

**SIMPLIFIED DESIGN OF MOMENT
END-PLATE CONNECTIONS**

by

Eric R. Ober

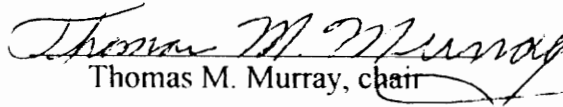
Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the degree of


MASTER OF SCIENCE


IN

CIVIL ENGINEERING

APPROVED:


Thomas M. Murray, chair


W. Samuel Easterling


Don A. Garst

July 1995

Blacksburg, Virginia

LD
3635
V855
1995
3247
012

SIMPLIFIED DESIGN OF MOMENT END-PLATE CONNECTIONS

by

Eric R. Ober

Thomas M. Murray, Chairman
Civil Engineering

(ABSTRACT)

Moment end-plate connections are widely used in the metal building industry and portal frame construction. This is due to the advantages of the field-bolted construction and excellent performance of moment end-plate connections. These connections have been studied extensively. Yield-line analysis has been used to predict the strength of the end-plate, and a modified Kennedy method has been used to predict the strength of the bolts. These analysis procedures have proven quite accurate. The major drawback is that the calculations involved in the modified Kennedy method are long and complex.

In this study, an alternative and simplified method of design for five configurations of moment end-plate is presented. To determine the connection capacity based on bolt strength, the concept of the number of effective bolts is employed. The connection capacity based on the end-plate strength is calculated by making use of the tee stub analogy. Comparisons are made between the simplified method and the yield-line and modified Kennedy based methods. Also, comparisons are made between the simplified method and a method proposed by Borgsmiller and Murray (1995). Design recommendations are given, and design examples are presented.

ACKNOWLEDGEMENTS

The most important people in my life are my family. It is to them that I owe everything that I have accomplished in my life. It is to them that I extend my greatest thanks. Also, I would like to acknowledge Dr. Thomas M. Murray, whose guidance and patience were essential to the completion of this study and my graduate studies. To the other members of my thesis committee, Dr. W. Samuel Easterling and Mr. Don A. Garst, I also extend my thanks for their participation and help. The help of Dr. Richard M. Barker is also appreciated.

TABLE OF CONTENTS

	<u>Page</u>
ABSTRACT.....	ii
ACKNOWLEDGMENTS	iii
TABLE OF CONTENTS.....	iv
LIST OF FIGURES.....	vii
LIST OF TABLES.....	x
CHAPTER I. INTRODUCTION.....	1
1.1 Background.....	1
1.2 Literature Review.....	6
1.3 Scope of Research.....	16
CHAPTER II. OVERVIEW OF PRESENT DESIGN PROCEDURE.....	17
2.1 Yield Line Analysis.....	17
2.1.1 General.....	17
2.1.2 Two-Bolt Flush Unstiffened.....	19
2.1.3 Four-Bolt Flush Unstiffened.....	22
2.1.4 Four-Bolt Extended Unstiffened.....	24
2.1.5 Four-Bolt Extended Stiffened.....	26
2.1.6 Multiple Row Extended Unstiffened 1/3.....	28
2.2 Modified Kennedy Method.....	31
2.2.1 General.....	31
2.2.2 Two Bolt Flush Unstiffened.....	31
2.2.3 Four Bolt Flush Unstiffened.....	33
2.2.4 Four Bolt Extended Unstiffened.....	35
2.2.5 Four Bolt Extended Stiffened.....	38

2.2.6 Multiple Row Extended Unstiffened 1/3.....	38
CHAPTER III. DEVELOPMENT OF SIMPLIFIED DESIGN PROCEDURE.....	42
3.1 General.....	42
3.2 Two-Bolt Flush Unstiffened.....	45
3.3 Four-Bolt Flush Unstiffened.....	45
3.4 Four-Bolt Extended Unstiffened.....	46
3.5 Four-Bolt Extended Stiffened.....	46
3.6 Multiple Row Extended Unstiffened 1/3.....	47
CHAPTER IV. COMPARISON OF RESULTS.....	48
4.1 General.....	48
4.2 Experimental Results.....	48
4.3 Determination of Predicted Capacity.....	52
4.4 Comparisons of Predicted and Experimental Capacities.....	54
4.5 Comparison of the Simplified Method to the Yield-Line and Kennedy Based Methods.....	57
CHAPTER V. SUMMARY, CONCLUSIONS AND RECOMMENDATIONS.....	75
5.1 Summary.....	75
5.2 Conclusions.....	76
5.3 Design Recommendations.....	78
5.4 Design Examples.....	79
REFERENCES.....	87
APPENDIX A. NOMENCLATURE.....	89

APPENDIX B. TWO BOLT FLUSH UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS.....	95
APPENDIX C. FOUR BOLT FLUSH UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS.....	99
APPENDIX D. FOUR BOLT EXTENDED UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS.....	102
APPENDIX E. FOUR BOLT EXTENDED STIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS.....	105
APPENDIX F. MULTIPLE ROW EXTENDED UNSTIFFENED 1/3 END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS.....	108
VITA.....	112

LIST OF FIGURES

<u>Figure</u>	<u>Page</u>
1.1 Typical Uses of Moment End-Plate Connections.....	2
1.2 Example of a Flush End-Plate Configuration.....	3
1.3 Example of an Extended End-Plate Configuration.....	3
1.4 Configurations for Simplified Method.....	5
1.5 Kennedy Split-Tee Analogy.....	7
1.6 Kennedy Split-Tee Behavior.....	8
1.7 Eight-Bolt Extended Stiffened Moment End-Plate.....	13
1.8 Tee Stub Analogy.....	14
2.1 Virtual Displacements in a Two-Bolt Flush Unstiffened End-Plate Configuration.....	20
2.2 Yield-Line Mechanism for Two-Bolt Flush Unstiffened Moment End-Plate.....	21
2.3 Yield-Line Mechanism for Four-Bolt Flush Unstiffened Moment End-Plate.....	23
2.4 Yield-Line Mechanism for Four-Bolt Extended Unstiffened Moment End-Plate.....	25
2.5 Yield-Line Mechanism for Four-Bolt Extended Stiffened Moment End-Plate.....	27
2.6 Yield-Line Mechanisms for Multiple Row Extended Unstiffened 1/3 Moment End-Plate.....	29

2.7	Modifications for Two-Bolt Flush Unstiffened Moment End-Plate.....	32
2.8	Modifications for Four-Bolt Flush Unstiffened Moment End-Plate.....	34
2.9	Modifications for Four-Bolt Extended Unstiffened Moment End-Plate.....	36
2.10	Modifications for Four-Bolt Extended Stiffened Moment End-Plate.....	39
2.11	Modifications for Multiple Row Extended Unstiffened 1/3 Moment End-Plate.....	40
4.1	Longitudinal Elevation of Laboratory Test Setup.....	50
4.2	Cross-section of Laboratory Test Setup.....	51
4.3	Sample of Applied Moment vs. End-Plate Separation Plots.....	53
4.4	Determination of Experimental Yield Moment.....	53
4.5	Two-Bolt Flush Unstiffened - Simplified vs Hendrick <i>et al.</i> (1985): Predicted Bolt Force and End-Plate Capacity.....	60
4.6	Two-Bolt Flush Unstiffened - Simplified vs Hendrick <i>et al.</i> (1985): Calculated Bolt Diameter and Plate Thickness.....	61
4.7	Two-Bolt Flush Unstiffened - Simplified vs Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness.....	62
4.8	Four-Bolt Flush Unstiffened - Simplified vs Hendrick <i>et al.</i> (1985): Predicted Bolt Diameter and End-Plate Capacity.....	63
4.9	Four-Bolt Flush Unstiffened - Simplified vs Hendrick <i>et al.</i> (1985): Calculated Bolt Diameter and Plate Thickness.....	64

4.10	Four-Bolt Flush Unstiffened - Simplified vs Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness.....	65
4.11	Four-Bolt Extended Unstiffened - Simplified vs Abel and Murray (1992): Predicted Bolt Force and End-Plate Capacity.....	66
4.12	Four-Bolt Extended Unstiffened - Simplified vs Abel and Murray (1992): Calculated Bolt Diameter and Plate Thickness.....	67
4.13	Four-Bolt Extended Unstiffened - Simplified vs Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness.....	68
4.14	Four-Bolt Extended Stiffened - Simplified vs Morrison <i>et al.</i> (1985): Predicted Bolt Force and End-Plate Capacity.....	69
4.15	Four-Bolt Extended Stiffened - Simplified vs. Morrison <i>et al.</i> (1985): Calculated Bolt Diameter and Plate Thickness.....	70
4.16	Four-Bolt Extended Stiffened - Simplified vs. Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness.....	71
4.17	Multiple Row Extended Unstiffened 1/3 - Simplified vs. Morrison <i>et al.</i> (1986): Predicted Bolt Force and End-Plate Capacity.....	72
4.18	Multiple Row Extended Unstiffened 1/3 - Simplified vs. Morrison <i>et al.</i> (1986): Calculated Bolt Diameter and Plate Thickness.....	73
4.19	Multiple Row Extended Unstiffened 1/3 - Simplified vs. Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness.....	74
5.1	Design Example 1.....	80
5.2	Design Example 2.....	83

LIST OF TABLES

<u>Table</u>	<u>Page</u>
4.1 Predicted versus Experimental Results.....	55
5.1 Summary of N_{eff} and p_{eff}	77

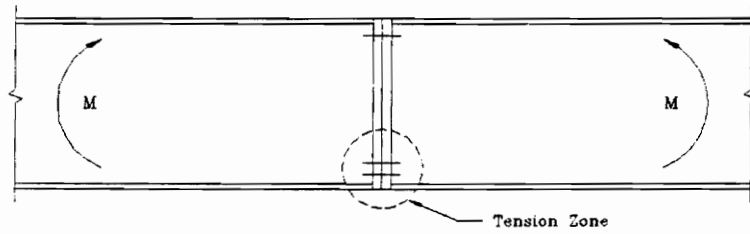
CHAPTER I

INTRODUCTION

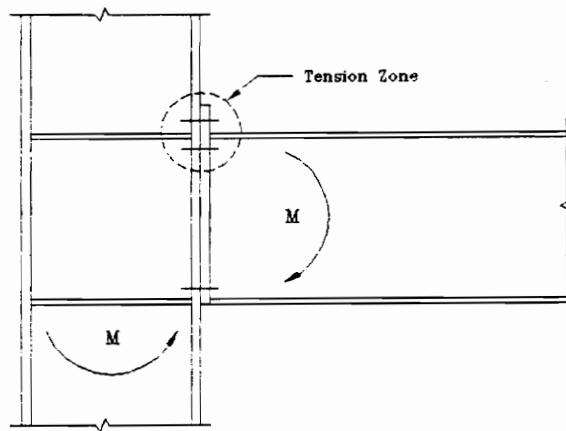
1.1 BACKGROUND

Moment end-plate connections are used extensively in the pre-engineered metal building industry and portal frame construction. They are designed as fully restrained (FR) moment connections. An end-plate is welded to the beam web and flanges and then bolted to the supporting member. These connections are all field bolted, with all welding performed during fabrication in the shop. Their use has also been increasing in the general building industry. This can be attributed to the ease of erection and excellent moment resistance of moment end-plate connections. In general, moment end-plates are used in beam splice connections, Figure 1.1(a), and beam-to-column connections, Figure 1.1(b).

Moment end-plates may be separated into two general classes, flush end-plates and extended end-plates. A flush end-plate has an end-plate that does not extend appreciably beyond the beam flanges, usually just enough for welding purposes (Figure 1.2). Extended end-plates are characterized by the end-plate being extended beyond the beam flanges (Figure 1.3). This allows placement of additional rows of bolts outside of the beam flanges.

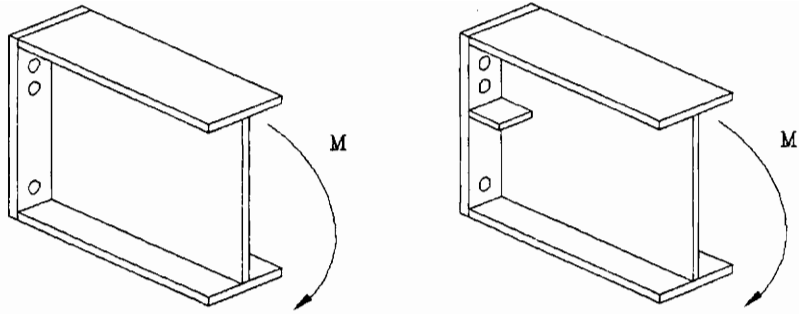


(a) Beam-to-Beam Splice Connection



(b) Beam-to-Column Connection

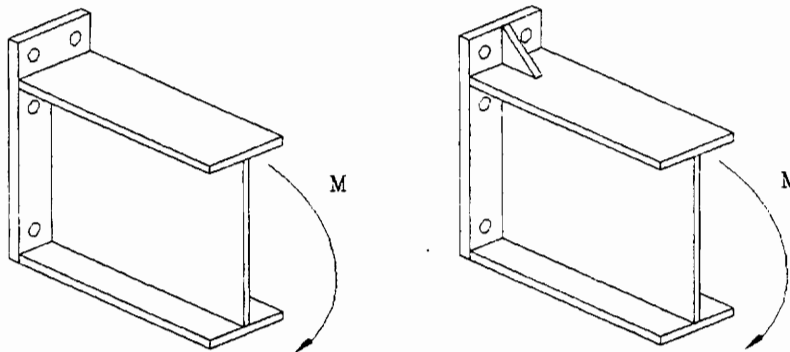
Figure 1.1 Typical Uses of Moment End-Plate Connections



(a) Unstiffened

(b) Stiffened

Figure 1.2 Example of a Flush End-Plate Configuration



(a) Unstiffened

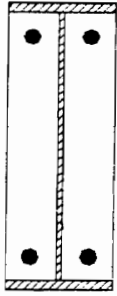
(b) Stiffened

Figure 1.3 Example of an Extended End-Plate Configuration

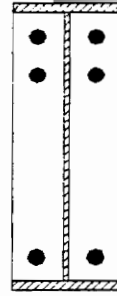
Moment end-plate connections may be further classified by whether they are stiffened or unstiffened and/or the number of bolts at the tension flange. Both classes of moment end-plate may be stiffened by placement of gusset plates. Stiffeners for flush end-plates are typically rectangular and welded to the end-plate and beam web (Figure 1.2(b)). For the extended end-plates, typically a triangular stiffener is welded to the plate extension and beam flange (Figure 1.3(b)). The stiffener should be approximately the thickness of the beam web.

Design of these connections is controlled by two limit states, end-plate yield and bolt rupture. Design procedures have been developed by various researchers in the past with a wide variation of results (Srouji *et al.* 1983b). A unified method of design was begun at the University of Oklahoma and continued at Virginia Tech. It is based on yield-line analysis to calculate the plate thickness and a modified Kennedy method (Kennedy *et al.* 1981) to calculate bolt forces. These require calculations that are most well suited to computer application. Due to this complexity, there seems to be a need for simplification, the objective of this study.

The literature used as background for the unified method of design will be reviewed. Following this, a simplified method of design making use of tee stub type calculations and the concept of a number of effective bolts will be presented for the configurations shown in Figure 1.4. Strength predictions using the simplified method will be compared to experimental data and the unified method.



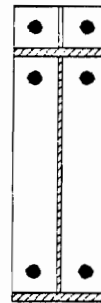
(a) Two-Bolt Flush Unstiffened



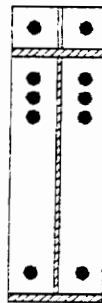
(b) Four-Bolt Flush Unstiffened



(c) Four-Bolt Extended Unstiffened



(d) Four-Bolt Extended Stiffened



(e) Multiple Row Extended Unstiffened 1/3

Figure 1.4 Configurations for Simplified Method

1.2 LITERATURE REVIEW

As stated previously, Srouji *et al.* (1983b) reviewed various design procedures that have been developed for moment end-plate connections. A wide variation in calculated required plate thickness was found between the different procedures. This led to a continuing effort to develop a unified approach to design using yield-line theory to predict the strength of the end-plate and a modification of a method proposed by Kennedy *et al.* (1981) to determine the bolt capacity.

The method proposed by Kennedy *et al.* (1981) is used to predict the forces in the bolts as a function of the applied flange force plus prying forces. It is based on the split-tee analogy, as shown in Figure 1.5. This analogy is composed of a plate bolted to a rigid support with an additional plate welded perpendicularly to form the tee. In the figure, $2F$ is the applied tension loading, Q is the prying force, and B is the resultant bolt force.

The Kennedy method has a basic assumption that the plate bolted to the support may go through three stages of behavior depending upon the level of applied load. **Thick plate behavior** (Figure 1.6(a)) is assumed at lower levels of applied load. Plastic hinges have not formed in the bolted plate, hence no prying forces are assumed to develop. The forces in the bolts are only functions of the applied load. As the level of applied load increases, two plastic hinges form at mid-depth of the bolted plate at the intersection of the two plates (Figure 1.6(b)). This initiation of yielding is termed the *thick plate limit* and denotes the beginning of **intermediate plate behavior**. Prying forces are assumed to

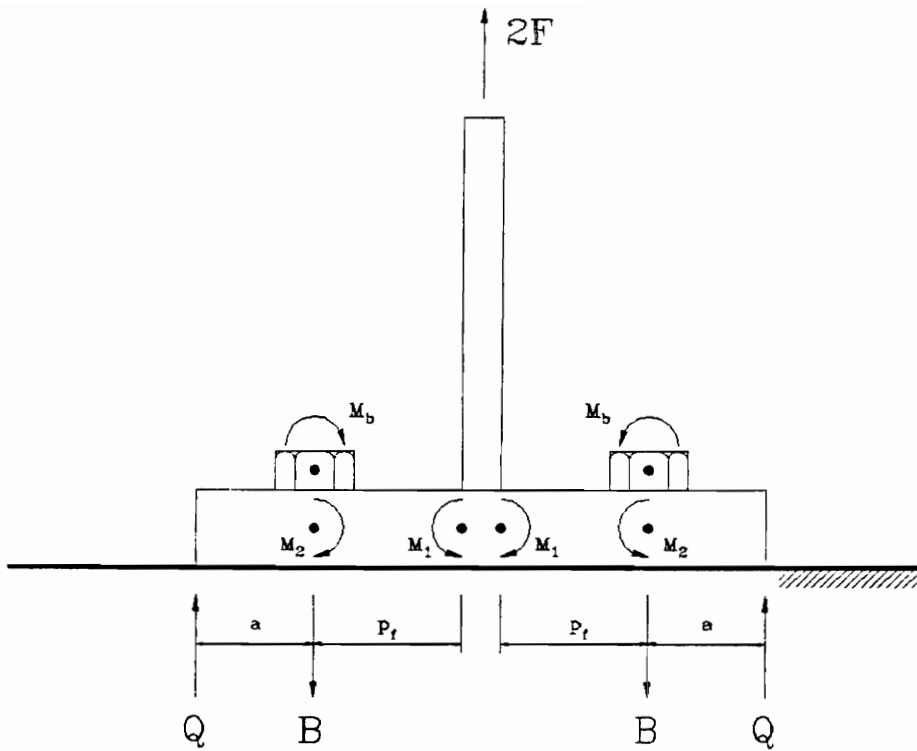
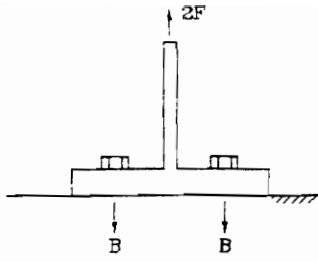
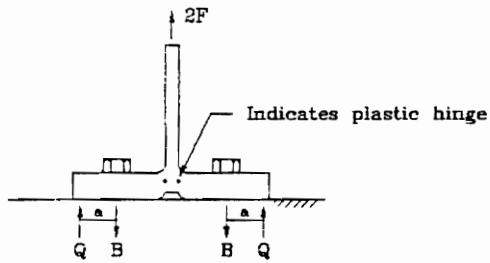


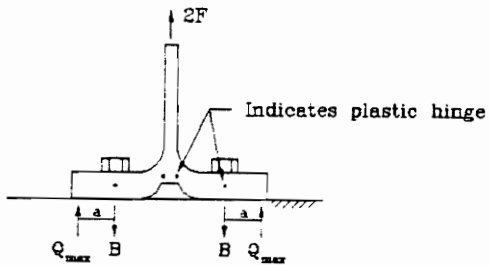
Figure 1.5 Kennedy Split-Tee Analogy
(after Kennedy *et al.* 1981)



(a) First Stage - Thick Plate Behavior



(b) Second Stage - Intermediate Plate Behavior



(c) Third Stage - Thin Plate Behavior

Figure 1.6 Kennedy Split-Tee Behavior
(after Morrison *et al.* 1985; 1986)

develop. Now the bolt forces are functions of the applied load and the prying forces. With an increase in applied load, two additional plastic hinges form in the bolted plate at the centerline of the bolts (Figure 1.6(c)). This is termed the *thin plate limit*. It is then assumed that the prying forces are at their maximum value. This stage of behavior is known as **thin plate behavior**.

To determine the stage of plate behavior, the plate thickness is compared with the thin and thick plate limits. The thick plate limit, denoted by t_1 , is found using an iterative process.

$$t_1 = \sqrt{\frac{2p_f(2F)}{b_f \sqrt{F_{py}^2 - 3\left(\frac{F}{b_f t_1}\right)^2}}} \quad (1.1)$$

The thin plate limit, t_{11} , is also found using iteration.

$$t_{11} = \sqrt{\frac{p_f(2F) - \pi d_b^3 F_t / 8}{\frac{b_f}{2} \sqrt{F_{py}^2 - 3\left(\frac{F}{b_f t_{11}}\right)^2} + w' \sqrt{F_{py}^2 - 3\left(\frac{F}{2w' t_{11}}\right)^2}}} \quad (1.2)$$

In these formulae, $2F$ = applied flange force, p_f = bolt pitch = distance from near face of the beam flange to the centerline of the bolts, b_f = beam flange width, F_{py} = nominal yield strength of plate material, w' = the width of the plate per bolt at the bolt line minus the bolt hole diameter, F_t = tensile strength of bolt material per Table J3.2 of AISC (1994). If $t_p > t_1$, then the end-plate is treated as thick, and the prying force (Q) equals zero. If $t_1 > t_p > t_{11}$ then the plate behaves as an intermediate plate. This state of behavior includes some prying force, which is calculated by the following equation.

$$Q = \frac{p_f F}{a} - \frac{\pi d_b^3 F_t}{32a} - \frac{b_f t_p^2}{8a} \sqrt{F_{py}^2 - 3 \left(\frac{2F}{b_f t_p} \right)^2} \quad (1.3)$$

In this equation, a = the distance from the bolt centerline to the line of action of Q , suggested to be between $2d_b$ and $3d_b$. If $t_p < t_{11}$ then the plate is in the thin plate stage of behavior, with maximum prying action. The maximum prying force is calculated using the following formula.

$$Q = Q_{\max} = \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3 \left(\frac{F'}{w' t_p} \right)^2} \quad (1.4)$$

The parameter, F' , is the flange force per bolt at the thin plate limit. It may be found from the following equation.

$$F' = \frac{t_p^2 F_{py} (0.85b_f / 2 + 0.8w') + \pi d_b^3 F_t / 8}{4p_f} \quad (1.5)$$

Once the prying force is determined, the bolt force may be determined.

$$B = F / 2 + Q \quad (1.6)$$

A word of caution is in order concerning the equations for the prying forces for intermediate and thin plate behavior: the term under the radical may become negative. If this occurs, it signifies that the plate is failing locally in shear before prying action can develop. Therefore the connection is inadequate.

Srouji *et al.* (1983b) reported yield-line methods for determining the strength of the end-plate for four different flush end-plate configurations. These included the two-bolt flush unstiffened, four-bolt flush unstiffened, four-bolt flush stiffened between the

bolt rows, and four-bolt flush stiffened outside the bolt rows. Bolt force predictions for the unstiffened configurations were also included using a modified Kennedy method. Experimental testing was conducted to verify the accuracy of the proposed methods. Srouji *et al.* (1983a) presented the results of eight, two-bolt flush unstiffened, tests. Six, four-bolt flush unstiffened, tests were reported in Srouji *et al.* (1984). From these results, it was concluded that yield-line analysis and the modified Kennedy method provided an accurate means for predicting the capacity of the connections.

Hendrick *et al.* (1985) presented prediction equations for the same configurations studied by Srouji *et al.* (1983b). The yield-line and modified Kennedy approach introduced by Srouji *et al.* (1983b) was followed. Two substantial changes were made in the modified Kennedy predictions however. The distance from the bolt centerline to the line of action of the prying force, a , was redefined by the following equation.

$$a = 3.682(d_b / t_p)^3 - 0.085 \quad (1.7)$$

Secondly, the fraction of the flange force carried by the most interior row of bolts for the four-bolt flush unstiffened configuration was changed from 1/6 to 1/8. These changes were applied successfully to the tests conducted by Srouji *et al.* (1983a; 1984). Additionally, eight tests of the four-bolt flush stiffened configurations were conducted. The prediction equations resulted in sufficient accuracy when compared to the tests.

Morrison *et al.* (1985) studied the four-bolt extended stiffened configuration. Yield-line analysis and the modified Kennedy method were again followed. With the extended configuration, it was necessary to determine the portion of the flange force to

be applied to the inner and outer rows of bolts. Six tests were conducted. From these tests, it was discovered that the inner row of bolts always reached proof load before the outer row, hence the inner row of bolts controls. Also, it was concluded that the inner row receives 60% of the flange force and the outer row 40%.

Morrison *et al.* (1986) conducted studies of the multiple row extended unstiffened 1/3 configuration. The “1/3” signifies one row of bolts outside the beam tension flange and three rows inside. Analysis was accomplished using yield-line theory and the modified Kennedy method. Six tests were conducted. From these tests, a distribution of the flange force was chosen for the bolt rows. The inner bolt rows always yielded before the outer rows in the tests, hence the inner bolts control the bolt capacity.

Murray and Kukreti (1988) reported a simplified design method for the eight-bolt extended stiffened configuration (Figure 1.7). A finite element model was formulated and then verified by testing six stiffened tee hangers and eight end-plate connections. The results of the model and the tests showed good agreement. Using the finite element model, twenty-five cases with variation of geometric parameters were analyzed. Regression analyses were performed on these results to develop strength prediction equations. To simplify these predictions, use was made of the tee stub analogy (Figure 1.8) and the number of effective bolts. The tee stub analogy assumes an inflection point at one-half of the bolt pitch ($p_f/2$) and two bolts per row. The plate moment is defined by the following equation.

$$M_{pl} = 2T(p_f / 2) = T(p_f) \quad (1.8)$$

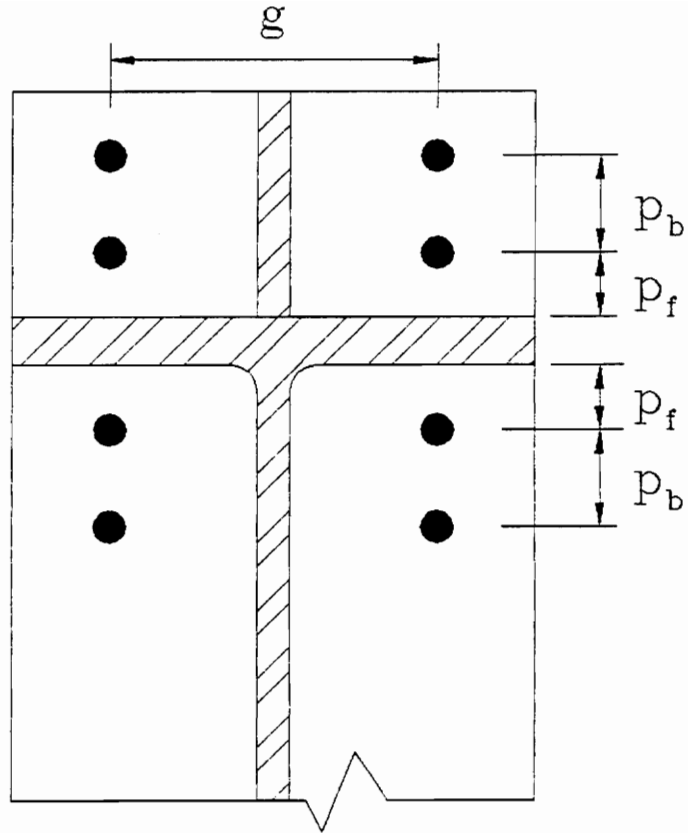


Figure 1.7 Eight-Bolt Extended Stiffened Moment End-Plate

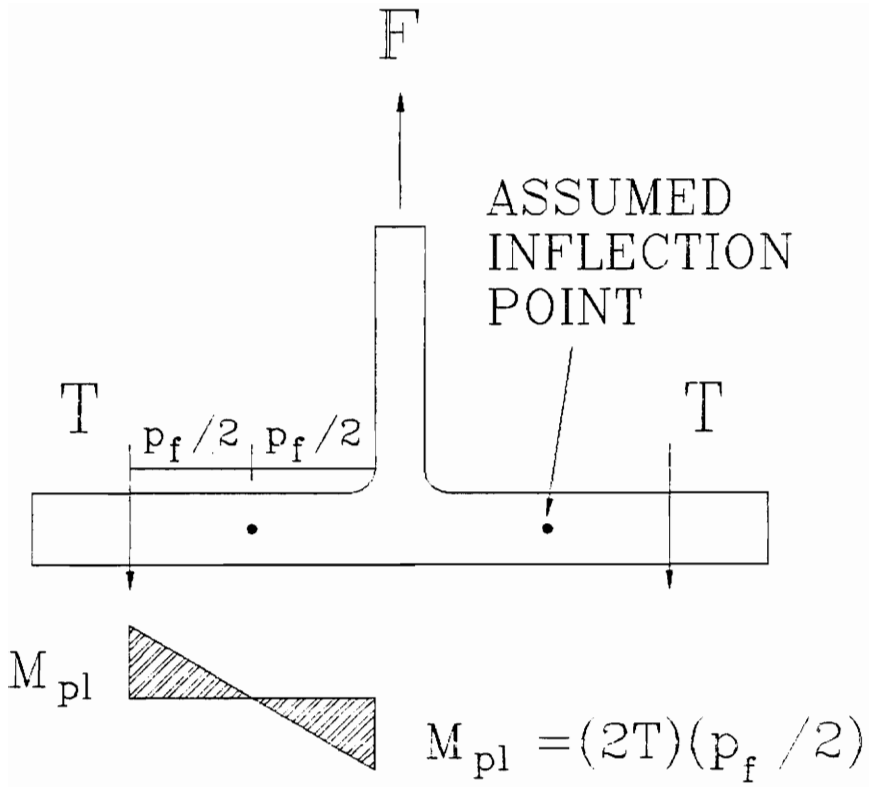


Figure 1.8 Tee Stub Analogy

where T = the force per bolt, and p_f = the bolt pitch. To use the analogy, an effective plate moment is determined by replacing p_f with an effective bolt pitch (p_{eff}) and T becomes the force per number of effective bolts. Various equations were tried in an attempt to define p_{eff} . The best equation for p_{eff} was determined by comparing the predicted capacities using the tee stub analogy and the regression equations.

$$p_{eff} = \frac{p_f}{5} \sqrt{g^2 + p_f^2} \quad (1.9)$$

Abel and Murray (1992) reported results of a study of the four-bolt extended unstiffened configuration. Yield-line analysis and the modified Kennedy method were used for predictions. Four tests were conducted. From these it was determined that the flange force is equally distributed between the inner and outer bolt rows. The tests also showed that there is no prying action in the outer bolts when the bolt prediction controls.

Borgsmiller (1995) reported a design method for the nine previously studied configurations which were analyzed using yield line theory and the modified Kennedy method. The yield-line analysis was retained, and a simplified Kennedy analysis was introduced. Thin plate behavior and maximum prying action are assumed to take place in the bolts at what is termed the prying threshold, 90% of the strength of the plate. At applied loadings less than this value, thick plate behavior is assumed. The method predicted results with sufficient accuracy for 52 previously reported tests.

1.3 SCOPE OF RESEARCH

This research has been conducted to develop a simplified design method for the five moment end-plate configurations included in the study. This simplified design is derived from existing design procedures based on yield line analysis and the modified Kennedy method. The design procedure will provide:

- Minimum required plate thickness using classical tee stub type calculations using given geometric and material properties of the end plate.
- Minimum required bolt diameter using a conservative estimate of the number of effective bolts.

This simplified method will differ from the simplified method developed by Borgsmiller (1995). He maintains the use of the yield-line equations to determine the plate thickness and uses a simplified Kennedy analysis to determine the bolt diameter.

CHAPTER II

OVERVIEW OF PRESENT DESIGN PROCEDURES

2.1 YIELD-LINE ANALYSIS

2.1.1 General

Yield-line analysis was developed for analysis of reinforced concrete slabs but has been successfully applied to steel plates. A yield-line is a continuous formation of plastic hinges along a straight or curved path. A failure mechanism is assumed to exist when the yield-lines form a kinematically valid collapse mechanism. Rigid plastic theory is applied in the analysis.

Typically, yield-line patterns are assumed to be in straight lines. The location of the yield-lines are found using the guidelines set forth by Srouji et al. (1983b):

- Axes of rotation generally lie along lines of support.
- Yield-lines pass through the intersection of the axes of rotation of adjacent plate segments
- Along a yield-line, the bending moment is assumed constant and equal to the plastic moment capacity of the plate.

Two methods may be used to analyze the yield-line mechanisms: the equilibrium method and the method of virtual work. Typically, due to simplicity, the method of virtual work is employed. The external work of the applied loads due to imposition of a

virtual displacement, and the internal work of the plate due to rotation at the yield lines, are set equal. From this, either the unknown applied loading or the unknown resisting moment may be found. Since yield-line analysis is an upper bound theory, the least upper bound must be found. Least upper bound means the mechanism giving the least failure load for a given plastic moment, or the mechanism giving the greatest plastic moment for a given load.

The internal work for a yield-line mechanism is the summation of the internal work for each individual yield-line in the mechanism. The internal work per unit of length for a yield-line is equal to the normal moment on the yield-line multiplied by the normal rotation of the yield-line. Therefore the work of the n^{th} yield-line is as follows:

$$w_i = \int_{L_n} m_p \theta_n ds = m_p \theta_n L_n \quad (2.1)$$

where m_p is the plastic moment capacity of the plate, θ_n is the normal rotation of yield-line n , and ds is the elemental length of line n . The total work for the entire mechanism is as follows:

$$W_i = \sum_{n=1}^N w_i = \sum_{n=1}^N m_p \theta_n L_n \quad (2.2)$$

where N is the total number of yield-lines in the mechanism.

Complicated patterns may be simplified by separating the internal work into its components in the x - and y - directions. Equation 2.2 may then be rewritten in the following form:

$$W_i = \sum_{n=1}^N (m_{px} \theta_{nx} L_{nx} + m_{py} \theta_{ny} L_{ny}) \quad (2.3)$$

where m_{px} and m_{py} are the x- and y- components of the plate plastic moment capacity per unit of length, θ_{nx} and θ_{ny} are the x- and y- components of the normal rotations of the plate, and L_{nx} and L_{ny} are the x- and y- components of the length of the n^{th} yield line.

The expressions for internal work for each configuration are different, but the expressions for external work have the same form.

$$W_e = M_u \theta = M_u \left(\frac{1}{h} \right) \quad (2.4)$$

M_u is the ultimate beam moment for which the connection is to be designed, θ is the virtual rotation of the connection, equal to $1/h$, where h is the height of the beam. This virtual rotation is shown in Figure 2.1 for the two-bolt flush unstiffened configuration.

2.1.2 Two-Bolt Flush Unstiffened

The controlling yield-line mechanism was found by Srouji *et al.* (1983b) to be that shown in Figure 2.2. The expression for the internal work for this mechanism is as follows:

$$W_i = \frac{4m_p}{h} (h - p_t) \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + \frac{2}{g} (p_f + s) \right] \quad (2.5)$$

The moment capacity of the end-plate may be expressed by equating the external and internal work.

$$M_{pt} = 4m_p (h - p_t) \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + \frac{2}{g} (p_f + s) \right] \quad (2.6)$$

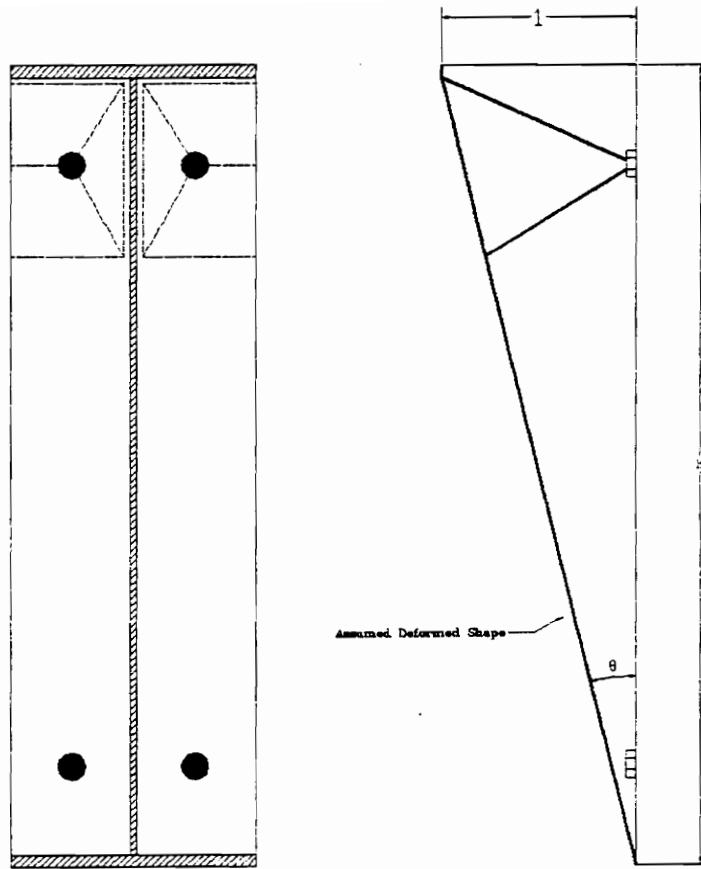


Figure 2.1 Virtual Displacements in a Two-Bolt Flush Unstiffened End-Plate Configuration

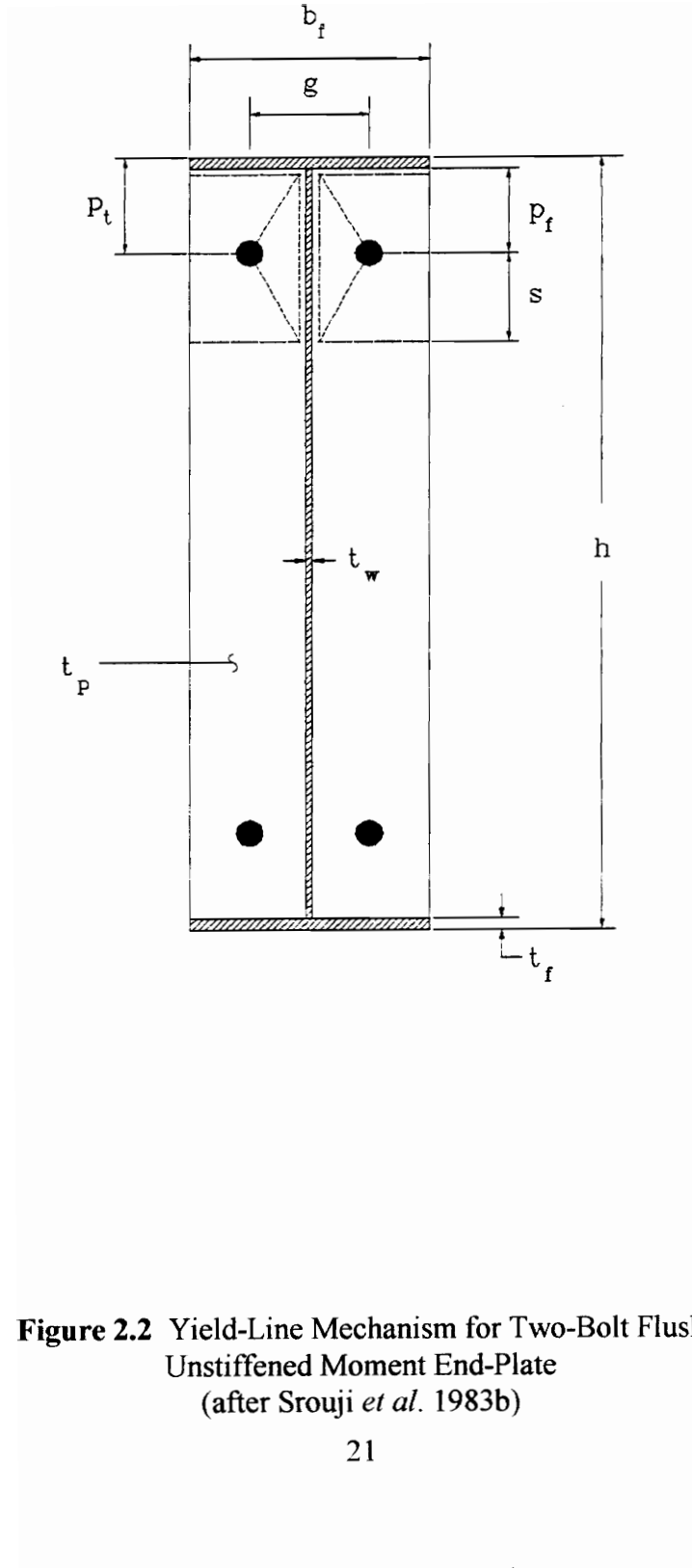


Figure 2.2 Yield-Line Mechanism for Two-Bolt Flush Unstiffened Moment End-Plate (after Srouji *et al.* 1983b)

This expression may be rearranged to solve for the plate thickness by equating it to the applied moment.

$$t_p = \left[\frac{M_u / F_{py}}{(h - p_t) \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + \frac{2}{g} (p_f + s) \right]} \right]^{1/2} \quad (2.7)$$

In this expression, s is an unknown dimension that is found by differentiating Equation 2.5 with respect to s and setting the resultant expression equal to zero. This results in the following expression for s .

$$s = \frac{1}{2} \sqrt{b_f g} \quad (2.8)$$

2.1.3 Four-Bolt Flush Unstiffened

Strouji *et al.* (1983b) also found the controlling yield-line mechanism for the four-bolt flush unstiffened configuration. It is shown in Figure 2.3. The expression for the moment capacity of the plate is found by equating the internal work and external work. The expression for the internal work and moment capacity of the plate are:

$$W_i = \frac{4m_p}{h} \left[\frac{b_f}{2} \left(\frac{h - p_t}{p_f} + \frac{h - p_{t2}}{u} \right) + 2(p_f + p_b + u) \left(\frac{h - p_t}{g} \right) \right] \quad (2.9)$$

$$M_{pl} = 4m_p \left[\frac{b_f}{2} \left(\frac{h - p_t}{p_f} + \frac{h - p_{t2}}{u} \right) + 2(p_f + p_b + u) \left(\frac{h - p_t}{g} \right) \right] \quad (2.10)$$

As before, Equation 2.10 may be rearranged to solve for the plate thickness by equating it to the applied moment.

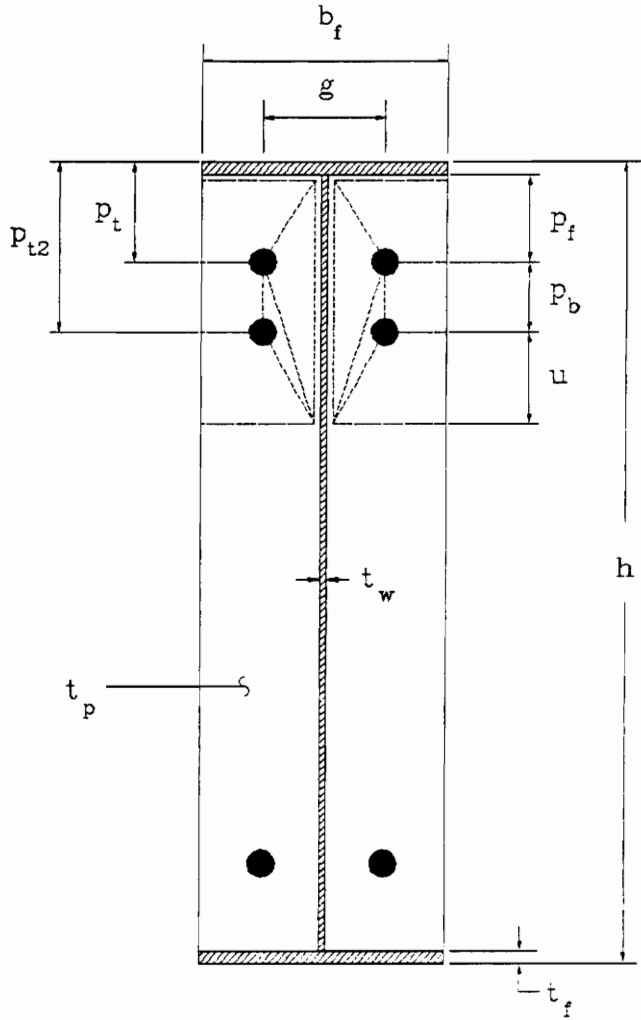


Figure 2.3 Yield-Line Mechanism for Four-Bolt Flush Unstiffened Moment End-Plate (after Srouji *et al.* 1983b)

$$t_p = \left[\frac{M_u / F_{py}}{\frac{b_f}{2} \left(\frac{h - p_t}{p_f} + \frac{h - p_{t2}}{u} \right) + 2(p_f + p_b + u) \left(\frac{h - p_t}{g} \right)} \right]^{1/2} \quad (2.11)$$

The unknown dimension, u , is derived by differentiating Equation 2.9 with respect to u and equating the resulting expression to zero. This results in the following expression.

$$u = \frac{1}{2} \sqrt{b_f g \left(\frac{h - p_{t2}}{h - p_t} \right)} \quad (2.12)$$

2.1.4 Four-Bolt Extended Unstiffened

The yield-line mechanism for this configuration was found by Srouji *et al.* (1983b) and later used by Abel and Murray (1992). It is shown in Figure 2.4. The internal work for this mechanism may be found from the following expression.

$$W_i = \frac{4m_p}{h} \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right] \quad (2.13)$$

Here, s is the distance between parallel yield-lines. The ultimate moment capacity of the end-plate may be found using the following expression, derived by equating the external work and internal work.

$$M_{pt} = 4m_p \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right] \quad (2.14)$$

This expression may be then solved for the required plate thickness by equating Equation 2.14 to the applied moment, M_u .

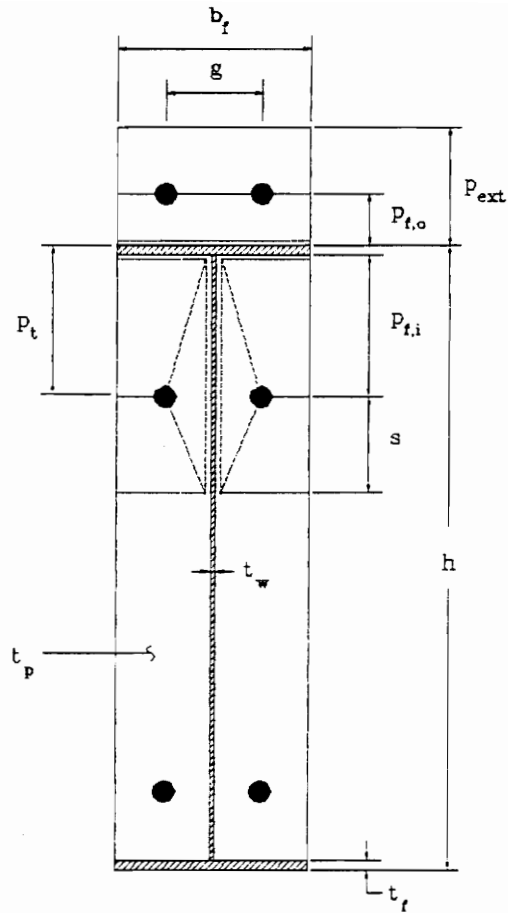


Figure 2.4 Yield-Line Mechanism for Four-Bolt Extended Unstiffened Moment End-Plate
(after Srouji *et al.* 1983b)

$$t_p = \left[\frac{M_u / F_{py}}{\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right)} \right]^{1/2} \quad (2.15)$$

An expression for s is found, as previously, by differentiating Equation 2.13 with respect to s and equating the resultant expression to zero.

$$s = \frac{1}{2} \sqrt{b_f g} \quad (2.16)$$

2.1.5 Four-Bolt Extended Stiffened

Strouji *et al.* (1983b) presented the yield-line mechanisms for this configuration. The mechanisms are shown in Figure 2.5. Two mechanisms can potentially control, depending on the geometry of the end-plate. These were later used by Morrison *et al.* (1985). Expressions for W_i , M_{pl} and t_p will be presented for both cases. As before, s is the distance between parallel yield-lines. The expression for s was derived as before and does not change, hence Equation 2.16 may be used. Expressions for t_p were found by equating Equation 2.18 and Equation 2.21 to the applied moment and rearranging.

Case 1, $s < d_e$:

$$W_i = \frac{4m_p}{h} \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)] \quad (2.17)$$

$$M_{pl} = 4m_p \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)] \quad (2.18)$$

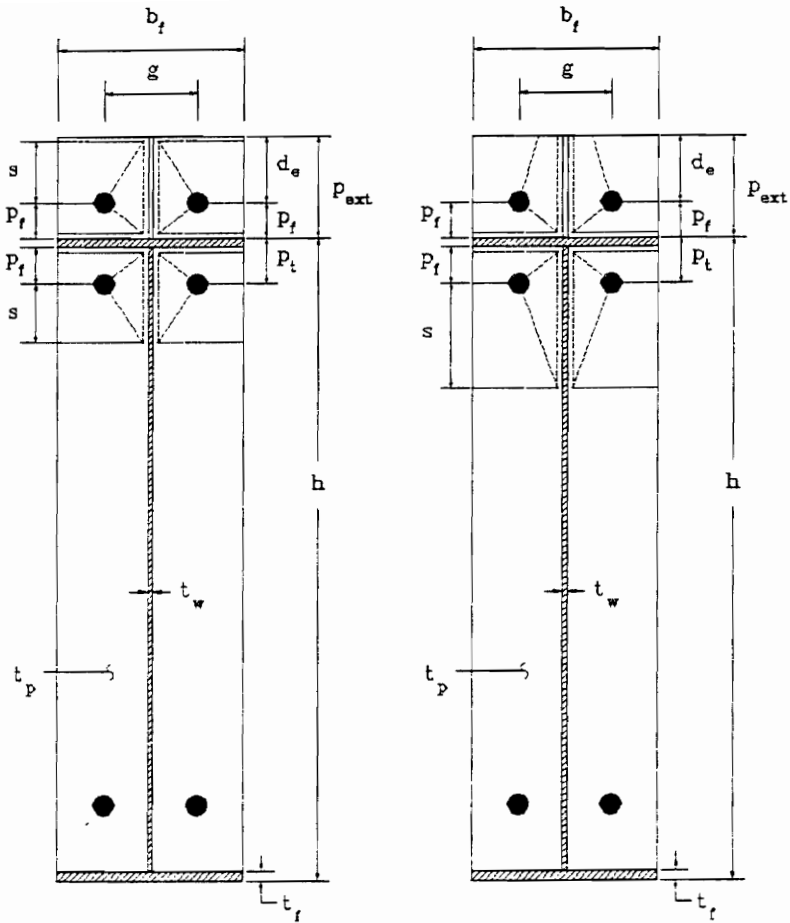


Figure 2.5 Yield-Line Mechanisms for Four-Bolt Extended Stiffened Moment End-Plate (after Srouji *et al.* 1983b)

$$t_p = \left[\frac{M_u / F_{py}}{\left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)]} \right]^{1/2} \quad (2.19)$$

Case 2, $s > d_e$:

$$W_i = \frac{4m_p}{h} \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{2s} \right) + (p_f + d_e) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)] \quad (2.20)$$

$$M_{pl} = 4m_p \left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{2s} \right) + (p_f + d_e) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)] \quad (2.21)$$

$$t_p = \left[\frac{M_u / F_{py}}{\left[\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{2s} \right) + (p_f + d_e) \left(\frac{2}{g} \right) \right] [(h - p_t) + (h + p_f)]} \right]^{1/2} \quad (2.22)$$

2.1.6 Multiple Row Extended Unstiffened 1/3

For this configuration, two yield-line mechanisms may control depending on the geometry of the end-plate. They were developed and used by SEI (1984) and Morrison *et al.* (1986) and are shown in Figure 2.6. Expressions for W_i , M_{pl} and t_p are given for each mechanism.

Mechanism I:

$$W_i = \frac{4m_p}{h} \left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h - p_t}{p_f} + \frac{h - p_{t3}}{u} \right) + 2(p_f + p_{b1,3} + u) \left(\frac{h - p_t}{g} \right) \right] \quad (2.23)$$

$$M_{pl} = 4m_p \left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h - p_t}{p_f} + \frac{h - p_{t3}}{u} \right) + 2(p_f + p_{b1,3} + u) \left(\frac{h - p_t}{g} \right) \right] \quad (2.24)$$

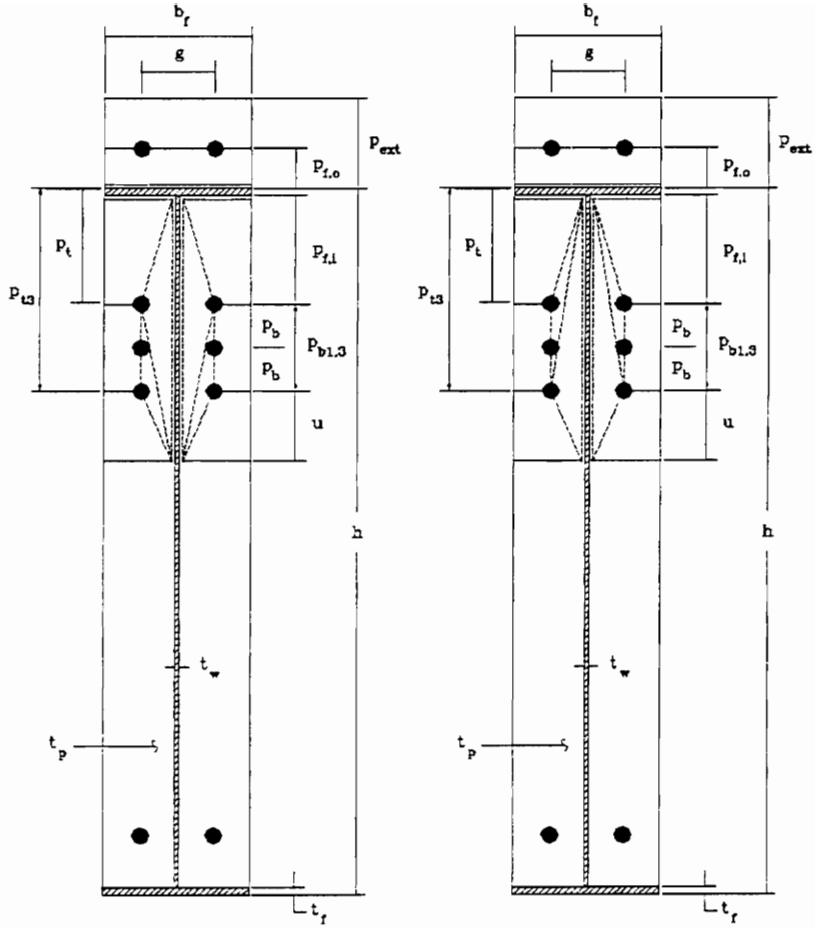


Figure 2.6 Yield-Line Mechanisms for Multiple Row Extended Unstiffened 1/3 Moment End-Plate (after SEI 1984)

$$t_p = \left[\frac{M_u / F_{py}}{\left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h-p_t}{p_f} + \frac{h-p_b}{u} \right) + 2(p_f + p_{b1,3} + u) \left(\frac{h-p_t}{g} \right) \right]} \right]^{1/2} \quad (2.25)$$

$$u = \frac{1}{2} \sqrt{b_f g \left(\frac{h-p_b}{h-p_t} \right)} \quad (2.26)$$

Mechanism II:

$$W_i = \frac{4m_p}{h} \left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h-p_t}{p_f} + \frac{h-p_b}{u} \right) + \frac{2}{g} (p_f + p_{b1,3}) (h-t_f) + \frac{2u}{g} (h-t_f) + \frac{g}{2} \right] \quad (2.27)$$

$$M_{pl} = 4m_p \left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h-p_t}{p_f} + \frac{h-p_b}{u} \right) + \frac{2}{g} (p_f + p_{b1,3}) (h-t_f) + \frac{2u}{g} (h-t_f) + \frac{g}{2} \right] \quad (2.28)$$

$$t_p = \left[\frac{M_u / F_{py}}{\left[\frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} + \frac{h-p_t}{p_f} + \frac{h-p_b}{u} \right) + \frac{2}{g} (p_f + p_{b1,3}) (h-t_f) + \frac{2u}{g} (h-t_f) + \frac{g}{2} \right]} \right]^{1/2} \quad (2.29)$$

$$u = \frac{1}{2} \sqrt{b_f g} \quad (2.30)$$

2.2 MODIFIED KENNEDY METHOD

2.2.1 General

Yield-line analysis, described previously, does not furnish any information about the forces in the bolts. A method proposed by Kennedy *et al.* (1981) was discussed in some detail in Chapter 1. This method of bolt analysis was adopted for the various moment end-plate configurations considered here. Modifications were necessary, however, to make use of the method. These modifications were to the basic model and also affect the basic equations. The discussion that follows will describe these modifications with respect to each configuration.

2.2.2 Two-Bolt Flush Unstiffened

Modifications to the method for the two-bolt flush unstiffened configuration were introduced by Srouji *et al.* (1983a; 1983b). Figure 2.7 shows the modifications. It is basically half of the original Kennedy model. Hendrick *et al.* (1985) introduced a new equation for a , the distance from the bolt centerline to the line of action of Q (Equation 1.7). Though the basic procedure remains the same, the modifications to the model result in some modifications to the basic equations used in the Kennedy analysis.

$$t_1 = \sqrt{\frac{4p_f F_f}{b_f \sqrt{F_{py}^2 - 3\left(\frac{F_f}{b_f t_1}\right)^2}}} \quad (2.31)$$

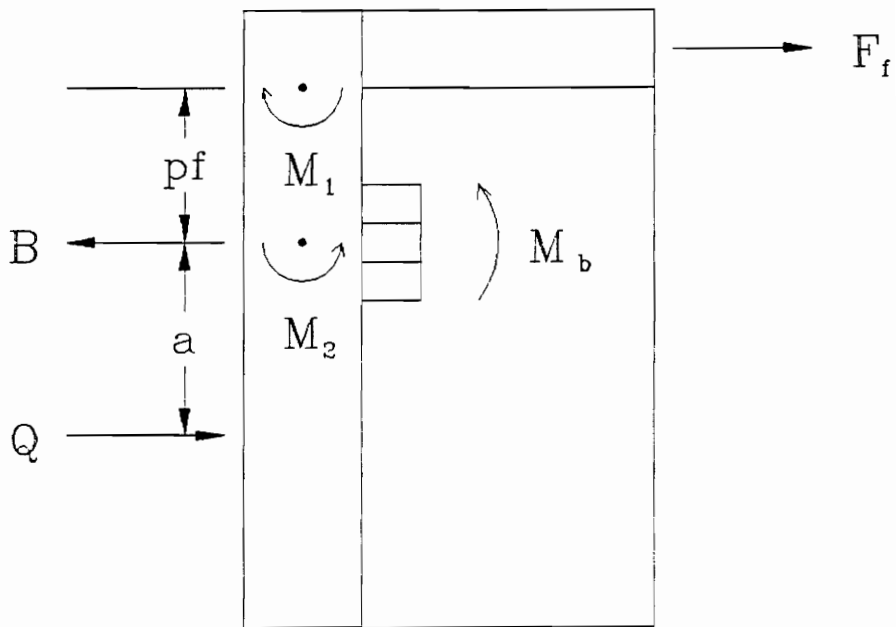


Figure 2.7 Modifications for Two-Bolt Flush Unstiffened Moment End-Plate

$$t_{11} = \sqrt{\frac{2(F_f p_f - \pi d_b^3 F_t / 16)}{\frac{b_f}{2} \sqrt{F_{py}^2 - 3 \left(\frac{F_f}{b_f t_{11}} \right)^2} + w' \sqrt{F_{py}^2 - 3 \left(\frac{F_f}{2w' t_{11}} \right)^2}}} \quad (2.32)$$

For $t_p > t_{11}$, there is thick plate behavior and Q equals zero. If $t_p > t_{11}$ the plate is within the intermediate stage of plate behavior, and Q is found by the following expression:

$$Q = \frac{F_f p_f}{2a} - \frac{b_f t_p^2}{8a} \sqrt{F_{py}^2 - 3 \left(\frac{F_f}{b_f t_p^2} \right)^2} - \frac{\pi d_b^3 F_t}{32a} \quad (2.33)$$

If $t_p < t_{11}$ then the plate behavior is thin, and Q is equal to Q_{\max} .

$$Q_{\max} = \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3 \left(\frac{F'}{w' t_p} \right)^2} \quad (2.34)$$

F' is equal to the lesser of F_{limit} (Equation 1.5) and $(F_f)_{\max} / 2$. The bolt force is then calculated as:

$$B = F_f / 2 + Q \quad (2.35)$$

2.2.3 Four-Bolt Flush Unstiffened

Srouji *et al.* (1983a; 1983b) also introduced the modifications used for this configuration, later used by Hendrick *et al.* (1985). The modifications are shown in Figure 2.8. The second row of bolts creates an indeterminate problem. Assumptions were made concerning the force in this additional row by using the experimental results. At thick plate behavior, the inner bolt force is assumed to be zero. For intermediate and thin plate behavior, it is assumed that the bolt force is only a function of the flange force, $F_f / 10$ for thin plate behavior and $F_f / 8$ for intermediate plate behavior. The bolt force

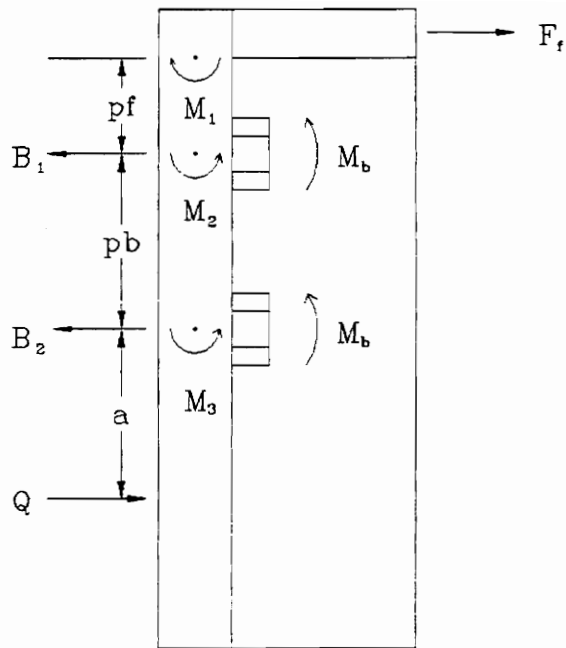


Figure 2.8 Modifications for Four-Bolt Flush Unstiffened Moment End-Plate

for the outermost row of bolts (closest to the tension flange) controls the connection capacity and is denoted by B_1 . The values for a , t_1 and t_{11} may be found using Equations 1.7, 2.31 and 2.32. The following are the modified equations for use in the Kennedy analysis. If $t_p > t_{11}$:

$$Q = \frac{F_f(p_f + 0.1p_b)}{2(a + p_b)} - \frac{b_f t_p^2}{8(a + p_b)} \sqrt{F_{py}^2 - 3 \left(\frac{F_f}{b_f t_p^2} \right)^2} - \frac{\pi d_b^3 F_t}{16(a + p_b)} \quad (2.36)$$

The bolt force is then equal to the following:

$$B_1 = 0.4 F_f + Q \quad (2.37)$$

If $t_p < t_{11}$ then there is thin plate behavior and Q equals Q_{max} . Q_{max} is found using Equation 2.34. The bolt force for thin plate behavior is:

$$B_1 = 3 F_f / 8 + Q_{max} \quad (2.38)$$

2.2.4 Four-Bolt Extended Unstiffened

This configuration was modeled as an inner end-plate and an outer end-plate by Abel and Murray (1992), as shown in Figure 2.9. The outer end-plate consists of the plate extension outside the beam tension flange and part of the beam tension flange. The inner end-plate consists of the rest of the beam tension flange and the part of the end-plate within the beam flanges. It should be noted that the inner bolt pitch, $p_{f,i}$, and outer bolt pitch, $p_{f,o}$, are equal. The factors α and β distribute the flange force between the inner and outer end-plates. These factors were determined experimentally to both be equal to 0.5. To determine the bolt forces, the inner and outer end-plate forces are

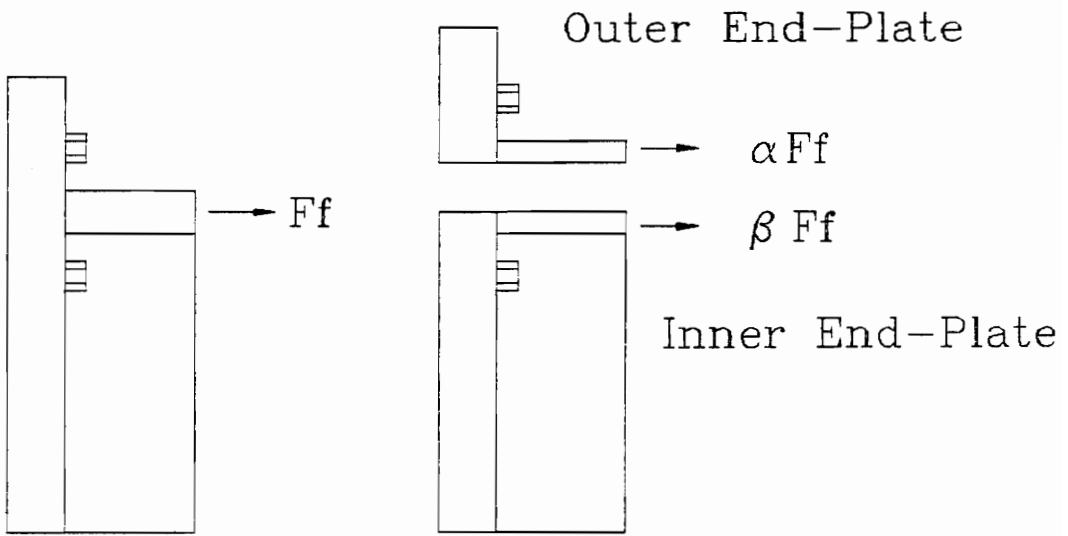


Figure 2.9 Modifications for Four-Bolt Extended Unstiffened Moment End-Plate

compared to the thick plate limit force, F_1 , and the thin plate limit force, F_{11} . The thick plate limit force is:

$$F_1 = \frac{b_f t_p^2 F_{py}}{4p_f \sqrt{1 + \left(\frac{3t_p^2}{16p_f^2}\right)}} \quad (2.39)$$

The thin plate limit force is:

$$F_{11} = \frac{t_p^2 F_{py} \left[0.85 \left(\frac{b_f}{2}\right) + 0.8w' \right] + \left[\pi d_b^3 F_t / 8 \right]}{2p_f} \quad (2.40)$$

When the outer or inner end-plate force is less than the thick plate limit force, there is thick plate behavior and the prying force, Q , is equal to zero. If the inner end-plate force or the outer end-plate force is greater than or equal to F_1 and less than or equal to F_{11} , then there is intermediate plate behavior. The prying force is equal to:

$$Q = \frac{\gamma F_f p_f}{2a} - \frac{\pi d_b^3 F_t}{32a} - \frac{b_f t_p^2}{8a} \sqrt{F_{py}^2 - 3 \left(\frac{\gamma F_f}{b_f t_p} \right)^2} \quad (2.41)$$

In this equation, γ is equal to α or β , depending upon which portion of end-plate, inner or outer, is being considered. If either end-plate force is less than the thin plate limit, then there is thin plate behavior and the prying force is equal to Q_{max} , found using Equation 2.34. In this equation, F' is equal to $F_{11} / 2$. The bolt force is calculated using the following.

$$\begin{aligned} B_I &= \beta F_f / 2 + Q \\ B_E &= \alpha F_f / 2 + Q \end{aligned} \quad (2.42)$$

2.2.5 Four-Bolt Extended Stiffened

Morrison *et al.* (1985) modified the basic Kennedy model for the four-bolt extended unstiffened configuration as shown in Figure 2.10. The connection is idealized as an inner end-plate and an outer end-plate. It should again be noted that the inner bolt pitch and outer bolt pitch are equal. The factors, α and β , distribute the flange force to the outer end-plate and the inner end-plate respectively. Values for these parameters were found experimentally, α equals 0.4 and β equals 0.6. The capacity of the connection was found to be controlled by the inner bolts. When $\beta F_f / 2 < F_1$, then there is thick plate behavior and Q is zero. Equation 2.41 and Equation 2.34 may be used to determine the prying force for intermediate and thin plate behavior here with the appropriate value of β substituted for γ . The force in the inner bolts is:

$$B_I = \beta F_f / 2 + Q \quad (2.43)$$

2.2.6 Multiple Row Extended Unstiffened 1/3

As in the other extended configurations, this configuration was modeled as an inner end-plate and an outer end-plate, shown in Figure 2.11 (SEI 1984; Morrison *et al.* 1986). As with the other extended configurations, the inner bolt pitch and outer bolt pitch are equal. Four factors, α , β_2 , β_3 and β_4 , were determined experimentally to distribute the flange force between the inner and outer end-plates. It was found from analysis of the experimental results that no prying action takes place in the outer row of

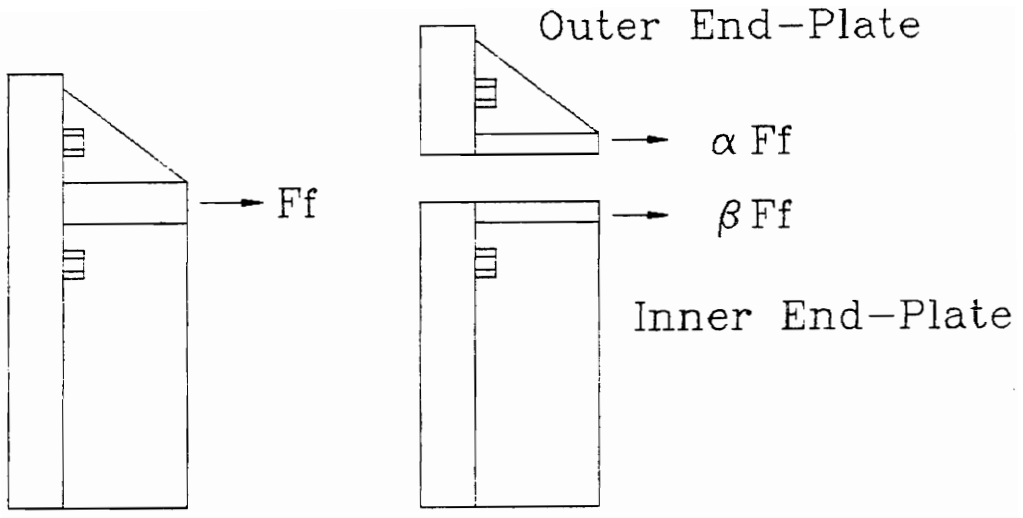


Figure 2.10 Modifications for Four-Bolt Extended Stiffened Moment End-Plate

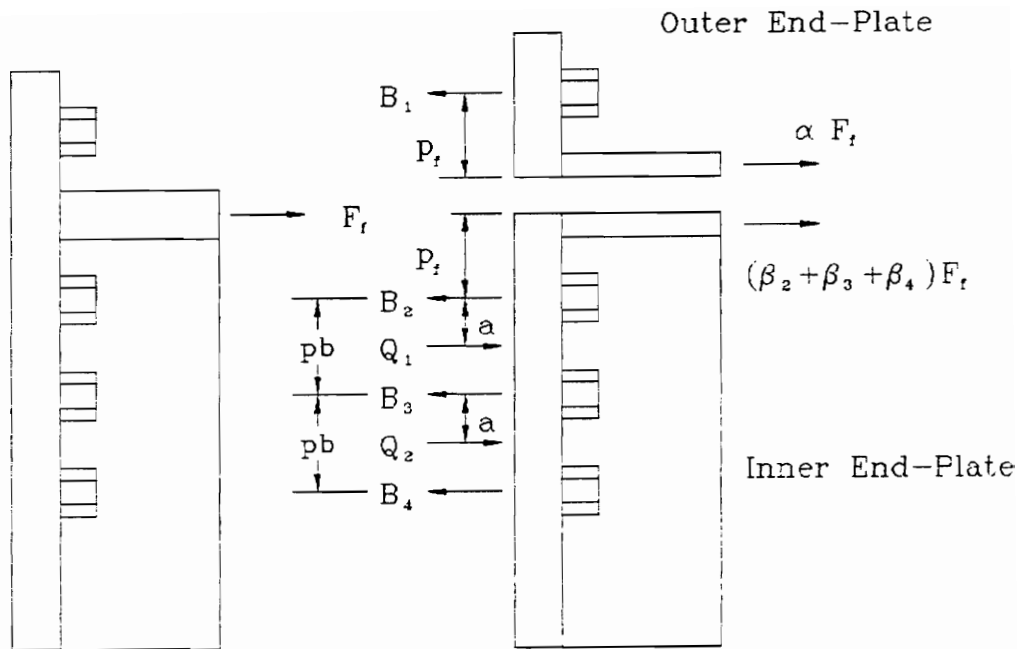


Figure 2.11 Modifications for Multiple Row Extended Unstiffened 1/3 Moment End-Plate

bolts, at any level of applied load. Hence the bolt force for the outer row, denoted by B_1 , is:

$$B_1 = \alpha F_f / 2 \quad (2.44)$$

The factor, α , is equal to 0.6. With respect to the inner end-plate, it was found that the force in the row of bolts closest to the beam tension flange, denoted by B_2 , is critical. A β_2 of 0.35 was chosen to best represent the experimental results. Equation 2.39 and Equation 2.40 may be used to determine the thick and thin plate limit forces respectively. If $\beta_2 F_f < F_1$ then there is thick plate behavior and Q is taken as zero. To determine the prying forces for intermediate and thin plate behavior, Equation 2.41 and Equation 2.34 may be used by substituting the appropriate value of β_2 for γ . The bolt force for the bolt row closest to the beam tension flange is:

$$B_2 = \beta_2 F_f / 2 + Q \quad (2.45)$$

The controlling bolt force is the maximum of B_1 and B_2 .

CHAPTER III

DEVELOPMENT OF A SIMPLIFIED DESIGN PROCEDURE

3.1 GENERAL

The design methods which make use of yield-line analysis and a modified Kennedy analysis have been described in the previous chapter. The calculations required are long, complex and make it difficult to develop a “feel” for the behavior of the connection. They are most well-suited to computer application. A simplified method has been developed so that hand calculations may be more easily conducted. It is based on the work of Murray and Kukreti (1988). Use is made of the tee-stub analogy (Figure 1.7) and the number of effective bolts.

The tee-stub analogy moment has been shown previously to be:

$$M_{pl} = 2T(p_f / 2) = T p_f \quad (1.8)$$

To use the analogy, T is replaced by the force per number of effective bolts, r_u , and p_f is replaced by an effective bolt pitch, p_{eff} . This results in the effective plate moment, M_{eu} .

$$M_{eu} = r_u p_{eff} \quad (3.1)$$

To determine the number of effective bolts, N_{eff} , and the effective bolt pitch, p_{eff} , the design procedures based on yield-line analysis and a modified Kennedy method were computer programmed. All W-shapes in the AISC manual normally used as beam

sections of size W10 and larger were used in the analyses. Levels of load equal to 50%, 75% and 100% of the plastic moment for each section were analyzed. Plate material yield stresses, F_{py} , of 36 ksi and 50 ksi were used. Bolt spacings, p_b , were limited to less than or equal to $3d_b$. The bolt pitch, p_f , was limited to be less than or equal to $d_b + 1$ in. Beam flange widths, b_f , were used as the plate widths, b_p , rounded to the next highest 1/2 in. The plate extension beyond the outer row of bolts for the extended configurations was limited to less than or equal to $2d_b$.

N_{eff} is a measure of the effect of the prying forces in reducing the effectiveness of the bolts. To determine N_{eff} , the computer programmed design procedures were used to calculate the maximum bolt force. N_{eff} is defined as the flange force divided by the maximum bolt force. The calculations over the range of the parameters described above resulted in some variation of the values for N_{eff} . For the simplified design procedure, a single value was chosen from the more conservative end of the range. The criterion used for the choice of N_{eff} was that the ratio of the simplified method bolt force prediction to the modified Kennedy method bolt force prediction must be less than or equal to 1.10.

Equations for p_{eff} were chosen using a trial and error process. An attempt was made to make the equations a function of only the gage, g , and the actual bolt pitch, p_f . This was achieved for four of the configurations, with a slight difference for the other configuration. To evaluate the adequacy of a particular p_{eff} expression, the predicted plate capacities using the yield-line equations and the simplified methods were compared. The yield-line prediction equations for M_{pl} are located in Chapter 2. The

simplified method prediction for M_{pl} is determined by using Equations 4.4 and 4.5 (see section 4.3). The plate thickness is found from the yield-line equations. The flange force, $F_{f,plate}$, is calculated by backsolving the tee-stub equations using the yield-line determined plate thickness. M_{pl} is then $F_{f,plate}$ multiplied by the distance center-to-center of the flanges. In all cases, the ratio of the simplified prediction to the yield-line prediction is less than or equal to 1.10; any value greater than 1.0 is unconservative.

To determine the required bolt diameter, use of N_{eff} is as follows:

$$r_u = F_f / N_{eff} \quad (3.2)$$

$$d_b = \sqrt{\frac{4r_u}{\pi\phi F_t}} \quad (3.3)$$

where r_u = force per number of effective bolts, F_f = factored beam flange force, ϕ = LRFD resistance factor for bearing type bolts in tension (AISC 1993), and F_t = tensile strength of the bolt material from LRFD Table J3.2 (AISC, 1993).

Determination of the required plate thickness makes use of the tee-stub type calculations. First, an effective plate moment is determined using the tee-stub type calculation. This effective plate moment is set equal to the plastic capacity of the plate, and the expression is then rearranged to solve for the plate thickness.

$$M_{eu} = r_u p_{eff} \quad (3.4)$$

$$t_p = \sqrt{\frac{4M_{eu}}{\phi F_{py} b_p}} \quad (3.5)$$

where M_{eu} = the effective plate moment, ϕ = LRFD resistance factor for flexure, F_{py} = plate material yield stress, and b_p = the plate width.

3.2 TWO-BOLT FLUSH UNSTIFFENED

The following are the number of effective bolts and effective bolt pitch expression determined for the two-bolt flush unstiffened configuration:

$$N_{eff} = 1.6 \quad (3.6)$$

$$P_{eff} = \frac{p_f}{1.3} \left(\frac{g}{g + p_f} \right)^2 \quad (3.7)$$

Based on the experimental regimen conducted by Srouji *et al.* (1983b) and Hendrick *et al.* (1985), the following limitations are placed on this procedure:

$$\begin{aligned} d_b &\leq 1 \text{ in.} \\ p_f &\leq d_b + 1 \text{ in.} \\ g &\leq 4 \text{ in.} \end{aligned}$$

3.3 FOUR-BOLT FLUSH UNSTIFFENED

For the four-bolt flush unstiffened configuration, the following are the number of effective bolts and effective bolt pitch expression:

$$N_{eff} = 2.0 \quad (3.8)$$

$$P_{eff} = \frac{p_f}{1.25} \left(\frac{g}{g + p_f} \right)^2 \quad (3.9)$$

Again, based on the experimental work of Srouji *et al.* (1983b) and Hendrick *et al.* (1985), the limitations on this procedure are as follows:

$$\begin{aligned} d_b &\leq 1 \text{ in.} \\ p_f &\leq d_b + 1 \text{ in.} \\ g &\leq 4 \text{ in.} \end{aligned}$$

$$p_b \leq 3d_b$$

3.4 FOUR-BOLT EXTENDED UNSTIFFENED

For the four-bolt extended unstiffened configuration, the following results were obtained from the analyses:

$$N_{\text{eff}} = 2.6 \quad (3.10)$$

$$p_{\text{eff}} = \frac{p_f}{2.2} \left(\frac{\sqrt{1.5g^2 + p_f}}{g + p_f} \right) \quad (3.11)$$

The limitations on the simplified procedure from the work of Abel and Murray (1992) are as follows:

$$\begin{aligned} d_b &\leq 1.5 \text{ in.} \\ p_f &\leq d_b + 1 \text{ in.} \\ g &\leq 7 \text{ in.} \\ p_{\text{ext}} &\leq p_f + 2d_b \end{aligned}$$

3.5 FOUR-BOLT EXTENDED STIFFENED

The number of effective bolts and effective bolt pitch for the four-bolt extended stiffened configuration are as follows:

$$N_{\text{eff}} = 2.6 \quad (3.12)$$

$$p_{\text{eff}} = \frac{p_f}{1.45} \left(\frac{g}{g + p_f} \right)^2 \quad (3.13)$$

The limitations for the simplified procedure from the work of Morrison *et al.* (1985) are as follows:

$$\begin{aligned} d_b &\leq 1.25 \text{ in.} \\ p_f &\leq d_b + 1 \text{ in.} \\ g &\leq 5.5 \text{ in.} \\ p_{\text{ext}} &\leq p_f + 2d_b \end{aligned}$$

3.6 MULTIPLE ROW EXTENDED UNSTIFFENED 1/3

The expressions for the number of effective bolts and the effective bolt pitch for the multiple row extended unstiffened 1/3 configuration are as follows:

$$N_{\text{eff}} = 3.33 \quad (3.14)$$

$$p_{\text{eff}} = \frac{2p_f}{h^{1/4}} \left(\frac{g}{g + p_f} \right)^2 \quad (3.15)$$

where h is the height of the beam. The limitations placed on this simplified procedure stemming from the work of SEI (1984) and Morrison *et al.* (1986) are as follows:

$$\begin{aligned} d_b &\leq 1.5 \text{ in.} \\ p_f &\leq d_b + 1 \text{ in.} \\ g &\leq 5.5 \text{ in.} \\ p_{\text{ext}} &\leq p_f + 2d_b \end{aligned}$$

CHAPTER IV

COMPARISON OF RESULTS

4.1 GENERAL

To gauge the adequacy of the simplified method, various comparisons were made. First, comparisons of the predicted capacities from the simplified method and experimental capacities of moment end-plate connection tests that have been reported in the literature will be shown. The predicted failure moments will be compared to the experimental failure moments. Also, comparisons will be made between the simplified methods and the methods based on yield-line analysis and the modified Kennedy method and between the simplified methods and the methods by Borgsmiller (1995). Predicted bolt load and end-plate strength as well as calculated bolt diameter and plate thickness will be compared graphically.

4.2 EXPERIMENTAL RESULTS

Calculations for end-plate connection tests reported in the literature are contained in Appendices B through F (Srouji *et al.* 1983a, 1983b, 1984; SEI 1984; Hendrick *et al.* 1985; Morrison *et al.* 1985, 1986; Abel and Murray 1992). The tests are designated using the following generic nomenclature:

XX - d_b - t_p - h

where d_b = the bolt diameter, t_p = the nominal plate thickness, h = the beam height, and XX = the configuration identification, defined as follows:

- F1 = two-bolt flush unstiffened
- F2 = four-bolt flush unstiffened
- EF_y = four-bolt extended unstiffened, F_y referring to the nominal plate material yield stress
- ES = four-bolt extended stiffened
- MRE1/3 = multiple row extended unstiffened 1/3

For all the tests, F_{py} was determined using standard ASTM coupon test procedures. The bolts used in the tests were A325, with a nominal tensile strength of 90 ksi, according to Table J3.2 (AISC 1994). Abel and Murray (1992) report bolt tensile strengths measured using a universal testing machine.

The typical end-plate connection test set-up used in the experiments is shown in Figure 4.1. All of the test specimens were splice moment connections tested under pure moment. This was accomplished by application of two equal concentrated loads, symmetrically placed. The load was applied by a hydraulic ram with a load cell and spreader beam as shown in Figure 4.2. Lateral brace mechanisms attached to the test frame provided support for the test specimen and spreader beam.

The failure moments for the experimental results are not necessarily equal to the maximum moment applied in the tests. The failure moment, M_{fail} , may be either M_y or M_u . M_y is the experimental yield moment, and M_u is the maximum applied moment in a

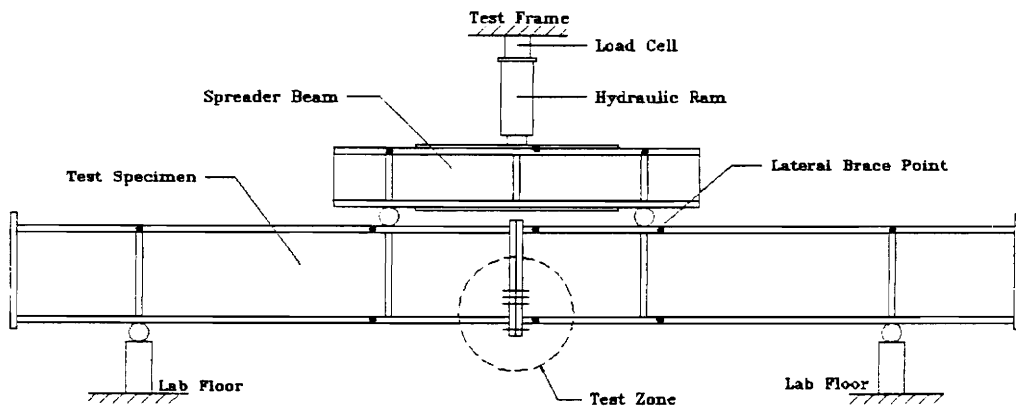


Figure 4.1 Longitudinal Elevation of Laboratory Test Setup

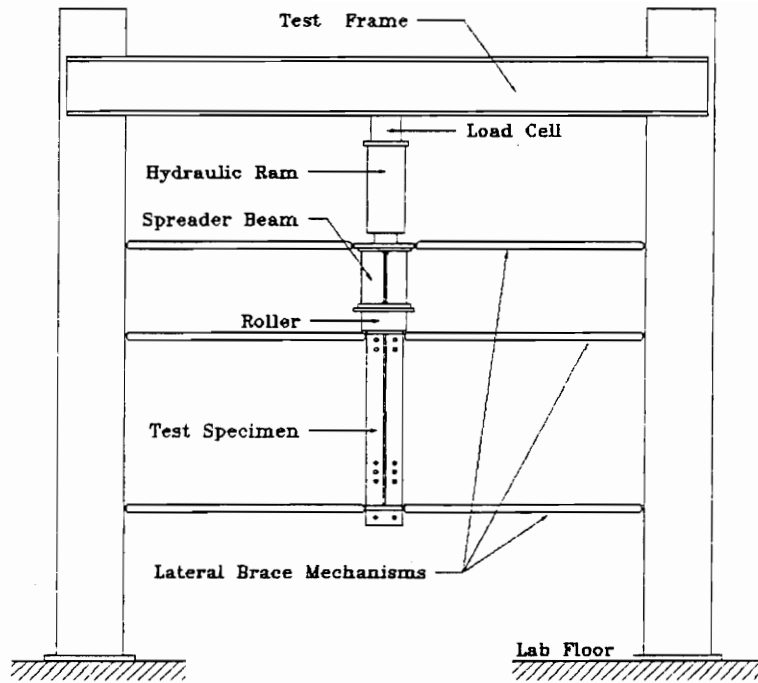


Figure 4.2 Cross-section of Laboratory Test Setup

test. Data from measuring devices placed at or near the tension flange of the test specimens were used to create applied moment versus end-plate separation plots for each test. The plots indicate whether or not the end-plate has yielded. If an approximately horizontal yield plateau is evident, then M_{fail} is taken as M_u since this level of load correlates well to end-plate yield. Excessive deformations are unlikely in a connection displaying this behavior. This is shown as Plot A in Figure 4.3. Plot B in Figure 4.3 shows a plot in which no clear yield point and a sloping yield plateau is evident. In this case, large deformations under service loads would occur if M_{fail} was taken as M_u . Therefore M_{fail} is taken as M_y , determined by bilinear approximation, as shown in Figure 4.4. M_y is located at or near the point where the end-plate yields.

In some of the tests, it is not clear whether the behavior of Plot A or Plot B of Figure 4.3 is displayed. Borgsmiller (1995) determined that the failure moment, M_{fail} , can be predicted based on the ratio of M_y / M_u . The following relationship was empirically determined:

$$M_{fail} = M_y \quad \text{if } M_y / M_u < 0.75 \quad (4.1)$$

$$M_{fail} = M_u \quad \text{if } M_y / M_u > 0.75 \quad (4.2)$$

This relationship is approximate, and if the ratio of M_y / M_u is equal to 0.75 ± 0.02 , then either M_y or M_u may be taken as M_{fail} .

4.3 DETERMINATION OF PREDICTED CAPACITY

The predicted capacities of the connection tests is taken as the minimum of the bolt and end-plate moment capacities. These are determined by using the given materials

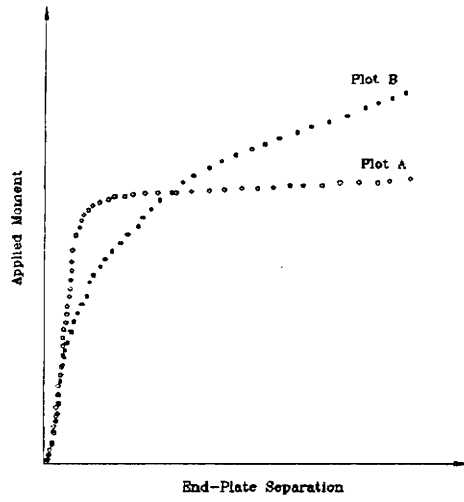


Figure 4.3 Sample of Applied Moment vs. End-Plate Separation Plots

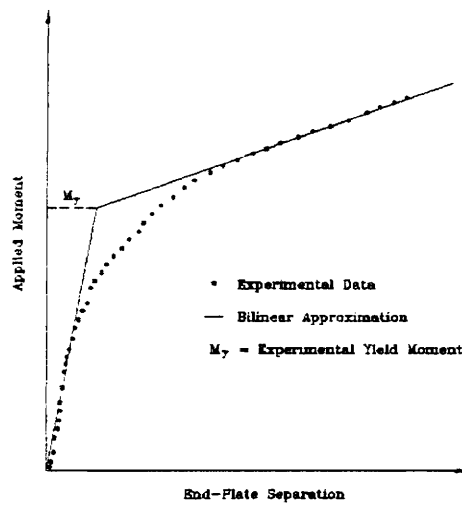


Figure 4.4 Determination of Experimental Yield Moment

and dimensions and backsolving for the flange force using Equations 3.2 and 3.3 for the bolt capacity, and Equations 3.4 and 3.5 for the end-plate capacity. The LRFD resistance factors, ϕ , are dropped from these equations. Once the flange forces are determined, the minimum of the two forces is chosen as critical and is designated as $F_{f,crit}$.

The bolt flange force is determined as follows:

$$F_{f,bolt} = \frac{\pi}{4} d_b^2 F_t N_{eff} \quad (4.3)$$

The end-plate flange force is computed as follows:

$$F_{f,plate} = \frac{F_{py} b_p t_p^2 N_{eff}}{4p_{eff}} \quad (4.4)$$

The predicted moment capacity, M_{pred} , is calculated as follows:

$$M_{pred} = F_{f,crit} (h - t_f) \quad (4.5)$$

4.4 COMPARISONS OF PREDICTED AND EXPERIMENTAL CAPACITIES

The predicted strength calculations for the connection tests referenced above are found in Appendices B through F. The results of the comparisons of the experimental results and the simplified method are tabulated in Table 4.1. The design ratios are shown for M_{pred} / M_y and M_{pred} / M_u . The gray-shaded ratios are those corresponding to the applicable failure load, determined as described previously. A design ratio greater than 1.0 is unconservative, while a ratio less than 1.0 is conservative.

The design ratios in Table 4.1 vary from 0.77 to 1.01 for the two-bolt flush unstiffened configuration. This would seem to indicate conservative results and fairly accurate agreement between the simplified method and the experimental results.

Table 4.1 Predicted versus Experimental Results

	Test	Moments (k-ft)						M_y/M_u	Design Ratios	
		M_{bolt}	M_{plate}	M_{pred}	M_y	M_u	M_{pred}/M_y		M_{pred}/M_u	
2-Bolt Flush Unstiff.	Srouji <i>et al.</i> (1983a)	83.50	78.83	78.83	75.00	97.50	0.77	1.05	0.81	
	Srouji <i>et al.</i> (1983a)	83.50	48.59	48.59	47.00	57.00	0.82	1.03	0.85	
	Srouji <i>et al.</i> (1983a)	57.98	68.45	57.98	60.00	75.00	0.80	0.97	0.77	
	Srouji <i>et al.</i> (1983a)	57.98	56.57	56.57	55.00	63.00	0.87	1.03	0.90	
	Srouji <i>et al.</i> (1983a)	35.90	45.27	35.90	32.00	39.47	0.81	1.12	0.91	
	Srouji <i>et al.</i> (1983a)	36.13	31.08	31.08	25.00	33.92	0.74	1.24	0.92	
	Srouji <i>et al.</i> (1983a)	125.91	122.05	122.05	110.00	120.25	0.91	1.11	1.01	
	Srouji <i>et al.</i> (1983a)	125.91	146.49	125.91	125.00	154.20	0.81	1.01	0.82	
	Srouji <i>et al.</i> (1983b, 1984)	72.48	108.77	72.48	85.00	108.00	0.79	0.85	0.67	
4-Bolt Flush Unstiff.	Srouji <i>et al.</i> (1983b, 1984)	72.48	68.52	68.52	85.00	85.50	0.99	0.81	0.80	
	Srouji <i>et al.</i> (1983b, 1984)	157.39	135.51	135.51	140.00	171.80	0.81	0.97	0.79	
	Srouji <i>et al.</i> (1983b, 1984)	157.39	109.47	109.47	110.00	144.70	0.76	1.00	0.76	
	Srouji <i>et al.</i> (1983b, 1984)	104.37	91.74	91.74	85.00	115.50	0.74	1.08	0.79	
	Srouji <i>et al.</i> (1983b, 1984)	104.37	56.22	56.22	62.00	73.20	0.85	0.91	0.77	
	Abel and Murray (1992)	173.92	396.03	173.92	250.00	259.70	0.96	0.70	0.67	
	Abel and Murray (1992)	173.92	526.39	173.92	275.00	275.10	1.00	0.63	0.63	
	Abel and Murray (1992)	333.96	302.27	302.27	230.00	337.80	0.68	1.31	0.89	
	Abel and Murray (1992)	419.74	370.66	370.66	350.00	503.00	0.70	1.06	0.74	
4-Bolt Extended Stiff.	Morrison <i>et al.</i> (1985)	92.89	102.54	92.89	80.00	114.90	0.70	1.16	0.81	
	Morrison <i>et al.</i> (1985)	132.41	143.20	132.41	130.00	163.40	0.80	1.02	0.81	
	Morrison <i>et al.</i> (1985)	167.64	193.48	167.64	180.00	235.10	0.77	0.93	0.71	
	Morrison <i>et al.</i> (1985)	167.87	143.70	143.70	150.00	203.00	0.74	0.96	0.71	
	Morrison <i>et al.</i> (1985)	360.93	249.28	249.28	260.00	349.50	0.74	0.96	0.71	
	Morrison <i>et al.</i> (1985)	355.02	331.18	331.18	280.00	379.40	0.74	1.18	0.87	
	Morrison <i>et al.</i> (1986)	581.12	280.52	280.52	270.00	404.90	0.67	1.04	0.69	
	Morrison <i>et al.</i> (1986)	324.79	253.92	253.92	300.00	425.10	0.71	0.85	0.60	
	Morrison <i>et al.</i> (1986)	683.30	593.40	593.40	520.00	866.10	0.60	1.14	0.69	
Multiple Row Extended Unstiff. 1/3	Morrison <i>et al.</i> (1986)	1130.26	929.21	929.21	975.00	975.10	1.00	0.95	0.95	
	Morrison <i>et al.</i> (1986)	1871.91	1226.19	1226.19	1200.00	1635.00	0.73	1.02	0.75	
	Morrison <i>et al.</i> (1986)	2689.55	1513.27	1513.27	1600.00	2329.60	0.69	0.95	0.65	
	Morrison <i>et al.</i> (1986)	673.05	1107.92	673.05	750.00	929.00	0.81	0.90	0.72	
	SEI (1984)	1196.53	2163.73	1196.53	1250.00	1364.00	0.92	0.96	0.88	

For the four-bolt flush unstiffened configuration, the design ratios vary from 0.67 to 1.08. All values are less than 10% percent unconservative. The more conservative values may be traced to some difficulty in finding an equation for p_{eff} and the range of values obtained for N_{eff} . The equation for the effective pitch is more conservative than wished for in some cases. Because a value for N_{eff} was chosen from the most conservative end of the range, it is not surprising to find these conservative ratios also.

For the four-bolt extended unstiffened configuration, the design ratio varies from 0.63 to 1.31. The most conservative ratios occur when the bolt prediction controls and are directly related to the choice of N_{eff} . The largest design ratio is the only one greater than 10% unconservative. A somewhat large range of values for N_{eff} was obtained for this configuration. Due to the conservative value chosen, it is not surprising that the design ratios in these cases are conservative.

The design ratio for the four-bolt extended stiffened configuration ranges from 0.81 to 1.16. In all but one case, the prediction is less than 10% unconservative. Again there was a wide range of values obtained for N_{eff} , but a conservative value was chosen and the equation for p_{eff} seems to predict the capacity of these tests quite well.

The final configuration examined is the multiple row extended unstiffened 1/3. The design ratio for this configuration varies from 0.72 to 1.14. Only one prediction is more than 10% unconservative.

4.5 COMPARISON OF THE SIMPLIFIED METHOD TO THE YIELD-LINE AND KENNEDY BASED METHODS

Additional comparisons were made between the simplified method and yield-line and Kennedy based methods, as well as, the simplified method and the method developed by Borgsmiller (1995). Borgsmiller's method uses the same yield-line equations shown in Chapter 2, but includes a simplified Kennedy analysis. The comparisons are made with respect to predicted bolt load and end-plate capacity and calculated required bolt diameter and plate thickness for all beam sections of W10 and larger, subject to the limitations of Chapter 3.

Figures 4.5 through 4.19 contain plots showing the relationships. The units in these plots are kips and inches. There are three figures for each of the five configurations. The first of the three figures contains two plots, one showing the comparison between predicted bolt load and the second showing the comparison between predicted end-plate capacity. These plots are made for the simplified method versus the unified yield-line and modified Kennedy based methods. The second and third figures also each contain two plots, one showing a comparison of calculated bolt diameter and the other calculated plate thickness. These figures are for the simplified method versus the unified methods and the Borgsmiller (1995) method respectively.

Within each plot are three lines. The solid line is an exact equivalency line, where the value calculated by both methods is equal. Points plotted below this line are conservative while points above are unconservative. Two additional dashed lines are

found on each plot. These are 10% lines, creating an envelope on either side of the exact equivalency line. If any point is within this envelope then the calculated values corresponding to that point differ by less than 10%.

Figures 4.5 through 4.7 relate to the two-bolt flush unstiffened configuration. For the simplified method versus both the yield-line and Kennedy based method (Hendrick *et al.*, 1985) and the Borgsmiller (1995) method, the data points in the plots generally fall within the 10% envelope. This is further confirmation that the method is rather accurate with respect to prediction of the connection capacity.

For the four-bolt flush unstiffened configuration, Figures 4.8 through 4.10 show the comparison plots. In both the simplified method versus the yield-line and Kennedy based method (Hendrick *et al.*, 1985) and versus Borgsmiller (1995), the data points for predicted bolt load, calculated bolt diameter and calculated plate thickness generally fall within the 10% envelope. The data points for predicted end-plate capacity range from within the envelope to more conservative.

The plots in Figures 4.11 through 4.13 relate to the four-bolt extended unstiffened configuration. Here data points fall within the 10% envelope and/or the conservative side for the simplified method versus both the yield-line and Kennedy based method (Abel and Murray, 1992) and the Borgsmiller (1995) method.

Figures 4.14 through 4.16 contain the plots relating to the four-bolt extended stiffened configuration. For the simplified method versus the yield-line and Kennedy based method (Morrison *et al.*, 1985), the data points generally fall within the

conservative side of the 10% envelope. A few points for predicted bolt load fall outside the 10% envelope. For the simplified versus Borgsmiller (1995) method, the data points also generally fall within the 10% envelope.

The last configuration is the multiple row extended unstiffened 1/3. The plots in Figures 4.17 through 4.19 relate to this configuration. For the simplified versus the yield-line and Kennedy based method (Morrison et al., 1986), the data points generally fall within the 10% envelope and to the conservative side. When the simplified method is compared to Borgsmiller (1995) method, the data points relating to bolt diameter are more conservative. The data points for end-plate capacity generally fall within the 10% envelope however. The trend for the bolt diameter data points for the simplified versus Borgsmiller (1995) method may not be unreasonable. Morrison *et al.* (1986) made assumptions as to the distribution of the flange force between the inner and outer end-plates. This feature is not incorporated into the method by Borgsmiller (1995). In addition, an examination of the results of Morrison *et al.* (1986) found that some of the predicted capacities were somewhat conservative when compared to experimental results. These may provide reasons for the more conservative results.

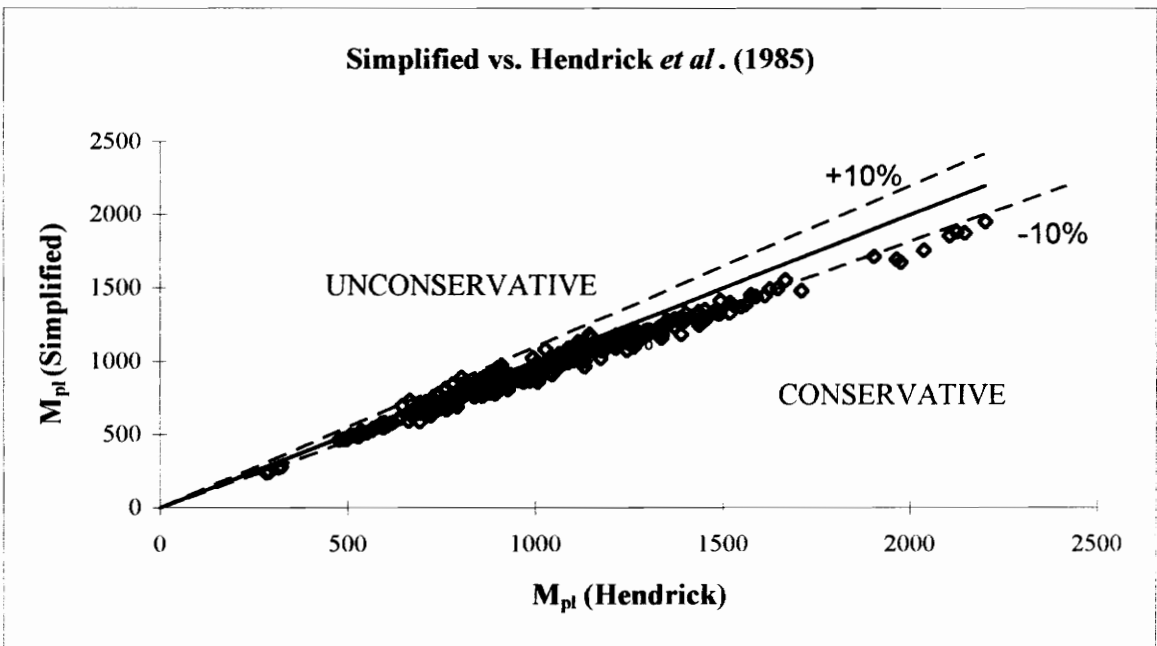
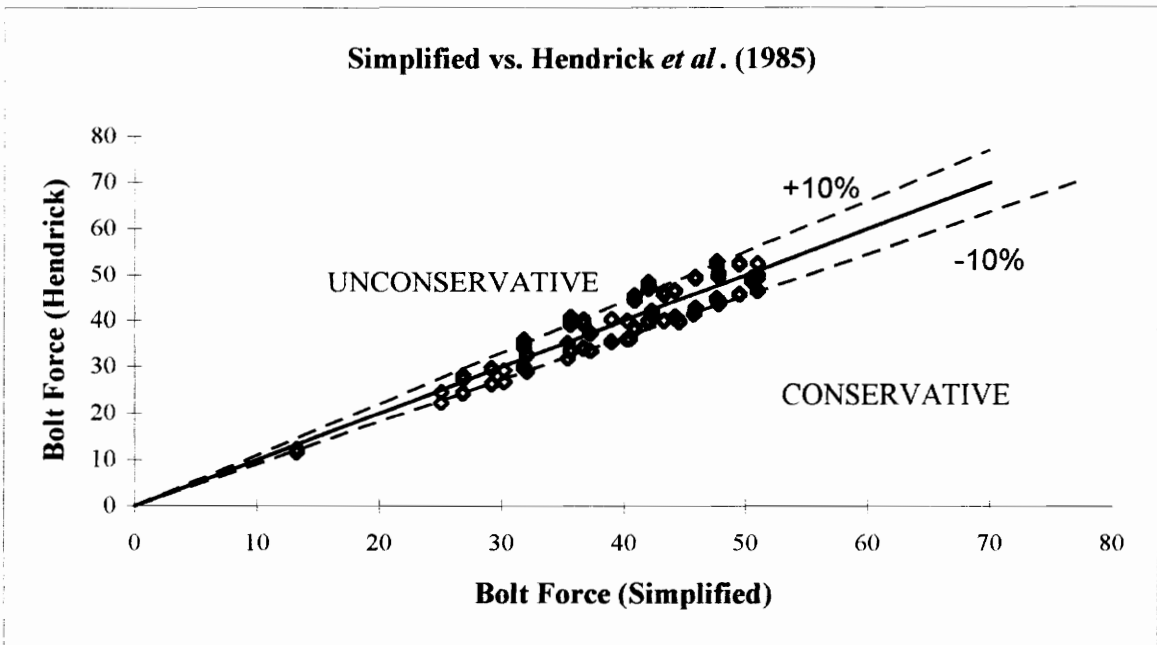


Figure 4.5 Two-Bolt Flush Unstiffened -
Simplified vs. Hendrick *et al.* (1985):
Predicted Bolt Force and End-Plate Capacity

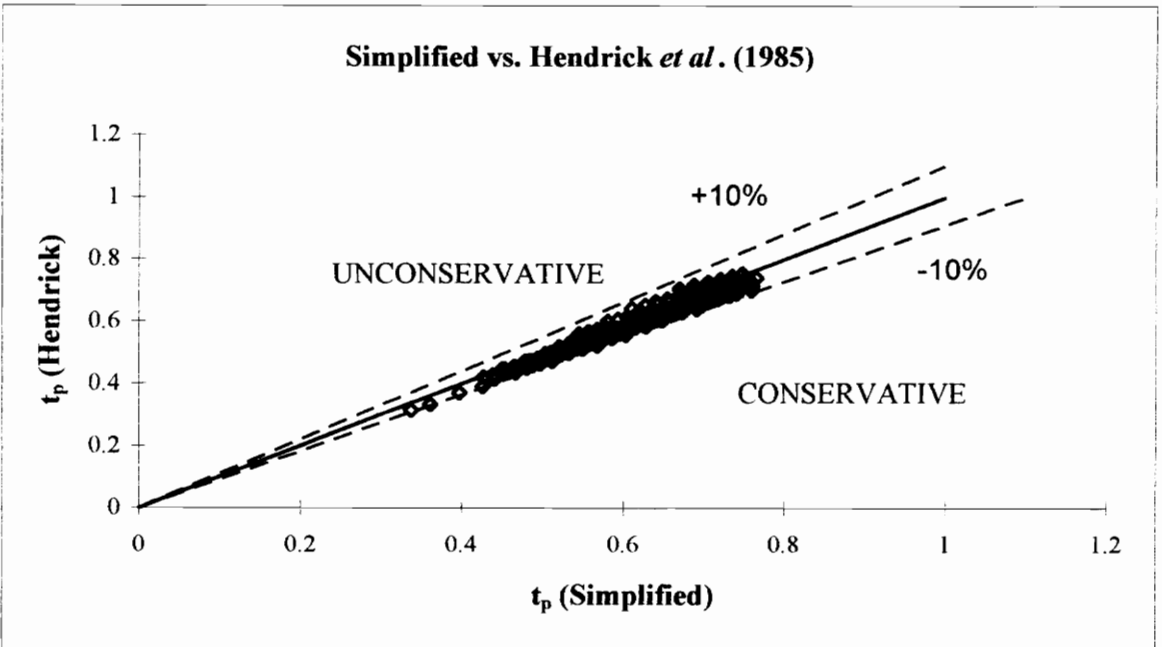
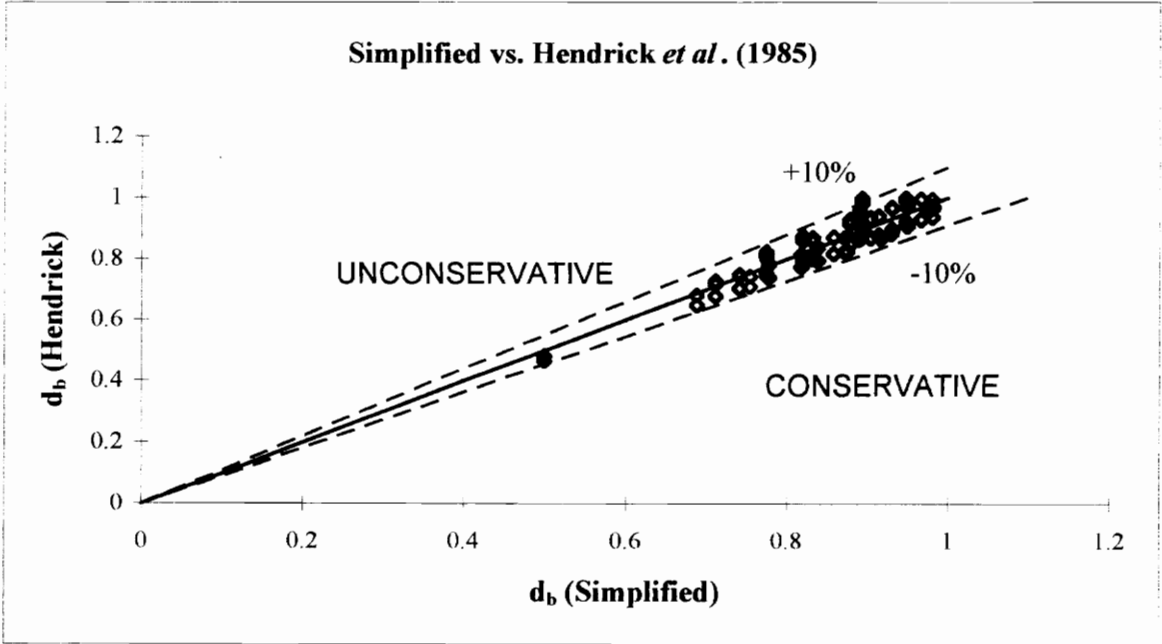


Figure 4.6 Two-Bolt Flush Unstiffened -
Simplified vs. Hendrick *et al.* (1985):
Calculated Bolt Diameter and Plate Thickness

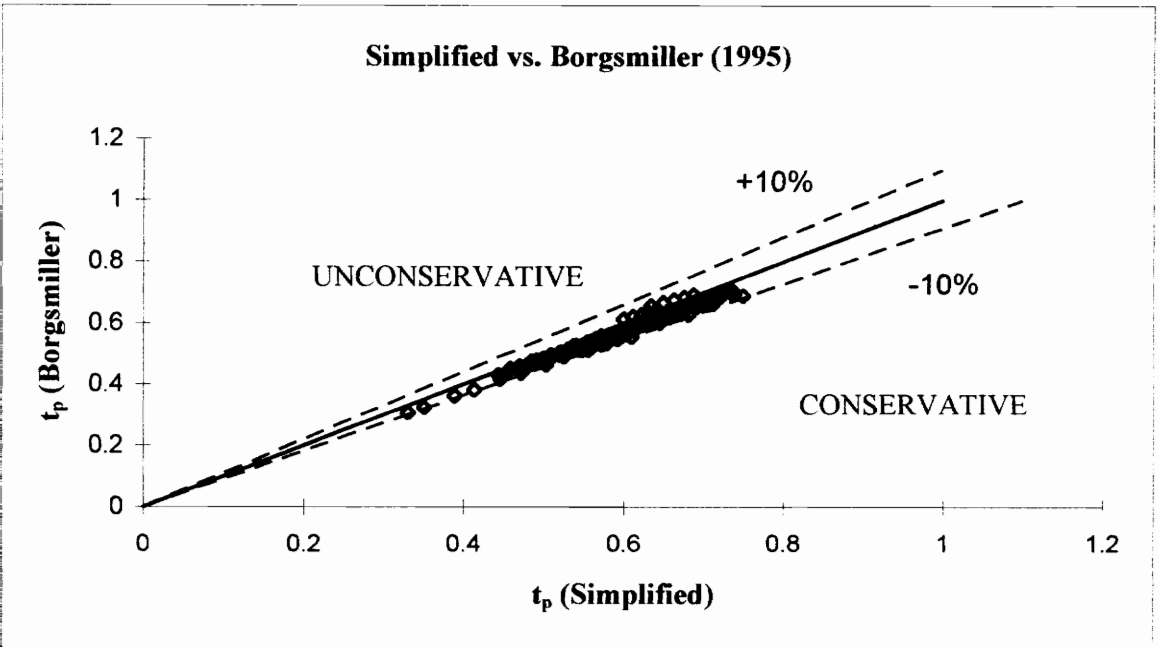
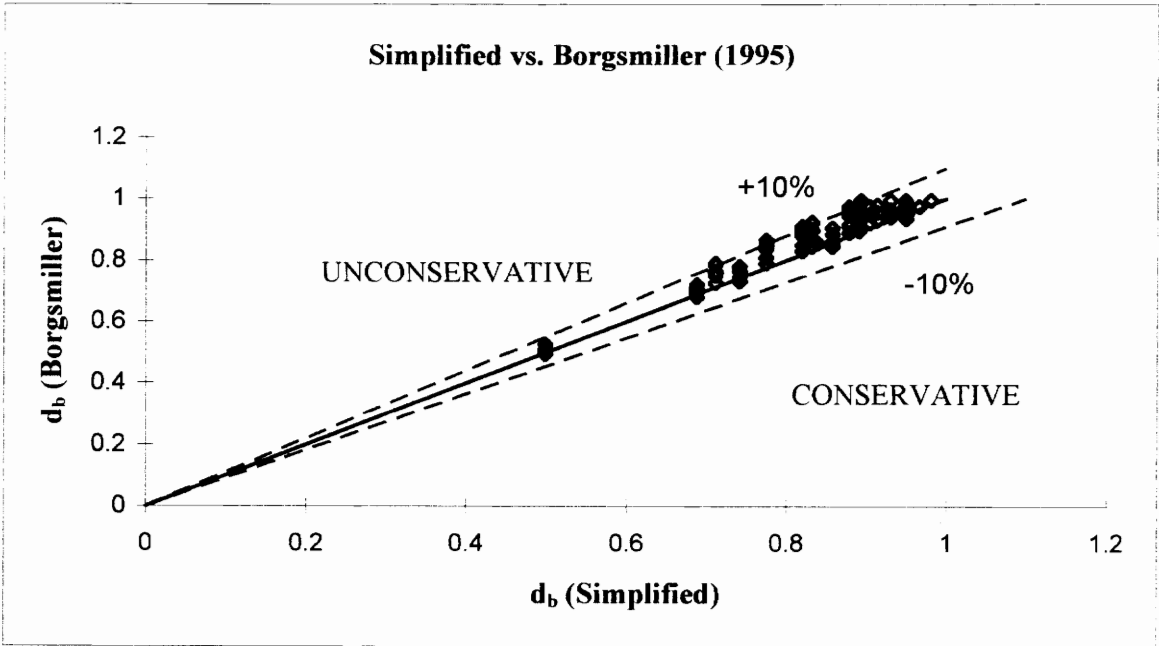


Figure 4.7 Two-Bolt Flush Unstiffened -
Simplified vs. Borgsmiller (1995):
Calculated Bolt Diameter and Plate Thickness

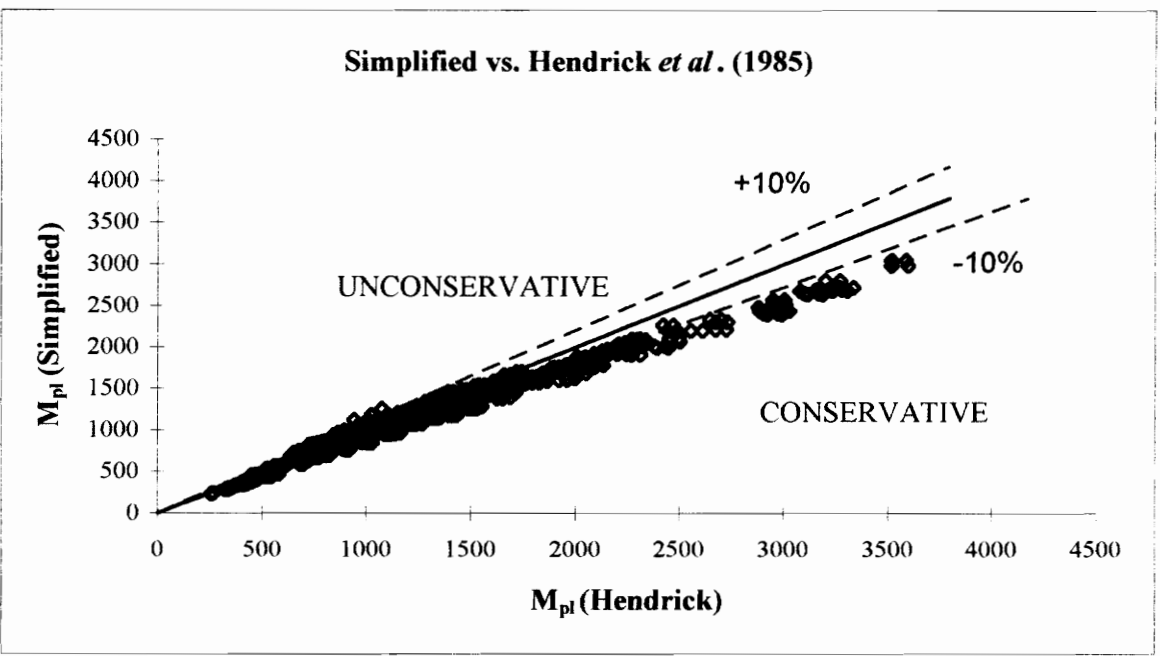
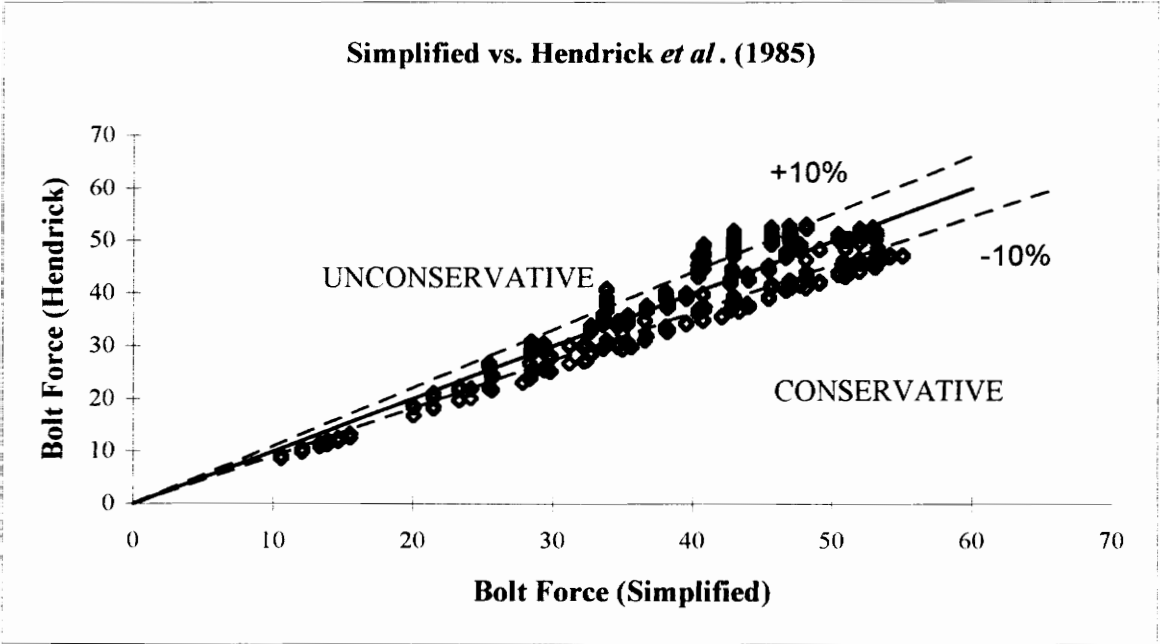


Figure 4.8 Four-Bolt Flush Unstiffened -
Simplified vs. Hendrick *et al.* (1985):
Predicted Bolt Force and End-Plate Capacity

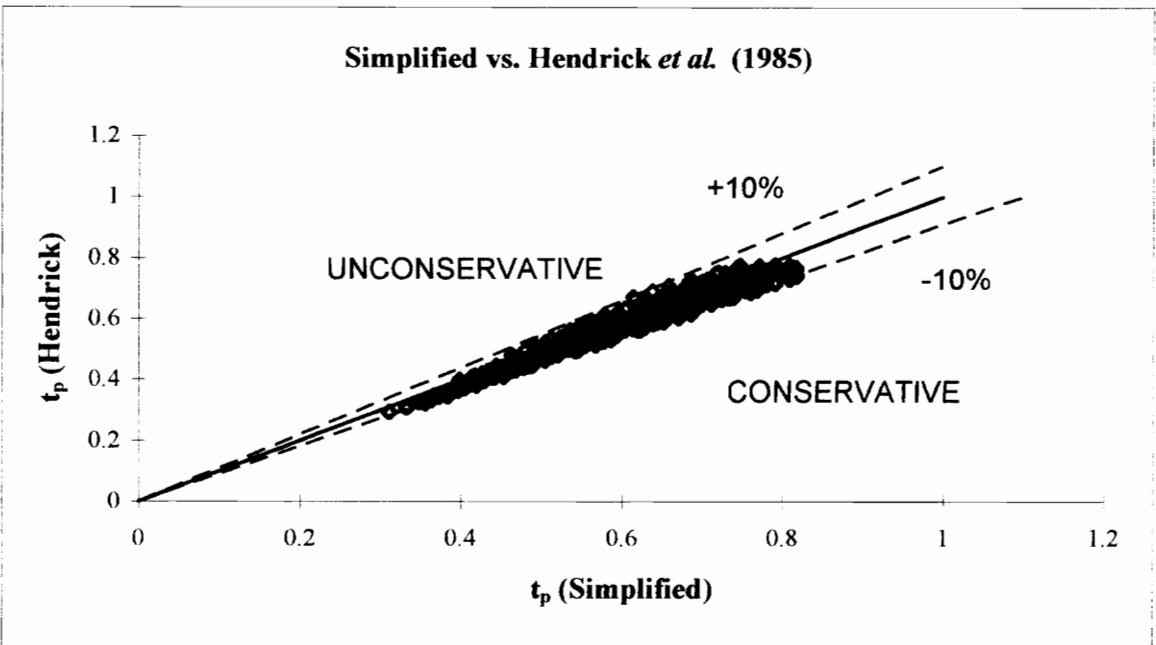
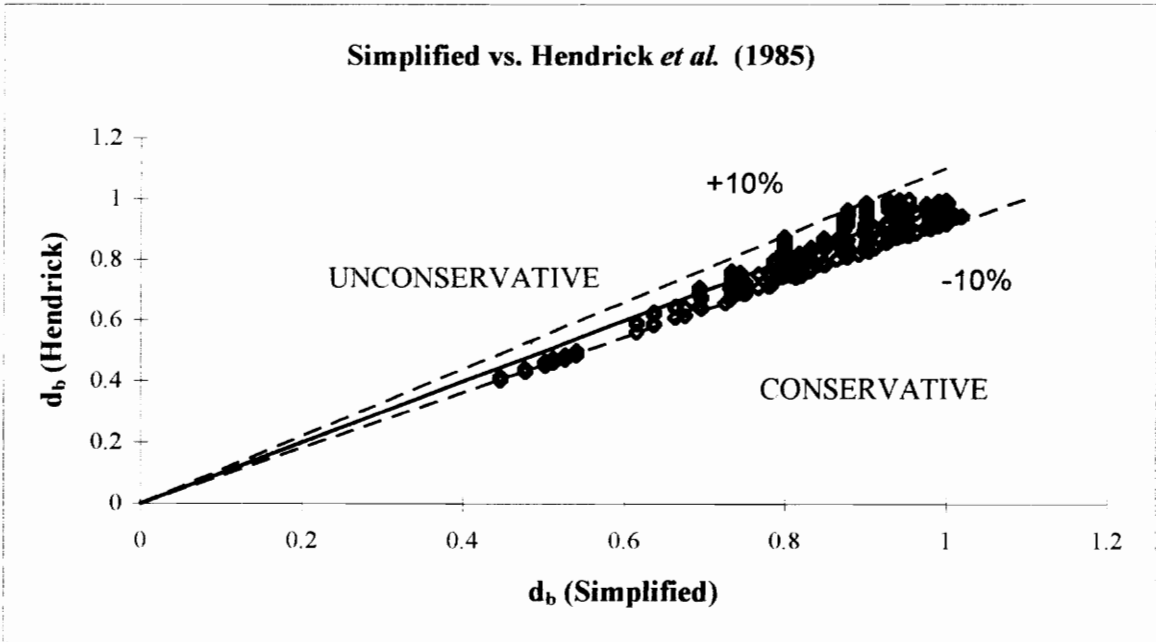


Figure 4.9 Four-Bolt Flush Unstiffened -
Simplified vs. Hendrick *et al.* (1985):
Calculated Bolt Diameter and Plate Thickness

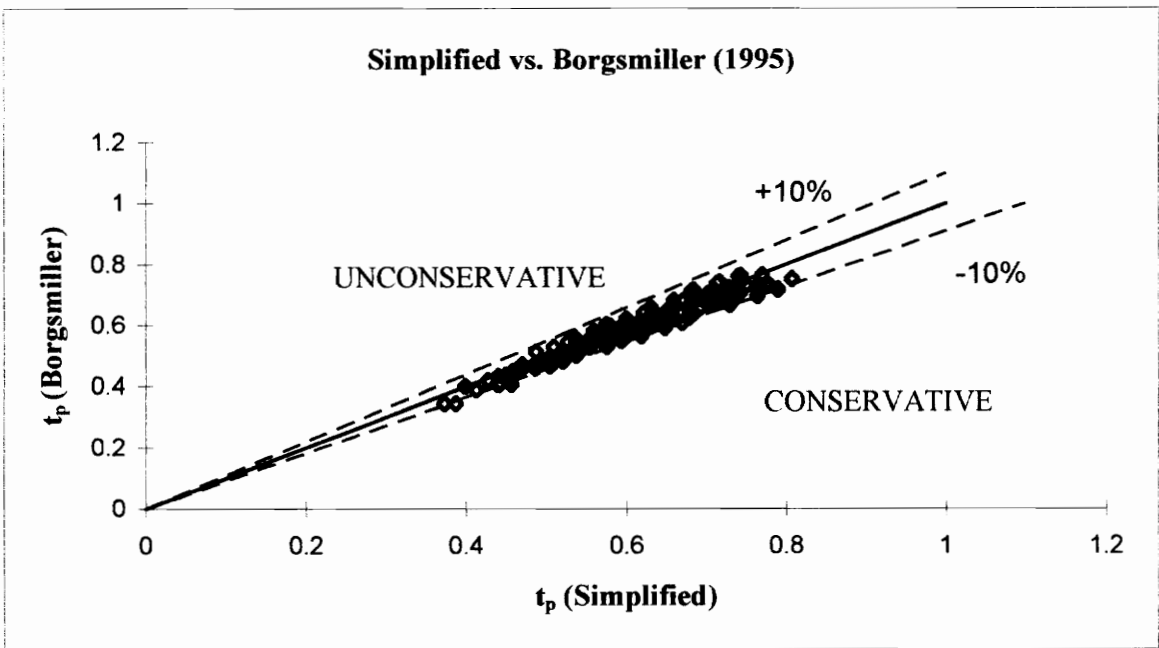
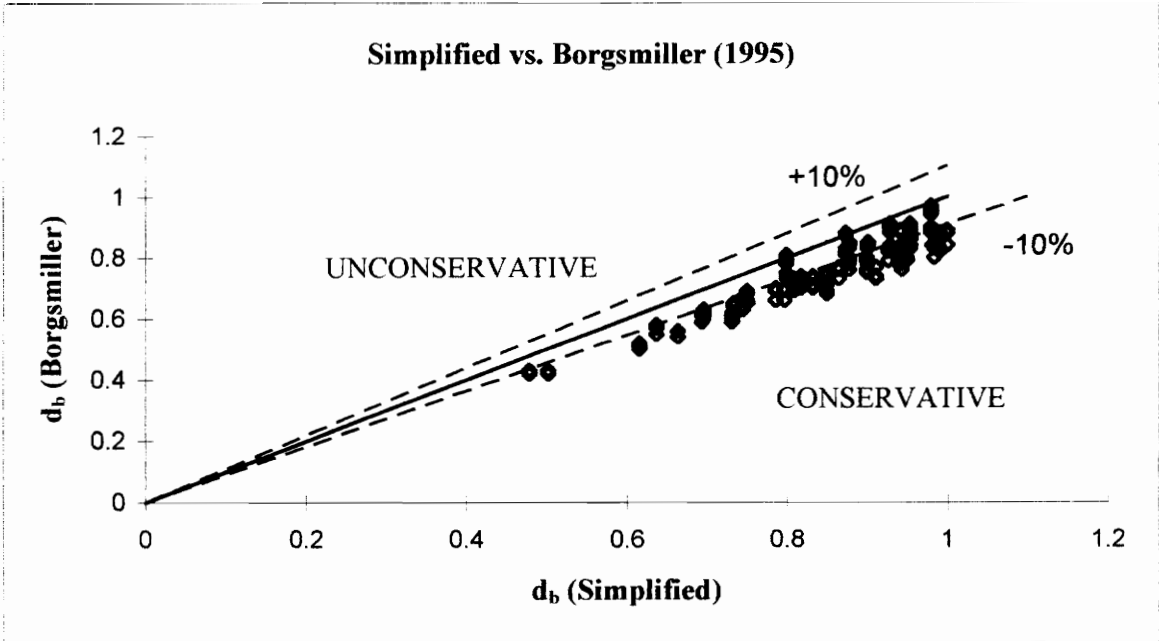


Figure 4.10 Four-Bolt Flush Unstiffened -
Simplified vs. Borgsmiller (1985):
Calculated Bolt Diameter and Plate Thickness

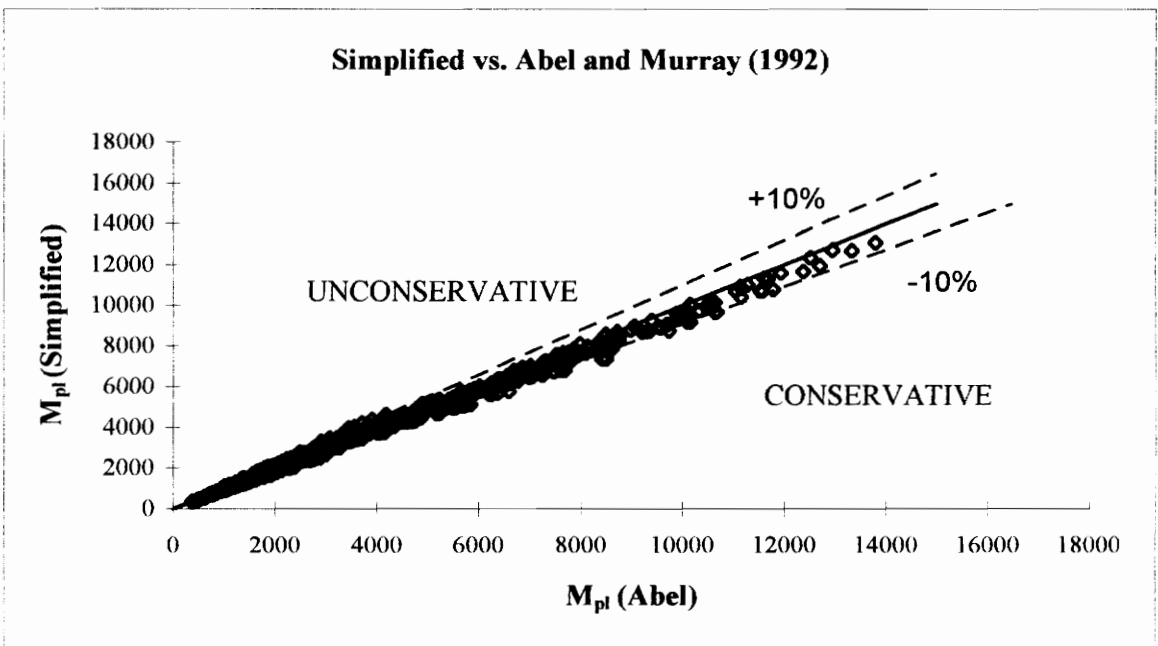
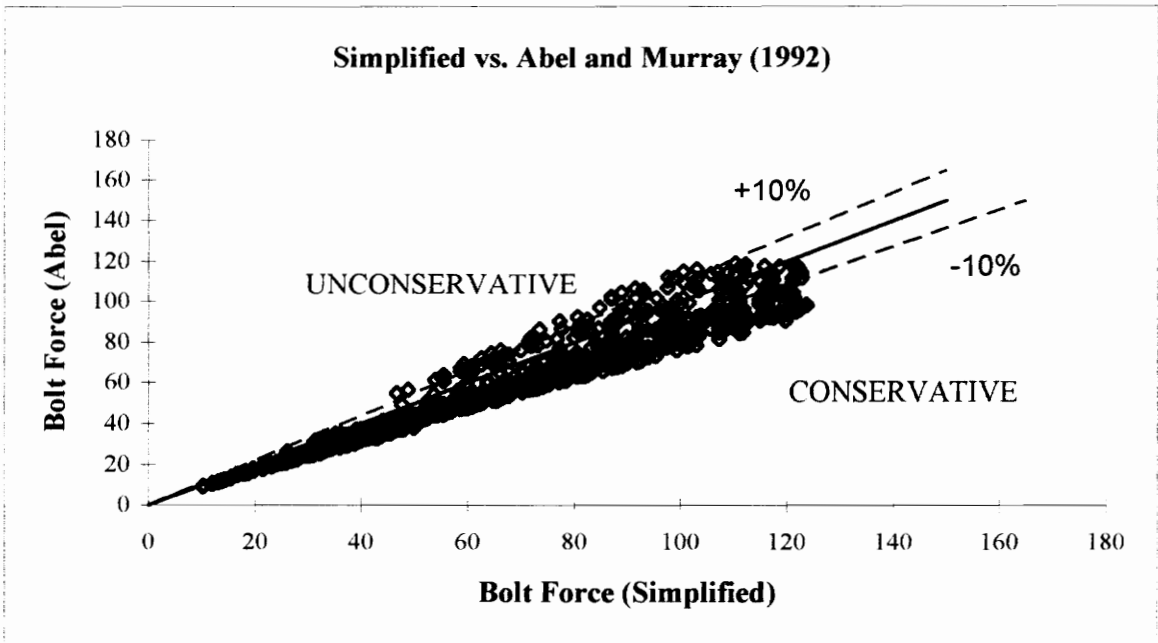


Figure 4.11 Four-Bolt Extended Unstiffened -
Simplified vs. Abel and Murray (1992):
Predicted Bolt Force and End-Plate Capacity

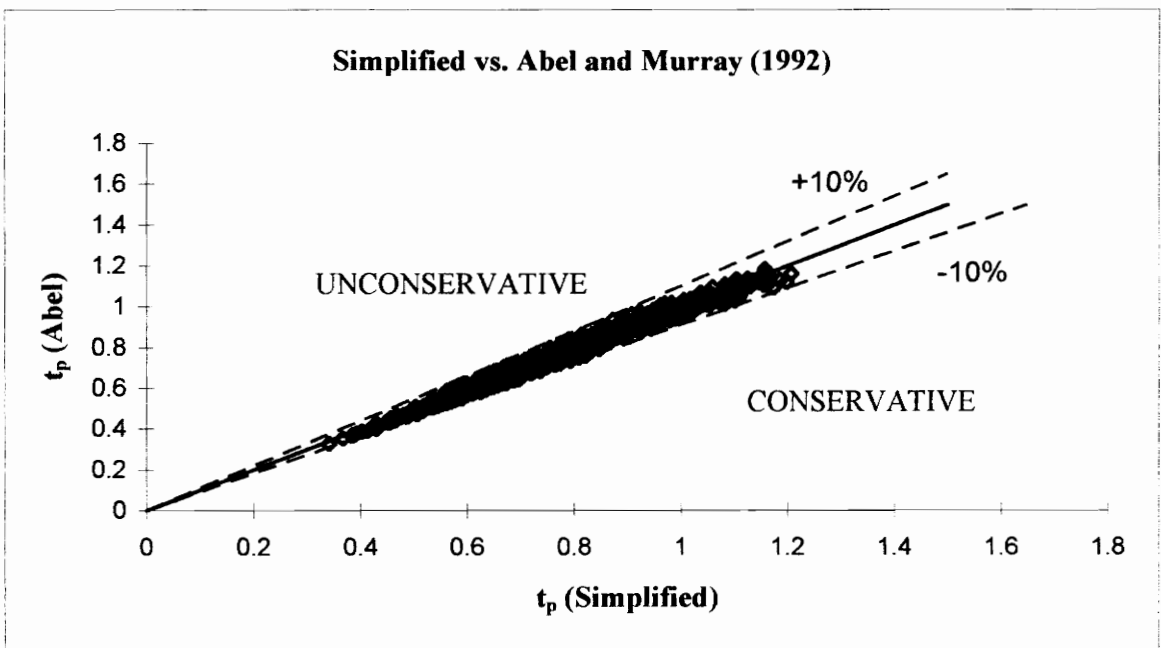
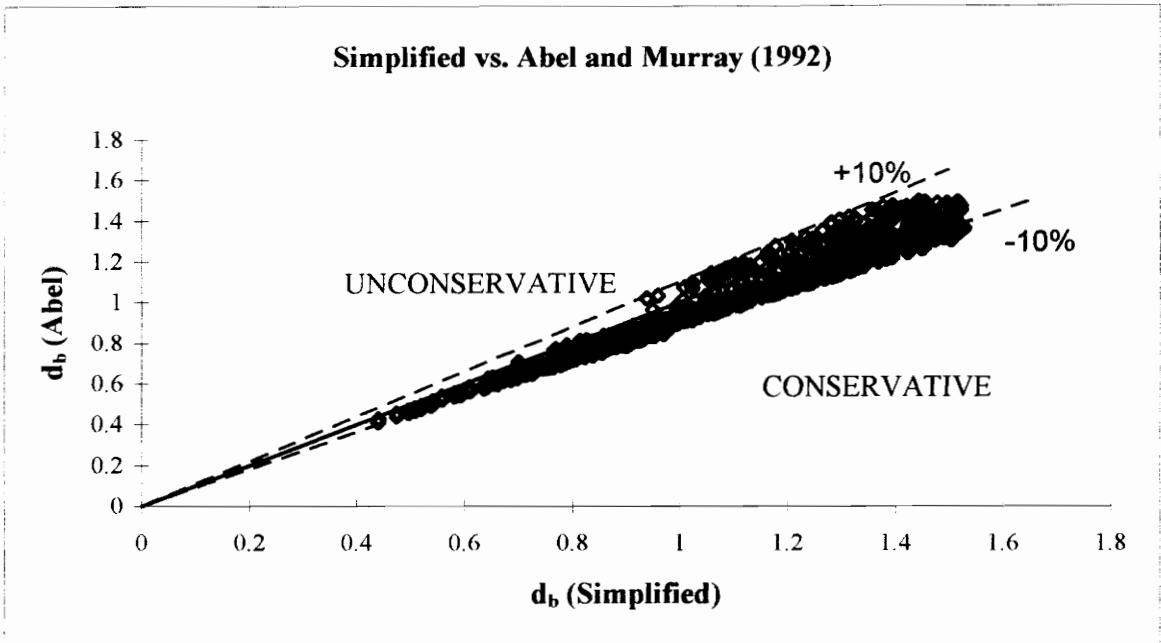


Figure 4.12 Four-Bolt Extended Unstiffened -
Simplified vs. Abel and Murray (1992):
Calculated Bolt Diameter and Plate Thickness

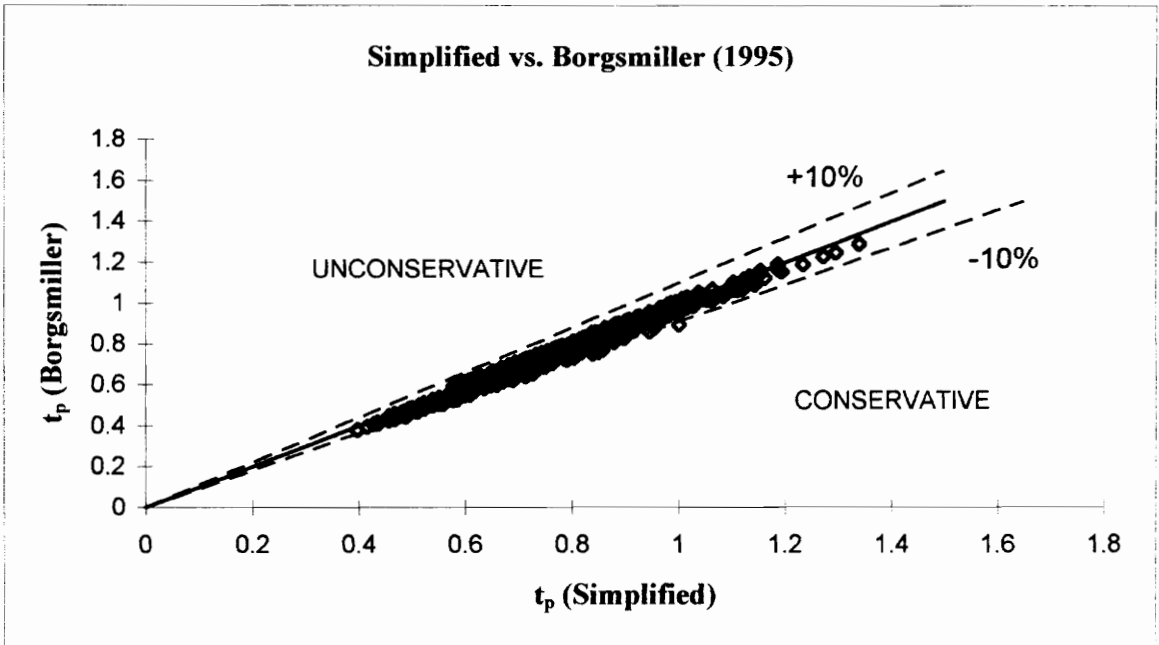
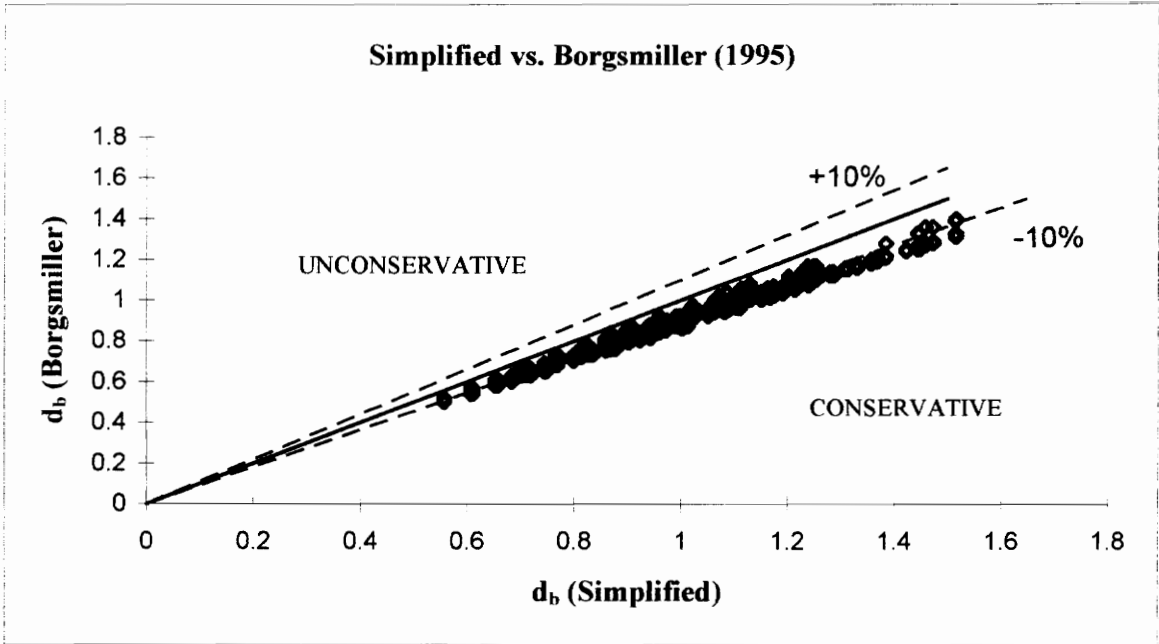


Figure 4.13 Four-Bolt Extended Unstiffened - Simplified vs. Borgsmiller (1995): Calculated Bolt Diameter and Plate Thickness

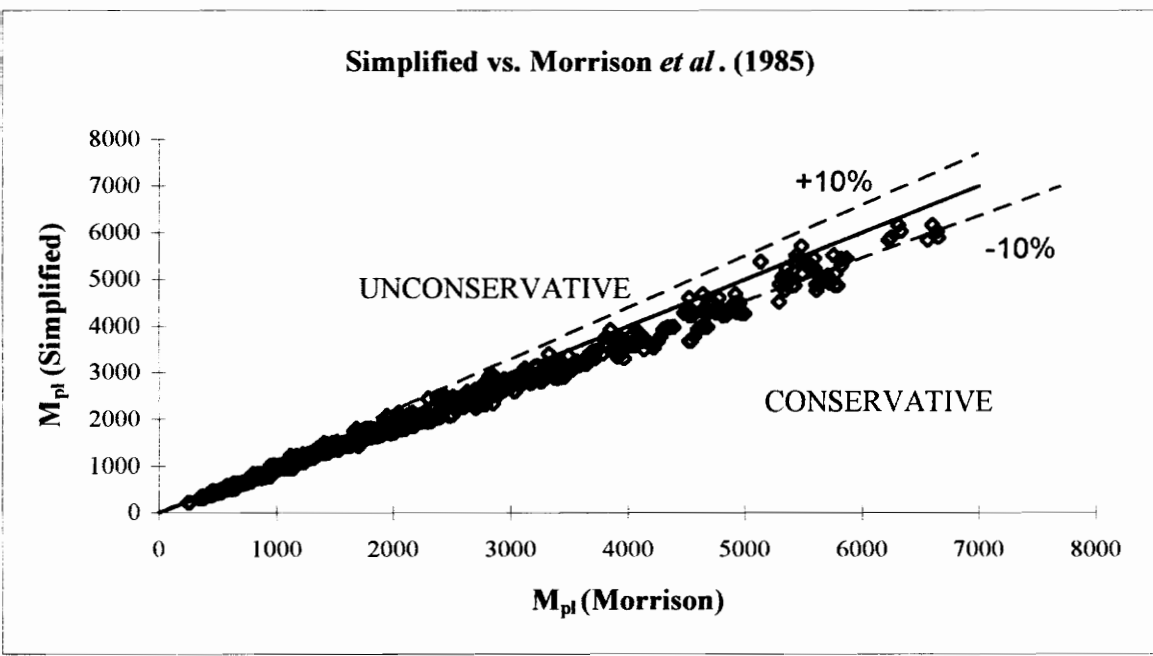
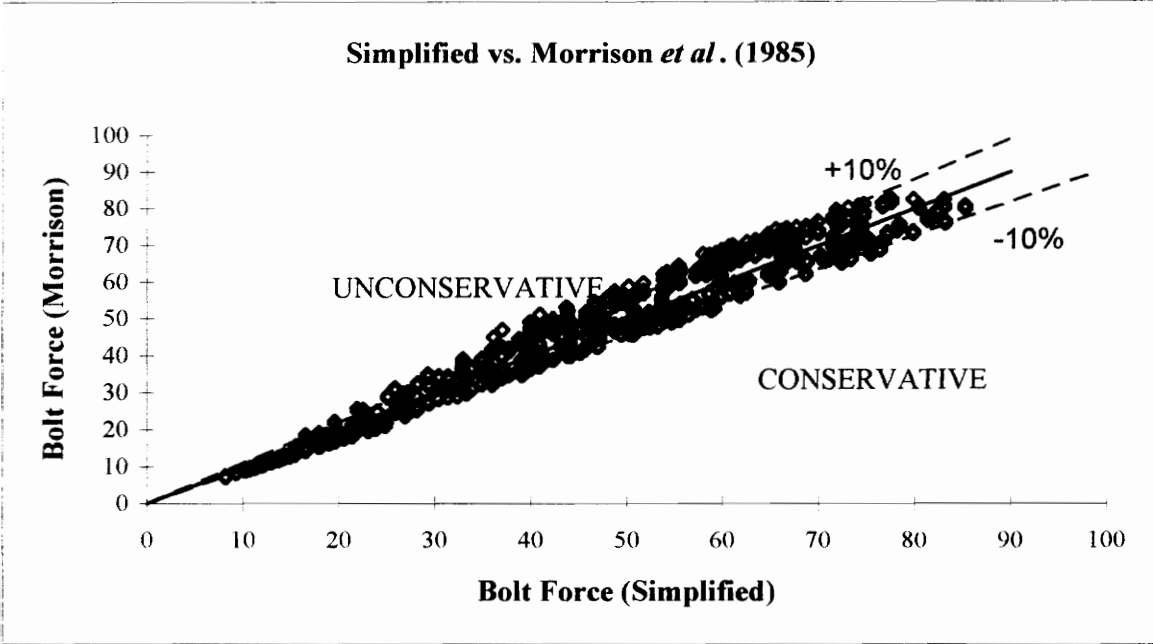


Figure 4.14 Four-Bolt Extended Stiffened - Simplified vs. Morrison *et al.* (1985); Predicted Bolt Force and End-Plate Capacity

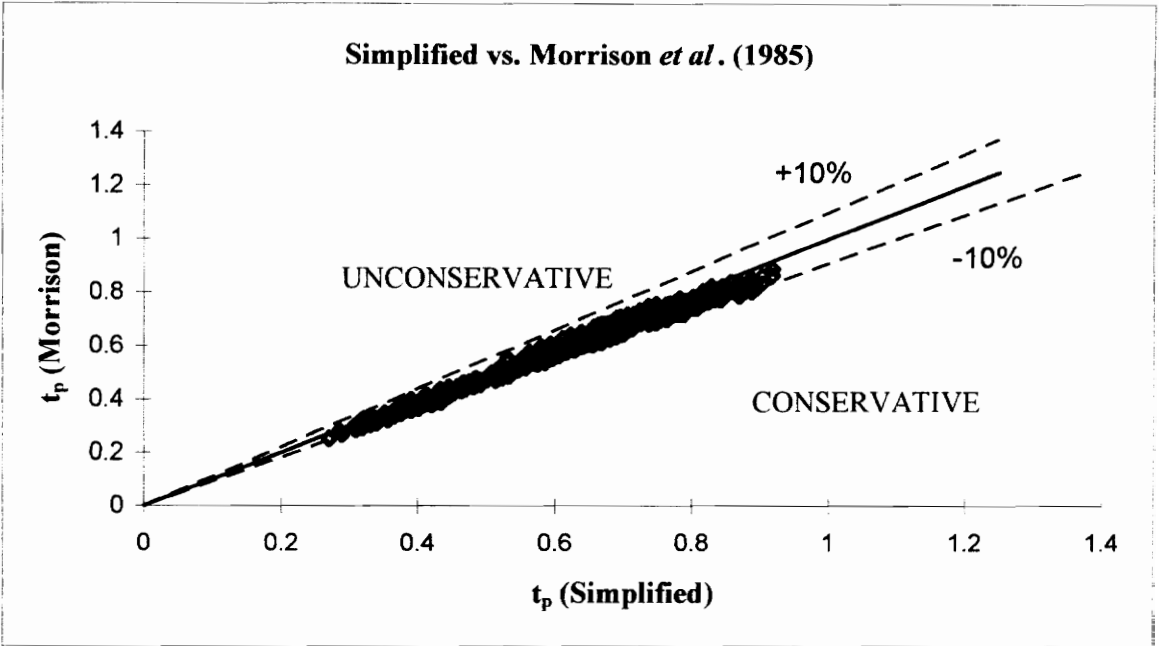
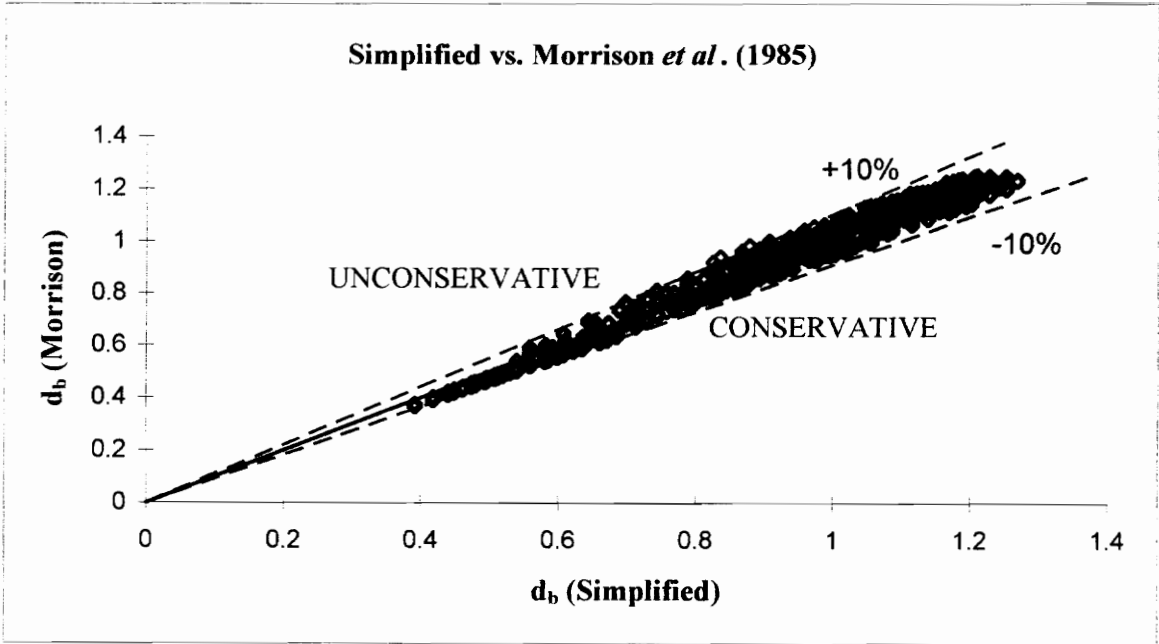


Figure 4.15 Four-Bolt Extended Stiffened -
Simplified vs. Morrison *et al.* (1985):
Calculated Bolt Diameter and Plate Thickness

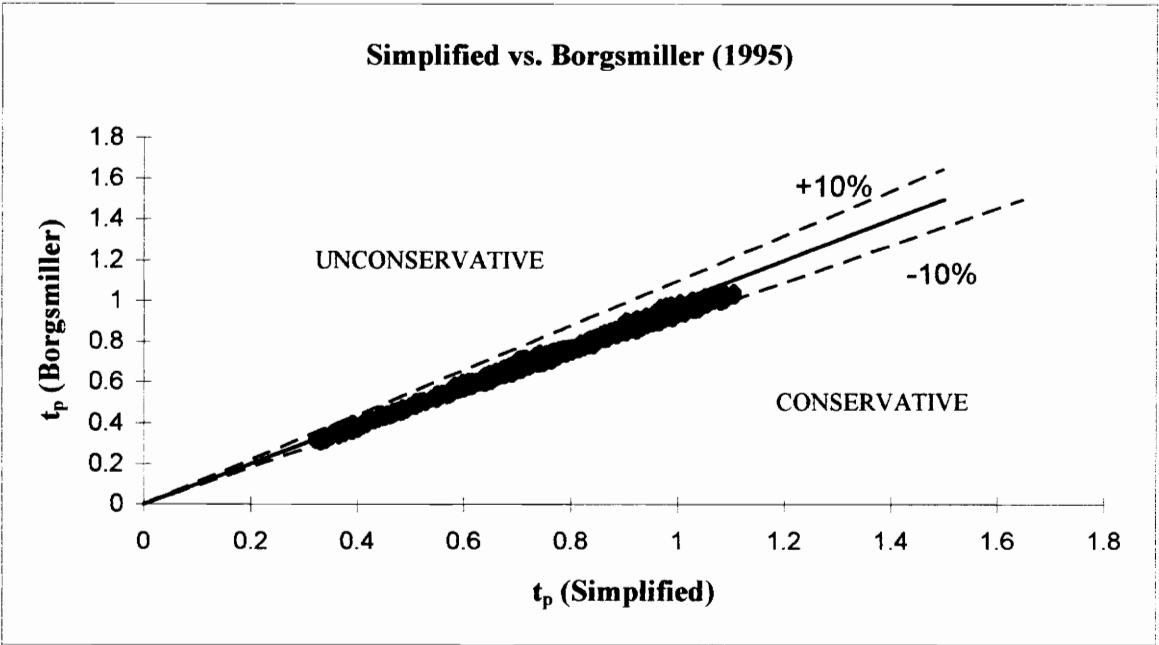
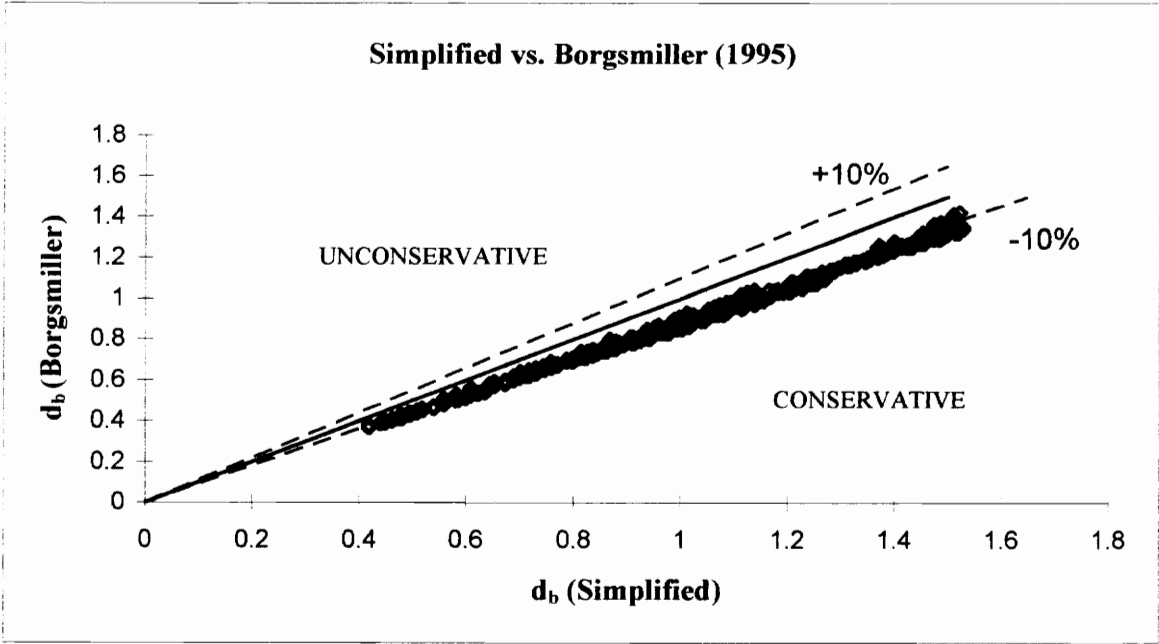
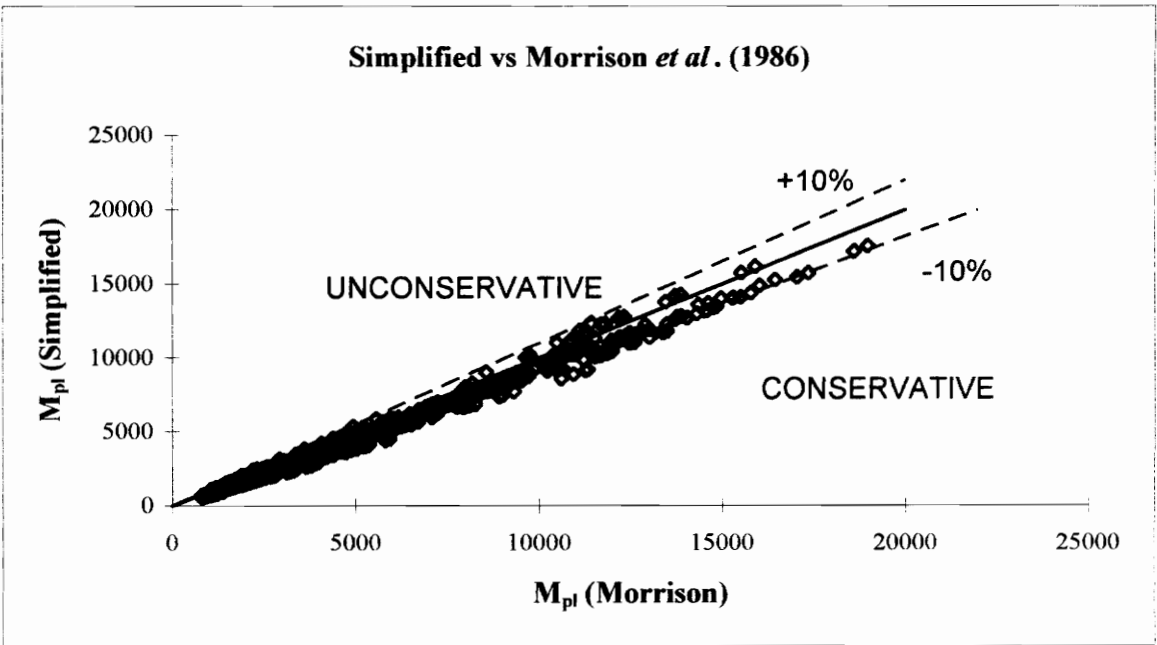
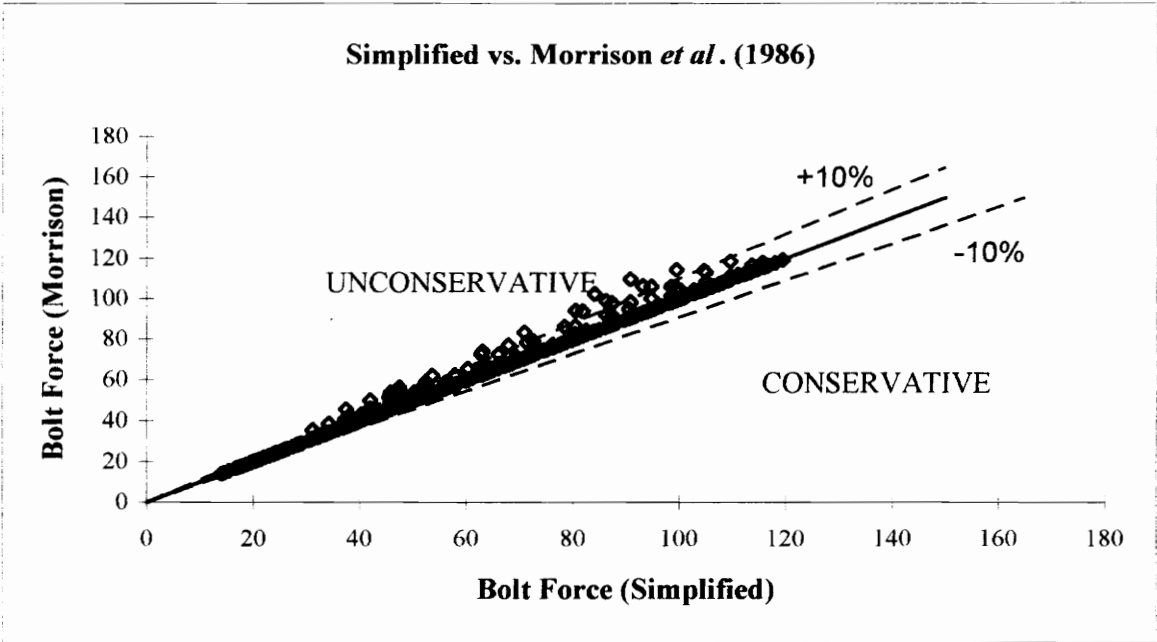


Figure 4.16 Four-Bolt Extended Stiffened -
Simplified vs. Borgsmiller (1995):
Calculated Bolt Diameter and Plate Thickness



**Figure 4.17 Multiple Row Extended Unstiffened 1/3 -
Simplified vs. Morrison *et al.* (1986):
Predicted Bolt Force and End-Plate Capacity**

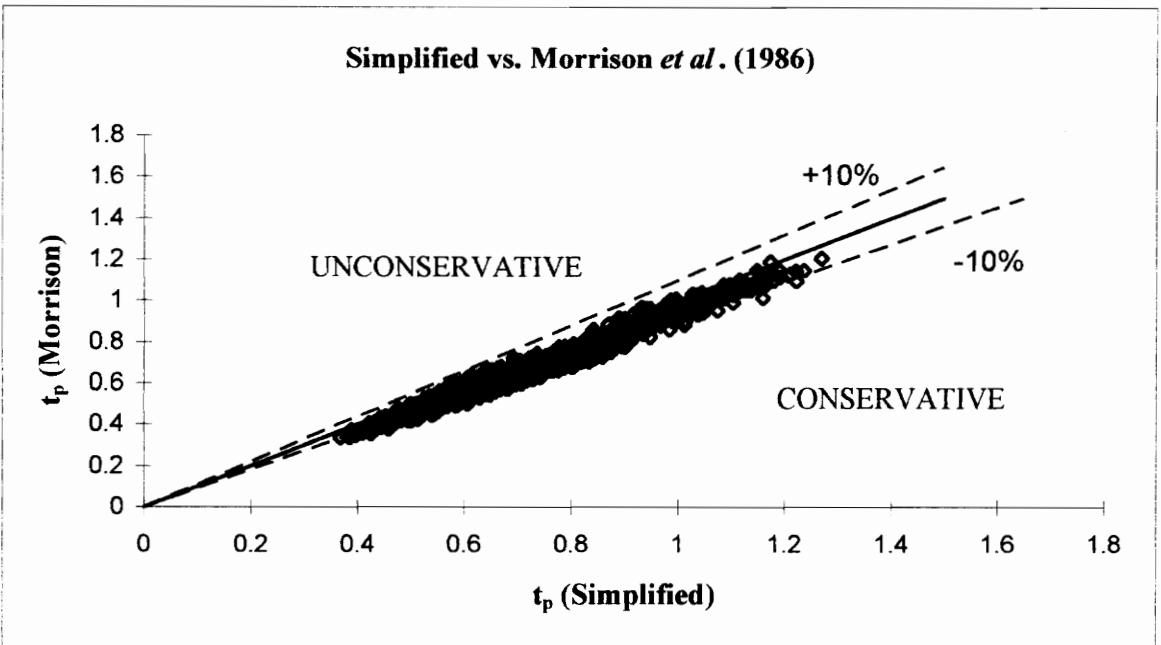
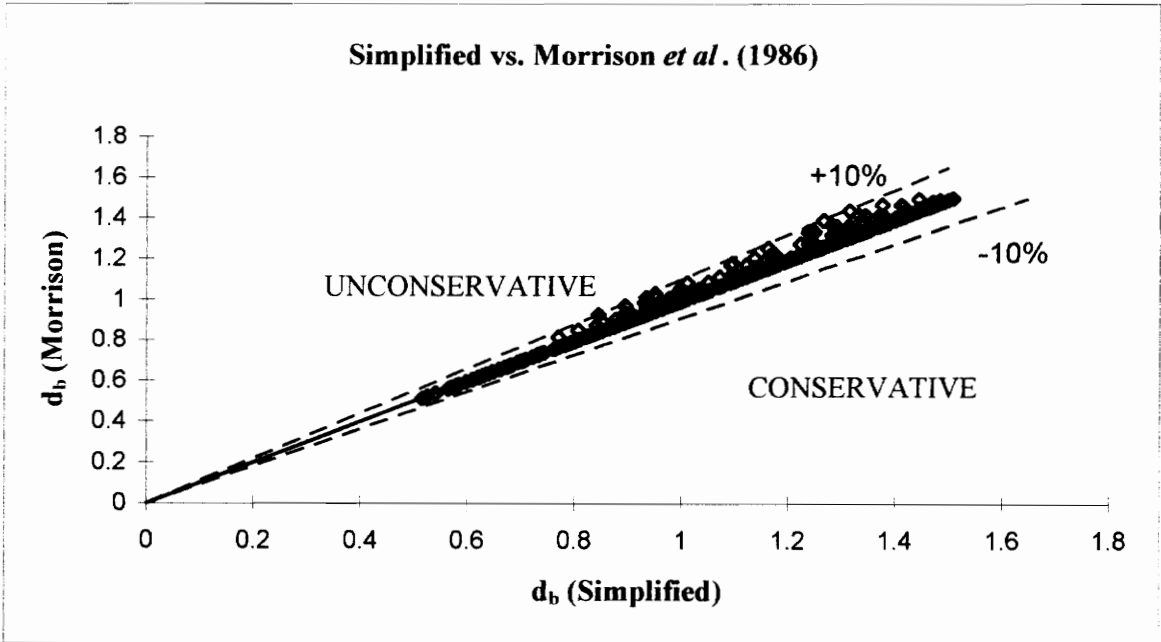


Figure 4.18 Multiple Row Extended Unstiffened 1/3 -
Simplified vs. Morrison *et al.* (1986):
Calculated Bolt Diameter and Plate Thickness

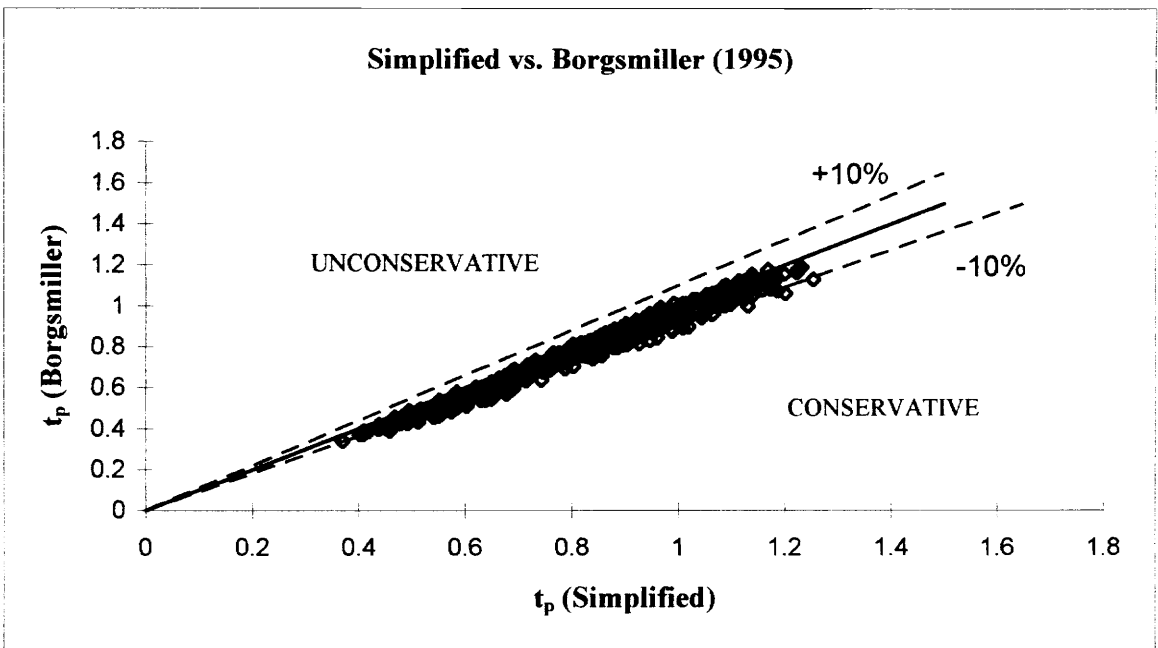
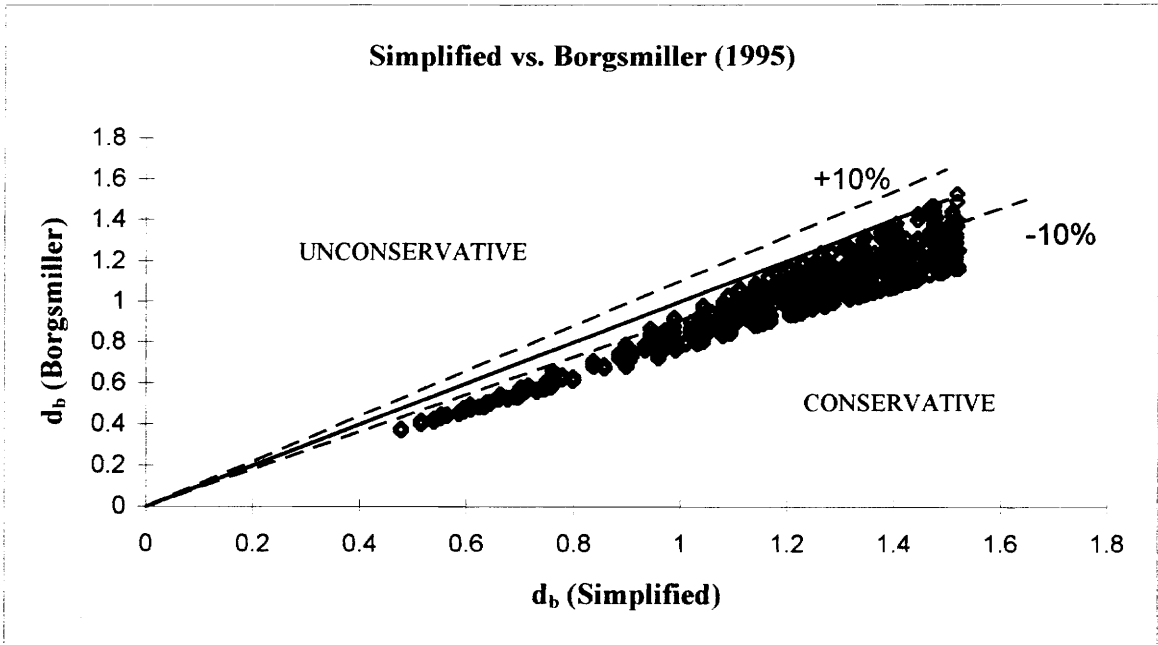


Figure 4.19 Multiple Row Extended Unstiffened 1/3 -
Simplified vs. Borgsmiller (1995):
Calculated Bolt Diameter and Plate Thickness

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 SUMMARY

Previously, design procedures for moment end-plates have been developed using yield-line theory to determine the required end-plate thickness and a modified Kennedy method to determine the required bolt diameter. These methods are not well-suited to hand calculations due to their length and complexity.

A simplified method of design has been presented in the previous chapters that is well-suited to hand calculations. The method was developed for use with the following five end-plate configurations:

- Two-Bolt Flush Unstiffened
- Four-Bolt Flush Unstiffened
- Four-Bolt Extended Unstiffened
- Four-Bolt Extended Stiffened
- Multiple Row Extended Unstiffened 1/3

The simplified design method makes use of the number of effective bolts to calculate the required bolt diameter and tee-stub type calculations to determine the required plate thickness.

The simplified method was used to predict the capacity of previously conducted experimental tests using the five connection configurations and reported in the literature. These predicted failure moments were then compared to the observed failure moments

for verification of the simplified method. Additional comparisons were made between calculated required bolt diameter and plate thickness and predicted bolt force and end-plate capacity for the simplified method versus the yield-line and Kennedy based methods and versus the method by Borgsmiller and Murray (1995).

A summary of the parameters N_{eff} and p_{eff} for use in the simplified method is shown in Table 5.1.

5.2 CONCLUSIONS

The simplified method has been used to predict the capacity of 32 connection tests previously conducted and reported in the literature. Only one case is excessively unconservative, for the four-bolt extended unstiffened configuration (Design Ratio = 1.31). Various values of the design ratio are less than 0.80, indicating somewhat conservative predictions. These are not unexpected due to the nature of the approximations made to create the simplified procedure.

Comparisons were also made between calculated bolt diameter and plate thickness and predicted bolt force and end-plate capacity for the simplified method versus the yield-line and Kennedy based procedures. Also calculated required bolt diameter and plate thickness for the simplified method versus the Borgsmiller and Murray (1995) procedure was compared. The error in bolt force prediction is smoothed when bolt diameter is calculated. This is because the bolt diameter varies with the square root of the bolt force. The predicted end-plate capacities for the simplified method are shown to be generally conservative and/or accurate with respect to the yield-line predictions. Figures 4.5 through 4.19 show these relationships. In general, the

Table 5.1 Summary of N_{eff} and p_{eff}

Configuration	N_{eff}	p_{eff}
Two-Bolt Flush Unstiffened	1.6	$\frac{p_f}{1.3} \left(\frac{g}{g + p_f} \right)^2$
Four Bolt Flush Unstiffened	2.0	$\frac{p_f}{1.25} \left(\frac{g}{g + p_f} \right)^2$
Four-Bolt Extended Unstiffened	2.6	$\frac{p_f}{2.2} \frac{\sqrt{1.5g^2 + p_f}}{g + p_f}$
Four-Bolt Extended Stiffened	2.6	$\frac{p_f}{1.45} \left(\frac{g}{g + p_f} \right)^2$
Multiple Row Extended 1/3 Unstiffened	3.33	$\frac{2p_f}{h^{1/4}} \left(\frac{g}{g + p_f} \right)^2$

majority of the data points lie within the 10% envelope and/or to the conservative side of the exact equivalency line.

Based on these observations, it can be concluded that the simplified procedure is adequate for design purposes. With respect to capacity prediction, the simplified method may be generally relied upon for conservative but not necessarily accurate results. When more accurate capacity prediction is required, it is recommended that the more rigorous yield-line and Kennedy based methods, or other accepted methods, are used.

5.3 DESIGN RECOMMENDATIONS

The design procedures that have been presented may be used in accordance with the limitations outlined in Chapter 3 for the five moment end-plate configurations listed below:

Two-Bolt Flush Unstiffened
Four-Bolt Flush Unstiffened
Four-Bolt Extended Unstiffened
Four-Bolt Extended Stiffened
Multiple Row Extended Unstiffened 1/3

The recommended LRFD design procedure is as follows:

1. Choose the parameters F_{py} , g and b_p in accordance with the criteria set forth in Chapter 3.
2. Determine N_{eff} and the equation for p_{eff} for the particular configuration using Table 5.1.
3. Calculate the required bolt diameter using Equations 3.2 and 3.3 and determine the nominal bolt diameter for the final design.

$$r_u = F_f / N_{\text{eff}} \quad (3.2)$$

$$d_b = \sqrt{\frac{4r_u}{\pi\phi F_t}} \quad (3.3)$$

4. Determine the parameters p_f , p_b and p_{ext} , which are dependent on the bolt diameter.

5. Calculate the required plate thickness using Equations 3.4 and 3.5 and determine the nominal plate thickness for the final design.

$$M_{\text{eu}} = r_u p_{\text{eff}} \quad (3.4)$$

$$t_p = \sqrt{\frac{4M_{\text{eu}}}{\phi F_{py} b_p}} \quad (3.5)$$

5.4 DESIGN EXAMPLES

Design Example 1:

Design a two-bolt flush unstiffened moment end-plate beam splice connection for the loading, materials and geometry given.

Given:

Beam: W16x36
 $h = 15.86$ in.
 $b_f = 6.985$ in.
 $t_f = 0.43$ in.

End-Plate: $F_{py} = 36$ ksi (A36)
 $g = 4$ in.
 $p_f = 2$ in. (chosen after calculated bolt diameter)
 $b_p = 7.0$ in. (b_f rounded up to next 1/2 in.)

Bolts: A325
 $F_t = 90$ ksi (LRFD Table J3.2)

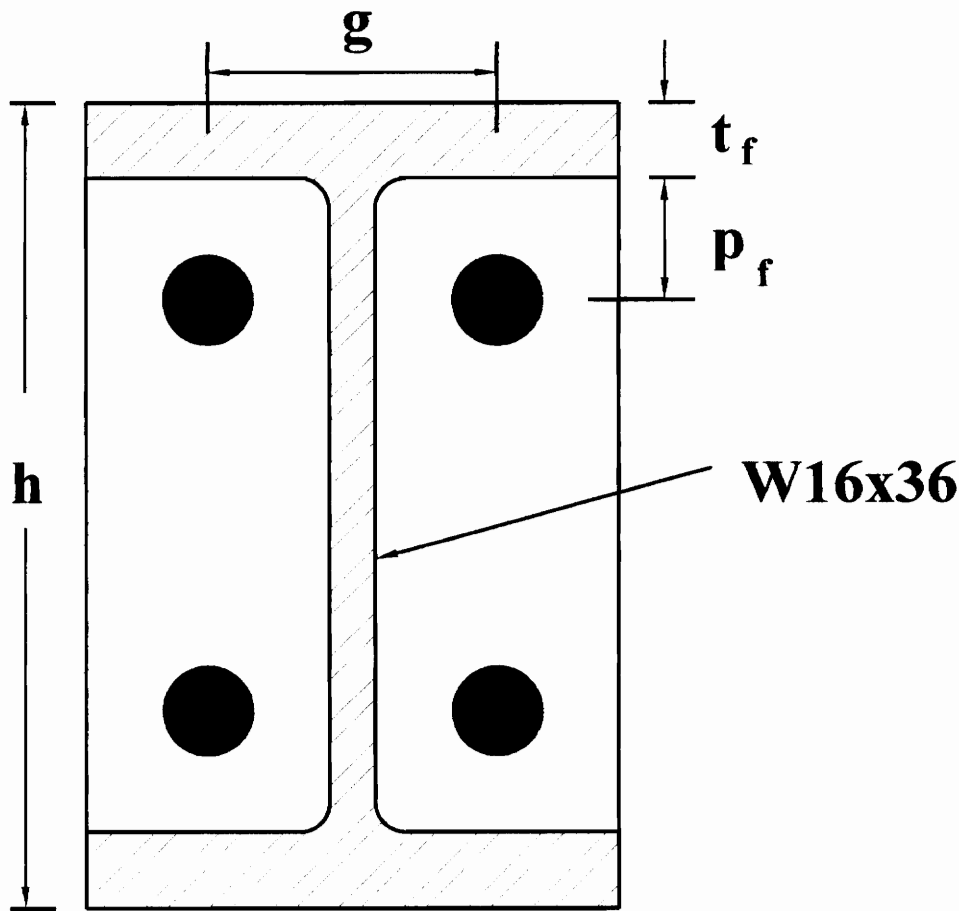


Figure 5.1 Design Example 1

$$M_u = 100 \text{ k-ft (factored moment)}$$

Solution:

1. Calculate the flange force.

$$\begin{aligned} F_f &= M_u / (h - t_f) \\ &= 100(12) / (15.86 - 0.43) \\ &= 77.8^k \end{aligned}$$

2. Calculate the bolt diameter using N_{eff} from Table 5.1 and Equation 3.2 and Equation 3.3.

$$N_{\text{eff}} = 1.6 \text{ (Table 5.1)}$$

$$\begin{aligned} r_u &= F_f / N_{\text{eff}} & (3.2) \\ &= 77.8 / 1.6 \\ &= 48.6^k \end{aligned}$$

$$\begin{aligned} d_b &= \sqrt{\frac{4r_u}{\pi\phi F_t}} & (3.3) \\ &= \sqrt{\frac{4(48.6)}{\pi(0.75)90}} \\ &= 0.957 \end{aligned}$$

Use 1 in. diameter A325 bolts

3. Calculate the plate thickness using p_{eff} from Table 5.1 and Equations 3.4 and 3.5.

$$p_{\text{eff}} = \frac{p_f}{1.3} \left(\frac{g}{g + p_f} \right)^2 \text{ (Table 5.1)}$$

$$= \frac{2}{1.3} \left(\frac{4}{4+2} \right)^2$$

$$= 0.684 \text{ in.}$$

$$M_{eu} = r_u p_{eff} \tag{3.4}$$

$$= 48.6(0.684)$$

$$= 33.2 \text{ k-in.}$$

$$t_p = \sqrt{\frac{4M_{eu}}{\phi b_p F_{py}}} \tag{3.5}$$

$$= \sqrt{\frac{4(33.2)}{0.9(7.0)36}}$$

$$= 0.75$$

Use $t_p = 3/4$ in.

4. Final Design

For given loading, materials and geometry use a 3/4 in. thick end-plate of A36 steel with 4 - 1 in. diameter A325 bolts (two at each flange).

Design Example 2:

Design a four-bolt extended stiffened moment end-plate beam to column connection for the loading, materials and geometry given.

Given:

Beam: W24x68

$h = 23.73$ in.

$b_f = 8.965$ in.

$t_f = 0.585$ in.

End-Plate: $F_{py} = 50$ ksi (A572 Gr 50)

$g = 5$ in.

$p_f = 2$ in. (chosen after calculated bolt diameter)

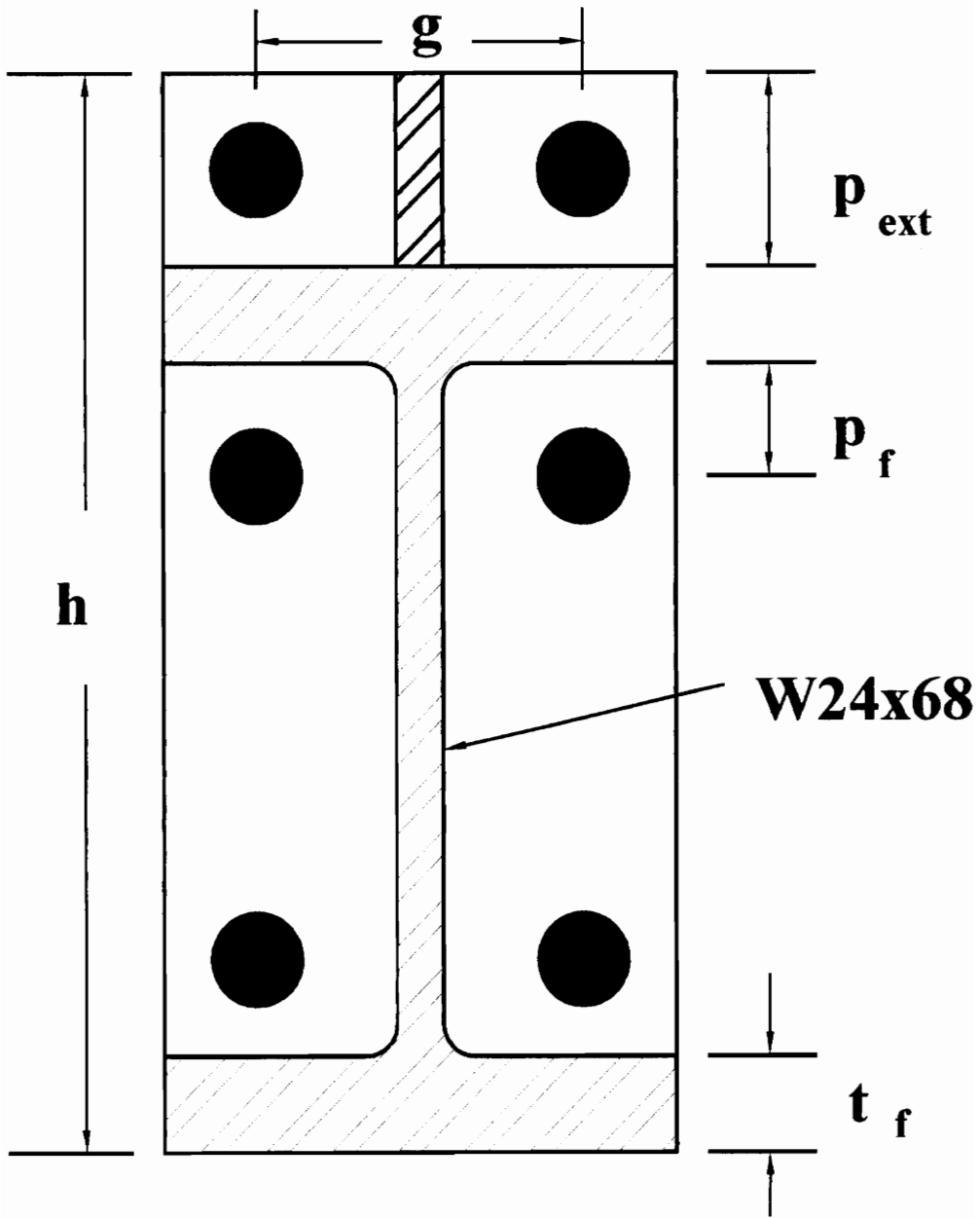


Figure 5.2 Design Example 2

$$b_p = 9.0 \text{ in. (} b_f \text{ rounded up to next } 1/2 \text{ in.)}$$

Bolts: A325

$$F_t = 90 \text{ ksi (LRFD Table J3.2)}$$

$$M_u = 300 \text{ k-ft (factored moment)}$$

Solution:

1. Calculate the flange force.

$$\begin{aligned} F_f &= M_u / (h - t_f) \\ &= 300(12) / (23.73 - 0.585) \\ &= 155.5^k \end{aligned}$$

2. Calculate the bolt diameter using N_{eff} from Table 5.1 and Equations 3.2 and 3.3.

$$N_{\text{eff}} = 2.6 \text{ (Table 5.1)}$$

$$\begin{aligned} r_u &= F_f / N_{\text{eff}} & (3.2) \\ &= 155.5 / 2.6 \\ &= 59.8^k \end{aligned}$$

$$\begin{aligned} d_b &= \sqrt{\frac{4r_u}{\pi\phi F_t}} & (3.3) \\ &= \sqrt{\frac{4(59.8)}{\pi(0.75)90}} \\ &= 1.06 \end{aligned}$$

Use 1 1/8 in. diameter A325 bolts

3. Calculate the plate thickness using p_{eff} from Table 5.1 and Equations 3.4 and 3.5.

$$p_{\text{eff}} = \frac{p_f}{1.45} \left(\frac{g}{g + p_f} \right)^2 \quad (\text{Table 5.1})$$

$$= \frac{2}{1.45} \left(\frac{5}{5+2} \right)^2$$

$$= 0.704 \text{ in.}$$

$$M_{\text{eu}} = r_u p_{\text{eff}} \quad (3.4)$$

$$= 59.8(0.704)$$

$$= 42.1 \text{ k-in.}$$

$$t_p = \sqrt{\frac{4M_{\text{eu}}}{\phi b_p F_{py}}} \quad (3.5)$$

$$= \sqrt{\frac{4(42.1)}{0.9(9)50}}$$

$$t_p = 0.645$$

Use $t_p = 11/16 \text{ in.}$

4. Final Design

For the given loading, materials and geometry use an 11/16 in. thick end-plate with 8 - 1 1/8 in. diameter A325 bolts (4 at each flange to account for load reversal).

Note: The yield-line and modified Kennedy method results in:

Design Example 1:

$$d_b = 0.932 \text{ in. (unrounded)}$$

$$t_p = 0.715 \text{ in. (unrounded)}$$

Design Example 2:

$$d_b = 1.05 \text{ in. (unrounded)}$$

$$t_p = 0.675 \text{ in. (unrounded)}$$

REFERENCES

- Abel, M. S. and T. M. Murray (1992), "Analytical and Experimental Investigation of the Extended Unstiffened Moment End-Plate Connection with Four Bolts at the Beam Tension Flange," Research Report CE/VPI-ST-93/08, Department of Civil Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, December, Revised October, 1994.
- AISC (1993), "Load and Resistance Factor Design Specification for Structural Steel Buildings," Second Edition, American Institute of Steel Construction, Chicago, IL.
- Borgsmiller, J. T. (1995), "Simplified Method for Design of Moment End-Plate Connections," M.S. Thesis, Department of Civil Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, March.
- Hendrick, D. M., A. R. Kukreti and T. M. Murray (1985), "Unification of Flush End Plate Design Procedures," Research Report FSEL/MBMA 85-01, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, March.
- Kennedy, N. A., S. Vinnakota and A. N. Sherbourne (1981), "The Split-Tee Analogy in Bolted Splices and Beam-Column Connections," *Joints in Structural Steelwork*, John Wiley and Sons, London-Toronto, 2.138-2.157.
- Morrison, S. J., A. Astaneh-Asl and T. M. Murray (1985), "Analytical and Experimental Investigation of the Extended Stiffened Moment End-Plate connection with Four bolts at the Beam Tension Flange," Research Report FSEL/MBMA 85-05, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, December.
- Morrison, S. J., A. Astaneh-Asl and T. M. Murray (1986), "Analytical and Experimental Investigation of the Multiple Row Extended Moment End-Plate Connection with Eight Bolts at the Beam Tension Flange," Research Report FSEL/MBMA 86-01, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, May.

Murray, T. M. and A. R. Kukreti (1988), "Design of 8-Bolt Stiffened End-Plates," *Engineering Journal*, American Institute of Steel Construction, 25(2), 45-52.

SEI (1984), "Multiple Row, Extended End-Plate Connection Tests," Research Report, Structural Engineers, Inc., Norman, OK, December.

Srouji, R., A. R. Kukreti and T. M. Murray (1983a), "Strength of Two Tension Bolt Flush End-Plate Connections," Research Report FSEL/MBMA 83-03, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, July.

Srouji, R., A. R. Kukreti and T. M. Murray (1983b), "Yield Line Analysis of End-Plate Connections with Bolt Force Predictions," Research Report FSEL/MBMA 83-05, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, December.

Srouji, R., A. R. Kukreti and T. M. Murray (1984), "Yield Line Analysis of End-Plate Connections with Bolt Force Predictions - Addendum," Research Report FSEL/MBMA 83-05A, Fears Structural Engineering Laboratory, University of Oklahoma, Norman, OK, August.

APPENDIX A
NOMENCLATURE

NOMENCLATURE

a	=	distance from centerline of bolt to line of action of prying force
B	=	bolt force
B_1	=	exterior bolt force in multiple row extended unstiffened 1/3 configuration
B_2	=	bolt force for first interior row in multiple row extended unstiffened 1/3 configuration
b_f	=	flange width
b_p	=	end-plate width
d_b	=	bolt diameter
d_e	=	end-plate extension from centerline of outermost bolt row to the edge of the plate extension
F	=	applied tensile force
F_f	=	flange force
$F_{f,bolt}$	=	flange force due to bolt failure for simplified method experimental predictions
$F_{f,crit}$	=	minimum of $F_{f,bolt}$ and $F_{f,plate}$
$F_{f,plate}$	=	flange force due to end-plate yield for simplified method experimental predictions
F_{py}	=	end-plate material yield stress

F_t	=	tensile strength of the bolt material from Table J3.2 (AISC 1993)
	=	90 ksi
F'	=	flange force per bolt at the thin plate limit
F_1	=	flange force at thick plate limit
F_{11}	=	flange force at thin plate limit
g	=	bolt gage
h	=	beam depth
L_n	=	length of yield-line n
L_{nx}	=	x-component of L_n
L_{ny}	=	y-component of L_n
M_b	=	bolt moment capacity
M_{bolt}	=	predicted capacity for bolt rupture using simplified method
M_{eu}	=	effective plate moment in tee stub analogy
M_{fail}	=	experimental failure moment
m_p	=	plastic moment capacity per unit of length of the end-plate
	=	$F_{py} t_p^2 / 4$
m_{px}	=	x-component of m_p
m_{py}	=	y-component of m_p
M_{pl}	=	plate moment
M_{plate}	=	predicted capacity for end-plate yield using simplified method
M_{pred}	=	minimum of M_{plate} and M_{bolt}

M_u	=	maximum moment applied in test
	=	factored LRFD applied moment
M_y	=	experimental yield moment
M_1	=	plastic plate moment at first plastic hinge line
M_2	=	plastic plate moment at second plastic hinge line
N	=	number of yield-lines in a mechanism
N_{eff}	=	number of effective bolts
p_b	=	distance centerline-to-centerline of bolts
$p_{b1,3}$	=	distance from first interior bolt centerline to innermost interior bolt centerline
	=	$2p_b$
p_{eff}	=	effective bolt pitch
p_{ext}	=	end-plate extension beyond exterior face of beam tension flange
p_f	=	bolt pitch
p_t	=	distance from centerline of bolt rows nearest to the tension flange to the far face of the tension flange
	=	$t_f + p_f$
p_{t2}	=	distance from the second interior bolt centerline to the far face of the beam tension flange
	=	$p_t + p_b$

p_{t3}	=	distance from innermost bolt centerline to the far face of the beam tension flange
	=	$p_t + p_{b1,3}$
Q	=	prying force
Q_{max}	=	maximum prying force
r_u	=	force per number of effective bolts
s	=	distance from innermost bolt centerline to innermost yield-line
T	=	force per bolt
t_f	=	beam flange thickness
t_p	=	end-plate thickness
t_w	=	web thickness
t_l	=	Kennedy thick plate limit
t_{l1}	=	Kennedy thin plate limit
u	=	distance from innermost bolt centerline to innermost yield-line
W_e	=	external work
W_i	=	internal energy of yield-line mechanism
w_i	=	internal energy of one yield-line
$w^?$	=	width of end-plate per bolt minus bolt hole diameter
	=	$b_f/2 - (d_b + 1/16)$
α	=	outer end-plate factor of past studies
β	=	inner end-plate factor of past studies

- β_2 = inner end-plate factor for multiple row extended unstiffened 1/3 of past studies
- β_3 = inner end-plate factor for multiple row extended unstiffened 1/3 of past studies
- β_4 = inner end-plate factor for multiple row extended unstiffened 1/3 of past studies
- γ = end-plate factor of past studies
- π = pi
- θ_n = relative normal plate rotation of yield-line n
- θ_{nx} = x-component of θ_n
- θ_{ny} = y-component of θ_n
- ϕ = LRFD resistance factor

APPENDIX B

TWO-BOLT FLUSH UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS

Srouji et al. (1983a)
F1-3/4-1/2-16

h= 16 in.
 bf= 6 in.
 tf= 0.25 in.
 tp= 0.505 in.
 pf= 1.5 in.
 g= 3.5 in.
 F_{py}= 55.48 ksi (measured)
 db= 0.75 in.
 d= 15.75 in.
 N_{eff}= 1.6
 peff= 0.5633846 in.
 F_{f,bolt}= 63.62 kips
 F_{f,plate}= 60.06 kips
 M_{bolt}= 83.50 kip-ft
 M_{plate}= 78.83 kip-ft

Mpred	78.83 kip-ft	Mpred/M
M _y	75 kip-ft	1.05
M _u	97.5 kip-ft	0.81

Srouji et al. (1983a)
F1-3/4-3/8-16

h= 16 in.
 bf= 6 in.
 tf= 0.25 in.
 tp= 0.383 in.
 pf= 1.5 in.
 g= 3.5 in.
 F_{py}= 59.45 ksi (measured)
 db= 0.75 in.
 d= 15.75 in.
 N_{eff}= 1.6
 peff= 0.5633846 in.
 F_{f,bolt}= 63.62 kips
 F_{f,plate}= 37.02 kips
 M_{bolt}= 83.50 kip-ft
 M_{plate}= 48.59 kip-ft

Mpred	48.59 kip-ft	Mpred/M
M _y	47 kip-ft	1.03
M _u	57 kip-ft	0.83

Srouji et al. (1983a)
F1-5/8-1/2-16

h= 16 in.
 bf= 6 in.
 tf= 0.25 in.
 tp= 0.508 in.
 pf= 1.875 in.
 g= 3.75 in.
 F_{py}= 53.98 ksi (measured)
 db= 0.625 in.
 d= 15.75 in.
 N_{eff}= 1.6
 peff= 0.6410256 in.
 F_{f,bolt}= 44.18 kips
 F_{f,plate}= 52.16 kips
 M_{bolt}= 57.98 kip-ft
 M_{plate}= 68.45 kip-ft

Mpred	57.98 kip-ft	Mpred/M
M _y	60 kip-ft	0.97
M _u	75 kip-ft	0.77

Srouji *et al.* (1983a)
F1-5-8-3/8-16

$h=$ 16 in.
 $bf=$ 6 in.
 $tf=$ 0.25 in.
 $tp=$ 0.385 in.
 $pf=$ 1.375 in.
 $g=$ 2.75 in.
 $F_{py}=$ 56.95 ksi (measured)
 $db=$ 0.625 in.
 $d=$ 15.75 in.
 $N_{eff}=$ 1.6
 $peff=$ 0.4700855 in.
 $F_{t,bolt}=$ 44.18 kips
 $F_{t,plate}=$ 43.10 kips
 $M_{bolt}=$ 57.98 kip-ft
 $M_{plate}=$ 56.57 kip-ft

$M_{pred}=$	56.57 kip-ft	M_{pred}/M
$M_y=$	55 kip-ft	1.03
$M_u=$	63 kip-ft	0.90

Srouji *et al.* (1983a)
F1-5-8-1/2-10

$h=$ 10 in.
 $bf=$ 5 in.
 $tf=$ 0.25 in.
 $tp=$ 0.506 in.
 $pf=$ 1.5 in.
 $g=$ 3 in.
 $F_{py}=$ 55.8 ksi (measured)
 $db=$ 0.625 in.
 $d=$ 9.75 in.
 $N_{eff}=$ 1.6
 $peff=$ 0.5128205 in.
 $F_{t,bolt}=$ 44.18 kips
 $F_{t,plate}=$ 55.72 kips
 $M_{bolt}=$ 35.90 kip-ft
 $M_{plate}=$ 45.27 kip-ft

$M_{pred}=$	35.9 kip-ft	M_{pred}/M
$M_y=$	32 kip-ft	1.12
$M_u=$	39.47 kip-ft	0.91

Srouji *et al.* (1983a)
F1-5-8-3/8-10

$h=$ 10 in.
 $bf=$ 5 in.
 $tf=$ 0.1875 in.
 $tp=$ 0.384 in.
 $pf=$ 1.3125 in.
 $g=$ 2.25 in.
 $F_{py}=$ 51.9 ksi (measured)
 $db=$ 0.625 in.
 $d=$ 9.8125 in.
 $N_{eff}=$ 1.6
 $peff=$ 0.4027275 in.
 $F_{t,bolt}=$ 44.18 kips
 $F_{t,plate}=$ 38.01 kips
 $M_{bolt}=$ 36.13 kip-ft
 $M_{plate}=$ 31.08 kip-ft

$M_{pred}=$	31.08 kip-ft	M_{pred}/M
$M_y=$	25 kip-ft	1.24
$M_u=$	33.92 kip-ft	0.92

Strouji *et al.* (1983a)
 F1-3.4-1/2-24A

$h^m = 24$ in.
 $bf^m = 6$ in.
 $tf^m = 0.25$ in.
 $tp^m = 0.504$ in.
 $pf^m = 1.75$ in.
 $g^m = 3.25$ in.
 $F_{py}^m = 57.53$ ksi (measured)
 $db^m = 0.75$ in.
 $d^m = 23.75$ in.
 $N_{eff}^m = 1.6$
 $peff^m = 0.56875$ in.
 $F_{t,bolt}^m = 63.62$ kips
 $F_{t,plate}^m = 61.67$ kips
 $M_{bolt}^m = 125.91$ kip-ft
 $M_{plate}^m = 122.05$ kip-ft

Mpred=	122.05 kip-ft	Mpred/M
My=	110 kip-ft	1.11
Mu=	120.25 kip-ft	1.01

Strouji *et al.* (1983a)
 F1-3.4-1/2-24B

$h = 24$ in.
 $bf = 6$ in.
 $tf = 0.25$ in.
 $tp = 0.502$ in.
 $pf = 1.375$ in.
 $g = 2.75$ in.
 $F_{py} = 57.53$ ksi (measured)
 $db = 0.75$ in.
 $d = 23.75$ in.
 $N_{eff} = 1.6$
 $peff = 0.4700855$ in.
 $F_{t,bolt} = 63.62$ kips
 $F_{t,plate} = 74.02$ kips
 $M_{bolt} = 125.91$ kip-ft
 $M_{plate} = 146.49$ kip-ft

Mpred=	125.91 kip-ft	Mpred/M
My=	125 kip-ft	1.01
Mu=	154.2 kip-ft	0.82

APPENDIX C

FOUR-BOLT FLUSH UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS

Strout et al. (1983b, 1984)
F2-5/8-1/2-16

h= 16 in.
bf= 6 in.
tf= 0.25 in.
tp= 0.5 in.
pf= 1.875 in.
g= 2.75 in.
Fpy= 58.6 ksi (measured)
db= 0.625 in.
h-tf= 15.75 in.
Neff= 2
peff= 0.5303 in.
Ft.bolt= 55.22 kips
Ff.plate= 82.88 kips
Mbolt= 72.48 kip-ft
Mplate= 108.77 kip-ft

Mpred=	72.48 kip-ft	Mpred/M
My=	85 kip-ft	0.85
Mu=	108 kip-ft	0.67

Strout et al. (1983b, 1984)
F2-5/8-3/8-16

h= 16 in.
bf= 6 in.
tf= 0.25 in.
tp= 0.375 in.
pf= 1.375 in.
g= 2.75 in.
Fpy= 60.5 ksi (measured)
db= 0.625 in.
h-tf= 15.75 in.
Neff= 2
peff= 0.4889 in.
Ft.bolt= 55.22 kips
Ff.plate= 52.21 kips
Mbolt= 72.48 kip-ft
Mplate= 68.52 kip-ft

Mpred=	68.52 kip-ft	Mpred/M
My=	85 kip-ft	0.81
Mu=	85.3 kip-ft	0.80

Strout et al. (1983b, 1984)
F2-3/4-1/2-24

h= 24 in.
bf= 6 in.
tf= 0.25 in.
tp= 0.5 in.
pf= 1.75 in.
g= 3.25 in.
Fpy= 54 ksi (measured)
db= 0.75 in.
h-tf= 23.75 in.
Neff= 2
peff= 0.5915 in.
Ft.bolt= 79.52 kips
Ff.plate= 68.47 kips
Mbolt= 157.39 kip-ft
Mplate= 135.51 kip-ft

Mpred=	135.51 kip-ft	Mpred/M
My=	140 kip-ft	0.97
Mu=	171.8 kip-ft	0.79

Strout *et al.* (1983b, 1984)
F2-3/4-3-8-24

h=	24 in.
bf=	6 in
tf=	0.25 in.
tp=	0.375 in.
pf=	1.375 in.
g=	2.75 in.
F _{py} =	64.1 ksi (measured)
db=	0.75 in.
h-tf=	23.75 in.
N _{eff} =	2
peff=	0.4889 in.
F _{f,bolt} =	79.52 kips
F _{f,plate} =	55.31 kips
M _{bolt} =	157.39 kip-ft
M _{plate} =	109.47 kip-ft

M _{pred} =	109.47 kip-ft	M _{pred} /M
M _y =	110 kip-ft	1.04
M _u =	144.7 kip-ft	0.76

Strout *et al.* (1983b, 1984)
F2-3/4-1/2-16

h=	16 in.
bf=	6 in
tf=	0.25 in.
tp=	0.5 in.
pf=	1.5 in.
g=	3.5 in.
F _{py} =	54.8 ksi (measured)
db=	0.75 in.
h-tf=	15.75 in.
N _{eff} =	2
peff=	0.5880 in.
F _{f,bolt} =	79.52 kips
F _{f,plate} =	69.90 kips
M _{bolt} =	104.37 kip-ft
M _{plate} =	91.74 kip-ft

M _{pred} =	91.74 kip-ft	M _{pred} /M
M _y =	85 kip-ft	1.08
M _u =	115.5 kip-ft	0.79

Strout *et al.* (1983b, 1984)
F2-3/4-3/8-16

h=	16 in.
bf=	6 in
tf=	0.25 in.
tp=	0.375 in.
pf=	1.5 in.
g=	3.5 in.
F _{py} =	59.7 ksi (measured)
db=	0.75 in.
h-tf=	15.75 in.
N _{eff} =	2
peff=	0.5880 in.
F _{f,bolt} =	79.52 kips
F _{f,plate} =	42.83 kips
M _{bolt} =	104.37 kip-ft
M _{plate} =	56.22 kip-ft

M _{pred} =	56.22 kip-ft	M _{pred} /M
M _y =	62 kip-ft	0.91
M _u =	73.2 kip-ft	0.77

APPENDIX D

FOUR-BOLT EXTENDED UNSTIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS

Abel and Murray(1994)
E:50-3/4-5/8-18

$h=$ 18 in.
 $bf=$ 8 in.
 $tf=$ 0.512 in.
 $tp=$ 0.633 in.
 $pf=$ 1.25 in.
 $g=$ 2.986 in.
 $F_{py}=$ 66.9 ksi (measured)
 $db=$ 0.75 in.
 $F_t=$ 103.9 ksi
 $h-tf=$ 17.488 in.
 $N_{eff}=$ 2.6
 $peff=$ 0.5129 in.
 $F_{f,bolt}=$ 119.34 kips
 $F_{f,plate}=$ 271.75 kips
 $M_{bolt}=$ 173.92 kip-ft
 $M_{plate}=$ 396.03 kip-ft

$M_{pred}=$	173.92 kip-ft	M_{pred}/M	
$M_y=$	250 kip-ft		0.70
$M_u=$	259.7 kip-ft		0.67

Abel and Murray(1994)
E:50-3/4-3/4-18

$h=$ 18 in.
 $bf=$ 8 in.
 $tf=$ 0.512 in.
 $tp=$ 0.788 in.
 $pf=$ 1.25 in.
 $g=$ 2.991 in.
 $F_{py}=$ 57.4 ksi (measured)
 $db=$ 0.75 in.
 $F_t=$ 103.9 ksi
 $h-tf=$ 17.488 in.
 $N_{eff}=$ 2.6
 $peff=$ 0.5131 in.
 $F_{f,bolt}=$ 119.34 kips
 $F_{f,plate}=$ 361.20 kips
 $M_{bolt}=$ 173.92 kip-ft
 $M_{plate}=$ 526.39 kip-ft

$M_{pred}=$	173.92 kip-ft	M_{pred}/M	
$M_y=$	275 kip-ft		0.63
$M_u=$	275.1 kip-ft		0.63

Abel and Murray(1994)
E:36-1 1/8-7/8-16

$h=$ 16.21 in.
 $bf=$ 10.25 in.
 $tf=$ 0.657 in.
 $tp=$ 0.886 in.
 $pf=$ 2.5 in.
 $g=$ 7 in.
 $F_{py}=$ 46.5 ksi (measured)
 $db=$ 1.125 in.
 $F_t=$ 99.7 ksi
 $h-tf=$ 15.553 in.
 $N_{eff}=$ 2.6
 $peff=$ 1.0428 in.
 $F_{f,bolt}=$ 257.67 kips
 $F_{f,plate}=$ 233.22 kips
 $M_{bolt}=$ 333.96 kip-ft
 $M_{plate}=$ 302.27 kip-ft

$M_{pred}=$	302.27 kip-ft	M_{pred}/M	
$M_y=$	230 kip-ft		1.31
$M_u=$	337.8 kip-ft		0.89

Abel and Murray(1994)
E:36-1 1/4-7/8-16

$h=$ 16.21 in.
 $b_f=$ 10.25 in.
 $t_f=$ 0.657 in.
 $t_p=$ 0.886 in.
 $p_f=$ 2 in.
 $g=$ 6 in.
 $F_{py}=$ 46.5 Ksi (measured)
 $d_b=$ 1.25 in.
 $F_t=$ 101.5 ksi
 $h-u=$ 15.553 in.
 $N_{eff}=$ 2.6
 $p_{eff}=$ 0.8504 in.
 $F_f, \text{bolt}=$ 323.85 kips
 $F_f, \text{plate}=$ 285.99 kips
 $M_{\text{bolt}}=$ 419.74 kip-ft
 $M_{\text{plate}}=$ 370.66 kip-ft

$M_{\text{pred}}=$	370.66 kip-ft	M_{pred}/M
$M_y=$	350 kip-ft	1.06
$M_u=$	503 kip-ft	0.74

APPENDIX E

FOUR-BOLT EXTENDED STIFFENED END-PLATES - PREDICTIONS FOR TESTED CONFIGURATIONS

Morrison *et al.* (1985)
ES-5-8-3-8-16

h= 15.907 in.
bf= 6 in.
tf= 0.38 in.
tp= 0.375 in.
pf= 1.089 in.
g= 2.734 in.
F_{py}= 55.5 ksi (measured)
db= 0.625 in.
F_t= 90 ksi
h-tf= 15.527 in.
N_{eff}= 2.6
p_{eff}= 0.3841 in.
F_t,bolt= 71.79 kips
F_t,plate= 79.24 kips
M_{bolt}= 92.89 kip-ft
M_{plate}= 102.54 kip-ft

M _{pred} =	92.89 kip-ft	M _{pred} /M
M _y	80 kip-ft	1.16
M _u	114.9 kip-ft	0.81

Morrison *et al.* (1985)
ES-3-4-1-2-16

h= 15.75 in.
bf= 6 in.
tf= 0.38 in.
tp= 0.481 in.
pf= 1.12 in.
g= 3.282 in.
F_{py}= 53.2 ksi (measured)
db= 0.75 in.
F_t= 90 ksi
h-tf= 15.37 in.
N_{eff}= 2.6
p_{eff}= 0.4294 in.
F_t,bolt= 103.38 kips
F_t,plate= 111.80 kips
M_{bolt}= 132.41 kip-ft
M_{plate}= 143.20 kip-ft

M _{pred} =	132.41 kip-ft	M _{pred} /M
M _y	130 kip-ft	1.02
M _u	163.4 kip-ft	0.81

Morrison *et al.* (1985)
ES-3-4-7-16-20

h= 19.938 in.
bf= 6.094 in.
tf= 0.479 in.
tp= 0.434 in.
pf= 1.037 in.
g= 2.766 in.
F_{py}= 60.5 ksi (measured)
db= 0.75 in.
F_t= 90 ksi
h-tf= 19.459 in.
N_{eff}= 2.6
p_{eff}= 0.3783 in.
F_t,bolt= 103.38 kips
F_t,plate= 119.31 kips
M_{bolt}= 167.64 kip-ft
M_{plate}= 193.48 kip-ft

M _{pred} =	167.64 kip-ft	M _{pred} /M
M _y	180 kip-ft	0.93
M _u	235.1 kip-ft	0.71

Morrison *et al.* (1985)
ES-3.4-1/2-20

$h=$ 19.969 in.
 $bf=$ 6 in.
 $tf=$ 0.483 in.
 $tp=$ 0.476 in.
 $pf=$ 1.58 in.
 $g=$ 3.5 in.
 $F_{py}=$ 51.8 ksi (measured)
 $db=$ 0.75 in.
 $Ft=$ 90 ksi
 $h-tf=$ 19.486 in.
 $N_{eff}=$ 2.6
 $peff=$ 0.5172 in.
 $Ff_{bolt}=$ 103.38 kips
 $Ff_{plate}=$ 88.49 kips
 $M_{bolt}=$ 167.87 kip-ft
 $M_{plate}=$ 143.70 kip-ft

$M_{pred}=$	143.7 kip-ft	M_{pred}/M
$M_y=$	150 kip-ft	0.96
$M_u=$	203 kip-ft	0.71

Morrison *et al.* (1985)
ES-1-1/2-24

$h=$ 24.063 in.
 $bf=$ 8.031 in.
 $tf=$ 0.496 in.
 $tp=$ 0.486 in.
 $pf=$ 1.692 in.
 $g=$ 3.218 in.
 $F_{py}=$ 51.6 ksi (measured)
 $db=$ 1 in.
 $Ft=$ 90 ksi
 $h-tf=$ 23.567 in.
 $N_{eff}=$ 2.6
 $peff=$ 0.5012 in.
 $Ff_{bolt}=$ 183.78 kips
 $Ff_{plate}=$ 126.93 kips
 $M_{bolt}=$ 360.93 kip-ft
 $M_{plate}=$ 249.28 kip-ft

$M_{pred}=$	249.28 kip-ft	M_{pred}/M
$M_y=$	260 kip-ft	0.96
$M_u=$	349.5 kip-ft	0.71

Morrison *et al.* (1985)
ES-1-5/8-24

$h=$ 23.938 in.
 $bf=$ 8 in.
 $tf=$ 0.496 in.
 $tp=$ 0.62 in.
 $pf=$ 1.723 in.
 $g=$ 4.5 in.
 $F_{py}=$ 52.7 ksi (measured)
 $db=$ 1 in.
 $Ft=$ 90 ksi
 $h-tf=$ 23.442 in.
 $N_{eff}=$ 2.6
 $peff=$ 0.6214 in.
 $Ff_{bolt}=$ 183.78 kips
 $Ff_{plate}=$ 169.53 kips
 $M_{bolt}=$ 359.02 kip-ft
 $M_{plate}=$ 331.18 kip-ft

$M_{pred}=$	331.18 kip-ft	M_{pred}/M
$M_y=$	280 kip-ft	1.18
$M_u=$	379.4 kip-ft	0.87

APPENDIX F

MULTIPLE ROW EXTENDED UNSTIFFENED 1/3 END-PLATES - PREDCTIONS FOR TESTED CONFIGURATIONS

Morrison *et al.* (1986)
MRF:1/3-1-1/2-30

h= 30.003 in.
bf= 8.063 in.
tf= 0.377 in.
tp= 0.501 in.
pf= 1.567 in.
g= 4.574 in.
F_{py}= 50.1 ksi (measured)
db= 1 in.
F_t= 90 ksi
h-tf= 29.626 in.
N_{eff}= 3.33
peff= 0.7428856 in.
F_tbolt= 235.38 kips
F_fplate= 113.62 kips
M_{bolt}= 581.12 kip-ft
M_{plate}= 280.52 kip-ft

M _{pred} =	280.52 kip-ft	M _{pred} /M
M _y =	300 kips-ft	0.94
M _u =	425.1 kip-ft	0.66

Morrison *et al.* (1986)
MRF:1/3-3/4-3/8-30

h= 29.813 in.
bf= 8 in.
tf= 0.377 in.
tp= 0.377 in.
pf= 1.105 in.
g= 2.72 in.
F_{py}= 52.3 ksi (measured)
db= 0.75 in.
F_t= 90 ksi
h-tf= 29.436 in.
N_{eff}= 3.33
peff= 0.4782619 in.
F_tbolt= 132.40 kips
F_fplate= 103.51 kips
M_{bolt}= 324.79 kip-ft
M_{plate}= 253.92 kip-ft

M _{pred} =	253.92 kip-ft	M _{pred} /M
M _y =	270 kips-ft	0.94
M _u =	404.9 kip-ft	0.63

Morrison *et al.* (1986)
MRF:1/3-7/8-7/16-46

h= 45.994 in.
bf= 8.112 in.
tf= 0.495 in.
tp= 0.445 in.
pf= 1.435 in.
g= 3.253 in.
F_{py}= 62.1 ksi (measured)
db= 0.875 in.
F_t= 90 ksi
h-tf= 45.499 in.
N_{eff}= 3.33
peff= 0.5306394 in.
F_tbolt= 180.22 kips
F_fplate= 156.50 kips
M_{bolt}= 683.30 kip-ft
M_{plate}= 593.40 kip-ft

M _{pred} =	593.4 kip-ft	M _{pred} /M
M _y =	520 kips-ft	1.14
M _u =	866.1 kip-ft	0.69

Morrison *et al.*, (1986)
MRE:1/3-1 1/8-5/8-46

h= 46.026 in.
 bf= 8.241 in.
 tf= 0.498 in.
 tp= 0.634 in.
 pf= 1.742 in.
 g= 3.949 in.
 F_{py}= 57.2 ksi (measured)
 db= 1.125 in.
 Ft= 90 ksi
 h-tf= 45.528 in.
 Neff= 3.33
 peff= 0.6440572 in.
 F_{f,bolt}= 297.91 kips
 F_{f,plate}= 244.91 kips
 Mbolt= 1130.26 kip-ft
 Mplate= 929.21 kip-ft

Mpred=	929.21 kip-ft	Mpred/M
My=	975 kip-ft	0.95
Mu=	975.1 kip-ft	0.95

Morrison *et al.*, (1986)
MRE:1/3-1 1/4-5/8-62

h= 62.08 in.
 bf= 9.941 in.
 tf= 1.004 in.
 tp= 0.626 in.
 pf= 2.403 in.
 g= 4.482 in.
 F_{py}= 53.9 ksi (measured)
 db= 1.25 in.
 Ft= 90 ksi
 h-tf= 61.076 in.
 Neff= 3.33
 peff= 0.7255739 in.
 F_{f,bolt}= 367.79 kips
 F_{f,plate}= 240.92 kips
 Mbolt= 1871.91 kip-ft
 Mplate= 1226.19 kip-ft

Mpred=	1226.19 kip-ft	Mpred/M
My=	1200 kip-ft	1.02
Mu=	1635 kip-ft	0.75

Morrison *et al.*, (1986)
MRE:1/3-1 1/2-3/4-62

h= 61.945 in.
 bf= 9.926 in.
 tf= 1.005 in.
 tp= 0.753 in.
 pf= 2.584 in.
 g= 5.559 in.
 F_{py}= 54.6 ksi (measured)
 db= 1.5 in.
 Ft= 90 ksi
 h-tf= 60.94 in.
 Neff= 3.33
 peff= 0.8585101 in.
 F_{f,bolt}= 529.61 kips
 F_{f,plate}= 297.99 kips
 Mbolt= 2689.55 kip-ft
 Mplate= 1513.27 kip-ft

Mpred=	1513.27 kip-ft	Mpred/M
My=	1600 kip-ft	0.95
Mu=	2329.6 kip-ft	0.65

SEI (1984)
MREI/3-3/4-1/2-62

$h=$ 62 in.
 $bf=$ 10 in.
 $tf=$ 1 in.
 $tp=$ 0.5 in.
 $pf=$ 1.375 in.
 $g=$ 3.5 in.
 $F_{py}=$ 52.9 ksi (measured)
 $db=$ 0.75 in.
 $F_t=$ 90 ksi
 $h-tf=$ 61 in.
 $N_{eff}=$ 3.33
 $peff=$ 0.5051515 in.
 $F_t, \text{bolt}=$ 132.40 kips
 $F_t, \text{plate}=$ 217.95 kips
 $M_{\text{bolt}}=$ 673.05 kip-ft
 $M_{\text{plate}}=$ 1107.92 kip-ft

$M_{\text{pred}}=$	673.05 kip-ft	M_{pred}/M
$M_y=$	750 kip-ft	0.90
$M_u=$	929 kip-ft	0.72

SEI (1984)
MREI/3-1-3/4-62

$h=$ 62 in.
 $bf=$ 10 in.
 $tf=$ 1 in.
 $tp=$ 0.75 in.
 $pf=$ 1.625 in.
 $g=$ 3.5 in.
 $F_{py}=$ 49.1 ksi (measured)
 $db=$ 1 in.
 $F_t=$ 90 ksi
 $h-tf=$ 61 in.
 $N_{eff}=$ 3.33
 $peff=$ 0.5401741 in.
 $F_t, \text{bolt}=$ 235.38 kips
 $F_t, \text{plate}=$ 425.65 kips
 $M_{\text{bolt}}=$ 1196.53 kip-ft
 $M_{\text{plate}}=$ 2163.73 kip-ft

$M_{\text{pred}}=$	1196.53 kip-ft	M_{pred}/M
$M_y=$	1250 kip-ft	0.96
$M_u=$	1364 kip-ft	0.88

VITA

Eric R. Ober was born on December 4, 1970, in Pittsburgh, PA, to Robert and Mary Ann Ober. After completing high school in June, 1989, in Pittsburgh, he enrolled at Virginia Tech and obtained a Bachelor of Science in Civil Engineering in December, 1993. Following that, he enrolled in the graduate program in Civil Engineering, Structures Division at Virginia Tech in January, 1994, in pursuit of a Master of Science degree.

A handwritten signature in cursive script that reads "Eric R. Ober". The signature is written in black ink and is centered on the page.