

RELIABILITY ALLOCATION AND APPORTIONMENT:

ADDRESSING REDUNDANCY AND LIFE-CYCLE COST

by

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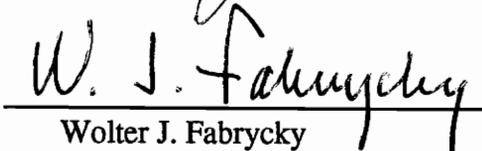
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(ABSTRACT)

Two reliability analysis techniques, allocation and apportionment, have the potential to influence a system's design (a distinction is made here between allocation and apportionment). Algorithms that account for the ever increasing design complexities are constructed here for both. As designs of aircraft, railway systems, automobiles and space systems continue to push the envelope in terms of their capabilities, the importance of performance criteria such as reliability and associated life-cycle cost (LCC) consequences become even more important. These interrelated criteria are the foundation for the reliability allocation and apportionment algorithms derived in this thesis.

Reliability allocation is the process of assigning reliability targets to lower-level assemblies to ensure the top-level assembly's goal is achieved. Reliability apportionment involves the analysis of an existing design configuration to determine the most cost-effective means of adding redundancy. In the apportionment problem, acquisition cost is the traditional cost-effectiveness measure. The apportionment algorithm defined herein expands the definition of cost-effectiveness to include downstream costs, thereby addressing LCC.

A well-behaved, allocation routine is derived to account for any combination of serial, parallel and partially redundant configurations. In addition, a closed-form analytic solution provides the framework for economically adding redundancy to a system's structure in order to achieve a system-level reliability goal. An Apportionment Criterion Ratio (ACR), which contrasts the incremental reliability benefits of adding redundant components with the corresponding incremental LCC, is used.

The Rate of Occurrence of Failure (ROCOF) is the reliability metric used in both the allocation and the apportionment routines. The formulation of the LCC model carefully distinguishes between failures and an allied measurement, demands.

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A special thanks to Mr. Robert Butler, my employer and respected colleague, who took the time to discuss many theoretical issues with me and who generously allowed a great deal of flexibility in my work schedule making the difficult task of completing my thesis a little easier.

The enormous amount of emotional support and encouragement I received from friends and family was greatly welcomed and contributed to the completion of this thesis.

DEDICATION

It is with great pride and honor that I dedicate my thesis to the memory of my father, Richard W. Nowicki. His dedication to my education and continual encouragement in my pursuits provided a strong educational foundation and most importantly, the confidence to actively pursue my goals. His memory continues to provide inspiration to me.

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1.0 INTRODUCTION

The fields of reliability modeling and logistics modeling have produced a body of results only tangentially useful for one another's application. While extensive research has been done to derive many sophisticated equations necessary to calculate different reliability metrics, there is a noticeable deficiency in applying these results in combination with cost measurements to produce useful information for making informed design decisions.

Two specific analytical methods are developed to improve the reliability of an equipment design -- reliability allocation and reliability apportionment. Reliability allocation, generally performed early in the design process, assigns target ROCOF values to lower-indenture level items based on a top-level goal. The top-level item is commonly referred to as the system-level with lower-indenture level items taking on a variety of names: sub-systems, assemblies, units, items or components. The allocation algorithm recursively applies known aggregation routines to assign a ROCOF value to each item down through the system's hierarchy. The allocation algorithm is robust and capable of handling simple to complex structures. The simplest structure is a strictly series configuration with the complexity increasing as different serial, parallel and partially redundant configurations are combined to produce a system structure.

Reliability apportionment ensures that a top-level reliability target is achieved by selectively adding redundancy within a system's configuration. An iterative approach is used to assign redundant components until the top-level reliability target is satisfied. The apportionment or selection decision is based on an Apportionment Criterion Ratio (ACR), defined as the ratio of incremental change in the subsystem's ROCOF to its corresponding change in LCC.

Reliability allocation and apportionment are important applications in reliability engineering whereby a system-level reliability target is achieved. During the conceptual phase of a design, reliability requirements are generally defined only at the system level because little is known about its detailed configuration. Even so, functional requirements are transformed into gross hardware aggregations, commonly referred to as subsystems. While reliability targets are seldom specified at the subsystem level, system-level targets exist. Reliability allocation and apportionment establish guidelines or a set of rules to transform the top-level target into values useful at lower indenture levels. Allocation is the appropriate method if a design is fixed, that is, additional redundant components can not be added. For a design that does not meet a specified system-level reliability requirement, assigning redundant components within subsystems is a means to improve the subsystem's reliability and therefore the reliability of the system. The idea is to determine the most cost effective method of apportioning redundancy within the system.

Far too often, what appear to be good decisions are not at all good when based only on short-term gains such as acquisition costs, or even worse, a collection of subjective measurements. Decisions based on production cost certainly do not reflect the often staggering downstream costs to operate and support the system. Including LCC in the decision process results in solutions that more accurately and comprehensively model the design, including its use and support.

As designs become increasingly more sophisticated the need to concentrate on LCC increases. LCC is a function of a system's propensity to fail and the associated demands generated with each failure. As such, LCC is fundamental to the field of logistics as well as the field of reliability. Many people would argue that reliability is part of logistics. In theory this is true, however, in practice the reliability formulations used in logistics models (e.g., LCC models) are relatively simplistic compared to the

sophistication that currently exists in the reliability field. Reliability allocation and apportionment are natural bridges between complex reliability formulations and LCC modeling.

For example, most system reliability computations used in LCC models have employed the constant failure rates that typify electronic components. The motive for doing so is analytical expediency. When redundancy is introduced, however, a constant failure rate assumption for the system can only be maintained under unrealistic conditions. The results show that a system failure rate increases with time even though each of its components exhibit a constant failure rate. Furthermore, a failure rate measurement has no value in a LCC model.

Ascher [2] postulates that there is a theoretical and practical difference between what it means to describe an item's ROCOF in contrast to the commonly used and often misused failure rate. The failure rate is a conditional probability measuring an item's propensity to fail at an instantaneous point in time given that it has not failed prior to that time. This is useful for examining the reliability characteristics of a design but it can not be transformed into a useful measurement for determining LCC estimates. Instead of examining the propensity of an item to fail, the times between successive failures is the more useful measure. Under perfect renewal, the ROCOF is simply the reciprocal of the average time between successive failures. Perfect renewal assumes that after a failed item is repaired its restored condition is as good as new. In other words, the chance of a second failure after one repaired failure is identical to the item's chance of a first failure given that it has yet to fail. In general, perfect renewal assumes that the times between any two successive failures are identically and independently distributed.

Deriving the ROCOF for a partially redundant system is only a part of the overall problem. For the LCC-based apportionment algorithm, a transformation of the reliability

concept is required to address the underlying problem of measuring the LCC consequences of changes in a design's reliability. Changes in reliability change the number of demands the system generates upon the infrastructure necessary to support the design. Therefore, the demand rate and the ROCOF are essential to a LCC model's accurate portrayal of downstream labor, equipment and spares costs. A suitable simplification used to derive the number of demands generated for a partially redundant structure is obtained here under the following assumptions:

- each component has a constant failure rate,
- the system fails only when $(n-k+1)$ components fail, and
- upon system failure $(n-k+1)$ components are replaced to restore system operation.

Reliability allocation and apportionment can now be performed in the presence of complex redundancy, handling many of the nuances this creates. Significant enhancements are now available with apportionment decisions based on LCC, failures distinguished from demands, and ROCOF distinguished from failure rate.

2.0 LITERATURE SEARCH

Examination of the literature reveals the existence of two related but distinct applications in the reliability allocation / apportionment field. The first application focuses on a fixed-design configuration, setting reliability goals for constituent subsystems. The second application addresses the problem of changing a design, through adding redundant components, to meet a specified system reliability goal.

The literature does not distinguish between these two applications as the terms reliability allocation and reliability apportionment are commonly interchanged as umbrella terminology for both applications. In this thesis, a distinction is made exclusively using reliability allocation to mean the process of assigning reliability goals to the components of a fixed design. In turn, reliability apportionment will mean the process of determining where redundancy should occur in the system in order to achieve a desired top-level reliability target.

Dhillon's survey [6] is an invaluable reference listing all publications relevant to reliability allocation and apportionment from its inception as a concept in 1957 until 1986. The concept of reliability allocation first appeared in the literature in 1957 with the introduction of the AGREE method [19]. In addition to the AGREE technique, Kapur and Lamberson [13] examine three other allocation approaches: the equal allocation technique, the ARINC technique, and a dynamic programming technique. These four approaches do not consider redundant configurations. They focus exclusively on the more simplistic, serial relationships. In fact, all other reliability allocation techniques found in the literature since the 1950s fail to address the complexities induced by the presence of redundancy:

- Graphical Methods [1],
- Geometric Programming Methods [11],
- Database Entropy Method [9],

- Dynamic Programming Methods [3, 16],
- Branch and Bound Methods [17],
- Fuzzy Set Theory Methods [18], and
- LaGrangian Multiplier Method [7, 12].

The allocation technique presented in this thesis not only accommodates serial relationships but also formulates a solution that handles parallel and, the more generic, partially redundant configurations.

Literature on reliability apportionment is sparse with the most notable contribution from Kettelle [14], who basis decisions on budget constraints and Gopal [10], who documents a comparative study of various heuristic redundancy allocation methods. The heuristics use only initial cost values, primarily production cost, in their respective decision policies, and ignore relevant downstream labor, equipment and spares costs. The reliability apportionment algorithm derived in this thesis measures the LCC consequences of incrementally adding a redundant component. This value, in part, is used to determine what and how many redundant components are necessary.

To summarize, this thesis contributes a technique to the reliability allocation literature that accommodates the presence of redundant structures and enhances the apportionment literature by deriving an algorithm based on LCC.

3.0 MODELING BACKGROUND

Extensive modeling is required to develop the decision-making metrics -- what ROCOF to assign to each item for allocation and the mixture of redundancy to add to a system's configuration for apportionment. Before the models are developed, background reliability and LCC concepts are presented. Reliability models are derived to account for any combination of serial, parallel or partially redundant configurations. In turn, these reliability results coupled with a demand metric, which is a function of the reliability, are used to derive a LCC measurement.

3.1 RELIABILITY DERIVATIONS FOR A (K OF N) SYSTEM

This section focuses on deriving reliability metrics in the presence of partially redundant structures. A partially redundant system is defined as a system consisting of n subsystems requiring at least k functional subsystems for the system to operate. Therefore, a system failure occurs when and only when $(n-k+1)$ subsystems fail.

A rocket engine igniter system is an example of a partially redundant system [15]. The rocket engine igniter system is designed to have n igniters connected in a parallel circuit. This design is intended to decrease the probability of failure by allowing ignition even if several igniter switches are "open". In fact, successful ignition will result if any group of k igniters work together.

It is important to recognize that the k of n system is the general case for both a strictly parallel system and a series system. A parallel system is nothing more than a k of n system where k equals one. The reliability, failure rate, MTTF, and MTBF calculations for a parallel system are well documented in Kapur and Lamberson [13]. However, the general k of n system is not described. The following describes the reliability characteristics of the partially redundant (i.e., k of n) system.

3.1.1 The Reliability Function

Reliability is defined as the probability that the system will adequately perform its intended function under stated environmental conditions for a certain time t . For a k of n system to perform its intended function, at least k components must be fully operational. Therefore, it is appropriate to use a binomial distribution to model the reliability function of a k of n system. All n components are identical, so all components have the same reliability function denoted by $R_C(t)$. The k of n reliability equation uses the component reliability, $R_C(t)$, assuming the system needs at least k of these components to operate. The k of n reliability equation is expressed below:

$$R_{k \text{ of } n}(t) = \sum_{j=k}^n \binom{n}{j} \{R_C(t)\}^j \{1 - R_C(t)\}^{n-j} \quad (1)$$

where,

$$\begin{aligned} R_{k \text{ of } n}(t) &= \text{reliability at time } t \text{ of a } k \text{ of } n \text{ system} \\ R_C(t) &= \text{reliability at time } t \text{ of a single component} \end{aligned}$$

The form of this equation is based upon the assumption that the time to failure random variables for the components are independently and identically distributed. Assuming the component's time to failure density functions is exponential, the reliability function can be written as:

$$R_{k \text{ of } n}(t) = \sum_{j=k}^n \binom{n}{j} (e^{-\lambda t})^j (1 - e^{-\lambda t})^{n-j} \quad (2)$$

This derivation will be useful in solving for the MTBF of a k of n system. The resultant MTBF is shown in Equation 3. The steps required to derive the MTBF equation are sequentially shown in Equations 4 through 6.

3.1.2 Expected Life and MTBF

For a k of n system the expected life (MTTF) and the MTBF are equivalent, if one assumes perfect renewal. The final expression for a k of n system's MTBF is shown in Equation 3:

$$MTBF_{k \text{ of } n} = \frac{1}{\lambda} \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (n-i)^{-1} \quad (3)$$

This expression holds true under the following conditions:

- each component has a constant failure rate,
- the system fails only when (n-k+1) components fails, and
- upon system failure (n-k+1) components are replaced to restore the system.

As a result, the system's mean time to first failure, mean time to second failure, mean time to third failure, and so on are identical. In other words, the system is fully restored to operation with each of its components having the same failure rate.

To derive MTBF in a k of n situation, note that by definition:

$$E(T) = \int_0^{\infty} R(t) dt \quad (4)$$

The result proving MTBF and MTTF equivalency for a k of n system under the previously stated condition is used as Equation 2 and is substituted into Equation 4 to yield:

$$MTBF = \int_0^{\infty} \sum_{j=k}^n \binom{n}{j} (e^{-\lambda t})^j (1 - e^{-\lambda t})^{n-j} dt \quad (5)$$

The binomial series expansion is used as an aid to solve the integral in Equation 5. The general form of the binomial expansion series is shown below:

$$(1 + e^{-\lambda t})^m = \sum_{i=0}^m \binom{m}{i} (-e^{-\lambda t})^{m-i} \quad (6)$$

The binomial series expansion is then used to solve the integral in Equation 5, as shown below:

$$\begin{aligned}
\text{MTBF} &= \int_0^{\infty} \sum_{j=k}^n \binom{n}{j} (e^{-\lambda t})^j \sum_{i=0}^{n-j} \binom{n-j}{i} (-1)^{n-j-i} (e^{-\lambda t})^{n-j-i} dt \quad (7) \\
&= \int_0^{\infty} \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (e^{-\lambda t})^{n-i} dt \\
&= \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} \int_0^{\infty} (e^{-\lambda t})^{n-i} dt \\
&= \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} \int_0^{\infty} e^{-\lambda(n-i)t} dt \\
&= \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} \left(-\frac{1}{\lambda(n-i)} \right) e^{-\lambda(n-i)t} \Big|_0^{\infty} \\
&= \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} \left(\frac{1}{\lambda(n-i)} \right) \\
&= \frac{1}{\lambda} \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (n-i)^{-1}
\end{aligned}$$

This result shows a constant MTBF for a partially redundant system because this is the mean for a renewal process. However, the system hazard rate increases with time.

3.1.3 The Failure Rate and Hazard Rate

Recall that the hazard rate is the instantaneous rate of failure. That is, it is the failure rate during the very small time interval $[t, t + \Delta t]$. In other words, the hazard rate is the probability of system failure during the time interval $t + \Delta t$ given that the system did not fail prior to time t . By definition the hazard rate is:

$$z(t) = \lim_{\Delta t \rightarrow 0} \frac{R(t) - R(t + \Delta t)}{\Delta t \cdot R(t)} = \frac{1}{R(t)} \left[-\frac{d}{dt} R(t) \right] = \frac{f(t)}{R(t)} \quad (8)$$

Using the reliability function described in Equation 2, the k of n hazard rate is as follows:

$$\begin{aligned}
z_{k \text{ of } n}(t) &= \frac{-\frac{d}{dt} R_{k \text{ of } n}(t)}{R_{k \text{ of } n}(t)} \\
&= \frac{\lambda \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (i-n) (-1)^{n-j-i} e^{-\lambda(n-i)t}}{\sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} e^{-\lambda(n-i)t}} \quad (9)
\end{aligned}$$

The k of n hazard rate describes the propensity of a system to fail with time. Specifically, if a system has not failed until now, the hazard function is the likelihood it will fail during a subsequent small interval of time? Unlike its individual components, this chance increases with time. For LCC modeling the most useful measure is the aggregate MTBF, that is, the mean of the renewal process. More specifically, the ROCOF is used. Assuming constant hazard rates for individual components and perfect renewal, the ROCOF can be written as the inverse of the MTBF:

$$\text{ROCOF}_{k \text{ of } n} = \frac{\lambda}{\sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (n-i)} \quad (10)$$

Using this transformation of MTBF the number of failures in any given time period can be derived. The time period must be measured in equipment operating hours and not elapsed calendar time. For example, if a system operates 10 hours per day, 5 days per week with a ROCOF of 1 failure per hour, on average 50 failures will occur per week.

When modeling LCC in the presence of redundancy, it not enough just to calculate the number of failures during a given time period. The number of demands created during this period is, at least, equally important.

3.1.4 Rate of Demand

Current LCC models tend to be careless in defining demands and failures, failing to make a distinction. The prevailing reliability assumption embedded in LCC models is that

all relationships are serial. As such, it is not necessary to make a distinction between failures and demands. However, in the presence of redundancy a significant difference exists.

In order to formulate a robust LCC model, a distinction is made between failures incurred and demands generated as a system is operated over its duration of intended use. For a serial system, there is no difference between the ROCOF and the rate of demand on the system's support infrastructure. For every failure there is a single demand on the support infrastructure, that is, a one-to-one relationship exists.

This simple one-to-one relationship disappears in a partially redundant system where $(n-k)$ is greater than zero. A system failure requires more than one component to fail, in fact a system failure occurs when $(n-k+1)$ components fail. This relationship is describe below.

$$\text{Demand rate } k \text{ of } n = \text{ROCOF}_{k \text{ of } n} \cdot (n-k+1) \quad (11)$$

In general, the number of demands effect labor, spares and facilities costs while the number of failures effect operational availability and support equipment costs. These relationships will be discussed in more detail when the LCC model is defined..

3.2 LIFE-CYCLE COST MODEL

The logistics community's continual struggle to convince designers around the world of the enormous benefits obtained from concentrating on a system's LCC cost, rather than adhering to the antiquated design-to-cost paradigm, is finally paying dividends. Even with these important strides in quantifying the economic consequences of design changes, additional work is needed to show how the ROCOF and demand rates impact the major life-cycle cost categories such as labor, spares, and support equipment. The LCC model presented in this section recognizes the important distinction between failures and

demands. System failures and component demands are the foundation of this simple LCC model constructed using concepts borrowed from an academic LCC model found in Blanchard and Fabrycky's *Systems Engineering and Analysis* textbook [4], Fabrycky and Blanchard's *Life-Cycle Cost and Economic Analysis* textbook [8] and from the commercially distributed Equipment Designer's Cost Analysis System EDCAS [5].

A Lotus 123 spreadsheet was developed incorporating the LCC model where nine (9) system-level and twelve (12) input parameters are required. The input variables are listed in Table 1 with a brief definition following each parameter. For a more complete description of input parameters refer to Appendix A.

Table 1. Listing of LCC's Input Parameters

System-Level Parameters	
Q	number of operating systems
t	yearly operating hours for an individual system
d	discount rate represented as a percentage
LC	effective operating life of an individual system
WHW	yearly hours a local repair technician is available to perform repairs
BNI	yearly cost of a local repair technician
dtc	daily training cost
tor	turnover rate for local repair technicians
mr	maintenance rate for all support equipment
Component-Level Parameters	
λ	failure rate (failures per million hours of operation)
n	components in the system
k	components required to operate for system to operate
UC	unit cost at a given lot size, LOT
LOT	lot size corresponding to the components UC
RR	reduction rate for producing this component
cond	condemnation rate
disp	disposal cost
mttr	mean time to repair
rmc	repair material cost
thc	classroom hours to train a repair technician
ste	cost for the different types of required support equipment

The objective of introducing the LCC model is to use it as part of a reliability apportionment routine. Therefore, all costs that are unaffected by the addition of a redundant component such as documentation costs and development costs are not included in the LCC model. Pertinent costs are those directly related to changes in the rate at which failures and demands occur when redundant components are introduced. In addition to streamlining the cost variables, the following simplifying assumptions are made:

- all systems are located in a single geographical location,
- all repairs are performed locally, thereby eliminating transportation costs,
- only two-indenture systems are analyzed -- the system level and its next lower indenture referred to as subsystems or components,
- sparing is done according to a 95% stockout criterion,
- spares are stocked once a year,
- all scheduled maintenance can be performed during scheduled system downtime and is therefore not included,
- redundancy does not exist at the system level, and
- constant repair times.

The pertinent cost categories for the LCC model are:

- Production Cost
- Spares Cost
- Manpower Cost
- Training Cost
- Repair Cost
- Support Equipment Cost

Preliminary calculations are required before the above aggregation values can be obtained. There are three basic preliminary calculations: subsystem failures per year, component demands per year and the stock quantity required to ensure against a 95% stockout criterion.

The quantity of subsystem failures per year (i.e., occurrences of $n - k + 1$ component failures) directly uses the general ROCOF results previously derived (refer to Equation 10).

$$\text{fails}_i = t \cdot \text{ROCOF}_{(k \text{ of } n)_i} \quad (12)$$

where,

fails_i = number of times $(n-k+1)$ i^{th} components fail per year

$\text{ROCOF}_{(k \text{ of } n)_i}$ = rate of occurrence of failure for the $(k \text{ of } n)$ group of the i^{th} components

Following the maintenance policy of waiting until $(n - k + 1)$ component failures occur before a repair action is generated, demands per year is the product of yearly failures and the number of demands generated for each failure $(n - k + 1)$.

$$\text{demands}_i = (n - k + 1) \cdot \text{fails}_i \quad (13)$$

where,

demands_i = number of i^{th} component demands per year

Determining the quantity of spares is a fairly elaborate procedure. The general idea is to calculate the number of spares in stock ensuring that there is a 95% chance a spare is available when needed. Let the variable x be defined as the number of spares stocked to meet the 95% criterion. The number of spares required is directly proportional to the number of component failures. Recall, component failures do not necessary cause system failures if redundancy exists. Therefore, let y denote the number of failures that can occur before the redundant components have failed and the components in stock are depleted. Since $(n - k)$ failures can be absorbed through redundancy, y can be expressed as equaling $(n - k + x)$.

The density function of y (number of failures in a time period) is assumed to be a Poisson random variable. With the stocking period set at one year, y can be expressed as follows:

$$f(y) = (n\lambda_i t)^y e^{-n\lambda_i t} / y! \quad (14)$$

To ensure against a stockout, the following expression must be solved in terms of y :

$$P(Y=y) \geq 0.95 \Rightarrow \sum_{y=n-k}^{n-k+x} (n\lambda_i t)^y e^{-n\lambda_i t} / y! \geq 0.95 \quad (15)$$

From this point forward, the quantity of spares is denoted as $sqty$ instead of x . With the spare quantity result, additional preliminary calculations can be performed: the number of components produced in the first year (tnp), the adjusted unit cost (auc) and the discounted life-cycle (dlc) factor.

The total number of components produced in the first year is a necessary calculation to determine the effects increased production lot sizes (i.e., learning) have on a component's effective unit cost. It is assumed that the spares produced during the first year have a positive benefit on the average unit cost.

$$tnp_i = sqty + n_i \cdot Q \quad (16)$$

The average unit cost is a function of a component's unit cost, the lot size at which the unit cost is observed and the reduction rate corresponding to doubling production lot sizes.

$$auc_i = UC_i \cdot \left(\frac{tnp_i}{LOT} \right)^{\ln(RR)/\ln(2)} \quad (17)$$

The discounted life-cycle factor is used to determine the relative impact of all downstream costs. Future cost are relatively uncertain compared to present-day costs with uncertainty increasing with time. As such, these future cost streams (e.g., manpower costs, support equipment maintenance, recurring training and replacement spares) are discounted according to the following formula:

$$dlc = \sum_{j=1}^{LC} (1 + d)^j \quad (18)$$

With the preliminary calculations complete, each major cost category's mathematical derivation is now described. The aggregating LCC measure is strictly the summation of each major cost category:

$$LCC_i = \sum_{j=1}^6 C_{i,j} \quad (19)$$

- j = 1 = production cost
- 2 = spares cost
- 3 = manpower cost
- 4 = training cost
- 5 = repair material cost
- 6 = support and test equipment cost

The production cost is a linear relationship between a component's average unit cost and the number of items in the systems purchased. This relationship is shown below:

$$C_{i,1} = auc_i \cdot n_i \cdot Q \quad (20)$$

Spares costs are comprised of two costs: the cost to purchase an initial stock and the cost to maintain that stock. The initial stock is determined based on meeting the stockout criterion. The recurring cost is a discounted cost stream dependent on repair demands generated and how often a component has to be condemned. A condemnation exists when a component is normally repaired, but damage is so severe that the component is thrown away. The components of the spares cost and their relationships are expressed below:

$$C_{i,2} = auc_i \cdot sqty_i + cond_i \cdot (auc_i + disp_i) \cdot dlc \cdot demands_i \quad (21)$$

The manpower cost is a function of the number of demands generated, the time to repair a demand, the cost of labor and the availability of a single repair technician. The BNI / WHW ratio represents the hourly labor cost of actual work delivered (i.e., performing a repair) in the manpower cost equation:

$$C_{i,3} = dlc \cdot demands_i \cdot mtrr_i \cdot BNI / WHW \quad (22)$$

Training cost considers the number of maintenance personnel initially needing training as well as the additional personnel requiring training in the future as the labor force turns over. The initial training requirements are a function of demands generated, time to accommodate a demand and a technicians availability to do the work. People are indivisible resources. Fractional people do not exist. As such, the number of people requiring training is represented by the ceiling function ($\lceil demands_i \cdot mtrr_i / WHW \rceil$) contained in the training cost equation below:

$$C_{i,4} = thc \cdot (drc/6 + BNI/1300) \cdot \lceil demands_i \cdot mtrr_i / WHW \rceil \cdot (1 + tor \cdot dlc) \quad (23)$$

Repair material cost is a discounted stream of recurring costs directly proportional to the number of yearly demands generated on a system's support infrastructure. The recurring cost for repair material is shown below:

$$C_{i,5} = dlc \cdot rmc_i \cdot demands_i \quad (24)$$

Support equipment calculations assume a suite of support equipment is required for each maintenance technician to adequately perform repairs. Therefore, initial support equipment cost is a function of number of maintenance technicians needed, the cost for a suite of support and test equipment. Maintaining properly functioning support equipment is a recurring cost represented by an annual maintenance rate discounted over the life of the system. The expression for the initial and recurring support equipment cost is as follows:

$$C_{i,6} = ste_i \cdot \lceil demands_i \cdot mtrr_i / WHW \rceil \cdot (1 + mr \cdot dlc) \quad (25)$$

These six cost categories comprise the major components of the LCC model used in the reliability apportionment algorithm.

3.3 ALLOCATION ROUTINE

The reliability allocation algorithm analyzes parent-child relationships as it sequentially assigns ROCOF values down a hierarchic tree. Looking at an item in a hierarchic structure, a child is any attached, next-lower indenture level item. Correspondingly, a parent is any next-higher indenture level item.

The allocation method depends on the aggregation algorithm applied in determining a parent's ROCOF based on all its children's values. In fact, the allocation routines are simply the reverse of the complementing aggregation routines. The objective of the allocation routine is to determine equivalent values for each child based its parent's value. Therefore, the goal of the allocation method is to recursively allocate target values equivalently throughout the tree structure. The general form of the aggregation algorithm is shown below:

$$\text{PartFail}_p = \sum_{c=0}^{\text{Type}(\text{child})} \text{ItemFail}_c \quad (26)$$

where,

PartValue_p: part value of parent
 ItemValue_c: item value of the cth child
 Type(child): types of children attached to the parent

The equation for the child's item failure rate differs depending on the presence of redundancy:

$$\text{ItemFail}_c = \begin{cases} n_{c,p} \cdot \text{PartFail}_c & \text{if serial} \\ \text{PartFail}_c \div \text{SumKof } N_{c,p} & \text{if redundant} \end{cases} \quad (27)$$

where,

PartFail_c: Part failure rate of the cth child
 n_{c,p}: Appearances of the cth child in the parent
 SumKof N_{c,p}: An internal calculation reflecting the combinatorics used to calculate the item failure rate as a function of its component's failure rate

The relationship between the item failure rate of a redundant child and the individual part failure rate is expressed below, derived from Equation 3.

$$\text{ItemFail}_c = \frac{\text{PartFail}_c}{\sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (n-i)^{-1}} \quad (28)$$

Equation 28 is simplified by defining the variable SumKofN:

$$\text{SumKofN}_{c,p} = \sum_{j=k}^n \sum_{i=0}^{n-j} \binom{n}{j} \binom{n-j}{i} (-1)^{n-j-i} (n-i)^{-1} \quad (29)$$

By substituting Equation 29 into Equation 28 the item failure rate for a partially redundant item is expressed in terms of its part failure rate.

$$\text{ItemFail}_c = \frac{\text{PartFail}_c}{\text{SumKofN}_{c,p}} \quad (30)$$

Describing the item failure rate of a group of children in terms of their part failure rate is an important step in determining the equivalent failure rate allocated to each child. The part failure rate of the parent is allocated according to Equation 26. Item failure rates can be derived from Equation 27. The next step is to set all the part failure rates of each child equal and solve for this equivalent failure rate.

To derive the equivalent failure rate, it is important to distinguish between items that are in series and items that contain redundancy. Therefore two variables are introduced: the number children in series (ncs) and number of children in redundant arrangements (ncr). In both cases, number refers to the number of part types.

Equations 31 through 33 display the equivalent part failure rate allocated to each child (i.e., FailAlloc). First the parent part value is expressed in terms of its serial and redundant children. Then FailAlloc is computed by setting the failure rate for each child equal and solving for FailAlloc.

$$\text{PartFail}_p = \sum_{c=0}^{nc_s} n_{c,p} \cdot \text{PartFail}_c + \sum_{c=0}^{nc_p} \frac{\text{PartFail}_c}{\text{SumKofN}_{c,p}} \quad (31)$$

$$\text{PartFail}_p = \text{PartFail}_c \cdot \left(\sum_{c=0}^{nc_s} n_{c,p} + \sum_{c=0}^{nc_p} \frac{1}{\text{SumKofN}_{c,p}} \right) \quad (32)$$

$$\text{FailAlloc} = \frac{\text{PartFail}_p}{\left(\sum_{c=0}^{nc_s} n_{c,p} + \sum_{c=0}^{nc_p} \frac{1}{\text{SumKofN}_{c,p}} \right)} \quad (33)$$

where,

- nc_s: types of children in series.
- nc_r: types of children in redundant structure
- FailAlloc: equivalent part failure rate allocated to each child of a given parent

The FailAlloc values are the foundation of the allocation routine. An example is presented to demonstrate how the allocation routine may be used in practice.

3.4 APPORTIONMENT ALGORITHM

A generalized method is presented that assigns redundant components to a system based on LCC. An Apportionment Criterion Ratio (ACR) is used to determine what and how many components are added to the system to meet a system-level reliability target. The apportionment algorithm is iterative in nature concluding when the system-level reliability goal is achieved.

Apportionment provides the greatest utility to a design if it is performed early in a design's life. At this time, ROCOF and other values necessary to formulate a LCC estimate are scarce, existing only in the face of uncertainty. As such, it is important to see the relative effects changes in these uncertain values have on apportionment decisions. Sensitivity analysis is easily accommodated by varying the values of the model's parameters and examining the effects on any apportionment decision. This is a necessary

step before any apportionment is complete. The LOTUS 123 model makes this function must easier to perform, automating the calculations and displaying the results.

Apportionment decisions are potentially swayed by the uncertainties in the various LCC parameters. It has been demonstrated that there exists a definitive relationship between most facets of a system's cost stream and the accuracy of an apportionment decision. Therefore, it is extremely important to consider both downstream as well as initial costs when making any apportionment decision.

The ACR is defined as the ratio of the change in the subsystem's ROCOF and its corresponding change in LCC. The ACR to LCC ratio is expressed below:

$$ACR = \frac{|\Delta ROCOF_{k \text{ of } n}|}{\Delta LCC} \quad (34)$$

As redundant components are added the $\Delta ROCOF_{k \text{ of } n}$ will always decrease and the ΔLCC will always increase. Given the complex interrelationship between the many LCC variables it is possible for the ΔLCC to decrease when redundant components are added, however, this possibility is eliminated by assuming there is only one repair policy, local repair. It is entirely possible that resources are more effectively utilized at one repair location versus another as failures and demands change with the introduction of redundancy. Given that this is not considered, the ΔLCC will always increase.

A good redundancy choice is one in which there is a large decrease in $ROCOF_{k \text{ of } n}$ with a relatively small increase in LCC. The ratio of these two variables can be difficult to interpret, so to reduce potential confusion the absolute value of the $\Delta ROCOF_{k \text{ of } n}$ is used. Therefore, a good redundancy choice is one in which the ACR is large.

The apportionment algorithm examines each subsystem's ACR with the potential addition of a single redundant component. The subsystem with the largest ACR is

assigned a redundant component. This process continues until the system-level ROCOF target is achieved. One exception does exist as the apportionment process is nearing completion. As the system's ROCOF approaches the target value, the incremental amount needed to satisfy the target may come from subsystem whose ACR is not the largest. For example, consider subsystem A and subsystem B which are competing for a redundant component. Assume the information in Table 2 is known:

Table 2. ACR Example

Subsystem	$ \Delta\text{ROCOF}_{k \text{ of } n} $	ΔLCC	ACR
A	10	\$100	0.10
B	50	200	0.25

Now suppose the system's target value is 500 fpmh and the current value is 510. According to the rule of choosing the subsystem with the largest ACR subsystem B would prevail, lowering the system value by 50 to 460 fpmh at a LCC of \$200. Upon further inspection, adding a redundant component to subsystem A achieves the system-level target value for only a \$100 LCC. This exception is defined as the closure criterion.

4.0 NUMERICAL EXAMPLES

A hypothetical example is presented to show how the allocation process is performed when redundancy exists throughout a system's structure. Another example is presented to demonstrate how LCC measures are used to decide what and how many items are added to a system's design to satisfy a system-level reliability goal.

4.1 ALLOCATION EXAMPLE

This example uses the Indenture Parts List (IPL) displayed in the Table 3. Part B has a 3 of 4 partially redundant structure and LRU 2 has a 2 of 3 partially redundant configuration.

The objective of this exercise is to allocate an equipment failure rate target of 1000 failures per million hours of operation (fpmh) down the branches of the hardware tree structure. As before, the allocation technique sequentially allocates the failure rate of a parent to each of its children until all items have been allocated a failure rate. The results of allocating a 1000 fpmh equipment-level target is shown in Table 3.

Table 3. Indentured Parts List with Allocated Values

Component	K	N	Part	Item
Eqmt	1	1	1000.0	1000.0
LRU1	1	1	454.5	454.5
SRU1	2	2	151.5	303.0
SRU2	1	1	151.5	151.5
Part A	1	1	55.8	55.8
Part B	3	4	55.8	95.7
LRU2	2	3	454.5	545.5

The sequential steps used to derive the results are described below.

STEP 1. Allocate the Equipment Target to the Two LRUs

The equivalent part failure rate of 454.5 (FailAlloc) assigned to each LRU is derived from Equations 31-33 and appears in the Part column.

STEP 2. Allocate the LRU part failure rates to the attached SRUs

For this step the parent failure rate is now the part failure rate derived in Step 1 (i.e., FailAlloc). Step 2 derives the FailAlloc for LRU1's attached SRUs. LRU1's attached SRUs all have serial arrangements. Therefore, only the first part of Equation 31 is used to derive the allocated failure rate. Using the values for this example the part failure rates for both SRU's are calculated as 151.5 fpmh.

STEP 3. Allocate the SRU part failure rate to the attached parts.

For this step the parent failure rate is now the equivalent part failure rate derived in Step 2 (i.e., FailAlloc). Step 3 derives the FailAlloc for SRU2's attached parts, employing the same algorithmic sequence as Step 1 to derive an equivalent part failure rate (FailAlloc) for Part A and Part B. SRU2 has components with both serial and redundant arrangements. As in Step 1, the aggregation algorithms for both arrangements are used to calculate FailAlloc (Equations 31-33). The result, 55.8 fpmh, is also recorded in the Part column.

The allocation process is complete now that all items have an allocated part failure rate. Refer to Table 3 for the part and item failure rates for each item in the hierarchical breakdown structure.

4.2 APPORTIONMENT EXAMPLE

The example under analysis is a system containing three subsystems generically labeled subsystem 1, subsystem 2 and subsystem 3. Subsystem 1 contains three identical components arranged in series. Subsystem 2 has only one component and subsystem 3 has 2 identical components in a series configuration. As such, the baseline configuration does not have any redundant components. The target system-level ROCOF is 900 fpmh. With the initial component (i) and corresponding subsystem (k of n) ROCOF values, the

baseline configuration has a 1550 ROCOF (refer to Table 4). Since this clearly exceeds the target value the design needs to be changed by introducing redundancy.

Table 4. Initial Configuration with ROCOF Values

	k	n	ROCOF _i	ROCOF _k of n
System	1	1	1550	1550
Subsystem 1	3	3	250	750
Subsystem 2	1	1	200	200
Subsystem 3	2	2	300	600

The first step is to use these ROCOF values to determine each subsystem's LCC. Again, a LOTUS 123 is used to assist in the LCC calculations. Refer to Equations 12 through 25 for the mathematical derivation of the LCC model. In addition to ROCOF, n and k values already defined, the LCC model requires nine additional component-level variables and nine system-level variables shown in Table 5.

Table 5. Additional LCC Input Parameters

System-Level Parameters									
Q	=	5 systems	BNI	=	\$35,000				
t	=	1000 hours	dtc	=	\$250/day				
d	=	10 %	tor	=	20 %				
LC	=	10 years	mr	=	15 %				
		WHW = 1500 hours							
Component-Level Parameters									
<u>Subsystem</u>	<u>UC</u>	<u>LOT</u>	<u>RR</u>	<u>cond</u>	<u>disp</u>	<u>mttr</u>	<u>rnc</u>	<u>thc</u>	<u>ste</u>
1	500	5	0.9	0.1	100	3	50	8	10,000
2	600	5	0.9	0.1	100	1	300	5	5,000
3	300	5	0.9	0.1	10	5	50	10	20,000

With the aid of the LOTUS 123 spreadsheet, the LCC values for each subsystem are calculated. Table 6 shows the initial values for both the ROCOF and LCC for each subsystem and the aggregate system values.

Table 6. Initial ROCOF and LCC Values

Subsystem	k	n	ROCOF _{k of n}	LCC
1	3	3	750	\$ 28,287.50
2	1	1	200	14,202.90
3	2	2	600	43,862.10
$ROCOF_{sys} = 1550 \text{ fpmh} > ROCOF_{target}$				
$LCC_{sys} = \$86,352.50$				

Since the $ROCOF_{sys}$ value exceeds the target value, redundant components must be added. ROCOF and LCC values are derived assuming that a redundant component is added (i.e., $n = n + 1$). After these values are calculated, their corresponding increments are analyzed to determine the most cost effective manner to add a redundant component. Table 7 shows the results of the changing LCC and ROCOF values. For additional insight into the LCC formulation the underlying calculations comprising subsystem 1's LCC derivation is illustrated in Appendix B. The row in Table 7 whose characters are in bold print represents the most desirable place to add a redundant component. For each iteration there is a subsystem whose values are in bold. This signifies which component to add to the system for that particular iteration.

Table 7. First Iteration

Subsystem	k	n	$ \Delta ROCOF_{k of n} $	ΔLCC	ACR
1	3	4	321.4	\$ 2,165.20	0.1484
2	1	2	66.7	2,504.90	0.0266
3	2	3	240.0	1,226.50	0.1957
$ROCOF_{sys} = 1310 \text{ fpmh} > ROCOF_{target}$					
$LCC_{sys} = \$87,352.50$					

The incremental benefit providing the greatest decrease in ROCOF for the smallest amount of LCC is subsystem 3. As such, a redundant component is added to subsystem 3 resulting in a system configuration containing three subsystem 1 components in series, a subsystem 2 component and three subsystem 3 components of which only two need to operate for the system to function. The system-level ROCOF target is still not achieved. Therefore another iteration of the apportionment algorithm is necessary. Subsystem 2 and 3 analyses remain the same and another redundant component is added to subsystem 3 for analysis.

Table 8. Second Iteration

Subsystem	k	n	$ \Delta\text{ROCOF}_{\text{kofn}} $	ΔLCC	ACR
1	3	4	321.4	\$ 2,165.20	0.1484
2	1	2	66.7	2,504.90	0.0266
3	2	4	83.1	1,368.60	0.0607
$\text{ROCOF}_{\text{sys}} = 988.6 \text{ fpmh} > \text{ROCOF}_{\text{target}}$					
$\text{LCC}_{\text{sys}} = \$89,744.20$					

A redundant component is added to subsystem 1's configuration. Another iteration is warranted since the resulting system ROCOF still exceeds the target value.

Table 9 Final Iteration and Closure Step

Subsystem	k	n	$ \Delta\text{ROCOF}_{\text{kofn}} $	ΔLCC	ACR
1	3	5	109.5	\$ 1,752.80	0.0625
2	1	2	66.7	2,504.90	0.0266
3	2	4	83.1	1,368.60	0.0607
$\text{ROCOF}_{\text{sys}} = 879.1 \text{ fpmh} < \text{ROCOF}_{\text{target}}$					
$\text{LCC}_{\text{sys}} = \$91,497.00$					

With the addition of a second redundant component assigned to subsystem 1's configuration the apportionment is complete. The final result is a system configuration failing 879.1 times per every one million hours of operation with an expected LCC of \$91,497 over a ten-year period.

The closure step would have changed if the ROCOF target value is set to 910 instead of 900. Examination of the last iteration reveals that adding a redundant component to subsystem 3 decreases the system's ROCOF to 905.5 fpmh, thus achieving the desired ROCOF objective at a cost of \$1,368.60. However, the ACR suggests an addition to subsystem 1. Herein lies the reason for the closure criterion. Either one additional component added to subsystem 1 or to subsystem 2 will meet the objective. Under these circumstance it is not necessary to use the ACR decision factor. The change in LCC is sufficient. The ACR represents the incremental utility of redundancy while simultaneously looking at costs. With the closure criterion, the incremental utility is the same, either addition satisfies the target. The question is, at what cost?

5.0 CONCLUSION

This research will likely alleviate many of the barriers preventing the adequate treatment of redundancy in reliability allocation and reliability apportionment. Apportionment can now be performed based on LCC instead of short-sighted production cost or worse yet, non-analytical measures subjectively obtained.

Solutions are now available to the allocation problem in the presence of redundancy. The closed-form allocation routine developed in this thesis is capable of handling any combination of items in series, parallel and partially redundant configurations.

The apportionment algorithm developed herein is robust and assigns redundancy based on LCC. It is believed LCC is a better measure of a design's utility than others used in previous apportionment routines (e.g., production cost, subjective weighting schemes, etc.). The LCC model embedded in the apportionment algorithm carefully distinguishes between system failures and item demands on the system's support infrastructure.

Allocating in the presence of redundancy, and apportioning with regard to LCC, are enhancements increasing the benefits of reliability allocation and apportionment. Designs may now be modeled to include their downstream cost consequences, thereby more accurately portraying the overall system cost. The result should lead to better design decisions. The benefits increase exponentially the earlier they are made. The reliability techniques derived in this thesis provide a means to make decisions early in the design process, thus having a beneficial influence on a design.

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APPENDIX A: DATA ELEMENT DICTIONARY

System-Level Parameters:

Q	Number of systems located at the single geographical operating site.
t	Yearly operating hours of an individual system.
d	Discount rate, a relative measure of the uncertainty of downstream costs.
LC	Number of years for which costs will be computed.
WHW	Number of hours per year maintenance personnel actually spend on direct maintenance work of the quality measured by mttr.
BNI	The undiscounted annual position cost of repair technician used to perform all repair activities on each component.
dtc	The average per student, daily training cost including instructor labor costs, space rent, materials and the consumption of goods and services.
tor	Fraction of repair technicians who change jobs over the course of one year. Includes such factors as reassignment, rotation and departure from the job.
mr	Maintenance rate measured as the fraction of support equipment's initial procurement cost.

Component-Level Parameters:

n_i	Number of i^{th} components in the system.
k_i	Number of i^{th} components required to operate to ensure that the system will function.
ROCOF_i	The rate at which an individual i^{th} component fails, measured in failures per million hours of operation.
$\text{ROCOF}_{(\text{kofn})_i}$	The rate at which a collection of n identical, partially redundant i^{th} component fail, measured in failures per million hours of operation. It is assumed that the system will operate until $(n-k+1)$ failures of the i^{th} component fail components occur resulting in $(n-k+1)$ demands are generated on the support infrastructure.
UC_i	The recurring cost to produce the i^{th} component, costs include materials, labor, assembly and test.
LOT_i	The lot size corresponding to the UC value. The model adjusts the UC to reflect the effects of learning.
RR_i	The reduction rate, often referred to as the learning curve slope, for an average unit cost learning curve. It is the reduction in

	unit cost which occurs when the production lot size for the i^{th} component doubles.
cond_i	The proportion of i^{th} component failures that ultimately must be thrown away, referred to as the condemnation rate.
disp_i	The cost to dispose of a failed i^{th} component given that the component can not be repaired.
mtr_i	Mean time to repair a failed i^{th} component including man-hours required to make-ready, gain access, fault isolate, remove and replace, test close, record data and put away.
rnc_i	Repair material cost incurred each time a repair action is incurred on the i^{th} component, includes all consumable materials.
thc_i	Number of special classroom training required before a maintenance technician is certified to perform repairs on the i^{th} component.
ste_i	Cost of any support and test equipment specifically intended for use in the repair of the i^{th} component.

APPENDIX B: SAMPLE LCC RESULTS

Preliminary Calculations:

Fails	= 0.4286	(refer to Equation 12)
Demands	= 0.8572	(refer to Equation 13)
tnp	= 23	(refer to Equation 16)
auc	= 396.5	(refer to Equation 17)
dlc	= 6.145	(refer to Equation 18)
sqty	= 3	(refer to Equation 15)

Top-Level Cost Categories:

Production Cost	= \$ 7,929.74	(refer to Equation 20)
Spares Cost	= \$ 1,450.97	(refer to Equation 21)
Labor Cost	= \$ 368.70	(refer to Equation 22)
Training Cost	= \$ 1,223.04	(refer to Equation 23)
Repair Cost	= \$ 263.36	(refer to Equation 24)
Support Equipment Cost	= \$19,216.85	(refer to Equation 25)
Life-Cycle Cost	= \$ 30,452.65	(refer to Equation 19)

Vita

Mr. David R. Nowicki is a senior analyst for Systems Exchange. He is Currently responsible for the theoretical development of EDCAS extensions (time-variant demands, life-cycle cost extensions and reliability modeling). David is also responsible for developing the analytical framework for ORCAS, an ordnance life-cycle model. He is a member of the Tools for Design (TFD) development team. In this capacity David has presented papers and participated in panel discussions at various symposia around the world. Before coming to Systems Exchange he held research positions in logistics, reliability and cost issues for the SDI, BSY-1 sonar and RAMCAD programs. Mr. Nowicki holds a B.S. from the University of Wisconsin and a M.S. degree from Virginia Tech.

A handwritten signature in black ink, appearing to read 'D. Nowicki', with a long horizontal stroke extending to the right.