

**Improvement of Structural Dynamic Models
via System Identification**

by

Peter A. Stiles

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of
Master of Science
in
Mechanical Engineering

APPROVED:

Dr. John B. Kosmatka, Chairman

Dr. Robert H. Fries

Dr. Raphael T. Haftka

Dr. Charles F. Reinholtz

December 1988
Blacksburg, Virginia

**Improvement of Structural Dynamic Models
via System Identification**

by

Peter A. Stiles

Dr. John B. Kosmatka, Chairman

Mechanical Engineering

(ABSTRACT)

Proper mathematical models of structures are beneficial for designers and analysts. The accuracy of the results is essential. Therefore, verification and/or correction of the models is vital. This can be done by utilizing experimental results or other analytical solutions. There are different methods of generating the accurate mathematical models. These methods range from completely analytically derived models, completely experimentally derived models, to a combination of the two. These model generation procedures are called System Identification. Today a popular method is to create an analytical model as accurately as possible and then improve this model using experimental results.

This thesis provides a review of System Identification methods as applied to vibrating structures. One simple method and three more complex methods, chosen from current engineering literature, are implemented on the computer. These methods offer the capability to correct a discrete (for example, finite element based) model through the use of experimental measurements. The validity of the methods is checked on a two degree of freedom problem, an eight degree of freedom example frequently used in the literature, and with experimentally derived vibration results of a free-free beam.

Acknowledgements

I would like to thank Drs. J. Kosmatka, R. Fries, R. Haftka, and C. Reinholtz for serving on my committee. Special thanks go to Dr. Kosmatka for providing insight, help, and guidance where needed. His methods of advising were well appreciated and respected. Additionally, I would like to thank; Ron Rorrer and Dr. A. Wicks for supplying the experimental data used in section 6.3, Alex Kilpatrick of MSC/NASTRAN for helping with DMAP problems, all those who helped solve SCRIPT and SAS problems, and NSWC/DL for supplying research funds for my first year.

My family and friends offered support, love, strength, and advice. They all helped keep me on course during my time in Virginia.

Thanks to all the people in Blacksburg who helped make this experience a beneficial one. The breaks from academia including; camping, volleyball, hiking, fishing, happy hours and lengthy discussions, were beneficial.

Thanks go to all the unmentioned people in my life who have inspired me to learn, as well as to pursue this degree. Final appreciation goes to the original WPI Plan.

Table of Contents

INTRODUCTION AND REVIEW OF PRESENT RESEARCH	1
1.1 Introduction	1
1.2 Review of Literature	4
1.2.1 Verification method	8
1.2.2 Modification method	11
1.2.3 Direct Identification Method	16
1.2.4 Related Topics	17
1.3 Organization of Thesis	19
METHOD BASED ON EIGENVALUE SENSITIVITY	21
2.1 Introduction	21
2.2 Development	22
2.3 Implementation	25
METHOD THAT UTILIZES ORTHOGONALITY EQUATIONS	29
3.1 Introduction	29
3.2 Development	31

3.3	Implementation	36
METHOD BASED ON CONNECTIVITY CONSTRAINT		39
4.1	Introduction	39
4.2	Development	40
4.3	Implementation	48
METHOD BASED ON RESIDUAL FORCE VECTOR		51
5.1	Introduction	51
5.2	Development	52
5.3	Implementation	57
NUMERICAL RESULTS		60
6.1	Verification Studies - 2 DOF model	60
6.2	Modification Studies using an 8 DOF Model	73
6.3	Application of Methods with data for a free-free beam	82
CONCLUSIONS AND RECOMMENDATIONS		105
7.1	Conclusions	105
7.2	Recommendations	107
REFERENCES		110
Modeling and Solution Sequences from MSC/NASTRAN		115
A.1	Solution Sequence for Sensitivity Based Method	116
A.2	Solution Sequence for Orthogonality Based Method	126
A.3	Solution Sequence for Connectivity Constraint Based Method	129
A.4	Solution Sequence for Residual Force Based Method	136

Vita 147

List of Illustrations

Figure 1.	2 DOF SPRING MASS SYSTEM USED FOR EXAMPLE 1	62
Figure 2.	8 DOF TEST PROBLEM, DEVELOPED BY KABE, USED FOR SECOND EXAMPLE	74
Figure 3.	ERROR IN PREDICTED FREQUENCY WITH SENSITIVITY METHOD FOR 8 DOF EXAMPLE	77
Figure 4.	ERROR IN PREDICTED FREQUENCY WITH ORTHOGONALITY METHOD FOR 8 DOF EXAMPLE	78
Figure 5.	ERROR IN PREDICTED FREQUENCY WITH CONNECTIVITY METHOD FOR 8 DOF EXAMPLE	79
Figure 6.	ERROR IN PREDICTED FREQUENCY WITH RES. FORCE METHOD FOR 8 DOF EXAMPLE	81
Figure 7.	ILLUSTRATION OF BEAM USED FOR EXAMPLE 3 AND THE FINITE ELEMENT MODEL	83
Figure 8.	ERROR IN FREQUENCY PREDICTION FOR BEAM USING SENSITIVITY METHOD	85
Figure 9.	CONVERGENCE OF FREQUENCIES WHEN USING SENSITIVITY METHOD	86
Figure 10.	ERROR IN FREQUENCY PREDICTION FOR BEAM USING ORTHOGONALITY METHOD	88
Figure 11.	EXTRANEOUS MODE INTRODUCED BY ORTHOGONALITY METHOD	89
Figure 12.	ERROR IN FREQUENCY PREDICTION FOR BEAM USING CONNECTIVITY METHOD	90
Figure 13.	FREQUENCY PREDICTION FOR THE CONNECTIVITY METHOD	91
Figure 14.	ERROR IN FREQUENCY PREDICTION FOR BEAM USING RES. FORCE METHOD	93

Figure 15. FIRST EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM	94
Figure 16. SECOND EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM . .	95
Figure 17. THIRD EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM	96
Figure 18. FOURTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM . .	97
Figure 19. FIFTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM	98
Figure 20. SIXTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM	99

List of Tables

Table 1.	TABLE OF VARIABLE VALUES FOR TESTS 1a - 1f	63
Table 2.	INITIAL PARAMETERS AND RESULTS OF TEST 1a	65
Table 3.	INITIAL PARAMETERS AND RESULTS OF TEST 1b	66
Table 4.	INITIAL PARAMETERS AND RESULTS OF TEST 1c	67
Table 5.	INITIAL PARAMETERS AND RESULTS OF TEST 1d	68
Table 6.	INITIAL PARAMETERS AND RESULTS OF TEST 1e	70
Table 7.	INITIAL PARAMETERS AND RESULTS OF TEST 1f	72
Table 8.	RELATIVE CHANGES TO MATRICES, # OF DIAGONAL TERMS CHANGED	101
Table 9.	RELATIVE CHANGES TO MATRICES, # OF OFF-DIAGONAL TERMS CHANGED	102
Table 10.	CPU TIME REQUIRED FOR THE FOUR METHODS FOR FREE-FREE BEAM (min:sec)	104

Nomenclature

a	Multiplier of element stiffness matrix
[A]	$= 4 [M_e][\phi_e][\mu_e]$
b	Multiplier of element mass matrix
[B]	$= [M_e][\Delta\phi][\mu_e] + [M_e][\phi_e][\Delta\mu] - [K_e][\Delta\phi]$
$\overline{[B]}$	[B] partitioned into a column
[D]	Matrix used in connectivity method, see eq (4.26)
DOF	Degree of Freedom
f	Frequency, $\frac{\text{cycles}}{\text{second}}$
$[H]_{ij}$	$= -[\overline{\phi}]_j [D]_i^T$
[I]	Identity Matrix
$\hat{[I]}$	Matrix used to update stiffness matrix where $\hat{l}_{ij} = 1$ if $K_{a_{ij}} \neq 1$ or else $\hat{l}_{ij} = 0$ if $K_{a_{ij}} = 0$
[k]	Element Stiffness Matrix
[K]	Global Stiffness Matrix
$[\Delta K]$	Change in stiffness matrix $= [K_u] - [K_e]$
l	Number of Design Variables
m	Number of Experimental Frequencies

$[m_e]$	$= [\phi_e]^T [M_e] [\phi_e]$
$[m]$	Element Mass Matrix
$[M]$	Global Mass Matrix
$[\Delta M]$	Change in mass matrix $= [M_v] - [M_e]$
n	Number of Analytical DOF
$[N]$	$= [M_e]^{1/2}$
r	Number of deficient elements in model
$[Q]$	Matrix obtained in residual force method, see eq (5.14)
$[R]$	Residual Force Matrix $= - [K_e] [\phi_e] + [M_e] [\phi_e] [\mu_e]$
$[X]$	Column Vector of Design Variables
$[Z]$	Column Vector of Unknown matrix multipliers, a and b

Greek Letters

$[\alpha]$	Matrix used in connectivity method, see eq (4.22)
$[\beta]$	Matrix used in connectivity method, see eq (4.23)
ε	Error function
δX_j	Change in jth design variable
$[\Delta]$	Matrix used to update mass matrix
$[\gamma]$	Matrix used to update stiffness matrix
λ	Lagrangian multiplier
$[\Lambda]$	Matrix of Lagrangian multipliers
$[\Lambda^i]$	ith row of $[\Lambda]$ with all other rows set to zero

μ	Eigenvalue
$[\mu]$	Matrix of Eigenvalues
$[\Delta\mu]$	Matrix of Change of Eigenvalues
ω_n	Circular frequency $\frac{\text{radians}}{\text{seconds}}$
$[\Omega]$	Diagonal Matrix of non-zero Eigenvalues of the equation $([\alpha] + [\beta])$
$[\phi]$	Eigenvector/Modal Matrix (Mass Normalized)
$[\Delta\phi]$	Change in Eigenvector/Modal Matrix
$[\bar{\phi}]$	Column of $[\phi_s]^T$
$[\Phi]$	Element by element multiplication of the stiffness matrix times the stiffness matrix
$[\hat{\Phi}]$	Diagonal Matrix whose diagonal is a row of $[\Phi]$
$[\rho]$	Transformation Matrix
ψ	Lagrangian function
Ψ	Eigenvectors associated with non-zero eigenvalues of $([\alpha] + [\beta])$
ζ	Pre-determined margin of error

Subscripts

a	Analytical
e	Experimental
i	ith Column
ij	ith Component of the jth column
λ	Denotes eigenvalue equation

O Denotes orthogonality equation
S Denotes symmetry condition
u Updated
, Partial Derivative

Superscripts

T Transpose

Chapter I

INTRODUCTION AND REVIEW OF PRESENT RESEARCH

1.1 Introduction

Analytical methods of problem solving have become more prominent with advances in technology. Computer modelling techniques, such as finite element methods, which predict the response of structures to loads, have become accepted. However, these approaches do not always yield accurate results, as the formulations are based on assumptions, that may not be completely valid for the problem being modelled. In addition to these errors, modelling errors can exist which are dependent on the model refinement, human errors, and solution techniques for large problems. There is an increasing need to verify the models of structures and to learn

how to compensate for the inaccuracies that exist. By doing this, discrete models will be able to predict the response of structures more accurately.

When properly used, finite element methods have been shown to properly model structures (Zienkiewicz, 1967).¹ Computer modelling techniques are invaluable in the aerospace industry. The space shuttle, satellites, and airplanes have all benefitted from the use of these techniques. Many modern structures are very complicated. They can be comprised of complex shapes, non-linear boundary conditions, stress concentrations, composite and non-linear material properties. Computer modelling offers an excellent opportunity to analyze a complex design, and to study the effect of design changes. Hence, it is usually both time and cost effective to utilize computer models.

If the developed analysis method properly represents the problem being analyzed, then an infinite number of elements using infinite precision will yield the exact results. However, a model with a large number of elements costs more than one with fewer elements. To save money and time, computationally smaller models are desired.

Alternatively, if the developed computer method does not represent the problem of interest, then "System Identification" may be able to generate a model which does. System Identification is the process by which a system is generated by examining the response as a function of the input. Modelling problems arise when the material properties are not known exactly, or when previously mentioned complex problems exist. It is also difficult to measure the stiffness of a sub region of a structure, as the structure is continuous and it is awkward to measure the stiffness of a section

¹ Authors names and dates in parenthesis indicate where to find the reference in the alphabetized reference list at the end of this thesis.

directly. A goal of System Identification is to keep the cost of analysis low while achieving accurate results.

In the field of vibrations, a popular System Identification method is to combine an analytical model with experimental data to generate a more accurate theoretical model. In dynamics, a discrete analytical model yields eigenvalues and eigenvectors. Eigenvalues correspond to the circular frequency squared, ω_n^2 . The eigenvectors correspond to the discretized mode shape vectors. For normal modes, these modal vectors are orthogonal to each other with respect to the mass matrix and measure relative deflection of the structure if it were vibrating only at the corresponding frequency. Using this procedure, the experimental data is used as a guide to change the analytical model.

Experimental data offers the ability to verify at least some results of a model. This data can be in the form of strain, deflection, natural frequencies, etc.. By utilizing experimental results, computer models can be verified and/or fine tuned to more accurately predict the response of a structure. A variety of methods exist and offer different benefits. Some attempt to extract different parameter values, while others have varying constraints which must be met while altering the model.

This thesis will review many of the current System Identification methods. Four methods are implemented on a VAX 11/780 within MSC/NASTRAN Version 65b (Macneal-Schwendler, 1984). They are methods based upon; (1) the sensitivity of the frequencies with respect to the design variables (classical approach), (2) requiring the experimental modes to be orthogonal to the analytical system matrices, (Berman and Nagy, 1983), (3) requiring the connectivity of the updated matrices to be the same as the original matrices (Kabe, 1985), and (4) locating errors in the analytical model using a residual force vector (Chen and Wang, 1988). A simple two DOF spring mass system is used to illustrate the capabilities of each method. The second problem,

developed by A. M. Kabe (1985), is an eight DOF, undamped, free vibration, model consisting of lumped masses and springs. All methods will be used to identify the modes of a free-free beam with experimentally derived mode shapes and natural frequencies. The experimental data contains the translational displacements obtained at 19 points along a 36 in. steel beam (Rorrer et al., 1989). Finally, conclusions and recommendations will be presented covering the basic capabilities and deficiencies of the four methods.

1.2 Review of Literature

Man has had a desire to study and analyze his surroundings ever since the beginning of time. Many times this desire begins with an theoretical model that needs to be modified or corrected, depending on experimental results. An accurate understanding of Halley's comet was developed utilizing System Identification techniques (as detailed by Lancaster-Brown (1985)). First, an understanding of Newtonian mechanics provided an a priori model of Halley's comet. The previous sightings of the comet were then used to alter the existing model. Halley applied what was known and utilized experimental data to obtain a more accurate understanding of Halley's comet. He was able to predict the return of his comet fifty years before it reappeared in 1758.

System Identification is used in many different fields. It is a means by which a set of equations is generated by examining the response of the system to the input, or given a response and an input, determine the system. A general interpretation of System Identification implies that a system of equations is developed or identified

which properly represents the system of interest. Often this set of equations or relationships developed needs to be utilized and interpreted as a system. However, it would be beneficial to be able to determine actual physical and geometric properties of a system.

The identification of parameters of a system is called "parameter estimation." Often there is not enough data available to verify and determine the parameters of a system. In such cases, it can be very difficult, if not impossible, to generate a unique system. For example, with a set of linear equations in order to uniquely identify a system, there must be the same number of independent unknowns as there are independent identifiable equations.

Depending on how a system is sub-divided and what types of assumptions are made, a system can be over-constrained or under-constrained. If every component of the mass and stiffness matrices are taken as unknowns, usually the number of unknowns are greater than the number of equations. In vibration analysis, the number of equations is related to the quantity of experimental modal data in the form of the mode shapes and natural frequencies. If a parameter used for generating the matrices (i.e. Young's modulus, density, thickness, etc.) is used as a variable which does not vary from element to element in the finite element model, then often the number of equations may be greater than the number of unknowns. These differences and how they are interpreted define the basis of different sub-classes of System Identification methods.

Many advances in methods for System Identification came with more accessible computer power. These methods rely on computationally intensive analytical techniques to create large models. A review of System Identification concepts as applied to a variety of fields can be found in "System Identification Techniques: A tutorial Review" (Jain and Dobeck, 1979). There have been many articles in

engineering literature which review a certain class of methods, or reference related articles; (Berman, 1979a), (Pakstys, 1982), (Heylan and Sas, 1986), (Ibrahim and Saafan, 1986), (Caesar, 1986). The review in this thesis gives an in depth overview of many types of methods. A review of concepts and classifications of system identification as applied to vibrating structures is presented by Berman (1975). Berman divides System Identification into three groups: verification, modification and direct identification.

Verification relies on the analyst and deals with varying the individual parameters, or design variables, in an analytical model until a correct model is generated. This can be done by an individual utilizing an iterative intuitive procedure or through the use of a procedure which systematically varies the parameters. Modification deals with altering the components of the global mass and stiffness matrices. Different procedures of matrix modification and utilization of constraint equations distinguish the many different methods which fall under the modification label. Direct identification deals with generating an analytical model directly from experimental data. This method, unlike the other two, does not rely on initial estimates for parameters of the model.

The mathematical model which results from applying System Identification techniques can be classified as either a 'correct' or 'physically correct' model. A correct model is one that predicts the response of the structure, but which may not yield realistic physical properties, i.e., new load paths or unreasonable values may have been introduced. In order for a model to be physically correct, it should yield the correct connectivity relationships, geometric properties, and material properties. This enables one to gain a better understanding of the modelling errors and other inherent problems in the original analytical model, as well as making the resulting model more understandable.

An analytical model consists of a mass and a stiffness matrix, $[M_a]$ and $[K_a]$. An experimental model usually consists of modal vectors, $[\phi_e]$, and frequencies, which are multiplied by 2π and squared in order to correspond to the eigenvalues $[\mu_e]$. The two different systems are related through the eigenvalue equation,

$$\mu_e [M_a][\phi_e]_i = [K_a][\phi_e]_i \quad (1.1)$$

In order to compare models generated experimentally and analytically, they both should be in the same form. Often when utilizing System Identification techniques it is desired to modify the analytical matrices through the use of experimental data. Therefore, there is some effort done to put the experimental data in the same form as the analytical data, i.e. mass and stiffness matrices versus frequencies and mode shapes.

Experimental testing usually yields natural frequencies and mode shapes. This data is often incomplete, i.e. measurements are taken at more points for each frequency than there are frequencies measured. An equal number of measurement points and frequencies is desired so that square matrices can be created and manipulated. A large number of data points is desired to best express the mode shape in the form of an accurate discrete vector. With fewer data points, the accuracy of the mode vector tends to decrease.

If the number of measurement points is equal to the to number of frequencies measured then a square modal matrix is created. With this square modal matrix, a mass and stiffness matrix can be created by utilizing the orthogonality of normal modes. If modal data is incomplete, then different steps can be taken to correct the problem. It is possible to use the psuedo-inverse method of generating mass and stiffness matrices (Luk, 1987). Another method would be to reduce the number of points at which data is taken in order to make the modal matrix square. An

alternative method is to add linearly independent mode shape vectors to the matrix while requiring that their associated frequencies are large. Another alternative would be to synthesize the flexibility matrix instead of the stiffness matrix and compare the flexibility matrices.

In an analytical analysis it is possible to generate eigenvalues and eigenvectors. This is similar to the form the experimental data is in, hence it is easy to compare the modal data, instead of the matrices. This is good for comparison of results, but does not enable one to directly modify the system matrices. In order to keep the matrices physically correct, one usually wants to make minimum changes to the system matrices. Minimizing the percent changes to the components of the matrices is difficult to do if the mode shape vectors are modified and the system matrices are determined through matrix manipulations.

Problems can result when trying to compare an analytical model and an experimental model, or experimental results. Analytical models tend to have data taken at more points than an experimental model would. Therefore, the analytical models are of greater dimension. In order to compare matrices of the same size, the analytical system matrices can be reduced to be of the same size as the experimental modal matrices. This can be done by a variety of methods (Guyan, 1965), (Kidder, 1973). Experimentally derived matrices can be expanded using linear interpolation to expand the modal vectors (Berman and Nagy, 1983).

1.2.1 Verification method

Intuition and experience are often used in the area of design. These concepts are often enough of a tool to provide good results. Much of what an analyst needs

to do is to combine past experiences with intuition, and make a choice. Picking the right bolt or sheet metal stock can often be done without any analysis. If the bolt breaks, get a new one. If it was too heavy, try a lighter one.

When a design gets more complex the analyst must resort to calculations and/or experiments to get a reliable answer. When modifying a complex or simple structure, often the analyst has analyzed the situation at hand and determined in which direction the changes should head. This is the basis for many of the methods developed in this section.

Early intuitive System Identification methods belonged to the verification group. This appears to be the simplest and most straight forward procedure. A basic approach would be to calculate the sensitivity of the response of the structure with respect to the design variables, and modify the structure accordingly. Many of these methods utilize a least squares approach to minimize an error function. The error function is composed of the sum of the square of the differences of the experimental and analytical data. In the case of vibrating structures, the data would be in the form of the square root of the eigenvalues and natural frequencies and/or eigenvectors and the modal matrix. For static analysis this results could be in the form of deflection or strain. The change in the parameters could be determined by minimizing the error function with respect to the design variables.

An early approach of this variety was derived by Hall et al., (1970). In order to estimate unknown mass and stiffness parameters, a least squares approach is used, which utilizes the difference of corresponding frequencies and modal vectors. For this method some parameters are assumed to be known while the others are to be estimated. During the minimization a higher emphasis is placed on low frequency correspondence of the model.

Collins et al. (1974) also developed a verification method utilizing sensitivity information. The purpose of this method is to generate linear estimators to approximate structural parameters. This is done by relating the modal data to the design variables through the use of a truncated Taylor's series expansion. Also included is the confidence level in the data. This is the level of confidence of the experimental or analytical data, and is determined by the analyst.

Chen and Garba (1979) assume that the two initial models are close and that through an iterative procedure the updated model will get better. They include methods of efficiently recomputing the analytical eigendata, and allow for the inclusion of other data such as modal forces, kinetic energy distribution and strain energy distribution. Yokoki et al. (1987) utilized a verification method which updates only the mass matrix in order to match the analytical eigendata to the natural frequency and modal matrix.

Blakely and Walton (1984) utilize Bayesian parameter estimation. This method is similar to other parameter updating methods which use sensitivity methods, but it utilizes confidence levels in the obtained data. By utilizing this technique unrealistic changes in parameter values can be avoided. If there is no or equal confidence in the analytical parameters then the method reduces to a weighted least squares fit. A weighted least squares fit uses a least squares approximation with a weighting constant for different data. If there is no confidence in the experimental data then the analytical model is not changed.

These methods follow intuitive methods and have the advantage of being able to extract parameters. It is also possible to examine the values chosen for the parameters to check the validity of the model. One problem that results from this method is that the identification procedure may modify the wrong variable. For instance the stiffness of a rod is dependant on the cross sectional area, the length,

and the modulus of elasticity. If the bar is too stiff, then any one or all of the design variables may be modified. Another difficulty arises if a parameter has increased or decreased too much from the initial value. Then the results may not be correct. This can be avoided by using constrained minimization in the problem formulation.

The method developed in Chapter 2 is a verification method. The updated model is obtained using the difference between the experimental and analytical frequencies. It utilizes an iterative Newton-Raphson approach to estimate new values for the design variables. The reason that this 'classical' method was chosen is that it is an intuitively simple method and is similar to methods the author has seen used in industry.

1.2.2 Modification method

Within the modification class, each component of the analytically generated matrices can be treated as a variable. In general, there are more variables than reliable data, making for an underconstrained problem. This means that there may not necessarily be a unique system which properly predicts the response of a structure. There may be a class of systems which represents the system accurately.

By utilizing different methods of matrix modification and utilizing different constraint equations, various methods were developed. Some methods minimize the change to each component of the matrices. Other methods attempt to keep the connectivity of the matrices the same as in the original matrices. Still other methods try to predict the exact response directly. The modification area of System Identification is currently receiving the most attention in literary journals.

A method which utilizes first a verification technique and then a modification technique is presented by Vanhonacker (1987). Initially he utilizes sensitivity methods to update the general structural parameters of the system. He utilizes both the sensitivity of the eigenvalue and the components of the eigenvector with respect to the design variables of interest. After updating the system matrices with verification techniques, Vanhonacker then modifies individual coefficients of the matrices where needed. This is to account for, among other things, boundary conditions which are often unrealistic, either completely clamped or completely free for a given DOF.

Another method which is a combination of verification and modification techniques was utilized by Sol et al. (1984). Using sensitivity methods they vary the design variables of the finite element model to obtain better correlation. Then utilizing forced response calculations, global changes to the system matrices are made.

The previous two methods attempt to first update the analytical model in a manner which results in a physically correct model. The methods then attempt to make minor changes to the model to create a system which correlates even better with the experimental data.

Some recent approaches try to determine the location of modelling errors within the finite element model. By locating the elements where discrepancies between experimental and analytical predictions exist, these elements can be altered and, as a result, force the correction of the model.

Chen and Wang (1988) developed an approach which utilizes the error location approach. They calculate a residual force vector by combining the experimental data with the system matrices using the eigenvalue equation. If any components of the residual vector are not zero, then this indicates either modelling or experimental

error. The properties around these locations are then modified to correct the model. Chen and Wang then examine different methods to update the system matrices.

Berman and Flannelly (1971) discuss topics related to the modification method as well as the development of a method. They discuss the importance of data at specific points of interest and within a limited frequency range. Their method improves an analytical mass matrix by finding the smallest changes which make a set of measured modes orthogonal. In addition, an improved stiffness matrix is formed by summing the contributions of the measured modes, and with the use of the improved mass matrix. This method does not iterate or use an initial eigensolution, but rather uses modal orthogonality and the eigenvalue equation to update the system matrices.

Baruch (1978) presents means of minimizing percent changes to the components of the stiffness matrix once the modal matrix and mass matrix are compatible. He also details different techniques for representing the modal data while using either the modal data, the mass matrix, or the stiffness matrix as a reference basis (1982).

Berman and Nagy (1983) developed the Analytical Model Improvement (AMI) method. It is built upon the work of Berman and Flannelly (1971) and Baruch (1978). They chose an interesting approach in that the method is computationally efficient, generates an accurate match for the experimental data, and can generate a set of expanded modal data which is of the same dimension of the analytical model. The expanded modal results are generated for each natural frequency from the incomplete results by partitioning the mass and stiffness matrices to correspond to the known data and then solving for the unknown data. This is similar to a Guyan reduction (Guyan, 1965), although the new modal data is a linear interpolation of the measured data.

An improved mass matrix is generated using the method of Lagrange multipliers. The mass matrix is required to be such that the modal vectors are orthogonal. An error function is generated which takes into account relative sizes of the coefficients of the analytical mass matrix. This function is then minimized with respect to the elements of the mass matrix. After the mass matrix is adjusted, the stiffness matrix is adjusted in a similar manner (Baruch,1978), (Wei,1980). The resulting system contains the modes and frequencies which were included in the adjustment procedure. This method requires no eigensolutions or iterative computations to generate the updated system.

Another method of modifying analytical models is to compare flexibility matrices. The flexibility matrix is the inverse of the stiffness matrix. This method was utilized by Dobson (1984). He generates both analytical and experimental flexibility matrices and compares both matrices.

O'Callahan and Leung (1985) utilize a generalized inverse technique in order to modify system matrices. Their method requires the orthogonality of the modal vectors to the mass matrix. The mass matrix is adjusted to minimize an error function and then the stiffness matrix is adjusted. This is done by requiring that the eigenvalue equation be satisfied and that the stiffness matrix is symmetric. This is an updated version of the method developed by O'Callahan et al. (1984). The original paper dealt with a generalized weighted inverse technique but had some problems. These problems were corrected with the use of the generalized inverse technique.

Kabe (1985) developed the Stiffness Matrix Adjustment method (KMA) method. He examines a new requirement for modifying matrices. Kabe utilizes the original connectivity of the analytical matrix as a requirement for the final matrix. This means that his method will not introduce any new load paths in the model by modifying stiffness coefficients that were originally zero. The mass matrix, Kabe assumes, is

initially correct and modifications are made to the stiffness matrix. The end result is that he modifies the stiffness matrix with modal data while utilizing the element connectivity as a constraint. Physical configuration is preserved, and the adjusted model better reproduces the modes used in identification.

Wang and Chen (1986) examine different modification methods and the pros and cons. They implement and compare the results of different methods with a simple circular cross section cantilever beam. From the conclusions of their analysis of the methods, they propose three different methods which are a combination of reviewed methods.

Modification methods usually result in a system which accurately reproduces the experimental data. This is due to the underconstrained problem that exists. There are more coefficients in the matrices than modal data. Therefore additional constraints are applied to help define a unique system. Some methods initially attempt to modify the matrices in a physically correct manner. Others attempt to minimize the changes to the coefficients, while others require that the connectivity remain the same. However methods that don't require physically correct updated matrices usually result in only correct models, or models which only predict the proper response of a structure.

Berman and Nagy's (1983) method is a popular and inexpensive method to use and will nearly always yield a model which exactly reproduces the modes used in the identification. Their method is developed in Chapter 3. Kabe's (1985) method will also yield the correct results if the analytical mass matrix is correct and the measured modes are normal modes. His method also attempts to minimize changes to the coefficients of the stiffness matrix while also introducing no new load paths to the model. Whatever stiffness coefficients were originally zero, must be zero in the updated model. Therefore his model will tend to be more physically correct than the

model developed by Berman and Nagy's method. Kabe's method is developed in Chapter 4. Chen and Wang's (1988) method utilizes a different approach to modification. It is similar to a verification method in that the element stiffnesses and masses are updated utilizing experimental data. However the new values are not derived from sensitivity methods, but rather from errors that exist when the analytical matrices and experimental data are used in the eigenvalue equation. The resulting system matrices will be physically meaningful, as they are derived only from local element matrices. Chen and Wang's method is developed in Chapter 5.

1.2.3 Direct Identification Method

If it were feasible, direct identification might be the ideal method of System Identification. Given complete and accurate experimental data, a unique analytical model can be constructed. However, it is rare that such data is available, so the accuracy of the generated model can vary greatly. Experimental data is rarely exact and contains errors due to; experimental method error, human error, and temperature fluctuations, among other things. In addition, the size of the model generated by this method is equal to the number of accurate experimental results.

Utilizing the direct identification method means relying on experimental data completely. A comparison between strictly theoretical modelling and experimental modal analysis is detailed by Pal and Schmidtberg (1982). This paper lists some of the pros and cons of modal modelling.

An example of direct identification is given by Luk (1987). His method identifies the system matrices using either a complete or incomplete set of normal modes utilizing a pseudo-inverse. His method utilizes modal matrices which are a function

of the frequency response function. Luk demonstrates his method by solving for the system matrices of a complete set of modal data of order three. He then eliminates one of the modal vectors and shows that the other two frequencies and modal vectors can be duplicated using the psuedo-inverse technique.

Another example of direct identification is given by Shye and Richardson (1987). They developed a curve fitting algorithm from which the system matrices are generated. They also utilize the assumptions that; 1) the stiffness matrix is symmetric, 2) the mass matrix can be approximated by a diagonal matrix, and 3) the total mass of the structure is known. The assumption that the mass matrix is a diagonal matrix can reduce the accuracy of the model. The method yielded good results when the frequency range was limited and the number of data points was small.

Due to the limitations of experimental measurements, it is difficult to generate reliable, error free, experimental data at enough data points to be used as the sole input to generating a mathematical model. When an analytical model is utilized as input in the identification process, the method can no longer be defined as direct identification. Due to these limitations there is no direct identification method developed in this thesis.

1.2.4 Related Topics

Besides actual System Identification methods there is research going on in related areas. These fields include; model error estimations, sensitivity calculations, mode shape orthogonalization methods, and matrix perturbation techniques.

Sensitivity calculations are important to most optimization techniques. Two books (Haug et al., 1984; Haftka and Kamat, 1985) offer insight to methods of sensitivity calculations along with hints and shortcomings. Nelson (1976) provides a technique for calculating the sensitivity of the eigenvectors. Barthelemy and Haftka (1987) discuss accuracy problems associated with semi-analytical methods of matrix derivative calculations.

Error estimation is a topic related to System Identification. Ojalvo and Pilon (1988) offer a few techniques, successful and unsuccessful, which compare analytically and experimentally generated matrices. Sidhu and Ewins (1984) introduce the Error Matrix Method (EMM). This method utilizes a Guyan reduction to make the analytical and experimental system matrices of the same dimension. They then compare differences in the matrices to locate modelling errors. Errors associated with linearly reduced models are examined by Fuh et al. (1986). These errors are introduced during Guyan reduction.

Many methods of System Identification utilize the property that the mass matrix is orthogonal to the modes. This is considered by Targoff (1976), by taking into account the reliability of the measured modes and the analytical mass matrix before making adjustments to either one. Baruch and Itzhack (1978) examine methods to modify measured modes to be orthogonal. Berman (1979) examines means of correcting the mass matrix with an incomplete set of measured modes. This is later adapted to his Analytical Model Improvement (AMI) method.

Manipulating large matrices instead of regenerating them from new parameters can be efficient when performing iterative techniques. A method of this nature is examined by Chen and Wada (1988), they update the mass and stiffness matrices efficiently after altering parameters. An alternative method of recalculating the

eigendata of a modified structure is presented by Wang and Chu (1981). They update the frequency estimation by utilizing modal parameters of the original system.

1.3 Organization of Thesis

System Identification can be well utilized when combined with discrete analysis techniques. Finite element methods are a subclass of discrete techniques, where a continuous structure is represented by a series of discrete linear algebraic equations where the properties are lumped instead of continuous.

The first two numerical studies utilize discrete lumped parameters to generate the system of equations. The third example problem utilizes an analytical model which was derived from finite elements. Other analytical techniques could have been chosen, i.e. boundary element methods or finite difference methods, but finite element methods can easily be used to model structures with geometric and material non-linearities.

Discrete models yield mass and stiffness matrices. They are equal in size to the number of DOF of the model. The matrices are assembled utilizing the connectivity of the nodes. The eigenvalue equation: $([K] - \mu_i[M])[\phi]_i = [0]$ is solved to determine the eigenvalues. From the resulting eigenvalues, the eigenvectors of the system can be determined.

Chapters 2 - 5 each detail the development of a chosen method. This includes the derivation, the assumptions utilized, and the requirements. The MSC/NASTRAN Direct Matrix Abstraction Programming (DMAP) alter source codes for each method are contained in the Appendices. MSC/NASTRAN not only supplies the source code

for their solution sequences, but also supplies the user with the ability to easily modify the program to accomplish calculations for the users purposes. MSC/NASTRAN was chosen for this reason, along with its ability to generate, save, store, and manipulate matrices.

Numerical results results and comparisons for all of the developed methods, are presented in Chapter 6. Examined will be computational requirements, ability to predict the response under given conditions, physical changes to the system matrices. Details of this along with results will be included. Chapter 7 contains the conclusions of the thesis and recommendations for further study.

Chapter II

METHOD BASED ON EIGENVALUE SENSITIVITY

2.1 Introduction

The method developed in this chapter is similar to a classical approach. It determines how to improve a mathematical model by utilizing sensitivity information. The improvement of the model is done in the same way an analyst might do it.

A design can be changed by determining the sensitivity of the response of a structure to changes in parameters of interest. A systematic method is developed which determines the direction and magnitude of change in the parameters that should be made in order to improve the objective function. An objective function, which is the sum of the squares of the difference between the experimental and analytical natural frequencies, is improved through the use of an iterative Newton-Raphson technique.

The development of the following method is based upon methods which the author has seen utilized in industry. This method will be examined for its own value. It will also be used as a method which will stand as a benchmark for the other developed methods. Comparison of experimental and analytical modal vectors will not be included in this method. An initial eigensolution will be run and the different experimental and analytical mode shapes will be matched graphically.

This method uses optimization techniques to generate a better model. The objective function in this procedure is the sum of the squares of the differences between the analytical and experimental frequencies. The goal is to minimize the objective function, or in turn minimize the difference between the experimental and analytical results. New parameter values will be chosen based upon sensitivity information.

2.2 Development

This method assumes that the original mathematical model is a reasonable representation of the structure of interest. Analytical modal vectors must be compared to experimental modal vectors and judged to determine how well they correlate. Assuming they correlate well, then this method can be used.

The sensitivity calculations in MSC/NASTRAN are done using the approach presented by Arora and Haug (1979). They calculate the derivative by taking the derivative of the eigenvalue equation,

$$\frac{d}{dX_j} (\mu_{a_i}[M_a][\phi_a]_i - [K_a][\phi_a]_i) = 0 \quad \text{for } i = 1 \text{ to } m \text{ and } j = 1 \text{ to } l \quad (2.1)$$

where m is the number of measured frequencies and l is the number of design variables. Rearranging, and pre-multiplying each term by $[\phi_a]^T$ results in,

$$[\phi_a]_i^T [M_a] [\phi_a]_i \frac{d\mu_{a_i}}{dX_j} = [\phi_a]_i^T \left(\frac{d[K_a]}{dX_j} - \mu_{a_i} \frac{d[\Delta M_a]}{dX_j} \right) [\phi_a]_i \quad (2.2)$$

Assuming that the eigenvectors are mass normalized the equation can be rewritten as,

$$\frac{d\mu_{a_i}}{dX_j} = [\phi_a]_i^T \left(\frac{d[K_a]}{dX_j} - \mu_{a_i} \frac{d[M_a]}{dX_j} \right) [\phi_a]_i \quad (2.3)$$

and utilized to calculate $\frac{d\mu_{a_i}}{dX_j}$.

The calculations of the derivatives of the matrices with respect to the design variables are done in one of two methods. The first approach consists of directly calculating the derivative of the matrix with respect of the design variables, i.e., if X_j is a proportional section property or material property, then the derivative of the matrix is equal to the matrix divided by the value of the design variable. If X_j is a variable which changes the location of a DOF within the matrix, then this method is not used.

The second approach, which is less accurate, uses the finite difference method of calculating derivatives

$$\frac{d[K_a]}{dX_j} = \frac{([K(X_j)]_a) - [K(X_j + \delta X_j)]_a)}{\delta X_j} \quad (2.4)$$

Problems can be encountered if the finite difference step is too large or too small. If it is too large, then an accurate representation of the derivative may not be calculated if the function is non-linear. If the step size is too small, then the precision

of the computer can introduce errors in the results. This occurs when the round-off error due to the limits of the computer become a large percentage of the change in the evaluation of the function. Reasonable results have been achieved when using a step size equal to 1% of the current variable.

In order to improve the model, new values for the design variables need to be determined. This is done by summing the derivatives of an individual frequency with respect to each design variable $\frac{\partial f_{a_i}}{\partial X_j}$, multiplying it by the change in the design variable and setting this equal to the difference between the analytical and experimental frequencies. Hence, the following equation is used

$$\sum_{j=1}^I \frac{\partial f_{a_i}}{\partial X_j} \times \Delta X_j \approx \Delta f_{a_i} \quad (2.5)$$

Where f_{a_j} is the j th analytical frequency, and I is the number of design variables.

For a nonlinear equation, this is not usually exact. However, it should yield a better solution if the initial guess is close to the correct answer. The same equation can be written for each measured frequency and in matrix form, ie.

$$\begin{bmatrix} \frac{\partial f_1}{\partial X_1} & \frac{\partial f_1}{\partial X_2} & \cdots & \frac{\partial f_1}{\partial X_i} \\ \frac{\partial f_2}{\partial X_1} & \frac{\partial f_2}{\partial X_2} & \cdots & \frac{\partial f_2}{\partial X_i} \\ \cdot & \cdot & \cdot & \cdot \\ \frac{\partial f_m}{\partial X_1} & \frac{\partial f_m}{\partial X_2} & \cdots & \frac{\partial f_m}{\partial X_i} \end{bmatrix} \begin{bmatrix} \Delta X_1 \\ \Delta X_2 \\ \cdot \\ \Delta X_i \end{bmatrix} = \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \cdot \\ \Delta f_m \end{bmatrix} \quad (2.6)$$

where; $\Delta f_j = f_{a_j} - f_{e_j}$, ΔX_i is the change required for the i th design variable, f_{e_j} is the j th experimental frequency, and m is the number of experimental frequencies

In the above equation, the sensitivity matrix is known, as is the vector, $[\Delta f]$. However, the sensitivity matrix may not be square. Depending on how the problem

is defined and what assumptions are made, the problem can be either under constrained or over constrained. If the problem is over constrained, then a least squares fit can be used to solve for $[\Delta X]$. If the problem is underconstrained, then there are different solution techniques that can be used. Equations can be added to the system of equations to make it square. These equations can be in the form of requiring that the sum of the mass be constant, that the ratio of the masses be constant, or the ratio of the individual components of the mass and stiffness matrices be constant.

The problem is nonlinear, therefore it requires an iterative solution. An initial eigensolution is performed, the sensitivity values are calculated, then a new $[X]$ vector is generated. This procedure is repeated with the new $[X]$ vector until the results fall within a predetermined margin of error, ζ ,

$$[X]_{i+1} = [X]_i - [\Delta X]_i \quad (2.7)$$

where, $[X]$ is the vector of design variables. The procedure is repeated with new design variables until

$$[\Delta f]^T [\Delta f] \leq \zeta \quad (2.8)$$

2.3 Implementation

The implementation of this method is fairly straight forward. However, it requires a good understanding of Direct Matrix Abstraction Programming (DMAP) and program flow in MSC/NASTRAN. The direct method of calculating matrix derivatives is used

instead of the finite difference approach. This makes the method quicker and is easy to do for simple problems. The solution sequence from MSC/NASTRAN's design sensitivity analysis (Solution 53) could have been used for a more difficult problem.

Once the individual sensitivities are calculated they are assembled into a sensitivity matrix. Additional equations are added to make the sensitivity matrix square. These equations depend on constraints supplied by the user and depend on the number of frequencies measured.

Appendix A.1 lists the program used to solve the free-free beam problem with 6 modes used in the identification process. Following is a step-by-step explanation of the alterations required for this method.

- 1.) The initial statements set up the NASTRAN run and initialize a database for storage.

- 2.) Store Matrix derivatives in database: These statements store the previously determined matrix derivatives in the database in a subscripted fashion. If they were not previously calculated this is where they could be determined using sensitivity analysis.

- 3.) Initialize and rename matrices: section renames matrices and internally sets some matrices to be able to be appended to, so that the dimensions of the matrices can change.

- 4.) Top of iterative loop: This is the top of the loop for iterative calculations of the objective function and Newton-Raphson iteration.

- 5.) Perform eigenvalue analysis: Performs eigenvalue analysis using current mass and stiffness matrices.

- 6.) Generate analytical and experimental frequencies: This section extracts the frequencies and eigenvalues and puts them in a useable form for later use.

7.) Exit loop if error is small: This section calculates the current value for the objective function and compares it to a pre-determined error value. If the value of the objective function is less than the error value then the program flow jumps to the label OUTLOOP, which is after the iterative loop.

8.) Loop to store analytical data: This loop extracts the eigenvectors, eigenvalues, and frequencies from the previous eigenvalue analysis and stores them. This data is then used when calculating frequency derivatives.

9.) Determine sensitivities of frequencies with respect to design variables: This statement begins two loops, one nested inside the other. The first loop extracts the derivatives of the mass and stiffness matrices with respect to a given design variable. This is done with the DBFETCH command. The second loop, which begins with LABEL KTOP3 extracts the eigendata for each frequency. This data is used to calculate the derivative of each frequency with respect to the current design variable. Within this nested loop the current row of the sensitivity matrix is calculated using the APPEND statement. Each time after exiting the nested loop the current row is appended to the previous one to generate the sensitivity matrix. These two loops end with the statement REPT KTOP,3, which means repeat the above loop 3 times after completing it the first time.

10.) Create square matrix: This section adds the required equations to the sensitivity matrix and to the $[\Delta F]$ to make the equations square. The matrices are put in the form of $AX = B$, where $A = AA$, $B = BB$, and $X = XX$. AA is the square matrix which contains the sensitivity information and extra equations. BB is the vector which contains the difference in analytical and experimental frequencies, and XX contains the changes in the design variables for a better solution. If additional equations are not used, then a least squares formulation is utilized to create a square matrix and solve for the unknowns.

11.) Solve system of equations: This statement solves for XX in the previously mentioned equation.

12.) Extract components of XX (Delta X) vector: This section extracts the values from the XX vector for use as multipliers when creating new design variables.

13.) Assemble new mass and stiffness matrices: This section generates new mass and stiffness matrices, by using the new design variables.

14.) Repeat iterative loop 5 times: This is the end of the iterative loop which tells the program to execute the loop 6 times, if the objective function has not obtained a low enough value.

The rest of the program is data used for setting up the analysis in NASTRAN.

This procedure assumes that the matrix derivatives are calculate directly. Step 9) would not be changed by calculating the derivatives by finite difference. The step-by-step procedure would need to be modified previous to step 1). This would involve utilizing the solution sequence 53 (sensitivity analysis) from MSC/NASTRAN. One could either do a previous run and calculate the derivatives and store them in the database, or one could include solution 53 (slightly modified) in the program listed in the appendix.

Chapter III

METHOD THAT UTILIZES ORTHOGONALITY EQUATIONS

3.1 *Introduction*

The previous chapter detailed a technique which utilized an iterative approach to update the system matrices. Berman and Nagy's (1983) modification method, which is developed in this chapter, yields a closed form solution. Their approach relies on utilizing constraint equations which are associated with normal modes analysis. Normal modes analysis implies a system which is undamped and unforced. They attempt to satisfy the orthogonality of the measured modes to the analytically derived system matrices, while minimizing the changes to the mass and stiffness matrices. Their paper is probably the most referenced of all of the papers dealing with the

existing methods. More recent techniques utilize some of its characteristics while others use it to compare new methods.

Often there are more coefficients in the analytical matrices than there are measured data. The experimental data consists of modal parameters; i.e. frequencies and modes shapes. If this is the case, then the natural frequencies and modes shapes can be represented by a new eigenvalue problem, which results from the updated mass and stiffness matrices. An advantage of this technique is that there is no initial eigensolution required, and the method offers a closed form solution, and the updated system matrices are symmetric.

There are also less desirable attributes of this method. The matrices are globally modified, meaning that all coefficients of the matrices can be modified even if they are already correct, or zero. Modifying matrix coefficients that were originally zero is similar to adding new members or elements to the model. This will usually result in non-physically correct models. In addition, new eigenvalues can be introduced when utilizing this technique (Ibrahim and Saafan,1988).

In order to modify the mass matrix an error function must be set up which utilizes the orthogonality condition of normal modes. This function is minimized using Lagrange multipliers. From this an updated mass matrix is developed. The stiffness matrix can also be modified using the eigenvalue equation with the updated mass matrix. An error function, comprised of changes to the stiffness matrix, is minimized while using Lagrange multipliers for the eigenvalue constraint equation.

3.2 Development

This method can be applied using a set of analytical system matrices along with a set of experimentally derived natural frequencies and corresponding modal vectors. Berman and Nagy assume that the mass matrix is more accurate than the stiffness matrix. It is usually easy to determine the total mass of a system, and it is only somewhat more difficult to determine the distribution of that mass throughout the structure. Whereas, it is more difficult to determine a lumped stiffness value for a continuous system. Hence, the mass matrix is corrected with the modal matrix, then the stiffness matrix is adjusted with the use of the frequencies, modal matrix, and the updated mass matrix.

The modification of the analytical mass matrix is accomplished assuming that the experimental modal matrix is correct, and that experimental data is available at every analytical DOF. If this is not the case, then the analytical matrices can be reduced to match the experimental data, or the experimental data can be expanded to match the analytical model.

The mass matrix is updated in order to make the matrices and experimental data compatible. All of the modal vectors are assumed to be mass normalized, i.e.

$$[\phi_e]_i^T [M_a] [\phi_e]_i = 1 \quad \text{for } i = 1 \text{ to } m \quad (3.1)$$

where, m is the number of measured modes. However, the experimental modes may not be orthogonal since;

$$[\phi_e]_i^T [M_a] [\phi_e]_j \quad \text{may not equal } 0 \text{ for } i \neq j$$

where, $[\phi_e]_i$ is the i th experimental modal vector, and $[M_a]$ is the analytical mass matrix.

Berman defines ΔM to be equal to the difference between the analytical mass matrix and the updated mass matrix.

$$[\Delta M] = [M_u] - [M_a] \quad (3.2)$$

Hence the orthogonality relationship can be expressed as

$$[\phi_e]^T ([M_a] + [\Delta M]) [\phi_e] = [I] \quad (3.3)$$

By separating the equation and rearranging it

$$[\phi_e]^T [\Delta M] [\phi_e] = [I] - [m_a] \quad (3.4)$$

where, $[m_a]$ is defined as

$$[m_a] = [\phi_e]^T [M_a] [\phi_e] \quad (3.5)$$

Equation (3.4) implies that $[\phi_e]^T [\Delta M] [\phi_e]$ contains only the negative of the off-diagonal terms defined by $[\phi_e]^T [M_a] [\phi_e]$.

An error function is set up such that utilizes a mass weighted minimization of the percent changes to the mass coefficients

$$\varepsilon = || [N]^{-1} [\Delta M] [N]^{-1} || \quad (3.6)$$

where $|| \quad ||$ denotes using the Einstein rule of summation

$$\varepsilon = \sum_{i=1}^n \sum_{j=1}^n ([N]^{-1} [\Delta M] [N]^{-1})_{ij}^2 \quad (3.7)$$

n is the number of DOF in the model, and N is defined as

$$[N] = [M_a]^{1/2} \quad (3.8)$$

A Lagrangian multiplier λ_{ij} , is used for each matrix coefficient of eq. (3.4). The Lagrange multipliers are used to incorporate constraints while minimizing the percent change to the components of the analytical mass matrix, while modelling the modal data correctly. The Lagrangian function is then set up

$$\psi = \varepsilon + \sum_{i=1}^m \sum_{j=1}^m \lambda_{ij} ([\phi_e]^T [\Delta M] [\phi_e] - [I] + [m_a])_{ij} \quad (3.9)$$

where m is the number of measure modes. Equation (3.9) is differentiated with respect to each component of $[\Delta M]$, and each derivative is set equal to zero, i.e.,

$$\frac{d\psi}{d\Delta M_{rs}} = 2 \sum_{i=1}^n \sum_{j=1}^n N_{is}^{-1} \left[\sum_{k=1}^n \left(\sum_{l=1}^n N_{il}^{-1} \Delta M_{lk} \right) N_{kj}^{-1} \right] N_{rj}^{-1} + \sum_{i=1}^m \sum_{j=1}^m \lambda_{ij} \phi_{si} \phi_{rj} \quad (3.10)$$

where, ΔM_{rs} corresponds to the coefficient of $[\Delta M]$ in the rth row and sth column.

This results in

$$2 [M_a]^{-1} [\Delta M] [M_a]^{-1} + [\phi_e] [\Lambda]^T [\phi_e]^T = 0 \quad (3.11)$$

or

$$[\Delta M] = -\frac{1}{2} [M_a] [\phi_e] [\Lambda]^T [\phi_e]^T [M_a] \quad (3.12)$$

Where Λ contains the Lagrangian multipliers λ_{ij} . Equations (3.4) and (3.12) contain only 2 matrix unknowns $[\Delta M]$ and $[\Lambda]$.

By substituting eq. (3.12) into eq. (3.4), $[\Lambda]$ is then determined to be

$$[\Lambda] = [\Lambda]^T = -2 [m_a]^{-1} ([I] - [m_a]) [m_a]^{-1} \quad (3.13)$$

Λ is symmetric, because of the symmetry of eq. (3.5), i.e., the off diagonal terms of $[\phi_e]^T [M_a] [\phi_e]$ are equal. Equation (3.13) requires the inversion of $[m_a]$. This matrix is a function of the analytical mass matrix and the experimental modal vectors. In general $[\mu_a]$ is not singular even if $[M_a]$ is singular. This is due to the arithmetic process of carrying out the triple product of the modal vectors and mass matrix (see eq. 3.5). If the modes are orthogonal, this is apparent as $[m_a]$ will be a diagonal matrix with no zero terms on the diagonal. However, if the quality of the modes is not high, $[m_a]$ is not guaranteed to be non-singular. By substituting eq. (3.13) into eq. (3.12), $[\Delta M]$ is determined to be

$$[\Delta M] = [M_a] [\phi_e] [m_a]^{-1} ([I] - [m_a]) [m_a]^{-1} [\phi_e]^T [M_a] \quad (3.14)$$

The updated mass matrix can be obtained by recalling from eq. (3.2) that

$$[M_u] = [M_a] + [\Delta M] \quad (3.15)$$

Thus a closed form method of updating the mass matrix can be performed. This method assumes that the modal matrix is correct and that the mass matrix should be modified to make the modes normal. This is done without an eigensolution and minimizes changes to components of the mass matrix. However, all of the coefficients of the mass matrix can change. Thus, the compatibility of the finite

element model is not conserved. The resulting modal matrix and mass matrix are compatible, in that the triple product of the modal matrix and the mass is equal to the identity matrix.

The updated mass matrix and the eigendata are used to modify the analytical stiffness matrix. This is accomplished using three constraint equations which are based on; the eigenvalue equation, the orthogonality relations, and the enforced symmetry of the stiffness matrix;

$$[K_u] [\phi_e] - [M_u] [\phi_e] [\mu_e] = 0 \quad (3.16)$$

$$[\phi_e]^T [K_u] [\phi_e] - [\mu_e] = 0 \quad (3.17)$$

$$[K_u] - [K_u]^T = 0 \quad (3.18)$$

An error function is set up similar to the one used for the mass matrix modification,

$$\varepsilon = \| [N]^{-1} ([K_u] - [K_a]) [N]^{-1} \| \quad (3.19)$$

Three Lagrangian matrices are required for the constraint equations ($[\Lambda_\mu], [\Lambda_o], [\Lambda_s]$). The Lagrangian equation can then be set up as

$$\begin{aligned} \psi &= \varepsilon + \sum_{i=1}^n \sum_{j=1}^m \Lambda_{\mu_{ij}} ([K_u] [\phi_e] - [M_u] [\phi_e] [\mu_e])_{ij} + \\ &\sum_{i=1}^m \sum_{j=1}^m \Lambda_{o_{ij}} ([\phi_e]^T [K_u] [\phi_e] - [\mu_e])_{ij} + \sum_{i=1}^n \sum_{j=1}^n \Lambda_{s_{ij}} ([K_u] - [K_u]^T)_{ij} \end{aligned} \quad (3.20)$$

In a similar manner to the derivation of the mass matrix modification, equation (3.20) is differentiated with respect to the components of the updated stiffness matrix and set equal to zero.

Thus results in an updated stiffness matrix of

$$[K_u] = [K_a] + ([\Delta] + [\Delta]^T) \quad (3.21)$$

where

$$[\Delta] = \frac{1}{2} [M_u][\phi_e] ([\phi_e]^T [K_a][\phi_e] + [\mu_e]) [\phi_e]^T [M_u] - [K_a][\phi_e][\phi_e]^T [M_u] \quad (3.22)$$

The stiffness matrix is modified without an eigensolution using the updated mass matrix and the eigendata. However all coefficients of the stiffness matrix can be modified.

3.3 Implementation

This method is straight-forward to implement within MSC/NASTRAN. The eigendata is entered into the program via Direct Matrix Input (DMI) cards in the bulk data deck in normalized form. The system matrices are assembled within solution 63 (Normal modes analysis). Appendix A.2 provides a listing of the program used in the identification process for the free-free beam using 6 modes. Following is a step-by-step explanation of the alterations of the program, required to implement this method.

1.) The first few lines set up the analysis in NASTRAN. The ALTER statement instructs the computer to put the following lines into the source code of solution 63 before running the analysis.

2.) Rename matrices: This section renames existing matrices for use in the identification.

3.) Extract original frequency values: This section extracts the frequency values predicted by the original finite element model. This is for comparison purposes only. An initial eigensolution is not required for this method, only the generation of the analytical system matrices.

4.) Generate LMA: This section generates the matrix $[m_a]$ for use in the analysis.

5.) Generate updated mass matrix: This section updates the mass matrix using the modal vectors.

$$[M_u] = [M_a] + [M_a] [\phi_e] [m_a]^{-1} ([I] - [m_a]) [m_a]^{-1} [\phi_e]^T [M_a]$$

Recalling that

$$[M_u] = [M_a] + [\Delta M]$$

6.) Generated updated stiffness matrix: This is done using all of the experimental data, and the updated mass matrix.

$$[K_u] = [K_a] + ([\Delta] + [\Delta]^T)$$

where

$$[\Delta] = \frac{1}{2} [M_u] [\phi_e] ([\phi_e]^T [K_a] [\phi_e] + [\mu_e]) [\phi_e]^T [M_u] - [K_a] [\phi_e] [\phi_e]^T [M_u]$$

7.) Extract newly calculated eigenvalues and eigenvectors: This section performs an eigenvalue analysis using the updated system matrices and extracts the values for comparison.

8.) Print results: This section prints the results extracted during the computation. The rest of the input is the required data to set up the NASTRAN analysis.

Chapter IV

METHOD BASED ON CONNECTIVITY CONSTRAINT

4.1 *Introduction*

A goal of System Identification is to generate a system which accurately predicts the response of a structure to an input. Of interest, is to also keep the mathematical system physically realistic. This can be done by applying additional constraints; i.e. no change in overall mass, similar connectivity relationship as the initial model, or making element by element change only.

The method developed in this section utilizes a closed form technique, and does not require a previous eigensolution in order to perform the matrix modification. Kabe's method (1985) utilizes a constraint relationship that implies that no new load paths should be introduced to the structure. This implies that all matrix coefficients that were originally null, must remain null in the final matrix.

Kabe's method minimizes the percent change to the coefficients of the stiffness matrix. The method assumes that the mass matrix is reasonably correct, because mass is something that can be directly measured. Therefore, it is more important to modify the stiffness matrix than the mass matrix.

His formulation assumes that the modes are orthogonal. However, results can be obtained if they are not. Different methods, similar to the one used for the orthogonality method can be used to make the modal vectors compatible with the mass matrix. Kabe utilizes Lagrange multipliers to minimize an error function comprised of relative changes to the original stiffness matrix, while adhering to certain constraints. This method yields a closed form solution.

4.2 *Development*

Kabe's method modifies the global stiffness matrix using a connectivity constraint and minimizing the relative change to the components of the analytical matrix. His method does not require an initial eigensolution, only the formulation of the analytical system matrices, and the experimental data.

Kabe assumes that the mass matrix is reasonably correct. He wants to base the modification method on defining the stiffness matrix as accurately as possible. This is due to the fact that the structures being modelled may eventually have added mass, due to loads they will be carrying. He requires that the updated stiffness matrix be defined as

$$[K_u] = [K_a] \cdot [\gamma] \quad (4.1)$$

Where \bullet implies that the components of both matrices be multiplied by each other, i.e.

$$K_{u_{ij}} = K_{a_{ij}} \gamma_{ij} \quad (4.2)$$

This constraint implies that no new load paths will be introduced to the model. If $K_{a_{ij}} = 0$ then $K_{u_{ij}} = 0$ as well.

An error function is set up which contains the relative change to each stiffness coefficient:

$$\varepsilon = \sum_{i=1}^n \sum_{j=1}^n (\hat{I}_{ij} - \hat{I}_{ij} \gamma_{ij}) \quad (4.3)$$

where n is the number of DOF in the model,

$$\hat{I}_{ij} = 1 \text{ if } K_{a_{ij}} \neq 0 \quad (4.4)$$

$$\hat{I}_{ij} = 0 \text{ if } K_{a_{ij}} = 0 \quad (4.5)$$

The adjusted stiffness matrix is determined by finding the $[\gamma]$ which minimizes ε while obeying the following constraints. The eigenvalue equation

$$- [M_a] [\phi_e] [\mu_e] + ([K_a] \bullet [\gamma]) [\phi_e] = 0 \quad (4.6)$$

and the symmetry of the updated stiffness matrix

$$[\gamma] - [\gamma]^T = 0 \quad (4.7)$$

The method of Lagrangian multipliers is used to minimize ε while adhering to the constraints defined by eq (4.6) and (4.7).

$$\psi = \varepsilon - [M] [\phi_e] [\mu_e] + \sum_{i=1}^n \sum_{j=1}^n \lambda_{s_{ij}} (\gamma_{ij} - \gamma_{ji}) + \sum_{i=1}^n \sum_{j=1}^m \lambda_{\mu_{ij}} \left(\sum_{l=1}^n K_{a_{il}} \gamma_{il} \phi_{e_{il}} \right) \quad (4.8)$$

where, m is the number of measured frequencies.

The partial derivative of ψ is taken with respect to each γ_{rs} , and these equations are set equal to zero.

Taking the derivative of ε with respect to γ_{rs} results in

$$\frac{\partial \varepsilon}{\partial \gamma_{rs}} = \sum_{i=1}^n \sum_{j=1}^n 2 \left(\hat{l}_{ij} - \hat{l}_{ij} \gamma_{ij} \right) \left(\hat{l}_{ij} \frac{\partial \gamma_{ij}}{\partial \gamma_{rs}} \right) \quad (4.9)$$

Noting that:

$$\frac{\partial \gamma_{ij}}{\partial \gamma_{rs}} = 0 \text{ if } i \neq r \text{ or } j \neq s$$

$$\frac{\partial \gamma_{ij}}{\partial \gamma_{rs}} = 1 \text{ if } i = r \text{ and } j = s$$

This results in

$$\frac{\partial \varepsilon}{\partial \gamma_{rs}} = -2 \left(\hat{l}_{rs} - \hat{l}_{rs} \gamma_{rs} \right) \quad (4.10)$$

Taking the derivative of

$$\sum_{i=1}^n \sum_{j=1}^m \lambda_{\mu_{ij}} \left(\sum_{l=1}^n K_{a_{il}} \gamma_{il} \phi_{e_{ij}} \right)$$

yields

$$\sum_{j=1}^m \lambda_{\mu_{rj}} K_{a_{rs}} \phi_{e_{sj}}$$

Taking the derivative of $[M] [\phi_e] [\mu_e]$ yields zero. Taking the derivative of

$$\sum_{i=1}^n \sum_{j=1}^n \lambda_{s_{ij}} (\gamma_{ij} - \gamma_{ji})$$

yields

$$\lambda_{s_{rs}} - \lambda_{s_{sr}}$$

The resulting equation in matrix form yields

$$-2([\hat{l}] - [\gamma]) + [K_a] \cdot ([\Lambda_\mu] [\phi_e]^T) + [\Lambda_s] = 0 \quad (4.11)$$

Noting that

$$[\Lambda_s] + [\Lambda_s]^T = 0 \quad (4.12)$$

from the symmetry condition, $[\Lambda_s]$ can be eliminated from eq (4.11) by adding the transpose of the equation to itself, yielding

$$-4([\hat{i}] - [\gamma]) + [K_a] \cdot ([\Lambda_s] [\phi_e]^T + [\phi_e] [\Lambda_s]^T) = 0 \quad (4.13)$$

Equation (4.13) can be pre-multiplied element by element by $\frac{1}{4} [K_a]$ and rearranged to form

$$[K_a] \cdot [\gamma] = [K_a] - \frac{1}{4} [\Phi] \cdot ([\Lambda_s] [\phi_e]^T + [\phi_e] [\Lambda_s]^T) \quad (4.14)$$

where,

$$[\Phi] = [K_a] \cdot [K_a] \quad (4.15)$$

and noting that

$$[K_a] \cdot [\hat{i}] = [K_a] \quad (4.16)$$

By combining eq. (4.6) and (4.14), these yield

$$[A] + ([\Phi] \cdot ([\Lambda_s] [\phi_e]^T)) [\phi_e] + ([\Phi] \cdot ([\phi_e] [\Lambda_s]^T)) [\phi_e] = 0 \quad (4.17)$$

where,

$$[A] = 4 [M_a] [\phi_e] [\mu_e] - [K_a] [\phi_e] \quad (4.18)$$

In order to solve eq (4.17) for Λ_s , we define $[\bar{A}]$ as a column vector containing all of the transposed rows of $[A]$. $[A]$ is a n by m matrix and $[\bar{A}]$ is an (n x m) by 1 matrix. For example, for the case when $[A]$ is a 3 by 2 matrix:

$$[\bar{A}] = \begin{bmatrix} A_{11} \\ A_{12} \\ A_{21} \\ A_{22} \\ A_{31} \\ A_{32} \end{bmatrix} \quad \text{where } [A] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \quad (4.19)$$

Similarly $[\bar{\Lambda}_s]$ as

$$[\bar{\Lambda}_s] = \begin{bmatrix} \lambda_{11} \\ \lambda_{12} \\ \lambda_{21} \\ \lambda_{22} \\ \lambda_{31} \\ \lambda_{32} \end{bmatrix} \quad \text{where } [\Lambda_s] = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \\ \lambda_{31} & \lambda_{32} \end{bmatrix} \quad (4.20)$$

Recalling that \bullet means element by element multiplication, $([\Phi] \bullet ([\Lambda_s] [\phi_e]^T)) [\phi_e]$ can be rewritten as

$$([\Lambda_s] [\phi_e]^T) \bullet [\Phi] [\phi_e]$$

or

$$\left(\sum_{i=1}^n ([\Lambda'_s] [\phi_e]^T [\hat{\Phi}_i]) \right) [\phi_e]$$

where $[\Lambda_i]$ is $[\Lambda_s]$ with everything but the i th row set equal to zero, and where $[\hat{\Phi}_i]$ is the i th row of $[\Phi]$ turned into a diagonal matrix.

Transposing this equation results in

$$[\phi_e]^T [\hat{\Phi}_i]^T [\phi_e] [\Lambda_s]^T$$

$([\Phi] \cdot ([\phi_e] [\Lambda_s]^T)) [\phi_e]$ can be rewritten as

$$([\Phi] \cdot ([\phi_e] [\Lambda_s]^T)) [\phi_e] = (([\phi_e] [\Lambda_s]^T) \cdot [\Phi]) [\phi_e]$$

or

$$\left(\sum_{i=1}^n ([\phi'] [\Lambda_s]^T [\hat{\Phi}_i]) [\phi_e] \right)$$

where $[\phi']$ is $[\phi_e]$ with everything but the i th row set equal to zero.

Thus, by manipulating eq (4.17), $[A]$ can be rewritten as

$$[\bar{A}] = ([\alpha] + [\beta]) [\bar{\Lambda}_s] \quad (4.21)$$

where $[\alpha]$ and $[\beta]$ are defined as

$$[\alpha] = \begin{bmatrix} [G_1] & 0 & 0 & \cdot & 0 \\ 0 & [G_2] & 0 & \cdot & 0 \\ 0 & 0 & [G_3] & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & [G_n] \end{bmatrix} \quad (4.22)$$

$$[\beta] = \begin{bmatrix} [H_{11}] & [H_{12}] & [H_{13}] & \cdot & [H_{1n}] \\ [H_{21}] & [H_{22}] & [H_{23}] & \cdot & [H_{2n}] \\ [H_{31}] & [H_{32}] & [H_{33}] & \cdot & [H_{3n}] \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ [H_{n1}] & [H_{n2}] & [H_{n3}] & \cdot & [H_{nn}] \end{bmatrix} \quad (4.23)$$

where

$$[G_i] = -[\phi_e]^T [\hat{\Phi}_i] [\phi_e] \quad (4.24)$$

and

$[\hat{\Phi}_i]$ is a diagonal matrix whose elements are the i th row of $[\Phi]$

and where

$$[H]_{ij} = -[\bar{\phi}]_j [D_j]_i^T \quad (4.25)$$

and

$$[D_j]_i = \text{ith column of } [\phi_e]^T [\hat{\Phi}_j] \quad (4.26)$$

$$[\bar{\phi}]_j = \text{jth column of } [\phi_e]^T \quad (4.27)$$

Equation (4.21) can be solved by defining the following transformation

$$[\bar{\Lambda}_s] = [\Psi] [\rho] \quad (4.28)$$

where the columns of $[\Psi]$ are the eigenvectors associated with the nonzero eigenvalues of $([\alpha] + [\beta])$. By substituting eq (4.28) into (4.21) and pre-multiplying this by $[\Psi]^T$

$$[\Psi]^T [\bar{A}] = [\Psi]^T ([\alpha] + [\beta]) [\Psi] [\rho] = [\Omega] [\rho] \quad (4.29)$$

where, $[\Omega]$ is a diagonal matrix whose terms are the nonzero eigenvalues of $([\alpha] + [\beta])$

$$[\rho] = [\Omega]^{-1} [\Psi]^T [\bar{A}] \quad (4.30)$$

Therefore

$$[\bar{\Lambda}_s] = [\Psi] [\Omega]^{-1} [\Psi]^T [\bar{A}] \quad (4.31)$$

Which leads to the updated stiffness matrix,

$$[K_u] = [K_a] - \frac{1}{4} [\Phi] \cdot ([\bar{\Lambda}_s] [\phi_e]^T + [\phi_e] [\bar{\Lambda}_s]^T) \quad (4.32)$$

4.3 Implementation

Kabe's method is somewhat more difficult to implement within MSC/NASTRAN. Operations that are simple in programming languages such as FORTRAN can be cumbersome in DMAP. This method requires the assemble of matrices using partitioning and merging commands. These can be very time consuming on the

computer. Due to the fact that only non-zero eigenvalues of $([\alpha] + [\beta])$ are used, and the number of these may be unknown, the analysis may need to be run for a different number of non-zero eigenvalues. Appendix A.3 contains a listing of the program used to solve the free-free beam using 6 modes in the identification process. Following is a step-by-step explanation of the alterations required to implement this method.

1.) The first few lines set up the initial analysis in NASTRAN. The ALTER statement tells the NASTRAN to include the next statements with the source code.

2.) Store matrices: This section renames current matrices for later use.

3.) Generate experimental eigenvalues: This section converts the experimental frequencies into eigenvalues.

4.) Generate A: This section generates the matrix $[A]$.

5.) Generate cap phi from original K matrix: This section computes $[\Phi]$ by multiplying the stiffness matrix element by element by itself.

6.) Generate partition vectors: In order to partition or merge certain matrices within NASTRAN, there must be a vector which defines how the vector should be partitioned or merged.

7.) Set up loop to generate matrices: This section starts a loop which executes the number of times there are DOF in the model.

8.) Extract rows from cap phi: This section extracts each row from $[\Phi]$ and converts it into a diagonal matrix.

9.) Generate A bar from A: This section converts matrix $[A]$ into $[\bar{A}]$.

10.) Generate G sub i: This section generates each $[G_i]$.

11.) Generate ephr#: This section extracts each row of the modal matrix and turns it into a column.

12.) Generate M###: This section generates the MMM matrix used in later calculations.

13.) Generate G partitioning vector: This section generates the partition vector necessary for assembling $[\alpha]$.

14.) Assemble alpha matrix: This loop assembles $[\alpha]$ using $[g_i]$ and the partitioning vectors.

15.) Generate Beta matrix: This section generates the $[\beta]$ matrix. It is done within two loops, the first loop extracts the MMM matrix, and the second loop assembles the i th row of $[\beta]$ with the appropriate matrices.

16.) Find eigendata of alpha + beta: This section sums $[\alpha]$ and $[\beta]$, and finds the eigenvalues of the sum.

17.) Purge modal matrices of zero eigenvalues and corresponding eigenvectors: This section extracts the data from the previous eigenvalue analysis which corresponds to the non-zero eigenvalues of $([\alpha] + [\beta])$,

18.) Generate lambda b: This section generates $[\lambda]$ using the existing data,

$$[\Lambda_s] = [\Psi] [\Omega]^{-1} [\Psi]^T [\bar{A}]$$

19.) Generate lambda from lambda b: This section converts $[\Lambda]$ a column vector, into $[\Lambda]$, a matrix.

20.) Generate new K matrix: This section generates a new K matrix using,

$$[K_u] = [K_a] - \frac{1}{4} [\Phi] \cdot ([\Lambda] [\phi_e]^T + [\phi_e] [\Lambda]^T)$$

21.) Perform final eigenvalue analysis: This section performs the final eigenvalue analysis, using the updated stiffness matrix, and the analytical mass matrix.

The rest of the program listing contains information pertinent to the NASTRAN analysis.

Chapter V

METHOD BASED ON RESIDUAL FORCE VECTOR

5.1 *Introduction*

In both experimental and analytical results, errors exist. The object of System Identification is to identify the best system. Finite Elements methods are developed based on certain assumptions. It is rare that these assumptions are 100% correct in the entire model. Through various techniques it is possible to determine where errors in the analytical model may exist. By correcting the model in this area only, one may be able to generate a good model which is also physically meaningful.

The method implemented in this chapter utilizes residual force vectors. It was developed by Chen and Wang (1988). A residual force vector is a vector which results from subtracting two quantities, which calculate forces, from each other. Due to the discrete nature and assumptions of finite elements, as well as errors in the experimental error, residual forces will usually result. By examining these forces one

should be able to locate modelling errors in a model. Once these locations are known, then one can modify these locations in order to update the analytical model using test data.

Chen and Wang introduce a method for calculating the residual force vector. They then offer different approaches to updating the analytical model using this information. The three approaches offered are based on; (1) the orthogonality approach, (2) the re-analysis approach, and (3) the statistical approach.

The method developed in this chapter is one which uses the orthogonality approach. Their method attempts to find matrices which when added to the analytical matrices will solve the eigen problem more accurately. The new matrices are comprised of contributions from elements around the location of modelling errors only. Therefore, this method enforces element connectivity, and matrix symmetry.

5.2 Development

The model refinement method described by Chen and Wang is divided into two parts. Their initial concern is to locate modelling errors in the analytical model. This is accomplished by finding locations in the model where forces do not sum to zero. Their second concern is to modify the matrices utilizing information obtained from the error location procedure.

The eigenvalue equation is manipulated in order to find the location of the errors. In theory

$$- [M_a] [\phi_e]_i \mu_e + [K_a] [\phi_e]_i = 0 \quad (5.1)$$

should be true. However, due to the inability to exactly measure stiffness properties of an element within a structure, complex geometries or material properties, or other reasons, errors can result. Therefore Chen and Wang chose to locate errors in the finite element model by seeing where equation (5.1) does not hold true.

They defined $[\Delta M]$ and $[\Delta K]$ as

$$[\Delta M] = [M_u] - [M_a] \quad (5.2)$$

and

$$[\Delta K] = [K_u] - [K_a] \quad (5.3)$$

Therefore

$$([K_a] + [\Delta K])[\phi_e]_i = ([M_a] + [\Delta M])[\phi_e]_i \mu_e \quad (5.4)$$

or

$$- [K_a] [\phi_e]_i + [M_a] [\phi_e]_i \mu_e = [\Delta K] [\phi_e]_i - [\Delta M] [\phi_e]_i \mu_e \quad (5.5)$$

$[R]_i$ is defined as the Residual force vector which is equal to either side of the above equation.

$$[R]_i = - [K_a] [\phi_e]_i + [M_a] [\phi_e]_i \mu_e \quad (5.6)$$

This is valid for each mode, so for all modes the equation can be changed from vector form to matrix form

$$[R] = - [K_a][\phi_e] + [M_a][\phi_e][\mu_e] \quad (5.7)$$

By examining locations in $[R]$, corresponding to degrees of freedom in the model, problem areas can be located. By modifying elements attached to those degrees of freedom one can make the analytical model more accurate while changing only problem areas of the model.

The orthogonality approach used by Chen and Wang also defines $[\Delta K]$ and $[\Delta M]$ as

$$[\Delta K] = \sum_{i=1}^r a_i [k_i] \quad (5.8)$$

where r is the number of deficient element matrices, and a_i is the coefficient multiplier for each individual matrix associated with a given element stiffness matrix $[k_i]$.

$$[\Delta M] = \sum_{i=1}^r b_i [m_i] \quad (5.9)$$

where b_i is the coefficient multiplier for each individual matrix associated with a given element mass matrix $[m_i]$.

They utilize the eigenvalue equation in this form

$$([K_a] + [\Delta K])([\phi_a] + [\Delta\phi]) = ([M_a] + [\Delta M])([\phi_a] + [\Delta\phi])([\mu_a] + [\Delta\mu]) \quad (5.10)$$

where

$$[\phi_e] = [\phi_a] + [\Delta\phi]$$

and

$$[\mu_e] = [\mu_a] + [\Delta\mu]$$

Expanding and rearranging

$$[\Delta K] [\phi_e] - [\Delta M] [\phi_e] [\mu] = [M_a] [\Delta\phi] [\mu] + [M_a] [\phi_a] [\Delta\mu] - [K_a] [\Delta\phi] \quad (5.11)$$

Substituting eqs (5.8) and (5.9) into eq (5.11) yields

$$\left(\sum_{i=1}^r a_i [k_i] \right) [\phi_e] - \left(\sum_{i=1}^r b_i [m_i] \right) [\phi_e] [\mu_e] = [M_a] [\Delta\phi] [\mu_e] + [M_a] [\phi_a] [\Delta\mu] - [K_a] [\Delta\phi] \quad (5.12)$$

The only unknowns in the above equation are a_i and b_i . They can be determined by solving a set of linear equations obtained from the above matrix equation. Each component of the matrices on each side of the equation are set equal to each other. This results in $n \times m$ equations, where m is the number of measured frequencies, and n is the number of DOF in the model. There are generally more equations than unknowns resulting from the above equation, where the unknowns are the local element multipliers. Therefore a least squares fit is used to find the set of variable values which best satisfies the equations.

The matrix $[B]$, resulting from the right hand side of the above equation is known. It is partitioned such that each consecutive row is extracted from the matrix. The row is then transposed and appended to the previous row. This results in a column vector of size $(n \times m)$, the number of coefficients in the original matrix. If $[B]$ were a 3 by 2 matrix, then $[\bar{B}]$ would be defined as

$$[\bar{B}] = \begin{bmatrix} B_{11} \\ B_{12} \\ B_{21} \\ B_{22} \\ B_{31} \\ B_{32} \end{bmatrix} \quad \text{where} \quad [B] = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \\ B_{31} & B_{32} \end{bmatrix} \quad (5.13)$$

Similarly, this is done for each variable, or unknown matrix multiplier on the left hand side of the equation. The resulting matrix $[Q]$ is then $(n \times m)$ rows by $(2r)$ columns, where r is the number of deficient elements.

The resulting equation is

$$[Q][Z] = [\bar{B}] \quad (5.14)$$

where $[Z]$ contains the unknown matrix multipliers, and $[Q]$ contains the columns associated with the deficient elements.

This equation can be solved by pre-multiplying each side of the equation by $[Q]^T$, and then pre-multiplying each side by $([Q]^T[Q])^{-1}$. This assumes that $([Q]^T[Q])^{-1}$ is not singular. The resulting equation is

$$[Z] = ([Q]^T[Q])^{-1}[Q]^T[\bar{B}] \quad (5.15)$$

It should be noted that this is an ill conditioned solution method which may not yield the desired the desired results for a large problem. The updated matrices are obtained by,

$$[K_u] = [K_a] + \sum_{i=1}^r a_i [k_i] \quad (5.16)$$

and

$$[M_u] = [M_a] + \sum_{i=1}^r b_i [m_i] \quad (5.17)$$

The eigenvalue equation is then solved with the updated matrices and a better prediction of the response is usually obtained.

5.3 Implementation

This method is very easy to implement. All of the required matrices for the error location procedure are already calculated within the original normal modes analysis. Determining the DOF with errors associated can be done by choosing the largest values in a particular residual force vector column.

The solution of the equations for the variables a_i and b_i , requires the manipulation of matrices. Appendix A.4 contains a listing of the program used for the analysis of the free-free beam using 6 modes in the identification. Following is a listing of the step-by-step alterations required for this identification process.

1.) The first few lines set up the analysis in NASTRAN. The ALTER statement tells the system to include the following statements with the source code.

2.) Rearrange matrices: This section rearranges, or renames matrices for use in the alteration.

3.) The following MMAT and KMAT ... : This section stores element matrices in the database for use later in the analysis. These matrices are input by the user.

4.) Generate or rename experimental and analytical data: This section puts the existing data in a form to be used later.

5.) Multiply each eigenvector by its eigenvalue: This section extracts each experimental eigenvector and its corresponding eigenvalue and multiplies them together. They are then reassembled into the matrix LPHI.

6.) Calculate residual force matrix: This section calculates the residual force matrix,

$$[R] = - [K_a][\phi_e] + [M_a][\phi_e][\mu]$$

The user must examine this matrix to determine where deficient elements exist in the model.

7.) Calculate matrix for unknowns: This section calculates $[B]$, from

$$[M_a][\Delta\phi][\mu] + [M_a][\phi_a][\Delta\mu] - [K_a][\Delta\phi]$$

8.) Generate A matrix: This section calculates all possible columns needed for $[Q]$, using deficient element matrices. There is one column for each possible unknown, a_i or b_j .

9.) Generate B matrix: This section calculates $[\bar{B}]$, from $[B]$.

10.) Extract columns corresponding to deficient elements: This section appends each of the columns needed for $[Q]$, using deficient element matrices. There is one column for each unknown, a_i or b_j .

11.) Solve for X using simple formulation: This section generates $[Z]$, the vector of unknown matrix multipliers. It then extracts the values and uses them to generate new mass and stiffness matrices. An eigenvalue analysis is then performed with the updated matrices.

12.) Print results: This section prints out the calculated data.

The rest of the program contains information pertinent to the NASTRAN analysis.

Chapter VI

NUMERICAL RESULTS

This chapter describes three numerical studies which were executed using the four System Identification methods. The first study was a simple 2-DOF problem that checked the validity of the developed methods. The second numerical study, which is an 8-DOF example, was used to correlate the procedures to published studies. The third study, utilizing experimental results from a free-free beam, details how the methods work for an engineering problem.

6.1 Verification Studies - 2 DOF model

In order to assure that the methods were programmed correctly, they were examined using a 2 DOF model. It consists of 2 springs and 2 masses connected in series, with the end of the first spring grounded (ref Figure 1 on page 62). These

verification studies include *engineering* problems as well as problems which examine the mathematical capabilities of the methods. For this thesis engineering problems are defined as those which contain errors in the stiffness matrix only.

The "experimental" data used for the identification processes in this section is generated from the 2-DOF model, and this data is referred to as the quasi-experimental data. The properties of the initial analytical matrices differ from the quasi-experimental system matrices, thus the analytical results did not match the quasi-experimental data exactly. By utilizing the four different System Identification methods, the initial analytical models were updated using the quasi-experimental data. These updated system matrices were then compared to the original matrices which yielded the quasi-experimental data. The frequencies and modal vectors that the updated matrices yielded were also compared to the quasi-experimental data.

Table 1 on page 63 lists the properties used for the springs and masses for the six 2-DOF test cases. The tests are called TEST 1a thru TEST 1f.

For each test there is a corresponding table (Tables 2 - 7) which contain the matrices and modal data corresponding to the quasi-experimental data, as well as the for the initial analytical model. Also included are the updated matrices resulting from each method as well as the corresponding modal data.

The first four tests (TESTS a-d) consist of engineering tests, or problems where only the stiffness matrix is modified. The mass matrix in most analytical models is assumed to be more accurate, as the mass of a structure can be determined accurately, and the distribution of that mass throughout the structure can usually be determined. Whereas, the stiffness of a sub region of a model is more difficult to specify.

The first two tests contain minor (10%) modifications to the springs. Table 2 on page 65 contains the results of the first test in which the initial analytical model is

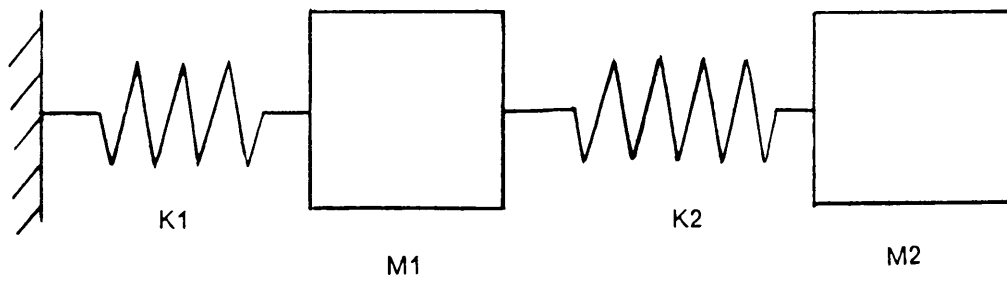


Figure 1. 2 DOF SPRING MASS SYSTEM USED FOR EXAMPLE 1

Table 1. TABLE OF VARIABLE VALUES FOR TESTS 1a - 1f

	Quasi-	Initial Analytical Models			
Parameter	Experimental	TEST 1a	TEST 1b	TEST 1c	TEST 1d
K 1	1000.	900.	900.	10.	10.
K 2	1000.	1000.	1100.	1000.	100.
M 1	.1	.1	.1	.1	.1
M 1	1.	1.	1.	1.	1.
	Quasi-				
Parameter	Experimental	TEST 1e	TEST 1f		
K 1	1000.	900.	900.		
K 2	1000.	900.	1100.		
M 1	.1	.09	.09		
M 1	1.	.9	1.1		

defined by defining K_1 as 10% lower than the model used to generate the quasi-experimental data. All four of the methods were able to correct the initial analytical matrices with a reasonable degree of accuracy (1%). Table 3 on page 66 contains the results for TEST 1b where both K_1 and K_2 had errors of 10% in the initial analytical model. Again all four methods were able to reproduce the desired stiffness matrix, the correct frequencies, and corresponding modal vectors. This indicates that the methods work well for a problem with few DOF and minor changes to the initial analytical stiffness matrix. Note that all of the modal vectors in this section are normalized such that the largest term is always 1.

In TEST 1c (ref Table 4 on page 67), the initial analytical model is defined assuming a very large error in one of the springs (K_1). Therefore the first frequency and modal vector resemble the data from a rigid body mode, i.e., the frequency becomes very small and the components of the modal vector approach uniformity. Again all methods were able to update the initial analytical stiffness matrix so that they yielded the proper frequencies and modal vectors.

For TEST 1d, the analytical model is defined as having very large errors in the values for both springs. All of the methods were able to perform the identification properly for this test, as can be seen in Table 5 on page 68.

It should be noted that all of the methods made no changes to the mass matrix. For the sensitivity method, only spring constants were used as design variables. The orthogonality method makes changes to the mass matrix only if the modal vectors are not orthogonal. These vectors were already orthogonal since the same mass matrix was used for the quasi-experimental and initial analytical cases. The connectivity method modifies only the stiffness matrix. The components of the mass matrix were not allowed to change in the residual force method, as a correct mass matrix was assumed for the three previous methods in this example.

Table 2. INITIAL PARAMETERS AND RESULTS OF TEST 1a

Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1900. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.416 22.25	$\begin{Bmatrix} .5393 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0539 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

Table 3. INITIAL PARAMETERS AND RESULTS OF TEST 1b

Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1100. \\ -1100. & 1100. \end{bmatrix}$	3.407 22.85	$\begin{Bmatrix} .5635 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0560 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

Table 4. INITIAL PARAMETERS AND RESULTS OF TEST 1c

Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 1010. & -1000. \\ -1000. & 1000. \end{bmatrix}$.4779 16.76	$\begin{Bmatrix} .9903 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0987 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

Table 5. INITIAL PARAMETERS AND RESULTS OF TEST 1d

Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 110. & -100. \\ -100. & 100. \end{bmatrix}$.4161 5.494	$\begin{Bmatrix} .9161 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0916 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

The results of these tests indicate that the four methods work well for simple systems. All four techniques were able to modify stiffness matrices that have minor and/or major discrepancies with the experimental equivalents. This implies that for a similar problem, small errors in the determination of material properties, geometric properties, etc., as well as large errors due to human error, can be corrected.

To examine the methods more in depth, two additional studies were considered. First the methods were used to identify the modes of the 2 DOF system where non-engineering changes have been made (TEST 1e and 1f). This implies that both the initial analytical mass and stiffness matrices differ from the quasi-experimental matrices. Secondly, the methods were also applied to a larger 8 DOF problem (section 6.2).

The initial analytical system for TEST 1e is defined such that all springs and masses are decreased by 10% from the quasi-experimental matrices. This system yields the same frequencies as the original system, however the modal vectors are not exactly the same due to round-off error. The sensitivity method does not alter the initial analytical matrices as the quasi-experimental frequencies are properly represented. The other three methods make minimal changes to the system matrices as the frequencies are correct and the modal vectors are accurate to within 1%. In other words, modal data from this system is not unique, which indicates all systems may not be identifiable. For these results, reference Table 6 on page 70.

For TEST 1f, the initial system is defined where such that errors are present for all of the springs and masses. The sensitivity method attempts to decrease both frequencies by increasing the terms of the stiffness matrix. The orthogonality method makes inappropriate, but minor changes to the mass matrix and also adds stiffness to all the components the stiffness matrix. The connectivity method never changes the mass matrix and changes the stiffness matrix to represent the system as well as

Table 6. INITIAL PARAMETERS AND RESULTS OF TEST 1e

Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 0.900 \end{bmatrix}$	$\begin{bmatrix} 1800. & -900. \\ -900. & 900. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5123 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 0.900 \end{bmatrix}$	$\begin{bmatrix} 1800. & -900. \\ -900. & 900. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5123 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 0.900 \end{bmatrix}$	$\begin{bmatrix} 1799. & -899. \\ -899. & 900. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5123 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 0.900 \end{bmatrix}$	$\begin{bmatrix} 1799. & -900. \\ -900. & 900. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5123 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 0.900 \end{bmatrix}$	$\begin{bmatrix} 1800. & -900. \\ -900. & 900. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5123 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

possible. The ratio of the components of the mass and stiffness matrices for the quasi-experimental and updated matrices are similar. The same is true for the residual force method. These results can be seen in Table 7 on page 72. The last three methods are able to better represent the modal data of the original system, but none are able to duplicate the original system matrices.

From these first six tests, it appears that all of the methods work well for engineering problems. When more difficult problems exist, such as changes in the mass matrix, then these methods generate a system which yields more accurate modal data. However, these systems do not necessarily represent the true structure, since the modal data is not unique.

When performing the identifications in this section, the experimental data has been assumed to be exact. When errors exist, these methods will most likely attempt to modify the matrices so that they generate modal data with error in it. This indicates the need for experimental data that is accurate.

Table 7. INITIAL PARAMETERS AND RESULTS OF TEST 1f

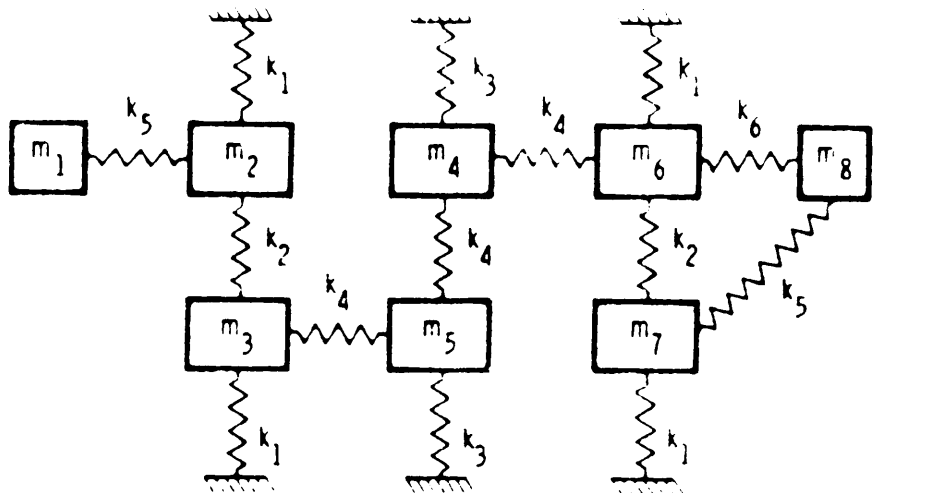
Method	Mass Matrix	Stiffness Matrix	Freq.	Mode Shapes
Quasi Experimental	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1000. \\ -1000. & 1000. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$
Initial Analytical	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 1.100 \end{bmatrix}$	$\begin{bmatrix} 2000. & -1100. \\ -1100. & 1100. \end{bmatrix}$	3.334 24.02	$\begin{Bmatrix} .5161 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0459 \end{Bmatrix}$
Systems after Modification				
Sensitivity Based	$\begin{bmatrix} .0901 & 0.000 \\ 0.000 & 1.101 \end{bmatrix}$	$\begin{bmatrix} 2101. & -1201. \\ -1201. & 1201. \end{bmatrix}$	3.540 24.66	$\begin{Bmatrix} .5840 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0521 \end{Bmatrix}$
Orthogonality Based	$\begin{bmatrix} .0911 & 0.010 \\ 0.010 & 1.090 \end{bmatrix}$	$\begin{bmatrix} 1810. & -900. \\ -900. & 995. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0516 \end{Bmatrix}$
Connectivity Based	$\begin{bmatrix} .0900 & 0.000 \\ 0.000 & 1.100 \end{bmatrix}$	$\begin{bmatrix} 1799. & -1081. \\ -1081. & 1100. \end{bmatrix}$	3.171 22.84	$\begin{Bmatrix} .5231 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0502 \end{Bmatrix}$
Residual Force Based	$\begin{bmatrix} .1000 & 0.000 \\ 0.000 & 1.000 \end{bmatrix}$	$\begin{bmatrix} 2200. & -1100. \\ -1100. & 1100. \end{bmatrix}$	3.514 22.79	$\begin{Bmatrix} .5125 \\ 1.000 \end{Bmatrix} \begin{Bmatrix} 1.000 \\ -.0512 \end{Bmatrix}$

6.2 Modification Studies using an 8 DOF Model


The first study helped establish the validity of the methods. It also indicated short-comings of the different methods. This section will yield the results for a larger problem which has been used in the literature (Kabe, 1985). (ref Figure 2 on page 74) (The figure is an enlarged copy from Kabe's 1985 paper.)

The problem developed by Kabe consists of 8 degrees of freedom and 14 load paths. It may be easiest to think of this problem as 8 masses lined up in series with springs connecting different masses, such that the system can move only axially. The springs range in magnitude of 1.5 to 1000., and the lumped masses range from .001 to 1.0. There are six unique spring constants and three possible lumped mass values. As the entire system is grounded there are no rigid body modes. The mass and stiffness matrices are 8 by 8 in dimension and there are 8 unique frequencies and corresponding modal vectors associated with the system. The frequencies range from 4.88 hz to 6.67 hz.

The original eigendata is used as the experimental data for the identification processes. For the perturbed matrices, spring constant 1 is changed to 1050 from 1000 and spring constant 6 is changed to 4 from 2. Figures 3 thru 6 contain 3-Dimensional plots which indicate the absolute value of the percent error of the frequency prediction after applying each method. Each mode number is plotted along the *Frequency Number* axis and the number of modes used in the identification process is plotted along the *Number of Modes* axis. What each plot indicates is the error in the frequency which the updated matrices yield for each set of modes used in the identification process. By plotting in this manner, the graphs indicate what happens to all of the modes of the system even if they are not all used in the



$k_1 = 1000$ $k_4 = 100$
 $k_2 = 10$ $k_5 = 1.5$
 $k_3 = 900$ $k_6 = 2.0$
 $m_1 = 0.001$ $m_8 = 0.002$
 $m_j = 1.0$ $j = 2, 7$

 LOAD PATH


 DEGREE-OF-FREEDOM i

Figure 2. 8 DOF TEST PROBLEM, DEVELOPED BY KABE, USED FOR SECOND EXAMPLE

identification process. This helps determine whether the method generates a better system or just a system which represents the modal data.

Figure 3 on page 77 shows the results when using the sensitivity method. The first set of pillars, which can be seen when viewing the graph, represent the error in the frequency corresponding to using zero modes, or the results before any method is applied. These errors all are less than 3% except for the last mode.

For all of the sensitivity analyses, the set of design variables chosen are done such that the problem will not be underconstrained. This implies that the number of design variables is less than the number of modes used in the identification process. Design variable linking is used between the springs, in order to modify more springs than there are modes used. When one mode is used, all springs are linked together as one design variable. When two modes are used, the grounded and ungrounded links are divided into separate groups. When three, four, or five modes are used, the springs are divided such that all springs below 5, below 250, and above 1500 are in separate groups. For six, seven, and eight modes, all of the springs are grouped according to the six individual stiffness values.

Examining the row which corresponds to the use of one mode for the sensitivity method, it appears that the first frequency is predicted correctly. However, the accuracy of most of the other frequencies decreases. As more modes are applied, the accuracy of the frequencies used in the identification process increases, but the modes not used in the identification process can become quite unrealistic. Eventually when all 8 modes are used in the identification process, the accuracy of all the frequencies becomes better than the corrupted finite element model. These results for the frequencies are what would be expected; by forcing the linking of spring constants when there are not enough measured frequencies, an exact solution is not guaranteed. However when individual spring constants can be used, the

results are much better. Since the modal vectors are not used in the identification process, the modal vectors from the updated matrices do not match the experimental data exactly. Some terms of the new eigenvectors have changed both magnitude and sign, while other terms have increased by over 25%. Initially what these results indicate is that the quality of any mode not used in the identification process will tend to decrease, and that the modal vectors may not be accurately represented.

Figure 4 on page 78 is similar to Figure 3 on page 77 except that the orthogonality method is used to update the matrices. The orthogonality method yields correct frequencies and corresponding modal vectors for each mode used in the identification process. It does this at the expense of not requiring the connectivity of the model to be the same, as well as allowing unrealistic changes to some components of the matrices in order to generate the correct results. For this problem, the orthogonality method correctly identifies the frequencies and mode shapes used in the identification process, including any associated error. The other frequencies are altered by negligible amounts.

The results of applying the connectivity method to this problem are presented in Figure 5 on page 79. When only one mode is used in the identification process, all of the frequencies become more accurate. The accuracy increases when two modes are used as well. This method requires the accurate determination of the eigenvalues and eigenvectors of a non-positive definite matrix of rank $(n \times m)$. For this type of matrix, MSC/NASTRAN recommends using the modified Givens eigensolution method. However, the author has found that not all of the higher eigendata found for the matrix in this method is exact, and this can lead to some errors in the resulting matrices. This method does perform well in correcting the frequencies and modal vectors used in the identification while increasing the accuracy of the other frequencies.

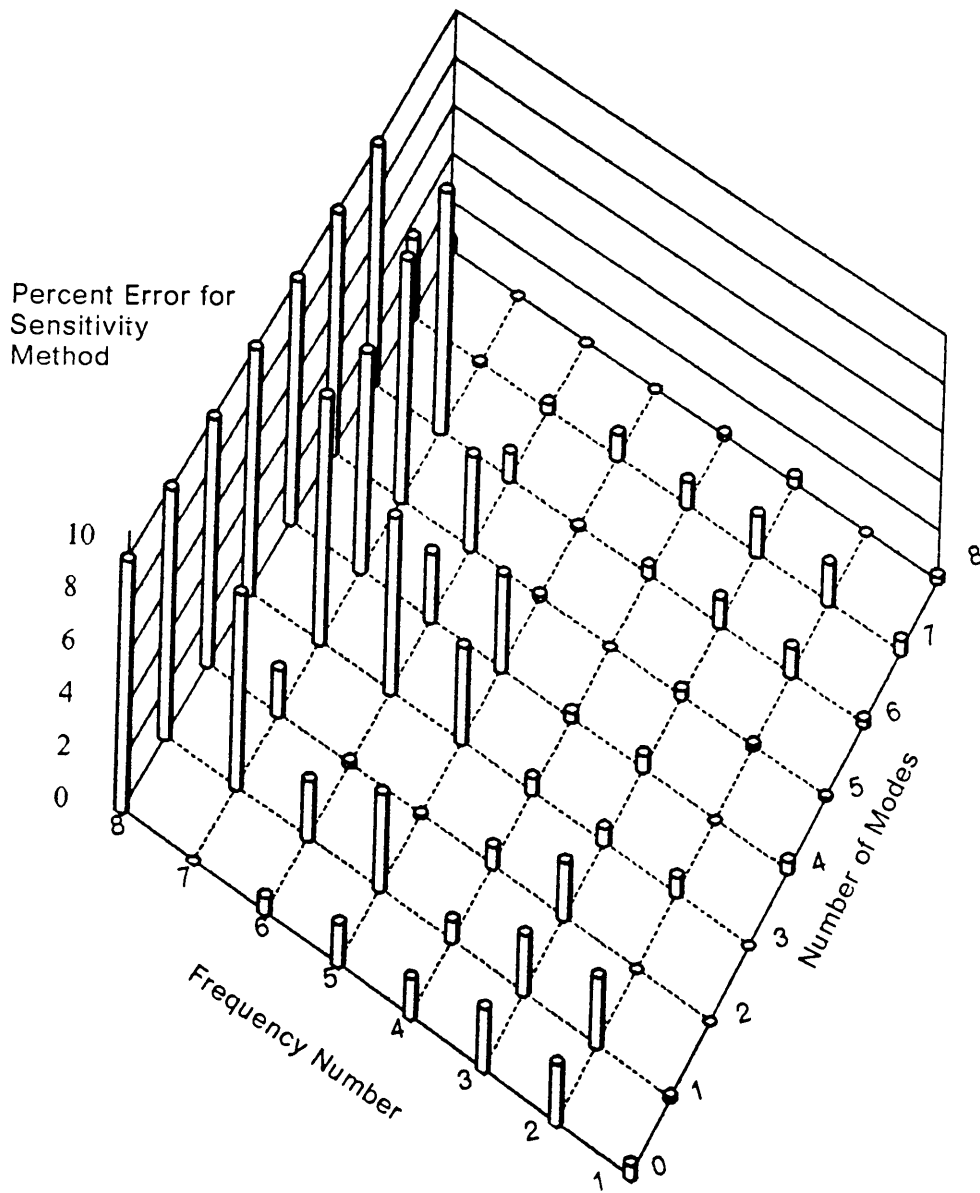


Figure 3. ERROR IN PREDICTED FREQUENCY WITH SENSITIVITY METHOD FOR 8 DOF EXAMPLE

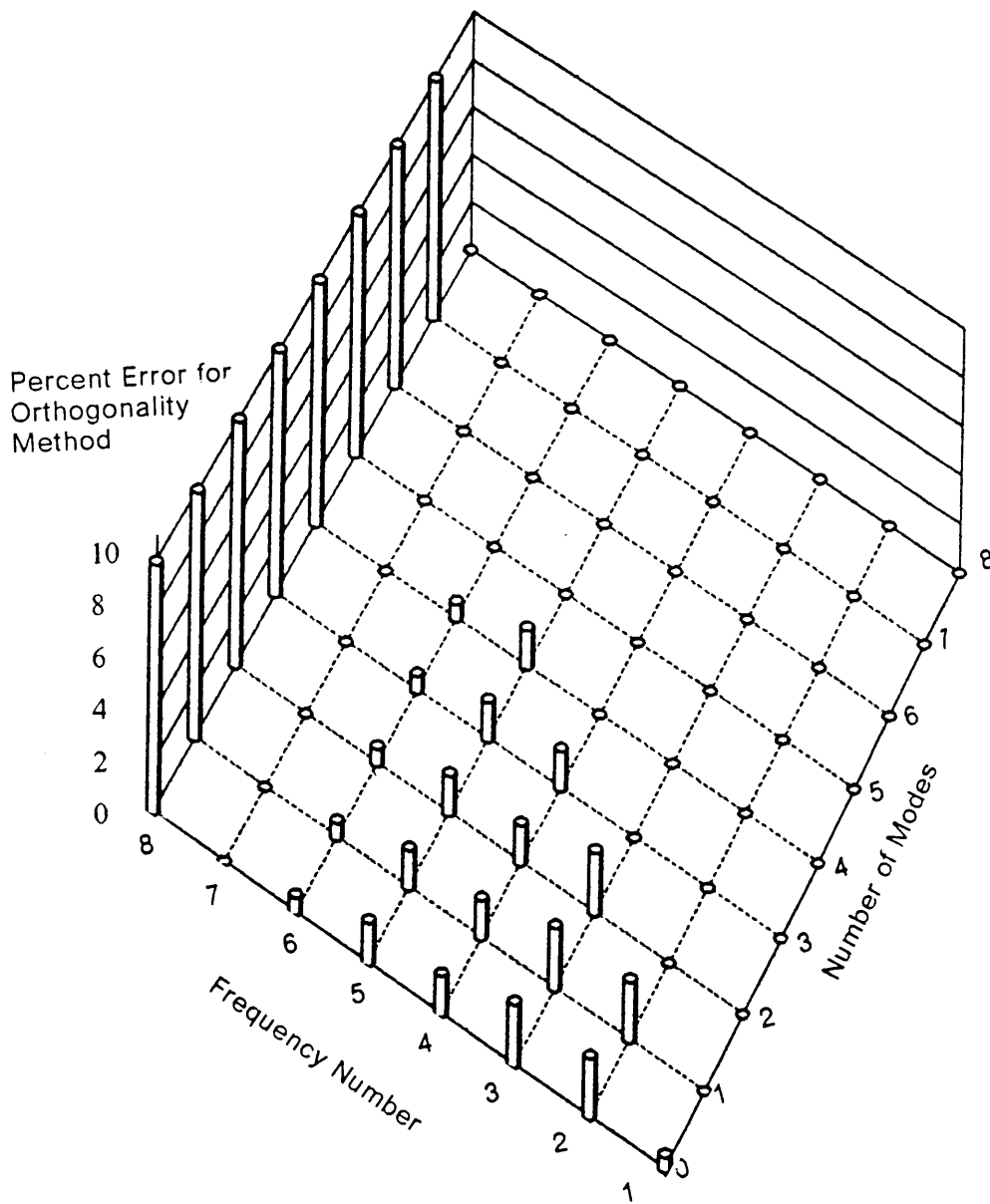


Figure 4. ERROR IN PREDICTED FREQUENCY WITH ORTHOGONALITY METHOD FOR 8 DOF EXAMPLE

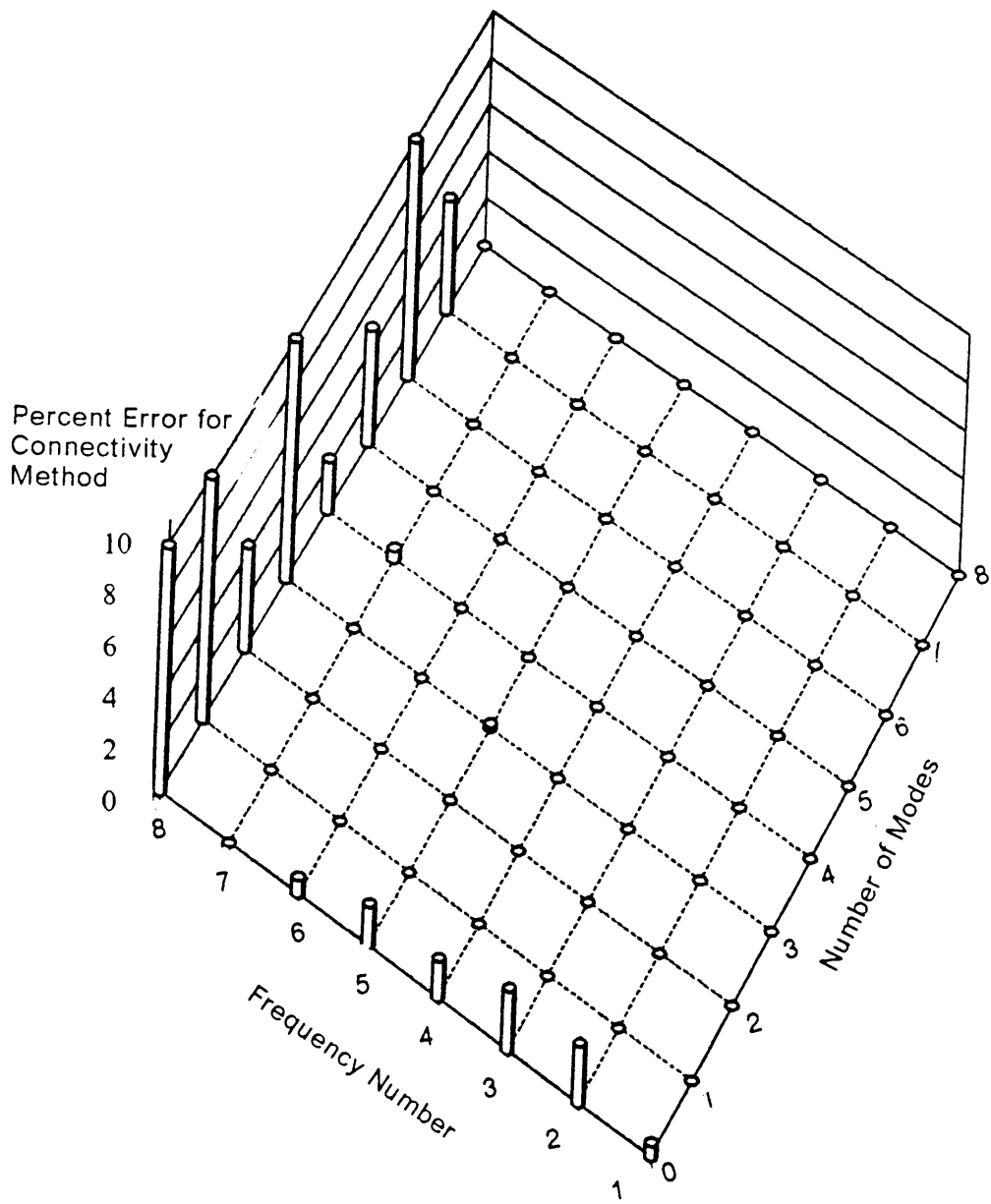


Figure 5. ERROR IN PREDICTED FREQUENCY WITH CONNECTIVITY METHOD FOR 8 DOF EXAMPLE

Figure 6 on page 81 indicates the results of utilizing the residual force approach. The residual force vector is calculated and examined to see which springs should be used in the identification processes. The figure indicates that this method improves the accuracy of the frequencies used in the identification process. However, this method is unable to consistently increase the accuracy of the modes not used in the identification.

All of the four methods seem to be able to improve the frequencies used in the identification process. The sensitivity method does not perform this function as well as the others. It tends to decrease the accuracy of the modes not used, and the modal vectors resulting from this method also are the least accurate. The orthogonality method yields proper frequencies and modal vectors for modes used in the identification process, but makes minimal changes to the frequencies not used. This method also globally modifies the stiffness matrix. When all the modes are used, the updated stiffness matrix matches the desired analytical matrix, however when fewer modes are used this method created terms which approached 5% of the diagonal terms in the stiffness matrix. The connectivity method improves the frequencies and modal vectors for all modes independent of their use in the identification process. The residual force method performed reasonable well, but the modal vectors are not as accurate as those from the orthogonality or connectivity methods.

It should be noted, that this example represents a complex problem, as the components of the original matrices vary greatly in magnitude. The perturbations to the stiffness matrix are also relatively large.

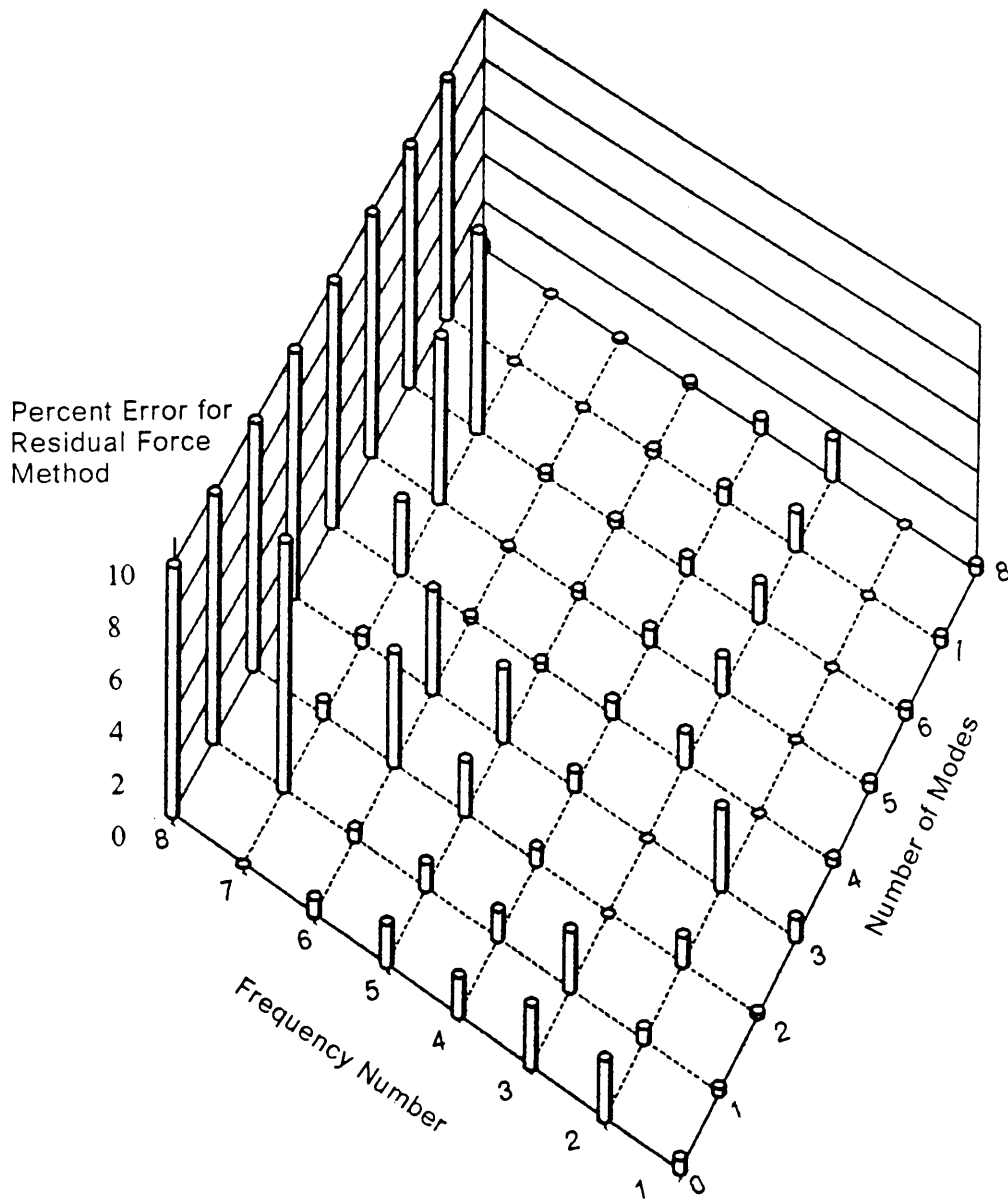
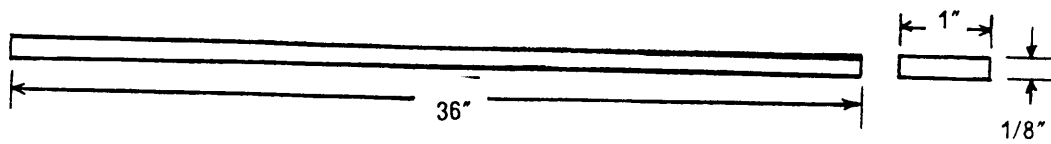


Figure 6. ERROR IN PREDICTED FREQUENCY WITH RES. FORCE METHOD FOR 8 DOF EXAMPLE

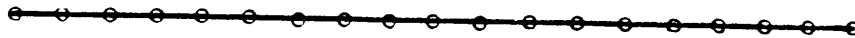
6.3 Application of Methods with data for a free-free beam

In this section all four of the methods will be compared to actual measured accelerometer data of a free-free beam (ref Figure 7 on page 83) (Rorrer et al, 1989). The beam is made of steel, is 36 inches long, with a constant cross section of 1/8 in. by 1. in.. In addition small weights were placed at 19 points along the beam. This was done so that a weight could be removed and replaced by an accelerometer, and not change the characteristics of the structure. Transverse accelerations were determined at each of the 19 stations along the beam, and frequencies and mode vectors were obtained for the first 6 elastic modes. The beam is supported vertically in space by shock cords to simulate free-free boundary conditions. The experimental modal vectors were modified by a minor shift. This was because the data was known to contain a DC offset. The shift was determined by comparing the experimental modal vectors with the finite element modal vectors. For an 18 element model, the finite element results approximate Euler beam theory very well. The DC offset was approximately 2% of twice the amplitude of the mode. Then the modal vectors were scaled such that the triple product with the mass matrix had all ones along the diagonal.

The original finite element model of the structure was reasonably accurate, within 5 % for the first 6 modes. This model was obtained by utilizing 19 nodes and 18 beam elements (CBAR) (based on Euler theory with no shear stiffness) with 2-DOF per node. In addition, 19 concentrated masses (CONM2) were added to the model to simulate the small weights. The model was reduced to match the translational experimental data by applying Guyan reduction to eliminate the rotational DOF's. This operation created fully populated system matrices.



a) Steel beam used for example 3



b) Finite element model of steel beam; 19 nodes with 18 elements

Figure 7. ILLUSTRATION OF BEAM USED FOR EXAMPLE 3 AND THE FINITE ELEMENT MODEL

Figure 8 on page 85 shows the results of the identification when using the sensitivity method. This figure is similar in format to the ones used in section 6.2. However, the scale for the percent error is smaller. For all of these identification steps, the only design variable used is Young's modulus. Either the mass density of the beam or the 19 lumped masses could have been chosen, but all would result in a linear scaling of either the mass or stiffness matrix. One design variable is chosen, as an analyst using this method would most likely assume that the properties of the beam were uniform throughout the length. The sensitivity method works well for this problem. Computationally it is fairly time consuming as the change in design variables is not accomplished in one step. However, the proper results are obtained. This is due to the fact that the finite element model yields a satisfactory initial solution, however perhaps too stiff. The stiffness matrix is linearly scaled down, in order to update the matrix.

The sensitivity method requires an iterative solution. Figure 9 on page 86 shows the convergence of the first three frequencies when applying this method to the free-free beam example using three modes in the identification process. This method is also dependent on the analyst, in that s/he must choose the design variables.

The orthogonality method was applied to this problem and the results are presented in Figure 10 on page 88. This method identified all of the modes used in the identification process, but tended to decrease the accuracy of the other modes. It should be noted that when two or more modes were used in the identification process, this method generated an extraneous mode between the second and third modes (ref Figure 11 on page 89). This mode is not recognizable as a standard mode of a beam. For identification purposes, this mode was ignored and the proper modes were matched to the experimental data, as the modes above the extraneous one correspond to the experimental mode numbers offset by one.

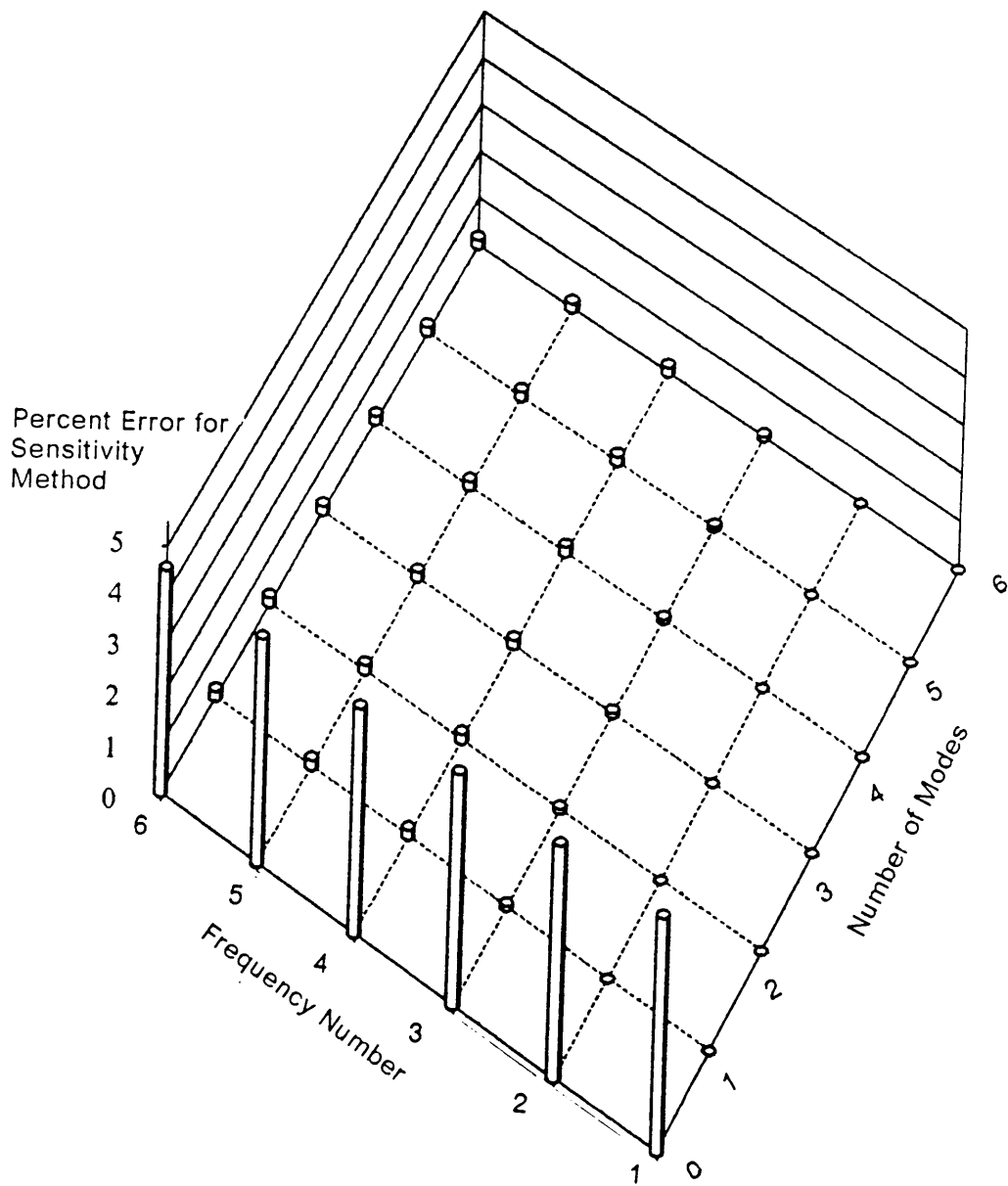


Figure 8. ERROR IN FREQUENCY PREDICTION FOR BEAM USING SENSITIVITY METHOD

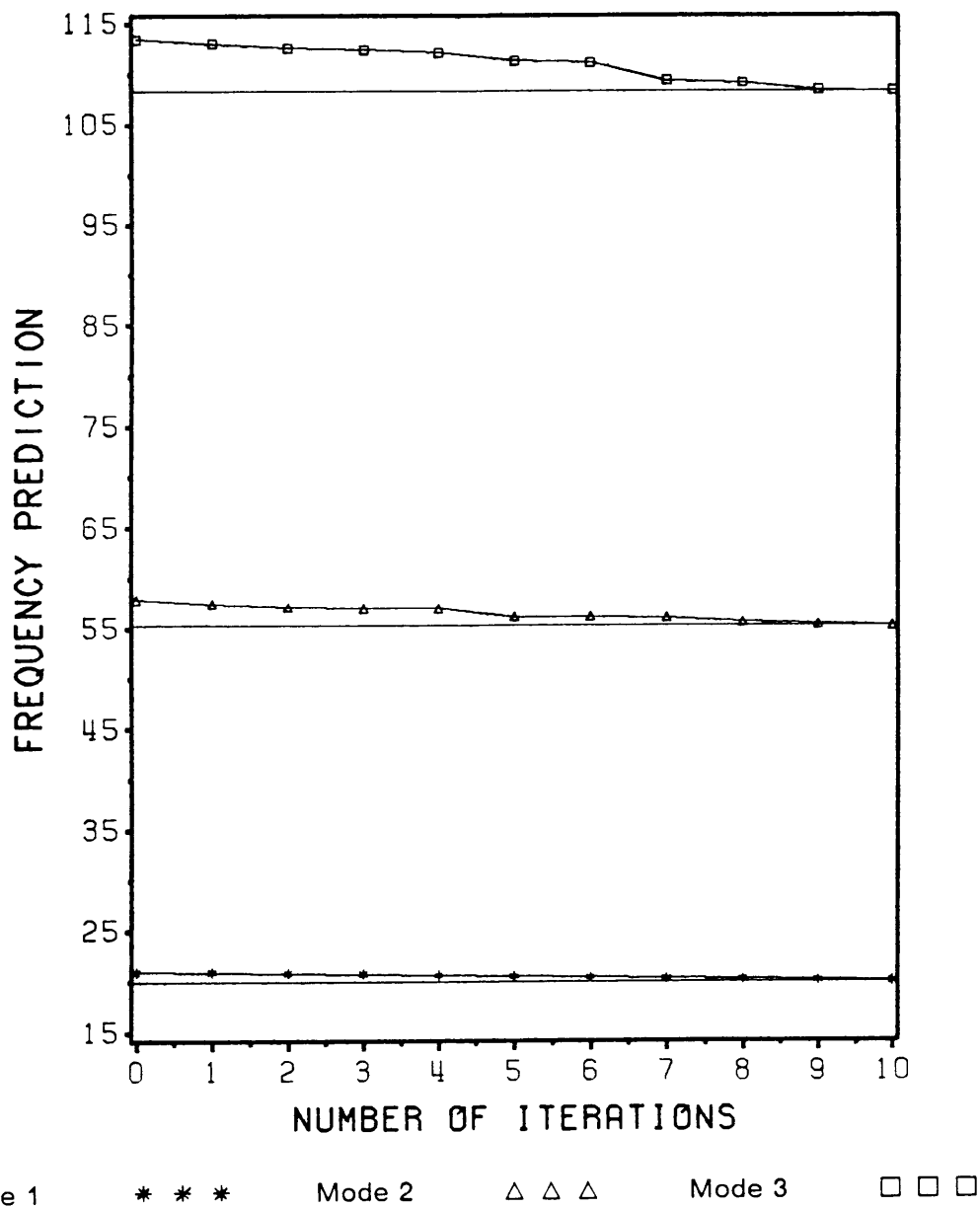


Figure 9. CONVERGENCE OF FREQUENCIES WHEN USING SENSITIVITY METHOD

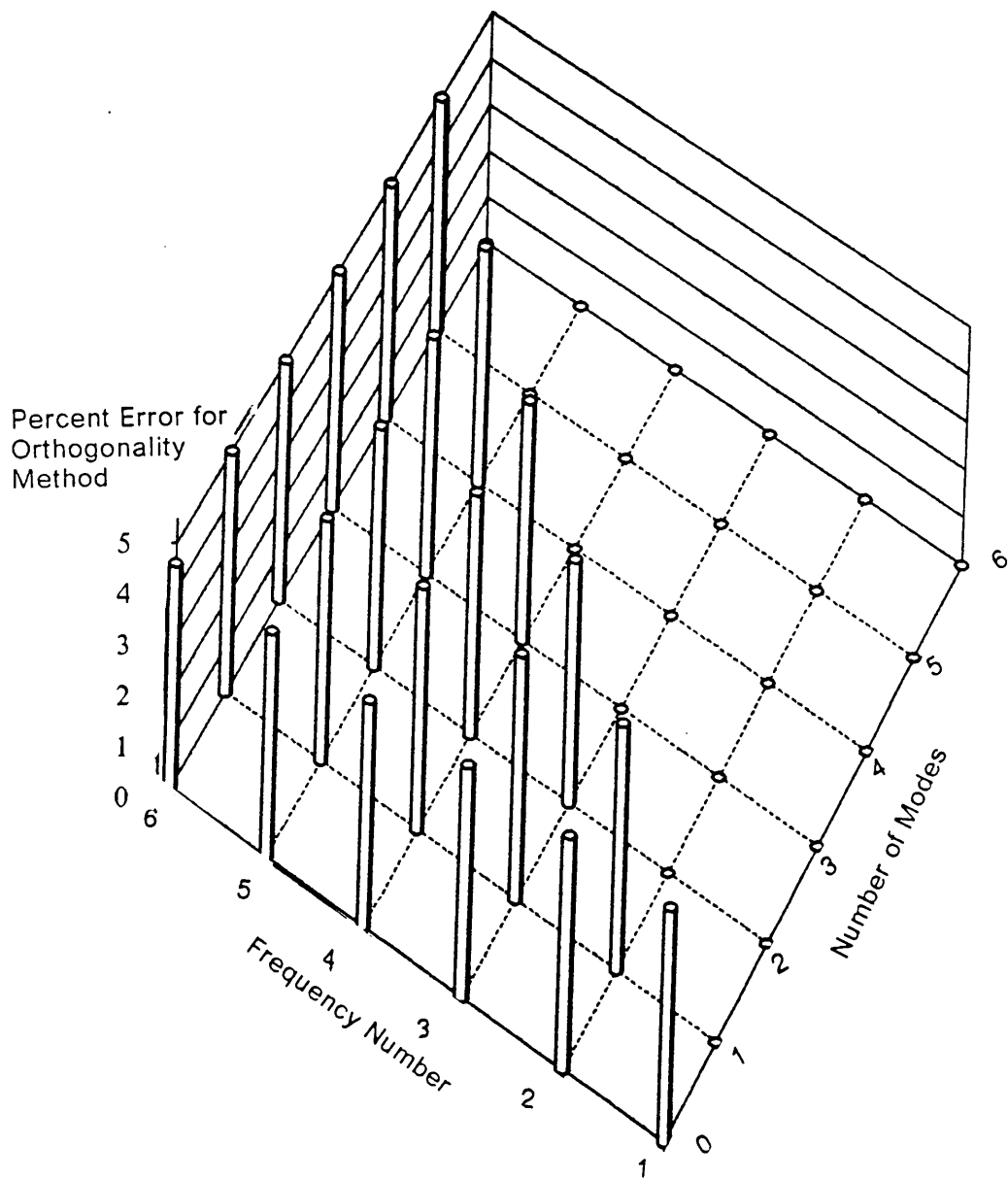


Figure 10. ERROR IN FREQUENCY PREDICTION FOR BEAM USING ORTHOGONALITY METHOD

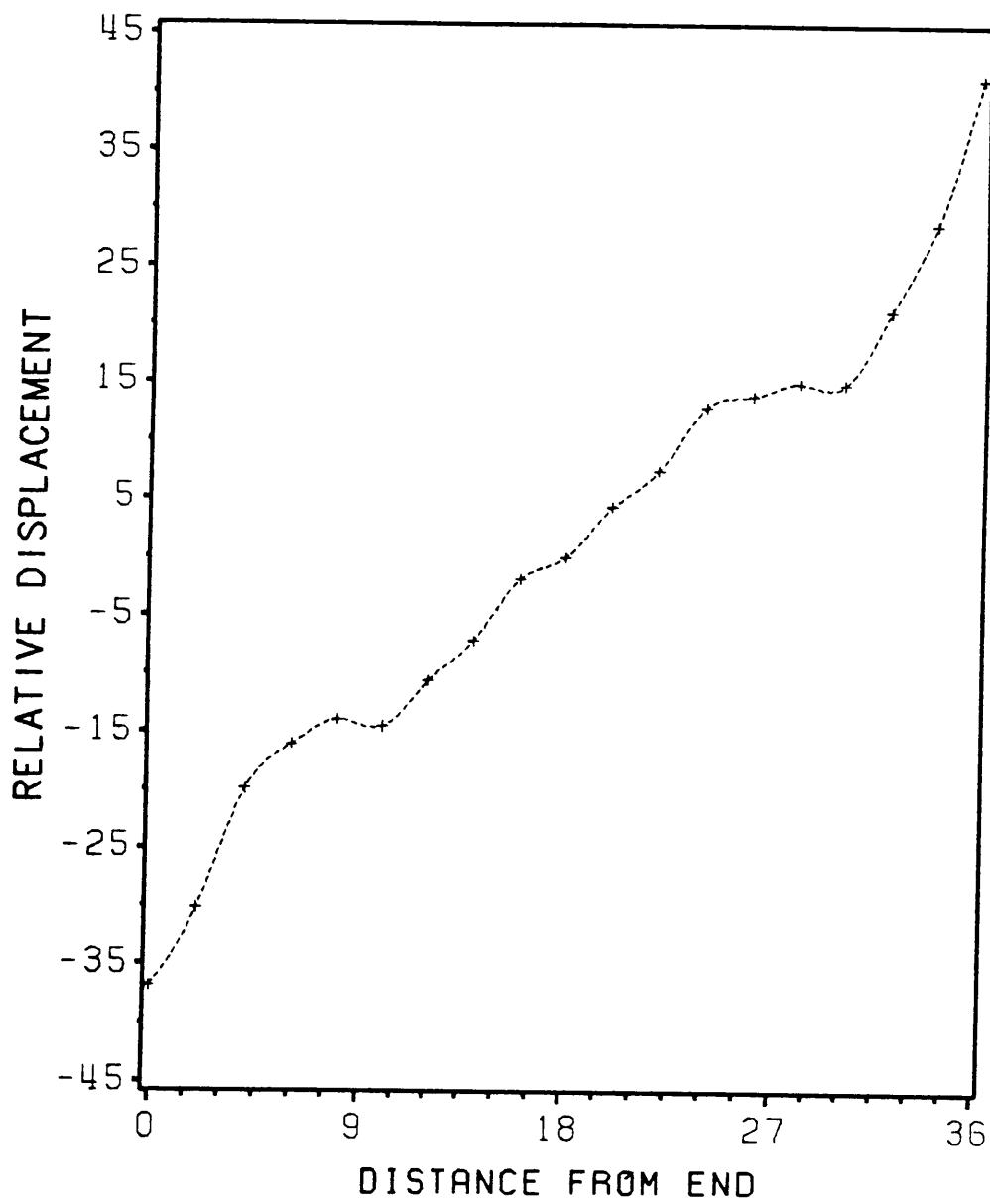


Figure 11. EXTRANEIOUS MODE INTRODUCED BY ORTHOGONALITY METHOD

The connectivity method (ref Figure 12 on page 90) performed well for this example problem. The mass matrix and experimental modal vectors were made compatible in a similar manner to that used for the orthogonality method. Errors existed in some of the frequencies when many modes were used in the identification process. This may be due to the fact that the connectivity method requires utilizing the non-zero eigenvalues of $([\alpha] + [\beta])$. However, due to the nature of certain problems and errors in the eigensolution methods, it is not always clear how many eigenvalues to use. For the problem in section 6.2 it is obvious. However for the free-free beam example it is not. Figure 13 on page 91 shows the values of the first three frequencies as a function of the number of eigenvalues used when applying the connectivity method using three modes in the identification process. The eigenvalues are extracted from $([\alpha] + [\beta])$ which is $(n \times m) \times (n \times m)$ in dimension. It is a square matrix equal in size to the number of DOF's times the number of modes used in the identification process. The vertical line at 67 represents the optimal choice.

The results of applying the residual force method are presented in Figure 14 on page 93. The method was unable to consistently increase the accuracy of the frequencies. This may be due to the fact that the method is trying to correct the system matrices to contain the experimental error in the mode vectors. In addition, the residual force vector tends to locate DOF's with errors as well as error free DOF's. This is because an improper stiffness value at one point can effect the relative displacement at another point, even if that point is correct. This method then attempts to update the system by correcting an already correct position in the model. In addition the least squares formulation used may bias certain parameters because there are many more equations than unknowns.

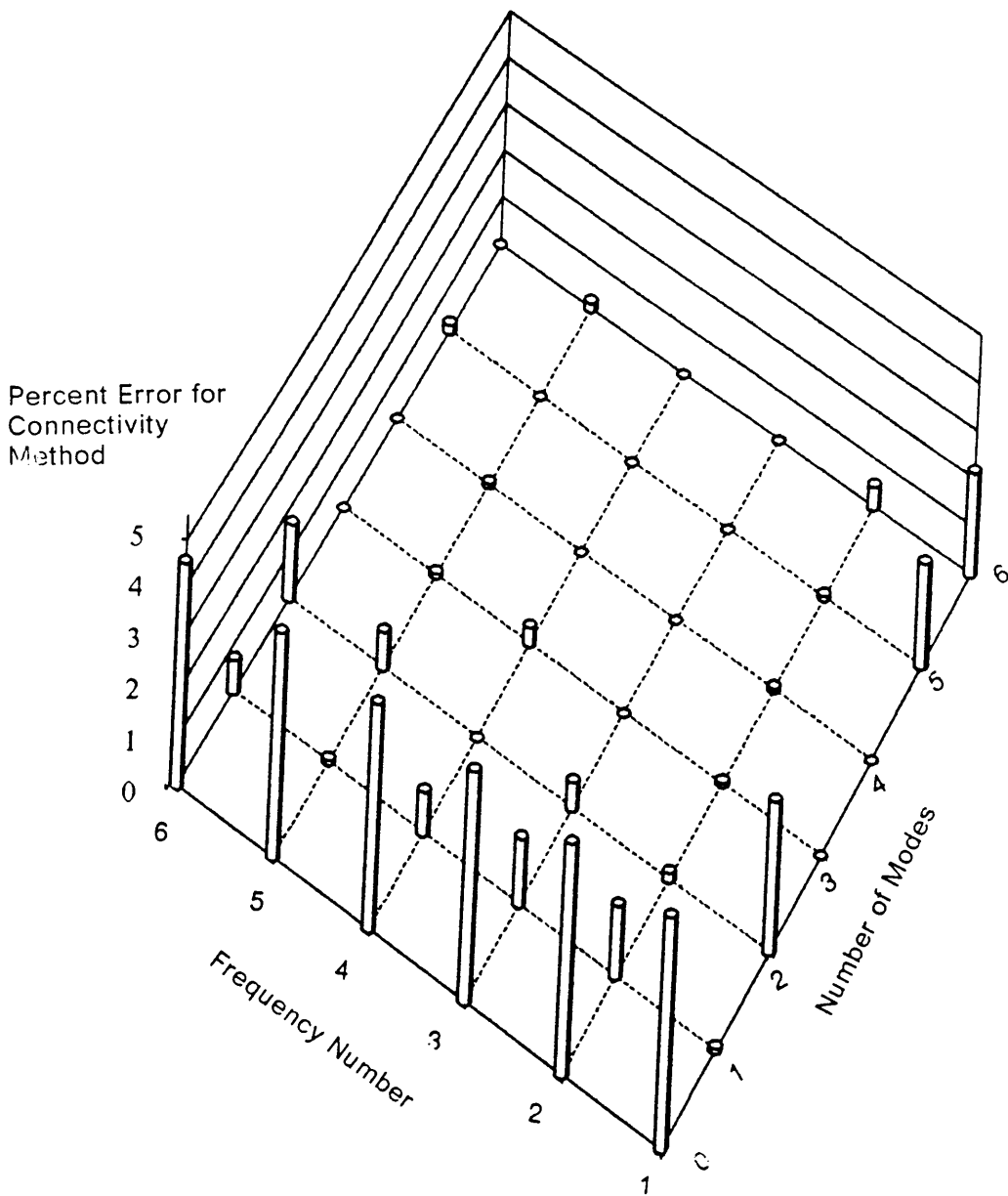
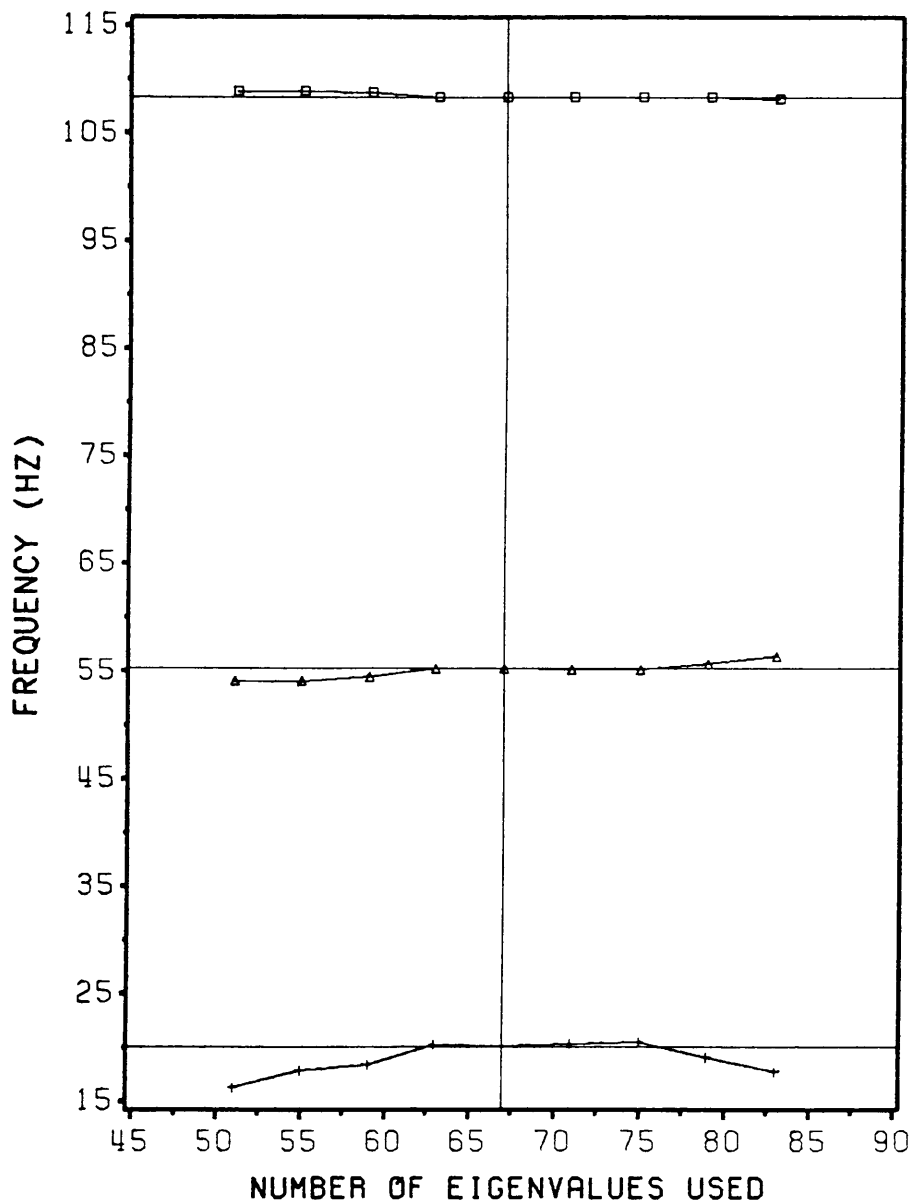


Figure 12. ERROR IN FREQUENCY PREDICTION FOR BEAM USING CONNECTIVITY METHOD



Mode 1 + + + Mode 2 Δ Δ Δ Mode 3 □ □ □

Figure 13. FREQUENCY PREDICTION FOR THE CONNECTIVITY METHOD:
As a function of the number of non-zero eigenvalues used

The residual force method requires the analyst to determine which elements are deficient. This is done through the use of the residual force vector, but depends on the analyst to determine which elements to modify. Usually all of the components of the residual force vector are not zero, so the analyst can chose all of the elements, or decide what size error is negligible. This could be an automated process, but the range of values in the residual force vector for different problems make it still problem dependent.

Figures 15 - 20 show the first six experimentally derived elastic modes of the free-free beam. There are six curves in each figure, and they represent modes determined from; experimental measurements, the initial finite element model, the sensitivity method, the orthogonality method, the connectivity method, and the residual force method. They are the result of using six modes in the identification process. Due to the nature of this problem we realize that the mode shapes should resemble sine waves, hence we realize that the experimental mode shape contain errors. The initial finite element predictions for the modes are accurate, and are only significantly modified by the connectivity and orthogonality method.

For the first two modes the orthogonality and connectivity methods change the mode shape. The orthogonality method matches the experimental data exactly while the connectivity method tends to make changes opposite to the experimental results. For the higher modes, the mode shapes from the updated matrices do not change much from the finite element modes, except for the orthogonality method.

There are many ways to measure the effectiveness of the methods. These include comparing the modal data from the updated system to the experimental modal data. Another method is to examine the relative changes to the components of the stiffness matrices. The greater the number of large percentage change, the more an analyst will need to examine the physics of the resulting matrices. The

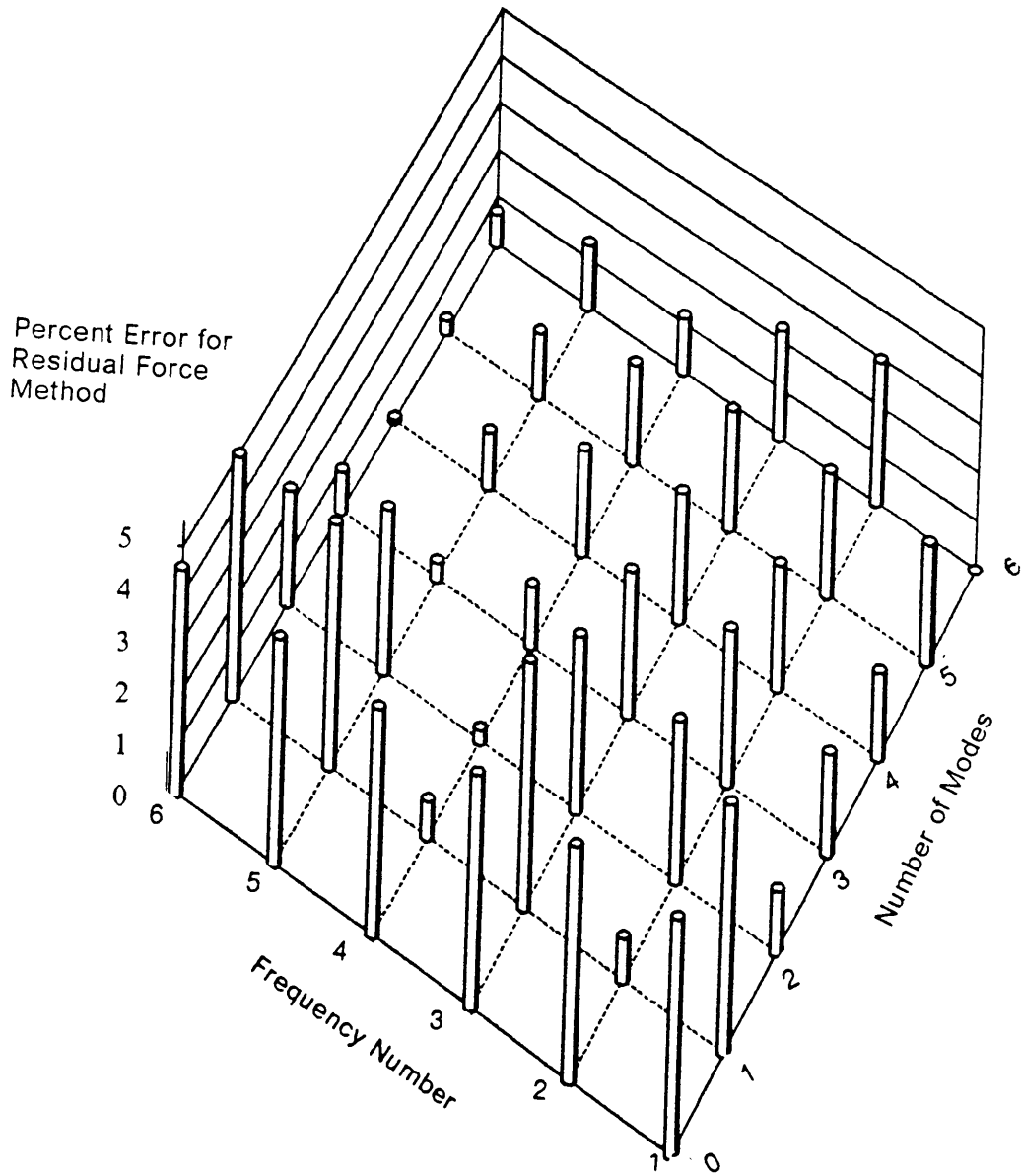


Figure 14. ERROR IN FREQUENCY PREDICTION FOR BEAM USING RES. FORCE METHOD

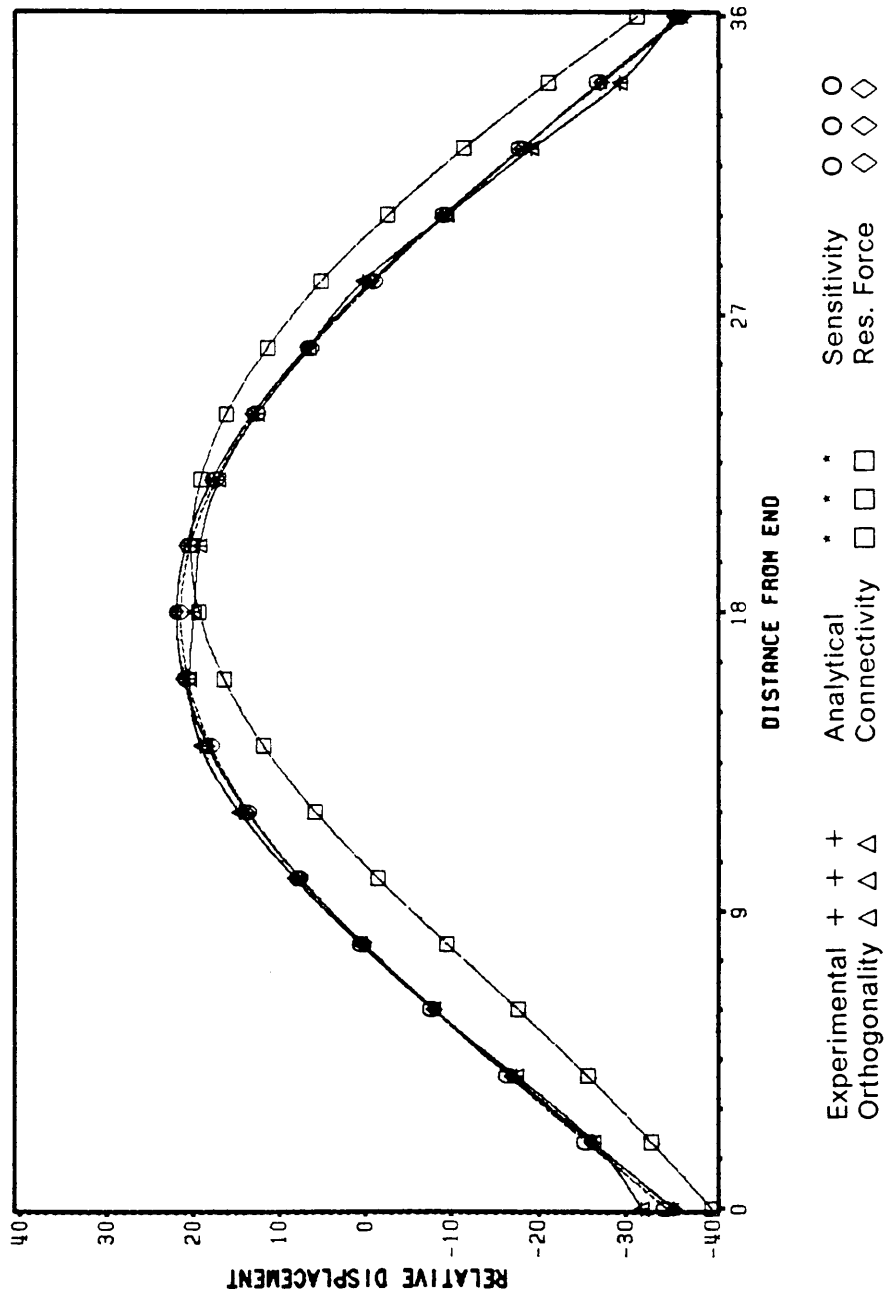


Figure 15. FIRST EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:
Including the four System Identification and finite element predictions

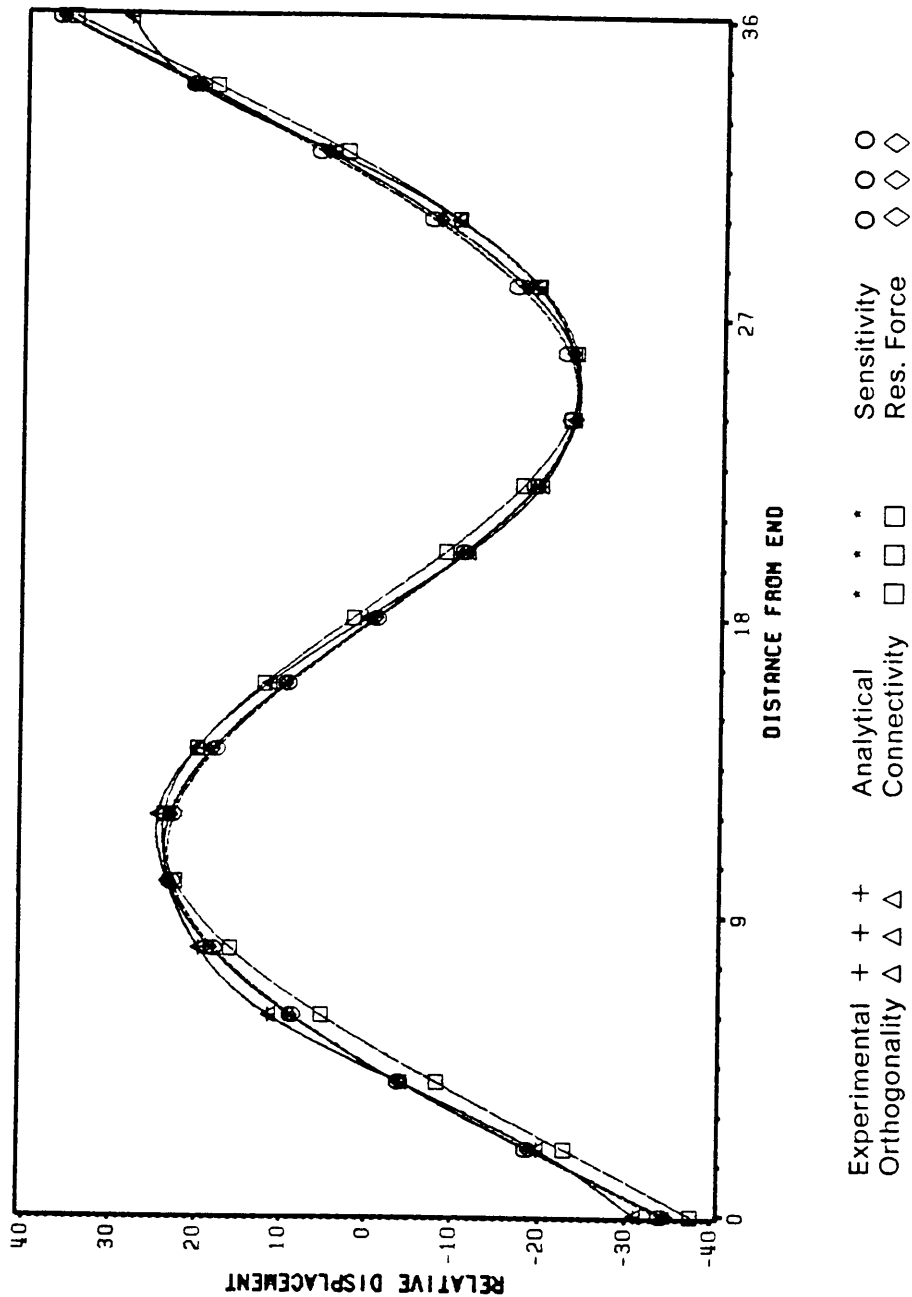


Figure 16. SECOND EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:
Including the four System Identification and finite element predictions

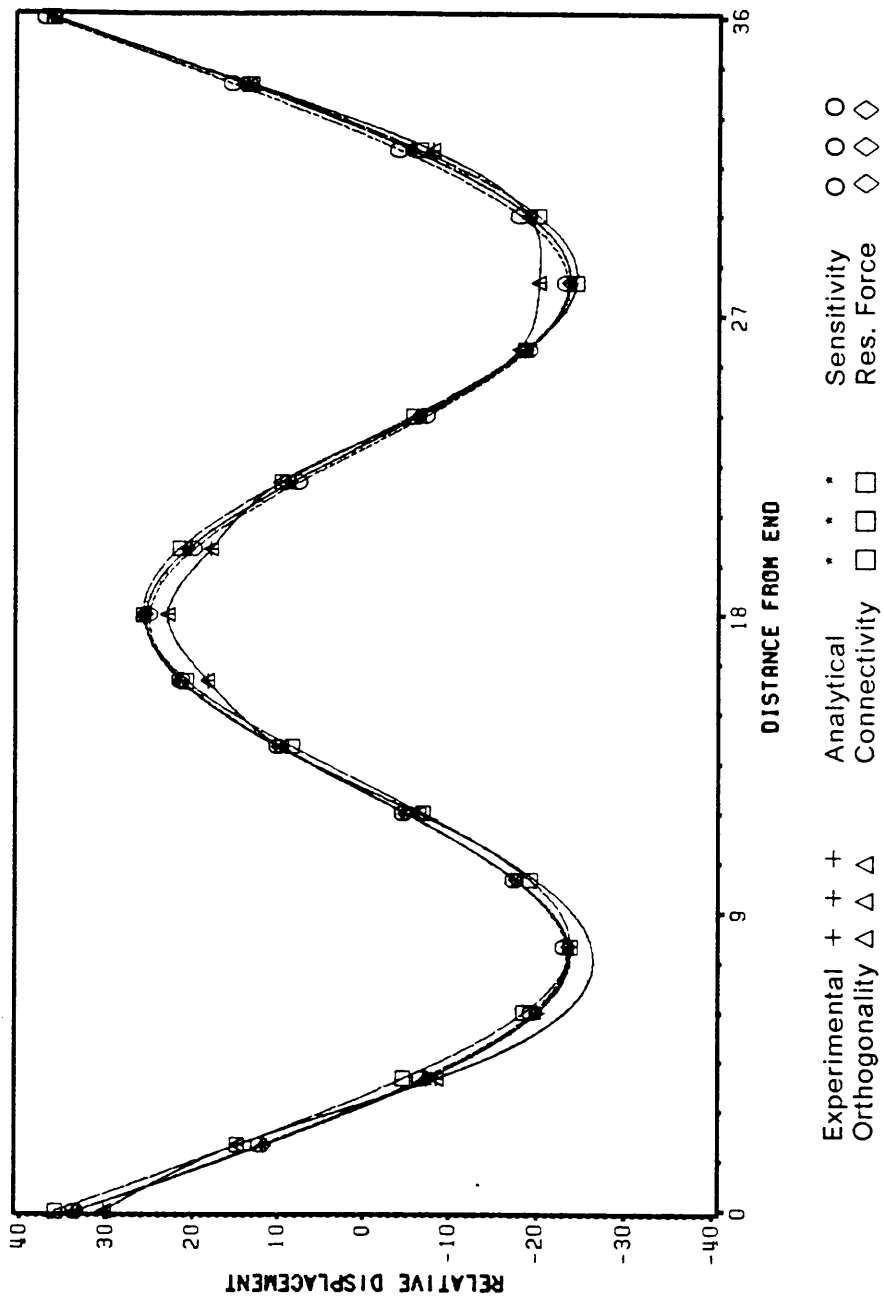
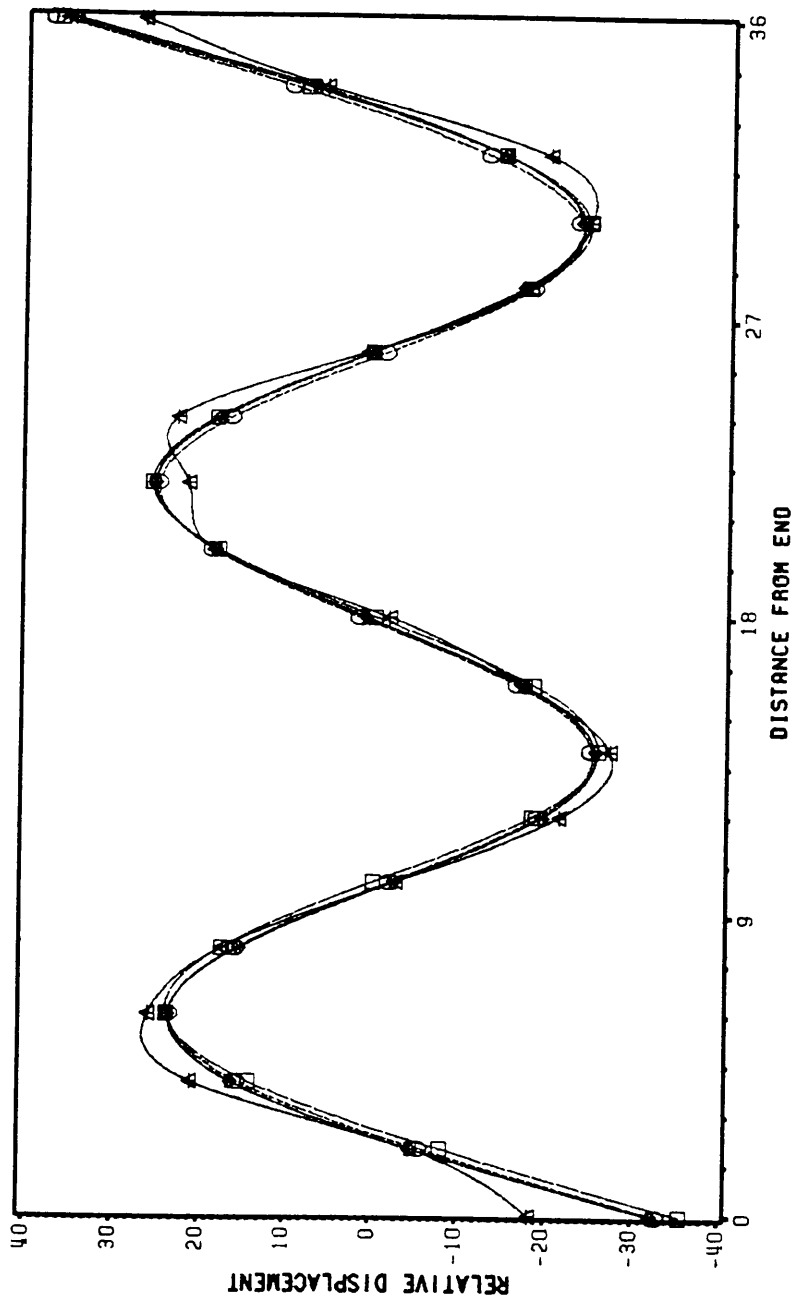


Figure 17. THIRD EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:
 Including the four System Identification and finite element predictions



Experimental + + + Analytical * * * Sensitivity ○ ○ ○
 Orthogonality Δ Δ Δ Connectivity □ □ □ Res. Force ◇ ◇ ◇

Figure 18. **FOURTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:**
 Including the four System Identification and finite element predictions

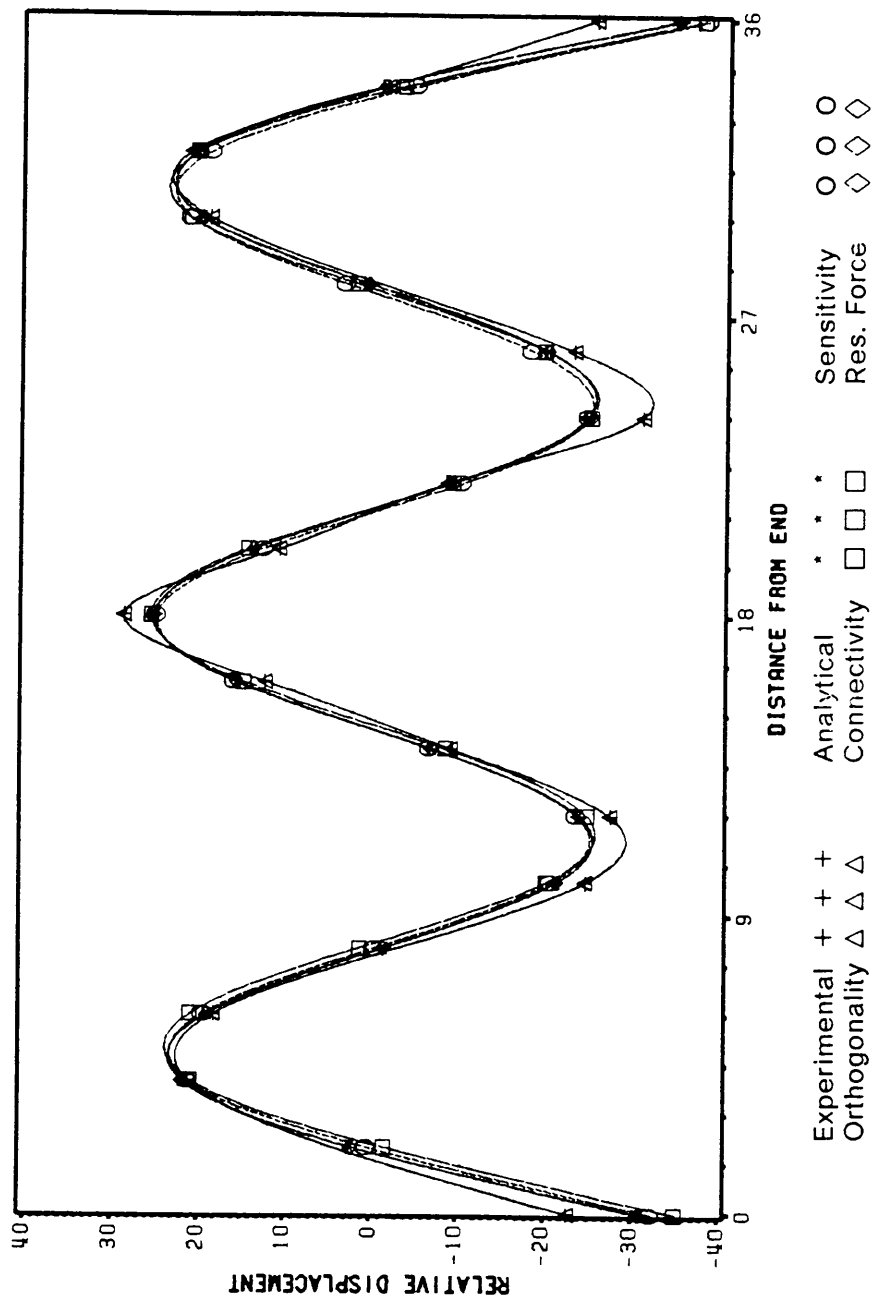


Figure 19. FIFTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:
 Including the four System Identification and finite element predictions

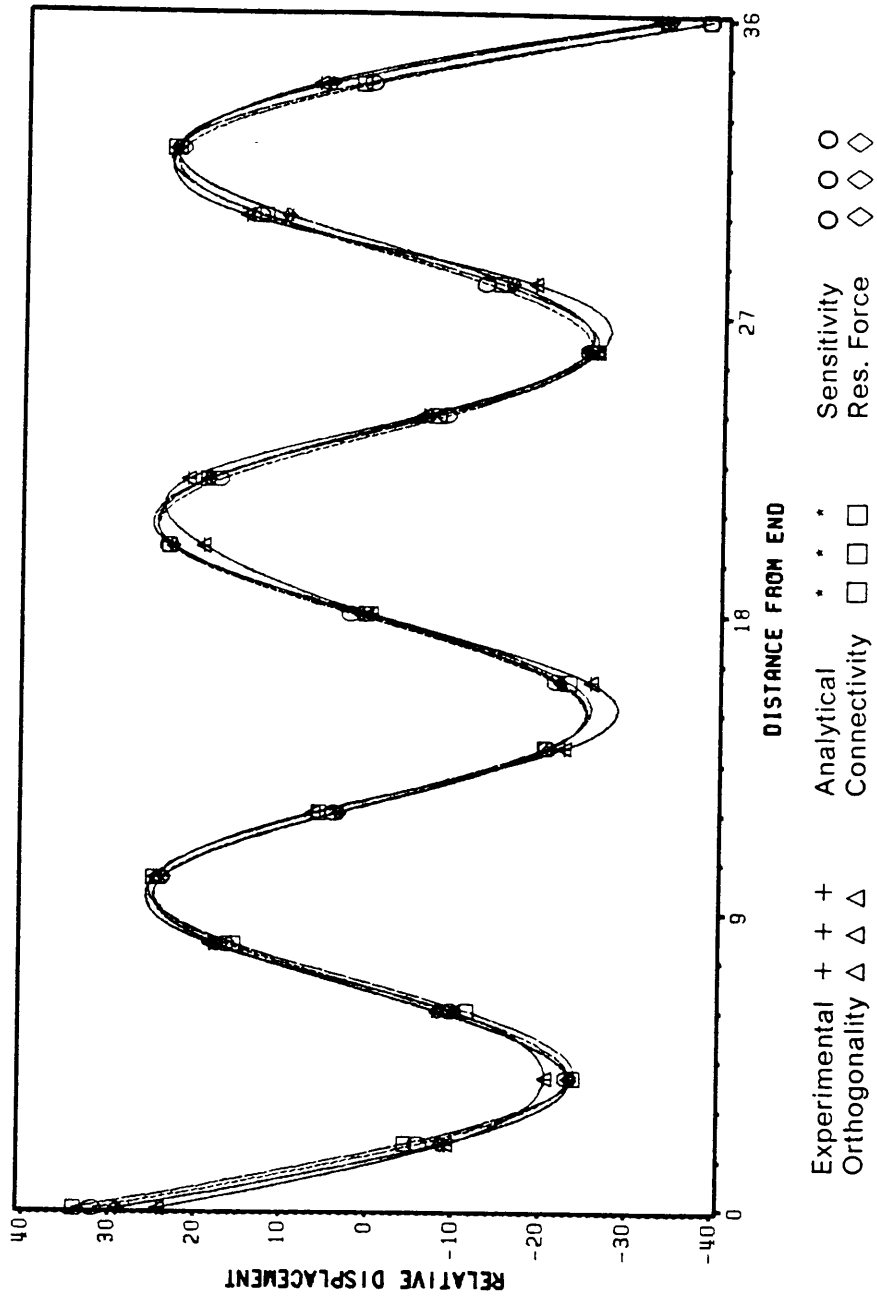


Figure 20. SIXTH EXPERIMENTAL MODE SHAPE OF THE FREE-FREE BEAM:
 Including the four System Identification and finite element predictions

orthogonality method makes many large relative changes to the stiffness matrix. This is because this method models the modal data exactly, including the error. The many high percentage changes are mostly due to the modification of stiffness coefficients with small initial values. All of the other methods make small changes to the stiffness matrix components. Table 8 on page 101 shows the relative changes to the diagonal components of the stiffness matrices for each method when analyzing the free-free beam using six modes. Table 9 on page 102 shows the relative changes to the off diagonal terms. Note that for the residual force method, better results were obtained by modifying the concentrated masses only, therefore there are only 19 terms which can be changed instead of the 361 terms in the full matrix.

The computational times are also of interest to an analyst when considering the usefulness of a method. Table 10 on page 104 shows the CPU time on a Micro-VAX II computer for each method when utilizing 1 thru 6 modes. These times can be compared to a time of 2:09 for the analysis of the finite element model only. All methods performed the finite element calculations, so the the modification time is the differential between 2:09 and the time listed. The time used by the orthogonality method is consistently lower than the other methods and the time is only slightly dependent on the number of modes used. The connectivity method is the most expensive to use for each set of modes used. This may be due to the fact that there is quite a bit of looping done for this method, and the way this was programmed with the DMAP language in MSC/NASTRAN may not be the most efficient method. The sensitivity method is highly dependent on the number of modes in the identification process, even though the same stiffness matrices are generated. The residual force method does not seem highly dependent on the number of modes used, this may be due to the fact that the method is programmed to generate the column vector associated with each element.

Table 8. RELATIVE CHANGES TO MATRICES, # OF DIAGONAL TERMS CHANGED

METHOD	# < 1%	# < 10%	# < 100	# > 100%
SENSITIVITY	0	19	0	0
ORTHOGONALITY	2	17	0	0
CONNECTIVITY	5	14	0	0
RESIDUAL FORCE	13	6	0	0

Table 9. RELATIVE CHANGES TO MATRICES, # OF OFF-DIAGONAL TERMS CHANGED

METHOD	# < 1%	# < 10%	# < 100	# > 100%
SENSITIVITY	0	342	0	0
ORTHOGONALITY	0	0	74	268
CONNECTIVITY	256	96	0	0
RESIDUAL FORCE	0	0	0	0

Table 10. CPU TIME REQUIRED FOR THE FOUR METHODS FOR FREE-FREE BEAM (min:sec)

METHOD	1 MODE	2 MODES	3 MODES	4 MODES	5 MODES	6 MODES
SENSITIVITY	4:12	6:12	10:52	12:45	14:43	17:10
ORTHOGONALITY	2:27	2:23	2:24	2:25	2:24	2:36
CONNECTIVITY	12:53	13:54	14:57	16:30	18:40	21:17
RESIDUAL FORCE	7:46	7:49	7:53	7:56	8:00	8:15

It is interesting to note the results from some of the methods change with small changes in the experimental data. The sensitivity method works independent of errors in the modal matrix, and it is dependent only on the frequencies. The orthogonality method models the experimental data very well, but the resulting updated system matrices can change for even small changes in the modal data. The connectivity method does not seem to be affected as much by changes in the modal data. The residual force method seems to be the most sensitive to changes in the modal data. This gives another indication of the difficulty in obtaining quality results. It is beneficial for the connectivity method in that it is less sensitive to errors in the experimental data.

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

7.1 *Conclusions*

This thesis reviewed many System Identification methods, and examined four methods in depth. All of the four methods appeared to be able to update analytical system matrices using modal data. However not all of them performed the identification as well as desired. They all exhibited pros and cons.

The sensitivity method was able to adequately modify the system matrices in order to generate an accurate match for the modes used in the identification process. However, modes not used were generally not improved, and the modal vectors from the updated model were not as accurate as the second and third methods.

However this method worked quite well for the free-free beam example. There were no large errors in the analytical model and this method could then scale the stiffness matrix to get a better representation of the frequencies. The resulting mode

shapes did not fit the experimental data exactly, which may be beneficial as all experimental data contains errors. Since this method performs element by element changes only, it retains the connectivity of the original model. The iterative nature of this method means that it is computationally expensive.

The orthogonality method was able to update the frequencies and mode shapes used in the identification process for the second example problem. However, it did not improve the accuracy of the modes that were not used. Similar results were obtained for the free-free beam. However, a new mode was introduced to the system, and large relative changes were made to the system matrices. This indicates that there must be high confidence in the experimental data for this method to be used. This method does not preserve the original connectivity of the model and results in a less physically meaningful model. The connectivity criteria is not important for a model which has been reduced to fewer degrees of freedom. However, this method is the least expensive to use and the updated matrices model the modal data used in the identification process very well.

The connectivity method did an excellent job of updating the modes for the 8 DOF example. It was able to improve the accuracy of modes that were not used in the identification. This is beneficial and indicates that the resulting matrices are likely to be correct. This method performed well for the third example. This method requires the accurate determination of all of the eigenvalues and eigenvectors of a large non-positive definite matrix. The recommended solution technique is the modified Givens, and the accuracy of higher eigenvalues may not be good. However, this method does retain the original connectivity of the model. This method is fairly computationally expensive to use, but makes minimal changes to the stiffness matrix.

The residual force method worked well for the second example. It was able to improve the accuracy of all modes in the identification process as well as modes that

were not included. This method did not work as well for the free-free beam example. This may be due to the difficulty of locating only the DOF's with errors within the model. The technique of solving for new variable values can result in a highly over constrained problem which may bias certain variables. This method does preserve the connectivity of the original model, and is not the most computationally expensive method to use.

The following is a list of the methods and when they are beneficial, in order of preference by the author. The connectivity method works well for many models, and the derivation lends itself to give the analyst confidence in the resulting model. The residual force method has great potential, and might improve if a better error location technique were developed, and if $[Q]$ could be reduced to be square by another means than a least squares fit. The orthogonality method requires very accurate modal data to yield physically meaningful system matrices. The sensitivity method works very well for problems where the analyst realizes that the model is physically correct, but there may be errors in only a few variables (for example, material properties).

7.2 Recommendations

Structural analysis techniques have developed to the point where confidence can be placed on their results. There will be minor errors though, and they should be corrected. But, any resulting modification is only as good as the experimental data that is used. And this data will contain errors. Therefore, a method which allows for a standard deviation in the experimental data, or a confidence level in the analytical

and experimental data would be beneficial. This could be accomplished by utilizing a Bayesian technique (Blakely and Walton, 1984).

Perhaps a combination of some of these methods would yield a better System Identification method. First, it would be beneficial to be able to have the mass matrix where the modal vectors are orthogonal. This could be done by using the technique utilized in the orthogonality method. Perhaps a better method would be to modify the modal vectors as well as the mass matrix, as there is probably a higher confidence level in the mass matrix than there is in the modal vectors. The confidence in frequencies is usually good, so the experimental frequencies should not be modified. Different variations of these techniques were presented by Targoff (1976) and Baruch (1982).

Once the mass matrix and modal vectors are compatible, then perhaps a sensitivity method should be used to improve the stiffness matrix. This should not necessarily be done on an element by element basis, but instead by linking elements in the model that are similar; i.e. elements in similar geometric regions with the same material properties. Once global changes are made to update the matrices, while retaining the connectivity, then element by element changes could be made. For an analytical model which has not been reduced, the connectivity method could be used to perform the final model modification. Or, if a better error location technique is developed, then the residual force technique could be used as well. If the model is reduced then the orthogonality means of updating could be used.

It should be noted that the orthogonality and connectivity methods are similar for a reduced model (fully populated matrices). However, the orthogonality method uses a mass weighted objective function in the optimization process. The connectivity minimizes the un-weighted percent change to the stiffness coefficients. If an analyst

has intuitive knowledge of the problem to be solved, s/he may prefer one method over the other.

This resulting method would utilize many of the benefits of the four methods. The resulting model should contain minimum changes to the system matrices while yielding good predictions of the modal data.

Chapter VIII

REFERENCES

- Barthelemy, B., and Haftka, R. T., "Physically Based Sensitivity Derivatives for Finite Element Programs," *Proceedings of AIAA/ASME 28th Structures, Structural Dynamics, and Materials Conference*, April 1987, pp. 642 - 650
- Baruch, M., and Itzhack, "Optimal Weighted Orthogonalization of Measured Modes," *AIAA Journal*, Vol. 16, No. 4, April 1978, pp. 346 - 351
- Baruch, M., "Optimization procedure to Correct Stiffness and Flexibility Matrices using Vibration Tests" *AIAA Journal*, Vol. 16, No. 11, Nov. 1978, pp. 1208 - 1210
- Baruch, M., "Methods of Reference Basis for Identification of Linear Dynamic Structures," *Proceedings of AIAA/ASME 23th Structures, Structural Dynamics, and Materials Conference*, May 1982, pp. 557 - 563
- Berman, A., and Flannely, W. G., "The Theory of Incomplete Models of Dynamic Structures," *AIAA Journal*, Vol. 9, No. 8, Aug. 1971, pp. 183 - 192
- Berman, A., "Determining Structural Parameters from Dynamic Testing," *Shock and Vibration Digest*, Vol. 7, Jan. 1975 pp. 10-17

- Berman, A., "Parameter Identification techniques for Vibrating Structures," *Shock and Vibration Digest*, Vol. 11, Jan 1979, pp. 13 - 16
- Berman, A., "Mass Matrix Correction Using an Incomplete Set of Measured Modes," *AIAA Journal*, Vol. 17, No. , 1979, pp. 1147 - 1148
- Berman, A., and Nagy, E. G., "Improvement of a Large Analytical Model Using Test Data," *AIAA Journal*, Vol. 21, No.8, Aug. 1983, pp. 1168 - 1173
- Blakely, K. D., and Walton, W. B., "Selection of Measurement and Parameter Uncertainties for Finite Element Model Revision," *2nd IMAC*, Orlando Florida, 1983, pp. 82 - 86
- Caesar, B., "Updating System Matrices using Modal Test Data," *5th IMAC*, Orlando Florida, 1986, pp. 453 - 459
- Chen, J. C., and Garba, J. A., "Analytical Model Improvement Using Modal Test Results," *AIAA Journal*, Vol. 18, No. 6, June 1980, pp. 684 - 690
- Chen, J. C., and Wada, B. K., "Matrix Perturbation for Structural Dynamic Analysis," *Proceedings of AIAA/ASME 29th Structures, Structural Dynamics, and Materials Conference*, April 1988, pp. 355 - 364
- Chen, T. Y., and Wang, B. P., "Finite Element Model Refinement Using Modal Analysis Data," *Proceedings of AIAA/ASME 29th Structures, Structural Dynamics, and Materials Conference*, April 1988, pp. 1219 - 1229
- Collins, J. D., Hart, G. C., Hasselman, T. K., and Kennedy, B., "Statistical Identification of Structures," *AIAA Journal*, Vol. 12, No. 2, Feb. 1974, pp. 187 - 190
- Dobson, B. J., "Modification of Finite Element Models using Experimental Modal Analysis," *2nd IMAC*, Orlando Florida, 1983, pp. 593 - 597
- Fuh, J., Gustavson, B., and Berman, A., "Error Estimation and Compensation in Reduced Dynamic Models of Large Space Structures," *Proceedings of AIAA/ASME 27th Structures, Structural Dynamics, and Materials Conference*, April 1986, pp. 1 - 9
- Haftka, R. T., and Kamat, M. P., *Elements of Structural Optimization*, Martinus Nijhoff Publishers, Boston, 1985

- Guyan, R. J., "Reduction of Stiffness and Mass Matrices," *AIAA Journal*, Feb. 1965, Vol. 3, No. 2, p. 380
- Hall, B. M., Calkin, E. D., Sholar, M. S., "Linear Estimation of Structural Parameters from Dynamic Test Data," *Proceedings of AIAA/ASME 11th Structures, Structural Dynamics, and Materials Conference*, April 1970, pp. 193 - 197
- Haug, E. J., Komkov, V., Choi, K. K., "Design Sensitivity Analysis of Structural Systems," Academic Press, 1984
- Heylan, W., and Sas, P., "Review of Model Optimization Techniques," *5th IMAC*, Orlando Florida, 1986, pp. 1177 - 1182
- Ibrahim, S. R., and Saafan, A. A., "Correlation of Analysis and Test in Modeling of Structures Assessment and Review," *5th IMAC*, Orlando Florida, 1986, pp. 1651 - 1660
- Jain, V. K., and Dobeck, G. J., "System Identification Techniques: a Tutorial Review," in *79-WA/DSC-20*, ASME, New York, 1979
- Kabe, A. K., "Stiffness Matrix Adjustment Using Mode Data," *AIAA Journal*, Vol. 23, No. 9, September 1985b, pp. 1431 - 1436
- Kidder, R. L., "Reduction of Structural Frequency Equations," *AIAA Journal*, Vol. 11, No. 2, Feb. 1973, p. 380
- Lancaster-Brown, Peter, "Halley & His Comet," Blanford Press, New York, NY, 1985
- Luk, Y. W., "Identification of Physical Mass, Stiffness and Damping Matrices Using Psuedo-Inverse," *5th IMAC*, Orlando Florida, 1986, pp. 679 - 685
- Macneal-Schwendler Corporation, "User Manuals. Vol 1 and 2," Los Angeles, CA, July, 1984
- Nelson, R. B., "Simplified Calculation of Eigenvector Derivatives," *AIAA Journal*, Vol. 14, 1976, p. 1201-1205

- O'Callahan, J. C., Avitabile, P., and Leung, R., "Development of Mass and Stiffness Matrices for an Analytical Model using Experimental Data," *2nd IMAC*, Orlando Florida, 1983, pp. 585 - 592
- O'Callahan, J. C. and Leung, R. K., "Optimization of Mass and Stiffness Matrices using a Generalized Inverse Technique on Measured Modes," *3rd IMAC*, Orlando Florida, 1984, pp. 75 - 79
- Ojalvo, I. U., and Pilon, D., "Diagnostics for Geometrically Locating Structural Math Model Errors from Modal Test Data," *Proceedings of AIAA/ASME 29th Structures, Structural Dynamics, and Materials Conference*, April 1988, pp. 1174 - 1186
- Pakstys, M. P., "Correlation of Experimental and Analytical Structural Dynamics Parameters - A 1982 Overview," *1st IMAC*, Orlando Florida, 1982, pp. 59 - 63
- Pal, T. G., and Schmidtberg, R. A., "Combining Analytical and Experimental Modal Analysis for Effective Structural Dynamic Modelling," *1st IMAC*, Orlando Florida, 1982, pp. 265 - 271
- Rorrer, R. A. L., Wicks, A. L., and Williams, J., "Angular Acceleration Measurements of a Free-Free Beam," To be presented at the 8th IMAC, 1989
- Shye, K., and Richardson, M., "Mass, Stiffness, and Damping Estimates from Structural Measurements," *5th IMAC*, Orlando Florida, 1986, pp. 756 - 761
- Sidhu, J., and Ewins, D. J., "Correlation of Finite Element and Modal Test Studies of a Practical Structure," *2nd IMAC*, Orlando Florida, 1983, pp. 756 - 762
- Sol, E., Verdonck, E., Snoeys, R., and Wilde, W. P., "Design Optimisation of Dynamic Structures by the Combined use of Modal Analysis (M.A.) and the Finite Element Method (F.E.M.)," *2nd IMAC*, Orlando Florida, 1983, pp. 769 - 776
- Targoff, W. P., "Orthogonality Check and Correction of Measured Modes," *AIAA Journal*, Vol. 14, No. 2, Feb. 1976, pp. 164 - 167
- Vanhonacker, P., "A Methodology for Combining Experimental and Mathematical Modal Analysis with Application to an Automotive Component," *5th IMAC*, Orlando Florida, 1986, pp. 406 - 408

- Wang, B. P., and Chu, F. H., "Effective Dynamic Reanalysis of Large Structures," *Shock and Vibration Bulletin*, No. 51, May 1981, pp. 73 - 79
- Wang, B. P., and Chen, T. Y., "Model Refinement using Test Data," *4th IMAC*, Orlando Florida, 1985, pp. 1052-1056
- Yokoki, K., Hayakawa, S., and Yoshino, H., "Optimization of Finite Element Modeling using Experimental Modal Analysis and Sensitivity Analysis," *5th IMAC*, Orlando Florida, 1986, pp.
- Wei, F. S., "Stiffnes Matrix Correction From Incomplete Test Data," *AIAA Journal*, Vol. 18, No. 10, October 1980, pp. 1274 - 1275
- Zienkiewicz, O. Z., "The Finite Element Method in Engineering Science," McGraw Hill, New York, 1967

Appendix A

Modeling and Solution Sequences from MSC/NASTRAN

Summary of solution methods available in MSC/NASTRAN for the System Identification of vibrating structures

The following are the MSC/NASTRAN Version (64/65) solution sequences for system identification methods.

A.1 Solution Sequence for Sensitivity Based Method

```
ID MSC,NASTRAN $PAS
TIME 100
$
$
$
$ THIS METHOD USES A NEWTON-RAPHSON ITERATIVE SCHEME TO
$ UPDATE THE SYTEM MATRICES BASED ON SENSITIVITY INFORMATION
$
$ M is the number of measured frequencies
$ R is the number of design variables
$
$
BEGIN
$
$ 1.)
$ INITIALIZE THE DATABASE
DBMGR //0/V,Y,DBCLEAN=0/V,Y,DBNBLNKS=40000//V,Y,DBINIT=0//
V,Y,DBNAME=DB01 $
$
$ 2.) STORE MATRIX DERIVATIVES IN DATABASE
ADD MXX./OMXX
ADD KXX./OKXX
DBSTORE OMXX,OKXX//1/1 $
ADD MXX./DMXXJ/(0.0,0.0)
ADD KXX./DKXXJ/(3.333E-8,0.0)
DBSTORE DKXXJ,DMXXJ//1/1
ADD MXX./DMXXJ/(1365.0,0.0)
ADD KXX./DKXXJ/(0.0,0.0)
DBSTORE DKXXJ,DMXXJ//2/1
$
$ 3.) INITIALIZE AND RENAME MATRICES
ADD KXX./KA
ADD MXX./MA
DBSTORE KXX.MXX,KA,MA// $
FILE SENSCT=APPEND
FILE SENS=APPEND
FILE AAT=APPEND
FILE BBT=APPEND
MATMOD WV,,,,/WM,/28
DIAGONAL WM/VEC1/C,Y,SQUARE/V,Y,POWER=0.
$
PARAM //NOP/V,Y,KLOOP=0 $
PARAM //ADD/V,Y,KLOOP1/V,Y,MM=0/1
PARAM //ADD/V,Y,KLOOP2/V,Y,RR=0/1
PARAM //ADD/V,Y,KLOOP3/V,Y,MM=0/1
PARAM //ADD/V,Y,MMM1/V,Y,MM=0/-1
PARAM //ADD/V,Y,MMP1/V,Y,MM=0/1
PARAM //ADD/V,Y,RRP1/V,Y,RR=0/1
PARAM //SUB/V,Y,NMM/0/V,Y,MM=0
PARAM //SUB/V,Y,NRR/0/V,Y,RR=0
PARAMR //NOP/V,Y,FOLD=10000.
$
```

```

$ 4.) TOP OF ITERATIVE LOOP
LABEL  TOPLOOP
PARAM  //ADD/V,Y,KLOOP/V,Y,KLOOP/1
PRTPARM //0/C,N, TOPLOOP
$
$ 5.) PERFORM EIGENVALUE ANALYSIS
DBFETCH /KA,MA,,,/ $
SETVAL //V,N,NEIGVS/-1 $
READ   KA,MA,,,DYNAMICS,,CASECC/LAMA,PHIX,MI,OEIGS/ V,N,
      READAPP=MODES/S,N,NEIGVS $
OFF    LAMA,OEIGS// $
$
$ 6.) GENERATE ANALYTICAL AND EXPERIMENTAL FREQUENCIES
PRTPARM //0/C,N,FREQEN
LAMX   ,,LAMA/LAMM1/-1
PARTN  LAMM1,,LPV/JUNK,LAMM,,/1
ADD    LAMM,/NLAMM/-1.
MATMOD LAMM,,,,/AF,/1/3
MATMOD NLAMM,,,,/NLAMBDA,/1/1
TRNSP  AF/AFT
TRNSP  NLAMBDA/NLAMBDAT
ADD    AF,EF/DELFREQ//-1.
TRNSP  DELFREQ/DELFREQT
$
$ 7.) EXIT LOOP IF ERROR IS SMALL OR DOES NOT DECREASE
PRTPARM //0/C,N,JUMPSHIP
MPYAD  DELFREQT,DELFREQ,/F
PARAML F//DMI/1/1/V,N,F
PRTPARM //0/C,N,F
PARAMR //LE//F/1.E-6///V,N,FLAG
COND   OUTLOOP,FLAG
PARAMR //SUB/V,N,FOMFN/FOLD/F
PRTPARM //0/C,N,FOLD
PRTPARM //0/C,N,FOMFN
PARAMR //LE//FOMFN/1.E-3///V,N,FLAG
COND   OUTLOOP,FLAG
PARAMR //ADD/V,N,FOLD/F/0.
$
$ 8.) LOOP TO STORE ANALYTICAL DATA
PRTPARM //0/C,N,STORE
PARAM   //ADD/V,Y,KLOOP1/V,Y,KLOOP1/NMM $ # CORRESPONDS TO (-M) CHANGE
LABEL  KTOP1
PRTPARM //0/C,N,KLOOP1
MATMOD  PHIX,,,,/UI,/1/KLOOP1
DBSTORE UI//KLOOP1/1
PARAM   //ADD/V,Y,KLOOP1/V,Y,KLOOP1/1
PARAM   //SUB/V,N,FLAG/V,Y,MM/V,Y,KLOOP1
COND    OUT1,FLAG
REPT    KTOP1,1000 $ # CORRESPONDS TO (M-1). CHANGE
LABEL   OUT1
$
$ 9.) DETERMINE SENSITIVITY OF FREQUENCIES W/RESPECT TO DESIGN VARIABLES
PARAMR  //ADDC///V,Y,MPYR/(.012665,0.0) $
PURGE   SENSM/NEVER
PARAM   //ADD/V,Y,KLOOP2/V,Y,KLOOP2/NRR $ (-R) CHANGE
LABEL   KTOP2

```

```

PRTPARM //0/C,N,KLOOP2
DBFETCH /DMXXJ,DKXXJ,,/KLOOP2/1
PURGE SENSCT/NEVER
PARAM //ADD/V,Y,KLOOP3/V,Y,KLOOP3/NMM $ # (-M) CHANGE
LABEL KTOP3
  PRTPARM //0/C,N,KLOOP3
  MATMOD PHIX,,,,/UI,/1/KLOOP3
  TRNSP UI/UIT
  SCALAR NLAMM//KLOOP3/1/S,Y,NLI
  SCALAR LAMM//KLOOP3/3/S,Y,FI
  ADD DKXXJ,DMXXJ/DXX//NLI
  SMPYAD UIT,DXX,UI,,/DLIDXJ/3
  PARAMR //DIVC///V,Y,MFI/V,Y,MPYR/V,Y,FI/V,N,FLAG
  ADD DLIDXJ,/DFIDXJ/MFI
  APPEND DFIDXJ,/SENSCT/2
  DBSTORE DFIDXJ//KLOOP2/KLOOP3
  PARAM //ADD/V,Y,KLOOP3/V,Y,KLOOP3/1
PARAM //SUB/V,N,FLAG/V,Y,MM/V,Y,KLOOP3
COND OUT3,FLAG
REPT KTOP3,1000 $ # CORRESPONDS TO (M-1). CHANGE
LABEL OUT3
TRNSP SENSCT/SENSC
APPEND SENSCT./SENSM/2
PURGE SENSCT/ALWAYS
PARAM //ADD/V,Y,KLOOP2/V,Y,KLOOP2/1
PARAM //SUB/V,N,FLAG/V,Y,RR/V,Y,KLOOP2
COND OUT2,FLAG
REPT KTOP2,1000 $ # CORRESPONDS TO (R-1). CHANGE
LABEL OUT2
$
$ 10.) CREATE SQUARE MATRIX
PURGE AAT.BBT/NEVER
TRNSP SENSM/AAT
PURGE SENSM/ALWAYS
TRNSP DELFREQ/BBT
TRNSP AAT/AA
TRNSP BBT/BB
FILE NAA=APPEND
FILE NBB=APPEND
PURGE NAA,NBB/NEVER
PARAM //NOP/V,Y,KLOOP4=1
LABEL KTOP4
  PRTPARM //0/C,N,KLOOP4
  SCALAR LAMM//KLOOP4/3/S,Y,FI
  MATMOD DELFREQT,,,,/DELF,/1/KLOOP4
  PARAMR //DIVC///V,Y,ONEOF/(1.0,0.0)/V,Y,FI/V,N,FLAG
  ADD DELF,/NDELF/ONEOF
  APPEND NDELF,/NBB/2
  MATMOD AAT,,,,/D1,/1/KLOOP4
  ADD D1,/ND1/ONEOF
  APPEND ND1,/NAA/2
  MATPRN NAA,NBB// $
  PARAM //ADD/V,Y,KLOOP4/V,Y,KLOOP4/1
  PARAM //SUB/V,N,FLAG/V,Y,MM/V,Y,KLOOP4
  COND OUT4,FLAG
REPT KTOP4,1000 $

```

```

LABEL   OUT4
PARAM   //SUB/V,Y,KLOOP4/V,Y,KLOOP4/MM
TRNSP   NAA/AA
TRNSP   NBB/BB
PURGE   NAA,NBB/ALWAYS
TRNSP   AA/AAT
MPYAD   AAT,AA/ATA
MPYAD   AAT,BB/ATB
$
$ 11.) SOLVE SYSTEM OF EQUATIONS
SOLVE   ATA,ATB/NXX
ADD     NXX,XX/(-3.0,0.0)
PURGE   AAT,BBT/ALWAYS
$
$ 12.) EXTRACT COMPONENTS OF XX (DELTA X) VECTOR
MATPRN  XX// $
SCALAR  XX//1/1/S,Y,X1
$
$ 13.) ASSEMBLE NEW MASS AND STIFFNESS MATRICES
DBFETCH /KXX,MXX,,/ $
DBFETCH /DKXXJ,,,/1/1
ADD     DKXXJ,/KADD1/(3.333E-3,0.0)
ADD     KXX,KADD1/KA//X1
ADD     KA,/KXX
ADD     MA,/MXX
DBSTORE KXX,MXX,KA,MA// $
PARAM   //SUB/V,N,FLAG/V,Y,NITER/V,Y,KLOOP
COND    OUT,FLAG
$ 14.) REPEAT ITERATIVE LOOP 5 TIMES
REPT    TOPLOOP,1000
LABEL   OUT
$
$ JUMP HERE IF ERROR IS SMALL
LABEL   OUTLOOP
PRTPARM //0/C,N,JUMPEDSHIP
$
$ PRINT RESULTS
PRTPARM //0/C,N,RESULTS
MATPRN  KA,MA// $
MATPRN  EPHI,PHIX// $
$
$
DBFETCH /OMXX,OKXX,,/1/1 $
DBFETCH /KA,MA,,/ $
ADD     OMXX,MA/MDIF//-1.
ADD     OMXX,MDIF/MPERC///2
ADD     OKXX,KA/KDIF//-1.
ADD     KDIF,OKXX/KPERC///2
MATPRN  MPERC,KPERC// $
MATPCH  PHIX// $
$
$ END PROGRAM
END
$
CEND
$

```

\$ INFORMATION PERTINENT TO ANALYSIS

METHOD = 10

DISPLACEMENT = ALL

SPC = 1

MODES = 3

\$

\$ ENTER PHYSICAL DATA BELOW

BEGIN BULK

\$

PARAM , NITER , 30

PARAM , MM , 6

PARAM , RR , 1

\$

DMI , LPV , 0 , 2 , 1 , 0 , , 8 , 1

DMI , LPV , 1 , 3 , 1 , , THRU , 8

\$

DMI , EQ1 , 0 , 2 , 1 , 0 , , 4 , 1

DMI , EQ1 , 1 , 1 , 0.000111111 , 0 , , 1 , , 0.

DMI , EQ2 , 0 , 2 , 1 , 0 , , 4 , 1

DMI , EQ2 , 1 , 1 , 0 , , .00111111 , 0 , , 1.

\$

\$

DMI , R1 , 0 , 2 , 1 , 0 , , 1 , 1

DMI , R1 , 1 , 1 , 0.

DMI , R2 , 0 , 2 , 1 , 0 , , 1 , 1

DMI , R2 , 1 , 1 , 0.

\$

\$DMI , KXX , 0 , 2 , 1 , 0 , , 2 , 2

\$DMI , KXX , 1 , 1 , 1900. , -900.

\$DMI , KXX , 2 , 1 , -900. , 900.

\$DMI , MXX , 0 , 2 , 1 , 0 , , 2 , 2

\$DMI , MXX , 1 , 1 , .1

\$DMI , MXX , 2 , 2 , 1.

\$

DMI , DKXX1 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DKXX2 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DKXX3 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DKXX4 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DMXX1 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DMXX2 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DMXX3 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DMXX4 , 0 , 2 , 1 , 0 , , 2 , 2

DMI , DKXX1 , 1 , 1 , 1.

DMI , DKXX2 , 1 , 1 , 1. , -1.

DMI , DKXX2 , 2 , 1 , -1. , 1.

DMI , DMXX3 , 1 , 1 , 1.

DMI , DMXX4 , 2 , 2 , 1.

\$

DMI , WV , 0 , 2 , 1 , 0 , , 2 , 1

DMI , WV , 1 , 1 , 1. , THRU , 2

\$

EIGR , 10 , MGIV , , , , 8

\$

DMI , EF , 0 , 2 , 1 , 0 , , 6 , 1

DMI , EF , 1 , 1 , 20.07 , 55.31 , 108.4 , 179.2 , 267.7 ,
, 374.0

\$

DMI , EPHI , 0 , 2 , 1 , 0 , , 2 , 2
 DMI , EPHI , 1 , 1 , .5059 , .9871
 DMI , EPHI , 2 , 1 , 3.122 , -.1600

\$

DMI	MXX	0	6	1	0	19	19
DMI*	MXX			1		1	7.03468759E-05
*		2.30934056E-05	-8.50973902E-06	2.97744327E-06	-9.84635335E-07		
*		3.13893651E-07	-9.75214718E-08	2.97250473E-08	-8.92787799E-09		
*		2.65027311E-09	-7.79284304E-10	2.27336136E-10	-6.58789620E-11		
*		1.89823747E-11	-5.44253921E-12	1.55310118E-12	-4.39147877E-13		
*		1.15298938E-13	-2.19724700E-14				
DMI*	MXX		2		1	2.30934056E-05	
*		1.65691046E-04	1.15399544E-05	-6.99536395E-06	2.99539283E-06		
*		-1.10298163E-06	3.76026520E-07	-1.22321495E-07	3.85544006E-08		
*		-1.18789538E-08	3.59783048E-09	-1.07520104E-09	3.17885801E-10		
*		-9.31582450E-11	2.70991753E-11	-7.83037697E-12	2.23802595E-12		
*		-5.92578287E-13	1.15298938E-13				
DMI*	MXX		3		1	-8.50973902E-06	
*		1.15399544E-05	1.58695679E-04	1.45353470E-05	-8.09834546E-06		
*		3.37141933E-06	-1.22530309E-06	4.14580938E-07	-1.34200448E-07		
*		4.21522337E-08	-1.29541542E-08	3.91571620E-09	-1.16835930E-09		
*		3.44984985E-10	-1.00988620E-10	2.93372028E-11	-8.42295504E-12		
*		2.23802595E-12	-4.39147877E-13				
DMI*	MXX		4		1	2.97744327E-06	
*		-6.99536395E-06	1.45353470E-05	1.57592702E-04	1.49113730E-05		
*		-8.22066704E-06	3.40997371E-06	-1.23718200E-06	4.18178757E-07		
*		-1.35275656E-07	4.24701163E-08	-1.30473126E-08	3.94281541E-09		
*		-1.17618959E-09	3.47223000E-10	-1.01581202E-10	2.93372028E-11		
*		-7.83037697E-12	1.55310118E-12				
DMI*	MXX		5		1	-9.84635335E-07	
*		2.99539283E-06	-8.09834546E-06	1.49113730E-05	1.57470378E-04		
*		1.49499274E-05	-8.23254595E-06	3.41357145E-06	-1.23825725E-06		
*		4.18496654E-07	-1.35368808E-07	4.24972164E-08	-1.30551427E-08		
*		3.94505362E-09	-1.17678223E-09	3.47223000E-10	-1.00988620E-10		
*		2.70991753E-11	-5.44253921E-12				
DMI*	MXX		6		1	3.13893651E-07	
*		-1.10298163E-06	3.37141933E-06	-8.22066704E-06	1.49499274E-05		
*		1.57458504E-04	1.49535254E-05	-8.23362097E-06	3.41388932E-06		
*		-1.23835036E-06	4.18523740E-07	-1.35376638E-07	4.24994546E-08		
*		-1.30557352E-08	3.94505362E-09	-1.17618959E-09	3.44984985E-10		
*		-9.31582450E-11	1.89823747E-11				
DMI*	MXX		7		1	-9.75214718E-08	
*		3.76026520E-07	-1.22530309E-06	3.40997371E-06	-8.23254595E-06		
*		1.49535254E-05	1.57457427E-04	1.49538437E-05	-8.23371465E-06		
*		3.41391660E-06	-1.23835821E-06	4.18525985E-07	-1.35377235E-07		
*		4.24994546E-08	-1.30551427E-08	3.94281541E-09	-1.16835930E-09		
*		3.17885801E-10	-6.58789620E-11				
DMI*	MXX		8		1	2.97250473E-08	
*		-1.22321495E-07	4.14580938E-07	-1.23718200E-06	3.41357145E-06		
*		-8.23362097E-06	1.49538437E-05	1.57457325E-04	1.49538710E-05		
*		-8.23372193E-06	3.41391865E-06	-1.23835878E-06	4.18525985E-07		
*		-1.35376638E-07	4.24972164E-08	-1.30473126E-08	3.91571620E-09		
*		-1.07520104E-09	2.27336136E-10				
DMI*	MXX		9		1	-8.92787799E-09	
*		3.85544006E-08	-1.34200448E-07	4.18178757E-07	-1.23825725E-06		
*		3.41388932E-06	-8.23371465E-06	1.49538710E-05	1.57457325E-04		

*	1.49538728E-05	-8.23372284E-06	3.41391865E-06	-1.23835821E-06
*	4.18523740E-07	-1.35368808E-07	4.24701163E-08	-1.29541542E-08
*	3.59783048E-09	-7.79284304E-10		
DMI*	MXX	10	1	2.65027311E-09
*	-1.18789538E-08	4.21522337E-08	-1.35275656E-07	4.18496654E-07
*	-1.23835036E-06	3.41391660E-06	-8.23372193E-06	1.49538728E-05
*	1.57457325E-04	1.49538728E-05	-8.23372193E-06	3.41391660E-06
*	-1.23835036E-06	4.18496654E-07	-1.35275656E-07	4.21522337E-08
*	-1.18789538E-08	2.65027311E-09		
DMI*	MXX	11	1	-7.79284304E-10
*	3.59783048E-09	-1.29541542E-08	4.24701163E-08	-1.35368808E-07
*	4.18523740E-07	-1.23835821E-06	3.41391865E-06	-8.23372284E-06
*	1.49538728E-05	1.57457325E-04	1.49538710E-05	-8.23371465E-06
*	3.41388932E-06	-1.23825725E-06	4.18178757E-07	-1.34200448E-07
*	3.85544006E-08	-8.92787799E-09		
DMI*	MXX	12	1	2.27336136E-10
*	-1.07520104E-09	3.91571620E-09	-1.30473126E-08	4.24972164E-08
*	-1.35376638E-07	4.18525985E-07	-1.23835878E-06	3.41391865E-06
*	-8.23372193E-06	1.49538710E-05	1.57457325E-04	1.49538437E-05
*	-8.23362097E-06	3.41357145E-06	-1.23718200E-06	4.14580938E-07
*	-1.22321495E-07	2.97250473E-08		
DMI*	MXX	13	1	-6.58789620E-11
*	3.17885801E-10	-1.16835930E-09	3.94281541E-09	-1.30551427E-08
*	4.24994546E-08	-1.35377235E-07	4.18525985E-07	-1.23835821E-06
*	3.41391660E-06	-8.23371465E-06	1.49538437E-05	1.57457427E-04
*	1.49535254E-05	-8.23254595E-06	3.40997371E-06	-1.22530309E-06
*	3.76026520E-07	-9.75214718E-08		
DMI*	MXX	14	1	1.89823747E-11
*	-9.31582450E-11	3.44984985E-10	-1.17618959E-09	3.94505362E-09
*	-1.30557352E-08	4.24994546E-08	-1.35376638E-07	4.18523740E-07
*	-1.23835036E-06	3.41388932E-06	-8.23362097E-06	1.49535254E-05
*	1.57458504E-04	1.49499274E-05	-8.22066704E-06	3.37141933E-06
*	-1.10298163E-06	3.13893651E-07		
DMI*	MXX	15	1	-5.44253921E-12
*	2.70991753E-11	-1.00988620E-10	3.47223000E-10	-1.17678223E-09
*	3.94505362E-09	-1.30551427E-08	4.24972164E-08	-1.35368808E-07
*	4.18496654E-07	-1.23825725E-06	3.41357145E-06	-8.23254595E-06
*	1.49499274E-05	1.57470378E-04	1.49113730E-05	-8.09834546E-06
*	2.99539283E-06	-9.84635335E-07		
DMI*	MXX	16	1	1.55310118E-12
*	-7.83037697E-12	2.93372028E-11	-1.01581202E-10	3.47223000E-10
*	-1.17618959E-09	3.94281541E-09	-1.30473126E-08	4.24701163E-08
*	-1.35275656E-07	4.18178757E-07	-1.23718200E-06	3.40997371E-06
*	-8.22066704E-06	1.49113730E-05	1.57592702E-04	1.45353470E-05
*	-6.99536395E-06	2.97744327E-06		
DMI*	MXX	17	1	-4.39147877E-13
*	2.23802595E-12	-8.42295504E-12	2.93372028E-11	-1.00988620E-10
*	3.44984985E-10	-1.16835930E-09	3.91571620E-09	-1.29541542E-08
*	4.21522337E-08	-1.34200448E-07	4.14580938E-07	-1.22530309E-06
*	3.37141933E-06	-8.09834546E-06	1.45353470E-05	1.58695679E-04
*	1.15399544E-05	-8.50973902E-06		
DMI*	MXX	18	1	1.15298938E-13
*	-5.92578287E-13	2.23802595E-12	-7.83037697E-12	2.70991753E-11
*	-9.31582450E-11	3.17885801E-10	-1.07520104E-09	3.59783048E-09
*	-1.18789538E-08	3.85544006E-08	-1.22321495E-07	3.76026520E-07
*	-1.10298163E-06	2.99539283E-06	-6.99536395E-06	1.15399544E-05

* 1.65691046E-04 2.30934056E-05
 DMI* MXX 19 1 -2.19724700E-14
 * 1.15298938E-13 -4.39147877E-13 1.55310118E-12 -5.44253921E-12
 * 1.89823747E-11 -6.58789620E-11 2.27336136E-10 -7.79284304E-10
 * 2.65027311E-09 -8.92787799E-09 2.97250473E-08 -9.75214718E-08
 * 3.13893651E-07 -9.84635335E-07 2.97744327E-06 -8.50973902E-06
 * 2.30934056E-05 4.09468739E-05
 DMI KXX 0 6 1 0 19 19
 DMI* KXX 1 9.81256714E + 02
 * -2.22544043E + 03 1.57756165E + 03 -4.22706390E + 02 1.13263832E + 02
 * -3.03489532E + 01 8.13197708E + 00 -2.17895675E + 00 5.83849728E-01
 * -1.56442061E-01 4.19185236E-02 -1.12320343E-02 3.00961407E-03
 * -8.06422264E-04 2.16074855E-04 -5.78771942E-05 1.54339195E-05
 * -3.85847989E-06 6.43079943E-07
 DMI* KXX 2 1 -2.22544043E + 03
 * 6.02844238E + 03 -5.80327002E + 03 2.53623828E + 03 -6.79583008E + 02
 * 1.82093719E + 02 -4.87918663E + 01 1.30737410E + 01 -3.50309825E + 00
 * 9.38652337E-01 -2.51511127E-01 6.73922077E-02 -1.80576853E-02
 * 4.83853370E-03 -1.29644922E-03 3.47263180E-04 -9.26035136E-05
 * 2.31508784E-05 -3.85847989E-06
 DMI* KXX 3 1 1.57756165E + 03
 * -5.80327002E + 03 8.56468066E + 03 -6.48285303E + 03 2.71833203E + 03
 * -7.28374878E + 02 1.95167465E + 02 -5.22949638E + 01 1.40123930E + 01
 * -3.75460935E + 00 1.00604451E + 00 -2.69568831E-01 7.22307414E-02
 * -1.93541348E-02 5.18579688E-03 -1.38905272E-03 3.70414054E-04
 * -9.26035136E-05 1.54339195E-05
 DMI* KXX 4 1 -4.22706390E + 02
 * 2.53623828E + 03 -6.48285303E + 03 8.74677441E + 03 -6.53164502E + 03
 * 2.73140576E + 03 -7.31877991E + 02 1.96106110E + 02 -5.25464745E + 01
 * 1.40797853E + 01 -3.77266717E + 00 1.01088309E + 00 -2.70865262E-01
 * 7.25780055E-02 -1.94467381E-02 5.20894770E-03 -1.38905272E-03
 * 3.47263180E-04 -5.78771942E-05
 DMI* KXX 5 1 1.13263832E + 02
 * -6.79583008E + 02 2.71833203E + 03 -6.53164502E + 03 8.75984863E + 03
 * -6.53514795E + 03 2.73234448E + 03 -7.32129456E + 02 1.96173508E + 02
 * -5.25645332E + 01 1.40846233E + 01 -3.77396345E + 00 1.01123035E + 00
 * -2.70957887E-01 7.26011544E-02 -1.94467381E-02 5.18579688E-03
 * -1.29644922E-03 2.16074855E-04
 DMI* KXX 6 1 -3.03489532E + 01
 * 1.82093719E + 02 -7.28374878E + 02 2.73140576E + 03 -6.53514795E + 03
 * 8.76078711E + 03 -6.53539990E + 03 2.73241187E + 03 -7.32147522E + 02
 * 1.96178345E + 02 -5.25658264E + 01 1.40849714E + 01 -3.77405620E + 00
 * 1.01125348E + 00 -2.70957887E-01 7.25780055E-02 -1.93541348E-02
 * 4.83853370E-03 -8.06422264E-04
 DMI* KXX 7 1 8.13197708E + 00
 * -4.87918663E + 01 1.95167465E + 02 -7.31877991E + 02 2.73234448E + 03
 * -6.53539990E + 03 8.76085449E + 03 -6.53541797E + 03 2.73241675E + 03
 * -7.32148804E + 02 1.96178680E + 02 -5.25659218E + 01 1.40849943E + 01
 * -3.77405620E + 00 1.01123035E + 00 -2.70865262E-01 7.22307414E-02
 * -1.80576853E-02 3.00961407E-03
 DMI* KXX 8 1 -2.17895675E + 00
 * 1.30737410E + 01 -5.22949638E + 01 1.96106110E + 02 -7.32129456E + 02
 * 2.73241187E + 03 -6.53541797E + 03 8.76085938E + 03 -6.53541895E + 03
 * 2.73241699E + 03 -7.32148926E + 02 1.96178711E + 02 -5.25659218E + 01
 * 1.40849714E + 01 -3.77396345E + 00 1.01088309E + 00 -2.69568831E-01
 * 6.73922077E-02 -1.12320343E-02

DMI* KXX 9 1 5.83849728E-01
 * -3.50309825E+00 1.40123930E+01 -5.25464745E+01 1.96173508E+02
 * -7.32147522E+02 2.73241675E+03 -6.53541895E+03 8.76085938E+03
 * -6.53541895E+03 2.73241699E+03 -7.32148926E+02 1.96178680E+02
 * -5.25658264E+01 1.40846233E+01 -3.77266717E+00 1.00604451E+00
 * -2.51511127E-01 4.19185236E-02
 DMI* KXX 10 1 -1.56442061E-01
 * 9.38652337E-01 -3.75460935E+00 1.40797853E+01 -5.25645332E+01
 * 1.96178345E+02 -7.32148804E+02 2.73241699E+03 -6.53541895E+03
 * 8.76085938E+03 -6.53541895E+03 2.73241699E+03 -7.32148804E+02
 * 1.96178345E+02 -5.25645332E+01 1.40797853E+01 -3.75460935E+00
 * 9.38652337E-01 -1.56442061E-01
 DMI* KXX 11 1 4.19185236E-02
 * -2.51511127E-01 1.00604451E+00 -3.77266717E+00 1.40846233E+01
 * -5.25658264E+01 1.96178680E+02 -7.32148926E+02 2.73241699E+03
 * -6.53541895E+03 8.76085938E+03 -6.53541895E+03 2.73241675E+03
 * -7.32147522E+02 1.96173508E+02 -5.25464745E+01 1.40123930E+01
 * -3.50309825E+00 5.83849728E-01
 DMI* KXX 12 1 -1.12320343E-02
 * 6.73922077E-02 -2.69568831E-01 1.01088309E+00 -3.77396345E+00
 * 1.40849714E+01 -5.25659218E+01 1.96178711E+02 -7.32148926E+02
 * 2.73241699E+03 -6.53541895E+03 8.76085938E+03 -6.53541797E+03
 * 2.73241187E+03 -7.32129456E+02 1.96106110E+02 -5.22949638E+01
 * 1.30737410E+01 -2.17895675E+00
 DMI* KXX 13 1 3.00961407E-03
 * -1.80576853E-02 7.22307414E-02 -2.70865262E-01 1.01123035E+00
 * -3.77405620E+00 1.40849943E+01 -5.25659218E+01 1.96178680E+02
 * -7.32148804E+02 2.73241675E+03 -6.53541797E+03 8.76085449E+03
 * -6.53539990E+03 2.73234448E+03 -7.31877991E+02 1.95167465E+02
 * -4.87918663E+01 8.13197708E+00
 DMI* KXX 14 1 -8.06422264E-04
 * 4.83853370E-03 -1.93541348E-02 7.25780055E-02 -2.70957887E-01
 * 1.01125348E+00 -3.77405620E+00 1.40849714E+01 -5.25658264E+01
 * 1.96178345E+02 -7.32147522E+02 2.73241187E+03 -6.53539990E+03
 * 8.76078711E+03 -6.53514795E+03 2.73140576E+03 -7.28374878E+02
 * 1.82093719E+02 -3.03489532E+01
 DMI* KXX 15 1 2.16074855E-04
 * -1.29644922E-03 5.18579688E-03 -1.94467381E-02 7.26011544E-02
 * -2.70957887E-01 1.01123035E+00 -3.77396345E+00 1.40846233E+01
 * -5.25645332E+01 1.96173508E+02 -7.32129456E+02 2.73234448E+03
 * -6.53514795E+03 8.75984863E+03 -6.53164502E+03 2.71833203E+03
 * -6.79583008E+02 1.13263832E+02
 DMI* KXX 16 1 -5.78771942E-05
 * 3.47263180E-04 -1.38905272E-03 5.20894770E-03 -1.94467381E-02
 * 7.25780055E-02 -2.70865262E-01 1.01088309E+00 -3.77266717E+00
 * 1.40797853E+01 -5.25464745E+01 1.96106110E+02 -7.31877991E+02
 * 2.73140576E+03 -6.53164502E+03 8.74677441E+03 -6.48285303E+03
 * 2.53623828E+03 -4.22706390E+02
 DMI* KXX 17 1 1.54339195E-05
 * -9.26035136E-05 3.70414054E-04 -1.38905272E-03 5.18579688E-03
 * -1.93541348E-02 7.22307414E-02 -2.69568831E-01 1.00604451E+00
 * -3.75460935E+00 1.40123930E+01 -5.22949638E+01 1.95167465E+02
 * -7.28374878E+02 2.71833203E+03 -6.48285303E+03 8.56468066E+03
 * -5.80327002E+03 1.57756165E+03
 DMI* KXX 18 1 -3.85847989E-06
 * 2.31508784E-05 -9.26035136E-05 3.47263180E-04 -1.29644922E-03

```

*      4.83853370E-03 -1.80576853E-02  6.73922077E-02 -2.51511127E-01
*      9.38652337E-01 -3.50309825E+00  1.30737410E+01 -4.87918663E+01
*      1.82093719E+02 -6.79583008E+02  2.53623828E+03 -5.80327002E+03
*      6.02844238E+03 -2.22544043E+03
DMI*   KXX           19           1  6.43079943E-07
*      -3.85847989E-06  1.54339195E-05 -5.78771942E-05  2.16074855E-04
*      -8.06422264E-04  3.00961407E-03 -1.12320343E-02  4.19185236E-02
*      -1.56442061E-01  5.83849728E-01 -2.17895675E+00  8.13197708E+00
*      -3.03489532E+01  1.13263832E+02 -4.22706390E+02  1.57756165E+03
*      -2.22544043E+03  9.81256714E+02
DMI   EPHI1         0         2         1         0           19         1
DMI*  EPHI1           1           1 -1.76166458E+01
*      -1.46499987E+01 -1.16833515E+01 -8.71670341E+00 -5.75005627E+00
*      -2.78340864E+00  1.83238924E-01  3.14988637E+00  6.11653376E+00
*      9.08318138E+00  1.20498285E+01  1.50164766E+01  1.79831238E+01
*      2.09497719E+01  2.39164181E+01  2.68830662E+01  2.98497143E+01
*      3.28163605E+01  3.57830086E+01
DMI   EPHI2         0         2         1         0           19         1
DMI*  EPHI2           1           1  3.05762062E+01
*      2.88765221E+01  2.71768379E+01  2.54771538E+01  2.37774715E+01
*      2.20777874E+01  2.03781033E+01  1.86784191E+01  1.69787350E+01
*      1.52790508E+01  1.35793676E+01  1.18796835E+01  1.01799994E+01
*      8.48031616E+00  6.78063202E+00  5.08094788E+00  3.38126421E+00
*      1.68158031E+00 -1.81036163E-02
ENDDATA

```

A.2 Solution Sequence for Orthogonality Based Method

```
ID MSC,NASTRAN $PAS
SOL 63
TIME 100
$
$
$ 1.)
$ ALTER 1134 $ IBM
ALTER 1159 $ VAX
$
$
$ 2.) RENAME MASS AND STIFFNESS MATRICES
MATMOD PHIX,,,,/EPHI1,/1/1
MATMOD PHIX,,,,/EPHI2,/1/2
MATPCH EPHI1,EPHI2// $
ADD MXX,/MA
ADD KXX,/KA
TRNSP EPHI/EPHIT
ADD EPHI,/NPHI
$
$ 3.) EXTRACT ORIGINAL FREQUENCY VALUES
LAMX ,,LAMA/LAMM/-1
MATMOD LAMM,,,,/OF,/1/3
$
$ 4.) GENERATE LMA
TRNSP NPHI/NPHIT $
SMPYAD NPHIT,MA,NPHI,,,/LMA/3 $
$
$ 5.) GENERATE UPDATED MASS MATRIX
MATMOD NPHIT,,,,/IT,/28
DIAGONAL IT/I/SQUARE/0.
ADD I,LMA/IMLMA/(-1.0,0.0) $
SOLVE LMA,/LMAI $
SMPYAD NPHI,LMAI,IMLMA,LMAI,NPHIT,/MTEMP/5 $
SMPYAD MA,MTEMP,MA,,,/MA/MNEW/3 $
$
$ 6.) GENERATE UPDATED STIFFNESS MATRIX
MATMOD EF,,,,/EFT,/28
ADD EFT,/EFM/(6.283185,0.0) $
DIAGONAL EFM/EOME2/SQUARE/2.
SMPYAD KA,NPHI,NPHIT,MNEW,./KT1/4 $
SMPYAD NPHIT,KA,NPHI,,,/EOME2/KT2/3 $
SMPYAD MNEW,NPHI,KT2,NPHIT,MNEW,./KT3/5 $
ADD KT3,KT1/DEL/(-1.0,0.0)/(-1.0,0.0) $
TRNSP DEL/DELT $
ADD5 DEL,DELT,KA,./KNEW $
$
$ 7.) EXTRACT NEWLY CALCULATED EIGENVALUES AND EIGENVECTORS
SETVAL //V,N,NEIGVS/-1 $
READ KNEW,MNEW,MR,DMX,EED,VXCOMPR,CASES/LAMA,NPHIX,MI,OEIGS/ V,N,
READAPP=MODES/S,N,NEIGVS $
OFF LAMA,OEIGS// $
LAMX ,,LAMA/LAMM/-1
```

```

MATMOD LAMM,,,,/NEWF,/1/3
$
$ 8.) PRINT ORIGINAL FE DATA, EXPERIMENTAL DATA, UPDATED DATA
ADD MXX,MNEW/DELM/-1.
ADD KXX,KNEW/DELK/-1.
MATPRN MA,MNEW,KA,KNEW// $
$ MATPRN PHIX,NPHI,NPHIX// $
MATPRN OF,EF,NEWF// $
$
$
DIAGONAL MXX/ALLONE/WHOLE/0.
ADD MXX.MNEW/MDIF//-1.
ADD MXX,MDIF/MPERC///2
ADD KXX,KNEW/KDIF//-1.
ADD KDIF,KXX/KPERC///2
MATPRN MDIF,KDIF// $
MATPRN MPERC,KPERC// $
MATPCH NPHIX// $
$
CEND
$
TITLE = IMPROVEMENT OF A LARGE ANALYTICAL MODEL USING TEST DATA
SUBTITLE = UTILIZING BERMAN AND NAGY'S METHOD
LABEL = CANTILEVER BEAM 10 x 1 x .1
$
$
METHOD = 10
SEALL = ALL
DISPLACEMENT = ALL
$
SPC = 1
$
BEGIN BULK
$
$
$
PARAM , COUPMASS , 1

$
EIGR , 10 , MGIV , , , , 10
ASET1 , 2 , 1 , THRU , 19
GRDSET , , , , , , 1345
GRID , 1 , , , 0. , 0. , 0.
= , *(1) , = , *(2) , = , =
=(17)
CBAR , 1 , 1 , 1 , 2 , 0.0 , 1.0
= , *(1) , = , *(1) , *(1) , = , =
=(16)
CONM2 , 101 , 1 , , , 0.0000294
= , *(1) , *(1) , = , =
=(16)
PBAR , 1 , 1 , .1 , 1.6276-4 , .01
MAT1 1 30.0+6 0.33 7.324-4
$
DMI , EF , 0 , 2 , 1 , 0 , , 8 , 1
DMI , EF , 1 , 1 , 0.00001 , 0.00001 , 20.07 , 55.31 , 108.4 ,

```

, 179.2 , 267.7 , 374.0

\$

DMI EPHI 0 2 1 0 19 8

\$

\$

\$ NORMALIZED TEPHIN.BULK TO NEW MASS MATRIX

```
DMI* EPHI 1 1 1.00000000E+01
* 0.00000000E+01 1.00000000E+01 0.00000000E+01 1.00000000E+01
* 0.00000000E+01 1.00000000E+01 0.00000000E+01 1.00000000E+01
* 0.00000000E+01 1.00000000E+01 0.00000000E+01 1.00000000E+01
* 0.00000000E+01 1.00000000E+01 0.00000000E+01 1.00000000E+01
DMI* EPHI 2 1 0.00000000E+01
* 1.00000000E+01 0.00000000E+01 1.00000000E+01 0.00000000E+01
* 1.00000000E+01 0.00000000E+01 1.00000000E+01 0.00000000E+01
* 1.00000000E+01 0.00000000E+01 1.00000000E+01 0.00000000E+01
* 1.00000000E+01 0.00000000E+01 1.00000000E+01 0.00000000E+01
DMI* EPHI 3 1 -3.18472538E+01
* -2.65386353E+01 -1.74418297E+01 -7.80900669E+00 7.23694801E-01
* 8.34104633E+00 1.47938519E+01 1.92597923E+01 2.05792141E+01
* 2.00870094E+01 1.93061790E+01 1.71260357E+01 1.27322531E+01
* 6.72526789E+00 4.22959059E-01 -9.35005283E+00 -1.91838799E+01
* -2.93217945E+01 -3.55065918E+01
DMI* EPHI 4 1 -3.10419502E+01
* -1.97756252E+01 -3.90104151E+00 1.16861610E+01 1.99744129E+01
* 2.36747532E+01 2.47047443E+01 2.03940392E+01 1.18374815E+01
* 3.20117235E-01 -1.13767452E+01 -1.96993294E+01 -2.32089310E+01
* -2.31072025E+01 -1.89236565E+01 -9.69569778E+00 4.95942736E+00
* 2.15766220E+01 2.88247108E+01
DMI* EPHI 5 1 3.03047791E+01
* 1.48218765E+01 -8.48413849E+00 -2.02269726E+01 -1.97451744E+01
* -1.71320267E+01 -6.27276182E+00 9.76706886E+00 1.84639530E+01
* 2.31594524E+01 1.80964794E+01 9.93039036E+00 -6.31930876E+00
* -1.79159698E+01 -2.01534786E+01 -1.90510578E+01 -7.85290003E+00
* 1.33846464E+01 4.39258118E+01
DMI* EPHI 6 1 -1.82504902E+01
* -4.54117727E+00 2.10175896E+01 2.59855595E+01 1.73300571E+01
* -2.73532486E+00 -2.14730549E+01 -2.72171612E+01 -1.73849392E+01
* -1.74101532E+00 1.88608665E+01 2.18974438E+01 2.31886177E+01
* 1.21079671E+00 -1.75494652E+01 -2.43987560E+01 -1.96883049E+01
* 6.26746941E+00 2.74698696E+01
DMI* EPHI 7 1 -2.27725697E+01
* 9.27494705E-01 2.12805443E+01 1.83374405E+01 -7.78031707E-01
* -2.46232395E+01 -2.74261951E+01 -9.07402515E+00 1.25015440E+01
* 2.88341484E+01 1.09958544E+01 -8.53858852E+00 -3.09406681E+01
* -2.30169296E+01 -5.80351472E-01 1.92681656E+01 2.15572453E+01
* -1.20835721E+00 -2.52521076E+01
DMI* EPHI 8 1 2.42963963E+01
* -9.11770344E+00 -2.06297493E+01 -9.88259411E+00 1.85433483E+01
* 2.50731907E+01 6.63844204E+00 -2.25553684E+01 -2.55435009E+01
* -2.15928182E-01 1.97695541E+01 2.15910053E+01 -5.65337563E+00
* -2.59304104E+01 -1.84511528E+01 1.51534271E+01 2.32457867E+01
* 6.85273027E+00 -3.25167999E+01
```

ENDDATA

A.3 Solution Sequence for Connectivity Constraint Based Method

```
ID MSC,NASTRAN $PAS
SOL 63
TIME 100
$
$ 1)
$ ALTER 509 $ SOL 3
ALTER 1159 $ SOL 63
$
PARAM //NOP/V,Y,KLOOP=1 $
PARAM //ADD/V,Y,KLOOP1/V,Y,NN=0/1
PARAM //ADD/V,Y,NNM1/V,Y,NN=0/-1
PARAM //ADD/V,Y,NNM2/V,Y,NN=0/-2
PARAM //ADD/V,Y,NNP1/V,Y,NN=0/1
PARAM //SUB/V,Y,NNN/0/V,Y,NN=0
PARAM //SUB/V,Y,NNNP1/0/V,Y,NNP1
PARAM //NOP/V,Y,KLOOP6=1
PARAM //ADD/V,Y,KLOOP7/V,Y,NNP1/0 $
PARAM //NOP/V,Y,KLOOP2=2
PARAM //NOP/V,Y,NEGONE=-1
PARAM //NOP/V,Y,KLOOP3=1
PRTPARM //0/C,N,NN
PRTPARM //0/C,N,NNM1
PRTPARM //0/C,N,NNM2
PRTPARM //0/C,N,NNP1
PRTPARM //0/C,N,NNN
PRTPARM //0/C,N,NNNP1
$
$ 2.) STORE MATRICES FROM DMI
ADD KXX,/KA
ADD MXX,/MA
ADD EPHI,/EPHIN
$
$ 3.) GENERATE EXPERIMENTAL EIGENVLAUES
PRTPARM //0/C,N,LAMMEXE
MATMOD EF,,,,/EF1,/28
ADD EF1,/EF2/(6.283185,0.0) $
DIAGONAL EF2/OMEGANE/SQUARE/2.
$
$ 4.) GENERATE A FROM SYSTEM DATA
PRTPARM //0/C,N,AGEN
MPYAD KXX,EPHI,/TEMP
SMPYAD MXX,EPHI,OMEGANE,,,TEMP/ATEMP/3// -1
ADD ATEMP,/A/4.
$
$ 5.) GENERATE CAP PHI FROM ORIGINAL K MATRIX
PRTPARM //0/C,N,CPHGEN
ADD KXX,/KXX2
ADD KXX,KXX2/CAPPHI///C,Y,IOPT=1
$
$ 6.) GENERATE PARTITIONING VECTORS
```

```

PRTPARM //0/C,N,CPPHART
TRNSP  CAPPHI/CAPPHIT
TRNSP  A/AT
TRNSP  EPHI/EPHIT
DIAGONAL KXX/NZTEMP/C,Y,COLUMN/V,Y,POWER=0.$
ADD    NZTEMP,/NZ/(0.0,0.0)
TRNSP  NZTEMP/NZTEMP
TRNSP  NZ/NZT
MATMOD  NZTEMP,,,/ONEONES,/1/1
ADD    ONEONES,/ONEONE
MATMOD  NZT,,,/ONEZERO,/1/1
DIAGONAL KXX/NONES/C,Y,COLUMN/V,Y,POWER=0.$
MPYAD  EPHIT,EPHI/MBYM
DIAGONAL MBYM/MONES/C,Y,COLUMN/V,Y,POWER=0.$
ADD    MONES,/MZEROS/(0.0,0.0)
TRNSP  MONES/MONEST
TRNSP  MZEROS/MZEROST
DBSTORE CAPPHI,EPHI,EPHIT,KXX,MXX//1/1 $
DBSTORE MONES,MZEROS,A,OMEGANE//1/1 $
DBSTORE KA,MA//1/1 $
FILE   ONEZEROS=APPEND
FILE   MZEROST=APPEND
FILE   ABART=APPEND
FILE   GPVTEMP=APPEND
FILE   VPGTEMP=APPEND
FILE   MMMTEMP=APPEND
$
$ 7.) SET UP LOOP TO GENERATE MATRICES
LABEL  KTOP1
PRTPARM //0/C,N,KLOOP
PARAM  //ADD/V,Y,KLOOPP1/V,Y,KLOOP/1
$
$ 8.) EXTRACT ROWS FROM CAPPHI
PRTPARM //0/C,N,Z1
MATMOD  CAPPHIT,,,,/T1,/1/KLOOP
MATMOD  T1,,,,/CPHI,/28
$
$ 9.) GENERATE ABAR FROM A
PRTPARM //0/C,N,Z2
MATMOD  AT,,,,/ACOL,/1/KLOOP
TRNSP  ACOL/ACOLT
APPEND  ACOLT,/ABART/2
$
$ 10.) GENERATE G SUP I
PRTPARM //0/C,N,Z3
SMPYAD  EPHIT,CPHI,EPHI,,,/NGG/3
$
$ 11.) GENERATE EPHR#
PRTPARM //0/C,N,Z4
MATMOD  EPHIT,,,,/EPHR,/1/KLOOP
$
$ 12.) GENERATE M##
PRTPARM //0/C,N,Z5
APPEND  ONEONES,/MMMTEMP/2

```

```

ADD   MMMTEMP,/MMM
$
$ 13.) GENERATE G PARTITIONING VECTOR
PRTPARM //0/C,N,GGG
APPEND MZEROST,/GPVTEMP/2
ADD   GPVTEMP,/GPV
APPEND MONEST,/VPGTEMP/2
ADD   VPGTEMP,/VPG
PRTPARM //0/C,N,BCD
DBSTORE GPV,MMM//KLOOPP1/1
DBSTORE VPG,CPhi,NGG,EPHR//KLOOP/1
PARAM //ADD/V,Y,KLOOP/V,Y,KLOOP/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP
COND   OUT,FLAG
REPT   KTOP1,1000 $ # CORRESPONDS TO (N-1). CHANGE
LABEL   OUT
TRNSP  ABART/ABAR
DBSTORE ABAR//1/1 $
$
$ 14.) ASSEMBLE ALPHA MATRIX
ADD   NZ,ONEONE/MNUM
DBSTORE MNUM,,,,/1/1
DBFETCH /NGG,,,,/1/1
ADD   NGG,/NALPHT1
LABEL   KTOP2
PRTPARM //0/C,N,KLOOP2
DBFETCH /NGG,GPV,GPVTEMP,MMM,MMMTEMP/KLOOP2/1
APPEND GPV,MONEST/GPV1T/0
TRNSP  GPV1T/GPV1
MERGE  NALPHT1,,NGG,GPV1,/NALPHT2/-1/0/V,Y,FORM
PURGE  NALPHT1/ALWAYS
PURGE  NALPHT1/NEVER
ADD   NALPHT2,/NALPHT1
PURGE  NALPHT2/ALWAYS
PURGE  NALPHT2/NEVER
ADD   MMM,/MMM1/(0.0,0.0)
APPEND MMM1,ONEONES/MNUM1/0
TRNSP  MNUM1/MNUM1T
ADD   NZ,MNUM1T/MNUM
DBSTORE MNUM//KLOOP2/1
PARAM //ADD/V,Y,KLOOP2/V,Y,KLOOP2/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP2
COND   OUT2,FLAG
REPT   KTOP2,1000 $ CORRESPONDS TO (N-2) CHANGE
LABEL   OUT2
ADD   NALPHT1,/ALPHA/(-1.0,0.0)
MATPRN ALPHA// $
$
$ 15.) GENERATE BETA MATRIX
PRTPARM //0/C,N,BGEN
FILE   HROW=APPEND
FILE   BETA=APPEND
LABEL   KTOP6
PRTPARM //0/C,N,KLOOP6
DBFETCH /MNUM,,,,/KLOOP6/1
MATMOD MNUM,,,,/MNUMM,/28

```



```

PARAM //ADD/V,Y,KLOOP7/V,Y,KLOOP7/V,Y,NNN $ CORRESPONDS TO (-N).
LABEL KTOP7
PRTPARM //0/C,N,KLOOP7
DBFETCH /EPHR,CPHI,,,/KLOOP7/1
TRNSP NONES/NONEST
SMPYAD EPHR,NONEST,MNUMM,CPHI,EPHIN,/HCN/5/-1
APPEND HCN,/HROW/2
PURGE HCN/ALWAYS
PURGE HCN/NEVER
PARAM //ADD/V,Y,KLOOP7/V,Y,KLOOP7/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP7
COND OUT7,FLAG
REPT KTOP7,100 $ # CORRESPONDS TO (N-1). CHANGE
LABEL OUT7
TRNSP HROW/HROWT
APPEND HROWT,/BETA/2
PURGE HROW/ALWAYS
PURGE HROW/NEVER
PARAM //ADD/V,Y,KLOOP6/V,Y,KLOOP6/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP6 $
COND OUT6,FLAG
REPT KTOP6,1000 $ CORRESPONDS TO (N-1). CHANGE
LABEL OUT6
DBSTORE BETA//1/1 $
MATPRN BETA// $
$
$ 16.) FIND EIGENDATA OF ALPHA + BETA
ADD ALPHA,BETA/AB $
SETVAL //V,N,NEIGV/-1 $
DIAGONAL ALPHA/IA/SQUARE/0.
DBSTORE AB,IA//1/1 $
ADD IA,/NIA/-1.
READ AB,IA,,,DYNAMICS,,CASECC/ABLA,ABPH,ABMI,ABOE/MODES/S,N,NEIGV $
OFF ABLA.ABOE// $
LAMX ,,ABLA/ABLAMM/-1
MATMOD ABLAMM,,,,/OMEGAV,/1/1
MATMOD OMEGAV,,,,/OMEGAB,/28
ADD ABLAMM,/NABLAMM/-1.
$
$ 17.) PURGE MODAL MATRICES OF ZERO EIGENVALUES AND CORRESPONDING E. VEC.
PRTPARM //0/C,N,MODMODAL
PARTN ABPH,ABPV,/,,ABPHN,/0
PARTN OMEGAB,ABPV,/,,,OMEGABN/-1
$
$ 18.) GENERATE LAMBDA B
PRTPARM //0/C,N,LAMBGEN
SOLVE OMEGABN,/OMEGAINV
TRNSP ABPHN/ABPHNT
SMPYAD ABPHN,OMEGAINV,ABPHNT,ABAR,,/LAMBDA B/4
MATPRN LAMBDA B// $
$
$ 19.) GENERATE LAMBDA FROM LAMBDA B
PRTPARM //0/C,N,LAMKGEN
FILE LAMBDA B = APPEND
ADD LAMBDA B,/LTEMP2

```

```

DBSTORE LTEMP2//1/1
LABEL KTOP3
  PRTPARM //0/C,N,KLOOP3
  PARAM //ADD/V,Y,KLOOP3NT/V,Y,NNNP1/V,Y,KLOOP3 $ TO -(N+1). CHANGE
  PARAM //ADD/V,Y,KLOOP3P1/V,Y,KLOOP3/1
  PRTPARM //0/C,N,HERE
  PARAM //MPY/V,Y,KLOOP3N/V,Y,KLOOP3NT/V,Y,NEGONE
  PRTPARM //0/C,N,KLOOP3NT
  PRTPARM //0/C,N,KLOOP3N
  DBFETCH /VPG,VPGTEMP,,/KLOOP3N/1
  TRNSP VPG/VPGT
  ADD VPGT,MONES/GPV2//(-1.0,0.0)
  DBFETCH /LTEMP2,,,/KLOOP3/1
  ADD LTEMP2,/LTEMP
  PARTN LTEMP,,GPV2/LLL,LTEMP2,,/0
  DBSTORE LTEMP2//KLOOP3P1/1
  APPEND LLL./LAMB DAT/2
  PRTPARM //0/C,N,HERE
  PARAM //ADD/V,Y,KLOOP3/V,Y,KLOOP3/1
  PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP3
  COND OUT3,FLAG
REPT KTOP3,1000 $ # CORRESPONDS TO (N-1) CHANGE
LABEL OUT3
$
TRNSP LAMB DAT/LAMBDA
MATPRN LAMBDA// $
DBSTORE LAMBDA// $
$
$ 20.) GENERATE NEW K MATRIX
PRTPARM //0/C,N,NEWKGEN
TRNSP LAMBDA/LAMB DAT
MPYAD EPHI,LAMB DAT,/KT1A
MPYAD LAMBDA,EPHIT,/KT1B
ADD KT1A,KT1B/KT2
ADD CAPPHI,KT2/KT3//C,Y,IOPT = 1
ADD KA,KT3/KNEW//(-.25,0.0)
$
$ 21.) PERFORM FINAL EIGENVALUES ANALYSIS
PRTPARM //0/C,N,FINALEIGEN
SETVAL //V,N,NEIGV/-1 $
READ KNEW,MXX,,,DYNAMICS,,CASECC/ABL,ABPH,ABMI,ABOE/MODES/S,N,NEIGV $
OFF ABL,ABOE// $
MATPRN EF,EPHI// $
MATPRN ABPH// $
MATPRN KNEW,MXX// $
$
$ END OF ALTERATIONS
CEND
$
$ ANALYSIS OPTIONS
TITLE = STIFFNESS MATRIX ADJUSTMEN USING MODE DATA (CONNECTIVITY)
SUBTITLE = UTILIZING A. M. KABE'S METHOD
METHOD = 10
SEALL = ALL
DISPLACEMENT = ALL
SPC = 1

```

```

$
$ ENTER PHYSICAL DATA
BEGIN BULK
$
$ # CHANGE ABOVE ENTRIES TO SUIT # OF NON-ZERO EIGENVALUES
$
$ EXPERIMENTAL DATA OBTAINED FROM UNPERTURBED DATA
$ FOR FREE FREE BOUNDARY CONDITIONS
$
$
$
EIGR , 10 , MGIV , , , 888
$
$
$
PARAM , COUPMASS , 1
$
ASET1 , 2 , 1 , THRU , 19
GRDSET , , , , , 1345
GRID , 1 , , 0. , 0. , 0.
= , *(1), = , *(2.) , = , =
=(17)
CBAR , 1 , 1 , 1 , 2 , 0.0 , 1.0
= , *(1), = , *(1), *(1), = , =
=(16)
CONM2 , 101 , 1 , , 0.00002
= , *(1), *(1), = , =
=(16)
PBAR , 1 , 1 , .1 , 1.6276-4 , .01
MAT1 1 30.0+6 0.33 7.324-4
$
DMI , ABPV , 0 , 2 , 1 , 0 , , 152 , 1
DMI , ABPV , 1 , 98 , 1. , THRU , 152
$
PARAM , NN , 19
$
$ CHANGE
DMI , EF , 0 , 2 , 1 , 0 , , 8 , 1
DMI , EF , 1 , 1 , 0.00001 , 0.00001 , 20.07 , 55.31 , 108.4 ,
, 179.2 , 267.7 , 374.0
$
DMI EPHI 0 2 1 0 19 8
$ CHANGE
DMI* EPHI 1 1 -1.72176323E+01
* -1.41148415E+01 -1.10120497E+01 -7.90925884E+00 -4.80646753E+00
* -1.70367646E+00 1.39911485E+00 4.50190592E+00 7.60469723E+00
* 1.07074881E+01 1.38102798E+01 1.69130707E+01 2.00158615E+01
* 2.31186543E+01 2.62214451E+01 2.93242359E+01 3.24270287E+01
* 3.55298195E+01 3.86326103E+01
DMI* EPHI 2 1 -3.46183472E+01
* -3.25617943E+01 -3.05052452E+01 -2.84486923E+01 -2.63921413E+01
* -2.43355885E+01 -2.22790375E+01 -2.02224865E+01 -1.81659336E+01
* -1.61093826E+01 -1.40528307E+01 -1.19962788E+01 -9.93972683E+00
* -7.88317537E+00 -5.82662344E+00 -3.77007198E+00 -1.71352017E+00
* 3.43031585E-01 2.39958334E+00
DMI* EPHI 3 1 3.86144180E+01

```

```

*      2.86342773E+01  1.87511196E+01  9.18296051E+00  2.37452328E-01
*      -7.73347664E+00 -1.43759937E+01 -1.93720779E+01 -2.24709320E+01
*      -2.35125999E+01 -2.24426174E+01 -1.93169231E+01 -1.42968159E+01
*      -7.63417530E+00  3.52272034E-01  9.30845547E+00  1.88827553E+01
*      2.87684174E+01  3.87489510E+01
DMI*   EPHI          4          1 -3.85642090E+01
*      -2.17269306E+01 -5.56334639E+00  8.57991505E+00  1.91425362E+01
*      2.48533192E+01  2.51050472E+01  2.01576328E+01  1.11488228E+01
*      -7.81321302E-02 -1.12895975E+01 -2.02546787E+01 -2.51377831E+01
*      -2.48122253E+01 -1.90302696E+01 -8.40984535E+00  5.77112436E+00
*      2.19516354E+01  3.87917709E+01
DMI*   EPHI          5          1 3.85213585E+01
*      1.50855913E+01 -5.99615622E+00 -2.06693840E+01 -2.55651627E+01
*      -1.99639053E+01 -6.45989990E+00  9.75792027E+00  2.26646786E+01
*      2.75165100E+01  2.25364819E+01  9.54851818E+00 -6.67327547E+00
*      -2.01011295E+01 -2.55710430E+01 -2.05314846E+01 -5.74531984E+00
*      1.53937120E+01  3.88399200E+01
DMI*   EPHI          6          1 -3.84950523E+01
*      -8.61501789E+00  1.54652910E+01  2.55439034E+01  1.84084969E+01
*      -5.35287321E-01 -1.96334839E+01 -2.74453926E+01 -1.92774658E+01
*      1.44958913E-01  1.94827061E+01  2.74463310E+01  1.94306545E+01
*      2.49772698E-01 -1.86044827E+01 -2.55246258E+01 -1.52180090E+01
*      8.99818039E+00  3.89052696E+01
DMI*   EPHI          7          1 -3.85018806E+01
*      -2.42385864E+00  2.20858669E+01  2.25441017E+01  1.78078961E+00
*      -2.12663708E+01 -2.64738045E+01 -9.22068691E+00  1.58506241E+01
*      2.73878517E+01  1.55598850E+01 -9.55444622E+00 -2.65665512E+01
*      -2.10404854E+01  2.12887764E+00  2.27075920E+01  2.18992424E+01
*      -2.86966300E+00 -3.90048752E+01
DMI*   EPHI          8          1 3.85655937E+01
*      -3.39209175E+00 -2.53591595E+01 -1.28830242E+01  1.61290245E+01
*      2.70523090E+01  6.91272497E+00 -2.11531601E+01 -2.47781010E+01
*      2.10803017E-01  2.49562874E+01  2.08820953E+01 -7.32034779E+00
*      -2.71267033E+01 -1.57869587E+01  1.32383242E+01  2.52917023E+01
*      2.89967895E+00 -3.91637192E+01
ENDDATA

```

A.4 Solution Sequence for Residual Force Based Method

```
ID MSC,NASTRAN $PAS
SOL 63
TIME 100
$
$
$ THIS SYSTEM IDENTIFICATION METHOD UTILIZES A RESIDUAL FORCE MATRIX
$ TO LOCATE MODELLING ERRORS IN THE FINITE ELEMENT MODEL
$ THE METHOD THEN CORRECTS DEFICIENT ELEMENTS ONLY TO UPDATE THE
$ SYSTEM MATRICES TO MATCH THE EXPERIMENTAL DATA
$
$ L IS THE NUMBER OF DEFICIENT ELEMENTS DETERMINED
$ M IS THE NUMBER OF EXPERIMENTAL FREQUENCIES
$ N IS THE NUMBER OF DOF'S MEASURED
$
$
$ 1.)
$
ALTER 1159
$
$ 2.) REARRANGE MATRICES
PRTPARM //0/C,N,BEGMOD
PARAM //ADD/V,Y,MMM1/V,Y,MM=0/-1
PARAM //ADD/V,Y,MMP1/V,Y,MM=0/1
PARAM //ADD/V,Y,NNP1/V,Y,NN=0/1
PARAM //ADD/V,Y,NNM1/V,Y,NN=0/-1
PARAM //NOP/V,Y,KLOOP=1
PARAM //NOP/V,Y,KLOOP4=1
PARAM //NOP/V,Y,KLOOP2=1
PARAM //NOP/V,Y,KLOOP3=1
PARAM //NOP/V,Y,KLOOP5=1
PARAM //ADD/V,Y,KNTR/V,Y,NN=0/1
MATMOD COM,,,,,/CNM,/28
ADD KXX,/KA
ADD MXX,/MA
ADD KXX,/OKXX
ADD MXX,/OMXX
DBSTORE OKXX,OMXX//1/1 $
$
$ 3.) THE FOLLOWING MMAT AND KMAT MATRICES NEED TO BE CHANGED FOR THE
$ ANALYSIS OF DIFFERENT ENTITIES
ADD KXX01,/KMAT
ADD MXX01,/MMAT
DBSTORE KMAT,MMAT//1/1
ADD KXX02,/KMAT
ADD MXX02,/MMAT
DBSTORE KMAT,MMAT//2/1
ADD KXX03,/KMAT
ADD MXX03,/MMAT
DBSTORE KMAT,MMAT//3/1
ADD KXX04,/KMAT
ADD MXX04,/MMAT
DBSTORE KMAT,MMAT//4/1
```

```

ADD KXX05,/KMAT
ADD MXX05,/MMAT
DBSTORE KMAT,MMAT//5/1
ADD KXX06,/KMAT
ADD MXX06,/MMAT
DBSTORE KMAT,MMAT//6/1
ADD KXX07,/KMAT
ADD MXX07,/MMAT
DBSTORE KMAT,MMAT//7/1
ADD KXX08,/KMAT
ADD MXX08,/MMAT
DBSTORE KMAT,MMAT//8/1
ADD KXX09,/KMAT
ADD MXX09,/MMAT
DBSTORE KMAT,MMAT//9/1
ADD KXX10,/KMAT
ADD MXX10,/MMAT
DBSTORE KMAT,MMAT//10/1
ADD KXX11,/KMAT
ADD MXX11,/MMAT
DBSTORE KMAT,MMAT//11/1
ADD KXX12,/KMAT
ADD MXX12,/MMAT
DBSTORE KMAT,MMAT//12/1
ADD KXX13,/KMAT
ADD MXX13,/MMAT
DBSTORE KMAT,MMAT//13/1
ADD KXX14,/KMAT
ADD MXX14,/MMAT
DBSTORE KMAT,MMAT//14/1
ADD KXX15,/KMAT
ADD MXX15,/MMAT
DBSTORE KMAT,MMAT//15/1
ADD KXX16,/KMAT
ADD MXX16,/MMAT
DBSTORE KMAT,MMAT//16/1
ADD KXX17,/KMAT
ADD MXX17,/MMAT
DBSTORE KMAT,MMAT//17/1
ADD KXX18,/KMAT
ADD MXX18,/MMAT
DBSTORE KMAT,MMAT//18/1
$

```

\$ 4.) GENERATE OR RENAME ANALYTICAL AND EXPERIMENTAL DATA

```

ADD PHIX,/APHIN
ADD EPHI,/EPHIN
LAMX ,,LAMA/LAMM/-1
MATMOD LAMM,,,,/ALAMV,/1/1
MATMOD ALAMV,,,,/ALAM,/28
MATMOD EF,,,,/EFT,/28
ADD EFT,/EFM/(6.283185,0.0) $
DIAGONAL EFM/ELAM/SQUARE/2.
ADD APHIN,EPHIN/DELPHI/-1.
ADD ALAM,ELAM/DELLAM/-1.
MATPRN DELPHI,EPHIN,APHIN// $
DBSTORE KA,MA,APHIN,EPHIN// $

```

```

DBSTORE ALAM,ELAM,DELPHI,DELLAM// $
MATMOD LAMM,,,,/AF,/1/3
DBSTORE AF,EF// $
$
$ 5.) MULTIPLY EACH EIGENVECTOR TO ITS CORRESPONDING EIGENVALUE
LAMX  ,,LAMA/LAMM/-1
PRTPARM //0/C,N,DEV
DBFETCH /EPHIN,,,/ $
FILE  LPHI=APPEND
LABEL  KTOP4
      MATMOD EPHIN,,,,/U,/1/KLOOP4
      SCALAR ELAM//KLOOP4/KLOOP4/S,Y,L
      ADD  U,/LU/L
      APPEND LU,/LPHI/2
      PARAM //ADD/V,Y,KLOOP4/V,Y,KLOOP4/1
      PARAM //SUB/V,N,FLAG/V,Y,MMM1/V,Y,KLOOP4
      COND  OUT4,FLAG
REPT  KTOP4,1000 $ # CORRESPONDS TO (M-1). CHANGE
LABEL  OUT4
DBSTORE LPHI// $
$
$ 6.) CALCULATE RESIDUAL FORCE MATRIX
PRTPARM //0/C,N,RFM
DBFETCH /EPHIN,KA,MA,/ $
MPYAD  KA,EPHIN./RMT1
MPYAD  MA,LPHI./RMT2
ADD  RMT1,RMT2/RM/-1.
MATPRN RM// $
DBSTORE RM// $
$
$ 7.) CALCULATE MATRIX FOR UNKNOWNNS
PRTPARM //0/C,N,MUNKGEN
DBFETCH /APHIN,KA,MA,ALAM,/ $
DBFETCH /DELLAM,DELPHI,,,/ $
MPYAD  KA,DELPHI./MT1
SMPYAD MA,DELPHI,ELAM,,,MT1/MT2/3// -1
SMPYAD MA,APHIN,DELLAM,,,MT2/MUNK/3
DBSTORE MUNK// $
$
$ 8.) GENERATE A MATRIX
PRTPARM //0/C,N,AGEN
FILE  AKXX=APPEND
FILE  AMXX=APPEND
LABEL  KTOP1
      PRTPARM //0/C,N,KLOOP
      DBFETCH /KMAT,MMAT,,,/KLOOP/1
      MPYAD  KMAT,EPHIN,/AL1
      TRNSP  AL1/AL1T
      SMPYAD  MMAT,EPHIN,ELAM,,,/AL2/3/-1
      TRNSP  AL2/AL2T
      PURGE  AKXX/NEVER
      PURGE  AMXX/NEVER
      PARAM  //SUB/V,Y,KNTR/V,Y,KNTR/NN $ # CORRESPONDS TO (N). CHANGE
      LABEL  KTOP2
      PRTPARM //0/C,N,KNTR
      MATMOD  AL1T,,,,/T1,/1/KNTR

```

```

TRNSP T1/T01T
APPEND T01T,/AKXX/2
MATMOD AL2T,,,,/T2,/1/KNTR
TRNSP T2/T02T
APPEND T02T,/AMXX/2
PARAM //ADD/V,Y,KNTR/V,Y,KNTR/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KNTR
COND KNTR1,FLAG
REPT KTOP2,1000 $ # CORRESPONDS TO (N-1). CHANGE
LABEL KNTR1
DBSTORE AKXX,AMXX//KLOOP/1
DBSTORE AKXX//KLOOP/1
PURGE AKXX/ALWAYS
PURGE AMXX/ALWAYS
PARAM //ADD/V,Y,KLOOP/V,Y,KLOOP/1
REPT KTOP1,17 $ # CORRESPONDS TO (L-1). CHANGE
LABEL TOHERE
SMPYAD CNM,EPHIN,ELAM,,,/AL2/3/-1
TRNSP AL2/AL2T
FILE CONMXXT=APPEND
LABEL KTOP5
MATMOD AL2T,,,,/T2,/1/KLOOP5
TRNSP T2/T02T
APPEND T02T,/CONMXXT/2
PARAM //ADD/V,Y,KLOOP5/V,Y,KLOOP5/1
PARAM //SUB/V,N,FLAG/V,Y,NNM1/V,Y,KLOOP5
COND OUT5,FLAG
REPT KTOP5,1000 $ # CORRESPONDS TO (N-1). CHANGE
LABEL OUT5
TRNSP CONMXXT/CONMXX
$
$ 9.) GENERATE B MATRIX
PRTPARM //0/C,N,BGEN
TRNSP MUNK/MUNKT
PRTPARM //0/C,N,BGEN
FILE B1T=APPEND
LABEL KTOP3
PRTPARM //0/C,N,KLOOP3
MATMOD MUNKT,,,,/T1,/1/KLOOP3
TRNSP T1/T01T
APPEND T01T,/B1T/2
PARAM //ADD/V,Y,KLOOP3/V,Y,KLOOP3/1
PARAM //SUB/V,N,FLAG/V,Y,NN/V,Y,KLOOP3
COND OUT3,FLAG
REPT KTOP3,1000 $ # CORRESPONDS TO (N-1). CHANGE
LABEL OUT3
TRNSP B1T/B1
DBSTORE B1//1/1 $
$
$ 10.) EXTRACT COLUMNS CORRESPONDING TO DEFICIENT ELEMENTS
PURGE AKXX/NEVER
PURGE AMXX/NEVER
FILE A=APPEND
DBFETCH /AKXX,,,,/1/1
TRNSP AKXX/AC01
DBFETCH /AKXX,,,,/2/1

```



```

TRNSP AKXX/AC02
DBFETCH /AKXX,,,/3/1
TRNSP AKXX/AC03
DBFETCH /AKXX,,,/4/1
TRNSP AKXX/AC04
DBFETCH /AKXX,,,/14/1
TRNSP AKXX/AC14
DBFETCH /AKXX,,,/15/1
TRNSP AKXX/AC15
DBFETCH /AKXX,,,/16/1
TRNSP AKXX/AC16
DBFETCH /AKXX,,,/17/1
TRNSP AKXX/AC17
DBFETCH /AKXX,,,/18/1
TRNSP AKXX/AC18
$APPEND AC01./A/2
$APPEND AC02./A/2
$APPEND AC03./A/2
$APPEND AC04./A/2
$APPEND AC14./A/2
$APPEND AC15./A/2
$APPEND AC16./A/2
$APPEND AC17./A/2
$APPEND AC18./A/2
DBFETCH /AMXX,,,/1/1
TRNSP AMXX/AC01
DBFETCH /AMXX,,,/2/1
TRNSP AMXX/AC02
DBFETCH /AMXX,,,/3/1
TRNSP AMXX/AC03
DBFETCH /AMXX,,,/4/1
TRNSP AMXX/AC04
DBFETCH /AMXX,,,/14/1
TRNSP AMXX/AC14
DBFETCH /AMXX,,,/15/1
TRNSP AMXX/AC15
DBFETCH /AMXX,,,/16/1
TRNSP AMXX/AC16
DBFETCH /AMXX,,,/17/1
TRNSP AMXX/AC17
DBFETCH /AMXX,,,/18/1
TRNSP AMXX/AC18
APPEND AC01./A/2
APPEND AC02./A/2
APPEND AC03./A/2
APPEND AC04./A/2
APPEND AC14./A/2
APPEND AC15./A/2
APPEND AC16./A/2
APPEND AC17./A/2
APPEND AC18./A/2
$ APPEND CONMXX./A/2
MATPRN A// $
DBSTORE A// $
$
$ 11.) SOLVE FOR X USING USING SIMPLE FORMULATION

```

```

PRTPARM //0/C,N,X1GEN
TRNSP A/AT
MPYAD AT,A,/ATA
MATPRN ATA// $
SOLVE ATA,/ATAI
SMPYAD ATAI,AT,B1,,,/X1/3
$
$
DBSTORE X1,A,B1,KXX,MXX//1/1
$
$
SCALAR X1//1/1/S,Y,X01
SCALAR X1//2/1/S,Y,X02
SCALAR X1//3/1/S,Y,X03
SCALAR X1//4/1/S,Y,X04
SCALAR X1//5/1/S,Y,X05
SCALAR X1//6/1/S,Y,X06
SCALAR X1//7/1/S,Y,X07
SCALAR X1//8/1/S,Y,X08
SCALAR X1//9/1/S,Y,X09
$SCALAR X1//10/1/S,Y,X10
$SCALAR X1//11/1/S,Y,X11
$SCALAR X1//12/1/S,Y,X12
$SCALAR X1//13/1/S,Y,X13
$SCALAR X1//14/1/S,Y,X14
$SCALAR X1//15/1/S,Y,X15
$SCALAR X1//16/1/S,Y,X16
$SCALAR X1//17/1/S,Y,X17
$SCALAR X1//18/1/S,Y,X18
$SCALAR X1//19/1/S,Y,X19
$ADD5 KXX01,KXX02,KXX03,KXX04,KXX05/KADD1/X01/X02/X03/X04/
$ (0.0,0.0)
$ADD5 KXX06,KXX07,KXX08,KXX09,KXX10/KADD2/(0.0,0.0)/(0.0,0.0)/
$ (0.0,0.0)/(0.0,0.0)/(0.0,0.0)
$ADD5 KXX11,KXX12,KXX13,KXX14,KXX15/KADD3/(0.0,0.0)/(0.0,0.0)/
$ (0.0,0.0)/X05/X06
$ADD5 KXX16,KXX17,KXX18,,/KADD4/X07/X08/X09
$
ADD5 MXX01,MXX02,MXX03,MXX04,MXX05/MADD1/X01/X02/X03/X04/
(0.0,0.0)
ADD5 MXX06,MXX07,MXX08,MXX09,MXX10/MADD2/(0.0,0.0)/(0.0,0.0)/
(0.0,0.0)/(0.0,0.0)/(0.0,0.0)
ADD5 MXX11,MXX12,MXX13,MXX14,MXX15/MADD3/(0.0,0.0)/(0.0,0.0)/
(0.0,0.0)/X05/X06
ADD5 MXX16,MXX17,MXX18,,/MADD4/X07/X08/X09
$
$ADD5 KADD1,KADD2,KADD3,KADD4,KA/KXX1
$ADD KXX1,/KXX/(.250,0.0)
$ADD MA,/MXX
$
ADD5 MADD1,MADD2,MADD3,MADD4,/MADD5
ADD MADD5,MA/MXX/(.01,0.0)
$ ADD CNM,MA/MXX/X10
PRTPARM //0/C,N,X01
$MATPRN CNM,KA,KXX,MA,MXX// $
PRTPARM //0/C,N,USINGX1

```

```

SETVAL //V,N,NEIGVS/-1 $
READ KXX,MXX,,,DYNAMICS,,CASECC/LAMA,PHIX,MI,OEIGS/ V,N,
  READAPP=MODES/S,N,NEIGVS $
OFF LAMA,OEIGS// $
MATPRN PHIX// $
LAMX ,,LAMA/LAMM/-1
MATMOD LAMM,,,,/FX1,/1/3
DBSTORE X1,FX1// $
ADD EPHIN,PHIX/DELPH/-1.
$MATPRN DELPH// $
$
$ 12.) PRINT RESULTS
MATPRN EF,EPHI,PHIX,KXX,MXX// $
OUTPUT2 CSTM,GPL,GPDT,EPT,GEOM2//0/S,Y,OP2UN = 51 $
OUTPUT2 OUGV1,,,,/0/OP2UN
OUTPUT2 ,,,,/-9/OP2UN $
PARAM //ADD/OP2UN/V,N,ONE = 1/OP2UN $
label tempend
DBFETCH /OMXX,OKXX,,,/1/1
MATPRN KXX,OKXX// $
ADD MXX,OMXX/MDIF//-1.
ADD MDIF,MA/MPERC///2
ADD KXX,OKXX/KDIF//-1.
ADD KDIF,KXX/KPERC///2
MATPRN KDIF// $
MATPRN MPERC,KPERC// $
MATPCH PHIX// $
$
$
$ END ALTERATIONS
CEND
$
TITLE = FINITE ELEMENT MODEL REFINEMENT USING MODAL ANALYSIS DATA (R. F.)
SUBTITLE = UTILIZING CHEN AND WANG'S METHOD
$
$ ANALYSIS OPTIONS
METHOD = 10
SEALL = ALL
DISPLACEMENT = ALL
ECHO = NONE
SPC = 1
$
$ PHYSICAL CARDS FOR ANALYSIS
BEGIN BULK
$ ELEMENT MASS AND STIFFNESS MATRICES
DMI , KXX01 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX02 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX03 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX04 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX05 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX06 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX07 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX08 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX09 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX10 , 0 , 2 , 1 , 0 , , 19 , 19
DMI , KXX11 , 0 , 2 , 1 , 0 , , 19 , 19

```

DMI , KXX12 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX13 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX14 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX15 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX16 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX17 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , KXX18 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX01 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX02 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX03 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX04 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX05 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX06 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX07 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX08 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX09 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX10 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX11 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX12 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX13 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX14 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX15 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX16 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX17 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , MXX18 , 0 , 2 , 1 , 0 , , 19 , 19
 DMI , COM , 0 , 2 , 1 , 0 , , 19 , 1

\$

DMI , KXX01 , 1 , 1 , 1.
 DMI , KXX01 , 2 , 1 , 1.
 DMI , KXX02 , 2 , 2 , 1.
 DMI , KXX02 , 3 , 2 , 1.
 DMI , KXX03 , 3 , 3 , 1.
 DMI , KXX03 , 4 , 3 , 1.
 DMI , KXX04 , 4 , 4 , 1.
 DMI , KXX04 , 5 , 4 , 1.
 DMI , KXX05 , 5 , 5 , 1.
 DMI , KXX05 , 6 , 5 , 1.
 DMI , KXX06 , 6 , 16 , 1.
 DMI , KXX06 , 7 , 16 , 1.
 DMI , KXX07 , 7 , 17 , 1.
 DMI , KXX07 , 8 , 17 , 1.
 DMI , KXX08 , 8 , 18 , 1.
 DMI , KXX08 , 9 , 18 , 1.
 DMI , KXX09 , 9 , 19 , 1.
 DMI , KXX09 , 10 , 19 , 1.
 DMI , KXX10 , 10 , 10 , 1.
 DMI , KXX10 , 11 , 10 , 1.
 DMI , KXX11 , 11 , 11 , 1.
 DMI , KXX11 , 12 , 11 , 1.
 DMI , KXX12 , 12 , 12 , 1.
 DMI , KXX12 , 13 , 12 , 1.
 DMI , KXX13 , 13 , 13 , 1.
 DMI , KXX13 , 14 , 13 , 1.
 DMI , KXX14 , 14 , 14 , 1.
 DMI , KXX14 , 15 , 14 , 1.
 DMI , KXX15 , 15 , 15 , 1.

DMI , KXX15 , 16 , 15 , 1.
DMI , KXX16 , 16 , 16 , 1.
DMI , KXX16 , 17 , 16 , 1.
DMI , KXX17 , 17 , 17 , 1.
DMI , KXX17 , 18 , 17 , 1.
DMI , KXX18 , 18 , 18 , 1.
DMI , KXX18 , 19 , 18 , 1.
DMI , MXX01 , 1 , 1 , 1.
DMI , MXX01 , 2 , 2 , 1.
DMI , MXX02 , 2 , 2 , 1.
DMI , MXX02 , 3 , 3 , 1.
DMI , MXX03 , 3 , 3 , 1.
DMI , MXX03 , 4 , 4 , 1.
DMI , MXX04 , 4 , 4 , 1.
DMI , MXX04 , 5 , 5 , 1.
DMI , MXX05 , 5 , 5 , 1.
DMI , MXX05 , 6 , 6 , 1.
DMI , MXX06 , 6 , 6 , 1.
DMI , MXX06 , 7 , 7 , 1.
DMI , MXX07 , 7 , 7 , 1.
DMI , MXX07 , 8 , 8 , 1.
DMI , MXX08 , 8 , 8 , 1.
DMI , MXX08 , 9 , 9 , 1.
DMI , MXX09 , 9 , 9 , 1.
DMI , MXX09 , 10 , 10 , 1.
DMI , MXX10 , 10 , 10 , 1.
DMI , MXX10 , 11 , 11 , 1.
DMI , MXX11 , 11 , 11 , 1.
DMI , MXX11 , 12 , 12 , 1.
DMI , MXX12 , 12 , 12 , 1.
DMI , MXX12 , 13 , 13 , 1.
DMI , MXX13 , 13 , 13 , 1.
DMI , MXX13 , 14 , 14 , 1.
DMI , MXX14 , 14 , 14 , 1.
DMI , MXX14 , 15 , 15 , 1.
DMI , MXX15 , 15 , 15 , 1.
DMI , MXX15 , 16 , 16 , 1.
DMI , MXX16 , 16 , 16 , 1.
DMI , MXX16 , 17 , 17 , 1.
DMI , MXX17 , 17 , 17 , 1.
DMI , MXX17 , 18 , 18 , 1.
DMI , MXX18 , 18 , 18 , 1.
DMI , MXX18 , 19 , 19 , 1.
DMI , COM , 1 , 1 , 1. , THRU , 19

\$

\$ EXPERIMENTAL DATA OBTAINED FROM UNPERTURBED DATA
\$ AND ELEMENT CARDS

\$

\$ \$ PARAM , COUPLASS , NO

\$

\$ PARAM , MM , 5
PARAM , NN , 19

\$

ASET1 , 2 , 1 , THRU , 19
GRDSET , , , , , 1345

\$
 \$
 EIGR , 10 , MGIV , , , , 8 \$ # CORRESPONDS TO (M). CHANGE
 GRID , 1 , , 0 , , 0 , , 0.
 = , *(1) , = , *(2) , = , =
 =(17)
 CBAR , 1 , 1 , 1 , 2 , 0.0 , 1.0
 = , *(1) , = , *(1) , *(1) , = , =
 =(16)
 CONM2 , 101 , 1 , , , 0.00002
 = , *(1) , *(1) , = , =
 =(16)
 PBAR , 1 , 1 , .1 , 1.6276-4 , .01
 MAT1 1 30.0+6 0.33 7.324-4
 \$
 \$
 PARAM , MM , 8
 \$
 \$
 DMI , EF , 0 , 2 , 1 , 0 , , 8 , 1 \$SECOND TO LAST # CORRESPONDS TO (M)
 \$ \$ CHANGE
 DMI , EF , 1 , 1 , 0.00001 , 0.00001 , 20.07 , 55.31 , 108.4 ,
 , 179.2 , 267.7 , 374.0
 \$ \$LAST # CORRESPONDS TO (M)
 DMI EPHI 0 2 1 0 19 8
 \$ \$ CHANGE
 \$ EIGENVALUES, NORMALIZED AND MINIMIZED TO .00002
 DMI* EPHI 1 1 3.57167015E+01
 * 3.30177765E+01 3.04058990E+01 2.78259907E+01 2.49399452E+01
 * 2.28970356E+01 1.98644390E+01 1.55503006E+01 1.05684566E+01
 * 5.20570707E+00 2.57466149E+00 7.46367812E-01 3.52458656E-01
 * -2.35017037E+00 -7.37524033E+00 -1.34586601E+01 -1.84211063E+01
 * -2.18565788E+01 -2.48404961E+01
 DMI* EPHI 2 1 7.15983057E+00
 * 6.06342173E+00 5.61678791E+00 7.34838772E+00 1.07304287E+01
 * 1.37291098E+01 1.49786186E+01 1.47920618E+01 1.52318544E+01
 * 1.68891201E+01 1.94311790E+01 2.17246723E+01 2.29907875E+01
 * 2.40129471E+01 2.54970360E+01 2.77868366E+01 2.93144779E+01
 * 3.01787758E+01 3.03296032E+01
 DMI* EPHI 3 1 3.21526451E+01
 * 2.66535816E+01 1.75208302E+01 7.70977736E+00 -1.16032064E+00
 * -9.21957684E+00 -1.67217293E+01 -2.25219746E+01 -2.45921383E+01
 * -2.44326077E+01 -2.26205788E+01 -1.94452190E+01 -1.34360981E+01
 * -7.13358402E+00 -1.03177488E+00 9.24126053E+00 2.03835926E+01
 * 3.30448761E+01 4.09507027E+01
 DMI* EPHI 4 1 3.12785187E+01
 * 1.97751923E+01 3.68843627E+00 -1.16805687E+01 -1.95057163E+01
 * -2.26965733E+01 -2.33740921E+01 -1.89882317E+01 -1.06356564E+01
 * 5.91088891E-01 1.21870327E+01 2.04515934E+01 2.38454208E+01
 * 2.34679909E+01 1.89977531E+01 9.47869682E+00 -5.16561365E+00
 * -2.14697285E+01 -2.82717915E+01
 DMI* EPHI 5 1 3.13220100E+01
 * 1.34863663E+01 -1.19593534E+01 -2.38389263E+01 -2.17605324E+01
 * -1.75234985E+01 -6.10287237E+00 9.63667011E+00 1.79388905E+01
 * 2.27219162E+01 1.94384117E+01 1.23664703E+01 -3.49145174E+00
 * -1.69003124E+01 -2.10529938E+01 -2.11135120E+01 -9.35707283E+00

```

*      1.48981800E + 01  4.89699249E + 01
DMI*  EPHI              6              1  1.79429245E + 01
*      3.08145905E + 00 -2.48424149E + 01 -2.99360313E + 01 -2.00538712E + 01
*      2.70642471E + 00  2.36876526E + 01  2.98448067E + 01  1.85193844E + 01
*      6.37351274E-01 -2.20714741E + 01 -2.50264091E + 01 -2.56713161E + 01
*      -9.86995399E-01  1.94862328E + 01  2.63514938E + 01  2.04945984E + 01
*      -8.78151417E + 00 -3.26646233E + 01
DMI*  EPHI              7              1  2.84253445E + 01
*      2.25800681E + 00 -2.12453003E + 01 -1.96493320E + 01 -2.22188666E-01
*      2.43530788E + 01  2.78198528E + 01  9.80179882E + 00 -1.20091457E + 01
*      -2.85931187E + 01 -1.08003969E + 01  8.68634701E + 00  3.04884052E + 01
*      2.17766685E + 01 -1.16737545E + 00 -2.12131138E + 01 -2.27934742E + 01
*      1.71394908E + 00  2.81306782E + 01
DMI*  EPHI              8              1  2.56874809E + 01
*      -1.07468987E + 01 -2.32455082E + 01 -1.13995495E + 01  1.98160591E + 01
*      2.70894985E + 01  7.12519407E + 00 -2.45235386E + 01 -2.75550919E + 01
*      3.31426322E-01  2.24564190E + 01  2.46059132E + 01 -5.05006886E + 00
*      -2.73418236E + 01 -1.92842770E + 01  1.74014549E + 01  2.64915276E + 01
*      9.20206356E + 00 -3.30909004E + 01

```

ENDDATA

**The vita has been removed from
the scanned document**