INVESTIGATION OF SUBSONIC BOUNDARY LAYER EFFECTS ON SUPERSONIC-TYPE AIRFOIL SECTIONS

by

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II. SYMBOLS

a	=	Velocity of Sound
c	=	Chord Length
C_p	=	Coefficient of Pressure = $p-p_o/q$
C_{pinc}	=	Coefficient of Pressure for Incompressible Case
E	=	Bulk Modulus = $dp/\frac{dv}{v}$
M	=	Mach Number = V/a
M_o	=	Free Stream Value of M
P	=	Pressure
P_o	=	Free Stream Value of P
q	=	Dynamic Pressure = $[1/2]\rho V^2$
R	=	Reynolds Number = $VL\rho/\mu$
t	=	Thickness of Airfoil
V	=	Velocity of Body
x	=	Linear Distance
γ	=	Ratio of Specific Heats
δ	=	Wedge Half-angle
μ	=	Coefficient of Viscosity
ϵ	=	Function Symbol
ρ	=	Density
ϕ	=	Function Symbol

III. INTRODUCTION

Before the advent of World War II, the problem of a body moving through a compressible fluid was essentially academic. At that time there was no reason for considering the problem from a practical viewpoint since it did not appear that the speed of aircraft would approach the point at which it would become necessary to consider the effects of compressibility. However, with the acceleration of scientific research which always accompanies a war, it became obvious that faster and more powerful aircraft were needed. More especially, when the Germans produced the V-1 and V-2 missiles which attained extremely high velocities, the need for even faster protective devices necessitated the design of aircraft and missiles with which to counter these weapons. Immediately, the designers found themselves in the position of having to design an instrument of flight which was not only in the subsonic compressible range but which might even transcend the speed of sound. The immediacy of the problem found the world lacking in theoretical knowledge of the subject. The fortunate end of the war relieved the necessity of having to produce aircraft which would have been intuitively designed since the aircraft engineer had very little theory with which to work.

The pressing need for a workable theory which would apply in the compressibility range of velocities has brought together a collection of approximate and exact theories. To the mathematician, the problem is one in which the type of partial differential equation of motion changes from elliptical, in the subsonic case, to hyperbolic, in the supersonic case, and ceases to have any solution at Mach Number of unity.

Although the subsonic flow (elliptic type) has no general exact solution, fortunately, the supersonic (hyperbolic type) does. That is, the fundamental equations of motion in the subsonic case are of such a non-linear character that no mathematical tools exist whereby a general exact solution may be extracted. As pointed out, there have been developed several approximate theories which have given some insight as to the nature of the physical phenomena that exist at high subsonic speeds.

There are three of these theories which are in common use and are of varying degrees of accuracy. The first is the Prandtl-Glauert rule which is a very good first approximation. The second, which is more accurate than the Prandtl-Glauert relation, is that developed by von Karman and Tsien (10). Recently, there has been developed a more accurate approximation by Truitt (16), and verified by Laitone (8).

Several investigations have been carried out recently (1, 3, 9) to verify the exactness of these theories at high subsonic and supersonic speeds. While the results are, in general, confirmative, there are several discrepancies between the theoretical pressure distributions and the actual results. Since the very nature of high-speed testing equipment necessitates using test conditions which result in very low Reynolds Numbers, the differences were thought to be due to a viscous phenomena rather than compressibility phenomena. This investigation was initiated in order to determine whether these phenomena were due to viscous effects (low Reynolds Number), completely below the range of compressibility, or to compressibility (Mach Number).

IV. REVIEW OF LITERATURE

Bryson, Arthur Earl, Jr., "An Experimental Investigation of Transonic Flow Past Two-dimensional Wedge and Circular-arc Section Using a Mach-Zehnder Interferometer", NACA TN 2560, November, 1951.

Interferometer measurements for the flow fields near two-dimensional wedge and circular-arc sections at zero angle of attack at high subsonic and low supersonic velocities are discussed. Pressure distributions as a function of the Mach Number are given and compared with the theoretical work of Guderley and Yoshihara, Vencenti and Wagoner, and Cole.

The experimental data gives evidence of an added overpressure which is not predicted by the theory nor truly accounted for by the Author. This phenomenon gives rise to a shift in the point $C_p = 0$.

(This discovery led to the present investigation on the assumption that this shift was due to boundary layer effect. It was discovered, however, that the boundary layer did not cause the phenomenon evident in the data but some additive effect due to Mach Number was deduced to be the cause.)

Griffith, Wayland, "Shock-Tube Studies of Transonic Flow over Wedge Profiles," Journal of the Aeronautical Sciences, Vol. 19, No. 4, April, 1952, pp. 249-257.

Steady flow fields around two-dimensional wedges for Mach Numbers between 0.85 and 1.80 are discussed. It is found that the pressure distributions over a wedge are identical in form over a large range of Mach Numbers. Data for the computation of drag throughout the entire transonic range is presented. The results are in excellent agreement with theoretical predictions within the limitations provided by such theories.

The same increase in overpressure and shift in $C_p = 0$ point is noted in this more recent report. Again, there is no explanation offered by the Author and no basis found for such a contradictory effect.

This report, while extending the range of the Bryson investigation, is much closer to discovery of the actual flow phenomena at a Mach Number of unity as far as published information is concerned.

Guduley, G. and Yoshihara, H., "The Flow Over a Wedge Profile at Mach Number 1," Journal of the Aeronautical Sciences, Vol. 17, No. 11, November, 1950, pp. 723-735.

The flow over a wedge-shaped profile at zero angle of attack and with a free-stream Mach Number of unity is computed using the hodograph method as simplified by the transonic law of similarity.

This theoretical analysis of the flow at the sonic speed serves to carry the Aerodynamic information through a range where testing is virtually impossible.

Stack, John; Lindsey, W. F.; and Littell, Robert E., "The Compressibility Burble and the Effect of Compressibility on Pressures and Forces Acting on an Airfoil," NACA TR 646, 1938.

The data discussed include the results of pressure distribution measurements and force tests for three low angles of attack for a speed range extending from one-tenth the speed of sound to speeds in excess of the critical values at which a breakdown of the flow, or compressibility burble, occurs.

(It was this report which was used to confirm that the $C_p = 0$ shift occurred for bodies of all shapes and expanded the field of investigation beyond that reported in the preceding papers.)

V. THEORY

The effect of compressibility on the pressure distribution of any airfoil has become a problem to which the aircraft industry is inexorably tied. It has long been known that compressibility increases the amount of overpressure existent on the surface of the airfoil, and since the drag is directly proportional to the overpressure, it is obvious that a costlier expenditure of energy is necessary to propel the wing. With the advent of near-sonic aircraft, the problem of predicting the pressures has become acute and several theories and approximations have been put forth which attempt to calculate this effect. A brief outline of these theories will be submitted here for comparison with the data obtained, and no attempt will be made to follow their development or limitation except as they relate to the investigation in this paper.

First, it may be shown that the pressure coefficient in subsonic, incompressible, inviscid, potential flow about a two-dimensional wedge is given by (1, 7):

$$C_{p_{inc}} = \frac{-2\delta}{\pi} \ln \left[\frac{x/c}{1-x/c} \right] \tag{1}$$

where δ is the angle of flow deviation and x/c is the percent chord. It will be noted that if this equation is solved for $C_p = 0$, then the pressure coefficient must be <u>always</u> zero at $x/c = 0.50$.

From the same considerations, the pressure coefficient for subsonic, incompressible, inviscid, potential flow about a two-dimensional, bi-convex section is given by (1, 7):

$$C_{p_{inc}} = \frac{-4}{\pi} \frac{t}{c} \left[(1-x/c) \ln \frac{x/c}{1-x/c} + 1 \right] \tag{2}$$

Solution of this equation for $C_p = 0$ shows that this condition <u>must always prevail</u> at $x/c = 0.22$.

A. <u>Prandtl-Glauert Rule</u>

By considering the small perturbation theory, i.e. that the disturbances generated by an airfoil moving at high speeds are very small, Prandtl and Glauert simultaneously arrived at a solution of the differential equations of motion. By use of a linear transformation of a known incompressible function, Prandtl and Glauert were able to show that the pressure coefficient for the compressible, inviscid, potential case could be written in an approximation as:

$$\frac{C_{p_{inc}}}{\sqrt{1 - M_o^2}}, \qquad M_o < 1 \tag{3}$$

where $C_{p_{inc}}$ is the pressure distribution obtained for incompressible flow (10).

This equation states that the coefficient of pressure (both positive and negative) at a given point on a body at a free stream $M = M_o$ is larger by the factor $\frac{1}{\sqrt{1 - M_o^2}}$. It will be noted that while the pressure coefficient is increased, the theory will not admit to a positive increase in the pressure "envelope"; more expressly, <u>there is no translation of the point $C_p = 0$ along the body.</u> This theory, then, does not predict the <u>added</u> growth in the overpressure region which is evident from the experimental data obtained in the subsonic, compressible range.

B. <u>Karman-Tsien</u>

The transformation of the equation of motion into the hodograph plane linearized the non-linear equation and from an idea advanced by von Karman (21), H. S. Tsien (20) was able to find a solution applicable to modern airfoil theory. The results of these investigations indicated that, as a better approximation than the Prandtl-Glauert rule, the pressure coefficient could be expressed as:

$$C_p = \frac{C_{pinc}}{\sqrt{1 - M_o^2} + \frac{M_o^2 \, C_{pinc}}{(\sqrt{1 - M_o^2} + 1)^2}} \tag{4}$$

This expression for small values of C_{pinc} is identical with the Prandtl-Glauert rule (10).

While this equation gives better agreement with actual experience, it is again evident that for a given point on a body, the compressible pressure is merely an expansion of the incompressible distribution. The theory, also, will not allow translation along the body of the point $C_p = 0$.

C. Truitt Sound-Space Theory

More recently, Truitt (16, 17, 18, 19), by use of a Lorentz-type transformation of time and space variables, was able to show that the pressure coefficient could be written as:

$$C_p = \frac{C_{pinc}}{\left[\frac{\gamma + 1}{\gamma - 1} - \left(M_o^2 + \frac{2}{\gamma - 1}\right)\left(1 + \frac{\gamma M_o^2}{2} C_{pinc}\right)^{-(\gamma-1/\gamma)}\right]^{1/2}} \tag{5}$$

This equation, more accurate than the two preceding ones, for a third time is merely an expansion of the incompressible pressure. And again, this theory will allow no translation of the $C_p = 0$ point along the body.

All of these theories, in everyday use, are applied in the prediction of the pressure distribution on high-speed airfoils, but each fails to predict the added growth in the overpressure region and consequently the shift in the $C_p = 0$ point.

D. Ackeret or Linear Theory

For the pure supersonic case, the linearized Ackeret Theory (11) is evidently very good and immediately gives the pressure distribution over the surface at $M_o > 1$. Actually, this theory assumes that the shock must be attached which condition never prevails at $M_o = 1$. The shock attachment is important in that the flow behind a shock wave is not supersonic unless the wave is attached to the body. The expression for the pressure coefficient as derived by Ackeret is:

$$C_p = \frac{2\delta}{\sqrt{M_o^2 - 1}} \qquad M_o > 1 \tag{6}$$

Since this equation represents a complete overpressure region for a wedge at zero angle of attack, there is no difficulty involved in its use except the determination of the point at which the shock becomes attached. If the shock is not attached, $M_o < 1$ and equation (6) has no meaning.

VI. APPARATUS

A. Wind Tunnel

This investigation was conducted in the Virginia Polytechnic Institute Wind-Tunnel. The tunnel is a single-return, open throat type with a 3-foot diameter working section. A detailed explanation of the tunnel is given in reference (2).

B. Airfoils

Two airfoils were made by the Virginia Polytechnic Institute Industrial Engineering Shops as directed by Figures (1) and (2).

One feature worth noting is that the circular-arc profile was determined from the restriction that the arc must become tangent to the straight portion at a point two inches from the nose. This restriction insured the correct radius of curvature and agreed with the theoretical assumption for the body shape.

The location of the orifices was determined by pure mechanical considerations. The first orifice was located as close to the nose as practicable and was then used as a base from which to lay off all other tap measurements.

One orifice was drilled into the bottom face of each model geometrically opposing the last one in the upper surface in order to facilitate maintainance of zero angle of attack.

Each model was fitted with two, 6 x 3 inch, sheet aluminum end plates to produce two dimensionality.

The models were mounted horizontally in the test section by supporting them between two, 2 foot lengths of 1/2 inch, streamlined, steel tubing.

Material: 24-ST Aluminum Stock Scale: Full

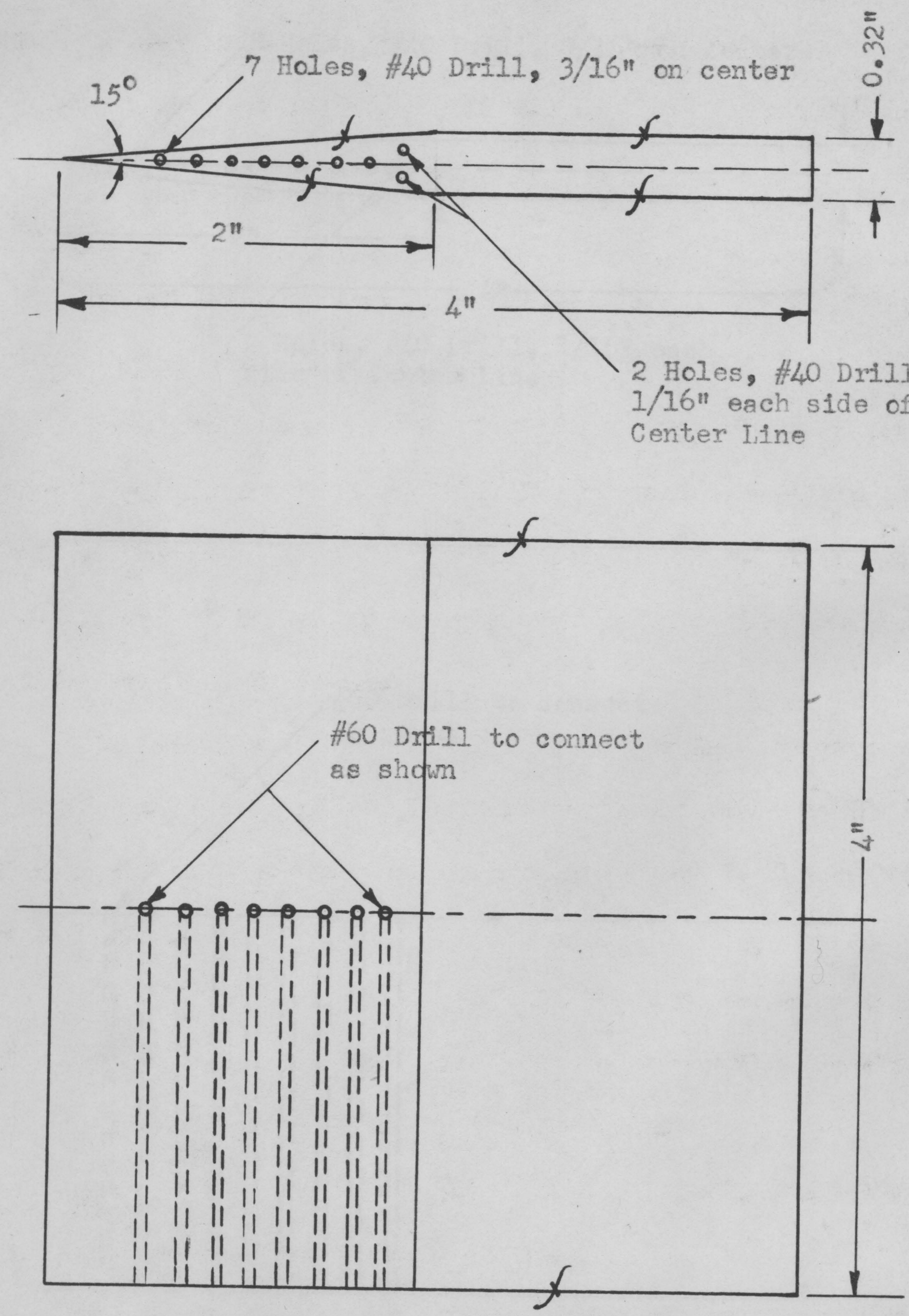
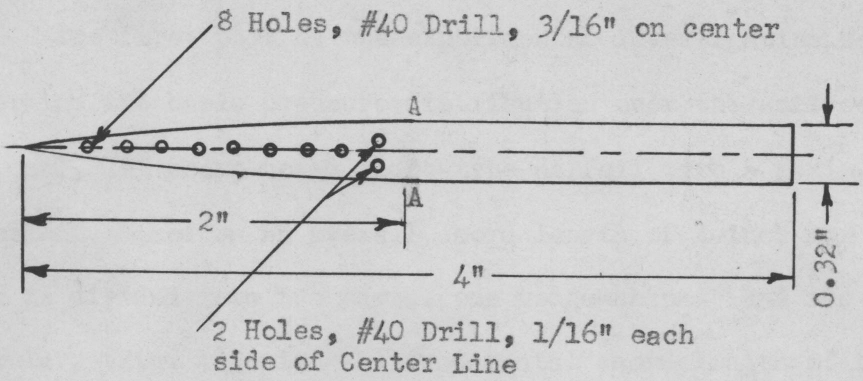


Fig. 1

15° Wedge Airfoil Section

Material: 24-ST Aluminum Stock Scale: Full

NOTE: Arc must be tangent at point A.



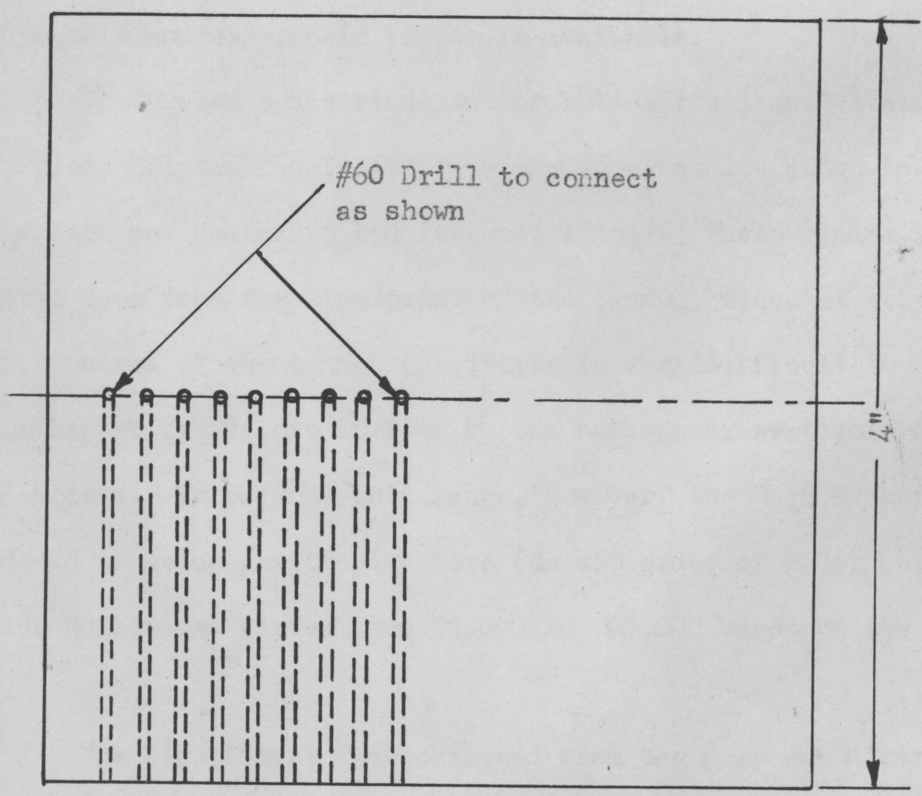


Fig. 2

Circular-Arc Airfoil Section

VII. INVESTIGATION AND DISCUSSION

A. Experimental Investigation of Fundamental Pressure Distribution

1- Two-dimensional 15° wedge Airfoil

The first part of the experimental investigation was undertaken to determine the basic pressure distribution over the surface of a two-dimensional, 15° sharp-nosed wedge-type airfoil with a maximum thickness of 8 percent, based on an overall chord length of 4-inches. The airfoil section is divided into two parts, one wedge-shaped and the remainder rectangular, (Fig. 1), with the fundamental chord length of the wedge section 2-inches and that of the rectangular section 2-inches. This particular configuration was chosen because it is one for which data in the transonic and supersonic ranges is available.

The pressure distribution for this airfoil was obtained in the Virginia Polytechnic Institute Wind-Tunnel at low subsonic speeds from 20 feet per second to 150 feet per second. These limits are practical ones from the standpoint of the tunnel, since at very low speeds, control of the tunnel velocities is very difficult to maintain and the higher speeds are limited by the horsepower available from the tunnel motors. Throughout this range, however, the Mach Number may be considered as being practically zero (on the order of 0.05) while the Reynolds Number was varied from 30,000 to 90,000 based on the 4-inch chord.

The significant fact obtained from the pressure distribution (Fig. 3) in this low-speed range, where viscous phenomena predominates, is that the shape of the pressure "envelope" (as evidenced by the

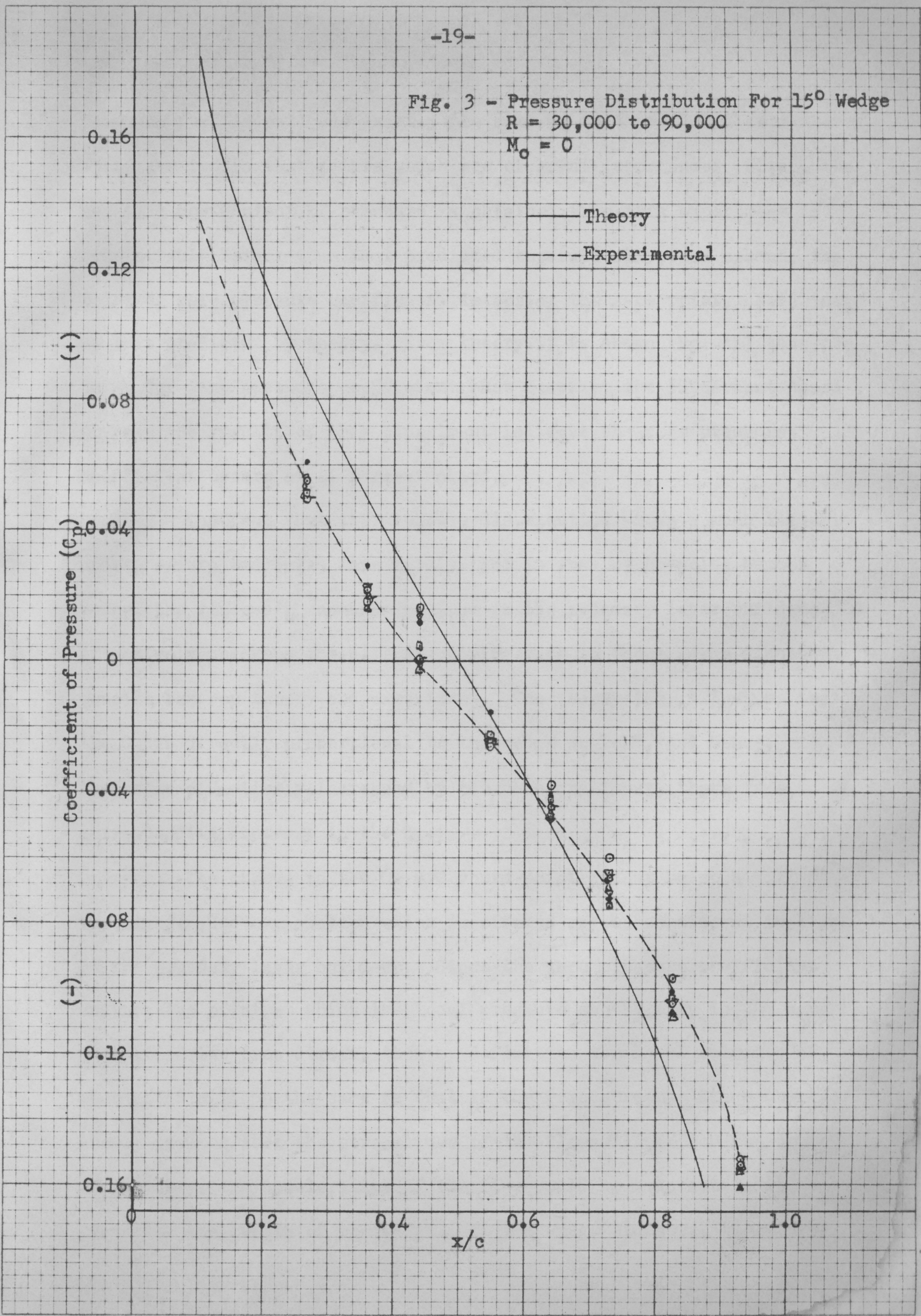


Fig. 3 - Pressure Distribution For 15° Wedge
R = 30,000 to 90,000
$M_o = 0$

pressure coefficient) was exactly the same as that predicted from theory. There are two further points which are of great importance: first, the negative coefficient of pressure as obtained from this experiment is considerably larger than that predicted theoretically, and second, the point of zero pressure coefficient is not at the same point, chordwise, as that predicted by theory. From theory it is shown (Pg. 11) that the point for zero pressure coefficient must occur at the 50 percent chord whereas the results of this experiment indicate the zero pressure coefficient occurs at 44 percent chord.

The fact that the pressure distribution shows a much larger underpressure indicates immediately that in the low Reynolds Number range, the viscous phenomena manifest in the boundary layer build-up effectively thickens the airfoil, giving rise to these large discrepancies between theory and experiment. The theory (Pg. 11) shows that the C_p is directly proportional to the wedge angle δ, and hence, with the angle effectively increased by the boundary layer, there is a corresponding increase in the C_p.

The change in the $C_p = 0$ chord point cannot be so easily explained. One explanation which is offered may be considered in light of the boundary layer build-up. Since the zero pressure coefficient defines the point at which the local pressure on the body is the same as that of the free stream, then by definition the local velocity at that point will be the same as that in the free stream. Since the theory assumes that the streamlines of the flow will follow the contour of the body, the acceleration of the flow would be constant as it traversed the length of the wedge. At the wedge midchord, therefore, the velocity would be exactly the same as that of the free stream. However, it is known that the boundary layer thickness is of the form:

$$\delta = 5.20\ (R)^{-1/2}\ (x) \tag{7}$$

where R is the free stream Reynolds Number and x is the linear distance along a flat plate at zero angle of attack. It seems reasonable to assume, then, that over an inclined flat plate (one face of the wedge) at a very small angle of inclination, the boundary layer thickness would be very nearly the same, or, sensibly, would be:

$$\delta = K\ f(R)^{y}\ x \tag{8}$$

where K is a constant. Assuming for the sake of argument that the boundary layer thickness may be expressed as:

$$\delta = 5.20\ (R)^{-1/2}\ x \tag{9}$$

then the thickness may be seen to increase parabolically along the plate. As a consequence, the airflow must be accelerated, not constantly, but parabolically, and would reach free stream velocity at a point on the airfoil ahead of that based on non-viscous flow consideration. Further, as the Reynolds Number is increased, then, the boundary layer would decrease in thickness and the $C_p = 0$ point would approach asymptotically the point ($x/c = 0.5$) predicted by the theory.

It is shown by Liepmann and Bryson (9) that the thickness of the boundary layer is $\delta = 0.007\ (R)^{-1/2}\ c^{1/2}$ where c is the chord length. This value is in good agreement with the results of this investigation and is very nearly the exact relation obtained by the Author. Since the difference was negligible, the Bryson value was assumed to be correct.

2- Two-dimensional Bi-convex Airfoil

The second part of the experimental investigation was the determination of the basic pressure distribution over the surface of a two-dimensional, sharp-nosed bi-convex type of airfoil 8 percent thick based on a chord length of 4-inches. Again, the section is divided into two parts with the bi-convex section 2-inches long followed by the rectangular section 2-inches long (Fig. 2). As with the wedge, this section was chosen in order to correlate the data obtained with the data already available in the high-speed ranges.

The same phenomena which exists for the wedge appears again for the bi-convex section. While the shape of the pressure "envelope" is, of course, different from that of the wedge, the same under-prediction by the theory is evident. The $C_p = 0$ point occurs experimentally at 10 percent of the chord while the theory predicts that it should fall at 22 percent of the chord, (Fig. 4).

The same arguments that were advanced in the preceding analysis for the wedge could as easily be applied to the bi-convex section. In fact, the visualization of the front part of the bi-convex section, as the superposition of the boundary layer on the wedge, will substantiate the explanation offered concerning the shift forward of the $C_p = 0$ point.

There is no known value for the boundary layer thickness on a curved section and the difficulties involved in testing or computation made such a determination prohibitive.

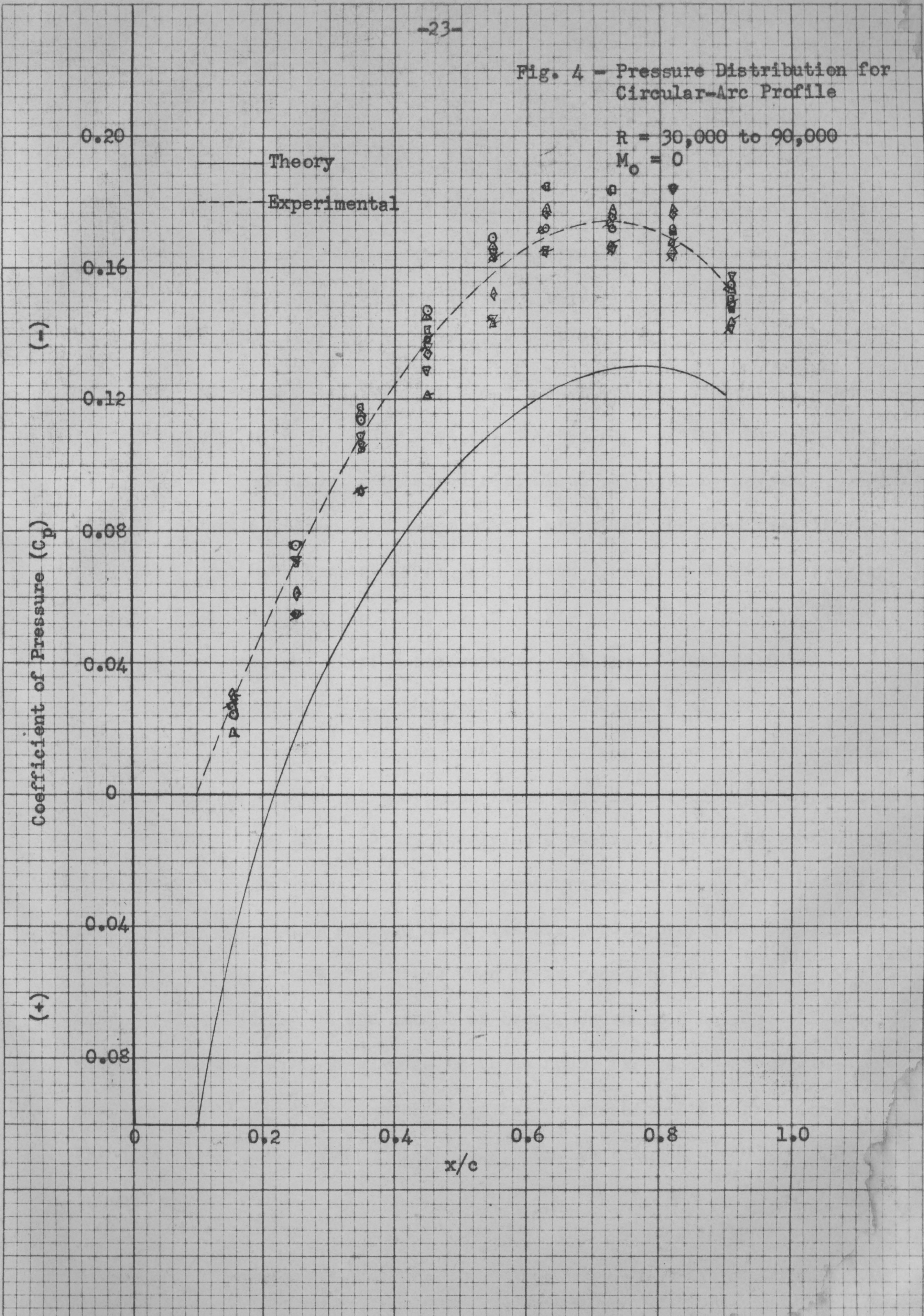


Fig. 4 - Pressure Distribution for Circular-Arc Profile
R = 30,000 to 90,000
M_o = 0
Theory
Experimental
0.20
0.16
0.12
0.08
0.04
0
0.04
0.08
(-)
(+)
Coefficient of Pressure (C_p)
0
0.2
0.4
0.6
0.8
1.0
x/c

B. Comparison of High-Speed Pressure Distributions with the Basic Subsonic Pressure Distribution as to Form.

Bryson (1, 9), in a series of tests on a 15^{o}, 8 percent thick wedge section has amply demonstrated the shape and growth of the pressure distributions. From comparison with the theory and with the experimental results, it is convincingly demonstrated that the pressure distribution will maintain essentially the same shape throughout the high-speed subsonic range $0.5 \leqslant M \leqslant 1.0$ as that established at a virtual Mach Number of zero. The theoretical pressure distribution as given by several different formulae (Pg. 12) shows that the growth of the pressures is a function of the Mach Number. That this growth is such a function is clear from the experiment, and in fact, is in fairly good agreement with theory. There is, however, one glaring discrepancy. At the beginning of the compressibility range $M = 0.5$ the point $C_p = 0$ begins a migratory movement backwards along the body and continues to move aft with ever-increasing Mach Number. It has already been shown that no additional overpressure is predicted by any known theory involving a compressibility correction. This shift then indicates that the overpressure region is grossly underestimated by the theory and the underpressure region overestimated. That this is a serious fault is obvious when it is recalled that the drag is directly proportional to the amount of overpressure on the body. If the pressure distribution for a wedge is to be found from a low-speed, incompressible test and then extended by the compressibility corrections, the drag as estimated will be much lower than that which would actually be encountered.

In a more recent paper, Griffith (2) has extended the range of the Bryson investigations by utilizing a shock tube method of flow generation. These experimental results truly cover the transonic range since the Mach Numbers investigated were from M = 0.909 to M = 1.108. This is the closest to a Mach Number of unity that data has ever been taken, so far as is known to this investigator.

As in the Bryson investigations, the form of the pressure distribution remained the same as that obtained in this paper and on comparison of the Bryson and Griffith reports, does so until the shock wave becomes attached at M = 1.37. The pressure "envelope" form changes rather abruptly at that point and very nearly assumes the pure supersonic distribution which has been well defined from theory and is almost exact.

In the Bryson investigation, the Reynolds Number was held approximately constant at a numerical value of 60,000. From practical consideration, this is an extremely low value since at standard atmospheric conditions, a body moving at M = 1, in free air, could be only 0.05 - feet in length to maintain R = 60,000. The extremely low values are inherent in high-speed wind-tunnel work because of the necessity of using very small models in a low-density, rapidly expanding flow. As a direct consequence of the low Reynolds Number, laminar flow is maintained, resulting in a large boundary layer build-up. The additional thickness due to the boundary layer exhibits the same effect in the transonic and supersonic regions as at the low speeds reported in this paper. Specifically, the overpressure is less than the theoretical prediction. While this would appear to negate some of the overpressure addition due to compressibility, there seems to be no correlation and it must be remembered that a full-size airfoil would be flown at much higher Reynolds Numbers than this.

This same difficulty is encountered by Griffith since his investigation was carried on at R = 100,000 which is still in the laminar flow region.

From comparison, therefore, it is evident that if pressure distributions are to be taken at a low speed for extension by the compressibility corrections to that of high speeds, then extreme care must be taken to insure that the Reynolds Number is high enough to remove the boundary layer effect. If not, the drag may be in error by a very large amount resulting in a disastrous calculation.

C. Projected Considerations

From the investigation reported in this paper and in the Bryson and Griffith reports, it appears that an additional overpressure of the Ackeret or square-type distribution begins to form at M = 0.5. It would seem feasible, then, that a method of superposition could be advantageously employed. That is, the pressure distribution shape remains essentially the same from $M_o = 0$ up to the shock attachment Mach Number and from there changes to the Ackeret-type of distribution. It would appear, therefore, that some distribution of the Ackeret-type which would be a function of the Mach Number and the shape variable, δ, could be added (superimposed) to the original equations to obtain the correct pressure distribution before shock attachment.

In view of the above considerations, the form of such an equation for a wedge would be:

$$C_p = \frac{-2\,\delta}{\pi}\;\frac{\ln}{\sqrt{1 - M^2}}\left[\frac{x/c}{1 - x/c}\right] + \varepsilon\,(M,\,\delta). \tag{10}$$

$\varepsilon(M, \delta)$ is of the Ackeret-type and is a function of the Mach Number and the shape function δ. By an extension of the same arguments, the pressure coefficient for the bi-convex section would be:

$$C_p = \frac{-4}{\pi}\;\frac{t/c}{\sqrt{1 - M_o^2}}\left[(1 - x/c)\,\ln\,\left(\frac{x/c}{1 - x/c}\right) + 1\right] + \varepsilon'(M,\,t/c) \tag{11}$$

where t/c is the fineness ratio for the body.

In order that the form of $\varepsilon(M, \delta)$ may be determined, a procedure of dimensional analysis could be easily followed. For example, it may be assumed that the pressure over a body can be defined by:

$$p = p\,(\rho,\,V,\,c,\,\mu\,,\,E,\,\delta,\,x) \tag{12}$$

where c is the chord length, E, the bulk modulus of elasticity, δ is the effective wedge angle to be defined later, and x any distance along the chord. Then it can be shown that the pressure will be:

$$p = \rho\,V^2 c\;\phi_1\,(M)\;\phi_2\,(R)\;\phi_3\,(\delta)\;\phi_4\,(x/c) \tag{13}$$

and it follows by definition that the pressure coefficient is of the same form. In the usual manner, the pressure distribution and coefficient may be found as a function of each of the variables taken one at a time. By careful analysis one could then determine the exact form of $\varepsilon(M)$.

It should be noted at this point that the Reynolds Number does not appear expressly in the expression for C_p but is indicated in the analysis. It has already been shown that the Reynolds Number is inherent in the shape variable term δ, and becomes obvious upon defining an effective wedge angle δ_e as:

$$\delta_e = \delta_a + \delta_b \tag{14}$$

where δ_a is the actual wedge angle and δ_b is the additional angle due to boundary layer thickness. Therefore, this effective angle could be used throughout in the expression for C_p. Consideration of the circular-arc profile would repeat these results if t/c is replaced by te/c where te is the effective thickness defined as:

$$t_e = t_a + t_b \tag{15}$$

The subscripts refer to the same quantities as equation (14).

By analysis of the very limited data available in the compressible range, this investigator was able to approximate one form of the pressure coefficient as:

$$C_p = \frac{-2\,\delta}{\pi}\,\frac{1}{\sqrt{1 - M^2}}\;\ln\left[\frac{x/c}{1 - x/c}\right] + \frac{2\,\delta}{\sqrt{1 - M^2}}\;\phi\,(M) \tag{16}$$

for the wedge section, where the last term is $\varepsilon(M, \delta)$. The form and inference of equation (16) is obviously valid at and beyond shock attachment since:

$$M_2^{\,2} - 1 = 1 - M_1^{\,2} \tag{17}$$

where M_2 denotes the Mach Number ahead of the normal shock wave and M_1 denotes the Mach Number behind. That is, the first term on the right of equation (16) becomes:

$$\frac{-2\,\delta}{\pi}\ \frac{1}{\sqrt{M^2 - 1}}\ \ln\left[\frac{x/c}{x/c - 1}\right] \tag{18}$$

which shows that it affects the shape of the pressure distribution curve even though its predominance decreases with increasing Mach Number in favor of the Ackeret-type distribution of the second term. The fact that the C_{pinc} term (Equation 18) does not lose its identity as the shape function beyond shock attachment, is readily apparent from actual experimental evidence. In other words, this term, which was shown to be affected by the boundary layer, maintains some control, although small, on the pressure distribution shape with increasing Mach Number. This explanation would account for the contradictory forms obtained by Bryson and Griffith after shock attachment.

VIII. CONCLUSIONS

It was found in this investigation that the testing of supersonic airfoils at subsonic speeds to obtain pressure distributions will result in considerable error if the models are tested at very low Reynolds Numbers. The thickness of the boundary layer is critical at low Reynolds Numbers and causes a decrease in the overpressure region. This, coupled with the fact that the overpressure region increases more with Mach Number than predicted by the theory, could lead to erroneous calculations of the drag.

With the present theories, it is impossible to predict the actual pressure distribution throughout the subsonic to supersonic range. A modification in the basic theory equations using the method of superposition of an Akeret-type pressure distribution, $\varepsilon(M, \delta)$, is suggested. This function was not determined by the Author for lack of high-speed test equipment.

It is further pointed out that, should the necessity arise for testing the models at low Reynolds Number, the equations should be modified to offset the effect of the boundary layer.

It is believed, then, that by using the function $\varepsilon(M, \delta)$ and including any effect due to boundary layer thickness, the present theories could be modified to give results more in keeping with actual experiment. As the matter stands, however, there is a great likelihood that the drag forces would be grossly underestimated, possibly resulting in structural failure at worst and certainly expensive power loss at best.

IX. ACKNOWLEDGMENTS

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