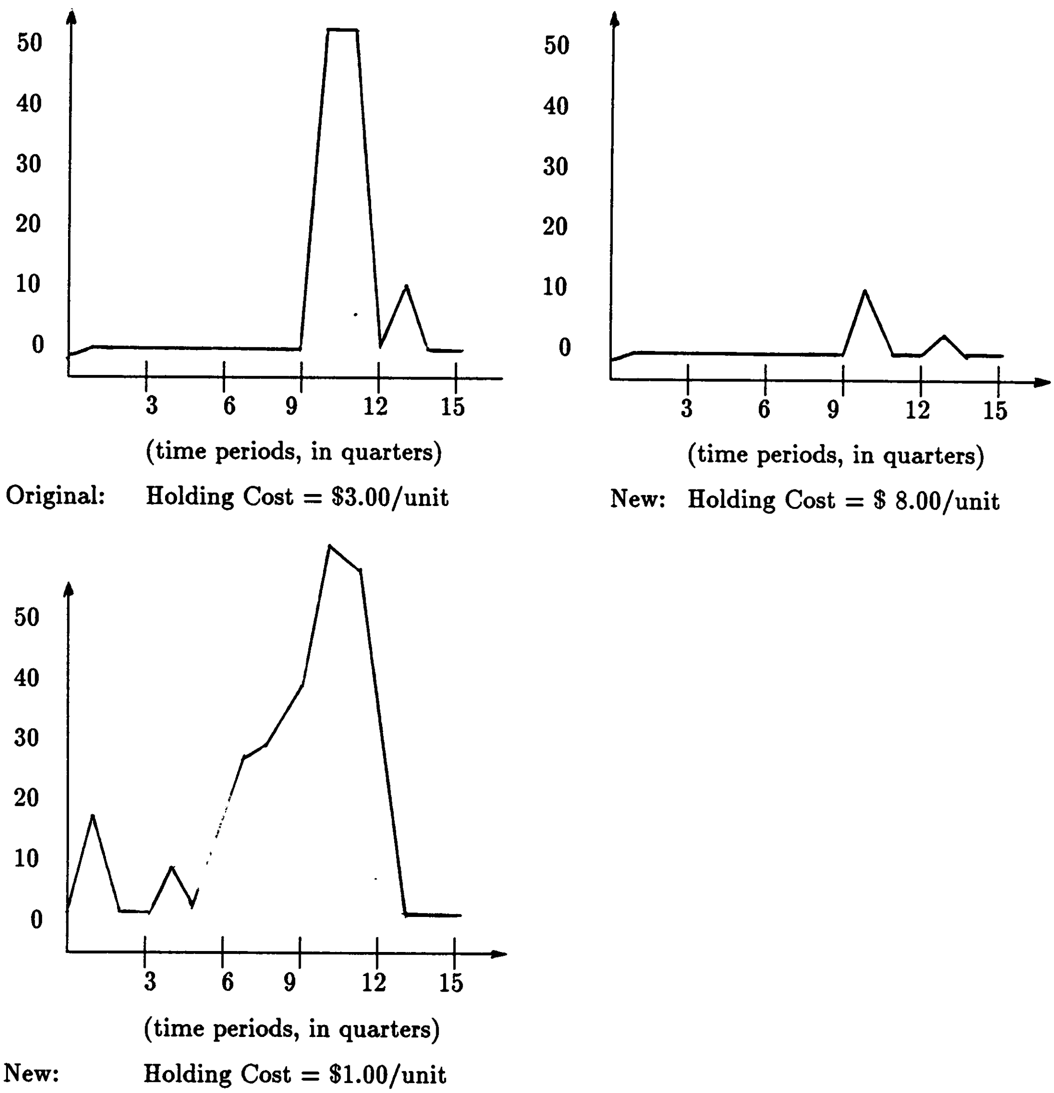
FIGURE 5.6 GROWTH IN CAPACITY--PLANNING HORIZON VARIED

the planning horizon. Here, the horizon was increased by one business quarter. The revised expansion strategy was to postpone an expansion originally made in the 9^{th} period until the 10^{th} period but at the same time increase the expansion from 5.4 units of capacity to 30.5 units of capacity. This change in strategy reflects an additional period of high automobile demand (746 automobiles at time 21, in thousands) from which costs incurred from additional expansion may be recovered.

Figure 5.7 demonstrates the sensitivity of inventory levels over time to changes in the inventory holding cost. When inventory holding costs were increased, the expansion path follow the demand curve more closely to meet periods of peak automobile demand without having to inventory large quantities of tires and incur additional cost. The holding costs for the three model run were increased to $8.00, left at the original $3.00, and decreased to $1.00 per period, respectively. Such changes in the expansion strategy (and likewise in the objective function--the present worth of the company) may be used to determine if expenditures to reduce holding cost (example, new inventory facilities) are worthwhile projects.

5.5 SUMMARY

The primary use of the model is to assist top level managers as they make decisions concerning capacity expansions in various industrial sectors. The optimum expansion strategy and the cost sensitivity results are very useful as tools for analyzing potential plant modernizations and/or the construction of new manufacturing facilities. The key to successfully implementating the model is to obtain reliable production, labor, material, and capital data for each manufacturer in the product tree. Given accurate production, capital, material, and labor data, the model identifies the optimal strategy for expanding operations.



Note: Safety Stock = 4

FIGURE 5.7 INVENTORY LEVELS--HOLDING COST VARIED

CHAPTER 6
SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 SUMMARY

It was stated in Chapter 1 of this thesis that a major weakness of current capacity expansion models is in their restriction to a single level of the product tree. The objective of this research was to develop an expansion model which would encompass the entire manufacturing process from raw materials to final products and which would account for market interactions with other sectors of the economy. The model would include labor, material, and capital contributions at each level of manufacture.

A literature search was conducted that summarized the types of modeling approaches which have been used for capacity expansion problems. Economic concepts of Leontief and Frisch were extended upon and used in this research.

The model development was presented in Chapter 3. All assumptions, decision variables, and equations were developed in detail. Data requirements, model reduction strategies, and computational limitations were described in Chapter 4, and the validity of the model was demonstrated in Chapter 5 by studying expansion decisions of a tire manufacturer. The case study also demonstrated how an analysis could be performed to test the optimal expansion strategy's sensitivity to variations in market estimates and model parameters.

6.2 CONCLUSIONS

Upon completion of this research, it does appear to model capacity expansion decisions and account for interactions with other industrial sectors of the economy. The model meets the objectives listed in Section 1.2. The concepts applied in this model proved to be useful in developing a generalized capacity expansion model. The extension of manufacturing operations to the entire conversion of raw materials to final products allows the model to be used by any industry for the purpose of finding optimal expansion decisions. The inclusion of material and capital resources in the model allows the manager to find the strategy that maximizes the present worth of the company. Furthermore, the sensitivity of the optimal solution to deviations in interest rates, market predictions, corporate tax rates, and other model parameters may be found as detailed in Section 5.4.

As the case for most research, several limiting assumptions were made to arrive at the present capacity expansion model. By relaxing these assumptions, the model's optimal strategy could be made even more reliable. Extensions of this research that would further enhance the model are detailed in the next section.

6.3 RECOMMENDATIONS

The research on capacity expansion decisions has only touched the surface of the issues important to capacity expansion decisions. Several extensions of this research to the modeling process are listed below. These extensions would allow the manager to more thoroughly test the expansion strategy identified by the model to economic factors which might influence the optimal decision.

1. Allow for improvements in manufacturing efficiency as a result of progress along the learning curve.
2. Allow for the cost of capacity expansions to be incurred throughout the planning, construction, and testing phases of the expansion.

3. Account for inflation in the currency rate over time.
4. Include domestic and non-domestic trade effects which would result from fluctuations in the currency rates.
5. Allow for a production delivery risk between each production level.
6. Include a capital budget risk for expansion costs.
7. Allow for backordering of manufactured products at all production levels.

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APPENDIX 1--LINDO INPUT FILE LISTING

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    0.595 N7K1  + 0.595 N8K1  + 0.595 N9K1  +
    0.595 N10K1 + 0.595 N11K1 + 0.595 N12K1 +
    0.595 N13K1 + 0.595 N14K1 + 0.595 N15K1 +
    0.595 N16K1 + 0.595 N17K1 + 0.595 N18K1 +
    0.595 N19K1 + 0.595 N20K1 + 0.595 N21K1
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+ D2M1  <      609.119
+ D3M1  <      625.246
+ D4M1  <      649.135
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		E7K1	-	E8K1	-	E9K1	<	585.00	
+ P10K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	<	585.00					
+ P11K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	<	585.00			
+ P12K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	<	585.00	
+ P13K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	<	585.00					
+ P14K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	-	E14K1	<	585.00			
+ P15K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	-	E14K1	-	E15K1	<	585.00	
+ P16K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	-	E14K1	-	E15K1	-		
		E16K1	<	585.00					
+ P17K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	-	E14K1	-	E15K1	-		
		E16K1	-	E17K1	<	585.00			
+ P18K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
		E4K1	-	E5K1	-	E6K1	-		
		E7K1	-	E8K1	-	E9K1	-		
		E10K1	-	E11K1	-	E12K1	-		
		E13K1	-	E14K1	-	E15K1	-		
		E16K1	-	E17K1	-	E18K1	<	585.00	

+	P19K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
			E4K1	-	E5K1	-	E6K1	-		
			E7K1	-	E8K1	-	E9K1	-		
			E10K1	-	E11K1	-	E12K1	-		
			E13K1	-	E14K1	-	E15K1	-		
			E16K1	-	E17K1	-	E18K1	-		
			E19K1	<	585.00					
+	P20K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
			E4K1	-	E5K1	-	E6K1	-		
			E7K1	-	E8K1	-	E9K1	-		
			E10K1	-	E11K1	-	E12K1	-		
			E13K1	-	E14K1	-	E15K1	-		
			E16K1	-	E17K1	-	E18K1	-		
			E19K1	-	E20K1	<	585.00			
+	P21K1	-	E0K1	-	E1K1	-	E2K1	-	E3K1	-
			E4K1	-	E5K1	-	E6K1	-		
			E7K1	-	E8K1	-	E9K1	-		
			E10K1	-	E11K1	-	E12K1	-		
			E13K1	-	E14K1	-	E15K1	-		
			E16K1	-	E17K1	-	E18K1	-		
			E19K1	-	E20K1	-	E21K1	<	585.00	
+	Q0K1	-	220.0 E0K1	=	0					
+	Q1K1	-	220.0 E1K1	=	0					
+	Q2K1	-	220.0 E2K1	=	0					
+	Q3K1	-	220.0 E3K1	=	0					
+	Q4K1	-	220.0 E4K1	=	0					
+	Q5K1	-	220.0 E5K1	=	0					
+	Q6K1	-	220.0 E6K1	=	0					
+	Q7K1	-	220.0 E7K1	=	0					
+	Q8K1	-	220.0 E8K1	=	0					
+	Q9K1	-	220.0 E9K1	=	0					
+	Q10K1	-	220.0 E10K1	=	0					
+	Q11K1	-	220.0 E11K1	=	0					
+	Q12K1	-	220.0 E12K1	=	0					
+	Q13K1	-	220.0 E13K1	=	0					
+	Q14K1	-	220.0 E14K1	=	0					
+	Q15K1	-	220.0 E15K1	=	0					
+	Q16K1	-	220.0 E16K1	=	0					
+	Q17K1	-	220.0 E17K1	=	0					
+	Q18K1	-	220.0 E18K1	=	0					
+	Q19K1	-	220.0 E19K1	=	0					
+	Q20K1	-	220.0 E20K1	=	0					
+	Q21K1	-	220.0 E21K1	=	0					
-	F0K1	+	31.270 D0K1	-	3.002 V0K1	-				
			19.897 P0K1	=	0					
-	F1K1	+	31.270 D1K1	-	3.002 V1K1	-				
			19.897 P1K1	=	0					
-	F2K1	+	31.270 D2K1	-	3.002 V2K1	-				
			19.897 P2K1	=	0					
-	F3K1	+	31.270 D3K1	-	3.002 V3K1	-				

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                19.897 P3K1     =  0
- F4K1     +    31.270 D4K1     -    3.002 V4K1     -
                19.897 P4K1     =  0
- F5K1     +    31.270 D5K1     -    3.002 V5K1     -
                19.897 P5K1     =  0
- F6K1     +    31.270 D6K1     -    3.002 V6K1     -
                19.897 P6K1     =  0
- F7K1     +    31.270 D7K1     -    3.002 V7K1     -
                19.897 P7K1     =  0
- F8K1     +    31.270 D8K1     -    3.002 V8K1     -
                19.897 P8K1     =  0
- F9K1     +    31.270 D9K1     -    3.002 V9K1     -
                19.897 P9K1     =  0
- F10K1    +    31.270 D10K1    -    3.002 V10K1    -
                19.897 P10K1    =  0
- F11K1    +    31.270 D11K1    -    3.002 V11K1    -
                19.897 P11K1    =  0
- F12K1    +    31.270 D12K1    -    3.002 V12K1    -
                19.897 P12K1    =  0
- F13K1    +    31.270 D13K1    -    3.002 V13K1    -
                19.897 P13K1    =  0
- F14K1    +    31.270 D14K1    -    3.002 V14K1    -
                19.897 P14K1    =  0
- F15K1    +    31.270 D15K1    -    3.002 V15K1    -
                19.897 P15K1    =  0
- F16K1    +    31.270 D16K1    -    3.002 V16K1    -
                19.897 P16K1    =  0
- F17K1    +    31.270 D17K1    -    3.002 V17K1    -
                19.897 P17K1    =  0
- F18K1    +    31.270 D18K1    -    3.002 V18K1    -
                19.897 P18K1    =  0
- F19K1    +    31.270 D19K1    -    3.002 V19K1    -
                19.897 P19K1    =  0
- F20K1    +    31.270 D20K1    -    3.002 V20K1    -
                19.897 P20K1    =  0
- F21K1    +    31.270 D21K1    -    3.002 V21K1    -
                19.897 P21K1    =  0
- T0K1   + 0.700 F0K1   + 1.000 Q0K1
- T1K1   + 0.700 F1K1   + 0.063 Q0K1   -
           1.000 Q1K1   = 0
- T2K1   + 0.700 F2K1   + 0.063 Q0K1   +
           0.063 Q1K1   - 1.000 Q2K1   = 0
- T3K1   + 0.700 F3K1   + 0.063 Q0K1   +
           0.063 Q1K1   + 0.063 Q2K1   - 1.000 Q3K1   = 0
- T4K1   + 0.700 F4K1   + 0.063 Q0K1   +
           0.063 Q1K1   + 0.063 Q2K1   + 0.063 Q3K1   -
           1.000 Q4K1   = 0
- T5K1   + 0.700 F5K1   + 0.063 Q0K1   +
           0.063 Q1K1   + 0.063 Q2K1   + 0.063 Q3K1   +
           0.063 Q4K1   - 1.000 Q5K1   = 0
```

```
- T6K1  + 0.700 F6K1  + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  - 1.000 Q6K1  = 0
- T7K1  + 0.700 F7K1  + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  -
          1.000 Q7K1  = 0
- T8K1  + 0.700 F8K1  + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  - 1.000 Q8K1  = 0
- T9K1  + 0.700 F9K1  + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  - 1.000 Q9K1  = 0
- T10K1 + 0.700 F10K1 + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  -
          1.000 Q10K1 = 0
- T11K1 + 0.700 F11K1 + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
          0.063 Q10K1 - 1.000 Q11K1 = 0
- T12K1 + 0.700 F12K1 + 0.063 Q0K1  +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
          0.063 Q10K1 + 0.063 Q11K1 - 1.000 Q12K1 = 0
- T13K1 + 0.700 F13K1 +
          0.063 Q1K1  + 0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
          0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 -
          1.000 Q13K1 = 0
- T14K1 + 0.700 F14K1 +
          0.063 Q2K1  + 0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
          0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
          0.063 Q13K1 - 1.000 Q14K1 = 0
- T15K1 + 0.700 F15K1 +
          0.063 Q3K1  +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
          0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
          0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
          0.063 Q13K1 + 0.063 Q14K1 - 1.000 Q15K1 = 0
- T16K1 + 0.700 F16K1 +
          0.063 Q4K1  + 0.063 Q5K1  + 0.063 Q6K1  +
```

```
                0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 -
                1.000 Q16K1 = 0
 - T17K1 + 0.700 F17K1 +
                0.063 Q5K1  + 0.063 Q6K1  +
                0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 +
                0.063 Q16K1 - 1.000 Q17K1 = 0
 - T18K1 + 0.700 F18K1 +
                0.063 Q6K1  +
                0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 +
                0.063 Q16K1 + 0.063 Q17K1 - 1.000 Q18K1 = 0
 - T19K1 + 0.700 F19K1 +
                0.063 Q7K1  + 0.063 Q8K1  + 0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 +
                0.063 Q16K1 + 0.063 Q17K1 + 0.063 Q18K1 -
                1.000 Q19K1 = 0
 - T20K1 + 0.700 F20K1 +
                0.063 Q8K1  + 0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 +
                0.063 Q16K1 + 0.063 Q17K1 + 0.063 Q18K1 +
                0.063 Q19K1 - 1.000 Q20K1 = 0
 - T21K1 + 0.700 F21K1 +
                0.063 Q9K1  +
                0.063 Q10K1 + 0.063 Q11K1 + 0.063 Q12K1 +
                0.063 Q13K1 + 0.063 Q14K1 + 0.063 Q15K1 +
                0.063 Q16K1 + 0.063 Q17K1 + 0.063 Q18K1 +
                0.063 Q19K1 + 0.063 Q20K1 - 1.000 Q21K1 = 0
  T0K1   + 1.000 B0K1 - 1.000 L0K1  = 0
  T1K1   + 1.000 B1K1  - 1.016 B0K1  -
           1.000 L1K1  + 1.010 L0K1  = 0
  T2K1   + 1.000 B2K1  - 1.016 B1K1  -
           1.000 L2K1  + 1.010 L1K1  = 0
  T3K1   + 1.000 B3K1  - 1.016 B2K1  -
           1.000 L3K1  + 1.010 L2K1  = 0
  T4K1   + 1.000 B4K1  - 1.016 B3K1  -
           1.000 L4K1  + 1.010 L3K1  = 0
  T5K1   + 1.000 B5K1  - 1.016 B4K1  -
           1.000 L5K1  + 1.010 L4K1  = 0
  T6K1   + 1.000 B6K1  - 1.016 B5K1  -
           1.000 L6K1  + 1.010 L5K1  = 0
  T7K1   + 1.000 B7K1  - 1.016 B6K1  -
           1.000 L7K1  + 1.010 L6K1  = 0
  T8K1   + 1.000 B8K1  - 1.016 B7K1  -
```

```
            1.000 L8K1  + 1.010 L7K1  = 0
 T9K1  +    1.000 B9K1  - 1.016 B8K1  -
            1.000 L9K1  + 1.010 L8K1  = 0
 T10K1 +    1.000 B10K1 - 1.016 B9K1  -
            1.000 L10K1 + 1.010 L9K1  = 0
 T11K1 +    1.000 B11K1 - 1.016 B10K1 -
            1.000 L11K1 + 1.010 L10K1 = 0
 T12K1 +    1.000 B12K1 - 1.016 B11K1 -
            1.000 L12K1 + 1.010 L11K1 = 0
 T13K1 +    1.000 B13K1 - 1.016 B12K1 -
            1.000 L13K1 + 1.010 L12K1 = 0
 T14K1 +    1.000 B14K1 - 1.016 B13K1 -
            1.000 L14K1 + 1.010 L13K1 = 0
 T15K1 +    1.000 B15K1 - 1.016 B14K1 -
            1.000 L15K1 + 1.010 L14K1 = 0
 T16K1 +    1.000 B16K1 - 1.016 B15K1 -
            1.000 L16K1 + 1.010 L15K1 = 0
 T17K1 +    1.000 B17K1 - 1.016 B16K1 -
            1.000 L17K1 + 1.010 L16K1 = 0
 T18K1 +    1.000 B18K1 - 1.016 B17K1 -
            1.000 L18K1 + 1.010 L17K1 = 0
 T19K1 +    1.000 B19K1 - 1.016 B18K1 -
            1.000 L19K1 + 1.010 L18K1 = 0
 T20K1 +    1.000 B20K1 - 1.016 B19K1 -
            1.000 L20K1 + 1.010 L19K1 = 0
-N21K1 +          T21K1 + 1.016 B20K1 -
                          1.010 L20K1 = 0
END
SUB P0K2     3600.000
SUB P1K2     3619.800
SUB P2K2     3639.709
SUB P3K2     3659.727
SUB P4K2     3679.856
SUB P5K2     3700.095
SUB P6K2     3720.446
SUB P7K2     3740.908
SUB P8K2     3761.483
SUB P9K2     3782.171
SUB P10K2    3802.973
SUB P11K2    3823.889
SUB P12K2    3844.921
SUB P13K2    3866.068
SUB P14K2    3887.331
SUB P15K2    3908.712
SUB P16K2    3930.209
SUB P17K2    3951.826
SUB P18K2    3973.561
SUB P19K2    3995.415
SUB P20K2    4017.390
SUB P21K2    4039.486
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SUB	P0K3	3750.000
SUB	P1K3	3767.812
SUB	P2K3	3785.710
SUB	P3K3	3803.692
SUB	P4K3	3821.759
SUB	P5K3	3839.913
SUB	P6K3	3858.152
SUB	P7K3	3876.478
SUB	P8K3	3894.892
SUB	P9K3	3913.392
SUB	P10K3	3931.981
SUB	P11K3	3950.658
SUB	P12K3	3969.424
SUB	P13K3	3988.278
SUB	P14K3	4007.223
SUB	P15K3	4026.257
SUB	P16K3	4045.382
SUB	P17K3	4064.597
SUB	P18K3	4083.904
SUB	P19K3	4103.303
SUB	P20K3	4122.793
SUB	P21K3	4142.377
SUB	P0L1	3970.000
SUB	P1L1	4018.632
SUB	P2L1	4067.861
SUB	P3L1	4117.692
SUB	P4L1	4168.134
SUB	P5L1	4219.193
SUB	P6L1	4270.879
SUB	P7L1	4323.197
SUB	P8L1	4376.156
SUB	P9L1	4429.764
SUB	P10L1	4484.028
SUB	P11L1	4538.958
SUB	P12L1	4594.560
SUB	P13L1	4650.843
SUB	P14L1	4707.816
SUB	P15L1	4765.487
SUB	P16L1	4823.864
SUB	P17L1	4882.957
SUB	P18L1	4942.773
SUB	P19L1	5003.322
SUB	P20L1	5064.612
SUB	P21L1	5126.654
SUB	P0M1	650.000
SUB	P1M1	657.637
SUB	P2M1	665.365
SUB	P3M1	673.183
SUB	P4M1	681.093
SUB	P5M1	689.096

SUB P6M1	697.192
SUB P7M1	705.384
SUB P8M1	713.673
SUB P9M1	722.058
SUB P10M1	730.543
SUB P11M1	739.126
SUB P12M1	747.811
SUB P13M1	756.598
SUB P14M1	765.488
SUB P15M1	774.482
SUB P16M1	783.583
SUB P17M1	792.790
SUB P18M1	802.105
SUB P19M1	811.530
SUB P20M1	821.065
SUB P21M1	830.713
SLB V0K1	8.0
SLB V1K1	4.0
SLB V2K1	4.0
SLB V3K1	4.0
SLB V4K1	4.0
SLB V5K1	4.0
SLB V6K1	4.0
SLB V7K1	4.0
SLB V8K1	4.0
SLB V9K1	4.0
SLB V10K1	4.0
SLB V11K1	4.0
SLB V12K1	4.0
SLB V13K1	4.0
SLB V14K1	4.0
SLB V15K1	4.0
SLB V16K1	4.0
SLB V17K1	4.0
SLB V18K1	4.0
SLB V19K1	4.0
SLB V20K1	4.0
SLB V21K1	4.0
SUB E0K1	240.0
SUB E1K1	240.0
SUB E2K1	240.0
SUB E3K1	240.0
SUB E4K1	240.0
SUB E5K1	240.0
SUB E6K1	240.0
SUB E7K1	240.0
SUB E8K1	240.0
SUB E9K1	240.0
SUB E10K1	240.0
SUB E11K1	240.0

```
SUB E12K1     240.0
SUB E13K1     240.0
SUB E14K1     240.0
SUB E15K1     240.0
SUB E16K1     240.0
SUB E17K1     240.0
SUB E18K1     240.0
SUB E19K1     240.0
SUB E20K1     240.0
SUB E21K1     240.0
SLB T0K1     - 450000.0
SLB T1K1     - 450000.0
SLB T2K1     - 450000.0
SLB T3K1     - 450000.0
SLB T4K1     - 450000.0
SLB T5K1     - 450000.0
SLB T6K1     - 450000.0
SLB T7K1     - 450000.0
SLB T8K1     - 450000.0
SLB T9K1     - 450000.0
SLB T10K1    - 450000.0
SLB T11K1    - 450000.0
SLB T12K1    - 450000.0
SLB T13K1    - 450000.0
SLB T14K1    - 450000.0
SLB T15K1    - 450000.0
SLB T16K1    - 450000.0
SLB T17K1    - 450000.0
SLB T18K1    - 450000.0
SLB T19K1    - 450000.0
SLB T20K1    - 450000.0
SLB T21K1    - 450000.0
SUB B0K1     450000.000
SUB B1K1     450000.000
SUB B2K1     450000.000
SUB B3K1     450000.000
SUB B4K1     450000.000
SUB B5K1     450000.000
SUB B6K1     450000.000
SUB B7K1     450000.000
SUB B8K1     450000.000
SUB B9K1     450000.000
SUB B10K1    450000.000
SUB B11K1    450000.000
SUB B12K1    450000.000
SUB B13K1    450000.000
SUB B14K1    450000.000
SUB B15K1    450000.000
SUB B16K1    450000.000
SUB B17K1    450000.000
```

SUB	B18K1	450000.000
SUB	B19K1	450000.000
SUB	B20K1	450000.000
SUB	B21K1	450000.000

APPENDIX 2--LINDO OUTPUT LISTING

LP OPTIMUM FOUND AT STEP 285

OBJECTIVE FUNCTION VALUE

1) 61973.6250

VARIABLE	VALUE	REDUCED COST
N21K1	111664.062000	0.000000
D0M1	610.808838	0.000000
D1M1	600.429932	0.000000
D2M1	628.002930	0.000000
D3M1	626.531982	0.000000
D4M1	633.157959	0.000000
D5M1	665.221924	0.000000
D6M1	644.871826	0.000000
D7M1	653.169922	0.000000
D8M1	649.777100	0.000000
D9M1	649.272949	0.000000
D10M1	622.218994	0.000000
D11M1	671.337891	0.000000
D12M1	654.261963	0.000000
D13M1	649.272949	0.000000
D14M1	649.272949	0.000000
D15M1	649.272949	0.000000
D16M1	649.272949	0.000000
D17M1	649.272949	0.000000
D18M1	649.272949	0.000000
D19M1	649.272949	0.000000
D20M1	649.272949	0.000000
D21M1	649.272949	0.000000
D0K1	610.808838	0.000000
D1K1	600.429932	0.000000
D2K1	628.002930	0.000000
D3K1	626.531982	0.000000
D4K1	633.157959	0.000000
D5K1	665.221924	0.000000
D6K1	644.871826	0.000000
D7K1	653.169922	0.000000
D8K1	649.777100	0.000000
D9K1	649.272949	0.000000
D10K1	622.218994	0.000000
D11K1	671.337891	0.000000

D12K1	654.261963	0.000000
D13K1	649.272949	0.000000
D14K1	649.272949	0.000000
D15K1	649.272949	0.000000
D16K1	649.272949	0.000000
D17K1	649.272949	0.000000
D18K1	649.272949	0.000000
D19K1	649.272949	0.000000
D20K1	649.272949	0.000000
D21K1	649.272949	0.000000
P0K1	613.808838	0.000000
V0K1	3.000000	4.537740
P1K1	602.900879	0.000000
V1K1	5.470947	0.000000
P2K1	626.531982	0.000000
V2K1	4.000000	2.487738
P3K1	626.531982	0.000000
V3K1	4.000000	1.229093
P4K1	649.106934	0.000000
V4K1	19.948975	0.000000
P5K1	649.272949	0.000000
V5K1	4.000000	2.464398
P6K1	649.272949	0.000000
V6K1	8.401123	0.000000
P7K1	649.272949	0.000000
V7K1	4.504150	0.000000
P8K1	649.272949	0.000000
V8K1	4.000000	4.693596
P9K1	649.272949	0.000000
V9K1	4.000000	1.354695
P10K1	649.272949	0.000000
V10K1	31.053955	0.000000
P11K1	649.272949	0.000000
V11K1	8.989014	0.000000
P12K1	649.272949	0.000000
V12K1	4.000000	1.438685
P13K1	649.272949	0.000000
V13K1	4.000000	1.410479
P14K1	649.272949	0.000000
V14K1	4.000000	1.382824
P15K1	649.272949	0.000000
V15K1	4.000000	1.355708
P16K1	649.272949	0.000000
V16K1	4.000000	1.329128
P17K1	649.272949	0.000000
V17K1	4.000000	1.303066
P18K1	649.272949	0.000000
V18K1	4.000000	1.277516
P19K1	649.272949	0.000000
V19K1	4.000000	1.252467

P20K1	649.272949	0.000000
V20K1	4.000000	1.227910
P21K1	649.272949	0.000000
V21K1	4.000000	11.412569
P0M1	610.808838	0.000000
P1M1	600.429932	0.000000
P2M1	628.002930	0.000000
P3M1	626.531982	0.000000
P4M1	633.157959	0.000000
P5M1	665.221924	0.000000
P6M1	644.871826	0.000000
P7M1	653.169922	0.000000
P8M1	649.777100	0.000000
P9M1	649.272949	0.000000
P10M1	622.218994	0.000000
P11M1	671.337891	0.000000
P12M1	654.261963	0.000000
P13M1	649.272949	0.000000
P14M1	649.272949	0.000000
P15M1	649.272949	0.000000
P16M1	649.272949	0.000000
P17M1	649.272949	0.000000
P18M1	649.272949	0.000000
P19M1	649.272949	0.000000
P20M1	649.272949	0.000000
P21M1	649.272949	0.000000
E0K1	28.808838	0.000000
E1K1	0.000000	0.000000
E2K1	12.723145	0.000000
E3K1	0.000000	0.000000
E4K1	22.574951	0.000000
E5K1	0.166016	0.000000
E6K1	0.000000	0.000000
E7K1	0.000000	0.000000
E8K1	0.000000	0.000000
E9K1	0.000000	0.000000
E10K1	0.000000	0.000000
E11K1	0.000000	0.000000
E12K1	0.000000	0.000000
E13K1	0.000000	0.000000
E14K1	0.000000	0.000000
E15K1	0.000000	0.000000
E16K1	0.000000	0.000000
E17K1	0.000000	0.000000
E18K1	0.000000	0.000000
E19K1	0.000000	0.000000
E20K1	0.000000	0.000000
E21K1	0.000000	129.412766
Q0K1	6914.121090	0.000000
Q1K1	0.000000	0.005397

Q2K1	3053.554690	0.000000
Q3K1	0.000000	0.001703
Q4K1	5417.988280	0.000000
Q5K1	39.843750	0.000000
Q6K1	0.000000	0.006690
Q7K1	0.000000	0.009670
Q8K1	0.000000	0.019081
Q9K1	0.000000	0.034796
Q10K1	0.000000	0.037559
Q11K1	0.000000	0.047280
Q12K1	0.000000	0.062282
Q13K1	0.000000	0.085237
Q14K1	0.000000	0.109495
Q15K1	0.000000	0.135072
Q16K1	0.000000	0.160877
Q17K1	0.000000	0.188037
Q18K1	0.000000	0.215459
Q19K1	0.000000	0.243714
Q20K1	0.000000	0.272264
Q21K1	0.000000	0.000000
F0K1	6878.031250	0.000000
F1K1	6763.101560	0.000000
F2K1	7159.535160	0.000000
F3K1	7113.539060	0.000000
F4K1	6823.679690	0.000000
F5K1	7870.894530	0.000000
F6K1	7221.335940	0.000000
F7K1	7492.515620	0.000000
F8K1	7387.937500	0.000000
F9K1	7372.171870	0.000000
F10K1	6444.980470	0.000000
F11K1	8047.164060	0.000000
F12K1	7528.179690	0.000000
F13K1	7372.171870	0.000000
F14K1	7372.171870	0.000000
F15K1	7372.171870	0.000000
F16K1	7372.171870	0.000000
F17K1	7372.171870	0.000000
F18K1	7372.171870	0.000000
F19K1	7372.171870	0.000000
F20K1	7372.171870	0.000000
F21K1	7372.171870	0.000000
T0K1	-2787.312500	0.000000
T1K1	4493.437500	0.000000
T2K1	1677.750000	0.000000
T3K1	4896.062500	0.000000
T4K1	-695.875000	0.000000
T5K1	5651.937500	0.000000
T6K1	5304.562500	0.000000
T7K1	5467.312500	0.000000

T8K1	5404.562500	0.000000
T9K1	5395.062500	0.000000
T10K1	4838.750000	0.000000
T11K1	5800.062500	0.000000
T12K1	5488.687500	0.000000
T13K1	4959.500000	0.000000
T14K1	4959.500000	0.000000
T15K1	4767.125000	0.000000
T16K1	4767.125000	0.000000
T17K1	4425.812500	0.000000
T18K1	4423.250000	0.000000
T19K1	4423.250000	0.000000
T20K1	4423.250000	0.000000
T21K1	4423.250000	0.000000
B0K1	2787.301760	0.000000
L0K1	0.000000	0.000825
B1K1	0.000000	0.000809
L1K1	1647.615480	0.000000
B2K1	0.000000	0.000793
L2K1	3358.323730	0.000000
B3K1	0.000000	0.000777
L3K1	8321.574220	0.000000
B4K1	0.000000	0.000762
L4K1	7792.187500	0.000000
B5K1	0.000000	0.000747
L5K1	13600.015600	0.000000
B6K1	0.000000	0.000733
L6K1	19176.621100	0.000000
B7K1	0.000000	0.000718
L7K1	25027.460900	0.000000
B8K1	0.000000	0.000704
L8K1	30932.566400	0.000000
B9K1	0.000000	0.000690
L9K1	36946.312500	0.000000
B10K1	0.000000	0.000677
L10K1	42524.015600	0.000000
B11K1	0.000000	0.000664
L11K1	49174.582000	0.000000
B12K1	0.000000	0.000651
L12K1	55646.765600	0.000000
B13K1	0.000000	0.000638
L13K1	61719.191400	0.000000
B14K1	0.000000	0.000625
L14K1	67913.062500	0.000000
B15K1	0.000000	0.000613
L15K1	74038.437500	0.000000
B16K1	0.000000	0.000601
L16K1	80286.312500	0.000000
B17K1	0.000000	0.000589
L17K1	86317.812500	0.000000

B18K1	0.000000	0.000578
L18K1	92467.437500	0.000000
B19K1	0.000000	0.000566
L19K1	98740.000000	0.000000
B20K1	0.000000	0.000555
L20K1	105138.062000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	2.931349
3)	0.000000	5.627532
4)	0.000000	3.838716
5)	0.000000	4.572711
6)	0.000000	4.082448
7)	0.000000	2.396804
8)	0.000000	3.208611
9)	0.000000	1.588420
10)	9.794678	0.000000
11)	0.000000	3.136317
12)	0.000000	2.964271
13)	0.000000	1.467461
14)	64.113037	0.000000
15)	14.993896	0.000000
16)	66.593018	0.000000
17)	53.464844	0.000000
18)	48.852051	0.000000
19)	60.088867	0.000000
20)	103.574951	0.000000
21)	106.950928	0.000000
22)	89.835938	0.000000
23)	96.950928	0.000000
24)	0.000000	-2.931349
25)	0.000000	-5.627532
26)	0.000000	-3.838716
27)	0.000000	-4.572711
28)	0.000000	-4.082448
29)	0.000000	-2.396804
30)	0.000000	-3.208611
31)	0.000000	-1.588420
32)	0.000000	0.000000
33)	0.000000	-3.136317
34)	0.000000	-2.964271
35)	0.000000	-1.467461
36)	0.000000	0.000000
37)	0.000000	0.000000
38)	0.000000	0.000000
39)	0.000000	0.000000
40)	0.000000	0.000000
41)	0.000000	0.000000
42)	0.000000	0.000000

43)	0.000000	0.000000
44)	0.000000	0.000000
45)	0.000000	0.000000
46)	0.000000	-12.866442
47)	0.000000	-9.845330
48)	0.000000	-11.330765
49)	0.000000	-10.299336
50)	0.000000	-10.497998
51)	0.000000	-11.897758
52)	0.000000	-10.805675
53)	0.000000	-12.151082
54)	0.000000	-13.470109
55)	0.000000	-10.069678
56)	0.000000	-9.982792
57)	0.000000	-11.225743
58)	0.000000	-12.444324
59)	0.000000	-12.200325
60)	0.000000	-11.961109
61)	0.000000	-11.726583
62)	0.000000	-11.496656
63)	0.000000	-11.271236
64)	0.000000	-11.050238
65)	0.000000	-10.833572
66)	0.000000	-10.621156
67)	0.000000	-10.412904
68)	0.000000	0.000000
69)	0.000000	0.000000
70)	0.000000	0.000000
71)	0.000000	0.000000
72)	0.000000	0.000000
73)	0.000000	0.000000
74)	0.000000	0.000000
75)	0.000000	0.000000
76)	0.000000	0.000000
77)	0.000000	0.000000
78)	0.000000	0.000000
79)	0.000000	0.000000
80)	0.000000	0.000000
81)	0.000000	0.000000
82)	0.000000	0.000000
83)	0.000000	0.000000
84)	0.000000	0.000000
85)	0.000000	0.000000
86)	0.000000	0.000000
87)	0.000000	0.000000
88)	0.000000	0.000000
89)	0.000000	0.000000
90)	0.000000	2.814362
91)	10.907959	0.000000
92)	0.000000	1.678476

93)	0.000000	0.836304
94)	0.000000	1.220510
95)	0.000000	2.802177
96)	0.000000	1.888433
97)	0.000000	3.408684
98)	0.000000	4.899125
99)	0.000000	1.666748
100)	0.000000	1.744620
101)	0.000000	3.149101
102)	0.000000	4.526043
103)	0.000000	4.437300
104)	0.000000	4.350296
105)	0.000000	4.264997
106)	0.000000	4.181374
107)	0.000000	4.099387
108)	0.000000	4.019010
109)	0.000000	3.940207
110)	0.000000	3.862950
111)	0.000000	3.787209
112)	0.000000	-0.281572
113)	0.000000	-0.269845
114)	0.000000	-0.269845
115)	0.000000	-0.262851
116)	0.000000	-0.259367
117)	0.000000	-0.254282
118)	0.000000	-0.242606
119)	0.000000	-0.234737
120)	0.000000	-0.220535
121)	0.000000	-0.200122
122)	0.000000	-0.193177
123)	0.000000	-0.185908
124)	0.000000	-0.172786
125)	0.000000	-0.153928
126)	0.000000	-0.135439
127)	0.000000	-0.117313
128)	0.000000	-0.099542
129)	0.000000	-0.082120
130)	0.000000	-0.065039
131)	0.000000	-0.048293
132)	0.000000	-0.031876
133)	0.000000	-0.555000
134)	0.000000	-0.505206
135)	0.000000	-0.494815
136)	0.000000	-0.485113
137)	0.000000	-0.475601
138)	0.000000	-0.466276
139)	0.000000	-0.457134
140)	0.000000	-0.448170
141)	0.000000	-0.439383
142)	0.000000	-0.430768

143)	0.000000	-0.422322
144)	0.000000	-0.414041
145)	0.000000	-0.405923
146)	0.000000	-0.397964
147)	0.000000	-0.390161
148)	0.000000	-0.382511
149)	0.000000	-0.375011
150)	0.000000	-0.367658
151)	0.000000	-0.360449
152)	0.000000	-0.353382
153)	0.000000	-0.346453
154)	0.000000	-0.339660
155)	0.000000	-0.333000
156)	0.000000	-0.842010
157)	0.000000	-0.824692
158)	0.000000	-0.808522
159)	0.000000	-0.792669
160)	0.000000	-0.777127
161)	0.000000	-0.761889
162)	0.000000	-0.746951
163)	0.000000	-0.732305
164)	0.000000	-0.717947
165)	0.000000	-0.703870
166)	0.000000	-0.690069
167)	0.000000	-0.676538
168)	0.000000	-0.663273
169)	0.000000	-0.650268
170)	0.000000	-0.637518
171)	0.000000	-0.625018
172)	0.000000	-0.612763
173)	0.000000	-0.600748
174)	0.000000	-0.588969
175)	0.000000	-0.577421
176)	0.000000	-0.566100
177)	0.000000	-0.555000
178)	0.000000	-0.842010
179)	0.000000	-0.824692
180)	0.000000	-0.808522
181)	0.000000	-0.792669
182)	0.000000	-0.777127
183)	0.000000	-0.761889
184)	0.000000	-0.746951
185)	0.000000	-0.732305
186)	0.000000	-0.717947
187)	0.000000	-0.703870
188)	0.000000	-0.690069
189)	0.000000	-0.676538
190)	0.000000	-0.663273
191)	0.000000	-0.650268
192)	0.000000	-0.637518

193)	0.000000	-0.625018
194)	0.000000	-0.612763
195)	0.000000	-0.600748
196)	0.000000	-0.588969
197)	0.000000	-0.577421
198)	0.000000	-0.566100
199)	0.000000	-0.555000

NO. ITERATIONS= 285

RANGES IN WHICH THE BASIS IS UNCHANGED:

VARIABLE		OBJ COEFFICIENT RANGES	
	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
N21K1	0.555000	INFINITY	0.554743
D0M1	0.000000	INFINITY	2.931349
D1M1	0.000000	INFINITY	5.627532
D2M1	0.000000	INFINITY	3.838716
D3M1	0.000000	INFINITY	4.572711
D4M1	0.000000	INFINITY	4.082448
D5M1	0.000000	INFINITY	2.396804
D6M1	0.000000	INFINITY	3.208611
D7M1	0.000000	INFINITY	1.588420
D8M1	0.000000	0.451565	1.045439
D9M1	0.000000	INFINITY	3.136317
D10M1	0.000000	INFINITY	2.964271
D11M1	0.000000	INFINITY	1.467461
D12M1	0.000000	0.338674	1.045439
D13M1	0.000000	1.354695	1.410479
D14M1	0.000000	1.354695	1.382824
D15M1	0.000000	1.354695	1.355708
D16M1	0.000000	1.354695	1.329128
D17M1	0.000000	1.329128	1.303066
D18M1	0.000000	1.303066	1.277516
D19M1	0.000000	1.277516	1.252467
D20M1	0.000000	1.252467	1.227910
D21M1	0.000000	1.227910	3.136317
D0K1	0.000000	INFINITY	2.931349
D1K1	0.000000	INFINITY	5.627532
D2K1	0.000000	INFINITY	3.838716
D3K1	0.000000	INFINITY	4.572711
D4K1	0.000000	INFINITY	4.082448
D5K1	0.000000	INFINITY	2.396804
D6K1	0.000000	INFINITY	3.208611
D7K1	0.000000	INFINITY	1.588420
D8K1	0.000000	0.451565	1.045439
D9K1	0.000000	INFINITY	3.136317
D10K1	0.000000	INFINITY	2.964271
D11K1	0.000000	INFINITY	1.467461

D12K1	0.000000	0.338674	1.045439
D13K1	0.000000	1.354695	1.410479
D14K1	0.000000	1.354695	1.382824
D15K1	0.000000	1.354695	1.355708
D16K1	0.000000	1.354695	1.329128
D17K1	0.000000	1.329128	1.303066
D18K1	0.000000	1.303066	1.277516
D19K1	0.000000	1.277516	1.252467
D20K1	0.000000	1.252467	1.227910
D21K1	0.000000	1.227910	3.136317
P0K1	0.000000	4.537740	2.931349
V0K1	0.000000	4.537740	INFINITY
P1K1	0.000000	0.408694	0.836304
V1K1	0.000000	0.408694	0.836304
P2K1	0.000000	0.836304	0.408694
V2K1	0.000000	2.487738	INFINITY
P3K1	0.000000	1.229093	2.487738
V3K1	0.000000	1.229093	INFINITY
P4K1	0.000000	1.605636	1.229093
V4K1	0.000000	1.605636	1.354695
P5K1	0.000000	1.354695	1.605636
V5K1	0.000000	2.464398	INFINITY
P6K1	0.000000	1.354695	1.888433
V6K1	0.000000	1.354695	1.888433
P7K1	0.000000	1.354695	3.136317
V7K1	0.000000	0.677348	1.568158
P8K1	0.000000	1.354695	3.136317
V8K1	0.000000	4.693596	INFINITY
P9K1	0.000000	1.354695	3.136317
V9K1	0.000000	1.354695	INFINITY
P10K1	0.000000	1.354695	1.744620
V10K1	0.000000	0.677348	1.744620
P11K1	0.000000	1.354695	3.136317
V11K1	0.000000	0.451565	1.568158
P12K1	0.000000	1.354695	3.136317
V12K1	0.000000	1.438685	INFINITY
P13K1	0.000000	1.354695	3.136317
V13K1	0.000000	1.410479	INFINITY
P14K1	0.000000	1.354695	3.136317
V14K1	0.000000	1.382824	INFINITY
P15K1	0.000000	1.354695	3.136317
V15K1	0.000000	1.355708	INFINITY
P16K1	0.000000	1.354695	3.136317
V16K1	0.000000	1.329128	INFINITY
P17K1	0.000000	1.354695	3.136317
V17K1	0.000000	1.303066	INFINITY
P18K1	0.000000	1.354695	3.136317
V18K1	0.000000	1.277516	INFINITY
P19K1	0.000000	1.354695	3.136317
V19K1	0.000000	1.252467	INFINITY

P20K1	0.000000	1.354695	3.136317
V20K1	0.000000	1.227910	INFINITY
P21K1	0.000000	1.354695	3.136317
V21K1	0.000000	11.412569	INFINITY
P0M1	0.000000	0.000000	2.931349
P1M1	0.000000	0.000000	5.627532
P2M1	0.000000	0.000000	3.838716
P3M1	0.000000	0.000000	4.572711
P4M1	0.000000	0.000000	4.082448
P5M1	0.000000	0.000000	2.396804
P6M1	0.000000	0.000000	3.208611
P7M1	0.000000	0.000000	1.588420
P8M1	0.000000	0.000000	1.045439
P9M1	0.000000	0.000000	3.136317
P10M1	0.000000	0.000000	2.964271
P11M1	0.000000	0.000000	1.467461
P12M1	0.000000	0.000000	1.045439
P13M1	0.000000	0.000000	1.410479
P14M1	0.000000	0.000000	1.382824
P15M1	0.000000	0.000000	1.355708
P16M1	0.000000	0.000000	1.329128
P17M1	0.000000	0.000000	1.303066
P18M1	0.000000	0.000000	1.277516
P19M1	0.000000	0.000000	1.252467
P20M1	0.000000	0.000000	1.227910
P21M1	0.000000	0.000000	3.136317
E0K1	0.000000	2.814362	2.931349
E1K1	0.000000	1.295244	INFINITY
E2K1	0.000000	0.836304	0.408694
E3K1	0.000000	0.408694	INFINITY
E4K1	0.000000	1.220510	0.614546
E5K1	0.000000	0.677348	0.802818
E6K1	0.000000	1.605636	INFINITY
E7K1	0.000000	2.320920	INFINITY
E8K1	0.000000	4.579475	INFINITY
E9K1	0.000000	8.351027	INFINITY
E10K1	0.000000	9.014182	INFINITY
E11K1	0.000000	11.347160	INFINITY
E12K1	0.000000	14.947715	INFINITY
E13K1	0.000000	20.456985	INFINITY
E14K1	0.000000	26.278748	INFINITY
E15K1	0.000000	32.417297	INFINITY
E16K1	0.000000	38.610474	INFINITY
E17K1	0.000000	45.128845	INFINITY
E18K1	0.000000	51.710068	INFINITY
E19K1	0.000000	58.491318	INFINITY
E20K1	0.000000	65.343369	INFINITY
E21K1	0.000000	129.412766	INFINITY
Q0K1	0.000000	0.011727	0.012214
Q1K1	0.000000	0.005397	INFINITY

Q2K1	0.000000	0.003485	0.001703
Q3K1	0.000000	0.001703	INFINITY
Q4K1	0.000000	0.005085	0.002561
Q5K1	0.000000	0.002822	0.003345
Q6K1	0.000000	0.006690	INFINITY
Q7K1	0.000000	0.009670	INFINITY
Q8K1	0.000000	0.019081	INFINITY
Q9K1	0.000000	0.034796	INFINITY
Q10K1	0.000000	0.037559	INFINITY
Q11K1	0.000000	0.047280	INFINITY
Q12K1	0.000000	0.062282	INFINITY
Q13K1	0.000000	0.085237	INFINITY
Q14K1	0.000000	0.109495	INFINITY
Q15K1	0.000000	0.135072	INFINITY
Q16K1	0.000000	0.160877	INFINITY
Q17K1	0.000000	0.188037	INFINITY
Q18K1	0.000000	0.215459	INFINITY
Q19K1	0.000000	0.243714	INFINITY
Q20K1	0.000000	0.272264	INFINITY
Q21K1	0.000000	0.539220	INFINITY
F0K1	0.000000	INFINITY	0.198163
F1K1	0.000000	0.036521	0.017848
F2K1	0.000000	0.020540	0.042032
F3K1	0.000000	0.125031	0.053675
F4K1	0.000000	0.059160	0.070118
F5K1	0.000000	0.080697	0.068085
F6K1	0.000000	0.082468	0.059160
F7K1	0.000000	0.121089	0.046347
F8K1	0.000000	0.018328	0.042433
F9K1	0.000000	0.235895	0.059160
F10K1	0.000000	0.076188	0.052303
F11K1	0.000000	0.121089	0.042818
F12K1	0.000000	0.012879	0.041978
F13K1	0.000000	0.046009	0.041155
F14K1	0.000000	0.045106	0.040349
F15K1	0.000000	0.044222	0.039557
F16K1	0.000000	0.043355	0.038782
F17K1	0.000000	0.042505	0.038021
F18K1	0.000000	0.041671	0.037276
F19K1	0.000000	0.040854	0.036545
F20K1	0.000000	0.040053	0.035828
F21K1	0.000000	0.039268	0.275769
T0K1	0.000000	0.012571	0.011727
T1K1	0.000000	0.060869	0.005397
T2K1	0.000000	0.001622	0.003319
T3K1	0.000000	0.055311	0.001602
T4K1	0.000000	0.002424	0.005085
T5K1	0.000000	0.003167	0.002672
T6K1	0.000000	0.137447	0.006294
T7K1	0.000000	0.248096	0.009162

T8K1	0.000000	0.022780	0.015649
T9K1	0.000000	0.110235	0.032734
T10K1	0.000000	0.126979	0.035525
T11K1	0.000000	0.229202	0.044720
T12K1	0.000000	0.017317	0.052740
T13K1	0.000000	0.061735	0.068592
T14K1	0.000000	0.061735	0.067248
T15K1	0.000000	0.027030	0.055311
T16K1	0.000000	0.061735	0.055311
T17K1	0.000000	0.036550	0.063369
T18K1	0.000000	0.069452	0.062126
T19K1	0.000000	0.068091	0.060908
T20K1	0.000000	0.066756	0.059714
T21K1	0.000000	0.065447	0.459615
B0K1	0.000000	0.000825	0.012571
L0K1	0.000000	0.000825	INFINITY
B1K1	0.000000	0.000809	INFINITY
L1K1	0.000000	0.000809	0.005397
B2K1	0.000000	0.000793	INFINITY
L2K1	0.000000	0.000793	0.003515
B3K1	0.000000	0.000777	INFINITY
L3K1	0.000000	0.000777	0.003675
B4K1	0.000000	0.000762	INFINITY
L4K1	0.000000	0.000762	0.005085
B5K1	0.000000	0.000747	INFINITY
L5K1	0.000000	0.000747	0.005457
B6K1	0.000000	0.000733	INFINITY
L6K1	0.000000	0.000733	0.005379
B7K1	0.000000	0.000718	INFINITY
L7K1	0.000000	0.000718	0.005265
B8K1	0.000000	0.000704	INFINITY
L8K1	0.000000	0.000704	0.006675
B9K1	0.000000	0.000690	INFINITY
L9K1	0.000000	0.000690	0.006588
B10K1	0.000000	0.000677	INFINITY
L10K1	0.000000	0.000677	0.006446
B11K1	0.000000	0.000664	INFINITY
L11K1	0.000000	0.000664	0.006254
B12K1	0.000000	0.000651	INFINITY
L12K1	0.000000	0.000651	0.009493
B13K1	0.000000	0.000638	INFINITY
L13K1	0.000000	0.000638	0.010959
B14K1	0.000000	0.000625	INFINITY
L14K1	0.000000	0.000625	0.013008
B15K1	0.000000	0.000613	INFINITY
L15K1	0.000000	0.000613	0.016074
B16K1	0.000000	0.000601	INFINITY
L16K1	0.000000	0.000601	0.021160
B17K1	0.000000	0.000589	INFINITY
L17K1	0.000000	0.000589	0.047975

B18K1	0.000000	0.000578	INFINITY
L18K1	0.000000	0.000578	0.061637
B19K1	0.000000	0.000566	INFINITY
L19K1	0.000000	0.000566	0.060908
B20K1	0.000000	0.000555	INFINITY
L20K1	0.000000	0.000555	0.059714

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	610.808838	7.389971	10.907959
3	600.429932	10.907959	241.451309
4	628.002930	10.907959	1.470947
5	626.531982	1.470947	10.907959
6	633.157959	0.166016	22.574951
7	665.221924	0.166016	15.948975
8	644.871826	0.504150	9.794678
9	653.169922	0.504150	9.794678
10	659.571777	INFINITY	9.794678
11	649.272949	3.264893	0.083008
12	622.218994	4.989014	64.113037
13	671.337891	4.989014	64.113037
14	718.375000	INFINITY	64.113037
15	664.266846	INFINITY	14.993896
16	715.865967	INFINITY	66.593018
17	702.737793	INFINITY	53.464844
18	698.125000	INFINITY	48.852051
19	709.361816	INFINITY	60.088867
20	752.847900	INFINITY	103.574951
21	756.223877	INFINITY	106.950928
22	739.108887	INFINITY	89.835938
23	746.223877	INFINITY	96.950928
24	0.000000	10.907959	7.389971
25	0.000000	241.451309	10.907959
26	0.000000	1.470947	10.907959
27	0.000000	10.907959	1.470947
28	0.000000	22.574951	0.166016
29	0.000000	15.948975	0.166016
30	0.000000	9.794678	0.504150
31	0.000000	9.794678	0.504150
32	0.000000	9.794678	649.777100
33	0.000000	0.083008	3.264893
34	0.000000	64.113037	4.989014
35	0.000000	64.113037	4.989014
36	0.000000	64.113037	654.261963
37	0.000000	14.993896	649.272949
38	0.000000	66.593018	649.272949
39	0.000000	53.464844	649.272949
40	0.000000	48.852051	649.272949
41	0.000000	60.088867	649.272949

42	0.000000	103.574951	649.272949
43	0.000000	106.950928	649.272949
44	0.000000	89.835938	649.272949
45	0.000000	96.950928	649.272949
46	0.000000	6.805266	10.907959
47	0.000000	10.907959	602.900879
48	0.000000	10.907959	1.470947
49	0.000000	1.470947	10.907959
50	0.000000	0.166016	22.574951
51	0.000000	0.166016	15.948975
52	0.000000	0.504150	9.794678
53	0.000000	0.504150	9.794678
54	0.000000	236.262863	9.794678
55	0.000000	3.264893	0.083008
56	0.000000	4.989014	64.113037
57	0.000000	4.989014	64.113037
58	0.000000	240.747742	64.113037
59	0.000000	235.758682	14.993896
60	0.000000	235.758682	66.593018
61	0.000000	235.758682	53.464844
62	0.000000	235.758682	48.852051
63	0.000000	235.758682	60.088867
64	0.000000	235.758682	103.574951
65	0.000000	235.758682	106.950928
66	0.000000	235.758682	89.835938
67	0.000000	235.758682	96.950928
68	0.000000	39.191162	610.808838
69	0.000000	57.207031	600.429932
70	0.000000	37.362061	628.002930
71	0.000000	46.650879	626.531982
72	0.000000	47.934814	633.157959
73	0.000000	23.874023	665.221924
74	0.000000	52.320068	644.871826
75	0.000000	52.213867	653.169922
76	0.000000	63.895752	649.777100
77	0.000000	72.784912	649.272949
78	0.000000	108.323975	622.218994
79	0.000000	67.788086	671.337891
80	0.000000	93.548828	654.261963
81	0.000000	107.324951	649.272949
82	0.000000	116.214844	649.272949
83	0.000000	125.208984	649.272949
84	0.000000	134.309814	649.272949
85	0.000000	143.516846	649.272949
86	0.000000	152.832031	649.272949
87	0.000000	162.256836	649.272949
88	0.000000	171.791992	649.272949
89	0.000000	162.256836	649.272949
90	585.000000	10.907959	7.166039
91	585.000000	INFINITY	10.907959

92	585.000000	1.470947	10.907959
93	585.000000	10.907959	1.470947
94	585.000000	22.574951	0.166016
95	585.000000	15.948975	0.166016
96	585.000000	9.794678	0.504150
97	585.000000	9.794678	0.504150
98	585.000000	9.794678	649.272949
99	585.000000	0.083008	3.264893
100	585.000000	64.113037	4.989014
101	585.000000	64.113037	4.989014
102	585.000000	64.113037	649.272949
103	585.000000	14.993896	648.216797
104	585.000000	66.593018	648.216797
105	585.000000	53.464844	648.216797
106	585.000000	48.852051	648.216797
107	585.000000	60.088867	648.216797
108	585.000000	103.574951	648.216797
109	585.000000	106.950928	648.216797
110	585.000000	89.835938	648.216797
111	585.000000	96.950928	648.216797
112	0.000000	1719.849120	2787.301760
113	0.000000	0.000000	3053.554690
114	0.000000	3358.323730	3053.554690
115	0.000000	0.000000	2617.910160
116	0.000000	7792.187500	5417.988280
117	0.000000	13600.015600	39.843750
118	0.000000	0.000000	19.921875
119	0.000000	0.000000	19.921875
120	0.000000	0.000000	19.921875
121	0.000000	0.000000	19.921875
122	0.000000	0.000000	3598.535160
123	0.000000	0.000000	3598.535160
124	0.000000	0.000000	3598.535160
125	0.000000	0.000000	3598.535160
126	0.000000	0.000000	11724.492200
127	0.000000	0.000000	11724.492200
128	0.000000	0.000000	11724.492200
129	0.000000	0.000000	14421.328100
130	0.000000	0.000000	21560.625000
131	0.000000	0.000000	21560.625000
132	0.000000	0.000000	21560.625000
133	0.000000	454423.250000	0.000000
134	0.000000	2689.545900	4645.500000
135	0.000000	2746.025880	INFINITY
136	0.000000	5597.203120	INFINITY
137	0.000000	7113.539060	INFINITY
138	0.000000	6823.679690	INFINITY
139	0.000000	7870.894530	INFINITY
140	0.000000	7221.335940	INFINITY
141	0.000000	7492.515620	INFINITY

142	0.000000	7387.937500	INFINITY
143	0.000000	7372.171870	INFINITY
144	0.000000	6444.980470	INFINITY
145	0.000000	8047.164060	INFINITY
146	0.000000	7528.179690	INFINITY
147	0.000000	7372.171870	INFINITY
148	0.000000	7372.171870	INFINITY
149	0.000000	7372.171870	INFINITY
150	0.000000	7372.171870	INFINITY
151	0.000000	7372.171870	INFINITY
152	0.000000	7372.171870	INFINITY
153	0.000000	7372.171870	INFINITY
154	0.000000	7372.171870	INFINITY
155	0.000000	7372.171870	INFINITY
156	0.000000	1613.727290	2787.301760
157	0.000000	1647.615480	INFINITY
158	0.000000	3358.323730	INFINITY
159	0.000000	7639.402340	INFINITY
160	0.000000	7792.187500	INFINITY
161	0.000000	13600.015600	INFINITY
162	0.000000	19176.621100	INFINITY
163	0.000000	25027.460900	INFINITY
164	0.000000	30932.566400	INFINITY
165	0.000000	36946.312500	INFINITY
166	0.000000	42524.015600	INFINITY
167	0.000000	49174.582000	INFINITY
168	0.000000	55646.765600	INFINITY
169	0.000000	61719.191400	INFINITY
170	0.000000	67913.062500	INFINITY
171	0.000000	74038.437500	INFINITY
172	0.000000	80286.312500	INFINITY
173	0.000000	86317.812500	INFINITY
174	0.000000	92467.437500	INFINITY
175	0.000000	98740.000000	INFINITY
176	0.000000	105138.062000	INFINITY
177	0.000000	454423.250000	INFINITY
178	0.000000	1613.727290	2787.301760
179	0.000000	1647.615480	INFINITY
180	0.000000	3358.323730	INFINITY
181	0.000000	7639.402340	INFINITY
182	0.000000	7792.187500	INFINITY
183	0.000000	13600.015600	INFINITY
184	0.000000	19176.621100	INFINITY
185	0.000000	25027.460900	INFINITY
186	0.000000	30932.566400	INFINITY
187	0.000000	36946.312500	INFINITY
188	0.000000	42524.015600	INFINITY
189	0.000000	49174.582000	INFINITY
190	0.000000	55646.765600	INFINITY
191	0.000000	61719.191400	INFINITY

192	0.000000	67913.062500	INFINITY
193	0.000000	74038.437500	INFINITY
194	0.000000	80286.312500	INFINITY
195	0.000000	86317.812500	INFINITY
196	0.000000	92467.437500	INFINITY
197	0.000000	98740.000000	INFINITY
198	0.000000	105138.062000	INFINITY
199	0.000000	561663.625000	INFINITY

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