MECHANICAL DESIGN OF THE CARPAL WRIST:
A PARALLEL-ACTUATED, SINGULARITY-FREE ROBOTIC WRIST

by

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(ABSTRACT) 

Parallel robotic manipulators offer potential advantages over the more conventional serial manipulators, such as a higher stiffness-to-weight ratio and more precise control. However, their use as base manipulators and robotic wrists has been limited by their more complicated kinematic solutions and, especially in the case of a wrist, severely limited workspaces. Due to these shortcomings, robotic wrists in existence today are typically built as serial kinematic chains that provide large, spherical workspaces but suffer from lower stiffness, higher mass, and interior singularities. A parallel mechanism consisting of three sets of serial-chains can be manufactured that possesses characteristics suitable for use as a robotic wrist. As a result of its parallel structure, it provides for the high strength and low weight of a parallel manipulator, while its novel configuration gives it a workspace comparable to that of serial-type wrists. Additionally, the configuration of the links allows the wrist to have a large open center through which cables or hoses supplying the end-effector can be routed. This tunnel-like similarity to a human wrist’s carpal tunnel leads to its name, the Carpal wrist. This thesis discusses the characteristics of the Carpal wrist and develops simple tools for its design. A simple force analysis is developed and used to solve for the required input forces and resulting internal forces in the wrist. Computer algorithms are developed that facilitate the solution of both the kinematic and force equations resulting from the analysis of the wrist. Using these tools, a prototype wrist is designed such that it may serve as a proof-of-concept model capable of demonstrating both the kinematic function and load-carrying capability of this device.
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I would like to express my sincere appreciation to the chairman of my advisory committee, Prof. Charles F. Reinholtz. In addition to simply being a graduate student advisor, he has been a mentor, a tutor, and a good friend. His open-door policy towards those with whom he is involved and his positive outlook have provided an excellent example to emulate. Due in large part to him, I will fondly remember my days in graduate school. I would also like to extend my appreciation to the other members of my committee, Dr. Robert J. Salerno and Prof. Harry H. Robertshaw. Their instruction throughout both my undergraduate and graduate careers has been invaluable.

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Chapter 1

Introduction

As alternatives to serial manipulators, parallel manipulators are the subject of much current research. The objective of this thesis is to describe the development of one such parallel device, the Carpal wrist. The basic kinematic configuration of this mechanism has been presented in previous literature; however, its use as a robotic wrist is a novel concept. The expectations and constraints imposed upon a robotic wrist, i. e., developing a large workspace, having a high strength-to-weight ratio, and easily solvable kinematics, create numerous design difficulties when practically implemented. While current wrist technologies do not meet all the design objectives, the parallel-structured Carpal wrist has the potential to meet these difficult requirements. However, this novel parallel wrist device requires new design tools; these are developed from the kinematic and static force analyses and used to aid in the conceptual design of practical and useful Carpal wrists. Further, the practical considerations related to extending the conceptual design to a physical model are addressed and the detailed design of a prototype, proof-of-concept model is described.

There are many different types of robotic manipulators in existence today. These can be divided into categories depending on their configuration, e. g., there are cartesian,
SCARA, spherical, cylindrical, and articulated, anthropomorphic robotic arms. While certain manipulators, such as the cartesian and SCARA types, are designed for specific tasks that do not require a robotic wrist, many manipulators are used for more general positioning and orienting tasks. Whether or not these general manipulators are serial or parallel devices, they typically belong to the wrist-partitioned class of manipulators. As shown in Fig. 1.1, manipulators in this class consist of two distinct parts, the base manipulator, sometimes called the regional or positioning structure, and the wrist. The base manipulator typically consists of three joints used to locate the wrist at a given position in space. The wrist is then expected to provide for the orientation of the tool fixture at that spatial point. For general spatial tasks, the overall manipulator is expected to be capable of providing five or six degrees-of-freedom. With the base manipulator providing the three degrees-of-freedom necessary for locating the wrist, the robotic wrist is expected to provide either the two or three degrees-of-freedom necessary to orient the end-effector.

Fig. 1.1: Wrist-partitioned articulated manipulator.
Most robots are serial manipulators, i.e., they consist of an open kinematic chain of links connected by actuated joints. Likewise, most wrists are also serially constructed. It is well known that parallel manipulators possess certain benefits over their serial counterparts. Parallel manipulators can be stronger, lighter, and more rigid than their serial counterparts; however, they typically suffer from the drawback of having a severely limited workspace. The Carpal wrist, so named because its geometric configuration creates an interior tunnel much like the carpal tunnel of the human wrist, maintains the benefits of parallel manipulators while still providing a large, dexterous workspace.

1.1 Robotic Wrists

The wrist of a robotic manipulator typically refers to the last \( n-3 \) joints of its overall structure (Craig, 1989). They are typically designed to have axes that intersect at a common point in order to simplify the overall kinematic solution for the manipulator. Manipulators having such an arrangement, in which a positioning structure is followed by an orienting structure or wrist, belong to the common wrist-partitioned class of mechanisms (Craig, 1989). The topics covered in this thesis focus solely on this wrist structure.

The most common wrist configurations consist of either two or three revolute joints arranged into orthogonal, intersecting axes. Within that classification, there are two types of wrists that are prevalent: roll-pitch-roll and pitch-yaw-roll wrists (Rosheim, 1989). The pitch and yaw movements are pivotal-type movements of the end of the wrist, and the roll movement refers to a rotation about an axis orthogonal to either end of the wrist. Schematics of these two types of arrangements are illustrated in Fig. 1.2. The roll-pitch-roll wrist is the more common of the two types due to a relatively simpler design and construction. However, all such wrists suffer from the problem of having a singularity exist within their workspace volumes whenever the two roll axes are aligned. This
singular position results in either an impossible motion, a degradation of dynamic performance, or a loss of dexterity. The pitch-yaw-roll wrist generally has greater dexterity than the roll-pitch-roll wrist and, as such, is growing in popularity.

Whether or not a wrist is classified as roll-pitch-roll or pitch-yaw-roll or it has two actuated joints or three, there are certain characteristics that are generally advantageous for it to have. According to Rosheim (1989), an industrial robot wrist should have at

**Introduction**
least three degrees-of-freedom and the same range of motion the human wrist has for the
dexterous, three-dimensional manipulations required in tasks such as welding, spray
painting, and insertion operations. In addition, the actuators, like the muscles that move
the human wrist, should be removed from the vicinity of the wrist itself. In this way,
larger and more powerful actuators can be used while the size and weight of the wrist
structure is reduced. Another important feature that is useful in a wrist is a singularity-
free interior workspace. A wrist possessing no singularities improves the design of the
overall manipulator, making it more adept at adjusting to varied environments since it
does not need to account for unattainable positions. Finally, enclosing the wrist and
providing a safe route for the end-effector cabling is necessary to increase the longevity
of the device. Robots are typically operated in harsh environments where the existence of
corrosive chemicals, abrasives, or harsh conditions could adversely affect the mechanical
parts of the wrist or the cables. For this reason, it is advisable to seal all the mechanical
components from the environment and also provide a safe, interior passage to protect the
cabling.

1.2 Literature Review

1.2.1 Review of Previous Wrist Design

Beginning with the roll-pitch-roll wrist designs of Raymond Goertz, named the father of
teleoperators by Rosheim (1989), he categorizes and describes numerous roll-pitch-roll
and pitch-yaw-roll wrist designs dating from the 1940’s to 1989. Although Rosheim
mainly covers serial-type wrists and does not specifically address parallel wrist design, he
does provide valuable insight to many key factors beneficial to wrist design. His text
describes workspace requirements, wrist dexterity needs, and the characteristics such as
open-center construction, lightweight and simple design, and location of actuators that
govern an effective wrist design. A summary of the characteristics for numerous existing
wrists is shown in Table 1.1. This table lists the specific type of wrist, its angular workspace limits, and a measurement of its load-carrying capability.

Table 1.1: Summary of characteristics of existing wrists (adapted from Rosheim, 1989).

<table>
<thead>
<tr>
<th>Name</th>
<th>Roll-Pitch Design</th>
<th>Pitch Range (°)</th>
<th>Yaw Range (°)</th>
<th>Roll 1 Range (°)</th>
<th>Roll 2 Range (°)</th>
<th>Load Capacity - N (lbs)</th>
<th>Wrist Diameter - cm (in.)</th>
<th>High Precision (0.013-0.025 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ex-Cell-O HYD-RO WRIST MO 1-1/2-1/2</td>
<td>X</td>
<td>180</td>
<td>180</td>
<td>280</td>
<td></td>
<td>667 (150)</td>
<td>36 (14)</td>
<td>X</td>
</tr>
<tr>
<td>Cincinnati Milacron T3 776-3-Roll Wrist</td>
<td>X</td>
<td>238</td>
<td>360</td>
<td>360</td>
<td></td>
<td>687 (150)</td>
<td>20 (8)</td>
<td>X</td>
</tr>
<tr>
<td>Devilbiss TR 3500 Flexiarm Wrist</td>
<td>X</td>
<td>176</td>
<td>176</td>
<td>420</td>
<td></td>
<td>67 (15)</td>
<td>8.9 (3.5)</td>
<td></td>
</tr>
<tr>
<td>Graco OM 5000 Slim Wrist</td>
<td>X</td>
<td>170</td>
<td>170</td>
<td>420</td>
<td></td>
<td>67 (15)</td>
<td>8.9 (3.5)</td>
<td></td>
</tr>
<tr>
<td>Martin Marietta Metr Wrist</td>
<td>X</td>
<td>180</td>
<td>90</td>
<td>360</td>
<td></td>
<td>111 (25)</td>
<td>14 (5.5)</td>
<td>X</td>
</tr>
<tr>
<td>Moog type A Wrist Mode 1</td>
<td>X</td>
<td>220</td>
<td>220</td>
<td>270</td>
<td></td>
<td>76 (17)</td>
<td>46 (18)</td>
<td>X</td>
</tr>
<tr>
<td>Ross-Hime Designs Omni-Wrist</td>
<td>X</td>
<td>180</td>
<td>180</td>
<td>360</td>
<td></td>
<td>111 (25)</td>
<td>14 (5.5)</td>
<td>X</td>
</tr>
<tr>
<td>Tokico Armstar Type E Wrist</td>
<td>X</td>
<td>270</td>
<td></td>
<td>260</td>
<td>210</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thermwood Paintmiser Wrist</td>
<td>X</td>
<td>180</td>
<td>180</td>
<td>270</td>
<td></td>
<td>67 (15)</td>
<td>10 (4)</td>
<td></td>
</tr>
<tr>
<td>Thermwood PR Series Wrist</td>
<td>X</td>
<td>90</td>
<td>90</td>
<td>460</td>
<td></td>
<td>67 (15)</td>
<td>15 (6)</td>
<td></td>
</tr>
<tr>
<td>Unimation Puma 762 Wrist</td>
<td>X</td>
<td>200</td>
<td></td>
<td>532</td>
<td>600</td>
<td>196 (44)</td>
<td>15 (6)</td>
<td>X</td>
</tr>
<tr>
<td>Yaskawa Motoman-L60</td>
<td>X</td>
<td>180</td>
<td>360</td>
<td>360</td>
<td></td>
<td>578 (130)</td>
<td>30 (12)</td>
<td>X</td>
</tr>
</tbody>
</table>
In more recent works, researchers have undertaken the design of parallel-structured robotic wrists. Agrawal, et al. (1994) present a three degree-of-freedom parallel wrist mechanism. The authors describe a pitch-yaw-roll mechanism similar in design to the Omni-Wrist by Rosheim (1989). However, as with many parallel devices, their wrist has a limited workspace and, as such, is used simply as a joint in a five-segment mechanical spine instead of as a true robotic wrist. Similarly, Smith and Nguyen (1991) present the design of a Stewart-platform-based robotic end-effector. As a result of its structure, it is rigid and strong but again, has a limited workspace, limiting its potential for application as a robotic wrist. Hashimoto and Imamura (1994) design a parallel link compliant wrist that, like the Carpal wrist, possesses two angular degrees-of-freedom and one translational degree-of-freedom. Other documented wrist designs include a three degree-of-freedom parallel-structured wrist designed for the United Parcel Service (Lee and Chang, 1992), a spherical three degree-of-freedom parallel mechanism which possesses the ability to function as an orientational device (Gosselin and Angeles, 1989), and an actuated double Cardan joint-based wrist (Milenkovic, 1987).

1.2.2 Parallel Manipulators

Beginning with the Stewart platform (Stewart, 1965), much has been proposed and written on parallel manipulators. The focus of this thesis is the design of a parallel-actuated wrist. With this in mind, only the subset of information that may aid in this task is presented.

Cleary and Arai (1991) present a prototype parallel manipulator with a structure similar to the Stewart platform. They include a description of their design which is based upon kinematic considerations. This leads to a discussion of the hardware and software selection and an overview of the workspace of the manipulator but does not focus upon the load-carrying ability of the device. The static and kinematic analyses of parallel
manipulators are further described by other authors. Agrawal and Roth (1992) use screw theory to solve for static force equilibrium of in-parallel manipulator systems in order to facilitate actuator selection. Ling and Huang (1994) extend the use of screw theory to solve both the static force and velocity problems for general parallel manipulator systems. Finally, general coordinates and spatial transformations are used to analyze the static equilibrium in a class of parallel manipulators resembling the Stewart platform (Pang and Shahinpoor, 1993). This work presents a useful technique towards developing the static force analysis used for designing the Carpal wrist.

1.2.3 Evolution of the Carpal Wrist

The general kinematic configuration of the Carpal wrist shares characteristics with an 1872 patent that introduces a constant-velocity universal joint coupling (Clemens, 1872). However, the application of this configuration towards a robotic manipulator was possibly first enumerated by Hunt (1972). In his review of possible kinematic mechanisms, he suggests the double tripod, as he calls the Clemens coupling, for use in robotic applications. The development of this device is further discussed as a constant velocity joint (Canfield and Reinholtz, 1995), and a kinematic design method is developed for designing a useful constant velocity coupling (Canfield, et al. 1994). Hertz and Hughes (1994) further develop the kinematic relations for both a double tripod and the variable-geometry truss as parallel manipulators.

Although not exactly a Clemens coupling, a similar manipulator is developed by Clavel (1988). The Delta parallel robot, as he refers to his device, is a manipulator with a kinematic configuration that is slightly different to the Carpal wrist. Also, the Delta robot is a high speed stand-alone manipulator designed for light payload, pick-and-place operations.
In a parallel set of developments, much work has been invested in the development of the variable geometry truss (VGT), specifically the double-octahedral VGT. Contrary to other parallel manipulators, this type of mechanism has been shown to possess a large angular workspace (Padmanabhan, 1992). In another work, Padmanabhan, et al. (1992) presents an inverse kinematic analysis for the double octahedral VGT and discusses practical design considerations for this type of manipulator. Salerno (1993) furthers the development of the kinematic solutions for this class of manipulators by describing the use of a canonical input set to aid in the development of closed-form kinematic relations. This canonical input set consists of the use of a secondary set of measurements as kinematic inputs; in the case of a double-octahedral VGT, these inputs for the kinematic solutions are the angular rotations of one or more triangular faces instead of the lengths of the actuated legs. This method, originally used to develop the kinematic relations, led to the notion of actuating these triangular faces just as they are measured. This is evident in the actuation of the Carpal wrist. Using this idea, the closed-form kinematic solutions that can be directly applied to the Carpal wrist are first presented by Salerno, et al. (1995).

1.3 Motivation and Objective

It is easy to see from an examination of the current technology that the field of robotics, in general, and robotic wrists, in particular, is dominated by serial chain designs. Although these designs have been proven capable, they have serious shortcomings, which for the most part have been accepted due to the lack of a better solution. In spite of their shortcomings and in defense of serial wrists, they have remained in use for a variety of reasons. The following section details that benefits of the serial wrists that have contributed to their acceptance but also describes the shortcomings that must be addressed in an improved wrist design.
Serial wrists typically have large workspaces -- a necessity for a good wrist. Yet, their dexterity, a measure of the singularity-free workspace of the wrist (Rosheim, 1989), is not ideal. All roll-pitch-roll wrists and many pitch-yaw-roll wrists suffer from singular positions within their respective workspaces. These present a serious drawback since these positions must be completely avoided during use. This increases the difficulty of controlling robots possessing such wrists. Dynamic problems resulting from an attempt to move near a singular position can lead to motor overloads or unacceptable motion.

Serial wrists can also be designed to possess a fairly high payload capability. However, these wrists suffer from a low strength-to-weight ratio, because actuated joints in a serial wrist are placed in line along an open kinematic chain which leads to a massive wrist regardless of its payload capacity. This creates a massive wrist regardless of payload capacity since the strength of each joint must be sufficient to support each joint that follows it in line. This serial design also tends to lead to complex arrangements of bearings, concentric shafts, and gearing necessary to transmit power to each of these joints. Figures 1.3 through 1.5 show three such serial wrists, the three-roll wrist used on Cincinnati Milacron manipulators, the Graco Robotics Slim-Wrist, and the Omni-Wrist by Ross-Hime Designs, to illustrate the complexity involved in these mechanisms. This complexity increases the mass of the wrist further. The added mass of the wrist components has more of a detrimental effect on the operation of the overall manipulator than the wrist itself. A massive wrist severely increases the overall inertia of the system, i. e. the robotic manipulator, which decreases the dynamic performance and increases the power needed to drive it.
Fig. 1.3: Cincinnati Milacron 3-Roll Wrist (reprinted from Rosheim, 1989).
Fig. 1.4: Graco Robotics Slim-Wrist (reprinted from Rosheim, 1989).

Fig. 1.5: Omni-Wrist by Ross-Hime Designs (reprinted from Rosheim, 1989).
The area of robotics dealing with parallel manipulators, particularly those used as robotic wrists, is in its infancy. Continuing advances in computational power and kinematic solution methods have cleared a path for the use of parallel manipulators in many applications. The Carpal wrist is a parallel manipulator that presents a possible solution to overcome many of the drawbacks of common robotic wrists. Its parallel structure is similar to that of the double or reflected tripod (Hertz and Hughes, 1994) shown in Fig. 1.6 and likewise similar to a double-octahedral variable-geometry truss (Padmanabhan, 1992) which is shown in Fig. 1.7. It has three sets of kinematic chains, or leg pairs, between a basal and distal plate. These serve to create multiple load paths that distribute the effects of the payload more evenly to the base. The result is a high strength-to-weight ratio and a lightweight, open-interior structure. The benefits of a lightweight wrist are twofold. Not only is this manipulator easier to move due to lower inertia, it adds less mass and inertia to the end of the base manipulator. This allows the entire robot to benefit from less powerful actuators resulting in lower weight and possibly higher operating speeds.

Fig. 1.6: Schematic diagram of a double (reflected) tripod constant-velocity joint.
Fig. 1.7: Double-Octahedral Variable Geometry Truss structure.

The Carpal wrist also facilitates the mounting of actuators farther back along the arm of the base manipulator. While typical serial wrists suffer from strict constraints limiting their actuation designs, this wrist has the unique ability to incorporate a variety of actuation schemes. Either linear actuators or rotary actuators can be effectively used.

Although parallel manipulators have been avoided due to their complex kinematics, both forward and inverse closed-form kinematic exist for the Carpal wrist (Salerno, et al. 1995). Also, the kinematic configuration of the Carpal wrist leads to a large angular workspace that is free from interior singularities (Salerno, et al. 1995), which makes it more suitable as a wrist than other proposed parallel manipulators. Its configuration gives it an open center which is ideal for the passage and protection of cables and hoses to the end-effector (Rosheim, 1989). Finally, even though its parallel-structure may at first seem more complex to manufacture and maintain than a comparable serial wrist, the Carpal wrist can actually be constructed of a few simple, identical components.
Furthermore, these components can be easily serviced due to its open, external design. Figure 1.8 shows the prototype Carpal wrist to illustrate its mechanical simplicity compared to the wrists shown in Figs. 1.3 through 1.5.

Having enumerated the possible use of the Carpal wrist as a new solution that overcomes some of the drawbacks of existing serial wrists, the purpose of this thesis is to describe the design of a working prototype of the Carpal wrist that can serve as a proof-of-concept model capable of handling a moderate payload, be realistically-sized, and demonstrate the

![Prototype Carpal Wrist](image-url)
workspace and kinematic control of the device. Tools are developed to aid in the design of the Carpal wrist which simulate the kinematic solutions for conceptual models of the wrist and also perform a static force analysis for these conceptual models. Before arriving at this result, the forward and inverse kinematic solutions described by Salerno, et al. (1995) are summarized, and the derivation of the static force analysis is described. A discussion of general design considerations for some of the main components of the wrist is included to present the design alternatives with their corresponding benefits and drawbacks. Finally, this thesis will wrap-up with a description of the prototype design process and the design tools that are developed in order to aid in actuator selection, stress/strain analyses, and component design. These tools are included in software that contains a module used to visualize a conceptual kinematic model of the wrist and simulate its kinematic solutions, and a module that performs the static force analyses for these conceptual models.
Chapter 2

Analysis of the Carpal Wrist

The kinematic and force analysis of serial wrists, and serial manipulators in general, are well developed and relatively simple. In contrast, parallel manipulators pose a much more difficult problem in the sense that each parallel manipulator requires its own unique solution to the kinematic problem. No solution method has been developed that can be generally applied to parallel manipulators, although much effort has been directed toward this goal. This is supported by the fact that numerous researchers have approached the position kinematics for a class or a specific type of parallel manipulator yet no general method has surfaced (Ling and Huang, 1994). Similarly, the force analysis of parallel mechanisms is equally difficult. However, recent research has focused upon complicated generalized techniques for finding the instantaneous kinematics, or velocity, of parallel manipulators (Ling and Huang, 1994) which lend themselves to force analyses based upon a virtual work solution. This chapter begins the development of design tools for the Carpal wrist by presenting the solution to the forward and inverse kinematic problems and then detailing a simple static force analysis of the mechanism to be used in design.
2.1 Kinematic Model

The Carpal wrist represents a special subset of the double-tripod mechanism. It is a symmetric, parallel mechanism consisting of three equivalent parallel leg pairs connecting two equivalent platforms, a basal and distal plate, as shown in Fig. 1.6. The symmetric configuration was chosen over a more general design in order to simplify the kinematics and ensure a closed-form solution. Due to this symmetry, this subset of the double-tripod mechanism can also be referred to as a reflected tripod arrangement (Hertz and Hughes, 1994).

The geometry of the basal and distal plates is characterized kinematically by the location and orientation of the revolute joints connecting the leg pairs to these plates. In this symmetric arrangement, these revolutes lie on the sides of an equilateral triangle. The leg pairs are revolute-spheric-revolute (RSR) kinematic chains in which all legs are of equal length. Due to this symmetry, the spheric joints connecting each set of legs lie on a fictitious midplane, which comprises a plane of symmetry between the basal and distal halves of the wrist. In certain cases, each spheric joint, also referred to as a midjoint, can be replaced by an equivalent joint consisting of three orthogonal, intersecting revolute joints such that they provide mobility equivalent to that of the spheric joint. Using this arrangement, the leg pairs consist of five revolute (5R) kinematic chains. These two arrangements are shown schematically in Fig. 2.1. Although the two forms are kinematically equivalent, the 3R midjoint configuration possesses practical benefits that are enumerated in Chapter 3. However, in subsequent discussions within this chapter, these joints will be shown as spheric joints for simplicity.

In either of these two arrangements, the mechanism provides three degrees-of-freedom between the basal and distal plates. This is proven using the Kutzbach equation for spatial mobility (Mabie and Reinholtz, 1987):
\[ M = 6(n - 1) - 5f_1 - 4f_2 - 3f_3 - 2f_4 - f_5 \]  \hfill (2.1)

where:

\[
n = \text{number of links (including ground)}
\]

\[
f_i = \text{number of } i \text{ degree-of-freedom joints}.
\]

For these specific kinematic configurations, equation 2.1 mathematically proves their equivalence with respect to mobility:

\[
\begin{align*}
\text{case } i: & \quad \text{(spheric midjoints)} \quad M = 6(8-1) - 5(6) - 3(3) = 3 \\
\text{case } ii: & \quad \text{(3R midjoints)} \quad M = 6(14-1) - 5(15) = 3.
\end{align*}
\]

When the basal plate is fixed to ground, these three degrees-of-freedom exist as two orientational and one translational motion of the distal plate, i.e., a pitch, a yaw, and a plunge, respectively. As a robotic wrist, these three motions are controlled by actuating the rotations of the three legs connected to the basal plate. However, many wrists require only two orientational degrees-of-freedom at the wrist; spray painting and welding are perfect examples of such applications. In fact, the lightweight, open-center structure, and singularity-free workspace of the Carpal wrist make it ideal for such tasks.
For other applications requiring general orienting capability, a fourth degree-of-freedom can be added as a roll axis located on the basal or distal plate and situated orthogonally to it. Although the design of such a wrist is not directly addressed in this thesis, the majority of the analyses and design issues presented are not altered. The addition of this extra joint simply requires minor extensions to most of the concepts presented. For example, the kinematic analysis for the four degree-of-freedom configuration does result in closed form solutions for both the forward and kinematic problems by simply including the effect of another rotation matrix (Salerno, et al. 1995), but the force analysis is unchanged by the addition of an added joint -- the results pertaining to the fourth actuator must simply be interpreted correctly.

2.2 Kinematic Analysis

A detailed diagram of the Carpal wrist is shown in Fig. 2.2 to describe the geometry needed to derive the kinematic solutions. For clarity, most vectors have been omitted. However, each point in the figure is representative of a vector originating at \( \mathbf{c}_b \), the center of the basal plate and extending to that specific point. As shown in this figure, the revolute joints of the basal and distal plates are located by vectors \( \mathbf{b}_i \) and \( \mathbf{d}_i \), respectively, while the orientation of each basal revolute will be denoted by the unit vectors, \( \mathbf{u}_i \) (i=1...3 throughout). Coordinate frames \{B\} and \{D\} are located at the centers of the basal and distal plates, \( \mathbf{c}_b \) and \( \mathbf{c}_d \). The z-axis of each frame is normal to its corresponding plane, and the x-axis points toward the first nodal point, \( \mathbf{b}_1 \) or \( \mathbf{d}_1 \). The geometric center of the wrist, \( \mathbf{c}_w \), lies on the midplane and is located at the intersection of the z-axes of frames \{B\} and \{D\}. This position is unique in that the distance from \( \mathbf{c}_w \) to either \( \mathbf{c}_d \) or \( \mathbf{c}_b \) is equal to the plunge distance, \( p_d \). The geometric center is important because at any given plunge distance, the motion of the distal coordinate frame describes a sphere of radius \( p_d \) about this point.
Fig 2.2: Vector representation of the Carpal wrist needed for the kinematic analyses.

The specific geometric configuration of the wrist is described by a base length \( b \), i.e., the distance from the center of either plate to each revolute joint and the length of each leg \( l \). The length of each leg is denoted as the distance between its associated revolute joint, \( b_i \), and its midjoint, \( m_i \).
2.2.1 Forward Kinematic Solution

The forward kinematic problem for the Carpal wrist is stated as follows: *given the fixed dimensions of the wrist and a set of basal revolute angles* \((\theta_1, \theta_2, \theta_3)\), *find the position and orientation of the tool coordinate frame*. The derivation of the solution to this problem is detailed in the following section.

For the forward kinematic analysis of the wrist, the location of its base is assumed known. Typically, in application, it is calculated from the kinematics of the base manipulator to which the wrist is attached. In this derivation, it is simply considered ground. This locates the basal revolutes, and the corresponding angles are given as the kinematic input parameters. Since the legs are a fixed length, the midjoints follow circular paths. Their locations are found as follows:

\[
\mathbf{m}_i = \mathbf{b}_i + \mathbf{R}_{[u_i, \theta_i]} l \mathbf{q}_i
\]

(2.2)

where

- \(l\) is the scalar length of the leg,
- \(\mathbf{R}_{[u_i, \theta_i]}\) represents a rotation about axis \(\mathbf{u}_i\) by an amount \(\theta_i\), and
- \(\mathbf{q}_i\) defines an inward pointing unit vector perpendicular to \(\mathbf{u}_i\).

The plane coefficients of the midplane are determined as:

\[
\begin{bmatrix}
A_m \\
B_m \\
C_m
\end{bmatrix}
= (\mathbf{m}_2 - \mathbf{m}_1) \times (\mathbf{m}_3 - \mathbf{m}_2)
\]

(2.3)

The perpendicular distance from each point, \(\mathbf{b}_n\), to the midplane is calculated as follows:

\[
\delta_i = n_m \cdot (\mathbf{m}_i - \mathbf{b}_i)
\]

(2.4)

where
\[
\mathbf{n}_m = \frac{[A_m \ B_m \ C_m]^T}{\sqrt{A_m^2 + B_m^2 + C_m^2}} \tag{2.5}
\]

Utilizing the inherent symmetry about the midplane, the distal revolute centers may be located by extending a distance \(2(b_i)\) from each basal revolute in the direction of the midplane normal, \(\mathbf{n}_m\). Thus,

\[
\mathbf{d}_i = \mathbf{b}_i + 2b_i \mathbf{n}_m \tag{2.6}
\]

The distal plane is defined by these points. The position of the distal coordinate frame \(\{D\}\) is determined from the average or centroid of the three distal revolutes.

\[
\mathbf{p}_D = \frac{\mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3}{3} \tag{2.7}
\]

The direction of the z-axis of \(\{D\}\) is identified from the vector cross-product:

\[
\mathbf{z}_D = \frac{\left(\mathbf{d}_3 - \mathbf{d}_1\right) \times \left(\mathbf{d}_2 - \mathbf{d}_1\right)}{\left\| \left(\mathbf{d}_3 - \mathbf{d}_1\right) \times \left(\mathbf{d}_2 - \mathbf{d}_1\right) \right\|} \tag{2.8}
\]

while the other two axes are similarly defined based on knowledge of the distal revolute positions.

\[
\mathbf{x}_D = \frac{\mathbf{d}_1 - \mathbf{p}_D}{\left\| \mathbf{d}_1 - \mathbf{p}_D \right\|} \tag{2.9}
\]

\[
\mathbf{y}_D = \mathbf{z}_D \times \mathbf{x}_D \tag{2.10}
\]

Using this information, the rotation matrix characterizing the relative rotation between \(\{B\}\) and \(\{D\}\) may be expressed as:

\[
^B_D \mathbf{R} = [\mathbf{x}_D \ \mathbf{y}_D \ \mathbf{z}_D] \tag{2.11}
\]

and the overall transformation is written:

\[
^B_D \mathbf{\Gamma} = \begin{bmatrix}
^B_D \mathbf{R} & \mathbf{p}_D \\
0 & 1
\end{bmatrix} \tag{2.12}
\]

Analysis of the Carpal Wrist
2.2.2 Inverse Kinematic Solution

The inverse kinematic problem for the Carpal wrist involves a more complicated solution than the forward problem. It may be stated as follows: *given a desired end orientation and plunge distance, find a set of input angles, \( \theta_1, \theta_2, \) and \( \theta_3 \), that will produce this desired result.* The primary complication results from the fact that the parallel structure of the wrist provides two orientational and one translational degrees-of-freedom instead of the more common three orientational degrees-of-freedom.

The solution to the inverse kinematic problem begins by determining that the specified position and orientation lie within the workspace of the wrist. This is simplified by restricting the goal specification to a consistent set of three variables. For this wrist, one must specify the orientation (two parameters) and plunge distance (one parameter). Since the architecture of the wrist prohibits relative rotation between the basal and distal plates (Canfield and Reinholtz, 1993), the goal specification must not include any such rotation. To better visualize this problem, at any plunge distance, \( p_d \), the wrist can be modeled as two rods of length \( p_d \) that are joined by a spheric joint. Figure 2.3 illustrates this simplified representation of the wrist. The location of the tool can then be described by a rotation about an axis, \( \mathbf{u}_{\text{bend}} \). This axis lies in a plane that is parallel to the basal plate, and its orientation depends upon a rotation about \( \mathbf{z}_B \) by an amount \( \beta \) starting from a line parallel to \( \mathbf{x}_B \). The two orientational degrees-of-freedom are now given by \( \mathbf{u}_{\text{bend}} \) (one degree-of-freedom since only its rotation is unspecified) and the angle, \( \gamma \), which specifies the amount of bending.

Once the goal is specified, \( \mathbf{u}_{\text{bend}} \) and \( \gamma \) can be evaluated:

\[
\mathbf{u}_{\text{bend}} = \frac{\mathbf{z}_B \times \mathbf{z}_D}{\| \mathbf{z}_B \times \mathbf{z}_D \|} \quad (2.13)
\]

\[
\gamma = \cos^{-1}(\mathbf{z}_D \cdot \mathbf{z}_B). \quad (2.14)
\]
Fig 2.3: Spherical model of Carpal wrist used in the inverse kinematic solution.

The location of the distal revolute joints are found by rotating a set of vectors that point to the undeflected distal revolute joints about the bend axis.

\[ \mathbf{d}_i = R_{\{w_{	ext{end, y}}\}} \mathbf{d}'_i + \mathbf{c}_w \]  \hspace{1cm} (2.15)

where

\[ \mathbf{d}'_i = \begin{bmatrix} \dot{b}_a \\ \dot{b}_y \\ P_d \end{bmatrix} \quad \text{and} \quad \mathbf{c}_w = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

From this, it is possible to identify three points which must lie on the plane of symmetry by bisecting the vector between the basal and distal revolutes.

\[ \mathbf{p}_i = \frac{\mathbf{d}_i + \mathbf{b}_i}{2} \]  \hspace{1cm} (2.16)
In general, these do not coincide with the midjoints, \( \mathbf{m}_i \), but since they also lie on the midplane, the coefficients for the equation of the midplane may be determined:

\[
\mathbf{n}_m = \begin{bmatrix} A_m \\ B_m \\ C_m \end{bmatrix} = \frac{(\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1)}{\| (\mathbf{p}_2 - \mathbf{p}_1) \times (\mathbf{p}_3 - \mathbf{p}_1) \|},
\]

(2.17)

\[
D_m = -\mathbf{n}_m \cdot \mathbf{p}_i
\]

(2.18)

where \( \mathbf{n}_m \) is the normal to the midplane. As described before, the basal and distal plates are joined by three RSR chains. Both the basal and distal legs form circles that intersect the midplane at the location of the midjoints. This is the basis for the following set of constraint equations:

\[
|\mathbf{m}_i - \mathbf{b}_i| = l
\]

(2.19)

\[
A_{ci} m_{ix} + B_{ci} m_{iy} + C_{ci} m_{iz} + D_{ci} = 0,
\]

(2.20)

\[
A_{mi} m_{ix} + B_{mi} m_{iy} + C_{mi} m_{iz} + D_m = 0.
\]

(2.21)

where

- \( \mathbf{m}_i \) is a vector locating the midplane node \( i \),
- \( m_{ix}, m_{iy}, \text{ and } m_{iz} \) are the components of the above vector, and
- \( A_{ci}, \ldots, D_{ci} \) represent the plane equation coefficients for each base arm circle.

Equations 2.20 and 2.21 are solved simultaneously for the \( y \)- and \( z \)-components of \( \mathbf{m}_i \):

\[
m_{iy} = R m_{ix} + S
\]

(2.22)

and

\[
m_{iz} = T m_{iy} + U
\]

(2.23)

where
\[ R = \frac{A_C C_m - C_m A_m}{C_c B_m - B_C C_m} \]
\[ S = \frac{D_C C_m - C_m D_m}{C_c B_m - B_C C_m} \]
\[ T = \frac{B_C A_m - A_m B_m}{C_c B_m - B_C C_m} \]
\[ U = \frac{B_D D_m - D_m B_m}{C_c B_m - B_C C_m} \]

These relations are substituted into the original distance formula (2.19) and expanded to form the equation:

\[ A m_{ix}^2 + B m_{ix} + C = 0 \]  
(2.24)

where

\[ A = R^2 + T^2 + 1 \]
\[ B = 2 \cdot (RS + TU - b_{ix} - Rb_{iy} - Tb_{iz}) \]
\[ C = S^2 + U^2 - 2Sb_{iy} - 2Ub_{iz} + b^2 - l^2 \]

This equation can be solved for \( m_{ix} \) using the quadratic formula, and the other two components of the vector, \( m_i \), can be found using equations 2.22 and 2.23. Once the midplane nodes are known, the required input angles are easily calculated as the angle between each leg and its associated unit vector, \( q_i \).

\[ \theta_i = \cos^{-1} \left[ \frac{(m_i - b_i) \cdot q_i}{\| (m_i - b_i) \| \| q_i \|} \right] \]  
(2.25)

It should be noted that equation 2.24 results in two solutions for each of the three legs, a total of eight \( (2^3) \) possible closures while equation 2.25 results in two solutions for each angle. These additional solutions arising from equation 2.25 result from the possible kinematic situation in which a specific closure is reflected about the base. This solution is physically impossible in practice; to avoid this, the solution providing a midplane normal with positive \( z \)-component must be chosen. The multiple closures resulting from equation 2.24 can be seen as combinations of each leg pair angling outward from the
center of the wrist or collapsing in towards the center of the wrist. By first solving for the x-component of \( m_x \), a convenient method of choosing a specific closure exists. Table 2.1 describes the criteria that can be used to choose the correct value of \( m_{ix} \) from equation 2.24 that results in the desired closure.

<table>
<thead>
<tr>
<th>Leg Pair</th>
<th>Outward Closure</th>
<th>Inward Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \max(m_{1x}) )</td>
<td>( \min(m_{1x}) )</td>
</tr>
<tr>
<td>2</td>
<td>( \min(m_{2x}) )</td>
<td>( \max(m_{2x}) )</td>
</tr>
<tr>
<td>3</td>
<td>( \min(m_{3x}) )</td>
<td>( \max(m_{3x}) )</td>
</tr>
</tbody>
</table>

### 2.3 Static Force Analysis

In Chapter 1, references were made to relate the evolution of the Carpal wrist to the development of variable geometry truss manipulators. In the previous section, the kinematic analyses focused on the basic wrist mechanism being a reflected tripod configuration. The static force analysis that follows is based on a truss-type model of the wrist. Even though the model is slightly different from the actual kinematic configuration of the wrist, this analysis yields accurate solutions for the internal reactions within the mechanism. The analysis is intended to be a general static-force solution that will aid in the design of such a wrist by providing an approximate measure of the internal reactions in the legs, certain bearing loads, and the required input forces or torques necessary to actuate the wrist when carrying a payload.

#### 2.3.1 Static Force Model

As mentioned before, the Carpal wrist is similar in kinematic design to the variable geometry truss. Specifically, it is kinematically equivalent to a symmetric, midplane-
actuated, double-octahedral truss that has its actuating members removed. By removing the actuated truss members shown in Fig. 1.7, the modified octahedral truss is similar to the structure of the Carpal wrist. This model is kinematically equivalent to the reflected tripod except each leg of the reflected tripod is replaced by two truss members (along with one truss member from the base) that form each triangular “leg”.

The static force analysis is simplified by beginning with this truss-like structure where all links act as two-force members. This requires the assumption that mass of the links are negligible compared to the forces developed by the external loads; however, this assumption is reasonable because the structure of the Carpal wrist is lightweight. By modeling the legs of the wrist as two truss members, the directions of the internal reactions in the legs are known. This simplifies the general problem and eliminates the effects of bending and shear stress during the static force analysis.

It is not necessary that the legs be constructed solely of two-force members in order for the truss-model to yield satisfactory results. For legs not resembling a truss frame, the force analysis of the modified truss-structure is used to yield a set of generalized internal reactions occurring at the midjoints resulting from an externally applied load. Since the midjoints of the reflected tripod exactly correspond to the joints within the truss structure, the generalized reaction forces occurring at this point are equivalent between the two configurations. Referring to Fig. 2.4, the distal half of the Carpal wrist is shown with general internal forces from the midjoint acting upon the distal legs. Since the spheric cannot support a moment, the force must lie within the plane of each respective leg; otherwise, without the ability to supply a resisting moment, the legs would rotate. This midjoint force is resolved into two components, chosen for convenience to lie along the imaginary two-force links of the distal leg. This choice of component directions allows the use of the methods of sections and joints for truss structures in the solution. In a similar fashion, the midjoint forces acting upon the basal leg are resolved into
components along its imaginary two-force members. However, the input torques are modeled as an additional force located at the midjoint and normal to the plane of the leg.

An important aspect of the modified truss model is the addition of instantaneous truss members. Without the actuated links, the variable-geometry truss is a statically indeterminate structure. In order to maintain a static equilibrium, additional truss links are needed to model the actuation forces present in the Carpal wrist. The placement of these additional links is vital to the modified truss model of the wrist; they simplify the solution for static equilibrium by modeling the required actuator input torques as forces acting at the midjoints. Figure 2.5 shows the necessary placement and orientation of the added members used to complete the structure and simulate the force that each actuator torque applies to its leg. These links, connected between ground and the midjoint, can be considered “instantaneous” truss members since they are always perpendicular to the two
truss members which make up the leg of the model. The required actuator torque is
directly calculated as the moment due to an applied force.

2.3.2 Solution Procedure

In order to describe the solution procedure for the static force analysis of the Carpal wrist,
a second set of vectors must be defined that correspond to the modified truss structure.
Figure 2.6 shows this new set of notation that will be referred to during this section. A
set of nine nodal points, \( n_i \) (\( i = 1...9 \)), define the joints of the truss, or endpoints of each
link. In a slight deviation from the convention used earlier, the vectors \( b_i \) and \( d_i \) (\( i = 1...6 \)) represent the basal and distal links making up the legs of the wrist. The
instantaneous links are represented by the vectors labeled \( t_i \) (\( i = 1...3 \)). As described
previously, these originate at the midjoints and are perpendicular to the plane of the two
links making up each basal, or actuated, leg. All of these vectors are used to describe the
orientation of the links. In this regard, they are unit vectors that are scaled by an
appropriate magnitude (\( F_{bt} \), \( F_{dt} \), or \( F_{tt} \)) to represent the forces developed by the links.
The coordinate axes, \{B\} and \{D\}, are defined similarly to the previous analysis. The applied load is located at the center of the distal plate and is shown as a force, \(F_D\), and a moment, \(M_D\). For convenience, six vectors, \(r_n\), are defined between \(c_D\) and each node on the distal plate (there is one vector for each distal link in order to simplify notation later). These are used as the moment arms during the analysis of the static equilibrium for the distal plate. Additional coordinate frames, \{b_i\} and \{d_i\} used solely for the force analysis.
label axes situated at the center of the revolute axis of each leg. These are oriented such that the x-axis coincides with the axis of revolution and the y-axis points towards the associated midjoint node. The geometry of each truss face is defined by the angle \( \psi \), due to the symmetry, this angle is equal for all basal and distal legs. Using previous notation for the dimensions of the Carpal wrist, this angle is calculated as

\[
\psi = \sin^{-1}\left(\frac{1}{b\sqrt{3}}\right).
\]  

(2.26)

One final addition to the notation used for this analysis concerns the rotation of the distal legs relative to the distal plate. Due to the symmetric configuration of the wrist, these angles are equivalent to their basal leg counterpart. For this reason, the same angle, \( \theta_i \) (\( i = 1...3 \)), is used to refer to the rotations of both the basal and distal legs in each set.

A brief outline of the static analysis for the modified truss model of the Carpal wrist is as follows:

1. Solve the forward/inverse kinematics to obtain \( \theta_i \) and \( \frac{D}{b}T \).

2. Use the method of sections (Meriam and Kraige, 1986) to solve for the internal reactions in the distal leg links.

3. Use the method of joints (Meriam and Kraige, 1986) at each midjoint node to solve for the internal reactions occurring in the basal leg and the instantaneous links.

4. Solve for the overall base reaction forces, if necessary.

5. The analysis is completed by using the internal reactions to develop a set of generalized midjoint forces and by using the instantaneous link forces to calculate the approximate input torques.

The initial step in this process is completed according to the previous discussions. The kinematic solution locates the vectors representing the leg links. Since the links are
modeled as two-force members the directions of the associated forces are also known. Thus, considering an analysis of the entire truss-type model and ignoring the links making up the basal and distal plates, there are fifteen unknown quantities. These are the magnitudes of the forces in the six basal, six distal, and three instantaneous links. Since the applied load is known, the method of sections is used to first solve for the six distal leg link forces by separating the distal half of the structure as shown in Fig. 2.7 and enforcing static equilibrium on that section.

The first step in completing this solution is a simple transformation of the leg links from the \( \{d_i\} \) frame to the distal frame.

\[
^D d_i = ^d_j R^{d_j} d_i; \quad i = 1..6, j = 1..3 \tag{2.27}
\]

where

\[
^d_j d_i = \begin{cases} 
[\cos(\psi) \sin(\psi) 0]^T; & i = 1,3,5 \\
[-\cos(\psi) \sin(\psi) 0]^T; & i = 2,4,6
\end{cases}
\]
and the rotation matrix \( \mathbf{R}_{d_i} \) is determined from two standard Euler rotations about the z-axis and x-axis of each frame \( \{d_i\} \),

\[
\begin{align*}
\mathbf{R}_{d_i} &= \mathbf{R}_z(90^\circ)\mathbf{R}_x(-\theta_1) \\
\mathbf{R}_{d_i} &= \mathbf{R}_z(210^\circ)\mathbf{R}_x(-\theta_2) \\
\mathbf{R}_{d_i} &= \mathbf{R}_z(330^\circ)\mathbf{R}_x(-\theta_3)
\end{align*}
\]

Now the equations for static equilibrium of the distal plate can be written in vector form by summing the forces and moments about \( \mathbf{e}_D \). Since all vectors are expressed in the distal frame, the descriptive superscripts are omitted.

\[
\sum F = 0 = \mathbf{F}_D + \sum_{i=1}^{6} \mathbf{F}_{d_i} \mathbf{d}_i \tag{2.28}
\]

and

\[
\sum \mathbf{M}_{e_D} = 0 = \mathbf{M}_D + \sum_{i=1}^{6} \mathbf{r}_i \times \mathbf{F}_{d_i} \mathbf{d}_i \tag{2.29}
\]

where

\[
\begin{align*}
\mathbf{r}_1 = \mathbf{r}_6 &= b \begin{bmatrix} 1 \\ -\sqrt{3} \\ 0 \end{bmatrix}; \\
\mathbf{r}_2 = \mathbf{r}_3 &= b \begin{bmatrix} 1 \\ \sqrt{3} \\ 0 \end{bmatrix}; \\
\mathbf{r}_4 = \mathbf{r}_5 &= b \begin{bmatrix} -2 \\ 0 \\ 0 \end{bmatrix}.
\end{align*}
\]

By separating the vectors into their components, equations 2.28 and 2.29 can be written in matrix form.
\[
\begin{bmatrix}
  d_{1x} & \cdots & d_{6x} \\
  d_{1y} & \cdots & d_{6y} \\
  d_{1z} & \cdots & d_{6z} \\
  (\mathbf{r}_1 \times \mathbf{d}_1)_x & \cdots & (\mathbf{r}_6 \times \mathbf{d}_6)_x \\
  (\mathbf{r}_1 \times \mathbf{d}_1)_y & \cdots & (\mathbf{r}_6 \times \mathbf{d}_6)_y \\
  (\mathbf{r}_1 \times \mathbf{d}_1)_z & \cdots & (\mathbf{r}_6 \times \mathbf{d}_6)_z \\
\end{bmatrix}
\begin{bmatrix}
  F_{d_1} \\
  F_{d_2} \\
  F_{d_3} \\
  F_{d_4} \\
  F_{d_5} \\
  F_{d_6} \\
\end{bmatrix}
= 
\begin{bmatrix}
  -F_{D_x} \\
  -F_{D_y} \\
  -F_{D_z} \\
  -M_{D_x} \\
  -M_{D_y} \\
  -M_{D_z} \\
\end{bmatrix}
\] (2.30)

Since all elements in the matrix on the left-hand side are known, this is a set of six linear equations in the six unknown magnitudes of the distal leg forces. Using any standard solving technique, these six forces can easily be calculated.

Once the distal leg link forces are known, the forces occurring in the basal links and instantaneous truss links are determined using method of joints at each midjoint node. A free body diagram for this model of a typical midjoint is shown in Fig. 2.8.

---

![Free-body diagram of midjoint used for method-of-joints solution.](image)

Fig. 2.8: Free-body diagram of midjoint used for method-of-joints solution.
Once again, the kinematic analysis provides the directions for all associated links so that the only unknowns at each node are the magnitudes of the forces within the two basal links and the imaginary, instantaneous link. Like the solution procedure before, the vectors at each midjoint must be expressed in the same coordinate frame. In this case, the basal frame is chosen for simplicity. The basal leg links and instantaneous links are transformed by a similar procedure using two Euler rotations about each \( \{b_i\} \) frame:

\[
{^B b_i = {^b_i R}^b b_i}, \quad i=1...6; \; j=1...3 \tag{2.31}
\]

\[
{^B t_i = {^b_i R}^b t_i}, \quad i=1...3 \tag{2.32}
\]

where

\[
{^b_i b_i} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\cos(\psi) & \sin(\psi) & 0 \end{bmatrix}^T; \quad i=1,3,5
\]

\[
{^b_i b_i} = \begin{bmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\cos(\psi) & \sin(\psi) & 0 \end{bmatrix}^T; \quad i=2,4,6
\]

and

\[
{^b_i R} = {R_z(90^\circ)}{R_z(\theta_1)}
\]

\[
{^b_i R} = {R_z(210^\circ)}{R_z(\theta_2)}
\]

\[
{^b_i R} = {R_z(330^\circ)}{R_z(\theta_3)}
\]

The distal leg vectors are easily transformed from their expressions in the distal frame to the basal frame using the rotation matrix calculated from the kinematic solution,

\[
{^B d_i = {^b_i R}^D d_i} \tag{2.33}
\]

With all vectors expressed in the basal frame of reference, the static equilibrium can be enforced by summing forces at each midjoint and setting the result equal to zero:

\[
\sum F = 0 = F_i t_1 - F_i b_1 - F_i b_2 - F_i t_1 - F_i t_2 \tag{2.34}
\]

\[
\sum F = 0 = F_i t_2 - F_i b_3 - F_i b_4 - F_i t_3 - F_i t_4 \tag{2.35}
\]

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\[
\sum F = 0 = F_{t_3} - F_{b_5} - F_{b_6} b_6 - F_{t_5} t_5 - F_{t_6} t_6
\] (2.36)

Each of these equations can be expanded into matrix form to form a set of three linear equations in three unknowns. This set of matrix equations is written as follows:

\[
\begin{bmatrix}
-b_{t_3} & -b_{t_{i+1}} & t_{i+1} \\
-b_{t_3} & -b_{t_{i+1}} & t_{i+1} \\
-b_{t_3} & -b_{t_{i+1}} & t_{i+1}
\end{bmatrix}
\begin{bmatrix}
F_{b_{t_3}} \\
F_{b_{t_{i+1}}} \\
F_{t_{i+1}}
\end{bmatrix}
= \begin{bmatrix}
F_{d_{t_3}} \\
F_{d_{t_{i+1}}} \\
F_{d_{t_{i+1}}}
\end{bmatrix}
+ \begin{bmatrix}
d_{t_3} \\
d_{t_{i+1}} \\
d_{t_{i+1}}
\end{bmatrix} ; \quad i=1,3,5; \quad j=1,2,3
\] (2.37)

Equation 2.37 represents three separate sets of three linear equations which can be solved for the three unknowns at each midjoint using any standard method. Once these unknowns, \( F_{b_i} \) (\( i=1..6 \)) and \( F_{d_i} \) (\( i=1..3 \)), are determined, all the internal forces within the truss legs are known. It should be noted that the external forces acting on each link will be identical to the forces acting on the equivalent truss subassemblies.

If the overall base reactions are needed, such as for a transfer of forces to the base manipulator, a simple summation of forces about the center of the basal plate is required. Since the links are assumed to be massless, these reactions are calculated simply as:

\[
\sum \mathbf{F} = 0 = \mathbf{R}_b + \mathbf{F}_D
\] (2.38)

and

\[
\sum \mathbf{M}_{\mathbf{c}_p} = 0 = \mathbf{M}_b + \mathbf{M}_D + \mathbf{c}_p \times \mathbf{F}_D
\] (2.39)

The static force analysis is completed by developing a set of generalized midjoint forces from the internal forces that have been previously calculated. The purpose of these generalized forces is to convert the tensile/compressive forces occurring in the truss members to a set of orthogonal forces that can then be applied to any leg design for an approximate stress analysis. A single leg is illustrated in Fig. 2.9 to show the directions of these generalized forces. The set consisting of the two forces shown and the pseudo-torque force resulting from the instantaneous truss member comprises the vector
generalized force, $G_b$ or $G_d$. A simple trigonometric relation is used to resolve the truss forces into these new reactions, so that the complete set of generalized forces are written as:

$$
G_{b_1} = \begin{bmatrix}
(F_{b_1} - F_{b_2}) \cos(\psi) \\
(F_{b_1} + F_{b_2}) \sin(\psi)
\end{bmatrix} ;
G_{b_2} = \begin{bmatrix}
(F_{b_1} - F_{b_2}) \cos(\psi) \\
(F_{b_1} + F_{b_2}) \sin(\psi)
\end{bmatrix} ;
G_{b_3} = \begin{bmatrix}
(F_{b_1} - F_{b_2}) \cos(\psi) \\
F_{b_3}
\end{bmatrix}
$$

and

$$
G_{d_1} = \begin{bmatrix}
(F_{d_1} - F_{d_2}) \cos(\psi) \\
(F_{d_1} + F_{d_2}) \sin(\psi)
\end{bmatrix} ;
G_{d_2} = \begin{bmatrix}
(F_{d_1} - F_{d_2}) \cos(\psi) \\
(F_{d_1} + F_{d_2}) \sin(\psi)
\end{bmatrix} ;
G_{d_3} = \begin{bmatrix}
(F_{d_1} - F_{d_2}) \cos(\psi) \\
F_{d_3}
\end{bmatrix}
$$

where

$G_b$ represents the generalized midjoint forces for the basal legs, and

$G_d$ represents those for the distal legs.

The required actuator input torque for each leg can be calculated directly from the component of these generalized forces normal to the leg member. Since the

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instantaneous member was always oriented perpendicular to the leg, the torque required is simply this force multiplied by its moment arm, or the length of the leg. In equation form, this is written as:

\[ T_{\text{act}} = F_i \cdot l. \]  \hspace{1cm} (2.42)

This completes the derivation of the equations governing static equilibrium of the modified truss model for the Carpal wrist. The analysis results in values for the forces that are generated at the midjoints and required actuator inputs necessary to statically balance an applied load at the distal plate. Results of this analysis are presented and used in Chapter 4 to aid in the design of critical components of the prototype wrist.
Chapter 3

General Design Considerations

This chapter discusses general design considerations for the Carpal wrist. A set of overall design objectives is presented followed by an analysis of possible alternatives for the design of the structural components of the Carpal wrist which meet these objectives. An enumeration of the benefits and drawbacks of each alternative is included to aid in future wrist design.

3.1 Design Objectives

The Carpal wrist is envisioned as a solution to certain problems that exist in serial wrist designs. Although simple in concept, serial wrists are typically mechanically complex devices due in part to the extensive gearing and complex arrangement of shafting required to implement the intersecting, actuated revolute joints. Not only does this complexity increase the overall weight of the joint, it nearly eliminates the possibility of providing an open passage through the center of the wrist, which Rosheim (1989) describes as an important attribute for routing the various cables, hoses, and wires needed for the end-effector through a safe, protected passageway. A serious drawback also arises from the inherent singular positions serial wrists possess within their workspaces. These
singularities reduce the effectiveness of these devices by requiring complicated path planning to avoid traversing these locations.

However, in spite of their shortcomings, serial wrists have several advantages that make them widely accepted and extensively used -- these must be maintained in the design of the Carpal wrist. Their major advantage is their large workspace, which provides general orientational capabilities, so long as the singular positions are avoided. Serial wrists also possess closed-form kinematic solutions, and more importantly, their three intersecting axes allow the use of methods such as Pieper's solution to easily solve the kinematics of the overall manipulators to which they are attached (Craig, 1989). Finally, much of their mechanical complexity arises from the design of their actuation schemes, i.e., incorporation of distantly located actuators controlling the axes of the wrist. Although the mass and complexity of these designs is a drawback, the placement of the actuators for the wrist closer to the base of the manipulator is a common trait of most serial wrists. This moves the overall center of mass for the wrist (structural components plus actuators) closer to the base of the manipulator, thus increasing the load capacity of the manipulator by decreasing its overall inertia.

The Carpal wrist, in order to be an attractive alternative, must possess these desirable characteristics that have prompted the use of serial wrists while improving upon the aforementioned drawbacks. Based upon these requirements, design criteria can be developed to govern the design of the overall wrist, its individual components, and actuation scheme. Additional criteria are included to create a flexible design that is easily adaptable to a variety of applications. The general design criteria include:

1. A relatively large, dexterous workspace.

2. High strength-to-weight ratio, i.e., a lightweight wrist with a large payload capability.
3. An actuation scheme that allows for actuators to be situated closer to the manipulator base than the wrist.

4. A design which provides an accurate and precise output using available sensor and control system technology.

5. Easily manufacturable design - The use of standard components makes the overall device simple to fabricate and reduces the cost of manufacture.

6. Interchangeable actuation methods - Since the basic structure of the Carpal wrist is lightweight and strong, similar structural designs could be used for radically different applications, i.e., a manipulator intended to move rapidly with light payloads vs. a slower moving robot carrying a heavy payload. These differing objectives can be achieved by simply modifying the actuation scheme and not the entire wrist.

7. An open passageway through the center of the wrist should be maintained that is of adequate size for typical bundles of cables and/or hoses necessary for the end-effector.

8. The overall size of the wrist must be practical - the specific size of the wrist is relative to the application, but it must realistically operate on the end of a suitable manipulator.

3.2 Structural Configuration

As described before, the basic overall design for the Carpal wrist is similar to both the reflected tripod geometric configuration and a symmetric double-octahedral variable geometry truss. The two structures both have characteristics that are advantageous to the design of this parallel-actuated wrist. The VGT configuration has an extremely high strength-to-weight ratio due to the characteristics gained from its truss structure. Although it has a large workspace compared to other parallel manipulators, its workspace
and functionality as a robotic wrist are limited by the linear actuators located within its midplane. The constrained travel of the actuators reduces the workspace while this method of actuation complicates the mechanical joints between the multiple truss members. Conversely, the reflected tripod arrangement produces a large kinematic workspace because the rotation of the legs in the lower half of the structure are directly actuated. This helps to satisfy the criteria that the actuators be placed farther back on the base manipulator. However, the conversion from the VGT to the reflected tripod requires the removal of the actuated truss members from the midplane. This changes the overall structure from a truss to a frame. The legs must now support bending which weakens their structure and increases the mass of the mechanism accordingly. However, since a robotic wrist is intended to function as a dexterous, orienting device, the benefits of the large workspace outweigh the disadvantages of a weaker structure. In addition, the constraints placed upon robotic wrists necessitate that they be compact devices, whereas trusses are typically large structures in which strength are weight are critical. Therefore, the typical size of a wrist lessens any differences that exist between the truss and frame side members that might comprise the structure. In light of this, the Carpal wrist is designed as a reflected tripod mechanism with actuation applied to the rotation of the basal legs.

3.2.1 Orientational Capability

This general orientational capability of a wrist is usually important for achieving any required position and orientation of the end-effector of the robotic manipulator. However, in many useful tasks, such as welding, laser cutting, and spray painting, the tool itself is symmetric about the roll axis and eliminates the need for a sixth degree-of-freedom. For these tasks, a two degree-of-freedom wrist having the capability for singularity-free pitch and yaw motion is ideal.
The basic kinematic configuration of the Carpal wrist results in a non-standard three degree-of-freedom device. The Carpal wrist is capable of the typical pitch and yaw motions, but instead of the final roll axis, it possesses a plunge-type, translational degree-of-freedom. This final, plunging degree of freedom can be eliminated or fixed through the kinematic control algorithm (Salerno, et. al 1995). In this case, the Carpal wrist functions as a two-degree of freedom spherical device. The spherical nature of the workspace is important because it ensures the existence of inverse kinematic solutions for certain manipulators to which this wrist is attached. A spherical wrist allows the inverse kinematic problem for the overall manipulator to be decomposed into two simpler problems (Wampler, 1989), one for the base manipulator and the other for the orientation of the wrist. Therefore, if a solution exists for the inverse kinematics of the base manipulator, a solution will exist for the overall manipulator since the kinematics of the Carpal wrist are solvable.

Although not included on the prototype design presented in Chapter 4, a fourth degree-of-freedom can be included by mounting a rotary actuator directly to the distal plate. By restraining the translational degree-of-freedom, the Carpal wrist can be used as a general three degree-of-freedom orientational device (Salerno, et. al 1995).

3.2.2 Workspace Considerations

According to Craig (1989), the existence or nonexistent of a kinematic solution defines the workspace of a manipulator. Geometrically, this produces a volume containing all the discrete points the origin of the outermost frame of the robotic manipulator can reach. For a typical serial wrist having general orientational capabilities and considered separately from its base manipulator, the workspace describes a section of a spherical shell representing two orientational degrees-of-freedom. The third degree-of-freedom, a roll-rotation, is superimposed on it. A useful quantitative measure of the workspace of a specific wrist is the angular range of these rotations. Referring back to Table 1.1, the roll
rotations (two for roll-pitch-roll wrists and one for pitch-yaw-roll wrists) can usually achieve at least one complete rotation (360°). However, the pitch and yaw motions are the more useful measure, since they describe the sectional size of the spherical shell representing the workspace. These rotations are typically limited to values between 170° and 220°.

Ignoring the fourth degree-of-freedom, or roll-axis, a two degree-of-freedom device providing pitch and yaw motion is created from the reflected tripod structure by kinematically fixing the plunge distance of the Carpal wrist. This results in a workspace consisting of a series of spherical shell-sections described about each fixed plunge distance, as shown in Fig 3.1. For the Carpal wrist to be successful, it must possess a spherical workspace comparable to those of serial wrists. An approximately hemispherical workspace, which corresponds to a 180° range-of-motion for both the pitch and yaw axes, is proposed as a reasonable design.

Fig. 3.1: Typical workspace of Carpal wrist shown for $R_b = 0.667$ and various values of $R_d$. 

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3.2.3 Kinematic Ratios

In their kinematic design of constant velocity couplings that utilize a similar reflected tripod structure, Canfield, et al. (1995) describe two convenient kinematic ratios that can be used in the design process. In their work, the plunge ratio, $R_p$, and the base-to-leg ratio, $R_b$, are used to develop joints which can continuously rotate when bent at a prescribed angle. By using the principle of kinematic inversion, i.e., by holding the rotation fixed and examining the rotation of the axis about which the device is bent, these ratios can be used to aid in the design of the Carpal wrist by providing insight into the effects that geometric changes have upon the orientational workspace capability.

The plunge ratio, which is equal to the plunge distance divided by the leg length, is a measure used to quantify the specific spherical shells within the volumetric workspace. As shown in Fig. 3.1, a typical volumetric workspace for one specific value for $R_b$ can be displayed as a series of shells corresponding to values for this plunge ratio. Each spherical shell, in turn, can be seen to decrease in overall size with an increase in the plunge ratio. It is easy to see that in order to maximize the available workspace of a specific design, the plunge ratio must be minimized.

The base-to-leg ratio is so named because it is equal to the base length divided by the leg length. Whereas the plunge ratio describes the center of the sphere about which the distal plate rotates, the base-to-leg ratio is a quantifiable measure of the geometric configuration of the Carpal wrist. Canfield, et al. (1995) describe a method for using this ratio and the plunge ratio to aid in the kinematic design of a reflected tripod constant velocity joint. A typical design chart developed from their method is reprinted in Fig. 3.2. This graph shows the influence of the base-to-leg ratio upon the kinematic workspace of a reflected tripod device for a series of plunge ratios. Once again, the resulting workspace is shown to decrease corresponding to an increase in the plunge ratio while the size of the workspace increases with corresponding decreases in the base-to-leg ratio. The figure

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Fig. 3.2: Graph showing the effects of the kinematic ratios upon the maximum workspace of the Carpal wrist (Adapted from Canfield and Reinholtz, 1994).

also illustrates that kinematically the reflected tripod structure can achieve the desired hemispherical workspace. In fact, theoretically, it can be designed to fold back upon itself and achieve workspaces with a solid angle much greater than 180°.

There are a few caveats to blindly using these ratios to design for a certain workspace. Figures 3.1 and 3.2 show that larger workspaces can be obtained from using a lower values of $R_b$ and $R_d$. However, Fig. 3.3 illustrates the structural differences obtained from varying these ratios by displaying a sectional side view of one leg pair from a wrist with each ratio altered. It is easy to see that designs providing larger workspaces and dexterity also lead to wrists with legs that protrude substantially and increase the overall size of the
wrist. This complicates the use of the wrist in applications with tight space requirements. Additionally, the longer legs effectively act as longer moment arms developing larger input torques needed to support the payload. Finally, the use of these ratios to design the exact configuration is limited because they consider only the kinematic design. In actuality, physical problems such as interference and strength requirements may limit the ability of a real device to achieve the dexterity proposed by these solutions. Although larger workspaces may be possible, the longer leg lengths and shorter plunge distances result in wrists that must account for smaller interior angles, \( \alpha \), during operations approaching the edges of the workspace. The following sections address how this complicates the design of certain components and weakens the overall structure.

### 3.3 Component Design

#### 3.3.1 Midjoint Design

The midjoints comprise the subassembly used to supply the required degrees-of-freedom between the two legs in each leg pair. Kinematically, it is a spheric joint but it can be any equivalent mechanism, the 3R joint was described in Chapter 2. The design of the midjoints is addressed first and foremost because it is the weak link in the design of the Carpal wrist. This individual component has the most conflicting set of constraints and
requirements. It must be capable of three passive degrees-of-freedom to give it the motion of a spherical joint, but it must produce a larger range-of-motion than typical spheric joints. Increasing the overall length of the midjoint aids in avoiding interference when the interior angle between the leg pairs is at a minimum, but in order to reduce bending stresses that are present, the length must be minimized to reduce the cantilevered loading.

The midjoint is kinematically modeled as a spheric joint, but spheric joints are typically difficult to lubricate and maintain, and their range-of-motion is limited due to inherent physical interferences. For this reason, it is helpful to replace the spheric joint with a joint consisting of three revolute axes intersecting at a point, as shown in Fig. 2.1. This joint is kinematically equivalent to a spheric joint and can be designed to deliver a larger range of motion. Revolute axes are typically easier to maintain through the use sealed bearings or simpler shielding. However, the three revolute replacement contains a singular position when the two roll axes are aligned, which can result in a physical locking of the joint. This presents the designer with the added requirement of limiting the range-of-motion of the joint such that the angle between axes of the first and third revolute joints, i.e., the interior angle, does not approach 180°.

In implementation, the three revolute design is most easily effected by a clevis-type joint, as shown in Fig. 3.4. For this joint to have a large range of motion, the length of these two links must be increased in order to avoid physical interference at the base of the forked link. This induces larger bending stresses due to the cantilevered-type supports at its two roll axes. For this reason, it is imperative to minimize the length of these links. With known loads and workspace requirements, the designer can make a compromise based on the necessity of a large range-of-motion and the strength needed in the midjoint itself.
3.3.2 Leg Design

The legs of the Carpal wrist must also be designed to accomplish a variety of functions. As with the other individual components they must efficiently support the payload; they must physically locate two revolute joints, one connected to either the distal or basal plate and another for the associated roll-axis of the midjoint; and for the basal legs, an attachment to the actuation scheme is needed. As with the midjoints, the physical design of the legs must also consider the physical interferences that can occur. The most critical area is at the midjoint interface, where the two adjacent legs fold together tightly.

Although a variety of shapes can be imagined, the legs exhibit a good compromise of attributes if they are designed as triangular links simulating the original truss faces on the
VGT. A flat, triangular shape, as shown in Fig. 3.5, simulates the truss structure and allows the bending moments occurring at the midjoint interface to be distributed along the sides of the triangle as approximately axial forces. As with a truss, this efficiently supports the loads and minimizes the overall mass of the leg. The loads upon the base revolute joint are also reduced since the base of the triangle creates a longer axis of revolution that distributes the transverse bending moment created in the plane of the leg from the generalized forces at the midjoint.

The triangular structure also creates a hollow opening in the center of the leg. This opening is imperative in order to create clearance for the revolute joint corresponding to the roll axis of the midjoint. Even though the midjoint subassembly protrudes from the outer surface of the leg, the roll axis must be securely fastened in such a way to prevent the midjoint from being “pulled out” of the leg. The interior surface of the leg provides a convenient location to secure the midjoint.

Fig. 3.5: Basic triangular leg shape.
Finally, the triangular shape helps with the problem of physical interference at the midjoint interface. In this regard, the thickness and taper of the leg is important. During operation, the legs not only fold together but also twist relative to each other; thus at the midjoint, interference on all sides of the legs must be accounted for. A steeper triangle and thinner leg minimize this type of interference problem. Although the triangular shape creates a problem of interference at the base due to adjacent legs connected to the same plate folding up near one another, the same design, i.e., a steeper triangle that is thinner and has a smaller base, eliminates the problem. For both of these interference problems, minimizing the thickness of the legs is advantageous in eliminating possible interference.

There is another alternative to improve the physical interference characteristics of the midjoint interface. By bending the side plane of the legs, the range of motion of the midjoints can be centered about the aligned position as shown in Fig. 3.6. This is effective in possibly increasing the strength of the midjoint by reducing its length since

Fig. 3.6: Bent-leg design showing minimized midjoint flexure.
the total bend is minimized. However, the legs are weakened by introducing bending stresses created by the new geometry. Perhaps most significantly, the midjoints must pass through their singular position. Since these joints must be free to rotate, this design risks attaining a physical locking of the midjoints if they rotate slightly about their roll axes when these two axes are aligned.

There is a final consideration in the design of the legs, which is independent of the triangular design or any other basic shape. A method of actuation or a transmission system for the actuators must be included with the basal legs. In order to improve the manufacturability and cost of the entire wrist, it is beneficial to design the legs to be a modular component in order to limit the number of different components in the wrist. The actuation scheme should be easily fastened to the basal leg so that it can be produced concurrently with the distal leg. For a triangular leg, the interior space can be used for mounting the actuation or transmission device. This means that the size of the legs and this area should be maximized. The other alternative -- mounting the actuation to the side of the leg -- requires a smaller leg with a narrower base. Neither choice is clearly superior; it must instead be factored into the overall design of the actuation scheme that is required.

### 3.3.3 Basal/Distal Plate Design

The design of the basal and distal plates is straightforward and, as with the legs, the design for both plates should be similar to improve manufacturability. The plates must be designed to support the loads required and maintain orientation of the revolute axes for the base of the legs. Other than maintaining these requirements, there are only a few considerations in the general design of these plates. Interference with the legs as they bend inwards must be avoided, and a passageway for cables and hoses should be included. The basal plate must be sized to accommodate the standard mounting
configuration on the base manipulator and the distal plate must accommodate the end-effector mounting configuration.

3.4 Actuation Methods

The design of the actuation scheme focuses upon the selection of the actuating power source and the design of the transmission system for this actuator. There are a variety of methods to consider in implementing an actuation scheme for the Carpal wrist. Due to the parallel structure and location of the actuated legs either rotary or linear actuation schemes are possible. Whichever method is chosen though, there are certain specific requirements it must satisfy. Most importantly, the actuation scheme must develop the required result -- controlled rotation of the basal legs with respect to the basal plate. In certain instances, it may be advantageous to have interchangeable actuation schemes; in that way, different design criteria can be satisfied using a single overall design. Finally, the importance of moving the mass of the actuators back along the base manipulator dictates that the actuation scheme should be such that the majority of its components and weight can be located inboard of the basal plate.

3.4.1 Rotary Actuation

A rotary actuation scheme refers to a system using a motor as an actuator that directly controls the rotation of the basal leg. In this form of actuation, power is transmitted along the arm of the base manipulator. It is probably the cleanest form of actuation in the sense that it directly controls the rotation of the legs and it is most like the commonly used form of actuation among typical serial wrists.

There are numerous choices for the type of motor to be used in this scheme. In this section, only those motors that are typically used in robotic applications or that are particularly suited to the characteristics of the Carpal wrist are described for the purpose of choosing a suitable type.
Either a stepper motor or a hybrid servo is an excellent choice for control of robotic manipulators. They are suited to the intermittent motion required by robotic devices and are a good choice for high torque, low speed applications (Compumotor Digiplan, 1991). The stepper motor can be controlled with simple open-loop control methods using a standard PC controller card, thus eliminating the need for encoders or continuous feedback devices. This makes them easy to implement and relatively inexpensive. Stepper motors have drawbacks, however, that preclude their use in high-performance systems. For a given output power, they are heavier than other types of motors, their maximum speeds are fairly low, they are relatively noisy, and under high loads, they can miss steps or lose track of their exact position (Andeen, 1988). This last drawback is especially harmful in maintaining precise control. A calibration method using a “home” or limit position is needed to occasionally reset its position. Otherwise, a closed-loop control system using sensors to provide feedback are needed to effectively use a stepper motor. If a closed-loop system is used, however, the hybrid servo is a better alternative. As opposed to the stepper motor, it requires a feedback control system, which can be based on positional, velocity, or torque feedback depending on the need. The added requirement improves its performance by increasing the available torque-speed characteristics and eliminating the potential for positional inaccuracy.

A good alternative to the stepper motor is the DC servo motor, which is similar to the hybrid servo. Like the hybrid servo, it requires a closed-loop control system for positioning and, as such, requires feedback devices, such as encoders, resolvers, or tachometers. However, these motors deliver greater continuous shaft power at higher speeds and are smaller in both size and weight than comparable stepper motors. The main disadvantage in using these motors is that they are limited in their ability to deliver frequent starts, stops and direction changes (Compumotor Digiplan, 1991), which is a characteristic of the actuation system of the wrist.
Hydraulic motors provide yet another alternative for powering the basal leg rotations. These devices provide the highest torque-to-weight ratios and may provide for the most direct transmission to the leg. Although accurate control of hydraulic actuators is possible, it is not practical for hydraulic motors, because they are not designed for intermittent, reversible motion. Additionally, the added equipment and maintenance needed to incorporate a hydraulic system is extensive, while the systems have the disadvantage of being noisy and inefficient.

### 3.4.2 Linear Actuation

There are three main forms of linear actuation to consider: electromechanical, hydraulic, and pneumatic. For the design of the Carpal wrist, only the electromechanical, i.e., rotary motion from a motor converted to linear motion through a lead screw, and hydraulic actuators are considered. Pneumatic actuators are excluded since they are not well-suited for precise position-controlled tasks.

Electromechanical linear use either stepper motors or DC servo drives can be used to turn the linear screw. These have the same benefits and drawbacks as the rotary actuators listed earlier. The linear actuators have the added option of being used with a ball-screw or lead-screw design. In most instances the ball-screw option is more advantageous, having less friction, more power transmission, and smoother operation. However, if cost is a factor, the lead screw offers a slight advantage.

The linear hydraulic actuators are similar to the hydraulic motors. In certain cases, they may be ideal. Their high torque input means the only limit on payload is the strength of the components comprising the wrist. However, just as with the hydraulic motor, they are difficult to control and require a lot of additional equipment.
There are certain problems common to both forms of linear actuation. They must be connected between ground and the basal legs with revolute joints since the actuation scheme kinematically acts as a slider-crank. This may create problems with electromechanical actuators because the entire assembly must be attached between pivots. The attachment of the linear actuators to the basal legs then must allow for the required range of motion such that they do not interfere with the any parts of the wrist. If the range-of-motion required by the input legs is large, an additional input arm attached to the leg, as shown in Fig. 3.7, may be required to avoid interference. This input arm becomes another design consideration. Since the torque transmission is directly related to the moment created by the force of the linear actuator, it is best to use the longest input arm possible. However, this length must be kept reasonable in order to limit the travel of the linear actuators and to maintain a practical size for the cylindrical envelope surrounding the wrist assembly. The use of linear actuators also limits the distance that the actuators can be placed inboard of the wrist because the push-rods or shafts are subject to buckling under high compressive loads.

Fig. 3.7: Section view of one leg showing added input arm needed for linear actuator.
3.4.3 Transmission Systems

The transmission system for the actuation system must transmit the power delivered from the actuator to the basal legs. To satisfy the criteria that the actuators be located inboard from the wrist, it must be able to transmit this power from a distant position and, at the base of the wrist, convert the direction of motion to rotate the leg at its base revolute. The transmission system for the linear actuators is direct, i.e., the actuators attach directly to the legs. However, this direct transmission also creates a non-linear input/output relationship which complicates the kinematic control algorithms. In contrast, in order to remove the rotary actuators from the base of the wrist, various types of transmission systems are possible. In this thesis, three types of transmissions are considered for fulfilling this purpose: a gear-type transmission, a cable/tendon drive, and a linkage transmission system. Examples of a few of these are shown in Fig. 3.8. While the geared and cable-type transmissions provide for a simple linear input/output relationship, the linkage-type transmission provides for numerous variations which allow it have a linear or non-linear relationship. The following sections describe these systems in greater detail.

3.4.3.1 Geared Transmissions

A worm gear transmission located at the base of each basal leg is an effective
transmission system for a variety of reasons. It not only changes the direction of motion from the driving shaft to the leg, but it includes a large reduction between the worm and the worm gear. Its combination of good precision with large inherent reduction make it the best choice for a high velocity ratio and right-angle drive (Stock Drive Products, 1992). However, it is limited to relatively larger wrist designs due to the strength of the gear teeth in smaller gears and the space required for the additional bearing supports that are needed within the basal plate to secure the worm in place.

A bevel gear transmission is similar to the worm-gear type. It is possible to incorporate bevel gears on slightly smaller wrist designs due to fewer required support bearings; however, the tooth strength may also be critical. Additionally, a bevel gear set can not deliver as high a gear-ratio in a small area as the worm gearing. Thus, additional gearing at the motor is likely required. This increases the addition of backlash into the system, which requires additional measures to ensure accurate positional control.

3.4.3.2 Cable/Tendon Driven Transmission
Similar to the bevel-gearing transmission, the cable or chain driven assembly suffers from a difficult compromise due to space requirements. In order to include a large speed reduction without additional gearing, a large sprocket is required at the base of the leg and a tensioning mechanism is required. Also, the cable driven assembly does not provide the change of direction of the rotary motion as do the gear methods. This requires the less attractive alternative of situating the motors perpendicular to the arm on which they are mounted or including a right-angle gearhead with the motor. On the positive side, though, the cable systems are lightweight and do not have a limit on the distance the actuators can be from the wrist.

3.4.3.3 Linkage Transmissions
The final type of transmission that is considered in the design of the Carpal wrist is a linkage-type actuation scheme. This actuation method combines some of the positive
features of both rotary and linear actuation. When a gear transmission can not be included for size or strength reasons, a four-bar linkage can be used to actuate the legs. The linkage acts much like the linear actuator shown in Fig. 3.7, but a rotary actuator is attached to a driving link instead of a linear screw. The actuator, in this case, is an electric motor selected from those described earlier.

The linkage provides some flexibility in design. An image of the input arm can be copied to the “actuator-plane” to create a parallelogram linkage. This has the benefit of moving the actuators to an area having added space for the actuators and sensors, and since the driving links are an exact positional-image of the basal legs, a linear input/output relationship is developed from the transmission system. The parallel linkage also maintains a parallel relationship to the manipulator arm, but if this can be sacrificed, the linkage can be rotated to create additional space for the actuator mounting.

In contrast to a parallelogram linkage, a four-bar linkage can be designed so as to provide some force-generating characteristics to improve the torque supplied to the basal legs. By designing a linkage which has an input to output rotation ratio other than one, a certain amount of mechanical advantage can be included in the system. However, this introduces non-linearity into the transmission system, thus complicating the control algorithms. The possible linkage-type actuation schemes are summarized graphically in Fig. 3.9.

The drawbacks of the linkage system are a combination of the drawbacks for the rotary and linear actuation schemes. The susceptibility of the connecting link, or pushrod, to buckling under compression increases with the distance the actuators are moved from the wrist, thus limiting the distance these actuators can be placed inboard if the internal forces are high. If the torque requirements are high, some form of transmission is required at the actuators to provide for an additional gear-reduction. As with the rotary transmissions
discussed earlier, this will introduce backlash and additional compliance into the actuation system which limits the accuracy of the positional control.

Even with its shortcomings, the linkage actuation scheme is a viable alternative for actuating the Carpal wrist. The variety of possible designs make it a flexible scheme while its simple, jointed attachment helps to overcome the physical space limitations present in the wrist. As with the other alternatives, its usefulness must be determined with regard to a specific problem for which the wrist is designed.

### 3.5 Design Summary

This chapter has enumerated several alternatives available to satisfy the general design considerations for the Carpal wrist. Although an infinite number of possibilities could have been addressed, a limited number of alternatives were presented that satisfy a set of

General Design Considerations

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objectives formulated to ensure the design of a generally useful robotic wrist. The information included in this chapter is used as a basis for detailed design of a wrist capable of performing within a specific application. In this regard, Chapter 3 is included as the “backbone” of information necessary to complete the design of the prototype Carpal wrist presented in Chapter 4.
Chapter 4

Proof-of-Concept Prototype Design

Chapter 3 presents general design considerations for the Carpal wrist. Using that information as a basis, this chapter describes the detailed design of a prototype wrist intended as a proof-of-concept model developed for a specific application. In developing this design, it is helpful to acknowledge that this design process is not serial. It consists of numerous iterations upon various concepts that require concurrent design and evaluation. Figure 4.1 shows a flowchart illustrating the iterative nature of the prototype design process. The horizontal connections in the figure represent the various subsystems in the mechanism that require concurrent consideration during the design process while, at any stage, new information that is discovered may require the design to begin anew from the initial stages. The new information may arise from results showing that failure is expected, a realization that the components will not fit in the available space, or even that a certain design is impossible to fabricate with available resources.

4.1 Proposed Application

The prototype model of the Carpal wrist is intended to demonstrate the general concept and help prove some of its claimed advantages, such as increased load-carrying capability and singularity-free operation. As a prototype, it is needed to examine the three-
Fig. 4.1: Iterative and concurrent design process used to develop the prototype Carpal wrist.

dimensional interaction of components during its motion in order to analyze problems with the conceptual model and original designs and to develop improved designs. As a proof-of-concept model, the prototype is also expected to demonstrate the kinematic control of the wrist and prove its load-carrying capability.

There is no specific application proposed for the Carpal wrist. However, the prototype is intended to demonstrate the possibility for its use in a variety of applications currently
occupied by serial wrists. For this reason, the general design objectives presented in the preceding chapter are addressed by the development of a set of quantitative specifications based upon those of currently available wrists. The design specifications are summarized as follows:

1. Size - Overall length (unrotated):  >14 cm
   <22 cm
   Diameter:  <21 cm
2. Load Capacity -  ≥ 89 N (20 lbs)
3. Full load speed  < 3 sec full workspace move.
4. Workspace - Plunge distance:  > 5 cm
   < 10 cm
   Spherical shell:  ≥ 180° solid angle
5. Output Resolution -  < 0.254 mm (0.01 in.)

The following sections describe in greater detail the bases for the selection of these criteria.

4.1.1 Size

Robotic wrists do not exist as free-standing entities; their basic purpose dictates they be attached to a robotic manipulator. Thus, it is necessary that the wrist be small in relation to the size of the arm to which it is attached. The limits on the size then are dictated by choice of manipulator. In this case, two manipulators, a Unimate PUMA 562 and a Cincinnati Milacron T3-726, located in the Robotics and Mechanisms Laboratory at the University provided the most suitable testbed. Their proximity allows for the future possibility of attaching the wrist to either manipulator in place of the existing wrist.

The manipulators are medium-sized, light-duty manipulators with wrist sizes as shown in Table 4.1. The values shown in this table are approximate dimensions for each wrist; the
length corresponds to the unrotated position, and the diameter represents an imaginary cylinder surrounding the physical size of the wrist. These dimensions create a convenient set of limiting values for the overall size of the prototype Carpal wrist, i.e., its length should be between 14 and 22 cm and the envelope encompassing its unflexed position should be no greater than 21 cm.

<table>
<thead>
<tr>
<th>Wrist</th>
<th>Length cm (in.)</th>
<th>Diameter cm (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unimate PUMA 562</td>
<td>14 (5.5)</td>
<td>15 (6)</td>
</tr>
<tr>
<td>Cincinnati Milacron T3-762</td>
<td>22 (8.5)</td>
<td>21 (8)</td>
</tr>
</tbody>
</table>

### 4.1.2 Load Capacity

The necessary payload capacity of the wrist is also dictated by the manipulator to which it is attached. In order to optimally design the wrist for strength, it is convenient to design it around the payload capacity of the manipulator. Once again, the robotic manipulators in place in the laboratory are used to specify the design. In this case, the 71 N (16 lbs) payload of the Cincinnati Milacron robot is used as a basis since it is larger than that of the PUMA. Based upon this value, a slightly larger payload of 89 N (20 lbs) is specified for the Carpal wrist. It is noted that the payload capacity of the manipulator is a function of its static strength, dynamic strength, and minimum acceptable deflection. Thus, the static payload specification for the wrist does not directly correlate to the overall capacity of the manipulator, but the two should be on the same order of magnitude. Referring back to Table 1.1 shows that this specification is consistent with the light duty wrists described by Rosheim.
4.1.3 Speed

This is an arbitrarily developed specification developed to ensure that the wrist moves sufficiently fast as a demonstration device. The term, full workspace move, in the specification simply refers to a complete 180° motion from one edge of the workspace to the opposite edge. Using this range, three seconds was chosen as an adequate speed when fully loaded to its maximum payload capacity.

4.1.4 Workspace

The workspace of the Carpal wrist has been described as a series of spherical shells about a fixed plunge dimension, each of which emulates the spherical workspace of a typical serial wrist. The objective of designing the wrist as a two degree-of-freedom device with a fixed plunge distance is described in detail earlier. In order to specify the workspace, it is necessary to specify this plunge distance and the solid angle describing the extents of the spherical shell.

It is convenient to again use the laboratory robots as a reference to help specify the plunge distance. The radius of the spherical workspaces described by these wrists is equal to the distance from the pitch axis to the end-plate. These dimensions on the PUMA and T3-762 are approximately 5 cm (2 in.) and 10 cm (5 in.) respectively. Once again, these values form a convenient range to specify the plunge distance of Carpal wrist.

The two degrees of freedom of the wrist describe the angular orientation of its distal plate. Since these interact to create the spherical workspace, they can both be described by specifying the solid angle of the workspace which the wrist must be able to achieve. Based on the typical workspace plots shown in Fig. 3.1, the actual workspace is a section of a spherical shell which contains three lobes corresponding to each leg pair. The lobes represent areas where the wrist can not complete a full rotation of a single bend angle.
For this reason, the specified solid angle of the workspace must correspond to the size of the workspace excluding the extended lobe areas.

The workspace of the Carpal wrist, defined by the preceding discussion, must be described by a solid angle of at least 180°. This corresponds to a minimum 90° bend of the wrist and is consistent with the limits of existing serial wrists shown in Table 1.1.

4.1.5 Resolution

Rosheim (1989) lists a low and high range to classify the precision of serial wrists. Low-precision wrists can resolve their output between 0.254 (0.01 in.) and 10.2 mm (0.4 in.), and high-precision wrists have resolutions on the order of 0.013 - 0.254 mm (0.0005 - 0.01 in.). The prototype Carpal wrist is expected to be a high precision wrist and as such is specified to have an output resolution in the range described by Rosheim.

4.2 Kinematic Design

The kinematic design of the prototype model focuses on the selection of suitable kinematic parameters to satisfy the size and workspace specifications. Specifically, the base length, leg length, and plunge distance must be selected. The solution to this problem required two separate approaches to consider not only the kinematic solution but also the possibility of link interference early in the design process.

The first approach relies upon the method developed by Canfield and Reinholtz (1994) in designing a constant-velocity coupling. Their method of kinematic synthesis is used to develop the ratios governing the kinematic parameters for achieving certain workspace sizes. Since it only considers the kinematic closure of the mechanism in its calculation of workspace, this method lacks any consideration of physical dimensions and possible interference problems between links. This is not a trivial effect, since it can be shown that this method results in mechanisms that are capable of full rotations about the bend
axis -- a physical impossibility. For this reason, the kinematic synthesis is used solely as a tool to develop the first iterations in selecting the actual kinematic parameters of the wrist.

In order to bridge the gap between the kinematic parameters and final physical dimensions, a second approach is employed that uses both graphical simulation and relies upon an analysis of the interior angles, α. This additional information helps to visualize the wrist during its motion and determine the worst-case scenarios regarding the kinematic closure. Using the graphical simulation, it is easier to predict possible problems that may occur due to physical link dimensions and interference and avoid these problems in the prototype design.

4.2.1 Kinematic Synthesis

The kinematic synthesis for the constant velocity couplings presented by Canfield and Reinholtz (1994) results in a useful design chart, which has been shown in Fig. 3.2 but is reproduced in Fig. 4.2 for convenience. Knowing the workspace must be at least 180°, a horizontal line can be found above which exist possible parameters sets. It is easy to see that the base-to-leg ratio, $R_b$, must be less than approximately 1.3. As described in Chapter 3, in order to maintain a suitable aspect ratio for the wrist a relatively large plunge ratio, $R_{ph}$, is desirable. Although any plunge ratio above 0.1 (with $R_b = 1.3$) will theoretically provide for the required workspace, no values below 0.5 are considered for the aforementioned reason. With this limit imposed on $R_{ph}$, a new limit for $R_b$ ($R_b \leq 0.875$) is created at the intersection of the line representing $R_d = 0.5$ and the horizontal line at 180°.

Noting that this design only accounts for kinematic closure, a small margin is included in the initial guess for these ratios to account for the additional problems arising from interference due to realistic link sizes. Thus, a suitable first iteration for the design of a
Fig. 4.2: Graph showing maximum workspace of Carpal wrist vs. the kinematic ratios (Adapted from Canfield and Reinholtz, 1994).

wrist capable of achieving a $180^\circ$ solid angle workspace has the following kinematic parameters: $R_b = 0.75$ and $R_d = 0.6$.

4.2.2 Kinematic Visualization

It has been repeatedly stressed the design of an acceptable wrist relies heavily on avoiding the problems associated with physical interference in the links. In particular, the midjoint is required to achieve acute angles when folded, and the twisting of adjacent legs at the midjoint can cause additional interference problems with legs that are non-cylindrical at the midjoint interface. The interior angles, $\alpha$, (Fig. 3.3), of the midjoints are useful parameters to use in an attempt to eliminate problems with these types of interference.
Without considering other factors, the best wrist possible in terms of interference at the midjoint can be designed by maximizing the minimum value of these angles. This creates a design in which the legs do not fold up quite so tightly and thus minimizes the effect of their thickness. For other problems associated with the extensive link motions and extreme positions, a graphical kinematic simulation that quickly allows an analysis of specific closures has been developed. This allows a designer to assess possible problems with the kinematic solutions without the expense of time consuming solid models.

The first of two design tools created for the Carpal wrist provides a graphical simulation of the kinematic solutions of this mechanism. The code listed in Appendix 1 is a segment of the computer algorithm written in Visual C++ that performs both a graphical simulation of the wrist and displays the kinematic variables, i.e., leg angles, output parameters, and interior angles, in both tabular and graphical form. It performs both the forward and inverse kinematic solutions presented in Chapter 2 and draws the wrist schematically with triangular legs to help assess possible interference with the various closures.

An example of the graphical output shows how this tool can be used. Figure 4.3 shows the schematic drawings of the wrist produced by the program in a progression of iterations towards the final choice of kinematic parameters. Each set of views is shown for a different set of kinematic ratios and the same orientation of the wrist ($\gamma = 90^\circ$, $\beta = 30^\circ$). These two angles specify the orientation of the distal frame: $\beta$ is the angular orientation of the axis about which the wrist is bent, and $\gamma$ is the amount of rotation about this axis. These images in Fig. 4.3 clearly illustrate the possibility for physical problems in valid kinematic solutions.
Fig. 4.3: Design iterations for various kinematic ratios with wrist oriented to $\gamma=90^\circ$, $\beta=30^\circ$.

A possible set of kinematic ratios is shown in the final image in Fig. 4.3. Using these parameters to refine the iterative search for an interference-free design, the minimum
interior angles resulting from similar configurations are analyzed to compare wrists not exhibiting such intersecting links. Table 4.2 shows a list of various designs and their associated minimum interior angle among the three midjoints. From this chart, it is seen that decreasing the base-to-leg ratio and increasing the plunge ratio improves the magnitude of the interior angle. Although the final row in this table represents the best choice based upon the minimum interior angle criteria, the midrange values, $R_b = 0.375$ and $R_d = 0.875$, is chosen from this directed trial-and-error search based upon additional compromises that arise in the static force analysis in the following section.

<table>
<thead>
<tr>
<th>$R_b$</th>
<th>$R_d$</th>
<th>$\alpha_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.375</td>
<td>.75</td>
<td>30.8°</td>
</tr>
<tr>
<td>.333</td>
<td>.777</td>
<td>36.6°</td>
</tr>
<tr>
<td>.375</td>
<td>.875</td>
<td>41.4°</td>
</tr>
<tr>
<td>.375</td>
<td>.9</td>
<td>43.58</td>
</tr>
<tr>
<td>.25</td>
<td>.875</td>
<td>52.46</td>
</tr>
</tbody>
</table>

All $\alpha_{min}$ values occur at $\gamma = 90^\circ$ and $\beta = 90^\circ$.

Once the ratios are developed, the actual size of the wrist is chosen. Although some iteration is involved in this process during the force analysis and actual component design stages, it is an arbitrary choice so long as the chosen dimensions fall within the specified bounds for the overall size. Based upon these specifications, the actual base length for the prototype model is 3 cm (1.18 in.). The chosen ratios lead to a plunge distance equal to 7 cm (2.76 in.) and leg length of 8 cm (3.15 in.).

### 4.3 Force Analysis Results

Before the selection of actuators and detailed design of components can be considered, it is necessary to determine the required input torques and internal forces that result from
the maximum applied load allowed for the wrist. This is not a trivial task since there are
infinite possibilities for applying loads to the wrist. A method is needed to quickly scan
the results from the static force analysis presented in Chapter 2. The objective is to
simplify the early stages of the design process and to estimate the maximum internal
loads in the wrist.

The second tool developed to aid in the process of design for the Carpal wrist is a module
that is included in the previously described simulation code. This module performs the
static force analysis and displays the results, i.e., the input torques and generalized
midplane forces, in graphical or tabular format over the entire workspace. A screen
capture of typical output from this program is shown in Fig. 4.4. For the analysis of a
single applied unit force and moment load ($F_b$ and $M_b$), this figure shows a typical plot
of the required input torques in each leg over the range of the workspace ($-90 \leq \gamma \leq 90$)
for one orientation of the bend axis ($\beta = 50^\circ$). The $\beta$ parameter can be quickly changed
through the dialog box on the right to quickly examine the values over the entire
workspace of the wrist without resorting to confusing three-dimensional surface plots.
The units on these graphs are arbitrary but consistent with the units chosen for the
kinematic parameters (length) and applied force.

The amount of data can become tremendous due to the number of independent loading
cases and the fact that the results occur over a three-dimensional workspace. Judgment
must be used to determine the possible load cases that may be applied to the wrist. Then,
in order to circumvent direct examination of the entire set of data, a grid search is applied
to the static force analysis to find the maximum resultant torques and forces due to these
load cases. By numerically calculating results for each load at a large number of
positions in the workspace, the estimated maximum values occurring over the entire
workspace and set of load cases can be found and stored. Since there are a number of
different internal forces to be considered in the design, the search procedure stores the

Proof-of-Concept Prototype Design
Fig. 4.4: Screen capture of simulation program demonstrating results from force analysis.
maximum value of each generalized midplane force with its associated load case and position. These values are used to determine three sets of maximum midplane loads corresponding to the results for each maximum torque, generalized x-force and generalized y-force. These midplane-force sets contain the information needed to design and analyze the actuation assembly and the various components of the wrist.

For the prototype model of the Carpal wrist, the numerical search is conducted for an arbitrarily oriented unit load that is applied to the center of the distal plate with a moment created by applying this force 5 cm (1.97 in.) from the distal plate. Once the maximum midplane forces are found for this unit load, they can be scaled for any magnitude of actual payload. The results from the numerical search are shown in Table 4.3 for the previously selected wrist, i.e., $b=3$ cm, $l=8$ cm, and $p_d = 7$ cm. This table shows the

Table 4.3: Maximum midplane forces resulting from numerical search over entire workspace and all possible orientations of a unit load (1 N).

<table>
<thead>
<tr>
<th>Generalized Force</th>
<th>Applied Loading(^*$)</th>
<th>Orientation</th>
</tr>
</thead>
</table>
| \begin{tabular}{c}
Midplane \\
Force
\end{tabular} | \begin{tabular}{c}
Max. \\
Value
\end{tabular} | \begin{tabular}{c}
$F_x$ (N) \ 
$F_y$ (N) \ 
$F_z$ (N)
\end{tabular} | \begin{tabular}{c}
$M_x$ (N) \ 
$M_y$ (N) \ 
$M_z$ (N)
\end{tabular} | \begin{tabular}{c}
$\beta$ (°) \ 
$\gamma$ (°)
\end{tabular} |
| GB\(_{x1}\) | 2.24 N | 0.63 | -0.75 | -0.17 | 3.77 | 3.17 | 0. | -20 | 90 |
| GB\(_{x2}\) | 2.24 N | 0.97 | -0.17 | 0.17 | 0.86 | 4.85 | 0. | -40 | 90 |
| GB\(_{x3}\) | -2.24 N | 0.34 | -0.93 | -0.17 | 4.63 | 1.68 | 0. | 80 | 90 |
| GB\(_{y1}\) | 1.80 N | 0.93 | -0.34 | -0.17 | 1.68 | 4.63 | 0. | 50 | 90 |
| GB\(_{y2}\) | -1.80 N | 0.75 | -0.63 | 0.17 | 3.17 | 3.77 | 0. | 70 | 90 |
| GB\(_{y3}\) | -1.80 N | 0.81 | 0.47 | 0.34 | -2.35 | 4.07 | 0. | -80 | 90 |
| GD\(_{x1}\) | 1.40 N | 0.99 | 0. | 0.17 | 0. | 4.92 | 0. | -30 | 90 |
| GD\(_{x2}\) | -1.40 N | 0.49 | -0.85 | -0.17 | 4.26 | 2.46 | 0. | -90 | -90 |
| GD\(_{x3}\) | -1.23 N | 0. | -0.99 | 0.17 | 4.92 | 0. | 0. | 90 | 90 |
| GD\(_{y1}\) | -2.67 N | 0.75 | -0.63 | -0.17 | 3.17 | 3.77 | 0. | -30 | 90 |
| GD\(_{y2}\) | 2.67 N | 0.93 | -0.34 | 0.17 | 1.69 | 4.63 | 0. | -30 | 90 |
| GD\(_{y3}\) | -2.67 N | 0.17 | -0.97 | -0.17 | 4.85 | 0.86 | 0. | -90 | -90 |
| $T_{act1}$ | -12.30 N-cm | 0.99 | 0.17 | 0. | -0.87 | 4.92 | 0. | -80 | 90 |
| $T_{act2}$ | 12.30 N-cm | 0.34 | -0.94 | 0. | 4.7 | 1.71 | 0. | 20 | -90 |
| $T_{act3}$ | -12.13 N-cm | 0. | -1. | 0. | 5. | 0. | 0. | -10 | 90 |

\(^*$\)Applied loads are specified in the distal frame.

Proof-of-Concept Prototype Design
maximum value for each generalized force using the notation developed in Chapter 2. Each force is listed along with the applied load and orientation of the wrist at which the maximum occurs. Although the other possible designs that were analyzed by this method are not presented here because of space limitations, it is noted that between the few comparison cases, this choice provided the best compromise of kinematic function and load carrying capacity.

It is interesting to note that each type of force results in the same maximum for each leg pair. This is due to the axial symmetry of the wrist created by the equilateral basal and distal plates. Since there were multiple maximum values, one load case is arbitrarily chosen to represent the maximum result occurring for each generalized midplane force. These three cases, one for each type of generalized force, are shown as shaded rows in the table. The complete set of midplane nodal forces is found from the simulation program for each load case. These results, listed in Table 4.4, are used in the following design discussions. After being scaled by an appropriate magnitude, these force sets are used to analyze the various designs for possible success or failure and ultimately determine the payload capacity of the wrist.

Table 4.4: Midplane nodal force sets corresponding to maximum values of $G_x$, $G_y$, and $T_{act}$ for an applied unit load.

<table>
<thead>
<tr>
<th>Set</th>
<th>Max. Nodal Force Type</th>
<th>$T_{act}$ (N·cm)</th>
<th>$GB_x$ (N)</th>
<th>$GB_y$ (N)</th>
<th>$GD_x$ (N)</th>
<th>$GD_y$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$G_x$</td>
<td>8.2</td>
<td>2.24</td>
<td>0.63</td>
<td>0.66</td>
<td>2.45</td>
</tr>
<tr>
<td>2</td>
<td>$G_y$</td>
<td>10.4</td>
<td>2.20</td>
<td>1.31</td>
<td>1.04</td>
<td>2.67</td>
</tr>
<tr>
<td>3</td>
<td>$T_{act}$</td>
<td>12.3</td>
<td>-0.23</td>
<td>-1.74</td>
<td>0.02</td>
<td>2.33</td>
</tr>
</tbody>
</table>

† Maximum forces correspond to a unit applied load to the distal plate of the wrist.
Although it was developed throughout the design process, it is convenient at this point to discuss the maximum design payload of the prototype wrist, i.e., the scale factor for the maximum forces presented above. Considering selection of actuators, design of the transmission, and selection of bearings, a static load maximum payload of 89 N (20 lbs) was selected to satisfy the design specifications. This value is used throughout the following discussions as a scale factor for the forces presented in Table 4.4. For example, this payload capacity leads the maximum required input torque of:

\[
T_{\text{act}} = (12.3 \text{ N·cm}) (89 \text{ N/1 N}) \quad \text{(for 89 N payload)}
\]

\[
T_{\text{act}} = 1090 \text{ N·cm} = 10.9 \text{ N·m}
\]

The other values are scaled in a similar fashion.

4.4 Actuation System Design

The design of the actuation system corresponds directly to the maximum required input torque calculated in the preceding section. With the maximum torque known, the viability of various transmissions and actuators can be determined. Although the choice of a parallel four-bar mechanism actuation scheme was ultimately made, this section describes the design choices involved with the selection of an actuator/transmission system that can satisfy the design specifications and safely provide an input torque consistent with the static force results.

4.4.1 Worm-gear Transmission

The transmission of the prototype Carpal wrist was originally chosen to be a worm-gear drive located at the base of each basal leg driven by a shaft to a remotely-located actuator. Due to size requirements of the prototype, this presented a challenge. The size of the base is small compared to standard, available worm drives while the required torque to actuate the leg is relatively high, which necessitates larger components. Figure 4.5 shows the top view of the basal plate for one conceptual design incorporating a worm gear transmission. The placement of the worm is a function of the size of the worm gear.
However, the size and strength of the worm gear are proportional to the number of teeth it has and consequently proportional to the gear ratio of the set. This creates a difficult compromise since the torque requirements mandate a large gear ratio. Even for a modestly sized worm gear, such as the one depicted in the figure, interference with the transmission system is imminent.

With the conceptual design shown in Fig. 4.5, it is evident that there is a possible problem that results from the worm interfering with the rotation of the adjacent leg. This problem is increased further when the bearing mounts for the worm are included. Finally, the base length of 3 cm is shown to add a sense of scale to this figure. This illustrates that the components needed in this transmission, i.e., the worm and worm gear, need to be sized on the order of 1 - 1.5 cm. Available components of this size are too small to adequately handle the required torques.

![Diagram](image)

**Fig. 4.5:** Top view of wrist base showing conceptual design of worm-gear transmission.
Although the worm gear transmission is possibly the best transmission to be used with electric motor actuation, they are probably more useful in a larger design that possesses adequate clearance for sufficiently sized components and rigid mounting. Due to this, other alternatives were explored for the transmission in the prototype wrist.

4.4.2 Bevel-gear Transmission

While a bevel-gear transmission is not as attractive as a worm-gear transmission due to lower attainable gear ratios, they do allow for a more convenient mounting in the small space available on the basal plate. The bevel gear mounted to the input shaft can be located below the plate, as shown in Fig. 4.6, thus moving it out of the center of the wrist. However, due to the lower gear ratios available from bevel gear sets, the bevel gear transmission is intended solely to transmit the rotation from the motors through the 90° angle to actuate the rotation of the basal legs. Therefore, a design incorporating the bevel-gear transmission needs additional gearing or some type of reduction at the motor to assist in providing sufficient torque.

This design was pursued by performing a failure analysis on the bevel gears. For this

![Diagram](image)

**Fig. 4.6:** Side view of wrist base showing conceptual design of bevel-gear transmission.
analysis, the largest possible bevel gear set was examined. By choosing the largest size available, the effect of the torque is minimized and thus if the analysis indicates failure, all available gears are eliminated. First, it is assumed that the largest available gear that can feasibly be used is a 2.54 cm (1.00 in.) diameter bevel gear with the following parameters: diametral pitch = 24 and face width = 0.71 cm (0.28 in). The bevel gear set can be analyzed for static failure according to Shigley and Mitchell (1989) as follows:

\[ W_i = \frac{T}{r} \] (4.1)

\[ W_i = \frac{10.9 \text{ N} \cdot \text{m}}{0.0127 \text{ m}} = 858.3 \text{ N (193 lbs)} \]

where

\[ W_i = \text{the tangential load carried by the gear.} \]

\[ r = \text{radius of the smaller gear} \]

and

\[ \sigma = \frac{W_i P}{K_v F J} \] (4.2)

\[ \sigma = \frac{(193 \text{ lbs})(24 \text{ teeth/in.})}{(1.0)(0.28 \text{ in.})(0.27)} = 61.3 \text{ kpsi (422 MPa)} \]

where

\[ K_v = \text{velocity factor (Shigley and Mitchell, 1993)} \]

\[ F = \text{tooth length} \]

\[ J = \text{Geometry factor (Shigley and Mitchell, 1993)} \]

Although certain steels will withstand this level of stress, it does indicate that failure is expected in normal, available materials due to the stress level in the gear. The effect of including a factor of safety within the design increases the chances of failure significantly. Thus, the bevel-gear transmission can not be used for the prototype model of the Carpal wrist.
4.4.3 Linear Transmission

Due to failure of both gear type transmissions, a completely different approach for actuating the basal legs was examined. By using linear actuators, the high input torques required by the basal leg can be more easily applied using available components and simply fabricated parts. While the linkage-type transmission was earlier described as a transmission system and not a linear actuator, it shares certain characteristics with the linear actuators. This section describes the relevant design issues concerning the linear actuators; however, much of this discussion is directly applicable to the linkage-type transmission system described in the next section.

Both linear actuators and linkage transmissions introduce additional concerns regarding the required input forces. Since each method actuates the wrist through a pushrod mounted to the leg and the leg rotates through a large angular range, the pressure angle, \( \phi \), shown in Figs. 4.8 and 4.9 as the angle between the pushrod and the basal leg must not approach \( 0^\circ \) or \( 180^\circ \). If the pressure angle approaches these extremes, either the required input force or the internal forces in the pushrod will approach infinity. The other difficulty associated with the linear-type actuation stems from the possibility that the basal legs may travel over the basal plate, i.e., \( \theta_j < 90^\circ \). As described in Chapter 5, this creates the need for an additional input arm that attaches to the basal leg in order to move the pushrod away from possible interference with the leg or basal plate.

The screw-type linear actuation when first examined seems a good choice. However, when attached to the basal leg and to ground, it is an inverted slider-crank mechanism, which means that the screw and pushrod must rotate relative to ground and the leg. Additional hardware is needed to mount the actuating motor to this system; either to decouple the motor from the rotation of the shaft or to allow the motors to rotate with the shaft. In either case, it was not an efficient use of resources to attempt to design this
system. For this reason, the use of commercially available electromechanical linear actuators was investigated. Although they offer the benefit of easy mounting and solid construction, the off-the-shelf items are too bulky and massive to consider.

4.4.4 Linkage-type Transmission

Finally, the use of the four-bar linkage actuation method on the Carpal wrist was investigated. Although it may seem it was chosen solely by the process of elimination, this method actually possesses numerous benefits for a prototype model. Nevertheless, several items must be addressed to obtain a feasible design. The angular travel of the basal legs is important for both determining the geometric configuration of the four-bar linkage and for design of an interference-free design. The length and location of the input arm, if needed, must be addressed, and the forces required in the links must be calculated for an accurate stress analysis.

Before any design choices can be made the angular range of the basal leg motion is required. For the kinematic configuration chosen, this is found simply by finding the maximum input angle and minimum input angle from the kinematic simulation. The necessary result from the simulation program is shown in Fig. 4.7; the maximum input angle is 204.3° and the minimum is 72.9°. Obviously, the unwanted condition of θ<90° is present, but this attribute was inevitable on all wrists examined which have a useful workspace of 180°. With the use of an input arm offset from the leg, it is more important that the overall range be less than 180 degrees. The angular range is simply the difference, or, in this case, 131.4°.

Since the mechanical advantage of a non-parallel four-bar mechanism is directly proportional to the maximum angular travel of the input and output links, the maximum mechanical advantage that is attainable on this mechanism can be calculated. We know the angular range of motion of the output link of the four-bar mechanism to be the same
Fig. 4.7: Graph of $\theta$ vs. $\gamma$ showing both the overall maximum and minimum values for $\theta_i$.

as the input arm on the basal leg. It is also seen that the input link of the four-bar can not travel more than $180^\circ$, otherwise, the mechanism input becomes a crank. Thus, the maximum mechanical advantage attainable is $180/131.4$, or 1.37. Applied to the required input torque, this torque can be reduced from 10.9 N·m (96.8 in·lb) to 8.0 N·m (70.7 in·lb). This may be substantial enough to warrant a force-generating mechanism, but the added complexity in the design of a joint capable of $180^\circ$ motion did not warrant its inclusion in the design of the prototype Carpal wrist.

A parallel four-bar mechanism was used instead of a force-generating linkage to actuate the wrist. With a parallel four-bar linkage, the input torque is exactly equal to the output torque. However, the force in the connecting link, or pushrod, is a function of this torque and the pressure angle, $\phi$. The best case, i. e., the maximum pressure angle, can be obtained by attaching the input arm to the basal leg in such an orientation to create an equal pressure angle at both extremes as shown in Fig. 4.8. For the angular travel on the prototype model, the input arm must be attached to the basal leg at an angle of $41.4^\circ$. 

Proof-of-Concept Prototype Design 85
This "centers" the angular travel of the input arm about the horizontal and creates a minimum pressure angle of 24.3°. Even though a good rule-of-thumb for the pressure angle in four-bar mechanisms is for $\phi_{min} > 30$, the force and stress analysis shows this value to be acceptable under the assumed loading.

The length of the input link of the four-bar and the length of the input arm on the basal leg must be equal to produce a parallel four bar mechanism. The lengths of these links must be determined based on two factors: the desirable outer radius of the envelope surrounding the pushrods and the maximum force that will occur in the connecting rod. This is a subjective choice; a longer length reduces the force but increases the envelope containing the actuation assembly, while a shorter length reverses the effect. As a compromise, a 4.5 cm (1.77 in.) link length was chosen for the prototype model.

Finally, the connecting link force must be calculated for design purposes. While this force is a function of both the input torque and the pressure angle, only the maximum value is important for the failure analysis so only the maximum torque and minimum pressure angle need be used. The solution for the link force results from a simple sum of
moments at either the input or output link of the four-bar mechanism when it is at an extreme position, as shown in the free-body diagram of Fig. 4.9. This results in a maximum connecting link force equal to 589 N (132 lbs). This result will be used later in the chapter to help develop the detailed design of the links.

### 4.4.5 Actuator Selection

A stepper motor with an attached gearhead was chosen as the actuator in the four-bar transmission system. The specific stepper motor chosen is described as follows:

- Cyber Research ORM-264K stepper motor
- NEMA 23 size frame
- 6 lead/2 phase operation
- 54.2 oz-in static holding torque
- 15,000 steps/sec @ 20% Load
- 2.0 Amps/phase (series)
- 3.6 VDC unipolar voltage
- Weight ~0.5 kg (1 lb)

A stepper motor was chosen for its ease of control, good characteristics at low speeds, and its suitability for intermittent motion. Even though this particular motor was chosen for its relatively high torque capability and small size, the holding torque of 0.38 N-m (1.7 in-lb) is not sufficient for the requirements. To provide the required torque, a gearhead is used to obtain a large gear reduction and increase the available torque. A smaller motor/gearhead combination was chosen over a larger motor for two reasons. The size and weight of the motor and gearhead together is much less than a similar-type motor capable of directly supplying the required torque. Also, the gearhead increases the resolution of the system by a factor equal to its gear ratio.

The gearhead specified for the actuator is described as follows:
- Bayside Controls NE 23-LB Low Backlash NEMA gearhead
- NEMA 23 size frame
- 50:1 ratio
- Overall size: 2.27 in. x 2.27 in. x 2.30 in.
- Rated Output Torque: 5.7 N-m (50 in-lbs)
- Peak Output Torque: 11.3 N-m (100 in-lbs)
- Rated Input Speed: 4000 RPM
- Backlash: 10 arcmin
- Average Efficiency: 92%
- Max. Weight: 0.5 kg (1.2 lbs)
- Radial Load: 89 N (20 lbs)
- Axial Load: 178 N (40 lbs)

The operation of the wrist does not utilize the motors in such a way that the continuous rated values are important. The peak output torque is used for comparison due to the
intermittent motion typical of robotic devices. In this regard, the gearhead maintains a slight safety factor when supplying the required 10.9 N·m (96.8 in-lbs) torque.

The gear ratio was chosen to satisfy the torque and speed requirements. Based on torque requirements, the gear ratio must be at least 28.6:1 (i.e. 10.9 N-m/0.38 N-m). The closest available gear-ratio to this is 50:1. To ensure the speed specification can be maintained, the specified three second full workspace move must be transformed to the leg rotation speed, and the torque-speed chart of the motor must be checked to see it can provide the necessary torque at that speed. Since a 132° rotation of the legs corresponds to a full workspace move, the average angular speed of the legs must be 132°/3 sec, or 0.122 rev/sec. The motor must travel 50 times this speed, or 6.1 rev/sec. Since the speed of a stepper is specified in half steps per second, this is further multiplied by 400 steps/rev to obtain a maximum motor speed of 2440 steps/sec. According to its chart, the specified motor is capable of delivering approximately 0.28 N-m (2.48 in-lbs) of torque at this speed. Multiplied through the gear-ratio, this corresponds to 14 N-m (124 in-lbs) output torque, which is sufficient to drive the wrist.

The gearhead also helps to satisfy the resolution requirement for positioning the wrist. Without microstepping control, a stepper motor operated in half-step mode is capable of 400 steps/rev of resolution. The gear ratio increases this to 400·50 = 2000 steps/rev. Assuming an approximately linear relationship between the input leg rotation and the rotation of the distal plate, this resolution can be transformed to a measure of the workspace resolution. Once again, the resolution at the driving link is multiplied by a ratio of driving link angular travel to workspace angular travel, or 132°/180°. With the gearhead attached, this results in a workspace resolution of 1467 steps/rev. Taking the inverse and converting to radians, the resolution is 0.00428 rad/step. In order to compare this to the specification, it must be multiplied by the radius of the workspace to obtain a resolution expressed in length units. A radius of 7 cm (2.76 in.) results in a resolution of
0.30 mm/step, which is close to the specification. To completely satisfy the specification, the stepper motor can be controlled with a microstepping driver that further increases its resolution.

### 4.4.6 Hardware

Neglecting the power source and amplifiers, two types of electronic hardware are required to control the stepper motors with open-loop positional control. The stepper motor controller is needed to coordinate the three actuators and develop the control pulses corresponding to the calculated moves, and a stepper motor driver is needed for each motor to convert the controller signals to the large currents needed by the motor.

A four-axis intelligent stepping controller (CyberResearch ESH 384E) that is compatible with a standard PC is specified as the controller. The board can control 4 axes simultaneously, sufficient for the three axes required by the wrist. The controller supplies each axis with the following:
- microstepping capability up to 50,000 steps/rev
- high speed pulse rates up to 524,000 steps/sec
- direction signal
- auxiliary output line
- 2 limit switch inputs
- home switch input

In addition to this, the controller maintains a 4-channel encoder input. This allows for the flexibility to include closed-loop positional control using the same hardware with only the addition of feedback sensors.

The specified drivers are Panther series integral driver/power supply systems (CyberResearch PANT-LD). They are capable of microstepping resolutions up to 51,200 steps/rev and step frequencies up to 10 MHz. Powered by standard wall current, they
provide up to 3 amps/phase for powering their associated stepper motor. The combination of a microstepping controller with the microstepping driver allows the described actuation design to obtain the required resolution while improving the dynamic performance of the stepper motors themselves.

4.5 Structural Design

![Diagram of structural design](image_url)

**Fig. 4.10:** CAD model of final design for the prototype Carpal wrist.
The remainder of this chapter summarizes the final detailed designs for the major components of the prototype wrist. A CAD model of the accepted design with labels indicating the specific components is shown in Fig. 4.10. For additional reference, the discussions that follow can be augmented by the detailed mechanical drawings produced for the manufacture of the prototype, which appear in Appendix 3. As described before, the process that is followed to arrive at this end-result is highly iterative and is based upon numerous concurrent choices. Only certain key points are addressed in the discussion that follows. For example, failure analyses for the final component designs are presented only for the recognized critical areas on each component. A description of the effort necessary to arrive at these acceptable designs is omitted.

4.5.1 Detailed Midjoint Design

The final design of the midjoint components is shown in Fig. 4.11. This figure illustrates a number of important features included in this joint. Detailed design drawings of the components are shown in the appendix. The basic design of the midjoint assembly is similar to a clevis joint, each complete joint having a “tongue” and a “fork” half. The top half of the figure shows one half of a complete joint. A sleeve bearing in the mating joint provides one revolute surface. The other two revolutes comprising the midjoint spheric equivalent are the shafts which rotate in each associated leg. The rotation of these shafts is provided by two flanged bearings mounted in the top of the leg. The sleeve portion of the flanged bearings provides the main bearing surface for the shaft. The flanged surfaces are necessary due to the axial loads, both compressive and tensile forces can arise in the revolute shaft. Finally, a locking nut and thrust washer (not shown) secures the assembly in place.

Both the sleeve and flange bearings are bronze oil-impregnated bearings. This choice was dictated by the extremely small clearances that are available once the components, both leg and midjoint, have been designed to account for the interference that exists at the
Midjoint when the interior angle approaches its minimum. A failure analysis based upon the static loads carried by these bearings is developed using the three load cases resulting from the maximum midplane nodal force search shown in Table 4.4. Figure 4.12 shows the free-body diagram used for the approximate static force analysis of the midplane bearing loads. The structure is not statically indeterminate because only one the flanged surfaces is subjected to the axial load \( (F_a) \) at a particular time. Using the equations for static equilibrium, the equations governing the bearing reactions are derived to be:
\[ \mathbf{R}_{m1} = \begin{bmatrix} F_x \left( \frac{l_m}{d_b} \right) \\ -F_y \\ F_z \left( \frac{l_m}{d_b} \right) \end{bmatrix} \quad \text{and} \quad \mathbf{R}_{m2} = \begin{bmatrix} -F_x \left( 1 + \frac{l_m}{d_b} \right) \\ -F_y \\ -F_z \left( 1 + \frac{l_m}{d_b} \right) \end{bmatrix} \] (4.3)

where

\( F_{x,y,z} \) are the midjoint nodal maximum forces that act in an equal and opposite sense on the midjoint

\( \mathbf{R}_{mi} \) = the vector reaction force occurring on midjoint bearing \( i \) \( (i=1,2) \)

\( l_m \) = the distance from the end of the leg to the midjoint node, and

\( d_b \) = the distance between bearing centers.

The bearing reactions, \( R_{mi} \), represent the worst case solution, i.e., it has been assumed for each that the axial load is included in the reaction. To be completely accurate, one reaction would have zero force in this component.

Since the motion imposed on the bearing is intermittent and oscillatory, a failure analysis based upon a static analysis using a \( P_{max} \) value of 31.0 Mpa (4500 psi) for sintered bronze

\[ \text{Fig. 4.12: Free-body diagram of midjoint used to complete the bearing analysis.} \]
(Shigley and Mitchell, 1993) is presented. A MathCad® document included in Appendix 2 details this analysis, which results in a minimum factor of safety of nearly two covering static failure of these bearings. Based upon this analysis, the distance between flange bearings, \( d_b \), has been maximized in order to limit the bearing loads and prevent failure. The length of this dimension represents a necessary compromise arising from conflicting requirements imposed upon the design of the leg. This decision to lengthen this distance decreases the inner area of the leg, while the actuating assembly described later requires additional space in this area.

The length of the clevis joint section of the midjoint components is a compromise between another set of conflicting issues. It should be the minimum possible length to reduce the bending moment created from transverse loading at the actual midjoint nodal location. However, as the midjoints and leg pairs approach the minimum interior angle (41.3°), interference in the clevis joint requires that the midjoint be lengthened. To eliminate as much of this required length as possible while still maintaining a certain amount of strength, the clevis joint halves were shaped with interference reliefs (see Appendix 3 for detail). As shown in the lower half of Fig. 4.11, these are designed to allow the midjoint to bend to 40° with a minimal separation between the leg-connected ends.

The previous paragraph brings attention to another critical location in the prototype wrist that must be analyzed for possible failure. The necessary construction of the midjoint leads to a relatively large bending moment and associated bending stresses in the small diameter (4.76 mm) shaft at the juncture between the midjoint and the end of the leg. Due to the loading at the actual midjoint node, the clevis half of the midjoint acts as cantilevered beam with additional axial loading; this induces axial, shear, and bending stresses at the juncture between this clevis joint and the shaft. Using BEAM VI (Mitchell, 1992), a beam-stress analysis program using the transfer matrix method, the
stresses resulting from the beam loads are analyzed. Due to the difference in cross-sectional area, only the shaft is modeled. The midjoint loads can be replaced by axial forces and a moment applied to the end of the shaft. The bearing supports for the shaft are modeled as distributed supports since their length is not much shorter than the overall length of the shaft. The completed model of the midjoint shaft is shown in Fig. 4.13. This model can only approximate the stress, because as soon as there is an infinitesimal deflection of the shaft, the flange of the bearing begins to support a portion of this load. Thus, the simpler load scenario produces a more conservative stress analysis.

As expected, the static stress analysis results from BEAM VI showed the maximum stress occurs at the boundary between the shaft and the clevis joint section. The combination axial-bending stress for this point is approximately 430 MPa (62 ksi). This is a relatively high stress, requiring either a design change or a choice of high strength material to maintain a reasonable safety margin. Changing the design is extremely difficult since no acceptable alternatives were found. There are, however, numerous mid-range steels capable of supporting over 860 MPa (124 ksi). Thus, out of necessity, the midjoints are specified to be fabricated from AISI 1050 steel drawn at 600°F, which has a

![Diagram of midjoint shaft](image)

**Fig. 4.13:** Beam model used for stress and failure analysis of midjoint shaft
yield stress of approximately 1240 MPa (180 kpsi). This results in a static factor of safety of approximately 2.9 based upon failure at this juncture.

This location and its associated bearing represent the “weak link” of the structure. Thus, the prototype model has an overall factor of safety against bearing failure of approximately two and a structural-component static factor of safety equal to approximately 2.9.

4.5.2 Detailed Leg Design

To simplify the fabrication, the legs were designed to be modular, i.e., both the basal and distal legs used the same basic design, shown in Fig. 4.14. This represents a compromise between the overall strength needed to support the loads, the clearance and interference requirements between the midjoints and adjacent legs, and the need to fasten the actuation
system to the basal leg.

The basic shape of the legs evolved out of the necessity to eliminate any interference with its associated distal leg when the interior angle approaches its minimum and with each adjacent basal leg when both basal legs fold upward. Based upon the discussion in Chapter 3, the semi-triangular shape is used to eliminate both these problems. By cropping the edges of the base of the triangle, interference with the base of the adjacent legs is eliminated. The stepped profile eliminates interference problems at the midjoint by reducing the thickness of the leg at this interface while allowing a slightly larger bearing at the base revolute joint.

Another benefit of the triangular shape arises from the internal space that is created. Due to the small size of these components, this area provides the best position to include the actuator insert, which creates the necessary offset input arm. These insert pieces are attached with screw fasteners for two reasons. The fabrication of the basal legs is much simpler and less expensive if these two components are manufactured separately, and these inserts must be removed in order to service the midjoint assembly, which is secured in its mounting by a locknut on the inside surface of the leg.

As with the midjoint assembly, the basal and distal revolute joints use bronze, oil-impregnated flanged bearings. As with the midjoint assembly, this type of bearing is chosen because the size limitations prohibit the use of more durable roller bearings. A flanged bearing is chosen to provide bearing surfaces for the revolute shafts and the interface between the legs and basal/distal plates. Similar to the midjoint assembly, a static force analysis is used to analyze the possibility of failure in these bearings by calculating the bearing loads and associated pressure imposed on them. The free-body diagram for this analysis is shown in Fig. 4.15. Since the maximum load profiles were
determined to exist at the extreme positions of the leg, this free-body diagram illustrates the actuator force as it applies through the minimum pressure angle.

The actuator rod force, $F_{act}$, is calculated as:

$$F_{act} = \frac{T_{act}}{l_a \sin(\phi_{min})}$$  \hspace{1cm} (4.4)

where

$l_a$ is the length of the input arm.
The bearing reactions are then derived from the equations for static equilibrium to be:

\[
R_1 = \begin{bmatrix}
-F_x \\
\frac{F_x}{l_x} - \frac{F_y}{2} \frac{F_z l \cos(\theta_{offset} - \phi_{min})}{2l_a \sin(\phi_{min})} \\
-\frac{F_x}{2} \left[ 1 + \frac{l \sin(\theta_{offset} - \phi_{min})}{l_a \sin(\phi_{min})} \right]
\end{bmatrix}
\]

\[
R_2 = \begin{bmatrix}
-F_x \\
-\frac{F_x}{l_b} \frac{F_y}{2} - \frac{F_z l \cos(\theta_{offset} - \phi_{min})}{2l_a \sin(\phi_{min})} \\
-\frac{F_x}{2} \left[ 1 + \frac{l \sin(\theta_{offset} - \phi_{min})}{l_a \sin(\phi_{min})} \right]
\end{bmatrix}
\]

(4.5)

Like the midjoint bearing analysis, the \( y \)-component for both these reactions includes the force exerted upon the flanged surface even though only one may support the load at each instant. This allows the worst-case scenario to be analyzed in a general fashion without trying to determine which bearing actually "feels" the load. These calculations are performed for each of the three maximum load cases described in Table 4.4, and the results are shown in a similar MathCad® document included in Appendix 2. This analysis results in a minimum factor of safety against static failure slightly greater than three.

With the exception of the length of the leg, which is dictated by the kinematic design, the dimensions of the legs are chosen to aid in eliminating interference and to accommodate the midjoint assembly, the fastening system for the actuators, and the various bearings needed for the two revolute axes. Once these requirements are factored into the design, the size of the leg is such that the strength of the leg is not critical. This is proven by examining the most critical area, which occurs at the base of the leg due to the thickness of the material surrounding the bearings. A simple tensile stress calculation shows that
the stress generated in this area is only 18.7 MPa (2.7 kpsi). A 6061 series aluminum alloy, the material chosen for the legs due to its strength-to-weight ratio and excellent machinability, provides a factor of safety against static failure between 3.0 and 14.8 depending upon the heat treatment of the alloy.

4.5.3 Detailed Design of Actuation Components

4.5.3.1 Actuator Inserts
The actuator insert fastens to the interior triangular space on each basal leg. The difficulty in mounting the actuators led to design of the component shown in the detailed drawings in Appendix 3. They are optimized for function and manufacturability, not stress distribution. In fact, the requirements for fastening and shaft mounting were such that the part needed to be highly overdesigned in terms of stress. Functionally, the main purpose of this component is to rigidly locate the shaft connected to the pushrod according to the input arm dimensions, as shown in Fig. 4.16. Additionally, the interference between the pushrod and the actuator insert as the leg approaches its angular limits must be eliminated. This has been accounted for by using another clevis-type joint with relatively deep channels (see Appendix 3). The clevis-type joint at the actuator insert/pushrod interface is also to used to provide a double-shear connection to limit the stress occurring in the connecting shaft. Finally, the actuator insert is designed to be easily assembled and disassembled from the basal leg to allow for maintenance or replacement of the leg and midjoint assemblies by fastening to the sides of each leg with countersunk screws.

Similar to the legs, the insert is designed to be constructed from a 6061-series aluminum alloy for its machinability since the stress in this component is not critical. However, in designing the insert to be easily manufactured using available tooling, a critical stress area develops in the revolute shaft connecting the pushrod. Because the pushrod end is narrower than the channel in the insert, the shaft is loaded much like a beam. Figure 4.17
Fig. 4.16: Kinematic link superimposed on actuator insert arm/basal leg assembly.

shows the configuration of the insert, shaft, and pushrod with its associated beam model. Like the midjoint shaft, this model was analyzed using BEAM VI, and the resulting maximum static stress due to bending is approximately 193 MPa (28 kpsi). In order to maintain a factor of safety against static failure above two, this relatively high stress dictates the use of either a medium strength stainless steel (S41400) or the equivalent of cold-drawn AISI 1050. Either of these will supply a factor of safety over three.

4.5.3.2 Actuator Pushrods
The pushrods are a simple component; they consist of a cylindrical rod with two male clevis fittings attached to both ends. Once again, a detail of the final design is shown in Appendix 3. Due to the nature of the actuation, these rods must be designed according to
the following requirements: they must be long enough to sufficiently remove the actuators from the base of the wrist, and they must withstand the tensile and compressive forces to which they are subjected. They should be designed for minimum weight once the other requirements are satisfied.
The pushrods are naturally chosen as cylindrical rods since the circular cross-section is most effective against buckling failure. Assuming the rods are to be made from aluminum, a buckling failure analysis using the Euler equation (Shigley and Mitchell, 1993) can be used to determine the permissible length. The Euler equation is given as follows:

$$\frac{P_{cr}}{A} = \frac{C\pi^2 E}{(l/k)^2}$$  \hspace{1cm} (4.6)

where

- $P_{cr}$ = the critical buckling load
- $A$ = the cross-sectional area of the column
- $C$ = an end-condition factor (in this case, $C = 1$)
- $l$ = the column length
- $k$ = the radius of gyration, and
- $E$ = the modulus of elasticity of the column material.

This equation must be rearranged since the only free choices are the geometric variables. The diameter is dictated to be 12.7 mm (0.5 in.) by the other components, so the free variables remaining are the inner diameter (for a hollow cylinder) and the length. The maximum design load has already been calculated to be 600 MPa (135 lbs). Including a modest factor of safety, the critical buckling load shall be 1200 MPa (270 lbs). Since aluminum has been chosen, the modulus of elasticity is known. By expanding the geometric quantities and rearranging the equation a more suitable form of the Euler equation is obtained:

$$d_i = \sqrt[4]{d_o - \frac{64P_{cr}l^2}{C\pi^2 E}}$$  \hspace{1cm} (4.7)

This equation can be used to develop a short table (Table 4.5) of maximum permissible length vs. inner diameters that is helpful for choosing the correct tubing size or column
length. Although the table shows that lengths above 0.75 m (29.5 in.) are permissible, the prototype uses a pushrod of approximately 0.30 m (12 in.). Thus, any standard size tubing with an inner diameter less than 11.5 mm (0.450 in.) in is sufficient.

Table 4.5: Maximum pushrod length vs. inner diameter for a 12.7 mm (1/2 in.) circular pushrod.

<table>
<thead>
<tr>
<th>Maximum permissible length m (in.)</th>
<th>Inner diameter, (d_i) mm (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25 (9.8)</td>
<td>12.4 (0.488)</td>
</tr>
<tr>
<td>0.50 (19.6)</td>
<td>11.5 (0.453)</td>
</tr>
<tr>
<td>0.75 (29.5)</td>
<td>8.9 (0.350)</td>
</tr>
<tr>
<td>1.00 (39.4)</td>
<td>failure imminent</td>
</tr>
</tbody>
</table>

4.5.3.3 Driving Links

For the prototype Carpal wrist, the input link of the four-bar mechanism that actuates the legs is referred to as the driving link. The design of this link is straightforward and is shown in Fig. 4.18. The connection to the pushrod end is a double-shear clevis joint similar to the pushrod connection to the actuator insert. The odd, bent shape of the link is

![Diagram of clevis joint and pushrod](image)

Fig. 4.18: Driving link used as the four-bar linkage input link.
necessary to prevent interference with the pushrod when the driving link rotates to its extreme position.

The only stress-wise critical location on this component is the attachment to the driving shaft where the input torque from the motor/gearhead must be transmitted. For this attachment, set screws are used. To ensure the selected set screw is sufficient to transfer the input torque, an analysis of the torsional holding power of suitably sized screws is presented. The torsional holding power is calculated according to Mark's Standard Handbook for Mechanical Engineers (1987) as:

\[ T_{HP} = \frac{1}{2} A_{HP} d_{shaft} \]  

(4.8)

where

\[ A_{HP} = \]  

the axial holding power of the screw, and

\[ d_{shaft} = \]  

the diameter of the shaft.

The required torsional holding power is equal to the maximum input torque calculated earlier. The diameter of the shaft is 9.5 mm (3/8 in.). Thus, the axial holding power must be 2300 N (516 lbs). From a list of the axial holding power of various sized set screws included in the handbook, two choices are available that are capable of providing a margin of safety of two on the maximum input torque: two size No. 10 set screws which can each hold 2400 N (540 lbs) or one 1/4" size set screw capable of holding up to 4450 N (1000 lbs). An analysis of the bearing stresses in the driving link produced by each selection shows the choice of two No. 10 screws to be superior by providing a greater safety factor against this type of failure.
Chapter 5

Conclusions and Recommendations

The Carpal wrist, a newly developed parallel-actuated robotic wrist, has been presented as a possible alternative to serial robotic wrists. While serial wrists are more commonly used compared to their parallel counterparts, they do have accepted limitations. These limitations include the presence of singular positions within the workspace, cantilevered designs that lead to a low strength-to-weight ratio, and complex power transmission elements. Their wide acceptance stems from the benefits they possess, which include large orientational workspaces, designs which allow the actuators to be placed towards the base of the manipulator, and the existence of closed-form kinematic solutions for all of the typical serial configurations.

The Carpal wrist, in contrast, is a parallel-actuated wrist. It may provide solutions to some of the problems inherent in existing serial wrists without comprising their positive attributes. The Carpal wrist can function as a two-degree of freedom spherical pointing device, a three degree-of-freedom device with an additional translational motion, or, by including an additional actuator, as a four degree-of-freedom wrist capable of general orientational capabilities. Its parallel-structure gives it a better strength-to-weight ratio than its serial counterparts, while its novel actuation method allows the actuators to be
located closer to the base of the manipulator. It also has a larger, more dexterous workspace than other proposed parallel wrists.

When confronted with the problem of designing such a wrist, several issues must be considered that relate to the kinematic design and the physical design of the mechanism. As an aid to this design process, a set of tools has been developed. This set includes a kinematic simulation package useful for visualization during the early stages of design and a static force analysis package useful for the screening of designs based upon failure analyses and input force requirements. The design process itself is highly iterative. A change to any one element of the design may require reconsideration of all associated elements. Without dwelling upon the numerous iterations, the essence of the steps taken during the design process have been addressed in this thesis.

The wrist is synthesized by selecting two kinematic ratios, the base-to-leg ratio and plunge ratio, to develop a mechanism capable of providing a specified workspace. Clearances and interference points resulting from physical dimensions must be accounted for to efficiently complete the kinematic design process. Once a suitable kinematic design is selected, the actuation scheme and design of the individual components must be considered. Several alternatives for actuating the Carpal wrist have been presented: rotary actuation schemes using cable drives or geared transmissions, linear actuation systems, and a linkage-type actuation scheme. The most important considerations arising during the selection of a specific actuation scheme were discussed with the intent being to aid the reader in selecting the best design for a specific application. With an actuation scheme chosen, the structural components, i.e., the basal and distal plate, the legs, and the midjoints along with their associated joining elements, are designed. Careful consideration of the kinematic and force requirements for each of these components must be taken into account while also solving the difficult problem of eliminating interference between these elements once they are assembled and functioning within the wrist.
Based on the discussion of the general considerations necessary for the design of a Carpal wrist, a prototype model, shown in Fig. 5.1, was fabricated. This will serve as a proof-of-concept device that can be used for testing and evaluation. Referring back to Fig. 4.1, this testing and evaluation is expected to lead to further refinement of the design, both in the context of general design requirements and specific design details.
The model was created to demonstrate the kinematic capability and control of the Carpal wrist as a two degree-of-freedom orientational device. For this reason, the model Carpal wrist has a spherical workspace capable of locating its distal plate anywhere within a 180° hemisphere. The wrist is actuated by a set of three four-bar linkages being driven distantly from the main structure of the wrist. While not fully developed, positional control of the wrist is provided by a simple open-loop control system driving three stepper motors through a PC-interface. The prototype is also expected to demonstrate the load carrying capability of the Carpal wrist. Using the tools developed for the static force analysis, the prototype was designed to safely carry a 90 N (20 lb) payload. Although the chosen size of the prototype had some effect upon the maximum payload, it was limited most significantly by a few key elements. One limiting factor was found to be need for relatively high input torques necessary to drive the basal legs. This caused the need for an extremely high reduction. Another weak point arises from the revolute joint bearings in the wrist. The small size of the prototype dictated the use of simple bronze bushing-type bearings which have limited strength compared to the forces developed from supporting the payload. This is most evident in the bearings for the midjoint, which maintain a factor of safety against static failure slightly less than two. The final weak point was found to be the bending stress that develops in the midjoint assemblies.

As Fig. 4.1 suggests, the design process never completely ends. The role of the engineer is to seek continual improvement over past designs. In this light, it is easy to see that future work should include testing and evaluation of the prototype in an ongoing effort to refine and improve the design. Specifically, the key weak points mentioned earlier should be addressed. It was seen that the size of the prototype led to numerous difficulties encountered designing the actuation scheme and when selecting bearings for the revolute joints. For this reason, limits upon the size and payload capabilities of future wrists can be determined to ensure reliable, useful designs. Additionally, improved design of the midjoint is necessary. Once again, the overall size of the prototype dictated

Conclusions and Recommendations
that certain compromises be made in the construction of this component. An improved
design, either by modifying the existing three revolute joint chain or by developing a
completely new spheric equivalent, may increase the structural strength of the Carpal
wrist.

There are numerous avenues to explore for continuing advancement of the Carpal wrist
concept other than simply refining the design presented herein. The wrist has been
described as singularity free. Work is underway to analyze the nature of the singularities
that exist at the boundaries of the workspace of the Carpal wrist. An effective means of
pursuing this result is the development of the Jacobian matrix for this manipulator. While
aiding in the singularity analysis, the Jacobian serves the multiple purpose of solving the
velocity and force-control problems. Finally, for future theoretical work, an analysis of
the dynamic characteristics for the Carpal wrist is needed to better understand the
velocity and acceleration that occur in the various components and, ultimately, to either
support or contradict the assumption made in this thesis that link masses truly are
negligible in the force analysis.

Another useful endeavor may be to explore optimization of the kinematic parameters to
provide the desired motion while improving the force transmission characteristics. In this
work, a trial-and-error approach was taken towards finding the best combination of
parameters. In future work, an optimization algorithm can be used to select designs based
upon a cost function that may include maximizing the interior angle, limiting the required
input torque, minimizing the basal leg angular range, etc.

There are many other design issues that can be pursued in order to push the development
of the Carpal wrist from the prototype stage to a truly industrial robotic device. The next
step in this developmental process is to use the Carpal wrist as it is intended -- an
orienting device attached to a base manipulator. For this, a means to retrofit the

Conclusions and Recommendations
prototype on the positional structure of an existing manipulator should be pursued. The control scheme may also be refined to include more sophisticated velocity control so that a certain level of path-planning may be effectively utilized.

In progressing from a theoretical kinematic model to an ideal design and on to the actual fabrication, numerous sources of positional inaccuracies accumulate. The design drawings in Appendix 3 show one major source of these errors, the numerous critical machining tolerances. It is necessary to know if the effect of these expected errors produces acceptable positional inaccuracies in the kinematic control of the Carpal wrist. A sensitivity analysis can be performed to determine the effect that the accumulation of errors has upon the positional accuracy of the Carpal wrist.

The overall design may be advanced by pursuing new designs which include the fourth degree-of-freedom that provides a final roll axis. While it may be difficult to locate components useful for such a task due to strict operating requirements, its inclusion would expand the usefulness of the Carpal wrist to any application currently in use for general six degree-of-freedom robotic manipulators.

The possibilities for future work on and improvement to the Carpal wrist are endless. While some work can focus on improvement of the details for one component, other work may find a radically different and possibly better solution to the overall design. Designs should evolve to reflect the new information gathered and learned; thus, this chapter does not “close the book” on the design of the Carpal wrist. A series of tools, suggestions, and guiding principles have been presented that represent the initial designer’s experience. It is hoped that the current prototype will find acceptance and that future designs will be directed toward industrial applications and adoption of the Carpal wrist.
References


20. Mitchell, L. D., 1992, *User's Guide -- BEAM VI, Ed. 6.1*, AMDF Publication 08.05.92, Department of Mechanical Engineering, Virginia Polytechnic Institute & State University, Blacksburg, VA.


Appendix 1: Listing of C++ Code

The following appendix contains a listing of the code developed to be used as a tool for the design of a Carpal wrist. It is written in C++ and compiled using the Microsoft Visual C++ v1.5 compiler. It represents a small subset of the complete set of files needed for the package described in the main text. For the sake of brevity, it only includes the C++ classes and function descriptions relevant to the analyses developed in the text of Chapter 2. It excludes all miscellaneous code related to the Microsoft Windows-based graphical user-interface.
// WRIST.H

// Wrist header file.
//
// Classes:
//
//   Vector
//   AxisRotation
//   Wrist
//
// Global Functions:
//   UnitCross
//   UnitSub
//   GaussElim
//   ForceCalc

#include <math.h>
#include <errno.h>

#define N 3
#define PI 3.141592654
#define DTR (3.141592654/180.0)
#define RTD (180.0/3.141592654)

class Vector
{
public:

double v[N];       // v[3] is the vector variable x,y,z
double magv;       // magv is the magnitude of v[3]
Vector();          // constructor
Vector(double x, double y, double z);    // overloaded constructor

Vector operator+(Vector & arg);          // vector addition
Vector operator-(Vector & arg);          // vector subtraction (vector - arg)
double operator%(Vector & arg);          // vector dotproduct (vector*dot*arg)
Vector operator*(Vector & arg);          // vector crossproduct
Vector operator*(double arg);            // vector scalar multiplication

    // inline functions
    double x() { return v[0]; }
    double y() { return v[1]; }
    double z() { return v[2]; }
    double Mag() { return (magv = sqrt(v[0]*v[0]+v[1]*v[1]+v[2]*v[2])); }   // calculate and return magnitude

    void SetV(double x, double y, double z);    // sets the v[3] vector variable with x, y, and z
};

Vector UnitCross(Vector & a, Vector & b);    // performs a (cross) b and divides by resulting magnitude
Vector UnitSub(Vector & a, Vector & b);     // performs a-b and divides by resulting magnitude

class AxisRotation
{
public:

double R[N][N];       // R[3][3] is a marix used for rotation operations
AxisRotation();       // constructor
void Rotate(Vector & a, double theta);     // fills R with elements created by theta rotation about a
Vector operator*(Vector & arg);           // matrix multiplication.
};

class Transform
{
public:

double T[4][4];        // T[4][4] is a general transform matrix
void operator=(Transform & arg); // overload the = operator
Vector operator*(Vector & arg);  // overload the * operator to perform matrix mult.
Transform operator*(Transform & arg);  // overload the * operator to perform matrix mult.

Appendix 1
void SetTransform(Vector& a, Vector& b, Vector& c, Vector& p); // fills T with 4 vector columns
void SetTransform(double, double, double, double, double, double); // fills T with a, b, c rotations and x, y, z location

class Wrist
{
public:
    double psi; // inclination of leg triangle
    double theta[2*N]; // basal and distal leg rotations
    double alpha[N]; // interiots angles
    double Pd; // plunge distance
    double base; // base lengths
    double leg; // leg length
    double beta, gamma; // inverse kinematic inputs -- gamma is the bend of the wrist, beta locates the bend axis
    double Am, Bn, Cn, Dm, Ac[3], Bc[3], Cc[3], Dc[3]; // midplane and leg plane-coefficients
    Vector b[N], d[N]; // b = base revolves, d = distal revolves
    Vector u[2*N], q[N], n[3*N], MidNorma, Cw; // u = revolute axis, q = perp. to rev., n = nodes, Cw = center point
    AxisRotation Ru[N]; // intermediate axis rotations for leg revolves
    Transform bT; // overall transform for wrist from base to tool

    Wrist(); // constructor
    void operator=(Wrist& mod); // overloaded =
    void SetL(double a1, double a2); // set base and leg length and associated base vectors
    void SetBaseRotations(double a[]); // sets the input leg angles
    void MidDirections(); // calculates the midplane coefficients
    void TopPhiCalc(Vector& P); // calculates distal leg rotations
    void CalcDistal(); // calculates the x, y, z axes and bT of distal frame
    void CalcAlphas(); // calculates the interior angles
    double AnglePsi(double d1, double d2); // calculates angle psi (angle of triangular leg link)
    double Distance(int ni, int nj); // distance between two nodes
    void Node(int node); // calculates the midnode vector from given leg rotations
    int ForwardSolve(double batters[]); // solves forward kinematics given three input leg rotations
    int InverseSolve(double beta, double gamma, double pd); // solve inverse kinematics given beta, gamma, pd
};

void GaussElim(int size, double coeff[10][10], double knowns[10], double result[10]); // gaussian elimination linear equation solver for up to 10 equations in 10 unknowns
void ForceCalc(); // calculates generalized midplane forces given a distal frame general force

// WRIST.CPP

#include "wrist.h"

// Vector class functions

// Default constructor
Vector::Vector()
{
    for (int i=0; i<3; i++)
        v[i] = 0.0;
        magv = 0.0;
}

// Overloaded constructor
Vector::Vector(double x, double y, double z)
{
    v[0] = x;
    v[1] = y;
    v[2] = z;
}

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void Vector::SetV(double x, double y, double z)
{
    v[0] = x;
    v[1] = y;
    v[2] = z;
}

// Define vector addition
Vector Vector::operator+(Vector& arg) {
    return Vector(v[0]+arg.x[0],v[1]+arg.y[0],v[2]+arg.z[0]);
}

// Define vector subtraction
Vector Vector::operator-(Vector& arg) {
    return Vector(v[0]-arg.x[0],v[1]-arg.y[0],v[2]-arg.z[0]);
}

// Define vector cross product
Vector Vector::operator*(Vector& arg) {
    double a,b,c;
    a = v[1]*arg.z[0] - v[2]*arg.y[0];
    b = v[2]*arg.x[0] - v[0]*arg.z[0];
    c = v[0]*arg.y[0] - v[1]*arg.x[0];
    return Vector(a,b,c);
}

// Define vector * a scalar
// Usage: Vector * k
Vector Vector::operator*(double k) {
    return Vector((v[0]*k,v[1]*k,v[2]*k);
}

// Define vector dot product
double Vector::operator%(Vector& arg) {
    return (arg.x[0]*v[0] + arg.y[0]*v[1] + arg.z[0]*v[2]);
}

// Define a unit cross product -- take cross product and divide by magnitude
Vector UnitCross(Vector& a, Vector& b)
{
    Vector temp;
    temp = a*b;
    return (temp * (1.0/temp.Mag()));
}

// Define a unit subtraction -- subtract and divide by magnitude
Vector UnitSub(Vector& a, Vector& b)
{
    Vector temp;
    temp = a - b;
    return (temp * (1.0/temp.Mag()));
}

// AxisRotation class functions

// Default constructor
AxisRotation::AxisRotation()
{
    // Initialize matrix to identity
    for (int r=0; r<N; r++)
        for (int c=0; c<N; c++)
R[r][c] = (r==c) ? 1.0 : 0.0;

// Overloaded constructor -- fill matrix with values resulting from theta rotation about a
// void AxisRotation::Rotate(Vector& a, double theta)
{ double v;
  v = 1 - cos(theta);
  R[0][0] = a.x() * a.x() * v + cos(theta);
  R[1][0] = a.x() * a.y() * v + a.z() * sin(theta);
  R[2][0] = a.x() * a.z() * v - a.y() * sin(theta);
  R[0][1] = a.x() * a.y() * v - a.z() * sin(theta);
  R[1][1] = a.y() * a.y() * v + cos(theta);
  R[2][1] = a.y() * a.z() * v + a.x() * sin(theta);
  R[0][2] = a.x() * a.z() * v + a.y() * sin(theta);
  R[1][2] = a.y() * a.z() * v - a.x() * sin(theta);
  R[2][2] = a.z() * a.z() * v + cos(theta);
}

// Define operation: matrix * Vector
Vector AxisRotation::operator*(Vector& arg)
{
  Vector temp;
  for (int i=0; i<3; i++)
  { temp.v[i] = 0.0;
    for (int j=0; j<3; j++)
      temp.v[i] += R[i][j] * arg.v[j];
  }
  return temp;
}

void Transform::operator=(Transform& arg)
{
  for (int i=0; i<4; i++)
  { for (int j=0; j<4; j++)
      T[i][j] = arg.T[i][j];
  }
}

// Define operation: matrix * matrix
Transform Transform::operator*(Transform& arg)
{
  int i,j,k;
  Transform result;
  for (i=0; i<4; i++)
  { for (j=0; j<4; j++)
      result.T[i][j] = 0.0;
    for (k=0; k<4; k++)
      result.T[i][j] += T[i][k] * arg.T[k][j];
  }
  return result;
}

// Define operation: matrix * Vector
Vector Transform::operator*(Vector& arg)
{
  double augx=aug[4],result[4];

Appendix 1
int i,j;

for (i=0; i<3; i++)
    augvector[i] = arg.v[i];

augvector[3] = 1;

for (i=0; i<4; i++)
{
    result[i] = 0;
    for (j=0; j<4; j++)
        result[i] += T[i][j] * augvector[j];
}

Vector temp(result[0], result[1], result[2]);
return temp;

} //fill the matrix with the input vectors placed in the columns of the matrix
void Transform::SetTransform(Vector& a, Vector& b, Vector& c, Vector& p)
{
    for (int i=0; i<3; i++)
    {
        T[i][0] = (fabs(a.v[i])>0.000005) ? a.v[i] : 0.0;
        T[i][1] = (fabs(b.v[i])>0.000005) ? b.v[i] : 0.0;
        T[i][2] = (fabs(c.v[i])>0.000005) ? c.v[i] : 0.0;
        T[i][3] = (fabs(p.v[i])>0.000001) ? p.v[i] : 0.0;
    }

    T[3][0]=T[3][1]=T[3][2]=0.0;
    T[3][3]=1.0;
}

} //fill the matrix with the results of an euler rotation of Rx(gamma), Ry(beta), Rz(alpha), position (x, y, z)
void Transform::SetTransform(double alpha, double beta, double gamma, double x, double y, double z) {

    // alpha = zrotation
    // beta = yrotation
    // gamma = xrotation
    T[0][0] = cos(alpha)*cos(beta);
    T[1][0] = sin(alpha)*cos(beta);
    T[2][0] = -sin(beta);
    T[3][0] = 0.0;
    T[0][1] = cos(alpha)*sin(beta)*sin(gamma)-sin(alpha)*cos(gamma);
    T[1][1] = sin(alpha)*sin(beta)*sin(gamma)+cos(alpha)*cos(gamma);
    T[2][1] = cos(beta)*sin(gamma);
    T[3][1] = 0.0;
    T[0][2] = cos(alpha)*sin(beta)*cos(gamma)+sin(alpha)*sin(gamma);
    T[1][2] = sin(alpha)*sin(beta)*cos(gamma)-cos(alpha)*cos(gamma);
    T[2][2] = cos(beta)*cos(gamma);
    T[3][2] = 0.0;
    T[0][3] = x;
    T[1][3] = y;
    T[2][3] = z;
    T[3][3] = 1.0;
}

} //Wrist class functions

//default constructor
Wrist::Wrist()
int i;
beta=gamma=0;
for (i=0; i<9; i++)
    n[i].SetV(0.0,0.0,0.0);
for (i=0; i<9; i++)
{
    b[i].SetV(0.0,0.0);
    d[i].SetV(0.0,0.0);
    theta[i] = theta[i+3] = 2.0*pi/3.0;
}
Ac[0] = 0;
Ac[1] = -0.5;
Ac[2] = -0.5;
Bc[0] = 1;
Bc[1] = 0.866;
Bc[2] = -0.866;

Ac[0]=0;
Ac[1]=-0.866;
Ac[2]=-0.866;
Bc[0]=1.0;
Bc[1]=-0.5;
Bc[2]=-0.5;

SetL(1.0,2.0,0);

// set the base and leg length and develop the associated basal vectors in their equilateral arrangement
void Wrist::SetL(double a1, double a2)
{
    base = a1;
    leg = a2;
    double tempb = sqrt(3.0)*base/2.0;

    b[0].SetV(base,0.0,0);
    b[1].SetV(-0.5*base,tempb,0);
    b[2].SetV(-0.5*base,-tempb,0);

    n[0].SetV(base,-2.0*tempb,0);
    n[1].SetV(base,2.0*tempb,0);
    n[2].SetV(-2*base,0.0,0.0);

    u[0] = UnitSub(n[1],n[0]);
    u[1] = UnitSub(n[2],n[1]);
    u[2] = UnitSub(n[0],n[2]);

    q[0] = UnitCross(u[0],u[1])*u[0];
    q[1] = UnitCross(u[0],u[1])*u[1];
    q[2] = UnitCross(u[0],u[1])*u[2];

    psi = AnglePsi(2.0*tempb,leg);
}

void Wrist::SetBaseRotations(double a[])
{
    for (int i=0; i<3; i++)
    {
        theta[i] = a[i];
    }
}

void Wrist::operator=(Wrist& mod)
{
    SetL(mod.base,mod.leg);
    psi = mod.psi;
    for (int i=0; i<9; i++)
    {
        u[i] = mod.u[i];
    }
}

Appendix 1
q[i] = mod.q[i];
theta[i] = mod.theta[i];

for (i=0; i<3; i++)
{
      b[i] = mod.b[i];
      d[i] = mod.d[i];
}
for (i=0; i<9; i++)
{   q[i] = mod.n[i];
}
for (i=0; i<4; i++)
    for (int j=0; j<4; j++)
        bT[i][j][j] = mod.bT[i][j][j];

//develop midplane normal and its plane coefficients
void Wrist::MidDirections()
{
    u[3] = UnitSub(n[4],n[3]);
    u[4] = UnitSub(n[5],n[4]);
    u[5] = UnitSub(n[3],n[5]);

    MidNormal = UnitCross(u[3],u[4]);
    if (MidNormal.z() < 0.0)      //make sure we use correct reflection of solution
        MidNormal = MidNormal * -1.0;
    Dm = -1.0*(MidNormal*n[3]);
}

//set distal leg rotations equal to basal leg rotations
void Wrist::TopPhiCalc(Vector& P)
{
    theta[3] = theta[0];
    theta[4] = theta[1];
    theta[5] = theta[2];
}

//calculate the interior angles
void Wrist::CalcAlphas()
{
    Vector b1,b2,t1,t2,ui,vi;
    double temp;

    b1 = UnitSub(n[3],n[0]);  //used to find ui
    b2 = UnitSub(n[3],n[1]);  //used to find vi
    t1 = UnitSub(n[3],n[6]);  //used to find vi
    t2 = UnitSub(n[3],n[7]);

    ui = b1 + b2;  ui = ui * 0.5;  //ui is the vector from the basal leg revolute to the midjoint
    vi = t1 + t2;  vi = vi * 0.5;  //vi is the vector from the distal leg revolute to the midjoint
    b1 = ui * vi;  //vector cross-product
    temp = b1 % u[0];  //vector dot product with orientation of revolute 1

    if (temp >= 0)
        //the following checks to see if the angle is greater or less than 180 deg.
        alpha[0] = acos((ui*vi)/(ui.Mag()*vi.Mag()));
    else if (temp < 0)
        alpha[0] = 2*PI - acos((ui*vi)/(ui.Mag()*vi.Mag()));

    b1 = UnitSub(n[4],n[1]);  //repeat above procedure for second set of legs
    b2 = UnitSub(n[4],n[2]);
    t1 = UnitSub(n[4],n[7]);
    t2 = UnitSub(n[4],n[8]);

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ui = b1 + b2; ui = ui * 0.5;
v1 = t1 + t2; vi = vi * 0.5;
b1 = ui * vi;
temp = b1 % u[1];
if (temp >= 0)
else if (temp < 0)

b1 = UnitSub(n[5],n[2]); // repeat above procedure for third set of legs
b2 = UnitSub(n[5],n[0]);
t1 = UnitSub(n[5],n[8]);
t2 = UnitSub(n[5],n[6]);
ui = b1 + b2; ui = ui * 0.5;
v1 = t1 + t2; vi = vi * 0.5;
b1 = ui * vi;
temp = b1 % u[2];
if (temp >= 0)
else if (temp < 0)

// calculate the midpoint nodal vector given the basal leg rotation
void Wrist::Node(int node) {
    Vector temp;
    switch (node+1) {
        case 4: {
            Ru[0].Rotate(u[0],theta[0]); // setup rotation matrix of basal leg, theta about u[i]
            n[3] = Ru[0]*q[0]; // Matrix * a Vector
            n[3] = n[3]*leg; // Vector *= a scalar
            n[3] = n[3] + b[0];
            break;
        }
        case 5: {
            Ru[1].Rotate(u[1],theta[1]); // setup rotation matrix of basal leg, theta about u[i]
            n[4] = Ru[1]*q[1]; // Matrix * a Vector
            n[4] = n[4]*leg; // Vector *= a scalar
            break;
        }
        case 6: {
            Ru[2].Rotate(u[2],theta[2]); // setup rotation matrix of basal leg, theta about u[i]
            n[5] = n[5]*leg; // Vector *= a scalar
            break;
        }
    }
}

double Wrist::AnglePsi(double d1, double d2) {
    return atan(d2/d1);
}

double Wrist::Distance(int a, int b) {
    return (sqrt((n[a].x()-n[b].x())*(n[a].x()-n[b].x())

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+(n[a].y()-n[b].y())*(n[a].y()-n[b].y())
+(n[a].z()-n[b].z())*(n[a].z()-n[b].z()));

//solve forward kinematics
int Wrist::ForwardSolve(double thetas[])
{
    double di;
    Vector temp1;
    SetBaseRotations(thetas);
    Node(3);
    Node(4);
    Node(5);
    MidDirections();

    // calculate location of distal nodes
    for (int i=0; i<3; i++)
    {
        di = -1.0*(MidNormal.x()*(temp1.x())+MidNormal.y()*(temp1.y())+MidNormal.z()*(temp1.z())+Dm)/MidNormal.Mag();
        temp1 = MidNormal*(2.0*di);
        d[i] = temp1 + b[i];
    }
    CalcDistal();
    CalcAlphas();

    return 1;
}

// calculate parameters of distal frame (x,y,z axes and point location) and fill transform bTt
void Wrist::CalcDistal()
{
    Vector Pt, Xt, Yt, Zt, temp1, temp2;
    double zeta;
    Pt = d[0] + d[1];
    Pt = Pt + d[2];
    Pt = Pt*(1.0/3.0);
    temp1 = d[1] - d[0];
    temp2 = d[2] - d[1];
    Zt = UnitCross(temp1,temp2);
    Xt = UnitCross(Zt,temp2);
    Yt = Zt * Xt;
    bTt.SetTransform(Xt, Yt, Zt, Pt);

    //calculate beta & gamma in case Forward Solve() called this function
    //this is redundant if InverseSolve() is used
    beta = atan2(-Xt.z(),Yt.z());
    gamma= acos(Zt.x());
    TopPhiCalc(Pt);

    //calculate plunge distance in case Forward Solve() called this function
    //this is redundant if InverseSolve() is used
    zeta = Pi-2*aocos(MidNormal.z());
    Pd = sqrt((Pt%Pt)*(2*(1-cos(zeta))));
    Cw.SetV(0.0,Pd);
    temp1 = Yt * (-1.0 * base * sqrt(3.0));

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n[6] = d[0] + temp1;
temp1 = temp1 * (-1.0);
n[7] = d[0] + temp1;
temp2 = Xt * (-2.0 * base);
n[8] = Pt + temp1;
}

// solve inverse kinematics
int Wrist: InverseSolve(double axrot, double bendrot, double pd)
{
    double R, S, T, U, A, B, C, RAD, x1, x2, mx;
    Vector Nm, temp1, temp2, p[3], ds;
    Wrist oldwrist;
    oldwrist = *this;

    // beta is rotation of bend axis from x-axis in ground frame
    // gamma is rotation about bend axis
    // pd is plunge distance (radius of sphere)
    gamma = bendrot;
    beta = axrot;

    double cB = cos(beta);
    double vG = 1-cos(gamma);
    double sB = sin(beta);
    double cG = cos(gamma);
    double sG = sin(gamma);

    temp1.SetV(0, 0, 0, pd);
    for (int i = 0; i < 3; i++)
    {
        ds.v[0] = (cB*cB*vG+cG)*b[i].x() + (cB*sB*vG)*b[i].y() + (sB*sG)*pd;
        ds.v[1] = (cB*sB*vG)*b[i].x() + (sB*cB*vG+cG)*b[i].y() + (cB*sG)*pd;
        ds.v[2] = (-sB*cG)*b[i].x() + (cB*sG)*b[i].y() + (cG)*pd;
        d[i] = ds + temp1;
        p[i] = d[i] + b[i];
        t[i] = p[i]*0.5;
    }

    temp2 = p[2]-p[0];
    temp2 = p[0]-p[1];
    Nm = temp1*temp2;
    Am = Nm.x();
    Bm = Nm.y();
    Cm = Nm.z();
    Dm = -1*Nm%p[0];

    for (i = 0; i < 3; i++)
    {
        R = (Ac[i]*Cm-Cc[i]*Am)/(Cc[i]*Bm-Bc[i]*Cm);
        S = (Dc[i]*Cm-Cc[i]*Dm)/(Cc[i]*Bm-Bc[i]*Cm);
        T = (Bc[i]*Am-Ac[i]*Bm)/Cc[i]*Bm-Bc[i]*Cm);
        U = (Bc[i]*Dm-Dc[i]*Bm)/Cc[i]*Bm-Bc[i]*Cm);

        A = RA + T*T + 1.0;
        C = S*S + U*U - 2*S*T*b[i].y() - 2*U*b[i].z() + base*base - leg*leg;

        RAD = B*B - 4*A*C;

        if (RAD < 0)
        {
            *this = oldwrist;
            return 0;
        }
    }

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else if (RAD == 0.0)
{
    mx = 1+(sqrt(RAD))/(2*A);
    n[i+3].SetV(mx, R*mx+S, T*mx+U);
    if (n[i+3].v[2] < 0.0)
        n[i+3].v[2] *= -1.0; //make sure niz > 0
}  
else
{
    x1 = (-B-sqrt(RAD))/(2*A);
    x2 = (-B-sqrt(RAD))/(2*A);

    switch (i) {

    case 0:
        mx = x1>x2 ? x1 : x2; //leg1:_max(x1,x2);
        break;
    case 1: //fail through
    case 2:
        mx = x1<x2 ? x1 : x2; //legs 2&3:_min(x1,x2);
        break;

    }

    n[i+3].SetV(mx,R*mx+S, T*mx+U);
}

    temp1 = b[i]*(-1.0);
    temp2 = n[i+3] + temp1;
    theta[i] = acos(temp1|temp2/(temp1.Mag()*temp2.Mag()));
    temp1 = temp1*temp2;
    if ((temp1|temp2)<0.0) theta[i] = 2*PI - theta[i];

MidDirections();
CalcDistal();
CalcAlphas();

return 1;


//general gaussian elimination linear equation solver for up to 10 equations in 10 unknowns
void GaussElim(int size, double coeff[10][10], double knowns[10], double result[10])
{
    double aug[10][11],temp[10],mult;
    int i,j,k;
    int singularflag = 1;

    for (i=0; i<size; i++) {
        for (j=0; j<size; j++) {
            aug[i][j] = coeff[i][j];
        }
        aug[i][size] = knowns[i];
    }

    for (i=0; i<size; i++) {
        if (aug[i][i] == 0.0) {
            for (j=i+1; j<size; j++) {
                if (aug[j][i] != 0.0) {
                    singularflag = 0;
                    for (k=0; k<(size+1); k++)
                    {
                        temp[k] = aug[i][k];
                        aug[i][k] = aug[j][k];
                        aug[j][k] = temp[k];
                    }
                }
            }
        }
    }
}
if (singularflag) AfxMessageBox("Singular Solution", MB_OK);  

for (i=j+1; i<size; j++)  
    mult = -aug[i][i]/aug[i][i];  
    for (k=i; k<size+1; k++)  
        aug[i][k] += mult * aug[i][k];  

//backsab routine  
result[size-1] = aug[size-1][size]/aug[size-1][size-1];  
for (j=size-2; j>0; j--)  
    result[j] = aug[j][size];  
    for (k=j+1; k<size; k++)  
        result[j] -= (-aug[j][k]*result[k]);  
    result[j] /= aug[j][j];  

//calculate the general midplane forces, input torques, interior angles and leg rotations from a given wrist, applied load, and //maximum workspace  
void ForceCalc(Wrist& wristmodule, float force[6], float solid_wkspce, float Gbbx[3][19][19], float Gby[3][19][19], float Gbx[3][19][19], float Gdy[3][19][19], Torq[3][19][19], alpha[3][19][19], thetas[3][19][19])  
{  
    Transform Rt[3], Rb[3], RBT;  
    Vector t[6], r[6], tT[6], tB[6], bB[6], mid[3], FT, MT, tempvec;  
    int i,j,k,xcounter, ycounter;  
    double temp1,temp2,beta,gaamma,plunge,temp[10];  
    double lowthetas[3],gaussceef[10][10],gausskknowns[10],FT[6],Fmid[3],Fb[6],torque[3];  
    wristmodule.ForwardSolve(lowthetas);  

    /define unit vectors pointing in directions of legs in the corresponding "leg" frame  
    temp1 = cos(wristmodule.psi);  
    temp2 = sin(wristmodule.psi);  
    for (i = 0; i<3; i++)  
        {  
            t[2*i].SetV(temp1,temp2,0);  
            t[2*i+1].SetV(-temp1,temp2,0);  
        }  

    //define r(base and top) and unit vector pointing to base and top nodes  
    temp1 = wristmodule.base * sqrt(3.0);  
    temp2 = wristmodule.base;  
    r[0].SetV(temp2,temp1,0);  
    r[5] = r[0];  
    r[1].SetV(temp2,temp1,0);  
    r[2] = r[1];  
    r[3].SetV(-temp2,-temp1,0);  
    r[4] = r[3];  
    plunge = plungevar;  

    //define distal frame applied load  
    FT.v[0] = force[0];  
    FT.v[1] = force[1];  

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FT.v[2] = force[2];
MT.v[0] = force[3];
MT.v[1] = force[4];
MT.v[2] = force[5];

// cycle through entire workspace divided into a 19x19 grid developed on equally spaced angles
// for(xcounter = 0; xcounter < 19; xcounter++)
//
// gamma = (xcounter-9) * solid_wkspce * DTR/18.0;
// for (ycounter = 0; ycounter < 19; ycounter++)
{
    beta = (ycounter-9) * solid_wkspce * DTR/18.0;
    if ((wristmodule.InverseSolve(seta,gamma,plunge)) return;

    // Create a rotation matrix from the full transform of the wrist
    RBT = wristmodule.bT;
    RBT.T[0][3]=RBT.T[1][3]=RBT.T[2][3] = 0.0;

    // Define the leg rotation matrices for the base legs (Rb) and the top legs (Rt)
    for (i=0; i<3; i++)
    {
        temp1 = PI/2.0 + 2.0*PI*i/3.0; // intermediate z-rotation in derivation
        Rb[i,].SetTransform(temp1,0,wristmodule.theta[i],0.0,0.0);
        Rt[i,].SetTransform(temp1,0,-1.0*wristmodule.theta[i],0.0,0.0);
    }

    // Define tT -> legvectors rotated to top frame
    for (i = 0; i<6; i++)
    {
        rT[i] = Rb[i,]*l2[1];
        rT[2+i] = Rb[i,]*l2[1+i];
    }

    // Define the coefficient matrix for solving Ft's
    for (i=0; i<6; i++)
    {
        tempvec = rT[i] * tT[i]; // crossproduct
        for (j=0; j<3; j++)
        {
            gausscoeff[i][j] = rT[i].v[j];
            gausscoeff[i+3][j] = tempvec.v[j];
        }
    }

    // Define knowns for solving Ft's
    for (i=0; i<3; i++)
    {
        gaussknowns[i] = -FT.v[i];
        gaussknowns[i+3] = -MT.v[i];
    }

    // solve for Ft's (distal leg "russ" forces)
    Gausselim(6,gausscoeff,gaussknowns,temp);
    for (i=0; i<6; i++)
    Ft[i] = temp[i];

    // Define bb -> base legvectors rotated to base frame
    // tB -> top legvectors (tT) rotated to base frame
    for (i = 0; i<3; i++)
    {
        bb[i] = Rb[i,]*l2[1];
        bb[i+1] = Rb[i,]*l2[1+i];
        tB[i] = RBT * tT[2+i];
        tB[2+i] = RBT*rT[2+i];
    }
// Define perpendicular pseudo-torque direction vectors at
// each midplane node
mid[0] = UnitCross(bB[0], bB[1]);
mid[1] = UnitCross(bB[2], bB[3]);
mid[2] = UnitCross(bB[4], bB[5]);

// Define coeff matrix and knowns for each midplane node
// and solve for Fb[i], Fb[i+1], and corresponding Fmid[i]
for (i = 0; i<3; i++)
{
    for (j=0; j<3; j++)
    {
        gausscoeff[i][j] = -bB[2*i + 0]*v[j];
        gausscoeff[i][1] = -bB[2*i + 1]*v[j];
        gausscoeff[i][2] = mid[i]*v[j];
    }
    GaussElim3(gausscoeff, gaussknowns, temp);
    Fb[2*i] = temp[0];
    Fb[2*i+1] = temp[1];
    Fmid[i] = temp[2];
    torque[i] = Fmid[i]*wristmodule.leg;
}

for (k= 0; k<3; k++)
{
    Gbx[k][xcounter][ycounter] = (float)((Fb[2*k]-Fb[2*k+1])*cos(wristmodule.psi));
    Gby[k][xcounter][ycounter] = (float)((Fb[2*k]+Fb[2*k+1])*sin(wristmodule.psi));
    Gdx[k][xcounter][ycounter] = (float)((Ft[2*k]-Ft[2*k+1])*cos(wristmodule.psi));
    Gdy[k][xcounter][ycounter] = (float)((Ft[2*k]+Ft[2*k+1])*sin(wristmodule.psi));
    Torq[k][xcounter][ycounter] = (float)torque[k];
    alphas[k][xcounter][ycounter] = (float)(wristmodule.alpha[k] * (float)RTD);
    thetas[k][xcounter][ycounter] = (float)(wristmodule.theta[k] * (float)RTD);
}

wristmodule.InverseSolve(0.0, plungevar);
Appendix 2: Bearing Failure Analyses

This appendix contains two MathCad® documents used during the analysis of the various designs for the prototype Carpal wrist. The initial document calculates the possibility of static failure among the bearings supporting the midjoint shaft, and the second document calculates the possibility of static failure for the bearings used in the revolute joint connecting the basal leg to the basal plate. The solution procedure within these documents follows from the main text using the three sets of maximum midplane forces as the load cases. The results represent the factor of safety present in the final choice of bearings used in the prototype.
Static load failure analysis for midjoint bearings

Insert geometric parameters of midplane joint:

\( l_m \): midjoint length from leg to mid-revolute
\( d_{mb} \): distance between midjoint shaft bearing centers

\[
\begin{align*}
l_m &= 2.0 \text{ cm} \\
d_{mb} &= 1.1 \text{ cm}
\end{align*}
\]

Three load cases resulting from the maxima search performed by the force analysis are entered into one matrix. Note: force analysis calculates the forces required to support the midnode. The forces acting upon each half of the midjoint are equal and opposite the generalized midnode forces.

\[
\begin{align*}
\text{Case: 1} & \quad 2 \quad 3 \\
F_T &= \begin{pmatrix} -2.2 & -2.2 & 0.22 \\ -0.64 & -1.3 & 1.74 \\ 1.02 & -1.3 & -1.54 \end{pmatrix} \\
F_T &= F_x \\
F_T &= F_y \\
F_T &= F_z = \frac{T_{act}}{\text{leg length}}
\end{align*}
\]

\( F_m \) is the required force vector at the midjoint (comes from force analysis program).\( FT_{\text{scale}} \) is the specified load capacity of the wrist.

\[
F_m = F_T \cdot \text{FT}_{\text{scale}} \cdot \text{FT}_{\text{newton}}
\]

\[
\begin{align*}
\text{FT}_{\text{scale}} &= 20 \text{ lb/ft} \\
\text{FT}_{\text{newton}} &= \begin{pmatrix} -44 & -44 & 4.4 \\ -12.8 & -26 & 34.8 \\ -20.4 & -26 & -30.8 \end{pmatrix} \text{ lb/ft}
\end{align*}
\]

The bearing forces are calculated using the forces acting on the midjoint. \( R_{m1} \) and \( R_{m2} \) are matrices of reaction force vectors of each midjoint bearing for the 3 loads cases.

\[
\begin{align*}
R_{m1x} &= \begin{pmatrix} F_m^T \end{pmatrix} \begin{pmatrix} l_m \\ d_{mb} \end{pmatrix} \\
R_{m1y} &= \begin{pmatrix} -F_m^T \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix} \\
R_{m1z} &= \begin{pmatrix} F_m^T \end{pmatrix} \begin{pmatrix} 1 \\ d_{mb} \end{pmatrix}
\end{align*}
\]

Columns of \( R_{m1} \) represent force vectors corresponding to each load case.

\[
R_{m1} = \text{augment}(R_{m1x} \text{augment}(R_{m1y} \text{ R}_{m1z}))^T \\
R_{m1} = \begin{pmatrix} -355.858 & -355.858 & 35.586 \\ 56.937 & 115.654 & -154.798 \\ -164.989 & -210.28 & -249.1 \end{pmatrix} \text{ newton}
\]
\[ R_{m2x} := \begin{pmatrix} F_m \\ T \\ \left< 0 > \right> \end{pmatrix} \begin{pmatrix} l_m \\ d_{mb} + 1 \end{pmatrix} \]

Columns of \( R_{m2} \) represent force vectors corresponding to each load case.

\[ R_{m2y} := \begin{pmatrix} F_m \\ T \end{pmatrix} \begin{pmatrix} \left< 1 > \right> \end{pmatrix} \]

\[ R_{m2z} := \begin{pmatrix} F_m \\ T \end{pmatrix} \begin{pmatrix} \left< 2 > \right> \end{pmatrix} \begin{pmatrix} l_m \\ d_{mb} + 1 \end{pmatrix} \]

\[ R_{m2} := \text{augment} \left( R_{m2x}, \text{augment} \left( R_{m2y}, R_{m2z} \right) \right)^T \]

\[ R_{m2} = \begin{pmatrix} 551.58 & 551.58 & -55.158 \\ 56.937 & 115.654 & -154.798 \\ 255.732 & 325.933 & 386.106 \end{pmatrix} \text{ newton} \]

The maximum radial and axial bearing loads for each midjoint bearing are developed. The radial loads equal the square root of the sum of the square of the x- and z-components, and the axial load is simply equal to the y-component nodal load.

\[ R_{m1rad} = \begin{pmatrix} \left( R_{m1} \right)^T \left< 0 > \right> \end{pmatrix}^2 \]

\[ R_{m2rad} = \begin{pmatrix} \left( R_{m2} \right)^T \left< 0 > \right> \end{pmatrix}^2 \]

\[ R_{m1ax} = \begin{pmatrix} R_{m1} \end{pmatrix} \begin{pmatrix} \left< 1 > \right> \end{pmatrix}^T \]

\[ R_{m2ax} = \begin{pmatrix} R_{m2} \end{pmatrix} \begin{pmatrix} \left< 1 > \right> \end{pmatrix}^T \]

\[ \max(R_{m1rad}) = 413.3 \text{ newton} \]

\[ \max(R_{m1ax}) = 115.654 \text{ newton} \]

\[ \max(R_{m2rad}) = 640.7 \text{ newton} \]

\[ \max(R_{m2ax}) = 115.654 \text{ newton} \]

\[ R_{rad_{max}} := \max(\max(R_{m1rad}), \max(R_{m2rad})) \]

\[ R_{rad_{max}} = 640.68 \text{ lbf} \]

\[ R_{rad_{max}} = 144.031 \text{ lbf} \]

\[ R_{ax_{max}} := \max(\max(R_{m1ax}), \max(R_{m2ax})) \]

\[ R_{ax_{max}} = 115.654 \text{ lbf} \]

\[ R_{ax_{max}} = 26 \text{ lbf} \]
According to Shigley and Mitchell (1993), the maximum bearing pressure for bronze bearings can be used to calculate the factor of safety against static failure for the chosen bearings.

\[
P_{\text{max}} = 4500 \text{ psi} \quad P_{\text{max}} = 3.10 \times 10^7 \text{ Pa}
\]

\[
do_{\text{brg}} = \frac{7}{16} \text{ in} \quad do_{\text{brg}} = 0.019 \text{ m}
\]

\[
di_{\text{brg}} = \frac{3}{16} \text{ in} \quad di_{\text{brg}} = 0.009 \text{ m}
\]

\[
l_{\text{brg}} = \frac{5}{16} \text{ in} \quad l_{\text{brg}} = 0.009 \text{ m}
\]

\[
A_{\text{sleeve}} = di_{\text{brg}} \cdot l_{\text{brg}}
\]

\[
n_{\text{sleeve}} := \frac{P_{\text{max}}}{\text{max} \left( \text{max} \left( \frac{R_{\text{m1rad}}}{A_{\text{sleeve}}} \right) \text{max} \left( \frac{R_{\text{m2rad}}}{A_{\text{sleeve}}} \right) \right)}
\]

\[
n_{\text{sleeve}} = 1.831
\]

\[
A_{\text{flange}} := do_{\text{brg}} \cdot l_{\text{brg}}
\]

\[
n_{\text{flange}} := \frac{P_{\text{max}}}{\text{max} \left( \text{max} \left( \frac{R_{\text{m1ax}}}{A_{\text{flange}}} \right) \text{max} \left( \frac{R_{\text{m2ax}}}{A_{\text{flange}}} \right) \right)}
\]

\[
n_{\text{flange}} = 23.663
\]
Static load failure analysis for leg/base bearings.

Insert geometric parameters of wrist:

- \( b \): base "radius" to revolute
- \( l \): leg length from center of base revolute to midjoint
- \( l_a \): actuator leg length
- \( l_b \): distance between base bearing centers
- \( \theta_{\text{offset}} \): offset angle of input arm

\[
\begin{align*}
\theta_{\text{offset}} & \approx 41.7 \text{ deg}
\end{align*}
\]

Three load cases resulting from the maxima search performed by the force analysis are entered in one matrix (columns vectors of x,y, and z components representing each case). After scaling by the maximum payload capacity, \( \vec{F}_m \) is the equal and opposite reaction force vector acting on the leg at the midjoint (FT comes from force analysis program).

**Case:** 1 2 3

\[
\begin{pmatrix}
-2.2 & 2.2 & 0.22 \\
-0.64 & 1.3 & -1.74 \\
1.02 & -1.3 & -1.54
\end{pmatrix}
\]

\[
\vec{F}_m := 20 \text{ lbf/Newton}
\]

\[
\vec{F}_m := \text{FTscale} \cdot \vec{F}_\text{newton}
\]

\[
\begin{pmatrix}
-44 & 44 & 4.4 \\
-12.8 & 26 & -34.8 \text{ lbf} \\
20.4 & -26 & -30.8
\end{pmatrix}
\]

\( \phi_{\text{min}} \) is the minimum pressure angle between the actuator leg and the actuator shaft. Using the minimum angle, the transmitted forces are maximized and a worst-case scenario is analyzed.

\[
\phi_{\text{min}} := 24.3 \text{ deg}
\]
The actuator pushrod force is calculated - this is the two force member which pushes/pulls on the input arm.

\[
F_a = \begin{pmatrix} F_m \\ F_m \end{pmatrix}^T \cdot \frac{1}{l_a \sin (\phi_{\text{min}})}
\]

\[
F_a = (88.13 \quad -112.322 \quad -133.059) \text{ lbf}
\]

The base bearing reactions are calculated. \( R_1 \) and \( R_2 \) are the matrices of reaction force vector for the 3 load cases for the bearings on the opposite sides of the leg.

\[
R_{1x} = \frac{1}{2} \begin{pmatrix} F_m \\ F_m \end{pmatrix}^{\text{T}} \\
R_{1y} = \frac{1}{2} \begin{pmatrix} F_m \\ F_m \end{pmatrix}^{\text{T}} \cdot \frac{1}{l_b} - \begin{pmatrix} F_m \\ F_m \end{pmatrix}^{\text{T}} \cdot \frac{\cdot \text{cos} (\theta_{\text{offset}} - \phi_{\text{min}})}{l_a \sin (\phi_{\text{min}})} \\
R_{1z} = \frac{1}{2} \begin{pmatrix} F_m \\ F_m \end{pmatrix}^{\text{T}} \cdot \left( \frac{\cdot \sin (\theta_{\text{offset}} - \phi_{\text{min}})}{l_a \sin (\phi_{\text{min}})} \right)
\]

\[
R_1 = \text{augment} \left( R_{1x} \text{augment} \left( R_{1y}, R_{1z} \right)^T \right)
\]

\[
R_1 = \begin{pmatrix} 97.861 & -97.861 & -9.786 \\ -550.016 & 572.002 & 398.939 \\ -103.987 & 132.532 & 157 \end{pmatrix} \text{ newton}
\]
\[ R_{2T} := R_1^T \]
\[ R_{2T}^{<1>} := \frac{1}{2} \left[ 2 \left( F_m^T \right)^{<1>} + \frac{l}{b} \left( F_m^T \right)^{<1>} + \left( F_m^T \right)^{<2>} \frac{\cdot \cos(\theta_{offset} - \phi_{min})}{l_a \cdot \sin(\phi_{min})} \right] \]
\[ R_2 := R_{2T}^T \]
\[ R_2 = \begin{pmatrix}
97.861 & -97.861 & -9.786 \\
232.871 & -210.885 & 320.65 \\
103.987 & 132.532 & 157
\end{pmatrix} \text{ newton} \]

The maximum radial and axial bearing loads for each base bearing are developed. The radial load equals the square root of the sum of the squares of the y- and z-components, and the axial load is equal to the x-component of the bearing reaction.

\[ R_{1rad} := \left( \sqrt{\left( R_1^T \right)^{<1>}^2 + \left( R_1^T \right)^{<2>^2}} \right)^T \]
\[ R_{2rad} := \left( \sqrt{\left( R_2^T \right)^{<1>}^2 + \left( R_2^T \right)^{<2>^2}} \right)^T \]
\[ R_{1ax} := \left( R_1^T \right)^{<0>}^T \]
\[ R_{2ax} := \left( R_2^T \right)^{<0>}^T \]

\[ \max(R_{1rad}) = 587.153 \text{ newton} \]
\[ \max(R_{1ax}) = 97.861 \text{ newton} \]

\[ \max(R_{2rad}) = 357.023 \text{ newton} \]
\[ \max(R_{2ax}) = 97.861 \text{ newton} \]
The maximum bearing pressure for the bronze bearings according to Shigley and Mitchell (1993) can be used to calculate the factor of safety against static failure for the chosen bearings:

\[ P_{\text{max}} = 4500 \text{ psi} \]

\[ d_{o\text{ brg}} = \frac{3}{8} \text{ in} \quad d_{o\text{ brg}} = 0.0114 \text{ m} \]

\[ d_{i\text{ brg}} = \frac{1}{4} \text{ in} \quad d_{i\text{ brg}} = 0.006 \text{ m} \]

\[ l_{\text{brg}} = \frac{3}{8} \text{ in} \quad l_{\text{brg}} = 0.0114 \text{ m} \]

\[ A_{\text{sleeve}} := d_{i\text{ brg}} \cdot l_{\text{brg}} \]

\[ n_{\text{sleeve}} := \frac{P_{\text{max}}}{\max\left(\frac{\max(R_{1\text{rad}})\max(R_{2\text{rad}})}{A_{\text{sleeve}}}\right)} \]

\[ n_{\text{sleeve}} = 3.196 \]

\[ A_{\text{flange}} := \pi \frac{d_{o\text{ brg}}^2 - d_{i\text{ brg}}^2}{4} \]

\[ n_{\text{flange}} := \frac{P_{\text{max}}}{\max\left(\frac{\max(R_{1\text{ax}})\max(R_{2\text{ax}})}{A_{\text{sleeve}}}\right)} \]

\[ n_{\text{flange}} = 19.176 \]
Appendix 3: Detailed Design Drawings of Prototype

The final appendix is the complete set of detailed mechanical design drawings for the prototype model described in Chapter 4 of the main text. The first four of the set are exploded isometric drawings that describe the correct assembly of the entire device and provide a list of the specific components required. Appropriate references are included to the detailed drawings of the individual components that follow.
3. Countersink holes are needed for only 3 of the 8 parts to be produced. Do not drill and countersink on 3 of 8 parts.

**Tolerance:** (Unless specified)
- X X 1/0.1
- X X X 1/0.05
- X X X X 1/0.005

**Note:**
- 1/001-16
- By:
- IDY Needed: 6
- Scale: None
- Date: 12/21/95
Thread 1/4"-20UNF x 3/4"

Use 1/4" St 303 Ground Shafting
Borg No. 54-xx or
ER Ground Shafting
Borg No. 320-3

\[ \phi \, 0.250 \]

\[ 0.07 \quad 0.375 \quad 0.06 \]

\[ 0.05 \]

Table:

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<th>Threaded Stud Shaft</th>
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<td>( \pm 0.05 )</td>
</tr>
<tr>
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<td>( \pm 0.005 )</td>
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Dwg No. 165-0004

Date: 1/24/05

By:
Use 1/2" x 0.049" wall tubing.
Note:
1. For all other dimensions - see Dwg No. 50F-0004
2. All holes drilled in corresponding bolt circles
3. Angular dimensions are symmetric within each 120° segment.

Tolerance: (unless specified)
x x 1/0.1
x xx 1/0.05
x xxx 1/0.005

Dwg No. 30F-0004b

Mod 1: All 6061-T6

Dry Needed: 1

Scale: Nine
Date: 1/10/95
Drill & Tap 2 holes
3/8" 15NC x 1" deep

Tolerance: (unless specified)
x.x ±0.01
x.xx ±0.005
x.xxx ±0.005

Grounded Support Columns

Dwg No: SLP-1004

Mat: A1 6061-T6

Dly Needed: 1

Scale: None | Date: 1/10/96
Vita

Anthony J. Ganino was born on February 3, 1971 in the “big, pink” hospital at Fort Shafter, Honolulu, Hawaii. He earned his Bachelor of Science degree in Mechanical Engineering from Virginia Tech in May, 1994. Fearing the real world due to his cooperative education employment with Dow Chemical and exhausted from his participation on the SAE Mini-Baja team during his senior year, he chose to pursue his Master of Science degree. His love for Virginia Tech kept him in Blacksburg during this time. He will be beginning his post-graduate career at Jet Propulsion Laboratory in Pasadena, California where he will be designing robotic manipulators for future planetary missions.