THE FREQUENCY DISTRIBUTION OF AVAILABILITY

by

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(ABSTRACT)

The use of availability measures is very informative when analyzing the performance of repairable components. The derivation and evaluation of such measures is usually focused on describing the status of the component over time. It is not generally acknowledged that the resulting availability measure is in fact an expected value with respect to frequency. In a population of n independent, identical components, the number functioning at any point in time is a random variable. The distribution of this random variable is determined and described here.

The intuitive view that this frequency distribution is binomial is verified. This is accomplished using direct analysis and Monte Carlo simulation. Two general cases are considered: (1) components with exponential life and repair time distributions, and (2) components with Weibull life distributions and exponential repair time distributions. The analysis leads to more accurate models of component behavior in terms of frequency including an exact confidence bound on the number of components functioning at any point in time. In addition, the time evolution of the frequency distribution is described and the relationships between the frequency distribution and the life and repair time distribution parameters are explored. Finally, the implications for availability decision analyses are shown. The overall result is a new perspective on availability which should prove to be very useful.

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CHAPTER 1 – INTRODUCTION

Mechanical, electrical, and other types of industrial systems typically consist of one or more components which are subject to failure. Reliability analysis is concerned with evaluating the statistical properties of the lifetimes of such components, and thus, the statistical properties of the lifetimes of the systems themselves. If these components are repaired upon failure, then the focus of reliability analysis likely becomes the availability of the components, and thus, the availability of the system. In the broadest sense, component (system) availability is the ratio of the component's (system's) operating time to its total lifetime.

1.1 Traditional Availability Modeling

Component and system availability have been a subject of interest for many years. The focus of most previous research has been to model the failure and repair mechanisms for the component in order to derive and evaluate some type of component availability measure and then to use the results to compute or estimate a corresponding system availability measure. The method used to compute or estimate the system availability measure depends upon the system structure. This system availability measure can then be used to evaluate system performance, system readiness, and system support requirements.

Barlow and Proschan (1975) define four component availability measures:

1. availability
2. limiting availability
3. average availability
4. limiting average availability.

Availability, $A(t)$, is defined to be the probability that the component is operating at time t, given that it starts operating at time $t = 0$. In this research, $A(t)$ is the availability measure of interest (from this point on, availability will refer to A(t)). The most common implementation of this availability measure is to compute $A(\infty)$, the limiting availability. The limiting availability has been shown in many cases to be a reasonable approximation to A(t). However, there are some cases in which limiting availability may not be representative of A(t), particularly for relatively small values of t.

Much work has been done in formulating and analyzing component availability models. The component availability models generate a probability $(0 \leq A(t) \leq 1)$ for any time t. For example, if a component has $A(100) = 0.98$, then the the component has probability 0.98 of being operational at $t = 100$. The probabilities for all the components in the system are combined (in a manner according to the system structure) to determine the system availability.

The simplest component to have been considered is the two-state component. This type of component operates for a certain period of time, fails, is repaired, operates for another period of time, fails, is repaired, and so on. The two states of the component can be classified as operating and failed (under repair). Availability models have been derived for the two-state component using various failure and repair models. For example, the component may function for an amount of time that is exponentially distributed with rate λ, and the time required to repair the component may be exponentially distributed with rate μ. Another example is a component that functions for an amount of time that is Weibull distributed with shape parameter β and scale parameter α. Obviously, there are components for which neither of these examples adequately portrays component behavior. For example, Blanchard (1992) states that repair times are often lognormally distributed. It is the responsibility of the analyst to determine the appropriate failure and repair mechanisms when selecting or constructing an availability model.

1.2 The Frequency Distribution of Availability

In this research, the system under consideration consists of a single component which is always in one of two states - operating (functioning properly) or failed (under repair). In practice, the situation may arise in which several identical components operate concurrently and independently. For example, a machine shop may have five identical milling machines (the machine as a whole is considered to be the component), a furniture factory may have ten identical sanders, and a large building may have 100 identical fire alarms. Therefore, it may be of interest to the practitioner to apply the concept of availability to a population of identical components. In particular, if the population consists of n such components, what is the probability that k of them are operating at a certain point in time?

1.3 Problem Statement

The purpose of this research is to investigate the frequency behavior of availability at any specific point in time. Authors who have mentioned this frequency behavior have neither examined the frequency behavior over time, nor have they related this behavior to the parameters of the life and repair time distributions.

Consider the fire alarm example. If $A(1000 \text{ hours}) = 0.926$ for a single alarm, then one would anticipate an average of 92.6 of the fire alarms to be operating at $t = 1000$ hours. Obviously, it is not possible for 92.6 alarms to be operating. In fact, if several buildings, each with a set of 100 fire alarms, were monitored, one would not expect to see the same number of alarms operating in each building. Therefore, the number of alarms operating at time t is an integer-valued discrete random variable. The distribution of this random variable is referred to as the frequency distribution of availability.

The goal of this research is to examine this frequency distribution. First, the time evolution of this distribution and its behavior are examined. Next, the relationships between the components' life and repair time distribution parameters and the frequency distribution are explored. Finally, this distribution is compared to previously suggested techniques in availability decision analyses.

1.4 Summary of Solution Approach

The first step in this research is the definition of the required algebraic notation. Second, some reasonable assumptions are made. Using probability theory, a hypothesis is then made as to the form of the frequency distribution. Monte Carlo simulation and statistical analysis of the simulation output are used to verify the hypothesis. Two general cases are considered: (1) exponential life distribution (rate λ) and exponential repair time distribution (rate μ), and (2) Weibull life distribution (shape parameter β and scale parameter α) and exponential repair time distribution (rate μ). Within each general case, parameter values and the size of the component population are varied in order to ascertain the robustness of the model.

Once the general form of the distribution is verified, analytical techniques and probability theory are used to examine the time evolution of this distribution and to characterize its behavior. Similar techniques are used to explore the relationship between the frequency distribution and the components' life and repair time distribution parameters. Finally, analytical techniques are used to compare this distribution to alternative decision making techniques.

CHAPTER 2 – LITERATURE REVIEW

As stated earlier, most previous analyses of availability have been concerned with modeling and computing A(t) or the limiting availability for components and then computing or estimating the corresponding availability measure for systems. The emphasis has been on modeling and understanding behavior measured in the time domain. Very little mention has been made of the frequency distribution of availability at a specific point in time.

Although little mention has been made of this frequency distribution, there has been much discussion regarding the random nature of availability. Butterworth and Nikolaisen (1973) compute bounds for A(t) for a component with exponentially distributed time to failure and generally distributed time to repair. Moore, Hobbs, and Hasaballa (1985) present a Monte Carlo method for obtaining confidence bounds on the limiting availability of systems. Shaw and Shooman (1976) develop bounds on the uncertainties in calculating the limiting availability of a system. Shooman (1985) describes bounds which can be utilized to simplify the calculation of a system's limiting availability.

Gray and Lewis (1967) develop a method for calculating an exact confidence interval for the limiting availability of a component with exponentially distributed time to failure and lognormally distributed time to repair. Thompson and Palicio (1975) give a method for computing Bayes confidence limits for the limiting availability of a series or parallel system of two-state components having exponential life and repair time distributions. Nelson (1970) gives a method for predicting and computing a statistical prediction interval for the average availability of a component during future failure and repair cycles. Nelson's method depends on data from previous cycles and assumes that the component has exponential life and repair time distributions.

Nelson (1968) develops an exact statistical test for testing the equality of the limiting availability of two components. Nelson assumes that both components have exponential life and repair time distributions. Schafer and Takenaga (1972) present a Generalized Sequential Probability Ratio Test for the limiting availability of a component which has exponential life and repair time distributions. The GSPRT tests the hypothesis that the limiting availability equals some specified value versus the alternative hypothesis that the limiting availability is less than the specified value. Rise (1979) presents the concepts of statistical compliance test plans for the limiting availability of a two-state component. Rise assumes that the component has Gamma life and repair time distributions.

Although no one has formally discussed the distribution of availability in the frequency domain, there has been some passing mention of the problem. Lynch (1974) and Campbell and Keller (1986) both describe the problem and state that this underlying frequency distribution is binomial with parameters $n =$ the number of components and $p = A(\infty)$. Lynch states that the binomial distribution determines the probability that X out of N independent systems will be working at a specific point in time, provided that $p = A(\infty)$. Campbell and Keller give an example of two identical and independent oil pipelines operating in parallel. They state: (1) the probability that both pipelines are operating is $(A(\infty))^2$, (2) the probability that only one pipeline is operating is $2 \cdot A(\infty) \cdot [1 - A(\infty)]$, and (3) the probability that neither pipeline is operating is $[1 - A(\infty)]^2$. These probabilities correspond to the binomial distribution with $n = 2$ and $p = A(\infty)$.

CHAPTER 3 – MODELING THE FREQUENCY DISTRIBUTION OF AVAILABILITY

In order to model the frequency distribution of availability, some algebraic notation must be defined and some reasonable assumptions are made. A hypothesis is made as to the form of the frequency distribution of availability. A detailed method involving Monte Carlo simulation and statistical analysis is then used to verify the hypothesis. After the form of the distribution is verified, the time evolution of this frequency distribution is examined and its behavior is characterized. Then, the relationship between the frequency distribution and the components' life and repair distribution parameters are explored. Finally, this distribution is compared to alternative techniques used in availability decision analyses.

3.1 Notation and Assumptions

Consider an environment which contains n identical, independent two-state components. The primary goal is to discover the probability that k of the n components are working at a specific time t. The notation used is the following:

t	time variable.
$A(t)$	availability at time t.
$A(\infty)$	limiting availability.
$X_i(t)$	component status variable (1 if component is operating at time t, 0 otherwise).
n	number of components in the population.
$W(t)$	number of the n components working at time t.
$p_k(t)$	$P[W(t) = k]$.
$F(t)$	each component's life distribution.
$R(t)$	each component's reliability function $= 1 - F(t)$.
$G(t)$	each component's repair time distribution.
$H(t)$	the convolution of $F(t)$ and $G(t)$.

$M_H(t)$	the renewal function based on H(t).
$m_H(t)$	the renewal density corresponding to $M_H(t)$.
λ	each component's failure rate.
μ	each component's repair rate.
β	shape parameter of a Weibull life distribution.
α	scale (modified) parameter of a Weibull life distribution.

The model developed in this research does not assume that the limiting availability is a reasonable approximation for A(t) at any value of t. The following assumptions are made:

1. at $t = 0$, each system has just completed repair (or is new);
2. times to failure for the components are independent and identically distributed;
3. repair times for the components are independent and identically distributed.

3.2 The Model of the Frequency Distribution of Availability

The first step in solving this problem is to formulate a hypothesis regarding the frequency distribution. Consider a population of n independent and identical components. A well known availability expression for any individual component (Barlow and Proschan, 1975) is:

$$A(t) = P[X_i(t) = 1]. \tag{1}$$

The number of functioning components at any time is

$$W(t) = \sum_{i=1}^{n} X_i(t), \tag{2}$$

for which the probability of occurrence is

$$p_k(t) = P[W(t) = k] = P[\sum_{i=1}^{n} X_i(t) = k]. \tag{3}$$

This is the probability that the sum of n independent Bernoulli random variables takes on a specific value. Each of these Bernoulli random variables has $p = A(t)$. Therefore, one would intuitively expect $p_k(t)$ to be binomial with parameters n and $p = A(t)$.

Formally, the hypothesized form of the frequency distribution of availability is:

$$p_k(t) = \binom{n}{k} [A(t)]^k \, [1 - A(t)]^{n-k}. \qquad (4)$$

3.3 Plan for Model Verification and Analysis

A Monte Carlo simulation is developed in order to verify this hypothesis. The Monte Carlo simulation required is very simple. Therefore, all simulation is accomplished using a general purpose language (Pascal). Three items are needed: (1) a reliable random number generator, (2) an exponential random variable generator, and (3) a Weibull random variable generator. All three of these items can be found in Law and Kelton (1991). They are explained in detail in Appendix A.

The simulation mimics the operation and repair cycles of a population of n components one at a time. The status of each component is checked at times which correspond to specific values of $A(t)$: 0.99, 0.95, 0.90, 0.85, 0.80, 0.75, and 0.70. The simulation records the number of the n components operating at each of these times. This experiment is then repeated 1000 times. In other words, 1000 random samples are taken from each of seven frequency distributions. Two general cases are considered: (1) exponential life distribution and exponential repair time distribution, and (2) Weibull life distribution and exponential repair time distribution. Within each general case, parameter values and population size are varied in order to ascertain the robustness of the hypothesized distribution. The simulation code can be found in Appendix A.

Consider the first case. If F(t) is exponential with rate λ, and G(t) is exponential with rate μ, then Ross (1989) shows that

$$A(t) = \frac{\mu}{\mu+\lambda} + \frac{\lambda}{\mu+\lambda} e^{-(\lambda+\mu)\,t} \tag{5}$$

In addition, for all life and repair time distribution models, Gray and Lewis (1967) state that

$$A(\infty) = \frac{\text{mean time to failure}}{\text{mean time to failure} + \text{mean time to repair}} \tag{6}$$

Using equation (6), three sets of life and repair time distribution parameter values (λ, μ) can be selected so that the limiting availability for each set is 0.65. Then, the specific values of t for which A(t) is 0.99, 0.95, and so on, are calculated using equation (5) for each set of parameter values. The resulting examples can be found in Table 3.3.1.

Note that for all three examples $A(t_1) = 0.99$, $A(t_2) = 0.95$, $A(t_3) = 0.90$, $A(t_4) = 0.85$, $A(t_5) = 0.80$, $A(t_6) = 0.75$, and $A(t_7) = 0.70$.

The simulation is then executed for each example. By executing these three simulations, 1000 random samples are taken from each of 21 frequency distributions. Each random sample is then subjected to a chi-square goodness of fit test. A computer software package is used to perform the required chi-square tests. This software package is Distribution Identification and Data Analysis (DIDA), developed by Schmidt (1991). DIDA will test the fit of the binomial distribution to a data set. The only input required is the parameters for the distribution and the level of significance for the test. All other decisions (number of intervals, interval width, etc.) are built into the software. The level of significance is 0.05 for all tests. The results of the chi-square tests verify the accuracy of the hypothesized distribution for this case.

Table 3.3.1 – Examples for Exponential Life Distribution and Exponential Repair Time Distribution

Example #1

$n = 100$	$\lambda = 0.1000$	$\mu = 0.1857$	
$t_1 = 0.1015$	$t_2 = 0.5395$	$t_3 = 1.1776$	$t_4 = 1.9586$
$t_5 = 2.9655$	$t_6 = 4.3845$	$t_7 = 6.8100$	

Example #2

$n = 10$	$\lambda = 0.1615$	$\mu = 0.3000$	
$t_1 = 0.062820$	$t_2 = 0.3341$	$t_3 = 0.7292$	$t_4 = 1.2129$
$t_5 = 1.8364$	$t_6 = 2.7154$	$t_7 = 4.2185$	

Example #3

$n = 50$	$\lambda = 0.5000$	$\mu = 0.9286$	
$t_1 = 0.02029$	$t_2 = 0.1079$	$t_3 = 0.2355$	$t_4 = 0.3917$
$t_5 = 0.5931$	$t_6 = 0.8770$	$t_7 = 1.3622$	

Consider the second case. If $F(t)$ is Weibull with shape parameter β and scale parameter α, and $G(t)$ is exponential with rate μ, then $A(t)$ must be approximated. The method for approximating $A(t)$ is as follows:

<u>Step 1</u>: Apply Simpson's rule for numerical integration to numerically compute values of the convolution of the Weibull life distribution, $F(t)$, and the exponential repair time distribution, $G(t)$.

<u>Step 2</u>: Construct a regression based fit to a hypothesized "adjusted" Weibull distribution on the duration of the combined operating and repair interval. Define the fitted distribution to be $H(t)$.

<u>Step 3</u>: Represent $M_H(t)$ using Lomnicki's (1966) approximation.

<u>Step 4</u>: Express $m_H(t)$ as the numerical derivative of $M_H(t)$.

<u>Step 5</u>: Use Simpson's rule to evaluate the resulting form of the integral

$$A(t) = R(t) + \int_0^t m_H(t)\ R(t-x)\ dx. \tag{7}$$

Three sets of life and repair distribution parameter values (α, β, μ) can be selected using equation (6) so that the limiting availability for each set is 0.65. The same values of μ are used as in the first case. For simplicity, all three sets have $\alpha = 1$. The specific values of t for which $A(t)$ is approximately 0.99, 0.95, 0.90, 0.85, 0.80, 0.75, and 0.70 are found using a search technique with the above approximation method. The resulting examples can be found in Table 3.3.2.

Note that for all three examples $A(t_1) \approx 0.99$, $A(t_2) \approx 0.95$, $A(t_3) \approx 0.90$, $A(t_4) \approx 0.85$, $A(t_5) \approx 0.80$, $A(t_6) \approx 0.75$, and $A(t_7) \approx 0.70$.

Table 3.3.2 – Examples for Weibull Life Distribution and Exponential Repair Time Distribution

Example #1

$n = 100$	$\beta = 0.2950$	$\alpha = 1$	$\mu = 0.1857$
$t_1 = 5 \times 10^{-8}$	$t_2 = 0.00003$	$t_3 = 0.00042$	$t_4 = 0.0024$
$t_5 = 0.0071$	$t_6 = 0.0138$	$t_7 = 0.0270$	

Example #2

$n = 10$	$\beta = 0.3306$	$\alpha = 1$	$\mu = 0.3000$
$t_1 = 0.0000003$	$t_2 = 0.00009$	$t_3 = 0.0010$	$t_4 = 0.0045$
$t_5 = 0.0120$	$t_6 = 0.0220$	$t_7 = 0.0420$	

Example #3

$n = 50$	$\beta = 0.5000$	$\alpha = 1$	$\mu = 0.9286$
$t_1 = 0.00004$	$t_2 = 0.0021$	$t_3 = 0.0110$	$t_4 = 0.0280$
$t_5 = 0.0555$	$t_6 = 0.0848$	$t_7 = 0.1460$	

The simulation is then executed for each example. Each of the 21 random samples is then subjected to a chi-square test. Since the t values are found using an approximation technique, the chi-square test uses an estimate of A(t) which is calculated from the simulation output. The results of the chi-square test verify the accuracy of the hypothesized distribution for this case.

After the form of the frequency distribution has been verified, the time evolution of the distribution and patterns of the distribution's behavior are analyzed. The first objective is to discover the overall patterns in the frequency distribution as t increases. This is accomplished using analytical techniques and probability theory. The next objective is to determine how the frequency distribution is related to the components' life distribution and repair time distribution parameters. For the case in which both the life and repair time distributions are exponential, changes in the distribution's mean and variance with respect to λ and μ are determined. For the case in which the life distribution is Weibull and the repair time distribution is exponential, changes in the distribution's mean and variance with respect to β and μ are determined.

The final stage in the analysis of the model is the comparison of the use of this frequency distribution to alternative models in availability decision analyses. The three alternative techniques which are compared to the frequency distribution are (1) assuming that $n \cdot A(t)$ components will be operating, (2) assuming that $n \cdot A(\infty)$ components will be operating, and (3) assuming a binomial distribution with parameters n and $p = A(\infty)$. The comparison is made among these three methods for the following examples:

1. $n = 5$, $A(t) = 0.99$, $A(\infty) = 0.95$
2. $n = 5$, $A(t) = 0.99$, $A(\infty) = 0.80$
3. $n = 10$, $A(t) = 0.95$, $A(\infty) = 0.90$
4. $n = 10$, $A(t) = 0.95$, $A(\infty) = 0.75$
5. $n = 10$, $A(t) = 0.90$, $A(\infty) = 0.65$

In addition, a method for obtaining exact confidence bounds on the number of components operating at any point in time is described.

3.4 Results of Model Verification and Analysis

The Monte Carlo simulation is first executed for each of the three examples in which both F(t) and G(t) are exponential. For each of the seven specified time values, a sample of 1000 is generated from the frequency distribution. Each sample is then compared to a binomial distribution with parameters n and $p = A(t)$ using a chi-square goodness-of-fit test. The results can be found in Table 3.4.1. For all 21 instances in which the life and repair time distributions are both exponential, the binomial distribution is verified as an appropriate model.

The Monte Carlo simulation is then executed for each of the three examples in which F(t) is Weibull and G(t) is exponential. For each of the seven specified time values, a sample of 1000 is generated from the frequency distribution. Each sample is then compared to a binomial distribution with parameters n and $p = A(t)$ using a chi-square goodness-of-fit test. Recall that in this case, the value of A(t) used in the chi-square test is estimated from the simulation output. The results can be found in Table 3.4.2.

For the instance in which the chi-square test was unable to execute, a comparison of sample frequencies to expected frequencies indicates that the binomial model is appropriate. Thus, the proposed binomial model has been verified for both general cases. Detailed results of the chi-square tests as well as frequency comparisons for all cases can be found in Appendix B.

Having confirmed the appropriateness of the binomial model, it is possible to analyze the behavior of the frequency distribution over time and in terms of the life and repair time distribution parameters.

Table 3.4.1 – Chi-Square Test Results for Exponential Life Distribution and Exponential Repair Time Distribution

Example #1

n = 100, $\lambda = 0.1000$, $\mu = 0.1857$

t	A(t)	Test Statistic	Critical Value	Conclusion
0.1015	0.99	2.238	9.488	accept
0.5395	0.95	8.430	19.68	accept
1.1776	0.90	16.72	25.00	accept
1.9586	0.85	14.78	28.87	accept
2.9655	0.80	17.40	32.67	accept
4.3845	0.75	17.59	33.92	accept
6.8100	0.70	16.36	35.17	accept

Example #2

n = 10, $\lambda = 0.1615$, $\mu = 0.3000$

t	A(t)	Test Statistic	Critical Value	Conclusion
0.06282	0.99	0.335	3.841	accept
0.3341	0.95	0.617	7.815	accept
0.7292	0.90	3.784	9.488	accept
1.2129	0.85	3.020	11.07	accept
1.8364	0.80	4.297	12.59	accept
2.7154	0.75	3.577	12.59	accept
4.2185	0.70	3.869	14.07	accept

Example #3

n = 50, $\lambda = 0.5000$, $\mu = 0.9286$

t	A(t)	Test Statistic	Critical Value	Conclusion
0.02029	0.99	4.760	7.815	accept
0.1079	0.95	2.855	14.07	accept
0.2355	0.90	11.74	19.68	accept
0.3917	0.85	16.03	22.36	accept
0.5931	0.80	21.18	25.00	accept
0.8770	0.75	18.69	26.30	accept
1.3622	0.70	15.44	27.59	accept

Table 3.4.2 – Chi-Square Test Results for Weibull Life Distribution and Exponential Repair Time Distribution

Example #1

n = 100, $\beta = 0.2950$, $\alpha = 1$, $\mu = 0.1857$

t	$\hat{A}(t)$	Test Statistic	Critical Value	Conclusion
5×10^{-8}	0.9923	0.850	7.815	accept
0.00003	0.9549	3.838	18.31	accept
0.00042	0.9040	14.06	23.68	accept
0.0024	0.8444	14.72	27.59	accept
0.0071	0.7927	20.23	30.14	accept
0.0138	0.7543	22.22	32.67	accept
0.0270	0.7089	25.20	33.92	accept

Example #2

n = 10, $\beta = 0.3306$, $\alpha = 1$, $\mu = 0.3000$

t	$\hat{A}(t)$	Test Statistic	Critical Value	Conclusion
0.0000003	0.9922	*	*	*
0.00009	0.9545	2.018	5.991	accept
0.0010	0.9011	0.656	7.815	accept
0.0045	0.8432	2.485	9.488	accept
0.0120	0.7936	5.505	11.07	accept
0.0220	0.7557	4.435	11.07	accept
0.0420	0.7063	6.094	12.59	accept

Example #3

n = 50, $\beta = 0.5000$, $\alpha = 1$, $\mu = 0.9286$

t	$\hat{A}(t)$	Test Statistic	Critical Value	Conclusion
0.00004	0.9933	0.038	3.841	accept
0.0021	0.9549	3.474	12.59	accept
0.0110	0.9008	3.636	18.31	accept
0.0280	0.8476	11.12	21.03	accept
0.0555	0.7957	14.42	22.36	accept
0.0848	0.7594	10.85	23.68	accept
0.1460	0.7051	10.33	25.00	accept

* – insufficient degrees of freedom to execute the Chi-square test

Since $W(t)$ is binomial with parameters n and $p = A(t)$,

$$E[W(t)] = n \cdot A(t) \tag{8}$$

and

$$Var[W(t)] = n \cdot A(t) \cdot [1 - A(t)]. \tag{9}$$

First, consider the case in which both $F(t)$ and $G(t)$ are exponential. For this case, $A(t)$ is monotonically decreasing in t. So as time is increased, $A(t)$ decreases. Therefore, as t increases the average number of components operating decreases, and the variance in the number of components operating increases (assuming $A(\infty) \geq 0.50$). This is especially important since repair rates are often set so that $A(\infty)$ exceeds 0.50. This can lead to variability in the number of components operating large enough that anticipated performance is not achieved. For example, suppose the repair rate is selected for a population of 100 components in order to obtain $A(\infty) = 0.65$ with an associated $A(40) = 0.75$. In this case, the average number of components operating at 40 hours is 75, the variance in $W(40)$ is 18.75, and $P[W(40) \leq 70] = 0.148$. The point is that the variance in the frequency domain can imply reasonably large deviations from anticipated performance and that this possibility should be included in availability decision analyses.

A corresponding result is that for components having $A(\infty) \geq 0.50$, reducing λ or increasing μ increases $A(t)$ for all values of t. Therefore, the average number of components operating is increased and the variance in the number of components operating is reduced. Thus, increasing $A(t)$ has the additional benefit of reducing the variance in the number of components operating. Accomplishing this makes the time domain availability model (equation (5)) more accurate.

Similar relationships apply for the case in which $F(t)$ is Weibull and $G(t)$ is exponential. As β increases, the expected operating time decreases, thereby decreasing $A(t)$. Also, for this case, availability declines and then increases to converge to $A(\infty)$. Thus, even for a component population for which $A(\infty)$ is reasonably large, values of t such that $A(t) \approx 0.50$ are possible. For these time values, the corresponding average number of components is small, and the corresponding variability in the number of components operating is large. For example, a component population having $A(\infty) = 0.65$ can have a minimum $A(t) \approx 0.40$ under plausible values of the life and repair time distribution parameters. This implies that for a considerable portion of the functional life of the component, $A(t) \approx 0.50$. At these times, the variance in the frequency distribution is relatively large, and the deviation from anticipated performance may be substantial.

The final stage in the analysis of the model is the comparison of the use of the frequency distribution to alternative models in availability decision analyses. Three alternative techniques which are sometimes used are (1) assuming that $n \cdot A(t)$ components will be operating, (2) assuming that $n \cdot A(\infty)$ components will be operating, and (3) assuming a binomial distribution with parameters n and $p = A(\infty)$. The binomial model is compared to these three methods for five simple examples. The results are given in Table 3.4.3.

It is apparent that using any of the three alternative methods can results in substantial errors. Therefore, the frequency distribution is the appropriate model for availability decision analyses, especially for small values of t.

Another consequence of the binomial model is the fact that exact confidence bounds on the number of components operating may now be defined easily. Since equation (4) provides the distribution of $W(t)$, any confidence bound may be computed either directly or using the Normal approximation to the binomial. For example, in a population of 50 components for which $A(\infty) = 0.65$ and $A(t) = 0.80$ at some point in time, the probability that 35 or more of the devices are functioning at that point in time is 0.9692. This is true regardless of the identities of the life and repair time distributions for the components.

Table 3.4.3 – Comparison to Alternative Models

Comparison #1 – $n = 5$, $A(t) = 0.99$, $A(\infty) = 0.95$

$$n \cdot A(t) = 4.95$$
$$n \cdot A(\infty) = 4.75$$

k	P[W(t) = k], binomial with n, A(∞)	P[W(t) = k], binomial with n, A(t)
2	0.001	0
3	0.021	0.001
4	0.204	0.048
5	0.774	0.951

Method	Mean	Variance
$n \cdot A(t)$	4.95	0
$n \cdot A(\infty)$	4.75	0
binomial with n and $p = A(\infty)$	4.75	0.2375
binomial with n and $p = A(t)$	4.95	0.0495

Comparison #2 – $n = 5$, $A(t) = 0.99$, $A(\infty) = 0.80$

$$n \cdot A(t) = 4.95$$
$$n \cdot A(\infty) = 4$$

k	P[W(t) = k], binomial with n, A(∞)	P[W(t) = k], binomial with n, A(t)
1	0.006	0
2	0.051	0
3	0.205	0.001
4	0.410	0.048
5	0.328	0.951

Method	Mean	Variance
$n \cdot A(t)$	4.95	0
$n \cdot A(\infty)$	4.00	0
binomial with n and $p = A(\infty)$	4.00	0.8000
binomial with n and $p = A(t)$	4.95	0.0495

Table 3.4.3 (continued)

Comparison #3 – $n = 10$, $A(t) = 0.95$, $A(\infty) = 0.90$

$$n \cdot A(t) = 9.5$$
$$n \cdot A(\infty) = 9$$

k	P[W(t) = k], binomial with n, A(∞)	P[W(t) = k], binomial with n, A(t)
5	0.002	0
6	0.011	0.001
7	0.057	0.010
8	0.194	0.075
9	0.387	0.315
10	0.349	0.599

Method	Mean	Variance
$n \cdot A(t)$	9.50	0
$n \cdot A(\infty)$	9.00	0
binomial with n and $p = A(\infty)$	9.00	0.9000
binomial with n and $p = A(t)$	9.50	0.4750

Comparison #4 – $n = 10$, $A(t) = 0.95$, $A(\infty) = 0.75$

$$n \cdot A(t) = 9.5$$
$$n \cdot A(\infty) = 7.5$$

k	P[W(t) = k], binomial with n, A(∞)	P[W(t) = k], binomial with n, A(t)
3	0.003	0
4	0.016	0
5	0.058	0
6	0.146	0.002
7	0.250	0.010
8	0.282	0.075
9	0.188	0.315
10	0.056	0.599

Method	Mean	Variance
$n \cdot A(t)$	9.50	0
$n \cdot A(\infty)$	7.50	0
binomial with n and $p = A(\infty)$	7.50	1.875
binomial with n and $p = A(t)$	9.50	0.475

Table 3.4.3 (continued)

Comparison #5 – $n = 10$, $A(t) = 0.90$, $A(\infty) = 0.65$

$$n \cdot A(t) = 9$$
$$n \cdot A(\infty) = 6.5$$

k	P[W(t) = k], binomial with n, A(∞)	P[W(t) = k], binomial with n, A(t)
1	0.001	0
2	0.004	0
3	0.021	0
4	0.069	0
5	0.154	0.002
6	0.238	0.011
7	0.252	0.057
8	0.176	0.194
9	0.072	0.387
10	0.013	0.349

Method	Mean	Variance
$n \cdot A(t)$	9.00	0
$n \cdot A(\infty)$	6.50	0
binomial with n and $p = A(\infty)$	6.50	2.275
binomial with n and $p = A(t)$	9.00	0.900

CHAPTER 4 – SUMMARY

Availability is probably the most informative indicator of performance for repairable components. The derivation and of evaluation of availability measures is usually focused on understanding the time evolution of component performance. It is not generally recognized that such an availability measure is actually an expected value with respect to frequency. At any point in time and with any associated value for availability, the number of components operating in a population of n independent, identical components is a random variable. The behavior of this random variable is described here.

4.1 Conclusions

The intuitive view that the frequency distribution of availability is binomial with parameters n and $p = A(t)$ is confirmed. This is done using a combination of direct analysis and Monte Carlo simulation. The two general cases considered are (1) exponential life and repair time distributions, and (2) Weibull life distribution and exponential repair time distribution. In the second case, a numerical method for approximating availability is developed. The implications of the analysis are that more accurate models of component behavior in terms of frequency are defined including exact confidence bounds for the number of components operating at any point in time. In addition, the time evolution of the frequency distribution is described and relationships between the frequency distribution and the components' life and repair time distribution parameters are explored. Finally, the implications for availability decision analyses are explained. The overall result is a new perspective on availability that should prove quite useful.

4.2 Areas for Further Development

Depending on the life and repair time distributions used to model component behavior, computing availability can be simple or quite difficult. For the case in which the life distribution is Weibull and the repair time distribution is exponential, the numerical approximation method defined here is efficient but somewhat inaccurate. However, such a method or one similar to it may be the only reasonable approach to computing values of availability. The need exists for alteration of this strategy or creation of a new method which provides more accurate availability calculations. Without considering the frequency distribution of availability, models of component population performance are incomplete. However, utilizing the frequency distribution with an inaccurate method of computing availability only results in a different wrong answer.

It is important to note that only two general cases are used in this analysis. However, the general form of the frequency distribution (binomial with parameters n and $p = A(t)$) should be the same for all cases in which the three underlying assumptions hold. There are other cases in which computing availability is difficult. There is a need for additional numerical methods for accurately computing availability for these cases.

As Seward, Nachlas, and Blanchard (1987) point out, the influence of probability in life-cycle cost analysis typically is not given appropriate attention. The frequency distribution of availability is an example of such incomplete analysis. They further state that an accurate life-cycle cost evaluation must incorporate the effects of random behavior. Steps should therefore be taken to include the frequency distribution of availability in reliability, maintainability, and life cycle cost analyses.

4.3 Closing Comments

An ability to compute availability values in the time domain is useful but treats only part of the issue of component behavior in that it only provides an expected value. The understanding of the frequency distribution of availability provides a basis for a thorough evaluation of component performance. It also provides the opportunity for more complete and thus more accurate models of population performance.

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APPENDIX A – MONTE CARLO SIMULATION

A.1 Random Number and Random Variable Generators

Law and Kelton (1991, p.451) provide a prime modulus multiplicative linear congruential random number generator based on Marse and Roberts' (1983) generator UNIRAN. This widely-accepted generator provides 100 independent random number streams and has a period of $2^{31}-1$. The algebraic representation of this random number generator is as follows:

$$u_i = 630{,}360{,}016\ u_{i-1} \mod 2{,}147{,}483{,}647 \qquad (10)$$

where u_i represents the $i^{\underline{th}}$ random number.

Law and Kelton (1991) also provide the necessary random variable generators. The exponential random variable generator (p. 486) works as follows:

1. Generate a random number u.
2. Return $x = -\frac{1}{\lambda}\ln(u)$.
3. x will be an exponential random variable with mean $1/\lambda$.

The Weibull random variable generator (p. 490) works as follows:

1. Generate a random number u.
2. Return $x = [-\frac{1}{\alpha}\ln(u)]^{1/\beta}$.
3. x will be a Weibull random variable with shape parameter β and scale parameter α.

Such a Weibull distribution has the distribution function

$$F(x) = 1 - e^{-\alpha x^{\beta}} \qquad (11)$$

A.2 Simulation Code for Exponential Life Distribution and Exponential Repair Time Distribution

```
program availability (input, output, av1data, av2data, av3data, av4data,
  av5data, av6data, av7data);

const
  lambda = xxxxxx;            { failure rate }
  mu = xxxxxxxxxx;            { repair rate }
  runs = xxxxxxxx;            { number of experiments }
  population = xx;            { sample size for each experiment }
  time1 = xxxxxxx;            { A(time1) = 0.99 }
  time2 = xxxxxxx;            { A(time2) = 0.95 }
  time3 = xxxxxxx;            { A(time3) = 0.90 }
  time4 = xxxxxxx;            { A(time4) = 0.85 }
  time5 = xxxxxxx;            { A(time5) = 0.80 }
  time6 = xxxxxxx;            { A(time6) = 0.75 }
  time7 = xxxxxxx;            { A(time7) = 0.70 }

var
  Zrng :array [1..100] of longint;    { array of random number streams }

  av1data,                    { output files }
  av2data,
  av3data,
  av4data,
  av5data,
  av6data,
  av7data
  :text;

  time,                       { current time }
  nexttime                    { time of next failure or repair completion }
  :real;

  t,                          { experiment number }
  i,                          { component number }
  w1,                         { number of components working at time1 }
  w2,                         { number of components working at time2 }
  w3,                         { number of components working at time3 }
  w4,                         { number of components working at time4 }
  w5,                         { number of components working at time5 }
  w6,                         { number of components working at time6 }
  w7                          { number of components working at time7 }
  :longint;

  working :boolean;           { status of component }
```

```
{ RANDOM NUMBER GENERATOR, see Law and Kelton (1991), p. 451 }

procedure Randdf;
forward;
function Rand(Stream :longint) :real;
forward;

procedure Randdf;

  begin { Randdf }

    { Set the seeds for all 100 streams. }

    Zrng[ 1]:=1973272912; Zrng[ 2]:= 281629770; Zrng[ 3]:=  20006270;
    Zrng[ 4]:=1280689831; Zrng[ 5]:=2096730329; Zrng[ 6]:=1933576050;
    Zrng[ 7]:= 913566091; Zrng[ 8]:= 246780520; Zrng[ 9]:=1363774876;
    Zrng[10]:= 604901985; Zrng[11]:=1511192140; Zrng[12]:=1259851944;
    Zrng[13]:= 824064364; Zrng[14]:= 150493284; Zrng[15]:= 242708531;
    Zrng[16]:=  75253171; Zrng[17]:=1964472944; Zrng[18]:=1202299975;
    Zrng[19]:= 233217322; Zrng[20]:=1911216000; Zrng[21]:= 726370533;
    Zrng[22]:= 403498145; Zrng[23]:= 993232223; Zrng[24]:=1103205531;
    Zrng[25]:= 762430696; Zrng[26]:=1922803170; Zrng[27]:=1385516923;
    Zrng[28]:=  76271663; Zrng[29]:= 413682397; Zrng[30]:= 726466604;
    Zrng[31]:= 336157058; Zrng[32]:=1432650381; Zrng[33]:=1120463904;
    Zrng[34]:= 595778810; Zrng[35]:= 877722890; Zrng[36]:=1046574445;
    Zrng[37]:=  68911991; Zrng[38]:=2088367019; Zrng[39]:= 748545416;
    Zrng[40]:= 622401386; Zrng[41]:=2122378830; Zrng[42]:= 640690903;
    Zrng[43]:=1774806513; Zrng[44]:=2132545692; Zrng[45]:=2079249579;
    Zrng[46]:=  78130110; Zrng[47]:= 852776735; Zrng[48]:=1187867272;
    Zrng[49]:=1351423507; Zrng[50]:=1645973084; Zrng[51]:=1997049139;
    Zrng[52]:= 922510944; Zrng[53]:=2045512870; Zrng[54]:= 898585771;
    Zrng[55]:= 243649545; Zrng[56]:=1004818771; Zrng[57]:= 773686062;
    Zrng[58]:= 403188473; Zrng[59]:= 372279877; Zrng[60]:=1901633463;
    Zrng[61]:= 498067494; Zrng[62]:=2087759558; Zrng[63]:= 493157915;
    Zrng[64]:= 597104727; Zrng[65]:=1530940798; Zrng[66]:=1814496276;
    Zrng[67]:= 536444882; Zrng[68]:=1663153658; Zrng[69]:= 855503735;
    Zrng[70]:=  67784357; Zrng[71]:=1432404475; Zrng[72]:= 619691088;
    Zrng[73]:= 119025595; Zrng[74]:= 880802310; Zrng[75]:= 176192644;
    Zrng[76]:=1116780070; Zrng[77]:= 277854671; Zrng[78]:=1366580350;
    Zrng[79]:=1142483975; Zrng[80]:=2026948561; Zrng[81]:=1053920743;
    Zrng[82]:= 786262391; Zrng[83]:=1792203830; Zrng[84]:=1494667770;
    Zrng[85]:=1923011392; Zrng[86]:=1433700034; Zrng[87]:=1244184613;
    Zrng[88]:=1147297105; Zrng[89]:= 539712780; Zrng[90]:=1545929719;
    Zrng[91]:= 190641742; Zrng[92]:=1645390429; Zrng[93]:= 264907697;
    Zrng[94]:= 620389253; Zrng[95]:=1502074852; Zrng[96]:= 927711160;
    Zrng[97]:= 364849192; Zrng[98]:=2049576050; Zrng[99]:= 638580085;
    Zrng[100]:=547070247
  end; { Randdf }
```

```
function Rand; { Generate the next random number. }

  { Define the constants. }

  const
    B2E15  =      32768;
    B2E16  =      65536;
    Modlus = 2147483647;
    Mult1  =      24112;
    Mult2  =      26143;

  var
    Hi15, Hi31, Low15, Lowprd, Ovflow, Zi :longint;

  begin { Rand }

    { Generate the next random number. }

    if (stream mod 100) = 0 then stream:=100 else stream:=stream mod 100;

    Zi     := Zrng[Stream];
    Hi15   := Zi DIV B2E16;
    Lowprd := (Zi - Hi15 * B2E16) * Mult1;
    Low15  := Lowprd DIV B2E16;
    Hi31   := Hi15 * Mult1 + Low15;
    Ovflow := Hi31 DIV B2E15;
    Zi     := (((Lowprd - Low15 * B2E16) - Modlus) +
              (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
    if Zi < 0 then Zi := Zi + Modlus;
    Hi15   := Zi DIV B2E16;
    Lowprd := (Zi - Hi15 * B2E16) * Mult2;
    Low15  := Lowprd DIV B2E16;
    Hi31   := Hi15 * Mult2 + Low15;
    Ovflow := Hi31 DIV B2E15;
    Zi     := (((Lowprd - Low15 * B2E16) - Modlus) +
              (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
    if Zi < 0 then Zi := Zi + Modlus;
    Zrng[Stream] := Zi;
    Rand   := (2 * (Zi DIV 256) + 1) / 16777216.0

  end; { Rand }

procedure failtime (var ftime :real);  { generates the time until failure }

{ In this case, the time until failure follows an exponential distribution   }
{ with rate lambda. The exponential random variable generator is taken       }
{ from Law and Kelton (1991), p. 490. Also, each experiment uses a            }
{ different random number stream.                                             }
```

```
  var
    r :real;                                { random number }

  begin

    r := Rand(t);                           { generate the random number  }
    ftime :=  (-1)/lambda*ln(r);            { generate the time           }
                                            { until failure               }
  end;

procedure repairtime (var rtime :real);     { generates the time until repair }
                                            { completion                      }

{ In this case, the time until repair completion follows an     }
{ exponential distribution with rate mu.                        }

  var
    r :real;

  begin

    r := Rand(t);
    rtime := (-1)/mu*ln(r)

  end;

begin

  { set up the output files }

  assign (av1data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av1data);
  assign (av2data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av2data);
  assign (av3data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av3data);
  assign (av4data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av4data);
  assign (av5data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av5data);
  assign (av6data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av6data);
  assign (av7data,'c:\xxxxxxxxxxxxxxxxx');
  rewrite (av7data);

  { initialize the random number streams }

  Randdf;
```

```
{ run the experiments }

for t := 1 to runs do begin

   { initialize the number working variables }

   w1 := 0;
   w2 := 0;
   w3 := 0;
   w4 := 0;
   w5 := 0;
   w6 := 0;
   w7 := 0;

   { test each component }

   for i := 1 to population do begin

      { initialize time and status }

      time := 0.0;
      working := true;

      { The component subsequently fails, gets repaired, fails, ...      }
      { For each time, time1 thru time7 the component's status is        }
      { checked. If the component is working at time"j" w"j" is          }
      { incremented.                                                     }

      while time < time1 do begin

         if working then begin             { if the component is working, a    }
            failtime (nexttime);           { failure time is generated         }
            working := false;
            time := time + nexttime
         end

         else begin                        { if the component is not working, }
            repairtime (nexttime);         { a repair time is generated.      }
            working := true;
            time := time + nexttime
         end;

      end;

      { w1 should be incremented if the first event after time1 is a     }
      { failure.                                                         }

      if not working then w1 := w1 + 1;
```

```
while time < time2 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
   end;

end;

if not working then w2 := w2 + 1;

while time < time3 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
   end;

end;

if not working then w3 := w3 + 1;

while time < time4 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
   end;
```

```
end;

if not working then w4 := w4 + 1;

while time < time5 do begin

  if working then begin
    failtime (nexttime);
    working := false;
    time := time + nexttime
  end

  else begin
    repairtime (nexttime);
    working := true;
    time := time + nexttime
  end;

end;

if not working then w5 := w5 + 1;

while time < time6 do begin

  if working then begin
    failtime (nexttime);
    working := false;
    time := time + nexttime
  end

  else begin
    repairtime (nexttime);
    working := true;
    time := time + nexttime
  end;

end;

if not working then w6 := w6 + 1;

while time < time7 do begin

  if working then begin
    failtime (nexttime);
    working := false;
    time := time + nexttime
  end
```

```
        else begin
          repairtime (nexttime);
          working := true;
          time := time + nexttime
        end;

      end;

      if not working then w7 := w7 + 1;

    end;

    { Write the results of the experiment to the output files. }

    writeln (av1data, w1:10);
    writeln (av2data, w2:10);
    writeln (av3data, w3:10);
    writeln (av4data, w4:10);
    writeln (av5data, w5:10);
    writeln (av6data, w6:10);
    writeln (av7data, w7:10);

  end;

close (av1data);
close (av2data);
close (av3data);
close (av4data);
close (av5data);
close (av6data);
close (av7data);

end.
```

A.3 Simulation Code for Weibull Life Distribution and Exponential Repair Time Distribution

```
program availability (input, output, av1data, av2data, av3data, av4data,
  av5data, av6data, av7data);

const
  alpha = xxx;          { life distribution scale parameter }
  beta = xxxx;          { life distribution shape parameter }
  mu = xxxxxx;          { repair rate }
  runs = xxxx;          { number of experiments }
  population = xx;      { sample size for each experiment }
```

```
  time1 = xxxxxxx;          { A(time1) = 0.99 }
  time2 = xxxxxxx;          { A(time2) = 0.95 }
  time3 = xxxxxxx;          { A(time3) = 0.90 }
  time4 = xxxxxxx;          { A(time4) = 0.85 }
  time5 = xxxxxxx;          { A(time5) = 0.80 }
  time6 = xxxxxxx;          { A(time6) = 0.75 }
  time7 = xxxxxxx;          { A(time7) = 0.70 }

var
  Zrng :array [1..100] of longint;    { array of random number streams }

  av1data,          { output files }
  av2data,
  av3data,
  av4data,
  av5data,
  av6data,
  av7data
  :text;

  time,                     { current time }
  nexttime                  { time of next failure or repair completion }
  :real;

  t,                        { experiment number }
  i,                        { component number }
  w1,                       { number of components working at time1 }
  w2,                       { number of components working at time2 }
  w3,                       { number of components working at time3 }
  w4,                       { number of components working at time4 }
  w5,                       { number of components working at time5 }
  w6,                       { number of components working at time6 }
  w7                        { number of components working at time7 }
  :longint;

  working :boolean;    { status of component }

{ RANDOM NUMBER GENERATOR, see Law and Kelton (1991), p. 451 }

procedure Randdf;
forward;
function Rand(Stream :longint) :real;
forward;

procedure Randdf;

  begin { Randdf }

    { Set the seeds for all 100 streams. }
```

```
    Zrng[ 1]:=1973272912; Zrng[ 2]:= 281629770; Zrng[ 3]:=  20006270;
    Zrng[ 4]:=1280689831; Zrng[ 5]:=2096730329; Zrng[ 6]:=1933576050;
    Zrng[ 7]:= 913566091; Zrng[ 8]:= 246780520; Zrng[ 9]:=1363774876;
    Zrng[10]:= 604901985; Zrng[11]:=1511192140; Zrng[12]:=1259851944;
    Zrng[13]:= 824064364; Zrng[14]:= 150493284; Zrng[15]:= 242708531;
    Zrng[16]:=  75253171; Zrng[17]:=1964472944; Zrng[18]:=1202299975;
    Zrng[19]:= 233217322; Zrng[20]:=1911216000; Zrng[21]:= 726370533;
    Zrng[22]:= 403498145; Zrng[23]:= 993232223; Zrng[24]:=1103205531;
    Zrng[25]:= 762430696; Zrng[26]:=1922803170; Zrng[27]:=1385516923;
    Zrng[28]:=  76271663; Zrng[29]:= 413682397; Zrng[30]:= 726466604;
    Zrng[31]:= 336157058; Zrng[32]:=1432650381; Zrng[33]:=1120463904;
    Zrng[34]:= 595778810; Zrng[35]:= 877722890; Zrng[36]:=1046574445;
    Zrng[37]:=  68911991; Zrng[38]:=2088367019; Zrng[39]:= 748545416;
    Zrng[40]:= 622401386; Zrng[41]:=2122378830; Zrng[42]:= 640690903;
    Zrng[43]:=1774806513; Zrng[44]:=2132545692; Zrng[45]:=2079249579;
    Zrng[46]:=  78130110; Zrng[47]:= 852776735; Zrng[48]:=1187867272;
    Zrng[49]:=1351423507; Zrng[50]:=1645973084; Zrng[51]:=1997049139;
    Zrng[52]:= 922510944; Zrng[53]:=2045512870; Zrng[54]:= 898585771;
    Zrng[55]:= 243649545; Zrng[56]:=1004818771; Zrng[57]:= 773686062;
    Zrng[58]:= 403188473; Zrng[59]:= 372279877; Zrng[60]:=1901633463;
    Zrng[61]:= 498067494; Zrng[62]:=2087759558; Zrng[63]:= 493157915;
    Zrng[64]:= 597104727; Zrng[65]:=1530940798; Zrng[66]:=1814496276;
    Zrng[67]:= 536444882; Zrng[68]:=1663153658; Zrng[69]:= 855503735;
    Zrng[70]:=  67784357; Zrng[71]:=1432404475; Zrng[72]:= 619691088;
    Zrng[73]:= 119025595; Zrng[74]:= 880802310; Zrng[75]:= 176192644;
    Zrng[76]:=1116780070; Zrng[77]:= 277854671; Zrng[78]:=1366580350;
    Zrng[79]:=1142483975; Zrng[80]:=2026948561; Zrng[81]:=1053920743;
    Zrng[82]:= 786262391; Zrng[83]:=1792203830; Zrng[84]:=1494667770;
    Zrng[85]:=1923011392; Zrng[86]:=1433700034; Zrng[87]:=1244184613;
    Zrng[88]:=1147297105; Zrng[89]:= 539712780; Zrng[90]:=1545929719;
    Zrng[91]:= 190641742; Zrng[92]:=1645390429; Zrng[93]:= 264907697;
    Zrng[94]:= 620389253; Zrng[95]:=1502074852; Zrng[96]:= 927711160;
    Zrng[97]:= 364849192; Zrng[98]:=2049576050; Zrng[99]:= 638580085;
    Zrng[100]:=547070247

  end; { Randdf }

function Rand; { Generate the next random number. }

  { Define the constants. }

  const
    B2E15  =      32768;
    B2E16  =      65536;
    Modlus = 2147483647;
    Mult1  =      24112;
    Mult2  =      26143;

  var
```

```
    Hi15, Hi31, Low15, Lowprd, Ovflow, Zi :longint;

  begin { Rand }

    { Generate the next random number. }

    if (stream mod 100) = 0 then stream:=100 else stream:=stream mod 100;

    Zi     := Zrng[Stream];
    Hi15   := Zi DIV B2E16;
    Lowprd := (Zi - Hi15 * B2E16) * Mult1;
    Low15  := Lowprd DIV B2E16;
    Hi31   := Hi15 * Mult1 + Low15;
    Ovflow := Hi31 DIV B2E15;
    Zi     := (((Lowprd - Low15 * B2E16) - Modlus) +
              (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
    if Zi < 0 then Zi := Zi + Modlus;
    Hi15   := Zi DIV B2E16;
    Lowprd := (Zi - Hi15 * B2E16) * Mult2;
    Low15  := Lowprd DIV B2E16;
    Hi31   := Hi15 * Mult2 + Low15;
    Ovflow := Hi31 DIV B2E15;
    Zi     := (((Lowprd - Low15 * B2E16) - Modlus) +
              (Hi31 - Ovflow * B2E15) * B2E16) + Ovflow;
    if Zi < 0 then Zi := Zi + Modlus;
    Zrng[Stream] := Zi;
    Rand   := (2 * (Zi DIV 256) + 1) / 16777216.0

  end; { Rand }

procedure failtime (var ftime :real);  { generates the time until failure }

{ In this case, the time until failure follows a Weibull distribution with    }
{ scale parameter alpha and shape parameter beta.  The Weibull random         }
{ variable generator is taken from Law and Kelton (1991), p. 490.  Also,      }
{ each experiment uses a different random number stream.                      }

  var
    r :real;                    { random number }

  begin

    r := Rand(t);                 { generate the random number }
    ftime :=  exp(ln(alpha)+(1/beta)*ln(-ln(r)));      { generate the time  }
                                                       { until failure      }
  end;

procedure repairtime (var rtime :real);        { generates the time until repair }
                                               { completion                      }
```

```
{ In this case, the time until repair completion follows a           }
{ exponential distribution with rate mu.                             }

  var
    r :real;

  begin

    r := Rand(t);
    rtime := (-1)/mu*ln(r)

  end;

begin

  { set up the output files }

  assign (av1data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av1data);
  assign (av2data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av2data);
  assign (av3data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av3data);
  assign (av4data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av4data);
  assign (av5data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av5data);
  assign (av6data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av6data);
  assign (av7data,'c:\xxxxxxxxxxxxxxx');
  rewrite (av7data);

  { initialize the random number streams }

  Randdf;

  { run the experiments }

  for t := 1 to runs do begin

    { initialize the number working variables }

    w1 := 0;
    w2 := 0;
    w3 := 0;
    w4 := 0;
    w5 := 0;
    w6 := 0;
    w7 := 0;
```

```
{ test each component }

for i := 1 to population do begin

   { initialize time and status }

   time := 0.0;
   working := true;

   { The component subsequently fails, gets repaired, fails, ...    }
   { For each time, time1 thru time7 the component's status is     }
   { checked. If the component is working at time"j" w"j" is       }
   { incremented.                                                  }

   while time < time1 do begin

      if working then begin          { if the component is working, a     }
         failtime (nexttime);        { failure time is generated.         }
         working := false;
         time := time + nexttime
      end

      else begin                     { if the component is not working,   }
         repairtime (nexttime);      { a repair time is generated.        }
         working := true;
         time := time + nexttime
      end;

   end;

   { w1 should be incremented if the first event after time1 is a  }
   { failure.                                                      }

   if not working then w1 := w1 + 1;

   while time < time2 do begin

      if working then begin
         failtime (nexttime);
         working := false;
         time := time + nexttime
      end

      else begin
         repairtime (nexttime);
         working := true;
         time := time + nexttime
      end;
```

```
end;

if not working then w2 := w2 + 1;

while time < time3 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
   end;

end;

if not working then w3 := w3 + 1;

while time < time4 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
   end;

end;

if not working then w4 := w4 + 1;

while time < time5 do begin

   if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
   end

   else begin
```

```
      repairtime (nexttime);
      working := true;
      time := time + nexttime
    end;

  end;

  if not working then w5 := w5 + 1;

  while time < time6 do begin

    if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
    end

    else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
    end;

  end;

  if not working then w6 := w6 + 1;

  while time < time7 do begin

    if working then begin
      failtime (nexttime);
      working := false;
      time := time + nexttime
    end

    else begin
      repairtime (nexttime);
      working := true;
      time := time + nexttime
    end;

  end;

  if not working then w7 := w7 + 1;

end;

{ Write the results of the experiment to the output files. }
```

```
    writeln (av1data, w1:10);
    writeln (av2data, w2:10);
    writeln (av3data, w3:10);
    writeln (av4data, w4:10);
    writeln (av5data, w5:10);
    writeln (av6data, w6:10);
    writeln (av7data, w7:10);

  end;

close (av1data);
close (av2data);
close (av3data);
close (av4data);
close (av5data);
close (av6data);
close (av7data);

end.
```

APPENDIX B – CHI-SQUARE TEST RESULTS

B.1 Chi-Square Test Results for Exponential Life Distribution and Exponential Repair Time Distribution

Table B.1.1 – Example #1, t_1

The distribution tested is the binomial.

$n = 100$ $\quad\quad$ $A(t_1) = 0.99$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 96$	24	18.37446
2	$x = 97$	62	61.00066
3	$x = 98$	181	184.8695
4	$x = 99$	377	369.7395
5	$x = 100$	356	366.0425

Test statistic value = 2.237776
Degrees of freedom = 4
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 9.488

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.2 – Example #1, t_2

The distribution tested is the binomial.

$n = 100$ $A(t_2) = 0.95$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 89$	12	11.46906
2	$x = 90$	13	16.71101
3	$x = 91$	33	34.89112
4	$x = 92$	71	64.85194
5	$x = 93$	98	105.9946
6	$x = 94$	151	149.9711
7	$x = 95$	165	179.9653
8	$x = 96$	196	178.0907
9	$x = 97$	140	139.535
10	$x = 98$	87	81.15809
11	$x = 99$	25	31.1516
12	$x = 100$	9	5.918803

Test statistic value = 8.430394
Degrees of freedom = 11
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 19.68

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.3 – Example #1, t_3

The distribution tested is the binomial.

$n = 100$ $A(t_3) = 0.90$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 82$	10	10.00532
2	$x = 83$	15	10.58946
3	$x = 84$	17	19.28794
4	$x = 85$	31	32.67605
5	$x = 86$	52	51.29378
6	$x = 87$	74	74.28753
7	$x = 88$	100	98.76861
8	$x = 89$	134	119.854
9	$x = 90$	129	131.8394
10	$x = 91$	122	130.3906
11	$x = 92$	106	114.8003
12	$x = 93$	93	88.87766
13	$x = 94$	50	59.56691
14	$x = 95$	31	33.85907
15	$x = 96$	20	15.87143
16	$x \geq 97$	16	7.834929

Test statistic value = 16.71808
Degrees of freedom = 15
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 25

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.4 – Example #1, t_4

The distribution tested is the binomial.

n = 100 A(t) = 0.85

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 75$	7	6.079886
2	$x = 76$	8	5.805827
3	$x = 77$	8	10.25445
4	$x = 78$	19	17.13457
5	$x = 79$	23	27.03937
6	$x = 80$	47	40.22108
7	$x = 81$	53	56.27644
8	$x = 82$	73	73.89146
9	$x = 83$	98	90.80639
10	$x = 84$	96	104.1391
11	$x = 85$	128	111.0818
12	$x = 86$	118	109.7902
13	$x = 87$	92	100.1152
14	$x = 88$	83	83.8086
15	$x = 89$	59	64.03355
16	$x = 90$	32	44.34921
17	$x = 91$	33	27.61673
18	$x = 92$	11	15.30927
19	$x \geq 93$	12	12.16422

Test statistic value = 14.77584
Degrees of freedom = 18
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 28.87

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.5 – Example #1, t_5

The distribution tested is the binomial.

n = 100   A(t)=0.8

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 69$	9	6.058676
2	$x = 70$	5	5.189079
3	$x = 71$	7	8.770274
4	$x = 72$	9	14.12989
5	$x = 73$	30	21.67872
6	$x = 74$	28	31.63923
7	$x = 75$	43	43.87307
8	$x = 76$	55	57.72773
9	$x = 77$	73	71.97227
10	$x = 78$	96	84.89035
11	$x = 79$	80	94.56142
12	$x = 80$	97	99.28947
13	$x = 81$	99	98.06366
14	$x = 82$	99	90.8883
15	$x = 83$	70	78.8429
16	$x = 84$	67	63.82522
17	$x = 85$	47	48.05664
18	$x = 86$	43	33.5279
19	$x = 87$	19	21.58119
20	$x = 88$	10	12.75252
21	$x = 89$	9	6.877765
22	$x \geq 90$	5	5.695778

Test statistic value = 17.40173
Degrees of freedom = 21
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 32.67

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.6 – Example #1, t_6

The distribution tested is the binomial.

n = 100 A(t) = 0 .75

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 63$	9	5.181821
2	$x = 65$	12	11.24404
3	$x = 66$	9	11.16722
4	$x = 67$	16	17.00084
5	$x = 68$	34	24.75123
6	$x = 69$	32	34.43649
7	$x = 70$	42	45.75135
8	$x = 71$	66	57.99469
9	$x = 72$	51	70.07696
10	$x = 73$	85	80.63646
11	$x = 74$	93	88.26425
12	$x = 75$	95	91.79481
13	$x = 76$	83	90.587
14	$x = 77$	88	84.70471
15	$x = 78$	67	74.93108
16	$x = 79$	64	62.60067
17	$x = 80$	50	49.29802
18	$x = 81$	39	36.51704
19	$x = 82$	25	25.38381
20	$x = 83$	16	16.51476
21	$x = 84$	13	10.02683
22	$x = 85$	7	5.662207
23	$x \geq 86$	4	5.420477

Test statistic value = 17.58512
Degrees of freedom = 22
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 33.92

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.7 – Example #1, t_7

The distribution tested is the binomial.

n = 100 A(t) = 0.7

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 58$	7	7.173673
2	$x = 59$	4	5.324919
3	$x = 60$	10	8.490287
4	$x = 61$	12	12.9906
5	$x = 62$	19	19.06686
6	$x = 63$	24	26.83483
7	$x = 64$	38	36.19906
8	$x = 65$	55	46.78034
9	$x = 66$	74	57.88475
10	$x = 67$	70	68.54012
11	$x = 68$	84	77.61163
12	$x = 69$	83	83.98553
13	$x = 70$	79	86.78505
14	$x = 71$	87	85.56274
15	$x = 72$	84	80.41315
16	$x = 73$	53	71.96793
17	$x = 74$	58	61.27001
18	$x = 75$	46	49.56062
19	$x = 76$	32	38.03994
20	$x = 77$	29	27.66542
21	$x = 78$	24	19.03476
22	$x = 79$	13	12.36858
23	$x = 80$	7	7.57575
24	$x \geq 81$	8	8.887329

Test statistic value = 16.36164
Degrees of freedom = 23
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 35.17

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.8 – Example #2, t_1

The distribution tested is the binomial.

n = 10 A(t) = 0 .99

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 9$	101	95.61774
2	$x = 10$	899	904.3814

Test statistic value = 0.3349848
Degrees of freedom = 1
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 3.841

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.9 – Example #2, t_2

The distribution tested is the binomial.

$n = 10$ $\quad$ $A(t) = 0.95$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 7$	14	11.50358
2	$x = 8$	73	74.63497
3	$x = 9$	312	315.1255
4	$x = 10$	601	598.7384

Test statistic value = 0.6171093
Degrees of freedom = 3
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 7.815

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.10 – Example #2, t_3

The distribution tested is the binomial.

$n = 10$ $A(t) = 0.9$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 6$	17	12.79522
2	$x = 7$	61	57.39571
3	$x = 8$	184	193.7105
4	$x = 9$	406	387.421
5	$x = 10$	332	348.6788

Test statistic value = 3.783683
Degrees of freedom = 4
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 9.488

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.11 – Example #2, t_4

The distribution tested is the binomial.

$n = 10$ $\quad$ $A(t) = 0.85$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 5$	12	9.874098
2	$x = 6$	34	40.09575
3	$x = 7$	135	129.8339
4	$x = 8$	281	275.897
5	$x = 9$	356	347.426
6	$x = 10$	182	196.8748

Test statistic value = 3.019837
Degrees of freedom = 5
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 11.07

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.12 – Example #2, t_5

The distribution tested is the binomial.

$n = 10$ $A(t) = 0.8$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 4$	4	6.369372
2	$x = 5$	19	26.42407
3	$x = 6$	96	88.08023
4	$x = 7$	200	201.3262
5	$x = 8$	313	301.9894
6	$x = 9$	261	268.4352
7	$x = 10$	107	107.3741

Test statistic value = 4.29679
Degrees of freedom = 6
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 12.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.13 – Example #2, t_6

The distribution tested is the binomial.

n = 10 A(t) = 0.75

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 4$	15	19.72771
2	$x = 5$	57	58.39922
3	$x = 6$	153	145.998
4	$x = 7$	259	250.2823
5	$x = 8$	273	281.5676
6	$x = 9$	195	187.7117
7	$x = 10$	48	56.31352

Test statistic value = 3.576967
Degrees of freedom = 6
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 12.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.14 – Example #2, t_7

The distribution tested is the binomial.

n = 10 A(t) = 0.7

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 3$	15	10.59208
2	$x = 4$	35	36.75694
3	$x = 5$	106	102.9194
4	$x = 6$	189	200.1211
5	$x = 7$	278	266.8281
6	$x = 8$	226	233.4746
7	$x = 9$	119	121.0609
8	$x = 10$	32	28.24755

Test statistic value = 3.869192
Degrees of freedom = 7
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 14.07

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.15 – Example #3, t_1

The distribution tested is the binomial.

n = 50 A(t) = 0.99

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 47$	20	13.81747
2	$x = 48$	78	75.6192
3	$x = 49$	322	305.5636
4	$x = 50$	580	605.0167

Test statistic value = 4.759821
Degrees of freedom = 3
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 7.815

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.16 – Example #3, t_2

The distribution tested is the binomial.

n = 50 A(t) = 0.95

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 43$	12	11.78624
2	$x = 44$	22	25.98928
3	$x = 45$	70	65.83951
4	$x = 46$	133	135.9729
5	$x = 47$	226	219.871
6	$x = 48$	268	261.0969
7	$x = 49$	203	202.4833
8	$x = 50$	66	76.94369

Test statistic value = 2.85532
Degrees of freedom = 7
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 14.07

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.17 – Example #3, t_3

The distribution tested is the binomial.

n = 50 A(t) = .9

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 39$	9	9.354169
2	$x = 40$	13	15.18263
3	$x = 41$	24	33.32772
4	$x = 42$	62	64.2749
5	$x = 43$	118	107.623
6	$x = 44$	172	154.0966
7	$x = 45$	196	184.9159
8	$x = 46$	178	180.8959
9	$x = 47$	117	138.5585
10	$x = 48$	73	77.93913
11	$x = 49$	34	28.63069
12	$x = 50$	4	5.153524

Test statistic value = 11.74216
Degrees of freedom = 11
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 19.68

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.18 – Example #3, t_4

The distribution tested is the binomial.

$n = 50$ $A(t) = 0.85$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 35$	3	5.28673
2	$x = 36$	8	7.879366
3	$x = 37$	19	16.89451
4	$x = 38$	29	32.75164
5	$x = 39$	69	57.10542
6	$x = 40$	91	88.98931
7	$x = 41$	136	122.9934
8	$x = 42$	148	149.3492
9	$x = 43$	173	157.4534
10	$x = 44$	127	141.9467
11	$x = 45$	93	107.2487
12	$x = 46$	66	66.05901
13	$x = 47$	20	31.85825
14	$x \geq 48$	18	14.18861

Test statistic value = 16.03337
Degrees of freedom = 13
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 22.36

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.19 – Example #3, t_5

The distribution tested is the binomial.

n = 50 A(t) = 0.8

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 32$	6	6.260878
2	$x = 33$	4	8.181011
3	$x = 34$	26	16.36202
4	$x = 35$	30	29.91913
5	$x = 36$	44	49.86523
6	$x = 37$	78	75.47174
7	$x = 38$	100	103.2771
8	$x = 39$	116	127.1103
9	$x = 40$	151	139.8213
10	$x = 41$	150	136.411
11	$x = 42$	119	116.9237
12	$x = 43$	74	87.01302
13	$x = 44$	44	55.37192
14	$x = 45$	41	29.53169
15	$x = 46$	13	12.83987
16	$x \geq 47$	4	5.65646

Test statistic value = 21.18132
Degrees of freedom = 15
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 25

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.20 – Example #3, t_6

The distribution tested is the binomial.

n = 50   A(t) = 0.75

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 29$	9	6.263059
2	$x = 30$	5	7.65489
3	$x = 31$	13	14.81592
4	$x = 32$	26	26.39086
5	$x = 33$	58	43.18504
6	$x = 34$	62	64.77757
7	$x = 35$	102	88.83781
8	$x = 36$	96	111.0473
9	$x = 37$	110	126.0537
10	$x = 38$	136	129.3708
11	$x = 39$	133	119.4193
12	$x = 40$	86	98.52087
13	$x = 41$	75	72.08846
14	$x = 42$	42	46.34259
15	$x = 43$	30	25.86563
16	$x = 44$	10	12.34496
17	$x \geq 45$	7	7.046407

Test statistic value = 18.68674
Degrees of freedom = 16
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 26.3

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.1.21 – Example #3, t_7

The distribution tested is the binomial.

$n = 50$ $\quad$ $A(t) = 0.7$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 26$	6	5.592196
2	$x = 27$	10	6.68399
3	$x = 28$	12	12.81097
4	$x = 29$	23	22.6769
5	$x = 30$	25	37.03892
6	$x = 31$	57	55.75751
7	$x = 32$	83	77.24739
8	$x = 33$	112	98.31489
9	$x = 34$	122	114.7007
10	$x = 35$	119	122.3475
11	$x = 36$	107	118.949
12	$x = 37$	100	105.018
13	$x = 38$	81	83.83014
14	$x = 39$	60	60.18575
15	$x = 40$	34	38.61919
16	$x = 41$	32	21.97841
17	$x = 42$	11	10.9892
18	$x \geq 43$	6	7.26424

Test statistic value = 15.4393
Degrees of freedom = 17
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 27.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

B.2 Chi-Square Test Results for Weibull Life and Exponential Repair Time Distributions

Table B.2.1 – Example #1, t_1

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = .99287$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 96$	5	5.894472
2	$x = 97$	26	29.30054
3	$x = 98$	131	124.9027
4	$x = 99$	353	351.3728
5	$x = 100$	485	489.2934

Test statistic value = 0.8503742
Degrees of freedom = 3
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 7.815

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.2 – Example #1, t_2

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = 0.95485$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 89$	7	5.558004
2	$x = 90$	9	9.528102
3	$x = 91$	22	22.14332
4	$x = 92$	46	45.81157
5	$x = 93$	91	83.3412
6	$x = 94$	125	131.2525
7	$x = 95$	168	175.3125
8	$x = 96$	200	193.103
9	$x = 97$	164	168.405
10	$x = 98$	108	109.0254
11	$x = 99$	46	46.58007
12	$x = 100$	14	9.850942

Test statistic value = 3.837712
Degrees of freedom = 10
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 18.31

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.3 – Example #1, t_3

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = 0.90398$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 82$	7	6.686126
2	$x = 83$	9	7.659301
3	$x = 84$	11	14.59338
4	$x = 85$	31	25.86153
5	$x = 86$	33	42.46629
6	$x = 87$	60	64.33546
7	$x = 88$	102	89.47636
8	$x = 89$	127	113.5787
9	$x = 90$	116	130.6906
10	$x = 91$	134	135.2073
11	$x = 92$	117	124.5237
12	$x = 93$	110	100.8455
13	$x = 94$	74	70.70071
14	$x = 95$	38	42.03864
15	$x = 96$	16	20.61316
16	$x \geq 97$	15	10.78886

Test statistic value = 14.06216
Degrees of freedom = 14
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 23.68

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.4 – Example #1, t_4

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = 0.84443$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 75$	5	9.610716
2	$x = 76$	8	8.45011
3	$x = 77$	13	14.29621
4	$x = 78$	21	22.88189
5	$x = 79$	35	34.58801
6	$x = 80$	50	49.28251
7	$x = 81$	64	66.05044
8	$x = 82$	72	83.07179
9	$x = 83$	106	97.78805
10	$x = 84$	115	107.422
11	$x = 85$	115	109.7571
12	$x = 86$	124	103.9114
13	$x = 87$	88	90.76327
14	$x = 88$	76	72.77943
15	$x = 89$	45	53.26448
16	$x = 90$	34	35.33671
17	$x = 91$	13	21.07767
18	$x = 92$	9	11.1922
19	$x \geq 93$	7	8.321457

Test statistic value = 14.71553
Degrees of freedom = 17
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 27.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.5 – Example #1, t_5

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = 0.79273$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 68$	7	5.504054
2	$x = 70$	8	12.71468
3	$x = 71$	10	12.91964
4	$x = 72$	17	19.90237
5	$x = 73$	29	29.19636
6	$x = 74$	33	40.74271
7	$x = 75$	61	54.01951
8	$x = 76$	68	67.96195
9	$x = 77$	77	81.01684
10	$x = 78$	81	91.36866
11	$x = 79$	120	97.31542
12	$x = 80$	109	97.70116
13	$x = 81$	96	92.26424
14	$x = 82$	80	81.764
15	$x = 83$	72	67.81806
16	$x = 84$	55	52.49331
17	$x = 85$	34	37.79147
18	$x = 86$	18	25.21015
19	$x = 87$	16	15.51575
20	$x = 88$	5	8.76642
21	$x \geq 89$	4	8.004076

Test statistic value = 20.22877
Degrees of freedom = 19
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 30.14

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.6 – Example #1, t_6

The distribution tested is the binomial.

$n = 100$ $\hat{A}(t) = 0.75432$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 64$	9	7.070549
2	$x = 65$	3	5.539763
3	$x = 66$	5	9.019883
4	$x = 67$	16	14.05371
5	$x = 68$	23	20.94024
6	$x = 69$	26	29.81731
7	$x = 70$	39	40.54321
8	$x = 71$	43	52.59775
9	$x = 72$	66	65.04569
10	$x = 73$	74	76.60193
11	$x = 74$	83	85.81397
12	$x = 75$	97	91.33894
13	$x = 76$	104	92.25041
14	$x = 77$	95	88.28256
15	$x = 78$	96	79.92709
16	$x = 79$	70	68.3401
17	$x = 80$	43	55.07959
18	$x = 81$	47	41.75627
19	$x = 82$	31	29.70622
20	$x = 83$	10	19.78008
21	$x = 84$	10	12.2909
22	$x = 85$	6	7.103469
23	$x \geq 86$	4	7.084075

Test statistic value = 22.21981
Degrees of freedom = 21
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 32.67

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.7 – Example #1, t_7

The distribution tested is the binomial.

n = 100 $\hat{A}(t) = .70886$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 59$	5	7.302951
2	$x = 60$	5	5.443077
3	$x = 61$	15	8.690269
4	$x = 62$	11	13.3096
5	$x = 63$	17	19.54638
6	$x = 64$	23	27.51354
7	$x = 65$	29	37.10174
8	$x = 66$	43	47.90453
9	$x = 67$	75	59.18877
10	$x = 68$	69	69.93634
11	$x = 69$	69	78.97005
12	$x = 70$	82	85.14999
13	$x = 71$	104	87.6004
14	$x = 72$	84	85.90733
15	$x = 73$	93	80.22762
16	$x = 74$	82	71.2713
17	$x = 75$	56	60.15685
18	$x = 76$	46	48.18036
19	$x = 77$	32	36.56363
20	$x = 78$	24	26.25071
21	$x = 79$	13	17.79898
22	$x = 80$	10	11.37582
23	$x = 81$	8	6.8389
24	$x \geq 82$	5	7.671466

Test statistic value = 25.19641
Degrees of freedom = 22
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 33.92

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.8 – Example #2, t_1

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.9922$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 9$	73	75.31849
2	$x = 10$	927	924.6846

Test statistic value = 0.07716671
Degrees of freedom = 0
Insufficient degrees of freedom to run the test.

Table B.2.9 – Example #2, t_2

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.9545$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 7$	7	8.880356
2	$x = 8$	72	64.18636
3	$x = 9$	286	299.223
4	$x = 10$	635	627.7109

Test statistic value = 2.018316
Degrees of freedom = 2
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 5.991

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.10 – Example #2, t_3

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.9011$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 6$	11	12.31043
2	$x = 7$	60	55.9991
3	$x = 8$	185	191.3326
4	$x = 9$	390	387.3942
5	$x = 10$	354	352.9634

Test statistic value = 0 .6555056
Degrees of freedom = 3
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 7.815

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.11 – Example #2, t_4

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.8432$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 5$	13	11.93835
2	$x = 6$	44	45.62333
3	$x = 7$	147	140.1953
4	$x = 8$	265	282.7153
5	$x = 9$	354	337.8481
6	$x = 10$	177	181.6795

Test statistic value = 2.485246
Degrees of freedom = 4
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 9.488

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.12 – Example #2, t_5

The distribution tested is the binomial.

n = 10 $\hat{A}(t) = 0.7936$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 4$	5	7.495933
2	$x = 5$	28	29.7137
3	$x = 6$	97	95.20674
4	$x = 7$	213	209.1807
5	$x = 8$	293	301.6095
6	$x = 9$	280	257.7061
7	$x = 10$	84	99.08703

Test statistic value = 5.504961
Degrees of freedom = 5
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 11.07

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.13 – Example #2, t_6

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.7557$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 4$	11	17.5998
2	$x = 5$	52	54.04629
3	$x = 6$	147	139.3191
4	$x = 7$	251	246.2626
5	$x = 8$	286	285.6641
6	$x = 9$	201	196.3673
7	$x = 10$	52	60.74283

Test statistic value = 4.435012
Degrees of freedom = 5
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 11.07

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.14 – Example #2, t_7

The distribution tested is the binomial.

$n = 10$ $\hat{A}(t) = 0.7063$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 3$	7	9.332815
2	$x = 4$	30	33.54284
3	$x = 5$	93	96.79802
4	$x = 6$	206	193.9861
5	$x = 7$	259	266.574
6	$x = 8$	263	240.3999
7	$x = 9$	115	128.4716
8	$x = 10$	27	30.89531

Test statistic value = 6.093983
Degrees of freedom = 6
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 12.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.15 – Example #3, t_1

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.99328$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 48$	46	44.72311
2	$x = 49$	241	241.4643
3	$x = 50$	713	713.8142

Test statistic value = 0.03827761
Degrees of freedom = 1
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 3.841

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.16 – Example #3, t_2

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.95492$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 43$	7	6.877435
2	$x = 44$	11	17.52304
3	$x = 45$	56	49.49156
4	$x = 46$	113	113.9531
5	$x = 47$	208	205.4334
6	$x = 48$	275	271.9783
7	$x = 49$	230	235.1532
8	$x = 50$	100	99.62401

Test statistic value = 3.474274
Degrees of freedom = 6
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 12.59

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.17 – Example #3, t_3

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.90076$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 39$	9	8.852822
2	$x = 40$	16	14.5514
3	$x = 41$	36	32.21391
4	$x = 42$	59	62.65546
5	$x = 43$	105	105.8041
6	$x = 44$	144	152.7813
7	$x = 45$	188	184.8976
8	$x = 46$	195	182.4172
9	$x = 47$	130	140.9127
10	$x = 48$	80	79.93787
11	$x = 49$	31	29.6148
12	$x = 50$	7	5.376024

Test statistic value = 3.636246
Degrees of freedom = 10
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 18.31

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.18 – Example #3, t_4

The distribution tested is the binomial.

n = 50 $\hat{A}(t) = 0.84758$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 35$	2	6.147147
2	$x = 36$	13	8.896945
3	$x = 37$	15	18.72001
4	$x = 38$	36	35.61266
5	$x = 39$	58	60.93402
6	$x = 40$	93	93.18185
7	$x = 41$	140	126.3823
8	$x = 42$	154	150.5977
9	$x = 43$	163	155.804
10	$x = 44$	116	137.836
11	$x = 45$	105	102.1975
12	$x = 46$	62	61.77195
13	$x = 47$	29	29.23428
14	$x \geq 48$	14	12.72302

Test statistic value = 11.1187
Degrees of freedom = 12
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 21.03

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.19 – Example #3, t_5

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.7957$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 32$	8	7.859292
2	$x = 33$	9	9.831257
3	$x = 34$	9	19.1452
4	$x = 35$	40	34.08731
5	$x = 36$	57	55.31752
6	$x = 37$	83	81.52111
7	$x = 38$	111	108.6202
8	$x = 39$	135	130.1692
9	$x = 40$	133	139.4191
10	$x = 41$	133	132.4401
11	$x = 42$	111	110.5334
12	$x = 43$	89	80.09332
13	$x = 44$	42	49.62755
14	$x = 45$	32	25.77167
15	$x \geq 46$	8	15.55786

Test statistic value = 14.42326
Degrees of freedom = 13
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 22.36

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.20 – Example #3, t_6

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.75936$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 30$	4	9.07515
2	$x = 31$	12	10.53973
3	$x = 32$	19	19.74755
4	$x = 33$	35	33.99004
5	$x = 34$	57	53.62923
6	$x = 35$	81	77.36305
7	$x = 36$	95	101.7191
8	$x = 37$	124	121.4531
9	$x = 38$	119	131.1138
10	$x = 39$	148	127.3048
11	$x = 40$	112	110.4733
12	$x = 41$	83	85.02638
13	$x = 42$	49	57.49459
14	$x = 43$	38	33.75426
15	$x = 44$	14	16.94548
16	$x \geq 45$	10	10.37923

Test statistic value = 10.84673
Degrees of freedom = 14
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 23.68

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

Table B.2.21 – Example #3, t_7

The distribution tested is the binomial.

$n = 50$ $\hat{A}(t) = 0.70506$

Interval Number	Interval Limits	Observed Frequency	Expected Frequency
1	$x \leq 27$	7	9.901256
2	$x = 28$	10	10.78038
3	$x = 29$	15	19.55021
4	$x = 30$	34	32.71462
5	$x = 31$	48	50.45482
6	$x = 32$	85	71.61412
7	$x = 33$	92	93.3791
8	$x = 34$	121	111.6123
9	$x = 35$	121	121.971
10	$x = 36$	108	121.4892
11	$x = 37$	106	109.8896
12	$x = 38$	103	89.86871
13	$x = 39$	59	66.10246
14	$x = 40$	45	43.4553
15	$x = 41$	27	25.33678
16	$x = 42$	11	12.97887
17	$x \geq 43$	8	8.892201

Test statistic value = 10.32789
Degrees of freedom = 15
Level of significance = 0.05
Critical chi-square value at the 0.05 level of significance = 25

At the stated level of significance the distribution tested should not be rejected as a descriptor of the random variable represented by the data.

B.3 Other Distributions

In order to investigate the power of the chi-square test, an attempt was made to fit several other discrete distributions to the 42 data sets. These distributions were the uniform, the negative binomial, and the Poisson. In all cases, the conclusion was to reject the specific distribution as an adequate representation of the data.

VITA

Charles Richard Cassady was born on August 21, 1970 in Martinsville, Virginia. He graduated as valedictorian from Bassett High School in Bassett, Virginia in June, 1988. In August, 1992, he received his Bachelor of Science summa cum laude in Industrial and Systems Engineering from Virginia Polytechnic Institute and State University in Blacksburg, Virginia. While completing his bachelor's degree, he was inducted into Tau Beta Pi, Alpha Pi Mu, Kappa Theta Epsilon, Phi Kappa Phi, Phi Eta Sigma, and the Golden Key National Honor Society. In addition, he participated in the Cooperative Education Program as an employee of the Management Systems Department of Carilion Health System in Roanoke, Virginia. In December, 1993, he will receive his Master of Science in Industrial and Systems Engineering with concentration in operations research from Virginia Polytechnic Institute and State University. He is currently working toward his Ph.D. in Industrial and Systems Engineering at Virginia Polytechnic Institute and State University. He is a member of the Institute of Industrial Engineers, the Operations Research Society of America, the Institute of Management Science, and the American Society for Quality Control.