PREDICTION OF FILM CONDENSATION AND AEROSOL FORMATION
IN A GAS-VAPOR MIXTURE FLOW THROUGH A VERTICAL TUBE

by

Samuel H. McGhee IV

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APPROVED:

[Signature]
Dr. Douglas J. Nelson, Chairman

[Signature]  [Signature]
Dr. Brian Vick  Dr. William C. Thomas

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Blacksburg, Virginia
A numerical analysis of laminar film condensation and the prediction of aerosol formation is presented for a gas-vapor mixture undergoing forced flow through a vertical tube. This analysis is useful for estimating the sizes and operating conditions of condensers used for removing vapors from gas-vapor mixtures. The ability to predict the possibility of aerosol formation without expensive experimental studies makes it practical to design condensers in which aerosol formation is impossible.

Three different vapors (water, mercury, and benzene) mixed with air are considered in the analysis at three different inlet vapor mass fractions (50% of saturated, 75% of saturated, and saturated). The results indicate that condensers could effectively be used to remove vapors from gas/vapor mixtures and therefore, that design estimates could be determined from the model presented here. The results also show that inlet vapor mass fractions as low as 50% of saturation for water and benzene and 25% of saturation for mercury could cause aerosol formation in the bulk flow field. Aerosol formation degrades the performance of a condenser in removing vapor because the vapor condenses in the bulk flow where it is not easily removed without the use of a filtering medium.
To John Galt
Acknowledgements

Above all else, thanks go to my advisor Dr. Doug Nelson, for his patience, encouragement, and guidance through this work. Thanks also to my other committee members, Dr. Brian Vick and Dr. Bill Thomas, for their suggestions and insight concerning my research.

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Many thanks also to all the graduate students whose paths I crossed during the last two years. We shared our cynicism and ideals; our frustrations and successes; our laughter and tears; as well as many late nights on campus together. The constant reminder of all the others struggling with me was motivation to work harder. Thanks.
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Nomenclature

\( c_p \)  specific heat \( \left( \frac{\text{J}}{\text{kg K}} \right) \)

\( D \)  diameter (m)

\( D_{12} \)  binary diffusion coefficient \( \left( \frac{\text{m}^2}{\text{s}} \right) \)

\( g \)  gravitational acceleration \( \left( \frac{\text{m}}{\text{s}^2} \right) \)

\( h \)  heat transfer coefficient \( \left( \frac{\text{W}}{\text{m}^2\text{K}} \right) \)

\( h_l \)  latent heat of vaporization \( \left( \frac{\text{J}}{\text{kg}} \right) \)

\( h_m \)  mass transfer coefficient \( \left( \frac{\text{m}}{\text{s}} \right) \)

\( k \)  thermal conductivity \( \left( \frac{\text{W}}{\text{m K}} \right) \)

\( m \)  mass fraction

\( \dot{m}_{\text{cond}} \)  condensate mass flow rate \( \left( \frac{\text{kg}}{\text{s}} \right) \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M$</td>
<td>molar mass (\frac{\text{kg}}{\text{kmol}})</td>
</tr>
<tr>
<td>$\text{Nu}$</td>
<td>Nusselt number (\frac{hD}{k})</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure (Pa)</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless pressure (\frac{P}{\nu D \rho \sigma_0})</td>
</tr>
<tr>
<td>$\text{Pe}$</td>
<td>Peclet number (\frac{\nu D \rho \sigma_0}{k_0})</td>
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<tr>
<td>$\text{Pr}$</td>
<td>Prandtl number (\frac{c_0 \nu}{k_0})</td>
</tr>
<tr>
<td>$r$</td>
<td>radial coordinate (m)</td>
</tr>
<tr>
<td>$R$</td>
<td>dimensionless radial coordinate (\frac{r}{D})</td>
</tr>
<tr>
<td>$R_u$</td>
<td>Universal gas constant (8314 \frac{\text{J}}{\text{kmol K}})</td>
</tr>
<tr>
<td>$\text{Re}$</td>
<td>Reynolds number (\frac{\rho_0 \nu D}{\mu_0})</td>
</tr>
<tr>
<td>$S$</td>
<td>dimensionless mass fraction (\frac{m - m_w}{m_u - m_w})</td>
</tr>
<tr>
<td>$\text{Sc}$</td>
<td>Schmidt number (\frac{\mu_0}{\rho_0 D_{120}})</td>
</tr>
<tr>
<td>$\text{Sh}$</td>
<td>Sherwood number (\frac{h_m D}{D_{12}})</td>
</tr>
<tr>
<td>$T$</td>
<td>absolute temperature (K)</td>
</tr>
<tr>
<td>$u$</td>
<td>axial velocity component (\frac{m}{s})</td>
</tr>
<tr>
<td>$U$</td>
<td>dimensionless axial velocity (\frac{u}{u_0})</td>
</tr>
</tbody>
</table>
\( v \)  radial velocity component ( \( \frac{m}{s} \) )

\( V \)  dimensionless radial velocity ( \( \frac{v}{U_s} \) )

\( x \)  axial coordinate (m)

\( X \)  dimensionless axial coordinate ( \( \frac{x}{D} \) )

**Greek Symbols**

\( \delta \)  film thickness (m)

\( \theta \)  dimensionless temperature ( \( \frac{T - T_w}{T_s - T_w} \) )

\( \mu \)  viscosity ( \( \frac{N \cdot s}{m^2} \) )

\( \rho \)  density ( \( \frac{kg}{m^3} \) )

\( \phi \)  ratio of inlet mass fraction to saturated mass fraction

**Subscripts**

0  reference property

1  of vapor

2  of gas

\( B \)  bulk quantity

\( e \)  at the inlet
\( i \) at the gas/liquid interface

\( l \) of liquid

\( sat \) saturation quantity

\( v \) of vapor

\( w \) at the wall

\( \chi \) axial location

Superscripts

* dimensional property

— average value
1.0 Introduction

The processes of waste incineration and nuclear waste processing generate off-gasses which are likely to contain hazardous vapors. These vapors must be partially removed from the gas before it may be safely released into the atmosphere. In other processes, vapors have traditionally been removed from gas-vapor mixtures by means of filters and scrubbers. The use of a filtering or absorption medium for vapor removal in waste processing applications adds to the total amount waste.

The use of a condenser for vapor removal would eliminate additional waste generated by a filtering or absorption medium. When a gas-vapor mixture passes through a condenser, the vapor will tend to condense on the wall forming a liquid film which can easily be removed from the off-gas stream. A drawback of this approach to vapor removal is the potential of aerosol formation in the bulk flow. An aerosol would tend to follow the bulk flow instead of adhering to the wall of the tube, thus negating the ease of vapor removal with a condenser. Vapor removal with a condenser is also limited by the condenser wall temperature because the minimum vapor mass fraction in the mixture is limited to the saturated mass fraction at the wall temperature.

Condensation of pure vapors and of vapor-gas mixtures has been widely researched over the last century. In 1916 Nusselt presented assumptions for film condensation on a vertical plate (Incropera
and DeWitt, 1985) which have been the basis for most later film condensation work. Numerical work by Denny, et. al. (1969, 1971, 1972), Minkowycz, et. al. (1966), and Sparrow, et. al. (1964) has demonstrated both methods of solution to film condensation problems and the effects of noncondensable gases on condensation. This numerical work has been verified by experimental work by Al-Diwany, et. al. (1973) and Henderson, et. al. (1969).

The work presented here applies the methods and proven assumptions from previous work to the specific problem of laminar film condensation of a gas-vapor mixture undergoing forced flow through a vertical tube. While previous work has concentrated on gas-vapor mixtures with small gas mass fractions, this work focuses on mixtures with small vapor fractions as would be found in waste processing off-gas streams. From the numerical results, predictions can be made concerning the potential for aerosol formation in a condenser. Design estimates can also be made to insure that an aerosol will not form and that a certain percent of vapor will be removed from the gas-vapor mixture.
2.0 Problem Description and Analysis

2.1 Problem Description

Condensers are widely used in industrial applications to change vapors into liquids. The most common condensers are composed of many tubes passing through a shell with condensation occurring either in the shell (outer surface of the tubes) or in the tubes (inner surface of the tubes). If condensation is to occur in the tubes, then the cooling fluid flows through the shell. This work only considers the case of condensation occurring on the inner surface of the tubes. Figure 1 shows a schematic of the condenser type for which this work is based.

Two assumptions are made to determine the necessary solution domain for this work. The first is that the fluid flow, heat transfer, and mass transfer rates are nearly equal in each of the many tubes allowing the analysis of only one tube. Off-gasses from waste processing tend to have low velocities, due to low pressure drop requirements, and therefore low Reynolds numbers. This provides the assumption that the flow is steady, laminar, and axisymmetric, allowing a two dimensional solution domain from the tube centerline to its inner radius (and from the inlet to
Figure 1. Schematic of a Condenser for Tubeside Condensation
outlet). Figure 2 illustrates the solution domain and coordinate system as well as the dimensions used in the analyses.

Because there are many different conditions under which the condenser considered here may operate, only the type of vapor in the mixture and the inlet vapor mass fraction were varied in order to keep the scope of this work reasonable. Three different vapors in air are considered (water, mercury, and benzene) each at three different inlet mass fractions (saturated, 75% of saturated, and 50% of saturated).

Inlet pressure and temperature; wall temperature; tube diameter and length; and Reynolds number remain the same for all cases. The mixture enters the tube at atmospheric pressure (101 kPa) and with a temperature of 313 K. Industrial chilled water is usually available at 278 K, therefore the tube wall temperature is assumed to be constant at this temperature. The length of the tube is 100 times its diameter. The inlet axial velocity is calculated such that the Reynolds number based on the tube diameter is 1000 for each mixture.

2.2 Governing Equations

The governing equations (and boundary conditions) are presented in dimensionless terms which are defined in the Nomenclature. Gas-vapor mixture transport properties are nondimensionalized by division by the reference transport property indicated by a subscript "0". For convenience the liquid transport properties and the latent heat of vaporization remain dimensional, indicated by a superscript "*". Velocity, pressure, mass fraction, and coordinate variables are dimensionless when capitalized (note that a lower case "m" refers to dimensional mass fraction while a capital "M" refers to molar mass). The Greek letter "θ" represents dimensionless temperature. A subscript "ν"
Figure 2. Solution Domain and Coordinate System
indicates the gas-vapor region and a subscript "f" indicates the liquid region. The subscripts "e", "i", and "w" indicate properties at the inlet, interface, and wall respectively.

In terms of the dimensionless variables, the equations governing conservation of mass, momentum, species and energy in axisymmetric coordinates for the gas-vapor phase are (Bird, et. al., 1960):

Conservation of Mass

\[
\frac{\partial}{\partial X} (\rho_v U) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V) = 0
\]  
(1)

Conservation of Momentum

\[
\frac{\partial}{\partial X} (\rho_v U^2) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v UV) = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left[ \frac{\partial^2 U}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) \right] + \frac{\varepsilon D}{u_e^2} (\rho_v - 1)
\]  
(2)

\[
\frac{\partial}{\partial X} (\rho_v UV) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V^2) = - \frac{\partial P}{\partial X} + \frac{1}{Re} \left[ \frac{\partial^2 V}{\partial X^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial V}{\partial R} \right) - \frac{V}{R^2} \right]
\]  
(3)

Conservation of Species

\[
\frac{\partial}{\partial X} (\rho_v U S_i) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V S_i) = \frac{\partial}{\partial X} \left( \frac{\rho_v}{Re Sc} \frac{\partial S_i}{\partial X} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\rho_v}{Re Sc} \frac{\partial S_i}{\partial R} \right)
\]  
(4)

Conservation of Energy

\[
\frac{\partial}{\partial X} (\rho_v U \theta) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V \theta) = \frac{\partial}{\partial X} \left( \frac{1}{c_p Pe} \frac{\partial \theta}{\partial X} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{R}{c_p Pe} \frac{\partial \theta}{\partial R} \right)
\]  

\[
+ \left( \frac{m_{i.e} - m_{i,w}}{Re Sc} \right) \left( \frac{c_{p,1} - c_{p,2}}{c_p} \right) \rho \frac{\partial m_{i,e} \theta}{\partial R}
\]  

\[
+ \frac{1}{c_p Pe} \frac{\partial \theta}{\partial R} \left( \ln c_p \right)
\]  
(5)
The last term in equation (2) is the buoyancy term formed from the combination of the force due to gravity and the hydrostatic pressure on the fluid (West, 1988).

Each species of the mixture may be governed by separate species equations, or by one species equation and the conservation of mass equation. Since the numerical procedure used for this work enforces conservation of mass and, in a binary mixture, the knowledge of the mass fraction of one species leads directly to that of the other, it is most efficient to employ only one species equation. Thermal diffusion (Soret effect) is ignored in equation (4) due to the results from Minkowycz, et. al. (1966).

Because only low velocity flow is under consideration, viscous dissipation and compressibility effects are omitted from equation (5). The third term on the right-hand-side of equation (5) is the heat flux due to mass diffusion. By moving specific heat from the convective side of the energy equation to inside the first differential of the diffusion terms, the last term in the energy equation is formed. Due to the "strength" of convection in the axial direction relative to the radial direction, these two extra terms are negligible in the axial direction. Diffusional thermo (Dufour effect) is ignored due to the results from Minkowycz, et. al. (1966).

Density is a function of both mass fraction and temperature and is calculated with the ideal gas equation of state:

\[ \rho_v = \frac{P_0}{\rho_{v,0} R_w [\theta (T_e - T_w) + T_w]} \left[ \frac{S_1 (m_{1,e} - m_{1,w}) + m_{1,w}}{M_1} + \frac{1 - S_1 (m_{1,e} - m_{1,w}) - m_{1,w}}{M_2} \right]^{-1} \]  

(6)

Specific heat is a function of mass fraction only and is determined from mixture laws:

\[ c_p = \frac{1}{c_{p,0}} \left[ [S_1 (m_{1,e} - m_{1,2}) + m_{1,w}] c_{p,1}^* + [1 - S_1 (m_{1,e} - m_{1,w}) - m_{1,w}] c_{p,2}^* \right] \]  

(7)
The other fluid properties are considered to be constant because of the small temperature range considered. Air properties are used for all the other transport properties due to the small vapor mass fractions.

2.3 **Boundary Conditions**

At the inlet \((X = 0)\) the boundary conditions are all constant:

\[
U = 1; \quad V = 0
\]

\[
S_1 = 1; \quad \theta = 1
\]

Axisymmetric flow indicates that there are no changes across the centerline of the tube, thus the centerline \((R = 0)\) boundary conditions are:

\[
\frac{\partial U}{\partial R} = 0; \quad V = 0
\]

\[
\frac{\partial S_1}{\partial R} = 0; \quad \frac{\partial \theta}{\partial R} = 0
\]

The domain is long enough that the flow is both hydrodynamically and thermally fully developed by the tube outlet. The outlet \((X = 100)\) boundary conditions are:

\[
\frac{\partial U}{\partial X} = 0; \quad \frac{\partial V}{\partial X} = 0
\]

\[
\frac{\partial S_1}{\partial X} = 0; \quad \frac{\partial \theta}{\partial X} = 0
\]
Because only small inlet vapor mass fractions are assumed, it is also assumed that the condensation film thickness is small. A small film thickness allows the assumptions that the mixture/liquid interface is at the inner edge of the tube and that the liquid velocity and temperature profiles are linear (Figure 3). These assumptions lead to the following interface (R = 0.5) boundary conditions:

**Continuity of Axial Velocity**

\[ U_l | _l = U_v | _l \]  \hspace{1cm} \text{(8)}

**Continuity of Temperature**

\[ \theta_l | _l = \theta_v | _l \]  \hspace{1cm} \text{(9)}

**Fick's Law**

\[ V_l | _v,l = -\frac{1}{\text{ReSc}} \frac{m_{1,e} - m_{1,w}}{1 - S_l(m_{1,e} - m_{1,w}) - m_{1,w}} \frac{\partial S_l}{\partial R} | _v,l \]  \hspace{1cm} \text{(10)}

**Interfacial Shear Balance**

\[ \mu^* \frac{\partial U}{\partial R} | _l,l = \mu_v \frac{\partial U}{\partial R} | _v,l \]  \hspace{1cm} \text{(11)}

**Interfacial Energy Balance**

\[ \frac{k_l}{k_v} \frac{\partial \theta}{\partial R} | _l,l = \frac{\partial \theta}{\partial R} | _v,l - \frac{h_{fg}}{\text{Pe}_v \frac{\partial \theta}{\partial R} | _v,l} \left( T_e - T_w \right) c_p | _v,l \]  \hspace{1cm} \text{(12)}

**Saturated Vapor**

\[ S_1 | _l = \left[ 1 + \frac{M_2}{M_1} \left( \frac{P_0}{P_{sat} | _v,l} - 1 \right) \right]^{-1} - m_{1,w} \]  \hspace{1cm} \text{(13)}
Figure 3. Diagram of Condensate Film
Six equations are provided for four interface boundary conditions, but equations (8) and (9) must be considered in order to numerically evaluate equations (11) and (12).

A mass balance on the film at each axial step closes the interface boundary condition set of equations by providing the film thickness, \( \delta \):

\[
\delta X + dX = \frac{\left. \delta X U_{i,X} + \frac{2 \rho_{\ast}}{\rho_l} \rho V dX \right|_{r_i,X}}{U_{i,x} + dX}
\]  

(14)

which is used for determining velocity and temperature gradients across the film:

\[
\frac{\partial U}{\partial R} \bigg|_{r_i} = \frac{U_w - U_i}{\delta}
\]

\[
\frac{\partial \theta}{\partial R} \bigg|_{r_i} = \frac{\theta_w - \theta_i}{\delta}
\]

The saturation pressure of each vapor is required to determine the interface vapor mass fraction (and later for prediction of aerosol formation). The Antoine equation is used for the saturation pressure of mercury and benzene vapors with constants from Ohe (1976). A similar equation presented by Wexler et. al. (1971) is used for the saturation pressure of water vapor.

### 2.4 Numerical Procedure

The numerical procedure SIMPLEC (Semi-Implicit Method for Pressure Linked Equation - Consistent) is employed to obtain solutions to the aforementioned governing equations and boundary conditions. SIMPLEC is a fully-implicit, control volume based, finite difference method (Patankar, 1980, Van Doormal, et. al. 1984).
A 200 x 40 grid is used for the numerical solution. The grid is stretched using a method outlined by Anderson, et. al. (1984) so that more control volumes are located near the inlet and interface where the gradients (velocity, temperature, and mass fraction) are the largest.

The heat and mass transfer results are extrapolated to zero grid size using a method presented by Churchill, et. al. (1981). Comparison of the extrapolated results (Table 1) with the results from the 200 x 40 grid show a maximum difference of only 2.1%. This difference is small enough to insure that the results from a 200 x 40 grid are independent of the grid. For details on the extrapolation procedure and results see Appendix D.
Table 1. Numerical and Extrapolated Results Comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Numerical Results</th>
<th>Extrapolated Results</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_B$</td>
<td>$8.6010 \times 10^{-2}$</td>
<td>$8.4676 \times 10^{-2}$</td>
<td>1.58</td>
</tr>
<tr>
<td>$\bar{N}_D$</td>
<td>4.3655</td>
<td>4.3418</td>
<td>0.55</td>
</tr>
<tr>
<td>$S_{l,s}$</td>
<td>$5.8000 \times 10^{-2}$</td>
<td>$5.6817 \times 10^{-2}$</td>
<td>2.08</td>
</tr>
<tr>
<td>$S_{l,D}$</td>
<td>4.2107</td>
<td>4.1881</td>
<td>0.54</td>
</tr>
</tbody>
</table>
3.0 Results

The numerical results for three different vapors (water, mercury, and benzene) mixed with air at three different inlet vapor mass fractions (saturated, 75% of saturated, and 50% of saturated) are presented and discussed. Table 2 summarizes the tube dimensions, inlet and wall temperatures, ambient pressure, and transport properties all of which remain the same for each case. Table 3 - Table 5 give both the parameters that vary between cases and the information necessary to make quantitative analyses from the results.

As the ratio of inlet mass fraction to saturated mass fraction, $\phi$, of water vapor changes from 1.00 to 0.50, less than a two percent change is noticed in all the parameters that vary with inlet mass fraction (Table 3). This is due to the low dimensional inlet mass fraction which indicates that water vapor accounts for at most five percent of the total inlet mass.

The dimensional inlet mass fraction of mercury vapor (Table 4) is nearly three orders of magnitude smaller than that of water vapor. Table 4 indicates that even a 50% change in the dimensional inlet mass fraction of mercury vapor as small as is considered here will not affect the parameters that may vary with inlet mass fraction.
### Table 2. Constant Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tbody>
<tr>
<td>D</td>
<td>0.02 m</td>
</tr>
<tr>
<td>L</td>
<td>2.00 m</td>
</tr>
<tr>
<td>$T_e$</td>
<td>313 K</td>
</tr>
<tr>
<td>$T_w$</td>
<td>278 K</td>
</tr>
<tr>
<td>$P_0$</td>
<td>101000 Pa</td>
</tr>
<tr>
<td>$c_p$</td>
<td>1007 J/kg K</td>
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<tr>
<td>$\mu$b</td>
<td>$1.85 \times 10^{-4} \frac{N\cdot s}{m^2}$</td>
</tr>
<tr>
<td>$k_b$</td>
<td>$2.62 \times 10^{-2} \frac{W}{m\cdot k}$</td>
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</table>
Table 3. Parameters for Water Vapor Cases

<table>
<thead>
<tr>
<th></th>
<th>$\phi$</th>
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<th>0.75</th>
<th>1.00</th>
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<td></td>
<td>0.722</td>
<td>0.729</td>
<td>0.736</td>
</tr>
<tr>
<td>$Sc$</td>
<td></td>
<td>0.675</td>
<td>0.680</td>
<td>0.685</td>
</tr>
<tr>
<td>$m_{\text{ha}}$</td>
<td></td>
<td>2.31x10^{-2}</td>
<td>3.47x10^{-2}</td>
<td>4.62x10^{-2}</td>
</tr>
<tr>
<td>$m_{\text{h,w}}$</td>
<td></td>
<td>5.32x10^{-3}</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$u_e$ (m/s)</td>
<td></td>
<td>0.833</td>
<td>0.839</td>
<td>0.845</td>
</tr>
<tr>
<td>$c_{p,0}$ (J/kg K)</td>
<td></td>
<td>1027</td>
<td>1037</td>
<td>1047</td>
</tr>
<tr>
<td>$c_{p,1}$ (J/kg K)</td>
<td></td>
<td></td>
<td>1868</td>
<td></td>
</tr>
<tr>
<td>$h_f$ (J/kg)</td>
<td></td>
<td></td>
<td>2.49x10^{6}</td>
<td></td>
</tr>
<tr>
<td>$\rho_{x,0}$ (kg/m^3)</td>
<td></td>
<td>1.11</td>
<td>1.10</td>
<td>1.09</td>
</tr>
<tr>
<td>$\rho_1$ (kg/m^3)</td>
<td></td>
<td></td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>$\mu_1$ (Ns/m^2)</td>
<td></td>
<td></td>
<td>1.52x10^{-3}</td>
<td></td>
</tr>
<tr>
<td>$k_1$ (W/m K)</td>
<td></td>
<td></td>
<td>0.580</td>
<td></td>
</tr>
<tr>
<td>$D_{11}$ (m^2/s)</td>
<td></td>
<td></td>
<td>2.47x10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Parameters for Mercury Vapor Cases

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_r$</td>
<td></td>
<td>0.708</td>
<td></td>
</tr>
<tr>
<td>$Sc$</td>
<td></td>
<td>1.274</td>
<td></td>
</tr>
<tr>
<td>$m_{te}$</td>
<td>2.96x10^{-5}</td>
<td>4.44x10^{-5}</td>
<td>5.92x10^{-5}</td>
</tr>
<tr>
<td>$m_{i,w}$</td>
<td></td>
<td>3.00x10^{-6}</td>
<td></td>
</tr>
<tr>
<td>$u_e$ (m/s)</td>
<td></td>
<td>0.822</td>
<td></td>
</tr>
<tr>
<td>$c_{p,0}$ (J/kg K)</td>
<td></td>
<td>1007</td>
<td></td>
</tr>
<tr>
<td>$c_{p,1}$ (J/kg K)</td>
<td></td>
<td>1036</td>
<td></td>
</tr>
<tr>
<td>$h_f$ (J/kg)</td>
<td></td>
<td>3.17x10^5</td>
<td></td>
</tr>
<tr>
<td>$\rho_{v,0}$ (kg/m^3)</td>
<td></td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$ (kg/m^3)</td>
<td></td>
<td>13,595</td>
<td></td>
</tr>
<tr>
<td>$\mu_1$ (Ns/m^2)</td>
<td></td>
<td>1.69x10^{-3}</td>
<td></td>
</tr>
<tr>
<td>$k_i$ (W/m K)</td>
<td></td>
<td>8.18</td>
<td></td>
</tr>
<tr>
<td>$D_{ij}$ (m^2/s)</td>
<td></td>
<td>1.29x10^{-5}</td>
<td></td>
</tr>
</tbody>
</table>
The dimensional inlet mass fraction of benzene (Table 5) is two orders of magnitude larger than the water inlet mass fraction. The benzene inlet mass fraction is so large that benzene vapor accounts for 25 to 50% of the total inlet mass which indicates that the assumption of a small inlet vapor mass fraction may not be valid for the benzene cases. This assumption is used for the interface boundary conditions and some of the gas-vapor transport properties and its violation may lead to erroneous results.

Even though the benzene vapor accounts for such a large portion of the inlet mass, most of the parameters based on inlet mass fraction vary by less than two percent with a 50% change in inlet vapor mass fraction. The Schmidt number, inlet velocity, and reference density all vary by about 20% as $\phi$ changes from 1.00 to 0.50. It is important to reiterate at this point that vapor density and specific heat are the only fluid properties that vary with mass fraction.

Because large inlet mass fraction changes only cause small variations of the parameters affecting the results, it is expected that the only noticeable differences between the cases of each vapor will be in the magnitude of the interface boundary conditions and in the potential of aerosol formation. The interface boundary conditions and the potential for aerosol formation are both strongly affected by the dimensional vapor mass fraction while the heat and mass transfer results are affected more by the Prandtl and Schmidt numbers.
Table 5. Parameters for Benzene Vapor Cases

<table>
<thead>
<tr>
<th>$\phi$</th>
<th>0.50</th>
<th>0.75</th>
<th>1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pr</td>
<td>0.716</td>
<td>0.720</td>
<td>0.724</td>
</tr>
<tr>
<td>Sc</td>
<td>1.607</td>
<td>1.471</td>
<td>1.335</td>
</tr>
<tr>
<td>$m_{l,c}$</td>
<td>0.230</td>
<td>0.346</td>
<td>0.461</td>
</tr>
<tr>
<td>$m_{l,v}$</td>
<td></td>
<td></td>
<td>0.115</td>
</tr>
<tr>
<td>$u_a$ (m/s)</td>
<td>0.703</td>
<td>0.643</td>
<td>0.584</td>
</tr>
<tr>
<td>$c_{p,0}$ (J/kg K)</td>
<td>1018</td>
<td>1023</td>
<td>1029</td>
</tr>
<tr>
<td>$c_{p,1}$ (J/kg K)</td>
<td></td>
<td></td>
<td>1054</td>
</tr>
<tr>
<td>$\dot{h}_0$ (J/kg)</td>
<td></td>
<td></td>
<td>4.34x10^5</td>
</tr>
<tr>
<td>$\rho_{n,0}$ (kg/m³)</td>
<td>1.32</td>
<td>1.44</td>
<td>1.58</td>
</tr>
<tr>
<td>$\rho_l$ (kg/m³)</td>
<td></td>
<td></td>
<td>884</td>
</tr>
<tr>
<td>$\mu_l$ (N·s/m²)</td>
<td></td>
<td></td>
<td>6.62x10^-4</td>
</tr>
<tr>
<td>$k_l$ (W/m·K)</td>
<td></td>
<td></td>
<td>0.148</td>
</tr>
<tr>
<td>$D_{l,1}$ (m²/s)</td>
<td></td>
<td></td>
<td>8.75x10^-6</td>
</tr>
</tbody>
</table>
3.1 Interface Results

The gas-liquid interface is the most significant of the four boundaries. While the other three boundaries are necessary to solve the problem and the inlet boundary conditions contain important information about the specific problem, the interface boundary conditions provide condensation rates which are needed for piping design. The converged interface boundary conditions are also indicative of the validity of the assumptions made at the interface boundary.

Knowledge of condensation rates in condensers is required in order to design the piping that will carry the condensate. The condensation rate for one condenser tube is calculated from the interface axial velocity, film density and thickness, and the tube diameter using:

$$\dot{m}_{\text{cond}} = \frac{u_i}{2} \rho_i \delta^* \pi D$$

(15)

where:

$$u_i = U_i \mu_e$$

$$\delta^* = \delta D$$

For the cases presented in this work $D$, $u_i$, and $\rho_i$ are found in Table 2 - Table 5. $U_i$ and $\delta$ can be determined from figures that are to follow. Multiplication of $\dot{m}_{\text{cond}}$ by the total number of tubes yields the condensation rate of the condenser. Examples of condensation rate calculations are found in Appendix E.
Figure 4 - Figure 12 show the interface axial velocity, film thickness, and interface temperature in dimensionless terms for each of the three water, mercury and benzene vapor cases. Failure of the converged interface boundary conditions to follow some general trends or growth of the interface boundary conditions to significant magnitudes may indicate a diminished validity of the assumptions made at the interface.

The spikes in the velocity curves at \( X = 0 \) are due to a singularity point. Notice that the film thickness is zero at this point and therefore the film can have no velocity at the tube inlet leading to a zero velocity gradient across the film:

\[
\left. \frac{\partial U}{\partial R} \right|_{U} = 0
\]

However the gas-vapor mixture has a very large axial velocity gradient with respect to the radial direction at the interface near the tube inlet:

\[
\left. \frac{\partial U}{\partial R} \right|_{U} \rightarrow \infty
\]

due to the constant inlet velocity (\( U_{i} \)) and the zero interface velocity at this point (see equation (11) in Chapter 2). As the gas-vapor mixture flows through the tube and condensation grows on the wall, it is expected that the interface axial velocity will increase due to the increase in film thickness.

The condensate film thickness is zero at the tube inlet and continues to increase through the tube as more vapor condenses. A large dimensional vapor inlet mass fraction will lead to a larger film thickness because more vapor is available to condense. If the film thickness becomes greater than a couple of percent of the tube diameter, the assumption that the film is located at the wall (film thickness does not affect the gas flow area) becomes less valid. This would also make the linear film velocity and temperature profile assumptions less valid.
The dimensionless interface temperature, $\theta_i$, is zero at the tube inlet because condensation, which releases latent heat, does not begin until just after this point. Immediately after the tube inlet, $\theta_i$ increases to its maximum value and then slowly decreases along the length of tube. The sudden jump in the interface temperature is due to the high local condensation rate at the tube inlet (see Heat and Mass Transfer Results). As more vapor condenses, more latent heat, $h_g$, is released and the interface temperature increases. The interface temperature decreases through the tube because less condensation takes place as $X$ increases and the cold tube wall continues to cool the film. If the interface temperature is large, the constant wall temperature and linear film temperature profile assumptions are no longer valid.

The interface boundary quantities should all increase with $\phi$. In the cases presented here, if there is more vapor available, then the interface axial velocity, film thickness, and interface temperature should all be greater than if there is less vapor available.

Figure 4 - Figure 6 show the interface results for an air-water vapor mixture at three different inlet vapor mass fractions. As expected, the interface velocity and film thickness both increase while the interface temperature decreases through the tube. It is easily seen that as the inlet water vapor mass fraction increases, each of the boundary quantities also increases. None of the interface boundary quantities, for the water vapor cases, are large enough to violate the assumptions that are made at the boundary.

After the initial spikes in the water vapor interfacial axial velocity curves (Figure 4), the general trends previously mentioned are followed. While it is true that as $\phi$ increases the interface axial velocity also increases, the interface velocity never exceeds 0.14% of the inlet velocity for $\phi = 1.0$ (the maximum interface velocity is $1.37 \times 10^{-3}$ and the inlet velocity is 1.0) supporting the film velocity linear profile assumption.

The film thickness in all the water vapor cases (Figure 5) follows the predicted trends and steadily increases through the tube as condensation continues. The maximum film thickness is at the tube
Figure 4. Interface Axial Velocity for Water Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
outlet for $\phi = 1.0$ and is about 1.6% of the pipe diameter. This is not large enough to significantly change the flow through the tube or to violate the linear film velocity and temperature profile assumptions.

The interface temperature for all the water vapor cases (Figure 6) also follows the expected trends. The maximum interface temperature is for $\phi = 1.0$ and is less than 1% of the inlet temperature. The constant wall temperature and linear film temperature profile assumptions are both supported by the low interface temperatures.

Figure 7 - Figure 9 show the interface boundary conditions for the three mercury vapor runs. As with the water vapor cases, all three runs follow expected trends and do not violate the assumptions made for the interface boundary. In fact, due to dimensional mercury vapor inlet mass fractions three orders of magnitude less than water vapor, the mercury interface results are a couple of orders of magnitude smaller than the water interface results. There is less mercury vapor than water vapor to condense, thus the mercury film thickness, velocity, and temperature results must be less than the water results.

Figure 10 - Figure 12 show the interface boundary conditions for the three benzene vapor runs. It should be noted that the dimensional benzene inlet mass fraction (Table 5) is two orders of magnitude greater than the water vapor inlet mass fraction (Table 3). In fact, benzene accounts for 25 to 50 percent of the total inlet mass in the air-benzene cases, while water and mercury vapor never account for more than five percent of the inlet mass. This large dimensional inlet mass fraction indicates that the benzene vapor is a very significant part of the mixture and that more detail should be used in determining fluid transport properties and interface boundary conditions for the air/benzene mixture.

The interface velocity (Figure 10) and temperature (Figure 12) for a saturated inlet benzene mass fraction ($\phi = 1.0$) do not follow the expected trends, but for $\phi = 0.75$ the expected trends are followed. This variation from expected trends indicates that the film assumptions (or interface
Figure 5. Film Thickness for Water Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 6. Interface Temperature for Water Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 7. Interface Axial Velocity for Mercury Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 8. Film Thickness for Mercury Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 9. Interface Temperature for Mercury Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 10. Interface Axial Velocity for Benzene Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 11. Film Thickness for Benzene Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
Figure 12. Interface Temperature for Benzene Vapor:
Solid: $\phi = 1.00$
Dash: $\phi = 0.75$
Dot: $\phi = 0.50$
boundary conditions) used in this work become less valid for situations of very large inlet mass fractions as in the saturated benzene vapor case. The benzene film thickness (Figure 11) for \( \phi = 1.0 \) grows to greater than five percent of the pipe diameter (nearly four times the greatest water film thickness). This large film thickness not only effects bulk flow cross-sectional area, but also makes the assumption of linear velocity and temperature profiles in the film less valid.

### 3.2 Heat and Mass Transfer Results

The heat and mass transfer results tell how much and how efficiently heat and vapor are removed from the mixture. The bulk temperature and mass fraction tell how much heat and vapor remain in the mixture and are useful for determining the tube length required to achieve a desired outlet temperature or vapor mass fraction. The Nusselt and Sherwood numbers tell how efficiently heat and vapor are removed from the flow. They also indicate the developing and fully-developed regions of heat and mass transfer.

#### 3.2.1 Bulk Temperature and Mass Fraction

Bulk temperature and bulk mass fraction are weighted averages of the temperature and mass fraction at a particular axial location. If the mixture at a particular axial location could be thoroughly stirred, the bulk temperature and mass fraction would be the temperature and mass fraction of the stirred mixture. In the cases considered here both the bulk temperature and bulk mass fraction will decrease through the tube. The wall temperature is less than the inlet temperature therefore heat is removed from the fluid and the bulk temperature must decrease as \( X \) increases. The wall vapor mass fraction is also less than the inlet vapor mass fraction and thus vapor is removed from the mixture and the bulk mass fraction must decrease as \( X \) increases.
The bulk temperature at each axial location is numerically calculated with:

$$\theta_B(X) = \frac{\int_{R=0}^{1/2} \frac{1}{c_p Pe} \rho \nu URdR}{\int_{R=0}^{1/2} \frac{1}{c_p Pe} \rho \nu URdR}$$  \hspace{1cm} (16)

and in a similar manner, the bulk mass fraction is numerically calculated with:

$$S_{1,B}(X) = \frac{\int_{R=0}^{1/2} \rho \nu US_1 RdR}{\int_{R=0}^{1/2} \rho \nu URdR}$$  \hspace{1cm} (17)

Empirical relationships for bulk temperature and mass fraction have been developed for various common situations (Incropera, et. al., 1985). Comparison of the numerical results with empirical results based on a similar situation will give some insight into the validity of the model presented here. This comparison will also show the usefulness of the model in determining bulk temperatures and mass fractions.

The empirical relationship that most closely resembles the phenomena considered in this work assumes simultaneously developing flow and a constant wall (interface) temperature for heat transfer or a constant wall (interface) mass fraction for mass transfer. While the constant wall temperature or mass fraction is not strictly met in this work, it is closely approximated. Condensation and the coupling of the species and energy equations are also not considered in the empirical relationships.
The empirical relationships (Incropera, et. al., 1985) used for comparison with the numerical bulk temperature and mass fraction are respectively:

\[
\ln[\theta_{b}(\chi)] = -\frac{4X}{RePr} \left[ 1.86 \left( \frac{RePr}{X} \right)^{1/3} \right]
\]  

\[
\ln[S_{1,b}(\chi)] = -\frac{4X}{ReSc} \left[ 1.86 \left( \frac{ReSc}{X} \right)^{1/3} \right]
\]  

If the desired outlet temperature or mass fraction is known, the bulk temperature or mass fraction results may be used to determine the required tube length up to 100 times the tube diameter for cases presented here. The model and empirical relationships may, of course, be used to determine the bulk temperature or mass fraction in tubes of any reasonable dimensions.

Figure 13 - Figure 18 show the numerical and empirical bulk temperature and mass fraction for each vapor for the saturated inlet mass fraction case. These results are indistinguishable from the cases of lower inlet mass fractions due to the small changes in the Prandtl and Schmidt numbers for varying inlet mass fractions.

The empirical bulk temperature and mass fraction results show less heat and vapor removal than the numerical results in the water and mercury vapor cases. Removal of vapor from the bulk flow through condensation also removes heat from the bulk flow. This loss of heat is not considered in the empirical relationships. Condensation also creates a radial velocity component at the interface which generates a mass flux which is also not considered in the empirical relationships. The effects of condensation over the length of the tube cause the greatest difference between the numerical and empirical results at the tube outlet for the water and mercury vapor cases.

The empirical bulk temperature (Figure 15) and mass fraction (Figure 18) for benzene are less than the numerical results for most of the tube. The large dimensional mass fraction in the benzene cases causes the species and energy equations to be more strongly coupled than in the water and mercury.
Figure 13. Bulk Temperature vs. Axial Location for Water Vapor
Solid: Numerical $\theta_d(X)$
Dot: Empirical $\theta_d(X)$
Figure 14. Bulk Temperature vs. Axial Location for Mercury Vapor
Solid: Numerical $\theta_d(X)$
Dot: Empirical $\theta_d(X)$
Figure 15. Bulk Temperature vs. Axial Location for Benzene Vapor
Solid: Numerical $\theta_s(X)$
Dot: Empirical $\theta_s(X)$
Figure 16. Bulk Mass Fraction vs. Axial Location for Water Vapor
Solid: Numerical $S_{1,a}(X)$
Dot: Empirical $S_{1,a}(X)$
Figure 17. Bulk Mass Fraction vs. Axial Location for Mercury Vapor
Solid: Numerical $S_{i,g}(X)$
Dot: Empirical $S_{i,g}(X)$
Figure 18. Bulk Mass Fraction vs. Axial Location for Benzene Vapor
Solid: Numerical $S_{1,n}(x)$
Dot: Empirical $S_{1,n}(x)$
cases. This coupling is not considered in the empirical relationships and is the cause of the discrepancies in the numerical and empirical results.

From the figures it is seen that, for the water vapor cases, the bulk mass fraction is always less than the bulk temperature (except, of course, at the inlet) indicating that water vapor is removed at a more rapid rate than heat in air-water vapor mixtures. This result could be predicted by the fact that the Prandtl number is greater than the Schmidt number and that diffusional heat transfer is inversely proportional to the Prandtl number just as diffusional mass transfer is to the Schmidt number. In the mercury and benzene cases these results show that heat is more easily removed than vapor from the mixtures. These trends are useful in determining the potential of aerosol formation (see Aerosol Formation Results).

If the desired exit gas-vapor temperature (or the vapor mass fraction) is known, the bulk temperature (or mass fraction) curves may be used to determine the necessary tube length. However, the bulk temperature and bulk mass fraction, in situations as presented here, can never be less than the wall temperature or the saturated mass fraction at the wall temperature.

While these empirical relationships are not strictly valid due to the varying wall temperature and mass fraction, condensation, and the coupling of the species and energy equations, they do provide results that are accurate enough for engineering analysis. The empirical relationships are also not limited by the Reynolds number or tube dimensions as the numerical results are. The numerical bulk temperature and mass fraction are, however, readily obtained from the model which is required to determine radial distribution information which, in turn, is required for potential aerosol formation information. The numerical bulk temperature and mass fraction results are also more accurate than the empirical results because the variation in wall temperature (or mass fraction), coupling of the species and energy equations, and condensation are considered in the numerical results.
3.2.2 Nusselt and Sherwood Numbers

While a bulk quantity indicates how much of that quantity is present, the Nusselt and Sherwood numbers correspond to heat and mass transfer conductance. They also distinguish between developing and fully-developed regions of heat and mass transfer. In the cases presented here, a large Nusselt (or Sherwood) number indicates that heat (or vapor) is readily removed from the bulk flow. If the Nusselt or Sherwood number is changing with X, the flow is considered to be developing. When the Nusselt or Sherwood numbers become constant the flow is considered fully-developed with respect to heat or mass transfer.

The local Nusselt and Sherwood numbers are indicative of the heat and mass transfer rates at that point. Average Nusselt and Sherwood numbers correspond to the heat and mass transfer rates from the tube inlet to a point downstream. The local Nusselt number evaluated at the mixture/liquid interface is numerically calculated with the following:

$$\text{Nu}_D(X) = -\frac{1}{\theta_B(X)} \left. \frac{\partial \theta}{\partial R} \right|_{v, \ell}$$

and in the same manner, the local Sherwood number is calculated with:

$$\text{Sh}_D(X) = -\frac{1}{S_{1, B}(X)} \left. \frac{\partial S_1}{\partial R} \right|_{v, \ell}$$

The average Nusselt number evaluated at the mixture/liquid interface up to an axial position is numerically calculated with:

$$\overline{\text{Nu}}_D(X) = -\frac{1}{X} \int_0^X \frac{1}{\theta_B(X)} \left. \frac{\partial \theta}{\partial R} \right|_{v, \ell} dX$$
and the average Sherwood number with:

\[
\overline{Sh_D}(\lambda) = -\frac{1}{X} \int_{0}^{X} \frac{1}{S_{1,R}(\lambda)} \frac{\partial S_{1}}{\partial R} \, dX \bigg|_{\nu_{t}}
\]  

(23)

As the flow develops, in terms of heat and mass transfer, and after it has become fully-developed, the values of the average Nusselt and Sherwood numbers approach the local Nusselt and Sherwood numbers because the local numbers reach a constant fully-developed value.

Empirical relations very similar to those for bulk temperature and mass fraction also exist for the average Nusselt and Sherwood numbers (Incropera, et. al., 1985). In fact equations (18) and (19) are derived from the empirical relations for the average Nusselt and Sherwood numbers, thus the same assumptions are made for these empirical relationships and they are presented for the same reasons as the bulk temperature and mass fraction empirical relationships. The empirical relationships for the average Nusselt and Sherwood numbers are:

\[
\overline{Nu_D}(\lambda) = 1.86 \left( \frac{RePr}{X} \right)^{1/3}
\]  

(24)

and likewise the average Sherwood number:

\[
\overline{Sh_D}(\lambda) = 1.86 \left( \frac{ReSc}{X} \right)^{1/3}
\]  

(25)

Figure 19 - Figure 24 show the local, average, and empirical average Nusselt and Sherwood numbers as functions of axial location for each vapor at saturated inlet conditions. Once again, these results are indistinguishable from the ones with less than saturated inlet mass fractions.

It is readily seen that as \( X \) increases, the rate of heat and vapor removal decreases. As the Nusselt and Sherwood numbers decrease, the change in bulk temperature or mass fraction with respect to axial location also decreases. This shows that most of the heat and vapor are removed while the flow is hydrodynamically, thermally, and diffusonally developing. The local Nusselt and Sherwood

Results

45
Figure 19. Nusselt Number vs. Axial Location for Water Vapor
Solid: Numerical $\text{Nu}_p(X)$
Dash: Numerical $\text{Nu}_k(X)$
Dot: Empirical $\text{Nu}_d(X)$
Figure 20. Nusselt Number vs. Axial Location for Mercury Vapor
Solid: Numerical $\text{Nu}_0(X)$
Dash: Numerical $\text{Nu}_g(X)$
Dot: Empirical $\text{Nu}_d(X)$
Figure 21. Nusselt Number vs. Axial Location for Benzene Vapor
Solid: Numerical $\text{Nu}_d(X)$
Dash: Numerical $\text{Nu}_d(X)$
Dot: Empirical $\text{Nu}_d(X)$
Figure 22. Sherwood Number vs. Axial Location for Water Vapor

Solid: Numerical $Sh_d(X)$
Dash: Numerical $Sh_d(X)$
Dot: Empirical $Sh_d(X)$
Figure 23. Sherwood Number vs. Axial Location for Mercury Vapor
Solid: Numerical Sh_d(X)
Dash: Numerical Sh_d(X)
Dot: Empirical Sh_d(X)
Figure 24. Sherwood Number vs. Axial Location for Benzene Vapor
Solid: Numerical $Sh_D(X)$
Dash: Numerical $Sh_D(X)$
Dot: Empirical $Sh_D(X)$
numbers change very little after the first 20 to 50% of the tube length indicating the fully-developed region for the last 50 to 80% of the tube. In all the cases, the average Nusselt and Sherwood numbers approach the local values as X increases.

The discrepancies between the numerical and empirical results are due to the same reasons mentioned for the bulk temperature and mass fraction results.

All the curves begin with an infinite value at X = 0, due to the singularity point at the tube inlet/wall corner (see Interface Results), and they quickly decrease as the flow becomes fully-developed. None of the curves, except for the benzene numerical average and local Nusselt number, pass below the empirical limit of 3.66 as X increases. The numerical average and local Nusselt numbers for benzene pass below this limit because the benzene heat transfer problem does not correspond to the empirical relationships as closely as the other vapors.

3.3 Aerosol Formation Results

Condensation rates determined from interface conditions and vapor removal determined from bulk mass fractions are based on the assumption that an aerosol does not form in the bulk flow. The formation of an aerosol can drastically change the hydrodynamics and thermodynamics of the flow thus yielding all the previously presented results invalid. An aerosol would also tend to follow the bulk motion of the flow instead of adhering to the tube walls making it very difficult to remove the condensed vapor from the flow without a filtering medium. If a condenser is to be employed for removing hazardous vapor from a gas-vapor mixture, the formation of an aerosol will greatly reduce its effectiveness.
In order for condensation to occur or an aerosol to form, the ratio of the vapor partial pressure to saturation pressure at the local temperature must be at least one. This ratio is often referred to as *supersaturation* and is determined from:

\[
\frac{p_1}{p_{1,\text{sat}}} = \frac{p_0}{p_{1,\text{sat}}} \left\{ 1 + \frac{M_1}{M_2} \left[ \frac{1 - S_1(m_{1,e} - m_{1,w}) - m_{1,w}}{S_1(m_{1,e} - m_{1,w}) + m_{1,w}} \right] \right\}^{-1}
\]  

(26)

Heterogeneous condensation occurs when a surface is available on which condensation may occur. Supersaturation values as low as one will cause heterogeneous condensation as on the condenser wall. If there exists particles (such as dust) in the bulk flow, heterogeneous condensation may also occur on the particles at a supersaturation of one. Heterogeneous condensation is the easiest way for an aerosol to form because supersaturation values of no greater than one are required and dust is likely in waste processing off-gasses.

Homogeneous condensation occurs when a vapor condenses without the help of nucleation sites such as the condenser wall or dust in the bulk flow. Homogeneous condensation is much more difficult than heterogeneous condensation and may require supersaturations much greater than one. While kinetic theory indicates that supersaturations on the order of $10^2$ or greater are required for homogeneous condensation, experimental work shows that supersaturations less than 10 and as low as 1.02 will often cause the formation of aerosols (Hinds, 1982).

In the cases presented here, aerosol formation is limited by heat and vapor removal from the mixture as well as how the saturation pressure changes with temperature. If vapor is removed from the gas at a faster rate than heat, then there is little likelihood of aerosol formation if the inlet mass fraction is less than saturated unless that saturation pressure significantly decreases with a small temperature drop. A saturated inlet mass fraction will enhance the possibility of aerosol formation, at least near the tube inlet, even if the vapor is removed from the mixture more readily than heat. Because the vapor at the interface must be saturated for condensation to occur, the possibility will always exist for aerosol formation at the interface. However, this possibility is not a concern
because condensation will occur more readily on the wall as a film than near the wall as an aerosol (heterogeneous vs. nearly homogeneous condensation).

Figure 25 show the maximum supersaturation in the domain as a function of inlet mass fraction ratio. For all three vapors, inlet mass fractions as low as 50% of saturated ($\phi = 0.50$) could cause the formation of an aerosol for the conditions studied here. Visually extrapolating these results show that for water and benzene, a $\phi$ just less than 0.50 will eliminate the possibility of aerosol formation. A $\phi$ less than 0.25 is required to insure that the mercury vapor will not condense in the bulk flow. These results may be used to determine the maximum inlet mass fraction for which aerosol formation is impossible. The supersaturation values for mercury and benzene are greater than those for water because the heat transfer rate is greater than the mass transfer rate ($\text{Sc} > \text{Pr}$) for these cases. Additionally, mercury has much greater supersaturation values because the saturation pressure of mercury decreases significantly with a slight temperature drop.

Figure 26 - Figure 34 are shaded contours showing the regions where aerosol formation is most likely for the nine cases. Due to the geometry of the tube, an aspect ratio of about 200 to 1 is used for these contour plots so that information may be more readily obtained from them.

The contours for water vapor (Figure 26 - Figure 28) show that as the inlet vapor mass fraction decreases, the area of potential aerosol formation moves away from the inlet and centerline and toward the condenser wall. If an aerosol does form only near the condenser wall, as may happen in the case shown in Figure 28, the low axial velocity, high radial mass flux, and slight radial velocity due to condensation will tend to move the aerosol toward the condenser wall. The formation of an aerosol in a case such as this should have little effect on condenser performance.

The supersaturation contours for mercury vapor (Figure 29 - Figure 31) show a much greater potential (based on supersaturation only) for aerosol formation than either water or benzene vapor. With $\phi = 1.00$ most of the domain near the tube outlet and centerline has supersaturation values greater than 3.0. Even without the presence of dust in the flow, the potential for aerosol in this case
Figure 25. Maximum Supersaturation vs. Inlet Mass Fraction Ratio
Squares: Water Vapor
Circles: Mercury Vapor
Triangles: Benzene Vapor

Results
Figure 26. Supersaturation Contours for Water Vapor (\( \phi = 1.00 \))
Figure 27. Supersaturation Contours for Water Vapor ($\phi = 0.75$)
Figure 28. Supersaturation Contours for Water Vapor ($\phi = 0.50$)

- 3.00 and above
- 1.00 to 3.00
- 1.00 and below
is high. With $\phi = 0.75$ there still exists a large region of supersaturation values greater than 3.0. Even with $\phi = 0.50$, more than 70% of the domain has supersaturations greater than one. Because so much of the domain has supersaturation values greater than one in this case, it would be advisable to assume, until more information is known about the flow, that an aerosol will form.

The supersaturation contours for benzene vapor (Figure 32 - Figure 34) are similar to those for water vapor. As $\phi$ decreases, the potential for aerosol formation moves away from the inlet and centerline. However, for benzene vapor, there still exists a rather large (more than 30%) portion of the domain where an aerosol could form. This area is located at the tube outlet and extends completely across the radius of the tube. If an aerosol did form in this area, it would surely be carried out of the condenser by the bulk flow.

The supersaturation contours show that, in most cases, even inlet mass fractions that are 50% of saturation may lead to the potential of aerosol formation in undesirable locations. They also show that as the inlet vapor mass fraction decreases, aerosol formation potential decreases. The region where aerosol formation is impossible begins at and grows from the tube inlet and centerline as the inlet vapor mass fraction decreases. It is possible to decrease the inlet vapor mass fraction to a point where a supersaturation value of one or greater exists only at the interface. The absence of supersaturation values greater than one away from the tube wall indicates that aerosol formation is impossible in the bulk flow.
Figure 29. Supersaturation Contours for Mercury Vapor ($\phi = 1.00$)
Figure 30. Supersaturation Contours for Mercury Vapor ($\phi = 0.75$)
Figure 31. Supersaturation Contours for Mercury Vapor (φ = 0.50)
Figure 32. Supersaturation Contours for Benzene Vapor ($\phi = 1.00$)
Figure 33. Supersaturation Contours for Benzene Vapor ($\phi = 0.75$)
Figure 34. Supersaturation Contours for Benzene Vapor ($\phi = 0.50$)
4.0 Conclusions and Recommendations

4.1 Conclusions

Off-gasses from nuclear waste processing and waste incineration may contain hazardous vapors which cannot be safely released into the atmosphere. Because the use of filters or scrubbers to remove these vapors increases the total amount of waste to be processed, it is desirable to remove the vapors with condensers. While a vapor in a gas-vapor mixture will readily condense on the tube wall of a condenser, the possibility exists that an aerosol will form in the bulk flow. The formation of an aerosol reduces the effectiveness of a condenser for the purpose of vapor removal.

A method for determining condensation rates and predicting aerosol formation in a gas-vapor mixture has been presented here. The results from this method for three different vapors at three different inlet mass fractions show that the model is valid for most cases and that it only falters for cases with high dimensional inlet mass fractions such as saturated benzene. It is also shown that the possibility of aerosol formation exists at high inlet mass fractions. This possibility of aerosol formation may be eliminated by lowering the inlet mass fraction, raising the condenser wall temperature, or by numerous other means. The ability to eliminate the risk of aerosol formation
and to determine the necessary condenser length for vapor removal makes the design of condensers for vapor removal feasible.

4.2 Recommendations

While the work presented here is useful for vapor removal condenser design estimates, a few modifications could improve the accuracy of the results and increase the number of situations where it is useful.

The off-gasses from nuclear waste processing and waste incineration will usually contain many more than two species making the number of situations where this work is directly applicable very small. The addition of more species conservation equations and the implementation of mass diffusion coefficients for mixtures of more than two species would greatly increase the applicability of the model.

The assumption of linear velocity and temperature profiles in the liquid film is only valid when the film thickness is small. If the inlet vapor mass fraction is large, as in the benzene case, these may assumptions cause the model to give inconsistent results. The use of parabolic liquid velocity and temperature profiles are more accurate assumptions and should help to eliminate this problem.

In cases where the inlet vapor mass fraction is large, the film thickness may become significant enough to effectively decrease the mixture flow area. Modification of the model to take this into consideration would make the results more realistic and may also improve the interface velocity and temperature results.

Simple knowledge of the possibility of aerosol formation is useful for designing so that it cannot happen, but more precise knowledge about the actuality of aerosol formation would be even more
useful. The addition of aerosol formation and aerosol flow dynamics to the model would provide more detailed information on the formation of aerosols and how they behave in the flow. This could allow the design of condensers with less stringent inlet mass fraction limits.
Bibliography


Appendix A. Additional Nomenclature for Appendix

\( S_c \) constant source term

\( S_r \) proportional source term

Greek Symbols

\( \beta \) grid stretching parameter

\( \Gamma \) diffusion coefficient

\( \phi \) dependent variable

Subscripts

\( o \) at the outlet

\( r \) in the radial direction

\( x \) in the axial direction
Appendix B. Literature Review

The study of condensation heat transfer began nearly 80 years ago with the work of Nusselt. His focus was the condensation of a pure vapor onto a flat surface (Incropera, et. al., 1985). This work of Nusselt in the area of condensation heat transfer has been the basis for most subsequent work in the same area.

The first numerical solutions showing the effect of non-condensible gases on condensation (Sparrow, et. al., 1964) helped to confirm the belief that heat transfer rates of a condensing vapor would be reduced by small fractions of non-condensible gases in the vapor. This work only considers the boundary layer region of the problem through the use of a similarity variable and integral equations. The agreement between the numerical results and experimental results from other investigators (but also presented by Sparrow) is satisfactory.

A comprehensive study of condensation heat transfer (Minkowycz, et. al., 1966) not only examines the effects of the presence of a noncondensable gas, but also looks into interfacial resistance, superheating, variable properties, and diffusion. This work shows that interfacial resistance, diffusional thermo, and thermal diffusion are negligible in most condensation heat transfer problems. All of the transport properties used for this work are variable, but no mention is made of the effect on the results.
The use of finite difference methods for condensation heat transfer showed no great discrepancies between analytical or experimental work (Denny, et. al., 1969). While this work employed variable liquid properties, the authors suggest that for most engineering applications the benefits derived from this are inconsequential.

A study of the additional effects of forced flow on laminar film condensation of a vapor-gas mixture (Denny, et. al., 1971) shows that condensation heat transfer increases as the axial velocity of the vapor-gas increases. The effects of different temperature levels and differences is also presented.

The results from the previously mentioned work are presented again with a strong focus on different vapors rather than different velocities and temperatures (Denny, et. al., 1972). This work is used to develop a semi-empirical calculation method method for similar problems. While the mass transfer results from the semi-empirical method are poor, the heat transfer results show the method to be adequate for engineering calculations.

Experimental results from a study of condensation in the presence of a noncondensable gas (Al-Diwany, et. al., 1973) show that the numerical boundary layer analyses are more accurate than earlier experimental work indicated. The earlier discrepancies are believed to be from an inability to precisely control all the test parameters in the experimental work.
Appendix C. Development of Governing Equations

A brief discussion of the governing equations and boundary conditions is presented here. First the dimensional form is presented with assumptions and unusual terms explained. Then the dimensionless form is presented with an explanation of any further assumptions. Finally, the general equation used in the numerical procedure is presented and the terms defined for each governing equation.

The balances that are used to indicate model convergence are also presented in dimensional and dimensionless forms.
C.1 Dimensional Form

C.1.1 Dimensional Governing Equations

The equations governing conservation of mass, momentum, species, and energy in two dimensional, axisymmetric coordinates are:

**Conservation of Mass**

\[
\frac{\partial}{\partial x}(\rho \cdot u) + \frac{1}{r} \frac{\partial}{\partial r}(r \cdot \rho \cdot u) = 0
\]

(C.1)

**Conservation of Momentum**

\[
\frac{\partial}{\partial x}(\rho \cdot u^2) + \frac{1}{r} \frac{\partial}{\partial r}(r \cdot \rho \cdot uu) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \rho \frac{\partial u}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \frac{\partial u}{\partial r} \right)
\]

\[
+ g(\rho \cdot - \rho \cdot u)
\]

(C.2)

\[
\frac{\partial}{\partial x}(\rho \cdot vv) = \frac{1}{r} \frac{\partial}{\partial r}(r \cdot \rho \cdot vv) = -\frac{\partial p}{\partial r} + \frac{\partial}{\partial x} \left( \rho \frac{\partial v}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \frac{\partial v}{\partial r} \right) - \frac{\mu v}{r^2}
\]

(C.3)

**Conservation of Species**

\[
\frac{\partial}{\partial x}(\rho \cdot \nu \cdot m) + \frac{1}{r} \frac{\partial}{\partial r}(r \cdot \rho \cdot \nu \cdot m) = \frac{\partial}{\partial x} \left( \rho \nu D_{12} \frac{\partial m}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho \nu D_{12} \frac{\partial m}{\partial r} \right)
\]

(C.4)
Conservation of Energy

\[
\frac{\partial}{\partial x} (\rho_v u_T) + \frac{1}{r} \frac{\partial}{\partial r} (r \rho_v v_T) = \frac{\partial}{\partial x} \left( \frac{k_v}{c_{p,v}} \frac{\partial T}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( \frac{k_v}{c_{p,v}} \frac{\partial T}{\partial r} \right) \\
+ \frac{c_{p,1}^* - c_{p,2}^*}{c_p^*} \rho_v D_{12}^* \left[ \frac{\partial m_1}{\partial x} \frac{\partial T}{\partial x} + \frac{\partial m_1}{\partial r} \frac{\partial T}{\partial r} \right] \\
+ \frac{k^*}{c_p^*} \left[ \frac{\partial T}{\partial x} \frac{\partial}{\partial x} (\ln c_p^*) + \frac{\partial T}{\partial r} \frac{\partial}{\partial r} (\ln c_p^*) \right]
\]  (C.5)

The forces on the fluid due to gravity and hydrostatic pressure are combined to form the last term in equation (C.2) which is the buoyancy term (West, 1988). The last term in equation (C.3) is an extra viscous dissipation term which arises from the transformation to axisymmetric coordinates from cartesian coordinates.

Because only binary mixtures are considered in this work and the numerical procedure employed enforces conservation of mass, only one species equation is required. The species equation represents the vapor mass fraction because the vapor is of most concern in this work. Mass fraction is chosen over mole fraction because no chemical reactions are considered and so that the results may easily be converted to related quantities (kg/s rather than kmol/s). Mathematically mass fraction and mole fraction are equivalent (Sparrow, et. al., 1960). Thermal diffusion (Soret effect) is ignored in equation (C.4) in light of the results of Minkowycz, et. al. (1966).

Viscous dissipation and compressibility effects are omitted from equation (C.5) because only low velocity flow is under consideration. Diffusional thermo (Dufour effect) is ignored due to the results from Minkowycz, et. al. (1966). The third term on the right hand side of the energy equation arises from a heat flux due to mass diffusion. The last term on the right hand side of equation (C.5) is necessary to algebraically move a variable specific heat into the partial derivatives of the first and second terms on the right hand side.

Appendix C. Development of Governing Equations
The two terms on the left hand side of the momentum, species, and energy equations are the standard convective terms for axisymmetric coordinates.

C.1.2 Dimensional Boundary Conditions

At the inlet \((x = 0)\) the boundary conditions are all constant:

\[
\begin{align*}
\vec{u} &= \vec{u}_i; \quad v = 0 \\
\rho I &= m_{1,i}; \quad T = T_e
\end{align*}
\]

Axisymmetric flow dictates that there are no changes across the tube centerline, thus the centerline \((r = 0)\) boundary conditions are:

\[
\begin{align*}
\frac{\partial u}{\partial r} &= 0; \quad v = 0 \\
\frac{\partial m_1}{\partial r} &= 0; \quad \frac{\partial T}{\partial r} = 0
\end{align*}
\]

The domain is long enough that the flow is both hydrodynamically and thermally fully developed by the tube outlet. The outlet \((x = 100D)\) boundary conditions are:

\[
\begin{align*}
\frac{\partial u}{\partial x} &= 0; \quad \frac{\partial v}{\partial x} = 0 \\
\frac{\partial m_1}{\partial x} &= 0; \quad \frac{\partial T}{\partial x} = 0
\end{align*}
\]
Only small inlet vapor mass fractions are assumed in this work allowing the assumption that the film thickness does not change the gas-vapor flow area. A thin film thickness also allows the assumptions that the liquid velocity and temperature profiles are linear. These assumptions lead to the following interface \((r = D/2)\) boundary conditions:

**Continuity of Axial Velocity**

\[
u |_l = \nu_v |_l \quad \text{(C.6)}
\]

**Continuity of Temperature**

\[
T |_l = T_v |_l \quad \text{(C.7)}
\]

**Fick’s Law**

\[
\nu |_l = - \frac{D_{12}^*}{1 - m_i} \frac{\partial m_i}{\partial r} |_{v,l} \quad \text{(C.8)}
\]

**Interfacial Shear Balance**

\[
\mu |_l = \mu_v |_{v,l} \quad \text{(C.9)}
\]

**Interfacial Energy Balance**

\[
k_i^* \frac{\partial T}{\partial r} |_{l,l} = k_v^* \frac{\partial T}{\partial r} |_{v,l} - \dot{m} h_{fg} |_l \quad \text{(C.10)}
\]

**Saturated Vapor**

\[
m_i |_l = \left[ 1 + \frac{M_2}{M_1} \left( \frac{P_0}{P_{sat,v,l}} - 1 \right) \right]^{-1} \quad \text{(C.11)}
\]
A mass balance on the film at each axial location closes the interface boundary conditions by providing the film thickness:

\[ \rho_l \frac{u_l x}{2} \delta_x + \rho_v \nu_v d \delta x = \rho_l \frac{u_l x + d \delta x}{2} \delta_x + d \delta x \]  

(C.12)

### C.1.3 Dimensional Mass, Momentum, Energy, and Species Balances

While global balances are not required for solutions to the aforementioned governing equations and boundary conditions, they are helpful for determining the convergence of a solution. The global mass, axial momentum, species, and energy balances are as follows:

**Mass Balance**

\[ \int_{r=0}^{D/2} \rho_v u r dr \bigg|_e - \int_{r=0}^{D/2} \rho_v u r dr \bigg|_o - \int_{x=0}^{100D} \rho_v \nu r dx \bigg|_l = 0 \]  

(C.13)

**Momentum Balance**

\[ \int_{r=0}^{D/2} \left( \rho_v u^2 + p - \mu_v \frac{\partial u}{\partial x} \right) r dr \bigg|_e - \int_{r=0}^{D/2} \left( \rho_v u^2 + p - \mu_v \frac{\partial u}{\partial x} \right) r dr \bigg|_o - \int_{x=0}^{100D} \left( \rho_v \nu u - \mu_v \frac{\partial u}{\partial r} \right) r dx \bigg|_l + \int_{r=0}^{D/2} \int_{x=0}^{100D} \left( \rho_v \nu - \rho_v \nu_0 \right) r dx dr = 0 \]  

(C.14)
Species Balance

\[
\int_{r=0}^{D/2} \left( \rho_v^* u m_1 - \rho_v^* D_{12}^* \frac{\partial m_1}{\partial x} \right) r dr \bigg|_e - \int_{r=0}^{D/2} \left( \rho_v^* u m_1 - \rho_v^* D_{12}^* \frac{\partial m_1}{\partial x} \right) r dr \bigg|_o
- \int_{x=0}^{100D} \left( \rho_v^* v m_1 - \rho_v^* D_{12}^* \frac{\partial m_1}{\partial r} \right) r dx \bigg|_l = 0
\]

\[(C.15)\]

Energy Balance

\[
\int_{r=0}^{D/2} \left( \rho_v^* u c_p^* T - k_v^* \frac{\partial T}{\partial x} \right) r dr \bigg|_e - \int_{r=0}^{D/2} \left( \rho_v^* u c_p^* T - k_v^* \frac{\partial T}{\partial x} \right) r dr \bigg|_o
- \int_{x=0}^{100D} \left( \rho_v^* v c_p^* T - k_v^* \frac{\partial T}{\partial r} \right) r dx \bigg|_l = 0
\]

\[(C.16)\]

C.2 Dimensionless Form

Application of the dimensionless parameters defined in the Nomenclature on the dimensional conservation equations, boundary conditions, and balances yields the dimensionless forms.
C.2.1 Dimensionless Governing Equations

The dimensionless conservation of mass, momentum, species, and energy are:

**Conservation of Mass**

\[
\frac{\partial}{\partial \chi} (\rho_v U) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V) = 0 \tag{C.17}
\]

**Conservation of Momentum**

\[
\frac{\partial}{\partial \chi} (\rho_v U^2) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v U V) = -\frac{\partial P}{\partial \chi} + \frac{1}{Re} \left[ \frac{\partial^2 U}{\partial \chi^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial U}{\partial R} \right) \right] + \frac{gD}{u_e^2} (\rho_v - 1) \tag{C.18}
\]

\[
\frac{\partial}{\partial \chi} (\rho_v U V) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V^2) = -\frac{\partial P}{\partial R}
\]

\[
+ \frac{1}{Re} \left[ \frac{\partial^2 V}{\partial \chi^2} + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial V}{\partial R} \right) - \frac{V}{R^2} \right] \tag{C.19}
\]

**Conservation of Species**

\[
\frac{\partial}{\partial \chi} (\rho_v U S_1) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V S_1) = \frac{\partial}{\partial \chi} \left( \frac{\rho_v}{ReSc} \frac{\partial S_1}{\partial \chi} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\rho_v}{ReSc} \frac{\partial S_1}{\partial R} \right) \tag{C.20}
\]
Conservation of Energy

\[\frac{\partial}{\partial X} (\rho_v U \theta) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_v V \theta) = \frac{\partial}{\partial X} \left( \frac{1}{c_p \mathrm{Pe}} \frac{\partial \theta}{\partial X} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{R}{c_p \mathrm{Pe}} \frac{\partial \theta}{\partial R} \right) \]

\[+ \left( \frac{m_{l,e} - m_{l,w}}{\mathrm{ReSc}} \right) \left( \frac{c_{p,1} - c_{p,2}}{c_p} \right) \rho \frac{\partial m_1}{\partial R} \frac{\partial \theta}{\partial R} \]

\[+ \frac{1}{c_p \mathrm{Pe}} \frac{\partial \theta}{\partial R} \frac{\partial}{\partial R} (\ln c_p) \]  
(C.21)

Equations (C.17), (C.18), (C.19), and (C.20) contain the same, yet dimensionless, terms as their dimensional counterparts.

The energy flux due to mass diffusion in the axial direction is overwhelmed by convection and therefore ignored in the dimensionless form of equation (C.21). The same is true for the last term in the energy equation.

### C.2.2 Dimensionless Boundary Conditions

At the inlet \((X = 0)\) the boundary conditions are:

\[U = 1; \quad V = 0\]

\[S_1 = 1; \quad \theta = 1\]

The centerline \((R = 0)\) boundary conditions are:

\[\frac{\partial U}{\partial R} = 0; \quad V = 0\]

\[\frac{\partial S_1}{\partial R} = 0; \quad \frac{\partial \theta}{\partial R} = 0\]
The outlet \((X = 100)\) boundary conditions are:

\[
\frac{\partial U}{\partial X} = 0; \quad \frac{\partial V}{\partial X} = 0
\]

\[
\frac{\partial S_1}{\partial X} = 0; \quad \frac{\partial \theta}{\partial X} = 0
\]

The interface \((R = 0.5)\) boundary conditions are:

**Continuity of Axial Velocity**

\[
U_l|_l = U_v|_l \quad \text{(C.22)}
\]

**Continuity of Temperature**

\[
\theta_l|_l = \theta_v|_l \quad \text{(C.23)}
\]

**Fick's Law**

\[
V_l|_l = -\frac{1}{ReSc} \frac{m_{1,e} - m_{1,w}}{1 - S_1(m_{1,e} - m_{1,w}) - m_{1,w}} \frac{\partial S_1}{\partial R} \bigg|_l \quad \text{(C.24)}
\]

**Interfacial Shear Balance**

\[
\mu_l^* \frac{\partial U}{\partial R} \bigg|_{l,l} = \mu_v^* \frac{\partial U}{\partial R} \bigg|_{v,l} \quad \text{(C.25)}
\]

**Interfacial Energy Balance**

\[
\frac{k_l^*}{k_{v,0}} \frac{\partial \theta}{\partial R} \bigg|_{l,l} = \frac{\partial \theta}{\partial R} \bigg|_{v,l} - Pe_{\beta, V} \frac{k_{\beta}^*}{(T_e - T_w)c_{p,0}} \bigg|_{v,l} \quad \text{(C.26)}
\]
Saturated Vapor

\[ S_1 |_i = \left[ 1 + \frac{M_2}{M_1} \left( \frac{P_0}{P_{sat}|_{i,j}} - 1 \right) \right]^{-1} - m_{1,w} \]  

\[ \delta X U_{i,X} + 2 \frac{\rho_v V}{\rho_i} \rho V dX \bigg|_{v_{i,X}} \delta X + dx = \frac{U_{i,X} + dx}{U_{i,X} + dx} \]  

The film thickness, \( \delta \), needed to calculate most of the interface boundary conditions is:

\[ (C.27) \]

\[ (C.28) \]

C.2.3 Dimensionless Mass, Momentum, Species, and Energy Balances

The dimensionless mass, momentum, species, and energy balances are:

**Mass Balance**

\[ \int_{R=0}^{1/2} \rho_v URdR |_e - \int_{R=0}^{1/2} \rho_v URdR |_o - \int_{X=0}^{100} \rho_v VRdX |_i = 0 \]  

\[ (C.29) \]

**Momentum Balance**

\[ \int_{R=0}^{1/2} \left( \rho_v U^2 + P - \frac{1}{Re} \frac{\partial U}{\partial X} \right) RdR |_e - \int_{R=0}^{1/2} \left( \rho_v U^2 + P \right) RdR |_o \]

\[ - \int_{X=0}^{100} \left( \rho_v UV - \frac{1}{Re} \frac{\partial U}{\partial R} \right) RdX |_i \]

\[ + \int_{R=0}^{1/2} \int_{X=0}^{100} \frac{gD}{\rho_v - 1} RdX |_o = 0 \]

\[ (C.30) \]
Species Balance

\[
\int_{R=0}^{1/2} \left( \rho_y U S_1 - \frac{\rho_y}{\text{ReSc}} \frac{\partial S_1}{\partial X} \right) RdR \int_{e}^{1/2} \rho_y U S_1 RdR|_o - \int_{R=0}^{100} \rho_y V S_1 - \rho_y \frac{1}{\text{ReSc}} \frac{\partial S_1}{\partial R} RdX|_l = 0
\]

(C.31)

Energy Balance

\[
\int_{R=0}^{1/2} \left( \rho_y U \theta - \frac{1}{c_p \text{Pe}} \frac{\partial \theta}{\partial X} \right) RdR \int_{e}^{1/2} \rho_y U \theta RdR|_o - \int_{X=0}^{100} \rho_y V \theta - \frac{1}{c_p \text{Pe}} \frac{\partial \theta}{\partial R} RdX|_l = 0
\]

(C.32)

The diffusion terms at the tube outlet in the momentum, species, and energy balances are omitted due to the outlet boundary conditions.

C.3 General Equation Form

The general equation in axisymmetric coordinates is (Patankar, 1980):

\[
\frac{\partial}{\partial X} (\rho_y U \phi) + \frac{1}{R} \frac{\partial}{\partial R} (R \rho_y V \phi) = \frac{\partial}{\partial X} \left( \Gamma X \frac{\partial \phi}{\partial X} \right) + \frac{1}{R} \frac{\partial}{\partial R} \left( R \Gamma R \frac{\partial \phi}{\partial R} \right) + S_p \phi + S_c
\]

(C.33)

The numerical procedure (SIMPLEC) automatically calculates the pressure gradients required for the fluid velocities, thus the pressure terms are not found in the general equation.
The source terms \( S_p \) and \( S_e \) are used to incorporate any terms additional to the standard convection and diffusion terms. These two terms are required for the axial and radial momentum equations as well as the energy equation. Table 6 gives the diffusion coefficients and source terms for each \( \phi \).

The evaluation of derivatives and integrals is required to find gradients and balances in this work. First-order accurate methods are used to remain consistent with the first-order accuracy obtained from a nonuniform grid. Forward and backward difference schemes are used for gradients along the boundaries as in \( \text{Nu}_D \), \( \text{Sh}_D \), etc. A central difference scheme is used for gradients in the domain as are found in the conservation of energy source term. A rectangle rule is used to evaluate integrals for the balances and results such as \( \theta_s \), \( S_{1,e} \), \( \overline{\text{Nu}}_D \), and \( \overline{\text{Sh}}_D \).
Table 6. Diffusion Coefficients and Source Terms for each Equation

<table>
<thead>
<tr>
<th>Φ</th>
<th>( \Gamma_x )</th>
<th>( \Gamma_R )</th>
<th>( S_p )</th>
<th>( S_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>( \frac{1}{\text{Re}} )</td>
<td>( \frac{1}{\text{Re}} )</td>
<td>0</td>
<td>( \frac{gD}{u^2} (\rho - 1) )</td>
</tr>
<tr>
<td>V</td>
<td>( \frac{1}{\text{Re}} )</td>
<td>( \frac{1}{\text{Re}} )</td>
<td>( -\frac{1}{\text{ReR}^2} )</td>
<td>0</td>
</tr>
<tr>
<td>S</td>
<td>( \frac{\rho}{\text{ReSc}} )</td>
<td>( \frac{\rho}{\text{ReSc}} )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>θ</td>
<td>( \frac{1}{c_p \text{Pe}} )</td>
<td>( \frac{1}{c_p \text{Pe}} )</td>
<td>0</td>
<td>( S'_c )</td>
</tr>
</tbody>
</table>

\[
S'_c = (m_{1,e} - m_{1,w}) \frac{c_{p,1} - c_{p,2}}{c_p} \frac{\rho}{\text{ReSc}} \frac{\partial}{\partial R} \frac{\partial S_1}{\partial R} \frac{\partial \theta}{\partial R} + \frac{1}{c_p \text{Pe}} \frac{\partial \theta}{\partial R} \frac{\partial}{\partial R} (\ln c_p)
\]
Appendix D. Demonstration of Grid Independence

The results from any numerical work must be independent of grid size to have significant meaning. Too coarse of a grid could introduce errors in a solution because of its inability to represent large gradients accurately. However, too fine a grid may increase the cost of solution to the point where it is no longer useful. For these reasons, it is important to determine a grid size which is fine enough to provide accurate solutions and coarse enough to keep the cost of solution to a minimum.

In order to demonstrate that the results obtained are independent of grid size, the problem was solved for an air-water vapor mixture on various grid sizes for three different stretching factors. Results representing the fluid flow, heat transfer, and mass transfer solutions (Table 7) are extrapolated to zero grid using the method presented by Churchill, et. al. (1981) and compared with results from the finest grid size used in the extrapolation. A small percent difference between the finest grid and extrapolated results indicates that the finest grid size used is indeed fine enough.

By stretching the grid such that more control volumes are placed where the largest gradients are located, an accurate solution can be obtained with less control volumes than needed with a uniform grid. The algorithm used in this work for generating a nonuniform grid is presented in Anderson, et. al. (1984). It employs a stretching factor, $\beta$, which indicates the amount of stretching applied to the grid. When $\beta = 10,000$ a uniform grid is generated, but as $\beta$ approaches one, more stretching
Table 7. Results for Various Grids and Stretching

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Number of Control Volumes</th>
<th>Average Grid Spacing</th>
<th>$\Delta P \times 10^2$</th>
<th>$\theta_0 \times 10^2$</th>
<th>$\bar{\text{Nu}}$</th>
<th>$S_{\text{f,g}} \times 10^2$</th>
<th>$\bar{\text{Sh}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>100 x 20</td>
<td>2.50E-2</td>
<td>3.5430</td>
<td>8.6030</td>
<td>4.4096</td>
<td>5.8173</td>
<td>4.2535</td>
</tr>
<tr>
<td>10,000</td>
<td>150 x 30</td>
<td>1.67E-2</td>
<td>3.8047</td>
<td>8.5887</td>
<td>4.3834</td>
<td>5.7956</td>
<td>4.2281</td>
</tr>
<tr>
<td>10,000</td>
<td>200 x 40</td>
<td>1.25E-2</td>
<td>3.9358</td>
<td>8.5774</td>
<td>4.3710</td>
<td>5.7813</td>
<td>4.2161</td>
</tr>
<tr>
<td>10,000</td>
<td>300 x 60</td>
<td>8.33E-3</td>
<td>4.0793</td>
<td>8.5609</td>
<td>4.3596</td>
<td>5.7626</td>
<td>4.2050</td>
</tr>
<tr>
<td>1.5</td>
<td>100 x 20</td>
<td>2.50E-2</td>
<td>3.6949</td>
<td>8.6712</td>
<td>4.3947</td>
<td>5.8728</td>
<td>4.2388</td>
</tr>
<tr>
<td>1.5</td>
<td>150 x 30</td>
<td>1.67E-2</td>
<td>3.9161</td>
<td>8.6266</td>
<td>4.3749</td>
<td>5.8260</td>
<td>4.2197</td>
</tr>
<tr>
<td>1.5</td>
<td>200 x 40</td>
<td>1.25E-2</td>
<td>4.0331</td>
<td>8.6010</td>
<td>4.3655</td>
<td>5.8000</td>
<td>4.2107</td>
</tr>
<tr>
<td>1.1</td>
<td>100 x 20</td>
<td>2.50E-2</td>
<td>3.8712</td>
<td>8.7801</td>
<td>4.3829</td>
<td>5.9649</td>
<td>4.2269</td>
</tr>
<tr>
<td>1.1</td>
<td>150 x 30</td>
<td>1.67E-2</td>
<td>4.0774</td>
<td>8.6841</td>
<td>4.3698</td>
<td>5.8747</td>
<td>4.2146</td>
</tr>
<tr>
<td>1.1</td>
<td>200 x 40</td>
<td>1.25E-2</td>
<td>4.1861</td>
<td>8.6369</td>
<td>4.3629</td>
<td>5.8303</td>
<td>4.2082</td>
</tr>
</tbody>
</table>
is applied. Figure 35 shows a typical 50 x 10 grid generated with $\beta = 1.5$. A 200 x 40 grid is used for the numerical work, but does not plot well in the area allowed. About a 200 to 1 aspect ratio is used to plot the grid, therefore the control volumes are short (in R) and wide (in X) rather than long and narrow as they appear in Figure 35.

The results for each different grid stretching are extrapolated using the following from Churchill, et. al. (1981):

$$x_0 = \left[ \frac{h_2}{h_1} \right]^n (x_1 - x_2) \quad (D.1)$$

where:

$$n = \frac{\log \left[ 1 - \left( \frac{x_1 - x_2}{x_1 - x_3} \right) \left[ 1 - \left( \frac{h_3}{h_1} \right)^n \right] \right]}{\log \left( \frac{h_2}{h_1} \right)} \quad (D.2)$$

$n =$ order of the method

$x_0 =$ value of $x$ at zero grid size

$x_1 =$ value of $x$ at finest grid size

$x_2 =$ value of $x$ at intermediate grid size

$x_3 =$ value of $x$ at coarsest grid size

$h_l =$ grid spacing at which $x_l$ was obtained
Figure 35. Illustration of Grid Stretching:
50 x 10 Control Volumes
\( \beta = 1.5 \)
Extrapolations were done for the pressure drop through the tube, dimensionless exit heat transfer results, and dimensionless exit mass transfer results. The percent difference is calculated with:

$$\text{\% Difference} = \left| \frac{x_1 - x_0}{x_c} \right| \times 100\%$$  \hspace{1cm} (D.3)

The pressure drop extrapolation results (Table 8) seem to indicate, when examined alone, that even the finest grid (300 x 60) did not capture changes in pressure very well (9\% difference). However, the numerical procedure uses a local $\Delta P$ to calculate the flow field while the $\Delta P$ presented here is a global pressure change. The much smaller percent differences in the extrapolated heat and mass transfer results indicate that the flow field was calculated more accurately than the extrapolated pressure drop results indicate at first glance.

Surprisingly, the extrapolation procedure is not able to extrapolate the exit bulk temperature for uniform grids. It is believed that this particular set of numbers generates a value of $n$ that is just too small for the extrapolation method to work. Because of this and due to the fact that the other heat and mass transfer results are very similar, the values of $n$ from the exit bulk mass fraction are used to extrapolate the exit bulk temperature to zero grid size (Table 9). For nonuniform grids the results were readily extrapolated and the exit bulk temperature from a 200 x 40 grid only has a 1.6 percent difference when compared to the extrapolated bulk temperature. The average Nusselt number at the tube exit has a percent difference less than one for all grids and stretchings when compared to the extrapolated values.

The extrapolated mass transfer results (Table 10) are very similar to the extrapolated heat transfer results. The extrapolation procedure is able to extrapolate the exit bulk mass fraction for uniform grids, but the order of the method, $n$, is very low (0.2) indicating that the results are not very accurate. For nonuniform grids, the percent difference is around 2\% for the exit bulk mass fraction and about 0.5\% for the exit average Sherwood number. These results are very similar to their heat transfer counterparts and, once again, show that the numerical results for the finest grids are very accurate when compared to the extrapolated results.
Table 8. Pressure Drop Extrapolation Results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Number of Radial CV's Used in Extrapolation</th>
<th>$n$</th>
<th>$\Delta P_{R \rightarrow 0} \times 10^2$</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>20, 30, 40</td>
<td>1.00</td>
<td>4.3308</td>
<td>9.12</td>
</tr>
<tr>
<td>10,000</td>
<td>20, 40, 60</td>
<td>0.84</td>
<td>4.4329</td>
<td>7.98</td>
</tr>
<tr>
<td>10,000</td>
<td>30, 40, 60</td>
<td>0.73</td>
<td>4.4935</td>
<td>9.22</td>
</tr>
<tr>
<td>1.5</td>
<td>20, 30, 40</td>
<td>0.84</td>
<td>4.4623</td>
<td>9.62</td>
</tr>
<tr>
<td>1.1</td>
<td>20, 30, 40</td>
<td>0.85</td>
<td>4.5788</td>
<td>8.58</td>
</tr>
</tbody>
</table>
### Table 9. Heat Transfer Extrapolation Results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Number of Radial CV's Used in Extrapolation</th>
<th>$n$</th>
<th>$\frac{\theta_{R \to 0}}{\times 10^2}$</th>
<th>% Difference</th>
<th>$n$</th>
<th>$\overline{Nu}_{R \to 0}$</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>20, 30, 40</td>
<td>0.20</td>
<td>8.3863</td>
<td>2.23</td>
<td>1.17</td>
<td>4.3400</td>
<td>0.71</td>
</tr>
<tr>
<td>10,000</td>
<td>20, 40, 60</td>
<td>0.22</td>
<td>8.3816</td>
<td>2.14</td>
<td>1.20</td>
<td>4.3413</td>
<td>0.42</td>
</tr>
<tr>
<td>10,000</td>
<td>30, 40, 60</td>
<td>0.23</td>
<td>8.3904</td>
<td>2.03</td>
<td>1.22</td>
<td>4.3417</td>
<td>0.41</td>
</tr>
<tr>
<td>1.5</td>
<td>20, 30, 40</td>
<td>0.61</td>
<td>8.4676</td>
<td>1.58</td>
<td>1.16</td>
<td>4.3418</td>
<td>0.55</td>
</tr>
<tr>
<td>1.1</td>
<td>20, 30, 40</td>
<td>1.04</td>
<td>8.5015</td>
<td>1.59</td>
<td>0.90</td>
<td>4.3399</td>
<td>0.53</td>
</tr>
</tbody>
</table>
Table 10. Mass Transfer Extrapolation Results

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>Number of Radial CV's Used in Extrapolation</th>
<th>$n$</th>
<th>$S_{L,R \to 0} \times 10^2$</th>
<th>$%$ Difference</th>
<th>$\overline{Sh}_{R \to 0}$</th>
<th>$%$ Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>10,000</td>
<td>20, 30, 40</td>
<td>0.20</td>
<td>5.5382</td>
<td>4.39</td>
<td>1.17</td>
<td>4.1861</td>
</tr>
<tr>
<td>10,000</td>
<td>20, 40, 60</td>
<td>0.22</td>
<td>5.5594</td>
<td>3.66</td>
<td>1.21</td>
<td>4.1875</td>
</tr>
<tr>
<td>10,000</td>
<td>30, 40, 60</td>
<td>0.23</td>
<td>5.5694</td>
<td>3.47</td>
<td>1.24</td>
<td>4.1880</td>
</tr>
<tr>
<td>1.5</td>
<td>20, 30, 40</td>
<td>0.69</td>
<td>5.6817</td>
<td>2.08</td>
<td>1.17</td>
<td>4.1881</td>
</tr>
<tr>
<td>1.1</td>
<td>20, 30, 40</td>
<td>1.05</td>
<td>5.7044</td>
<td>2.21</td>
<td>0.88</td>
<td>4.1359</td>
</tr>
</tbody>
</table>
The extrapolation results show that a 300 x 60 uniform grid is required to generate solutions as accurate as with a 200 x 40 non-uniform grid. They also show that changing $\beta$ from 1.5 to 1.1 does not produce significant improvement in the extrapolated results. Based on these results, a stretching factor of 1.5 on a 200 x 40 grid is employed for the balance of the results.
Appendix E. Analysis Examples

The following examples are based on air-vapor mixtures flowing through a tube with a length to diameter ratio of 100. The wall temperature of the tube is 278 K and the mixture inlet temperature is 313 K. A Reynolds number of 1000 is used for the flow.

While bulk temperatures and mass fractions are calculated using the numerical results, it is shown in the Heat and Mass Transfer Results that empirical relationships exist that are accurate and not limited by tube dimensions or Reynolds number. However, determination of condensation rates and prediction of aerosol formation as presented are both of significance.

E.1 Desired Outlet Bulk Temperature

Consider the case involving a saturated water vapor inlet condition and suppose an outlet bulk temperature of 290 K is desired. In order to determine the required tube length the dimensionless bulk temperature is found:
\[ \theta_B = \frac{T_B - T_w}{T_e - T_w} = \frac{290 - 278}{313 - 278} = 0.343 \]

The numerical results (Figure 13 on page 37) show that the tube should be at least 35 times its diameter in length in order to achieve an outlet temperature of 290 K. The local Nusselt number (Figure 19 on page 46) at this location is changing with \( X \) very slowly indicating that heat transfer is nearly fully-developed by this point.

The outlet dimensionless mass fraction (Figure 16 on page 40) for a tube of this length is about 0.3 or:

\[ m_{1,B} = S_{1,B}(m_{1,e} - m_{1,w}) + m_{1,w} = 0.3(4.62 \times 10^{-2} - 5.32 \times 10^{-3}) + 5.32 \times 10^{-3} = 1.76 \times 10^{-2} \]

in dimensional terms (the inlet and wall mass fractions are found in Table 3 on page 17). This indicates that over 60\% of the vapor is removed from the mixture in this length.

The condensation rate under these conditions in a tube of length 35 times its diameter (and assuming a diameter of 0.02 m) is found using equation (15) and information from Table 3 on page 17, Figure 4 on page 24, and Figure 5 on page 26:

\[ \dot{m}_{\text{cond}} = \frac{(U\mu_e)}{2} \rho_1 (\delta D) \pi D = \frac{[(1.25 \times 10^{-3})(0.845 \text{ m/s})]}{2} (1000 \text{ kg/m}^3) [(1.25 \times 10^{-2})(0.02\text{m})] \pi (0.02\text{m}) = 8.3 \times 10^{-6} \frac{\text{kg}}{\text{s}} \]
Multiplication of this by the number of tubes in the condenser will give an estimate of the condensate mass flow rate.

It should be noted that the contour plot of supersaturation for this case (Figure 26 on page 56) shows that the supersaturation is never less than one or greater than three in the region under consideration. This indicates a slight chance of aerosol formation and the need for more information (such as the presence of dust in the flow) before any conclusions are made.

### E.2 Desired Outlet Mass Fraction

Consider the case involving a 50\% of saturation mercury vapor inlet condition and suppose a dimensional outlet bulk mass fraction of $1.20 \times 10^{-5}$ is required. In order to determine the tube length the dimensionless bulk mass fraction is found:

$$S_{1,B} = \frac{m_{1,B} - m_{1,w}}{m_{1,e} - m_{1,w}}$$

$$= \frac{1.20 \times 10^{-5} - 3.00 \times 10^{-6}}{2.96 \times 10^{-5} - 3.00 \times 10^{-6}}$$

$$= 0.338$$

The numerical results (Figure 17 on page 41) show that the tube should be at least 60 times its diameter in order to achieve an outlet vapor mass fraction of $1.20 \times 10^{-5}$. The local Sherwood number (Figure 23 on page 50) at this location is not changing with X indicating that vapor removal is fully-developed at this point. The outlet dimensionless temperature (Figure 14 on page 38) for a tube of this length is about 0.2 or:
\[ T_B = \theta_B(T_e - T_w) + T_w \]
\[ = 0.2(313 - 278) + 278 \]
\[ = 285\text{K} \]

in dimensional terms.

If a tube diameter of 0.01 m is desired, the inlet velocity must be found from the definition of the Reynolds number rather than from Table 4 on page 18. The reference viscosity and density are taken from Table 2 on page 16 and Table 4 on page 18 to calculate the inlet velocity:

\[ u_e = \frac{\text{Re} \mu_e \rho}{\rho \nu D} \]
\[ = \frac{(1000)(1.85 \times 10^{-5} \text{ N s m}^{-2})}{\left(1.13 \text{ kg m}^{-3}\right)(0.01\text{m})} \]
\[ = 1.64 \text{ m s}^{-1} \]

The condensation rate under these conditions in a 0.01 m diameter tube of length 60 times its diameter is found using equation (15) and information from Table 4 on page 18, Figure 7 on page 28, and Figure 8 on page 29:

\[ \dot{m}_{\text{cond}} = \frac{(U \mu_e) \rho_f}{2} \rho_f (\delta D) \pi D \]
\[ = \left[ (0.76 \times 10^{-5})(1.64 \text{ m s}^{-1}) \right] \left(13.95 \text{ kg m}^{-3}\right) \left[ (0.88 \times 10^{-4})(0.01\text{m})\right] \pi(0.01\text{m}) \]
\[ = 2.34 \times 10^{-9} \text{ kg s}^{-1} \]

Multiplication of this by the number of tubes in the condenser will give an estimate of the condenser mass flow rate.
The contour plot (Figure 31 on page 62) shows that while some of the region under consideration has supersaturations less than one, most of the region has supersaturations greater than one. This indicates that the possibility exists for aerosol formation in this case.
Vita

Mack McGhee was born on Columbus Day, 1965, to Sam and Sara McGhee. He grew up in Roanoke, Virginia, where he graduated from Patrick Henry High School in 1984. Mack received his Bachelor's degree in Mechanical Engineering at Virginia Tech in May, 1989. Soon after graduation he began working for Westinghouse at the Savannah River Site. In August, 1990, Mack returned to Virginia Tech and began work on a Master of Science in Mechanical Engineering. Upon completion of his Master's degree, Mack and his new bride are moving to Bryan, Ohio, where he will begin working for Ingersoll-Rand.

[Signature]

Samuel H. McGhee