Static Two-dimensional Calculation of the Capacitance and Impedance of Open Microstrip-like Structures Using Variational Methods

by

Vassilios A. Papageorgiou

Thesis submitted to the faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE in Electrical Engineering

Vassilios A. Papageorgiou and VPI & SU 1993

APPROVED:

I. M. Besieris, Co-chairman

A. Elshabini-Riad, Co-chairman

Sedki Riad

June, 1993

Blacksburg, Virginia
Static Two-dimensional Calculation of the Capacitance and Impedance of Open Microstrip-like Structures Using Variational Methods

by

Vassilios A. Papageorgiou

Committee Co-chairmen:
Ioannis M. Besieris
Electrical Engineering Department

and

Aicha Elshabini-Riad
Electrical Engineering Department

(ABSTRACT)

This work examines and implements two different techniques for the estimation of the capacitance and impedance of microstrip-like open structures. Both theories, one developed by Yamashita and Mittra and the other by Itoh and Hebert are based on variational methods. The results for the capacitance and impedance of a microstrip-like structure are calculated numerically and compared with measurements taken using a sample. The results presented in this thesis indicate that the first method produces results with large error and it can be used for microstrip structures with only one strip. The second method produces very accurate results for the microstrip structure under consideration and is the one recommended.
ACKNOWLEDGEMENTS

The author would like to express his gratitude to Dr. Ioannis M. Besieris and Dr. Aicha Elshabini-Riad for their guidance and advice. Many thanks are also due to Dr. Besieris for the time he devoted for corrections of this thesis and to Dr. Elshabini-Riad for the help she provided regarding the experimental part of the work.

Sincere thanks also to Dr. Sedki Riad for his assistance as a committe member.

The author would also like to thank Eric B. Ellis and Jimmy Perdue for manufacturing the sample of the measurements, as well as Dr. Wansheng Su for his assistance during the measurements.

Finally, the author would like to express his love and heartfelt gratitude to his parents Andreas and Dimitra and his brother Spiros for all their love and support.
Contents

Abstract

Acknowledgements

I INTRODUCTION 1

II STATIC ESTIMATION OF THE LOWER BOUND VALUE OF THE CAPACITANCE OF TRANSMISSION LINES 3

II.1 Introduction. 3

II.2 Development of the Variational Expression for a Lower Bound Capacitance Value 4

II.3 Summary. 7

III ANALYSIS OF A SIMPLE MICROSTRIP LINE 8

III.1 Introduction. 8

III.2 Analysis of a Simple Microstrip. 9

III.3 Summary. 13
IV ANALYSIS OF A MORE COMPLEX MICROSTRIP-LIKE STRUCTURE IN THE FOURIER TRANSFORM DOMAIN 14

IV.1 Introduction ........................................... 14

IV.2 Formulation of the Problem in the Space Domain and the Fourier Transform Domain .................. 15

IV.3 Summary .................................................. 19

V CALCULATION OF THE CAPACITANCE USING YAMASHITA’S AND GALLERKIN’S METHOD 20

V.1 Introduction ........................................... 20

V.2 Yamashita’s Method ..................................... 21

V.3 Gallerkin’s Method ...................................... 21

V.4 Summary .................................................. 24

VI MEASUREMENTS AND NUMERICAL RESULTS 25

VI.1 Introduction ........................................... 25

VI.2 Measurements .......................................... 26

VI.3 Numerical Results ..................................... 26

VI.4 Manufacturing Process of the Sample ................. 30

VI.5 Comments on the Program ............................. 30

VI.6 Summary .................................................. 31

v
VII SUMMARY OF WORK AND CONCLUSION

Appendix

Program Listing. ........................................................... 34

References

Vita

vi
List of Tables

Table 1. Measurement Results. .............................................. 27

Tables 2.1-2.9. Calculation Results Using Galkerkin's Method. ..... 27

Tables 3.1-3.5. Calculation Results Using the Method of Yamashita
and Mittra. .......................................................... 29
List of Figures

Figure 1. A Simple Two-Conductor Configuration and Its Electrostatic Equivalent in Two Dimensions. ........................................ 5

Figure 2. Simple Microstrip Line. ........................................ 10

Figure 3. Charge Distribution on a Conducting Strip with Respect to Its Width. ..................................................... 10

Figure 4. Microstrip-like structure. ................................. 16
Chapter 1

INTRODUCTION

The idea for this thesis was motivated by the need for a better way of analyzing the structure of a new package for Microwave and Millimeter-wave Integrated Circuits (MMICs), designed at Virginia Tech, in order to minimize losses associated with packaging. Originally, it was attempted to analyze the microstrip-like structure in three dimensions by solving for the propagation coefficient. However, the method presented in the paper by Itoh and Mittra [8] required the application of elaborate theories, which were hard to implement. As a consequence, it was decided to limit the work to a static two-dimensional analysis of the structure by solving for its capacitance and impedance.

The first papers dealing with the problem of microstrip static analysis were written by Yamashita and Mittra in 1968 ([2] and [3]). These two papers dealt with the analysis of a simple microstrip line using a variational method developed by Collin [1]. The disadvantage of that method was that the
microstrip-like structure analyzed had to be simple. Papers for more complex structures written in 1978 and 1986 ([4], [5] and [6]), were based on Gallerkin's method. The structures analyzed in these papers were shielded.

The work in this thesis has several objectives: First, the two methods are combined into one which can be used to solve complex microstrip-like structures that are not shielded. Second, the results obtained by the two methods separately are compared, so that the most accurate method can be determined. Finally, the results from the two methods are compared with experimental data obtained from measurements of a sample.

Chapter I of this thesis is a general introduction. Chapter II contains a derivation of the variational expression for the calculation of the capacitance of a two-conductor transmission line. The static analysis of the simple microstrip line in two dimensions by means of the method of Yamashita and Mittra is given in chapter III. In Chapter IV, the analysis of a more complex microstrip-like structure is developed in the Fourier transform domain. In Chapter V the calculation of the capacitance of this structure is carried out first using the technique Yamashita and Mittra and then Gallerkin's. Chapter VI contains a comparison of the results obtained from the two techniques with experimental data; also, it explains how the sample was manufactured, how the measurements were made, and it gives a description of the algorithm used for the computation of the results. Finally, a summary of the work is provided in Chapter VII.
Chapter II

STATIC ESTIMATION OF THE LOWER BOUND VALUE OF THE CAPACITANCE OF TRANSMISSION LINES

II.1 Introduction
In this chapter, a method will be developed and explained by which we can obtain a variational formula for the lower-bound estimation of the capacitance of a transmission line by static two-dimensional analysis. This method is based on the Green's function technique for solving boundary-value problems. After deriving the lower bound value of the capacitance by utilizing the variational behavior of this quantity, we can try different basis functions, and choose the one yielding the highest capacitance value.
II.2 Development of the Variational Expression for a Lower-bound Capacitance Value

We have to solve the electrostatic problem of finding the capacitance of the structure shown in Fig. 1. The conductors $S_1$ and $S_2$ are assumed to be perfect ($R=0$) and of infinite length in the direction of the $z$-axis. They are oriented so as to be parallel to the $z$-axis. Let the potential of $S_1$ be equal to 0 and that of $S_2$ be $V_o$. This requires that a distribution of positive charge exists on the surface of $S_2$, which makes its presence "known" to $S_1$ by means of the electric field it creates; the latter is equal to

$$-\epsilon E_n = -\epsilon \frac{\partial}{\partial n} \phi(x,y)$$ (II.01)

where $E_n$ signifies the electric field perpendicular to the surface of the conductor $S_2$ and $\phi(x,y)$ is the potential function on the surface of $S_2$. We can remove the conductor $S_2$ and keep the charge of its surface at the same position without disturbing any previous effect of the electric field. We therefore arrive at the problem of finding the potential function $\phi(x,y)$ which will satisfy Poisson's equation

$$\nabla^2 \phi(x,y) = -\frac{1}{\epsilon} \rho(x,y)$$ (II.02)

and the boundary conditions

$$\phi(x,y) = 0 \quad \text{on } S_1$$ (II.03a)

$$\phi(x,y) = V_o \quad \text{on } S_2$$ (II.03b)

$$\lim_{x \to \pm \infty} \phi(x,y) = \lim_{y \to \pm \infty} \phi(x,y) = 0.$$ (II.03c)

The first step is to define a unit charge located at $(x', y')$ and find a solution to (II.02) in the presence of $S_1$, such that it vanishes on $S_1$. A way of representing a unit charge mathematically is by means of the delta function, so that the equation to be solved becomes

$$\nabla^2 G(x,y|x',y') = -\frac{1}{\epsilon} \delta(x-x') \delta(y-y')$$ (II.04)
Figure 1. A simple two-conductor configuration and its electrostatic equivalent in two dimensions.
subject to (II.03a-c). \( G(x,y|x',y') \) is a two-dimensional Green's function defined for the given boundary. It follows that the solution to (II.02) has the form

\[
\phi(x,y) = \int_{S_2} G(x,y|x',y') \rho(x',y') \, ds'.
\]  

(II.05)

From (II.03b) and (II.05) we obtain

\[
\phi(x,y) = V_o = \int_{S_2} G(x,y|x',y') \rho(x',y') \, ds' \quad x,y \in S_2
\]

(II.06)

Equation (II.06) is an integral equation involving the unknown charge distribution \( \rho(x',y') \), whose solution would yield \( \rho(x',y') \) and thus determine the function \( \phi(x,y) \) everywhere by (II.05). Now, we can multiply (II.06) by the charge \( Q \) on \( S_2 \) and integrate over \( S_2 \). Hence,

\[
QV_o = V_o \int_{S_2} \rho(x,y) \, ds = \int_{S_2} \int G(x,y|x',y')\rho(x',y')\rho(x,y) \, ds' \, ds
\]

(II.07)

Also,

\[
C_o = \frac{Q}{V_o} \Rightarrow QV_o = \frac{Q^2}{C_o}
\]

(II.08)

Therefore, from (II.07) and (II.08) we get the final variational expression

\[
\frac{1}{C_o} = \frac{\int_{S_2} \int G(x,y|x',y') \rho(x',y') \rho(x,y) \, ds \, ds'}{\left\{ \int_{S_2} \rho(x,y) \, ds \right\}^2}.
\]

(II.09)

The final step is to show that (II.09) is stationary for arbitrary first-order changes in the functional form of \( \rho(x,y) \). Let \( \rho \) change by \( \delta \rho \); then,

\[
\left\{ \int_{S_2} \rho(x,y) \, ds \right\}^2 \delta \left( \frac{1}{C_o} \right) + \frac{2}{C_o} \left\{ \int_{S_2} \int \rho(x',y') \delta \rho(x,y) \, ds' \, ds \right\}
\]

\[
= \int_{S_2} \int G(x,y|x',y') \rho(x,y) \delta \rho(x',y') \, ds \, ds' + \int_{S_2} \int G(x,y|x',y') \delta \rho(x,y) \rho(x',y') \, ds \, ds'
\]

\[= \]
\[
2 \left( \int_{s_2} G(x, y | x', y') \rho(x', y') \delta \rho(x, y) \, ds \, ds' \right)
\]
since \(G, \rho\) and \(\delta \rho\) are functions symmetrical with respect to \(x, y, x'\) and \(y'\). This expression can be rewritten as

\[
Q^2 \delta \left( \frac{1}{C_o} \right) = 2 \left\{ \int_{s_2} \delta \rho(x, y) \left\{ - \frac{Q}{C_o} + \int_{s_2} G(x, y | x', y') \rho(x', y') \, ds' \right\} \, ds \right\}
\]

which shows that the variation in \(1/C_o\) vanishes by use of (II.05). So, (II.09) is a stationary expression for \(1/C_o\) and, hence, if an approximate solution for \(\rho(x', y')\) can be found, it is possible to compute a value for \(1/C_o\) which will be correct to the second order. Since \(G\) is symmetrical, the integral in (II.09) is always positive and \(1/C_o\) is stationary with respect to \(\rho(x, y)\). Therefore, the stationary value is an absolute minimum, so the approximate value \(1/C\) is too large, which makes \(C < C_o\), thus obtaining a lower bound for the capacitance.

### II.3 Summary

A variational expression for the calculation of the capacitance of a transmission line is derived using a Green's function. It is also shown that this expression is free of the first-degree error introduced by the selection of the charge distribution function \(\rho(x, y)\) and it yields a lower bound value for the capacitance of the transmission line.
Chapter III

ANALYSIS OF A SIMPLE MICROSTRIP LINE

III.1 Introduction
In this chapter the method for obtaining the capacitance of a simple microstrip structure is developed. The boundary conditions are going to be written in the Fourier transform domain (FTD), so that the final computation of the results will be made easier.

The main difficulty in obtaining the capacitance of a microstrip line in a quasi-static analysis of the boundary conditions in the Fourier transform domain, consists in obtaining the inverse Fourier transform integral which is required in order to express the results in the space domain. The novelty of the method used by Yamashita and Mittra [2], [3], and later by Itoh and Herbert [4] and [5] is that no such inverse Fourier transform is required because, as will be seen in this chapter, by using Parseval’s Formula, only the evaluation of the integrals in the FTD will be necessary.
III.2 Analysis of a Simple Microstrip

First, the analysis of a simple microstrip line (Fig. 2) will be presented. The static potential distribution $\phi(x,y)$ in the dielectric slab of the microstrip line satisfies Poisson's equation

$$\nabla^2 \phi(x,y) = -\frac{1}{\epsilon} \rho(x,y)$$  \hspace{1cm} (III.01)

where $\rho(x,y)$ is the charge distribution on the suface of the conducting strip. The main assumptions for the analysis are the following:

i. All conducting surfaces (i.e. the strip and the ground plane) are perfect conductors.

ii. The conducting strips are infinitely thin.

iii. The dielectric material is lossless and nonmagnetic.

iv. The substrate extends infinitely in the direction of the y-axis.

These assumptions are also valid for all the work done in this thesis.

The solution function can be expressed as two functions, namely $\phi_1(x,y)$, valid for $0 \leq y \leq h$ and $\phi_2(x,y)$, valid for $y \geq h$. The following boundary conditions have to be satisfied

$$\phi_1(x,0) = 0$$ \hspace{1cm} (III.02a)

$$\phi_2(x,\infty) = 0$$ \hspace{1cm} (III.02b)

$$\phi_1(x,h) = \phi_2(x,h)$$ \hspace{1cm} (III.02c)

$$\frac{\partial}{\partial y} \phi_2(x,h) - \epsilon_r \frac{\partial}{\partial y} \phi_1(x,h) = \begin{cases} -\frac{1}{\epsilon_0} f(x) & |x| \leq W/2 \\ 0 & |x| > W/2 \end{cases}$$ \hspace{1cm} (III.02d)

where $f(x)$ is a function satisfying

$$\rho(x,y) = f(x) \delta(y-h)$$ \hspace{1cm} (III.03)

The solution for the microstrip line capacitance can be obtained by inserting the solution of equation (III.01) into the variational expression

$$\frac{1}{C} = \frac{1}{Q^2} \int \rho(x,y) \phi(x,y) \, ds$$ \hspace{1cm} (III.04)
Figure 2. Simple Microstrip Line (region 1: dielectric, region 2: air)

Figure 3. Charge distribution on a conducting strip with respect to its width
where

\[ Q = \int_{s} \rho(x,y) \, ds \quad \text{(III.05)} \]

and the integrals in (III.04) and (III.05) are to be evaluated throughout the surface over which the charge \( \rho(x,y) \) is distributed.

Next, the characteristic impedance transmission line in the TEM mode in free space can be calculated from

\[ Z_o = \frac{1}{C_o c} \quad \text{(III.06)} \]

where \( C_o \) is the capacitance of the structure without the dielectric slab and \( c \) is the speed of light in free space. If a slab of dielectric exists between the strip and the ground plane, then the impedance is given by

\[ Z = \sqrt{\frac{C^2}{C_o}} \, Z_o \quad \text{(III.07)} \]

Therefore, the basic properties of the microstrip line, whether loaded with a dielectric medium or not, can be derived from the knowledge of the structure’s line capacitance.

Now the definition of the Fourier transform to be used is given as

\[ \tilde{\phi}(\beta,y) = \int_{-\infty}^{\infty} \phi(x,y) \cos(\beta x) \, dx \quad \text{(III.08)} \]

Here the cosine transform is used since the structure is symmetrical about the \( y \)-axis, and the potential as well as the charge distributions on the strip are even functions of \( x \). So, \( \tilde{\phi}(\beta,y) = \tilde{\phi}(-\beta,y) \), which follows from \( \phi(x,y) = \phi(-x,y) \).

By applying definition (III.08), equation (III.01) can be written in the FTD as follows:

\[ (-\beta^2 + \frac{d^2}{dy^2}) \tilde{\phi}(\beta,y) = 0. \quad \text{(III.09)} \]

in regions where charge is absent. The boundary conditions in the FTD are given as follows:
\[ \tilde{\varphi}_1(\beta, 0) = 0 \]  

(III.10a)

\[ \tilde{\varphi}_2(\beta, \infty) = 0 \]  

(III.10b)

\[ \tilde{\varphi}_2(\beta, h^+) = \tilde{\varphi}_1(\beta, h^-) \]  

(III.10c)

\[ \frac{d}{dy} \tilde{\varphi}_2(\beta, h^+) - \epsilon_r \frac{d}{dy} \tilde{\varphi}_1(\beta, h^-) = -\frac{1}{\epsilon_0} \tilde{f}(\beta) \]  

(III.10d)

where, \( \tilde{f}(\beta) = \int_{-w/2}^{w/2} f(x) \cos(\beta x) \, dx \).

In the region \( 0 \leq y \leq h \) the solution of (III.01) is a linear combination of \( e^{-\beta y} \) and \( e^{\beta y} \), whereas for \( y > h \) the solution of (III.01) is analogous to \( e^{-|\beta|y} \) since the field approaches 0 as \( y \) approaches infinity. So, the potential is given by the following functions:

\[ \phi_1(\beta, y) = A e^{-\beta y} + B e^{\beta y} \]  

(III.11a)

\[ \phi_2(\beta, y) = C e^{-|\beta|y}. \]  

(III.11b)

The variational aspect of the method is based on the fact that equation (III.01) is free of the first-order error which is introduced by the selection of the basis function \( \tilde{f}(\beta) \). The inverse Fourier transform function \( f(x) \) is chosen in a way, so that it best represents the charge distribution on the strip. This means that it must be constant near the middle of the strip and increase toward the edges of the strip (Fig. 3).

By solving the system of equations (III.10), the following result is obtained for the potential, evaluated at the interface \( y = h \)

\[ \tilde{\varphi}(\beta, h) = \frac{\tilde{f}(\beta)}{\epsilon_0 |\beta| [1+\epsilon_r \coth(|\beta| h)]}. \]  

(III.12)

The variational expression (III.02) can be rewritten as

\[ \frac{1}{C} = \frac{1}{Q^2} \int_{-W/2}^{W/2} f(x) \phi(x, h) \, dx. \]  

(III.13)
By using Parseval’s theorem which states that

\[
\int_{-\infty}^{\infty} f(x) \, g(x) \, dx = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{f}(\beta) \, \tilde{g}(\beta) \, d\beta
\]  \hspace{1cm} (III.14)

we can estimate the capacitance from (III.13) without having to carry out the cumbersome inverse Fourier transform. Therefore, (III.13) becomes

\[
\frac{1}{C} = \frac{1}{2\pi Q^2} \int_{-\infty}^{\infty} \tilde{\phi}(\beta, h) \, d\beta.
\]  \hspace{1cm} (III.15)

Finally, substituting (III.12) into (III.15) we obtain

\[
\frac{1}{C} = \frac{1}{\pi \varepsilon_0 Q^2} \int_{0}^{\infty} \frac{\left\{ \tilde{f}(\beta) \right\}^2}{\beta \left[ 1 + \varepsilon_0 \coth(|\beta| h) \right]} \, d\beta
\]  \hspace{1cm} (III.16)

where use has been made of the evenness of the functions \(\tilde{f}(\beta)\) and \(\tilde{\phi}(\beta, h)\). Since (III.16) is variational, it is free of the first-degree error introduced by the selection of an approximate function \(\tilde{f}(\beta) \leftrightarrow f(x)\).

**III.3 Summary**

In this chapter, a simple microstrip configuration was analyzed. Poisson’s equation and the boundary conditions were given first in the space domain and then in the Fourier transform domain. The potential function was solved on the positive potential strip and, by application of formula (II.09), the final variational expression for the capacitance was derived.
Chapter IV

ANALYSIS OF A MORE COMPLEX MICROSTRIP-LIKE STRUCTURE IN THE FOURIER TRANSFORM DOMAIN

IV.1 Introduction

In this chapter, the groundwork for the solution of the Poisson equation for a more complex microstrip-like structure is developed. First, the boundary conditions have to be determined in the space domain and then be transformed in the FTD. Then, from solving a system of equations derived from these boundary conditions, we can determine the potential functions for the microstrip-like structure.
IV.2 Formulation of the Problem in the Space Domain and the Fourier

Transform Domain

The problem consists of solving Poisson's equation and thus determining a potential function for the dielectric region which will also satisfy the boundary conditions dictated by the physical structure shown in Fig. 2. To this end, we can separate the potential function $\phi(x,y)$ into three distinct functions $\phi_1(x,y)$, $\phi_2(x,y)$, $\phi_3(x,y)$ which are valid in the regions $0 \leq y \leq h$, $h < y \leq 2h$, and $2h < y$ respectively. Poisson's equation is given by

$$\nabla^2 \phi(x,y) = -\frac{1}{\varepsilon} \rho(x,y)$$  \hspace{1cm} (IV.01)

and the boundary conditions are

zero potential at $y=0$: \hspace{1cm} $\phi_1(x,0)=0$ \hspace{1cm} (IV.02a)

continuity of potential functions
at the interface $y=h$: \hspace{1cm} $\phi_1(x,h) = \phi_2(x,h)$ \hspace{1cm} (IV.02b)

estimation of the value of potential
at the interface $y=h$: \hspace{1cm} $\phi_1(x,h) = \begin{cases} 0 & |x| \leq W/2 \\ v(x) & |x| > W/2 \end{cases}$ \hspace{1cm} (IV.02c)

continuity of the electric flux density
at the interface $y=h$:
\[ \varepsilon_r \frac{\partial}{\partial y} \phi_2(x,y) \mid_{y=h} - \varepsilon_r \frac{\partial}{\partial y} \phi_1(x,y) \mid_{y=h} = \begin{cases} -\frac{\rho_m(x)}{\varepsilon_o} & |x| \leq W/2 \\ 0 & |x| > W/2 \end{cases} \] \hspace{1cm} (IV.02d)

continuity of potential functions
at the interface $y=2h$: \hspace{1cm} $\phi_2(x,2h) = \phi_3(x,2h)$ \hspace{1cm} (IV.02e)

estimation of the value of potential
at the interface $y=2h$: \hspace{1cm} $\phi_3(x,2h) = \begin{cases} V_o & |x| \leq W/2 \\ \phi_o(x) & |x| > W/2 \end{cases}$ \hspace{1cm} (IV.02f)

continuity of the electric flux density
at the interface $y=2h$:
\[ \frac{\partial}{\partial y} \phi_3(x,y) \mid_{y=2h} - \varepsilon_r \frac{\partial}{\partial y} \phi_2(x,y) \mid_{y=2h} = \begin{cases} -\frac{\rho(x)}{\varepsilon_o} & |x| \leq W/2 \\ 0 & |x| > W/2 \end{cases} \] \hspace{1cm} (IV.02g)
Figure 4. Microstrip-like structure (regions 1 & 2: dielectric, region 3: air)
The functions \( v(x) \) and \( \phi_0(x) \) are unknown, but, as it will be seen later, they will not have to be evaluated. The potential \( V_0 \) represents the voltage on the top strip and the charge distribution functions \( \rho(x,y) \) and \( \rho_m(x,y) \) are to be determined variationally.

The next step is to apply the Fourier transform with respect to \( x \) to all relations above, so that the solution will be obtained more easily. The Fourier transform is defined by

\[
F\{f(x)\} \equiv \tilde{f}(\beta) = \int_{-\infty}^{\infty} f(x) \cos(\beta x) \, dx \quad (IV.03)
\]

Using it in conjunction with Poisson's equation (III.01) we get the following ordinary differential equation

\[
\frac{d^2}{dy^2} \tilde{\phi}(\beta,y) = \beta^2 \tilde{\phi}(\beta,y) \quad y \neq h, 2h \quad (IV.04)
\]

Also, using the Fourier Transform, the boundary conditions (IV.02a-g) become

\[
\tilde{\phi}_1(\beta,0) = 0 \quad (IV.05a)
\]

\[
\tilde{\phi}_1(\beta,h) = \tilde{\phi}_2(\beta,h) \quad (IV.05b)
\]

\[
\tilde{\phi}_1(\beta,h) = \tilde{v}(\beta) \quad (IV.05c)
\]

\[
\epsilon_r \frac{d}{dy} \tilde{\phi}_2(\beta,y) |_{y=h} - \epsilon_r \frac{d}{dy} \tilde{\phi}_1(\beta,y) |_{y=h} = -\frac{\rho_m(\beta)}{\epsilon_0} \quad (IV.05d)
\]

\[
\tilde{\phi}_2(\beta,2h) = \tilde{\phi}_3(\beta,2h) \quad (IV.05e)
\]

\[
\tilde{\phi}_3(\beta,2h) = \tilde{\phi}_o(\beta) + \tilde{\phi}_o(\beta) \quad (IV.05f)
\]

\[
\frac{d}{dy} \tilde{\phi}_3(\beta,y) |_{y=2h} - \epsilon_r \frac{d}{dy} \tilde{\phi}_2(\beta,y) |_{y=2h} = -\frac{\rho(\beta)}{\epsilon_0} \quad (IV.05g)
\]

where

\[
\tilde{v}(\beta) = \int_{-\infty}^{-W/2} v(x) \cos(\beta x) \, dx + \int_{W/2}^{\infty} v(x) \cos(\beta x) \, dx \quad (IV.06)
\]

17
\[ \tilde{\rho}_m(\beta) = \int_{-W/2}^{W/2} \tilde{\rho}(x) \cos(\beta x) \, dx \] (IV.07)

\[ \tilde{\phi}_o(\beta) = \int_{-W/2}^{W/2} V_o \cos(\beta x) \, dx \] (IV.08)

\[ \tilde{\phi}_o(\beta) = \int_{-\infty}^{-W/2} \phi_o(x) \cos(\beta x) \, dx + \int_{W/2}^{\infty} \phi_o(x) \cos(\beta x) \, dx \] (IV.09)

\[ \tilde{\rho}(\beta) = \int_{-W/2}^{W/2} \rho(x) \cos(\beta x) \, dx \] (IV.10)

The best way to obtain a solution for all the above conditions is to guess the form of solution for each region of the microstrip-like structure, and then solve for the coefficients; specifically, we assume

\[ \tilde{\phi}_1(\beta,y) = A \sinh(\beta y) \] (IV.11a)

\[ \tilde{\phi}_2(\beta,y) = B \sinh[\beta(y-h)] + C \cosh[\beta(y-h)] \] (IV.11b)

\[ \tilde{\phi}_3(\beta,y) = D e^{-|\beta|(w-2h)} \] (IV.11c)

As the next step, the expressions in (IV.11a-c) are substituted into (IV.05b,d,e,g) [(IV.05a) is satisfied automatically], in order to obtain a system of four equations with four unknowns. The unknowns are the coefficients A, B, C and D, but only A and D need to be solved for. Also, the numerator and denominator of each coefficient is divided by \(-\beta \cosh^2(\beta h)\) in order to make the numerical calculations easier and faster. After this procedure the coefficients are given by

\[ A = \frac{\tilde{\rho}_m(\beta)}{\epsilon_o \epsilon_r \det \left\{ \text{sgn}(\beta) \frac{\tanh(\beta h)}{\cosh(\beta h)} + \frac{\epsilon_r}{\epsilon_o} \right\}} + \frac{\tilde{\rho}(\beta)}{\epsilon_o \det \frac{1}{\cosh^2(\beta h)}} \] (IV.12a)

\[ D = \frac{\tilde{\rho}_m(\beta)}{\epsilon_o \det \cosh(\beta h)} + \frac{\tilde{\rho}(\beta)}{\epsilon_o \det 2 \tanh(\beta h)} \] (IV.12b)
where $\text{sgn}(\beta) = -1$ when $\beta < 0$ and $\text{sgn}(\beta) = 1$ when $\beta \geq 0$ and

$$\det = \tanh(\beta h) \left\{ |\beta| + \epsilon_r \beta \tanh(\beta h) \right\} + \left\{ |\beta| \tanh(\beta h) + \epsilon_r \beta \right\} \quad (IV.12c)$$

Finally, we plug these expressions into (IV.05c) and (IV.05f) and obtain the following system of equations

$$\tilde{\phi}_1(\beta, h) = \tilde{\nu}(\beta) \Rightarrow \tilde{G}_{11}(\beta, h) \tilde{\rho}_m(\beta) + \tilde{G}_{12}(\beta, h) \tilde{\rho}(\beta) = \tilde{\nu}(\beta) \quad (IV.13a)$$

$$\tilde{\phi}_3(\beta, 2h) = \tilde{\phi}_o(\beta) + \tilde{\phi}_e(\beta) \Rightarrow \tilde{G}_{21}(\beta, 2h) \tilde{\rho}_m(\beta) + \tilde{G}_{22}(\beta, 2h) \tilde{\rho}(\beta) = \tilde{\phi}_o(\beta) + \tilde{\phi}_e(\beta) \quad (IV.13b)$$

where

$$\tilde{G}_{11} = \frac{1}{\epsilon_o \epsilon_r \det} \left\{ \text{sgn}(\beta) \tanh^2(\beta h) + \epsilon_r \tanh(\beta h) \right\} \quad (IV.14a)$$

$$\tilde{G}_{12} = \tilde{G}_{21} = \frac{1}{\epsilon_o \det} \frac{\tanh(\beta h)}{\cosh(\beta h)} \quad (IV.14b)$$

$$\tilde{G}_{22} = \frac{1}{\epsilon_o \det} \frac{2 \tanh(\beta h)}{\cosh(\beta h)} \quad (IV.14c)$$

$\tilde{G}_{11}, \tilde{G}_{12}, \tilde{G}_{21}, \tilde{G}_{22}$ represent the Fourier transforms of the Green's functions. The first subscript represents the location in terms of $y$ where the function is evaluated (1 for $y=h$ and 2 for $y=2h$), and the second subscript represents the location in terms of $y$ of the unit source (1 for $y=h$ and 2 for $y=2h$).

### IV.3 Summary

In this chapter, the formulation of the problem of the microstrip-like structure is developed in the Fourier transform domain and the basic work for the computation of the capacitance was completed. In the next chapter we shall use formula (II.09) to calculate the capacitance and develop a second more elaborate mathematical method, Gallerkin's method ([4]-[7]), for the calculation of the capacitance of the particular structure under consideration.
V.1 Introduction
In the previous chapter we formulated the problem in the Fourier transform domain. In this chapter we shall develop and use for the solution the method used by Yamashita and Mittra ([2], [3]) and the method used by Itoh [4], Itoh and Hebert [5] and Sawicki and Sachse [6]. The first method, although easier computationally, does not yield very accurate results. The advantage of the second technique, although more difficult to implement, can be used for structures with several secondary strip lines, like suspended septums, strips connected to ground and other kinds. The mathematical method implemented for the second technique is called Gallerkin’s Method (see Harrington [7]). This method allows one to expand the unknown charge distribution functions in terms of known basis functions and solve for the coefficients of the expansion. Using these coefficients the capacitance can be found using a formula containing the energy of the structure.
V.2 Method of Yamashita and Mittra

Yamashita and Mittra in their papers ([2] and [3]), use the variational technique described in chapter II to solve for the capacitance of simple microstrip structures. According to this method, one has to solve for the potential on the interface of the positive potential strip, i.e. $y=2h$ (see Fig. 4) and then use formula (III.15) in order to obtain a result for the capacitance.

From (IV.11c) and (IV.12b), it is seen that

$$\tilde{\varphi}_3(\beta,2h) = D e^{-|\beta|(2h-2h)} = D = \frac{\tilde{\varrho}_m(\beta)}{\epsilon_o \det \cosh(\beta h)} \times \frac{\tilde{\varrho}(\beta)}{\epsilon_o \det 2 \tanh(\beta h)}$$

Equation (III.15) is then used to obtain the capacitance. In this case, two basis functions have to be used, namely $\tilde{\varrho}(\beta)$ for the charge distribution on the top strip and $\tilde{\varrho}_m(\beta)$ for the lower strip. By making this modification, we obtain

$$\frac{1}{C} = \frac{1}{2\pi Q^2} \int_{-\infty}^{\infty} \left\{ \frac{\tilde{\varrho}(\beta) \tilde{\varrho}_m(\beta) \tanh(\beta h)}{\epsilon_o \det \cosh(\beta h)} + \frac{\tilde{\varrho}^2(\beta)}{\epsilon_o \det 2 \tanh(\beta h)} \right\} d\beta$$

which readily gives a value for the capacitance.

Of course, test functions for $\tilde{\varrho}(\beta)$ and $\tilde{\varrho}_m(\beta)$ have to be found. These will have to represent in a suitable way the distribution for the charges on the two strips. It is known that the charge on a conducting strip has an almost constant concentration in the middle and then increases significantly towards the edges. Specific results will be presented in chapter VI.

V.3 Galerkin’s Method

Instead of using the direct approach of solving for the potential at $y=2h$, as it was done in the last chapter, now we shall attempt to obtain a solution by solving the system of equations given by (IV.13a,b), presented here also for convenience:

$$\tilde{\varphi}_1(\beta,h) = \tilde{\varrho}(\beta) \Rightarrow \tilde{G}_{11}(\beta,h) \tilde{\varrho}_m(\beta) + \tilde{G}_{12}(\beta,h) \tilde{\varrho}(\beta) = \tilde{\varrho}(\beta) \quad \text{(IV.13a)}$$

$$\tilde{\varphi}_3(\beta,2h) = \tilde{\varphi}_o(\beta) + \tilde{\varphi}_o(\beta) \Rightarrow \tilde{G}_{21}(\beta,2h) \tilde{\varrho}_m(\beta) + \tilde{G}_{22}(\beta,2h) \tilde{\varrho}(\beta) = \tilde{\varphi}_o(\beta) + \tilde{\varphi}_o(\beta) \quad \text{(IV.13b)}$$
This system contains four unknowns, namely \( \tilde{\rho}_m(\beta) \), \( \tilde{\rho}(\beta) \), \( \tilde{v}(\beta) \) and \( \tilde{\phi}_o(\beta) \). However, the last two unknowns can be eliminated as will be shown.

The charge distribution functions are expanded in terms of known basis functions as follows:

\[
\tilde{\rho}_m(\beta) = \sum_{k=1}^{K} a_k \tilde{\rho}_{mk}(\beta) \quad k=1,2,3,...,K \tag{V.03a}
\]

\[
\tilde{\rho}(\beta) = \sum_{l=1}^{L} b_l \tilde{\rho}_l(\beta) \quad l=1,2,3,...,L \tag{V.03b}
\]

Here \( a_k \) and \( b_l \) are coefficients to be determined. The inverse Fourier transform of the basis functions \( \tilde{\rho}_{mk}(\beta) \) and \( \tilde{\rho}_l(\beta) \) must be equal to zero outside the strips and take some value within the strip width. In general the finite expansions given in (V.03a,b) will not be exact since most charge distribution functions will require infinite expansions.

The expressions (V.03a,b) are substituted into (IV.13a,b) to yield

\[
\sum_{k=1}^{K} \tilde{G}_{11}(\beta) a_k \tilde{\rho}_{mk}(\beta) + \sum_{l=1}^{L} \tilde{G}_{12} b_l \tilde{\rho}_l(\beta) = \tilde{v}(\beta) \tag{V.04a}
\]

\[
\sum_{k=1}^{K} \tilde{G}_{21}(\beta) a_k \tilde{\rho}_{mk}(\beta) + \sum_{l=1}^{L} \tilde{G}_{22} b_l \tilde{\rho}_l(\beta) = \tilde{\phi}_v(\beta) + \tilde{\phi}_o(\beta) \tag{V.04b}
\]

Next, the weighing functions \( \tilde{\rho}_1'(\beta) \) and \( \tilde{\rho}_m'(\beta) \) are introduced and the following inner products are formed:

\[
\int_{-\infty}^{\infty} \rho_{mj}(\beta) \tilde{G}_{11}(\beta) \sum_{k=1}^{K} a_k \tilde{\rho}_{mk}(\beta) \, d\beta + \int_{-\infty}^{\infty} \rho_{mj}(\beta) \tilde{G}_{12}(\beta) \sum_{l=1}^{L} b_l \tilde{\rho}_l(\beta) \, d\beta =
\]

\[
= \int_{-\infty}^{\infty} \tilde{v}(\beta) \rho_{mj}(\beta) \, d\beta \tag{V.05a}
\]

\[
\int_{-\infty}^{\infty} \tilde{\rho}_1'(\beta) \tilde{G}_{21}(\beta) \sum_{k=1}^{K} a_k \tilde{\rho}_m'(\beta) \, d\beta + \int_{-\infty}^{\infty} \tilde{\rho}_1'(\beta) \tilde{G}_{22}(\beta) \sum_{l=1}^{L} b_l \tilde{\rho}_l(\beta) \, d\beta =
\]

\[
= \int_{-\infty}^{\infty} \tilde{\phi}_v(\beta) \tilde{\rho}_1'(\beta) \, d\beta + \int_{-\infty}^{\infty} \tilde{\phi}_o(\beta) \tilde{\rho}_1'(\beta) \, d\beta \tag{V.05b}
\]

The weighing functions \( \tilde{\rho}_1'(\beta) \) and \( \tilde{\rho}_m'(\beta) \), like the basis functions in charge
distribution expansions, must also vanish outside the strips and have a nonzero value within the strips. They can be different from \( \tilde{\rho}_{m_j}(\beta) \) and \( \tilde{\rho}_i(\beta) \), but it is more convenient to choose, \( \tilde{\rho}_i'(\beta) = \tilde{\rho}_i(\beta) \) and \( \tilde{\rho}'_{m_j}(\beta) = \tilde{\rho}_{m_j}(\beta) \). By applying Parseval's theorem, given in (III.14) it can be explained why the potentials \( \tilde{v}(\beta) \) and \( \tilde{\phi}_o(\beta) \) do not have to be estimated. First, we note that, by (III.14)

\[
\int_{-w/2}^{w/2} \tilde{v}(\beta) \tilde{\rho}_{m_j}(\beta) \, d\beta = \frac{1}{2\pi} \int_{-w/2}^{w/2} v(x) \rho_{m_j}(x) \, dx \quad (V.06)
\]

\[
\int_{-w/2}^{w/2} \tilde{\phi}_o(\beta) \tilde{\rho}_i(\beta) \, d\beta = \frac{1}{2\pi} \int_{-w/2}^{w/2} \phi_o(x) \rho_i(x) \, dx \quad (V.07)
\]

Since, as mentioned earlier, the weighing functions must be equal to zero outside the strips and since the potential functions \( \phi_o(x) \) and \( v(x) \) are zero on the strips, it is concluded that the integrals (V.06) and (V.07) are equal to zero. With this conclusion, the system of equations (V.05a,b) can be rewritten as

\[
\sum_{k=1}^{K} K_{jk}^{11} a_k + \sum_{l=1}^{L} K_{jl}^{12} b_l = 0 \quad (V.08a)
\]

\[
\sum_{k=1}^{K} K_{ik}^{21} a_k + \sum_{l=1}^{L} K_{il}^{22} b_l = P_i \quad (V.08b)
\]

where, by application of Parseval's theorem, \( P_i \) is given by

\[
P_i = \int_{-w/2}^{w/2} \tilde{\phi}_o(\beta) \tilde{\rho}_i(\beta) \, d\beta = \frac{1}{2\pi} \int_{-w/2}^{w/2} V_o \rho_i(x) \, dx = \frac{V_o}{2\pi} \int_0^{w/2} \rho_i(x) \, dx \quad (V.09)
\]

since, by symmetry, \( \rho_i(x) \) is even. Also,

\[
K_{jk}^{11} = \int_{-w/2}^{w/2} \tilde{\rho}_{m_j}(\beta) \tilde{G}_{11}(\beta) \tilde{\rho}_{m_k}(\beta) \, d\beta \quad (V.10a)
\]

\[
K_{jl}^{12} = \int_{-w/2}^{w/2} \tilde{\rho}_{m_j}(\beta) \tilde{G}_{12}(\beta) \tilde{\rho}_l(\beta) \, d\beta \quad (V.10b)
\]

\[
K_{ik}^{21} = \int_{-w/2}^{w/2} \tilde{\rho}_i(\beta) \tilde{G}_{21}(\beta) \tilde{\rho}_{m_k}(\beta) \, d\beta \quad (V.10c)
\]

\[
K_{il}^{22} = \int_{-w/2}^{w/2} \tilde{\rho}_i(\beta) \tilde{G}_{22}(\beta) \tilde{\rho}_l(\beta) \, d\beta \quad (V.10d)
\]
The final step is to solve for $b_l$ and then the capacitance of the structure is given by the following formula [6]

$$C = \frac{1}{V^2} \left( \int_{-W/2}^{W/2} \phi(x, 2h) \rho(x, 2h) \, dx - \frac{1}{2} \int_{-W/2}^{W/2} \phi_m(x, h) \rho_m(x, h) \, dx \right)$$

$$C = \sum_{l=1}^{L} b_l P_l = \frac{V^2}{W} \sum_{l=1}^{L} b_l \int_{0}^{W/2} \rho_l(x) \, dx \quad (V.11)$$

### V.4 Summary

The methods of Yamashita-Mittra and Gallerkin have been presented and adapted for the calculation of the capacitance of the structure shown in Fig. 4. The first method is easier to implement, but it can only be used for simple structures. Gallerkin's method is more complicated than the one described in the last chapter, but it can yield results for more elaborate cases, where many secondary strips exist. In such cases it is not easy to solve explicitly for the potential functions at each interface; however, it is straightforward to solve a matrix equation.
Chapter VI

MEASUREMENTS AND NUMERICAL RESULTS

VI.1 Introduction
In this chapter a comparison between numerical results and actual measurements of the capacitance and impedance of the structure in Fig. 4 is made. The numerical results are obtained by the methods of Yamashita-Mittra and Gallerkin, and a comparison between the two techniques is made. It is explained how the calculation of the numerical results is performed. Also, the manufacturing process of the sample used for measurements is described briefly and an explanation of the program written to facilitate the calculations is provided.
VI.2 Measurements

Table 1 contains the measurements for the capacitance and impedance of the microstrip-like structure shown in Fig. 4. The capacitance measurements were performed using a HP 4276 LCZ meter with a voltage of 1 Volt on the top strip. Although the theories developed in the last four chapters assume static conditions, the meter yields more stable results at 10 kHz than at 100 Hz, and thus, a spot frequency of 10 kHz was used. The test signal level was selected to be HIGH and the measurement speed LOW at a capacitive auto circuit mode. For the impedance measurements a 7854 Tektronix oscilloscope was used with a 50-Ω probe in the TEM mode. The oscilloscope yielded a reflection coefficient, r, for each microstrip structure and the following formula was used to calculate the impedance

\[
Z = 50 \frac{1 - r}{1 + r} \tag{VI.01}
\]

VI.3 Numerical Results

Tables 2.1-2.9 and 3.1-3.4 contain the numerical results obtained by the method of Gallerkin and the method of Yamashita and Mittra, respectively. The functions \( f_0, f_1, f_2, f_3 \) given for \( \rho_m(x) \) and \( \rho(x) \) under each table, describe the basis functions selected in the variational method for the calculation of the capacitance. These basis functions are

\[
f_0(x) = \frac{1}{9.6} W \quad \text{(constant)} \tag{VI.02a}
\]

\[
f_1(x) = \frac{|x|}{4.8 \ W^2} \tag{VI.02b}
\]

\[
f_2(x) = 1 + \frac{8}{4.3} \frac{|x|^3}{\pi \ W^4} \tag{VI.02c}
\]

\[
f_3(x) = \frac{1}{4.8 \ \pi \ \sqrt{\frac{W^2}{4} - x^2}} \tag{VI.02d}
\]

As it can be seen from Tables 2.1-2.9 the best results for the capacitance, using Gallerkin’s method, are obtained with \( \rho(x) = \rho_m(x) = f_3 \) and for the impedance with \( \rho(x) = \rho_m(x) = f_0 \). The impedance is calculated using (III.05) and (III.06).
<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>11.380</td>
<td>N/A</td>
<td>N/A</td>
<td>63.636</td>
</tr>
<tr>
<td>0.1016</td>
<td>16.910</td>
<td>N/A</td>
<td>N/A</td>
<td>42.593</td>
</tr>
<tr>
<td>0.1524</td>
<td>21.760</td>
<td>N/A</td>
<td>N/A</td>
<td>36.207</td>
</tr>
</tbody>
</table>

Table 1. Measurement Results

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>10.500</td>
<td>1.554</td>
<td>163.479</td>
<td>62.886</td>
</tr>
<tr>
<td>0.1016</td>
<td>15.686</td>
<td>2.171</td>
<td>117.020</td>
<td>43.531</td>
</tr>
<tr>
<td>0.1524</td>
<td>20.903</td>
<td>2.751</td>
<td>92.339</td>
<td>33.498</td>
</tr>
</tbody>
</table>

Table 2.1 Calculation Results using Gallerkin's method with $\rho(x) = f_0$ and $\rho_m(x) = f_0$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>10.072</td>
<td>1.5834</td>
<td>160.416</td>
<td>61.663</td>
</tr>
<tr>
<td>0.1016</td>
<td>15.660</td>
<td>2.185</td>
<td>116.251</td>
<td>43.426</td>
</tr>
<tr>
<td>0.1524</td>
<td>20.262</td>
<td>2.725</td>
<td>93.215</td>
<td>34.183</td>
</tr>
</tbody>
</table>

Table 2.2 Calculation Results using Gallerkin's method with $\rho(x) = f_1$ and $\rho_m(x) = f_1$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>10.083</td>
<td>1.695</td>
<td>149.832</td>
<td>59.284</td>
</tr>
<tr>
<td>0.1016</td>
<td>14.635</td>
<td>2.177</td>
<td>116.654</td>
<td>44.996</td>
</tr>
<tr>
<td>0.1524</td>
<td>18.521</td>
<td>2.633</td>
<td>96.462</td>
<td>36.371</td>
</tr>
</tbody>
</table>

Table 2.3 Calculation Results using Gallerkin's method with $\rho(x) = f_2$ and $\rho_m(x) = f_2$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>11.073</td>
<td>1.6265</td>
<td>156.64</td>
<td>59.855</td>
</tr>
<tr>
<td>0.1016</td>
<td>16.497</td>
<td>2.2754</td>
<td>110.254</td>
<td>40.947</td>
</tr>
<tr>
<td>0.1524</td>
<td>21.628</td>
<td>2.8664</td>
<td>88.613</td>
<td>32.263</td>
</tr>
</tbody>
</table>

Table 2.4 Calculation Results using Gallerkin's method with $\rho(x) = f_3$ and $\rho_m(x) = f_3$
<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance (C pF)</th>
<th>Capacitance in air (Co pF)</th>
<th>Impedance in air (Zo ohms)</th>
<th>Impedance (Z ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>11.056</td>
<td>1.6227</td>
<td>156.529</td>
<td>59.967</td>
</tr>
<tr>
<td>0.1016</td>
<td>16.712</td>
<td>2.2905</td>
<td>110.892</td>
<td>41.054</td>
</tr>
<tr>
<td>0.1524</td>
<td>22.322</td>
<td>2.9204</td>
<td>86.974</td>
<td>31.459</td>
</tr>
</tbody>
</table>

Table 2.5 Calculation Results using Gallerkin's method with $\rho(x) = f_0$ and $\rho_m(x) = f_1$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance (C pF)</th>
<th>Capacitance in air (Co pF)</th>
<th>Impedance in air (Zo ohms)</th>
<th>Impedance (Z ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>11.132</td>
<td>1.632</td>
<td>155.637</td>
<td>59.592</td>
</tr>
<tr>
<td>0.1016</td>
<td>16.911</td>
<td>2.311</td>
<td>109.909</td>
<td>40.630</td>
</tr>
<tr>
<td>0.1524</td>
<td>22.699</td>
<td>2.957</td>
<td>85.898</td>
<td>31.003</td>
</tr>
</tbody>
</table>

Table 2.6 Calculation Results using Gallerkin's method with $\rho(x) = f_3$ and $\rho_m(x) = f_0$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance (C pF)</th>
<th>Capacitance in air (Co pF)</th>
<th>Impedance in air (Zo ohms)</th>
<th>Impedance (Z ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>11.070</td>
<td>1.626</td>
<td>156.202</td>
<td>59.867</td>
</tr>
<tr>
<td>0.1016</td>
<td>16.486</td>
<td>2.274</td>
<td>111.680</td>
<td>41.481</td>
</tr>
<tr>
<td>0.1524</td>
<td>21.648</td>
<td>2.868</td>
<td>88.563</td>
<td>32.235</td>
</tr>
</tbody>
</table>

Table 2.7 Calculation Results using Gallerkin's method with $\rho(x) = f_2$ and $\rho_m(x) = f_1$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance (C pF)</th>
<th>Capacitance in air (Co pF)</th>
<th>Impedance in air (Zo ohms)</th>
<th>Impedance (Z ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>9.392</td>
<td>1.397</td>
<td>181.831</td>
<td>70.125</td>
</tr>
<tr>
<td>0.1016</td>
<td>14.138</td>
<td>1.955</td>
<td>129.897</td>
<td>48.308</td>
</tr>
<tr>
<td>0.1524</td>
<td>19.117</td>
<td>2.503</td>
<td>101.499</td>
<td>36.723</td>
</tr>
</tbody>
</table>

Table 2.8 Calculation Results using Gallerkin's method with $\rho(x) = f_1$ and $\rho_m(x) = f_2$

<table>
<thead>
<tr>
<th>Width of strips (W cm)</th>
<th>Capacitance (C pF)</th>
<th>Capacitance in air (Co pF)</th>
<th>Impedance in air (Zo ohms)</th>
<th>Impedance (Z ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>9.299</td>
<td>1.396</td>
<td>183.327</td>
<td>70.764</td>
</tr>
<tr>
<td>0.1016</td>
<td>13.929</td>
<td>1.932</td>
<td>131.443</td>
<td>48.958</td>
</tr>
<tr>
<td>0.1524</td>
<td>18.774</td>
<td>2.467</td>
<td>102.967</td>
<td>37.324</td>
</tr>
</tbody>
</table>

Table 2.9 Calculation Results using Gallerkin's method with $\rho(x) = f_3$ and $\rho_m(x) = f_2$
<table>
<thead>
<tr>
<th>Width of strips W (cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>0.868</td>
<td>0.133</td>
<td>1904.288</td>
<td>746.497</td>
</tr>
<tr>
<td>0.1016</td>
<td>1.103</td>
<td>0.164</td>
<td>1552.082</td>
<td>597.910</td>
</tr>
<tr>
<td>0.1524</td>
<td>1.324</td>
<td>0.191</td>
<td>1331.705</td>
<td>505.524</td>
</tr>
</tbody>
</table>

Table 3.1 Calculation Results using Yamashita's method with $\rho(x) = f_0$ and $\rho_{m}(x) = f_0$

<table>
<thead>
<tr>
<th>Width of strips W (cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>0.886</td>
<td>0.136</td>
<td>1870.312</td>
<td>732.228</td>
</tr>
<tr>
<td>0.1016</td>
<td>1.138</td>
<td>0.168</td>
<td>1509.538</td>
<td>580.352</td>
</tr>
<tr>
<td>0.1524</td>
<td>1.376</td>
<td>0.197</td>
<td>1286.196</td>
<td>487.216</td>
</tr>
</tbody>
</table>

Table 3.2 Calculation Results using Yamashita's method with $\rho(x) = f_1$ and $\rho_{m}(x) = f_1$

<table>
<thead>
<tr>
<th>Width of strips W (cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>0.769</td>
<td>0.122</td>
<td>2077.732</td>
<td>828.150</td>
</tr>
<tr>
<td>0.1016</td>
<td>3.059</td>
<td>0.476</td>
<td>533.337</td>
<td>210.448</td>
</tr>
<tr>
<td>0.1524</td>
<td>7.301</td>
<td>1.116</td>
<td>227.593</td>
<td>88.891</td>
</tr>
</tbody>
</table>

Table 3.3 Calculation Results using Yamashita's method with $\rho(x) = f_2$ and $\rho_{m}(x) = f_2$

<table>
<thead>
<tr>
<th>Width of strips W (cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>3.603</td>
<td>0.122</td>
<td>461.044</td>
<td>180.288</td>
</tr>
<tr>
<td>0.1016</td>
<td>4.651</td>
<td>0.685</td>
<td>370.823</td>
<td>142.302</td>
</tr>
<tr>
<td>0.1524</td>
<td>5.651</td>
<td>0.807</td>
<td>314.901</td>
<td>118.975</td>
</tr>
</tbody>
</table>

Table 3.4 Calculation Results using Yamashita's method with $\rho(x) = f_3$ and $\rho_{m}(x) = f_3$

<table>
<thead>
<tr>
<th>Width of strips W (cm)</th>
<th>Capacitance C (pF)</th>
<th>Capacitance in air Co (pF)</th>
<th>Impedance in air Zo (ohms)</th>
<th>Impedance Z (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0508</td>
<td>5.845</td>
<td>0.919</td>
<td>276.454</td>
<td>109.620</td>
</tr>
<tr>
<td>0.1016</td>
<td>8.341</td>
<td>1.252</td>
<td>202.842</td>
<td>78.588</td>
</tr>
<tr>
<td>0.1524</td>
<td>10.691</td>
<td>1.548</td>
<td>164.130</td>
<td>62.446</td>
</tr>
</tbody>
</table>

Table 3.5 Calculation Results using Yamashita's method with $\rho(x) = f_3$ and $\rho_{m}(x) = f_2$
This is the case, because the reflection coefficient is read from the oscilloscope, which has an analog display. The line on the oscilloscope was not very well defined and therefore it was difficult to determine the exact value of the reflection coefficient. The best results are obtained when $\rho(x) = \rho_m(x)$ because of the structure geometry, i.e. the charge distribution on the lower strip is very similar to that on the top strip simply because the lower strip is directly below the top strip.

The results calculated using the method of Yamashita and Mittra are not as accurate as those obtained by Gallerkin's method. The reason for this is that formula (II.09) was developed for a transmission line with one main signal conductor and the ground plane. The structure of Fig. 4, which is analyzed here, contains an additional strip which is grounded, which can be taken into account in the development of the theory only by using a formula containing the energy of the structure, like (V.11). We can thus conclude that the best method to use is Gallerkin's.

In obtaining the numerical results, the method if Yamashita and Mittra was easier, since there was only one integral to be calculated [see (V.02)]. On the other hand, Gallerkin’s method required the calculation of three integrals, (since $G_{12}=G_{21}$) (V.10a-d) as well as a matrix inversion for the solution of (V.09). For the calculations performed in this work, the final matrix was 2x2. A bigger matrix would not improve the accuracy, because, as mentioned before, the most accurate results are given when the two basis functions $\rho(x)$ and $\rho_m(x)$ are equal. By creating a larger matrix for the calculation of the capacitance we would have to use different basis functions for the two strips, which would yield worse results and increase the numerical calculation time considerably.

VI.4 Manufacturing process of the sample
The sample used for the measurements was manufactured using Duroid 6010 material with a dielectric constant of 10.2. There were two 1x3-inch substrates cut, each containing three strips of widths 20, 40 and 60 mils (0.0508, 0.1016 and 0.1524 cm). The two outer strips of the substrates have a distance of 300 mils to the middle strip and 140 mils to the edge of the substrate. The ground plane of
one of the two substrates was etched, so that this substrate could be laminated on top of the other to obtain the structure of Fig. 4 with minimum air gap. The lower strips were connected to the ground plane using copper tape and the two substrates were put together using WA-adhesive to minimize the presence of an air gap. Although the dielectric constant of the adhesive is about 2.9, the sample yields better results than with an air gap.

VI.5 Explanation of the program
The program used for the evaluation of the integrals for the calculation of the capacitance using equations (V.02) and (V.09) was written in Turbo Pascal. It contains a routine from the book "Numerical Recipes in Pascal" which evaluates the Bessel function $J_0(\beta \frac{W}{2})$, which was used for the evaluation of the Fourier transform of the basis function $f_3$. The integration algorithm is a simple one using the trapezoidal rule with unequal partitions. Since the functions needed to be integrated are even, an evaluation of the integral from 0 to $+\infty$ is required. The lower limit of integration is $\beta=10^{-14}$ and the higher limit is $\beta=178700$. The higher limit number is dictated by the fact that the function $e^{\beta h}$ cannot be evaluated for higher values of $\beta h$ for $h=0.0635\text{cm} = 25\text{mils}$. Nevertheless, a relative accuracy of $10^{-5}$ is achieved. The time needed to compile and run the program is about 4 seconds per integral evaluated on a Gateway2000 PC with an Intel 80486 microprocessor running at 33 MHz.

VI.6 Summary
In this chapter, a description of the measurements performed was given. Then the numerical results and were compared with the experimental data. The best method to use was found to be Gallerkin’s. Finally, a brief description of the sample manufacturing process was given and some characteristics of the program used were explained.
Chapter VII

SUMMARY OF WORK AND CONCLUSIONS

In this thesis, the theoretical background and techniques for calculating the capacitance and impedance of a microstrip-like structure were presented. Then, these techniques were applied to calculate the desired quantities numerically; finally, experimental data obtained by using a sample which was manufactured were compared with the numerical results.

Extensive literature exists on the static, quasi-static, and full-wave analysis of microstrips. The portion dealing with static and quasi-static analysis was developed in the 1960s and 1970s and 1980s was limited to open single-strip or shielded multiple-strip structures. The author was not able to find a paper containing the analysis of a structure with an open structure with more than one strip. Therefore, it became one of the primary purposes of this work to combine the existing theories in order to solve more complicated open structures.

A time-consuming part of the work in this thesis was the procedure of writing
the program for the algorithm used in the integration of the functions in Chapters IV-VI. After trying a number of programs given in the book "Numerical Recipes in Pascal" [9], it was decided that the best algorithm to be used was a very simple implementation of the trapezoidal rule, as explained in the previous chapter.

The main conclusion regarding the results is that the method of Yamashita and Mittra can be used only for structures with one strip. Based on the results obtained by this method the lower bound results yielded are far too low. However, the second method developed in this thesis can yield good results for the value of the capacitance and impedance of the microstrip-like structure analyzed statically. The method recommended is Galerkin's since it yields more accurate results and can be used for more complicated structures.

The work presented in this thesis could be expanded to analyze the high frequency dispersion model of microstrip-like structures by following the method of Itoh and Mittra [8]. In that case, the set-up of the problem and the calculations become more elaborate, since the analysis is extended to three dimensions.
Appendix

PROGRAM LISTING
program charge(input,output);

(NAME: Vassilios Papageorgiou SSN#: 990-10-4220)
(Program for M.S. Thesis)

const
  h = 0.0635; {Dielectric thickness of one layer, in cm.}
e0 = 8.85418e-14; {Permittivity of vacuum - in F/cm,}
er = 10.2; {Relative dielectric constant of the dielectric.}
wi = 0.0508; {Width of the strip in cm.}

var
  it: integer; {An iteration integer used in the evaluation of the integrals.}

*************** BESSEL FUNCTION J0 ***************

(This routine, taken from the book "numerical Recipes in Pascal", calculates the value of the Bessel function J0(x) for a given real value of x. It is later used, since it represents the Fourier Transform of the function 1/sqrt(a^2-x^2), which is used as a basis function to find the charge distribution on the plates.)

function bessj0(x: real): extended;

var
  ax, xx, z: extended;
  y, ans, ans1, ans2: extended; {Accumulate polynomials in double precision}

begin
  if abs(x) < 8.0 then begin
    y := sqr(x);
    ans1 := 57568490574.0+y*(-13362590354.0+y*(651619640.7
      +y*(-11214424.18+y*(77392.33017+y*(-184.9052456))))));
    ans2 := 57568490411.0+y*(1029532985.0+y*(9494680.718
      +y*(59272.64853+y*(267.8532712+y*1.0))));
    bessj0 := ans1/ans2
  end
  else begin
    ax := abs(x);
    z := 8.0/ax;
    y := sqr(z);
    xx := ax-0.785398164;
    ans1 := 1.0+y*(-0.1098628627e-2+y*(0.2734510407e-4
      +y*(-0.2073370639e-5+y*0.2093887211e-6)));
    ans2 := -0.1562499995e-1+y*(0.1430488765e-3
      +y*(-0.6911147651e-5+y*(0.7621095161e-6
        -y*0.934945152e-7)));
    ans := sqrt(0.636619772/ax)*(cos(xx)*ans1-z*sin(xx)*ans2);
    bessj0 := ans
  end
end;
{FUNCTION SELECTION}
(This routine determines which of the basis functions
and which G function will be used.)

function func(b: real): extended;
var
  sgn : integer; {This is a function which is equal
to 1 when the argument is >0 and to
-1 when the argument is <0.)
  q,f0,f1,f2,f3,f5,del,g11,g12,g21,g22,
  sq,cq,eb,emb,tanh: extended;
begin
  q := b*wi/2;
  if b >= 0 then sgn := 1 {Define sgn.}
  else sgn := -1;
  sq := sin(q); {Use variables for the sines, cosines
  cq := cos(q) {and exponentials, so that they will not
  have to be evaluated more than once.)
  eb := exp(b*h);
  emb := exp(-b*h);
  tanh := (eb-emb)/(eb+emb); {Define the hyperbolic
tangent.)
  det := tanh*(abs(b)+er*b*tanh) + (abs(b)*tanh+er*b);
  g22 := tanh*(er+sgn*tanh) / (e0*er*det);
  g12 := (2*tanh) / (e0*det*(eb+emb));
  g11 := tanh*2 / (e0*det);
  f0 := sq/(9.6*q); {Fourier transforms of the basis
functions.)
  f1 := (wi*sq/b + 2*(cq-1)/sqr(b))/(wi*wi*4.8);
  f2 := (2*sq/b + 12/(b*b*wi))*(cq-2*sq/q +
  sqr{sin(q/2)/(q/2)})/(wi*pi*4.3);
  f3 := bessj0(q)/9.6;
  if b < 0 then f1 := -f1;
  if b < 0 then f2 := -f2;
  if it = 1 then func := g11*f0*f3
  else if it = 2 then func := g12*f0*f3
  else if it = 3 then func := g22*f0*f3;
end;
(************************** MAIN PROGRAM *************************)
var
  c,o : real;
res,res1,res2,res3,det,cap: extended;
begin
  for it := 1 to 3 do begin  (Evaluate three integrals
    in order to find the  capacitance.)
    c := 1e-14;  (Beginning value of the integration
    parameter.)
    o := 1e-14;
    res := 0;    (The result is zero at this point.)
    repeat
      res := res + 0.5*(c-o)*(func(c)+func(o));
      (Trapezoidal formula for the evaluation of the
      integral.)
      o := c;   (Old parameter takes value of last one.)
      if (c < 1e-13) then c := c+1e-15  (Integration parameter
      is increased according to its value.)
      else if (c >= 1e-13) and (c < 1e-12) then c := c+1e-14
      else if (c >= 1e-12) and (c < 1e-11) then c := c+1e-13
      else if (c >= 1e-11) and (c < 1e-10) then c := c+1e-12
      else if (c >= 1e-10) and (c < 1e-9) then c := c+1e-11
      else if (c >= 1e-9) and (c < 1e-8) then c := c+1e-10
      else if (c >= 1e-8) and (c < 1e-7) then c := c+1e-9
      else if (c >= 1e-7) and (c < 1e-6) then c := c+1e-8
      else if (c >= 1e-6) and (c < 1e-5) then c := c+1e-7
      else if (c >= 1e-5) and (c < 1e-4) then c := c+1e-6
      else if (c >= 1e-4) and (c < 1e-3) then c := c+1e-5
      else if (c >= 1e-3) and (c < 1e-2) then c := c+1e-4
      else if (c >= 1e-2) and (c < 0.1) then c := c+1e-3
      else if (c >= 0.1) and (c < 1) then c := c+1e-2
      else if (c >= 1) and (c < 10) then c := c+0.1
      else if (c >= 10) and (c < 100) then c := c+1
      else if (c >= 100) and (c < 1000) then c := c+10
      else if (c >= 1e3) and (c < 1e5) then c := c+100
      else if (c >= 1e5) then c := c+1000
      until (c >= 178700);  (This is the upper limit for b.)
    if it = 1 then res1 := 2*res
    else if it = 2 then res2 := 2*res
    else if it = 3 then res3 := 2*res
  end;
  det := res1*res3 - res2*res2;
cap := (7.62*res3/(det*pi));  (Final formula for
  capacitance calculation.)
  writeln (cap);
  readln;
end.
REFERENCES


Vita

Mr. Vassilios A. Papageorgiou was born in Athens, Greece in 1969. He went to Athens College high-school in Athens. He received his B. Sc. degree in Electrical Engineering from Brown University, Providence, R.I. in 1990. He came to Virginia Polytechnic Institute and State University in August 1991.

From September 1990 until April 1991 he worked as a design engineer for Texas Instruments Deutschland GmbH, in Freising, Germany.

Vassilios enjoys music, guitar playing, and reading. He is planning to get married by middle 1994.