

OPTIMAL CLASS SCHEDULING
SUBJECT TO PROFESSORS' PREFERENCES

by

Edward Hirschman

Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

MASTER OF ARTS

in

Economics

APPROVED:

Robert J. Mackay, Chairman

David I. Meiselman

Alan N. Freiden

March, 1989

Blacksburg, Virginia

OPTIMAL CLASS SCHEDULING
SUBJECT TO PROFESSORS' PREFERENCES

by

Edward Hirschman

Committee Chairman: Robert J. Mackay
Economics

(ABSTRACT)

The focus of this thesis is to present the theory and application of a new form of multiattribute utility optimization by way of an illustrative example, the optimal scheduling of classes subject to professors' preferences. This new form of multiattribute utility optimization is based on ordinal as opposed to cardinal utility and is defined from a corresponding integer programming model in operations research which (1) is solved for ordinal cost factors and (2) serves as the problem's theoretical starting point.

It is suggested herein that one start with a mathematical formulation that if solved in an acceptable or--preferably--best manner would yield a satisfactory or possibly best solution to the problem. Then, that mathematical formulation and its solution technique defines the multiattribute utility problem and its solution at issue. This is the reverse of what is usually done; and as will be shown, doing this can be quite fruitful.

The illustrative example concerns a mathematical formulation based on operation research's assignment problem. As will be argued, the cost factors must be ordinal, which essentially corresponds to using ordinal utility; hence the technique will be framed in the realm of ordinal utility.

The technique for solving the illustrative example's mathematical formulation is to achieve a premium mix of operations research solution properties. From this perspective, some sticky issues in multiattribute utility theory when the attributes involve the preferences of distinct persons are not included in the philosophical base for the multiattribute utility problem and its solution thusly defined.

TABLE OF CONTENTS

<u>Chapter</u>	<u>Page</u>
LIST OF TABLES	vi
LIST OF FIGURES	vii
1. INTRODUCTION	1
1.1 Preliminary Statements	1
1.2 Comments on the Proposed Approach	1
1.3 Introductory Comments on the Illustrative Example	3
2. THE CLASS SCHEDULING PROBLEM AS AN APPLICATION OF THE ASSIGNMENT PROBLEM IN OPERATIONS RESEARCH	6
2.1 Introduction	6
2.2 The Proposed Class Scheduling Model	7
2.3 A Discussion Regarding the Class Scheduling Model	9
2.4 Establishing That the Cost Factors Must Be Ordinal	15
2.5 Summary	21
3. ORDINAL COST FACTOR THEORY	22
3.1 Introduction	22
3.2 A Drawback Following From Utilizing Ordinal Cost Factors	22
3.3 Desirable Properties That the Class Scheduling Ordinal Cost Factors Have	27
3.4 Summary	34
4. PROCEDURE TO ASSIGN THE ORDINAL COST FACTORS	37
4.1 Preliminary Statements	37
4.2 A Concrete Discussion Regarding Assignment of the Cost Factors	39
5. COMPUTER IMPLEMENTATION OF SOLUTION TO THE CLASS SCHEDULING PROBLEM AND A TEST CASE	42
5.1 Preliminary Statements	42
5.2 On the Use of SUPER-LINDO	43
5.3 The Basic Framework and the Data Surrounding the Test Case	43

6.	EVALUATION OF THE CLASS SCHEDULING PROBLEM'S MODEL	53
6.1	Preliminary Statements	53
6.2	Results of the Test	54
6.3	Man-hours and Computer Time Involved in the Test	59
6.4	Concluding Remarks	61
	SELECTED REFERENCES	64
	VITA	65

LIST OF TABLES

<u>Table</u>	<u>Page</u>
1. An Array of Clusters with Associated Ordinal Cost Factors	38
2. A Tabulation of Professors' Preferences	48
3. Course Time Slots as Published in the Schedule of Classes	49
4. Untitled	50
5. Optimal Class Schedules	55
6. Untitled	56

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1.	Untitled	45
2.	Untitled	45

CHAPTER 1
INTRODUCTION

1.1 Preliminary Statements

The focus of this thesis is to present the theory and application of a new form of multiattribute utility optimization by way of an illustrative example, the optimal scheduling of classes subject to professors' preferences. This new form of multiattribute utility optimization is based on ordinal utility as opposed to cardinal utility and is defined from a corresponding integer programming model in operations research which (1) is solved for ordinal cost factors and (2) serves as the problem's theoretical starting point; to be noted is that these considerations are new to this thesis.

Section 1.2 will present comments on the proposed approach. Section 1.3 will then present introductory remarks on the illustrative example.

1.2 Comments on the Proposed Approach

In multiattribute utility optimization--particularly for the additive form which is at the heart of the analysis in this thesis--the problem at issue is defined and then a corresponding mathematical formulation is sought through which standard algorithms can be applied to yield the

desired answer. The mathematical formulation usually lies in the area of operations research methodology.

Traditionally, multiattribute utility theory requires the use of cardinal utility for the problem and its solution to be well-defined. And without the problem and its solution being well-defined no mathematical formulation follows.

However, as a new approach to defining the multiattribute utility problem and its solution, it is suggested herein that one start with a mathematical formulation that if solved in an acceptable or--preferably--best manner would yield a satisfactory or possibly best solution to the problem. In such a case, that mathematical formulation and its solution technique would define the multiattribute utility problem and its solution at issue. It is to be emphasized that this is the reverse of what is usually done; and as will be shown, doing this can be quite fruitful.

The illustrative example which is introduced in Section 1.3 concerns a mathematical formulation based on operation research's assignment problem, which is an integer programming problem. As will be argued, the cost factors for that formulation must be ordinal, which essentially corresponds to using ordinal utility; and hence the technique will be framed in the realm of ordinal utility.

The technique for solving the illustrative example's mathematical formulation is to achieve a premium mix of operations research solution properties.

With acceptance of the mathematical formulation and its solution technique, not only is a corresponding multiattribute utility problem and its solution well-defined, but it is well-defined for ordinal utility. And because the approach is developed from the perspective of achieving a premium mix of operations research solution properties, some sticky issues in multiattribute utility theory when the attributes involve the preferences of distinct persons are not included in the philosophical base for the multiattribute utility problem and its solution thusly defined.

1.3 Introductory Comments on the Illustrative Example

Ostensibly, the university schedules classes mainly to accommodate the needs of its students. For example, for graduate courses the classes are generally scheduled in the evening time slots. Furthermore, effort is made so that schedule conflicts between courses do not occur; i.e., classes which students can take simultaneously during a semester are not scheduled at the same time. The university must accommodate the students to those extents, and in some other ways, to remain competitive with other universities

that also seek to draw the most desirable bodies from the same pool of talent.

But, for the cream of pickings with respect to the professors from whom the university can draw to provide the bulk of its services, the university must also make accommodations to the professors, and it does this by granting perquisites to the professors. For example, a long established perquisite is the allocating of parking spaces to professors in such a way that the professors park closer to the university--or in other ways realize a parking advantage--over the students.

Seen in the above light, the problem of scheduling classes optimally is a people problem. Both students and professors need to be accommodated for any sense of optimality to be achieved. Previously, the needs of students have been explicitly taken into consideration. However, in no case of which this researcher is aware are the preferences of professors explicitly taken into account, except secretly and almost certainly suboptimally.

A goal of this thesis is to determine an efficient model (or way) of taking professors' preferences into account that yields chances of having professors being maximally happy--as a group--with the time slots to which classes they teach are assigned, without detracting from the scheduling concessions the university must make to the students to be competitive in filling those classes with

qualified students. The method must not be limited to any one particular set of concessions the university should or will make to the students, and in that sense will be very general.

CHAPTER 2

THE CLASS SCHEDULING PROBLEM AS AN APPLICATION OF THE ASSIGNMENT PROBLEM IN OPERATIONS RESEARCH

2.1 Introduction

In the Spring 1986 semester at George Mason University (GMU), a group of students in the OR-743 Applications of Management Science class taught by Dr. Hoffman performed a study on class scheduling from the perspective of professors. Their efforts revolved around an application of what is termed, in operations research, an assignment problem. As a part of their efforts, the concept of ordinal cost factors--i.e., cost factors whose value is determined largely by their ordinal ranking--was introduced as an important step in arriving at a solution to the class scheduling problem.¹

The purpose of this chapter is to show (1) a variant (modification) of the assignment problem formulation is the best know model for addressing the class scheduling problem, and (2) the cost factors for that formulation must be ordinal.

¹The concept of using ordinal cost factors in conjunction with the assignment problem was developed by Edward Hirschman in 1986.

Section 2.2 of this chapter will present the variant of the assignment formulation--constituting the class scheduling model--proposed by this document.

Section 2.3 will place the variant proposed in Section 2.2 into the context of the general class of problems to which it belongs and will present an argument why this can be considered the best model the current state-of-the-art in operations research has to offer for solving the class scheduling problem.

Section 2.4 will argue why the cost factors for the class scheduling model must be ordinal.

Section 2.5 will summarize this chapter.

2.2 The Proposed Class Scheduling Model

The class scheduling model proposed by this document is:

$$\text{Minimize} \quad \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = 1, \quad i=1, 2, \dots, m \quad (1)$$

$$\sum_{i=1}^m x_{ij} \leq k_j, \quad j=1, 2, \dots, n \quad (2)$$

$$x_{ij} + x_{pk} \leq 1 \quad (3)$$

Where:

$$x_{ij} = \begin{cases} 1 & \text{if course } i \text{ is given during period } j \\ 0 & \text{otherwise} \end{cases}$$

m is the number of courses in a given semester.

n is the number of time periods available for classes.

k_j is the maximum number of classes given during time period j .

c_{ij} is the cost factor determined in accordance with each faculty member's stated preferences.

Constraints (1) require that each course be taught exactly once during the semester. If two or more sections of a given course are to be offered, each section is to be regarded as a distinct course. Constraints (2) limit the number of courses which can be given during each time period (this number is not necessarily the same for each time period). A constraint of type (3) is required for each pair of courses, i and p , which should not be given during conflicting time periods j and k , where j may or may not equal k . This last set of constraints reduces or, possibly, eliminates the probability of schedule conflicts for the students and professors. The constraints of type (3) used for the students would typically have $j=k$. However, j need not equal k for the constraints of type (3) used to avoid conflicts for the professors; e.g., j and k could be neighboring periods (we might not want professors to teach two classes back-to-back). A fourth type of constraint will on occasion be added to the above list of three constraint types, because it will sometimes be necessary for ensuring that a particular class will, independent of all other

considerations, not be taught at a particular time; this constraint takes the form:

$$x_{1q} = 0 \quad (4)$$

for which course 1 cannot be taught during period q .

The cost coefficients are derived from the faculty's stated preferences for time periods with regard to specific courses. Assigning values to the cost coefficients will be addressed in detail in Chapter 4.

It is mentioned here--so as to minimize future chapter shock--that the parameter n and indices i and j are used differently in Chapter 3 from here. One reason for the difference is that the presentation in Chapter 3 need not be limited to a variant of the transportation formulation, as is the case here.

To be noted is that the class scheduling model is the starting point of all analysis and theory. When solved, this model and its solution technique will define a corresponding multiattribute utility optimization problem and solution.

2.3 A Discussion Regarding the Class Scheduling Model

According to Luenberger (1984: second edition), the assignment problem is a special case of the transportation problem, arising for two reasons. The first is that the application areas are usually quite different from those

areas for which the more general transportation problem is utilized. Second, the structure of the problem is of theoretical importance.

Classically, the assignment problem is concerned with optimally assigning n workers to n jobs. Should worker i be assigned to job j , a cost of c_{ij} accrues. Each worker is to be assigned to exactly one job, and each job requires that one worker be assigned to it.

According to Luenberger (1984: second edition), the assignment problem's formulation--as determined by motivating examples--is to find x_{ij} , $i=1, 2, \dots, n$; $j=1, 2, \dots, n$, to

$$\text{Minimize} \quad \sum_{j=1}^n \sum_{i=1}^n c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} = 1, \quad i=1, 2, \dots, n$$

$$\sum_{i=1}^n x_{ij} = 1, \quad j=1, 2, \dots, n$$

$$x_{ij} \in [0, 1] \quad \forall i, j.$$

Other books discussing the assignment problem and presenting a definition for it include Cooper-Steinberg (1974), Bazaraa-Jarvis (1977), and Taha (1982: third edition). That the assignment problem is fundamentally known and of basic importance to the operations research

discipline is attested to by the fact that the problem is addressed by and defined in Bronson (1982) as part of a book included in Schaum's Outline Series.

To be noted is that the number of workers is assumed to be equal to the number of jobs.

As presented by Cooper-Steinberg (1974), the general transportation problem is as follows:

$$\text{Minimize} \quad \sum_{j=1}^n \sum_{i=1}^m c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^n x_{ij} \leq a_i, \quad i=1, 2, \dots, m$$

$$\sum_{i=1}^m x_{ij} \geq b_j, \quad j=1, 2, \dots, n$$

$$x_{ij} \geq 0 \quad i=1, 2, \dots, m; \text{ and } j=1, 2, \dots, n.$$

According to Taha (1982: third edition), before the assignment problem model is able to be solved by the technique used for the transportation problem the problem must be balanced; i.e., the number of workers must equal the number of jobs. If such is not the case, then both Taha (1982: third edition) and Bronson (1982) discuss adding fictitious workers or jobs as a means of balancing the problem.

There exist efficient, specialized techniques for solving the classical assignment problem--such as the

Hungarian technique [see Luenberger (1984: second edition)]--but these techniques also tend to trade-off on the problem being balanced.

To make the class scheduling problem conform to a classical assignment problem formulation, both fictitious time periods and fictitious classes would have to be created. This has been avoided, and the problem greatly reduced in size, by slightly modifying the formulation. Therefore, a decision was made to use generalized integer programming solution techniques and not techniques peculiar to efficiently solving the assignment problem, because with the modifications, the solution effort is expected to be more efficient for the variant than going through what is necessary to make the classical assignment problem's peculiar solution techniques applicable.

By modifying the problem, the analysis is in part reverted back to the basic transportation formulation. However, the spirit of the assignment problem is maintained even though the basic structure is not, because the class scheduling application area is still typical of the areas handled by the assignment problem. Indeed, as mentioned earlier in this section, the class scheduling problem can be formulated as a classical assignment problem; unfortunately, the classical formulation for the class scheduling problem could be three or four times the size of the variant's formulation.

The class scheduling problem has been identified as to the class of problems to which it belongs; and therefore expectations arise as to what can be done to solve the problem. The modifications made to the classical assignment problem formulation are simple and straightforward. And, with the exception of the cost factors, everything can be known in exactly the form and manner which is desired. A model can be better for the class scheduling problem only if either of two events occur: (1) the formulation is made simpler (i.e., written more succinctly or employing a more efficient solution technique), or (2) a better approach to the use of cost factors is made or the cost factors are done away with altogether.

With regard to the first event, no simpler formulation is currently known. In fact, the problem is already so succinctly written that it is difficult to even imagine how it could be better written. That a formulation may be found for a more efficient solution technique is minimized in importance by the fact that the associated computer program was successfully run on an IBM PC (with 512K RAM and math coprocessor) using SUPER-LINDO²--which was done by the group of students doing the initial study in the OR-743 Spring

²SUPER-LINDO is a version of LINDO having capability for handling larger problems than LINDO can handle. LINDO is a computer program that solves linear, integer, and quadratic programming problems. For more information on LINDO--and hence on SUPER-LINDO--see Schrage (1986).

1986 semester class at George Mason University. Hence, with regard to the first event, the class scheduling model can, by default, be considered the best formulation according to the current state-of-the-art in operations research.

With regard to the second event, it must be observed that the cost factors as used in the initial study (and as will be used in the currently proposed procedure) reflect the disutility of assigning a class to a particular time slot. Some measure of cost is needed; otherwise there could be no objective function. The problem, as originally conceptualized in 1986, looked at the professors' utility of having certain classes assigned to certain time periods. There is no problem here, because if considering utility is feasible, a measure of disutility can be constructed from it. Note, however, that considering utility cannot be avoided because the purpose of the project is to formulate a schedule according to the wants and desires of the professors. Wants and desires necessarily involve utility. Consideration of cost is necessary for obtaining an objective function, and using utility (or disutility) for cost is inherently mandated by the problem's goal.

Therefore, a formulation employing one and only one--and unfortunately unavoidable--weak element is proposed as being the best model for addressing the class scheduling problem. It is a simple and straightforward variant of the assignment problem, arrived at by slightly modifying a

structure that is capable of being expressed in a classical assignment problem formulation. The preponderance of evidence is that the basic model proposed by this class scheduling study is the best that operations research's current state-of-the-art has to offer.

It cannot be emphasized strongly enough that the class of operations research problems to which the class scheduling problem belongs is known. From this knowledge, there arises reasonable expectations on the limitations of what can be done. That the class scheduling model can be regarded as the best that can be done in part arises from taking these expectations into consideration.

2.4 Establishing That the Cost Factors Must Be Ordinal

In Section 2.3, it was forwarded that the best model for class scheduling that the current state-of-the-art in operations research has to offer must utilize cost factors that involve utility. Therein, it was alluded to that the utility must be those of professors. Herein, that assumption will be gradually relaxed, and the possible source for the utility scale to be used for determining cost factors will include that scale of or assigned by anyone.

Economists, today, are surprisingly in strong agreement with regard to cardinal utility--that is, utility expressed in units reflecting strength of preference. It does not exist as a measurable entity. Gisser (1981) writes, simply,

"utility cannot be measured cardinally. At least no method has been invented to date." Similar sentiments are echoed in Hirschleifer (1980: second edition), Maurice-Phillips-Ferguson (1982: fourth edition), and McCloskey (1982). It would be an easy effort to find dozens more economics books taking the same position.

For a strong argument, a tolerant attitude will be manifest in this section, and though utility will not be assumed to be measurable cardinally, cardinal utility for a single individual will be assumed to exist at a point in time, at least to the extent of being known to that individual. Furthermore, the numbers the individual states as being his cardinal utility will be assumed to be able to be manipulated cardinally, as long as two or more cardinal sets of utility numbers from two or more individuals are not blended into a single weighted set of cardinal utility numbers.

The latter condition is assumed because interpersonal comparison of utility numbers are currently not possible. This is seen by observing that while a set of cardinal utility numbers may be stated by one person, and another set of cardinal utility numbers be given by another person, to be cardinally combined the scales underlying both sets must be related to one another. But there is no known way a person can express what his cardinal scale is such that it can be related cardinally to the scale of another.

According to present knowledge, interpersonal comparison of utilities is not possible.

As an example of why interpersonal comparison of utilities is not possible, consider the following. Suppose an individual states the utility he derives from drinking one cup of tea by itself is five utils, and one cup of tea and one doughnut is nine utils--both on his personal cardinal utility scale. Suppose that another individual--on his own personal cardinal utility scale--derives five utils from one cup of tea and eleven utils from one cup of tea and one doughnut. If the second individual's utilities were to be put on the same cardinal utility scale as that for the first individual, he might be said to derive seven utils from one cup of tea and eight utils from one cup of tea and one doughnut; we cannot know this however. All we can know is that the first person gave personal cardinal utilities of five and nine utils, respectively; and the second individual gave personal cardinal utilities of five and eleven. If each person had one cup of tea only, on the first person's cardinal scale the aggregate utility is actually twelve utils; but how is that to be determined from the cardinal numbers stated by both individuals? As can be appreciated from the preceding, there is no way to measure the underlying scale--which is what is really meant by cardinal utility not existing as a measurable entity--and therefore, the two fives, measured on different and nonmeasurable (with

respect to an outside observer) scales, cannot be combined to yield the twelve utils to which they would aggregate in terms of the first person's scale if the underlying scales were known.

Now, on the class scheduling project, it will be assumed that each professor would want "equal" weight given to his utility preferences with respect to those of his peers. However, there is more than one professor, and on the basis that their scales are not cardinally relatable to one another, any blending of their expressed utilities would not be cardinal or else interpersonal comparison of utilities would be sanctioned as a possibility, which would contradict the fact that such has been ruled out. Therefore, cost factors reflecting utility cannot be cardinal when the cardinal utilities of two or more professors are considered simultaneously.

To be contemplated, a way around this dilemma would be to have the analysts assign cardinal cost factors based on what the professors' stated preferences are as made known to the analysts. But where there are two or more analysts, interpersonal comparison of assigned (or imputed) utilities would be assumed; and the scale underlying the cardinal assignment by one analyst cannot be, by any known means, related to the scale underlying the cardinal assignment by another analyst. Again, any blending of utilities must be ordinal.

To be considered, another way around this dilemma might be to set up a dictator to impose cardinal cost factors. This can be a dictator such as the head of the college department in question (a professor with powers and authority over and above the professors expressing cardinal utilities); and this dictator can state what cardinal cost factors he wants to assign based on what the professors' stated preferences be. Or, the dictator can be from the group of analysts, or anyone else--but that dictator's cost factors would be cardinal as his assignment based on what the professors' stated cardinal preferences be. (Remember, what the professors' stated cardinal preferences are is what is driving the whole mechanism.)

There are two problems with the dictator way out. First, how is the dictator to be chosen? Politics and maneuvering on choosing a dictator--one whose assignment schedule is anticipated as part of the process of resolving who the dictator will be--will in essence mean that the dictator will have an assignment schedule best satisfying a group and not that of a single individual. This blending invokes criticism that an interpersonal comparison of utilities has been imposed as part of the appointment--if the assignment is regarded as cardinal. Hence, this chosen dictator's scale must be considered ordinal, not cardinal.

Second, suppose that there has always been a dictator to handle such matters as an assignment of cardinal cost

factors (the dictator is not to be chosen). Again problems are run into. Hirschman (1986) has rigorously proven that utility--particularly utility manifest in the marketplace--is affected by opportunity cost. This dictator will be in a situation where he will have his assignment affected by opportunities that are present. For example, if the dictator were the head of the college department, he would take into consideration keeping professors under him happy. If the dictator were head of the team of analysts, that dictator would want to keep his team happy. Because opportunities perceived by an individual (the dictator, for instance) are constantly changing, the assignment of cardinal cost factors is constantly changing. In conjunction with this, the fatal blow is finally delivered by James Buchanan of the Public Choice Center located at George Mason University. In his work attempting to rigorously define an individual, he has strongly substantiated that an individual today is not the same as an individual tomorrow. Therefore, at two points in time, a single individual must be regarded as two different, distinct individuals. Therefore, not only are the cardinal assignments constantly changing, but the underlying scale is changing, as well. A dictator over time, with varying scales, may state what his cardinal assignment is at any point in time, but that assignment does not have cardinal content when considered over a nonzero-lengthed period of

time--or else an interpersonal comparison of utilities (with regard to the dictator's utility at one point in time being compared to that dictator's utility at another point in time) would again be invoked, reaching a contradiction.

All the possibilities have been considered, and even with a permissive and tolerant set of assumptions the cost factors must be regarded as being ordinal, not cardinal.

2.5 Summary

In Section 2.2 of this chapter, the class scheduling model proposed by this thesis for adoption and implementation was presented.

In Section 2.3, the operations research class scheduling model was discussed in detail, explaining why this variant of the assignment problem can be regarded as the best method that the current state-of-the-art in operations research has to offer.

In Section 2.4, all possible manners of assigning cost factors for the class scheduling model were considered, and even with a permissive and tolerant set of assumptions it was found that the cost factors must be regarded as being ordinal, not cardinal.

CHAPTER 3

ORDINAL COST FACTOR THEORY

3.1 Introduction

In this chapter, the properties of ordinal cost factors will be investigated.

Section 3.2 of this chapter will present a drawback following from utilizing ordinal cost factors.

Section 3.3 will present what is an accepted way in the literature of operations research of addressing the drawback with regard to a similar situation and how the class scheduling situation can be related to that situation. Then, the desirable properties that the class scheduling ordinal cost factors have will be discussed and elaborated upon.

Section 3.4 will summarize this chapter.

3.2 A Drawback Following From Utilizing Ordinal Cost Factors

To make the problem of considering the properties of ordinal cost factors more tractable, a restriction will be imposed on the type of problem to be considered (it will be restricted to being a zero-one integer programming problem) and a restriction will be imposed upon the set of feasible solutions (all feasible solutions will have the same

cardinality; that is, all feasible solutions have a constant, specified number of ones). Without question, the class scheduling problem satisfies both restrictions.

For ease of exposition, the following notation will be introduced:

$$\begin{pmatrix} c_1 & c_2 & \dots\dots\dots & c_n \\ x_{11} & x_{21} & \dots\dots\dots & x_{n1} \\ x_{12} & x_{22} & \dots\dots\dots & x_{n2} \end{pmatrix}$$

For the above, $(x_{1j}, x_{2j}, \dots, x_{nj})$ is a specific feasible solution for the complete set of n variables; and we consider only two feasible solutions. x_{ij} are assigned only the numbers 0 or 1. The subscript i is used to denote the i th variable--from the complete set of n variables. The subscript j is used to differentiate between feasible solutions; and in what follows, only two feasible solutions are considered--meaning $j=1$ or 2 . A value of zero for x_{ij} means the variable i does not occur "as a happening" in the j th feasible solution. A value of one for x_{ij} means the variable i does occur "as a happening" in the j th feasible solution. Occurring "as a happening" for a variable means that the event represented by the variable takes place (e.g., the class is taught in that particular time slot). For simplicity, the event associated with x_{kj} is at least as preferred to (is as desirable as) the event associated with x_{ij} if $i > k$. In that case, the ordinal number associated

with x_{ij} -- c_i --will be greater than or equal to the ordinal number associated with x_{kj} -- c_k ; that is, $c_i \geq c_k$. If x_{ij} and x_{kj} are equally preferred events, they are grouped as immediate neighbors into a cluster of events indifferently ranked and $c_i = c_k$. The value of the objective function for feasible solution $(x_{1j}, x_{2j}, \dots, x_{nj})$ is $\sum_{i=1}^n c_i x_{ij}$.

Now, with the matter of notation taken care of, the substance of this section will be addressed.

Consider the problem:

$$\begin{pmatrix} c_1 & c_2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

where the two feasible solutions depicted above are the only two feasible solutions admissible to the problem.

Because c_2 , by convention, is always greater than or equal to c_1 , $\sum_{i=1}^2 c_i x_{i1}$ is always greater than or equal to $\sum_{i=1}^2 c_i x_{i2}$. Therefore, as long as the ordinal ranking is preserved, here is a case where any ordinal set of numbers will give the same result as the unique cardinal set of numbers which is the desired set.

That result can be shown to be true even for more complex problems, such as:

$$\begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

where the two feasible solutions depicted above are the only two feasible solutions admissible to the problem.

In the immediately above problem, it is noted that the one in question (i.e., the one associated with c_4) "travels" in only one direction (towards being the one associated with c_2).

That many ones may travel in only one direction and the solution arrived at for any order preserving set of ordinal numbers be the same as the desired set of cardinal numbers is next revealed through the following example:

$$\begin{pmatrix} c_1 & c_2 & c_3 & c_4 & c_5 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{pmatrix}$$

It would seem reasonable, therefore, to conclude that if each feasible solution can be related to all other feasible solutions in the set such that the ones "travel" in only one direction when comparing one feasible solution to another, it would be largely demonstrated that all the advantages of working with cardinal numbers would be preserved when working with order preserving ordinal numbers. That hope, however, is dashed when the following example is considered:

$$\begin{pmatrix} c_1 & c_2 & c_3 & c_4 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

To get from the feasible solution associated with $j=1$ to the feasible solution associated with $j=2$, it is found that the ones must travel in an opposing, not the same, direction.

To see what the possible impact of this could be, various values for the c_i 's were tried. For example, consider:

$$\begin{pmatrix} 1 & 2 & 8 & 12 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Here, $\sum_{i=1}^4 c_i x_{i1} = 13$ and $\sum_{i=1}^4 c_i x_{i2} = 10$. Therefore, the first feasible solution yields a higher objective function value than the second feasible solution.

Now, consider the following problem, where only the cost factors have been changed from the immediately preceding problem:

$$\begin{pmatrix} 1 & 2 & 3.5 & 4 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Here, $\sum_{i=1}^4 c_i x_{i1} = 5$ and $\sum_{i=1}^4 c_i x_{i2} = 5.5$. Therefore the second feasible solution yields a higher objective function value than the first feasible solution.

Only the cost factors have changed, yet an alternative feasible solution has been selected as yielding a higher (or

lower) value for the objective function. Therefore, the ordinal numbers that are assigned as cost factors will determine what the optimal solution will be. Therefore, there can be competing optimal solutions, and they not be substitutes for one another.

An alert reader will now realize why an effort was made to show that the model suggested by this thesis and associated ordinal cost factors are the best that can be done for the class scheduling problem--at least, the best that can be done in accordance with the current state-of-the-art in operations research. The reason for that effort is that it becomes apparent that there is no conventional operations research solution possible for the class scheduling problem. New criteria of arriving at and for accepting an optimal solution must be established.

3.3 Desirable Properties That the Class Scheduling Ordinal Cost Factors Have

While the drawback cited in Section 3.2 of this chapter is severe, all is not lost. Resorting to decision theory, that body of operations research occasionally uses what is known as the "principle of insufficient reason" in analysis. That principle is named the Laplace Criterion. (See Taha, 1982: third edition; pp. 434-5.) While the situation to which the Laplace Criterion is classically applied is not exactly comparable to the situation of the class scheduling

problem, striking similarities do exist. Taha (1982: third edition) writes: "Since the probabilities associated with the occurrence of [states] $\theta_1, \theta_2, \dots, \text{ and } \theta_n$ are unknown, we do not have enough information to conclude that these probabilities [associated with those states] will be different.... [Because] of insufficient reason to believe otherwise, the states $\theta_1, \theta_2, \dots, \text{ and } \theta_n$ are [regarded as] equally likely to occur." With that precedent to fall back upon, similar reasoning applied here would have cost factors selected such that no cost factor is weighted greater than any other cost factor. To do this would require equal spacing between all adjoining cost factors for which a difference in preference is to be registered, and identical values for adjoining cost factors for which there is no difference in preference.

But two questions immediately come to mind: (1) does the starting value (that is, the value assigned to the smallest cost factor) affect which feasible solution is selected as optimal, and (2) does the size or length of the space between adjoining distinct cost factors affect which feasible solution is selected as optimal? As the reader shall see, the questions are not moot. However, for a suitably restricted problem the answer is, fortunately, "no" for both questions, and therefore the considered adaption of the Laplace Criterion is feasible, reasonable, and effective

for the class scheduling problem. A "suitably restricted problem" is herein regarded as one that is both (1) a zero-one integer programming problem and (2) a problem for which all feasible solutions have a constant cardinality (that is, they all have the same number of ones).

First to be considered is whether the lowest value for cost factors affects which feasible solution is selected as optimal. It is to be recalled that the spacing between all adjoining cost factors is equal and nonzero if there is a difference in preference rankings for the two events (and the cost factor values are identical for cost factors when there is an associated indifference in preference); and for the analysis here the length of the space is held constant under translation (shifting). Therefore, if the starting value is shifted 5 units to the left (made smaller by 5 units), all the cost factors are made smaller by 5 units. Each feasible solution has the same number of ones; and for each 1, the objective value is reduced by 5. If, for instance, there are three ones in each feasible solution, then the overall objective value will be reduced by a constant $3*5=15$ for each and every feasible solution. Clearly, the feasible solution selected as optimal will not change, because the rankings of objective function values associated with the feasible solutions will not change.

The reader is asked to note that constant cardinality for all elements of the set of feasible solutions is

important in the immediately preceding argument. He/she will now note that if, for instance, one feasible solution has three ones and another feasible solution has four ones and furthermore that these are the only two feasible solutions to the problem, the aforementioned result need not hold. The demonstration is as follows: For a specified lowest value of the cost factors, let the feasible solution with three ones have higher objective function value than the feasible solution with four ones; and suppose the difference is y units. Now, translate the cost factors to the right by $y+1$; that is, make each of the cost factors larger by $y+1$. The feasible solution with four ones will have objective function value increase by $4y+4$, but the feasible solution with only three ones will have objective function value increase by only $3y+3$. Therefore, the feasible solution with four ones will now have a higher objective function value than the feasible solution with three ones (and the difference will be one unit).

Therefore, a different feasible solution can be selected as being optimal from the set if the feasible solutions do not have constant cardinality over the entire set of feasible solutions and the cost factors are translated similarly as indicated above.

Second to be considered is whether the size of the spacing between adjoining cost factors affects which

feasible solution is selected as optimal in a suitably restricted problem.

Using the notation of Section 2 of this chapter, consider the problem:

$$\begin{pmatrix} c_1 & c_2 & \dots & c_n \\ x_{11} & x_{21} & \dots & x_{n1} \\ x_{12} & x_{22} & \dots & x_{n2} \end{pmatrix}$$

with equal spacing between adjoining c_i 's having an associated difference in preference ranking (and adjoining c_i 's are identical if there is indifference associated in preference ranking); this will hereinafter be referred to as uniform spacing.

More specifically with regard to uniform spacing (and utilizing the notation in Section 3.2), for all adjoining events for which it is known or assumed those events are equally preferred to one another, have those events all have the same cost factor. There is no uncertainty involved herein and therefore there should be no detectable difference in the value assigned to these cost factors. This will lead to clusters of events, one cluster strictly preferred to another, and there be uncertainty how the clusters actually relate to one another in magnitude. Here, we want to have cost factors associated with clusters be equally spaced; i.e., equal spacing between all adjoining distinct cost factors. An example of this

might be:

$$\begin{pmatrix} \alpha_1 & 2\alpha_1 & 3\alpha_1 & 3\alpha_1 & 4\alpha_1 & 5\alpha_1 & 5\alpha_1 & 5\alpha_1 & 6\alpha_1 \\ x_{11} & x_{21} & x_{31} & x_{41} & x_{51} & x_{61} & x_{71} & x_{81} & x_{91} \\ x_{12} & x_{22} & x_{32} & x_{42} & x_{52} & x_{62} & x_{72} & x_{82} & x_{92} \end{pmatrix}$$

where $x_{31} \sim x_{41}, x_{61} \sim x_{71} \sim x_{81}$, and $x_{91} \prec x_{81} \prec x_{51} \prec x_{41} \prec x_{21} \prec x_{11}$.

Now, consider two distinct spacings: α_1 and α_2 . The property that the suitably restricted problem can be translated without affecting which feasible solution will be selected as optimal leads us to consider the following two problems which shall be compared:

$$\begin{pmatrix} \beta_1\alpha_1 & \beta_2\alpha_1 & \beta_3\alpha_1 & \dots & \beta_n\alpha_1 \\ x_{11} & x_{21} & x_{31} & \dots & x_{n1} \\ x_{12} & x_{22} & x_{32} & \dots & x_{n2} \end{pmatrix} \quad \begin{array}{l} \text{formulation} \\ \text{\#1} \end{array}$$

and

$$\begin{pmatrix} \beta_1\alpha_2 & \beta_2\alpha_2 & \beta_3\alpha_2 & \dots & \beta_n\alpha_1 \\ x_{11} & x_{21} & x_{31} & \dots & x_{n1} \\ x_{12} & x_{22} & x_{32} & \dots & x_{n2} \end{pmatrix} \quad \begin{array}{l} \text{formulation} \\ \text{\#2} \end{array}$$

where α_1 and α_2 are two distinct spacings, $\beta_1=1$ and for $i=2, \dots, n$:

$$\beta_i = \begin{cases} \beta_{i-1}, & \text{if event } x_{i,1} \text{ is equally preferred} \\ & \text{with event } x_{i-1,1} \\ \beta_{i-1}+1, & \text{if event } x_{i,1} \text{ is strictly preferred} \\ & \text{to event } x_{i-1,1}. \end{cases}$$

The reader is asked to note that any suitably restricted problem with uniform spacing equal to α_1 can be

translated to formulation #1 above where both the translation and any original set of c_i 's will yield the same feasible solution as being optimal. Furthermore, any suitably restricted problem with uniform spacing equal to α_2 can be translated to formulation #2 above where both the translation and any original set of c_i 's will yield the same feasible solution as being optimal.

Now, note that there exists a z such $\alpha_2 = z*\alpha_1$.

Therefore, formulation #2 can be rewritten as formulation #3 below:

$$\begin{pmatrix} z*\beta_1\alpha_1 & z*\beta_2\alpha_1 & z*\beta_3\alpha_1 & \dots\dots\dots & z*\beta_n\alpha_1 \\ x_{11} & x_{21} & x_{31} & \dots\dots\dots & x_{n1} \\ x_{12} & x_{22} & x_{32} & \dots\dots\dots & x_{n2} \end{pmatrix} \quad \begin{matrix} \text{formulation} \\ \#3 \end{matrix}$$

where α_1 and β_i are defined as for formulations #1 and #2, and where z is defined such that $\alpha_2 = z*\alpha_1$.

From formulations #1 and #3, it is readily seen that all feasible solutions with c_i 's having spacing α_1 here have objective function value $\sum_{i=1}^n c_i x_{ij}$, whereas all feasible solutions with c_i 's having spacing α_2 here have objective function value $z*\sum_{i=1}^n c_i x_{ij}$, where c_i is defined the same for both formulations #1 and #3. Therefore, as long as z is greater than zero, formulations #1 and #3 will lead to the same feasible solution selected as being optimal. (The concept of length requires that z be nonnegative.)

The reader should now take stock of what is done regarding uniform spacing of the c_i 's. First, any suitably restricted problem with uniform cost factor spacing α_1 is to be considered. The problem is then to be translated to formulation #1. Then formulation #1 is to be compared to formulation #3 (which is identical to formulation #2). Then formulation #2 is to be translated to any suitably restricted problem with uniform cost factor spacing α_2 . At all pairwise comparable steps, the same feasible solution is selected as being optimal for both elements of the intersystem pair. Therefore, under the suitable restrictions, all positive uniform spacings between the cost factors yield the same feasible solution as being optimal.

The reader is asked to note that the use of translations was important to proving the property associated with regard to spacing. In those situations where the desirable properties for translations do not hold, nothing has been said with regard to how changing the spacing affects selection of the optimal solution from the set of feasible solutions.

3.4 Summary

In Section 3.2 of this chapter, it was shown that the ordinal numbers that are assigned as cost factors will determine what the optimal solution will be. Therefore,

there can be competing optimal solutions, and they not be substitutes for one another.

In Section 3.3, it was pointed out that while the drawback cited in the immediately preceding section is severe, all is not lost. Resorting to decision theory, that body of operations research occasionally uses what is known as the "principle of insufficient reason" in analysis, otherwise known as the Laplace Criterion. With that precedent to fall back upon, similar reasoning applied here has cost factors selected such that no cost factor is weighted greater than any other cost factor. To do that requires equal spacing between all adjoining cost factors for which there is a difference in preference, and no spacing between adjoining cost factors for which there is no difference in preference; this is when the corresponding events are ranked in ascending order of preference, for example--or, alternatively, in descending order of preference.

But two questions immediately came to mind: (1) does the starting value (that is, the value assigned to the smallest cost factor) affect which feasible solution is selected as optimal, and (2) does the size or length of the space between adjoining cost factors affect which feasible solution is selected as optimal? As the reader saw, the questions were not moot. However, for a suitably restricted problem the answer was, fortunately, "no" for both

questions, and therefore the considered adaption of the Laplace Criterion is feasible, reasonable, and effective for the class scheduling problem. A "suitably restricted problem" is herein regarded as one that is both (1) a zero-one integer programming problem and (2) a problem for which all feasible solutions have a constant cardinality (that is, they all have the same number of ones).

CHAPTER 4

PROCEDURE TO ASSIGN THE ORDINAL COST FACTORS

4.1 Preliminary Statements

In this section, the discussion will be general and abstract. In the next section of this chapter, the discussion will deal more concretely with the class scheduling problem which is the focus of this thesis.

In assigning ordinal cost factors to events, all events first must be assembled into an overall array of ascending or descending preference. In this array, collections of events which are indifferent to one another by assumption are then grouped into clusters (of events) for which neighboring clusters depict adjoining, nonzero-differenced levels of preference.

Next, starting with the most (or least) preferred cluster, assign that cluster a "1" as the ordinal cost factor for the events of which it is comprised. Then, a "2" is assigned to the next most (or least) preferred cluster as the ordinal cost factor for the events of which it is comprised. Continuing down the array, then assign a "3," "4," and so on until all clusters have been assigned a cost factor for the events of which they are comprised.

In Table 1, an array of clusters with associated ordinal cost factors assigned to them is portrayed.

Table 1An Array of Clusters With Associated Ordinal Cost Factors

<u>The Ordinal Cost Factor to Assign to Events in the Respective Cluster</u>	<u>The Respective Cluster</u>
1	Cluster #1
2	Cluster #2
3	Cluster #3
.	.
.	.
.	.
n	Cluster #n

Where, for a minimization problem, Cluster #i \succ Cluster #j when $i < j$. For a maximization problem, Cluster #i \succ Cluster #j when $i > j$. A cluster may be comprised of one or more events.

4.2 A Concrete Discussion Regarding Assignment of the Cost Factors

Let a particular class being taught at a particular time be a specific event. These events will be assigned a level of preference according to the satisfaction--in ordinal terms--that the professor derives from the teaching that class in the time period during the semester in question.

For each class in the test of the model described in the next chapter, the professor assigned to teach that class will state the time period he would most prefer to teach that class for the particular semester which is involved; then the second most preferred time period; and so forth until the five most preferred time periods for that class for that professor for the particular semester which is involved have been stated and ranked.³ Note that the stating and ranking of the five most preferred time periods is arbitrary, and any number could be satisfactory.

Once the professors have each stated and ranked the five most preferred time periods according to their personal preferences, these may be assembled into clusters. An example of an appropriate collection of clusters is:

Cluster #1: the set comprising the most preferred choice of time slot for a professor, over all professors.

³Assignment of values to ordinal cost factors based upon such rankings was first done by Larry Haynes, now an Operations Research and Management Science, M.S. graduate of GMU.

Cluster #2: the set comprising the second most preferred choice of time slot for a professor, over all professors.

Cluster #3: the set comprising the third most preferred choice of time slot for a professor, over all professors.

Cluster #4: the set comprising the fourth most preferred choice of time slot for a professor, over all professors.

Cluster #5: the set comprising the fifth most preferred choice of time slot for a professor, over all professors.

The n-th cluster--here, Cluster #6--is the set comprising the sixth through last choice of time slot for a professor, over all professors; this is because it does not matter which of these possibilities occur--they are all equally undesirable as far as the problem considered in this thesis is concerned. Also included in the n-th cluster are all time periods associated with classes having no designated teacher or associated with a teacher who does not state his/her preferences; this is because no preferential treatment is to be accorded to any of these events, and preferential treatment of some sort would be accorded if they were not included in the n-th (and least preferred) cluster.

In Cluster #5, the following is an event:

Event #i: the fifth most preferred choice of time slot for a professor (i.e., for the course in question, this is the fifth most preferred time period in which to teach that class that that professor desires); a separate event exists for each professor for each course he/she teaches.

Each event included in Cluster #5 will have a cost factor of 5 in the test of the model described in the remainder of this thesis. More specifically, for the variable associated with the fifth most preferred choice of time slot for a particular professor, that variable would have a cost of "5" appearing as its coefficient in the objective function. The variable itself could be a zero-one variable representing whether, for example, the course in question is taught from 4:30P.M.-7:10P.M. on Tuesdays. (In general, each event included in Cluster #i will have a cost factor of i in the test of the model described in the remainder of this thesis.)

CHAPTER 5

COMPUTER IMPLEMENTATION OF SOLUTION TO THE CLASS SCHEDULING PROBLEM AND A TEST CASE

5.1 Preliminary Statements

The computerization of the model as done for the test case in this chapter and the next is via SUPER-LINDO. That is the most appropriate package of which this researcher is aware for a problem of this nature and size.

For those who wish to replicate the experiment or duplicate the test, a "trick" employed for computerization via the SUPER-LINDO computer package is discussed in Section 5.2.

For data to test the class scheduling model, professors in (1) the Operations Research and Applied Statistics Department and (2) the Systems Engineering Department at GMU were polled regarding the five most preferred times--with ranking--each professor would have wanted with regard to the graduate level course(s) they were named as teaching by the Schedule of Classes for the semester involved in the test. The Schedule of Classes was published before the professors were polled.

The basic framework for the test case is addressed, and the data is given, in Section 5.3.

5.2 On the Use of SUPER-LINDO

SUPER-LINDO can handle linear and integer programming problems involving 800 variables and 2,000 constraints on an IBM PC or an IBM clone that has at least 512K RAM and a math coprocessor.

Because SUPER-LINDO does not have subscript capability, the variable names are to be changed from x_{ij} to A_j , B_j , C_j , etc. Thus, $x_{1,1}$ becomes A_1 ; $x_{3,4}$ becomes C_4 ; and so on.

5.3 The Basic Framework and the Data Surrounding the Test Case

The courses involved in the test case are all the graduate courses at the 500 and 600 level offered by (1) the Operations Research and Applied Statistics Department and (2) the Systems Engineering Department at GMU--except one--listed in the Schedule of Classes published by George Mason University in the semester immediately prior to the semester involved in this test. The information in this schedule was known to this researcher and all professors involved in the test before any polling took place.

A decision was made to not include an introductory course--OR-540--as part of the test because that course does not apply towards any degree offered by the Department of Operations Research and Applied Statistics or the Department of Systems Engineering and therefore when it is to be scheduled is independent of considerations driving the study at hand.

Between the time the Schedule of Classes was published and the schedule was actually implemented in the involved semester, several basic changes took place: two professors did not teach the course they were listed to teach (but these two courses were taught) and one course was cancelled. For the test case, however, those changes were ignored. The idea was to test against the original schedule utilizing only information that was or could have been available at the time that schedule was initially formulated and presented for publication.

Polling of the professors took place via a questionnaire which consisted of an instruction page and a page containing a grid similar to that in Figure 1, which was requested to be filled in. Some additional information was requested on the page containing the grid. An example of a filled in grid was provided on the instruction page, and a facsimile of that example presented to the professors is presented in Figure 2.

On the instruction sheet given to the professors concerning filling in the grid was the information:

For the course listed on the attached page, you are to put a "1" in the box corresponding to the time period in which you would most like (most prefer) to teach that particular course this semester. Then put a "2" in the box corresponding to the second most preferred time period in which you would most like to teach that particular course this semester. Then put a "3" in the box corresponding to the third most preferred time period; and so on for the first five (5) most preferred time periods. Please, do NOT enter numbers six through eight, as these higher numbers will not be used.

	Monday	Tuesday	Wednesday	Thursday
4:30P.M.- 7:10P.M. (1630-1910)				
7:20P.M.- 10:00P.M. (1920-2200)				

Figure 1

	Monday	Tuesday	Wednesday	Thursday
4:30P.M.- 7:10P.M. (1630-1910)		1	3	
7:20P.M.- 10:00P.M. (1920-2200)	4	2	5	

Figure 2

The reason "this semester" was used in the instructions is that the semester involved in the test case was in progress at the time the polling took place.

In the example presented in Figure 2, Tuesday at 4:30P.M.-7:10P.M. is the most preferred time slot in which to teach the course in question for the semester in question.

To additionally clarify the nature of the information gathered from the questionnaire and its grid, the numbers under the headings 4:30P.M.-7:10P.M. and 7:20P.M.-10:00P.M. for the indicated day of the week depict the ordinal ranking of preferences for teaching the course in question during the time period in question. A "1" indicates that that is the most preferred time period. A "2" indicates that that is the second most preferred time period. And so forth. No number under the time period indicated that the preference is ranked somewhere between sixth through eighth, inclusively.

One professor taught two classes in the involved semester. He was given two questionnaires--each carefully labeled as to the course it concerned--for him to complete. He was advised that he did not have to give the same preference rankings for both courses; the explanation given to him for this instruction was that his preference rankings may, among other things, be dependent upon the course involved.

In Table 2, a tabulation of the professors' preferences as stated by them on the grid concerning the respective course is presented.

With respect to the x_{ij} mentioned in Section 5.2, the "i" is listed under heading "Course Index Number i" in Table 2; and in that table, the j's associated with the respective time periods and as used for the x_{ij} mentioned in Section 5.2 are specifically given.

In Table 3, the courses and the time slots published in the Schedule of Classes as allocated to them are presented. This correspondence is considered to be the allocation of time periods to the courses which is "actually observed."

In Table 4, the course pairs that cannot--or should not--be taught concurrently (i.e., should not be taught at the same time) are associated with a "1"; and the course pairs that can be taught concurrently are associated with a "0." To get a "worst-case" test for the case study being undertaken here, more pairs of courses were listed in Table 4 as "should not be taught concurrently" than might otherwise be desirable. Table 4 circumscribes constraints designed to accommodate the needs of the students. Other constraints of a similar nature may be imposed on the problem to accommodate the needs of the professors; and in the current test, Course #3 and Course #8 were taught by the same professor and hence were required to not be taught on

Table 2
A Tabulation of Professors' Preferences

Course Index Number i	4:30P.M.-7:10P.M.				7:20P.M.-10:00P.M.			
	M	T	W	R	M	T	W	R
	j=1	j=2	j=3	j=4	j=5	j=6	j=7	j=8
1				5	4	3	2	1
2	5				1	2	3	4
3	4	2	3	1		5		
4	4	2	3	1				5
5	3	1	2	4		5		
6			1	2	5		4	3
7	4	1	3	2				5
8	4	2	3	1		5		
9	4	1	3	2	5			
10	4	1	2	3		5		
11		4		5	3	2		1
12	1	2		3		4		5
13								
14	4		1	2	5		3	
15		1	3	2		4	5	
16	5				1	2	3	4

⁴For course #13, no professor was listed as teaching that course in the Schedule of Classes published for the semester involved in this test.

Table 3Course Time Slots as Published in the Schedule of Classes

Course Index Number	Published Time Slot for Course	
	Day	Time Period
1	Monday	4:30P.M.-7:10P.M.
2	Wednesday	7:20P.M.-10:00P.M.
3	Thursday	4:30P.M.-7:10P.M.
4	Thursday	4:30P.M.-7:10P.M.
5	Monday	4:30P.M.-7:10P.M.
6	Wednesday	4:30P.M.-7:10P.M.
7	Tuesday	4:30P.M.-7:10P.M.
8	Wednesday	4:30P.M.-7:10P.M.
9	Tuesday	4:30P.M.-7:10P.M.
10	Tuesday	7:20P.M.-10:00P.M.
11	Thursday	7:20P.M.-10:00P.M.
12	Wednesday	4:30P.M.-7:10P.M.
13	Thursday	7:20P.M.-10:00P.M.
14	Wednesday	7:20P.M.-10:00P.M.
15	Thursday	4:30P.M.-7:10P.M.
16	Tuesday	7:20P.M.-10:00P.M.

Table 4

	Course #1	Course #2	Course #3	Course #4	Course #5	Course #6	Course #7	Course #8	Course #9	Course #10	Course #11	Course #12	Course #13	Course #14	Course #15	Course #16
Course #1																
Course #2	1															
Course #3	1	1														
Course #4	1	1	1													
Course #5	0	0	0	1												
Course #6	0	0	0	1	1											
Course #7	0	0	0	1	1	1										
Course #8	0	0	0	1	1	1	1									
Course #9	0	0	0	0	0	0	0	0								
Course #10	1	1	1	0	0	0	0	0	0							
Course #11	0	0	0	1	1	1	1	1	0	0						
Course #12	0	0	0	1	1	1	1	1	0	0	1					
Course #13	1	1	1	0	0	0	0	0	1	0	0	0				
Course #14	0	0	0	0	0	0	0	0	0	0	0	0	0			
Course #15	1	1	1	0	0	0	0	0	0	1	0	0	0	1		
Course #16	1	1	1	0	0	0	0	0	0	1	0	0	0	1	0	

the same day so that he would not teach both classes at the same time or back-to-back. (To be noted is that for constraints of the type $x_{ij} + x_{pk} \leq 1$ --see Section 2.2--only the restrictions of classes not to be taught concurrently as specified in Table 4 and the restriction that Course #3 and Course #8 were not to be taught on the same day created constraints of this nature for the test case, and where allowed such constraints were created.)

A condition imposed in the test was that at most two courses would be taught in the same time slot. Since there were sixteen courses and eight time slots, that meant each time slot had two classes being taught in it. It is to be noted that with the extensive designation of course pairs which should not be taught concurrently, in actual practice allowance probably would be made for three or four courses being permissible to be taught in the same time slot so that the professors would be more likely to get their first or second choice for time period in which to teach the course they will be teaching. Again, however, the objective here is to put as many and the strongest restrictions on the problem as might ever be used so that a "worst-case" test is performed and evaluated. Furthermore, a reason for having at most two courses taught per time slot is so that the courses are distributed over the time periods as widely as possible in order to accommodate the exceptional student who

might desire to take a course pair concurrently when such normally would not be desirable; e.g., one course is the prerequisite for the other (such a circumstance might arise when a student already has expertise in the subject area and is merely trying to amass credits to apply towards a degree).

Considered were 128 events each comprising a distinct x_{ij} (sixteen courses times eight time slots equals 128 events). A distinct x_{ij} constitutes a distinct Course i coupled with a distinct time slot j . The coefficients of the x_{ij} for the objective function are determined from Table 2 in agreement with the example instructions in Section 4.2.

In addition to the objective function, there were 392 "subject to" constraints involved in the test case. There were 128 zero-one variables x_{ij} as described in the immediately preceding paragraph.

The results of the test case are presented in Chapter 6.

CHAPTER 6

EVALUATION OF THE CLASS SCHEDULING PROBLEM'S MODEL

6.1 Preliminary Statements

The class scheduling model forwarded in Section 2.2 was loaded and run via SUPER-LINDO utilizing a Leading Edge Model D micro computer with 640K RAM and a math coprocessor.

The test case revolved around polling the professors in (1) the Operations Research and Applied Statistics Department and (2) the Systems Engineering Department at GMU regarding the five most preferred times--with ranking--each professor would have wanted with regard to the graduate level course(s) they taught during the semester involved in the test. Then, in conjunction with other restrictions on the problem imposed by this analyst, the model was run.

Section 6.2 will present the results of the run, which will be examined against the schedule published by GMU for the semester involved in the test.

Section 6.3 will present the man-hours and computer time involved in the test.

Section 6.4 will then present concluding remarks for this thesis.

6.2 Results of the Test

The optimal class schedule determined from the run-- hereinafter referred to as the optimal schedule--is presented in Table 5. The objective function value is 37.

Table 6 presents a course-by-course comparison of how the schedule forwarded in the Schedule of Classes for the semester involved in the test (referred to in Table 6 as "Preference Rank Actually Observed") stacks up against the optimal schedule contained in Table 5. The numbers given under the respective schedule depict the level of preference, according to the course's professor designated in the Schedule of Classes, that the course's time slot allocation achieves in the respective schedule; i.e., a "1" indicates that the time slot allocated to the course in that schedule is the professor's most preferred, a "2" indicates that the time slot allocated to the course in that schedule is the professor's second most preferred, and so on. The professors' preferences are taken from Table 2. Note that Course #13 is missing from Table 6; this is because no professor was assigned to it at the time the Schedule of Classes for the semester involved in the test was published.

In comparing the optimal schedule to the published schedule cited as "actually observed," first noted is that the objective function value is 37 for the optimal schedule and a comparable value would be 45 for the published schedule. Therefore, the optimal schedule represents

Table 5
Optimal Class Schedule

	Monday	Tuesday	Wednesday	Thursday
4:30P.M.- 7:10P.M. (1630-1910)	Course #3 Course #12	Course #7 Course #9	Course #5 Course #10	Course #4 Course #15
7:20P.M.- 10:00P.M. (1920-2200)	Course #13 Course #16	Course #2 Course #8	Course #6 Course #14	Course #1 Course #11

Table 6

Course Index Number	Preference Rank for Optimal Schedule	Preference Rank Actually Observed
1	1	>5
2	2	3
3	4	1
4	1	1
5	2	3
6	4	1
7	1	1
8	5	3
9	1	1
10	2	5
11	1	1
12	1	>5
14	3	3
15	2	2
16	1	2

approximately a twenty percent decrease in objective function value.

More meaningful statistics are as follows: in the published schedule cited as "actually observed" there are two professors who were assigned a time slot in their sixth-through-eighth ranking region. There were six professors receiving their most preferred time period, and two professors receiving their second most preferred time period.

In contrast, however, in the optimal schedule there were no professors who were assigned a time slot in their sixth-through-eighth ranking region. There were seven professors receiving their most preferred time period, and four professors receiving their second most preferred time period.

In going from the published schedule to the optimal schedule six professors were made better off, six remained equally satisfied, and three were made worse off. Of those made worse off, two were moved from their most preferred time period to their fourth most preferred time period, and one was moved from his third most preferred time period to his fifth most preferred time period. Counterbalancing this, two professors were moved from the sixth-through-eighth most preferred time period region to the most preferred time period, and one was moved from his fifth most preferred time period to his second most preferred time

period; at the risk of involving interpersonal comparison of utilities, an argument could be made that these three professors more than offset the losses of the three professors who were made worse off--and then there is a net gain of three other professors who were made better off.

It should be emphasized that in the optimal schedule, no professor receives a time slot ranked in the sixth-through-eighth most preferred time period region; in the published schedule, however, there were two professors in this region.

The optimal schedule could have satisfied the professors' preferences even more fully had the large number of constraints protecting the students' needs and the organizational and institutional requirements been relaxed. Indeed, many of the constraints imposed in determining the optimal schedule were not upheld in the published schedule. For example, in the published schedule there were three classes taught in the time period comprised of 4:30P.M.-7:10P.M. on Thursday whereas the problem formulation for the optimal schedule required only two classes be taught during each time period. Had the optimal schedule been determined when allowing three classes to be taught during a time period, an improvement in the optimal schedule almost certainly would have been noted. Then, there were instances of pairs of courses being taught at the same time in the published schedule that were not allowed to be taught at the

same time in the constraints that yielded the optimal schedule; by relaxing these constraints imposed on determining the optimal schedule to the extent that they were relaxed in determining the published schedule, again there almost certainly would be the result of an optimal schedule which even more would have been superior to the published schedule.

It is to be noted that even with imposing as many and the strongest constraints as might ever be necessary to support students' needs and the organizational and institutional requirements--constraints which were partially violated in the published schedule--a superior result was achieved by using the operations research class scheduling model recommended in Chapter 2.

6.3 Man-hours and Computer Time Involved in the Test

Part of the feasibility of a method lies not only in achieving superior results, but in doing so using an efficient level of resources. This section lists the resources employed in running the test upon which evaluation of the class scheduling method proposed by this thesis is based.

First, the questionnaires were initially distributed on October 22. Approximately half of them were returned within the ten days immediately following October 22. Most of the rest straggled in over the two week period following the end

of the first ten days; and the last questionnaire was not returned until November 19. Preparing the questionnaire required about three man-hours; but in any event, once the technique has been implemented and established, preparing an appropriate questionnaire would not be a recurring expenditure of time. Regardless, getting the professors to fill out the questionnaires is another matter, and with follow-ups a total of about four man-hours was expended in getting the professors to fill out the questionnaires.

Determining the constraints to utilize in the problem specification phase took approximately four hours.

Loading the program comprising the test, checking what was input to be sure it was correct, and making the run consumed approximately six man-hours.

As far as the computer time goes, once the program was fully loaded the run took approximately sixteen minutes total time of a Leading Edge Model D computer having 640K RAM and math coprocessor--during which time a hard-copy printout of the run was made on an Epson LQ-800 printer.

Therefore, fourteen man-hours were required to fully execute the test if the time for preparing an appropriate questionnaire is not included, and seventeen man-hours were required if it is. Computer time was negligible.

6.4 Concluding Remarks

Section 2.2 proposed a mathematical model that defines a multiattribute utility optimization problem. As a digression, Section 2.3 argued that the mathematical model had a strong and valid connection with the application receiving primary focus of attention in this thesis--optimal class scheduling subject to professors' preferences. From the intended application, we then saw that we were interested in solving the mathematical model for ordinal cost factors.

In Chapters 2 through 4, the mathematical model and its application were presented and developed simultaneously. That was done to promote visibility of underlying motivations and to cut down on the extreme amount of "dryness" that otherwise would be present. However, the power of the philosophical base for the multiattribute utility method espoused by this thesis is derived from first developing the mathematical model in its bare essentials and then applying the mathematical model to a multiattribute utility problem which the mathematical model defines.

According to the theoretical thrust of this thesis, the starting point of all analysis in this thesis is in the putting forth of the mathematical model for study and declaring that that model is to be solved for ordinal cost factors. This mathematical model and its solution technique

subsequently well-defines a multiattribute utility problem and its solution.

Once the mathematical model/multiattribute utility optimization problem pair is established, then an application area for the pair is sought to ensure that the pair is useful. And for the pair established in this thesis, the optimal scheduling of classes subject to professors' preferences is such an application area.

The assignment model is the best model that current state-of-the-art operations research methodology has to offer for addressing the problem of optimal class scheduling subject to professors' preferences. And as shown in the test performed in Chapter 5 and evaluated in this chapter, using a variant of the assignment problem in conjunction with ordinal cost factor theory yields a superior result to methods currently used, and did so utilizing a reasonable amount of labor and capital resources.

By employing the theoretical thrust of this thesis, multiattribute utility theory has been expanded and aspects of it are better understood. Now, not only can ordinal utility be soundly used to solve multiattribute utility optimization problems, but is seen that the nature and significance of all utility manipulations underlying multiattribute utility optimization is more easily and fully evaluated by placing emphasis on the mathematical model--symbolizing what can be done--than on the problem--what is

desired to be done. As always, science is better rooted on the ground--e.g., its mathematics--than in the clouds--the "science fiction" of a billion pipe-dreamers.

SELECTED REFERENCES

- Bazaraa, Mokhtar S.; and John J. Jarvis. Linear Programming and Network Flows. New York: John Wiley & Sons, 1977.
- Bronson, Richard. Operations Research. New York: McGraw-Hill Book Company, 1982.
- Cooper, Leon; and David Steinberg. Methods and Applications of Linear Programming. Philadelphia: W.B. Saunders Company, 1974.
- Gisser, Micha. Intermediate Price Theory: Analysis, Issues, and Applications. New York: McGraw-Hill Book Company, 1981.
- Hirschman, Edward. Prospect Theory, Regret Theory, and Related Findings From a Study on the Determination of Individual Indifference Curves and Maps. Annandale, Va.: The DeLethain Press, 1986.
- Huber, George P. Managerial Decision Making. Glenview, Illinois: Scott, Foresman and Company, 1980.
- Keeney, Ralph L.; and Howard Raiffa. Decision with multiple objectives: Preferences and value tradeoffs. New York: John Wiley, 1976.
- Luenberger, David G. Linear and Nonlinear Programming. Second edition. Reading, Massachusetts: Addison-Wesley Publishing Company, (1973) 1984.
- McCloskey, Donald N. The Applied Theory of Price. New York: Macmillan Publishing Co., Inc., 1982.
- Maurice, Charles S.; Owen R. Phillips; and C. E. Ferguson. Economic Analysis: Theory and Application. Fourth edition. Homewood, Illinois: Richard D. Irwin, Inc., (1970) 1982.
- Schrage, Linus. Linear, Integer, and Quadratic Programming With Lindo. Third edition. Palo Alto, California: The Scientific Press, 1986.
- Taha, Hamdy A. Operations Research: An Introduction. Third edition. New York: Macmillan Publishing Co., Inc., (1968) 1982.

**The vita has been removed from
the scanned document**