A SYSTEM IDENTIFICATION TECHNIQUE FOR PREDICTING TRANSIENT OPERATION OF GAS TURBINE ENGINES

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Thesis submitted to the Graduate Faculty of the Virginia Polytechnic Institute and State University in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE in Mechanical Engineering

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November, 1996
Blacksburg, Virginia

Key Words: Gas Turbine, System Identification, Transient
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(ABSTRACT)

A method for developing transient, predictive models of gas turbine engine performance using system identification techniques in conjunction with test cell data has been successfully demonstrated. Test cell data were obtained for both transient and steady-state operation from two F402-RR-406A USMC AV-8B engines at the Naval Aviation Depot (NADEP), Cherry Point, North Carolina.

One engine was run to gather a single set of steady-state data consisting of 24 subsets of five seconds of data. The other engine was run to obtain two sets of transient data, recorded at three different rates of engine acceleration for each set. The acceleration rates of 3, 25, and 100 degrees of throttle per second were preset in the test cell controls. These rates correspond to the angular velocity of the fuel throttle during an acceleration. Each of these six transient test runs consisted of 25 seconds of data. Data were captured at a rate of five Hertz over the engine operating range from idle (26% Low Pressure spool speed) to full military power (105% LP spool speed) for all cases.
A BASIC code developed at the NADEP required significant modification before it could be used to reduce the data. The modified code generated engine operating points consisting of mass flow rate, total pressure ratio, spool speed, and rate of acceleration for the inner fan, outer fan, and high pressure compressor. Finally, a multivariate regression technique using the SAS language was developed in cooperation with the Virginia Tech Statistical Consulting Center. This technique was used to generate a closed-form model of each component capable of predicting operating points at spool speeds and acceleration rates intermediate to those measured.
ACKNOWLEDGEMENTS

I would like to express my sincere thanks to all of the people who have helped bring this project to completion. In particular, I owe special thanks to my advisor, Dr. Walter F. O'Brien, for giving me the opportunity to pursue this degree and the guidance to finish it. Also, I want to thank Drs. Larry D. Mitchell and Wing F. Ng for their invaluable advice. This body of research would not have been possible without the generous support of the personnel at the Naval Aviation Depot Cherry Point, NC. The entire engineering team, lead by Wayne Stanley, made me feel welcome at the NADEP. I want to give special thanks to John Gatt for his patience and unselfish willingness to help me. Also, Paul Hamlin provided me with vital assistance in modifying the computer codes used in my research. I would like to express my appreciation to Rebecca Hanna for her constant help and friendship. Additionally, I owe thanks to the F402 test cell personnel. Ed Powers, the F402 test cell software consultant, was a real asset to me throughout my experiments. The F402 test cell operators, Shawn Thompson and David Schmidt, familiarized me with the operating characteristics of the F402-RR-406A engine. John Morris, Dave Martin, and Nigel Schofield of Rolls Royce Plc. were always able to provide me with any technical support I needed. Dr. Eric Smith and Tim Robinson of the VPI & SU statistical consulting center really helped me pull my experimental data together into a statistically sound model.
# TABLE OF CONTENTS

1. INTRODUCTION .............................................................................................................. 1

2. BACKGROUND AND LITERATURE REVIEW ................................................................. 3
   2.1. Methods of Transient Gas Turbine Engine Simulation .............................................. 3
   2.2. Transient Compressor Performance ....................................................................... 4

3. TEST CELL OVERVIEW AND DATA COLLECTION ...................................................... 9
   3.1. Testing Facilities ..................................................................................................... 9
   3.2. Overview of Data Collection .................................................................................. 15
   3.3. Steady-state Test Runs ......................................................................................... 28
   3.4. Transient Test Runs ............................................................................................. 28

4. DATA REDUCTION METHOD ....................................................................................... 30
   4.1. Data Reduction Overview .................................................................................... 30
   4.2. Steady-state Test Cell Data Reduction .................................................................. 40
   4.3. Transient Test Cell Data Reduction ...................................................................... 48

5. DEVELOPMENT OF SYSTEM IDENTIFICATION MODEL ............................................ 51
   5.1. Description of System Identification Technique and Model .................................... 51
   5.2. Methodology of System Identification Model Calculations .................................... 54
   5.3. Uncertainty of System Identification Model .......................................................... 62

6. DISCUSSION OF RESULTS ......................................................................................... 64
   6.1. Overview of System Identification Model Predictions ............................................. 64
   6.2. F402-RR-406A Outer Fan Model ......................................................................... 70

7. CONCLUSIONS AND RECOMMENDATIONS .............................................................. 102
   7.1. Conclusions ........................................................................................................... 102
7.2. Recommendations

REFERENCES

APPENDIX A: GRAPHICAL PRESENTATION OF SYSTEM IDENTIFICATION MODELING RESULTS IN COMPARISON WITH TEST CELL DATA

APPENDIX A: Part 1 - F402-RR-406A INNER FAN MODEL IN COMPARISON WITH TEST CELL DATA

APPENDIX A: Part 2 - F402-RR-406A HIGH PRESSURE COMPRESSOR MODEL IN COMPARISON WITH TEST CELL DATA

VITA
LIST OF FIGURES

Figure 3.1.1: F402-RR-406A Engine Carrier Assembly in the Engine Preparation Room of Building 4188................................................................. 11
Figure 3.1.2: F402-RR-406A Engine mounted in Carrier Assembly in the Engine Test Enclosure of Building 4188....................................................... 12
Figure 3.1.3: Test Cell Operator's Station in the Control Room of Building 4188 14
Figure 3.2.1: Pegasus F402-RR-406A Engine Station Numbering Identification Diagram......................................................................................... 16
Figure 3.2.2: Four Inlet Temperature Thermocouples (T2) on Bell Mouth F.O.D. Screen - Station 0 ................................................................. 22
Figure 3.2.3: Venturi Static Pressure Tap (VSP-A) on Calibrated Bell Mouth - Station 2 ....................................................................................... 23
Figure 3.2.4: Intermediate Case Bypass Flow Thermocouple (T4) - Station 13 . 24
Figure 3.2.5: Cold Nozzle Static Pressure Probe (PS17) - Station 17 ............. 25
Figure 3.2.6: HP Compressor Discharge Temperature Thermocouple (T7) - Station 3 . ..................................................................................... 26
Figure 3.2.7: Exhaust Duct Static Pressure Probe (P7) - Station 7 ................ 27
Figure 4.1.1: F402-RR-406A HP Compressor Discharge Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3) ....................... 32
Figure 4.1.2: F402-RR-406A HP Compressor Discharge Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3) ................. 33
Figure 4.1.3: F402-RR-406A Cold Nozzle Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)................................. 34
Figure 4.1.4: F402-RR-406A Cold Nozzle Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)................................. 35
Figure 4.1.5: F402-RR-406A Hot Nozzle Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)................................. 36
Figure 4.1.6: F402-RR-406A Hot Nozzle Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)................................. 37
Figure 4.1.7: F402-RR-406A Bell Mouth Venturi Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)................................. 38
Figure 4.1.8: F402-RR-406A Bell Mouth Venturi Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)................................. 39
Figure 6.1.1: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A Outer Fan ......................... 67
Figure 6.1.2: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A Inner Fan ......................... 68
Figure 6.1.3: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A HP Compressor ......................... 69
Figure 6.2.1: F402-RR-406A Outer Fan Operating Line Prediction and Data - Steady-state (Alpha = 0) ................................................................. 71
Figure 6.2.2: F402-RR-406A Outer Fan Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0) ................................................................. 73
Figure A.1.1: F402-RR-406A Inner Fan Operating Line Prediction and Data - Steady-state (Alpha = 0) ........................................................................... 107
Figure A.1.2: F402-RR-406A Inner Fan Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0) ........................................................................... 108
Figure A.1.3: F402-RR-406A Inner Fan Total Pressure Ratio Prediction and Data - Steady-state (Alpha = 0) ........................................................................... 109
Figure A.1.4: F402-RR-406A Inner Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 3) ........................................................................... 110
Figure A.1.5: F402-RR-406A Inner Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 3) ........................................................................... 111
Figure A.1.6: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 3) ........................................................................... 112
Figure A.1.7: F402-RR-406A Inner Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 3) ........................................................................... 113
Figure A.1.8: F402-RR-406A Inner Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 3) ........................................................................... 114
Figure A.1.9: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 3) ........................................................................... 115
Figure A.1.10: F402-RR-406A Inner Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 3) ........................................................................... 116
Figure A.1.11: F402-RR-406A Inner Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 3) ........................................................................... 117
Figure A.1.12: F402-RR-406A Inner Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 3) ........................................................................... 118
Figure A.1.13: F402-RR-406A Inner Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 25) ........................................................................... 119
Figure A.1.14: F402-RR-406A Inner Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 25) ........................................................................... 120
Figure A.1.15: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 25) ........................................................................... 121
Figure A.1.16: F402-RR-406A Inner Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 25) ........................................................................... 122
Figure A.1.17: F402-RR-406A Inner Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 25) ........................................................................... 123
Figure A.1.18: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 25) ........................................................................... 124
Figure A.1.19: F402-RR-406A Inner Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 25) ........................................................................... 125
Figure A.1.20: F402-RR-406A Inner Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 25) ........................................................................... 126
Figure A.1.21: F402-RR-406A Inner Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 25) ........................................................................... 127
Figure A.1.22: F402-RR-406A Inner Fan Operating Line Prediction and Data - Transient Operation (Alpha = 100) ........................................................................... 128
Figure A.1.23: F402-RR-406A Inner Fan Mass Flow Rate Prediction and Data - Transient Operation (Alpha = 100) .................................................. 129
Figure A.1.24: F402-RR-406A Inner Fan Total Pressure Ratio Prediction and Data - Transient Operation (Alpha = 100) .................................................. 130
Figure A.2.1: F402-RR-406A High Pressure Compressor Operating Line Prediction and Data - Steady-state (Alpha = 0) .................................................. 132
Figure A.2.2: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0) .................................................. 133
Figure A.2.3: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction and Data - Steady-state (Alpha = 0) .................................................. 134
Figure A.2.4: F402-RR-406A High Pressure Compressor Operating Line Prediction #1 and Data - Transient Operation (Alpha = 3) .................................................. 135
Figure A.2.5: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 3) .................................................. 136
Figure A.2.6: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 3) .................................................. 137
Figure A.2.7: F402-RR-406A High Pressure Compressor Operating Line Prediction #2 and Data - Transient Operation (Alpha = 3) .................................................. 138
Figure A.2.8: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 3) .................................................. 139
Figure A.2.9: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 3) .................................................. 140
Figure A.2.10: F402-RR-406A High Pressure Compressor Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 3) .................................................. 141
Figure A.2.11: F402-RR-406A High Pressure Compressor Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 3) .................................................. 142
Figure A.2.12: F402-RR-406A High Pressure Compressor Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 3) .................................................. 143
Figure A.2.13: F402-RR-406A High Pressure Compressor Operating Line Prediction #1 and Data - Transient Operation (Alpha = 25) .................................................. 144
Figure A.2.14: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 25) .................................................. 145
Figure A.2.15: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 25) .................................................. 146
Figure A.2.16: F402-RR-406A High Pressure Compressor Operating Line Prediction #2 and Data - Transient Operation (Alpha = 25) .................................................. 147
Figure A.2.17: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 25) .................................................. 148
Figure A.2.18: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 25) .................................................. 149
Figure A.2.19: F402-RR-406A High Pressure Compressor Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 25) .................................................. 150
Figure A.2.20: F402-RR-406A High Pressure Compressor Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 25) .................................................. 151
LIST OF TABLES
Table 3.2.1: Test Cell Data used in Data Reduction Calculations..................19
Table 5.1.1: SAS System Identification Routine Executable Code Filenames..53
Table 5.1.2: SAS System Identification Routine macro Code Filenames........53
Table 5.2.1: F402-RR-406A Outer Fan Corrected Mass Flow Rate Models........56
Table 5.2.2: F402-RR-406A Outer Fan Total Pressure Ratio Models............57
Table 5.2.3: F402-RR-406A Inner Fan Corrected Mass Flow Rate Models........58
Table 5.2.4: F402-RR-406A Inner Fan Total Pressure Ratio Models.............59
Table 5.2.5: F402-RR-406A HP Compressor Corrected Mass Flow Rate Models.................................................................60
Table 5.2.6: F402-RR-406A HP Compressor Total Pressure Ratio Models........61
Table 6.1.1: R-Square Values for Corrected Mass Flow Rate Models...........66
Table 6.1.2: Adjusted R-Square Values for Corrected Mass Flow Rate Models.................................................................66
Table 6.1.3: R-Square Values for Total Pressure Ratio Models.................66
Table 6.1.4: Adjusted R-Square Values for Total Pressure Ratio Models......66
1. INTRODUCTION

Virtual engines or computer models are often generated within the gas turbine industry as a practical and economic means for analyzing and optimizing engine designs. These computer models simulate the complex mechanical, aerodynamic, and thermodynamic interactions within a specific gas turbine engine. In the past, researchers have used analog computers, digital computers, or hybrids of both to develop such models (Fawke and Saravanamuttoo, 1971). These models generally require inputs of specific design information, such as blade geometry and component characteristic maps (Kurzke, 1995). However, the design information is often restricted and unavailable to the manufacturer. It would be useful to have an alternate, simplified model for use in other applications for predicting engine performance. For example, military or commercial refit facilities often only have access to the engine designer's assembly instructions and design point specifications. However, such facilities have access to a wealth of test cell data from the large number of engines serviced by them.

A method is described for developing models from test cell data which should be useful to such facilities for predicting the performance of an engine over a variety of operating conditions. Such a model could be used in pinpointing problems or for diagnosing rebuilt engine problems. Furthermore, a model of this type could be used to expand the amount of engine test data
beyond the data actually measured in the test cell. By predicting engine or component operating points, such a model would provide researchers with an inexpensive means for broadening the number of test cases used to help validate more complex engine simulation models.

In subsequent sections the test cell data is described, the mathematical model is described, and finally the results are discussed. Chapter 2 centers on a review of literature pertinent to the development of transient gas turbine engine models, as well as an overview of engine component behavior during transient operation. The focus of Chapter 3 involves the test facilities used and the experimental procedures followed in the course of the research. Next, Chapter 4 is a discussion of the methods and computer codes involved in the reduction of the experimental test cell data. An explanation of the system identification model, including an explanation of the model uncertainty analysis follows in Chapter 5. The resulting transient, predictive models developed from the combination of the system identification models with the test cell data are discussed in Chapter 6. Finally, conclusions based upon the analysis of the results, and recommendations for future directions of research in this topic are outlined in Chapter 7.
2. BACKGROUND AND LITERATURE REVIEW

2.1. Methods of Transient Gas Turbine Engine Simulation

Theoretical models of the transient behavior of gas turbine engines were first developed during the 1950's. Initially, models were created from sets of equations calculated by hand. As the complexity of analysis required to accurately model the complex interactions between components of gas turbine engines during transient evolutions increased, hand calculations were abandoned in favor of computer models (Fawke and Saravanamutto, 1971). The advent of mainframe digital computers in the late 1950's allowed engineers a much greater ability to analyze transient gas turbine engine operation. The transition from vacuum tube to transistor provided the gas turbine industry with a more reliable computer which needed less maintenance. By the early 1960's, computers were used by the majority of gas turbine engine manufacturers for analysis and design. Increased computing power, combined with decreased costs, further integrated computer modeling into all segments of the gas turbine industry. Modern supercomputers have given engineers enough computing capability to use Computational Fluid Dynamics (CFD) models to more accurately simulate transient engine performance. These CFD models can perform both two-dimensional and three-dimensional working fluid analysis (Platt, 1996). It is reasonable to assume that as computing technology improves,
the gas turbine industry will continue to take advantage of these advances to improve the state-of-the-art of gas turbine engines.

2.2. Transient Compressor Performance

There are three major types of aerodynamic components found in gas turbine engines. These component types are compressors, turbines, and nozzles. The other major gas turbine engine component is the combustor. However, for the purpose of this discussion, combustors are only indirectly involved in the aerodynamic interaction between the other types of components (Cohen et al., 1987).

Compressors and turbines operate on similar principles. A difference in total pressure across the component results in a transfer between the energy of rotational motion and that of total pressure. Turbines operate in a favorable pressure gradient, converting higher upstream total pressure into rotational work by the expansion of the operating fluid. The motivating force behind this process is diffusion, making turbine operation inherently stable. On the other hand, compressors operate in an adverse pressure gradient, forcing the operating fluid from a region of lower total pressure to one of higher total pressure. Rotational work to perform this task is supplied to the compressor from the corresponding turbine at the opposite end of the same spool. This mode of operation is inherently unstable. Nozzles located behind these two types of components merely affect the downstream total pressure, particularly when the flow is
choked. Therefore, most gas turbine component performance analysis is concentrated in the area of compression systems (Curnock, 1993b).

Transient operation of a gas turbine engine is the system response to a change in operating conditions from one steady-state point to another. A steady-state operating point for a gas turbine engine is described as a combination of mass flow rates, total pressure ratios, and corrected speeds for all components which satisfy several criteria (Cohen et al., 1987). First, the mass flow rate into the engine and between components must be conserved. The momentum equation must apply to the energy in the flow (Curnock, 1993b). Assuming no energy losses, the work generated by a turbine equals the work required by the respective compressor on the same spool. Rotational speed must be equivalent for all components located on the same spool. Finally, the various component characteristic maps must be adhered to (Flack, 1986).

The transient change in operating conditions from one steady-state condition to another can be initiated by a variety of means. Examples of several causes of transient operation are: fluctuations in inlet atmospheric conditions, physical obstruction of engine flow paths, altered inlet flow angles due to sudden maneuvers at high speeds, and changes in fuel flow to the engine (Curnock, 1993a).

This discussion will concentrate on the fourth case, fuel flow, and will focus on the example of a single spool gas turbine engine. Engine accelerations are caused by increasing the amount of fuel sent to the combustion system. In
turn, the increased fuel flow causes an almost instantaneous increase in the total temperature at the inlet to the turbine. This temperature rise causes an increase in the energy transferred to the turbine. However, this process happens so quickly that there is no immediate change in the total pressure downstream of the compressor. Therefore, the energy requirement of the corresponding compressor to overcome the adverse pressure gradient across it is unchanged. At this point, there is more turbine energy available than that required by the compressor, and the steady-state energy equation for the two components is no longer satisfied. This excess turbine energy is converted to torque which accelerates the shaft, increasing the rotational speed of the spool (Curnock, 1993c). A re-matching of the engine component operating points follows, in accordance with their respective characteristic maps. This unsteady process continues until the fuel flow rate reaches a new steady-state value. The mechanism for initiating a deceleration is very similar, with the exception that a turbine energy deficit is created by reducing the fuel flow. An excellent and more in-depth discussion of this process, including supporting equations and sample calculations, can be found in Curnock’s lecture “Transient Performance,” or the text *Gas Turbine Theory* by Cohen, et al.

Calculations based on the previous explanation predict interesting results concerning the behavior of the operating line of multi-spool gas turbine engine components during an acceleration. The compressor attached to the spool which encloses the combustion system is designated as the leading compressor.
Compressors on other spools are referred to as lagging compressors. This nomenclature refers to the fact that compressors on different spools are normally only aerodynamically linked, not mechanically linked. Therefore, there is a time lag between the response of the components located on different spools to an increase in the fuel flow.

The behavior of a leading spool compressor during an acceleration is identical to that of a compressor on a single-spool engine. Transient compressor operating lines for accelerations tend to shift above the corresponding steady-state operating line between the same starting and stopping spool speeds. However, the end points of the transient occur at the same position as those on a steady-state characteristic map. This observation makes sense because a transient evolution is a process which occurs between two steady-state end points. The upward shift in the transient operating line can be attributed to an increase in the pressure ratio across the leading compressor as the spool accelerates (Pilidis and MacCallum, 1985).

The lagging compressor behaves much differently during an acceleration, because its performance is indirectly affected by the behavior of the leading compressor. The transient operating line of a lagging compressor shifts below that of the corresponding steady-state operating line on a characteristic map. This downward shift is due to a combination of factors. During the process, the total pressure ratio across the lagging compressor is increased due to the acceleration of the lagging spool. However, the total pressure ratio across the
leading compressor is increasing also. This greater rate of increase can be attributed to the fact that the leading compressor is located closer to the source of the acceleration, the combustor. Furthermore, the downstream total pressure of the leading compressor is the same as the upstream total pressure of the lagging compressor. As the leading compressor increases in rotational speed, it draws in mass flow at a greater rate than the lagging compressor can supply. In turn, the upstream total pressure on the lagging compressor is relieved, or reduced. This reduction is the dominant factor of the two and drops the operating line (Curnock, 1993b).
3. TEST CELL OVERVIEW AND DATA COLLECTION

3.1. Testing Facilities

All experimental work for this thesis was performed at the Naval Aviation Depot (NADEP) located in Cherry Point, North Carolina. The mission of this facility is to provide periodic maintenance and complete overhaul of Marine Corps fixed-wing and rotary-wing aircraft. The majority of NADEP engineering personnel and technicians are civilian employees of the Department of the Navy. The experiments were completed in cooperation with the F402 In-Service Support Team of the Turbojet/Turbofan Engine Division.

The goal of the experimental phase of this thesis was to measure the operating points of several engine components over a broad range of speeds at various rates of acceleration. The F402-RR-406A Pegasus engine was selected as the subject of this experiment for several reasons. First, a large number of F402-RR-406A engines are overhauled and tested at the NADEP. This volume of engines facilitated collection of large amounts of test cell data with relative ease. Secondly, the F402 Test Cell, located in Building 4188, is a new, state-of-the-art facility specifically created to capture both steady-state and transient performance data. Finally, the F402 engineers were able to provide a data reduction code capable of calculating individual engine component operating points from test cell data.
All of the experimental testing of the engines was carried out at the New NADEP Test Cell in Building 4188. This building is a large, two story structure containing an engine preparation bay, a test enclosure, a control room, and several rooms to house testing instrumentation. Figure 3.1.1 illustrates a side view of the Engine Carrier Assembly without an engine mounted in it. This assembly contains connections for the fuel supply and all test instrumentation. There are two of these assemblies in Building 4188, in order to meet time constraints for final engine performance checkout testing. The assemblies are connected to an overhead rail system to facilitate transportation of engines between the preparation bay and the engine test enclosure.

The performance testing of the engines occurs in the engine test enclosure. This enclosure has a set of louvered doors, which straighten the airflow, positioned upstream of the area where the engine is mounted in the carrier. There is a hydraulically operated engine service platform located in the test enclosure beneath the engine and Carrier Assembly. By raising this platform, test cell technicians are able to access all engine components and instrumentation. Figure 3.1.2 shows a view of the test enclosure with a F402-RR-406A Engine in the Carrier Assembly, looking through to the exhaust stack. This photograph was taken upstream of the engine near the louvered doors. The technicians in the lower left portion of the photo provide a sense of scale. Also, the engine service platform can be seen at the bottom of this photograph in the lowered position, which is the normal configuration for test runs.
Figure 3.1.1: F402-RR-406A Engine Carrier Assembly in the Engine Preparation Room of Building 4188.
Figure 3.1.2: F402-RR-406A Engine mounted in Carrier Assembly in the Engine Test Enclosure of Building 4188.
All of the testing procedures and recording of data are orchestrated from the Control Room. This room is adjacent to the Engine Test Enclosure and the Instrumentation Room. The Control Room houses user interfaces with the Automated Data Acquisition and Processing System (ADAPS). Figure 3.1.3 shows a photograph of the Test Cell Operator's Station in the Control Room. The window located beyond the left-most computer monitor looks out into the engine test enclosure, and the engine service platform is visible in the raised position. ADAPS is a suite of controls, sensors, data processors, data storage devices, and output devices configured specifically to analyze the F402-RR-406A engine. A self-monitoring and self-correcting instrument calibration system is integral to ADAPS. The central processing unit of ADAPS is configured from two HP 9000 Series 825 Computers connected to a HP 9263B Removable Winchester RAM Disk Drive. Additional information concerning the hardware and software used in this experiment, including instrumentation specifications, may be obtained from the F402 In-Service Support Team, Turbojet/Turbofan Engine ISE (4.4.8.1), Naval Aviation Depot, Cherry Point, NC.
Figure 3.1.3: Test Cell Operator’s Station in the Control Room of Building 4188.
3.2. **Overview of Data Collection**

Test cell data were obtained for both transient and steady-state operation from two F402-RR-406A engines at the NADEP, Cherry Point, North Carolina. The F402-RR-406A Pegasus is a twin-spool, unmixed bypass flow turbofan engine manufactured by Rolls Royce, Plc. Figure 3.2.1 provides a cross-sectional view of the F402-RR-406A Engine component layout from an overhead perspective. On this diagram, and subsequent engine-layout diagrams, the direction of airflow is from left to right. For ease of reference when discussing various engine components, this diagram includes engine station designation numbering. The station numbering follows standard Rolls Royce nomenclature. Currently, this engine is one of several variants of the powerplant used in the United States Marine Corps AV-8B Harrier II. Of particular note, the fan bypass airflow is unmixed, and exits the engine through two forward variable pitch nozzles. These nozzles are located at Station 17, and are referred to as the cold nozzles. The engine core airflow travels through variable inlet guide vanes (IGV) at Station 21, into the high pressure compressor (HPC) and then enters the burner. After the combustion of the compressed air/fuel mixture, the exhaust gases are expanded in turn through the high pressure turbine (HPT) and the low pressure turbine (LPT) before entering the exhaust duct. From the exhaust duct, the hot exhaust gases exit the engine through a rear pair of variable pitch nozzles, designated the hot nozzles (Station 7). By design, the thrust generated by the cold nozzles is nearly identical to the thrust developed by the hot nozzles.
Figure 3.2.1: Pegasus F402-RR-406A Engine Station Numbering Identification Diagram.
This design specification, combined with the variable pitch of both sets of nozzles, gives the Harrier II a vertical/short takeoff and landing (V/STOL) capability. This ability allows the Harrier II to provide close-air fire support to other Marine Corps units from forward deployed airfields without the need for conventional runways. Currently, the Harrier II is the only fixed-wing aircraft in the United States military arsenal with this unique and vital ability.

Similar procedures were followed for both the steady-state and transient experimental test runs. All test runs for this thesis were conducted immediately following regularly scheduled final performance checkout tests on the engines. The test cell Facility and Engine Control System (FECS) automatically operated the engine and recorded the data during all tests. FECS is a subsystem of ADAPS. Prior to running any tests, ADAPS was programmed to initiate a data recording program when prompted by the soft keys located at the Test Cell Operator's Station (Figure 3.1.3). Ed Powers, the F402 Test Cell Software Consultant, assisted in programming the soft keys to initiate recording data from specific sensors. In order to meet the goals of the experimental portion of this thesis, enough data had to be gathered from various locations in the engine to calculate individual component operating points during the test runs. The specific data which needed to be recorded were found by examining the required inputs of the “Pegasus F402 Engine Efficiency Calculation Program” test cell data reduction code. The original version of this code generates operating points for
all major components in the F402-RR-406A engine from steady-state test cell data. This code requires 21 different readings from sensors located throughout the engine. However, only 13 of these values are actually used by the code to calculate these operating points. These values are listed in Table 3.2.1.

It is important to note that the units for each sensor reading from the above list are consistent with standard NADEP and Rolls Royce test practice for the F402-RR-406A engine. These units do not comply with the regular policies covering the use of scientific units in reports for the Department of Mechanical Engineering at the Virginia Polytechnic Institute and State University. These policies dictate that all units used in both the recording of data and subsequent calculations should be in the SI system alone, or with both SI and US Consistent Units. However, it was decided for this project that it was best to use the mix of US Consistent Units and SI Units established by the NADEP and Rolls Royce for compatibility with the existing Test Cell and Analysis software.

At the time of the experiment, the test cell was equipped with only nine (9) simultaneous data recording channels. This limitation required that data from these 13 different sensors had to be measured in two (2) separate parts, one part for each test trial. For the purpose of this discussion, these two separate data recording runs will be referred to as Part A and Part B. During the first Part (A) of each test trial, values three through ten from Table 3.2.1 were recorded. Values 11 through 13 from Table 3.2.1 were captured during Part B,
<table>
<thead>
<tr>
<th>QUANTITY MEASURED</th>
<th>DESCRIPTION AND LOCATION OF SENSOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. SG</td>
<td>fuel specific gravity</td>
</tr>
<tr>
<td>2. $P_{02}$</td>
<td>atmospheric pressure (in Hg) at Station 0</td>
</tr>
<tr>
<td>3. TOF</td>
<td>atmospheric temperature ($^\circ$F) at Station 0</td>
</tr>
<tr>
<td>4. TCD</td>
<td>pressure drop across test cell inlet doors (in H2O)</td>
</tr>
<tr>
<td>5. $N_F$</td>
<td>LP spool speed (rpm)</td>
</tr>
<tr>
<td>6. $N_H$</td>
<td>HP spool speed (rpm)</td>
</tr>
<tr>
<td>7. FF</td>
<td>fuel flow rate (lbm/hr)</td>
</tr>
<tr>
<td>8. JPTOF</td>
<td>jet pipe temperature ($^\circ$F) at Station 5</td>
</tr>
<tr>
<td>9. $T_{04}$</td>
<td>bypass duct temperature ($^\circ$F) at Station 13</td>
</tr>
<tr>
<td>10. $T_{07}$</td>
<td>HP compressor discharge temperature ($^\circ$F) at Station 3</td>
</tr>
<tr>
<td>11. VSP</td>
<td>average of four Venturi static pressure taps (in. H$_2$O) located on the calibrated bell mouth at Station 2</td>
</tr>
<tr>
<td>12. CLDNOZ</td>
<td>average of the static pressures of the two cold nozzles (psia) at Station 17</td>
</tr>
<tr>
<td>13. CDPRESS</td>
<td>HP compressor discharge static pressure (psig) at Station 3</td>
</tr>
</tbody>
</table>
the second portion of each test trial. Both the LP and HP spool speeds were also recorded during Part B in order to facilitate matching similar points of operation with Part A data.

The use of spool speeds alone is sufficient to match data from Part A with data from Part B. Thrust is the generally accepted parameter used for evaluating performance of gas turbines used in fixed-wing aircraft applications. Performance of individual components in the engine can be directly linked to the overall thrust of the engine. For fixed-wing aircraft applications, thrust is a function of the atmospheric pressure and temperature, properties of the working fluid, flight speed of the engine, fuel/air combustion temperature, and engine design geometry. Equation 3.2.1 illustrates this functional relationship.

\[ T = f(P_a, T_a, R, \gamma, \mu, \text{velocity}, T_{0a}, \text{design}) \]  \hspace{1cm} [3.2.1]

Provided the engine remains the same, the design geometry is constant and may be omitted from this function. Similarly, if the atmospheric testing conditions remain the same, the working fluid properties and inlet pressure and temperature will remain constant. Furthermore, the fuel/air combustion temperature can be shown to be a function of atmospheric conditions and spool speed alone. From these assumptions, it can be shown that the thrust equation can be reduced to Equation 3.2.2, a function of corrected spool speed and Mach number alone. Therefore, it is reasonable to use only spool speed as a matching value between Part A and Part B data, provided the spool speeds are
corrected for inlet temperature. A more in-depth discussion of this functional relationship may be found in Chapter 6 of *Mechanics and Thermodynamics of Propulsion* (Hill et al., 1970).

\[
\frac{T}{\delta} = f\left(M, \frac{N}{\sqrt[3]{\theta}}\right) \quad [3.2.2]
\]

The necessity for taking the data in two sequential test runs compromised the relationship between the two data sets; that is, conditions could not possibly be exactly the same for both tests. However, it is assumed in the following that the assembled data from Part A and Part B are a single set of data. For the tests reported, every effort was made to insure that the two test runs were as similar as possible, and improvements to the test cell instrumentation have been suggested for future work.

Fuel specific gravity was automatically measured and displayed at the Test Cell Operator’s Console at the beginning of each engine test session. Prior to the commencement of each testing session, the atmospheric pressure was obtained from the Weather Center in the Control Tower at the Marine Corps Air Station, Cherry Point, NC. The data recording rate was set at five (5) Hertz for each trial. Data were captured over the entire operating range from idle (approximately 26% Low Pressure spool speed) up to full military power (105% LP spool speed) for all cases.

Figures 3.2.2 through 3.2.7 illustrate several of the locations of the probes on the engine used in the F402 Test Cell to measure this data.
Figure 3.2.2: Four Inlet Temperature Thermocouples (T2) on Bell Mouth F.O.D. Screen - Station 0.
Figure 3.2.3: Venturi Static Pressure Tap (VSP-A) on Calibrated Bell Mouth - Station 2.
Figure 3.2.4: Intermediate Case Bypass Flow Thermocouple (T4) - Station 13.
Figure 3.2.5: Cold Nozzle Static Pressure Probe (PS17) - Station 17.
Figure 3.2.6: HP Compressor Discharge Temperature Thermocouple (T7) - Station 3.
Figure 3.2.7: Exhaust Duct Static Pressure Probe (P7) - Station 7.
3.3. Steady-state Test Runs

One of the two engines was run to gather a single set of steady-state data. This steady-state test run contained 24 sets of data recorded for a period of five (5) seconds each. As soon as the Part A data set was recorded for each speed, the Part B data set was collected. No pause in between Parts A and B was required, because the engine was operating in steady-state with no change in the operating conditions from one data set to the next.

3.4. Transient Test Runs

A second engine was run to obtain two sets of transient data, recorded at three different rates of engine acceleration for each set. Under ideal conditions both the transient and steady-state tests would have been performed on each engine available for testing. However, due to testing schedule restraints during the initial planning phases of this experimental procedure, this ideal situation was not possible. Instead, an engine separate from the one used to obtain steady-state data had to be used to gather the transient data. Steady-state testing occurred on 30 January 1996, and transient testing could not commence until another F402-RR-406A engine was scheduled for the testing on 6 February 1996. This limitation is acknowledged, and should be considered during the following discussion of results. Furthermore, correction of this limitation will be mentioned in the list of recommendations for future work on this project contained in Chapter 7.
The acceleration rates of 3, 25, and 100 degrees of throttle per second were preset in the test cell controls. These rates correspond to the angular velocity of the fuel throttle during the acceleration, and are referred to as alpha (\(\alpha\)) values in later discussion. It is important to understand that these alpha (\(\alpha\)) values are simply a form of notation used by the NADEP to differentiate between different rates of fuel throttle demand controlled by the Test Cell Computers. The numerical values of alpha should be used only for purposes of comparison of the acceleration rates with one another in terms of ordered fuel demand. No other significance is implied or should be given to this convention for describing acceleration rate.

Each of these six transient test runs consisted of 25 seconds of data. In every instance, the engine was returned to idle following Part A data collection. Immediately following the return of all sensors to steady-state conditions at idle, an acceleration of the same rate in Part A was initiated and the corresponding Part B data was captured.
4. DATA REDUCTION METHOD

4.1. Data Reduction Overview

Two different codes written in the BASIC programming language were used to reduce the test cell data, before it could be used in the System Identification models. The "Pegasus F402 Engine Efficiency Calculation Program - Version 2.1 - Steady-state Version (8 April 1996)" was used for calculating thermodynamic properties, air flows, and efficiencies throughout the F402-RR-406A Pegasus Turbofan Engine from test cell data obtained during steady-state operation. This code is a significantly modified version of an older code originally written by Paul Hamlin, Lead F402 Engineer at the NADEP, Cherry Point, North Carolina. The original code was based upon a hand calculation method prescribed by the Rolls Royce Performance Department.

The code allows the user several operational options. The code can perform calculations for a single operating point and display complete output to either the screen or a hard copy print out. In this mode, the user may select the data file upon which to perform calculations, and then view all valid data points in that file listed by corrected Low Pressure spool speed (expressed in percent). Following each calculation, the user is prompted to either continue calculations on another engine operating point, or return to the main engine efficiency calculation operations menu.
The other main mode of operation of this code creates a data file containing the operating points for the Low Pressure Fan and High Pressure Compressor. The other code, “Pegasus F402 Low Pressure Fan & High Pressure Compressor Transient Operating Line Generation Program - Version 1.0.” was written to operate in this mode only. This chapter will focus on the methods of calculation used in this mode for both codes. This output is the basis for the creation of the component operating line models. More specifically, these output files contain the data upon which the System Identification models are based.

Before the data reduction codes could be used, the test cell data from Part A and Part B of each data set had to be combined into a single data set. Values for all parameters were required to be listed at the same spool speed for this combined data set. The two parts were combined by using a cubic spline fit of Part B values referenced to their measured LP spool speeds to generate estimated values at the corresponding LP spool speeds in Part A of the same data set. Figures 4.1.1 through Figure 4.1.8 illustrate the estimated values used in the combined data sets in relation to the values measured in part B of the transient data sets. Graphs have been made only for accelerations with alpha values of to illustrate the validity of this method. Results for other rates of acceleration (α = 25 and 100 degrees of throttle / sec) are similar. The coefficients for the cubic spline fits were calculated using AMTEC Engineering’s Tecplot software, Version 6.0.
Figure 4.1.1: F402-RR-406A HP Compressor Discharge Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)
Figure 4.1.2: F402-RR-406A HP Compressor Discharge Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)
Figure 4.1.3: F402-RR-406A Cold Nozzle Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)
Figure 4.1.4: F402-RR-406A Cold Nozzle Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)
F402-RR-406A Hot Nozzle Static Pressure (Station 7) vs. LP Spool Speed
Transient Operation - Alpha = 3 degrees throttle / sec

Figure 4.1.5: F402-RR-406A Hot Nozzle Static Pressure Test Run #1
Measured and Interpolated Values (Alpha = 3)
Figure 4.1.6: F402-RR-406A Hot Nozzle Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)
Figure 4.1.7: F402-RR-406A Bell Mouth Venturi Static Pressure Test Run #1 Measured and Interpolated Values (Alpha = 3)
Figure 4.1.8: F402-RR-406A Bell Mouth Venturi Static Pressure Test Run #2 Measured and Interpolated Values (Alpha = 3)
4.2. Steady-state Test Cell Data Reduction

The “Pegasus F402 Engine Efficiency Calculation Program - Version 2.1 - Steady-state Version (8 April 1996)” was used to reduce the steady-state test cell data. This code automatically generates an output file containing seven (7) reduced values for each operating point provided in the input file from each test run. These values are listed below.

**REDUCED VALUES CALCULATED FROM MEASURED TEST CELL DATA:**

1. $N_F (\%\text{Corr.})$ percent corrected LP spool speed (100% $N_F = 6500$ rpm)
2. $N_H (\%\text{Corr.})$ percent corrected HP spool speed (100% $N_H = 11000$ rpm)
3. $W_F 0^{\frac{1}{2}} \text{lbm}^{-1}$ corrected mass flow rate through the Fan (lbm/sec)
4. $W_H 0^{\frac{1}{2}} \text{lbm}^{-1}$ corrected mass flow rate through the HP Compressor (lbm/sec)
5. $P_{013}/P_{02}$ total pressure ratio across the outer fan
6. $P_{021}/P_{02}$ total pressure ratio across the inner fan
7. $P_{03}/P_{021}$ total pressure ratio across the HP compressor

In order to calculate these values, the code requires that the input file for each test run contain test cell data for each operating point to be evaluated. The required measurements from the test cell instrumentation are listed below.

**TEST CELL DATA USED IN DATA REDUCTION CALCULATIONS:**

1. SN Engine serial number - maintained for record keeping
2. TD test date - maintained for record keeping
3. SG fuel specific gravity
4. $P_{02}$ atmospheric pressure (in Hg) at Station 0
5. TOF atmospheric temperature ($^\circ$F) at Station 0
6. TCD pressure drop across test cell inlet doors (in H2O)
7. $N_F$ LP spool speed (rpm)
8. $N_H$ HP spool speed (rpm)
9. FF fuel flow rate (lbm/hr)
10. JPTOF jet pipe temperature ($^\circ$F) at Station 5
11. $T_{04}$ bypass duct temperature ($^\circ$F) at Station 13
TEST CELL DATA USED IN DATA REDUCTION CALCULATIONS (Continued):

12. $T_{07}$  
   HP compressor discharge temperature ($^\circ$F) at Station 3

13. VSP  
   average of four Venturi static pressure taps (in. H$_2$O) located on the calibrated bell mouth at Station 2

14. CLDNOZ  
   average of the static pressures of the two cold nozzles (psia) at Station 17

15. CDPRESS  
   HP compressor discharge static pressure (psig) at Station 3

The equations used by the code in this data reduction scheme follow. It is important to note that several of the values are found by referencing tables. Where applicable, the source of the table and the known variable are listed. These tables are stored in data files as individual data pairs. The code employs a linear interpolation method to find the desired unknown variable from these tabular files. The final equations used to calculate each of the seven (7) reduced values is highlighted with a box in the calculation steps listed below.

Calculate $\delta$ and $\theta$, the standard day pressure and temperature correction factors:

$$P_{o2} = [P_{o2} + (TCD \times 0.07355)] \times 0.489467 \text{ (psia)} \quad [4.2.1]$$

$$\delta = \frac{P_{o2}}{14.696} \quad [4.2.2]$$

$$T_{o2} = \frac{5}{9}(TOF - 32) + 273.15 \text{ (K)} \quad [4.2.3]$$
\[
\theta = \frac{T_{02}}{288.15} \tag{4.2.4}
\]

Solve for the percent corrected spool speeds using the temperature correction factor:

\[
N_f(\%\text{Corr.}) = \frac{N_f / \sqrt{\theta}}{6500} \times 100\% \tag{4.2.5}
\]

\[
N_h(\%\text{Corr.}) = \frac{N_h / \sqrt{\theta}}{11000} \times 100\% \tag{4.2.6}
\]

Find \(W_F \theta^{1.25} \delta^{-1}\) (lbm/sec) using graph TED 92717 and TED 92718 “Fan Air Mass Flow Chart for USMC Intake Flare & Stone Guard” for VSP/\(\delta\).

Calculate the observed Fan mass flow rate by factoring out the temperature and pressure correction terms:

\[
W_F = W\text{f}_{\text{corr}} \times \frac{\delta}{\sqrt{\theta}} \text{ (lbm/sec)} \tag{4.2.7}
\]

Calculate the total temperature at Station 5 from the observed JPTOF value:

\[
T_{05} = \left[ \frac{5}{9}(\text{JPTOF} - 32) + 273.15 + 9 \right] \text{ (K)} \tag{4.2.8}
\]

Find EHV (hp sec/lbm) using graph B.218412 “Effective Heating Value of Liquid Fuel” for \(T_{05} \) (K).
Find LCV (CHU/lbm) using graph TED 9500 “Lower Calorific Value Vs. Specific Gravity for Aviation Fuels” for SG.

Calculate the correct EHV of the fuel by accounting for the deviation of the internal energy of the fuel used, with that of Standard Fuel (LCV = 26240 hp sec/lbm):

\[
EHV = EHV + (LCV \times 2.545 - 26240) \text{ (hp sec/lbm)} \quad [4.2.9]
\]

Using the correct EHV value, solve for the heat input (energy) by the combustion of the fuel. Assume 100% combustion efficiency:

\[
H_{IN} = \frac{FF \times EHV \times LCV}{3600 \times 10308} \text{ (hp)} \quad [4.2.10]
\]

Convert the total temperature at Station 5 to Rankine:

\[
T_{05} = \left[\frac{5}{9}(JPTOF - 32) + 273.15 + 9\right] \times 1.8 \text{ (^R)} \quad [4.2.11]
\]

Find \( H_5 \) (Btu/lbm), the enthalpy at Station 5, using Cengel & Boles, *Thermodynamics: An Engineering Approach*, McGraw Hill, Inc. 1994, 2nd Ed., Table A-2E (c) for \( T_{05} \) (^R).

Convert the total temperature at Station 13 to Rankine:

\[
T_{04} = \left[\frac{5}{9}(T_{04} - 32) + 273.15\right] \times 1.8 \text{ (^R)} \quad [4.2.12]
\]

Find \( G_{13} \) (hp sec/lbm), the enthalpy at Station 13, using Gas Tables for Air for \( T_{04} \) (^R).
Convert the total temperature at Station 2 to Rankine:

\[ T_{o2} = \left[ \frac{5}{9}(TOF - 32) + 273.15 \right] \times 1.8 \ (°R) \]  \[ 4.2.13 \]

Find \( G_2 \) (hp sec/lbm), the enthalpy at Station 2, using Gas Tables for Air for \( T_{o2} \) (°R).

From the energy equation with a control volume around the Fan, calculate \( \text{Work}_F \), the work done by the Fan. Neglect energy losses and use the observed Fan mass flow rate, not the corrected value:

\[ \text{Work}_F = W_F (G_{13} - G_2) \ (hp) \]  \[ 4.2.14 \]

\[ W_H = \frac{(H_{IN} - \text{Work}_F)}{(G_5 - G_{13})} \ (lbm/sec) \]  \[ 4.2.15 \]

Correct the HP Compressor mass flow rate for temperature and pressure:

\[ W_{Hcorr} = \frac{W_H \sqrt{\theta}}{\delta} \ (lbm/sec) \]  \[ 4.2.16 \]

Calculate the mass flow rate through the bypass duct from the continuity equation at the flow split:

\[ W_{BYPASS} = W_F - W_H \ (lbm/sec) \]  \[ 4.2.17 \]

Calculate \( M_{13} \) using the flow parameter/Mach number relationship. Mach number is solved for by iteration, until the difference in the flow parameter is less than 0.000001:
Specific heat ratio: \( \gamma = 1.395 \)
Flow area at Station 13: \( A_{13} = 432 \) (in.\(^2\))
Specific heat: \( C_p = 0.2421 \) (Btu/lbm \(^0\)R)
Universal Gas Constant: \( R = 96.031 \)
\( g_c = 32.174 \)

\[
\frac{W_{\text{BYPASS}} \sqrt{T_{04}}}{(\text{CLDNOZP} \times A_{13})} = M_{13} \sqrt{\frac{\gamma g_c}{R}} \sqrt{1 + \frac{(\gamma - 1)}{2} M_{13}^2}
\]

[4.2.18]
Calculate the total to static pressure ratio at Station 13, using the value for $M_{13}$:

$$\frac{P_{013}}{P_{S13}} = \left(1 + \frac{(\gamma - 1) \times M_{13}^2}{2}\right)^{\frac{\gamma}{\gamma - 1}}$$  \[4.2.19\]

$$\frac{P_{013}}{P_{02}} = \frac{P_{013} \times CLDNOZP}{P_{S13} \times P_{02}}$$  \[4.2.20\]

Find $P_{02}/P_{02}$ using TED 79215 “Pegasus II Engine 7201 Build 20, $P_{021}/P_{02}$ Vs. $P_{013}/P_{02}$” from $P_{013}/P_{02}$.

$$P_{021} = \frac{P_{021}}{P_{02}} \times P_{02}$$  \[4.2.21\]

$$T_{07} = \frac{5}{9}(T_{07} - 32) + 273.15 + 8$$  \[4.2.22\]

$$SV = P_{02} \times 0.489467$$  \[4.2.23\]

$$P_{S3} = 78 \times \frac{(CDPRESS + SV - 110)}{80} + 102.7$$

Calculate $M_3$ using the flow parameter/Mach number relationship. Mach number is solved for by iteration, until the difference in the flow parameter is less than 0.000001:

Specific heat ratio:

$$\gamma = 1.395$$
Flow area at Station 3: \( A_3 = 106 \text{ (in.}^2) \)

Specific heat: \( C_p = 0.2421 \text{ (Btu/lbm °R)} \)

Universal Gas Constant: \( R = 96.031 \)

\( g_c = 32.174 \)

\[
\frac{W_H \sqrt{T_{07}}}{(P_{s3} * A_3)} = M_3 \sqrt{\frac{\gamma \cdot g_c}{R}} \sqrt{1 + \frac{(\gamma - 1) \cdot M_3^2}{2}} \tag{4.2.24}
\]

Then, the following equation is solved to find the total to static pressure ratio at Station 3:

\[
\frac{P_{s3}}{P_{s3}} = \left(1 + \frac{(\gamma - 1) \cdot M_3^2}{2}\right)^{\frac{\gamma}{\gamma - 1}} \tag{4.2.25}
\]

From this pressure ratio and the values of \( P_{021} \) and \( P_{s3} \) calculated previously, \( P_{03}/P_{021} \), the total pressure ratio across the HP Compressor, can be found:

\[
\begin{array}{c}
P_{03} = \frac{P_{03}}{P_{021}} \times \frac{P_{s3}}{P_{021}} \\
P_{021} = P_{s3} \quad P_{021}
\end{array} \tag{4.2.26}
\]
4.3. **Transient Test Cell Data Reduction**

The transient test cell data was reduced using the "Pegasus F402 Low Pressure Fan & High Pressure Compressor Transient Operating Line Generation Program - Version 1.0." This BASIC code produces output in the same format as the steady-state data reduction code. Additionally, this code uses a similar input data file format.

The calculation to find the mass flow rate into the HP Compressor in the steady-state code requires solution of the steady-state energy equation for the entire engine. Obviously, this assumption will not yield correct results for transient test cell data. Instead, an iterative routine was developed to solve for the HP Compressor mass flow rate in the transient reduction code, which uses the steady-state solution as an initial guess. All other reduced variables are calculated exactly the same way as in the steady-state code. The iterative solution method for the HP Compressor mass flow rate follows.

The flow bypass ratio is calculated from a third-order polynomial fit of steady-state bypass ratio values, given a corrected $N_H$ (rpm). Tecplot Version 6.0 was used to create this polynomial fit of the bypass ratio to the NH spool speed:

$$\text{BPRATIO} = \frac{W_{\text{BYPASS}}}{W_H} \quad [4.3.1]$$
This bypass ratio is used to calculate an initial estimate of the mass flow rate through the HP Compressor:

\[ W_H = \frac{W_F}{(\text{BPRATIO} + 1)} \]  \hspace{1cm} [4.3.2]

Calculate mass flow rate through the bypass duct:

\[ W_{\text{BYPASS}} = W_F - W_H \quad (\text{lbm/sec}) \]  \hspace{1cm} [4.3.3]

Calculate \( M_{13} \) using the flow parameter/Mach number relationship. Mach number is solved for by iteration, until the difference in the flow parameter is less than 0.000001:

Specific heat ratio: \( \gamma = 1.395 \)
Flow area at Station 13: \( A_{13} = 432 \quad (\text{in.}^2) \)
Specific heat: \( C_p = 0.2421 \quad (\text{Btu/lbm} \quad ^\circ \text{R}) \)
Universal Gas Constant: \( R = 96.031 \)
\( g_c = 32.174 \)

\[ \frac{W_{\text{BYPASS}} \sqrt{T_{04}}}{(\text{CLDNOZP} \times A_{13})} = M_{13} \sqrt{\frac{\gamma}{R}} \sqrt{1 + \frac{\gamma - 1}{2} \frac{M_{13}^2}{}} \]  \hspace{1cm} [4.3.4]

Then, the following equation is solved to find the total to static pressure ratio at Station 13:

\[ \frac{P_{013}}{P_{s13}} = \left(1 + \frac{\gamma - 1}{2} \frac{M_{13}^2}{\gamma - 1}\right)^\frac{1}{\gamma - 1} \]  \hspace{1cm} [4.3.5]
Find the total to static temperature ratio at Station 13 using \( \frac{P_{013}}{P_{S13}} \) and the isentropic law:

\[
\frac{T_{013}}{T_{S13}} = \frac{P_{013}}{P_{S13}} \left( \frac{\gamma - 1}{\gamma} \right) \quad [4.3.6]
\]

Solve for the density at Station 13 using the ideal gas law:

\[
\rho_{13} = \frac{\text{CLDNOZP} \times 144 \times \frac{T_{013}}{T_{S13}}}{T_{04} \times 1.8 \times 53.34} \quad [4.3.7]
\]

Calculate the velocity at Station 13:

\[
\text{VEL}_{13} = \sqrt{2 \times 53.34 \times 1.8 \times 32.174 \times \left( \frac{\gamma}{\gamma - 1} \right) \times T_{04} \times \left( 1 - \frac{T_{S13}}{T_{013}} \right)} \quad [4.3.8]
\]

Then, using the above values, calculate the bypass duct mass flow rate:

\[
W_{\text{BYPASS}} = \frac{\rho_{13} \times \text{VEL}_{13} \times 432}{144} \quad [4.3.9]
\]

An iteration is performed by comparing the \( W_{\text{BYPASS}} \) calculated in Equation 4.3.9 with the \( W_{\text{BYPASS}} \) value found in Equation 4.3.4. The \( W_{\text{BYPASS}} \) value found in Equation 4.3.9 is substituted back into Equation 4.3.4, and the calculations are carried out to solve for a new \( W_{\text{BYPASS}} \) value using Equation 4.3.9. This iterative process is continued until the percent difference between the two is less than 0.0001.
5. DEVELOPMENT OF SYSTEM IDENTIFICATION MODEL

5.1. Description of System Identification Technique and Model

The overall goal of this thesis research project was to develop a method for creating transient, predictive models of gas turbine engine performance using system identification techniques in conjunction with test cell data. System Identification is a process whereby a model is improved through the application of a transfer function derived from experimental data (Juang, 1994). For this particular project, the initial performance model is the steady-state operating line. A correlation procedure is then applied to the transient test cell data and this initial performance model to create a group of closed form mathematical models capable of predicting operating points intermediate to those measured.

These predictive System Identification models were developed in cooperation with the Virginia Tech Statistical Consulting Center. The models were written in the SAS programming language, which includes intrinsic functions for performing complex statistical analysis of data. The actual codes, which generate the models, are composed of two parts: an executable program and a macro subroutine.

One set of codes was developed to generate models of mass flow rate and total pressure ratio for both the outer fan and inner fan components. Another set was created to model the high pressure compressor mass flow rate, and total pressure ratio. Table 5.1.1 lists the names of the system identification
executable codes used for the various components and dependent variables. Table 5.1.2 provides the filenames of the SAS macro subroutines called by the executable codes.
Table 5.1.1: SAS System Identification Routine Executable Code Filenames

<table>
<thead>
<tr>
<th>Component</th>
<th>Predicted Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass Flow Rate</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>F402FAN.SAS</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>F402FAN.SAS</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>F402HPC.SAS</td>
</tr>
</tbody>
</table>

Table 5.1.2: SAS System Identification Routine Macro Code Filenames

<table>
<thead>
<tr>
<th>Component</th>
<th>Predicted Dependent Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mass Flow Rate</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>FANMACRO.SAS</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>FANMACRO.SAS</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>HPCMACRO.SAS</td>
</tr>
</tbody>
</table>
5.2. Methodology of System Identification Model Calculations

The SAS executable codes call the macro codes as subroutines. Four different SAS intrinsic functions are used in these two codes to generate the System Identification models. Initially, the executable code defines the location of the reduced performance data file and the associated macro code. Also, the dependent variable and the two independent variables in the performance data file are designated at this step. Additionally, the terms to be used in the closed form System Identification equation are defined.

Next, the executable code invokes the macro code, which uses the intrinsic function “PROC IML.” This function creates an array of the data and model terms to be used in the regression step. Following this step, the executable code applies “PROC REG” to the defined model terms and data. This function, “PROC REG,” performs a multivariate regression on the reduced data, using a least-squares method to fit the defined model terms. At this point, the desired values to be output are defined. For this research project, these terms were alpha (ω), the appropriate spool speed, the observed dependent variable, the predicted dependent value, and the 95 percent confidence limits. The next step uses “PROC SORT” to sort these outputted values first by alpha (ω) and then by spool speed. Lastly, “PROC PRINT” automatically generates a tabular file containing the output values and the “Analysis of Variance table.”
Table 5.2.1 through Table 5.2.6 define the form of the closed form equation of the models as well as the terms generated by the regression analysis. These tables are divided by component and dependent variable. The dependent variables used were corrected mass flow rate and total pressure ratio of the corresponding component. The independent variables used in all cases were alpha (\(\alpha\)) and the appropriate spool speed. In order to generate a component performance operating line, both dependent variables were cross referenced according to the two independent variables. This process yielded plots of pressure ratio versus mass flow rate at various rates of acceleration for each component. The reduced performance data sets were broken up into six (6) sets containing values for only two adjacent rates of acceleration. This step gave better results for individual operating lines at single rates of acceleration while retaining the ability to predict performance at intermediate alpha (\(\alpha\)) values. Because of this division of the data, two separate models were created to define operating lines at alpha (\(\alpha\)) values of 3 and 25 degrees throttle/sec. These models are referred to as Prediction Model #1 and Prediction Model #2 for the appropriate alpha (\(\alpha\)).
Table 5.2.1: F402-RR-406A Outer Fan Corrected Mass Flow Rate Models

\[ W_{\theta}^{1/2}\delta^{-1} = \beta_{0n} + \beta_{1n} \alpha + \beta_{2n} N_L + \beta_{3n} N_L^2 + \beta_{4n} N_L^3 + \beta_{5n} N_L^4 + \beta_{6n} N_L^5 + \beta_{7n} N_L^6 + \beta_{8n} (\alpha^p N_L) \]

- \( \alpha \): Corrected Mass Flow Rate through the Fan (lbm / sec)
- \( \beta \): Model Coefficients
- \( n \): Set of Coefficients used (in relation to ALPHA range)
- \( \alpha \): ALPHA - Rate of Acceleration (degrees of throttle / sec)
- \( N_L \): Corrected Percent Spool Speed (6500 RPM = 100% \( N_L \))

**MODEL COEFFICIENTS ( 0 <= \( \alpha <= 3 \) )**

<table>
<thead>
<tr>
<th>( \beta_{01} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{31} )</th>
<th>( \beta_{41} )</th>
<th>( \beta_{51} )</th>
<th>( \beta_{61} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{81} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1654.65</td>
<td>11.87096</td>
<td>177.6212</td>
<td>-7.38879</td>
<td>0.162277</td>
<td>-0.00194</td>
<td>1.19E-05</td>
<td>-3E-08</td>
<td>-0.08364</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 3 < \( \alpha < 25 \) )**

<table>
<thead>
<tr>
<th>( \beta_{02} )</th>
<th>( \beta_{12} )</th>
<th>( \beta_{22} )</th>
<th>( \beta_{32} )</th>
<th>( \beta_{42} )</th>
<th>( \beta_{52} )</th>
<th>( \beta_{62} )</th>
<th>( \beta_{72} )</th>
<th>( \beta_{82} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2204.15</td>
<td>0.581786</td>
<td>244.0392</td>
<td>-16.4633</td>
<td>0.23435</td>
<td>-0.00284</td>
<td>1.77E-05</td>
<td>-4.4E-08</td>
<td>0.003415</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 25 <= \( \alpha <= 100 \) )**

<table>
<thead>
<tr>
<th>( \beta_{03} )</th>
<th>( \beta_{13} )</th>
<th>( \beta_{23} )</th>
<th>( \beta_{33} )</th>
<th>( \beta_{43} )</th>
<th>( \beta_{53} )</th>
<th>( \beta_{63} )</th>
<th>( \beta_{73} )</th>
<th>( \beta_{83} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1432.96</td>
<td>-0.0271</td>
<td>155.4704</td>
<td>-6.35021</td>
<td>0.136357</td>
<td>-0.00157</td>
<td>9.15E-06</td>
<td>-2.1E-08</td>
<td>0.000104</td>
</tr>
</tbody>
</table>
### Table 5.2.2: F402-RR-406A Outer Fan Total Pressure Ratio Models

\[ \frac{P_{013}}{P_{02}} = \beta_{0n} + \beta_{1n} \cdot \alpha + \beta_{2n} \cdot N_L \]

- **\( P_{013}/P_{02} \)**: Total Pressure Ratio across the Outer Fan
- **\( \beta \)**: Model Coefficients
- **\( n \)**: Set of Coefficients used (in relation to ALPHA range)
- **\( \alpha \)**: ALPHA - Rate of Acceleration (degrees of throttle / sec)
- **\( N_L \)**: Corrected Percent Spool Speed (6500 RPM = 100% \( N_L \))

#### MODEL COEFFICIENTS ( \( 0 \leq \alpha \leq 3 \) )

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{31} )</th>
<th>( \beta_{41} )</th>
<th>( \beta_{51} )</th>
<th>( \beta_{61} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{81} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{01} )</td>
<td>-4.17897</td>
<td>0.02198</td>
<td>0.574157</td>
<td>-0.02536</td>
<td>0.000576</td>
<td>7E-06</td>
<td>4.43E-08</td>
<td>-1.1E-10</td>
</tr>
</tbody>
</table>

#### MODEL COEFFICIENTS ( \( 3 < \alpha < 25 \) )

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \beta_{12} )</th>
<th>( \beta_{22} )</th>
<th>( \beta_{32} )</th>
<th>( \beta_{42} )</th>
<th>( \beta_{52} )</th>
<th>( \beta_{62} )</th>
<th>( \beta_{72} )</th>
<th>( \beta_{82} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{02} )</td>
<td>-6.07391</td>
<td>0.001804</td>
<td>0.80168</td>
<td>-0.03597</td>
<td>0.000824</td>
<td>1E-05</td>
<td>6.35E-08</td>
<td>-1.6E-10</td>
</tr>
</tbody>
</table>

#### MODEL COEFFICIENTS ( \( 25 \leq \alpha \leq 100 \) )

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \beta_{13} )</th>
<th>( \beta_{23} )</th>
<th>( \beta_{33} )</th>
<th>( \beta_{43} )</th>
<th>( \beta_{53} )</th>
<th>( \beta_{63} )</th>
<th>( \beta_{73} )</th>
<th>( \beta_{83} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{03} )</td>
<td>-2.03352</td>
<td>-7.2E-05</td>
<td>0.321521</td>
<td>0.01295</td>
<td>0.000258</td>
<td>2.6E-06</td>
<td>1.19E-08</td>
<td>-2.6E-11</td>
</tr>
</tbody>
</table>

57
Table 5.2.3: F402-RR-406A Inner Fan Corrected Mass Flow Rate Models

\[ W_{\theta}^{1/2} = \beta_0 + \beta_1 \alpha + \beta_2 n + \beta_3 N_L^2 + \beta_4 n^2 + \beta_5 N_L^3 + \beta_6 n^3 + \beta_7 N_L^4 + \beta_8 n^4 (\alpha^* N_L) \]  

- \( W_{\theta}^{1/2} \): Corrected Mass Flow Rate through the Fan (lbm / sec)  
- \( \beta \): Model Coefficients  
- \( n \): Set of Coefficients used (in relation to ALPHA range)  
- \( \alpha \): ALPHA - Rate of Acceleration (degrees of throttle / sec)  
- \( N_L \): Corrected Percent Spool Speed (6500 RPM = 100% \( N_L \) )

**MODEL COEFFICIENTS ( 0 \( \leq \alpha \leq 3 \))**

<table>
<thead>
<tr>
<th>( \beta_{01} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{31} )</th>
<th>( \beta_{41} )</th>
<th>( \beta_{51} )</th>
<th>( \beta_{61} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{81} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1654.65</td>
<td>11.87096</td>
<td>177.6212</td>
<td>-7.38879</td>
<td>0.162277</td>
<td>-0.00194</td>
<td>1.19E-05</td>
<td>-3E-08</td>
<td>-0.08364</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 3 < \( \alpha < 25 \))**

<table>
<thead>
<tr>
<th>( \beta_{02} )</th>
<th>( \beta_{12} )</th>
<th>( \beta_{22} )</th>
<th>( \beta_{32} )</th>
<th>( \beta_{42} )</th>
<th>( \beta_{52} )</th>
<th>( \beta_{62} )</th>
<th>( \beta_{72} )</th>
<th>( \beta_{82} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2204.15</td>
<td>0.581786</td>
<td>244.039</td>
<td>-10.4633</td>
<td>0.23435</td>
<td>-0.00284</td>
<td>1.77E-05</td>
<td>-4.4E-08</td>
<td>0.003415</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 25 \( \leq \alpha \leq 100 \))**

<table>
<thead>
<tr>
<th>( \beta_{03} )</th>
<th>( \beta_{13} )</th>
<th>( \beta_{23} )</th>
<th>( \beta_{33} )</th>
<th>( \beta_{43} )</th>
<th>( \beta_{53} )</th>
<th>( \beta_{63} )</th>
<th>( \beta_{73} )</th>
<th>( \beta_{83} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1514.39</td>
<td>-0.01001</td>
<td>164.5638</td>
<td>-6.75868</td>
<td>0.145724</td>
<td>-0.00168</td>
<td>9.88E-06</td>
<td>-2.3E-08</td>
<td>3.33E-05</td>
</tr>
</tbody>
</table>
Table 5.2.4: F402-RR-406A Inner Fan Total Pressure Ratio Models

\[ P_{021}/P_{02} = \beta_{0n} + \beta_{21n} \alpha + \beta_{2n} N_L \]

- \( P_{021} / P_{02} \): Total Pressure Ratio across the Inner Fan
- \( \beta \): Model Coefficients
- \( n \): Set of Coefficients used (in relation to ALPHA range)
- \( \alpha \): ALPHA - Rate of Acceleration (degrees of throttle / sec)
- \( N_L \): Corrected Percent Spool Speed (6500 RPM = 100% \( N_L \))

**MODEL COEFFICIENTS ( 0 <= \( \alpha <= 3 \) )**

<table>
<thead>
<tr>
<th>( \beta_{01} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{31} )</th>
<th>( \beta_{41} )</th>
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<th>( \beta_{61} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{81} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5.49488</td>
<td>0.025091</td>
<td>0.711492</td>
<td>-0.03116</td>
<td>0.000703</td>
<td>-8.5E-06</td>
<td>5.29E-08</td>
<td>-1.3E-10</td>
<td>-0.00011</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 3 < \( \alpha < 25 \) )**

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<tr>
<th>( \beta_{02} )</th>
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<th>( \beta_{42} )</th>
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<th>( \beta_{62} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{82} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7.34703</td>
<td>0.002128</td>
<td>0.936024</td>
<td>-0.04174</td>
<td>0.000954</td>
<td>-1.2E-05</td>
<td>7.3E-08</td>
<td>-1.8E-10</td>
<td>8.13E-06</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 25 <= \( \alpha <= 100 \) )**

<table>
<thead>
<tr>
<th>( \beta_{03} )</th>
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<th>( \beta_{23} )</th>
<th>( \beta_{33} )</th>
<th>( \beta_{43} )</th>
<th>( \beta_{53} )</th>
<th>( \beta_{63} )</th>
<th>( \beta_{71} )</th>
<th>( \beta_{83} )</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.419615</td>
<td>-0.01749</td>
<td>0.000371</td>
<td>-4.1E-06</td>
<td>2.21E-08</td>
<td>-4.7E-11</td>
<td>-3.7E-07</td>
</tr>
</tbody>
</table>
Table 5.2.5: F402-RR-405A HP Compressor Corrected Mass Flow Rate Models

\[ W_\theta^{1/2} \delta^{-1} = \beta_0 n^\alpha + \beta_1 n^\alpha + \beta_2 n^\gamma + \beta_3 n^\gamma + \beta_4 n^\gamma + \beta_5 n^\gamma + \beta_6 n^\gamma + \beta_7 n^\gamma (\alpha^\gamma N_H) \]

**W_\theta^{1/2} \delta^{-1}**: Corrected Mass Flow Rate through the HPC (lbm/sec)

**\beta**: Model Coefficients

**n**: Set of Coefficients used (in relation to ALPHA range)

**\alpha**: ALPHA - Rate of Acceleration (degrees of throttle / sec)

**N_H**: Corrected Percent Spool Speed (11000 RPM = 100% N_H)

### MODEL COEFFICIENTS (0 <= \alpha <= 3)

<table>
<thead>
<tr>
<th>\beta_{01}</th>
<th>\beta_{11}</th>
<th>\beta_{21}</th>
<th>\beta_{31}</th>
<th>\beta_{41}</th>
<th>\beta_{51}</th>
<th>\beta_{61}</th>
<th>\beta_{71}</th>
</tr>
</thead>
<tbody>
<tr>
<td>7583.275</td>
<td>4.002102</td>
<td>-567.271</td>
<td>16.70854</td>
<td>-0.24162</td>
<td>0.00172</td>
<td>-4.8E-06</td>
<td>-0.03137</td>
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</table>

### MODEL COEFFICIENTS (3 <= \alpha <= 25)

<table>
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<tr>
<th>\beta_{02}</th>
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<th>\beta_{42}</th>
<th>\beta_{52}</th>
<th>\beta_{62}</th>
<th>\beta_{72}</th>
</tr>
</thead>
<tbody>
<tr>
<td>5800.321</td>
<td>0.060632</td>
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<td>-0.18782</td>
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<td>0.001553</td>
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</tbody>
</table>

### MODEL COEFFICIENTS (25 <= \alpha <= 100)

<table>
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<tr>
<th>\beta_{03}</th>
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<th>\beta_{43}</th>
<th>\beta_{53}</th>
<th>\beta_{63}</th>
<th>\beta_{73}</th>
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<tr>
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<td>0.028061</td>
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<td>-0.00578</td>
<td>8.35E-05</td>
<td>-3.6E-07</td>
<td>-1.5E-05</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.2.6: F402-RR-406A HP Compressor Total Pressure Ratio Models

\[ P_{02}/P_{01} = \beta_0 + \beta_1 \alpha + \beta_2 N_H + \beta_3 N_H^2 + \beta_4 N_H^3 + \beta_5 N_H^4 + \beta_6 N_H^5 + \beta_7 (\alpha^2 N_H) \]

- **\( P_{02}/P_{01} \)**: Total Pressure Ratio across the HPC
- **\( \beta \)**: Model Coefficients
- **\( n \)**: Set of Coefficients used (in relation to ALPHA range)
- **\( \alpha \)**: ALPHA - Rate of Acceleration (degrees of throttle / sec)
- **\( N_H \)**: Corrected Percent Spool Speed (11000 RPM = 100% \( N_H \))

**MODEL COEFFICIENTS ( 0 <= \( \alpha <= 3 \) )**

<table>
<thead>
<tr>
<th>( \beta_{01} )</th>
<th>( \beta_{11} )</th>
<th>( \beta_{21} )</th>
<th>( \beta_{31} )</th>
<th>( \beta_{41} )</th>
<th>( \beta_{51} )</th>
<th>( \beta_{61} )</th>
<th>( \beta_{71} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-101.713</td>
<td>0.228977</td>
<td>0.450326</td>
<td>0.15965</td>
<td>0.001952</td>
<td>1.2E-05</td>
<td>2.68E-08</td>
<td>-0.0016</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 3 < \( \alpha < 25 \) )**

<table>
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<tr>
<th>( \beta_{02} )</th>
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<th>( \beta_{32} )</th>
<th>( \beta_{42} )</th>
<th>( \beta_{52} )</th>
<th>( \beta_{62} )</th>
<th>( \beta_{72} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-164.727</td>
<td>-0.00822</td>
<td>1.083209</td>
<td>-0.27765</td>
<td>0.003497</td>
<td>2.1E-05</td>
<td>5.11E-08</td>
<td>0.000209</td>
</tr>
</tbody>
</table>

**MODEL COEFFICIENTS ( 25 <= \( \alpha <= 100 \) )**

<table>
<thead>
<tr>
<th>( \beta_{03} )</th>
<th>( \beta_{13} )</th>
<th>( \beta_{23} )</th>
<th>( \beta_{33} )</th>
<th>( \beta_{43} )</th>
<th>( \beta_{53} )</th>
<th>( \beta_{63} )</th>
<th>( \beta_{73} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-159.171</td>
<td>0.001616</td>
<td>10.5132</td>
<td>-0.27008</td>
<td>0.003394</td>
<td>2.1E-05</td>
<td>4.78E-08</td>
<td>-1.6E-05</td>
</tr>
</tbody>
</table>

61
5.3. **Uncertainty of System Identification Model**

The SAS codes output several statistical values in addition to the System Identification model coefficients. These statistical values are listed in the "Analysis of Variance table," which is automatically generated by the SAS executable codes. This table contains statistical values which provide the user with information regarding the fit of the model to the data and the importance of individual terms in the model.

Included in the "Analysis of Variance table" is the "Summary of Fit table" subsection. This portion of the table lists statistical values which reflect the amount of variation in the overall fit of the model. Of particular importance are the R-square and the Adjusted R-square values. The R-square value shows the amount of deviation present in the dependent variable which the model can accurately fit. In other words, this value gives the user a means by which to compare how good the model is in relation to other models. The Adjusted R-square value reflects the same deviation, but also accounts for the number of terms in the model as well as the number of observations. For this reason, the Adjusted R-square term is the more useful of the two values for comparing the validity of models created from differently sized observation sets and degrees of freedom. These terms are defined in the equations below.
The R-Square value automatically calculated by the SAS codes is defined as:

\[ R^2 = 1 - \left( \frac{\text{SSE}}{\text{CSS}} \right) \quad [5.3.1] \]

The parameters CSS and SSE are defined in the equations below:

\[ \text{CSS} = \text{corrected sum of squares} \]
\[ \text{CSS} = \sum_{i=1}^{n} \left[ (y_i - \text{mean}(y_i))^2 \right] \quad [5.3.2] \]

\[ \text{SSE} = \text{sum of squares for error} \]
\[ \text{SSE} = \sum_{i=1}^{n} \left[ (y_i - \hat{y}_i)^2 \right] \quad [5.3.3] \]

\( y_i = \) observed dependent variable values
\( \hat{y}_i = \) predicted dependent variable values

\[ \text{Adj.R}^2 = \text{Adjusted R - square value} \quad [5.3.4] \]

\[ \text{Adj.R}^2 = 1 - \left[ \left( \frac{\text{SSE}}{n-p} \right) \right] \quad [5.3.5] \]

\[ \left( \frac{\text{CSS}}{n-1} \right) \]

\( n = \) number of observations
\( p = \) number of parameters in model

This equation reduces to the following in terms of R-square:

\[ \text{Adj.R}^2 = 1 - \left[ \left( \frac{n-1}{n-p} \right)(1-R^2) \right] \quad [5.3.6] \]
6. DISCUSSION OF RESULTS

6.1. Overview of System Identification Model Predictions

Overall, the predictive System Identification models do a good job of following the general trends of the operating lines at the various rates of acceleration. Furthermore, both the reduced test cell data and the models agree well with other examples of transient behavior in gas turbine engine components. For instance, the transient operating line exhibited greater departures from the steady-state operating line for rates of acceleration of greater magnitudes. That is, the faster the rate of acceleration, the further the transient operating line moved away from that of the steady-state case. This phenomenon was predicted both by Curnock in a von Karman Institute Lecture Series from 1993 and by Cohen, et al. in the text, Gas Turbine Theory. This observation is evident in Figure 6.1.1 through Figure 6.1.3, which show the operating line System Identification models. The order of presentation of the components shown in these figures is the Outer Fan, the Inner Fan, and the HP Compressor.

It should be noted, that for all components in the previously mentioned figures. Prediction Model #1 \((0 \leq \alpha \leq 3)\) was selected as the best fit for alpha \((\alpha)\) values of 3 degrees throttle/sec. Also, Prediction Model #2 \((25 \leq \alpha \leq 100)\) was chosen as the best representation of accelerations occurring at alpha \((\alpha)\) values of 25 degrees of throttle/sec, in all three cases. These models were picked as the most nearly correct of the two possible selections for each of these two rates.
of acceleration by comparing the appropriate R-square and Adjusted R-square values. Table 6.1.1 and Table 6.1.3 summarize the System Identification model R-square values for the two dependent variables. Similarly, Table 6.1.2 and Table 6.1.4 summarize the Adjusted R-square values. The closer each of these statistical values is to unity, the better the model is at describing the variance in the reduced test cell performance data.
Table 6.1.1: R-Square Values for Corrected Mass Flow Rate Models

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate of Acceleration (α) Range of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ≤ α ≤ 3</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>0.9975</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>0.9990</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

Table 6.1.2: Adjusted R-Square Values for Corrected Mass Flow Rate Models

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate of Acceleration (α) Range of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ≤ α ≤ 3</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>0.9974</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>0.9990</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

Table 6.1.3: R-Square Values for Total Pressure Ratio Models

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate of Acceleration (α) Range of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ≤ α ≤ 3</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>0.9961</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>0.9989</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>0.9990</td>
</tr>
</tbody>
</table>

Table 6.1.4: Adjusted R-Square Values for Total Pressure Ratio Models

<table>
<thead>
<tr>
<th>Component</th>
<th>Rate of Acceleration (α) Range of Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0 ≤ α ≤ 3</td>
</tr>
<tr>
<td>HP Compressor</td>
<td>0.9959</td>
</tr>
<tr>
<td>Inner Fan</td>
<td>0.9989</td>
</tr>
<tr>
<td>Outer Fan</td>
<td>0.9989</td>
</tr>
</tbody>
</table>
F402-RR-406A Outer Fan Mass Flow Rate vs. Total Pressure Ratio
Steady State and Transient Operation - System ID Predictions

Figure 6.1.1: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A Outer Fan
Figure 6.1.2: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A Inner Fan
Figure 6.1.3: System Identification Predictions of Steady-state and Transient Operating Lines for the F402-RR-406A HP Compressor
6.2. F402-RR-406A Outer Fan Model

The results of the Outer Fan Model predictions in relation to the measured test cell data will be discussed in depth in this section. The presentation of the results for the Inner Fan and HP Compressor Models exactly follow the format used for the Outer Fan. These graphs illustrate clearly the results for all cases of both transient and steady-state operation over the full operating speed ranges used in this experiment. Specific discussion of each type of graph will be limited to the results for the Outer Fan Map only.

Figure 6.2.1 illustrates how well the Outer Fan operating line data and predicted values generated by the model compare. Additionally, this graph includes four operating points of test cell data from a third F402-RR-406A engine which failed to meet NADEP performance specifications during initial checkout testing following engine overhaul. This failed engine data was obtained for steady-state operation only, that is, Alpha (α) values of 0. Including test cell data from an out of specification engine in addition to the data from the two passing engines in the comparison with the model predictions provides an indication of the validity of the model as a diagnostic tool. For example, a significant departure in the failed engine operating point location from both the passing engine data and the predicted data would indicate a problem with the specific engine component modeled. Good correlation between the passing engine
Figure 6.2.1: F402-RR-406A Outer Fan Operating Line Prediction and Data - Steady-state (Alpha = 0)
operation points and the model predictions demonstrates the soundness of the model. However, to provide a quantitative measure of this correlation, established deviation limits must be applied to the model. These deviation limits are incorporated into the model automatically and have been fully explained in Chapter 5. The specific deviation limits on the models are a 95% confidence intervals, meaning that approximately 95% of the data points used to generate the model will fall within these limits. There is a direct correlation between the confidence limit interval and the accuracy of the model. This discussion will encompass an evaluation of the models for spool speed versus mass flow rate, spool speed versus total pressure ratio, and mass flow rate versus total pressure ratio for the outer fan.

Figure 6.2.2 illustrates mass flow rate prediction and data for an $\alpha$ value of zero. This graph appears deceiving because 7 out of 24 test cell data points from the passing engine fall outside of the confidence interval. This percentage is obviously greater than the 5% allowed. However, this graph illustrates only a two-dimensional (mass flow rate, $N_l$) section of the three-dimensional (mass flow rate, $N_l$, $A$) model prediction. The overall model was actually generated from test cell data for $\alpha$ values of both 0 and 3. Table 5.2.1 provides a summary of the actual terms and inputs used in this model. Approximately 125 data points contributed to the model from $\alpha = 3$ and 24 contributed from $\alpha = 0$ for a total of 149 data points. Figure 6.2.5 shows the same model at $\alpha = 3$. No test cell data falls outside of the confidence interval for this two-dimensional section.
Figure 6.2.2: F402-RR-406A Outer Fan Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0)
Therefore, 7 out of a total of 149 data points outside the interval is well within the expected 95% confidence interval.

Figure 6.2.3 illustrates the total pressure ratio as a function of spool speed at $\alpha = 0$. Again, 7 out of 24 data points fall outside of the confidence interval. The remaining data points which went into this model are found on Figure 6.2.6 for $\alpha = 3$. Table 5.2.2 provides a summary of the actual components of this model.

Figure 6.2.4 shows the outer fan total pressure ratio prediction #1 plotted versus the mass flow rate prediction #1 compared to the test cell operating points. Prediction #1 models were developed using $\alpha = 0$ and $\alpha = 3$ data. It is important to note that no confidence interval appears on this graph. This is due to the fact that the confidence intervals were based on individual models of both total pressure ratio and mass flow rate versus $N_L$. Therefore, when cross-plotting total pressure ratio and mass flow rate, the $N_L$ term cancels out and the confidence intervals become meaningless. However, it is apparent that the model operating line provides an excellent fit of the test cell operating points.

Figure 6.2.5 and Figure 6.2.6 illustrate the mass flow rate and total pressure ratio predictions, respectively, for $\alpha = 3$. Figure 6.2.5, as previously discussed, correlates to Figure 6.2.2 and is the same model as described in Table 5.2.1. No data points fall outside of the confidence limits. Figure 6.2.6 has the same correlation to Figure 6.2.3, as does Figure 6.2.5 to Figure 6.2.2.
Figure 6.2.3: F402-RR-406A Outer Fan Total Pressure Ratio Prediction and Data - Steady-state (Alpha = 0)
Figure 6.2.4: F402-RR-406A Outer Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure 6.2.5: F402-RR-406A Outer Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure 6.2.6: F402-RR-406A Outer Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure 6.2.7 illustrates the outer fan operating line prediction #2 for $\alpha = 3$ and again, shows good correlation between predicted and actual data. Near the design point, or full military power region, prediction model #2 does not perform as well as expected due to the fact that it was developed using $\alpha = 3$ and $\alpha = 25$ data. As will be apparent later in the discussion, there is greater correlation between $\alpha = 0$ and $\alpha = 3$ operating lines than with the $\alpha = 25$ operating line. Therefore, model #2 must cope with predicting operating lines between $\alpha = 3$ and $\alpha = 25$. As expected, model #2 does not perform as well as model #1 for generating $\alpha = 3$ predictions.

Figure 6.2.8 shows mass flow rate prediction model #2 versus NL for $a = 3$. It is readily apparent that the confidence interval is much broader than for model #1. All data points do fall within the confidence interval of model #2, but model #2 does not capture their behavior as closely as model #1. It is important to reiterate that model #1 is better in performing predictions in the $a = 3$ regime than model #2 due to the nature of the data from which the two models were generated. The behavior of $a = 0$ and $a = 3$ operating lines correspond more closely because they are both relatively slow accelerations when compared to $a = 25$ data. Table 6.1.1 supports this conclusion by providing R-Square values for corrected mass flow rate models. The R-Square value for prediction model #1 designed for the region $a = 0$ to $a = 3$ is higher than that for model #2 for the region $a = 3$ to $a = 25$. Table 6.1.2 provides a comparison of adjusted R-Square
Figure 6.2.7: F402-RR-406A Outer Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure 6.2.8: F402-RR-406A Outer Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 3)
values for mass flow rate models. These adjusted R-Square values account for
differently sized data sets and allow a more fair comparison of their validity.
Again, the adjusted R-Square value for model prediction #1 is higher than for
model #2, proving model #1 is best in predicting $\alpha = 3$ values.

Figure 6.2.9 illustrates total pressure ratio prediction model #2 versus $N_L$
for $\alpha = 3$. Similarly, all data points fit within the confidence interval, but this
model does not capture the trend of the data as well as model #1 for the same
reason as described regarding Figure 6.2.8. Table 6.1.3 provides R-Square
values for the total pressure ratio models. It is apparent that model #1 designed
for the region from $\alpha = 0$ to $\alpha = 3$ is better than model #2 because its R-Square
value is higher. Table 6.1.4 shows adjusted R-Square values for total pressure
ratio models. These values are adjusted to fairly compare models developed
from differently sized data sets. Again, the prediction model #1 has a higher
adjusted R-Square value.

Figure 6.2.10 provides a direct comparison of prediction models #1 and
#2. In general, these two predictions are very similar with the exception of some
divergence in the full military power region. Again, two models were used to
create the three-dimensional model capable of predicting operating points in the
range from $\alpha = 0$ to $\alpha = 25$. A single three-dimensional model was desired.
Figure 6.2.9: F402-RR-406A Outer Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure 6.2.10: F402-RR-406A Outer Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 3)
However, in order to have the best two-dimensional fit possible, six submodels were used to create an entire three-dimensional surface for each component (refer to Table 6.1.1 for a breakdown of model inputs).

Figure 6.2.11 and Figure 6.2.12 illustrate mass flow rate and total pressure ratio predictions, respectively, for models #1 and #2. In both cases, there is close agreement between both models, except within the full military power region in the upper right hand portion of the graph.

Figure 6.2.13 illustrates the prediction model #1 operating line versus test cell data for $\alpha = 25$. The prediction follows the data trend in general, however, again it fails to properly capture the behavior in the full military power region.

Figure 6.2.14 and Figure 6.2.15 illustrate the prediction model #1 versus $N_c$ for $\alpha = 25$ for both mass flow rate and total pressure ratio, respectively. Prediction model #1 for $\alpha = 25$ was developed using both $\alpha = 3$ and $\alpha = 25$ test cell data, and was intended to generate predictions in the region between these two $\alpha$ values. The confidence limits on these graphs are not as tight as previous models, and a large number of data points fall outside of the confidence limits. Both of these characteristics make prediction model #1 for $\alpha = 25$ to $\alpha = 100$ not as useful for diagnostic purposes.

Figure 6.2.16 shows the prediction model #2 operating line in conjunction with test cell data at $\alpha = 25$. When compared to Figure 6.2.13, it is readily apparent that model #2 provides a better fit of the data, particularly in the full military power region. Prediction model #2 is derived from test cell data.
Figure 6.2.11: F402-RR-406A Outer Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure 6.2.12: F402-RR-406A Outer Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure 6.2.13: F402-RR-406A Outer Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure 6.2.14: F402-RR-406A Outer Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure 6.2.15: F402-RR-406A Outer Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure 6.2.16: F402-RR-406A Outer Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 25)
taken at $\alpha = 25$ and $\alpha = 100$. Because both of these rates of acceleration are relatively fast, the behavior of the engine during these test runs is similar. In contrast, prediction model #1 does not do as good a job at capturing the behavior of the engine performance at $\alpha = 25$ because it was derived from $\alpha = 3$ and $\alpha = 25$ data. $\alpha = 3$ accelerations occur much more slowly than do the $\alpha = 25$ and $\alpha = 100$ accelerations, therefore it is reasonable that model #2 is the best model. Tables 6.1.1, 6.1.2, 6.1.3, and 6.1.4 provide both R-Square and adjusted R-Square values which further support this analysis.

Figure 6.2.17 illustrates outer fan mass flow rate versus $N_L$ for prediction model #2 at $\alpha = 25$. When compared with Figure 6.2.14, it is evident that model #2 provides a much tighter tolerance and better fit of data. Almost all of the test cell data fits within the tolerance, making this model an ideal candidate for implementation in diagnostic analysis of test cell data.

Similarly, Figure 6.2.18 compares with Figure 6.2.15 and illustrates outer fan total pressure ratio for prediction model #2 at $\alpha = 25$. Again, model #2 predictions have much narrower tolerance limits and show better agreement with test cell data at full military power. For example, for prediction model #2 only 3 out of 298 data points fall outside of the confidence interval as opposed to 11 out of 298 for model #1. This evidence lends further support to the superiority of model #2 for performance prediction in the $\alpha = 25$ regime and implies that it possesses excellent potential for use in diagnostic analysis.
Figure 6.2.17: F402-RR-406A Outer Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure 6.2.18: F402-RR-406A Outer Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure 6.2.19 provides a side-by-side comparison of prediction model #1 and prediction model #2 operating lines at $\alpha = 25$. As previously discussed, both models capture the broad behavior of engine performance satisfactorily. In general, both predictions follow very similar trends but do not fully agree in the full military power region. Furthermore, the component models of mass flow rate and total pressure ratio for these two predictions show differing quality with regard to the tightness of their tolerance intervals.

Figures 6.2.20 and 6.2.21, respectively, provide side-by-side comparisons of prediction models #1 and #2 for mass flow rate versus $N_L$ and total pressure ratio versus $N_L$. In both figures, there is evidence of divergence between both predictions, particularly in the full military power region. Notwithstanding, the general trend between the two models is in agreement. Therefore, this lends credibility to the overall outer fan model (see Tables 5.2.1 and 5.2.2) in predicting general performance over the $\alpha = 3$ to $\alpha = 25$ operating range. However, for predictions in the region between these two values, an improvement is necessary in the model before it can be applied in a diagnostic role.

Figure 6.2.22 shows operating line prediction and test cell data at $\alpha = 100$. This figure shows an excellent fit of data with the performance prediction, especially in the full military power region. It is particularly interesting to note that this operating line is convex as discussed in Section 6.1. This prediction was calculate using the same model as prediction model #2 for $\alpha = 25$, substituting $\alpha$ values of 100. This model was derived using both $\alpha = 25$ and $\alpha = 100$ test cell
data. Agreement between prediction and data can be attributed to the similarity of engine performance for $\alpha = 25$ and $\alpha = 100$ accelerations.

Figure 6.2.19: F402-RR-406A Outer Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure 6.2.20: F402-RR-406A Outer Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure 6.2.21: F402-RR-406A Outer Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure 6.2.22: F402-RR-406A Outer Fan Operating Line Prediction and Data - Transient Operation (Alpha = 10substituting α values of 100. This model was derived using both α = 25 and α = 100 test cell data. Agreement between prediction and data can be attributed to the similarity of engine performance for α = 25 and α = 100 accelerations.
Figures 6.2.23 and 6.2.24, respectively, show outer fan mass flow rate versus $N_L$ and total pressure ratio versus $N_L$. Predictions provide an excellent fit of data with tight tolerance limits and minimal data points outside of the tolerance interval. Both of these predictions are readily applicable to diagnostic use. 

The results for the inner Fan Model are illustrated in Appendix A. The organization of the figures in Appendix A follows that of the previous discussion of the Outer Fan Model figures.

**Figure 6.2.23: F402-RR-406A Outer Fan Mass Flow Rate Prediction and Data - Transient Operation (Alpha = 100)**
F402-RR-406A Outer Fan LP Spool Speed vs. Total Pressure Ratio

Transient Operation - Alpha = 100 degrees throttle / sec

+ Test Cell Data (PASSED ENGINE)

- System ID Prediction

- - Lower 95% Confidence Limit

- - Upper 95% Confidence Limit

Figure 6.2.24: F402-RR-406A Outer Fan Total Pressure Ratio Prediction and Data - Transient Operation (Alpha = 100)
7. CONCLUSIONS AND RECOMMENDATIONS

7.1. Conclusions

Several conclusions can be drawn from the analysis of this thesis experiment. First, an experimental method for capturing F402-RR-406A engine performance data during both steady-state and transient operation has been developed and tested. Second, computer codes have been written to reduce both the steady-state and transient test cell data to operating point values for several engine components. Also, this project has successfully demonstrated a method for creating transient, predictive models of several gas turbine engine components using System Identification techniques in combination with engine performance data. Furthermore, the utility of these predictive models in a diagnostic role has been demonstrated.

7.2. Recommendations

This project has led to the development and successful implementation of a method for generating transient, predictive models of gas turbine engine components. As the demonstration phase of this project ends, future effort must be spent improving the method and fully integrating it into a predictive diagnostic tool. There are several directions in which future work must focus.
• The F402 Test Cell Facilities need four additional data recording channels so that all test cell data necessary for the engine performance analysis can be captured simultaneously.

• The diagnostic capabilities of the System Identification models need to be validated for more test cases, including transient operation regimes.

• More data should be taken in the transient region in between rates of acceleration ($\alpha$ values) of 3 degrees of throttle / sec and 25 degrees of throttle / sec. This additional data is necessary to more accurately characterize the nature of the component operating lines in this region.

• Models should be generated from data sets recorded from a larger number of F402-RR-406A engines. This step should improve the ability of the System Identification models to serve in a diagnostic role.

• The process of developing the System Identification models from test cell data, including data reduction methods, needs to be streamlined. This step is necessary in order to make the diagnostic aspect of the models more useful to the production engineers at the NADEP.
REFERENCES


Appendix A: GRAPHICAL PRESENTATION OF SYSTEM IDENTIFICATION MODELING RESULTS IN COMPARISON WITH TEST CELL DATA
Appendix A: Part 1 - F402-RR-406A INNER FAN MODEL IN COMPARISON WITH TEST CELL DATA
Figure A.1.1: F402-RR-406A Inner Fan Operating Line Prediction and Data - Steady-state (Alpha = 0)
Figure A.1.2: F402-RR-406A Inner Fan Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0)
F-402-RR-406A Inner Fan LP Spool Speed vs. Total Pressure Ratio
Steady State Operation - Alpha = 0 degrees throttle / sec

Figure A.1.3: F402-RR-406A Inner Fan Total Pressure Ratio Prediction and Data - Steady-state (Alpha = 0)
Figure A.1.4: F402-RR-406A Inner Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure A.1.5: F402-RR-406A Inner Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 3)
F402-RR-406A Inner Fan LP Spool Speed vs. Total Pressure Ratio (MODEL #1)
Transient Operation - Alpha = 3 degrees throttle / sec

Figure A.1.6: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 3)
F402-RR-406A Inner Fan Mass Flow Rate vs. Total Pressure Ratio (MODEL #2)
Transient Operation - Alpha = 3 degrees throttle / sec

Figure A.1.7: F402-RR-406A Inner Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.1.8: F402-RR-406A Inner Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.1.9: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.1.10: F402-RR-406A Inner Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.1.11: F402-RR-406A Inner Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.1.12: F402-RR-406A Inner Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.1.13: F402-RR-406A Inner Fan Operating Line Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.1.14: F402-RR-406A Inner Fan Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.1.15: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.1.16: F402-RR-406A Inner Fan Operating Line Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure A.1.17: F402-RR-406A Inner Fan Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure A.1.18: F402-RR-406A Inner Fan Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure A.1.19: F402-RR-406A Inner Fan Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure A.1.20: F402-RR-406A Inner Fan Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure A.1.21: F402-RR-406A Inner Fan Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 25)
F402-RR-406A Inner Fan Mass Flow Rate vs. Total Pressure Ratio
Transient Operation - Alpha = 100 degrees throttle / sec

Test Cell Data (PASSED ENGINE)
System ID Prediction

Figure A.1.22: F402-RR-406A Inner Fan Operating Line Prediction and Data
- Transient Operation (Alpha = 100)
Figure A.1.23: F402-RR-406A Inner Fan Mass Flow Rate Prediction and Data - Transient Operation (Alpha = 100)
Figure A.1.24: F402-RR-406A Inner Fan Total Pressure Ratio Prediction and Data - Transient Operation (Alpha = 100)
Appendix A: Part 2 - F402-RR-406A HIGH PRESSURE COMPRESSOR MODEL IN COMPARISON WITH TEST CELL DATA
Figure A.2.1: F402-RR-406A High Pressure Compressor Operating Line Prediction and Data - Steady-state (Alpha = 0)
**Figure A.2.2: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction and Data - Steady-state (Alpha = 0)**
Figure A.2.3: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction and Data - Steady-state (Alpha = 0)
F402-RR-406A HP Compressor Mass Flow Rate vs. Total Pressure Ratio (MODEL #1)
Transient Operation - Alpha = 3 degrees throttle / sec

Figure A.2.4: F402-RR-406A High Pressure Compressor Operating Line Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure A.2.5: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure A.2.6: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 3)
Figure A.2.7: F402-RR-406A High Pressure Compressor Operating Line Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.2.8: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.2.9: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 3)
Figure A.2.10: F402-RR-406A High Pressure Compressor Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.2.11: F402-RR-406A High Pressure Compressor Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.2.12: F402-RR-406A High Pressure Compressor Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 3)
Figure A.2.13: F402-RR-406A High Pressure Compressor Operating Line Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.2.14: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.2.15: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #1 and Data - Transient Operation (Alpha = 25)
Figure A.2.16: F402-RR-406A High Pressure Compressor Operating Line Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure A.2.17: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction #2 and Data - Transient Operation (Alpha = 25)
Figure A.2.18: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction #2 and Data - Transient Operation (Alpha = 25)
F402-RR-406A HP Compressor Mass Flow Rate vs. Total Pressure Ratio (MODELS #1 and #2)
Transient Operation - Alpha = 25 degrees throttle / sec

Figure A.2.19: F402-RR-406A High Pressure Compressor Operating Line Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure A.2.20: F402-RR-406A High Pressure Compressor Mass Flow Rate Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure A.2.21: F402-RR-406A High Pressure Compressor Total Pressure Ratio Predictions #1 and #2 - Transient Operation (Alpha = 25)
Figure A.2.22: F402-RR-406A High Pressure Compressor Operating Line Prediction and Data - Transient Operation (Alpha = 100)
Figure A.2.23: F402-RR-406A High Pressure Compressor Mass Flow Rate Prediction and Data - Transient Operation (Alpha = 100)
Figure A.2.24: F402-RR-406A High Pressure Compressor Total Pressure Ratio Prediction and Data - Transient Operation (Alpha = 100)
VITA

Michael David Grose was born on June 23, 1972 in Hampton, Virginia. He was raised in York County on the outskirts of historic Williamsburg, Virginia. Following graduation from Bruton High School in the Spring of 1990, he matriculated to the Virginia Military Institute. On May 20, 1994 he was commissioned an Ensign in the United States Navy. The following day he graduated with distinction, earning a Bachelor of Science Degree in Mechanical Engineering. During the summer of 1994, he was selected for the Navy Scholarship Program and received orders to attend Virginia Polytechnic Institute & State University. In November of 1996, he received a Master of Science Degree in Mechanical Engineering. Following completion of this degree, he reported for Surface Warfare Officer's School in Newport, Rhode Island.