Laser Scanning Imaging for Increased Depth-Of-Focus

by

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(ABSTRACT)

Throughout the decades, different techniques have been proposed to improve the depth-of-focus in optical microscopy. Common techniques like optical sectioning microscopy and scanning confocal microscopy have innate problems. By simply modifying the pupil function in microscope imaging system, we can also extend the depth-of-focus. The scanning system with a thin annular pupil has a high depth-of-focus and can scan the whole object, but the output light is too dim to be detected well by a photodetector.

In this thesis, we propose a scanning technique employing an optical heterodyne scanning system using a difference-of-Gaussians (DoG) pupil. The object is illuminated by the combined beam which consists of two Gaussian beams with different waists, frequencies, and amplitudes. This system does not block most light like the annular pupil system and can obtain high depth-of-focus. The main objective of the thesis is to extend the depth-of-focus using the proposed system. The depth-of-focus characteristics of the DoG pupil function are examined and compared with those of well-known functions such as the circular, annular, and Gaussian pupils.
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Chapter 1
Introduction

1.1 Motivation

In microscope imaging, it is difficult to accomplish both lateral and axial resolution simultaneously. Different techniques have been proposed over the years to improve the depth-of-focus with high lateral resolution [1,2,3]. Two common microscope imaging techniques are optical sectioning microscopy [4] and scanning confocal microscopy [5,6]. In optical sectioning microscopy, a series of 2-D images at various focal planes throughout the 3-D specimen is recorded to collect 3-D information. Since each 2-D image contains both in-focus and out-of-focus information, it is required to extract the in-focus information from the 2-D image. The difficulty of this technique is the alignment of each 2-D images; during the reconstruction, it is required that exact focal spacing between adjacent 2-D images is accurately controlled and the 2-D images are precisely aligned. In scanning confocal microscopy, an focused objective lens and a small pinhole are used to image only a single point throughout the 3-D specimen. 3-D information is obtained by 3-D scanning in this technique. It is time consuming and the instrumental tolerances required to achieve high-resolution imaging are difficult to obtain.

Another way to extend the depth-of-focus is to use a pupil function. The imaging system with an annular pupil has a large depth-of-focus, but most of the light is blocked and the image shows poor contrast and high noise. In this thesis, we propose a scanning technique that employs an optical heterodyne scanning system using a difference-of-Gaussians (DoG) pupil that has been used in a real-time optical image system to generate difference-of-Gaussians wavelets [7]. In the proposed system, no light is blocked and a single 2-D scan achieves inspection of a 3-D
In the implementation of the optical heterodyne scanning system, two Gaussian beams with different temporal frequencies are generated by two acousto-optic modulators and are then combined by a beamsplitter. As the combined beam illuminates the object, the scattered or reflected light is collected by a photodetector, resulting in the electrical output signal. The system has several parameters that can change the depth-of-focus performance. Our discussion is focused on the data from simulations, and it also includes other related techniques, such as 2-D optical Fourier transform and numerical analysis.

1.2 Overview of material

The rest of this thesis is organized as follows. Chapter 2 presents the basic mathematics and the background of some concepts used in this thesis. It defines four pupil functions: circular, annular, Gaussian, and DoG. The chapter also defines the depth-of-focus quantitatively. Chapter 3 presents the analysis of each pupil function. It shows the optical implementation of an optical heterodyne scanning system using a DoG pupil and derives the point spread function (PSF) and the optical transfer function (OTF). The chapter then compares the PSF and OTF of each pupil function. Chapter 4 presents the characteristics of the DoG system. Some parameters of the system are explained and analyzed. Concluding remarks and future directions of this work are presented in Chapter 5. The MATLAB codes used in this thesis are presented in the Appendix A.
Chapter 2
Background

2.1 Mathematical Preliminaries

The following definition for the 2-D Fourier transform will be used:

\[ \mathcal{F}[f(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) \exp(jk_x x + jk_y y) dx dy = F(k_x, k_y) \]  \hspace{1cm} (2.1-1)

where \( k_x \) and \( k_y \) represent the frequencies related to \( x \) and \( y \) in the spatial domain. The 3-D Fourier transform is defined as:

\[ \mathcal{F}_{3D}[f(x, y, z)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) \exp(jk_x x + jk_y y + jk_z z) dx dy dz = F(k_x, k_y, k_z) \]  \hspace{1cm} (2.1-2)

where \( k_x, k_y, \) and \( k_z \) represent the frequencies related to \( x, y, \) and \( z \) in the spatial domain. The convolution operator "\(*\)" is defined as:

\[ f(x, y) * g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y') g(x-x', y-y') dx' dy'. \]  \hspace{1cm} (2.1-3)

2.2 Depth-Of-Focus

Figure (2.2-1) depicts the definition of the depth-of-focus. The horizontal axis represents the transversal direction. The depth-of-focus is the distance between two points that have half values of intensity in the intensity pattern. That is called full-width half max (FWHM). Though there are several definitions for the depth-of-focus, this definition will be used from now on.
2.3 Pupil Functions

Since all functions are circularly symmetric, a cylindrical coordinate is used instead of a rectangular coordinate. Note that \( r^2 = x^2 + y^2 \) and that \( R \) is the maximum radius of each pupil.

- **Circular pupil**
  \[
p(x, y) = \begin{cases} 
  1 & r \leq R \\
  0 & \text{otherwise} 
\end{cases}
\]  
  (2.3-1)

- **Annular pupil**
  \[
p(x, y) = \begin{cases} 
  1 & R_i \leq r \leq R \\
  0 & \text{otherwise} 
\end{cases}
\]  
  (2.3-2)

  where \( R_i \) is the inner radius. An obscuration ratio \( \varepsilon \) is defined as:

  \[
  \varepsilon = \frac{R_i}{R}
  \]
  (2.3-3)

- **Gaussian pupil**
  \[
p(x, y) = \begin{cases} 
  \exp\left(-\frac{(x^2 + y^2)}{w_u^2}\right) & r \leq R \\
  0 & \text{otherwise} 
\end{cases}
\]
  (2.3-4)

  where \( w_u \) is the waist or width of the Gaussian profile. Literally, the DoG pupil consists of two Gaussian pupils. Therefore, it is required that the depth-of-focus of the DoG pupil should be larger than that of a Gaussian pupil with a bigger waist between two Gaussian pupils consisting of the DoG pupil.

- **DoG pupil**
  \[
p(x, y) = \begin{cases} 
  a \times \exp\left(-\frac{(x^2 + y^2)}{w_u^2}\right) - b \times \exp\left(-\frac{(x^2 + y^2)}{w_v^2}\right) & r \leq R \\
  0 & \text{otherwise} 
\end{cases}
\]
  (2.3-5)

  where \( w_u \) and \( w_v \) are the waists of each Gaussian function and \( w_u > w_v \). \( a \) and \( b \) are determined by the assumption that two Gaussian functions consisting of the DoG pupil are generated by the
same laser source and their intensities are the same. The DoG pupil function, \( p(x,y) \) is rewritten as follows:

\[
p(x,y) = \begin{cases} 
  u(x,y) - v(x,y) & r \leq R \\
  0 & \text{otherwise}
\end{cases}
\]  

(2.3-6)

where

\[
u(x,y) = a \times \exp(- (x^2 + y^2)/w_u^2)
\]  

(2.3-7)

and

\[
v(x,y) = b \times \exp(- (x^2 + y^2)/w_v^2).
\]  

(2.3-8)

Since we assumed that the intensities of each Gaussian function are the same,

\[
\int \int |u(x,y)|^2 \, dx \, dy = \int \int |v(x,y)|^2 \, dx \, dy
\]

(2.3-9)

After some manipulations, \( a \) and \( b \) are given by

\[
a = n \times \sqrt{2}/(w_u \sqrt{\pi})
\]

(2.3-10)

and

\[
b = n \times \sqrt{2}/(w_v \sqrt{\pi})
\]

(2.3-11)

where \( n \) is constant. For simplicity's sake, \( n \) is set to unity. \( a \) and \( b \) are cast into the following form:

\[
a = \sqrt{2}/(w_u \sqrt{\pi})
\]

(2.3-12)

and

\[
b = \sqrt{2}/(w_v \sqrt{\pi}.
\]

(2.3-13)

2.4 The point spread function and the optical transfer function

Figure (2.4-1) shows a conventional laser scanning system using a single lens. Assume that an incident wave to the pupil is a plane wave. Note that when both the distance between a pupil and a lens and the distance between a lens and an object are a focal length of the lens, the field at the object plane is the Fourier transform of a pupil function. Therefore, the 3D point spread function (PSF) of the pupil function, \( p(x,y) \), is given by
\[ PSF(x, y; z) = |\Im[p(x, y)]|_{k_z = k_{0x}/f, k_y = k_{0y}/f} * h(x, y; z)|^2 \quad (2.4-1) \]

where \( h(x, y; z) \) is the free-space impulse response [9] and, aside from some phase constant, is given by

\[ h(x, y; z) = \frac{j k_0}{2 \pi z} \exp[-j \frac{k_0}{2z} (x^2 + y^2)] \quad (2.4-2) \]

with \( k_0 \) denoting the wave number of the light. The optical transfer function (OTF) is defined as:

\[ OTF(k_x, k_y, k_z) = \Im[PSF(x, y; z)] \quad (2.4-3) \]
Figure (2.2-1): Definition of the depth-of-focus
Figure (2.4-1): Conventional single lens imaging system
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Chapter 3
Analysis of pupil functions

3.1 Implementation of the proposed DOG pupil system

Figure (3.1-1) shows an optical implementation for the proposed DoG pupil system. The laser beam is divided into two Gaussian beams by a beamsplitter. Using acoustooptic modulators (AOM), two beams with different temporal frequencies ($w_0$ and $w_0 + \Omega$) are generated. Two lenses, which are used as a collimator, make two beams having different waists ($w_u$ and $w_v$). The optical beams are then joined spatially by a beamsplitter and used to scan the object. The scattered or reflected light from the object is then collected by a photodetector. The output signal has DC and AC current. The DC current is properly amplified by a DC amplifier and the AC current is also dealt with an AC amplifier and an envelope detector. The final signal is sent to a computer for display.

After the collimators, the two beams have different temporal frequencies and different waists. Therefore, the 3D-PSF of a DoG pupil at the object plane is given by

$$PSF(x, y; z) = |[\mathfrak{I}(u(x, y))]_{k_x=k_0x/f, k_y=k_0y/f} * h(x, y; z) \times \exp(jw_0t) + [\mathfrak{I}(v(x, y))]_{k_x=k_0x/f, k_y=k_0y/f} * h(x, y; z) \times \exp(j(w_0 + \Omega)t)|^2$$

$$= |P_1 \times \exp(jw_0t) + P_2 \times \exp(j(w_0 + \Omega)t)|^2 \tag{3.1-1}$$

where

$$P_1 = \mathfrak{I}[u(x, y)]_{k_x=k_0x/f, k_y=k_0y/f} * h(x, y; z) \tag{3.1-2}$$

and

$$P_2 = \mathfrak{I}[v(x, y)]_{k_x=k_0x/f, k_y=k_0y/f} * h(x, y; z)$$

$u(x, y)$ and $v(x, y)$ are given by eq. (2.3-7) and eq. (2.3-8). By expanding eq. (3.1-1), we have
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

\[ PSF(x, y; z) = |P_1|^2 + |P_2|^2 + P_1^*P_2 \exp(j\Omega t) + P_1P_2^* \exp(-j\Omega t) \]
\[ = |P_1|^2 + |P_2|^2 + 2|P_1P_2| \cos[\Omega t + \text{arg}(P_1) - \text{arg}(P_2)] \]
\[ = PSF_{DC} + PSF_{AC} \cos[\Omega t + PSF_{AC} \cos[\Omega t + \text{arg}(P_1) - \text{arg}(P_2)] \]  

(3.1-3)

where \( \text{arg}(.) \) stands for the argument of. The transmitted or reflected light, spatially collected by a photodetector, has the form of an output electrical signal as follows [8]:

\[ I_o = \int_D I_i(x, y)PSF(x_i - x, y_i - y; z)dxdy \]  

(3.1-4)

where the integration is done for the photodetector area \( D \); \( x_i(t) \) and \( y_i(t) \) are determined by the xy-driver’s motion. By substituting eq. (3.1-3) into eq. (3.1-4), we have

\[ I_o = I_i(x, y) * PSF(x, y; z) \]
\[ = I_i(x, y) * [PSF_{DC} + PSF_{AC} \cos[\Omega t + \text{arg}(P_1) - \text{arg}(P_2)] \]
\[ = (I_o(x, y))_{DC} + (I_o(x, y))_{AC} \]

(3.1-5)

The AC part of \( I_i(x, y) \) is an amplitude-modulated signal with temporal frequency \( \Omega \).

To demodulate the signal, we use an envelope detector. Now, by subtracting two output AC and DC signals, we can obtain the final output \( I_o \) as [7]

\[ I_o = (I_o(x, y))_{DC} - \beta(I_o(x, y))_{AC} \]
\[ = I_i * [PSF_{DC} - \beta PSF_{AC}] \]
\[ = I_i * (|P_1|^2 + |P_2|^2 - 2\beta|P_1P_2|) \]
\[ = I_i * PSF_{DOG} \]

(3.1-6)

where \( \beta \) is the gain of the AC amplifier. The PSF of the proposed system, \( PSF_{DOG} \), is given by

\[ PSF_{DOG} = |P_1|^2 + |P_2|^2 - 2\beta|P_1P_2| \]  

(3.1-7)
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Figure (3.1-1): Implementation of the proposed DOG pupil system employing optical heterodyne scanning technique.
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3.2 Simulation

3.2.1 Mathematical expression for simulation

The spatial frequency response is given by [9]

$$H(k_x, k_y; z) = \Im[h(x, y; z)] = \exp\left[j\frac{z}{2k_0}(k_x^2 + k_y^2)\right] \quad (3.2-1)$$

eq (3.1-2) can be rewritten as

$$P_1 = \Im\{u(x, y)\}|_{k_x = k_0x/f, k_y = k_0y/f} \ast h(x, y; z)$$

$$= \Im\{u(x, y) \times g(x, y; z)\}|_{k_x = k_0x/f, k_y = k_0y/f} \quad (3.2-2)$$

where $\Im\{g(x, y; z)\}|_{k_x = k_0x/f, k_0y/f} = h(x, y; z)$. Let's assume

$$g(x, y; z) = C \times H(a_1x, a_2y; z) \quad (3.2-3)$$

where $C$, $a_1$, and $a_2$ are constants. Two properties of the Fourier transform used in the proceeding derivation are as follows:

$$\Im\{P(x, y)\}|_{k_x = k_0x/f, k_y = k_0y/f} = 4\pi^2 p(-\frac{k_0x}{f}, -\frac{k_0y}{f}) \quad (3.2-4)$$

where $P$ is the Fourier transform of $p$ and

$$\Im\{p(a_1x, a_2y)\} = \frac{P(\frac{k_0x}{a_1f}, \frac{k_0y}{a_2f})}{|a_1a_2|} \quad (3.2-5)$$

From eq. (3.2-4) and eq. (3.2-5), we know that

$$\Im\{g(x, y; z)\}|_{k_x = k_0x/f, k_0y/f} = C\Im\{H(a_1x, a_2y; z)\}|_{k_x = k_0x/f, k_y = k_0y/f} = \frac{4\pi^2 C}{|a_1a_2|} h(-\frac{k_0x}{a_1f}, -\frac{k_0y}{a_2f}; z) \quad (3.2-6)$$

If $a_1 = a_2 = -\frac{k_0}{f}$ and $C = \frac{k_0^2}{4\pi^2 f^2}$, the following equation is satisfied;

$$\Im\{g(x, y; z)\}|_{k_x = k_0x/f, k_0y/f} = h(x, y; z) \quad (3.2-7)$$

Also, $g(x,y;z)$ is given by

$$g(x, y; z) = \frac{k_0^2}{4\pi^2 f^2} e^{j\frac{k_0z}{f}(x^2 + y^2)} \quad (3.2-8)$$

The reason why we use eq. (3.2-2) instead of eq. (3.1-2) is to reduce the number of
calculations. If we use eq. (3.1-2) to simulate, we take the Fourier transform of a pupil function and then convolve it with the impulse response function, \( h(x, y; z) \). However, we don't need to do a convolution operation if eq. (3.2-2) is used. Since the MATLAB code has several thousand loops to calculate the PSF, including a fast-Fourier-transform (FFT) operation in each loop, a smaller number of calculations is required.

Table (3.1-1) shows the MATLAB code used to calculate PSF. \( q(x_i,:,:) \) in line 5 represents \( \text{pupilFunction} \times g(x, y; z) \) without any convolution operation. After the multiplication of the pupil function and \( g(x, y; z) \), the Fourier transform operation is executed.

### 3.2.2 Normalization of the figures

Normally, the depth-of-focus of a circular pupil is the smallest among other pupils. The circular pupil could be used as a reference. For a circular aperture,

\[
\psi_{o0}(x, y) = \psi_{o0}(r) = \text{circ}(r / R)
\]

(3.2-9)

where \( R \) is the maximum radius of the aperture and \( r = \sqrt{x^2 + y^2} \). The Fourier transform of a circular aperture is [9]

\[
\mathcal{F}_p(r) = \psi_p(x, y) = \frac{2\pi f}{k_0 r} J_1\left(\frac{R k_0}{f} r\right)
\]

(3.2-10)

where \( J_1(x) \) is the first-order Bessel function. Figure (3.2-1) is called an Airy pattern of which the equation is [9]

\[
[2 J_1(\pi \xi) / \pi \xi]^2
\]

(3.2-11)

At \( \xi = 1.22 \), the plot has a first zero-crossing point. When eq. (3.2-10) is compared to eq. (3.2-11) and some constants are ignored, we have

\[
\xi = \frac{R k_0 f}{\lambda f} = \frac{2 R r}{\lambda f}
\]

(3.2-12)

at a first zero-crossing point. The intensity of eq. (3.2-10) is proportional to \( |\psi_p(x, y, z)|^2 \).

Therefore, the intensity has a first zero-crossing point at
\[
\Delta r = \frac{0.61 \lambda f}{R} \times 1.22 = \frac{0.61 \lambda}{NA}
\]

(3.2-13)

where \( NA = R / f \) if \( R \ll f \) (paraxial approximation). This normalization factor, \( \Delta r \), is for
the lateral space (r-direction).

Now consider the z-direction. The field at the observation plane is

\[
\psi_p(x_1, y_1; z) = \Re[\psi_{e0}(x, y)]^* h(x, y; z) = \Re[\psi_{e0}(x, y) \times g(x, y; z)]
\]

(3.2-14)

where \( \psi_{e0}(x, y) = \psi_{e0}(r) = \text{circ}(r / R) \). Note that \( g(x, y; z) \) is given by eq. (3.2-9). Using the
Fraunhofer approximation,

\[
\psi_p(x_1, y_1; z) = \Re[\psi_{e0}(x, y) \times g(x, y; z)]_{k_x = k_x / f, k_y = k_y / f} = \frac{k_0^2}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \psi_{e0}(x, y) e^{j-k_z^2(r^2+y^2)} e^{j(k_x x + k_y y)} dxdy | \frac{k_0}{f} k_x x + \frac{k_0}{f} k_y y | = \frac{k_0}{f} \pi \theta
\]

(3.2-15)

Note that \( x = r \cos \theta \) and \( y = r \sin \theta \).

\[
k_x x + k_y y = k_x r \cos \theta + k_y r \sin \theta = \sqrt{k_x^2 + k_y^2} \times r \cos(\theta - \phi)
\]

(3.2-16)

where \( k_r = \sqrt{k_x^2 + k_y^2} \), \( \cos \phi = \frac{k_x}{k_r} \), and \( \sin \phi = \frac{k_y}{k_r} \). Change the rectangular coordinate
system into the circular coordinate system.

\[
\psi_p(r; z) = \frac{k_0^2}{4\pi^2} \int_{0}^{2\pi} \text{circ}(r / R) \int_{0}^{\pi} e^{j-k_z^2(r^2+y^2)} e^{j(k_r r \cos(\theta-\phi))} rdrd\theta | \frac{k_0}{f} k_r r | = \frac{k_0}{f} \pi \theta
\]

(3.2-17)

Since the field along the z-direction on the observation plane is considered,

\( r_1 = 0 \) and \( k_r = 0 \)

Now eq. (3.2-17) becomes

\[
\psi_p(0; z) = \frac{k_0^2}{4\pi^2} \int_{0}^{\infty} 2\pi e^{j-k_z^2r^2} dr = \frac{k_0}{f} \left( e^{j-k_z^2z^2 / 2} - 1 \right)
\]

(3.2-18)

After several mathematical manipulations, the intensity of eq. (3.2-18) becomes
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\[
|\psi_p(0; z)|^2 = \left(\frac{k_0 R}{2f}\right)^4 \sin c^2 \left(\frac{k_0 R^2 z}{4f^2}\right)
\]  \hfill (3.2-19)

This intensity has a zero-crossing point at

\[
\Delta z = \frac{2\lambda}{(N A)^2}
\]  \hfill (3.2-20)

where \( N A = R / f \) if \( R << f \) (paraxial approximation). From now on, axes of every plot will be normalized by eq. (3.2-13) and eq. (3.2-20).

### 3.2.3 PSF and OTF of pupil functions

Here, the PSF and the OTF are plotted and explained. Each plot has its own MATLAB code and parameters that will be specified. Let us introduce parameters required for the following figures:

\[ R = 1 \text{ mm} \]
\[ z\text{Num} = 64 \]
\[ zm = 0.6 \text{ mm} \]
\[ x\text{Num} = 256/2 \]
\[ x\text{m} = 2 \text{ mm} \]
\[ NA = 0.0995 \]
waists for a DoG pupil = 0.3 mm and 0.2 mm
waist for a Gaussian pupil = 0.2 mm
\[ \lambda = 0.6 \text{ um} \]
\[ f=10 \text{ mm} \]
\[ \beta = -1 \]
\[ \varepsilon = 0.95 \]

\( R \) is the maximum radius of the pupil. \( zm \) and \( xm \) are the ranges of the each coordinate variable and \( z\text{Num} \) and \( x\text{Num} \) are the numbers of samples. \( w_u \) and \( w_v \) are the waists of the
Gaussian beams in the DoG pupil, respectively. $\lambda$ is the wavelength of the laser and $f$ is the focal length of the lens in front of the object. From eq. (3.2-13) and eq. (3.2-20), we have
\[
\Delta z = 0.12 \text{mm} \\
\Delta r = 3.68 \mu\text{m}
\]

From figure (3.2-2a) to figure (3.2-2c), the PSF and OTF for a circular pupil are presented. Figure (3.2-2a) presents the PSF of a circular pupil. $\zeta$ and $\rho$ are the normalized axes ($\zeta = z / \Delta z$ and $\rho = r / \Delta r$). Figure (3.2-2b) shows the OTF of a circular pupil that has the same axes as figure (3.2-2a). Figure (3.2-2c) shows the cross-sections of PSF for a circular pupil. The upper one shows the cross-section of the PSF vs. a normalized $\zeta$ and the lower one shows the cross-section of the PSF vs. a normalized $\rho$ for a circular pupil. Note that the PSF in the $z$-direction has the shape of the square of a $\text{sinc}$ function and the PSF in the $r$-direction is similar to figure (3.2-1), as was explained in chapter 3.2.2.

The lower figure in figure (3.2-2c) has a bad resolution. It can be improved by modifying some parameters, but that deteriorates the all the others. (The MATLAB file for a circular aperture is “circularplot.m” and lies in Appendix. A1.)

From figure (3.2-3a) to figure (3.2-3c), the PSF and OTF for an annular pupil with $\mathcal{E} = 0.95$ are presented. Figure (3.2-3a) and figure (3.2-3b) show the PSF and OTF of an annular pupil, respectively. Figure (3.2-3c) shows the cross-sections of the PSF. The upper one shows a cross-section of the PSF vs. a normalized $\zeta$ and the lower one shows a cross-section of the PSF vs. a normalized $\rho$. (The MATLAB file for the annular aperture is “annularplot.m” and lies in Appendix. A2.)

From figure (3.2-4a) to figure (3.2-4c), the PSF and OTF for a Gaussian pupil with $w_u = 0.3$ mm are presented. Figure (3.2-4a) and figure (3.2-4b) show the PSF and OTF of a Gaussian pupil, respectively. Figure (3.2-4c) shows the cross-sections of PSF. (The MATLAB file for a Gaussian pupil is “gaussplot.m” and lies in Appendix. A3.)

From figure (3.2-5a) to figure (3.2-5c), the PSF and OTF for a DoG pupil with $w_u = 0.3$ mm, $w_v = 0.2$ mm, and $\beta = -1$ are presented. Figure (3.2-5a) and figure (3.2-5b) show the PSF and OTF of a DOG pupil, respectively. Figure (3.2-5c) shows the cross-sections of the PSF. (The MATLAB file for DoG is “dogplot.m” and lies in Appendix. A4.)

Figure (3.2-6a), figure (3.2-6b), figure (3.2-7a), and figure (3.2-7b) show the cross-sections
of the PSF along $\zeta$, the PSF along $\rho$, the OTF along $\zeta$, and the OTF along $\rho$ for all pupils, respectively. (The MATLAB file is “plotall.m” and lies in Appendix. A5. For this, every MATLAB program for all pupils should be run first.)

From figure (3.2-6a), the order of the depth-of-focus in the PSF from the smallest to the largest is

circular < DOG < Gaussian < annular

As we expected, the depth-of-focus of an annular pupil is largest, but the depth-of-focus of a DoG pupil is smaller than that of Gaussian pupil. Since our goal is to make the depth-of-focus of DoG pupil be largest, except annular pupil, Gaussian pupil will be mainly compared with DoG pupil in the thesis. And the order of the lateral resolution from the smallest to the largest is

annular < circular < DOG < Gaussian,

but the order of the waist may be changed if the number of the samples is changed. Table (3.2-1) is obtained from figure (3.2-6a). It shows the depth-of-focus for each pupil.
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Table (3.1-1): MATLAB code to calculate PSF

```matlab
zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        q(xi,:) = p2(xRange/xm,x/xm,eta,xm,R,2) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x.^2));
        xi = xi + 1;
    end
    h = abs(fft2(q)).^2;
    h = fftshift(h);
    psf(zi,:) = h(xNum/2+1,:);
    zi = zi + 1;
end
```

Table (3.2-1): depth-of-focus obtained by simulation for each pupil in the FWHM format

<table>
<thead>
<tr>
<th>pupil</th>
<th>depth-of-focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>Circular(R=1mm)</td>
<td>112.5 um</td>
</tr>
<tr>
<td>Annular (ε = 0.95)</td>
<td>1100 um</td>
</tr>
<tr>
<td>Gaussian (waist: 0.2 mm)</td>
<td>937.5 um</td>
</tr>
<tr>
<td>DoG (waists: 0.2mm, 0.3mm)</td>
<td>525 um</td>
</tr>
</tbody>
</table>
Figure (3.2-1): Plot of $\left[ 2 J_1(\pi \xi / \pi \xi_0) \right]^2$ that is called an Airy pattern.
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Figure (3.2-2a): PSF for a circular pupil

Figure (3.2-2b): OTF for a circular pupil
Figure (3.2-2c): Cross-sections of the PSF for a circular pupil
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Figure (3.2-3a): PSF for an annular aperture with $\varepsilon = 0.95$

Figure (3.2-3b): OTF for an annular aperture with $\varepsilon = 0.95$
Figure (3.2-3c): Cross-sections of the PSF for an annular pupil with $\varepsilon = 0.95$
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Figure (3.2-4a): PSF for a Gaussian pupil with $w_w = 0.2$ mm

Figure (3.2-4b): OTF for a Gaussian pupil with $w_w = 0.2$ mm
Figure (3.2-4b): Cross-sections of PSF for a Gaussian pupil with $w_u = 0.2$ mm
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Figure (3.2-5a): PSF for a DoG pupil
\((w_u=0.3 \text{ mm}, \ w_v=0.2 \text{ mm}, \ \text{and} \ \beta = -1)\)

Figure (3.2-5b): OTF for a DoG pupil
\((w_u=0.3 \text{ mm}, \ w_v=0.2 \text{ mm}, \ \text{and} \ \beta = -1)\)
Figure (3.2-5c): cross-sections of the PSF for a DoG pupil

\( w_u = 0.3 \text{ mm}, \quad w_v = 0.2 \text{ mm}, \quad \beta = -1 \)
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Figure (3.2-6a): PSF along $\zeta$ for all pupils

Figure (3.2-6b): PSF along $\rho$ for all pupils
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Figure (3.2-7a): OTF along $k_z$ for all pupils

Figure (3.2-7b): OTF along $k_r$ for all pupils
3.3 Equations of PSF for each pupil

The equations for the PSF of each pupil in the r-axis and the z-axis from the definition are here derived and compared with the simulations to check whether the modeling for the simulations is proper, before we analyze the DoG system in detail.

3.3.1 PSF in the r-axis for a circular pupil

The intensity of the PSF in the r-axis for a circular pupil is the square of the absolute value of eq. (3.2-10) and is given by

\[ |\psi_p(x, y)|^2 \propto \left( \frac{2\pi R f}{k_0 r} \right)^2 J_1^2 \left( \frac{Rk_0}{f} r \right) \]  

(3.3-1)

In figure (3.3-1), the results are slightly different because of the resolution error in MATLAB. As \( x_m \) is relatively smaller, we would see the detailed graph for the simulation. For example, when \( x_m \) is 1mm, we couldn’t see the 1st peak in the sidelobe and others. As xNum is larger, we would see the detailed graph with the 1st peak. This resolution error for the simulation can be solved if the sampling number (xNum and zNum) becomes bigger. But, this takes too much time to simulate. (MATLAB file: circular.m, Appendix. A1; \( R = 50 \) um, xNum = 512, \( x_m = 0.2 \) mm)

3.3.2 PSF in the z-axis for a circular pupil

From eq. (3.2-19), the intensity of the PSF in the z-axis for a circular aperture is given by

\[ |\psi_p(0; z)|^2 = \left( \frac{k_0 R}{2 f} \right)^4 \sin^2 \left( \frac{k_0 R^2 z}{4 f^2} \right) \]  

(3.3-2)

Figure (3.3-2) shows that the two results are the same and that the simulation is right. (MATLAB file: circular.m, Appendix. A1; \( R = 1 \) mm, xNum = 128, \( x_m = 2 \) mm)

3.3.3 PSF in the r-axis for an annular pupil
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The defocused amplitude pattern for an annular pupil is the convolution of the Fourier transform of an annular function and $h(x, y; z)$ [9]:

$$U(x, y; z) = \mathcal{F}(\text{annularFunction}) \ast h(x, y; z)$$  \hfill (3.3-3)

Note that the convolution is not required to obtain the PSF in the r-axis for an annular pupil. From the definition, an annular pupil function is given by

$$\text{annularFunction} = \text{circ}(\frac{r}{R}) - \text{circ}(\frac{r}{R_i})$$  \hfill (3.3-4)

Since an annular function and its Fourier transform are circularly symmetrical, the intensity is considered in Cylindrical coordinates. From eq. (3.210) and eq. (3.3-4), the PSF in the r-axis for the annular aperture is given by

$$U(r; 0) = \frac{2\pi f}{k_0 r} \left[ R^2 J_1 \left( \frac{R k_0}{f} r \right) - R_i J_1 \left( \frac{R_i k_0}{f} r \right) \right]$$  \hfill (3.3-5)

where $R_i$ is the radius of the inner side of an aperture. The intensity pattern becomes

$$|U(r; 0)|^2 = \frac{2\pi f}{k_0 r} \left[ R^2 J_1 \left( \frac{R k_0}{f} r \right) - R_i J_1 \left( \frac{R_i k_0}{f} r \right) \right]^2$$  \hfill (3.3-6)

Figure (3.3-3) shows the two results are the same. (MATLAB file: annuplot.m, Appendix. A2; $R = 0.03\text{mm}$, $\varepsilon = 0.95$, $zm=3\text{mm}$, $xNum = 512$, $xm = 0.2\text{mm}$)

3.3.4 PSF in the z-axis for an annular pupil

From eq. (3.2-18), eq. (3.3-3), and eq. (3.3-4), the PSF in the z-axis for an annular pupil is given by

$$U(0; z) = \frac{k_0}{j^2 z} \left[ e^{-\frac{j k_0 R_i^2 z}{2f z^2}} - 1 - e^{\frac{j k_0 R_i^2 z}{2f z^2}} \right]$$ \hfill (3.3-7)

After several manipulations, the intensity pattern becomes
\[ |U(0; z)|^2 = \frac{k_0^2 R^4 (1 - \varepsilon^2)}{16 f^4} \sin^2 \left( \frac{\pi R^2}{2 f^2 \lambda} (1 - \varepsilon^2) z \right) \]  
\tag{3.3-8}

where \( \varepsilon \) is the obscuration ratio. Figure (3.3-4) shows that the two results are the same. (MATLAB file: annuplot.m, Appendix. A2; R = 1mm, xNum = 256, xm = 2mm)

### 3.3.5 PSF in the z-axis for a DoG pupil

From eq. (3.1-7), the PSF for a DoG pupil is given by

\[ \text{PSF}_{\text{DOG}} = |P_1|^2 + |P_2|^2 - 2 \beta |P_1 P_2|, \]  
\tag{3.3-9}

where

\[ P_1 = \Im[u(x, y)]_{k_x = k_0 x / f, k_y = k_0 y / f} * h(x, y; z) \]  
\tag{3.3-10}

\[ P_2 = \Im[v(x, y)]_{k_x = k_0 x / f, k_y = k_0 y / f} * h(x, y; z) \]  
\tag{3.3-11}

\[ u(x, y) = a \times \exp(-x^2 + y^2) / w_u^2 \]  
\tag{3.3-12}

\[ v(x, y) = b \times \exp(-x^2 + y^2) / w_v^2. \]  
\tag{3.3-13}

Using the scaling property of the Fourier transform, the Fourier transform of \( u(x, y) \) is given by

\[ \Im[u(x, y)]_{k_x = k_0 x / f, k_y = k_0 y / f} = w_u \sqrt{2 \pi} e^{\frac{k_0^2 w_u^2}{4 f^2 (x^2 + y^2)}} \]  
\tag{3.3-14}

\[ \Im[P_1(x, y; z)]_{k_x = k_0 x / f, k_y = k_0 y / f} = \Im[\Im[u(x, y)]_{k_x = k_0 x / f, k_y = k_0 y / f} * h(x, y; z)]_{k_x = k_0 x / f, k_y = k_0 y / f} \times H(k_x, k_y; z) \]  
\tag{3.3-15}

After some manipulations, eq. (3.3-10) becomes

\[ P_1(x, y; z) = \frac{2 \sqrt{2 \pi f} e^{\frac{x^2 + y^2}{w_1^2(z)}}}{k_0 w_1(z)} \frac{k_0 (x^2 + y^2)}{2 R_1(z)} \phi(z) \]  
\tag{3.3-16}

where

\[ w_1^2(z) = \frac{4 f^2}{k_0^2 w_u^2} [1 + \left( \frac{z}{z_{R1}} \right)^2] \]  
\tag{3.3-17}

\[ z_{R1} = \frac{2 f^2}{k_0^2 w_u^2} \]  
\tag{3.3-18}
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\[
R_1(z) = \frac{z^2 + z_R z^2}{2} \quad (3.3-19)
\]

\[
\phi_1(z) = -\tan^{-1}\left(\frac{z}{z_R}\right) \quad (3.3-20)
\]

In a similar way,

\[
P_2(x, y; z) = \frac{2\sqrt{2}\pi f}{k_0 w_2(z)} e^{\frac{x^2+y^2}{w_2^2(z)}} e^{\frac{j\alpha(x^2+y^2)}{2R_2(z)}} e^{-j\phi_2(z)} \quad (3.3-21)
\]

where

\[
w_2^2(z) = \frac{4f^2}{k_0^2 w_v^2} [1 + \left(\frac{z}{z_{R2}}\right)^2] \quad (3.3-22)
\]

\[
z_{R2} = \frac{2f^2}{k_0 w_v^2} \quad (3.3-23)
\]

\[
R_2(z) = \frac{z^2 + z_{R2}^2}{2} \quad (3.3-24)
\]

\[
\phi_2(z) = -\tan^{-1}\left(\frac{z}{z_{R2}}\right) \quad (3.3-25)
\]

From eq. (3.3-9), the PSF for a DoG pupil is given by

\[
PSF_{DoG} = \frac{8a^2\pi f^2}{k_0^2 w_1^2(z)} \exp\left[ -\frac{2r^2}{w_1^2(z)} \right] + \frac{8b^2\pi f^2}{k_0^2 w_2^2(z)} \exp\left[ -\frac{2r^2}{w_2^2(z)} \right] - \frac{16ab\pi f^2}{k_0^2 w_1^2(z) w_2^2(z)} \exp\left[ -r^2 \left(\frac{1}{w_1^2(z)} + \frac{1}{w_2^2(z)}\right)\right] \quad (3.3-26)
\]

This equation will be compared with the simulation results. First, consider the PSF in the z-axis for a DoG pupil. From eq. (3.61a), the equation becomes

\[
PSF_{DoG} \big|_{z=0} = \frac{8a^2\pi f^2}{k_0^2 w_1^2(z)} + \frac{8b^2\pi f^2}{k_0^2 w_2^2(z)} - \frac{16ab\pi f^2}{k_0^2 w_1^2(z) w_2^2(z)} \quad (3.3-27)
\]

Figure (3.3-5) shows that the two results are the same. (MATLAB file: dogplot.m, Appendix. A4; R = 1mm, xNum = 128, xm = 1mm)

3.3.6 PSF in the r-axis for a DoG pupil
Consider the PSF in the r-axis for a DoG pupil. From eq. (3.3-26), the equation becomes

\[
PSF_{\text{DOG}} \mid_{z=0} = \frac{8a^2 \pi^2}{k_0^2 w_1^2(0)} \exp \left[-\frac{2r^2}{w_1^2(0)}\right] + \frac{8b^2 \pi^2}{k_0^2 w_2^2(0)} \exp \left[-\frac{2r^2}{w_2^2(0)}\right] - \frac{16ab \beta \pi^2}{k_0^2 w_1(0)w_2(0)} \exp \left[-r^2 \left(\frac{1}{w_1^2(0)} + \frac{1}{w_2^2(0)}\right)\right]
\]  

(3.3-28)

Figure (3.3-6) shows that the two results are the same. (MATLAB file: dogplot.m, Appendix. A4; R = 0.1mm, xNum = 512, xm = 0.2mm)
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.3-1): Cross-section of the PSF along $\rho$ for a circular pupil

Figure (3.3-2): Cross-section of the PSF along $\zeta$ for a circular pupil
Figure (3.3-3): Cross-section of the PSF along $\rho$ for an annular pupil

Figure (3.3-4): Cross-section of the PSF along $\zeta$ for an annular pupil
Figure (3.3-5): Cross-section of the PSF along $\zeta$ for a DoG pupil

Figure (3.3-6): Cross-section of the PSF along $\rho$ for a DoG pupil
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

3.4 The effects of some important parameters

3.4.1 The obscuration ratio ($\epsilon$)

As $\epsilon$ is close to 1, the depth-of-focus of an annular pupil becomes larger. Though the resolution is not good, figure (3.4-1) shows that as $\epsilon$ is larger, the depth-of-focus becomes higher. (MATLAB file: plotGaussAnnulGraph.m, Appendix A6 and annuplot.m, Appendix A2; R=1mm, zNum=256, zm=1mm, xNum=64, xm=1mm)

3.4.2 The waist in a Gaussian pupil

It is well-known that a Gaussian beam with a smaller spot size expands faster as it goes through a free space. Figure (3.4-2) shows that the waist of a Gaussian pupil is almost inversely proportional to the depth-of-focus. (Matlab code: gaussplot.m and plotGaussAnnulGraph.m; R=1mm, zNum=256, zm=1mm, xNum=64, xm=1mm)

3.4.3 Numerical aperture (NA)

If a higher NA is used, we obtain a high lateral resolution and a reduced depth-of-focus. In the previous chapter, where a small NA is used, the output will be changed with different R or NA. R is changed from 0.2mm to 1.2mm for every aperture. From the definition of NA,

$$NA = \frac{R}{\sqrt{f^2 + R^2}}$$ (3.4-1)

Figure (3.4-3) through figure (3.4-10) show that the depth-of-focus is almost inversely proportional to NA, except for some regions in figure (3.4-8) and figure (3.4-10). A Gaussian beam has its own waist. If the radius of the pupil is more than twice the size of the waist, most of the light can pass through the pupil. This property explains that figure (3.4-8) and figure (3.4-10) have flat lines at a large NA. (MATLAB file: each aperturePlot(annuplot.m, circularplot.m, dogplot.m, gaussplot.m and plotWithChangedNA.m, Appendix A7; method: 1.with different R,
simulate each aperture plot 2. change *psf.dat to psf1,2,3,4,5, and 6.m according to each R 3. run plotWithChangedNA.m; zNum = 64, zm = 2 or 3 mm, xNum = 64, xm = 2 mm, $\varepsilon = 0.95$, $\beta = -1$, $w_u = 0.3 mm$, $w_v = 0.2 mm$ )
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.4-1): Depth-of-focus vs. obscuration ratio in an annular pupil

Figure (3.4-2): Depth-of-focus vs. waist of a Gaussian pupil
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.4-3): PSF along z for an annular pupil in different NA

Figure (3.4-4): Depth-of-focus vs. NA in an annular pupil
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.4-5): PSF along z for a circular pupil in different NA

Figure (3.4-6): Depth-of-focus vs. NA in a circular pupil
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Figure (3.4-7): PSF along z for a Gaussian pupil in different NA

Figure (3.4-8): Depth-of-focus vs. NA in a Gaussian pupil
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.4-9): PSF along z for a DoG pupil in different NA

Figure (3.4-10): Depth-of-focus vs. NA in a DoG pupil
3.5 The uncertainty principle

From the uncertainty principle, the depth-of-focus for an annular pupil has been found to be [10]

\[
\Delta z = \frac{\lambda}{\sqrt{1 - N A^2}} - \sqrt{1 - N A^2}
\]

Eq. (3.5-1) can be written as

\[
\Delta z = \frac{2\lambda}{N A^2 (1 - e^2)}
\]

if small angle approximations are used, which means that the NA is very small.

Figure (3.5-1) shows the relation between the obscuration ratio (\(\epsilon\)) and the depth-of-focus, which is obtained by eq. (3.5-1) and eq. (3.5-2). At a large NA, the two uncertainty principles don’t match well, since a red line in the lower figure is obtained by eq. (3.5-2), which is valid only if the NA is small. Two definitions at a low NA are almost the same. (MATLAB file: annuUncertaion.m, Appendix A8 and getDelzAnnuAlongObscuration.m, Appendix A9)

Figure (3.5-2) shows the results of two different uncertainty principle techniques and the simulation. The term 'simulation' means that the result is obtained by the MATLAB code using eq. (2.4-1). The figure shows that they are almost the same and that the two uncertainty principles matches well with another definition at a low NA. (Matlab code : annuUncertain.m, Appendix A8 and getDelzAnnuAlongObscuration.m, Appendix A9)
CHAPTER 3. ANALYSIS OF PUPIL FUNCTIONS

Figure (3.5-1): Depth-of-focus obtained by uncertainty principle at two NAs

Figure (3.5-2): Depth-of-focus vs. $\varepsilon$ by simulation and two uncertainty principle techniques
Chapter 4

Difference-of-Gaussians pupil function

As I said earlier, the DoG pupil system has 5 parameters. As was seen in figure (3.2-1), the parameters $a$ and $b$ are associated with the intensity of laser light, the parameters $w_u$ and $w_v$ are the waists of the two Gaussian beams, and the parameter $\beta$ is the gain of the AC amplifier. Chapter 4.1 presents the relation among the parameters of the DoG pupil where $a$ and $b$ parameters are given by eq. (2.3-12) and eq. (2.3-13) respectively. The plots of $w_v$ and the depth-of-focus show some patterns at fixed $\beta$ and $w_u$. As $\beta$ and $w_u$ are changed, those plots show different patterns. From these whole plots, optimized parameters to extend the depth-of-focus are obtained. Larger depth-of-focus of DoG pupil system, however, can be achieved by changing $a$ and $b$ parameters. Chapter 4.2 presents the DoG pupil system with non-fixed $a$ and $b$ parameters. The chapter shows plots of the depth-of-focus and each parameter.

4.1 Fixed $a$ and $b$ parameters

The term 'fixed $a$ and $b$ parameters' means that the values of $a$ and $b$ are given by eq. (2.3-12) and eq. (2.3-13), respectively. Figure (4.1a) through figure (4.1e) show that the depth of focus for the DoG pupil is peak at some values of parameters and is largest when the DoG pupil has the parameters of beta=-1 and the smallest $w_u$, i.e., when beta=-1, $w_u=0.3\text{mm}$, and $w_v=0.2\text{mm}$, the depth-of-focus is about 560 um, which is largest value in the range of the selected parameters. (MATLAB file: dogPlotBofA.m, Appendix A10 and findDepthOfFocus.m, Appendix A11; $R = 1\text{mm}$, $z\text{Num}=256$, $z\text{m}=1\text{mm}$, $x\text{Num}=64$, $x\text{m}=2\text{mm}$)

But, as is seen in figure (4.1-1f), the depth-of-focus with $\beta=-10$ is larger than that with another $\beta$. Though the magnitudes of all the plots for the PSF are the normalized ones, they
Table (4.1-1) shows that each PSF has a different maximum value as $\beta$ changes.

$$3D \text{ PSF} = ACpsf + DCpsf \tag{4.1-1}$$

Assume that PSF is represented as eq. (4.1-1). MaxPsf in table (4.1-1) is a maximum value in the PSF, literally. The AC Part and DC part are the maximum values in $ACpsf$ and $DCpsf$, respectively. As $\beta$ goes to minus infinity, the AC part becomes much larger than the DC Part. When $\beta$ is $-1$, the ratio between the AC part and the DC part is 0.75, but when $\beta$ is $-10$, the ratio is 7.45. Because the noise in the AC signal can be considered relatively significant in the large ratio, a large number for $\beta$ isn’t a good choice. Actually, when $\beta$ is $-1$, the PSF has the addition of the DC and the non-magnified AC. The ratio between AC and DC is 0.75. Therefore, that value $-1$ would be a better choice, but not the optimal one.
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.1-1a): waist vs. depth-of-focus for a DoG pupil
\( (\beta = 1, \text{ fixed } a \text{ and } b) \)

Figure (4.1-1b): waist vs. depth-of-focus for a DoG pupil
\( (\beta = 0.5, \text{ fixed } a \text{ and } b) \)
Figure (4.1-1c): waist vs. depth-of-focus for a DoG pupil
($\beta = 1$, fixed $a$ and $b$)

Figure (4.1-1d): waist vs. depth-of-focus for a DoG pupil
($\beta = -0.5$, fixed $a$ and $b$)
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.1-1e): waist vs. depth-of-focus for a DoG pupil
$\beta = -1$, fixed $a$ and $b$

Figure (4.1-1f): waist vs. depth-of-focus for a DoG pupil
$\beta = -10$, fixed $a$ and $b$
Table (4.1-1): maximum magnitude according to $\beta$ in the PSF

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>MaxPsf($10^4$)</th>
<th>Ac Part($10^4$)</th>
<th>Dc Part($10^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25</td>
<td>0.75</td>
<td>-0.49</td>
</tr>
<tr>
<td>0.5</td>
<td>0.63</td>
<td>0.37</td>
<td>0.26</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>-0.5</td>
<td>1.37</td>
<td>0.37</td>
<td>1</td>
</tr>
<tr>
<td>-1</td>
<td>1.75</td>
<td>0.75</td>
<td>1</td>
</tr>
<tr>
<td>-5</td>
<td>4.73</td>
<td>3.73</td>
<td>1</td>
</tr>
<tr>
<td>-10</td>
<td>8.46</td>
<td>7.45</td>
<td>1</td>
</tr>
</tbody>
</table>
4.2 Non-fixed \(a\) and \(b\) parameters

A depth-of-focus around 100 um is commonly required in microscopy for 3-D imaging. Since DOG pupil consists of two Gaussian beams, we first should find the waist of one Gaussian pupil that makes the depth-of-focus around 100 um.

Figure (4.2-1) shows that the depth-of-focus of a Gaussian pupil lies in the range of several hundred microns, using the following parameters. (MATLAB file: gaussBetaVsDof.m, Appendix A12; zNum=128, zm=0.3 mm, xNum=128, xm=6mm, R=6mm) So the waist around 0.6 mm will used for the following plots.

Figure (4.2-2a) through (4.2-2h) show the relation between \(w_u\) and \(w_v\) at a different \(\beta\).

Unlike figures (4.1-1), at some values for \(\beta\), it is difficult to find a value for \(w_u\) that gives the depth-of-focus a peak. (MATLAB file: dogPlotBofA.m, Appendix A10 and findDepthOfFocus.m, Appendix A11.)

In figure (4.2-3), the depth-of-focus of Gaussian and DOG pupil are drawn simultaneously. The upper straight line is the depth-of-focus for a Gaussian pupil with the waist \(w_v\) (=0.4mm) and the lower straight one is the depth-of-focus for a Gaussian pupil with the waist \(w_u\) (=0.6mm).

Figure (4.2-3) shows that the depth-of-focus for a DOG pupil is lower than that of the Gaussian pupil at any \(\beta\). Since the objective of this thesis is that the depth-of-focus of DoG pupil is largest among other pupil functions, except annular pupil, we should consider \(a\) and \(b\) parameters that had not been changed yet. (MATLAB file: dogOptimizedBeta.m, Appendix A13; R = 6 mm, zNum = 128, zm =0.3mm, xNum = 128, xm = 6mm, \(w_u = 0.6\)mm, \(w_v = 0.4\)mm)

Figure (4.2-4) shows an acoustooptic modulator (AOM) in Bragg regime. In Bragg regime, the AOM generates two scattered orders (\(\psi_0\), \(\psi_1\)), which are given by [9]

\[
\psi_0 \propto \cos(\hat{\alpha})
\]  (4.2-1)

and

\[
\psi_1 \propto \sin(\hat{\alpha})
\]  (4.2-2)

where \(\hat{\alpha}\) represents the peak phase delay of the light through the acoustic medium. In the proposed system, the 0\(^{th}\) order is blocked and the 1\(^{st}\) order is used. Figure (4.2-5) shows the intensity of two diffracted orders. Note that \(\hat{\alpha}\) is proportional to the amplitude of the sine wave for the AOM and \(|\psi_1|^2\) represents the square of the parameters \(a\) and \(b\). Therefore, the
amplitude of the sine wave can change the parameters $a$ and $b$. We can easily change the amplitude of the sine wave, because the sound wave is generated from an RF amplifier.

Figure (4.2-6) shows that the modification of $a$ and $b$ makes the depth-of-focus for the DoG pupil higher than that of the Gaussian pupil and that the depth-of-focus is at its peak when $b$ is twice as much as $a$. (MATLAB file: dogOptimizedAandBforOneBeta.m, Appendix A 14; $R = 6\text{mm}$, $z\text{Num} = 128$, $zr = 0.3\text{mm}$, $x\text{Num} = 128,xm = 6\text{m}$, $wu = 0.6\text{mm}$, $wv = 0.4\text{mm}$, $\beta = 0.7$)

Figure (4.2-7) shows that the PSF with the following parameters is similar to a Gaussian profile:

$$wu = 0.6\text{mm}, wv = 0.4\text{mm}, \beta = 0.7,a = wu \times \sqrt{\pi/2}, b = a*2$$

While $\beta$ is almost inversely proportional to the depth-of-focus in figure (4.2-3), figure (4.2-8) shows a totally different pattern. As $\beta$ increases, the depth-of-focus goes up. The depth-of-focus has a peak when $b$ is twice as much as $a$ and $\beta$ is $\sim 0.78$. (MATLAB file: dogOptimizedBeta.m, Appendix A13; $R = 6\text{mm}$, $z\text{Num} = 128$, $zr = 0.3\text{mm}$, $x\text{Num} = 128,xm = 6\text{m}$, $wu = 0.6\text{mm}$, $wv = 0.4\text{mm}$, $a = wu \times \sqrt{\pi/2}, b = a*2$)

Figure (4.2-9) shows that the pattern goes back to the previous one with fixed $a$ and $b$. For example, the depth-of-focus reaches a peak now with $wu=0.6 \text{ mm}$ and $wv=0.32\text{mm}$. But as is seen in figure (4.2-10), the PSF with the above parameters does not have a Gaussian profile with the following parameters:

$$wu = 0.6 \text{ mm}, wv = 0.32 \text{ mm}, \beta = 0.78,a = wu \times \sqrt{\pi/2}, b = a*2$$

We do not know whether the depth-of-focus calculated by the simulation is obtained by the PSF that has a Gaussian profile and the PSF that does not have a Gaussian profile can image the object as we expect. Keep in mind, however, that there is a range of parameters that can make the PSF stay in a tolerable Gaussian profile. The characteristics of the depth-of-focus should be analyzed and explained carefully with the parameters of a non-fixed $a$ and $b$ to find the desirable range of parameters.
Figure (4.2-1): depth-of-focus vs. waist in a Gaussian pupil
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.2-2a): depth-of-focus vs. waist for a DoG pupil 
\( \beta = 1, \text{ fixed } a \text{ and } b \)

Figure (4.2-2b): depth-of-focus vs. waist for a DoG pupil 
\( \beta = 0.9, \text{ fixed } a \text{ and } b \)
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.2-2c): depth-of-focus vs. waist for a DoG pupil
( $\beta = 0.7$, fixed $a$ and $b$ )

Figure (4.2-2d): depth-of-focus vs. waist for a DoG pupil
( $\beta = 0.5$, fixed $a$ and $b$ )
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.2-2e): depth-of-focus vs. waist for a DoG pupil 
\( (\beta = 0, \text{fixed } a \text{ and } b) \)

Figure (4.2-2f): depth-of-focus vs. waist for a DoG pupil 
\( (\beta = -0.5, \text{fixed } a \text{ and } b) \)
CHAPTER 4. Difference-of-Gaussians pupil function

Figure (4.2-2g): depth-of-focus vs. waist for a DoG pupil ($\beta = -1$, fixed $a$ and $b$)

Figure (4.2-2h): depth-of-focus vs. waist for a DoG pupil ($\beta = -10$, fixed $a$ and $b$)
Figure (4.2-3): depth of focus vs. $\beta$ for a DoG pupil with fixed $a$ and $b$

(upper straight line: depth of focus in a Gaussian pupil with $w=0.4\text{mm}$,
lower straight line: depth of focus in a Gaussian pupil with $w=0.6\text{mm}$)
Figure (4.2-4): Interaction of sound and light in AOM
Figure (4.2-5): intensity of two diffracted orders vs. $\hat{\alpha}$ in Bragg regime
Figure (4.2-6): depth of focus vs. $b/a$ for a DoG pupil with $\beta=0.7$
(upper straight line: depth of focus in a Gaussian pupil with $w=0.4\text{mm}$,
lower straight line: depth of focus in a Gaussian pupil with $w=0.6\text{mm}$)
Figure (4.2-7): Cross-sections of the PSF for a DoG pupil
(wu = 0.6mm, wv = 0.4mm, \( \beta = 0.7, a = wu \cdot \sqrt{\pi/2}, b = a^2 \))
Figure (4.2-8): depth of focus vs. $\beta$ for a DoG pupil with $b/a = 2$

(upper straight line: depth of focus in a Gaussian pupil with $w=0.4\text{mm}$, lower straight line: depth of focus in a Gaussian pupil with $w=0.6\text{mm}$)
Figure (4.2-9): depth-of-focus vs. waist for a DoG pupil
\( (\beta = 0.78, \ b/a = 2 ) \)
Figure (4.2-10): cross-sections of the PSF for a DoG pupil
($\beta = 0.78$, $b/a = 2$, $w_u = 0.6$ mm, $w_v = 0.32$ mm)
Chapter 5
Conclusion

We have presented a scanning technique employing an optical heterodyne scanning system using a DoG pupil. The proposed system with a DoG pupil has been compared with the system of other pupils, and has been explained quantitatively and qualitatively. Some other interesting properties have been also presented.

To achieve our main goal that is to increase the depth-of-focus, we have speculated that the depth-of-focus of DoG pupil should be largest among other pupil functions being investigated, except annular pupil. However, as was seen in table (3.2-1) or figure (3.2-6a), the depth-of-focus of DoG pupil is smaller than that of Gaussian pupil when arbitrary parameters are used. It is required that the DoG pupil offer moderate improvement over the Gaussian pupils, since the depth-of-focus of the annular pupil is very long and that of the circular pupil is smallest. The important factors in this approach are the parameters. The parameters $a$ and $b$ are associated with the intensity of laser light, the parameters $w_u$ and $w_v$ are the waists of the two Gaussian beams, and the parameter $\beta$ is the gain of the AC amplifier.

When $a$ and $b$ have the fixed values controlled by eq. (2.3-12) and eq. (2.3-13), those were called ‘fixed $a$ and $b$ parameters’. The term ‘fixed $a$ and $b$ parameters’ means that those parameters are not arbitrary, but specified ones, since those are determined by the assumption that the two Gaussian functions consisting of the DoG pupil have the same intensity. With fixed $a$ and $b$ parameters and in some range of $\beta$, a typical $w_v$ gives the depth-of-focus a peak at the specified $w_u$ [See figure (4.2-2e) to figure (4.2-2h)]. That property makes it easy to pick optimized values of $w_u$ and $w_v$. When $a$ and $b$ parameters are fixed, figure (4.2-3) shows an intensity result. As $\beta$ increases, the depth-of-focus becomes smaller. For $\beta \geq 0.6$, the proposed system might work like the scanning confocal microscopy system that has a very small depth-of-
focus. But, note that PSF does not have a Gaussian profile any more if $\beta$ is close to or larger than 1. Therefore, it is better to choose $\beta$ that is close to but smaller than 1 in order to make the proposed system work like the scanning confocal microscopy system.

In chapter 4.2, $a$ and $b$ parameters are varied. Figure (4.2-6) shows that the modification of $a$ and $b$ makes the depth-of-focus of the DoG pupil larger than that of the Gaussian pupil and that the depth-of-focus is at its peak when $b/a$ is $\sim 2$ and other parameters are arbitrarily selected. With the value of 2 for $b/a$, $\beta$ and the relation between $w_u$ and $w_v$ also give the depth-of-focus a peak. Therefore, when the following parameters are used, we have obtained the largest depth-of-focus among other pupil systems, except annular pupil.

$$b/a = 2, \quad \beta = 0.78, \quad w_u = 0.6 \text{ mm}, \quad w_v = 0.32 \text{ mm}$$

However, as was seen in figure (4.2-10), the PSF does not have a Gaussian profile with the above parameters. However, there is a range of parameters that can make the PSF stay in a tolerable Gaussian profile.

There are two natural extensions of this work. The first is that the characteristics of the depth-of-focus should be analyzed and explained carefully with the parameters of a non-fixed $a$ and $b$ to find the desirable range of parameters. In the second extension, the proposed scanning system should be optically implemented with the present results. Though it is difficult to modify $w_u$ and $w_v$ in the optical setup, we can easily change $a$, $b$, and $\beta$ parameters and tune the depth-of-focus in that those parameters are associated with the electrical equipments.
Bibliography

2. G. Hausler, "A method to increase the depth of focus by two step image processing," *Optics Communications*, 6, 38-42 (1972)
Appendix A

Program Listings

MATLAB codes are included on the following pages. To obtain the proper output or figures, each program needs its own parameters which are mentioned beforehand in the context.
APPENDIX A. PROGRAM LISTINGS

A.1  circularplot.m

=====================================================================%
Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
For normalization
R = 1.2e-3;
NA = R/sqrt(f^2+R^2)
delX = 0.61*lambda/NA;
delZ = 2*lambda/NA^2;

wu = 0.3e-3;
zNum = 64;
zm = 2e-3;
xNum = 256/4;
xm = 2e-3;
dz = 2*zm/zNum;
zRange = -zm:dz:zm;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2)/(dx*xNum)*lambda*f;

psf = [];
q = [];
zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        q(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,1) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x.^2));
        xi = xi + 1;
    end
    h = abs(fft2(q)).^2;
    h = fftshift(h);
    psf(zi,:) = h(xNum/2+1,:);
    zi = zi + 1;
end

psf = psf/max(max(abs(psf)));
otf = abs(fft2(psf)).^2;
otf = fftshift(otf);
otf = otf/max(max(abs(otf)));

=====================================================================
APPENDIX A. PROGRAM LISTINGS

fzr1=fzr/delZ;fxr1=fxr/delX;

figure;
% %psf' means the transpose matrix of the psf.
% pcolor(fzr1,fxr1,psf'.^0.3);
colormap('gray');
shading interp; % It shows smooth shading.
xlabel('zeta (z/Deltaz)');ylabel('rho (r/Deltax)');title('PSF');

figure;
pcolor(fzr1,fxr1,otf'.^0.3);
colormap('gray');
shading interp; % It shows smooth shading.
xlabel('zeta (z/Deltaz)');ylabel('rho (r/Deltax)');title('OTF');

figure;
subplot(2,1,1);
plot(fzr1,psf(:,xNum/2+1)');
xlabel('zeta (z/Deltaz)');ylabel('psf');
grid on;

subplot(2,1,2);
plot(fxr1,psf(zNum/2+1,:));
xlabel('rho (r/Deltax)');ylabel('psf');
grid on;

[m2 num2] = max(psf(:,xNum/2+1));
z0 = abs(psf(:,xNum/2+1) - m2 * exp(-1));
[zw1 num] = min(z0);
zwCirE1 = abs(fzr(num))

z0 = abs(psf(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwCirAtHalf = abs(fzr(num))*2

[m2 num2] = max(psf(zNum/2+1,:));
x0 = abs(psf(zNum/2+1,:) - m2/2);
[xw1 num] = min(x0);
xwCir = abs(fxr(num))*2

% % to save specific variables in mat-format file
APPENDIX A. PROGRAM LISTINGS

```matlab
% %savefile = 'test.mat';
%p = rand(1,10);
%q = ones(10);
%save(savefile,'p','q')
%
%
% 1. psf(0,z)
%
% % SINC Sin(pi*x)/(pi*x) function.
% % different from the form used usually.
% % argument inside sinc fn should be divided by pi.
% psfEqAlongZ = (sinc(zRange*R^2/(2*f^2*lambda))).^2;

figure;
h1=plot(fzr1,psf(:,xNum/2+1)','r',fzr1,psfEqAlongZ,'b');
legend(h1,'by simulation','by equation');xlabel('zeta   (z/Deltaz)');grid;
title('PSF along z for circular aperture');

% 2. psf(r,0)
% besselj(a,b) : a means order(if a=0, zeroth order), b is a variable
% See intro. E-optics, p. 105 Fig 3.19.
% The value at jeta = 0 becomes (some constant / the value at jeta = 1).

k=1;
for x = xRange,
    if x == 0
        psfEqAlongR(k) = 1;
    else
        psfEqAlongR(k) = (2*besselj(1,2*pi*R*x/lambda/f) / (2*pi*R*x/lambda/f) )^2;
    end
    k = k+1;
end

figure;
h1=plot(fxr1,psf(zNum/2+1,:),r',xRange/delX,psfEqAlongR,'b');
legend(h1,'by simulation','by equation');xlabel('rho    (r/Deltax)');grid;a=axis;axis([-5 5 a(3:4)]);
title('PSF along r for circular aperture');

% %to make Airy pattern
```
%  
k = 1;
xRan = linspace(-3,3,300);  
for x = xRan,  
    if x == 0  
        psfEqAlongR2(k) = 1;  
    else  
        psfEqAlongR2(k) = (2*besselj(1,pi*x) / (pi*x) )^2;  
    end  
    k = k+1;  
end  

figure;  
plot(xRan,psfEqAlongR2);  
title('Airy pattern');xlabel('\zeta');grid;  

save cPsf.dat psf -ascii;  
save cOtf.dat otf -ascii;  
%save fzr.dat fzr -ascii;
A.2  annuplot.m

=====================================================================  
% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%
% For normalization
R = 0.03e-3; % 1.2e-3
NA = R/sqrt(f^2+R^2)
delX = 0.6*lambda/NA;
delZ = 2*lambda/NA^2;
eta = 0.95;
%
wu = 0.3e-3;

zNum = 64;
zm = 3e-3;

xNum = 256;
xm = .2e-3;

dz = 2*zm/zNum;
zRange = -zm:dz:zm;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2)/(dx*xNum)*lambda*f;

psf = [];
q = [];

zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        q(xi,:) = p2(xRange/xm,x/xm,eta,xm,R,2).* exp(i*k0*z/(2*f^2)*(xRange.^2 + x.^2));
        xi = xi + 1;
    end
    h = abs(fft2(q)).^2;
h = fftshift(h);
psf(zi,:) = h(xNum/2+1,:);
zi = zi + 1;
end

psf = psf/max(max(abs(psf)));
.otf = abs(fft2(psf)).^2;
.otf = fftshift(otf);
.otf = otf/max(max(abs(otf)));

fzr1=fzr/delZ;fxr1=fxr/delX;

figure;

% %psf' means the transpose matrix of the psf.
% pcolor(fzr1,fxr1,psf'.^0.3);
colormap('gray');
shading interp; % It shows smooth shading.
xlabel('\zeta   (z/\Deltaz)');ylabel('\rho    (r/\Deltax)');title('PSF');

figure;
pcolor(fzr1,fxr1,otf'.^0.3);
colormap('gray');
shading interp; % It shows smooth shading.
xlabel('\zeta   (z/\Deltaz)');ylabel('\rho    (r/\Deltax)');title('OTF');

figure;
subplot(2,1,1);
plot(fzr1,psf(:,xNum/2+1)');
xlabel('\zeta   (z/\Deltaz)');
ylabel('psf');
grid on;

subplot(2,1,2);
plot(fxr1,psf(zNum/2+1,:).^0.3);
xlabel('\rho    (r/\Deltax)');
ylabel('psf');
grid on;

[m2 num2] = max(psf(:,xNum/2+1));
z0 = abs(psf(:,xNum/2+1) - m2 * exp(-1));
[zw1 num] = min(z0);
zwAnnuE1 = abs(fzr(num))
z0 = abs(psf(:,xNum/2+1) - m2/2);
zw1 num = min(z0);
zwAnnuAtHalf = abs(fzr(num))*2

m2 num2 = max(psf(zNum/2+1,:));
x0 = abs(psf(zNum/2+1,:) - m2 /2);
xw1 num = min(x0);
xwAnnu = abs(fxr(num))*2

psfEqAlongZ = (sinc(R^2*(1-eta^2)*zRange/(2*f^2*lambda))).^2;
psfEqAlongZ = psfEqAlongZ/max(psfEqAlongZ);

figure;
h1=plot(fzr1,psf(:,xNum/2+1)','r',fzr1,psfEqAlongZ,'b');
legend(h1,'by simulation','by equation');xlabel('\zeta   (z/\Delta z)');grid;

% besselj(a,b) : a means order(if a=0, zeroth order), b is a variable
k=1;
% See the Aiery pattern in the intro. E-optics, p. 105 Fig 3.19.
% The value at jeta = 0 becomes (some constant / the value at jeta = 1).
% I choose 1 at x==0 according to the expression in the textbook.
% See middleReportJuly232001.doc
for x = xRange,
    if x == 0
        psfEqAlongR(k) = (1-eta^2)^2;
    else
        temp1 = 2*besselj(1,2*pi*R*x/lambda/f) / (2*pi*R*x/lambda/f);
        temp2 = eta^2*2*besselj(1,2*pi*R*eta*x/lambda/f) / (2*pi*eta*R*x/lambda/f);
        psfEqAlongR(k) = (temp1-temp2)^2;
    end
    k = k+1;
end
psfEqAlongR = psfEqAlongR/max(psfEqAlongR);

figure;
h1=plot(fxr1,psf(zNum/2+1,:),'r',xRange/delX,psfEqAlongR,'b');
legend(h1,'by simulation','by equation');xlabel('\rho    (r/\Delta x)');grid;
a=axis;axis([-6 6 a(3:4)]);

save aPsf.dat psf -ascii;
save aOtf.dat otf -ascii;
save fzr.dat fzr -ascii;

=====================================================================
A.3 gaussplot.m

close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%

% For normalization
R = 6e-3;
NA = R/sqrt(f^2+R^2);
delX = 0.6*lambda/NA;
delZ = 2*lambda/NA^2;
%

wu = .4e-3;

zNum = 64*2;
zm = 0.3e-3;

xNum = 256/2;
xm = 6e-3;

dz = 2*zm/zNum;
zRange = -zm:dz:zm;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2)/(dx*xNum)*lambda*f;

psf = [];
q = [];

zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        % In stead of using 'p2' fn,
        % Start of substitute of 'p2'
        %
\%rho2 = xRange.^2 + x^2;
\%q(xi,:) = sqrt(2/pi)/wu* exp(-rho2/wu^2).* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));

\%
\% End of substitute of 'p2'
\%

q(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,3) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
xi = xi + 1;
end

h = abs(fft2(q)).^2;
h = fftshift(h);
psf(zi,:) = h(xNum/2+1,:);
zi = zi + 1;
end

maxPsf = max(max(abs(psf)))
psf = psf/max(max(abs(psf)));

otf = abs(fft2(psf)).^2;
otf = fftshift(otf);
otf = otf/max(max(abs(otf)));

fzr1=fzr/delZ;fxr1=fxr/delX;

figure;
%
\%psf' means the transpose matrix of the psf.
%
pcolor(fzr1,fxr1,psf'.^0.3);
colormap('gray');
shading interp;  \% It shows smooth shading.
xlabel("\zeta   (z/\Deltaz)'");
ylabel("\rho    (r/\Deltax)'");title('PSF');

figure;
pcolor(fzr1,fxr1,otf'.^0.3);
colormap('gray');
shading interp;  \% It shows smooth shading.
xlabel("\zeta   (z/\Deltaz)'");ylabel("\rho    (r/\Deltax)'");title('OTF');

figure;
subplot(2,1,1);
plot(fzr1,psf(:,xNum/2+1)');
xlabel('\(\zeta\) (z/\Delta z)');
ylabel('psf');
grid on;

subplot(2,1,2);
plot(fxr1,psf(zNum/2+1,:).^0.3);
xlabel('\(\rho\) (r/\Delta x)');
ylabel('psf');
grid on;

[m2 num2] = max(psf(:,xNum/2+1));
z0 = abs(psf(:,xNum/2+1) - exp(-1));
[zw1 num] = min(z0);
zwGaussE1 = abs(fzr(num))

z0 = abs(psf(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwGaussAtHalf = abs(fzr(num))*2

[m3 num3] = max(psf(zNum/2+1,:));
x0 = abs(psf(zNum/2+1,:) - m3*0.5);
[xw1 num] = min(x0);
xwGauss = abs(fxr(num))*2

save gPsf.dat psf -ascii;
save gOtf.dat otf -ascii;
%save fzr.dat fzr -ascii;
close all;
clear all;

%  
% 1. R=4e-3, zNum=64, zm=0.05e-3, xm=4e-3, xNum=256*2  
% wu=2e-3, wv=1.5e-3 beta=0.5 for enough x-resolution  
%  
% 2. R=1e-3, zNum=64, zm=0.6e-3, xm=2e-3, xNum=256/2  
% wu=0.3e-3, wv=0.2e-3, beta=-1: previous parameters.  
%  
% Basic coefficient  
lambda = 0.6e-6;  
k0 = 2*pi/lambda;  
f = 10e-3;  
%  
% For normalization  
R = 6e-3;  
NA = R/sqrt(f^2+R^2);  
delX = 0.61*lambda/NA;  
delZ = 2*lambda/NA^2;  
%  
zNum = 64*2;  
zm = 0.3e-3;  

xNum = 256/2;  
xm = 6e-3;  

wu = 0.6e-3;  
wv = 0.32e-3;  

beta = 0.78;  
a = wu * sqrt(pi/2);  
%b = wv * sqrt(pi/2);  
b = a*2;  

dz = 2*zm/zNum;  
zRange = -zm:dz:zm;
%fzr = (-zNum/2:zNum/2-1) * (1/dz) / zNum * f * lambda;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2) * (1/dx) / xNum * f * lambda;

psf = []; q = []; v1 = []; u1 = []; flag = 3; % This '3' means dog.

zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        % In stead of using 'p2' fn,
        % Start of substitute of 'p2'
        %
        %rho2 = xRange.^2 + x^2;
        %u1(xi,:) = sqrt(2/pi)/wu * exp(-rho2/wu^2) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
        %v1(xi,:) = sqrt(2/pi)/wu * exp(-rho2/wv^2) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
        
        %
        % End of substitute of 'p2'
        %
        u1(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
        v1(xi,:) = p2(xRange/xm,x/xm,wv,xm,R ,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
    xi = xi + 1;
end
u1_1 = fftshift(fft2(u1));
v1_1 = fftshift(fft2(v1));
acPart = 2*a*b*beta*abs(u1_1) .* abs(v1_1);
h = a^2*(abs(u1_1)).^2 + b^2*(abs(v1_1)).^2 - acPart;
psf(zi,:) = h(xNum/2+1,:);
psfAc(zi,:) = acPart(xNum/2+1,:);
zi = zi + 1;
end
maxPsf = max(max(abs(psf)))
maxPsfAc = max(max(abs(psfAc)))
maxPsfDc = maxPsf - maxPsfAc
psf = psf/maxPsf;
otf = abs(fft2(psf)).^2;
otf = fftshift(otf);
otf = otf/max(max(abs(otf)));

fzr1=fzr/delZ;fxr1=fxr/delX;

figure;

%psf' means the transpose matrix of the psf.
%pcolor(fzr1,fxr1,real(psf.^0.3));
colormap('gray');
shading interp;  % It shows smooth shading.
xlabel('\zeta   (z/\Deltaz)');
ylabel('\rho    (r/\Deltax)');title('PSF');

figure;
mesh(fzr1,fxr1,psf');
shading interp;
xlabel('\zeta   (z/\Deltaz)');
ylabel('\rho    (r/\Deltax)');title('PSF');

figure;
 pcolor(fzr1,fxr1,otf'.^0.3);
colormap('gray');
shading interp;  % It shows smooth shading.
xlabel('\zeta   (z/\Deltaz)');
ylabel('\rho    (r/\Deltax)');title('OTF');

figure;
 subplot(2,1,1);
 plot(fzr1,psf(:,xNum/2+1)');
xlabel('\zeta   (z/\Deltaz)');
ylabel('psf');
grid on;

subplot(2,1,2);
 plot(fxr1,psf(zNum/2+1,:).^0.3);
xlabel('\rho    (r/\Deltax)');
ylabel('psf');
grid on;

[m2 num2] = max(psf(:,xNum/2+1));
 z0 = abs(psf(:,xNum/2+1) - m2 * exp(-1));
 [zw1 num] = min(z0);
zwDogE1 = abs(fzr(num));
z0 = abs(psf(:,xNum/2+1) - m2/2);  
zw1 num] = min(z0);  
zwDogAtHalf = abs(fzr(num)) / 2

[m2 num2] = max(psf(zNum/2+1,:));  
x0 = abs(psf(zNum/2+1,:) - m2 / 2);  
xw1 num] = min(x0);  
xwDog = abs(fxr(num)) / 2

% to verify that the result by this simulation and the result by the equation are same.
%

% 1. psf(r,0)
%

w10 = 2*f/k0/wu;  
w20 = 2*f/k0/wv;  
zr1 = 2*f^2/k0/wu^2;  
zr2 = 2*f^2/k0/wv^2;  
R1 = zr1^2/2;  
R2 = zr2^2/2;

% xRange may be replaced by fxr1 and xRange/delX in plot fn may be also replaced by fxr1.
% U_r = 2*sqrt(2*pi)*f/k0/w10 * exp(-xRange.^2/w10^2-i*k0*xRange.^2/2/R1);  
V_r = 2*sqrt(2*pi)*f/k0/w20 * exp(-xRange.^2/w20^2-j*k0*xRange.^2/2/R2);

psfEqAlongR = a^2*(abs(U_r)).^2 + b^2*(abs(V_r)).^2 - 2*a*b*beta*abs(U_r).*abs(V_r);  
psfEqAlongR = psfEqAlongR/max(psfEqAlongR);

figure;
h1=plot(fxr1,psf(zNum/2+1,:),'r',xRange/delX,psfEqAlongR,'b');  
legend(h1,'by simulation','by equation');xlabel('rho (r/Deltax)');grid;  
aa=axis;axis([-5 5 aa(3:4)]);  
title('PSF along r for DOG');

% 2. psf(0,z)
%

w1z = w10*sqrt(1+(zRange/zr1).^2);  
w2z = w20*sqrt(1+(zRange/zr2).^2);  
U_z = 2*sqrt(2*pi)*f/k0./w1z;
APPENDIX A. PROGRAM LISTINGS

\[ V_z = 2\sqrt{2\pi} f' k_0 / w_2 z; \]

\[
\text{psfEqAlongZ} = a^2 \cdot (\text{abs}(U_z))^2 + b^2 \cdot (\text{abs}(V_z))^2 - 2 \cdot a \cdot b \cdot \beta \cdot (\text{abs}(U_z)) \cdot (\text{abs}(V_z));
\]

\[
\text{psfEqAlongZ} = \frac{\text{psfEqAlongZ}}{\max(\text{psfEqAlongZ})};
\]

figure;
\text{h1} = \text{plot}(fzr1, \text{psf(:, xNum/2+1)}', 'r', fzr1, \text{psfEqAlongZ}, 'b');
\text{legend(h1,} \text{by simulation', 'by equation');xlabel('} \zeta (z/Deltaz');grid;
title('PSF along z for DOG');

save dPsf.dat psf -ascii;
save dOtf.dat otf -ascii;
% save fzr.dat fzr -ascii;
close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%

% For normalization
R = 1e-3;

NA = R/sqrt(f^2+R^2)
delX = 0.61*lambda/NA;
delZ = 2*lambda/NA^2;
%

wu = 0.3e-3;

zNum = 64;
zm = 0.6e-3;

xNum = 256/2;
xm = 2e-3;
%

dz = 2*zm/zNum;
zRange = -zm:dz:zm;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2)/(dx*xNum)*lambda*f;

fzr1 = fzr/delZ; fxr1 = fxr/delX;

load dPsf.dat;
load dOtf.dat;
load gPsf.dat;
load gOtf.dat;
load aPsf.dat;
load aOtf.dat;
load cPsf.dat;
load cOtf.dat;

figure;
h1=plot(fzr1,dPsf(:,xNum/2+1)',fzr1,gPsf(:,xNum/2+1)',fzr1,aPsf(:,xNum/2+1)',fzr1,cPsf(:,xNum/2+1)');
legend(h1,'DOG','Gaussian','Annular','Circular');
xlabel('\zeta   (z/\Delta z)');ylabel('PSF');grid on;title('PSF along z');

figure;
h2=plot(fxr1,dPsf(zNum/2+1,:),fxr1,gPsf(zNum/2+1,:),fxr1,aPsf(zNum/2+1,:),fxr1,cPsf(zNum/2+1,:));
legend(h2,'DOG','Gaussian','Annular','Circular');
xlabel('\rho    (r/\Delta x)');ylabel('PSF');grid on;title('PSF along r');
a = axis; axis([-6 6 a(3:4)]);
grid on;

figure;
h3=plot(fzr1,dOtf(:,xNum/2+1)',fzr1,gOtf(:,xNum/2+1)',fzr1,aOtf(:,xNum/2+1)',fzr1,cOtf(:,xNum/2+1)');
legend(h3,'DOG','Gaussian','Annular','Circular');
xlabel('\zeta   (z/\Delta z)');ylabel('OTF');grid on;title('OTF along z');
a = axis; axis([-4 4 a(3:4)]);
grid on;

figure;
h4=plot(fxr1,dOtf(zNum/2+1,:),fxr1,gOtf(zNum/2+1,:),fxr1,aOtf(zNum/2+1,:),fxr1,cOtf(zNum/2+1,:));
legend(h4,'DOG','Gaussian','Annular','Circular');
xlabel('\rho    (r/\Delta x)');ylabel('OTF');grid on;title('OTF along r');
%a = axis; axis([-4 4 a(3:4)]);
grid on;
A.6  plotGaussAnnulGraph.m

\begin{verbatim}
gauss = [421.9 234.4 156.25 140.6 125 109.4];
annul = [172 188 203 234 297 391 563 1100 1700];
gRange = 0.3:0.1:0.8;
aRange = [0.6:0.05:0.95 0.97];

figure;
plot(gRange, gauss);
xlabel('w (mm)'); ylabel('depth of focus(um)'); grid on;
title('DOF vs. w in gaussian aperture');

figure;
plot(aRange, annul);
xlabel('\epsilon'); ylabel('depth of focus(um)'); grid on;
title('DOF vs. \epsilon in annular aperture');
\end{verbatim}
A.7  plotWithChangedNA.m

```matlab
xNum = 256/4;
zNum = 64;

R = 0.2:0.2:1.2;
NA = [0.02 0.04 0.0599 0.0797 0.0995 0.1191];

load psf1.dat;
load psf2.dat;
load psf3.dat;
load psf4.dat;
load psf5.dat;
load psf6.dat;
load fzr.dat;

figure;

h1 = plot(fzr*10^6,psf1(:,xNum/2+1)',fzr*10^6,psf2(:,xNum/2+1)',fzr*10^6,psf3(:,xNum/2+1)',fzr*10^6,psf4(:,xNum/2+1)',fzr*10^6,psf5(:,xNum/2+1)',fzr*10^6,psf6(:,xNum/2+1)');
legend(h1,'R=0.2mm  NA=0.02','R=0.4mm  NA=0.04','R=0.6 mm  NA=0.0599','R=0.8mm  NA=0.0797','R=1mm   NA=0.0995','R=1.2mm  NA=0.1191');
xlabel('z (um)');ylabel('psf');grid on;title('DOG');

[m2 num2] = max(psf1(:,xNum/2+1));
z0 = abs(psf1(:,xNum/2+1) - m2/2);
zwCirAtHalf(1) = abs(fzr(num))*2;

[m2 num2] = max(psf2(:,xNum/2+1));
z0 = abs(psf2(:,xNum/2+1) - m2/2);
zwCirAtHalf(2) = abs(fzr(num))*2;

[m2 num2] = max(psf3(:,xNum/2+1));
z0 = abs(psf3(:,xNum/2+1) - m2/2);
zwCirAtHalf(3) = abs(fzr(num))*2;

[m2 num2] = max(psf4(:,xNum/2+1));
z0 = abs(psf4(:,xNum/2+1) - m2/2);
zwCirAtHalf(4) = abs(fzr(num))*2;
```

[m2 num2] = max(psf5(:,xNum/2+1));
z0 = abs(psf5(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwCirAtHalf(5) = abs(fzr(num))*2;

[m2 num2] = max(psf6(:,xNum/2+1));
z0 = abs(psf6(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwCirAtHalf(6) = abs(fzr(num))*2;

figure;
plot(NA,zwCirAtHalf*10^6);
xlabel('NA');ylabel('depth of focus (um) as FWHM');title('NA vs. FWHM in DOG');grid on;
A.8 annuUncertaintion.m

```matlab
lambda = 0.6;
eps = 0.1:0.01:0.97;
f = 10e-3;
flag = 1;
NA = 0.0995*flag;
R = sqrt(NA^2*f^2/(1-NA^2));

disp(['NA = ',num2str(NA),'             R = ',num2str(R*10^3),'  (mm)']);

figure;
subplot(2,1,1);
dzann1 = lambda./((sqrt(1-NA^2)./sqrt(1-(NA^2)*(1-eps.^2)))-sqrt(1-NA^2));
plot(eps,dzann1,'r-');
title(["NA='",num2str(NA)]);xlabel('\epsilon(Obscuration ratio)');ylabel('\Delta z for annular aperture (\mum)');
grid on;hold on;
dzanns1 = 2*lambda./((NA.^2)*(1-eps.^2));
plot(eps,dzanns1,'b-');

subplot(2,1,2);
NA2 = 0.995;
dzann2 = lambda./[(sqrt(1-NA2^2))./(sqrt(1-(NA2^2)*(1-eps.^2)))-sqrt(1-NA2^2)];
plot(eps,dzann2,'r-');
title(["NA='",num2str(NA2)]);xlabel('\epsilon(obscuration ratio)');ylabel('\Delta z for annular aperture (\mum)');
grid on;hold on;
dzanns2 = 2*lambda./((NA2.^2)*(1-eps.^2));
plot(eps,dzanns2,'b-');

figure;
annul = [109.375 109.375 125 125 140.625 171.875 203.125 296.875 562.5 1062.5 1656.25];
aRange = [0.1:0.1:0.9 0.95 0.97];

h1=plot(aRange, annul,'r',eps,dzann1,'b');
legend(h1,'simulation','uncertainty principle');
xlabel('epsilon');ylabel('depth of focus(\mum) as FWHM');grid on;
title(['FWHM vs. \epsilon in annular aperture at NA=','num2str(NA)]);

N = 50;
aRange2 = 0.1:(0.97-0.1)/(N-1):0.97;
zNum = 64*64;
```

zm = 2e-3/flag;
dz = 2*zm/zNum;
zRange = -zm:dz:zm;

delzAnnu = getDelzAnnuAlongObscuration(R,N,f,zRange);

figure;
h2=plot(aRange2, delzAnnu,'r',eps,dzanns1,'b');
legend(h2,'equation','uncertainty principle');
xlabel('epsilon');ylabel('depth of focus(\mum) as FWHM');grid on;
title(['FWHM vs. \epsilon  in annular aperture at NA=',num2str(NA)]);

figure;
h3=plot(aRange2,delzAnnu,'r',aRange,annul,'b',eps,dzanns1,'k');
legend(h3,'equation','simulation','uncertainty principle');
xlabel('epsilon');ylabel('depth of focus(\mum) as FWHM');grid on;
a=axis;axis([a(1:2) 0 2000]);
title(['FWHM vs. \epsilon  in annular aperture at NA=',num2str(NA)]);
function psf = getDelzAnnuAlongObscuration(R,N,f,zRange)

lambda = 0.6e-6;
NA = R/sqrt(f^2+R^2);

eta = 0.1:(0.97-0.1)/(N-1):0.97;

ki = 1;
for k=eta,
    temp = (sinc(R^2*(1-k.^2)*zRange/(2*f^2*lambda))).^2;
    temp = temp/max(temp);
    z0 = abs(temp - 0.5);
    [zw1 num] = min(z0);
    psf(ki) = abs(zRange(num))*2*10^6;
    ki = ki+1;
end
A.10  dogPlotBofA.m

```matlab
% dogPlotNew.m
% plot graphs of wu vs dof at different beta.
%
close all;
clear all;
N=5;
wu = 0.3e-3:.4e-3/(N-1):0.7e-3;
wv = [0.1e-3:(wu(1)*0.9-0.1e-3)/(N-1):wu(1)*0.9;...
   0.1e-3:(wu(2)*0.9-0.1e-3)/(N-1):wu(2)*0.9;...
   0.1e-3:(wu(3)*0.9-0.1e-3)/(N-1):wu(3)*0.9;...
   0.1e-3:(wu(4)*0.9-0.1e-3)/(N-1):wu(4)*0.9;...
   0.1e-3:(wu(5)*0.9-0.1e-3)/(N-1):wu(5)*0.9];
beta = 0.78;
range = 1:N;
temp=0;
for mm = range,
    for kk =range,
        zwd(mm,kk) = findDepthOfFocus(wu(mm),wv(mm,kk),beta);
        temp = temp + 1;
    end
end
h1 = plot(wv(1,:)*10^3,zwd(1,:)*10^6,wv(2,:)*10^3,zwd(2,:)*10^6,wv(3,:)*10^3,zwd(3,:)*10^6,wv(4,:)*10^3,zwd(4,:)*10^6,wv(5,:)*10^3,zwd(5,:)*10^6);
xlabel('w_v [mm]');ylabel('depth of focus [um]');title(['beta = ',num2str(beta)]);grid;
legend(h1,['w_u = ',num2str(wu(1)*10^3),'mm' ],[' w_u = ',num2str(wu(2)*10^3),'mm' ],['w_u = ',num2str(wu(3)*10^3),'mm' ],
    ['w_u = ',num2str(wu(4)*10^3),'mm' ],['w_u = ',num2str(wu(5)*10^3),'mm' ]);```

---

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A.11  findDepthOfFocus.m

```matlab
% This fn is used by dogPlotNew.m
% calculate the dof of dog
%

function z = findDepthOfFocus(wu,wv,beta)

lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;

zNum = 64*2;
zm = 0.3e-3;  % 1e-3
xNum = 256/2;
xm = 6e-3;

R = 6e-3;
NA = R/sqrt(f^2+R^2);

a = wu * sqrt(pi/2);
%b = wv * sqrt(pi/2);
b = a*2;

dz = 2*zm/zNum;
zRange =  -zm:dz:zm;
fzr = (-zNum/2:zNum/2-1) * (1/dz) / zNum * f * lambda;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2) * (1/dx) / xNum * f * lambda;

psf = [];
q = [];
v1 = [];
u1 = [];
ans = [];

zi = 1;
for z = zRange,
    xi = 1;
    for x = xRange,
        u1(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,3) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x.^2));
    end
    zi = zi + 1;
end
```

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v1(xi,:) = p2(xRange/xm,x/xm,wv,xm,R,3).*exp(j*k0*z/(2*f^2)*(xRange.^2 + x.^2));  
    xi = xi + 1;  
end  
u1_1 = fftshift(fft2(u1));  
v1_1 = fftshift(fft2(v1));  
h = a^2*abs(u1_1).^2 + b^2*abs(v1_1).^2 - 2*a*b*beta*abs(u1_1).*abs(v1_1);  
    %h = fftshift(h);  
psf(zi,:) = h(xNum/2+1,:);  
    zi = zi + 1;  
end  
psf = psf/max(max(abs(psf)));  

[m2 num2] = max(psf(:,xNum/2+1));  
z0 = abs(psf(:,xNum/2+1) - m2/2);  
[zw1 num] = min(z0);  

z = abs(fzr(num))*2;
A.12  gaussBetaVsDof.m

close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%

% For normalization
R = 6e-3;
NA = R/sqrt(f^2+R^2);
delX = 0.6*lambda/NA;
delZ = 2*lambda/NA^2;
%

wu = .2e-3:0.1e-3:1e-3;
zNum = 64*2;
zm = 0.3e-3;

xNum = 256/2;
xm = 6e-3;

dz = 2*zm/zNum;
zRange = -zm:dz:zm;
fzr = zRange;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
fxr = (-xNum/2:xNum/2)/(dx*xNum)*lambda*f;

psf = [];
q = [];

wi = 1;
for wuTemp = wu,
  zi = 1;
  for z = zRange,
    xi = 1;
    for x = xRange,
      % In stead of using 'p2' fn,
      % Start of substitute of 'p2'

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%rho2 = xRange.^2 + x^2;
q(xi,:) = sqrt(2/pi)/wu * exp(-rho2/wu^2).* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));

% End of substitute of 'p2'
q(xi,:) = p2(xRange/xm,x/xm,wuTemp,xm,R,3) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));

xi = xi + 1;
end
h = abs(fft2(q)).^2;
h = fftshift(h);
psf(zi,:) = h(xNum/2+1,:);
zi = zi + 1;
end
maxPsf = max(max(abs(psf)));
psf = psf/max(max(abs(psf)));
[m2 num2] = max(psf(:,xNum/2+1));
z0 = abs(psf(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwGaussAtHalf(wi) = abs(fzr(num))*2;
wi = wi + 1
end

plot(wu*10^3,zwGaussAtHalf*10^6);
xlabel('w (mm)');ylabel('depth of focus(um)');grid on;
title('DOF vs. w in gaussian aperture');
A.13 dogOptimizedBeta.m

```matlab
% dogOptimizedBeta.m
% a and b are fixed and beta is changed.
% beta vs dof
% 

close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%
% For normalization
R = 6e-3;
NA = R/sqrt(f^2+R^2);
delX = 0.61*lambda/NA;
delZ = 2*lambda/NA^2;
%

zNum = 64*2;
zm = 0.3e-3;

xNum = 256/2;
xm = 6e-3;

wu = 0.6e-3;
wv = 0.4e-3;

a = wu * sqrt(pi/2);
b = a*2;

dz = 2*zm/zNum;
zRange = -zm:dz:zm;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;

flag = 3; % This '3' means dog.

bMin = -1;
```
bMax = 1;
bNum = 10;
bRange = bMin:(bMax-bMin)/(bNum-1):bMax;
yi = 1
for beta = bRange,
    zi = 1;
    for z = zRange,
        xi = 1;
        for x = xRange,
            u1(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
            v1(xi,:) = p2(xRange/xm,x/xm,wv,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
            xi = xi + 1;
        end
        u1_1 = fftshift(fft2(u1));
        v1_1 = fftshift(fft2(v1));
        h = a^2*(abs(u1_1)).^2 + b^2*(abs(v1_1)).^2 - 2*a*b*beta*abs(u1_1) .* abs(v1_1);
        psf(zi,:) = h(xNum/2+1,:);
        zi = zi + 1;
    end
    psf = psf/max(max(abs(psf)));
    [m2 num2] = max(psf(:,xNum/2+1));
    z0 = abs(psf(:,xNum/2+1) - m2/2);
    [zw1 num] = min(z0);
    zwDogAtHalf(yi) = abs(zRange(num))*2;
    yi = yi + 1
end
ga1 = 103*ones(size(bRange));
ga2 = 234*ones(size(bRange));
plot(bRange,zwDogAtHalf*10^6,bRange,ga1,bRange,ga2);;
xlabel('\beta');ylabel('depth of focus as FWHM [um]');
title('\beta vs depth of focus');
A.14  dogOptimizedAandBforOneBeta.m

```matlab
% dogOptimizedAandBforOneBeta.m
% a and beta are fixed and b is changed.
% b/a vs dof at specific beta
%
close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%
% For normalization
R = 6e-3; %1e-3 1.2e-3
NA = R/sqrt(f^2+R^2);
delX = 0.61*lambda/NA;
delZ = 2*lambda/NA^2;
%
zNum = 64*2;
zm = 0.3e-3;
xNum = 256/2;
xm = 6e-3;
wu = .6e-3;
wv = .4e-3;

beta = 0.7;
a = wu * sqrt(pi/2);

dz = 2*zm/zNum;
zRange = -zm:dz:zm;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;

flag = 3; % This '3' means dog.

bMin = a/4;
```
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bMax = a*20;
bNum = 20;
bRange = bMin:(bMax-bMin)/(bNum-1):bMax;

yi = 1
for b = bRange,
  zi = 1;
  for z = zRange,
    xi = 1;
    for x = xRange,
      u1(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
      v1(xi,:) = p2(xRange/xm,x/xm,wv,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
      xi = xi + 1;
    end
    u1_1 = fftshift(fft2(u1));
    v1_1 = fftshift(fft2(v1));
    h = a^2*(abs(u1_1)).^2 + b^2*(abs(v1_1)).^2 - 2*a*b*beta*abs(u1_1) .* abs(v1_1);
    psf(zi,:) = h(xNum/2+1,:);
    zi = zi + 1;
  end
  psf = psf/max(max(abs(psf)));
  [m2 num2] = max(psf(:,xNum/2+1));
  z0 = abs(psf(:,xNum/2+1) - m2/2);
  [zw1 num] = min(z0);
  zwDogAtHalf(yi) = abs(zRange(num))*2;
  yi = yi + 1
end

ga1 = 103*ones(size(bRange));
ga2 = 234*ones(size(bRange));
plot(bRange/a,zwDogAtHalf*10^6,bRange/a,ga1,bRange/a,ga2);

xlabel('b/a');ylabel('depth of focus as FWHM [um]');
title('b/a vs depth of focus');
A.15 dogOptimizedAandB.m

```matlab
% dogOptimizedAandB.m
% 'a' is fixed and beta and 'b' are changed.
% plot graph of b/a vs dof at different beta.
%

close all;
clear all;

% Basic coefficient
lambda = 0.6e-6;
k0 = 2*pi/lambda;
f = 10e-3;
%

% For normalization
R = 1e-3; %1e-3 1.2e-3
NA = R/sqrt(f^2+R^2);
delX = 0.61*lambda/NA;
delZ = 2*lambda/NA^2;
%

zNum = 64*4;
zm = 1.5e-3;

xNum = 256/4;
xm = 2e-3;

wu = 0.3e-3;
wv = 0.2e-3;

betaNum = 11;
betaRange = -1:2/(betaNum-1):1;

a = wu * sqrt(pi/2);
%b = wv * sqrt(pi/2);

dz = 2*zm/zNum;
zRange = -zm:dz:zm;

dx = 2*xm/xNum;
xRange = -xm:dx:xm;
```
flag = 3; % This '3' means dog.

bMin = a/4;
bMax = a*20;
bNum = 20;
bRange = bMin:(bMax-bMin)/(bNum-1):bMax;

for beta = betaRange,
yi = 1
for b = bRange,
  zi = 1;
  for z = zRange,
    xi = 1;
    for x = xRange,
      u1(xi,:) = p2(xRange/xm,x/xm,wu,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
      v1(xi,:) = p2(xRange/xm,x/xm,wv,xm,R,flag) .* exp(i*k0*z/(2*f^2)*(xRange.^2 + x^2));
      xi = xi + 1;
    end
  end
  u1_1 = fftshift(fft2(u1));
  v1_1 = fftshift(fft2(v1));
  h = a^2*(abs(u1_1)).^2 + b^2*(abs(v1_1)).^2 - 2*a*b*beta*abs(u1_1) .* abs(v1_1);
  psf(zi,:) = h(xNum/2+1,:);
  zi = zi + 1;
end
psf = psf/max(max(abs(psf)));
[m2 num2] = max(psf(:,xNum/2+1));
z0 = abs(psf(:,xNum/2+1) - m2/2);
[zw1 num] = min(z0);
zwDogAtHalf(yi) = abs(zRange(num))*2;
yi = yi + 1
end
ga1 = 421*ones(size(bRange));
ga2 = 960*ones(size(bRange));
plot(bRange/a,zwDogAtHalf*10^6,bRange/a,ga1,bRange/a,ga2);
hold on;
pause;
end

xlabel('b/a');ylabel('depth of focus as FWHM [um]');
title('b/a vs depth of focus');
Vita

Dong-Ik Shin was born in Daegu, South Korea, on June 28, 1972. He received the B.S. degree in Electronics Engineering from Kyungpook National University, Daegu, South Korea in 1998. He then joined optical image processing lab, where he has been a research assistant. He is currently pursuing a Master's of Science degree in Electrical Engineering at Virginia Polytechnic Institute and State University.