FUZZY LOGIC AND UTILITY THEORY FOR MULTIOBJECTIVE OPTIMIZATION OF AUTOMOTIVE JOINTS

by

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Fuzzy Logic and Utility Theory for Multiobjective Optimization of Automotive Joints

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(ABSTRACT)

In the early design stage of automotive joints, fuzziness is omnipresent because designers reason in non-quantitative terms and deal with imprecise data. Consequently, they need a design methodology that accounts for vagueness. Fuzzy sets and utility theory are appropriate tools because they link the vagueness in a problem formulation and the precise nature of mathematical models.

Fuzzy multiobjective optimizations are performed on an automotive joint to maximize the overall designer’s satisfaction. Several methods that account for all the attributes and the fuzziness in the goals are used. Three multiobjective fuzzy approaches, namely, the conservative, the aggressive and the moderate methods are investigated. Utility theory is also considered to optimize the joint. One of the performance attributes of the joint, the stiffness, is evaluated rapidly using approximate tools (neural networks and response surface polynomials) to overcome the high computational cost of FEA, which is traditionally used to calculate the stiffness.
This research compares fuzzy set methods and utility theory in design of automotive components. These methods are applied on two examples where the same B-pillar to rocker joint of an actual car is optimized.

Fuzzy set based methods and utility theory appear to be suitable for optimizing automotive joints because they allow for trading conflicting objectives. Fuzzy set based methods avoid trading objectives to the point of having a level of satisfaction equal to zero. When using the fuzzy set based methods investigated in this research, the trade-offs among the attributes are not explicitly defined by the user. Utility theory requires the user to quantify precisely the trade-offs among the attributes. When using utility theory, the overall satisfaction of a design can be non zero even if one or more attributes has a level of satisfaction equal to zero.

The approximate tools enable us to perform the optimization efficiently by reducing considerably the computational cost.
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1. Introduction

1.1. Multiobjective Optimization

In today’s highly competitive world, it is essential for an engineer to design the best system rather than find a system that performs the required task satisfactorily. This system has to perform well, to be reliable, durable and cost-effective. This is why optimization methods are becoming increasingly popular.

It is recognized that most real-world problems involve typically multiple conflicting objectives, which should be considered simultaneously. Usually, any improvement of one objective can be achieved only at the expense of another. An example of multiobjective optimization problem is job selection. A person who is looking for a job has to find the best compromise among the location, the salary, the job security, the company reputation, the advancement opportunities and the work conditions. It seems natural to consider decision-making as a process that requires multiobjective approaches rather than single objective ones.

1.1.1 Multiobjective Optimization Using Fuzzy Sets to Combine Objectives

The human brain is said to think and reason in imprecise, non-quantitative, vague terms. This vagueness is due to the decision maker’s ambiguous understanding and determination of the variables and constraints in a problem and the fuzziness of the goals. For instance, considering the case of job selection, the definition of high salary is a matter of degree: this definition depends on the cost of living of the job location and the social position of the person who is looking for a job. The fuzzy set approach has been
developed to account for vagueness [34, 35].Considering the fuzzy nature of human judgment, fuzzy set theory appears to be very promising for multiobjective optimization.

Fuzzy sets and utility theory treat multiobjective optimization problems as a generalization of traditional single-objective optimization problems. In fact, they combine several objectives into a single one that measures the overall merit of an alternate solution. Once the objective is assessed, fuzzy theory and utility theory seek to maximize it. On the other hand, traditional approaches optimize one objective treating the remaining objectives as constraints [2]. Then, traditional approaches use sensitivity analysis to deal with the trade-offs that must be made among conflicting objectives. In this analysis, the sensitivities of the optimum value of the objective function are calculated with respect to the constraints. Traditionally, the designer gets an optimum solution that is not the best combination of attribute levels for a particular application.

Using fuzzy set theory or utility theory, a design alternative, that represents the best compromise of attributes, is obtained because these methods account for all attributes simultaneously.

1.1.2 Using Approximate Tools for Rapid Analysis to Perform Optimization Efficiently

Fuzzy sets require many analyses of the performance of a system compared to traditional crisp optimization problems. Typically, the objective function when using fuzzy sets is more complex than the one considered when using traditional crisp optimization procedures. Generally, fuzzy sets also need the determination of a set of Pareto optima, which requires several optimization runs [23]. All studies in the open literature have performed fuzzy multiobjective optimizations using complete analysis, such as FEA. To overcome the problem of high computational cost, fuzzy set approach can be applied using approximate tools for analysis instead of complete analysis. Performance measures
are obtained as outputs from these approximate tools, which require minimal computational resources and allow to perform the optimization efficiently. These tools were developed by Zhu [36]. To the best of our knowledge, this is the first study in which performance characteristics of a system will be estimated using approximate tools to optimize a design using fuzzy set methods.

1.2 Design of Automotive Joints

1.2.1 Description of Automotive Joints

A joint is an important component of the body structure of a car because it affects significantly the static and dynamic behavior of automobile structures [7].

An automotive joint is a thin-walled subassembly made of structural steel tubes called branches that intersect and that are spot welded together. The branches of a T-joint do not have to meet at right angle: one branch can be inclined to another by an angle $\alpha$ as shown in Figure 1. The B-pillar to rocker joint is a simple T-shaped joint as shown in Figure 2.

![Figure 1: Oblique B-pillar to rocker joint whose branches meet at oblique angle $\alpha$.]
Figure 2: B-pillar to rocker joint.

Joints are flexible [7, 10], in the sense that their branches can rotate relative to each other. The stiffness of a joint is defined as the ratio of the moment applied to a branch over the rotation of that branch, while the ends of the other branches are fixed. Usually, only the rotation in the direction in which the moment is applied is investigated. A joint is usually considered rigid in translation. In most practical applications, all joint branches except for the most flexible branch are assumed to be rigidly connected. For example, the horizontal branches of the T-shaped joint in Figure 1 are considered rigidly connected. Only the vertical branch can rotate relative to the horizontal ones.

1.2.2 Multiobjective Optimization of Automotive Joints

Design of automotive joints is a case where there are several conflicting objectives. Specially, during the design process, engineers try to:

- maximize the stiffness,
- maximize the cross-sectional properties,
- minimize the mass,
- keep the dimensions of the joint within certain limits dictated by packaging constraints,
- ensure that the joint design is easy to manufacture.

Most of the above objectives are fuzzy, in the sense the boundary that separates acceptable from unacceptable design alternatives is non sharp. For example, it does not make sense to state that a joint is too heavy if its weight is 8.01 kg but acceptable if its weight is 7.99 kg. Actually, it is a matter of degree how heavy a joint is.

Although joint design involves multiple objectives, to the best of our knowledge, all studies in the open literature have solved this problem using one objective only. For example, Zhu minimized the mass subject to constraints on the stiffness and the cross-sectional properties [15, 36]. Because only one objective was considered, we do not believe that Zhu’s approach can find the joint that maximizes the designer satisfaction when all attributes (efficiency, performance, manufacturability) are taken into account.

1.3 Review of Previous work

1.3.1 Fuzzy Sets for Multiobjective Optimization

Fuzzy set theory is a tool for handling vagueness of concepts expressed using linguistic terms such as “tall man”, “high salary” and “reliable car”. The outstanding feature of fuzzy sets is the ability to express the amount of ambiguity in human thinking in a comparatively undistorted manner. The idea of fuzzy set theory is quite intuitive: a fuzzy set allows no sharply defined boundaries as in an ordinary set because of a generalization of a characteristic function to a membership function. In fact, a fuzzy set can be defined mathematically by assigning to each possible entity a value representing the grade of membership of the entity in the fuzzy set. This grade corresponds to the degree to
which that entity is compatible with the concept represented by the fuzzy set. The membership in a fuzzy set is not a matter of affirmation or denial, as in classical set theory, but rather a matter of degree. The supports of a membership are the extreme bounds of this membership. In the following paragraph, the literature review focuses on two different approaches to multiobjective decision making: the max-min and the product rules [17, 20].

Research on the theory of fuzzy sets has been growing steadily since the presentation of the theory by Prof. L. A. Zadeh [34]. In his paper, Zadeh introduced a theory whose objects - fuzzy sets - are sets with imprecise boundaries. In this approach, the best alternative is chosen based on the max-min rule. This rule implies that the overall performance of a system is determined by its weakest or poorest attribute. This method is applicable when the decision maker is assumed to have a pessimistic nature about decision making situation: the decision maker selects the alternative with the best value of the poorest attribute. As a result, only one attribute is used to represent the system, all other attributes being ignored in the process. In other words, the trade-off among attributes is non compensatory. This method is referred as the conservative strategy. Zimmermann first used Zadeh's max-min operator to resolve conflicts between objectives [37]. Many researchers applied this theory to design problems.

However, the minimum operator is not always appropriate when combining fuzzy goals, constraints and variables because in many problems, we can trade off attributes. For example, a heavy design alternative that is easy to manufacture can have the same engineer's level preference as a light design that is very difficult to manufacture. The use of the product function allows higher performing attributes to compensate for others. One of the main characteristics of this metric is that the trade-off between attributes is compensatory. In this case, the overall performance of a system is determined by all the attributes. This metric trades off the attributes to cooperatively improve the design. Antonsson, et.al., referred to this method as an aggressive strategy [17].
In this study, both methods, the conservative and aggressive, will be applied to the design optimization of an automotive joint. A third method, called moderate method, will also be discussed in this thesis.

1.3.1.1 Formulation of the Conservative Method Using Rao’s Method to Define the Memberships of the Objectives

Formulation of a multiobjective optimization problem:

Let $\bar{x}$ be the vector of design variables.

$$\bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

The objective functions are defined by:

$$f(\bar{x}) = \begin{bmatrix} f_1(\bar{x}) \\ f_2(\bar{x}) \\ \vdots \\ f_k(\bar{x}) \end{bmatrix}$$

where $f_j(\bar{x})$ are the single objective functions.

The constraints are defined by: $g_j(\bar{x}) \leq b_j \quad j = 1 \text{ to } m \quad (1)$
Assuming that all the objective functions have to be minimized, the problem can be described as follows:

Minimize: \( f(\tilde{x}) \) \hspace{1cm} (2)

Subject to: \( g_j(\tilde{x}) \leq b_j \) \hspace{1cm} (3)

**Formulation of a fuzzy multiobjective optimization problem:**

For a multiobjective fuzzy optimization, the fuzzy feasible domain \( D \) is:

\[
D = \left( \bigcap_{i=1}^{k} D_i \right) \cap \left( \bigcap_{j=1}^{m} D_{g_j} \right) \tag{4}
\]

where \( D_i \) and \( D_{g_j} \) denote the supports of the membership functions of the \( i \)th objective function, \( f_i(\tilde{x}) \) and the \( j \)th constraint, \( g_j(\tilde{x}) \).

Let \( y(\tilde{x}) \) be a vector which components are:

\[
y(\tilde{x}) = \begin{bmatrix}
f_1(\tilde{x}) \\
\vdots \\
f_k(\tilde{x}) \\
g_{k+1}(\tilde{x}) \\
\vdots \\
g_{k+m}(\tilde{x})
\end{bmatrix}
\]

A feasible set solution \( \tilde{x} \) is characterized by its membership function \( \mu_D(\tilde{x}) \):

\[
\mu_D(\tilde{x}) = \min_{p=1,...,k+m} \left\{ \mu_{y_p}(\tilde{x}) \right\} = \min_{i=1,...,k} \left\{ \min_{j=1,...,m} \{ \mu_{f_i}(\tilde{x}), \mu_{g_j}(\tilde{x}) \} \right\} \tag{5}
\]
where $\mu_{i_1}(\bar{x})$ and $\mu_{g_j}(\bar{x})$ denote respectively the membership functions of the $i$th objective function and the $j$th constraint. $\mu_{y_p}(\bar{x})$ denotes the membership of the $p$th component of $y(\bar{x})$. This component is the objective function $f_p(\bar{x})$ if $p$ is smaller or equal than $k$ and is the constraint $g_p(\bar{x})$ for $p$ larger than $k$.

$\mu_D(\bar{x})$ measures the degree to which design $\bar{x}$ satisfies the designer when all objectives and constraints are considered simultaneously.

The optimum design vector $\bar{x}^*$ is the one for which $\mu_D$ is maximum.

$$
\mu_D(\bar{x}^*) = \max_{\text{feasible domain}} \mu_D(\bar{x}) = \max_{\text{feasible domain}} \left[ \min_{i=1,\ldots,k} \{ \mu_{i_1}(\bar{x}), \mu_{g_j}(\bar{x}) \} \right]
$$

(6)

**Establishment of membership functions:**

An important problem when using fuzzy sets is to define membership functions of the objectives. The membership can be defined based on judgment or using Rao’s method [20].

Rao suggested the following technique: using single objective optimization procedure (crisp optimization), minimize each individual objective function $f_i(\bar{x})$ subjected to the constraints $g_i(\bar{x}) \leq b_i$ and assign a membership value of one to these values. Note that during the optimization process, no constraints on the remaining objectives are considered. This optimization procedure will determine optimum vectors $\bar{x}_{i^*}$ associated with the minimization of the corresponding objective function $f_i(\bar{x})$, $i = 1, \ldots, k$.

Let $P$ be a $k \times k$ matrix.
\[
[P] = \begin{bmatrix}
  f_1(\bar{x}_1^\ast) & f_2(\bar{x}_1^\ast) & \cdots & f_k(\bar{x}_1^\ast) \\
  f_1(\bar{x}_2^\ast) & f_2(\bar{x}_2^\ast) & \cdots & f_k(\bar{x}_2^\ast) \\
  \vdots & \vdots & \ddots & \vdots \\
  f_1(\bar{x}_k^\ast) & f_2(\bar{x}_k^\ast) & \cdots & f_k(\bar{x}_k^\ast)
\end{bmatrix}
\]  

(7)

The optimum of each objective function \( f_i \) is element \( P_{ii} \) of matrix \( P \).

The minimum and maximum possible values of the objective function \( f_i \) can be identified from equations (8) and (9):

\[
f_i^{\text{min}} = \min_{j=1,\ldots,k} f_i(\bar{x}_j^\ast) = f_i(\bar{x}_i^\ast) \quad i = 1, 2, \ldots, k
\]  

(8)

and

\[
f_i^{\text{max}} = \max_{j=1,\ldots,k} f_i(\bar{x}_j^\ast) \quad i = 1, 2, \ldots, k
\]  

(9)

Once the maximum and minimum values \( f_i^{\text{max}} \) and \( f_i^{\text{min}} \) are defined for each objective function, respectively, Rao proposes to build the membership functions as shown in Figure 3.

The discussion in the following paragraph and Figure 3 refers to an attribute that the decision maker wants to minimize.

![Figure 3: Membership function of the objective function \( f_i(\bar{x}) \).](image)

10
The membership function of the fuzzy objective \( f_i(\bar{x}) \) is constructed as:

\[
\mu_{f_i(\bar{x})} = \begin{cases} 
0 & \text{if } f_i(\bar{x}) \geq f_i^{\text{max}} \\
\frac{f_i^{\text{max}} - f_i(\bar{x})}{f_i^{\text{max}} - f_i^{\text{min}}} & \text{if } f_i^{\text{min}} \leq f_i(\bar{x}) \leq f_i^{\text{max}} \\
1 & \text{if } f_i(\bar{x}) \leq f_i^{\text{min}}
\end{cases}
\]  
(10)

The membership functions of the constraint \( g_j(\bar{x}) \) are defined by:

\[
\mu_{g_j(\bar{x})} = \begin{cases} 
0 & \text{if } g_j(\bar{x}) \geq b_j + d_j \\
\frac{b_j + d_j - g_j(\bar{x})}{d_j} & \text{if } b_j \leq g_j(\bar{x}) \leq b_j + d_j \\
1 & \text{if } g_j(\bar{x}) \leq b_j
\end{cases}
\]  
(11)

where \( d_j \) is the maximum acceptable deviation for the constraint \( g_j \).

The assessment of the memberships of the objectives using equation (10) is justified in the following paragraphs.

The lower boundary \( f_i^{\text{min}} \) of the membership is equal to the best possible value the attribute \( f_i \) can ever achieve if we try to minimize the \( i \)th objective and neglect the others. It seems reasonable to assign the maximum degree of designer satisfaction to \( f_i^{\text{min}} \).

The upper boundary \( f_i^{\text{max}} \) of the membership is assessed by substituting the optimum design variables obtained when deriving \( f_j^{\text{min}} \) (\( j \) different from \( i \)) in the attribute \( f_i \) and choosing the largest value of \( f_i \). The membership corresponding to this largest value of \( f_i \) is zero. Note that the value \( f_i^{\text{max}} \) is not equal to the maximum or worst value of the objective \( f_i \). In theory, the worst value of the attribute can be larger than \( f_i^{\text{max}} \). It can be assessed by performing a maximization of the attribute \( f_i \). Due to the presence of conflicting criteria, when the engineer computes \( f_i^{\text{max}} \) using equation (9) without any constraints on the other attributes, it is likely that \( f_i^{\text{max}} \) will be close to the worst value of \( f_i \). Using Rao’s approach, the engineer avoids running two optimizations, a minimization
and a maximization, to build the membership of each attribute. The engineer needs to perform one minimization per attribute. For instance, Rao considered a three-bar truss with two conflicting attributes: the weight and the vertical deflection of the loaded joint. There is no need to perform a maximization to find the maximum vertical deflection that has membership zero because we know that this is the deflection of the beam that has minimum mass.

In conclusion, Rao’s approach to assess the boundaries of the memberships makes sense. Furthermore, the membership functions are assumed to have linear boundaries given by equations (10) and (11). A linear interpolation between the boundaries of a membership is chosen because in absence of further information, designers can use the simplest possible form of membership that satisfies the boundary conditions. This linear interpolation means that the degree of satisfaction decreases linearly from one to zero as the attribute $f_i$ increases from $f_i^{\min}$ to $f_i^{\max}$.

**Solution of optimization problem:**

The implementation of this method can be facilitated by introducing an auxiliary design variable $\lambda$. To solve the multiobjective optimization problem numerically, $\lambda$ is employed to transform the max-min problem (equation (6)) into the following equivalent conventional single objective optimization problem:

Maximize: $\lambda$

Subject to: $\lambda \leq \mu_i(\tilde{x})$ \quad i=1,2,...,k \quad (12)

and $\lambda \leq \mu_j(\tilde{x})$ \quad j=1,2,...,m \quad (13)

$\lambda$ is maximized toward unity. In this case, the design vector has $n+1$ components, which are the variable $\lambda$ and the design variables of the original optimization problem (equation
(6)). The value of $\lambda$ represents the overall level of satisfaction of the design. The higher this value is, the happier the designer will be.

The arithmetic average is not considered in this study because using the arithmetic average operator, the attributes can always be traded off, even to the point of achieving a zero level of satisfaction in some objectives. This is not acceptable in most engineering problems because, if the degree of satisfaction with respect to one attribute is zero, the overall satisfaction has to be zero.

### 1.3.1.2 Formulation of the Aggressive Method

The aggressive method allows trading between the attributes. Using the aggregator product, this approach is stated as follows:

$$
\mu_{D} (\bar{x}^*) = \max_{\text{feasible region}} \left[ \prod_{p=1}^{k+m} \mu_{\gamma_p} (\bar{x}) \right]^{\frac{1}{(k+m)}} = \max_{\text{feasible region}} \left[ \prod_{i=1}^{k} \mu_{f_i} (\bar{x}) \cdot \prod_{j=1}^{m} \mu_{g_j} (\bar{x}) \right]^{\frac{1}{(k+m)}} \tag{14}
$$

where $\mu_{f_i} (\bar{x})$ and $\mu_{g_j} (\bar{x})$ represent the membership functions for the $i$th objective function and the $j$th constraint. $\mu_{\gamma_p} (\bar{x})$ denotes the membership of the $p$th component of $y(\bar{x})$. These membership functions are defined in the same manner as the conservative method. Note that if one of the memberships of the constraints or attributes is zero, the overall customer satisfaction is zero.

### 1.3.1.3 Formulation of the Moderate Method

The moderate approach is a weighted sum of the conservative and the aggressive methods. The aggregator combining the objectives is a weighted average of the conservative and the aggressive aggregator:
\[ \mu_\alpha(x) = \max_{\text{feasible region}} \left\{ (1 - w) \left( \prod_{p=1}^{k+m} \mu_{y_p}(\bar{x}) \right)^{1/(k+m)} \right\} \]
\[ = \max_{\text{feasible region}} \left\{ (1 - w) \left( \prod_{i=1}^{k} \mu_{f_i}(\bar{x}) \cdot \prod_{j=1}^{m} \mu_{g_j}(\bar{x}) \right)^{1/(k+m)} \right\} + w \cdot \min_{p=1,...,k+m} \left( \mu_{y_p}(\bar{x}) \right) \]  

(15)

where \( w \) varies from zero to one. The membership functions \( \mu_{f_i}(\bar{x}), \mu_{g_j}(\bar{x}) \) and \( \mu_{y_p}(\bar{x}) \) are defined in the same manner as the conservative method.

When \( w \) is equal to one, the expression above becomes the max-min approach. If \( w \) is equal to zero, the moderate method reduces to the aggressive method. For low values of \( w \) (close to zero), the product of the degrees of satisfaction is slightly offsetting the dominance of the max-min function and vice-versa.

The advantages of the aggregator of the moderate method are two-fold for values of \( w \) different from zero and one. First, it will be able to consider the contributions of attributes other than the one with minimum degree of satisfaction. Yet, it will still penalize a design alternative if one attribute has a very low degree of satisfaction. Second, it will allow trade-offs between objectives. The moderate method seems to combine the advantages of the two methods mentioned in sections 1.3.1.1 and 1.3.1.2.

### 1.3.2 Utility Theory for Multiobjective Optimization

Utility theory is an analytical method for decision making under uncertainty, given a set of conflicting attributes upon which the decision is based. The method does not eliminate uncertainty. It takes into account the effect that uncertainty has on the desirability of design alternatives using the certainty equivalence method and lottery questions as described at the end of this section.
Utility theory was originally developed for the management of decision making by von Neumann and Morgenstern followed by developments of Savage [22, 32]. Most of the applications of utility analysis did not concern engineering design optimization until recently because utility theory was originally developed for economic decision making. Thurston was among the first to apply utility theory to iterative design process such as material selection problems for automotive structural frames or automobile bumper systems [27, 28, 29, 30].

Utility theory is based on a group of well proven metric axioms, which are reviewed in Appendix A [18]. The annihilation condition is not satisfied by utility theory. This restriction implies that if the level of preference in one or more attributes is zero, then the overall satisfaction with the design is also zero. Once the axioms in Appendix A are accepted by designers and engineers, utility theory can be employed. Then, an evaluation function called overall utility is developed based on the decision maker’s preference and willingness to trade off design attributes. The overall utility of an alternate design is a function of a set of performances attributes and represents an overall measure of the total worth. In fact, engineers compare the overall utility of alternative designs as a function of the level of several performance characteristics: the alternate designs are evaluated as a function of their performances in different areas. This evaluation function, which is similar to the membership function \( \mu_D(\bar{x}) \) (equation (6)) in fuzzy set theory, is non-linear as shown in equation (16) [30]. The total number of attributes and constraints treated as attributes in the problem formulation is \( k+m \).

The overall utility \( U(y) \) is written in mathematical form as follows:

\[
U(y) = \frac{1}{K} \left[ \prod_{p=1}^{k+m} \left( K_{k,p} U_p(y_p) + 1 \right) - 1 \right] \tag{16}
\]

where \( U(y) \) is the overall utility of an alternative characterized by \( y(\bar{x}) \), \( y_p \) is the performance level for attribute \( p \), and \( U_p(y_p) \) is a single attribute utility function of \( p \).
p varies from 1 to k+m. There is a similarity between $U_p(y_p)$ of the utility theory and $\mu_{y_p}(\bar{x})$ of the fuzzy sets methods.

Finally, the overall utility is maximized with respect to the design variables. The optimization results in an optimal combination of attribute levels that provides the greatest overall utility.

Preferential and utility conditions are both required to apply equation (16). Preferential independence means that the decision maker always wants to increase the utility of an attribute regardless of the levels of other attributes. In other words, preferential independence between two attributes, such as stiffness and mass, signifies that the designer consistently prefers higher stiffness to lower stiffness when the weight is constant. The utility independence states that the engineer’s behavior when facing risk in one attribute remains unchanged when the levels of another attributes change. This implies that the shape of a single attribute utility function $U_l$ of the attribute l as shown in Figure 4, is not altered by changes in the levels of other attributes. Let us consider an example with two attributes a and b. The designer wants to maximize both attributes a and b. Figures 5 and 6 show how the overall utility changes when the two attributes are independent and dependent, respectively.

![Figure 4: Single attribute utility function $U_l(y_l)$ of the attribute l.](image-url)
In equation (16), $k_p$ and $K$ are scaling constants. The value of $K$ is obtained by solving the following equation:

$$1 + K = \prod_{p=1}^{k+m} (1 + K_{k_p})$$  \hspace{1cm} (17)

Equation (17) means that the overall utility of a design, whose attributes have all utilities equal to one, must be one.
A condition, called additive independence condition is satisfied if the $k_p$'s add up to one:

$$\sum_{p=1}^{k+m} k_p = 1$$  \hfill (18)

If the additive independence condition is satisfied, then the attributes are independent. In this case, the overall utility function can be simplified as shown in equation (19):

$$U(y) = \sum_{p=1}^{k+m} k_p U_p(y_p)$$  \hfill (19)

The additive independence condition (equation (18)) described by Fishburn is too restrictive [9]. Therefore, $U(y)$ in equation (19) is not valid even for simple decision making problems. As a result, the more general form, which is also called non-linear multiplicative form and is described in equation (16), will be used in this study.

As mentioned earlier, the single attribute utility functions and the scaling constants are evaluated by means of the certainty equivalent method, also called lottery method [28]. This method is briefly described below. In the following discussion, the engineer always tries to minimize $k+m$ attributes.

First, the best and worst values, respectively $y_p \text{ best}$ and $y_p \text{ worst}$ of each single attribute $p$ are assessed.

Second, the single attribute utility functions are determined. An iterative procedure is employed to find the point at which the designer is indifferent between two alternatives: a “certain” alternative and an “uncertain” (or “lottery”) one. Only one attribute $p$ is considered at a time when deriving the utility function $U_p(y_p)$. The “certain” alternative has a performance level for attribute $U_p$ that takes a certain value $h$. The “uncertain” design alternative is a design for which there is uncertainty in the level of performance of the attribute: the performance of the attribute of the “lottery” design will either be $y_p \text{ best}$ with a probability $\text{Prob}$ or $y_p \text{ worst}$ with a probability $1 - \text{Prob}$. In fact, the engineer decides
between designs with uncertain performance levels described in a sequence of questions, such as:

*Which option do you prefer? The “certain” option, the “uncertain” option or are you indifferent?*

If the engineer prefers the “certain” alternative, the attribute $p$ from this alternative is increased by a small increment. If the engineer chooses the “lottery” alternative, then the attribute $p$ is decreased by a small increment. Then, the iterative process is resumed. Once the engineer is indifferent, the iterative procedure is stopped. Then, the single attribute utility function is assumed to be:

$$U_p(y_p) = \text{Prob} \; U_p(y_{p \text{ best}}) + (1 - \text{Prob}) \; U_p(y_{p \text{ worst}})$$  \hspace{1cm} (20)

The utility function $U_p$ of attribute $p$ at its most desirable value is equal to one and at its worst value is assumed to be zero:

$$\begin{cases} U_p(y_{p \text{ best}}) = 1 \\ U_p(y_{p \text{ worst}}) = 0 \end{cases} \hspace{1cm} (21)$$

As a result, equation (20) reduces to:

$$U_p(y_p) = \text{Prob} + (1 - \text{Prob}) \cdot 0 = \text{Prob}$$ \hspace{1cm} (22)

Then, the value of the probability $\text{Prob}$ is changed and the iterative procedure is started again to find the value of the attribute which corresponds to an indifferent answer from the designer. This procedure enables an assessment of the single attribute utility functions.

Third, scaling constants $k_p$ are assessed using the lottery method but considering all the attributes simultaneously. As it will shown below, the value of $k_p$ is assumed to be equal to the overall utility $U(y)$ where attribute $p$ is at its most desirable level and all the other attributes are at their worst values.

$$U(y_{1 \text{ worst}}, y_{2 \text{ worst}}, \ldots, y_{p \text{ best}}, \ldots, y_{k+m \text{ worst}}) = k_p$$ \hspace{1cm} (23)
The overall utility of a design where all the attributes are equal to their best values is one, and the overall utility of a design where all the attributes are equal to their worst values is zero as shown in equation (24):

\[
\begin{align*}
U(y_{1 \text{ best}}, y_{2 \text{ best}}, \ldots, y_{k+m \text{ best}}) &= 1 \\
U(y_{1 \text{ worst}}, y_{2 \text{ worst}}, \ldots, y_{k+m \text{ worst}}) &= 0
\end{align*}
\] (24)

Then, the overall utility function is assumed to be:

\[
U(y_{1 \text{ worst}}, \ldots, y_{p \text{ best}}, \ldots, y_{k+m \text{ worst}}) = \text{Prob} \ U(y_{1 \text{ best}}, \ldots, y_{k+m \text{ best}}) + (1 - \text{Prob}) \ U(y_{1 \text{ worst}}, \ldots, y_{k+m \text{ worst}})
\] (25)

Plugging equation (24) in equation (25) yields equation (23).

An iterative procedure, similar to the one described in the previous paragraph, is employed to determine \(k_p\). The designer has to choose between two alternatives, one with certainty and another one with uncertainty towards the performances levels. The certain alternative has performance levels \(y_{1 \text{ worst}}, \ldots, y_{p \text{ best}}, \ldots, y_{k+m \text{ worst}}\), while the uncertainty alternative consists of two designs whose performance levels are set at the most desirable values and worst values, respectively.

An example with two objectives to minimize, the weight and the cost, is studied in detail as an application of the lottery method. This example is illustrated by Figures 7 and 8. The values of the weight and the cost are represented by the variables \(y_1\) and \(y_2\), respectively.

First, the best and worst values of attributes are determined:

\[
\begin{align*}
y_{1 \text{ best}} &= 100 \text{ lb} \\
y_{1 \text{ worst}} &= 400 \text{ lb}
\end{align*}
\] (26)

\[
\begin{align*}
y_{2 \text{ best}} &= $10,000 \\
y_{2 \text{ worst}} &= $90,000
\end{align*}
\]

Second, single attribute utility function, \(U_1(y_i)\), is assessed. Suppose the engineer is completely satisfied if the weight is less or equal than 100 lb and totally unhappy if the weight is greater or equal to 400 lb. We assign a value of one to \(U_1(100 \text{ lb})\) and zero to \(U_1(400 \text{ lb})\). The engineer has the choice between two alternatives as shown in Figure 7.
The weight of the “certain” alternative in Figure 7 will be 150 lb whereas the weight of the “lottery” design will either be 100 lb with a probability Prob of 0.6 or 400 lb with a probability 1 - Prob of 0.4. As long as the “certain” alternative is preferred, the weight of the “certain” alternative is increased by an increment of 15 lb. For instance, if the designer chose the “certain” design with a weight of 150 lb, then the next question is:

*Which option do you prefer? The “certain” option (y1 = 175 lb), the “uncertain” option (probability of 0.6 and weight of 100 lb, probability of 0.4 and weight of 400 lb) or are you indifferent?*

The iterative procedure is stopped as soon as the engineer is indifferent. For this example, assume that the engineer is indifferent for a weight equal to 175 lb. Then, the single attribute utility of 175 lb is, according to equation (20):

\[ U_1(175 \text{ lb}) = \text{Prob} \cdot U_1(y_{1 \text{ best}}) + (1 - \text{Prob}) \cdot U_1(y_{1 \text{ worst}}) \]  

(27)

or equation (21) states that:

\[
\begin{align*}
U_1(y_{1 \text{ best}}) &= U_1(100 \text{ lb}) = 1 \\
U_1(y_{1 \text{ worst}}) &= U_1(400 \text{ lb}) = 0
\end{align*}
\]  

(28)

Finally, equation (27) is equivalent to:

\[ U_1(175 \text{ lb}) = \text{Prob} + (1 - \text{Prob}) \cdot 0 = \text{Prob} = 0.6 \]  

(29)

Three values of \( U_1 \) have been determined so far: \( U_1(y_{1 \text{ best}}) \), \( U_1(y_{1 \text{ worst}}) \) and \( U_1(175 \text{ lb}) \). Then, the probability Prob is modified and the value of the weight corresponding to an indifferent answer is found. Next the single attribute utility function for the cost is determined in a similar way.
**Figure 7:** Lottery to assess the single attribute utility function $U_1(y_1)$ for weight.

Then, the scaling constants are assessed. We use the lottery method to find $k_1$ but, this time, all the attributes are considered simultaneously. Figure 8 shows the two design alternatives which have the best and worst possible weight and cost, respectively.

$$U(y_{1\text{ best}}, y_{2\text{ worst}}) = \text{Prob } U(y_{1\text{ best}}, y_{2\text{ best}}) + (1 - \text{Prob}) U(y_{1\text{ worst}}, y_{2\text{ worst}})$$  \hspace{1cm} (30)

$$\begin{aligned}
U(y_{1\text{ best}}, y_{2\text{ best}}) &= U(100 \text{ lb}, $100000) = 1 \\
U(y_{1\text{ worst}}, y_{2\text{ worst}}) &= U(400 \text{ lb}, $90000) = 0
\end{aligned}$$ \hspace{1cm} (31)

Substituting equation (31) into equation (30), the overall utility is:

$$U(y_{1\text{ best}}, y_{2\text{ worst}}) = \text{Prob } U(100 \text{ lb}, $10,000) + (1 - \text{Prob}) U(400 \text{ lb}, $90,000)$$

which is equal to

$$U(y_{1\text{ best}}, y_{2\text{ worst}}) = \text{Prob} = k_1$$ \hspace{1cm} (32)

Then, the scaling constant $k_2$ is assessed by considering the certain alternative $y_{1\text{ worst}}, y_{2\text{ best}}$.

Note that the scaling constants $k_p$ do not represent weighting factors and do not reflect the importance of an attribute. They are related to the trade-offs between attributes the designer is disposed to accept.
**Figure 8:** Lottery to assess the scaling constant $k_1$ for weight.

### 1.3.3 Comparison of Utility Theory with Fuzzy Sets and Crisp Optimization

Many publications have compared utility theory, fuzzy sets and crisp optimization. In the following sections, the literature review focuses on which technique is more meaningful and more consistent to perform multiobjective optimization under uncertainty.

Antonsson, et.al., compared utility theory with the method of imprecision, which is the same as the fuzzy set method [18]. They explained in detail the metrics axioms used by both techniques as mentioned in Appendix A. According to these authors, utility theory is made of well proven metrics and has a more complete axiomization due to strong theoretical foundations. Fuzzy set theory has a lower strength of axiomization because it has been less studied than utility theory from a pure theoretical point of view. One major difference between the two methods is due to the annihilation condition. This restriction states that the overall utility or satisfaction is zero if the degree of satisfaction with respect to one or more attributes is zero. This annihilation restriction is not satisfied by the multiplicative form of the utility theory described in equation (16). Clearly, this annihilation condition is acceptable in many cases of engineering design according to Otto and Antonsson [18]. For example, according to utility theory, a decrease in cost can
always compensate an increase of the material stress. However, the material stress may exceed the stress limit of this material and cause fracture of the specimen. In this case, utility theory is not appropriate because one attribute, the material stress, can not be always traded off.

Carnahan and Thurston compared utility analysis and fuzzy sets for multiattribute design problems [29]. Utility analysis requires a large amount of quantitative inputs from the engineer. These inputs allow the assessments of each single utility function and each scaling constant using the lottery method. Carnahan and Thurston proposed two approaches for the determination of the membership functions which are similar to the single utility functions of the performance characteristics. The first one is common with utility theory: this is the lottery method. The second one doesn’t require quantitative inputs but needs expressions such as “very low”, “low”, “high”, “very high”. This method converts automatically these fuzzy inputs into numerical inputs. In that case, Carnahan and Thurston observed that fuzzy sets are appropriate at the very beginning of the iterative design process because they can accommodate linguistic variables. Group meetings are making extensive use of these variables. These gatherings are very frequent at the earliest stage of design when engineers deal with overall concepts of design. As a result, Carnahan and Thurston concluded that fuzzy sets should be used at the early stage of design whereas utility theory should be limited to later stages.

Thurston and Sun studied the advantages of utility analysis over crisp optimization [31]. Traditionally, the solution to a crisp optimization problem is a Pareto optimum which is, in fact, a set of solutions. A Pareto optimum is obtained once it is no longer possible to improve one attribute without worsening at least another one. It is located along the boundaries of the feasible region. Then, the decision maker has two options. The first one is to select a preferred solution among this set. In that case, one attribute will be fully satisfied while the levels of satisfaction of other attributes will typically achieve low values, specially in case of conflicting attributes. The second option is to determine a solution that will partially satisfy each objective. This implies that the
designer should find a compromise solution by fixing the values of the trade-offs between the objectives. In utility analysis, the concept of one design attribute being less important than another is meaningless. The utility theory presents a detailed methodology for finding these trade-offs. The solution is obtained by the decision making itself and not from the engineer because the utility analysis generates only one optimum solution. Furthermore, Thurston also focused on the issue of the assessment of these trade-offs between conflicting attributes with utility theory [28]. Because the overall utility function is not linear, the trade-offs are not linear. The willingness to trade one performance characteristic for another changes as the performances are modified. Let us consider an example with two attributes \( a \) and \( b \) which levels of performance are \( y_a \) and \( y_b \), respectively. The designer is trying to maximize both attributes. When the attribute \( a \) is very high, the designer is ready to accept a smaller improvement in the attribute \( b \) than if \( a \) were small. This property is shown in Figure 9. The non-linearity of the utility function allows such phenomenon to happen during the iterative design process.

![Graph showing iso-utility curves and trade-offs between attributes.](image)

**Figure 9:** Iso-utility curves and trade-offs between attributes.

The weighted methods from fuzzy set theory, as mentioned in equation (33), require the assessment of weight factors which reflect the trade-offs [8].

25
\[
\mu_D(\bar{x}) = \gamma \min_{i=1,\ldots,k} \left\{ \mu_{f_i}(\bar{x})^{w_i}, \mu_{g_j}(\bar{x})^{w_j} \right\} + \frac{(1-\gamma)}{\sum_{i=1}^{k} w_i + \sum_{j=1}^{m} w_j} \left( \sum_{i=1}^{k} w_i \mu_{f_i}(\bar{x}) + \sum_{j=1}^{m} w_j \mu_{g_j}(\bar{x}) \right)
\]

where \( w_i, w_j \) and \( \gamma \) are weight factors. \( \mu_{f_i}(\bar{x}) \) and \( \mu_{g_j}(\bar{x}) \) are membership functions.

The coefficients \( w_i \) and \( w_j \) are very often thought as expressing the relative importance of the attributes associated with them. But they are fixed by the decision maker before the iterative design optimization is started. They are dictated by the design under consideration before the optimization. As a result, they reflect the designer's perception of the strength or weakness of the initial design guess. During the optimization, the trade-off among the attributes remain the same whatever the values of the attributes are.

Osyczka asserted that utility analysis is worth performing despite its relatively high computational cost compared to traditional single-objective optimization [16]. He acknowledged that the utility functions are not trivial to attain using the lottery method and request an important effort from the decision maker. The designer has to answer to a sequence of questions that reveal his willingness to trade off attributes. Nevertheless, the advantages of getting the values for the trade-offs using a detailed methodology instead of intuition or empiricism overcome the high computational cost according to Osyczka.

### 1.3.4 Rapid Analysis of Complex Systems Using Approximate Tools

In optimization, we need to analyze many design alternatives. For example, in design of T-shaped joints, one needs to evaluate the I/O stiffness at each step of the iterative design optimization. Traditionally, stiffness is found either by performing experiments or using finite element analysis (FEA). But these two methods are not well suited for optimization problems because there are very time consuming. As a result, approximate tools will be used in this study to estimate the stiffness of the B-pillar to
rocker joint: the stiffness will be an output of these tools. Zhu developed a response surface polynomial and a neural network for this purpose [36]. Because these tools are approximate, the value of the stiffness will be checked with FEA for some design alternatives such as the Pareto set and the final optimum designs.

Before employing these two tools, it is necessary to describe briefly the set of design variables used in this study. Then, the neural networks and the response surface polynomials will be presented in detail.

### 1.3.4.1 Parameterization of the Joint

The determination of the design variables used to represent the B-pillar to rocker joint is an important step prior to the optimization process. These variables should be independent as much as possible so that the optimization can be performed efficiently. Given a set of design parameters, the joint should also be defined uniquely.

The parameterization of the joint in this study will be the same as the one used by Zhu [36]. The joint is defined with 35 design parameters as shown in Figures 10 to 12 and Table 1.

![Diagram of a joint](image)

**Figure 10:** Parameters defining the joint side view.
The dotted line represents the door outboard panel.

**Figure 11:** Parameters defining the rocker cross section.

**Figure 12:** Parameters defining the transition section from rocker to B-pillar.
Table 1: List of the design variables that define the joint.

<table>
<thead>
<tr>
<th>#</th>
<th>Variable name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T1 (mm)</td>
</tr>
<tr>
<td>2</td>
<td>T2 (mm)</td>
</tr>
<tr>
<td>3</td>
<td>T3 (mm)</td>
</tr>
<tr>
<td>4</td>
<td>T4 (mm)</td>
</tr>
<tr>
<td>5</td>
<td>T5 (mm)</td>
</tr>
<tr>
<td>6</td>
<td>T6 (mm)</td>
</tr>
<tr>
<td>7</td>
<td>C1 (mm)</td>
</tr>
<tr>
<td>8</td>
<td>C2 (mm)</td>
</tr>
<tr>
<td>9</td>
<td>C3 (mm)</td>
</tr>
<tr>
<td>10</td>
<td>C4 (mm)</td>
</tr>
<tr>
<td>11</td>
<td>B1/B2</td>
</tr>
<tr>
<td>12</td>
<td>B2 (mm)</td>
</tr>
<tr>
<td>13</td>
<td>B3 (mm)</td>
</tr>
<tr>
<td>14</td>
<td>B4/B1</td>
</tr>
<tr>
<td>15</td>
<td>H1 (mm)</td>
</tr>
<tr>
<td>16</td>
<td>α₁ (°)</td>
</tr>
<tr>
<td>17</td>
<td>H3 (mm)</td>
</tr>
<tr>
<td>18</td>
<td>H4 (mm)</td>
</tr>
<tr>
<td>19</td>
<td>H5 (mm)</td>
</tr>
<tr>
<td>20</td>
<td>H6 (mm)</td>
</tr>
<tr>
<td>21</td>
<td>α₂ (°)</td>
</tr>
<tr>
<td>22</td>
<td>AU (mm)</td>
</tr>
<tr>
<td>23</td>
<td>BU/B4</td>
</tr>
<tr>
<td>24</td>
<td>HL (mm)</td>
</tr>
<tr>
<td>25</td>
<td>RP (mm)</td>
</tr>
<tr>
<td>26</td>
<td>RPL (mm)</td>
</tr>
<tr>
<td>27</td>
<td>X_{ini} (mm)</td>
</tr>
<tr>
<td>28</td>
<td>ZZZU (mm)</td>
</tr>
<tr>
<td>29</td>
<td>ZZL (mm)</td>
</tr>
<tr>
<td>30</td>
<td>ZZM (mm)</td>
</tr>
<tr>
<td>31</td>
<td>W (mm)</td>
</tr>
<tr>
<td>32</td>
<td>α₀ (°)</td>
</tr>
<tr>
<td>33</td>
<td>PF1 (mm)</td>
</tr>
<tr>
<td>34</td>
<td>RL (mm)</td>
</tr>
<tr>
<td>35</td>
<td>PL (mm)</td>
</tr>
</tbody>
</table>
1.3.4.2 Neural Networks

Early work on neural networks was motivated by studies of the human brain. Scientists attempted to create a tool that could reproduce human functions such as learning.

The idea of neural network is to construct a tool that represents the mapping from the space of design variables to the performance of the joint [4]. Inputs to the neural network are selected design variables and outputs are the performance characteristics. Neural networks are learning from examples like humans and need to be trained. Once a neural network has been trained, it can predict the response of a system to inputs that are not in the training set.

The neural network used in this study was developed by Zhu. Zhu trained the neural network using sets of inputs and outputs that were known [36]. The training of the neural network was accomplished using 57 joints whose stiffness was estimated with FEA. The designs used for training were determined using Box-Behnken experimental designs [5]. Zhu performed several studies to rank the design variables based on their effect on I/O stiffness [36]. Then, he selected the seven most important variables: B1/B2, T1, RPL, B4/B1, B2, H3 and α1 (Figures 10 to 12 and Table 1). Even when using this small set of parameters, the neural network was found to predict stiffness with good accuracy [36].

1.3.4.3 Response Surface Polynomials

The response surface polynomial determined by Zhu is used in this study [36]. The stiffness of the joint is approximated with a second order polynomial. This polynomial is a function of the same set of design parameters as the neural networks. The same data are also used for the fitting of the polynomial.
1.4 Objectives

The objective of the present research is to find a new design of an automotive joint which maximizes the overall designer satisfaction. In order to achieve this goal, we need to develop a method that accounts for all the objectives and the fuzziness in these objectives. This is the subject of this thesis.

In this study, three multiobjective fuzzy approaches, i.e., the conservative, the aggressive and the moderate methods will be considered to solve this particular problem. The fuzzy set methods will be applied using a neural network and a response surface polynomial. These approximate tools estimate the stiffness of a joint rapidly compared to FEA.

In addition, a simple optimization problem, which has two objectives, will be solved using the above three methods as well as utility theory. This simple example will allow for a comparison of these methods.

1.5 Thesis Organization

This section reviews the arrangement of the contents of this thesis.

In chapter 2, the detailed formulation of a multiobjective fuzzy optimization problem is discussed as well as the formulation of a multiattribute utility analysis problem.

In chapter 3, the methodology described in chapter 2 is applied to two examples of the same automobile joint. The first example, which has only two fuzzy objective functions and crisp constraints, is studied using fuzzy sets methods and utility theory. The second example, which considers a larger number of objectives and some constraints fuzzy, is studied using fuzzy sets only.

In Chapter 4, the results are presented. First, the results from the simple example are considered. Second, the optimum design obtained from the optimization of the second example is investigated. The comparisons of all the methods and of the costs of
optimization using neural network as an approximate tool and FEA are made in this chapter.

Finally, the concluding remarks of this study are given.
2 Formulation of a Multiobjective Optimization Problem Using Fuzzy Sets and Utility Theory

The methodology for formulating a multiobjective optimization problem is reviewed in this chapter.

First, in section 2.1, we describe the formulation of a multiobjective fuzzy optimization problem which consists of four tasks. These tasks are the assessment of design variables, constraints, objective functions and memberships for all these entities.

Second, in section 2.2, the formulation of a multiattribute utility analysis problem is reviewed. The determination of design variables, single attribute utility functions, scaling constants and overall utility function is explained.

The design of engineering systems is a complex process. There are many possibilities and factors that can be considered during the problem formulation phase. As a result, a problem can be formulated in several ways. The decision maker’s task is to find the appropriate way to do it by following well-defined procedures.

2.1 Formulation of a Multiobjective Fuzzy Optimization Problem

It is critical to formulate a multiobjective fuzzy optimization problem correctly. Typically, the decision maker needs to transcribe a fuzzy verbal description of the problem into a well-defined mathematical model.

If we assume that all the objective functions have to be maximized, a crisp optimization problem can be stated as follows:
Maximize: \[ f(\bar{x}) \] (34)

Subject to: \[ g_j(\bar{x}) \leq b_j \] (35)

where \( f(\bar{x}) \) is the objective function and \( g_j(\bar{x}) \leq b_j \) are the constraints.

The procedure for formulating a multiobjective optimization problem using fuzzy sets has four stages. First, the engineer identifies a set of design variables, a set of constraints for the system, the objective functions to be minimized and the membership functions for all the entities. The problem formulation process begins by identifying the design variables. This first step will be reviewed in section 2.1.1. Constraints will be considered in the formulation procedure because all systems are designed to perform within a given set of constraints. This will be the subject of section 2.1.2. Then, the engineer needs to define an objective function to judge whether or not a given design is better than another. This part will be discussed in section 2.1.3. Once the set of design variables, the constraints and the objective function are set up, the conversion to fuzzy sets takes place. This is done by gradation of the boundaries of the design variables, the constraints and the objectives. As a result, the last step of the formulation problem is to find expressions of the membership functions for all these entities as shown in paragraph 2.1.4.

Finally, a multiobjective fuzzy optimization problem can be expressed as:

Find: \[ \bar{x} \] (36)

which

Maximize: \[ \mu_D(\bar{x}) \] (37)

where \( \bar{x} \) is a design vector and \( \mu_D(\bar{x}) \) measures the degree to which \( \bar{x} \) satisfies the decision maker when all objectives and constraints are considered simultaneously.
2.1.1 Design Variables

It is clear that the formulation of an optimum design problem hinges on proper identification of the design variables.

First, the decision maker parameterizes the structure. The design variables should be independent. The property of independence means that one design variable can not be expressed as a function of one or several other design variables. Variables that are meaningful to the engineer, such as length of beams, thickness of plates, should be selected. This will make the assessment of the constraints and the objective function much easier. Once the parameterization is achieved, i.e. all the design variables are determined, lower and upper bounds of each variable are defined. These values are dictated by manufacturing, packaging and styling limitations.

2.1.2 Constraints

All systems are designed to perform within a set of constraints imposed by a given environment. A design that satisfies all the requirements is called a feasible design. The constraints limit and restrict the alternative set of design variables by defining a feasible domain.

Each constraint must be influenced by one or more design variables so that it is meaningful and has influence on the optimum system.

If an initial constraint is forgotten in the formulation, the optimum solution would most likely violate it because optimization methods tend to exploit the mistakes in the model used in design. Therefore, it is critical to identify all constraints.
2.1.2.1 Types of Constraints

A multiobjective optimization problem has several types of constraints.

A distinction among the constraints can be done from a mathematical point of view.

Generally, a problem possesses inequality as well as equality constraints. A feasible design must satisfy precisely all equality constraints. The feasible region for an equality constraint is much smaller than the one for the same constraint expressed as an inequality.

Typically, a design problem can also have linear as nonlinear constraints. If there are only first-order terms in design variables, then the constraint is linear.

There can also be crisp and fuzzy constraints. Very few constraints are crisp in real life problem. For instance, a crisp constraint on the tip deflection of a cantilever beam can be expressed as follows. A structure that deflects by 4.99 mm is acceptable but unacceptable if it deflects by 5.01 mm. When the design is acceptable, the level of satisfaction is one. When it is unacceptable, the level of satisfaction is equal to zero. This is clearly unrealistic. Most constraints should be described using fuzzy sets. Fuzzy constraints have continuous grades of satisfaction between zero and one. Fuzzy sets are suitable for modeling the uncertainty in the formulation of multiobjective optimization problems which is partly due to the fact that satisfaction of constraint is a matter of degree.

Another distinction among the constraints can be done from a physical point of view.

For instance, the side constraints define explicitly the upper and lower bounds of the design variables. These constraints are necessary in most applications. For instance, if the objective is the reduction of the mass of a structure, the thickness of all the elements
will tend to zero. Therefore, side constraints on the thicknesses are needed to avoid design alternatives that are impossible to build.

Styling constraints affect the appearance of the product. An example of a such a constraint is a requirement specifying that the lower rocker flange should be invisible to someone standing on the side of the car. These constraints are usually established at very early stage of the design.

Packaging constraints deal with the arrangement and the interaction of the components of a system in space. An example can be stated as follows: the dimensions of the rocker of a B-pillar to rocker joint should be small enough to accommodate for the seat mechanism.

The last category of constraints is manufacturing constraints that is established by manufacturing limitations. These constraints include welding and stamping constraints. For instance, there is a minimum length between bends of a plate. Another example of a manufacturing constraint is that a plate cannot be bent at a very sharp angle.

2.1.3 Objective Function

A criterion is required to evaluate the overall worth of an alternate design. This is needed to label one design as being worst or better than another. This criterion is called the objective function. It is a scalar function whose value is computed once a design alternative is specified. It must be influenced by one or more design variables because only then it can be optimized. Optimization problems will be formulated in a way that objective functions will always be minimized in this study.

Traditional approaches select the most important criterion as an objective function and the remaining criteria as constraints. However, fuzzy logic considers multiobjective optimization as a generalization of traditional single-objective optimization. Specifically, it
uses fuzzy set calculus to combine several objectives into a single one. This combination can be done using the normalized product metric from the aggressive method, the min function from the conservative method or the weighted sum aggregator.

The choice of the method to combine the preferences is dictated by the design strategy. The most crucial problem is the identification of an appropriate aggregation function which represents well the overall designer's satisfaction with respect to all attributes of the design. One approach is to choose an aggregator operator on an experimental basis. The operator must be an appropriate model of a real system behavior which can only be proven by performing empirical tests. This approach is difficult to put into practice because when a change is needed in the problem formulation phase, the designer must perform tests to check the accuracy of the fuzzy set operator. This inconvenience shows the lack of generality of the different methods. No operators are general enough for all contexts. The aggregating behavior of the operator should also be taken into consideration. The value of the membership function depends very frequently on the number of sets combined. For instance, combining fuzzy sets by the simple product operator (equation (38)) will decrease the membership value each time a new fuzzy set is introduced.

\[
\mu_D(\vec{x}^*) = \max_{\text{feasible region}} \left[ \prod_{i=1}^{k} \mu_i(\vec{x}) \cdot \prod_{j=1}^{m} \mu_j(\vec{x}) \right]
\] (38)

This behavior should usually be avoided. This is why the normalized product operator which is the operator used in the aggressive method, is employed instead of the simple product operator. Finally, the operator should be reasonably easy to compute. All these factors should be taken into consideration when choosing the appropriate aggregator operator.
2.1.4 Membership Functions

The objective function as well as the constraints are characterized by their membership functions. Building the membership function for each entity is a crucial part of the formulation problem.

The grade of a membership function indicates a subjective degree of satisfaction within given tolerances which are also called the supports of the membership. The values of these tolerances are important as well as the shape of the line connecting the level of satisfaction zero to the level equal to one. For instance, a fuzzy trapezoidal membership function is fully determined by three parameters: the value that corresponds to a level of satisfaction of zero, the value that corresponds to a level of satisfaction of one and the shape of the line connecting these two values. This connecting line can be a straight line, a convex function or a concave function.

Any form of membership function is possible: crisp, bell-shaped, exponential, hyperbolic, trapezoidal, triangular, linear, piecewise linear membership functions. The crisp and the fuzzy membership functions will be studied in detail in the following two sections.

2.1.4.1 Crisp Membership Functions

A crisp membership function is a step function: the degree of satisfaction is either zero or one. If all membership functions used in optimization are crisp, there are sharply defined boundaries for the design variables, the constraints and the objective function.
2.1.4.2 Fuzzy Membership Functions

The grade in a fuzzy set can be anything from zero to one, and this range is what makes it different from a crisp set. There is a continuous grade of satisfaction between zero and one as shown in Figure 3. These functions can be symmetric as well as non-symmetric. Instead of using a sharp boundary to separate goodness from not goodness, fuzzy sets describe ranges of soft boundaries by degrees of membership.

2.2 Formulation of a Multiattribute Utility Optimization Problem

The formulation follows four steps. First, the decision maker needs to define the design variables. Then, he evaluates the single attribute utility functions as shown in section 2.2.2. The following step consists of the assessment of the scaling constants as described in section 2.2.3. Finally, the overall utility function is computed as mentioned in section 2.2.4.

The multiattribute utility optimization problem can be stated as follow:

\[
\text{Find:} \quad \bar{x} \quad \quad (39)
\]

\[
\text{which}
\]

\[
\text{Maximize:} \quad U(y(\bar{x})) \quad \quad (40)
\]

where \( U(y(\bar{x})) \) is the overall utility of an alternative characterized by \( y(\bar{x}) \), \( y_p \) being the performance level for attribute \( p \). Note the similarity of \( U(y(\bar{x})) \) of the utility theory and \( \mu_D(\bar{x}) \) of the fuzzy sets methods.
2.2.1 Design Variables

The formulation of a multiattribute utility optimization problem requires the determination of design variables. These design variables have exactly the same meaning as the one defined during the formulation of a multiobjective fuzzy optimization problem.

2.2.2 Single Attribute Utility Functions

The single attribute utility function is evaluated by means of the certainty equivalence method with lottery questions as explained in section 1.3.2. Once the lottery questions have been answered, the single attribute utility function can be build.

2.2.3 Scaling Constants

The assessment of the scaling constants follows the same procedure as the determination of a single attribute utility function.

2.2.4 Overall Utility Function

The utility theory couples the physical relationships between design decision variables and the multiple attributes in a rigorously determined nonlinear evaluation function, called the overall utility function. This function represents the decision maker preferences. The metric that is employed to aggregate the single attribute utility function has a strong theoretical foundation: it is unique and well-defined.
3 Using Fuzzy Sets and Utility Theory to Design Automotive Joints: Problem Formulation

The methodology described in the previous chapter will be applied to the automotive joint. Two problems will be investigated: one in which only stiffness and mass are optimized and another one in which several objectives (stiffness, mass, cross sectional properties) are optimized simultaneously. Fuzzy multiobjective optimization is performed using three methods: the conservative, the aggressive and the moderate methods. The first problem is solved using both fuzzy multiobjective optimization and utility theory. The second problem is solved using fuzzy multiobjective optimization.

This chapter presents the formulation of both problems. In section 3.1, the different types of membership functions used in this study are reviewed in detail. In section 3.2, we describe the formulation of the example with two fuzzy memberships. The design variables, the constraints, the objectives and the utility functions (single attribute utility functions and overall utility function) are listed in section 3.2. In section 3.3, the example with a large number of fuzzy memberships is discussed.

In the following chapter, the terms “baseline optimum” and “baseline model” are employed. The first term refers to the optimum design that Zhu obtained by minimizing the mass considering crisp constraints on the other performance attributes (stiffness, cross sectional properties: moment of inertia and twist rigidity) [36]. The second term “baseline model” refers to a B-pillar to rocker joint of an existing car model from Ford Motor Co.
3.1 Types of Membership functions Used in Multiobjective Optimization

For this study, only linear membership functions are adopted because they are simple. Four different shapes of membership functions are used: crisp, triangular and trapezoidal (2 different shapes) as shown in Figures 13 to 16.

In the mathematical expressions and figures of the memberships, terms, such as “Min_entity” and “Max_entity” are employed. The entity can be a design variable, a constraint or an attribute. The prefixes “Min_” and “Max_” correspond to the minimum and maximum values of the entity.

If the engineer wants to consider the parameter as crisp, then the shape of the membership function is the following:

![Crisp membership function](Attachment)

**Figure 13:** Crisp membership function.

The membership of a parameter described in Figure 13 can be stated mathematically as follows:

\[
\text{Membership} = \begin{cases} 
0 & \text{if } X < \text{Min}_\text{parameter} \\
i & \text{if } \text{Min}_\text{parameter} \leq X \leq \text{Max}_\text{parameter} \\
0 & \text{if } X > \text{Max}_\text{parameter}
\end{cases}
\]
If the engineer wants to keep a parameter as close as possible to its values in the baseline model, then the membership is:

$$
\begin{align*}
\text{ST}(\text{parameter}, \text{Tol}) &= \begin{cases} 
0 & \text{if } X \leq \text{parameter}_{\text{baseline}} - \text{Tol} \\
\frac{X - \text{Min}_{\text{parameter}}}{\text{Tol}} & \text{if } \text{parameter}_{\text{baseline}} - \text{Tol} \leq X \leq \text{parameter}_{\text{baseline}} \\
\frac{X - \text{Max}_{\text{parameter}}}{-\text{Tol}} & \text{if } \text{parameter}_{\text{baseline}} \leq X \leq \text{parameter}_{\text{baseline}} + \text{Tol} \\
0 & \text{if } X \geq \text{parameter}_{\text{baseline}} + \text{Tol}
\end{cases}
\end{align*}
$$

(42)

Figure 14: Symmetric triangular membership function.

The membership of a parameter as shown in Figure 14 is given by:

In equation (42), ST stands for symmetric triangular. This type of membership function is also employed when the engineer wants to minimize the difference between two design variables. In that case, Figure 14 and equation (42) are still valid. However, the following changes are needed: X becomes the difference of two parameters, parameter_{baseline} is zero, Min_{parameter} is equal to -Tol and Max_{parameter} is equal to Tol. In fact, these changes are equivalent graphically to a horizontal translation of Figure 14.

If the engineer wants to minimize the value of an attribute, then the membership function is build as follows:
Figure 15: Trapezoidal membership function modeling an attribute for which the degree of satisfaction increases as the value of this attribute decreases up to a certain value.

The membership of an attribute as shown in Figure 15 can be written as:

\[
\mu(X, \text{Min}_\text{attribute}, \text{Max}_\text{attribute}) = \begin{cases} 
1 & \text{if } X \leq \text{Min}_\text{attribute} \\
\frac{\text{Max}_\text{attribute} - X}{\text{Max}_\text{attribute} - \text{Min}_\text{attribute}} & \text{if } \text{Min}_\text{attribute} \leq X \leq \text{Max}_\text{attribute} \\
0 & \text{if } X \geq \text{Max}_\text{attribute}
\end{cases}
\]  

(43)

Equation (43) applies also to a constraint or a parameter that the engineer tries to minimize. In Figure 15, the degree of satisfaction of the attribute is equal to one for small values of the attribute and starts to decrease to zero for higher values.

If the engineer wants to maximize the value of an attribute, then the following membership is chosen:

Figure 16: Trapezoidal membership function modeling an attribute a for which the degree of satisfaction increases as the value of this attribute increases up to a certain value.
The membership function is expressed as follows:

$$\beta(X, \text{Min\_attribute}, \text{Max\_attribute}) = \begin{cases} 0 & \text{if } X \leq \text{Min\_attribute} \\ \frac{\text{Min\_attribute} - X}{\text{Min\_attribute} - \text{Max\_attribute}} & \text{if } \text{Min\_attribute} \leq X \leq \text{Max\_attribute} \\ 1 & \text{if } X \geq \text{Max\_attribute} \end{cases} \quad (44)$$

Equation (44) applies also to a constraint or a parameter that the engineer wants to minimize. Figure 16 shows that the degree of satisfaction of the attribute increases up to the one and stays equal to one for higher values of the attribute.

### 3.2 First Problem: Application of Fuzzy Sets and Utility Theory to an Example with Two Fuzzy Objective Functions

In this example, we optimize a T-shaped joint (Figure 1) considering two objectives: mass and stiffness under Inboard/Outboard bending. Other requirements on packaging, styling and manufacturability are treated as crisp constraints. The joint is optimized using both fuzzy sets and utility theory to model the degree of satisfaction with respect to the stiffness and the mass.

#### 3.2.1 Design Variables

The design variables were chosen during the parameterization procedure. 35 independent design variables defined completely the geometry of the B-pillar-to-rocker joint as shown in Figures 10 to 12 [36]. Among these 35 design variables, 14 are fixed to the values of the baseline design as shown in Table 2. These fixed parameters are either global parameters that are chosen at the early stage of design or constants, such as the width of the flanges. The rest parameters can vary within ranges dictated by the manufacturing and the packaging limitations as shown in Table 3.
Table 2: List of the design variables that are kept constant in this study.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Value for the baseline design</th>
</tr>
</thead>
<tbody>
<tr>
<td>T4 (mm)</td>
<td>1.35</td>
</tr>
<tr>
<td>T5 (mm)</td>
<td>0.80</td>
</tr>
<tr>
<td>T6 (mm)</td>
<td>2.20</td>
</tr>
<tr>
<td>AU (mm)</td>
<td>163.6</td>
</tr>
<tr>
<td>BU/B4</td>
<td>0.95</td>
</tr>
<tr>
<td>X Ini (mm)</td>
<td>618.7</td>
</tr>
<tr>
<td>ZZU (mm)</td>
<td>18.61</td>
</tr>
<tr>
<td>ZZL (mm)</td>
<td>24.94</td>
</tr>
<tr>
<td>ZZM (mm)</td>
<td>31.09</td>
</tr>
<tr>
<td>W (mm)</td>
<td>17.80</td>
</tr>
<tr>
<td>(\alpha_p) (°)</td>
<td>73.80</td>
</tr>
<tr>
<td>RL (mm)</td>
<td>1350</td>
</tr>
<tr>
<td>PL (mm)</td>
<td>697.6</td>
</tr>
<tr>
<td>PF1 (mm)</td>
<td>81.82</td>
</tr>
</tbody>
</table>

In the first problem, all the parameters are considered crisp. The characteristics of the membership function of each design variable are given in Table 3; specifically this table shows the value of the baseline model, the range of variation, the lower and the upper bounds and the shape of the membership function. In Table 3, the lower bounds and upper bounds correspond to the value of \(\text{Min\_parameter}\) and \(\text{Max\_parameter}\) from equation (41), respectively.
Table 3: List of the crisp design variables with the first example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Value for the baseline model</th>
<th>% of variation</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Shape of the membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1 (mm)</td>
<td>0.80</td>
<td>30</td>
<td>0.56</td>
<td>1.04</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>T2 (mm)</td>
<td>0.91</td>
<td>30</td>
<td>0.637</td>
<td>1.18</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>T3 (mm)</td>
<td>1.00</td>
<td>30</td>
<td>0.70</td>
<td>1.30</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>C1 (mm)</td>
<td>28.99</td>
<td>10</td>
<td>26.09</td>
<td>31.89</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>C2 (mm)</td>
<td>86.12</td>
<td>10</td>
<td>77.51</td>
<td>94.73</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>C3 (mm)</td>
<td>78.61</td>
<td>10</td>
<td>70.76</td>
<td>86.47</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>C4 (mm)</td>
<td>52.51</td>
<td>10</td>
<td>47.26</td>
<td>57.76</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.723</td>
<td>15</td>
<td>0.651</td>
<td>0.832</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B2 (mm)</td>
<td>78.62</td>
<td>10</td>
<td>70.76</td>
<td>86.48</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>38.21</td>
<td>10</td>
<td>34.39</td>
<td>42.03</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.06</td>
<td>5.7</td>
<td>1.00</td>
<td>1.12</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>65.17</td>
<td>10</td>
<td>58.65</td>
<td>71.69</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>α1 (°)</td>
<td>72.61</td>
<td>4</td>
<td>69.71</td>
<td>75.52</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H3 (mm)</td>
<td>121.3</td>
<td>5</td>
<td>115.2</td>
<td>127.3</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H4 (mm)</td>
<td>62.26</td>
<td>10</td>
<td>56.03</td>
<td>68.49</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>16.08</td>
<td>10</td>
<td>14.47</td>
<td>17.69</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>6.319</td>
<td>10</td>
<td>5.69</td>
<td>6.95</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>α2 (°)</td>
<td>71.51</td>
<td>10</td>
<td>64.36</td>
<td>78.66</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>HL (mm)</td>
<td>130.3</td>
<td>10</td>
<td>117.2</td>
<td>143.3</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>217.4</td>
<td>10</td>
<td>195.7</td>
<td>239.14</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>RPL (mm)</td>
<td>300.0</td>
<td>18.75</td>
<td>243.7</td>
<td>356.2</td>
<td>Crisp, Figure 13</td>
</tr>
</tbody>
</table>

3.2.2 Constraints

This section presents all the constraints in the problem statement. There are 19 constraints which are considered crisp. These crisp constraints express styling, packaging and manufacturing requirements. Because these requirements are taken into account using crisp constraints, a design either satisfies fully or violates these constraints.
Mathematically, a constraint is represented by a variable $c_i$ (i=1 to 19) which depends on the design variables defining the joint. If this variable is non negative, then the design satisfies the constraint. Otherwise, the design violates it.

The constraints are listed below. Zhu has already determined most of them in his thesis [36]. The constraints are normalized with respect to some nominal values of the parameters involved. All the variables are shown in Figures 10 to 12.

**Styling constraints:**

- The width of the door should be smaller or equal to (B2+C1),

$$c_1 = \frac{B2 + C1}{Door\_width} - 1 \geq 0$$  \hspace{1cm} (45)

where $Door\_width$ is the width of the door.

- There should be a continuity between the shape of the door and the rocker outboard panel,

$$c_2 = -\frac{|\alpha_3 - \alpha_4|}{\Delta \alpha} + 1 \geq 0$$  \hspace{1cm} (46)

where $\alpha_4$ is a fixed angle defining the shape of the door. $\Delta \alpha$ is the tolerance between the two angles $\alpha_3$ and $\alpha_4$.

**Packaging constraints:**

- (B2-B4) should be sufficiently large so that step H5 in the outboard side of the rocker can accommodate the door edge,

$$c_3 = \frac{B2 - B4}{Door\_edge\_width} - 1 \geq 0$$  \hspace{1cm} (47)
where Door_edge_width is the width of both the lower edge of the door and the door sealant.

- The slope of step H5 in the outboard side of the rocker should not exceed a maximum value, Max_H5_slope,

\[
c_4 = \frac{B1 - B4}{H5} + 1 \geq 0
\]

(48)

- The slope of the step (H2-H1) should be greater than a minimum value, \( tan(\text{Min\_step\_angle}) \), to allow for water runoff,

\[
c_5 = \frac{H2 - H1}{B2 - B4} - \frac{1}{tan(\text{Min\_step\_angle})} \geq 0
\]

(49)

- The slope of the step (H2-H1) should not exceed a maximum value, \( tan(\text{Max\_step\_angle}) \), to accommodate for the door sealant,

\[
c_6 = -\frac{H2 - H1}{B2 - B4} \frac{1}{tan(\text{Max\_step\_angle})} + 1 \geq 0
\]

(50)

- Point D should be below point E,

\[
c_7 = \frac{B3}{\tan(\alpha_2)} + \frac{H1 + H5}{\tan(\alpha_1)} + \frac{B1}{H3} \geq 0
\]

(51)

- The part of the section between the points D and F must be concave,

\[
c_8 = D(z) + \frac{(F(z) - D(z)) \cdot (E(y) - D(y))}{F(y) - D(y)} - E(z) \geq 0
\]

(52)
where \( E(y) \) and \( E(z) \) are the \( y \) and \( z \) coordinates of the point \( E \). The same notation is used for the points \( D \) and \( F \).

- The stepout height should be large enough to accommodate the seat mechanism,
  \[
  c_9 = \frac{H3 + \frac{C2}{\tan(\alpha_2)} - H6 - H4}{\text{Stepout_height}} - 1 \geq 0
  \]  
  (53)
  where \( \text{Stepout_height} \) is the minimum value for the stepout height.

- \((B2+C2)\) should not exceed a maximum value,
  \[
  c_{10} = -\frac{B2 + C2}{\text{Max_width_rock}} + 1 \geq 0
  \]  
  (54)

- \((B2+C3)\) should not exceed a maximum value,
  \[
  c_{11} = -\frac{B2 + C3}{\text{Max_width_rock}} + 1 \geq 0
  \]  
  (55)

- The height of the rocker should not exceed a maximum value,
  \[
  c_{12} = -\frac{H3 + \frac{C2}{\tan(\alpha_2)} - H6}{\text{Max_height_rock}} + 1 \geq 0
  \]  
  (56)

**Manufacturing constraints:**

- The difference of thicknesses of two plates connected by spot welds should be small,
  \[
  c_{13} = -\frac{|T1 - T2|}{\Delta T} + 1 \geq 0
  \]  
  (57)
where $\Delta T$ is the maximum value of the difference of the thickness between two plates connected by spot welds.

\begin{align*}
\bullet \ c_{14} &= -\frac{|T_2 - T_3|}{\Delta T} + 1 \geq 0 & (58) \\
\bullet \ c_{15} &= -\frac{|T_3 - T_4|}{\Delta T} + 1 \geq 0 & (59) \\
\bullet \ c_{16} &= -\frac{|T_1 - T_5|}{\Delta T} + 1 \geq 0 & (60) \\
\bullet \ c_{17} &= -\frac{|T_1 - T_6|}{2 \Delta T} + 1 \geq 0 & (61) \\
\bullet \ c_{18} &= -\frac{|T_1 - T_7|}{\Delta T} + 1 \geq 0 & (62) \\
\bullet \ c_{19} &= -\frac{|T_1 - T_3|}{\Delta T} + 1 \geq 0 & (63)
\end{align*}

3.2.3 Application of Fuzzy Sets

Using fuzzy set calculus, the objective function is made of membership functions of the design variables, the constraints and the requirements on the performance targets. The membership functions for the design variables and the constraints were defined in the two previous sections.

There are 2 fuzzy objectives on the performance targets of the joint in this particular example.

- One fuzzy objective on the performance targets is the mass. The engineer wants to reduce the mass of the B-pillar to rocker joint. Any improvement in the mass of the joint
will have direct consequences on many other characteristics of a car. The gas mileage will increase as well as performance (acceleration, top speed, braking and road handling). Using equation (43), we can write:

\[ \mu_{\text{Mass}} = \mu(\text{Mass\_joint, Min\_mass, Max\_mass}) \]  \hspace{1cm} (64)

Min\_mass and Max\_mass are the minimum and maximum value of the mass respectively. These values are calculated using Rao’s method. The membership function of this objective is trapezoidal as shown in Figure 15.

- The second fuzzy objective is on the stiffness of the joint, which is obtained using a neural network to approximate the stiffness. Zhu also developed a response surface polynomial and used it as an alternate approximate tool. This tool is not employed in the first example but it will be used in the second example. In general, the higher the stiffness, the better the design is. The membership function of this objective is trapezoidal as shown in Figure 16. Using equation (44), the membership is:

\[ \mu_{K\_nn} = \beta(K\_nn, \text{Min\_K\_nn, Max\_K\_nn}) \]  \hspace{1cm} (65)

Min\_K\_nn and Max\_K\_nn are the minimum and the maximum values of the stiffness using Rao’s approach.

Finally, the objective function is calculated. The conservative, aggressive and moderate method are used to combine the two objectives (equations (64) and (65)) into a single one as described below.

3.2.3.1 Conservative Method

The formulation of the problem using the conservative method is:

Maximize: \[ \mu_D(\bar{x}) = \min\{ \mu_{\text{Mass}}, \mu_{K\_nn} \} \]  \hspace{1cm} (66)
Subject to: side constraints from Table 3 and constraints described in equations (45) to (63).

3.2.3.2 Aggressive Method

The formulation of the problem using the aggressive approach can be stated as follows:

Maximize: \[ \mu_D(\bar{x}) = \sqrt{\mu_{\text{Mass}} \cdot \mu_{K_{nn}}} \] \hspace{1cm} (67)
subject to: side constraints from Table 3 and constraints described in equations (45) to (63).

3.2.3.3 Moderate Method

Considering the moderate method, the problem to solve is:

Maximize: \[ \mu_D(\bar{x}) = (1 - w) \cdot \sqrt{\mu_{\text{Mass}} \cdot \mu_{K_{nn}}} + w \cdot \min \{ \mu_{\text{Mass}}, \mu_{K_{nn}} \} \] \hspace{1cm} (68)
subject to: side constraints from Table 3 and constraints described in equations (45) to (63).

3.2.4 Application of Utility Theory

This approach uses two single attribute utility functions. As a result, only three scaling constants \( k_1, k_2 \) and \( K \) (equation (17)) are assessed in the formulation procedure. The final step is the determination of the overall utility function.
3.2.4.1 Single Attribute Utility Function

The single attribute utility functions $U_1$ and $U_2$ are usually obtained by the certainty equivalent method and lottery questions. For the purpose of this work, they are assumed to be the same as the two membership functions mentioned in equations (64) and (65) in section 3.2.3: $U_1$ is the same as $\mu_{K_{mm}}$ and $U_2$ is the same as $\mu_{\text{Mass}}$. This strategy will allow a fair comparison of fuzzy sets and utility theory.

3.2.4.2 Scaling Constants

The constants $k_1$ and $k_2$ can be assessed by the certainty equivalent method. This method is not used in this study because of its complexity. It requires the engineer to answer a series of questions about the performance targets of the joint that can only be answered by very experienced designer. This method appears to be difficult to put into practice in this specific example. The following technique is employed to assess the constants. The decision maker is asked to grade two design alternatives, the sum of the grades being different than one. If the sum is one, then $K$ is equal to zero according to equation (17). As a result, the denominator of equation (16) becomes zero. The first grade is equal to $k_1$. It represents the level of satisfaction of a design alternative that has the highest mass and the highest stiffness possible. A high value for $k_1$ means that the engineer is pleased with the design. The second grade is equal to $k_2$. It reflects the degree of satisfaction of a design with the smallest mass and the smallest stiffness possible. The normalizing constant $K$ is obtained from equation (17), which can be stated as follows for this particular case:

$$K = \frac{1 - k_1 - k_2}{k_1 \cdot k_2}$$  \hspace{1cm} (69)
3.2.4.3 Overall Utility Function

Once the single attribute utility function and the scaling constant are assessed, the overall utility function is calculated from the following equation:

$$U(y) = \frac{1}{K} \left[ \left( K k_1 U_1(y_1) + 1 \right) \left( K k_2 U_2(y_2) + 1 \right) - 1 \right]$$  (70)

3.3 Second Problem: Application of Fuzzy Sets to an Example with a Large Number of Fuzzy Objective Functions and Fuzzy Constraints

Fuzzy multiobjective optimization will be investigated using the conservative, the aggressive and the moderate fuzzy approaches already described. In the following, we describe the design variables, constraints and objective functions.

3.3.1 Design Variables

The same design variables as in the previous example (section 3.2) are used. Ten variables are considered crisp. It is believed that the engineer is indifferent if these values are decreased or increased as long as these values are kept within certain limits. Furthermore, the engineer doesn’t require these design variables to be close to the values of the baseline model. These variables are global variables defining the pillar, such as the height of the pillar or dimensions that are not important, such as the height of a bend of the outboard cell of the rocker. These crisp design variables and their characteristics are listed in Table 4.
Table 4: List of the crisp design variables with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Value for the baseline model</th>
<th>% of variation</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Shape of the membership function</th>
</tr>
</thead>
<tbody>
<tr>
<td>C4 (mm)</td>
<td>52.51</td>
<td>10</td>
<td>47.26</td>
<td>57.76</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.723</td>
<td>15</td>
<td>0.651</td>
<td>0.832</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>38.21</td>
<td>10</td>
<td>34.39</td>
<td>42.03</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.06</td>
<td>5.7</td>
<td>1.00</td>
<td>1.12</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>65.17</td>
<td>10</td>
<td>58.65</td>
<td>71.69</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>$\alpha_1$ ($\degree$)</td>
<td>72.61</td>
<td>4</td>
<td>69.71</td>
<td>75.52</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>16.08</td>
<td>10</td>
<td>14.47</td>
<td>17.69</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>6.319</td>
<td>10</td>
<td>5.687</td>
<td>6.95</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>$\alpha_2$ ($\degree$)</td>
<td>71.51</td>
<td>10</td>
<td>64.36</td>
<td>78.66</td>
<td>Crisp, Figure 13</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>217.4</td>
<td>10</td>
<td>195.7</td>
<td>239.1</td>
<td>Crisp, Figure 13</td>
</tr>
</tbody>
</table>

If a design variable is fuzzy, then the membership function can be symmetric triangular as shown in Figure 14, or trapezoidal as shown in Figure 15. The membership functions for the thickness of the plates are modeled using symmetric triangular memberships, centered at the values of the thickness of the plates of the baseline design. We use triangular memberships because we assume that the decision maker wants the thicknesses to be as close as possible to those of the baseline design because the cost of retooling increases as a new design departs more and more from a current baseline design. The membership function of the height of the transition part from the rocker to the pillar is trapezoidal as shown in Figure 14. This choice is dictated by the engineer’s willingness to reduce this height in order to facilitate access in the car. The notation for the fuzzy membership functions of the parameters is $\mu_i$. The characteristics of the memberships of the fuzzy design variables are given in Table 5.
Table 5: List of the fuzzy design variables with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Value of the baseline model</th>
<th>% of variation</th>
<th>Lower bound</th>
<th>Upper bound</th>
<th>Shape of the membership function</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.80</td>
<td>30</td>
<td>0.56</td>
<td>1.04</td>
<td>Triangular, Figure 14</td>
<td>(\mu_1)</td>
</tr>
<tr>
<td>T2</td>
<td>0.91</td>
<td>30</td>
<td>0.637</td>
<td>1.183</td>
<td>Triangular, Figure 14</td>
<td>(\mu_2)</td>
</tr>
<tr>
<td>T3</td>
<td>1.00</td>
<td>30</td>
<td>0.70</td>
<td>1.30</td>
<td>Triangular, Figure 14</td>
<td>(\mu_3)</td>
</tr>
<tr>
<td>C1</td>
<td>28.99</td>
<td>10</td>
<td>26.09</td>
<td>31.89</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_4)</td>
</tr>
<tr>
<td>C2</td>
<td>86.12</td>
<td>10</td>
<td>77.51</td>
<td>94.73</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_5)</td>
</tr>
<tr>
<td>C3</td>
<td>78.61</td>
<td>10</td>
<td>70.75</td>
<td>86.47</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_6)</td>
</tr>
<tr>
<td>B2</td>
<td>78.62</td>
<td>10</td>
<td>70.76</td>
<td>86.48</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_7)</td>
</tr>
<tr>
<td>H3</td>
<td>121.28</td>
<td>5</td>
<td>115.2</td>
<td>127.3</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_8)</td>
</tr>
<tr>
<td>H4</td>
<td>62.26</td>
<td>10</td>
<td>56.03</td>
<td>68.49</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_9)</td>
</tr>
<tr>
<td>HL</td>
<td>130.26</td>
<td>10</td>
<td>117.2</td>
<td>143.3</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_{10})</td>
</tr>
<tr>
<td>RPL</td>
<td>300.0</td>
<td>18.75</td>
<td>243.7</td>
<td>356.2</td>
<td>Trapezoidal, Figure 15</td>
<td>(\mu_{11})</td>
</tr>
</tbody>
</table>

3.3.2 Constraints

Among the 19 constraints required to define the feasible region, eight are assumed to be crisp and eleven are considered fuzzy.

3.3.2.1 Crisp Constraints

Eight crisp constraints from section 3.2.2 are considered. These constraints are given by equations (45) and (47) to (53). We chose to keep these constraints crisp in the problem formulation because these constraints are dictated by strict styling or packaging requirements. For instance, it doesn’t not make sense to have a door of a car overhung, so the constraints on the width of the door (equation (47)) is considered crisp.
3.3.2.2 Fuzzy Constraints

Eleven crisp constraints from section 3.2.2, expressing packaging, styling and manufacturing requirements are converted to fuzzy constraint because uncertainty is present during the formulation of these constraints. For instance, a constraint on the height of the rocker should be considered as fuzzy because the designer’s satisfaction with respect to the rocker height gradually increases as the rocker height reduces.

The meaning and the values of “Min_entity” and “Max_entity” are the same as the one’s defined in section 3.1.

Packaging constraints:

Considering the fact that the dimensions of the baseline joint are already important, the following three constraints (equations (72) to (74)) were tightened instead of being relaxed when becoming fuzzy. Using Rao’s notation, let us consider the crisp constraint \( g_j: g_j(\tilde{x}) \leq b_j \). This constraint becomes fuzzy by reducing the value of \( b_j \) instead of enlarging it as suggested by Rao.

\[
g_j(\tilde{x}) \leq b_j - d_j \quad (71)
\]

The first three packaging constraints define the overall dimensions of the rocker (width, height, stepout height). Engineers try to reduce these dimensions because it is desirable to reduce the size of the joint. The following three fuzzy constraints are built as follows:

- (B2+C2), which is related to the rocker width, should not exceed a maximum value. Using equation (43), this constraint is expressed by membership function measuring the designer’s satisfaction with respect to the rocker width.

\[
\mu_{12} = \mu(B2 + C2, \text{Min\_width\_rock}, \text{Max\_width\_rock}) \quad (72)
\]
The crisp and fuzzy constraints related to the rocker width (equations (54) and (72), respectively) are both represented in Figure 17.

![Diagram showing crisp and fuzzy membership functions of (B2+C2)](image)

**Figure 17:** Fuzzy and crisp membership functions of the rocker width.

- (B2+C3), which is also related to the rocker width, should be small. The following membership is used for this constraint.
  \[ \mu_{13} = \mu(B2 + C3, \text{Min\_width\_rock}, \text{Max\_width\_rock}) \]  
  (73)

- The height of the rocker should not exceed a maximum value. Here, we use the following membership function:
  \[ \mu_{14} = \mu\left(H3 + \frac{C2}{\tan(\alpha_2)} - H6, \text{Min\_height\_rock}, \text{Max\_height\_rock}\right) \]  
  (74)

**Styling constraint:**

- There should be continuity between the shape of the door and the rocker outboard panel. The engineer tries to minimize the difference between the two angles \(\alpha_3\) and \(\alpha_4\). Using equation (42), the membership for this constraint is:
  \[ \mu_{15} = ST(\alpha_3 - \alpha_4, \Delta\alpha) \]  
  (75)
Manufacturing constraints:

The following manufacturing constraints specify that the difference of the thicknesses of two plates connected by spot welds should be small. Consequently, the memberships of these constraints are build as follows:

- $\mu_{16} = ST(T1 - T2, \Delta T)$ \hspace{1cm} (76)
- $\mu_{17} = ST(T2 - T3, \Delta T)$ \hspace{1cm} (77)
- $\mu_{18} = ST(T3 - T4, \Delta T)$ \hspace{1cm} (78)
- $\mu_{19} = ST(T1 - T5, \Delta T)$ \hspace{1cm} (79)
- $\mu_{20} = ST\left(T1 - \frac{T6}{2}, \Delta T\right)$ \hspace{1cm} (80)
- $\mu_{21} = ST(T1 - T7, \Delta T)$ \hspace{1cm} (81)
- $\mu_{22} = ST(T1 - T3, \Delta T)$ \hspace{1cm} (82)

3.3.3 Objective Function

This application uses six fuzzy objectives on the performance of the joint. Two memberships of the performance are similar to the one’s used in the previous case: a trapezoidal membership (Figure 15) for the mass $\mu_{mass}$ and also a trapezoidal membership (Figure 16) for the stiffness $\mu_{K_{nn}}$.

- An additional fuzzy objective on the stiffness of the joint is considered because the stiffness is obtained using a response surface polynomial.

\[
\mu_{K_{poly}} = \beta(K_{poly}, Min_{K_{poly}}, Max_{K_{poly}})
\] \hspace{1cm} (83)

$K_{poly}$ is the output of the response surface polynomial. $Min_{K_{poly}}$ and $Max_{K_{poly}}$ are the minimum and the maximum values of the stiffness obtained using Rao’s method.
• The last 3 fuzzy objectives are related to the cross sectional properties of the joint: moments of inertia of the rocker section in horizontal \( I_y \) and vertical \( I_z \) directions, twist rigidity \( GJ \) of the rocker section. The minimum and maximum values used in the membership functions of this three objectives are calculated using Rao's approach.

\[
\mu_y = \beta(I_y, \text{Min}_y, \text{Max}_y) \tag{84}
\]

\( \text{Min}_y \) and \( \text{Max}_y \) are the minimum and the maximum values of the moment of inertia of the rocker section in the horizontal \( I_y \) direction.

• \( \mu_z = \beta(I_z, \text{Min}_z, \text{Max}_z) \tag{85} \)

\( \text{Min}_z \) and \( \text{Max}_z \) are the minimum and the maximum values of the moment of inertia of the rocker section in the vertical \( I_z \) direction.

• \( \mu_J = \beta(J, \text{Min}_J, \text{Max}_J) \tag{86} \)

\( \text{Min}_J \) and \( \text{Max}_J \) are the minimum and maximum values of the torsional constant of the rocker.

Finally, the objective function, which is a single membership function measuring the overall designer's satisfaction with respect to all attributes of the design is calculated using the conservative, the aggressive and the moderate methods.

The objective function for the conservative method is:

\[
\mu_D(\vec{X}) = \min \{ \mu_{\text{Mass}}, \mu_{K_{\text{poly}}}, \mu_{K_{\text{ns}}}, \mu_{I_y}, \mu_{I_z}, \mu_J, \mu_1, \ldots, \mu_{22} \} \tag{87}
\]

For the aggressive method, the objective function is:

\[
\mu_D(\vec{X}) = \prod \{ \mu_{\text{Mass}}, \mu_{K_{\text{poly}}}, \mu_{K_{\text{ns}}}, \mu_{I_y}, \mu_{I_z}, \mu_J, \mu_1, \ldots, \mu_{22} \} \tag{88}
\]
Finally, for the moderate method, the objective function is a weighted sum of the objective functions obtained using the conservative and aggressive methods:

\[
\mu_D(\bar{x}) = (1 - w) \sqrt[28]{\mu_{\text{mass}} \cdot \mu_{K_{\text{poly}}} \cdot \mu_{K_{\text{nn}}} \cdot \mu_{\text{by}} \cdot \mu_{Iz} \cdot \mu_J \cdot \mu_1 \ldots \mu_{22}} + \]

w \cdot \min(\mu_{\text{mass}}, \mu_{K_{\text{poly}}}, \mu_{K_{\text{nn}}}, \mu_{\text{by}}, \mu_{Iz}, \mu_{GJ}, \mu_1, \ldots, \mu_{22})

(89)

3.3.4 Optimization Problem Formulation

The optimization problem formulations when using the methods for combining objectives are the same except for the objective function:

Find: \[\bar{x}\]

which

Maximize: \[\mu_D(\bar{x})\]

Subject to: 10 crisp side constraints in Table 4 and 8 crisp constraints (equations (45) and (47) to (53)) expressing packaging, styling and manufacturing limitations (section 3.2.2.1).

\[\mu_D(\bar{x})\] is calculated using equations (87), (88) and (89) depending on the method employed to combine the objectives into a single objective function.
4 Results and Discussion

In this chapter, the results of optimizations are presented and discussed. First, the set of Pareto optima that is employed to build the memberships of the objectives is presented and analyzed in section 4.1. Then, the results obtained by optimization of the example of the automotive joint with two objectives are introduced and explained in section 4.2. In section 4.2, fuzzy methods, crisp optimization (Zhu's results) and utility theory are compared. Finally, the results obtained by optimizing the example with a large number of objectives are given in section 4.3. These results are compared with results from crisp optimization. In section 4.4, the costs of optimization when using a neural network and FEA to predict the performance characteristic of the joint are compared.

4.1 Determination of the Membership Functions Using Rao's Method

Rao's method is used to build memberships of the attributes. This method requires the determination of a set of Pareto optima. This set results from the optimization of each objective without considering any constraints on the remaining objectives. The technique and the software of optimization employed in this study are reviewed in section 4.1.1. Then, the results of the optimizations are examined and discussed in section 4.1.2.

4.1.1 Optimization Technique and Software Employed

The design of automotive joint is a non-linear optimization problem. The constraints as well as the objectives are non-linear functions of the design variables. Numerical methods are required to solve non-linear optimization problems.
In this thesis, the optimization was performed using a sequential quadratic programming algorithm from an IMSL subroutine. This algorithm is based on a FORTRAN code which was developed by Schittkowski [24, 25].

During the optimization process, the optimizer may stop at a non critical point because of the non linearity of the problem. A non critical point can be a local optimum for instance. Consequently, every optimization was performed using ten different initial guesses. This technique should reduce the risk of getting a local optimum design instead of a global one. Furthermore, once an optimum was achieved, small perturbations of some design variables were performed to check the validity of the optimum.

4.1.2 Determination of the Parameters Used in the Constraints and the Supports of Membership Functions of the Constraints

Before running the optimization, the parameters used in the formulation of the constraints need to be assessed. In this section, these parameters are chosen and the supports of the membership functions of the constraints are determined. Table 7 depicts the values of parameters defining the ranges in which various quantities associated with the constraints can vary and the supports of memberships of the constraints.

The parameters, which are Δα, ΔT, Door_width, Door_edge_width, Max_H5_slope are equal to the values that Zhu used in his thesis [36].

The supports of the memberships of the constraints are assessed using the following approach. The values of the width and the height of the rocker were determined by studying the cross sectional dimensions of B-pillar to rocker joints from different cars and from the baseline design. The joints from other cars were provided by Ford. Their dimensions are presented in Table 6. It is observed that the baseline design has the largest width of the rocker among all the cars studied. In Table 6, car 5 has a rocker width which is 39.4 % less than the one of the baseline design. Consequently, the minimum width of
the rocker which corresponds to the low boundary of the membership function of the rocker width should be much smaller than the baseline design. However, this value shouldn’t be too small so that our feasible domain still has a reasonable size. The rocker height of the baseline design is between the smallest and the highest rocker height of the eight cars presented in Table 6. Consequently, the minimum and maximum values of the rocker height are chosen equal to the smallest and the highest values of the rocker height from Table 6, respectively.

Table 6: Cross section dimensions of other B-pillar to rocker joints.

<table>
<thead>
<tr>
<th>Baseline design</th>
<th>Width of the rocker (mm)</th>
<th>Height of the rocker (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car 1</td>
<td>115</td>
<td>120</td>
</tr>
<tr>
<td>Car 2</td>
<td>120</td>
<td>123</td>
</tr>
<tr>
<td>Car 3</td>
<td>122</td>
<td>125</td>
</tr>
<tr>
<td>Car 4</td>
<td>150</td>
<td>165</td>
</tr>
<tr>
<td>Car 5</td>
<td>100</td>
<td>140</td>
</tr>
<tr>
<td>Car 6</td>
<td>130</td>
<td>125</td>
</tr>
<tr>
<td>Car 7</td>
<td>110</td>
<td>175</td>
</tr>
</tbody>
</table>
Table 7: Parameters defining the ranges in which various quantities associated with the constraints vary and the supports of the memberships of the constraints.

<table>
<thead>
<tr>
<th>Constraint Type</th>
<th>Constraint Criteria</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Styling</td>
<td>Door-width (mm)</td>
<td>95.13</td>
</tr>
<tr>
<td>Styling</td>
<td>$\alpha_4$ (°)</td>
<td>63.20</td>
</tr>
<tr>
<td>Styling</td>
<td>$\Delta\alpha$ (°)</td>
<td>10</td>
</tr>
<tr>
<td>Packaging</td>
<td>Door_edge_width (mm)</td>
<td>16.50</td>
</tr>
<tr>
<td>Packaging</td>
<td>Max_H5_slope</td>
<td>0.2376</td>
</tr>
<tr>
<td>Packaging</td>
<td>Min_step_angle (°)</td>
<td>6</td>
</tr>
<tr>
<td>Packaging</td>
<td>Max_step_angle (°)</td>
<td>15</td>
</tr>
<tr>
<td>Packaging</td>
<td>Min_width_rock (mm)</td>
<td>140</td>
</tr>
<tr>
<td>Packaging</td>
<td>Max_width_rock (mm)</td>
<td>170</td>
</tr>
<tr>
<td>Packaging</td>
<td>Min_height_rock (mm)</td>
<td>120</td>
</tr>
<tr>
<td>Packaging</td>
<td>Max_height_rock (mm)</td>
<td>175</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Stepout_height (mm)</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>$\Delta T$ (mm)</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4.1.3 Determination of a Set of Pareto Optima

The engineer needs to determine the set of Pareto optima to build the memberships of the mass, the cross sectional properties and the stiffness. This set of optimum solutions is the result of successive optimizations of each objective considering the set of constraints defined in equations (45) to (63) and side constraints from Table 3. No constraints on the objectives are considered.

Table 8 presents the set of Pareto optima; this table shows the best and the worst values that the attributes can attain. The best values of the objectives are calculated according to equation (9). The diagonal elements of Table 8 are the best values the attributes can ever take. Each row in Table 8 represents the best design that the engineer can obtain if he optimizes the design to get the
best value of one objective neglecting the values of the other objectives. The worst values of the objectives are obtained when the engineer minimizes the mass.

The design variables associated with each design of the set of Pareto optima are listed in Table 9.

**Table 8:** Values of the attributes of the set of Pareto optima.

<table>
<thead>
<tr>
<th></th>
<th>Mass (kg)</th>
<th>$I_x (\text{mm}^4) \times 10^{-4}$</th>
<th>$I_z (\text{mm}^4) \times 10^{-4}$</th>
<th>$J (\text{mm}^4) \times 10^{-4}$</th>
<th>$K_{\text{poly}} (\text{Nm/rad})$</th>
<th>$K_{\text{nn}} (\text{Nm/rad})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimization of mass M</td>
<td>6.16</td>
<td>109</td>
<td>87</td>
<td>97</td>
<td>1,380 (1,000**)</td>
<td>970 (1,000**)</td>
</tr>
<tr>
<td>Maximization of $I_x$</td>
<td>11.09</td>
<td>315</td>
<td>228</td>
<td>241</td>
<td>10,800</td>
<td>10,600</td>
</tr>
<tr>
<td>Maximization of $I_z$</td>
<td>11.09</td>
<td>313</td>
<td>229</td>
<td>243</td>
<td>11,100</td>
<td>10,900</td>
</tr>
<tr>
<td>Maximization of $J$</td>
<td>10.59</td>
<td>248</td>
<td>224</td>
<td>253</td>
<td>10,800</td>
<td>10,900</td>
</tr>
<tr>
<td>Maximization of $K_{\text{poly}}$</td>
<td>9.42</td>
<td>210</td>
<td>185</td>
<td>183</td>
<td>12,900 (12,950**)</td>
<td>13,200 (12,950**)</td>
</tr>
<tr>
<td>Maximization of $K_{\text{nn}}$</td>
<td>9.42</td>
<td>210</td>
<td>185</td>
<td>183</td>
<td>12,900 (12,950**)</td>
<td>13,200 (12,950**)</td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)

The results of the stiffness evaluated using FEA agree quite well with the approximate stiffnesses obtained using the neural network. The relative error in Table 8 is 3% when minimizing the mass and 2% when maximizing the stiffness. Consequently, the use of approximate tools to estimate the stiffness of a given joint is appropriate. FEA method can be employed to map one particular set of design variables to stiffness with good accuracy but it cannot be used in iterative design process because it is very time consuming.

The error of the stiffness approximation of the response surface polynomial when the engineer minimizes the mass is large (relative error of 38%). In this case, the response
surface polynomial predicts the stiffness of a design that is significantly different than those used for estimating the polynomial parameters. Actually, the stiffness of the joint with minimum mass (first row in Table 8) is almost an order of magnitude smaller than the stiffnesses of all actual joints. To minimize the error described earlier, the value of the minimum approximate stiffness obtained with the response surface polynomial and used to assess the membership of stiffness will be fixed at 1,000 Nm/rad instead of 1,380 Nm/rad.

From Table 8 and from the previous remark, the values that are employed in equations (64), (65), (83) to (86) to determine the memberships of the stiffness, the mass and the cross sectional properties are assessed:

\[
\begin{align*}
\text{Min}_\text{mass} &= 6.16 \text{ kg} & \text{Min}_I &= 109 \times 10^4 \text{ mm}^4 & \text{Min}_I_z &= 87 \times 10^4 \text{ mm}^4 \\
\text{Max}_\text{mass} &= 11.09 \text{ kg} & \text{Max}_I_I &= 315 \times 10^4 \text{ mm}^4 & \text{Max}_I_z &= 229 \times 10^4 \text{ mm}^4 \\
\text{Min}_J &\approx 97 \times 10^4 \text{ mm}^4 & \text{Min}_K_{\text{poly}} &= 1,000 \text{ Nm/rad} & \text{Min}_K_{\text{nn}} &= 970 \text{ Nm/rad} \\
\text{Max}_J &\approx 253 \times 10^4 \text{ mm}^4 & \text{Max}_K_{\text{poly}} &= 12,900 \text{ Nm/rad} & \text{Max}_K_{\text{nn}} &= 13,200 \text{ Nm/rad}
\end{align*}
\]
Table 9 a: Values of the design variables that defined the set of Pareto optima.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Optimization of M</th>
<th>Optimization of $I_y$</th>
<th>Optimization of $I_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>0.60</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>T2</td>
<td>0.638</td>
<td>1.183</td>
<td>1.183</td>
</tr>
<tr>
<td>T3</td>
<td>0.85</td>
<td>1.30</td>
<td>1.30</td>
</tr>
<tr>
<td>C1</td>
<td>26.09</td>
<td>31.89</td>
<td>31.89</td>
</tr>
<tr>
<td>C2</td>
<td>77.51</td>
<td>87.84</td>
<td>86.47</td>
</tr>
<tr>
<td>C3</td>
<td>70.75</td>
<td>86.47</td>
<td>86.47</td>
</tr>
<tr>
<td>C4</td>
<td>47.26</td>
<td>57.76</td>
<td>57.76</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.6147</td>
<td>0.7992</td>
<td>0.8025</td>
</tr>
<tr>
<td>B2</td>
<td>70.76</td>
<td>82.16</td>
<td>83.53</td>
</tr>
<tr>
<td>B3</td>
<td>34.39</td>
<td>42.03</td>
<td>42.03</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.079</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>H1</td>
<td>71.39</td>
<td>71.69</td>
<td>70.44</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>69.71</td>
<td>75.52</td>
<td>75.52</td>
</tr>
<tr>
<td>H3</td>
<td>115.2</td>
<td>127.3</td>
<td>127.3</td>
</tr>
<tr>
<td>H4</td>
<td>56.03</td>
<td>68.49</td>
<td>68.49</td>
</tr>
<tr>
<td>H5</td>
<td>14.47</td>
<td>16.79</td>
<td>17.69</td>
</tr>
<tr>
<td>H6</td>
<td>6.950</td>
<td>5.687</td>
<td>5.687</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>78.66</td>
<td>64.36</td>
<td>64.36</td>
</tr>
<tr>
<td>HL</td>
<td>117.2</td>
<td>130.3</td>
<td>130.3</td>
</tr>
<tr>
<td>RP</td>
<td>195.7</td>
<td>217.4</td>
<td>217.4</td>
</tr>
<tr>
<td>RPL</td>
<td>243.7</td>
<td>300.0</td>
<td>300.0</td>
</tr>
</tbody>
</table>
Table 9 b: Values of the design variables that defined the set of Pareto optima.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Optimization of $J$</th>
<th>Optimization of $K_{\text{poly}}$</th>
<th>Optimization of $K_{\text{nn}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>1.04</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>T2</td>
<td>1.183</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>T3</td>
<td>1.30</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>C1</td>
<td>31.89</td>
<td>28.99</td>
<td>28.99</td>
</tr>
<tr>
<td>C2</td>
<td>86.47</td>
<td>86.12</td>
<td>86.12</td>
</tr>
<tr>
<td>C3</td>
<td>86.47</td>
<td>78.61</td>
<td>78.61</td>
</tr>
<tr>
<td>C4</td>
<td>57.76</td>
<td>52.51</td>
<td>52.51</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.8025</td>
<td>0.8033</td>
<td>0.8033</td>
</tr>
<tr>
<td>B2</td>
<td>83.53</td>
<td>83.88</td>
<td>83.88</td>
</tr>
<tr>
<td>B3</td>
<td>42.03</td>
<td>38.21</td>
<td>38.21</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>H1</td>
<td>71.69</td>
<td>65.17</td>
<td>65.17</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>69.71</td>
<td>69.71</td>
<td>69.71</td>
</tr>
<tr>
<td>H3</td>
<td>127.3</td>
<td>120.7</td>
<td>120.7</td>
</tr>
<tr>
<td>H4</td>
<td>68.49</td>
<td>62.26</td>
<td>62.26</td>
</tr>
<tr>
<td>H5</td>
<td>17.69</td>
<td>16.08</td>
<td>16.08</td>
</tr>
<tr>
<td>H6</td>
<td>6.950</td>
<td>6.319</td>
<td>6.319</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>78.22</td>
<td>71.51</td>
<td>71.51</td>
</tr>
<tr>
<td>HL</td>
<td>118.9</td>
<td>130.3</td>
<td>130.3</td>
</tr>
<tr>
<td>RP</td>
<td>206.4</td>
<td>217.4</td>
<td>217.4</td>
</tr>
<tr>
<td>RPL</td>
<td>243.7</td>
<td>356.2</td>
<td>356.2</td>
</tr>
</tbody>
</table>

4.1.4 Comparison of the Set of Pareto Optima with the Baseline Design and the Baseline Optimum

The baseline design has a mass equal to 8.375 kg. The stiffness of the baseline design evaluated using FEA and is equal to 4,440 Nm/rad.
The baseline optimum which is obtained by minimizing the mass and considering constraints on the performance targets has a mass of 7.237 kg and a stiffness using FEA of 4,200 Nm/rad.

Figures 18 to 22 show the cross sections of each solution of the set of Pareto optima except the solution obtained when maximizing the stiffness using the response surface polynomial. The dotted line represents the original design, also called the baseline design. Figures 23 and 24 show the cross sections of the set of Pareto optimum design and the baseline optimum that Zhu obtained by minimizing the mass with constraints on the performance targets.

From Tables 8 and 9, we can conclude that:

1. The minimization of the mass which is the first solution of the set of Pareto optimum leads to a design with a much lower mass and a much lower stiffness than the baseline design and Zhu’s baseline optimum. The absence of constraints on the performance targets during the minimization of the mass allows to get a smaller mass than the baseline optimum. Consequently, this Pareto optimum has a smaller sheet metal surface (hence volume and mass) than the baseline optimum as shown in Figure 24. Figure 17 indicates the cross section of the optimum design.

2. The maximization of the cross section properties (moments of inertia and twist rigidity) reveals a particular type of results for the attributes: high values for the mass and the stiffness. The stiffness is equal to two to three times the stiffness of the baseline optimum. When the engineer maximizes the cross section properties, the designs obtained after optimization will have cross sections larger than the one of baseline design as shown in Figures 19 to 21. Figure 23 compares the cross sections of the optimum designs. It is observed that the maximization of the moments of inertia in horizontal y and vertical z directions gives results that are very close.
3. The maximization of the stiffness results in a very high stiffness (3 times the value of the baseline optimum) and a large mass. This optimization of the stiffness using the response surface polynomial and the neural network presents the same results. Figure 22 illustrate the case when the engineer maximizes the approximate stiffness using the neural network.

4. The mass and the other attributes (cross section properties and stiffness) are conflicting attributes. The worst values for the cross section properties and the stiffness are achieved when the engineer minimizes the mass and vice versa.

The mass is the only objective that is an explicit function of all the design variables. When the engineer minimizes the mass, all the design variables are active and are fixed by the optimizer. The cross section properties are functions of design variables that described the cross sections only. Consequently, the values describing the pillar (dimensions, position with respect to the rocker) are determined arbitrarily during the optimization. The constraints defined in equations (45) to (63) are functions only of design variables that affect the cross sections. As a results, there is no need to fix the values describing the pillar. The stiffness obtained from the two approximate tools is a function of seven design variables only: T1, α1, H3, B1/B2, B4/B1, B2, RPL. When optimizing the stiffness, the rest of the design variables were fixed to their values at the baseline design.
Figure 18: Rocker sections of the baseline design versus the design obtained when minimizing the mass M.

Figure 19: Rocker sections of the baseline design versus the design obtained when maximizing the moment of inertia $I_y$ in the horizontal $y$ direction.
Figure 20: Rocker sections of the baseline design versus the design obtained when maximizing the moment of inertia $I_z$ in the vertical $z$ direction.

Figure 21: Rocker sections of the baseline design versus the design obtained when maximizing the twist rigidity $J$. 

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Figure 22: Rocker sections of the baseline design versus the design obtained when maximizing the approximate stiffness using the neural network.

Figure 23: Rocker sections of two solutions from the set of Pareto optima (maximization of $I_y$ and $I_z$) versus the baseline optimum.
Figure 24: Rocker sections of two solutions from the set of Pareto optima (minimization of the mass and maximization of $K_{nn}$) versus the baseline optimum.

4.2 Results from the Example with two Fuzzy Objectives

The results obtained when optimizing the automotive joint with two objectives are presented in this section. The first objective, which is to be minimized, is the mass as mentioned in equation (64). The second objective, which is to be maximized, is the stiffness approximated with the neural network as mentioned in equation (65). The constraints are defined by the equations (45) to (63). The conservative method was applied to solve this problem. Then, the aggressive and moderate approaches were investigated.
4.2.1 Optimum Design Obtained Using Conservative Approach

The first example solved in this study is an automotive joint with two objectives. This example is optimized using the conservative method. The overall objective function is defined by equation (66). Table 10 depicts the values of the attributes of the optimum design. The constraint that are active are listed in Table 11.

The overall level of satisfaction of the design is equal to:
\[
\mu_D(\hat{x}) = \mu_{\text{Mass}} = \mu_{\text{K,nn}} = 0.74
\]  

(90)

The levels of satisfaction of each attribute are the same. Typically, the optimizer tries to equalize the values for all the fuzzy memberships considered in the problem formulation because the membership function measuring the overall satisfaction is the minimum of the membership functions of the mass and the stiffness.

The cross section of the optimum design is showed in Figure 25 as well as the cross sections of the baseline optimum and the baseline design. The values of the design variables are listed in Table 12.

**Table 10:** Values of the attributes of the optimum design obtained using the conservative method with the first example.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Value of the Attribute</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>7.434</td>
<td>( \mu_{\text{Mass}} = 0.74 )</td>
</tr>
<tr>
<td>Stiffness neural net. app. (Nm/rad)</td>
<td>10,000 (8,900**)</td>
<td>( \mu_{\text{K,nn}} = 0.74 )</td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)**
Table 11: List of active constraints with the first example.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Constraint description</th>
<th>Equation number</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Styling</td>
<td>Door alignment</td>
<td>42</td>
<td>Yes</td>
</tr>
<tr>
<td>Packaging</td>
<td>Door edge width</td>
<td>43</td>
<td>Yes</td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope to accommodate the door sealant</td>
<td>46</td>
<td>Yes</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T3 &amp; T4</td>
<td>55</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Figure 25: Cross sections of the design obtained using the conservative method, the baseline optimum and the baseline design.
Table 12: Values of the design variables that defined the optimum design obtained using the conservative approach with the first example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Values</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1 (mm)</td>
<td>0.861</td>
<td></td>
</tr>
<tr>
<td>T2 (mm)</td>
<td>0.637</td>
<td>Lower bound</td>
</tr>
<tr>
<td>T3 (mm)</td>
<td>0.85</td>
<td></td>
</tr>
<tr>
<td>C1 (mm)</td>
<td>26.09</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C2 (mm)</td>
<td>77.51</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C3 (mm)</td>
<td>70.75</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C4 (mm)</td>
<td>47.26</td>
<td>Lower bound</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.809</td>
<td></td>
</tr>
<tr>
<td>B2 (mm)</td>
<td>86.48</td>
<td>Upper bound</td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>39.73</td>
<td></td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.00</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>62.48</td>
<td></td>
</tr>
<tr>
<td>α₁ (°)</td>
<td>69.71</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H3 (mm)</td>
<td>115.2</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H4 (mm)</td>
<td>56.03</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>14.47</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>6.95</td>
<td>Upper bound</td>
</tr>
<tr>
<td>α₂ (°)</td>
<td>78.66</td>
<td>Upper bound</td>
</tr>
<tr>
<td>HL (mm)</td>
<td>117.2</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>195.7</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RPL (mm)</td>
<td>356.2</td>
<td>Upper bound</td>
</tr>
</tbody>
</table>

4.2.2 Optimum Design Obtained Using Aggressive Approach

The second method employed to solve the example of the automotive joint with two objectives is the aggressive method. The overall objective function is expressed in equation (67).
The overall level of satisfaction of the optimum design using the aggressive method is equal to:

\[ \mu_D(\bar{x}') = 0.81 \]  \hspace{1cm} (91)

The levels of each attribute are different as shown in Table 13. The design variables of the optimum design obtained when using the aggressive method are the same as the one obtained with the conservative method except for the thickness T1 of the outboard cell of the rocker. When the engineer performs the optimization of the joint using the aggressive method, the thickness T1 of the optimum design is equal to 0.995 mm. The active constraints are the same as in the conservative method and are defined in Table 11.

**Table 13:** Values of the attributes of the optimum design obtained using the aggressive method with the first example.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Value of the Attribute</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>7.886</td>
<td>( \mu_{\text{Mass}} = 0.65 )</td>
</tr>
<tr>
<td>Stiffness neural net. app.</td>
<td>13,200 (11,600**)</td>
<td>( \mu_{K_{\text{nn}}} = 1.00 )</td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)

### 4.2.3 Optimum Design Obtained Using Moderate Approach

The third method used in this study is the moderate method. The mathematical expression of the overall objective function is given by equation (68). The moderate method requires the determination of a weight factor \( w \) as mentioned in equation (15). This factor represents the degree to which the designer prefers the conservative method to the aggressive method. Different values of \( w \) have been investigated. For low values of
w, the optimum solution is the same as the one obtained using the aggressive method. For \( w \) equal to 0.41, the level of satisfaction of optimum designs obtained using the conservative and the aggressive methods are the same. For high values of \( w \), the optimum design is the solution obtained using the conservative method. These results will be analyzed and explained in section 4.2.5.4. The different results are presented in Table 14. The active constraints remain unchanged compared to the conservative method and are listed in Table 11.

**Table 14:** Optimum designs using the moderate method with the first example.

<table>
<thead>
<tr>
<th>Value of ( w )</th>
<th>Solution same as</th>
<th>Overall satisfaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>aggressive method</td>
<td>( \mu_D(\vec{x}^* ) = 0.81 )</td>
</tr>
<tr>
<td>0.2</td>
<td>aggressive method</td>
<td>( \mu_D(\vec{x}^* ) = 0.77 )</td>
</tr>
<tr>
<td>0.41</td>
<td>conservative method</td>
<td>( \mu_D(\vec{x}^* ) = 0.74 )</td>
</tr>
<tr>
<td>0.6</td>
<td>conservative method</td>
<td>( \mu_D(\vec{x}^* ) = 0.74 )</td>
</tr>
<tr>
<td>0.8</td>
<td>conservative method</td>
<td>( \mu_D(\vec{x}^* ) = 0.74 )</td>
</tr>
<tr>
<td>0.999</td>
<td>conservative method</td>
<td>( \mu_D(\vec{x}^* ) = 0.74 )</td>
</tr>
</tbody>
</table>

For \( w \) equal to 0.6, 0.8 and 0.999, the memberships of the mass and the stiffness, \( \mu_{\text{Mass}} \) and \( \mu_{K_{nd}} \), are equal for the optimum designs. If we substitute \( \mu_{\text{Mass}} = \mu_{K_{nd}} \) in equation (92) for the overall degree of satisfaction of a design, the degree of overall satisfaction is constant, equal to that of the conservative optimum.

\[
\mu_D(\vec{x}) = \max_{\text{feasible domain}} \left\{ (1 - w) \cdot \sqrt{\mu_{\text{Mass}} \cdot \mu_{K_{nd}}} + w \cdot \min\{\mu_{\text{Mass}}, \mu_{K_{nd}}\} \right\}
\]  

(92)
We find that the degree of overall satisfaction is equal to the membership of the mass and the stiffness.

4.2.4 Comparison of the Results from Fuzzy Methods with the Baseline Design and the Baseline Optimum

The three fuzzy methods enable the engineer to get alternate designs with relatively small masses but very high stiffnesses compared to the baseline optimum and the baseline joint. Due to the very low stiffness of the baseline joint (4,440 Nm/rad), it is easy to increase the stiffness significantly. Note that it is possible to get a design with a better stiffness and a smaller mass than the baseline design.

Furthermore, Zhu’s constraint on the stiffness is not restrictive. The requirement specified that the stiffness should be larger than 4,400 Nm/rad. Consequently, the optimum design found by Zhu still has a relatively low stiffness which could be improved considerably. This is the reason why the three fuzzy methods reveal alternative designs with high stiffness compared to the optimum design.

The conservative method leads to a design with a small mass but a high stiffness. This solution considers both attributes in the evaluation of the overall satisfaction of the design by equalizing the level of satisfaction of each attribute. If the levels of satisfaction of the attributes were not equal, the min operator would consider only one attribute in the overall evaluation of the design. The engineer doesn’t know a priori if the design will be evaluated, based on the poorest attribute or several poor attributes that have equal memberships.

The aggressive method improves the stiffness of the joint dramatically. The design alternative obtained using the aggressive method has a very high stiffness but a reasonably
low mass. The very high value of the stiffness is equal to the maximum stiffness the joint can ever have. This maximum value of the stiffness is also determined by the set of Pareto optimum. The aggressive method allows trade-offs between the attributes. The level of satisfaction is maximum and equal to one. However, the level of satisfaction of the mass is satisfactory only. This means that there are trade-offs between the stiffness and the mass: the large value of the stiffness compensates for the poor performance level of the mass.

The moderate method gives either the results of the conservative or the aggressive methods. Depending on the results, the remarks made in one of the two previous paragraphs applied to the moderate approach.

4.2.5 Comparison of the Results Obtained Using Fuzzy methods and Utility Theory

The results of fuzzy methods and utility theory are compared in this section. The example with two objectives allows valuable comparison of the different methods.

4.2.5.1 Pareto Optima to Determine a Combination of Masses and Stiffnesses

To understand how the conservative approach optimizes a design, we find a set of feasible designs (Pareto optima) which for given mass have maximum stiffness. To get these designs, crisp optimizations are performed in which we maximize the stiffness subject to the constraints expressed by equations (45) to (63) and an additional equality constraint on the mass. Any multiobjective optimization method chooses a design in this set. The results are listed in Table 15.
Table 15: Results obtained when maximizing the stiffness with one equality constraint on the mass.

<table>
<thead>
<tr>
<th>Equality constraint on the mass M (kg)</th>
<th>Maximum value of the stiffness, neural net. app. K_nn (Nm/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.6</td>
<td>5,300</td>
</tr>
<tr>
<td>6.8</td>
<td>6,300</td>
</tr>
<tr>
<td>7.0</td>
<td>7,400</td>
</tr>
<tr>
<td>7.2</td>
<td>8,600</td>
</tr>
<tr>
<td>7.4</td>
<td>9,800</td>
</tr>
<tr>
<td>7.6</td>
<td>11,100</td>
</tr>
<tr>
<td>7.8</td>
<td>12,600</td>
</tr>
</tbody>
</table>

4.2.5.2 Comparison of the Results Considering the Metric of the Conservative Method

In this section, the results of the conservative method are compared with the results obtained using the aggressive method and crisp optimizations.

2-D graphs with lines joining points of same levels of overall satisfaction are constructed. The x axis represents the stiffness of the joint whereas the y axis represents the mass. Considering the min metric of the conservative method (equation (6)), curves representing constant levels of satisfaction are plotted as shown in Figure 26. These curves indicate combinations of stiffness and mass that would result in designs with the same level of designer preference (that is the same $\mu_0(\bar{x})$). Since the engineer prefers designs with low mass and high stiffness, the area in the lower right corner has the greatest level of satisfaction. The curves representing constant levels of satisfaction are made of two segments of straight lines that intersect at a right angle. Let us consider the vertical
segment AB of curve representing a constant level of satisfaction as shown in Figure 26. The stiffness along AB is equal to a low value of 2,100 Nm/rad which is associated with a low membership of the stiffness. This membership is lower than the membership of the mass. Consequently, the overall satisfaction measured by \( \mu_d(\bar{x}) \) is equal to the membership of the stiffness. The value of the mass can vary in a large range from 6.16 to 10.8 kg on the straight line AB without affecting the overall satisfaction. This situation shows a limitation of the conservative method because although \( \mu_d(\bar{x}) \) is constant along AB, in reality the engineer would prefer the design corresponding to B, because it has the lowest mass. Therefore, the metric that the conservative method uses does not represent the engineer choices in this case.

The locus of feasible designs, which for given mass has maximum stiffness is the curve marked with stars in Figure 26. The stiffness and mass values corresponding to this curve were presented in Table 15.

The best design is at the intersection of the horizontal and vertical lines described earlier. Note that the optimizer equalizes the values of the two memberships of the attributes. Figure 26 shows that the crisp optimizations as well as the aggressive method give worst results according to the conservative metric that is employed to grade the designs.
Figure 26: Curves representing constant levels of overall satisfaction according to the min metric and Pareto optimum points.

4.2.5.3 Comparison of the Results Considering the Metric of the Aggressive Method

In this paragraph, the results of multiobjective optimization methods are compared using the metric (equation (14)) of the aggressive method. Figure 27 shows Pareto optimum points, the results of the aggressive and conservative methods as well as lines joining points with same levels of overall satisfaction. The area in the lower right corner has the highest level of satisfaction possible. The optimum solution is reached when the stiffness of the joint is maximum.
**Figure 27:** Curves representing constant levels of overall satisfaction according to the normalized product metric and Pareto optimum points.

### 4.2.5.4 Comparison of the Results Considering the Metric of the Moderate Method

In this section, the results of the moderate method are studied. The optimum design of the moderate method is either the design obtained using the conservative or the aggressive approaches. 2-D graphs with lines representing constant levels of overall satisfaction are considered. The axes are the membership values of the stiffness and the mass. The upper right corner has the greatest overall design satisfaction. Figures 29 to 33 illustrate the results of the fuzzy methods, Pareto optimum points and curves representing constant levels of satisfaction according to the metric of the moderate method for different values of w.

The mathematical formulation of the moderate method tends to the formulation of the aggressive method as the factor w tends to zero. When w is equal to 0.001, the highest overall satisfaction is reached with the solution of the aggressive method as shown in Figure 28.
For $w$ equal to 0.2, the importance of the conservative method increases compared to the previous case. Each of the lines joining points with same levels of satisfaction of Figure 29 has a singularity point. At this point, the memberships of the mass and the stiffness are equal. The best design is obtained by the aggressive method.

For $w$ equal to 0.41, the levels of satisfaction of the optimum designs obtained using the aggressive and the conservative methods are the same. Figure 30 illustrates this case.

For $w$ equal to 0.6 and 0.8, the importance of the aggressive method decreases and the best design is the design obtained using the conservative method. Figures 31 to 32 shows that the curves of same levels of satisfaction have singularities that are becoming sharper as $w$ increases.

For $w$ equal to 0.999, the metric for overall satisfaction is practically equal to that of the conservative method. The optimum solution is the same as the one obtained with the conservative method as shown in Figure 33.

**Figure 28:** Curves representing constant levels of overall satisfaction according to metric of the moderate method ($w = 0.001$) and Pareto optimum points.
**Figure 29:** Curves representing constant levels of overall satisfaction according to metric of the moderate method (w = 0.2) and Pareto optimum points.

**Figure 30:** Curves representing constant levels of overall satisfaction according to metric of the moderate method (w = 0.41) and Pareto optimum points.
Figure 31: Curves representing constant levels of overall satisfaction according to metric of the moderate method \((w = 0.6)\) and Pareto optimum points.

Figure 32: Curves representing constant levels of overall satisfaction according to metric of the moderate method \((w = 0.8)\) and Pareto optimum points.
Figure 33: Curves representing constant levels of overall satisfaction according to metric of the moderate method (w = 0.999) and Pareto optimum points.

4.2.5.5 Comparison of the Results Considering the Metric of the Utility Theory

Utility theory is an approach that allows the engineer to quantify the trade-offs between the attributes that he is willing to accept. These trade-offs are represented by the two scaling constants $k_1$ and $k_2$. $k_1$ is the utility of a design that has the largest stiffness and also the largest mass and $k_2$ is the utility of a design that has the smallest stiffness and the smallest mass. Two different values for the scaling constants are investigated in this study: a low value equal to 0.1 and a high value equal to 0.8. Consequently, there are four possible combinations:

$k_1 = k_2 = 0.8$, \hspace{2cm} case 1

$k_1 = k_2 = 0.1$, \hspace{2cm} case 2

$k_1 = 0.8$ and $k_2 = 0.1$, \hspace{2cm} case 3

$k_1 = 0.1$ and $k_2 = 0.8$. \hspace{2cm} case 4

92
The single attribute utility $U_1$ is associated with the stiffness. The single attribute utility $U_2$ is associated with the mass.

The two first cases studied in this work have dependent attributes whereas the last two cases have almost independent attributes. When the attributes are dependent, the rate at which the degree of satisfaction of a design increases with respect to one attribute depends on the value of other attributes. When the attributes are independent, a large value of the scaling constant $k_1$ means that a small improvement in the stiffness will result in a large increase of the overall utility. Consequently, the engineer focuses on the stiffness and pays little attention to the value of the mass. A small value for $k_1$ means that the overall satisfaction is slightly affected by any changes in the stiffness. Similar conclusions can be drawn with the mass and the value of $k_2$.

Eight figures (34 to 41) illustrate these four cases. In Figures 35, 37, 39 and 41, iso-utility curves are plotted with feasible combinations of stiffness and mass. In these figures, the overall utility is plotted as a function of the stiffness when the mass is constant. The mass is equal to several discrete values and varies from the worst value (11.09 kg) to the most desirable value (6.16 kg). Three remarks can be made about Figures 34, 36, 38 and 40:

1. The straight line which represents the best mass the joint can have intersects the y axis at $k_2$. This line goes through the point corresponding to a maximum level of satisfaction of one when the stiffness is at the maximum level. When both attributes are at their best value, the overall level of satisfaction is one.

2. The straight line which represents the worst mass the joint can ever have goes through zero when the stiffness is at its worst level. When both attributes are equal to their worst value, the overall utility is zero. This line goes through $k_1$ when the stiffness is at its most desirable value.
3. Finally, the overall utility is different from zero as long as one attribute is not at its worst value. This characteristic is because the annihilation condition does not hold as described in Appendix A.

case 1:

The first case is illustrated by Figures 34 and 35. If one attribute has a good performance level, then the overall satisfaction of the design is high regardless of the value of the other attribute. It is observed from Figure 34 that when the stiffness is very low, the engineer is willing to accept a smaller improvement in the mass than if the stiffness were high. Figure 35 shows that the best design according to utility theory is the design obtained using the aggressive method.

case 2:

The second case is illustrated by Figures 36 and 37. If one attribute has a poor level of performance, then the overall degree of satisfaction is low. When the stiffness is low, the engineer trade off a larger improvement in the mass for a given reduction in stiffness than if the stiffness were high. Again, the greatest utility is reached with the design obtained using the aggressive method as shown in Figure 37.

case 3:

In the third case considered in this study, the two attributes are almost independent. Indeed, whatever the stiffness is, the engineer is ready to trade the same amount of mass to increase the utility of a certain value. In this case, the engineer focuses on the stiffness more than on the mass. Figures 38 and 39 depict this case. The optimum design is obtained with the aggressive method.
**case 4:**

In the fourth case, the attributes are almost independent again. The mass is more important in this example than the stiffness: a small improvement in the mass results in a larger increase of the overall satisfaction of the design than an improvement in the stiffness. The optimum design is a Pareto optimum that has a very low mass compared to the baseline design. Figures 40 and 41 illustrate this case.

![Graph of Influence of the scaling constant k1 and k2](image)

**Figure 34:** Overall utility of a design as a function of the stiffness when the mass is equal to discrete values and $k_1$ and $k_2$ are equal to 0.8.
Figure 35: Iso-utility curves and feasible combinations of mass and stiffness when \( k_1 \) and \( k_2 \) are equal to 0.8.

Figure 36: Overall utility of a design as a function of the stiffness when the mass is equal to discrete values and \( k_1 \) and \( k_2 \) are equal to 0.1.
**Figure 37:** Iso-utility curves and feasible combinations of mass and stiffness when $k_1$ and $k_2$ are equal to 0.1.

**Figure 38:** Overall utility of a design as a function of the stiffness when the mass is equal to discrete values and $k_1$ is equal to 0.8 and $k_2$ is equal to 0.1.
**Figure 39:** Iso-utility curves and feasible combinations of mass and stiffness when $k_1$ is equal to 0.8 and $k_2$ is equal to 0.1.

**Figure 40:** Overall utility of a design as a function of the stiffness when the mass is equal to discrete values and $k_1$ is equal to 0.1 and $k_2$ is equal to 0.8.
**Figure 41:** Iso-utility curves and feasible combinations of mass and stiffness when \( k_1 \) is equal to 0.1 and \( k_2 \) is equal to 0.8.

### 4.3 Results from the Example with a Large Number of Fuzzy Memberships

The second example studied in this thesis also deals with an automotive joint. This example has 28 objectives modeled using fuzzy sets. The results obtained using the conservative, the aggressive and the moderate methods are presented in this section. The memberships that are related to fuzzy objectives are described by equations (64), (65) and (83) to (86) and the ones related to constraints are given by equations (72) to (82). Table 5 depicts the memberships of the fuzzy design variables. The crisp constraints are expressed by equations (45) and (47) to (53).
4.3.1 Optimum Design Obtained Using Conservative Approach

The conservative method is applied to solve the second example. The overall objective function is expressed in equation (87). Tables 16 and 17 illustrate the values of the attributes and the memberships of the fuzzy constraints of the optimum design, respectively. It is observed that at the optimum, all the memberships of the objectives except for the stiffness from the response surface polynomial have the same value. The stiffness in parentheses in Table 16 is calculated using FEA. The values of the design variables of the optimum design are listed in Table 18. Table 19 depicts the constraints considered as crisp. Figure 42 illustrates the cross sections of the optimum design versus the baseline design and the baseline optimum. It is observed that the width and the height of the rocker of the design obtained with the conservative method are smaller than ones of the two other designs in Figure 42. The outboard ceil of the rocker contributes significantly in the stiffness of a joint. The design obtained with the conservative method has an outboard rocker cell that will accommodate the door sealant much better than the two other designs.

The overall design preference is equal to:

$$\mu_{D}(\hat{x}^*) = 0.49$$

(93)
Table 16: Values of the memberships of the fuzzy performance objectives with the second example.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Value of the Attribute</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>8.675</td>
<td>( \mu_{\text{Mass}} = 0.49 )</td>
</tr>
<tr>
<td>Torsional constant ( J ) (*10^4 mm^4)</td>
<td>174</td>
<td>( \mu_J = 0.49 )</td>
</tr>
<tr>
<td>Moment of inertia ( I_y ) (*10^4 mm^4)</td>
<td>210</td>
<td>( \mu_{I_y} = 0.49 )</td>
</tr>
<tr>
<td>Moment of inertia ( I_z ) (*10^4 mm^4)</td>
<td>157</td>
<td>( \mu_{I_z} = 0.49 )</td>
</tr>
<tr>
<td>Stiffness polynomial app. (Nm/rad)</td>
<td>7,120 (7,200**)</td>
<td>( \mu_{K_{\text{poly}}} = 0.51 )</td>
</tr>
<tr>
<td>Stiffness neural net. app. (Nm/rad)</td>
<td>6,990 (7,200**)</td>
<td>( \mu_{K_{\text{nn}}} = 0.49 )</td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)
Table 17: Values of the memberships of the fuzzy constraints with the second example.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Constraint description</th>
<th>Equation #, notation</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packaging</td>
<td>Width of the rocker (B2+C2)</td>
<td>69</td>
<td>( \mu_{12} = 0.49 )</td>
</tr>
<tr>
<td>Packaging</td>
<td>Width of the rocker (B2+C3)</td>
<td>70</td>
<td>( \mu_{13} = 0.50 )</td>
</tr>
<tr>
<td>Packaging</td>
<td>Height of the rocker</td>
<td>71</td>
<td>( \mu_{14} = 0.50 )</td>
</tr>
<tr>
<td>Styling</td>
<td>Door alignment</td>
<td>72</td>
<td>( \mu_{15} = 0.93 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T2</td>
<td>73</td>
<td>( \mu_{16} = 0.83 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T2 &amp; T3</td>
<td>74</td>
<td>( \mu_{17} = 0.49 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T3 &amp; T4</td>
<td>75</td>
<td>( \mu_{18} = 0.49 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T5</td>
<td>76</td>
<td>( \mu_{19} = 0.75 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T6/2</td>
<td>77</td>
<td>( \mu_{20} = 0.64 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T7</td>
<td>78</td>
<td>( \mu_{21} = 0.76 )</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T3</td>
<td>79</td>
<td>( \mu_{22} = 0.66 )</td>
</tr>
</tbody>
</table>
Table 18: Values of the design variables that defined the optimum design obtained using the conservative approach with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Values</th>
<th>Value of the membership</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1 (mm)</td>
<td>0.92</td>
<td>$\mu_1 = 0.49$</td>
<td></td>
</tr>
<tr>
<td>T2 (mm)</td>
<td>0.84</td>
<td>$\mu_2 = 0.74$</td>
<td></td>
</tr>
<tr>
<td>T3 (mm)</td>
<td>1.09</td>
<td>$\mu_3 = 0.68$</td>
<td></td>
</tr>
<tr>
<td>C1 (mm)</td>
<td>29.05</td>
<td>$\mu_4 = 0.49$</td>
<td></td>
</tr>
<tr>
<td>C2 (mm)</td>
<td>79.11</td>
<td>$\mu_5 = 0.91$</td>
<td></td>
</tr>
<tr>
<td>C3 (mm)</td>
<td>78.77</td>
<td>$\mu_6 = 0.49$</td>
<td></td>
</tr>
<tr>
<td>C4 (mm)</td>
<td>47.26</td>
<td>$\mu_7 = 1.00$ Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.783</td>
<td>$\mu_8 = 1.00$ Crisp</td>
<td></td>
</tr>
<tr>
<td>B2 (mm)</td>
<td>76.19</td>
<td>$\mu_9 = 0.65$</td>
<td></td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>42.03</td>
<td>$\mu_{10} = 1.00$ Crisp</td>
<td>Upper bound</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.00</td>
<td>$\mu_{11} = 1.00$ Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>69.71</td>
<td>$\mu_{12} = 1.00$ Crisp</td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$ (°)</td>
<td>75.52</td>
<td>$\mu_{13} = 1.00$ Crisp</td>
<td></td>
</tr>
<tr>
<td>H3 (mm)</td>
<td>121.4</td>
<td>$\mu_{14} = 0.49$</td>
<td></td>
</tr>
<tr>
<td>H4 (mm)</td>
<td>62.39</td>
<td>$\mu_{15} = 0.49$</td>
<td></td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>17.69</td>
<td>$\mu_{16} = 1.00$ Crisp</td>
<td>Upper bound</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>5.69</td>
<td>$\mu_{17} = 1.00$ Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>$\alpha_2$ (°)</td>
<td>68.16</td>
<td>$\mu_{18} = 1.00$ Crisp</td>
<td></td>
</tr>
<tr>
<td>HL (mm)</td>
<td>117.2</td>
<td>$\mu_{19} = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>195.7</td>
<td>$\mu_{20} = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RPL (mm)</td>
<td>243.7</td>
<td>$\mu_{21} = 1.00$</td>
<td>Lower bound</td>
</tr>
</tbody>
</table>
**Table 19:** List of crisp constraints with the second example.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Constraint description</th>
<th>Equation #, notation</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Styling</td>
<td>Width of the door</td>
<td>41, $c_1$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Door edge width</td>
<td>43, $c_3$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope of H5</td>
<td>44, $c_4$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope to accommodate for water runoff</td>
<td>45, $c_5$</td>
<td>Yes</td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope to accommodate the door sealant</td>
<td>46, $c_6$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Point D be below point E</td>
<td>47, $c_7$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Point E above line DF</td>
<td>48, $c_8$</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Stepout height</td>
<td>49, $c_9$</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 42:** Cross sections of the design obtained using the conservative method, the baseline optimum and the baseline design.
4.3.2 Optimum Design Obtained Using Aggressive Approach

The second method employed to solve the example of the automotive joint with a large number of fuzzy memberships is the aggressive method. The overall satisfaction is given by equation (88). The fuzzy performance objectives and the memberships of the fuzzy constraints are listed in Tables 20 and 21 respectively. Table 22 illustrates the design variables of the optimum solution. Finally, the active constraints are listed in Table 23. Figure 43 shows the rocker sections of the optimum design versus the baseline design and the baseline optimum. The optimum design obtained using the aggressive method has a smaller cross section than the baseline design and the baseline optimum (hence volume and mass). The overall level of satisfaction of the optimum design using the aggressive method is equal to:

\[ \mu_0(\tilde{x}^*) = 0.69 \]  

(94)

**Table 20:** Values of the memberships of the fuzzy performance objectives with the second example.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Value of the Attribute</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>8.410</td>
<td>( \mu_{\text{Mass}} = 0.54 )</td>
</tr>
<tr>
<td>Torsional constant ( J ) (*10^4 mm^4)</td>
<td>145</td>
<td>( \mu_J = 0.31 )</td>
</tr>
<tr>
<td>Moment of inertia ( I_y ) (*10^4 mm^4)</td>
<td>164</td>
<td>( \mu_{I_y} = 0.27 )</td>
</tr>
<tr>
<td>Moment of inertia ( I_z ) (*10^4 mm^4)</td>
<td>130</td>
<td>( \mu_{I_z} = 0.30 )</td>
</tr>
<tr>
<td>Stiffness polynomial app. (Nm/rad)</td>
<td>6,810 ( \text{(6,410**)} )</td>
<td>( \mu_{K_{\text{poly}}} = 0.49 )</td>
</tr>
<tr>
<td>Stiffness neural net. app. (Nm/rad)</td>
<td>6,810 ( \text{(6,410**)} )</td>
<td>( \mu_{K_{\text{nn}}} = 0.48 )</td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)**
Table 21: Values of the memberships of the fuzzy constraints with the second example.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Constraint description</th>
<th>Equation number</th>
<th>Value of the Membership</th>
</tr>
</thead>
<tbody>
<tr>
<td>Packaging</td>
<td>Width of the rocker (B2+C2)</td>
<td>69</td>
<td>$\mu_{12} = 0.72$</td>
</tr>
<tr>
<td>Packaging</td>
<td>Width of the rocker (B2+C3)</td>
<td>70</td>
<td>$\mu_{13} = 0.95$</td>
</tr>
<tr>
<td>Packaging</td>
<td>Height of the rocker</td>
<td>71</td>
<td>$\mu_{14} = 0.89$</td>
</tr>
<tr>
<td>Styling</td>
<td>Door alignment</td>
<td>72</td>
<td>$\mu_{15} = 1.00$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T2</td>
<td>73</td>
<td>$\mu_{16} = 1.00$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T2 &amp; T3</td>
<td>74</td>
<td>$\mu_{17} = 0.66$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T3 &amp; T4</td>
<td>75</td>
<td>$\mu_{18} = 0.51$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T5</td>
<td>76</td>
<td>$\mu_{19} = 0.74$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T6/2</td>
<td>77</td>
<td>$\mu_{20} = 0.66$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T7</td>
<td>78</td>
<td>$\mu_{21} = 0.74$</td>
</tr>
<tr>
<td>Manufacturing</td>
<td>Difference between T1 &amp; T3</td>
<td>79</td>
<td>$\mu_{22} = 0.66$</td>
</tr>
</tbody>
</table>
Table 22: Values of the design variables that defined the optimum design obtained using the aggressive approach with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Values</th>
<th>Value of the membership</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1 (mm)</td>
<td>0.93</td>
<td>$\mu_1 = 0.45$</td>
<td></td>
</tr>
<tr>
<td>T2 (mm)</td>
<td>0.93</td>
<td>$\mu_2 = 0.92$</td>
<td></td>
</tr>
<tr>
<td>T3 (mm)</td>
<td>1.10</td>
<td>$\mu_3 = 0.65$</td>
<td></td>
</tr>
<tr>
<td>C1 (mm)</td>
<td>26.10</td>
<td>$\mu_4 \approx 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C2 (mm)</td>
<td>77.51</td>
<td>$\mu_5 = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C3 (mm)</td>
<td>70.75</td>
<td>$\mu_6 = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>C4 (mm)</td>
<td>57.76</td>
<td>1.00 Crisp</td>
<td>Upper bound</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.767</td>
<td>1.00 Crisp</td>
<td></td>
</tr>
<tr>
<td>B2 (mm)</td>
<td>70.83</td>
<td>$\mu_7 = 1.00$</td>
<td></td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>38.40</td>
<td>1.00 Crisp</td>
<td></td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.00</td>
<td>1.00 Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>64.20</td>
<td>1.00 Crisp</td>
<td></td>
</tr>
<tr>
<td>$\alpha_1$ (°)</td>
<td>69.71</td>
<td>1.00 Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H3 (mm)</td>
<td>115.2</td>
<td>$\mu_8 = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H4 (mm)</td>
<td>56.05</td>
<td>$\mu_9 = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>17.69</td>
<td>1.00 Crisp</td>
<td>Upper bound</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>6.95</td>
<td>1.00 Crisp</td>
<td>Upper bound</td>
</tr>
<tr>
<td>$\alpha_2$ (°)</td>
<td>77.07</td>
<td>1.00 Crisp</td>
<td></td>
</tr>
<tr>
<td>HL (mm)</td>
<td>117.2</td>
<td>$\mu_{10} = 1.00$</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>195.7</td>
<td>1.00 Crisp</td>
<td>Lower bound</td>
</tr>
<tr>
<td>RPL (mm)</td>
<td>243.7</td>
<td>$\mu_{11} = 1.00$</td>
<td>Lower bound</td>
</tr>
</tbody>
</table>
Table 23: List of crisp constraints with the second example.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Constraint description</th>
<th>Equation #, notation</th>
<th>Active</th>
</tr>
</thead>
<tbody>
<tr>
<td>Styling</td>
<td>Width of the door</td>
<td>41, c₁</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Door edge width</td>
<td>43, c₃</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope of H5</td>
<td>44, c₄</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope to accommodate for water runoff</td>
<td>45, c₅</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Slope to accommodate the door sealant</td>
<td>46, c₆</td>
<td>Yes</td>
</tr>
<tr>
<td>Packaging</td>
<td>Point D be below point E</td>
<td>47, c₇</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Point E above line DF</td>
<td>48, c₈</td>
<td></td>
</tr>
<tr>
<td>Packaging</td>
<td>Stepout height</td>
<td>49, c₉</td>
<td></td>
</tr>
</tbody>
</table>

Figure 43: Cross sections of the design obtained using the aggressive method, the baseline optimum and the baseline design.
4.3.3 Optimum Design Obtained Using Moderate Approach

The third method considered in this study is the moderate method. Two values of the factor $w$ are considered: 0.25 and 0.75. The overall design preference is given by equation (89). Tables 24 and 25 present the fuzzy attributes and the memberships of the fuzzy constraints for the two values of $w$. The fuzzy and crisp design variables of the optimum solutions are listed in Tables 26 and 27 respectively. The active constraints are mentioned in Table 28. Note that the constraint to accommodate the sealant of the door is not active when $w$ is equal to 0.75.

The overall satisfaction of the design is equal to:

$$\mu_D(\bar{x}^{'}) = 0.60 \quad \text{when} \quad w = 0.25 \quad (95)$$

$$\mu_D(\bar{x}^{'}) = 0.52 \quad \text{when} \quad w = 0.75 \quad (96)$$

Figure 44 shows the rocker sections of the optimum designs for $w$ equal to 0.25 and 0.75. The shapes of the rocker cross section are very similar, only the inboard cell of the rocker is slightly different between the two optimum designs.
**Table 24:** Values of the memberships of the fuzzy performance objectives with the second example.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Value of the Attribute w = 0.25</th>
<th>Membership w = 0.25</th>
<th>Value of the Attribute w = 0.75</th>
<th>Membership w = 0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>9.021</td>
<td>0.42</td>
<td>8.696</td>
<td>0.486</td>
</tr>
<tr>
<td>Torsional constant $I$</td>
<td>162</td>
<td>0.42</td>
<td>173</td>
<td>0.486</td>
</tr>
<tr>
<td>(*$10^4$ mm$^4$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment of inertia $I_y$</td>
<td>195</td>
<td>0.42</td>
<td>209</td>
<td>0.486</td>
</tr>
<tr>
<td>(*$10^4$ mm$^4$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moment of inertia $I_z$</td>
<td>147</td>
<td>0.42</td>
<td>156</td>
<td>0.486</td>
</tr>
<tr>
<td>(*$10^4$ mm$^4$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stiffness polynomial app.</td>
<td>7,800</td>
<td>0.57</td>
<td>7,200</td>
<td>0.52</td>
</tr>
<tr>
<td>(Nm/rad)</td>
<td>(8,010**)</td>
<td></td>
<td>(7,250**)</td>
<td></td>
</tr>
<tr>
<td>Stiffness neural net. app.</td>
<td>7,700</td>
<td>0.55</td>
<td>7,100</td>
<td>0.50</td>
</tr>
<tr>
<td>(Nm/rad)</td>
<td>(8,010**)</td>
<td></td>
<td>(7,250**)</td>
<td></td>
</tr>
</tbody>
</table>

(** Stiffness calculated using FEA as explained in Appendix B)
Table 25: Values of the memberships of the fuzzy constraints with the second example.

<table>
<thead>
<tr>
<th>Constraint description</th>
<th>Equation number</th>
<th>Value of the Membership w = 0.25</th>
<th>Value of the Membership w = 0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of the rocker (B2+C2)</td>
<td>69</td>
<td>0.55</td>
<td>0.486</td>
</tr>
<tr>
<td>Width of the rocker (B2+C3)</td>
<td>70</td>
<td>0.77</td>
<td>0.486</td>
</tr>
<tr>
<td>Height of the rocker</td>
<td>71</td>
<td>0.70</td>
<td>0.51</td>
</tr>
<tr>
<td>Door alignment</td>
<td>72</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>Difference between T1 &amp; T2</td>
<td>73</td>
<td>1.00</td>
<td>0.85</td>
</tr>
<tr>
<td>Difference between T2 &amp; T3</td>
<td>74</td>
<td>0.54</td>
<td>0.486</td>
</tr>
<tr>
<td>Difference between T3 &amp; T4</td>
<td>75</td>
<td>0.64</td>
<td>0.51</td>
</tr>
<tr>
<td>Difference between T1 &amp; T5</td>
<td>76</td>
<td>0.72</td>
<td>0.75</td>
</tr>
<tr>
<td>Difference between T1 &amp; T6/2</td>
<td>77</td>
<td>0.68</td>
<td>0.65</td>
</tr>
<tr>
<td>Difference between T1 &amp; T7</td>
<td>78</td>
<td>0.72</td>
<td>0.75</td>
</tr>
<tr>
<td>Difference between T1 &amp; T3</td>
<td>79</td>
<td>0.54</td>
<td>0.63</td>
</tr>
</tbody>
</table>
Table 26: Values of the fuzzy design variables that defined the optimum design obtained using the moderate approach with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Values w =</th>
<th>Value of the membership w =</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>T1 (mm)</td>
<td>0.94</td>
<td>0.92</td>
</tr>
<tr>
<td>T2 (mm)</td>
<td>0.94</td>
<td>0.85</td>
</tr>
<tr>
<td>T3 (mm)</td>
<td>1.17</td>
<td>1.11</td>
</tr>
<tr>
<td>C1 (mm)</td>
<td>26.10</td>
<td>26.10</td>
</tr>
<tr>
<td>C2 (mm)</td>
<td>77.51</td>
<td>78.84</td>
</tr>
<tr>
<td>C3 (mm)</td>
<td>70.75</td>
<td>78.84</td>
</tr>
<tr>
<td>B2 (mm)</td>
<td>76.01</td>
<td>76.60</td>
</tr>
<tr>
<td>H3 (mm)</td>
<td>119.6</td>
<td>121.5</td>
</tr>
<tr>
<td>H4 (mm)</td>
<td>56.05</td>
<td>60.86</td>
</tr>
<tr>
<td>HL (mm)</td>
<td>117.2</td>
<td>117.2</td>
</tr>
<tr>
<td>RPL (mm)</td>
<td>283.7</td>
<td>243.7</td>
</tr>
</tbody>
</table>

Table 27: Values of the crisp design variables that defined the optimum design obtained using the moderate approach with the second example.

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Values w =</th>
<th>Memberships of the crisp parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>0.75</td>
</tr>
<tr>
<td>C4 (mm)</td>
<td>57.76</td>
<td>47.26</td>
</tr>
<tr>
<td>B1/B2</td>
<td>0.783</td>
<td>0.785</td>
</tr>
<tr>
<td>B3 (mm)</td>
<td>40.62</td>
<td>42.03</td>
</tr>
<tr>
<td>B4/B1</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>H1 (mm)</td>
<td>70.23</td>
<td>68.99</td>
</tr>
<tr>
<td>α_1 (°)</td>
<td>75.52</td>
<td>75.52</td>
</tr>
<tr>
<td>H5 (mm)</td>
<td>17.69</td>
<td>17.05</td>
</tr>
<tr>
<td>H6 (mm)</td>
<td>6.14</td>
<td>5.69</td>
</tr>
<tr>
<td>α_2 (°)</td>
<td>73.65</td>
<td>68.34</td>
</tr>
<tr>
<td>RP (mm)</td>
<td>212.7</td>
<td>195.7</td>
</tr>
</tbody>
</table>
Table 28: List of crisp constraints the second example.

<table>
<thead>
<tr>
<th>Constraint description</th>
<th>Equation # notation</th>
<th>Active w = 0.25</th>
<th>Active w = 0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width of the door</td>
<td>41, $c_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Door edge width</td>
<td>43, $c_3$</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Slope of H5</td>
<td>44, $c_4$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope to accommodate for water runoff</td>
<td>45, $c_5$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Slope to accommodate the door sealant</td>
<td>46, $c_6$</td>
<td>Yes</td>
<td></td>
</tr>
<tr>
<td>Point D be below point E</td>
<td>47, $c_7$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Point E above line DF</td>
<td>48, $c_8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stepout height</td>
<td>49, $c_9$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 44: Cross sections of the designs obtained using the moderate method when $w$ is equal to 0.25 and 0.75.
4.3.4 Comparison of the Results Obtained Performing Optimization on the Second Example

4.3.4.1 Comparison of the results of the second example with the results of the baseline design and the baseline optimum

Table 29 presents a comparison of the results of the fuzzy methods versus the baseline design and baseline optimum.

The conservative method leads to a design with a mass and a stiffness, respectively 19.9 % and 71.4 % higher than the values of the baseline optimum. The small increase in the mass is compensated by an important gain in the stiffness.

The moderate method gives results that are similar for the different values of w investigated in this work: high mass and stiffness. When w is equal to 0.25, the optimum design has a higher mass than the baseline design (7.7 %) and a stiffness that is 90.7 % higher than the stiffness of the baseline optimum. Note that the stiffness of the optimum design is 80.4 % higher than the one of the original joint of the actual car studied. When w is equal to 0.75, the design has a high mass but a high stiffness also.

The aggressive approach improves significantly the stiffness of the joint (44.4 %) and worsens the mass (0.4 %) compared to the baseline optimum. The optimum design obtained using the aggressive method is lighter than the baseline design but heavier than the baseline optimum. Because of the trade offs among the attributes, the stiffness of this optimum design is not as high as the stiffness of the design obtained with the conservative method.

From Table 29, we conclude that it is possible to design a joint with a much higher stiffness and a slightly higher mass than the baseline design. None of the methods
investigated have lead to a joint with a lower mass than the one of the baseline optimum because the mass is not the only objective considered in our study.

**Table 29:** Comparison with the mass and the stiffness of the baseline design and the baseline optimum.

<table>
<thead>
<tr>
<th></th>
<th>Conservative method</th>
<th>Moderate method ( w = 0.25 )</th>
<th>Moderate method ( w = 0.75 )</th>
<th>Aggressive Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( M ) (kg)</td>
<td>8.675</td>
<td>9.021</td>
<td>8.696</td>
<td>8.410</td>
</tr>
<tr>
<td>Change(_{relative \ to \ Baseline \ Design} (M))</td>
<td>3.6</td>
<td>7.7</td>
<td>3.8</td>
<td>0.4</td>
</tr>
<tr>
<td>Change(_{relative \ to \ Baseline \ Optimum} (M))</td>
<td>19.9</td>
<td>24.7</td>
<td>20.2</td>
<td>16.2</td>
</tr>
<tr>
<td>Stiffness, ( K_{FEM} ) (N(\text{m}/\text{rad}))</td>
<td>7,200</td>
<td>8,010</td>
<td>7,250</td>
<td>6,410</td>
</tr>
<tr>
<td>Change(<em>{relative \ to \ Baseline \ Design} (K</em>{FEM}))</td>
<td>62.2</td>
<td>80.4</td>
<td>63.3</td>
<td>44.4</td>
</tr>
<tr>
<td>Change(<em>{relative \ to \ Baseline \ Optimum} (K</em>{FEM}))</td>
<td>71.4</td>
<td>90.7</td>
<td>72.6</td>
<td>52.6</td>
</tr>
</tbody>
</table>

The Change\(_{relative \ to \ Baseline \ Design}\) and the Change\(_{relative \ to \ Baseline \ Optimum}\) are defined as follows:

\[
\text{Change}_{relative \ to \ Baseline \ Design}(X) = \frac{X - X \text{ of the Baseline Design}}{X \text{ of the Baseline Design}} \times 100\% \quad (97)
\]

\[
\text{Change}_{relative \ to \ Baseline \ Optimum}(X) = \frac{X - X \text{ of the Baseline Optimum}}{X \text{ of the Baseline Optimum}} \times 100\% \quad (98)
\]

**4.3.4.2 Discussion of the results**

Figure 45 shows the levels of satisfaction of the performance targets of the baseline design which are calculated using the memberships define in the second example. The performance targets are: mass, twist constant, moments of inertia and stiffness
estimated by two approximate tools. Figure 46 shows the levels of satisfaction of the six performance targets of the optimum designs obtained using the three fuzzy sets based methods.

The baseline design is illustrated in Figure 45. It is observed in this Figure that the memberships of the stiffness obtained with the two approximate tools are significantly lower than the memberships of the others attributes.

The design obtained using the conservative method has a relatively high mass and a high stiffness. The value of the membership that measured the overall satisfaction is equal to the minimum of all the memberships. The metric takes into account the value of eleven membership functions that are equal to grade the optimum design. Note that the minimum value of all the memberships is 0.49. It implies that the memberships of the attributes are equal or greater to 0.49 which corresponds to acceptable values for the attributes. No particular membership function drives the design because no membership is significantly lower than the others as can be observed from Figure 46.

The aggressive method \((w = 0)\) allows trade-offs among the attributes. As a result, some memberships have low values whereas others have high values. For instance, the membership of the moment of inertia \(I_y\) is equal to 0.27 as shown in Figure 46. Meanwhile, many memberships have high values; for example the membership of the mass is 0.54. The low membership of \(I_y\) is compensated by other memberships.

For \(w\) equal to 0.25 and 0.75, several memberships of the optimum design obtained using the moderate method are equal to the poorest level of satisfaction of all the memberships considered in the problem formulation. As it can be observed from equation 89, the importance of the memberships that have the lowest values increases with \(w\). The lowest memberships when \(w\) is equal to 0.25 are related to: the mass, the twist constant, the moments of inertia, the design variable \(T1\). When \(w\) is equal to 0.75, the lowest membership are the following: mass, twist constant, moments of inertia, width of the rocker, difference of thickness between \(T2\) and \(T3\), design variables \(T1, C3\) and \(H3\).
Figure 45: Memberships of the objectives of the baseline design.

Figure 46: Memberships of the objectives when using the conservative, the aggressive and the moderate methods.

Finally, an additional check of the results was performed. The optimum designs obtained using the three fuzzy methods are graded using the metrics of the conservative, the aggressive and moderate approaches as shown in Table 30. For instance, the design obtained using the conservative method has an overall level of satisfaction of 0.4898 using
the metric of the conservative method. But the degree of satisfaction of this optimum design is equal to 0.60 if we consider the metric of the aggressive method to evaluate it. This value of 0.60 is less than the value we obtained when optimizing the joint using the aggressive method.

**Table 30:** The use of several metrics to grade the optimum designs obtained with the conservative, the aggressive and moderate methods.

<table>
<thead>
<tr>
<th>Results of conservative meth.</th>
<th>Results of aggressive meth.</th>
<th>Results of moderate meth., w=0.25</th>
<th>Results of moderate meth., w=0.75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min metric used to grade the designs, (eq. 6)</td>
<td>0.4898</td>
<td>0.266</td>
<td>0.4197</td>
</tr>
<tr>
<td>Normalized product metric used to grade the designs, (eq. 14)</td>
<td>0.600</td>
<td>0.6940</td>
<td>0.6944</td>
</tr>
<tr>
<td>Metric of moderate meth. used to grade the designs, w = 0.25 (eq. 15)</td>
<td>0.5725</td>
<td>0.5870</td>
<td>0.5977</td>
</tr>
<tr>
<td>Metric of moderate meth. used to grade the designs, w = 0.75 (eq. 15)</td>
<td>0.5174</td>
<td>0.3730</td>
<td>0.4769</td>
</tr>
</tbody>
</table>

In the following, we summarize the results from the optimization of the second example:

1. All the results obtained using the three fuzzy methods have a higher mass than the original design and than the baseline design. Because several objectives require a high
mass to be satisfied (cross section properties, stiffness), the mass of the optimum joint obtained by all methods is high.

2. The optimum designs obtained by the fuzzy methods have stiffnesses that are 40 to 90% larger than the stiffnesses of the baseline design and baseline optimum. Note that it is possible to increase significantly the stiffness without increasing a lot the mass using the aggressive method.

3. The mass and the stiffness are conflicting objectives. To increase stiffness, we need to increase mass. However a small increase in the mass permits a significant increase of the stiffness. Using the fuzzy multiobjective methods, an engineer can decide precisely which trade-offs among the attributes are the best.

4.3.5 Importance of Using Realistic Membership Functions

This section demonstrates the importance of using realistic memberships functions for the objectives and the constraints. For this purpose, some key values defining the memberships of the rocker width will be changed and the effects of these changes on the results will be studied using the conservative and the aggressive methods. We want to determine which of these two methods is the most sensitive to a poorly built membership function.

It is important to invest time to formulate the problem correctly, and identify which memberships are the most critical in the sense that errors (or poor guesses) in these memberships result in poor final designs. In this section, only one value will be changed in the problem formulation. This value is the minimum width of the rocker that the joint can ever achieve, called Min_width_rock. There are two fuzzy memberships related to the rocker width as described in equations (72) and (73). Originally, Min_width_rock was equal to 140 mm. In Table 6, it is observed that one car has a rocker width equal to 100
mm. Based on this observation, an engineer who wants to reduce the dimensions of the actual joint can select this value as the minimum rocker width. Consequently, for the purpose of this work, Min_width_rock is fix at 100 mm. This change can be interpreted as a tightening of the constraint related to the width of the rocker. If the rocker width is 100 mm, the engineer will be completely satisfied. This implies that reducing the rocker width under the value of 100 mm does not affect the engineer’s satisfaction with the design. Then, we optimize the joint using the conservative and the aggressive approaches.

Tables 31 and 32 show how the attributes of the optimum designs change when the memberships of the rocker width change. Figure 47 illustrates the modification of the cross sections of the rocker when using the conservative method.

From Tables 31 and 32, we conclude that:

1. The conservative method is dramatically affected by theses changes. The cross sectional properties as well as the stiffness drop dramatically. The stiffness decreases by more than 30% compared to the stiffness of the joint obtained using the original memberships whereas the minimum rocker width decreases by 29%. It is important to have a design with a high stiffness because the stiffness of other cars in the same class is several times larger than the stiffness of the baseline design. Therefore, the design obtained with the modified membership functions using the conservative is actually poor.

Because of the side constraints of the design variables, the maximum value that the membership of the rocker width can take is low. Consequently, this membership drives the design and the optimizer focuses on decreasing the rocker width. A decrease in the rocker width will result in an increase in its membership which is the smallest membership of all the memberships. Of course, this observation must be examined with precaution because many entities in the problem formulation are
correlated. Figure 47 shows the rocker cross sections of the optimum designs obtained using the conservative method and the original memberships for the rocker width and when the rocker width decreases from 140 to 100 mm.

2. The aggressive method is not affected by the changes in the membership function of the rocker width. The optimum solution is the same as the one without any changes in the shape of the memberships of the rocker width. The reason is that when using the aggressive method and the original memberships, the levels of satisfaction of the constraints of the rocker width are already equal to the maximum values they can achieve. These maximum values are not equal to one as shown in Table 21 because of the side constraints of the design variables that define the rocker width. Indeed, the levels of satisfaction of these design variables (B2, C2 and C3) are equal to one. Consequently, even if the designer wants to reduce the rocker width by changing the shape of its memberships, the optimizer cannot find a better solution than the one obtained with the original memberships. Even if having a joint with a small rocker width is very important, the optimum design cannot have a smaller rocker width than the solution of the optimum design obtained with the original memberships.

3. In this case, we conclude that the conservative method is very sensitive to a poorly built membership function whereas the aggressive method is not.
Table 31: Effects of the change of Min_width_rock on the other attributes using the conservative method.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Attribute of optimum design obtained using the conservative meth. and the original memberships of the rocker width</th>
<th>Attribute of the optimum design when the minimum width of the rocker decreases from 140 to 100 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>8.675</td>
<td>8.52</td>
</tr>
<tr>
<td>Torsional constant J (*10⁴ mm⁴)</td>
<td>174</td>
<td>145</td>
</tr>
<tr>
<td>Moment of inertia Iₓ (*10⁴ mm⁴)</td>
<td>210</td>
<td>173</td>
</tr>
<tr>
<td>Moment of inertia Iᵧ (*10⁴ mm⁴)</td>
<td>157</td>
<td>131</td>
</tr>
<tr>
<td>Stiffness polynomial app. (Nm/rad)</td>
<td>7,120</td>
<td>4,660</td>
</tr>
<tr>
<td>Stiffness neural net. app. (Nm/rad)</td>
<td>6,990</td>
<td>4,850</td>
</tr>
</tbody>
</table>
Table 32: Effects of the change of Min_width_rock on the other attributes using the aggressive method.

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Attribute of optimum design obtained using the aggressive meth. and the original memberships of rocker width</th>
<th>Attribute of the optimum design when the minimum width of the rocker decreases from 140 to 100 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>8.410</td>
<td>8.409</td>
</tr>
<tr>
<td>Twist constant J (*10^4 mm^4)</td>
<td>145</td>
<td>145</td>
</tr>
<tr>
<td>Moment of inertia I_y (*10^4 mm^4)</td>
<td>164</td>
<td>164</td>
</tr>
<tr>
<td>Moment of inertia I_z (*10^4 mm^4)</td>
<td>130</td>
<td>129</td>
</tr>
<tr>
<td>Stiffness polynomial app. (Nm/rad)</td>
<td>6,810</td>
<td>6,790</td>
</tr>
<tr>
<td>Stiffness neural net. app. (Nm/rad)</td>
<td>6,810</td>
<td>6,790</td>
</tr>
</tbody>
</table>
Figure 47: Rocker cross sections of the optimum designs obtained using the conservative method and the original memberships of the rocker width and when the rocker width decreases from 140 to 100 mm.

From the results obtained in this section, the first example appears to be easier to implement correctly than the second one because only two fuzzy memberships need to be built correctly, such that the design won’t be ruined. We know that the conservative method ruins a design if a membership is poorly built. Consequently, the second example, with its 28 fuzzy memberships, requires a lot of rigor, such that none of the memberships is poorly established.

When optimizing the first example, the two memberships (mass and stiffness), which are primordial for the decision maker, have “high” priority during the optimization process. The optimum solutions using the conservative and aggressive methods achieve values for the mass and the stiffness that the second optimization problem can not reach. The optimum design obtained using the conservative method appears to be an excellent
compromise between the mass and the stiffness. This solution has a relatively small mass and a very high stiffness.

When optimizing the second example, important objectives such as the mass and the stiffness are compensated by memberships of factors with little importance such as the design variable B4/B1.

4.4 Comparison of Optimization Costs When Using a Neural Network and FEA to Predict Performance Characteristics

In the present research, the optimization of the automotive joint is performed using approximate tools to predict the stiffness of the joint to avoid the high computational cost associated with the use of FEA in optimization. These tools were developed by Zhu [36]. Recent advances in FEA and computers enable engineers to use numerical methods for optimization where the performance of the system is evaluated using FEA. Many FEA software programs include design optimization routines [19]. This section compares the costs of the optimization of the joint using a neural network and FEA.

For both optimization strategies, the cost is expressed in minutes of real time. It includes the time it takes to modify a finite element model using a parametric modeling program, such as Pro-Engineer, and to mesh and analyze the model using FEA. These modifications are done automatically when using FEA and optimization. These modifications need to be done also when optimizing using the neural network. Indeed, in our case, the neural network needs to be trained using a set of know input-output data which requires several FEA.

The time required to optimize the joint using neural network was estimated on the basis of our experience whereas the time required when using FEA programs for optimization was estimated based on discussion with Leiva from VMA Engineering/GENESIS and Wong from MSC/NASTRAN [11, 12, 14].
4.4.1 Optimization Costs When Using a Neural Network

The cost of optimization when using a neural network should consider the amount of time required to build a neural network. It is unrealistic to consider only the costs of the use and to neglect the costs of development of such a tool. The procedure to build a neural network consists of two steps: selection of the architecture and training/checking of the neural network. Table 33 presents the costs of optimization when using a neural network.

First, the engineer selects the architecture of the neural network. In this step, the engineer has to choose the network inputs and outputs, the number of hidden layers and the number of neurons in each hidden layer and the form of the transfer function of each neuron. The idea when determining the number of neurons in each hidden layers is that the number of unknowns should not exceed the number of equations. There are many strategies for making these choices [6, 21]. The estimated time for accomplishing this first step in the development of a neural network is 12 hours (720 minutes) as shown in Table 33 [12, 36].

Once the architecture has been selected for a particular problem, we need to train and check the neural network. First, we generate a set of training and checking data whose associated outputs are known using FEA. In developing the neural network used in this study, a known input-output data set consisting of 57 designs determined by the Box-Benkhen theory, was used to train the neural network [5, 36]. The neural network was checked using another set of 50 designs. As a result, a total of 107 known input-output data set was required to train and check the neural network. The first model of the structure built using a parametric CAD software, such as Pro-Engineer, required about 12 man weeks (28,800 minutes). This includes the time necessary to parameterize the structure, establish the constraints and develop the model on the computer. Once the first model was created, it took approximately five minutes to change the dimensions, one minute to mesh the structure using a parametric preprocessor, such as Pro/Mesh and
another five minutes to run the FEA. We assume that no crash happens during the FEM run. Consequently, the time to develop the training data set is 29,977 minutes, which consists of 28,800 minutes to develop the parametric model and 11*107 to analyze the performance of 107 designs used for training and testing as shown in Table 33. Next, the 107 designs are presented to the neural network and its parameters are adjusted to obtain the desired output values using the toolbox “Neural Network” from MATLAB [13]. The time required for this procedure is 120 minutes. The total time to build a neural network is estimated to 30,817 minutes.

Once a neural network is build, its major advantage is that its usage is almost free because it predicts the output in very small fraction of second [21]. Indeed, in this research, each optimization run lasts less than a minute. Consequently, this cost can be neglected in this study.

**Table 33:** Optimization costs when using a neural network.

<table>
<thead>
<tr>
<th>Task</th>
<th>Estimated time (minutes)</th>
<th>Source of estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selection of an architecture</td>
<td>720</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Creation of the first model of the structure using a parametric CAD software</td>
<td>28,800 (12 weeks)</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Change of the dimensions</td>
<td>5*107</td>
<td>our knowledge</td>
</tr>
<tr>
<td>New mesh of the improved design using a parametric preprocessor</td>
<td>1*107</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Run the FEA</td>
<td>5*107</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Training and checking of the neural network</td>
<td>120</td>
<td>our knowledge</td>
</tr>
<tr>
<td><strong>Total time</strong> required to develop and use a neural network in optimization</td>
<td><strong>30,817</strong></td>
<td>our knowledge</td>
</tr>
</tbody>
</table>
4.4.2 Optimization Costs When Using FEA

Many FEA programs have routines for optimization that automate the task of revising, remeshing and reevaluating models in order to achieve design goals as shown in Figure 48. Table 34 presents the optimization costs when using FEA. Approximation concepts are used to efficiently couple the numerical optimizer with the structural analysis code. These approximations consist of:

- Constraints screening. A typical optimization problem involves many constraints, some of which are not active during some iterations of the optimization procedure and are deleted temporarily.
- Developing an approximate model of the objective function and the constraints if these responses are implicit functions of the design variables. These approximations, which are based on simple first-order Taylor series expansions, are usually constructed automatically and are used to avoid the high cost associated with repeated FEA during design optimization.

The first step of the optimization method using FEA consists of defining the initial design, calculating the objective function, checking the constraints and performing sensitivity analysis. The user needs to input a preliminary design and to specify the design variables, the objective function and the constraints. Next, the user has to build the first model of the structure on a parametric CAD or a preprocessor. These tasks require 12 man weeks (28,800 minutes) as mentioned in Section 4.4.1. Then, the user runs the model once to get the values for the objective function and the constraints. The estimated time for this task is five minutes. Then, the constraint screening is performed, the sensitivities are evaluated and the approximate models are developed. The time required for these tasks is estimated to be five minutes. Note that the sensitivity analyses do not require FEA because they are computed either analytically or semi-analytically using the
results already available from the FEA’s performed to obtain the value of the objective function and the constraints.

The second step consists of the use of the approximate models by the optimizer. Typically, the optimizer uses the approximate models one or two dozens of times to find an improved design alternative. The cost of this step can be neglected because it lasts a couple of seconds only. Indeed, the optimizer is using approximate models and sensitivity analyses are already performed.

The third step consists of changing the mesh of the structure to check the design goals of the improved design. The automated procedure used by MSC/NASTRAN updates the mesh of the structure every time an improved design is proposed, reducing the risks of mesh distortion for large shape changes. This step requires about one minute.

If the new design does not satisfy the convergence criteria, then a new analysis cycle starts. In all cases, the last task is a FEA to check the properties of the final improved design as shown in Table 34. Because the number of design analysis revision cycles needed for an optimization can not be estimated, we considered this number as a variable in the comparison. Typically, 10 to 40 design analysis revision cycles are required for each optimizing simple structures such as a torque and a steering arms [1, 3, 11].

The total cost for this method is:

\[ 28,800 + (5 + 5 + 1) \times n + 5 \]

where \( n \) is the number of design analysis revision cycles using a complete FEA.
**Figure 48:** Algorithm explaining the approximation concepts to perform simultaneously optimization and FEA.

**Table 34:** Optimization costs when using FEA.

<table>
<thead>
<tr>
<th>Task</th>
<th>Estimated time</th>
<th>Source of estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Creation of the first model of the structure using a parametric CAD software</td>
<td>28,800 (12 weeks)</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Run the FEA</td>
<td>$5 \times n$</td>
<td>our knowledge</td>
</tr>
<tr>
<td>Constraint screening, sensitivities analyses and development of the approximate models</td>
<td>$5 \times n$</td>
<td>reference [11]</td>
</tr>
<tr>
<td>New mesh of the improved design using a parametric preprocessor</td>
<td>$1 \times n$</td>
<td>reference [11]</td>
</tr>
<tr>
<td>Check the properties of the improved design using FEA</td>
<td>5</td>
<td>our knowledge</td>
</tr>
<tr>
<td><strong>Total time required to develop and use FEA in optimization</strong></td>
<td>$28,805 + 11 \times n$</td>
<td>our knowledge and reference [11]</td>
</tr>
</tbody>
</table>

(where $n$ is the number of design analysis revision cycles using a complete FEA)
4.4.3. Comparison of Costs

The comparison of costs of the two approaches described earlier is illustrated in Figure 49. The initial cost to perform optimization using a neural network is relatively high (30,817 minutes) compared to the corresponding cost of optimization using FEA (28,805+11*10 = 28,915 minutes to 28,805+11*40 = 29,245 minutes). Consequently, if the optimization is performed only once, it is not advantageous to use a neural network to predict the performance of the system because of the initial cost of developing such a tool. However, the cost of use of a neural network is negligible once the neural network is built as shown in Figure 49. Thus, if we perform the optimization several times, then the cost of optimization remains constant, which is represented by the horizontal line on Figure 49. When optimizing using FEA, the initial cost is relatively low but the cost of each optimization run is between 110 and 440 minutes which corresponds to 10 to 40 design analysis revision cycles [1, 3, 11]. If we optimize the structure several times, then the cost of this strategy increases significantly. For approximately 180 design analysis revision cycles (four to eighteen optimization runs), the method using neural network is more advantageous than the method using FEA. Note that, in real life, we are likely to perform optimization of automotive joints more than eighteen times. One reason is that, once the structure had been optimized, we may realize that some constraints were not correctly defined and decide to modify them. These modifications required to run a new optimization. Another reason is that if the performance of the optimum design is unacceptable, then the joint must be redesigned using more resources.
Figure 49: Comparison of costs of optimization methods using neural network and FEA.

In the above discussion, we assume that no mesh distortion happens when performing optimization using FEA [26]. We neglect also the influence of the noise related to FE techniques. This noise is illustrated in Figure 50. When using FEA, the rate of change of the performance characteristic of the joint with respect to one design variable (segment AB in Figure 50) may not be correct because of the noise. Consequently, the optimizer may perform a large number of iterations. When using a neural network to perform optimization, if we assume that the neural network gives accurate results then the noise is reduced significantly as shown in Figure 50 because it is averaged. The segment AB’ reflects the overall tendency of the change of the performance characteristic when a design variable changes. In real life problem, mesh distortion and noise problem happen. As a result, optimization using a neural network is worth considering as an alternative to the use of FEA in optimization.
Figure 50: Representation of the noise when using neural network and FEA.
5 Conclusions

In this chapter, we give the concluding remarks. In section 5.1, the summary of the thesis is presented. Then, some suggestions for future work are presented in section 5.2.

5.1 Summary

This thesis focused on the comparison of three fuzzy set based methods and utility theory to design of an automotive joint using many objectives. The fuzzy set based methods were the conservative, the aggressive and the moderate methods.

The first objective of this research was to optimize the joint using fuzzy sets theory and utility theory to maximize the overall designer's satisfaction by considering several objectives simultaneously. Two problems involving multiobjective optimization of a joint were studied in order to understand the characteristics of fuzzy sets based methods and utility theory. The first problem involved two objectives: minimization of the mass and maximization of the stiffness. The optimization results were compared with the joint design of an existing car. This design was called baseline design. It was found that:

- The optimum designs obtained using the three fuzzy set methods have a small mass and a very high stiffness compared to the baseline design.
  1. The conservative method leads a design that has a small mass and a high stiffness compared to the baseline optimum.
2. The aggressive method allows trade-offs among the attributes. As a result, the degree of satisfaction of the optimum design with respect to some attributes can be very high while the degree of satisfaction with other attributes can be low. In the first example, the optimum design has extremely high stiffness and high mass compared to the baseline design.

3. The results obtained using the moderate methods are the same with the results obtained either with the aggressive method or with the conservative method depending on the discrete values of the weight factors considered.

- Utility theory requires the decision maker to quantify precisely the trade-offs that he is willing to accept among the attributes as opposed to fuzzy sets based methods studied in this thesis, which do not express the trade-offs explicitly. This characteristic made utility theory an appropriate method for problems where engineers can specify the trade-offs they wish to make.

Another important difference between fuzzy set based methods and utility theory is that attributes can always be traded even if the degree of satisfaction with respect to one or more attributes is zero using utility theory because the annihilation condition does not hold with utility theory.

- For certain values of scaling constants measuring the engineer's willingness to trade off various objectives, the optimum design was the design obtained with the conservative method. For other values of the scaling constants, the optimum was the same as the one obtained with the aggressive method. One case studied in this thesis showed that the optimum design was a Pareto optimum that has a mass considerably lower than the one of the baseline design.
In the second problem, we optimized the same joint considering many fuzzy objectives and fuzzy constraints.

- Three fuzzy methods led to design alternatives with a high mass and a relatively high stiffness compared to the baseline optimum.
  1. The mass of the optimum designs is slightly higher than the mass of the baseline design.
  2. The stiffness of the optimum designs is significantly higher than the stiffness of the baseline design.

- The optimum design obtained using the conservative method has more uniform levels of satisfaction with respect to the objectives and constraints than the designs obtained using the aggressive method. Actually, most of the memberships are equal at the optimum design. When using the conservative method, the optimizer focuses on increasing the value of the lowest membership.

- The optimum designs appear to be better than the baseline design and also a design obtained by minimizing the mass of the joint (baseline optimum obtained by Zhu) because most competitive joint designs have considerably higher stiffness than the one of the baseline design.

- The results of the optimization of this second example need to be interpreted with care. There are so many fuzzy memberships in the problem formulation that it is easy to ruin the optimum design because of a poorly built membership function as shown in section 4.3.5.
Fuzzy sets based methods seems to be more suitable for optimizing automotive joints than traditional crisp optimization approaches in the early design stage. Indeed, they handle the vagueness of concepts expressed using linguistic terms that are employed in the description of the problem. Another characteristic of these methods is that they optimize several attributes simultaneously whereas traditional single objective crisp optimizations consider only one attribute at a time. Consequently, they are more appropriate to model real life problems than traditional methods. Finally, these methods don’t allow the attributes to be traded to the point of reducing a degree of satisfaction of zero. This characteristic is necessary in most engineering problems.

Utility theory is a method that can be used for decision making under vagueness. However, this approach is different than fuzzy sets based methods. First, it allows engineers to decide the trade-offs among the attributes. This property is primordial in many applications. Second, when using utility theory, if the degree of satisfaction of one or more attributes is zero, the overall designer’s satisfaction can still be positive. This characteristic limits the use of utility theory to particular engineering problems where attributes can always be traded off.

Finally, the use of approximate tools to predict the performance of the system reduces significantly the computational time needed to analyze the design alternatives generated during the optimization process. The advantage of neural networks becomes obvious in cases where we need to repeat the optimization several times. This happens in many real life design problems where after performing an optimization, we find that we need to redesign the system by increasing the available resources or changing some constraints. Optimization using FEA was too expensive to be applied in this research because of the large number of optimizations performed. As a result, the method using approximate tools allowed us to conduct this study within a reasonable amount of time.
5.2 Future Work

Several new areas can be investigated:

- The methodology developed could be used to optimize other joints or parts of a car body.
- Other metrics could be investigated to combine the objectives into a single one. Weighted product and weighted additive metrics are among them.
- Methods for establishing membership functions, besides Rao's method, which was used in this study should be examined.

Utility theory can be studied with more than two single utility functions. These two functions can be estimated by means of the lottery method instead of assuming that they are equal to the membership functions of the associated attributes used in fuzzy set based optimization.
Appendix A:

Metric Axioms of Fuzzy Methods and Utility Theory
Antonsson [18]
Antonsson described the metric axioms of several fuzzy methods and utility theory [18]. In the following tables, \( P \) is the metric which combines various membership functions or single attribute utility functions.

**Conservative method:** \( \text{min metric (equation (6))} \)

The metric axioms are listed in Table A-1.

**Table A-1: Metric axioms of the conservative method.**

| \( P(0,...,0) = 0 \) & \( P(1,...,1) = 1 \) | boundary conditions |
| \( P(\mu^1,...,\mu^k,...,\mu^n) \leq P(\mu^1,...,\mu^k,...,\mu^n) \) iff \( \mu^k \leq \mu^k \) | monotonicity |
| \( P(\mu^1,...,\mu^k,...,\mu^n) = \lim P(\mu^1,...,\mu^k,...,\mu^n) \) when \( \mu^k \) tends to \( \mu^k \) | continuity |
| \( P(\mu^1,...,0,...,\mu^n) = 0 \) | annihilation |
| \( P(\mu^1,...,\mu,...,\mu) = \mu \) | idempotency |
| \( P(\mu^1,...,\mu,...,\mu) = P(\mu^1,...,\mu,...,\mu) \) | commutativity |
| \( P(P(\mu^1,...,\mu^k),\mu^k) = P(\mu^k,P(\mu^1,...,\mu^{n-1})) \) | associativity |
| \( P(1,...,1,\mu^k,1,...,1) = \mu^k \) | identity |

The first restriction, called boundary conditions, states that if the degree of satisfaction of all the attributes is zero, then the overall design preference is also zero. This restriction also implies that if the degree of satisfaction of all the attributes is one, then the overall design preference is one.

The second restriction is a monotonicity requirement. If a degree of satisfaction of one attribute is raised then the overall design preference either remains constant or is raised in the same direction.
The third restriction of continuity implies that if the level of satisfaction of one attribute changes slightly, then the overall satisfaction of the design remains either constant or changes slightly.

The fourth axiom is the annihilation restriction. It states that if the level of preference is zero in one or more attributes then the overall design preference is zero. This property is necessary in many engineering applications. For instance, an increase of the stress material shouldn’t always be compensated by a decrease in cost because the stress limit of the material can be exceeded.

The fifth axiom means that the metric under consideration doesn’t combine the attributes in a pessimistic or optimistic manner. If a design has n attributes and these n attributes have all the same level of satisfaction i, the overall design preference should also be equal to i and not be equal to a greater or smaller value than i.

The last three axioms present nothing new in terms of the way the conservative method is inferencing mechanisms under uncertainty.

**Aggressive method:** normalized product metric (equation (14))

The metric axioms are listed in Table A-2.
**Table A-2: Metric axioms of the aggressive method.**

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(0, ..., 0) = 0$</td>
<td>boundary conditions</td>
</tr>
<tr>
<td>$P(1, ..., 1) = 1$</td>
<td></td>
</tr>
<tr>
<td>$P(\mu^1, ..., \mu^k, ..., \mu^n) \leq P(\mu^1, ..., \mu^k, ..., \mu^n)$</td>
<td>monotonicity</td>
</tr>
<tr>
<td>$\mu^k \leq \mu^{k'}$</td>
<td></td>
</tr>
<tr>
<td>$P(\mu^1, ..., \mu^k, ..., \mu^n) = \lim P(\mu^1, ..., \mu^k, ..., \mu^n)$</td>
<td>continuity</td>
</tr>
<tr>
<td>when $\mu^k$ tends to $\mu^{k'}$</td>
<td></td>
</tr>
<tr>
<td>$P(\mu^1, ..., 0, ..., \mu^n) = 0$</td>
<td>annihilation</td>
</tr>
<tr>
<td>$P(\mu, ..., \mu, ..., \mu) = \mu$</td>
<td>idempotency</td>
</tr>
<tr>
<td>$P(\mu^1, ..., a, ..., b, ..., \mu^n) = P(\mu^1, ..., b, ..., a, ..., \mu^n)$</td>
<td>commutativity</td>
</tr>
<tr>
<td>$P(P(a, b), P(c, d)) = P(P(a, c), P(b, d))$</td>
<td>associativity</td>
</tr>
<tr>
<td>$P(\mu^1, ..., \mu^k, ..., \mu^n) &lt; P(\mu^1, ..., \mu^k, ..., \mu^n)$</td>
<td>strictness</td>
</tr>
<tr>
<td>$\mu^k \leq \mu^{k'}$</td>
<td></td>
</tr>
</tbody>
</table>

The first seven axioms are the same as the ones described earlier. The last condition is strictness. It implies that if a degree of satisfaction of one attribute is raised then the overall satisfaction of the design is raised in the same direction. This restriction is not always appropriate in engineering design. For instance, if one attribute has a very low level of satisfaction, the overall design preference should express directly this weakness in the design by being equal to this poor value. In this case, the overall design preference should not be affected by the level of performance of the remaining attributes.

Utility theory: non-linear multiplicative form (equation (16))

The metric axioms are listed in Table A-3 and have already been explained earlier. Note that the annihilation restriction is not satisfied by utility theory: a level of satisfaction of zero in one attribute doesn’t mean that the overall design preference is zero.
**Table A-3:** Metric axiom of the utility theory.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(0, ..., 0) = 0$</td>
<td>boundary conditions</td>
</tr>
<tr>
<td>$P(1, ..., 1) = 1$</td>
<td></td>
</tr>
<tr>
<td>$P(\mu^1, ..., \mu^k, ..., \mu^n) \leq P(\mu^1, ..., \mu^{k'}, ..., \mu^n) \quad \text{iff } \mu^k \leq \mu^{k'}$</td>
<td>monotonicity</td>
</tr>
<tr>
<td>$P(\mu^1, ..., \mu^k, ..., \mu^n) = \lim P(\mu^1, ..., \mu^{k'}, ..., \mu^n) \quad \text{when } \mu^k \text{ tends to } \mu^{k'}$</td>
<td>continuity</td>
</tr>
</tbody>
</table>
Appendix B

Determination of the stiffness of the joint using FEA,
Zhu [36]
The stiffness of an automotive joint is defined as the ratio of the moment applied to a branch over the rotation of that branch due to the joint flexibility, while the other branches are clamped. The rotation due to the joint flexibility is obtained if we subtract from the rotation of the branch the component of the rotation due to the flexibility of the attached beams. The stiffness under Inboard/Outboard (I/O) bending is investigated because it affects significantly the torsional rigidity of the car body.

The stiffness of the joint in the I/O direction was checked using FEA. A detailed model of the joint is very complex and expensive to develop. Consequently, its application is limited to late stages of design. We used a program developed by Zhu to convert the design variables defining a joint into a FEM model [36]. This model is called generic model and is illustrated in Figure B-1. It uses plate elements (CQUAD4) and gives reasonably accurate results compared to a detailed FEM model of the joint.

An MSC/NASTRAN generic model is investigated. An I/O bending moment $M_s$ is applied at the tip of the B-pillar (Figure 10 in the thesis). Both the front and the rear ends of the rocker are clamped. The different components of a real joint are connected to each other by appropriate welding or spot welds. In the generic model, rigid beam elements are used between nodes to model spot welds.

The total deflection at the tip of the pillar under I/O bending moment is given by the following equation:

$$\phi_{tip} = \phi_s + \phi_t + \phi_p + \phi_r$$

where:

- $\phi_{tip}$ is total deflection at the tip of the pillar,
- $\phi_s$ is shear deflection of the rocker,
- $\phi_t$ is twist deflection of the rocker (can be calculated using strength of materials),
- $\phi_p$ is bending deflection of the B-pillar (can be calculated using strength of materials),
$\phi_o$ is deflection due to weld access holes,
$\phi_r$ is residual deformation.

The I/O stiffness of the joint is defined as follows:

$$K_{I/O} = \frac{M_x}{\phi_{tip} - (\phi_I + \phi_p)} \quad \text{(B-2)}$$

where $K_{I/O}$ is the I/O stiffness of the joint.

In equation B-2, $\phi_I$ approximately is equal to two percents of the total deflection $\phi_{tip}$ for the detailed FEM model [36]. Consequently, instead of calculating this value which accounts for only two percent of the final deflection of the tip of the pillar, we consider:

$$\phi_I = 0.02 \times \phi_{tip} \quad \text{(B-3)}$$

Furthermore, the bending deflection of the pillar $\phi_p$ can be evaluated using strength of materials:

$$\phi_p = \frac{M_x L_p}{EI} \quad \text{(B-4)}$$

where:

$M_x$ is the bending moment (24,500 Nmm),
$L_p$ is the length of the pillar (500 mm),
$E$ is the elasticity modulus (2.1*10^5 N/mm²),
$I$ is the I/O bending moment of inertia (487.48 mm⁴).

**Application:**

Using equation B-4, we found that $\phi_p$ is equal to 0.1197*10⁻³ rad.
section 4.1:
The results are given in Table B-1. The stiffness is calculating using equation B-2.

**Table B-1:** Tip deflections of optimum joints measured using FEA (Pareto set of optimum).

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip deflection of the B-pillar $\phi_{up}$ ($10^3$ rad)</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimization of the mass set of Pareto optimum</td>
<td>24.745</td>
<td>1,000</td>
</tr>
<tr>
<td>Maximization of the stiffness set of Pareto optimum</td>
<td>2.0536</td>
<td>12,950</td>
</tr>
</tbody>
</table>

section 4.2:
The results are presented in Table B-2.

**Table B-2:** Tip deflections of optimum joints measured using FEA (conservative and aggressive methods applied to the first example).

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip deflection of the B-pillar $\phi_{up}$ ($10^3$ rad)</th>
<th>Stiffness</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum design conservative method</td>
<td>2.9291</td>
<td>8,900</td>
</tr>
<tr>
<td>Optimum design aggressive method</td>
<td>2.2738</td>
<td>11,600</td>
</tr>
</tbody>
</table>
section 4.3:

Table B-3 depicts the results.

**Table B-3:** Tip deflections of optimum joints measured using FEA (second example).

<table>
<thead>
<tr>
<th>Case</th>
<th>Tip deflection of the B-pillar $\phi_{tip}$ ($10^{-3}$ rad)</th>
<th>Stiffness $K$ (Nm/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum design conservative method</td>
<td>3.5954</td>
<td>7,200</td>
</tr>
<tr>
<td>Optimum design aggressive method</td>
<td>4.0253</td>
<td>6,410</td>
</tr>
<tr>
<td>Optimum design moderate method $w=0.25$</td>
<td>3.2429</td>
<td>8,010</td>
</tr>
<tr>
<td>Optimum design moderate method $w=0.75$</td>
<td>3.5709</td>
<td>7,250</td>
</tr>
</tbody>
</table>
Figure B-1: Generic model of the joint subassembly.
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