ACOUSTIC PREDICTION AND NOISE CONTROL
OF A REFRIGERATION COMPRESSOR

by

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(ABSTRACT)

In this study, the prediction and control of the acoustic radiation from a Bristol H25A refrigeration compressor are investigated. For the acoustic prediction, a modal decomposition approach is used. To this end, a boundary element model of the shell is created, and it is used to compute the modal radiation efficiency curves of the shell. These radiation efficiencies are then used in conjunction with the experimentally measured spring forces to obtain the acoustic power radiated by the compressor. Of twenty-three structural modes included in the analysis, it is found that eight have high radiation efficiency and six contribute significantly to the total radiated power. The analytically predicted overall radiated sound power of 82.2 dBA agrees very well with the 82 dBA experimentally measured.

For the noise control of the compressor, three approaches are investigated to reduce the forces transmitted to the shell and thus the radiation. (a) The spring mounts are moved to various locations on the shell, (b) dynamic vibration absorbers (DVAs) are added to the mounts, and (c) low modulus materials are inserted between the mounts and the springs to create an impedance mismatch. For all three approaches, efficient analytical methods to compute the radiated acoustic power upon the system modifications are developed. The most promising approach is the insertion of the low modulus materials, which yields a reduction of 6.4 dBA on the total radiated acoustic power. The addition of DVAs and the relocation of the mounts yield a reduction of 5.5 dBA and 1.7 dBA in the total radiated acoustic power, respectively.
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To my parents and

and my sister
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INTRODUCTION

1.1 MOTIVATION

During the last decade, the environmentalist movement has been growing very rapidly. As a result, various countries are creating government offices that have the specific task of looking after the environment. These offices are involved in a wide range of issues from the pollution on cities to the protection of wild life. One such issue is the concern about noise in industry. People are becoming aware of the negative effects of noise on human health and behavior, and the government offices are implementing rigorous regulations in industry that would guarantee low noise levels for the working environment [1]. Therefore, new mechanical systems are being designed so as to comply with these regulations, and acoustic control is now counted as one of the design variables. In this trend, Bristol Compressors Co. is making an effort to design refrigeration compressors with low sound radiation levels.

1.2 LITERATURE REVIEW

A typical refrigeration compressor consists of two major components: the compression mechanism and the hermetic shell. The compression mechanism is surrounded by the hermetic shell, to which it is connected through suspension springs and discharge/inlet pipes. For the case of the reciprocating compressor, the most important components of the compression mechanism are the cylinder housing (where the compression takes place), an electric motor that powers the pistons, a discharge/inlet pipe, and a muffler.
There are two types of reciprocating compressors: low-side and high-side compressors. In low-side compressors, the refrigerant gas fills the space between the compressor mechanisms and the shell at suction conditions. Upon compression, the refrigerant gas leaves the compressor through the discharge pipe. In contrast, in high-side compressors, the refrigerant gas enters the compression mechanisms directly through the inlet pipe. After it is compressed, it fills the space between the compression mechanisms and the shell before it leaves the compressor. The lower part of the shell in both low-side and high-side compressors is filled with oil, which is used to lubricate the mechanisms.

The prediction and noise control of sound radiated by these compressors have been carried out by many researchers. In 1987, Roys and Soedel [2] reviewed the noise control literature on small, high speed compressors. They divided the material in three topics: noise sources, transmission paths, and radiation. Under noise sources, physical phenomena such as the impact of the valves with their seats and stops, varying cylinder pressures, and unbalanced rotations are considered. Under transmission paths, the suspension springs, the discharge pipe and the fluid contained between the compressor and its housing are studied. Finally, under sound radiation, the shell of the compressor is considered since it is the major contributor to the radiated sound.

The fact that the shell is the major sound radiator has been the motivation for various researchers to address the effects of modifying the shell properties. Ingalls [3] divided the sound radiated by the shell in two frequency regions. The first region consists of the sound radiated below 500 Hz. Ingalls observed that this sound radiation was due to the rigid body motion of the entire shell. The second region corresponds to the radiation above 500 Hz. The radiated sound in this region is due to local deformations of the shell. Ingalls also studied the effects of the refrigerant gas on the shell parameters. He maintains that increasing the pressure of the fluid contained between the shell and the compressor results in increased natural frequencies of the shell. On the other hand, increasing the density of this fluid lowers the natural frequencies of the shell. The effect of these modifications on the radiated sound was not addressed.
Shell geometry modifications have yielded satisfactory results. Saito and Okubo [4] investigated the effect of having a shell with an asymmetric cross-section rather than a shell with a symmetric cross-section. They observed that in shells with a symmetric cross section, i.e., circular cross-section, the antinodes of the modes of the shell will tend to align themselves with the direction of the exciting forces. By making the cross-section of the shell asymmetric, they obtained a 6 dBA reduction in the sound pressure level.

Tojo and Saegusa [5] obtained a noise reduction by improving the characteristics of the transmission paths and the structure of the shell. From sound pressure measurements, they observed that sound was radiated mainly in the 500 Hz and 2000 Hz regions. Moreover, they observed a resonance of the shell at around 2000 Hz, which was responsible for a high amplitude vibration. Therefore, they decided to increase the natural frequencies of the shell by changing its geometry from cylindrical to semi-spherical. They also made a short bend in the discharge pipe, near the exiting point, to shift the resonance frequencies of the pipe to regions in the spectrum that did not include harmonics of the rotational frequency. With these changes, they obtained a reduction of 2 dBA in sound pressure level. A similar approach was taken by Simpson and Sisson [6] who changed the metal discharge pipe to a flexible automotive air-conditioning hose. This change caused a reduction of the forces transmitted to the shell.

The effect of the shell thickness was studied by Bucciarelli, et al. [7]. They found out that an increase of 25% in shell thickness resulted in an increase of the fundamental frequency by 8.6%, and an average of 10.7% in the natural frequencies of the first 20 modes. A further increase of shell thickness by 50% caused the fundamental frequency to increase by 14.6% and the natural frequencies of the first 20 modes to increase by an average of 18.9%. Even though the aim of this work was to optimize the acoustic behavior of the shell, an acoustic prediction after changing its thickness was not performed.
Price [8] studied experimentally the effect of adding a vibration absorber to the shell of a Tecumseh compressor. From sound radiation tests, Price found that most of the sound was radiated between 1100 and 1600 Hz. Based on the radiation efficiency of the modes in this frequency range, he determined that the top mode and the second circumferential mode of the shell radiate significant energy. Therefore, to control these modes, Price installed a vibration absorber to the top of the shell. The absorber had a twin cantilever geometry with damping materials attached to the end of the beams. With the installed vibration absorber, Price obtained a 4 dB reduction in the total acoustic power radiated by the compressor.

With respect to modifications of the shell damping, Waser [9] observed that increasing the damping of the shell results in a lowered response at the resonance frequencies; however, the response at frequencies close to the resonance frequencies was increased. Therefore, Waser concluded that the overall sound reduction would not be significant.

1.3 THE BRISTOL PROJECT

The Bristol Project is a joint effort between Bristol Compressors Company and Virginia Tech to investigate the radiation characteristics of the H25A compressor and to evaluate noise control options to reduce its sound radiation. The project is comprised of four teams at Virginia Tech. The four teams are the finite element team, the experimental analysis team, the analytical force prediction team, and the acoustic analysis team.

The task of the finite element team is to develop a Finite Element (FE) model of the H25A compressor, and to use this model to predict its dynamic behavior, i.e., natural frequencies, mode shapes, and forced response. The purpose of the experimental team is to measure the dynamic characteristics of the compressor system as well as the internal forcing functions to provide support to the other teams. For instance, updating of the finite element model can be carried out with the experimentally measured natural
frequencies. The objective of the analytical force prediction team is to develop an analytical model to predict the internal forces generated within the compressor, as well as the forces transmitted to the shell through the suspension springs and the discharge pipe. The task of the acoustics team is to develop an analytical model to predict the acoustic behavior of the H25A compressor to understand the radiation mechanisms, and to use this model to study noise control techniques that would result in a quieter compressor.

1.4 THE BRISTOL H25A COMPRESSOR

The Bristol H25A is a reciprocating, two cylinder, hermetically sealed compressor. It is used for refrigeration purposes and it can deliver up to 17.9 KW of cooling capacity. A general drawing of this compressor system is presented in Fig.1.1 and it shows the major components which are referred to throughout this work.

For the acoustic analysis that was carried on the compressor and described in this thesis, the H25A can be considered as being comprised of three major components: (a) the compression mechanisms, also known as the internal components, (b) the suspension system which consists of the three coil springs and their respective mounts, and (c) the compressor housing, also known as the shell. The internal components are connected to the shell through the three suspension springs and the discharge pipe, known as the shock loop tube.

The shell of the compressor has a cylindrical shape with an oval cross section; thus, it has four sides. Two of the sides are wider and are known as the "long sides"; the narrow sides are referred to as the "short sides". It is made out of steel with a wall thickness of 0.1196 in. It consists of a top and bottom portions which are welded in the middle, along the circumference; this weld is known as the belt line. It rests on two feet which run parallel to, and are located underneath of, the short sides; and, it has an electric box attached to the top portion on one of the long sides.

INTRODUCTION
From Fig.1.1 it can be observed that of the three suspension springs, one is attached to the top and the other two are attached to the long sides, below the belt line. When the compressor is running, various mechanical vibrations and pressure fluctuations are generated within the compressor itself and between the housing and the compressor, respectively. The mechanical vibrations originate from unbalanced rotations, impacting of the valves with their seats and stops as they open and close, pressure variations within the cylinders, and other physical phenomena. The pressure fluctuations are originated by the suction into the shell as well as by the vibration of various mechanical components. The spring forces and moments as well as the internal pressure fluctuations set the shell into vibration and, it is this vibration of the shell that generates most of the sound radiated by the compressor.

1.5 PREVIOUS WORK ON THE H25A COMPRESSOR

Finite Element models of the shell and of the major components of the internal mechanisms of a standard H25A compressor were constructed [10]. Table 1.1 presents a summary of the FE models of the various components of the compressor as well as the results of a dynamic analysis of these models. The finite element model of the shell was used to obtain the shell’s modal characteristics (i.e., natural frequencies and mode shapes). This analysis yielded 23 shell modes between 0 and 2000 Hz, with the fundamental frequency at 650 Hz.

To create a full model of the compressor assembly, all the internal components, except the springs and the discharge pipe, were assumed to be rigid. The inertial properties of these components were computed and assigned to a lumped-mass element located at the center of gravity (C.G.) of the compressor. This lumped-mass element which represented all the internal components was connected to the models of the springs through rigid elements and to the discharge pipe. In turn, the models of the springs and of the discharge pipe were connected to the FE model of the shell. This model, which is known as the compressor assembly, yielded 93 natural frequencies below 2000 Hz.
The noise propagation paths of the compressor were experimentally investigated using a multiple-input/single-output (MISO) modeling technique [11]. The primary sound transmission paths that were considered are the suspension springs, the shock-loop tube, and the housing cavity pressure. Also, the contribution of the suction and exhaust pressures to the total sound output was determined. To measure the forces transmitted to the shell through the springs, triaxial piezoelectric force gauges were inserted between each spring and its mount. The moments at the mounts were not measured due to the lack of transducers to measure triaxial forces and moments simultaneously. To measure the vibration of the shock loop, an accelerometer was placed on the shock loop tube 3.5 in. below the point where the tube is attached to the exhaust muffler. To measure the housing cavity pressure as well as the suction and exhaust pressures, piezoelectric pressure transducers were placed inside the shell at the suction and discharge ports, respectively. The far-field sound pressure was measured using an omnidirectional microphone located 6 ft away from the compressor.

The advantage of the MISO technique is that it allows an investigation the contribution to a response of an individual source without having to physically isolate it from the other sources. Thus, using the MISO technique, the contribution from each of the studied paths (inputs) to the far-field pressure (output) was obtained. However, the contribution from the shock-loop tube to the total sound output was not determined because it was totally coherent with the exhaust pressure. From the MISO model, it was determined that the forces transmitted to the shell through the suspension springs are the dominant contributors to the total sound radiated. The contribution from these spring forces to the far-field pressure for a running H25A compressor is shown in Fig.1.2, which agrees very well with the recorded sound pressure in the 0-2000 Hz frequency range. From this figure, it is seen that most of the radiation occurs between 1000 and 2000 Hz, i.e., many of the dominant harmonics corresponding to the fundamental frequency of 57.8 Hz lie in this frequency range. Moreover, from Fig.1.2, it can be argued that the contribution from the spring moments to the far-field pressure can be assumed to be negligible.

INTRODUCTION
1.6 SCOPE OF THE INVESTIGATION

The present work is a continuation of the previous research performed on the H25A compressor. It can be divided into two stages. In the first stage, analytical models are developed to predict the acoustic behavior of the H25A. In the second stage, these analytical models are used to investigate various noise control approaches to reduce the sound radiated by the compressor.

For the acoustic prediction of the H25A, a Boundary Element (BE) model is created based on the FE model of the shell previously developed [10]. Then the radiation efficiency of the first 23 modes of the shell is computed. These radiation efficiencies are used in conjunction with the experimentally measured spring forces to compute the radiated acoustic power by means of a modal expansion. The most important radiating modes and the spring force components driving them are identified.

For the noise control of the H25A, various approaches were investigated: (a) the location of the side-spring mounts was changed to affect the forces transmitted to the modes (i.e., modal forces); (b) dynamic vibration absorbers were added to the spring mounts to reduce the net forces transmitted to the shell; and (c) a low modulus mismatch material was inserted between the mounts and the springs to also reduce the forces transmitted to the shell, i.e., vibration isolation.
### Table 1.1 Summary of the Finite Element models [10]

<table>
<thead>
<tr>
<th>Model</th>
<th>Number of DOF</th>
<th>Mode Shapes (0-2000 Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shell</td>
<td>7992</td>
<td>23</td>
</tr>
<tr>
<td>Crankcase</td>
<td>6378</td>
<td>2</td>
</tr>
<tr>
<td>Springs</td>
<td>216</td>
<td>17</td>
</tr>
<tr>
<td>Shock Loop Tube</td>
<td>612</td>
<td>10</td>
</tr>
<tr>
<td>Compressor Assembly</td>
<td>8700</td>
<td>93</td>
</tr>
</tbody>
</table>
Figure 1.1 The Bristol H25A Compressor.
(Drawing Courtesy of Bristol Compressors Co.)

INTRODUCTION
Figure 1.2 Contribution from the spring forces to the far-field pressure spectrum for a running H25A compressor (after Matthew Craun [11])
ACOUSTIC PREDICTION

This chapter presents the theory and the results of the acoustic prediction of the H25A compressor. The acoustic prediction process is carried out in two steps. First, radiation efficiency curves of the shell modes are computed using an acoustic Boundary Element (BE) formulation. Then, the radiation efficiency curves are used in conjunction with the experimentally measured spring forces to compute the radiated acoustic power. Therefore, an overview of the boundary element formulation for acoustic problems is first presented, followed by a description of the boundary element model of the shell. Then, the concept of the acoustic radiation efficiency is introduced together with the radiation efficiencies of the first 23 modes of the shell. Finally, the expression for the modal expansion of the acoustic power is derived and the radiated acoustic power by the modes of the shell is presented.

2.1 THE BOUNDARY ELEMENT METHOD FOR ACOUSTIC ANALYSIS

The mathematical relations that govern the acoustic phenomena, i.e., the acoustic pressure distribution on the surface of a vibrating body and on its immediate surroundings, depend largely on the surface geometry of the vibrating body as well as on the velocity distribution of this surface. For simple geometries such as spherical or planar circular surfaces, with a constant surface velocity distribution, closed-form solutions have been obtained [12,13]. However, when the surface geometry and velocity distribution are more complex, it becomes extremely difficult to obtain a closed-form solution, and thus it is required to seek a numerical solution. A powerful numerical approach to solve these acoustic relations is the boundary element method.
The major advantage of the boundary element method is that for exterior and interior acoustic problems only the surface of the vibrating structure needs to be discretized. Other formulations such as the finite element method would require the discretization of the exterior field for external problems. This discretization would involve an extremely large amount of memory and thus would be impractical. However, for interior problems the finite element method is a competitive alternative to the boundary element method.

2.1.1 THEORETICAL BACKGROUND

The acoustic wave equation which relates the temporal variations to the spatial variations of the acoustic pressure has the form:

\[ \nabla^2 p(x,y,z,t) = \frac{1}{c^2} \frac{\partial^2 p(x,y,z,t)}{\partial t^2} \]  \hspace{1cm} (2.1)

where \( p(x,y,z,t) \) is the acoustic pressure, \( c \) is the phase speed of the acoustic wave, and \( \nabla^2 \) is the Laplace operator which for rectangular coordinates has the form:

\[ \nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \]  \hspace{1cm} (2.2)

Assuming a harmonic variation of the pressure, i.e., \( p(x,y,z,t) = P(x,y,z)e^{i\omega t} \), Eq. (2.1) reduces to:

\[ (\nabla^2 + k^2)P(x,y,z) = 0 \]  \hspace{1cm} (2.3)

which is known as the Helmholtz equation, where \( k \) is the wave number \( k = \omega/c \), with \( \omega \) as the excitation circular frequency. The objective now is to compute the acoustic pressure field due to the vibration of a structure which is given in terms of the surface velocity.
distribution. To this end, the relation between the acoustic pressure and particle velocity fields is required. This is given by Euler’s relation as:

$$\rho \frac{\partial v(x,y,z,t)}{\partial t} + \nabla p(x,y,z,t) = 0$$

(2.4)

where \(\rho\) is the density of the acoustic medium, \(v(x,y,z,t)\) is the particle velocity vector and \(\nabla\) is the gradient vector given by:

$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

(2.5)

where \(\hat{i}\), \(\hat{j}\), and \(\hat{k}\) are the unit directional vectors. Again, assuming a harmonic behavior of the pressure and velocity, Eq. (2.4) reduces to:

$$v(x,y,z,t) = -\frac{1}{i\rho \omega} \nabla p(x,y,z,t)$$

(2.6)

where \(i\) is the imaginary number \(\sqrt{-1}\).

Therefore, Eq. (2.6) evaluated on the surface of the structure is the boundary condition used to solve Eq. (2.1) where \(v\) is the velocity at the structure surface. Using Green’s theorem which relates a surface integral to a volume integral, the pressure \(p(x,y,z)\) at any point in volume \(V\), bounded by surfaces \(S_0\) and \(S_1\) as shown in Fig.2.1, is given by:

$$\int_S p(R_0) \frac{\partial G}{\partial n} - G \frac{\partial p(R_0)}{\partial n} dS = C(R)p(R)$$

(2.7)

where:

$$C(R) = 1 \quad \text{for } R \in V$$

(2.8a)

$$C(R) = 0.5 \quad \text{for } R \in S$$

(2.8b)
and $G(r)$ is the Green's function:

$$G(r) = \frac{e^{-ir}}{4\pi r}$$

where $r = |R - R_0|$ is the distance between any two points within volume $V$. This surface integral was derived by Helmholtz and is known as the Surface Helmholtz Integral Equation [12].

The task of the boundary element method is to solve Eq. (2.7) numerically. To this end, the surface of the vibrating structure is discretized and the Helmholtz Integral Equation is converted to a system of equations of the form:

$$[A][p] = [B][v]$$

This system of equations and the boundary condition in Eq. (2.6) are first solved for the pressure on the surface. Then, the acoustic pressure at any point in volume $V$ is given by Eq. (2.7).

### 2.1.2 BOUNDARY ELEMENT MODEL OF THE H25A COMPRESSOR

For the acoustic prediction of the H25A compressor, the SYSNOISE boundary element code for acoustic analysis was used. The finite element mesh of the shell, created in CAEDS [10] and used to carry out the dynamic analysis of the shell, will be referred to throughout this work as the Dynamic Mesh (see Fig. 2.2). The mesh that was used for the acoustic analysis, which will be referred to as the Acoustic Mesh (see Fig. 2.3), was constructed from the Dynamic Mesh. To this end, the Dynamic Mesh was subjected to various changes.
First, the legs and the electric box were removed from the Acoustic Mesh. Due to their relative size and higher stiffness, as compared to the housing, their contribution to the total radiated sound would be insignificant. Moreover, to have their presence on the acoustic mesh would just result in unnecessarily higher computational costs.

The second change consisted of closing a gap that existed between the top and the bottom of the housing. The modeling of the girth between the top and the bottom of the housing was not straightforward, for there is not yet any element that models welded joints. Therefore, various attempts to model this joint were made by the Finite Element team. At the end, line-type elements called 'rigid elements' in CAEDS modeled the girth to match the results to those obtained experimentally. As a result, the top and the bottom of the Dynamic Mesh of the housing were connected only at nodal locations leaving a gap. However, for the acoustic analysis, this gap would change the acoustic behavior of the system and thus needed to be closed. Therefore, the gap on the Acoustic Mesh was closed using quadrilateral shell elements.

To perform an acoustic analysis, it is recommended that the Boundary Element model has at least 6 linear elements per acoustic wavelength, i.e., $\lambda=\frac{c}{f}$ with $f$ frequency in Hz. Based on this criteria, the maximum frequency up to which the dynamic mesh could yield satisfactory results was 1150 Hz. Therefore, to increase this maximum frequency, the order of the elements of the acoustic mesh was changed from linear to parabolic. Upon this change, the maximum frequency for the acoustic mesh to yield satisfactory results was found to be 2300 Hz.

Finally, several elements were placed inside the mesh of the housing. The purpose of these elements was to eliminate a mathematical problem that arises when solving exterior problems and the surface of the vibrating structure is closed. In these type of problems, if the excitation frequency coincides with (or is near to) the natural frequency of the acoustic modes inside the cavity, the exterior acoustic characteristics are overestimated. These frequencies are also known as irregular frequencies and can be avoided by defining a zero impedance on the inner elements [14].

ACOUSTIC PREDICTION
2.2 RADIATION EFFICIENCY OF STRUCTURAL MODES

Using the Boundary Element model of the shell, described in the previous section, the radiation efficiency of the shell modes was computed as a function of frequency. The importance of these modal radiation efficiency curves is that they provide a measure of the effectiveness of each shell mode to radiate sound. If only a few modes in the frequency range of interest are efficient radiators, noise control approaches could focus on shifting the input vibrational energy from these efficient radiating modes, to modes that poorly couple into the acoustic field. The modal radiation efficiency curves of a structure can also be used to compute the acoustic power radiated by the structure.

2.2.1 THEORETICAL BACKGROUND

Radiation efficiency of a structure is defined as the ratio of the acoustic power radiated by the structure to the acoustic power that would be radiated by a piston of the same area that vibrates with the same average mean square velocity. In other words, radiation efficiency is a normalization of the radiated acoustic power for comparison purposes. Therefore, since radiation efficiency is the ratio of the power radiated by two different structures, i.e., the structure and the piston, it is possible for a structure to have a radiation efficiency greater than unity. The radiation efficiency \( \sigma_n(\omega) \) of the \( n^{th} \) mode at frequency \( \omega \) is given by:

\[
\sigma_n(\omega) = \frac{\Pi_{nn}(\omega)}{\rho c A \langle \left( v_n^2 \right)_n \rangle(\omega)}
\]  

(2.11)

where \( \Pi_{nn} \) is the power radiated by mode \( n \), \( \rho c \) is the characteristic impedance of the acoustic medium, \( A \) is the surface area of the structure, and \( \langle \left( v_n^2 \right)_n \rangle(\omega) \) is the average mean square velocity of the \( n^{th} \) mode. To compute these modal radiation efficiencies, SYSNOISE converts the mode shape data into normal velocity data by multiplying the
normal modal displacement vectors by $i\omega$. The modal velocity vector is then used as the boundary condition to solve the Helmholtz Integral Equation as explained in section 2.1.

To describe their general behavior, the radiation efficiency curves from the modes of a square panel are shown in Fig.2.4. These curves were evaluated analytically by Wallace [15]. Observe that at low frequencies, i.e., when the acoustic wave number $k$ is much smaller than the structural wave number $k_b$ ($k_b^2 = (p\pi/a)^2 + (q\pi/b)^2$, where $p$ and $q$ are the mode order, and $a$ and $b$ are the dimensions of the panel), the radiation efficiency varies from mode to mode significantly. At around the critical frequency, i.e., when the acoustic and the structural wave numbers are equal, the radiation efficiency curves of all modes reach their maximum values. At high frequencies, the radiation efficiency of all modes approaches unity, which implies that given the same average surface velocity all modes are efficient radiators.

2.2.2 MODAL RADIATION EFFICIENCY OF THE SHELL

The radiation efficiency for each of the first 23 modes of the shell was computed between 600 to 2000 Hz. For the sake of clarity, the radiation efficiency curves of only two modes are shown in Fig.2.5. The radiation efficiency curves for all the other modes can be found in appendix I.

From Fig.2.5, it can be seen that mode 1220 has a higher radiation efficiency and thus will radiate significantly more than mode 1357 in most of the 600-2000 Hz frequency range. These radiation efficiency curves do not take into account either the dynamic response of the modes or how strongly the excitation forces (spring forces) are driving each mode. For structures of low modal damping, most of the sound radiation will occur at the resonant frequency of the modes [16]. Thus, the radiation efficiency at the resonant frequency is the critical or most important value. Since the experimental damping values of the shell modes vary between 0.02 and 0.25 %, the radiation efficiencies at each of the
resonant frequencies were computed and are presented in Table 2.1. From this table, the eight modes marked with an asterisk are identified as the most efficient sound radiators.

2.3 RADIATED ACOUSTIC POWER

As discussed previously in section 2.2, the radiation efficiency is a parameter that compares the ability of the modes to radiate sound. However, the radiation efficiency does not take into account the dynamic response of the modes. For instance, the excitation forces may not drive all the efficient radiating modes but only a few of them. Thus, for noise control purposes it is important to determine the radiated power by each mode including their dynamic response. The importance of the acoustic power is that, unlike the acoustic pressure, it is a non-directional measure of sound radiation (i.e., it does not depend on the directivity of the source).

2.3.1 THEORETICAL BACKGROUND

The acoustic power that is radiated by a structural mode can be obtained directly from the radiation efficiency of the mode. This modal power can then be used to perform a modal expansion to predict the power radiated by the structure at any given frequency. In what follows, the expression for the power radiated by a structure is derived.

The acoustic intensity is the rate of flow of energy per unit area and is given by:

\[ I = \frac{1}{2} \text{Re}[pv^*] \]  

(2.12)

where \( p \) is the acoustic pressure and \( v^* \) is the complex conjugate of the particle velocity. Integrating this intensity over an imaginary spherical surface (see Fig.2.6), an expression for the acoustic power is obtained:
\[ \Pi = \frac{1}{2} \int_{S_1} \text{Re}[\rho v^*] dS \]

(2.13)

In the far-field, the acoustic pressure and the particle velocity are directly related by the characteristic impedance of the acoustic medium:

\[ \rho c = \frac{P}{v} \]

(2.14)

where \( \rho \) and \( c \) are the density and the phase speed of the medium respectively. Substituting the above relation into Eq. (2.13) we obtain:

\[ \Pi = \frac{1}{2} \int_{S_1} \frac{|p|^2}{\rho c} dS \]

(2.15)

The acoustic pressure generated by the structure can be expressed as a linear combination of the modes:

\[ p(\omega) = \sum_{n=1}^{N} F_n(\omega) H_n(\omega) p_n(\omega) \]

(2.16)

where \( F_n(\omega) \) is the modal force given by:

\[ F_n(\omega) = \{F(\omega)\}^T \{\phi_n\} \]

(2.17)

where \( \{F(\omega)\} \) is the external force vector and \( \{\phi_n\} \) is the \( n^{th} \) mode shape vector. The frequency response function \( H_n(\omega) \) in Eq. (2.16) is given as:

\[ H_n(\omega) = \frac{1}{m_n(\omega_n^2 - \omega^2 + 2i\beta_n \omega_n \omega)} \]

(2.18)
where \( \omega \) is the excitation frequency, and \( m_n, \omega_n \) and \( \beta_n \) are the modal mass, natural frequency and damping ratio of mode \( n \), respectively. The term \( p_n(\omega) \) is the acoustic pressure radiated due to the \( n^{th} \) mode. Substituting Eq. (2.16) into Eq. (2.15), an expression for the power radiated by the structure is obtained as:

\[
\Pi(\omega) = \sum_{n=1}^{N} |F_n(\omega)|^2 |H_n(\omega)|^2 \Pi_{nn}(\omega) + \sum_{n=1}^{N} \sum_{m=1}^{M} F_n(\omega) F_m^*(\omega) H_n(\omega) H_m^*(\omega) \Pi_{nm}(\omega) \tag{2.19}
\]

Therefore, from the above equation we can observe that the acoustic power radiated by a structure at a frequency \( \omega \) is the contribution of two terms. The first term (i.e., single summation) is the sum of the power radiated by the individual modes, i.e., direct terms, where \( \Pi_{nn}(\omega) \) is the power radiated by the \( n^{th} \) mode. The second term (i.e., double summation) represents the power radiated by the interaction between modes, i.e., cross terms, where \( \Pi_{nm}(\omega) \) is the power due to the interaction of the \( n^{th} \) and \( m^{th} \) modes. It is generally the case that the magnitude of the cross terms is not significant compared to that of the diagonal terms except at low frequencies, i.e., below the critical frequency [17]. Therefore, the power radiated by the structure can be approximated by the first term on the right-hand side of Eq. (2.19). That is:

\[
\Pi(\omega) \equiv \sum_{n=1}^{N} |F_n(\omega)|^2 |H_n(\omega)|^2 \Pi_{nn}(\omega) \tag{2.20}
\]

where the modal radiated power \( \Pi_{nn}(\omega) \) is obtained from the expression for the radiation efficiency in Eq. (2.11) as:

\[
\Pi_{nn}(\omega) = \sigma_n(\omega) pcA \left( \left( v_{rms} \right)_n(\omega) \right)^2 \tag{2.21}
\]

Substituting Eq. (2.21) into Eq. (2.20), yields:
\[ \Pi(\omega) = \sum_{n=1}^{N} |F_n(\omega)|^2 |H_n(\omega)|^2 \sigma_n(\omega) \rho c A \langle \nu_{rms}^2 \rangle_n(\omega) \]  

(2.22)

Furthermore, for structures with low damping only the value of the radiation efficiency of the mode at its natural frequency is important [16]. Thus, Eq. (2.22) can be further approximated as:

\[ \Pi(\omega) \equiv \sum_{n=1}^{N} |F_n(\omega)|^2 |H_n(\omega)|^2 \sigma_n(\omega) \rho c A \langle \nu_{rms}^2 \rangle_n(\omega) \]  

(2.23)

where \( \sigma_n(\omega_n) \) is the value of the radiation efficiency for the \( n^{th} \) mode at its natural frequency \( \omega_n \). By having to compute only one value of radiation efficiency per mode, the computational efforts decrease even further. However, it should be remembered that this simplification is valid only for cases of low structural damping.

Another approach to compute the power radiated by a structure is to obtain the forced response of the structure as a function of frequency and to use these forced response shapes, i.e., one response shape per frequency, to evaluate the power radiated by the structure. The advantage of using this approach is that the cross terms of the acoustic power are taken into account. However, in a general case a structure is excited by various forces at several locations. If the contribution of each force component to the total sound output is desired (as in this investigation), this method of obtaining the radiated power is not computationally efficient since it would require the computation of the forced response as a function of frequency for each single force component. On the other hand, if either Eq. (2.22) or Eq. (2.23) is used, these equations can also be used to obtain the power radiated by each force component as well as the power radiated by each mode. This is a the main advantage of using Eqs. (2.22) and (2.23).

Finally, one of the uses that can be given to Eq. (2.22) is that for an experimental determination of the radiation efficiency of a structure. To this end, the acoustic power
radiated by the structure as well as its average mean square velocity need to be measured experimentally. The acoustic power can be computed from intensity or far-field pressure measurements over an imaginary semi-spherical surface. The average mean square velocity normal to the surface can be measured using accelerometers, or for more accurate measurements a Laser Doppler Vibrometer (LDV) can be employed [18]. The radiation efficiency of the structure can then be obtained by normalizing the total radiated power on the average mean square velocity and on the surface area of the structure.

2.3.2 TOTAL RADIATED ACOUSTIC POWER

The acoustic power radiated by the shell was first computed using Eq. (2.22). In this equation, \( |F_n|^2 \) is computed using the experimentally measured forces and the mode shapes \( \{\phi_n\} \) from the FE model of the shell, \( H_n(\omega) \) is computed using the natural frequencies \( \omega_n \) and the modal masses \( m_n \) from the FE model and the experimentally measured damping \( \beta_n \) of the shell. Figure 2.7 shows the radiated power by the shell as a function of frequency. Observation of Fig.2.7 shows that there are two frequency regions at around 1200 and 1600 Hz at which significant power is radiated. Integrating the spectrum in Fig.2.7, the total acoustic power radiated by the H25A compressor is 82.2 dBA. From this point on, the values of predicted total radiated acoustic power will be for the 600 to 2000 Hz frequency range. The experimentally measured total radiated acoustic power (i.e., average of ten compressors tested) at the Bristol facility yielded 82 dBA. The good agreement between the experimental and the analytical prediction validates the approach used in this work.

The radiated acoustic power was also computed using Eq. (2.23) where the radiation efficiency function was approximated by the value at resonance \( \sigma_n(\omega) = \sigma_n(\omega_n) \). This predicted power spectrum is also plotted in Fig.2.7 and it is observed that both approaches yield very similar results. The major advantage of approximating the radiation efficiency function by the value at resonance is that only one SYSNOISE run per mode is needed to compute the total radiated acoustic power.
As explained in section 2.3.1, there exists a third approach to compute the radiated acoustic power. This approach consists on first computing the forced response of the shell using the FE model at a particular excitation frequency. This force response is then input into the BE model to predict the radiated power at that frequency. This process is then repeated for each frequency of interest. The advantage of this approach is that it takes into account the cross terms of the acoustic power, implicitly. However, for the H25A compressor there are 25 excitation frequencies in the 600-2000 Hz range. Since each run of the BE model takes about 1.25 hours at the VPI&SU system, this approach clearly requires a large computational effort. Therefore, this approach was only used to compute the radiated acoustic power at a few frequencies, to validate the assumption that for the present case the acoustic radiated power can be approximated using the direct terms only. Figure 2.8 compares the radiated acoustic power computed using the forced response shapes at a few frequencies, to the radiated acoustic power that was computed using Eq. (2.23). From this figure, it is seen that for the case of the compressor shell, Eq. (2.23) represents a good approximation to compute the radiated acoustic power.

2.3.3 MODAL RADIATED ACOUSTIC POWER

In this section, the modal radiated acoustic power was computed for each of the first 23 modes of the housing as a function of frequency. Each term of the summation in Eq. (2.22) represents the radiated acoustic power by the modes. For example, the spectrum of the acoustic power radiated by mode 1193 is shown in Fig.2.9. Figure 2.10 presents the cumulative radiated power as a function of frequency for the same mode, and it is obtained by integrating the curve in Fig.2.9. A sudden rise in the curve shown in Fig.2.10 occurs at the frequency at which significant sound power is radiated. As it would be expected, the largest rise occurs at, or near, the resonant frequency of the mode. The total acoustic power radiated by this mode is 77.6 dB. The power spectrum curves of the radiated acoustic power by the first 23 shell modes can be found in appendix II.
The total acoustic power radiated by each of the 23 shell modes is summarized in Table 2.2. This table also shows the fractional contribution of each mode to the total radiated power by the housing. From this table, it is observed that there are only six modes (indicated with an asterisk) that have a significant contribution to the total acoustic power radiated by the compressor. Notice that three of these modes are clustered around 1200 Hz, whereas the other three lie in the vicinity of 1600 Hz. It should also be observed that of these six modes, only five were identified as efficient radiating modes in Table 2.1, i.e., modes 1193 (top mode), 1220, 1558, 1616, and 1683.

Figures 2.11 and 2.12 show the six modes identified with an asterisk in Table 2.2. From the set of modes clustered around 1200 Hz, mode 1164 has a deformation on the back (long side), similar to the (2,1) mode on a plate. Mode 1220 resembles mode 1164 but with the same deformation pattern occurring in the front rather than in the back. Mode 1193 corresponds to the motion of the top as a piston. During experimental measurements, the natural frequencies of these three modes were found to be very close. In fact, modes 1164 and 1220 have some displacement of the top. Therefore, it is believed that there is a strong coupling between the motion of the long sides (front and back) and the top of the shell at around 1200 Hz. These three modes are strongly excited by the top spring forces, in particular, the z component as it will be discussed later.

Of the modes lying around 1600 Hz and shown in Fig.2.12, mode 1616 has the largest deformation on the front somewhat similar to a (2,2) mode on a plate. On the other hand, mode 1683 has the (2,2) like shape on the back rather than on the front of the shell. Finally, the deformation pattern of the front and back of mode 1558 resembles a (2,3) mode on a plate. No significant displacement of the top in these modes occurs.

2.4 RADIATION MECHANISMS

To perform noise control of the compressor, knowledge of the contribution of the forces on each spring mount to the acoustic power radiation is very important. The
contribution to the total radiated power by each of the nine experimentally measured spring forces was obtained by using Eq. (2.23) in conjunction with each force component separately. The acoustic power radiated by each force component was then compared to the total radiated power, i.e., all forces acting on the shell.

The contributions to the total radiated power from the forces on the x, y, and z directions on the top mount are given in Figures 2.13, 2.14, and 2.15, respectively. From these figures, it is observed that the most important force component from the top spring is that on the z direction. Figure 2.15 also shows that the force on the top mount in the z direction is the main force component responsible for the radiation at around 1200 Hz. On the other hand, the forces on the top mount in the x and y directions have negligible contributions to the total radiated power compared to the contribution from the z component.

The contributions to the total radiated power from the side spring forces are presented in Figures 2.16 through 2.21. From these figures it is observed that the x and y force components are the most important contributors to the total radiated power in both springs, specially at around 1600 Hz. The x force components of both side springs have a relatively important contribution to the radiation at around 1200 Hz.

Using the x, y, and z components of the force acting on one spring mount at a time, the contribution from each spring mount to the total sound radiation was obtained. The contribution from the forces acting on the top mount is given in Fig.2.22. From this figure it can be seen that the forces on this mount have a significant contribution at around 1200 Hz, which is the result of exciting modes 1164, 1193, and 1220. Similarly, Figures 2.23 and 2.24 show the contributions from the forces on the side mounts to the total radiated power. From these figures it is observed that these forces contribute significantly at around 1600 and also at around 1200 Hz.
The predicted radiation due to each force component does not include the coupling effect between forces, i.e., relative phases among them. However, when the radiation due to each spring is computed, the coupling between the three force components on the spring is included.

2.5 SUMMARY

The acoustic prediction of the H25A compressor was carried in two stages. In the first stage, the radiation efficiency function of each of the 23 shell modes was computed using the Boundary Element code for acoustic problems SYSNOISE. From this analysis, eight modes were identified as efficient sound radiators. The radiation efficiency of these modes at their respective natural frequency is given in Table 2.1, where these modes are marked by an asterisk.

In the second stage of the acoustic prediction, the total and the modal radiated acoustic power were computed. The power radiated by each mode is presented in Table 2.2. From this analysis, only six modes were found to have a significant contribution to the total radiated acoustic power. These modes are shown in Figures 2.11 and 2.12. It was observed that only four of these six modes have high radiation efficiencies in Table 2.1. Thus, it was shown that not only the radiation efficiency but also the dynamic response of the modes and the excitation forces are important factors when computing the total radiated acoustic power.

Finally, it was found that the radiation at around 1200 Hz is mainly due to the \( z \) component of the force on the top mount, and due to the \( x \) and \( y \) components of the forces on the side mounts. At around 1600 Hz, the acoustic radiation is mainly due to the \( x \) and \( y \) components of the force on the side mounts.
Table 2.1 Radiation efficiency of the first 23 modes of the shell at their natural frequency.

<table>
<thead>
<tr>
<th>Natural Frequency (Hz)</th>
<th>Radiation Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>650</td>
<td>0.05</td>
</tr>
<tr>
<td>717</td>
<td>0.54</td>
</tr>
<tr>
<td>740</td>
<td>0.157</td>
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<tr>
<td>889</td>
<td>0.077</td>
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<tr>
<td>975</td>
<td>0.292</td>
</tr>
<tr>
<td>1108</td>
<td>0.837</td>
</tr>
<tr>
<td>1164</td>
<td>0.607</td>
</tr>
<tr>
<td>1193</td>
<td>1.720 *</td>
</tr>
<tr>
<td>1220</td>
<td>1.138 *</td>
</tr>
<tr>
<td>1357</td>
<td>0.194</td>
</tr>
<tr>
<td>1382</td>
<td>1.307 *</td>
</tr>
<tr>
<td>1390</td>
<td>0.084</td>
</tr>
<tr>
<td>1477</td>
<td>1.980 *</td>
</tr>
<tr>
<td>1558</td>
<td>0.954 *</td>
</tr>
<tr>
<td>1616</td>
<td>1.258 *</td>
</tr>
<tr>
<td>1644</td>
<td>1.816 *</td>
</tr>
<tr>
<td>1683</td>
<td>1.791 *</td>
</tr>
<tr>
<td>1757</td>
<td>1.067</td>
</tr>
<tr>
<td>1805</td>
<td>0.896</td>
</tr>
<tr>
<td>1909</td>
<td>0.583</td>
</tr>
<tr>
<td>1943</td>
<td>0.704</td>
</tr>
<tr>
<td>1963</td>
<td>0.719</td>
</tr>
<tr>
<td>1985</td>
<td>0.693</td>
</tr>
</tbody>
</table>
Table 2.2 Total radiated power by the first 23 shell modes.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Total Radiated Power (dB)</th>
<th>Fraction of Total Power</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>650</td>
<td>52.3</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>717</td>
<td>59.1</td>
<td>0.006</td>
</tr>
<tr>
<td></td>
<td>740</td>
<td>53.3</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>889</td>
<td>55.9</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>975</td>
<td>45.5</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>1108</td>
<td>60.1</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>1164</td>
<td>72.8 *</td>
<td>0.134</td>
</tr>
<tr>
<td></td>
<td>1193</td>
<td>77.6 *</td>
<td>0.406</td>
</tr>
<tr>
<td></td>
<td>1220</td>
<td>70.2 *</td>
<td>0.074</td>
</tr>
<tr>
<td></td>
<td>1357</td>
<td>46.0</td>
<td>0.000</td>
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<tr>
<td></td>
<td>1382</td>
<td>61.0</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>1390</td>
<td>52.8</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>1476</td>
<td>66.6</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td>1558</td>
<td>71.2 *</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>1616</td>
<td>72.5 *</td>
<td>0.126</td>
</tr>
<tr>
<td></td>
<td>1644</td>
<td>55.0</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>1683</td>
<td>67.5 *</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td>1757</td>
<td>64.5</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>1812</td>
<td>54.3</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>1908</td>
<td>60.9</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>1943</td>
<td>62.9</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>1966</td>
<td>59.7</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>1986</td>
<td>61.6</td>
<td>0.010</td>
</tr>
</tbody>
</table>
Figure 2.1 Boundaries and vectors used in Helmholtz Integral Equation.
Figure 2.2 Dynamic Mesh (after Anand Ramani [10])

Figure 2.3 Acoustic Mesh.
Figure 2.4 Radiation efficiency curves of a flat panel [16].
Figure 2.5 Typical radiation efficiency curves of the H25A shell.
Figure 2.6 Spherical surface to compute the radiated acoustic power.
Figure 2.7 Radiated power by the first twenty three housing modes using the radiation efficiency curves, and the value of the radiation efficiency at the natural frequency.

Figure 2.8 Radiated acoustic power computed using the forced response shapes, and using the direct terms only.
Figure 2.9 Acoustic power radiated by mode 1193 (ref. $10^{-12}$ W).

Figure 2.10 Total acoustic power radiated by mode 1193.
Figure 2.10 Modes with natural frequencies close to 1200 Hz.
(a) Mode 1164, (b) Mode 1193, (c) Mode 1220.

Figure 2.11 Modes with natural frequencies close to 1600 Hz.
(a) Mode 1558, (b) Mode 1616, (c) Mode 1683.
Figure 2.13 Radiated power due to the force on the top mount in the $x$ dir. (ref. $10^{12}$ W)

Figure 2.14 Radiated power due to the force on the top mount in the $y$ dir. (ref. $10^{12}$ W)

Figure 2.15 Radiated power due to the force on the top mount in the $z$ dir. (ref. $10^{12}$ W)
Figure 2.16 Radiated power due to the force on the right mount in the $x$ dir. (ref. $10^{-12}$ W)

Figure 2.17 Radiated Power due to the force on the right mount in the $y$ dir. (ref. $10^{-12}$ W)

Figure 2.18 Radiated power due to the force on the right mount in the $z$ dir. (ref. $10^{-12}$ W)
Figure 2.19 Radiated power due to the force on the left mount in the \( x \) dir. (ref. \( 10^{-12} \) W)

Figure 2.20 Radiated power due to the force on the left mount in the \( y \) dir. (ref. \( 10^{-12} \) W)

Figure 2.21 Radiated power due to the force on the left mount in the \( z \) dir. (ref. \( 10^{-12} \) W)
Figure 2.22 Radiated power due to the forces on the top mount. (ref. $10^{-12}$ W)

Figure 2.23 Radiated power due to the forces on the right mount. (ref. $10^{-12}$ W)

Figure 2.24 Radiated power due to the forces on the left mount. (ref. $10^{-12}$ W)
This chapter is concerned with the noise control approaches that were investigated to reduce the sound radiated by the H25A compressor. The reduction of structurally radiated sound can be achieved by one of three approaches: (a) attenuation of the disturbance source, (b) reduction of the vibration transmitted to the shell, and (c) change the radiation characteristic of the system (i.e., reduce the radiation efficiency of the structure). In this project, the radiated sound by the housing is mainly due to the spring forces acting on the compressor housing. Thus, any effective noise control solution will have to attempt to reduce or modify these excitation forces.

3.1 SIDE-SPRING MOUNTS RELOCATION

The first modification of the system considered in this study was to change the position of the side-spring mounts. By repositioning the spring mounts, the forces transmitted to the modes (modal forces) that radiate significant sound could be reduced. In this analysis, it was assumed that the experimentally measured spring forces do not change as the mounts are moved. Due to this assumption, the further the mounts are moved from their original position, the less accurate the results will be. To approximate the value of the mode shapes at the mounts (where the spring forces were measured), the mounts are assumed to behave like rigid bodies. Thus, using the mode shapes at the nodes which represent the four weld spots that join the mounts to the shell, simple kinematic equations were used to compute the relative displacements, i.e., mode shapes of the points on the mounts where the spring forces act.

The side-spring mounts were moved simultaneously, horizontally and vertically, to the same relative location in the front and back. The various locations where the mounts...
were moved are shown in Fig.3.1 for the front of the shell and in Fig.3.2 for the back. In these figures, the corners of each square that indicates a position of the mounts, represent the four nodes at which the mounts are attached to the shell.

3.1.1 ACOUSTIC RESULTS

The radiated acoustic power for each of the studied positions of the mounts, shown in Figures 3.1 and 3.2, was computed. These values are presented in Table 3.1. From this table, it can be seen that when the mounts are moved in the vertical direction, either to a lower or to a higher position (i.e., close to the belt line) the total radiated power decreases. This decrease can be explained by observing Fig.2.10. Note that the three efficient radiating modes shown in this figure have circumferential nodal lines at the belt line and at the bottom of the shell. Thus, when the mounts are moved to these regions, the excitation forces drive these modes to a lesser extent. It was also found that the present location of the mounts is the worst in terms of sound radiation. Again, from Fig.2.10, it can be seen that the present locations of the side mounts lie on the antinodes of these modes.

The maximum reduction in the radiated power of 2.7 dBA, was obtained when the mounts were moved to a lower location and towards one of the short sides (i.e., position P25). The spectrum of the radiated power with the mounts in this position is shown in Fig.3.3. From this spectrum, it is seen that a significant local reduction was achieved at around 1600 Hz. However, this position may not be physically feasible, i.e., the top spring is not aligned with the side spring. An alternate position, more physically attainable, is position P18 where the side mounts are moved in the vertical direction towards the belt line. The spectrum of the radiated power for this position is given in Fig.3.4. From this figure, it is observed that a reduction in the 1600 Hz region is obtained. The total radiated power with the mounts at this position is 80.5 dBA, for a net reduction of 1.7 dBA.
Thus, a reduction in the radiated power was obtained by moving the side mounts to various positions on the lower half of the shell. However, at all the studied positions, only the 1600 Hz region of the power spectrum was affected. This behavior is expected since the most important contributors to the sound radiated at around 1600 Hz are the \(x\) and \(y\) components of the force in the side mounts. The radiation at around 1200 Hz was not affected significantly because the radiation at around this frequency is primarily controlled by the \(z\) component of the force on the top mount.

### 3.2 VIBRATION ABSORBERS

The second system modification considered to reduce the radiation was the addition of dynamic vibration absorbers (DVAs) to the spring mounts. The objective of these DVAs was to reduce the main forces transmitted by the springs to the housing.

In this analysis, the experimentally measured spring forces are again assumed to remain unchanged. Two analysis techniques were considered. First the models of the absorbers could be added to the FE model to obtain the new eigenproperties (i.e., mode shapes, frequencies and damping ratios). Then, these modes could be used to compute the total radiated power by the compressor in a similar analysis as that carried in Chapter II. This option, however, requires a large computational effort since the radiation efficiency curves for each mode should be computed again at various frequencies. Thus, this approach was not pursued.

The second approach consisted of first predicting the forces on the mounts generated by the DVAs. These forces are then combined with the original spring forces to determine the net forces acting on the housing. Thus, the acoustic modal properties of the original housing computed in Chapter II can be directly used in this analysis, i.e., neither a new FE nor a BE model is required. The theoretical formulation used in this analysis is presented in the next section.
3.2.1 THEORETICAL FORMULATION

Consider the \( N \) degree-of-freedom system shown in Fig.3.5. The external forces acting on this system are denoted by \( \mathbf{F}_n \). It is considered that \( M \) single-degree-of-freedom (SDOF) dynamic vibration absorbers of mass \( m_m \), damping coefficient \( c_m \), and stiffness \( k_m \) are added to the system as shown in Fig.3.6. It is assumed that the \( m^{th} \) DVA is acting in the direction of the \( j^{th} \) DOF of the original system in Fig.3.6. The equations of motion of the system with the added absorbers can be written as follows:

\[
\begin{pmatrix}
[D] & [0] \\
[0]^T & [A]
\end{pmatrix}
+ \sum_{m=1}^{M} (k_m + i\omega c_m) \begin{pmatrix}
\{v_{jm}\} \\
\{v_{jm}\}^T
\end{pmatrix}
\begin{pmatrix}
\{\ddot{x}\} \\
\{x_m\}
\end{pmatrix}
= \begin{pmatrix}
\{F\} \\
\{0\}
\end{pmatrix}
\]  

(3.1)

where: 
- \([D]\) is the \( N \times N \) dynamic stiffness matrix of the original system shown in Fig.3.5 which is obtained from the FE model as \([D] = [K] + i\omega [C] - \omega^2 [M]\) where \([K]\), \([C]\) and \([M]\) are stiffness, damping and mass matrices, respectively.
- \([0]\) is a \( N \times M \) matrix whose elements are all zeros.
- \([A]\) is a diagonal \( M \times M \) matrix whose diagonal elements are given by:
  \[A_{mm} = -\omega^2 m_m; \text{ for } m = 1, \ldots, M\]
- \(\{v_{jm}\}\) is a \((N+M)\) vector that contains zeros except at the \( j^{th} \) and \((N+m)^{th}\) positions where it has a value of 1 and -1, respectively:
  \(\{v_{jm}\} = \{0, \ldots, 0, 1, 0, \ldots, 0, -1, 0, \ldots, 0\}\), the \( m^{th} \) absorber is coupled to the \( j^{th} \) DOF
- \(\{\ddot{x}\}\) is the displacement vector of the original structure shown in Fig.3.5.
- \(\{x_m\}\) is the displacement vector of the vibration absorbers, where \( m = 1, \ldots, M \)
- \(\{F\}\) is the vector that contains the external forces applied to the structure shown in Fig.3.5.

and the term:

\[
\sum_{m=1}^{M} (k_m + i\omega c_m) \begin{pmatrix}
\{v_{jm}\} \\
\{v_{jm}\}^T
\end{pmatrix}
\]  

(3.2)
represents a \((N+M) \times (N+M)\) matrix that contains the stiffness and damping coupling terms. Let this coupling matrix be called matrix \([G]\) as:

\[
[G] = \sum_{m=1}^{M} (k_m + i\omega c_m) \{v_{jm}\} \{v_{jm}\}^T
\]  

(3.3)

Taking the diagonal \(G_{ii}\) terms of matrix \([G]\), where \(i = N+1, N+2, ..., N+M\), into matrix \([A]\) of Eq. (3.1) leads to:

\[
\begin{pmatrix}
[D] & [0] \\
[0]^T & [\Lambda]
\end{pmatrix} + \sum_{m=1}^{M} (k_m + i\omega c_m) \{v_{jm}\} \{v_{jm}\}^T - \{u_m\} \{u_m\}^T \begin{pmatrix}
\{\hat{x}\} \\
\{x_m\}
\end{pmatrix} = \begin{pmatrix}
\{F\} \\
\{0\}
\end{pmatrix}
\]  

(3.4)

where \([\Lambda]\) is a \(M \times M\) diagonal matrix whose elements are given by \(\Lambda_m = k_m - \omega^2 m_m + i\omega c_m\) for \(m = 1, ..., M\), and \(\{u_m\}\) is a \((N+M)\) vector whose elements are all zeros except at the \((N+m)\)th position where it has a value of 1.

For the sake of clarity in the presentation, it is convenient to consider Eq. (3.4) for the special case of \(M=1\), i.e., only one DVA is added to the system. Without loss of generality, the single DVA is attached to the \(N\)th DOF of the system. Thus, Eq. (3.4) becomes:

\[
\begin{pmatrix}
D & [0] \\
[0] & \Lambda
\end{pmatrix} + (k + i\omega c) \begin{pmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 1
\end{pmatrix} \begin{pmatrix}
\{\hat{x_i}\} \\
\{\hat{x_N}\} \\
\{x_a\}
\end{pmatrix} = \begin{pmatrix}
\{F_i\} \\
\{F_N\} \\
\{0\}
\end{pmatrix}
\]  

(3.5)

where \(x_a\) is the displacement of the DVA. Notice that the far right matrix inside the parenthesis has non-zero elements only at the \((N,N)\), \((N,N+1)\) and \((N+1,N)\) positions.

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Premultiplying Eq. (3.5) by the inverse of the first matrix on this equation leads to:

\[
\begin{pmatrix}
[I] + (k + i\omega) & D^{-1} & 0 & \cdots & 0 \\
0 & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 1 \\
0 & \vdots & \ddots & \ddots & \ddots \\
\end{pmatrix}
\begin{pmatrix}
\hat{x}_1 \\
\vdots \\
\hat{x}_N \\
\frac{x_1}{x_o} \\
\end{pmatrix} =
\begin{pmatrix}
0 \\
\cdots \\
0 \\
1 \\
0 \\
\end{pmatrix}
\]

where \([I]\) is the identity matrix and the \((i,j)\) element of the \([D]^{-1}\) matrix is the transfer function between the \(i^{th}\) and \(j^{th}\) DOFs of the original system given by:

\[
T_{ij}(\omega) = \sum_{n=1}^{S} \phi_n(i) \phi_n(j) H_n(\omega) \quad \text{for } i = 1, \ldots, N \text{ and } j = 1, \ldots, N
\]  

(3.7)

where \(S \leq N\) is the number of modes included in the analysis; \(\phi_n(i)\) and \(\phi_n(j)\) are the \(n^{th}\) mode shapes of the original structure evaluated at the \(i^{th}\) and \(j^{th}\) DOFs, respectively; and \(H_n(\omega)\) is the \(n^{th}\) frequency response function of the original structure given by:

\[
H_n(\omega) = \frac{1}{m_n \left( \omega_n^2 - \omega^2 + i2\beta_n\omega_n \omega \right)}
\]

(3.8)

where \(\omega_n, \beta_n\) and \(m_n\) are the \(n^{th}\) natural frequency, damping ratio and modal mass of the original structure, respectively. The natural frequencies \(\omega_n\), mode shapes \(\phi_n\) and the modal masses \(m_n\) are obtained from the shell FE model. The modal damping ratios \(\beta_n\) and the forces \(F_n\) are obtained experimentally. The components of the right-hand side vector in Eq. (3.6) are given by:

\[
x_n = \sum_{m=1}^{S} T_{nm} F_m \quad n = 1, \ldots, N
\]

(3.9)

where \(x_n\) represents the response at the \(n^{th}\) DOF of the original structure (without the DVAs) due to the external forces.

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It can be readily seen that Eq. (3.6) represents a \((N+1)\times(N+1)\) system of equations. The \(N^{th}\) and \((N+1)^{th}\) rows are the only coupled equations in this linear system. The \(N^{th}\) equation is:

\[
[1+T_{NN}(\omega)(k+i\omega c)]\hat{x}_N - T_{NN}(\omega)(k+i\omega c)x_a = x_N
\]  

(3.10)

while the \((N+1)^{th}\) equation is:

\[-(k+i\omega c)\Lambda^{-1}\hat{x}_N + x_a = 0\]

(3.11)

The unknowns in Eqs.(3.10) and (3.11) are the displacements of the \(N^{th}\) node \(x_N\) (where the DVA is attached) and the displacement of the added DVA, \(x_a\). The first equation represents the compatibility condition of the displacement at the \(N^{th}\) DOF, while the second is the equilibrium equation of the absorber’s mass. Writing Eqs.(3.10) and (3.11) in matrix form leads to:

\[
\begin{bmatrix}
1+T_{NN}(\omega)(k+i\omega c) & -T_{NN}(\omega)(k+i\omega c) \\
-(k+i\omega c)\Lambda^{-1} & 1
\end{bmatrix}
\begin{bmatrix}
\hat{x}_N \\
x_a
\end{bmatrix}
= \begin{bmatrix}
x_N \\
0
\end{bmatrix}
\]  

(3.12)

The solution of Eq. (3.12) yields the two displacements \(\hat{x}_N\) and \(x_a\). These displacements can then be used to compute the force exerted by the DVA on the original structure at the \(N^{th}\) position, \(F_N^a\). This force is given by:

\[F_N^a(\omega) = (k+i\omega c)(x_a - \hat{x}_N)\]

(3.13)

Finally, this force \(F_N^a\) can be used in conjunction with the original set of excitation forces \(\{F\}\) to compute the new acoustic power radiated after the DVA has been added to the original structure.
Equation (3.12) is for the special case of one DVA (i.e., $M=1$). For the general case of multiple DVAs, Eq. (3.12) is a $2M$ system of equations with $2M$ unknowns. This system of equations has the form:

\[
\begin{bmatrix}
[I] + \begin{bmatrix} [U] & -[U] \end{bmatrix} & [V] \\
[V] & [0]
\end{bmatrix}
\begin{bmatrix}
\{ \dot{x}_m \} \\
\{ x_a \}
\end{bmatrix} = \begin{bmatrix}
\{ x_m \} \\
\{ 0 \}
\end{bmatrix}
\tag{3.14}
\]

where $[V]$ is a $M \times M$ diagonal matrix whose elements are given by:

\[
v_{mm} = (k_m + i\omega c_m) \cdot \Lambda_m^{-1} \quad \text{for } m=1, \ldots, M
\tag{3.15}
\]

and the elements of matrix $[U]$ are:

\[
u_{mn} = (k_n + i\omega c_n) \cdot T_{mn} \quad \text{for } m=1, \ldots, M \text{ and } n=1, \ldots, M
\tag{3.16}
\]

Vector $\{ \dot{x}_m \}$ contains the displacements of the DOFs of the structure to which the DVAs are attached, and vector $\{ x_a \}$ contains the displacements of the DVAs.

### 3.2.2 ANALYTICAL RESULTS

Using the formulation presented in the previous section, various combinations of DVAs and the sensitivity of the DVAs parameters on the radiated power were studied.

First, two SDOF DVAs were added to each side spring mount for a total of four DVAs. In each mount, the DVAs were acting in the $x$ and $y$ directions, respectively, i.e., horizontal plane. The four DVAs had a stiffness of 70,964.61 lbf/in, a damping coefficient of 1.4118 lbf s/in (i.e., 10% damping), and a mass of $7.022 \times 10^{-4}$ lbf s$^2$/in. (i.e., this mass is equivalent to a cubic inch of steel). The natural frequencies of these DVAs were tuned to 1600 Hz. This frequency was chosen because from Fig.2.7 it can be seen that significant sound radiation occurs at around this frequency. Moreover, Figures 2.22 and 2.23 show
that at this frequency, most of the radiated power is due to the $x$ and $y$ components of the spring forces on the side mounts.

The forces exerted on the shell by the four DVAs were computed. These forces were then combined with the original forces, i.e., the experimentally measured forces, to obtain a new set of net forces to carry out an acoustic analysis. The original and the new forces (i.e., the forces exerted by the absorbers added to the original forces) in the directions where the absorbers were attached are shown in Figures 3.7 through 3.10. From these figures it is observed that the addition of the four absorbers tuned to 1600 Hz, in the $x$ and $y$ directions of the side mounts, significantly reduces the magnitude of the forces at around the tuning frequency. However, these absorbers also cause an increase in the amplitude of the forces at around 1030 Hz, particularly in the $x$ components. The forces on the directions where no absorbers were added remained unchanged.

Using this new set of forces, the acoustic power radiated by the shell was obtained, and it is compared to the original system in Fig.3.11. From this figure, it is observed that the addition of the four DVAs to the side spring mounts resulted in a decrease in the radiated power at around 1600 Hz. It is interesting to observe that the radiation at around 1200 Hz was not affected by these DVAs. The total radiated acoustic power is 81.2 dBA, which resulted in a reduction of 1 dBA from the original radiated power of 82.2 dBA.

In view of the above results, it was decided to add a fifth absorber to affect the radiation at around 1200 Hz. Thus, the fifth DVA was attached to the top spring mount in the $z$ direction since this is the main force component responsible for the radiation around this frequency. The natural frequency of this absorber was chosen to be 1200 Hz since it was found that most of the sound radiation at around this frequency is due to the forces on the top spring, especially the $z$ component. The stiffness and damping coefficients of the top absorber were 39,917.6 lbf/in and 1.059lbf s/in, respectively. The side absorbers had a stiffness of 70,964.6 lbf/in and a damping coefficient of 1.412 lbf s/in. All five DVAs had a mass of $7.022 \times 10^{-4}$ lbf s$^2$/in. and a damping ratio of 10%. Again, a new set of forces was
obtained. These forces are shown in Figures 3.12 through 3.16. Observe from Figures 3.7 and 3.12, and from Figures 3.9 and 3.14, that the large amplitude of the forces at around 1030 Hz was not affected on the x nor the y directions of the side mounts. However, the magnitude of the forces on the top mount in the z direction was effectively reduced. The radiated power was computed and it is shown in Fig.3.17. From this figure, it is observed that the radiation was reduced at around 1200 Hz and also at around 1600 Hz. The total radiated power was found to be 76.7 dBA, for a net reduction of 5.5 dBA in the acoustic power.

3.2.3 PARAMETRIC STUDY

The results shown above indicate that to obtain a significant reduction in the radiated power, five absorbers are required to reduce the most significant force component driving the housing from the radiation point of view. To this end, the total radiated power was computed for various combinations of damping and undamped natural frequency of the side-spring-mount absorbers while the mass and stiffness of the top spring-mount absorber were kept unchanged. Figure 3.18 shows the results of this study as a function of the side-mount absorbers undamped natural frequency for various damping ratios. To vary the undamped natural frequency of the side absorbers, the mass was kept constant while the stiffness was changed. It can be seen that a damping of up to 30% results in a decrease of the total radiated power for all frequencies. However, as the damping is increased above 30%, the effect of the absorber becomes self-defeating. For practical purposes, it was determined that for further study of the effect of DVAs on the acoustic behavior of the compressor, a damping of 10% would be assumed. Higher damping values would probably be difficult to achieve in practice.

The next study consisted of investigating the influence of the undamped natural frequency of the top and side absorbers. For this purpose, various curves of total radiated power as a function of the side mount absorbers' undamped natural frequency were computed for different top mount absorber's undamped natural frequencies. Again, to
vary the undamped natural frequency of the absorbers, the mass was kept constant while the stiffness was changed. Also, 10% damping was used for all the absorbers. The results from this analysis are shown in Fig.3.19. From this figure, it is observed that the total radiated power is not very sensitive to the natural frequency of the absorbers. In other words, if the natural frequency of the top absorber lies in the range of 1000 Hz to 1200 Hz and the natural frequency of the side-mount absorbers lies in the 1200 to 1600 Hz range, the total radiated power will be reduced from 81.5 dB to around 76.1 dB.

3.2.4 IMPLEMENTATION ISSUES

It is important to remark that in actual practice a single absorber device with 2 DOFs would be implemented at the side mounts. However, in the numerical model this 2 DOFs absorber system is modeled as two SDOF uncoupled DVAs. This is illustrated in Fig.3.20 where the single absorber device with 2 DOFs consists of two concentric rings where the inner ring acts as a spring and as a damper, and can be made out of a low modulus material such as rubber. The outer ring would be analog to the mass of the numerical SDOF system. Therefore, for the second case studied in which 5 SDOF absorbers were added to the shell, only three absorbers would be needed in practice.

3.3 ADDING LOW MODULUS MATERIALS TO THE SUSPENSION

The third modification to the suspension system consisted of inserting a low modulus mismatch material between the springs and the spring mounts. The objective of these low modulus materials was to reduce the forces transmitted to the shell by creating an impedance mismatch. As in the previous section, there were two analysis alternatives. The first approach would be to modify the FE model, obtain the new eigenproperties, compute the radiation efficiency of each new mode, and finally compute the total radiated acoustic power. The second approach is to use the modes and the radiation efficiencies of the standard model. Again, the second approach was chosen because of its computational
advantages, and because it takes into account the effect of the mismatch material by correcting the spring force. The theoretical formulation used for this analysis will now be presented.

### 3.3.1 THEORETICAL FORMULATION

In this analysis, the housing-internal compressor assembly is considered as illustrated in Fig.3.21a. In this figure, the forces $F_c$ represent the forces originated internally in the compressor. Since the internal forces developed at the spring-shell mount interface are known (i.e., experimentally measured), the assembly system can be separated at these interfaces.

Consider the $N$-DOF system shown in Fig.3.21b and the $Q$-DOF system shown in Fig.3.21c. These systems represent the housing and the compressor of the H25A separately. The subscript $j$ indicates the DOFs common to both models, i.e., DOFs where the shell and the springs are attached. The subscript $s$ refers to the remaining DOFs in the shell, and the subscript $c$ refers to the remaining DOFs of the internal mechanisms of the compressor. The discrete equations of motion of the housing shown in Fig.3.21b can be written as:

$$
\begin{bmatrix}
D_{ss} & D_{sj} \\
D_{js} & D_{jj}
\end{bmatrix}
\begin{bmatrix}
x_s \\
x_j
\end{bmatrix}=
\begin{bmatrix}
0 \\
F_j
\end{bmatrix}
$$

where the matrix in Eq. (3.18) is the dynamic stiffness matrix of the shell.

Similarly, the discrete equations of motion of the internal compressor system shown in Fig.3.21c are:

$$
\begin{bmatrix}
D_{ii} & D_{ij} \\
D_{ji} & D_{cc}
\end{bmatrix}
\begin{bmatrix}
x_j \\
x_c
\end{bmatrix}=
\begin{bmatrix}
-F_j \\
F_c
\end{bmatrix}
$$

NOISE CONTROL APPROACHES 53
The vectors \(\{x_s\}\) and \(\{x_e\}\) are the displacement of the shell and internal system DOFs, respectively, while \(\{x_j\}\) is the displacement of the DOFs at the interface of the shell and the springs. Note that \(\{x_j\}\) is the same in both the shell and the internal compressor system, i.e., displacement continuity condition. In these equations, the vector \(\{F_j\}\) represents the internal spring forces generated at the interface defined by the \(j^{th}\) DOFs. The moments at these interface DOFs, which were not measured experimentally, are assumed to be negligible in this analysis.

Combining Eqs. (3.18) and (3.19), we can write:

\[
\begin{bmatrix}
D_{ss} & D_{sj} & 0 & 0 \\
D_{js} & D_{jj}^s & 0 & 0 \\
0 & 0 & D_{jj}^e & D_{je} \\
0 & 0 & D_{cj} & D_{ce}
\end{bmatrix}
\begin{bmatrix}
x_s \\
x_j \\
x_j \\
x_e
\end{bmatrix}
=
\begin{bmatrix}
0 \\
F_j \\
-F_j \\
F_c
\end{bmatrix}
\]  
(3.20)

The systems shown in Figures 3.21b and 3.21c are now assembled using the coupling element as indicated in Fig.3.22. The discrete equations of motion of the new coupled system can be written as:

\[
\begin{bmatrix}
D_{ss} & D_{sj} & 0 & 0 \\
D_{js} & D_{jj}^s & 0 & 0 \\
0 & 0 & D_{jj}^e & D_{je} \\
0 & 0 & D_{cj} & D_{ce}
\end{bmatrix}
\begin{bmatrix}
0 & 0 & 0 & 0 \\
[A_j] & -[B_j] & 0 & 0 \\
0 & -[B_j] & [A_j] & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_s \\
\dot{x}_j \\
\dot{x}_j \\
\dot{x}_c
\end{bmatrix}
=
\begin{bmatrix}
0 \\
0 \\
0 \\
F_c
\end{bmatrix}
\]  
(3.21)

where \([A_j]\) and \([B_j]\) are matrices that represent the dynamic characteristics of the coupling elements; and, vectors \(\{\dot{x}_s\}\) and \(\{\dot{x}_c\}\) are the displacements of the \(j^{th}\) DOFs of the shell and of the compressor, respectively. In Eq. (3.21), it is assumed that the internal forces \(\{F_c\}\) developed by the compressor do not change due to the addition of the coupling terms.

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The objective now is to obtain the new forces \( \{ \tilde{F}_j \} \) exerted by the compressor on the shell through the added coupling elements as indicated in Fig. 3.22. These forces have the form:

\[
\{ \tilde{F}_j \} = [A_j] \{ \dot{x}_s \} - [B_j] \{ \dot{x}_c \}
\]  

(3.22)

Therefore, we are interested in computing vectors \( \{ \dot{x}_c \} \) and \( \{ \dot{x}_s \} \). These vectors can be obtained from Eq. (3.21) as follows:

Premultiplying Eq. (3.21) by the inverse of matrix \([D]\), we obtain:

\[
[I] + \begin{bmatrix}
D_{ss} & D_{sj} & 0 & 0 \\
D_{js} & D_{jj} & 0 & 0 \\
0 & 0 & D_{jj} & D_{jc} \\
0 & 0 & D_{cj} & D_{cc}
\end{bmatrix}^{-1} \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & [A_j] & -[B_j] & 0 \\
0 & -[B_j] & [A_j] & 0 \\
0 & 0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\dot{x}_s \\
\dot{x}_s \\
\dot{x}_c \\
\dot{x}_c
\end{bmatrix} = \begin{bmatrix}
D_{ss} & D_{sj} & 0 & 0 \\
D_{js} & D_{jj} & 0 & 0 \\
0 & 0 & D_{jj}^c & D_{jc} \\
0 & 0 & D_{cj} & D_{cc}
\end{bmatrix}^{-1} \begin{bmatrix}
0 \\
0 \\
F_c \\
0
\end{bmatrix}
\]  

(3.23)

where

\[
\begin{bmatrix}
D_{ss} & D_{sj} & 0 & 0 \\
D_{js} & D_{jj} & 0 & 0 \\
0 & 0 & D_{jj}^c & D_{jc} \\
0 & 0 & D_{cj} & D_{cc}
\end{bmatrix}^{-1} = \begin{bmatrix}
T_{ss} & T_{sj} & 0 & 0 \\
T_{js} & T_{jj} & 0 & 0 \\
0 & 0 & T_{jj}^c & T_{jc} \\
0 & 0 & T_{cj} & T_{cc}
\end{bmatrix}
\]  

(3.24)

where the submatrices \( [T_{ss}] \), \( [T_{sj}] \), etc., in Eq. (3.24) are the transfer functions among the various DOFs.

To solve Eq. (3.23), the vector of internal forces \( \{ F_c \} \) is needed. However, it is not required to know this force vector explicitly. To this end, Eq. (3.23) is rewritten as:

NOISE CONTROL APPROACHES
The first term on the right hand side of Eq. (3.25) represents the displacements vector of the original system as given by Eq. (3.20), i.e., when the housing and compressor systems are coupled at the $j^{th}$ nodes rigidly. Thus, an explicit knowledge of $\{F_c\}$ is not required in the present formulation.

To compute vectors $\{\hat{x}_{js}\}$ and $\{\hat{x}_{jc}\}$, we only need the equations of motion of the $j^{th}$ DOFs of the shell and of the $j^{th}$ DOFs of the compressor. From Eq. (3.25), these equations are:

\[
(I + [T^e_{jj}] [A_j]) \{\hat{x}_{js}\} - [T^e_{jj}] [B_j] \{\hat{x}_{jc}\} = \{x_{j}\} - [T^e_{jj}] \{F_j\} 
\]  
\[ (3.26) \]

and

\[
-[T^e_{jj}] [B_j] \{\hat{x}_{js}\} + (I + [T^e_{jj}] [A_j]) \{\hat{x}_{jc}\} = \{x_{j}\} - [T^e_{jj}] \{-F_j\} 
\]  
\[ (3.27) \]

where the displacements vector $\{x_{j}\}$ can be simply obtained from Eq. (3.18). Finally, Eqs. (3.26) and (3.27) can be solved for the displacements vectors $\{\hat{x}_{js}\}$ and $\{\hat{x}_{jc}\}$.
3.3.2 MISMATCH ELEMENT MODELING

In the previous section, the formulation to model the insertion of the mismatch materials was presented. In this investigation, it is assumed that the \( j^{th} \) DOF of the shell is only coupled to the \( j^{th} \) DOF of the internal compressor, i.e., springs. Furthermore, the rotational DOFs are not considered. The elements of matrices \([A_j]\) and \([B_j]\) contain the point and cross receptance of the coupling elements, respectively. By assuming that these matrices are diagonal, the models of the low modulus materials are uncoupled in the \( x, y, \) and \( z \) directions. For example, if a force is applied to one end of the model in the \( x \) direction, the displacement of the other end in the \( y \) direction is zero.

3.3.2.1 DISCRETE MODEL OF THE MISMATCH MATERIAL

The first mismatch material considered here was simply modeled as a stiffness-damper system. For this particular case (i.e., no inertia is associated with the mismatch material), matrices \([A_j]\) and \([B_j]\) are identical with elements given by:

\[
A_{jj} = B_{jj} = k_m + i\omega c_m
\]

where \( \omega \) is the excitation frequency, and \( k_m \) and \( c_m \) are the mismatch material stiffness and damping coefficient, respectively.

3.3.2.2 ANALYTICAL RESULTS WITH THE DISCRETE MODELS

Using the formulation presented in section 3.3.1, discrete models of the mismatch materials were inserted to the top and side mounts in the \( x, y \) and \( z \) directions. The axial stiffness and damping coefficient of these models was 50 lbf/in and 0.01 lbf s/in, respectively. The shear stiffness and damping coefficient was 250 lbf/in and 0.05 lbf s/in, respectively. These parameters were selected based on available vibration isolation mounts.
whose shear stiffness is greater than their axial stiffness by a factor of 5. For reference purposes only, note that the axial and shear stiffness of the current side suspension springs is 478 lbf/in and 500 lbf/in, respectively.

The new set of forces on the housing after inserting the models described above was computed. These forces are shown in Figures 3.23 through 3.25. From these figures, the following was observed: The forces on the top mount decreased over the entire spectrum except at around 1100 Hz where the change was insignificant. The forces on the side mounts were also reduced overall except at around 1200 Hz in the x and y directions, and at around 1100 Hz in the z directions where they remained very close to the original forces.

Using this new set of forces on the shell, the spectrum of the radiated power was computed. This spectrum is presented in Fig. 3.26. From this figure, it is seen that although there is a reduction in the 1200 Hz and in the 1600 Hz regions, the reduction at around 1200 Hz is much smaller than that at 1600 Hz. The new total radiated power was found to be 69.4 dBA, for a net reduction of 12.8 dBA.

Thus far, the results obtained with this approach are very satisfactory. However, by modeling the acoustic mismatch materials using spring and damper elements only, the dynamics of these materials are not accounted for. To check if the mismatch material models will have resonant frequencies within the range of interest (i.e., 600 to 2000 Hz) the properties of rubber mounts available in the market can be assumed to compute the first natural frequency of the model. Assuming a mass density and modulus of elasticity of 5.5115x10-4 lbf s^2/in^4 and 400 psi, respectively, the fundamental frequency of a fix-fix beam with a length of 1 in. and a cross sectional area of 1 in^2 was found to be around 200 Hz. Thus, there is a need to account for the dynamics of the mismatch material models. To this end, a continuous model of the low modulus mismatch materials was developed using a wave analysis. The formulation used for this purpose will now be presented.
3.3.2.3 CONTINUOUS MODEL OF THE MISMATCH MATERIALS

The acoustic mismatch inserts can be modeled as a continuous rod shown in Fig.3.27. To account for the damping of the rod, a complex form of the Modulus of Elasticity is used as follows:

\[ \tilde{E} = E(1 + i\eta) \]  
(3.28)

where \( E \) is the Modulus of Elasticity and \( \eta \) is the loss factor.

From a wave analysis which can be found in appendix III, the elements of the dynamic stiffness matrices \([A_i]\) and \([B_i]\) have the form:

\[ A_{ij} = kS\tilde{E} \cdot \cot(kL) \]  
(3.29)

\[ B_{ij} = \frac{-kS\tilde{E}}{\sin(kL)} \]  
(3.30)

where \( S \) and \( L \) are the cross sectional area and the length of the rod, respectively; \( k \) is the wave number \( k=\alpha/c \), with \( \alpha \) the excitation frequency and \( c \) the phase speed of the structural wave. For longitudinal vibration, the longitudinal phase speed \( c_l \) is given by:

\[ c_l = \sqrt{\frac{\tilde{E}}{\rho}} \]  
(3.31)

where \( \rho \) is the mass density of the rod. For the present case, it is assumed that the transverse vibration of the rod is controlled by shear. Thus, for shear controlled transverse vibration, the shear phase speed \( c_s \) has the form:

\[ c_s = \sqrt{\frac{\tilde{G}}{\rho}} \]  
(3.32)
Therefore, to account for the dynamic behavior of the mismatch materials, the same formulation as that presented in section 3.3.1 can be used. The only difference is that now the elements of the dynamic stiffness matrices are given by Eqs.(3.29) and (3.30).

3.3.2.4 ANALYTICAL RESULTS WITH THE CONTINUOUS MODELS

Using the formulation presented in the previous section, continuous models of the mismatch materials with a cylindrical geometry were inserted to the top and side mounts. The cross sectional area of these models was equivalent to the cross sectional area of the present coil springs (i.e., 0.9545 in²), and their length was selected to be 1 in. The mass density, modulus of elasticity, and loss factor for all the models was $5.5115 \times 10^{-4}$ lbf s²/in⁴, 400 psi, and 0.25, respectively. Again, these values were selected from available vibration isolation mounts.

The new forces exerted on the mounts were computed and are shown in Figures 3.28 through 3.30. From these figures, it is seen that the nine force components were reduced over the entire spectrum. The total radiated power corresponding to these new forces is shown in Fig.3.31. From this figure, it is seen that after inserting the continuous models of the mismatch materials, a significant sound power reduction is obtained at around 1600 Hz. However, the reduction at around 1200 Hz was not as large as that obtained when the spring and damper models were used. The total radiated power after inserting these continuous models of the mismatch materials was 75.8 dBA, for a net reduction of 6.4 dBA.

The effect of the length of the mismatch models was also studied. Keeping constant values for the mass density, modulus of elasticity and cross sectional area of $5.5115 \times 10^{-4}$ lbf s²/in⁴, 400 psi, and 0.9545 in², respectively, the length of the models was varied. It was found that when the length is 0.75 in., the total radiated power is reduced by 3.45 dB. When the length is 0.5 in., the total radiated power remains practically unchanged.
3.4 SUMMARY

Three noise control approaches were investigated. First, the side spring mounts were moved to various positions on the lower part of the shell. The objective of relocating the mounts was to reduce the excitation forces on the most efficient radiating modes. In this approach, the spring forces were assumed to be unchanged. The maximum physically attainable sound power reduction with this approach was around 1.7 dBA.

The second approach consisted of adding DVAs to the mounts to reduce the net forces transmitted to the shell. Again, the spring forces were assumed to be unchanged. It was found that a reduction of 5.5 dBA in the total radiated power is obtained when using 5 DVAs (i.e., one on the top in the z direction, and two on each side mount in the x and y directions, respectively).

In the third approach, low modulus materials were inserted between the mounts and the springs. The objective of inserting these low modulus materials was to reduce the spring forces by creating an impedance mismatch. In this approach, the effect of the low modulus materials on the spring forces was accounted for implicitly. This approach yielded the most promising results, with a reduction of 6.4 dBA in the total radiated power.
Table 3.1 Total radiated power for various side mount locations.

<table>
<thead>
<tr>
<th>Position</th>
<th>Total Radiated Power (dB)</th>
<th>Position</th>
<th>Total Radiated Power (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>79.41</td>
<td>P16</td>
<td>80.89</td>
</tr>
<tr>
<td>P2</td>
<td>79.65</td>
<td>P17</td>
<td>80.37</td>
</tr>
<tr>
<td>P3</td>
<td>80.16</td>
<td>P18 *</td>
<td>79.94</td>
</tr>
<tr>
<td>P4</td>
<td>80.41</td>
<td>P19</td>
<td>79.35</td>
</tr>
<tr>
<td>P5</td>
<td>80.45</td>
<td>P20</td>
<td>80.00</td>
</tr>
<tr>
<td>P6</td>
<td>80.19</td>
<td>P21</td>
<td>80.94</td>
</tr>
<tr>
<td>P7</td>
<td>79.32</td>
<td>P22</td>
<td>80.47</td>
</tr>
<tr>
<td>P8</td>
<td>80.39</td>
<td>P23</td>
<td>80.41</td>
</tr>
<tr>
<td>P9</td>
<td>80.48</td>
<td>P24</td>
<td>80.20</td>
</tr>
<tr>
<td>P10</td>
<td>80.40</td>
<td>P25 **</td>
<td>78.92</td>
</tr>
<tr>
<td>P11</td>
<td>80.17</td>
<td>P26</td>
<td>79.76</td>
</tr>
<tr>
<td>P12</td>
<td>79.97</td>
<td>P27</td>
<td>80.80</td>
</tr>
<tr>
<td>P13</td>
<td>79.83</td>
<td>P28</td>
<td>81.00</td>
</tr>
<tr>
<td>P14</td>
<td>81.12</td>
<td>P29</td>
<td>80.87</td>
</tr>
<tr>
<td>P15</td>
<td>81.25</td>
<td>P30</td>
<td>80.36</td>
</tr>
</tbody>
</table>

* Best Physically Realizable Position

** Best Position Probably not Physically Realizable.
Figure 3.1 Studied mount locations in the front.

Figure 3.2 Studied mount locations in the back.
Figure 3.3 Acoustic power radiated with the mounts at position P25.
(Total radiated power 79.5 dBA, ref. $10^{-12}$ W)

Figure 3.4 Acoustic power radiated with the mounts at position P18.
(Total radiated power 80.5 dBA, ref. $10^{-12}$ W)
Figure 3.5 N degree-of-freedom system.

Figure 3.6 N-DOF system with dynamic vibration absorbers.
Figure 3.7 Forces on the right mount in the x direction (4 absorbers).

Figure 3.8 Forces on the right mount in the y direction (4 absorbers).
Figure 3.9 Forces on the left mount in the $x$ direction (4 absorbers).

Figure 3.10 Forces on the left mount in the $y$ direction (4 absorbers).
**Figure 3.11** Radiated power after adding 4 DVAs to the side mounts.

(Total radiated power 81.2 dBA, ref. $10^{-12}$ W)
**Figure 3.12** Forces on the right mount in the x direction (5 absorbers).

**Figure 3.13** Forces on the right mount in the y direction (5 absorbers).
Figure 3.14 Forces on the left mount in the \( x \) direction (5 absorbers).

Figure 3.15 Forces on the left mount in the \( y \) direction (5 absorbers).
Figure 3.16 Forces on the top mount in the z direction (5 absorbers).
Figure 3.17 Radiated power after adding 5 DVAs.
(Total radiated power 76.7 dBA, ref. $10^{-12}$ W)
Figure 3.18 Effect of the side DVAs natural frequency and damping on the total radiated power.

Figure 3.19 Effect of the absorbers' natural frequency on the radiated power.
Figure 3.20 Models of the DVAs: (a) Numerical, and (b) physical implementation.
Figure 3.21 Models representing: (a) housing-internal compressor assembly, (b) the housing, and (c) the shell.

Figure 3.22 Models representing the compressor assembly with added coupling elements.
Figure 3.23 Forces on the top mount (with spring and damper models).
Figure 3.24 Forces on the right mount (with spring and damper models).
Figure 3.25 Forces on the left mount (with spring and damper models).
Figure 3.26 Radiated power after inserting the spring and damper models of the mismatch materials. (Total radiated power 69.4 dBA, ref. $10^{-12}$ W)
Figure 3.27 Continuous model of the low modulus material.
Figure 3.28 Forces on the top mount (with the continuous models).
Figure 3.29 Forces on the right mount (with the continuous models).
Figure 3.30 Forces on the left mount (with the continuous models).
Figure 3.31 Radiated power after inserting the continuous models of the mismatch materials. (Total radiated power 75.8 dBA, ref. $10^{-12}$ W)
CONCLUSIONS AND RECOMMENDATIONS

This work was concerned with the acoustic prediction and noise control of the H25A compressor. The general approach was to use the FE model together with the experimentally measured spring forces to compute the radiated power by the compressor. Various noise control approaches to reduce the sound radiated by the compressor were investigated. For each approach, efficient analytical models to study the effect of design modifications on the radiated sound power were developed. In this chapter, a brief review of the most important results obtained from the acoustic prediction and the investigated noise control approaches will be made. Then, some recommendations for future work will be given.

4.1 CONCLUSIONS

From the acoustic prediction of the H25A compressor, the following conclusions can be drawn. It was observed that of the 23 modes included in the analysis, 8 had high radiation efficiencies and only 6 radiated significant acoustic power. It is believed that the other two modes do not significantly contribute to the total sound radiated because they are not excited by the spring forces acting on the mounts. Of the six modes that contribute significantly to the total radiated power, three have natural frequencies around 1200 Hz, and the other three have natural frequencies around 1600 Hz. The spectrum of the total radiated power shows maximum values at around these two frequencies.

It was also observed that of the 9 experimentally measured forces on the mounts, five are responsible for most of the sound radiated. These are the forces on the top mount in the z direction at around 1200 Hz, and the forces on the side mounts in the x and y
directions at around 1600 Hz. Therefore, attempts to control the sound radiated by the H25A compressor should focus on these force components.

In all noise control approaches there are three areas that affect the system: (a) control at the source, (b) control at the paths, and (c) changing the acoustic characteristics of the source (i.e. changing the radiation efficiency). In this work, the second approach was investigated. The most promising modifications are the addition of vibration absorbers and the insertion of low modulus materials between the mounts and the springs; both modifications are aimed at reducing the forces transmitted to the shell (i.e., control at the paths).

The addition of the dynamic vibration absorbers to the mounts yielded a reduction of 5.37 dB on the total radiated acoustic power. It was found that to obtain a significant sound attenuation, the absorbers should be added to the top and side mounts (i.e., one absorber on the top mount acting in the z direction, and two absorbers on each side mount acting in the x and y directions, respectively). Moreover, the top absorber’s natural frequency was tuned to 1200 Hz (i.e., at around this frequency, most of the sound radiation is due to the force on the top mount in the z direction), while the side absorber’s natural frequencies were tuned to 1600 Hz (i.e., at around this frequency, most of the sound radiation is due to the forces on the side mounts in the x and y directions, respectively). In this analysis, it was assumed that the internal forces acting on the mounts are not affected by the vibration absorbers.

The insertion of low modulus materials between the mounts and the springs yielded a reduction of 6.23 dB in the total radiated acoustic power. In this analysis, the effect of the low modulus materials on the spring forces was taken into account implicitly. It was also assumed that the low modulus materials are not fully coupled in the x, y, and z directions.

CONCLUSIONS AND RECOMMENDATIONS
4.2 RECOMMENDATIONS

Due to time constraints, combinations of the various noise control approaches were not investigated. Therefore, a study involving two or more of the approaches presented in this work should be carried out. For example, even though relocating the mounts did not produce a significant reduction in the radiated sound, a combination of this approach with the addition of DVAs should be considered.

It is also recommended to perform an experimental testing of the most promising approaches. As pointed out earlier, these approaches are the addition of vibration absorbers and the insertion of acoustic mismatch materials to the mounts. Due to space constraints, addition of the absorbers may not be easily attainable in practice. However, to gain a feel for the efficiency of the absorbers to reduce the transmitted forces, they could initially be attached to the shell from outside at the mount locations.

An investigation regarding the dynamic behavior of the suspension springs is also encouraged. This dynamic behavior is believed to have a major role on the forces transmitted to the shell because the result of inserting the mismatch materials was to basically change the dynamics of the springs. Thus, it was found analytically that a significant reduction of the total radiated power could be attained by modifying the suspension springs.

Finally, the noise control approaches presented in this work were not directed to affect the source. For instance, the difference of the internal and external pressures on the motor cap may be responsible for the forces on the top mount. Therefore, it is recommended that noise control approaches to attack the noise problem directly at the source be investigated.
APPENDIX I

Radiation Efficiency Curves of the H25A Shell
Figure A1.1 Radiation efficiency of modes 1 through 6.

Figure A1.2 Radiation efficiency of modes 7 through 12.
Figure A1.3 Radiation efficiency of modes 13 through 18.

Figure A1.4 Radiation efficiency of modes 19 through 23.
APPENDIX II

Acoustic Power Radiated by the Modes of the H25A Shell

Using all the Experimentally Measured Forces

(For all figures, the reference acoustic power is $10^{-12}$ W.)
**Figure A2.1** Acoustic power radiated by mode 1 (650 Hz)

**Figure A2.2** Acoustic power radiated by mode 2 (717 Hz)
Figure A2.3 Acoustic power radiated by mode 3 (740 Hz)

Figure A2.4 Acoustic power radiated by mode 4 (889 Hz)
Figure A2.5 Acoustic power radiated by mode 5 (975 Hz)

Figure A2.6 Acoustic power radiated by mode 6 (1108 Hz)
Figure A2.7 Acoustic power radiated by mode 7 (1164 Hz)

Figure A2.8 Acoustic power radiated by mode 8 (1193 Hz)
Figure A2.9 Acoustic radiated power by mode 9 (1220 Hz)

Figure A2.10 Acoustic power radiated by mode 10 (1357 Hz)
Figure A2.11 Acoustic power radiated by mode 11 (1382 Hz)

Figure A2.12 Acoustic power radiated by mode 12 (1390 Hz)
Figure A2.13 Acoustic power radiated by mode 13 (1476 Hz)

Figure A2.14 Acoustic power radiated by mode 14 (1558 Hz)
Figure A2.15 Acoustic power radiated by mode 15 (1616 Hz)

Figure A2.16 Acoustic power radiated by mode 16 (1644 Hz)
Figure A2.17 Acoustic power radiated by mode 17 (1683 Hz)

Figure A2.18 Acoustic power radiated by mode 18 (1757 Hz)
Figure A2.19 Acoustic power radiated by mode 19 (1812 Hz)

Figure A2.20 Acoustic power radiated by mode 20 (1908 Hz)
Figure A2.21 Acoustic power radiated by mode 21 (1943 Hz)

Figure A2.22 Acoustic power radiated by mode 22 (1966 Hz)
Figure A2.23 Acoustic power radiated by mode 23 (1985 Hz)
APPENDIX III

Wave Analysis of the Mismatch Material Model
Consider the rod shown in Figure 3.27. To account for the damping of the rod, a complex form of the Modulus of Elasticity is used as follows:

\[ E(\omega) = E(1+i\eta) \]  

(A3.1)

where \( E \) is the Modulus of Elasticity and \( \eta \) is the loss factor.

The equation of motion for the axial vibration of this rod is given by:

\[ \frac{\partial^2 u(x,t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u(x,t)}{\partial t^2} \]  

(A3.2)

where \( u(x,t) \) is the axial displacement and \( c \) is the phase speed of the structural wave:

\[ c_i = \frac{\sqrt{E}}{\rho} \]  

(A3.3)

Equation (A3.2) has a solution of the form:

\[ u(x,t) = (A_+ e^{-ikx} + A_- e^{ikx}) e^{i\omega t} \]  

(A3.4)

where \( A_+ \) and \( A_- \) are the amplitudes of the waves traveling in the positive and negative \( x \) direction, respectively, and \( k \) is the wave number given by:

\[ k = \frac{\omega}{c_i} \]  

(A3.5)

with \( \omega \) as the excitation angular frequency.
The objective now is to obtain the dynamic stiffness matrix of the model. For the present case this is a 2x2 matrix, i.e. one degree-of-freedom at each end of the rod. The diagonal terms of this matrix represent the force required to induce a unit displacement on the DOF, where the force is applied while the other DOF is prevented from moving. For example, the first diagonal element in this matrix is obtained by imposing the following boundary conditions:

At \( x = 0 \), continuity of force:

\[
F_1 = -\vec{E}S \frac{\partial u}{\partial x} \bigg|_{x=0}
\]

(A3.6)

At \( x = L \), zero displacement:

\[
u(L,t) = 0
\]

(A3.7)

substituting Eq.(A3.4) into Eq.(A3.6) and Eq.(A3.7) gives:

\[
0 = A_+ e^{-ikL} + A_- e^{ikL}
\]

(A3.8)

\[
ike^L A_+ - ikeL A_- = F_1
\]

(A3.9)

Writing Eqs. (A3.8) and (A3.9) in matrix form leads to:

\[
\begin{bmatrix}
ike^L & -ike^L \\
e^{-ikL} & e^{ikL}
\end{bmatrix}
\begin{bmatrix}
A_+ \\
A_-
\end{bmatrix} = 
\begin{bmatrix}
F_1 \\
0
\end{bmatrix}
\]

(A3.10)

Solution of Eq.(A3.10) yields the amplitude of the waves as:
\[ A_{+} = \frac{e^{iAL}}{i2k\tilde{E}\cos(kL)} \cdot F_{1} \]  

(A3.11)

and

\[ A_{-} = \frac{-e^{-iAL}}{i2k\tilde{E}\cos(kL)} \cdot F_{1} \]  

(A3.12)

On the other hand, the displacement of the left end of the rod is given by:

\[ x_{1} = A_{+} + A_{-} \]  

(A3.13)

Thus, substituting Eqs.(A3.11) and (A3.12) into Eq.(A3.13), the displacement of the left end of the rod is:

\[ x_{1} = \frac{e^{iAL} - e^{-iAL}}{i2k\tilde{E}\cos(kL)} \cdot F_{1} \]  

(A3.14)

By rearranging Eq.(A3.14) and using Euler’s identity, we obtain an expression for the force \( F \) needed to cause a displacement \( x_{1} \) to the left end of the rod when the right end is kept fixed:

\[ \frac{F_{1}}{x_{1}} = k\tilde{E} \cdot \cot(kL) \]  

(A3.15)

This is the term \( A_{ii} \) needed in Eqs.(3.25) and (3.26).

The cross terms of the dynamic stiffness matrix represent the force required to keep fixed the DOF where the force is applied while the other DOF undergoes a unit displacement. This cross term can be obtained by imposing the following boundary conditions:

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At $x = 0$, zero displacement:

$$u(0,t) = 0 \quad (A3.16)$$

At $x = L$, prescribed displacement $x_2$:

$$u(L,t) = x_2 \quad (A3.17)$$

Substituting Eq. (A3.4) into Eqs. (A3.16) and (A3.17) gives:

$$0 = A_+ + A_- \quad (A3.18)$$

and

$$x_2 = A_+ e^{-iL} + A_- e^{iL} \quad (A3.19)$$

Substituting Eq. (A3.18) into Eq. (A3.19) we obtain:

$$x_2 = A_- (e^{iL} - e^{-iL}) \quad (A3.20)$$

On the other hand, the force on the left end of the rod is given by:

$$F_i = -S\hat{E} \cdot (-ikA_+ + ikA_-) \quad (A3.21)$$

Substituting Eq. (A3.18) into Eq. (A3.21) we obtain:

$$F_i = -i2kS\hat{E}A_- \quad (A3.22)$$

Substitution of Eq. (A3.19) into Eq. (A3.22) yields the expression of the force $F$ needed on the left end of the rod to prevent this end from moving when the right end undergoes a displacement $x_2$.
\[ \frac{F_1}{x_2} = -\frac{kS\bar{E}}{\sin(kL)} \] (A3.23)

By the reciprocity principle, the reaction force on the right end of the rod required to keep this end from moving when the left end undergoes a displacement \( x_1 \) is:

\[ \frac{F_2}{x_1} = -\frac{kS\bar{E}}{\sin(kL)} \] (A3.24)

and the force needed to induce a displacement \( x_2 \) to the right end of the rod when the left end is prevented from moving has the form:

\[ \frac{F_2}{x_2} = kS\bar{E} \cdot \cot(kL) \] (A3.25)

This is the \( B_{ii} \) term needed in Eqs.(3.25) and (3.26).

Therefore, the dynamic stiffness matrix of the rod shown in Fig.3.27 is given by:

\[
\begin{bmatrix}
    kS\bar{E} \cdot \cot(kL) & -\frac{kS\bar{E}}{\sin(kL)} \\
    -\frac{kS\bar{E}}{\sin(kL)} & kS\bar{E} \cdot \cot(kL)
\end{bmatrix}
\] (A3.26)

The formulation presented thus far was developed for the axial vibration of the rod. However, for the present analysis it is assumed that the transverse vibration can be approximated using transverse shear waves. Moreover, the assumed solution for shear waves has exactly the same form as Eq.(A3.2). The only difference is in the phase speed, which for shear waves is given by:

\[ c_s = \sqrt{\frac{G}{\rho}} \] (A3.27)

APPENDIX III
REFERENCES


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