SHORT-TERM TOOL LIFE TESTS USING RESPONSE SURFACES

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(ABSTRACT)

In the past, tool life tests have been performed using a conventional Taylor testing technique. This methodology is expensive and time-consuming. It requires wearing a number of tools until the tool failure criterion has been reached. A number of short-term tests designed to replace the Taylor test have been proposed but they suffer from a number of drawbacks. Many of these tests are performed under non-standard cutting conditions or require special workpiece preparation or equipment. As a result, tool life models developed from these tests are of limited usefulness in predicting tool failure times for conventional machining operations.

A methodology is required which combines the time and cost advantages of non-conventional tests with statistical validity and robustness. In this research, two short-term tests are presented which are based on the Taylor test. Response surface models are used to develop the parameters of Taylor's tool life equation. The tests are shortened by using regression equations of flank wear data to predict the tool failure time without wearing the tool to failure. The two methods, abbreviated conventional testing and sequential composite testing, are statistically validated and compared with the full Taylor test. The results show that these tests can accurately predict tool life and the resulting Taylor models are not significantly different from those estimated by conventional means.

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CHAPTER ONE
INTRODUCTION

1.1 Background and Importance

Tool life testing is an important step in the planning of manufacturing operations. It can be defined as the determination of how long a tool can be used while still providing satisfactory performance in terms of dimensional tolerances, surface finish, and economics [1]. Information from these tests is used in machinability indices, for selection and categorization of tool materials, and for determining the economics and optimal parameters of metal cutting operations [2]. Cutting-tool life is one of the most important economic considerations in metal cutting [3]. Tooling, labor costs, and the part production rate all depend greatly on tool life. Research has focused primarily on identifying mathematical models for tool life and the creation of short-term testing techniques to provide more accurate information while reducing the time, cost, and workpiece material required.

1.2 Previous Approaches

The first scientific investigation of tool life was performed by Taylor in 1906 [4]. His work resulted in the first empirical equation relating cutting speed to tool life. Subsequent efforts have investigated the relationship of tool life to machining parameters (speed, feed, depth of cut), cutting temperatures, and cutting forces. More complete mathematical models have been developed using statistical techniques and experimentally-designed tests [2, 5, 6, 7, 8, 9]. Attempts at developing short-term tests have focused on variable-speed tests using facing operations or tapered workpieces, [11, 12, 13, 14], accelerated wear tests under extreme machining conditions [15], abbreviated length tests [16], and more exotic techniques such as radioactive tracer analysis [3].

Regardless of the mathematical model or testing technique, most tool life experiments consist of machining a workpiece with a tool until a failure criterion (such as amount of tool flank or crater wear) is reached. The resulting tool life data is combined with the machining parameters in an empirical equation fitted using statistical techniques. This "conventional" methodology is tedious, lengthy, and consumes a large number of tools and a large quantity of workpiece material. Often a single tool can have a life exceeding 100 minutes. Short-term tests developed to address these problems usually require specially prepared workpieces and/or special equipment to perform. The techniques used seldom resemble standard cutting practices and hence the results of these tests often do not agree with conventional tests.

1.3 Problem Statement

Research into tool life testing has produced a number of useful empirical models and tests. However, almost without exception these tests suffer from the problems of time, expense, and poor correlation with actual production results. The cost and complexity of these tests along with the limited applicability of the results has restricted the application of tool life testing in industry [3]. A test is needed that will produce a statistically robust tool life model from a shortened test using a conventional machining process.

1.4 Research Objective and Significance

The objective of this research is to develop a short-term tool life test which is statistically valid, applicable to common production environments, and can be performed using a conventional turning operation. The methodology will be developed using accepted experimental design and curve-fitting techniques and the results will be compared with data from a conventional test.

No tool life testing methodology to date has combined statistical validity with a conventional testing setup in a short-term test. The development of such a test will lead to the availability of faster, easier, and more robust information on cutting tool life and facilitate more accurate estimation of production costs and machining times. This will increase the application of tool life testing in industry and provide time, cost, and material savings to its users.

1.5 Report Organization

Chapter Two presents a review of the relevant literature in the area of tool life testing. Specific topics covered are Short-Term Testing methodologies, including variable speed and accelerated tests; and Response Surface Methods for tool-life model development, including conventional experimental design models and advanced response surface techniques. An overview of each author's work is presented followed by a critique. The chapter concludes with a summary of the work related to this research and a discussion of research needs.

Chapter Three describes the testing methodology developed in this research. An overview of the test is presented first. The foundations and assumptions of the technique are then discussed and justified. Each step in the procedure is then explained. Finally, the statistical analysis techniques for analyzing the results are presented and discussed.

Chapter Four presents the results of the experiment, the statistical analysis of the results, and the comparison with the conventional test. The validity of the developed model is discussed and deficiencies in the methodology are explained. Finally, speed/material removal rate curves are developed for use in machine parameter selection.

Chapter Five presents future research opportunities resulting from this work. Areas addressed include the inclusion of additional parameters in the model, using a different experimental approach for acquiring the data, and expanding the area of inquiry to other tool and workpiece materials and to other machining operations.

CHAPTER TWO
LITERATURE REVIEW

This literature review will provide an overview of conventional and short-term tool life tests and the application of statistical techniques to tool life model construction. Testing efforts have taken a variety of approaches, from conventional Taylor-type tests [4, 17] to variable speed tests [11, 12, 13, 14], to more exotic tests such as micro-wear tests and shortened conventional methods [15, 16]. Tool life models have been constructed using basic response surface techniques [5, 6, 8] and experimental designs augmented with special techniques such as weighted regression [9] and Monte Carlo approaches [7]. Some research has occurred in the area of optimal model selection and experimental strategy development [18, 2].

2.1 Short-Term Tests

2.1.1 Taylor-Type Tests

The pioneering work of F.W. Taylor was responsible for the development of formalized tool life tests for relating tool life to cutting speed on a lathe [4]. These tests are useful for comparing tool wear data for different cutting conditions and for determining the cutting speed for a given tool life [19]. Tool wear is usually indicated by the mean width of the flank wear land or the maximum width of the land when it is irregularly worn. Other indicators include the depth of crater wear, nose wear, or the surface roughness of the part. Figure 2.1 indicates the locations of crater and flank wear on a conventional carbide cutting tool.

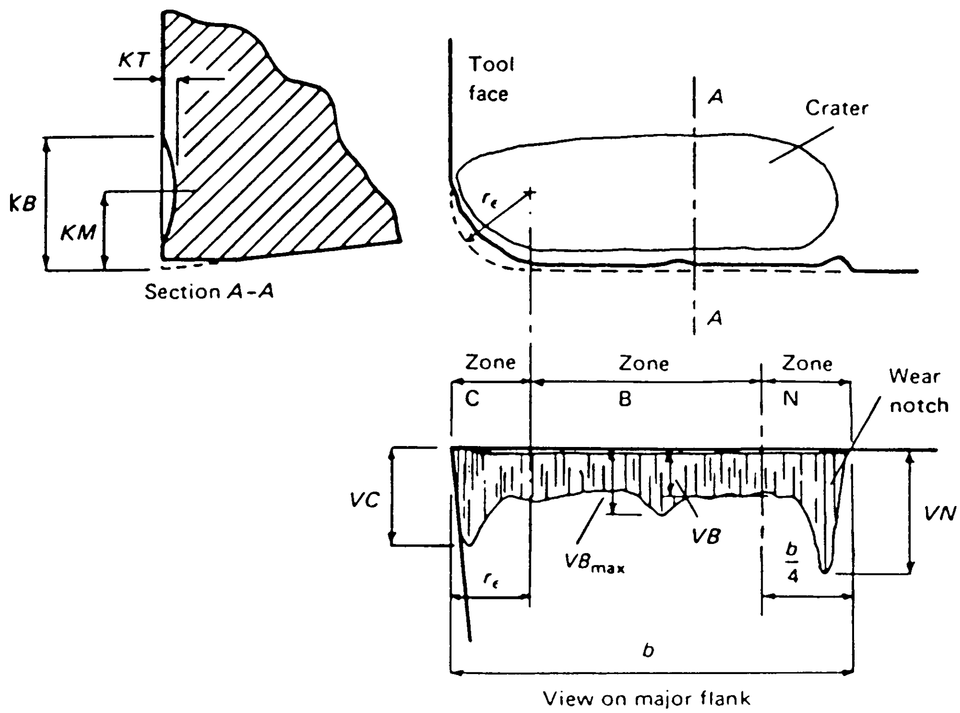

Figure 2.1. Flank and Crater Wear [3]

Taylor's work assumes that the relationship between tool life and cutting speed follows the form

$$VT^{n} = C,$$

where V is the cutting speed in surface feet per minute, T is the tool life in minutes, and n and C are constants to be determined. The equation is solved by obtaining tool wear data for different cutting speeds and selecting a wear criterion, usually some level of flank or crater wear. Tool life T is then defined as the time at which the wear criterion is reached and the constants are found using simultaneous equations or curve-fitting techniques. Determining the amount of tool wear requires the use of a tool microscope to indirectly measure the width or depth of the wear region. Figure 2.2 shows a selection of typical tool wear curves in the time domain at different cutting speeds. It can be seen that the wear curve is divided into three distinct sections or stages. Stage I is the region of initial wear or tool break-in. This wear phase quickly transitions into Stage II, a region of gradual and predominantly linear wear. In the final Stage III zone, the tool wear rate increases at an increasing rate as the tool nears catastrophic failure. If the Taylor model is transformed logarithmically, the result is usually linear within the range of speeds tested.

The classic Taylor test presents several problems [19]. First, when using the equation to estimate tool life, the desired speed for a given tool life must fall within the range of tool lives tested. Second, the tool being tested must be run until the failure criterion is reached; this constitutes a single data sample. A large number of samples, and hence tools, would be required. This would tend to make the tests lengthy and expensive. Third, there is a problem with the assumption of linearity. Three or more data points

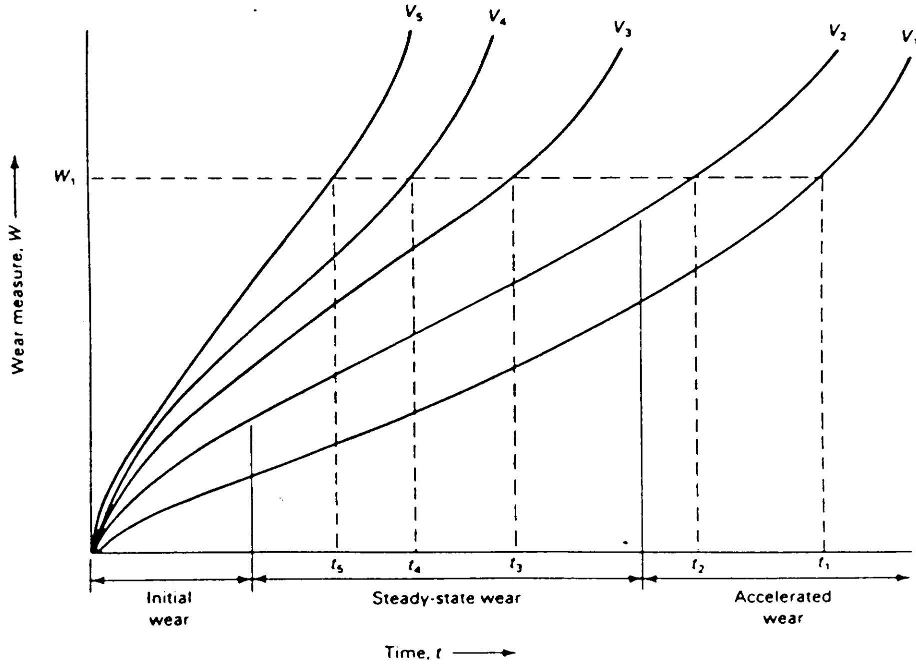

Figure 2.2. Typical Tool Wear Curves [1]

would be required to establish that the V-T curve is linear within the test region. A further problem highlighted by Wu, Ermer, and Hill [18] is that the logarithmic transformation of Taylor's equation may be inappropriate for some tool materials.

The tool life for a given cutting speed can be estimated from the Taylor equation graphs or by fitting a model using regression analysis. Then confidence intervals and estimates of parameter variance could be established. Unfortunately, this degree of statistical robustness would require several replications of the experiment to provide the necessary data for accurate analysis. This makes the test more expensive and time consuming. For these reasons, the Taylor test is generally considered to be a conventional long-term test.

2.1.2 Standardized Conventional Tests

In an attempt to standardize tool testing procedures, the International Standards Organization (ISO) published a tool-life testing procedure in 1977 [20]. The objective of this procedure is to reduce the dispersion among tool test results and to make the results comparable with one another. The ISO test is not widely used, even though it is the only internationally accepted testing procedure. There are several reasons for its lack of application: (1) it is a conventional long-term test, (2) it limits the flexibility of choice among workpiece materials, tool geometry, and machining conditions to achieve comparability among tests, and (3) it does not make any provision for measuring the effects of feed rate on tool life. Additionally, a lathe with a continuously variable-speed drive system is usually required.

The ISO procedure has been used to evaluate the machinability of low carbon free-machining steels [17]. Fourteen casts of steel were tested according to the standard. It was observed that the width of the flank wear land for an M2 high-speed steel tool decreases with an increase in cutting speed or flank wear rate, near the catastrophic failure point of the tool. A short-term test was formulated which predicts catastrophic tool failure by measurement of a portion of the gradual wear zone of the tool.

Due to the time and expense involved in applying conventional Taylor and ISO tests, several types of short-term tests have been developed. These tests attempt to duplicate the results of the Taylor test while reducing the amount of time and material required.

2.1.3 Facing Tests

Facing tests were conceived as an alternative to conventional longitudinal machining tests. In this test, tool wear is promoted by performing a facing operation on a cylindrical workpiece. Conventional facing tests utilize a regular facing operation in which the tool starts cutting from the outer diameter (O.D.) and feeds inward towards the center of the workpiece to an inner diameter (I.D.) [19]. The majority of facing tests, however, are nonconventional, where the facing operation starts from the inside diameter and feeds outward [14]. The assumption behind this test is that results obtained from a facing cut can be related to an equivalent longitudinal test cut.

Lorenz and Gibson [14] constructed a multiple pass facing test using different tool materials and tool life criteria. It was assumed in these tests that the slopes of the log-transformed time vs. wear curves were equal for both the facing and conventional tests and that the intercepts differed by some displacement factor k. The results of a conventional test were compared to those of an equivalent facing test and it was found that the assumptions were valid for that experiment. The k value was used as a correction factor to transform the wear equation developed from the facing test to an equation which would predict tool wear for conventional longitudinal turning processes.

Although these tests do not consume as much material as a conventional test, the testing time can be nearly as long as a Taylor test depending on the tool life criterion and cutting conditions. The minimum workpiece O.D. is restricted by the available spindle speed of the test lathe and by the type of tool being used. For tooling inserts, the I.D. must be large enough to allow clearance for the tool holder [19]. Also, cold-drawn bars cannot be tested using facing operations. The facing test does not correspond with the cutting conditions present in conventional longitudinal machining, namely, steady-state conditions of temperature and force are not established as rapidly during a facing operation [21]. Since cutting temperature has a strong affect on tool wear rate, the results of a facing test may not reflect steady-state cutting conditions. Due to these limitations, the facing test is not a reasonable replacement for the Taylor tests. However they can be used as short-term tests within narrow testing domains where it has been established that they have a good correlation with conventional tests [19].

2.1.4 Taper and Variable Rate Turning Tests

A tapered workpiece turning test was developed by Heginbotham and Pandey [11] to overcome some of the disadvantages of the facing test. The principle of the taper test is the same as the non-conventional facing tests, a linear increase in cutting speed with time. A tapered workpiece must be prepared for the test, however, the taper can be chosen so that the rate of speed variation occurs between two cutting speed limits. In addition, the variations in work material properties across the taper will be much less than when facing a cross-section [19]. However, there are still limitations on the taper angle for small workpiece diameters and cold-drawn bars cannot be tested using this method.

The taper test does overcome the problem of assuring that the acting tool wear mechanism is stable. Therefore, the results should correspond well with those of a conventional test. Taylor's constants were determined by taper turning at two different spindle speeds at the same taper angle to provide a constant gradual speed increase. The results indicated that pre-worn tools should be used for this test, as non-constant tool wear during the initial cutting phase significantly affected the outcome. Heginbotham and Pandey indicate that their results correspond well with those of steady-state tests. However, there was some question as to whether or not the constant change in cutting speed would produce inaccurate results due to a lag or lead in cutting temperature change.

To eliminate the need for special workpiece preparation and the possibility of radial variations in workpiece properties, the Variable Rate Test was developed [12]. This test uses the same principle as the taper-turning test, that of gradually increasing the speed, but eliminates the need for a tapered workpiece. Thus, this test requires less time and less material than the earlier taper test. A disadvantage is that the lathe used for the test must be equipped with a special drive system to provide a continuous, gradual increase in speed as soon as the cutter is engaged. A standard production lathe cannot be used for this test without being equipped with this continuous drive system.

Total testing times for each test were estimated and ratios for each test were established. The taper test was found to require approximately one-half the time of a conventional Taylor test, while the variable speed test requires one-fourth the time, or nearly a 4 : 2 : 1 relationship. Although requiring less time and material , the conventional test still has several advantages over these tests [19]. In conventional testing there are no limitations on workpiece geometry or tool life criteria, nor are there requirements for special drive systems or specially-prepared workpieces. Additionally, a break-in wear period for the tool is not required. Questions about temperature changes over time are not a concern in conventional testing, as steady-state cutting conditions are reached soon after the tool is engaged in the work. Therefore, it can be concluded that these tests suffer from severe limitations and are not replacements for the Taylor test or the facing test. To be used as short-term tests, a valid correlation with Taylor test results must be identified. This has already been accomplished for facing tests.

2.1.5 Step Turning Tests

To address the shortcomings of the taper and variable speed tests (namely the requirements for special workpiece preparation and drive systems), step turning tests were developed. The idea behind this test is to approximate the continual, gradual speed increase with a series of discrete increases. Kiang and Barrow [13] proposed the use of five or more steps to improve the quality of the approximation. At a very high number of steps (20-30) the frequency of tool disengagement and reengagement becomes a problem.

Kiang and Barrow used a cutting time of two minutes at each step so that most of the interval would represent a steady-state condition. Temperature tests indicated that this interval was sufficient. Results obtained indicated that pre-worn edges should be used for this test, as with the previous tests. The test results are based on two assumptions: (1) flank wear versus time occurs in the Stage II, or gradual wear, zone, and (2) the transformation of log T versus log V is linear. Only tool life criteria based on flank wear can be used with this test. The use of this test was recommended only in cases where conventional tests were impossible. Analyses of the results showed good correlation with Taylor test results, once a relationship was established with conventional test results.

2.1.6 Micro-Wear Tests

These tests are also a variant of conventional Taylor tests. A cutting tool with precise geometry and good surface finish is pre-worn to the completion of the primary (Stage I) wear zone. Short-duration tests are then carried out at various speeds in the gradual wear zone and the small amount of flank wear produced is recorded. The Taylor constants can then be determined from the wear data. These tests require little material and only a few minutes of cutting time. However, special equipment is required to measure the amounts of flank wear produced during the short-duration cuts (1-1.5 minutes). Additionally, flank wear is the only wear criterion which can be applied, and special care must be taken to properly prepare the tools to ensure that primary wear has completely occurred.

2.1.7 Shortened Conventional Tests

One of the fundamental assumptions of the Taylor model is that the gradual wear process is predominantly linear. If this assumption is true, linear regression techniques can be used to estimate the tool failure time without running the experiment until tool failure. Analysis of several data points from the gradual wear portion of the T versus wear curve can establish a trend line which can be used to predict the point at which the tool failure criterion will be reached. This approach was used by Thomas and Lambert [16] and was based on the additional assumption that the linear portion of the tool wear curve would remain linear for longer cutting periods. At very low and high cutting speeds this assumption does not hold [22]. However, for a broad range of cutting speeds this method could be used to shorten the testing time and reduce the amount of work material required.

Thomas and Lambert indicate that the results of the shortened conventional test correspond well to those of a conventional test using the same data set [16]. (The shortened test model was constructed using a subset of the full Taylor test data.) However, their analysis does not indicate whether the two models are statistically similar. They constructed confidence limits for the original Taylor model and superimposed the trend line for the shortened model, noting that its trend line fell within the confidence limits for the range of values tested. However, the two lines appear to have different slopes when plotted in log T versus log V space. Additionally, the confidence limits are shown as linear bounds when in fact the standard form for confidence limit lines is curvilinear [23]. A better approach would be to test the hypotheses that the parameters of each model are equal. The variability and predictive capabilities of each model should also be compared.

In summary, none of the above mentioned short-term tests is a replacement to the Taylor-type test. The Taylor model is based on certain assumptions which may or may not hold true for these short-term tests. Additionally, most of these tests are not conducted under conventional cutting conditions. Therefore the results obtained from them will indicate only general trends in tool life which may or may not correspond to conventional conditions [19]. These models should only be used in conjunction with Taylor test results for a given testing region.

2.2 Response Surface Analyses of Tool Life

2.2.1 Tool Life Response Models

In a two-part paper, Wu [5, 24] applies statistical experimentation techniques to tool life modeling. Using an experimental design known as a Central Composite (CCD), a first-order linear equation is fitted to tool life data collected at different values of cutting speed (V), feed rate (f), and depth of cut (d). Data is collected from a 10 horsepower Reed-Prentice lathe cutting six-inch SAE 1018 steel using Carboloy 162pR 78B carbide inserts and with a tool life criterion of 0.030 in. of flank wear. The objective is to reduce the variation associated with the parameter estimates in the tool life model and to estimate the precision of the prediction equation. In the experiment, 12 runs are made at different combinations of V, f, and d and the tool lives for each run are identified. Using linear regression, the data is fit to a three-parameter linear equation of the form

$$\mathbf{Y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \varepsilon},$$

where Y is the response value, x_1, x_2, and x_3 are the transformations of V, f, and d, respectively, the b-values are coefficients which are estimated by the regression, and ε

represents non-quantifiable experimental error. This model is the result of a logarithmic transformation of the extended Taylor equation

$$VT^n f^m d^p = K,$$

which includes the effects of feed rate and depth of cut. The linear model can be transformed back into the exponential form by applying the set of transformations equations used to code the levels of the input variables. The transformation is performed in order to allow the use of simple linear regression and to stabilize the variance associated with the tool life values [24].

An analysis of the fitted model indicates that it adequately fits the experimental data. Since the Central Composite Design is orthogonal (meaning that the X'X matrix for the regression is completely diagonal) the parameter estimates are uncorrelated and of minimum variance. The model lack of fit was not significant at a 95% level of confidence. However, since only 12 tests were performed, the confidence intervals for the tool life estimates are too large to be of practical value, ranging from 20-80% variation. Additional tests would be required to reduce the size of these intervals. Additionally, a careful examination of the residuals for the model reveals a positive average of the residual values, possibly indicating the presence of higher-order effects [5]. Wu does not test the homogeneity of tool life variances over time, a possible source of model error in tool life prediction [25]. Also, the experimental data is fit using a standard Taylor test in which the tool is worn until the failure criterion is reached. For even a small number of tests, the amount of time and material required would be considerable. No attempt is made to use a smaller experimental design, requiring fewer tests, for collecting data. The Central

Composite Design is not required for the first-order model as it contains redundant runs not required for estimation of a linear relationship.

In the second paper [10], the issue of possible higher-order effects is addressed by extending the CCD to include 16 additional points and fitting a full second order model of the form

$$\mathbf{Y = b_o + b_1x_1 + b_2x_2 + b_3x_3 + b_{11}x_1^2 + b_{22}x_2^2 + b_{33}x_3^2 + b_{12}x_{12} + b_{13}x_{13} + b_{23}x_{23} + \varepsilon},$$

where the x_ix_j terms represent interactions effects between two model variables and x_i^2 indicates a quadratic effect from a model variable. The second-order model was also shown to fit the data adequately, however, the quadratic and interaction effects were not found to be significant. In other words, the first-order model was adequate for tool-life prediction. The confidence intervals for the larger model were narrower, corresponding to the effect of the increased number of sample data points. It was noted that for wider ranges of cutting values that the higher-order effects could be significant.

Again, no effort was made to fit the model with a smaller experimental design. A simplex design consisting of 4 runs would be sufficient for estimating the linear model. Significant reduction in the model's predictive variation could be achieved by replicating this design and the total number of runs would still be fewer than that used by Wu (8 simplex runs versus 12 CCD runs). The replications allowed by a smaller design would reduce the width of the model confidence intervals and result in better predictive capability.

The response surface techniques used by Wu [5, 10] can be used to model the effects of cutting parameters on other responses besides tool life. Taraman and Lambert [6, 8] use response surface modeling to fit first-order equations to data collected on tool life, surface roughness and cutting force. The general form of the model equation is

$$\mathbf{R = CV^{p} f^{m} d^{n}},$$

where R is the response (tool life (min.), surface roughness (μin.), or cutting force (lbs.) and C is a scaling constant. Again, this model is transformed into a three-parameter linear equation of the type used by Wu and a Central Composite Design with 12 runs is used to collect data for all three responses. A flank wear criterion of 0.005 in. is specified, and carbide inserts with an SAE 1018 workpiece are used. Taraman and Lambert do not provide any analyses of their models so their is no indication of their adequacy or predictive capability. Their approach is nearly identical to that used by Wu, with the addition of different response to the modeling process. Therefore, the same problems exist for these models.

A procedure for utilizing these models for parameter selection was presented by Taraman and Lambert, however. Plots were developed showing tool life and material removal rate isobars in the cutting speed-feed plane. By identifying a desired material removal rate, the V-f combination resulting in maximum tool life can be identified and vice versa. An example of these curves is show in Figure 2.3. This represents an attempt at using the fitted models for cutting parameter optimization and shows the usefulness of comprehensive tool life models incorporating multiple machining parameters. These fitted equations could be used in machining economics models to identify cutting parameters

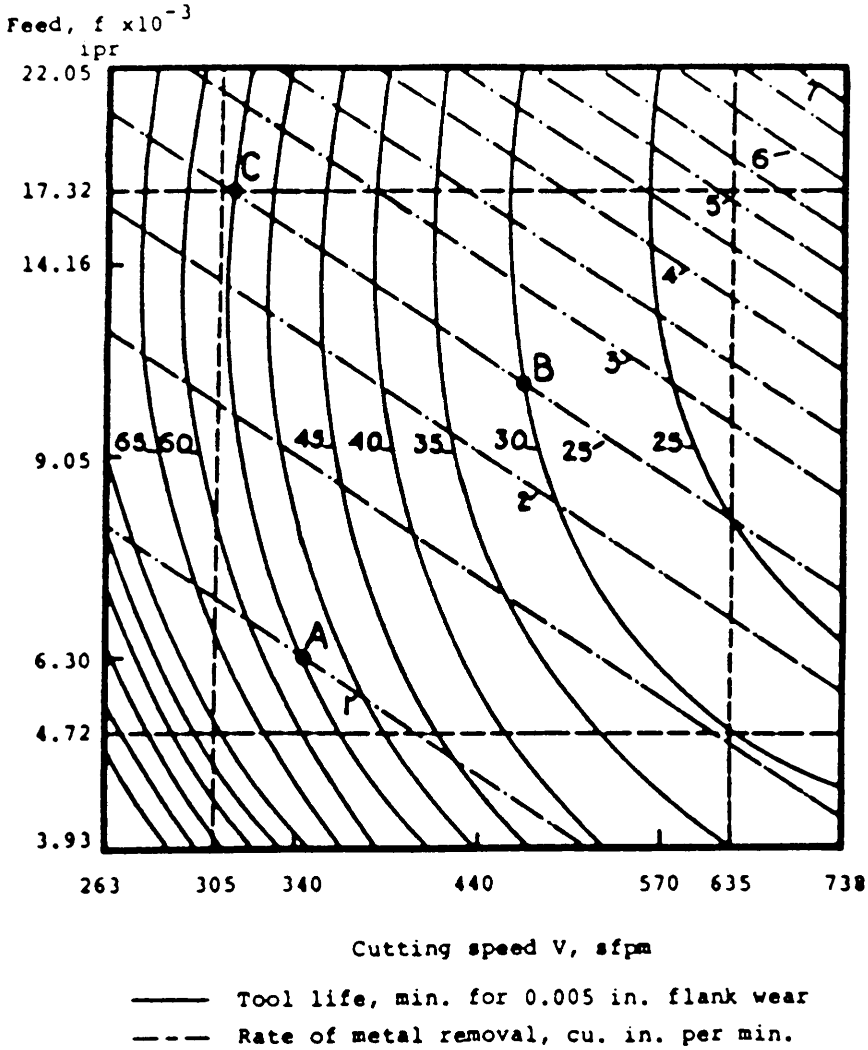

Figure 2.3. Tool life/MRR Contours [6]

which minimize machining cost while meeting constraints on machining time, surface finish, etc.

Miller and DeVor [27] present a comparative study of tool life models for a variety of work materials and tool inserts. The objective of their analysis is to compare the performance of different tools on several aluminum alloys and to develop mathematical models for tool life, with a view towards process selection and model optimization. Tool life testing was carried out using six different aluminum alloys and three tool inserts (tungsten carbide, Compax synthetic diamond, and Megadiamond natural diamond) both with and without cutting fluid. A Central Composite Design was used to select the experimental levels for speed, feed, and depth of cut. A number of performance measures were defined for each tool, including tool wear rate, surface roughness and integrity of finished workpieces, and the formation of built-up edge (BUE). Wear data was collected on a 30 horsepower variable speed lathe.

After completing the full CCD experiment, a subset of the initial test data was used to construct a more comprehensive database of tool wear over a narrower range of machining parameters. According to Miller and DeVor, certain unspecified optimization techniques were used to select additional test points to improve the prediction capability of the final models. These techniques are explained in a later article [2]. Based on the results of the experimentation, three families of wear curves were identified (see Figure 2.4). The group I curves indicate very low wear rates and are difficult to include in tool life model construction due to the requirement for extrapolating tool life. These wear curves were used as a basis of comparison for different tools and coolant types. Group III curves show very high wear rates which are again useful to comparative analysis. These

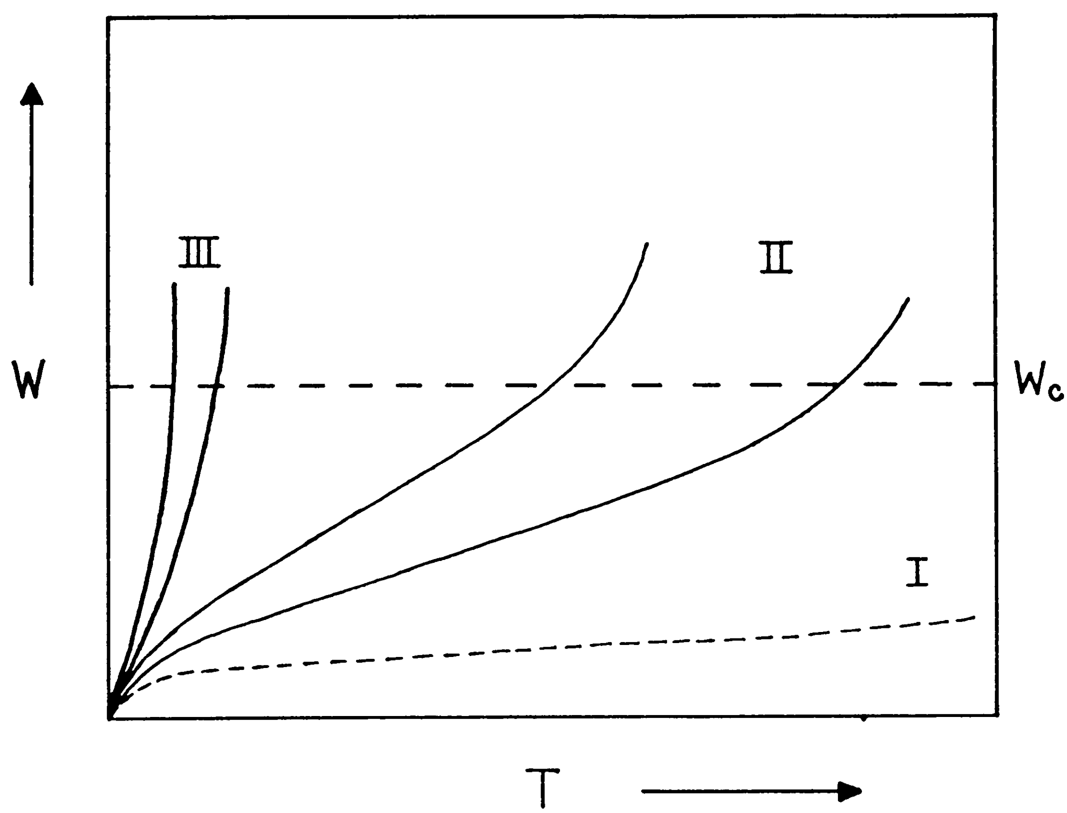

Figure 2.4. Families of Wear Curves [27]

curves are difficult to include in tool life models since the wear occurs too rapidly to allow a sufficient number of wear measurements to be taken. The group II wear curves were used for modeling the tool life equations; the ranges of cutting conditions which produce these curves are those most likely to be encountered in practice and therefore to benefit from process optimization.

A wear prediction equation was derived for each V-f-d combination so that tool wear versus time could be directly estimated. The equation used was a third-order equation of the form

$$\mathbf{W} = \mathbf{b}_0 + \mathbf{b}_1\mathbf{t} + \mathbf{b}_2\mathbf{t}^2 + \mathbf{b}_3\mathbf{t}^3 + \varepsilon,$$

where W is the amount of flank wear in inches, t is time in minutes, and b_1, b_2, and b_3 are coefficients. The models were fitted using regression techniques. It was noted that for most wear curves, only first- or second-order equations were required to adequately describe the relationship. Results published in [27] show that a linear model was effective in representing the time-wear relationship, based on high R^2 values (> 0.90). An analysis of the residuals indicates an excellent fit to the experimental data. Tool life equations were also developed, following the form used by Taraman and Lambert [6, 8]. The models were fitted using linear regression on the log-transformed equation. First-order models were used and they show an excellent correlation to the experimental data over a wide range of common cutting conditions. The tool life curves were also compared with material removal rates in a format useful for optimal parameter selection, similar to the graphical tools produced by Taraman.

The approach used by Miller and DeVor highlights the usefulness of experimental design techniques to tool life modeling. By constructing optimal designs they were able to construct models with excellent correlation and predictive capability, although the confidence limits on their models were not specified. However, the optimality of the design comes at the expense of additional testing time and material use for more experimental runs. An objective of tool life testing should be to reduce the amount of experimentation required while still developing models of acceptable predictive capability. This was not addressed in [27]. However, their development of time versus wear equations shows a possible technique for accomplishing a reduction in experimentation. By wearing tools partially and then fitting the flank wear-time equations to the partial data set, the tool life could be predicted without necessarily wearing the tool until failure. Earlier evidence that linear models could successfully represent this relationship indicates that such a method may be possible. This would result in a dramatic reduction in tool testing time and material use.

Vilenchich, Strobele, and Venter [7] attempt to improve the predictive ability of a fitted tool-life model by computer-generating additional random samples based on the initial experimental data. A linear model and CCD experiment similar to those used by Wu [5] are selected and 24 experimental runs are carried out. Based on the pure experimental error of the results, the standard deviation of the tool life data is estimated. A normal random variate generator is used to create random variates based on the mean response for each replicated test point and on the estimate of σ. These additional "responses" are then used along with the original data to estimate the parameters of the linear model.

This method of variance reduction is not valid. Since the additional data points are not experimentally determined, they may not accurately reflect the behavior of the actual system. No test of the experimental data is performed to confirm the assumption of normally-distributed tool lives and the amount of data collected is insufficient for such a test. Also, it is assumed that the tool life variance is homogeneous across all test points which may be incorrect [26]. Although the parameters of the predictive equation given in [7] do not show any significant change when the additional computer-generated samples are included in the model, biasing of the parameters may occur.

The above experimental approaches are not suitable for short-term tool life testing. All the designs used require large numbers of experimental runs and depend on wearing the tool past the failure criterion. Miller and DeVor's technique of fitting an equation to the time versus flank wear curve [27] does lend itself to abbreviated length tests, however. Using this approach, it may be possible to estimate the tool failure time without wearing the tool until the criterion is reached.

2.2.2 Modeling and Experimental Designs

In conjunction with his work on response models for tool life, Wu investigated the effect of different power transformations on the extended Taylor formula [18]. The researchers attempt to determine the linearizing transformations which provide the best model fit to experimental data. Tool life data is collected on both high-speed steel and tungsten carbide inserts. First-order models are then fitted to the data using linear regression. These models are based on the generalized tool life equation

$$VT^n f^m d^p = K,$$

and the power transformation on the dependent variable T is of the form

$$T^{(\lambda)} = \left\{ \begin{array}{ll} \dfrac{T^{\lambda} - 1}{\lambda\left(\dot{T}\right)^{\lambda - 1}}, & \lambda \neq 0 \\ \dot{T} \ln T \quad, & \lambda = 0 \end{array} \right\},$$

where $\dot{T}$ is the geometric mean of the observations. For the independent variables in the linearized models, the transformations is

$$U_i = \left\{ \begin{array}{ll} x_i^{\alpha_i}, & \alpha \neq 0 \\ \ln x_i, & \alpha = 0 \end{array} \right\}.$$

These are known as Box-Cox and Box-Tidwell transformations, respectively. These transformations were designed for efficiently linearizing sets of multi-variate data which exhibit strong nonlinear behavior [30]. The α and λ parameters can be varied to improve the fit of the transformed data to a regression line.

Each data set is fitted using several different power transformations. The fits are compared using the residual sum of squares (RSS) for each model. A smaller RSS indicates a better fit to the actual data. For the high-speed steel tool, the traditional logarithmic transformation is found to be the best when compared to several other transformations. However, the results for carbide inserts indicate that the straight logarithmic transformation may not effectively linearize the experimental data. Table 2.1 shows the RSS values for the different transformations that were analyzed.

The optimal transformation was found by plotting RSS contours in (λ, α_1) space and identifying the values of each parameter at the minimum RSS. The linear transformation which best fits the data (RSS = 20.10) is of the form

$$\mathbf{E}\left[\frac{\mathbf{T}^{0.25}-\mathbf{1.0}}{(-\mathbf{0.25})(\dot{\mathbf{T}})^{-\mathbf{1.25}}}\right]=\beta_0+\beta_1\mathbf{V}^{0.75}+\beta_2\,\mathbf{ln\,f}\,.$$

This model differs significantly from the standard log-transform model. However, the RSS value for this complex model does not differ significantly from the first model in Table 2.1. The slightly better fit of the optimal model is offset by its computational complexity. However, since it is based on a general transformation, the parameters can easily be optimized by varying them to find the minimum RSS. This will guarantee the best possible fit to the experimental data. Also, if computerized, the computational complexity would be eliminated.

Several additional data sets were analyzed and the best-fit models were found to be of the same form. Wu, Ermer, and Hill show that for all cases, the log-transform model does not fall within the 95% confidence limits surrounding the optimal model. These results indicate that a significant amount of lack-of-fit error can be eliminated by the judicious choice of transformation equations and the optimization of the resulting equation's parameters. This would improve the predictive capability of the tool life model and narrow the confidence limits for a given amount of experimental data. However, the

Table 2.1. Comparison of Transformations [18]

Tool Life Equation	Standardized RSS
$\hat{T}^{(0.2)} = -29.42 - 0.32V - 13.24 \ln f$	20.99
$\dot{T} \ln \hat{T} = 69.22 - 18.9 \ln V - 13.8 \ln f$	24.82
$\dot{T} \ln \hat{T} = -33.16 - 0.03V - 14.16 \ln f$	31.25

linearized model is not easily converted back into the standard Taylor equation. The equation would have to be used in its transformed state, requiring more in-depth calculations to arrive at a prediction of tool life.

In another effort to improve the fit of tool life models, Friedman and Zlatin [26] study the problem of nonhomogeneous variance of tool life with respect to its mean value. In an analysis of several sets of data, the variation in tool life is found to increase as the amount of wear increases based on an increase in the coefficient of variation and Cochran's test. This result was true for both HSS and carbide tools across a range of common engineering materials. DeVor, Anderson, and Zdeblick [9] confirm the problem of nonhomogeneous variance and demonstrate that the logarithmic transformation does not stabilize the tool life variance. Both papers show that the method of weighted least squares regression can be used to correct for nonhomogeneity. This method weights the influence of each data point on the model according to the amount of variation observed. Points with greater variation are weighted less than those with smaller variation. The weights chosen are of the form

$$\hat{\omega} = \frac{s_p^{\,2}}{s_i^{\,2}},$$

where $s_p^{\,2}$ is the estimate of the pooled variance over the design and $s_i^{\,2}$ is the variance estimate for the i^{th} test point. The weight matrix **V** is then included in the standard least squares estimation equation. In several comparisons [26, 9] the weighted regression models exhibit a better fit to the data based on higher R^2 values, smaller RSS values, and narrower confidence limits than the unweighted models.

A comprehensive methodology for addressing the problems of variance nonhomogeneity and experimental optimization is presented by Zdeblick and DeVor [2]. In an effort to identify the best experimental design to use for tool life modeling, the Central Composite Design (CCD) is compared to D-optimal designs for first- and second-order linear models and for a nonlinear model. D-optimal designs are those for which the determinant of the **X'X** matrix is maximized. These designs posses several desirable properties, including: (1) the volume of the confidence regions is minimized, (2) the variance of the parameter estimates is minimized, and (3) the design is invariant to changes in parameter scale. This class of designs is also useful for dealing with situations of nonhomogeneous variance [2].

D-optimal models for Wu's first- and second-order models [5] were identified as standard 2-factorial and 3-factorial designs. Experiments were then run using these designs and the CCD. Using weighted regression as specified in [9], models were fitted to the results of each design. The results indicated that the D-optimal models have less parameter variability and require less than half the number of experimental runs to achieve the same confidence limit percentage range as the Central Composite models (see Table 2.2).

The use of optimal experimental designs and variance minimization techniques present another opportunity for reducing model lack-of fit and experimental variation. By using such techniques in concert with optimized transformations the number of experimental data runs can be significantly reduced. The result would be faster, less costly tool life experiments using fewer tools and less workpiece material that would provide statistically robust models with low variability and good predictive capabilities.

Table 2.2. Number of Runs to Achieve Equivalent Model Precision [2]

Tool Life Model	Experimental Design	Number of Runs
Wu's First-Order	D-optimal (2^k factorial)	17
	CCD	47
Wu's Second-Order	D-optimal (3^k factorial)	27
	CCD	60

2.3 Summary

Conventional Taylor tool life tests are expensive, time-consuming, and require a large quantity of tools and workpiece material. Unfortunately, although a great deal of research has been accomplished, no short-term tool life tests have been created which match the utility of the Taylor test. Facing, variable rate, and step turning tests suffer from special material and equipment requirements, and micro-wear tests demand high-resolution measurement equipment and well-prepared tools. These deficiencies limit the usefulness of such tests in the machine shop environment. The shortened conventional test proposed by Thomas and Lambert [16] approximates the Taylor test but eliminates the need for wearing the tool until failure. Unfortunately, no further work was performed to analyze and validate these tests.

The use of experimental design and response modeling has been for the most part restricted to multivariate modeling using Taylor testing methods. No attempt has been made to model tool life using models based on shortened tests. Miller and DeVor [27] suggest fitting an equation to the time versus wear curve and using this equation to predict flank wear for a given cutting speed and feed. It may be possible to fit a model to an abbreviated set of testing data and use it to predict tool life, without having to wear the tool to the failure criterion. This would significantly reduce testing time and material requirements, making the test less expensive and more attractive for use in industry. However, due to the reduction in quantity of data, robust experimental designs and modeling techniques would be required to minimize experimental error and maintain the model's predictive capabilities.

A tool life test is needed which minimizes the time and expense involved in testing and provides results similar to those of a conventional test. A testing methodology must be developed for data collection and modeling which will result in robust, low-variability models of tool life with good predictive characteristics. This test should not require special workpiece preparation or equipment and should be able to be performed using conventional machining conditions in a production environment.

CHAPTER THREE
METHODOLOGY

3.1 Objective and Summary

The objective of this research is to create an abbreviated tool life test. This test is based on the traditional Taylor method but does not require wearing the cutting tool to the failure criterion. Instead, the tool life is predicted based on the relationship between flank wear and time. A linear model is fitted to this relationship and the predicted failure time is used to construct a tool life model of the form

$$T = KV^n f^m.$$

This abbreviated model is compared with a similar model constructed with a conventional Taylor test. The results of the comparison indicate whether a shortened test can predict tool failure time with sufficient accuracy to construct a robust tool life model. A sequential composite test is also be conducted and compared to the conventional test. In this test, two or three tools are worn for short periods at each set of cutting conditions until the failure criterion is reached. The wear rates for each (V, f) combination are averaged and used to predict the tool failure time. If valid, this test would reduce testing time and cost by an order of magnitude or more. All three tests are then be compared by testing time, material, and tooling requirements.

3.2 Experimental Setup

Experimentation is conducted on a Mazak high-speed precision lathe with a 7.5 kW (10 hp.) motor. The material tested is 4140 steel bar stock with an initial O.D. of 8 in; the material conforms to ANSI standards for composition and heat treatment. VR Wesson Fansteel CNMG 432 C-6 tungsten carbide inserts are used for cutting. These are 80° diamond-shaped inserts with a -5.0° rake angle and a nose radius of 1/32". A failure criterion of 0.025 in. FW is used. No cutting fluid is used in this experiment. Cutting speed is verified with a Mitutoyo 982-521 digital tachometer with a 2.0" registration wheel. Measurements of wear are taken on a Mitutoyo tool maker's microscope with a resolution of 0.0001" ± 0.0001".

3.3 Experimental Design

3.3.1 Conventional and Abbreviated Tests

Data for the conventional and abbreviated Taylor tests are obtained using a two-factor, two-level D-optimal experimental design. In this case, the design is a conventional 2^k factorial design with two levels of each factor. Two replicates are run and two additional center points are tested to allow for testing of model curvature. The independent variables are cutting speed (V) in surface feet per minute and feed rate (f) in inches per revolution. The values for cutting speed and feed were selected from [6] and are presented in Table 3.1. Depth of cut will remain constant at 0.080 in. Measurements of wear are taken every 2.5 minutes until 30 minutes or the failure criterion is reached. The time at tool failure is also recorded.

Table 3.1. Experimental Speed and Feed Levels

Level	Cutting Speed (sfpm)	Coded Speed Level (x_1)	Feed Rate (ipr)	Coded Feed Level (x_2)
Low	340	-1	0.009	-1
Center	440	0	0.012	0
High	570	1	0.016	1

The levels of each variable must be coded for analysis of the design. The coding equations used are taken from Taraman [6]:

$$x_1 = \frac{2(\ln V - \ln 570)}{(\ln 570 - \ln 340)} + 1,$$

$$x_2 = \frac{2(\ln f - \ln 0.016)}{(\ln 0.016 - \ln 0.009)} + 1.$$

These coding equations perform a logarithmic transformation on factors V and f to create the linear independent variables x_1 and x_2. The equations are designed so that the low level of each factor will be coded -1 and the high level of each factor +1. The center point values are selected so that their coded level equals 0. This allows the effects of each variable to be easily calculated without the use of complex multiple regression techniques and assures that the model will be orthogonal. Use of an orthogonal D-optimal model will minimize the experimental variance and the variance of the parameter estimates [28].

To reduce the possibility of model bias due to external effects, the order of experiments is randomized. A total of ten experimental runs are conducted; two runs at each combination of speed and feed (low and high) and two runs at the center point (see Table 3.2). All runs are performed on the same piece of bar stock to eliminate variation among different workpieces. The cutting tools used are from the same production batch and order. All four edges of each tool are used in testing to reduce the number of tools required. Due to the tight tolerances specified by the manufacturer on tool geometry and hardness, this should not introduce any model bias.

Table 3.2. Experimental Runs in Randomized Order

Run	Speed (sfpm)	Feed (ipr)	x_1	x_2
1	340	0.009	-1	-1
2	570	0.009	1	-1
3	440	0.012	0	0
4	570	0.016	1	1
5	440	0.012	0	0
6	570	0.009	1	-1
7	340	0.016	-1	1
8	340	0.009	-1	-1
9	570	0.016	1	1
10	340	0.016	-1	1

3.3.2 Sequential Composite Test

The sequential composite test was first proposed by Chen [29] as a short-term testing method which does not depend on the validity of the Taylor equation. In this test, two identical tools are worn for short periods (0.5-1 minutes) at cutting speed V_1. The speed is then changed to V_2 and the tools are worn at this speed. This sequence of tests repeats until the tool wear criterion is reached, in a wear process shown in Figure 3.1. The average wear rate at each speed is used to construct a linear equation relating time to flank wear for that speed. This equation can then be used to predict flank wear and tool failure times.

In this research, the test is extended to include feed rate. Four tool edges are tested at two levels of speed and feed, corresponding to the low and high levels for the Taylor experiment. Each test will be run for 1.5 minutes. The testing procedure is shown in Table 3.3. In order to eliminate external bias, the tests are randomized. However, each combination of speed and feed is used as a starting point on a single edge. This will facilitate calculation of the intercept parameter of the wear equation. Data on the wear rate for each (V, f) combination is collected until each edge reaches the failure criterion.

A concern in such a test is the lack of independence among successive measurements of wear rate. This may bias the result and render them invalid. A basic assumption is that the relationship between tool wear and time is linear in the gradual wear zone. If this assumption is true, then the wear rate of measurement n should not be biased by any of the preceding 1, 2, ..., n-1 measurements. In other words, the wear rate is independent of the amount of wear, and the wear rate predictions should be valid.

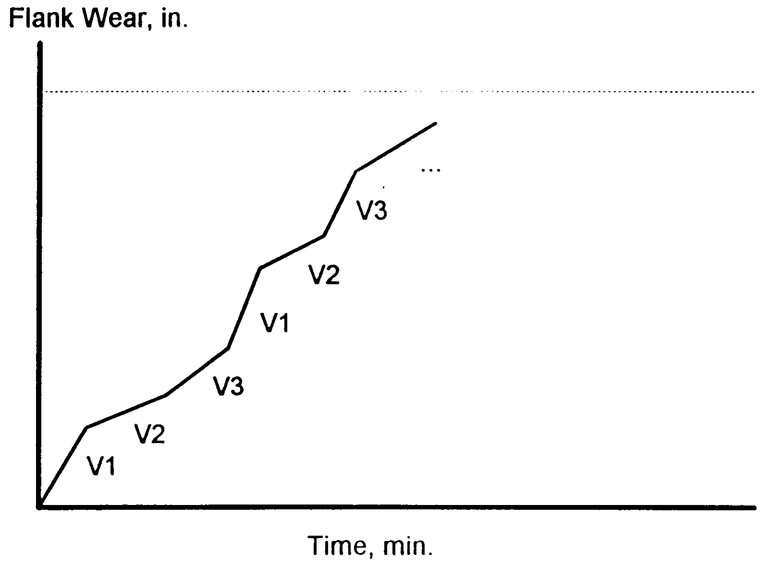

Figure 3.1. Wear Process for Sequential Composite Test [29]

Table 3.3. Sequential Composite Testing Procedure (one set)

Feed (l)	f_1		f_2	
Tool (j)	T_1	T_2	T_1	T_2
Speed (k)				
V_1	W_{ijkl}			
V_2				
Set # (i)	1			

3.4 Analysis of Results

The analysis of data from the experiment consists of: (1) model construction, (2) verification of model and its requisite assumptions, (3) determination of the predictive capabilities of each model, and (4) comparison of the conventional, abbreviated, and sequential composite models on the bases of predictive capability and the amount of time and material required by each test. Numerical analyses are conducted using MINITAB statistical analysis software and NONLIN regression package for analysis of variance and model construction, and Microsoft Excel for data tabulation and comparison of results.

3.4.1 Conventional Taylor Test

The results of the 2^k factorial experiment consist of 10 response values which are experimentally determined tool lives. Each set of cutting conditions (V and f) have two responses as the experiment was replicated twice. The center point responses are not included in the model since they are used for determination of model fit and curvature. Model estimation is done using the eight remaining points corresponding to the original factorial design.

The model to be fitted is of the form used by Taraman and Lambert [6] and represents a linear transformation of the Taylor equation extended to include feed rate effects:

$$\mathbf{y} = \mathbf{b}_0 + \mathbf{b}_1\mathbf{x}_1 + \mathbf{b}_2\mathbf{x}_2 + \varepsilon,$$

The error term ε is assumed to be normally-distributed with a mean of 0 and standard deviation σ. The terms b_0, b_1, and b_2 are estimated using the general linear model to form the equation

$$\hat{y} = \hat{b}_0 + \hat{b}_1 x_1 + \hat{b}_2 x_2 .$$

The method used to calculate the values of the parameters is standard orthogonal linear regression. The standard error of each estimated coefficient is calculated and confidence intervals established for the parameter estimates. T-tests are performed to determine the significance of each coefficient. An analysis of variance is used to determine whether model lack-of-fit and or curvature are significant. If fit and curvature are significant, a second model is fitted which includes the interaction term $\hat{b}_{12} x_1 x_2$ and the analysis is repeated. Appendix B contains the data used for these analyses.

The problem of homogeneity of variance is addressed by analyzing the residuals from the fitted model. If the residuals show a non-random trend or a non-zero mean, weighted regression is used to re-fit the model. The weights used are the reciprocals of the variance of responses within each treatment combination [30]. The significance analysis of model parameters and fit is then repeated.

Confidence limits and the residual sum of squares (RSS) for the final model are calculated for comparison of the three models. The full Taylor experiment model is the reference model and all comparisons are performed relative to it.

3.4.2 Abbreviated Taylor Test

In addition to the tool failure times, the experimental results include measurements of tool wear for each set of cutting conditions at 2.5 minute intervals until 30 minutes or tool failure are reached. These wear measurements are used to create an equation relating flank wear rate to a function of time. The equation is linear, following the assumption that the relationship between time and flank wear is linear during the gradual wear phase [27]. Linear regression is used to estimate the parameters of this equation for the entire set of data and for three subsets of the data; these subsets are: (1) 2.5 thru 15 minutes, (2) 2.5 thru 10 minutes, and (3) 5 and 15 minutes. Flank wear models are created for each set of machining conditions; the data replicates for each set of V and f are combined to increase the number of sample points in the regression. The parameters of the abbreviated equations are then compared to those of the full equation to determine if they differ significantly. Details of the testing procedure are found in Appendix A.

These "subset equations" are used to predict the tool failure times (the time at which flank wear reaches 0.025 in.) for each parameter set. The equations which predict the actual failure time with the fewest number of data points are identified. The predicted tool lives are then used to construct an "abbreviated" tool life model similar to the one created using the experimental data. The complete and abbreviated models are tested to determine if their parameters differ significantly. Confidence intervals are constructed for the abbreviated model and compared to those of the full model. The predicted tool lives for a set of speeds and feeds are calculated for each model and compared. The results of this analysis show whether an abbreviated test using predicted failure times can be used to construct a model with equivalent predictive capability and robustness to the conventional Taylor test model.

3.4.3 Sequential Composite Tests

The results of this test are used to develop time versus flank wear models similar to those created using the abbreviated test. In this test, however, the parameters of the linear equation are calculated directly. Statistical methods are not required which reduces the computational complexity of the model. The models are used to predict tool failure time and Taylor tool life equations are constructed using those results.

The flank wear prediction models are linear equations relating flank wear to time of the form

$$\mathbf{w}_t = \mathbf{w}_o + \omega \mathbf{t},$$

where w_t is the tool wear at time t, w_o is the initial tool wear resulting from the nonlinear wear occurring during the initial wear phase, and ω is the average wear rate for a given combination of speed and feed. The average wear rate is determined by taking the arithmetic average of the individual wear rates from the experiment using the equation

$$\omega_{V,f} = \frac{\sum_{i=1}^{n} \omega_{V,f(i)}}{\sum_{i=1}^{n} \mathbf{t}_i},$$

where n is the number of data points for a (V, f) set and t is the wear time for each point. The intercept parameter w_0 is calculated by substituting the initial tool wear (measured after the first test on the edge) for w_t and solving for w_0. The initial tool wear must occur at the same speed and feed that the wear equation is being constructed for. Since only one data point is available to estimate w_0, additional edges may be tested to generate further data and reduce the variability of the estimate.

The results of this test are compared to those of the abbreviated Taylor test. The predictive ability of each model is contrasted and the sequential composite test results are evaluated to determine whether this test can produce reliable tool wear prediction equations. If the results of this method are valid and reliable, the data required for generating the extended Taylor tool life equations can be acquired using a much shorter test with less material waste and tool cost than incurred with conventional or abbreviated tests.

CHAPTER FOUR
ANALYSIS AND RESULTS

4.1 Summary of Results

The conventional factorial and sequential composite experiments were conducted successfully. The results were analyzed and carefully tested for validity. A comparison was then performed among the three tests to determine if the results were equivalent. Table 4.1 shows the tool lives for five combinations of speed and feed which were predicted by each method along with the average tool life obtained experimentally. As can be seen, the three methods compare favorably. With one exception, all predicted values are within three minutes of the actual average tool lives and the residual sum of squares is small for all three methods.

A transformed linear model was found to be sufficient to portray the relationship among speed, feed, and tool life. Some curvature in the model was found but it was insignificant at a 0.05 level of significance. However, if the range of speeds and feeds was increased, the higher order effects would be significant according to DeVor [9]. The relationship between time and flank wear in the gradual wear zone was found to be highly linear, corresponding with the assumption made by Taylor's model. Homogeneity of variance was not indicated by an analysis of residuals and response variances, so weighted regression analysis was not required. The standard logarithmic transformations were applied to all data sets and produced models with very high variance correlation coefficients. The Box-Cox transformations used by Wu [18] were not used as they would not have significantly improved model linearity.

Table 4.1. Comparison of Experimental & Fitted Model Predicted Tool Lives (residuals)

Speed (sfpm)	Feed (ipr)	Experimental	Taylor	Abbreviated Taylor	Sequential Composite
340	0.009	41.15	43.18 (-2.03)	39.74 (1.41)	41.66 (-0.51)
340	0.016	37.25	35.49 (1.76)	34.90 (2.35)	32.22 (5.03)
440	0.012	20.65	18.78 (1.87)	18.50 (2.15)	19.19 (1.46)
570	0.009	10.45	9.95 (0.50)	9.81 (0.64)	11.43 (-0.98)
570	0.016	7.80	8.18 (-0.38)	8.61 (-0.81)	8.84 (-1.04)
Residual Sum of Squares			11.1098	13.1988	29.7364
Standard Error of Regression Model			0.07565	0.03961	0.07969

4.2 Conventional Taylor Test

The results of the factorial experiment were fitted to the linear model and subjected to an analysis of variance test to determine if model lack-of-fit and curvature were significant. Each coefficient estimate was tested using a t-test to establish the significance of the model variables. Studentized residuals were computed, plotted, and examined to determine if homogeneity of variance was present. Finally, model fits and their confidence intervals were calculated for the speeds and feeds used in the experiment. These fits were then compared to the experimental results. Detailed calculations for the testing procedures can be found in Appendix A. Printed output from MINITAB and NONLIN are given in Appendix B. The data collected during the experiment is presented in Table 4.2.

4.2.1 Model Fit and Parameter Significance Tests

The model fitted to the experimental data by MINITAB was

$$\mathbf{y = 2.93328 - 0.73404\ x_1 - 0.0981\ x_2}.$$

The standard error of this linear model is 0.07565. This model explains 99.1% of the variation found in the experimental data. Curvature and lack-of-fit were tested by calculating the F-statistics for each and finding their p-value, or level at which the statistics become significant. Both curvature and lack of fit were found to be insignificant at a 0.05 significance level, indicating that a linear model was adequate to describe the relationship among the dependent and independent variables. However, a possible interaction between

Table 4.2. Results of Experiment

Speed (sfpm)	340		340		440		570		570	
Feed (ipr)	0.019		0.016		0.012		0.009		0.016	
Time										
2.5	0.0052	0.0049	0.0056	0.0058	0.0076	0.0073	0.0119	0.0112	0.0138	0.0124
5.0	0.0057	0.0066	0.0063	0.0079	0.0108	0.0105	0.0160	0.0178	0.0185	0.0177
7.5	0.0070	0.0084	0.0083	0.0085	0.0137	0.0127	0.0210	0.0200		0.0228
10.0	0.0084	0.0087	0.0095	0.0100	0.0151	0.0145	0.0244			
12.5	0.0107	0.0099	0.0109	0.0121	0.0172	0.0167				
15.0	0.0129	0.0105	0.0127	0.0132	0.0201	0.0198				
17.5	0.0136	0.0121	0.0135	0.0134	0.0210	0.0215				
20.0	0.0153	0.0137	0.0147	0.0141	0.0230					
22.5	0.0164	0.0143	0.0166	0.0160						
25.0	0.0165	0.0164	0.0172	0.0182						
27.5	0.0174	0.0172	0.0177	0.0203						
30.0	0.0181	0.0195	0.0186	0.0212						
Tool Life	41.7	40.6	36.6	37.9	21.3	20.0	10.9	10.0	7.4	8.2
Average	41.15		37.25		20.65		10.45		7.80	
Standard Dev.	0.77782		0.91921		0.91924		0.63640		0.56569	

speed and feed is just barely insignificant ($p=0.051$), indicating that a minor 2^{nd} order effect may be present. T-tests were then performed on the model coefficients (b_o, b_1, and b_2). All three were found to be significant, indicating that the intercept of the equation is non-zero and both speed and feed have an effect on the rate of tool wear. An analysis of variance showed that the magnitude of experimental error was small when compared to the model error, indicating a good correlation between the model and the experimental data.

4.2.2 Analysis of Residuals

The residual differences between the actual and predicted results ($\mathbf{e}_i = \mathbf{y}_i - \hat{\mathbf{y}}_i$) were standardized using the transformation

$$\mathbf{r}_i = \frac{\mathbf{e}_i}{s\sqrt{1 - \mathbf{h}_i}},$$

where r_i is the "Studentized" residual, s is the model standard error, and h_i is s times the standard error of prediction for point i. The quantity r_i closely follows a t-distribution and is scale-free, removing scaling bias from the residuals plot. The mean of the residuals should be zero and the variance near one if the variance of the model is homogeneous. For the Taylor experimental results, $\bar{r}$ equals 0.00125 and σ_r is 1.07. These near-ideal values indicate that model variance is nearly homogeneous. An examination of the plot in Figure 1 reveals no trend in the residuals which would indicate model lack-of-fit. Therefore, weighted regression analysis will not be required to develop a satisfactory model.

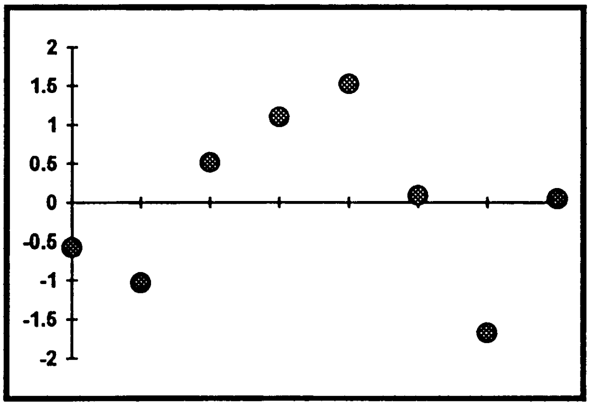

Figure 4.1. Plot of Studentized Residuals for Conventional Model

Table 4.3. Comparison of Actual and Predicted Tool Lives

Speed	Feed	Actual Tool Life	Fitted Tool Life	LCL	UCL
340	0.009	41.15	43.18	38.33	48.65
340	0.016	37.25	35.49	31.5	39.98
440	0.012	20.65	18.80	17.54	20.13
570	0.009	10.45	9.95	8.83	11.20
570	0.016	7.80	8.18	7.24	9.21

4.2.3 Fits and Confidence Limits

The preceding analysis indicates that the postulated model is adequate and sufficient to represent the machining parameter-tool life relationship. However, the usefulness of the model lies in its ability to robustly predict tool life when given a set of machining parameters. This predictive capability can be examined by constructing confidence intervals around the tool life estimates from the model. The width of the confidence limit indicates the range in which the true mean tool life lies with a $(1-\alpha)100\%$ probability. Narrow limits indicate robust predictive capability.

Confidence intervals were computed for the estimated tool lives at the experimental (V, f) values. The results show that the actual mean values for tool life consistently fall within the confidence limits. The confidence limits at the experimental speeds and feeds vary less than ±12% from the fitted estimates. Both these results indicate a model with very good predictive capability. The conventional model fits and confidence limits are shown in Table 4.3. If we apply the transformations used to code the design variables, the conventional Taylor form of the equation is

$$\mathbf{T = 134{,}963{,}914\ V^{-2.841313} f^{-0.341106}}.$$

This formula can be used to estimate tool life directly from the machining parameters without having to code the values of each variable.

4.3 Abbreviated Taylor Test

The time versus wear data collected during the factorial experiment were used to construct linear models relating flank wear to time. A complete model was created by linear regression using all the data points. Subset models were also created using shortened sets of data. For the data sets with a speed of 340 sfpm three subset equations were developed; for the data sets with a speed of 570 sfpm only one subset equation was regressed due to the short tool life at this speed. The parameters of each subset equation were tested against those of the complete model with a t-test to determine whether they differed significantly. One subset equation for each speed and feed combination was used to estimate the tool failure time at that set of parameters. These estimated tool lives were then used to create a Taylor tool life model similar to that derived from the conventional test. The parameters of the two models were compared using a t-test to determine whether they differ significantly. Calculations for the t-tests can be found in Appendix A and MINITAB printouts are given in Appendix B. Table 4.4 contains the abbreviated test estimates of tool life.

The time versus flank wear equations derived using the complete data set were tested for lack of fit and non-linearity. It was found that the relationship between time and flank wear was very linear, with no detectable higher-order effects. In general, the complete and abbreviated models accounted for greater than 90% of the variation in the experimental data. When the parameters of the abbreviated models were tested against those of the complete model, a significant difference in the parameters was found for only one equation. The models chosen to estimate failure time for the abbreviated Taylor model were constructed using data taken at 5 and 15 minutes (for V=340) and 2.5 and 7.5

Table 4.4. Estimated & Fitted Tool Lives Obtained from Selected Abbreviated Equations

Speed	Feed	Actual Tool Life	Estimated Tool Life	Fitted Tool Lives	Prediction Equations
340	0.009	41.15	38.96	39.74	**FW = 0.00338 + 0.000555 T**
340	0.016	37.25	35.60	34.9	**FW = 0.00417+ 0.000585 T**
570	0.009	10.45	10.01	9.81	**FW = 0.00708 + 0.00179 T**
570	0.016	7.80	8.45	8.61	**FW = 0.00810 + 0.00200 T**

minutes (for V=570). The linearized Taylor model derived from these tool life estimates is

$$y = 2.91816 - 0.69928\ x_1 - 0.06490\ x_2 .$$

The parameters of this equation were found not to differ significantly from those in the conventional model. This equation accounts for 99.8% of the variation found in the experimental data and the model standard error is 0.03961. The fitted tool lives and prediction equations are found in Table 4.4.

Based on these tests it can be concluded that adequate tool life equations can be constructed using predicted failure time values from data on flank wear versus time. The RSS and model standard error for the abbreviated test equation compare favorably with those of the conventional equation. Confidence limits for the abbreviated Taylor equation are not a valid basis of comparison since the number of data points used in this regression was four as opposed to the eight used in the conventional model. The robustness of this model could be improved by running an additional replicated of the abbreviated test. This should result in a model with roughly the predictive capability of the conventional Taylor model.

4.4 Sequential Composite Test

Wear data was collected using the sequential composite experiment on four tool edges. The initial wear on each edge was produced at a different V and f combination. The average wear rate and intercept parameters were calculated for each speed-feed combination. The resulting equations were used to estimate the tool failure time for each set of machining parameters. A Taylor tool life model was constructed using these failure time estimates and the model parameters were tested to determine if they differ significantly from those in the conventional model. The data collected from the experiment and the parameters estimated are presented in Table 4.5. The parameter t-tests are similar to those shown in Appendix A.

4.4.1 Derivation and Parameter Testing of Sequential Composite Models

Each tool edge was worn at one set of parameters for 1.5 minutes and the wear was measured. This process was repeated until the tool edge reached the failure criterion. When the wear values were tabulated, the assumption of independence among successive wear measurements appeared to be valid. No variation in the wear rate was noted based on the time at which the wear occurred. The Taylor equation derived using the estimated failure times is

$$\hat{y} = 2.95423 - 0.64693\ x_1 - 0.12856\ x_2 .$$

The parameters of this equation do not differ significantly from those of the conventional Taylor model. This model accounts for 98.9% of the variation in the experimental data and the model standard error is 0.07969. The fitted tool lives are found in Table 4.5.

Based on these results it appears the sequential composite experiment can be successfully used to develop Taylor tool life models. The estimated tool failure times are similar to those developed from the abbreviated test and the model error is no greater than that for the conventional testing method.

Table 4.5. Sequential Composite Experimental Data and Results

(shaded measurements used to compute intercept)

Speed	340	570	340	570
Feed	0.009	0.016	0.009	0.016
Edge 1	0.0053	0.0030	0.0008	0.0031
	0.0009	0.0032	0.0009	0.0033
	0.0006	0.0025	0.0007	0.0030
	0.0006			
Edge 2	0.0009	0.0068	0.0007	0.0038
	0.0007	0.0021	0.0008	0.0040
	0.0007	0.0023	0.0008	0.0032
	0.0009			
Edge 3	0.0006	0.0026	0.0085	0.0031
	0.0009	0.0029	0.0006	0.0035
	0.0009	0.0024	0.0008	0.0034
	0.0007			
Edge 4	0.0009	0.0025	0.0009	0.0098
	0.0009	0.0026	0.0007	0.0024
	0.0006	0.0028	0.0008	0.0031
	0.0007			
ω	0.000511	0.001752	0.000515	0.002176
w_0	0.004533	0.004173	0.007727	0.006536
Predicted Failure Time	40.04	11.89	33.53	8.49
Fitted Tool Life	41.66	11.43	32.22	8.84

4.4.2 Comparison of Abbreviated and Sequential Composite Tests

The Taylor equations developed using the abbreviated and sequential composite test results can be directly compared by their standard errors and confidence intervals since each was developed with four data points. Table 4.6 lists the confidence limits for both models. Each model has very wide confidence limits, ranging from 55% for the abbreviated model to 140% for the sequential model. The standard error and interval widths are much larger for the sequential composite results. This may be due to the method by which the time versus flank wear equations were computed. The abbreviated test equations were developed using linear regression while the sequential composite test equation parameters were estimated using averages. This may have introduced additional error into the tool life predictions. Additionally, the intercept parameters for the sequential test was calculated with only a single data point.

Results obtained from the abbreviated tests are more robust. However, the reduction in testing time and material consumption offered by the sequential test may offset the lack of robustness since the results obtained are still generally adequate and comparable to the other methods.

Table 4.6. Confidence Limits for Sequential Composite and Abbreviated Tests

Speed	Feed	Sequential Composite UCL	Sequential Composite LCL	Abbreviated Taylor UCL	Abbreviated Taylor LCL
340	0.009	17.33	100.14	30.77	61.45
340	0.016	13.41	77.43	22.57	53.97
570	0.009	4.75	27.46	6.35	15.18
570	0.016	3.67	21.23	5.57	13.33

4.5 Comparison of Time and Material Required

In addition to the predictive capability of the resulting models the three tests can be compared by the amount of time and material they require. The amount of machining time and material consumed by each test (conventional w/o center points, abbreviated, and sequential) was calculated. The results are listed in Table 4.7. Material consumed was calculated as the sum of the material removal rates (MRR) at each set of cutting conditions multiplied by the corresponding machining times. Time required was restricted to actual cutting time; measurement and setup time was ignored.

The conventional Taylor test is clearly inefficient, consuming twice as much material as the abbreviated test and requiring over 2.5 times the cutting time. However, the abbreviated and sequential tests are similar, with the sequential test requiring slightly less time and material than the abbreviated test. The sequential test is clearly more efficient in terms of tool usage, requiring only four edges versus eight for the other two tests. However, tool costs should not be a major factor in the total testing cost. These marginal benefits of the sequential test must be weighed against the robustness of the abbreviated test in order to judge which test is superior.

Table 4.7. Comparison of Time and Material Requirements for Each Test

Test	Cutting Time (min.)	Material Consumed (in^3)	Tool Edges Required
Conventional	235	870	8
Abbreviated Taylor	90	450	8
Sequential Composite	78	411	4

CHAPTER FIVE
SUMMARY AND CONCLUSIONS

5.1 Summary

In this research two short-term tool life testing methodologies have been developed, tested, and compared. The results indicate that the abbreviated conventional and sequential composite tests can be used in place of the conventional Taylor test for the development of an extended Taylor equation relating tool life to cutting speed and feed rate. Both tests are conducted under normal cutting conditions and do not require special workpiece preparation or equipment. Additionally, each test offers considerable savings in testing time and the amount of work material required. Both short-term tests offer similar savings, but the abbreviated Taylor test model showed greater predictive robustness in this experiment. Because of this fact, the abbreviated test may be the better choice among the two for actual applications.

The results of such tests are important for manufacturing and industrial engineers who are involved in process planning and operations scheduling. Accurate estimates of tool life are required for scheduling tool changes on machining equipment and for estimating machining costs. The resulting tool life equations can be used to develop contour graphs for selecting combinations of tool life and material removal rate which meet operational requirements. Short, reliable, and inexpensive tool life tests like the ones presented herein reduce the cost of tool life testing and make the procedure more attractive for use by industrial practitioners and researchers.

5.2 Further Research

Many opportunities exist for expanding and improving the testing methodologies described in this research. Beneficial research would include efforts at increasing the robustness of the developed models and reducing the amount of testing required. More specifically, four areas could be addressed:

1. Modification of the models and testing procedures to include depth of cut (d) as an independent variable. Although d has a much smaller effect on tool life than V or f, its effect can be significant at heavy cuts or across wide ranges of d [3].

2. Identification of the point at which interaction and quadratic terms should be included in the model. Miller and DeVor [27] indicate that as the range of cutting parameters increases the higher-order effects become significant. However, no indication is given as to how wide the range (or the cutting parameter magnitudes) must be in order for this to happen. Information is needed in this area so that proper models and experimental designs can be chosen for the situation being tested.

3. For narrower ranges of cutting conditions, smaller experimental designs may be useful for fitting purely linear models. Designs such as the simplex or fractional factorials require fewer test points but can provide the same information for estimating linear coefficients.

4. Modification of the sequential composite design to improve its robustness by reducing the variation associated with the parameter estimates of the flank wear prediction equations. It may be possible to shorten the increment of time during which each cut is taken, thereby providing more sample data for parameter estimation. Linear regression might also be applied to estimating the model coefficients.

Many interesting and challenging research opportunities exist in the area of tool life testing. These suggestions represent only a few of the possible directions for research. As machining technology, materials science, and statistical theory advance even more areas of research will be opened.

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APPENDIX A

REGRESSION PARAMETER TESTS

The general form of the regression parameter tests is:

$$H_0: \ \beta_i = \beta_{i,0}$$
$$H_1: \ \beta_i \neq \beta_{i,0}$$

where β_1 is the value of the regression parameter and $\beta_{1,0}$ is some constant. For these tests, the constant value will be equal to the corresponding parameter value in the reference model. The simple linear test of the slope parameter is a t-test with the statistic

$$t = \frac{b_1 - \beta_{1,0}}{s/\sqrt{S_{xx}}},$$

where b_1 is the estimate of parameter β_1, s is the model standard error, and S_{XX} is the sum of the squared deviation of the x_i's from the average. The value of the t-statistic is compared with the value of the T-distribution at n-p degrees of freedom and the desired two-tailed significance level (p is the number of independent variables). The intercept test is conducted in the same manner with the statistic

$$t = \frac{b_0 - \beta_{0,0}}{s\sqrt{(1/n) + \left(\bar{x}^2 / S_{xx}\right)}}.$$

For multi-variable linear models created using multiple regression, the test statistic for all parameters is

$$t = \frac{b_j - \beta_{j,0}}{s\sqrt{c_{jj}}},$$

where cjj is the jth diagonal element of $(\mathbf{X}'\mathbf{X})^{-1}$, the matrix of variances and covariances from the regression calculations. Using these equations and choosing a level of significance for each statistic, all model parameters can be tested to determine whether they are statistically different than the reference model parameters.

APPENDIX 2
MINITAB PRINTOUTS

Conventional Taylor Model Regression

The regression equation is
loglife = 2.93 - 0.734 speed - 0.0981 feed

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	2.93328	0.02675	109.67	0.000	
speed	-0.73404	0.02675	-27.44	0.000	1.0
feed	-0.09813	0.02675	-3.67	0.014	1.0

s = 0.07565 R-sq = 99.4% R-sq(adj) = 99.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	4.3875	2.1938	383.29	0.000
Error	5	0.0286	0.0057		
Total	7	4.4162			

SOURCE	DF	SEQ SS
speed	1	4.3105
feed	1	0.0770

Obs.	speed	loglife	Fit	Stdev.Fit	Residual	St.Resid
1	-1.00	3.7305	3.7654	0.0463	-0.0349	-0.58
2	-1.00	3.7038	3.7654	0.0463	-0.0617	-1.03
3	-1.00	3.6000	3.5692	0.0463	0.0309	0.52
4	-1.00	3.6349	3.5692	0.0463	0.0658	1.10
5	1.00	2.3888	2.2974	0.0463	0.0914	1.53
6	1.00	2.3026	2.2974	0.0463	0.0052	0.09
7	1.00	2.0015	2.1011	0.0463	-0.0996	-1.67
8	1.00	2.1041	2.1011	0.0463	0.0030	0.05

Fit	Stdev.Fit	95% C.I.	95% P.I.
3.7654	0.0463	(3.6463, 3.8846)	(3.5373, 3.9935)
3.5692	0.0463	(3.4501, 3.6883)	(3.3411, 3.7973)
2.9333	0.0267	(2.8645, 3.0021)	(2.7270, 3.1396)
2.2974	0.0463	(2.1782, 2.4165)	(2.0693, 2.5255)
2.1011	0.0463	(1.9820, 2.2202)	(1.8730, 2.3292)

Lack of fit test
Possible interactions with variable speed (P = 0.051)
Overall lack of fit test is significant at P = 0.051

Pure error test - F = 7.51 P = 0.0519 DF(pure error) = 4

Estimated Effects and Fit for Conventional Taylor Model

Term	Effect	Coef	Std Coef	t-value	P
Constant		2.9521	0.02528	116.79	0.000
Speed	-1.4681	-0.7340	0.02826	-25.98	0.000
Feed	-0.1963	-0.0981	0.02826	-3.47	0.010

Analysis of Variance for LogLife

Source	DF	Seq SS	Adj SS	Adj MS	F	P
Main Effects	2	4.38753	4.38753	2.19376	343.38	0.000
Residual Error	7	0.04472	0.04472	0.00639		
Curvature	1	0.01412	0.01412	0.01412	2.77	0.147
Lack of Fit	1	0.01867	0.01867	0.01867	7.82	0.038
Pure Error	5	0.01193	0.01193	0.00239		
Total	9	4.43225				

Abbreviated Taylor Model Regression

The regression equation is
loglife = 2.92 - 0.699 speed - 0.0649 feed

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	2.91816	0.01981	147.33	0.004	
speed	-0.69928	0.01981	-35.30	0.018	1.0
feed	-0.06490	0.01981	-3.28	0.189	1.0

s = 0.03961   R-sq = 99.9%   R-sq(adj) = 99.8%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	1.97283	0.98642	628.58	0.028
Error	1	0.00157	0.00157		
Total	3	1.97440			

SOURCE	DF	SEQ SS
speed	1	1.95598
feed	1	0.01685

Obs.	speed	loglife	Fit	Stdev.Fit	Residual	St.Resid
1	-1.00	3.6625	3.6823	0.0343	-0.0198	-1.00
2	-1.00	3.5723	3.5525	0.0343	0.0198	1.00
3	1.00	2.3036	2.2838	0.0343	0.0198	1.00
4	1.00	2.1342	2.1540	0.0343	-0.0198	-1.00

Fit	Stdev.Fit	95% C.I.	95% P.I.
3.6823	0.0343	(3.2464, 4.1183)	(3.0165, 4.3482)
3.5525	0.0343	(3.1166, 3.9884)	(2.8867, 4.2184)
2.2838	0.0343	(1.8479, 2.7197)	(1.6179, 2.9496)
2.1540	0.0343	(1.7181, 2.5899)	(1.4881, 2.8198)

Sequential Composite Taylor Model Regression

The regression equation is
loglife = 2.95 - 0.647 speed - 0.129 feed

Predictor	Coef	Stdev	t-ratio	p	VIF
Constant	2.95423	0.03984	74.15	0.009	
speed	-0.64693	0.03984	-16.24	0.039	1.0
feed	-0.12856	0.03984	-3.23	0.191	1.0

s = 0.07969   R-sq = 99.6%   R-sq(adj) = 98.9%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	2	1.74020	0.87010	137.03	0.060
Error	1	0.00635	0.00635		
Total	3	1.74655			

SOURCE	DF	SEQ SS
speed	1	1.67409
feed	1	0.06611

Fit	Stdev.Fit	95% C.I.	95% P.I.
3.7297	0.0690	(2.8529, 4.6066)	(2.3903, 5.0691)
3.4726	0.0690	(2.5958, 4.3494)	(2.1332, 4.8120)
2.4359	0.0690	(1.5590, 3.3127)	(1.0965, 3.7753)
2.1787	0.0690	(1.3019, 3.0556)	(0.8393, 3.5181)

Flank Wear Vs. Time for V=340, f=0.009, 5 and 15 minutes

The regression equation is
FW = 0.00338 +0.000555 Time

Predictor	Coef	Stdev	t-ratio	p
Constant	0.003375	0.001433	2.36	0.143
Time	0.0005550	0.0001282	4.33	0.049

s = 0.001282 R-sq = 90.4% R-sq(adj) = 85.5%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.000030803	0.000030803	18.75	0.049
Error	2	0.000003285	0.000001643		
Total	3	0.000034088			

Obs.	Time	FW	Fit	Stdev.Fit	Residual	St.Resid
1	5.0	0.005700	0.006150	0.000906	-0.000450	-0.50
2	15.0	0.012900	0.011700	0.000906	0.001200	1.32
3	5.0	0.006600	0.006150	0.000906	0.000450	0.50
4	15.0	0.010500	0.011700	0.000906	-0.001200	-1.32

Fit	Stdev.Fit	95% C.I.	95% P.I.	
0.025658	0.003917	(8.8E-03,4.3E-02)	(7.9E-03,4.3E-02)	XX

X denotes a row with X values away from the center
XX denotes a row with very extreme X values

Flank Wear Vs. Time for V=340, f=0.016, 5 and 15 minutes

The regression equation is
FW = 0.00417 +0.000585 Time

Predictor	Coef	Stdev	t-ratio	p
Constant	0.0041750	0.0009371	4.46	0.047
Time	0.00058500	0.00008382	6.98	0.020

s = 0.0008382   R-sq = 96.1%   R-sq(adj) = 94.1%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.000034222	0.000034222	48.72	0.020
Error	2	0.000001405	0.000000702		
Total	3	0.000035627			

Obs.	Time	FW	Fit	Stdev.Fit	Residual	St.Resid
1	5.0	0.006300	0.007100	0.000593	-0.000800	-1.35
2	15.0	0.012700	0.012950	0.000593	-0.000250	-0.42
3	5.0	0.007900	0.007100	0.000593	0.000800	1.35
4	15.0	0.013200	0.012950	0.000593	0.000250	0.42

Fit	Stdev.Fit	95% C.I.	95% P.I.	
0.025966	0.002322	(1.6E-02,3.6E-02)	(1.5E-02,3.7E-02)	XX

X denotes a row with X values away from the center
XX denotes a row with very extreme X values

Flank Wear Vs. Time for V=570, f=0.009, 5 and 15 minutes

The regression equation is
FW = 0.00708 + 0.00179 Time

Predictor	Coef	Stdev	t-ratio	p
Constant	0.0070750	0.0006824	10.37	0.009
Time	0.0017900	0.0001221	14.66	0.005

s = 0.0006103   R-sq = 99.1%   R-sq(adj) = 98.6%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.000080102	0.000080102	215.04	0.005
Error	2	0.000000745	0.000000373		
Total	3	0.000080847			

Obs.	Time	FW	Fit	Stdev.Fit	Residual	St.Resid
1	2.50	0.011900	0.011550	0.000432	0.000350	0.81
2	7.50	0.021000	0.020500	0.000432	0.000500	1.16
3	2.50	0.011200	0.011550	0.000432	-0.000350	-0.81
4	7.50	0.020000	0.020500	0.000432	-0.000500	-1.16

Fit	Stdev.Fit	95% C.I.	95% P.I.	
0.025780	0.000732	(2.3E-02,2.9E-02)	(2.2E-02,3.0E-02)	X

X denotes a row with X values away from the center

Flank Wear Vs. Time for V=570, f=0.016, 5 and 15 minutes

The regression equation is
FW = 0.00810 + 0.00200 Time

Predictor	Coef	Stdev	t-ratio	p
Constant	0.008100	0.001275	6.35	0.024
Time	0.0020000	0.0003225	6.20	0.025

s = 0.0008062   R-sq = 95.1%   R-sq(adj) = 92.6%

Analysis of Variance

SOURCE	DF	SS	MS	F	p
Regression	1	0.000025000	0.000025000	38.46	0.025
Error	2	0.000001300	0.000000650		
Total	3	0.000026300			

Obs.	Time	FW	Fit	Stdev.Fit	Residual	St.Resid
1	2.50	0.013800	0.013100	0.000570	0.000700	1.23
2	5.00	0.018500	0.018100	0.000570	0.000400	0.70
3	2.50	0.012400	0.013100	0.000570	-0.000700	-1.23
4	5.00	0.017700	0.018100	0.000570	-0.000400	-0.70

Fit	Stdev.Fit	95% C.I.	95% P.I.	
0.023700	0.001367	(1.8E-02,3.0E-02)	(1.7E-02,3.1E-02)	XX

X denotes a row with X values away from the center
XX denotes a row with very extreme X values