AN INVESTIGATION OF SEVERAL VARIABLES IN THE STRENGTH OF SHELL MOLDS

by

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III. Introduction

One of the more recent developments of technique in the foundry industry has been the development of the shell molding process, which was a process kept secret by the German government until the end of World War II. The patent[13] for this process, being declared in the public domain at the end of World War II, was of such immediate practical value that within four months after patent application had been filed, the process was being used in production.

The process is used for making hollow cores and mold shells with thin and approximately uniform wall thickness, to be used in metal casting as a replacement for the more cumbersome sand molds. The ingredients for making these shell molds in present practice are dry sand or other refractory material, dry phenol-formaldehyde resin, and dry hexamethylene tetramine.

Shell molding, as it is used today, is essentially a process for making small precision castings on a production basis. A typical sequence of operations in making a shell mold would be:

1. Preheat pattern to an appropriate temperature; for example, 500 degrees fahrenheit.
2. Place the hot pattern so that a sand-resin mixture may be dumped on it.
3. Allow the sand-resin mixture to "dwell" on the pattern for a length of time dependent upon pattern temperature and thickness of shell desired.
4. Remove the excess sand-resin mixture.
5. Place the pattern and adhering shell mold in an oven to "cure".

6. Remove the mold and pattern from the curing oven and separate the mold from the pattern
7. The pattern may be immediately re-used for several cycles before reheating is necessary.

Heat is the force which causes the shell mold to form from the sand-resin mixture. The heat supplied by the hot pattern does two things to the two-step resin; it causes the phenol-formaldehyde resin to flow and it breaks the hexamethylene tetramine down into formaldehyde and ammonia. The formaldehyde causes the resin to cross-link, or set; the ammonia is a catalyst for this reaction.

For several years after the shell molding process was adopted in this country the information that could be found in open literature concerning the process was only general. Recently, results of experimentation with the process have become available, and much literature can be found concerning shell molds.

A recent report[12] published was concerned with:

1. Bonding efficiency of various phenolic resins, and effects of variations in amount and size of resin.
2. Effects of variations in grain size, grain size distribution, purity and angularity of sand.
3. Effectiveness of other molding refractories than silica sand in making shell molds.
4. Surface treatment of the sand to improve resin-sand adhesion.
5. Adaptability of vibration packing for preparing shell molds.
6. Performance of various sand-resin mixtures used in making shell molds.

7. Surface characteristics of aluminum and copper base alloys cast against shells of various mixtures of sand and resins.
8. Heat transfer in shell molds and breakdown time of molds (to accommodate contraction of castings).
9. Means to prevent segregation of binders in practice.
10. Mechanism of bonding of sands with resin binders.

In this report several conclusions were made which were of importance to the foundry industry. In particular, one of interest was, "the effect of grain size distribution upon packing density and the strength of a shell is small when making shell molds by the dumping method". To this date, there have been no experiments to check the interaction effects of the variables affecting the strength of shell molds.

This thesis presents experimental work done to show the single effects and combined effects of curing temperature, dwelling time, amount of resin used, average sand grain size, and distribution of grain size on the flexural strength of shell molds; and the correlation between bulk density and flexural strength. The importance of continuing the investigation of the several variables presented in this thesis is that possibly the cost of making shell molds may be reduced by decreasing resin consumption without having subsequent decrease in their strength. The correlation of bulk density and flexural strength provides a means of measuring the strength of shell molds without destructive testing. If such correlations for various shell molding mixtures and treatments were available, the flexural strength of molds made from any mixture could be inferred from a measurement of bulk density.

IV. Review of Literature

In reviewing the literature on shell molding, much information was found concerning the effects of various variables on the properties of shell molds. A few of the later publications were quite comprehensive and gave a good survey of the status of shell molding to date. It was evident, however, that there are several factors or combinations of factors relating to the strength of shell molds which are little understood and in some cases essentially uninvestigated.

The strength of shell molds may be inferred from the results of one of several different tests. The two which have been used most frequently are the tensile test, which uses the "dog-bone" type specimen[11,12] and the flexural test, which requires a flexural test bar[11,12]. It appears, however, that a flexural test more closely simulates stress conditions in a shell mold while casting metal. There is a standard flexural test[2] used for ceramic materials, which can also be used for shell molds because of the similarity between the physical structure of ceramic materials and those of shell molds.

Much work has been done on the strength of shell molds as a function of numerous variables. It appears that the most comprehensive coverage of these variables to date was done by investigations at M.I.T.[12]. They conclude that:

1. In shell molding proper choice of sand and resin is important from the point of view of performance in making shell molds and in production of castings.
2. Resin is wasted in coating the surfaces of sand grains.

3. Vibration packing produces stronger shells.
4. Separation of resin from the surface of sand is the mechanism of fracture of shell molds.
5. Zircon yields an unusually strong shell, apparently due to favorable affinity for resin.
6. Breakdown time of a shell mold is a function of the rate of heat transfer, initial strength, and shell thickness.

According to Sprinkle[15], the strength of shell molds is a function of the work of adhesion for phenol-formaldehyde resin and the metallic oxide in question.

Some disagreement was found between investigators concerning the influence of grain size distribution on density and strength. Anspach[1] mentions that greater grain size distribution yields significantly greater bulk density and strength in green sand molds; the investigators at M.I.T.[12] conclude that the effect of grain size distribution is small when making shell molds by the dumping method. Even though one is considering the effect of grain size distribution on green sand molds and the other, on shell molds, the effect should be generally the same in each case. Dallavalle[8] shows that the bulk density increases with wider grain size particles in an idealized situation.

Braybrook and Waters[4,5] also consider the effect of grain size. They write, "It may be mentioned that none of the work described here or carried out recently by others supports any suggestion that the grain size of a sand alone is a primary factor in determining the amount of resin needed to give a particular strength". They state further that more important factors are sand contamination and resin segregation.

Precoating sand with resin has been mentioned by several authors as a means of minimizing segregation.

In considering appropriate curing temperatures, Braybrook and Waters[4,5] indicate that curing temperature is important. Childs and Hyman[6] have investigated the variation in temperature of shell molds during the dwelling and curing cycles. They show that appreciable burnout of resin on the shell surface is probably at the curing temperatures that are used.

No information on the synergistic effects of variables affecting the strength of shell molds was disclosed in the literature.

V. Plan of Research

A. Construction and Adaptation of Equipment

To investigate the effects of the variables on the flexural strength of shell molds in this work, it was necessary to contruct equipment or adapt existing equipment to make and test the flexural test bar. This included adaptation of a furnace, construction of a table and metal wagon to facilitate the handling of the mold, construction of a flexural testing device, and construction of a mold.

B. Influence of Several Variables on Flexural Strength

It was planned to investigate the effects of dwelling time, curing temperature, sand composition, and amount of resin by using a statistical factorial design for the experiment. In this design two levels of dwelling time, two levels of curing temperature, two levels of resin amount, and four levels of sand composition were planned. The reson that the factorial design was used in this investigation was to show the interaction effects of the variables, if any existed. It was found after the experimental work was started that the temperature of the curing furance interior decreased from the front to the rear of the muffle. This was evidenced by the difference in color of different areas on the test bar. Therefore, to decrease the experimental error another variable, approximate position of the test bar area in the curing furnace, was added to the experimental design.

C. Influence of Grain Size Distribution

It was planned to investigate the effect of grain size distribution on the strength of shell molds by using a separate experiment in which the sand distributions had been prepared artificially. The other variable in this part of the investigation was the effect of the variation of temperature in the curing furnace.

D. Correlation of Density and Flexural Strength

To correlate density and the flexural strength of shell molds, a density measurement was planned for each test bar.

VI. Description of Apparatus and Materials

A. Mixing Apparatus

Mixing of the various mixtures was accomplished by using a standard ball mill with the balls removed to prevent the sand from being crushed. The ball mill container was eight inches in diameter and ten inches in length. Better mixing would probably have resulted had a mulling machine been used for mixing, but the volumes of the mixtures used were too small to be used in the mulling machines which were available. One attempt to get better mixing in the ball mill was the use of baffles. However, it was found that a greater amount of segregation resulted from the use of baffles than without them.

B. Flexural Test Bar Mold

Figure 1 illustrates the flexural test bar mold, which is similar to one used to make flexural test specimens in a previous investigation. It was designed to make a specimen 13 inches long, 1 inch wide, and .25 inch thick. Machined from .25 inch steel plate, it consisted of three pieces, which were held in position by a clamp at either end. The outside dimensions of the mold were 2" x 1/2" x 14" when it is assembled. To prevent the test bars from sticking to the mold a thin coating of Dow silicone grease was used as a parting agent.

C. Furnace, Wagon, and Table

For all heating done in this investigation a Hoskins muffle type furnace, having a temperature range up to 2300°F, was used. It has a muffle measuring 15" x 7.5" x 5.5", and its temperature was controlled with a Leeds and Northrup recording pyrometer.

The principal difficulty encountered with the furnace was the fluctuation in temperature of ± 15°F. In view of the total heat content of the mold, this variation in temperature probably contributed little to the experimental error. Another difficulty was the variation in temperature inside the furnace, but this error was removed by accounting for it in the design of the experiment. A small wagon was built to facilitate handling the mold. A metal table was also constructed and placed on a level even with the furnace door so that the wagon containing the mold could be rolled in and out of the furnace easily.

D. Flexural Testing Device

A device which was designed and constructed for making flexural tests on the test bars was quite similar to the standard flexural testing device for ceramic materials[2]. This device is shown in Figure 2. It consisted of a steel plate mounted on four legs. On top of the steel plate were two steel blocks whose distance apart could be adjusted by removing the two bolts in each and moving the blocks to the desired holes in the plate. The blocks were then secured by inserting the bolts through the holes in the blocks and plate. A piece of tubing, which was two inches long and one-half inch in outside diameter, was used as the bearing surface. The bucket for loading was connected to the tubing by a chain which passed through holes in the steel plate. In actual testing, the test bar was placed on the steel blocks with the tubing or bearing surface on top of the bar and in the center between the two blocks. It was then loaded with lead shot until the bar failed.

E. Silica Sand

The sand used in this investigation was round silica sand obtained from Wedron Sand Company in Wedron, Illinois. It contained at least 99.9%

silica and less than 0.02% iron. The sand from Wedron was separated into fractions, using a Ro-Tap Sieve Shaker, according to screen sizes which are shown in Tables 2 and 3. All sand mixtures were than compounded from these fractions. All sand sizes presented in this investigation are expressed in microns because such measurements of grain size are seemingly easier to interpret and use than the familiar AFS Grain Fineness number. For purposes of comparison, the approximate AFS grain fineness numbers are shown in Tables 2 and 3. In order to find the average grain size of a sand, the following procedure was used:

1. Average the size of openings of the last screen through which the sand passed and the screen on which the sand was retained.
2. Complete step 1 for all screens in the series which are used.
3. Weight the sand on each screen and find the fractional portion of the total sample retained on each.
4. Multiply the decimal fractions found in step 3 by the average size retained on each screen and add the products to obtain the overall average grain size.

F. Shell Molding Resin

The resin used in this investigation was a commercial shell molding resin, "Durex" 736 Resin Compound, Serial 6825, from the Monsanto Chemical Company. This is a two-step resin, the chemistry of which was discussed in the Introduction.

VII. The Experimental Design

A. Design to Determine the Effects of Several Variables

The statistical factorial design shown in Table 1 was used for determination of the effects of dwelling time, curing temperature, amount of resin used, and sand composition on the flexural strength of shell molds.

Table 1. Statistical Factorial Design of One Replication

Curing Temp., °F		T_1 (500)						T_2 (600)					
Dwelling Time, Sec.		t_1 (20)			t_2 (40)			t_1 (20)			t_2 (40)		
Furnace Position		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
Sand Composition	Resin Amount	Specimen Number											
C_1	R_1 (6%)	1			2			5			6		
	R_2 (4%)	3			4			7			9		
C_2	R_1 (6%)	9			10			13			14		
	R_2 (4%)	11			12			15			16		
C_3	R_1 (6%)	17			18			21			22		
	R_2 (4%)	19			20			23			24		
C_4	R_1 (6%)	25			26			29			30		
	R_2 (4%)	27			28			31			32		

Table 2 is a tabulation of sand composition. The sign, (-), indicates the screen through which the sand passed, while the sign, (+), indicates the screen on which the sand was retained. D_1, D_2, and D_3

represent respectively that part of the specimen which was located in the middle of the mold and hence in the middle of the muffle furnace during curing, that part of the specimen which was located near the front or opening of the furnace, and that part of the specimen located near the rear of the furnace.

Table 2. Sand Compositions for the Factorial Design

U.S. Sieve Number	Average Size Microns	Sand Composition C_1	C_2	C_3	C_4
		Per Cent Used			
- 20 + 30					
- 30 + 40	505	50	50		
- 40 + 50	359				50
- 50 + 70	254				
- 70 + 100	180			100	
-100 + 140	127		50		50
-140 + 200	90	50			
-200 + Pan	26				
Average Grain Size in Microns		298	316	180	243
Approximate AFS Grain Fineness Number		50	50	100	70

Three replications were made for each possible combination of the variables shown in Table 1. One density and three flexural strength measurements were taken on each of the 32 specimens in each replication.

B. Design for Determination of Effects of Grain Size Distribution

To determine the effect of grain size distribution on the strength of shell molds, five different distributions were used, all having

approximately the same average grain size. These different distributions were obtained by plotting the available grain sizes on probability paper and drawing straight distribution lines through the chosen average grain size, which is located on the 50% probability line.[9] This plot is shown in Figure 3. Probability paper was used because commerical sand distributions plotted on this paper results in a nearly linear curve, a curve quite similar to that obtained by plotting on log-log graph paper. Also, it is most difficult to find the average grain size on log-log graph paper. The composition of each distribution was easily read from the distribution line. Each of these compositions is shown in Table 3.

For all of these specimens the curing temperature was 500°F., the dwelling time was 20 seconds, and 5% resin was used. For this part of the investigation, two replications were used.

Table 3. Compositions of the Sand Distributions.

U.S. Sieve Number		Average Size Microns	Distribution Number 1		2		3		4		5	
			C%F	%U	C%F	%U	C%F	%U	C%F	%U	C%F	%U
-20	+30	715	100	1								
-30	+40	505	99	9	100	2						
-40	+50	359	90	21	98	19	100	7				
-50	+70	254	69	19	79	29	93	43	100	50		
-70	+100	180	50	14	50	20	50	34	50	49	100	100
-100	+140	127	36	10	30	15	16	15	1	1		
-140	+200	90	26	12	15	10	1	0.9				
-200	+Pan	26	14	14	5	5	0.1	0.1				
Average Grain Size in Microns			229		217		226		217		180	
Approximate AFS Grain Fineness Number			70		70		70		70		70	

C%F - Cumulative Per Cent Finer
%U - Per Cent Used

VIII. Experimental Procedure

A. Weighing and Mixing

For each replication of the experiment, individual samples weighing 3000 grams each, of appropriate sand and resin mixtures were made. The compositions of the mixtures used are given in Tables 2 and 3.

These mixtures were each mixed one-half hour in the ball mill which yielded nearly homogeneous samples. Visual inspection indicated that longer mixing periods did not improve the homogenuity. After mixing, the samples were stored in closed containers until they were made into test bars. It should be pointed out that the time required to make each test bar made it impossible to use the mixtures immediately upon mixing. It is probable that this error in the experiment had little effect upon the results since all the test bars of an individual replicate were made in a short period of time.

B. Preparation of the Flexural Test Bars

To make the flexural test bars the cold flexural test bar mold was placed on the wagon, which had been preheated to the curing temperature, and rolled into the furnace which was set for the appropriate curing temperature. After remaining in the furnace for 15 minutes, which was sufficient time for the mold to reach the curing temperature, the wagon containing the mold was removed. The sand and resin mixture was poured from a flask onto the mold immediately, and the appropriate dwelling time was allowed. During the dwelling period, the mixture in the mold was leveled so that it was approximately 0.25 inch above the top of the mold. This made the overall depth of the mixture approximately 0.50

inch. After dwelling, the mold containing the "set" mixture was rolled into the furnace and cured at the appropriate curing temperature for one minute. The mold containing the flexural test bar specimen was than removed from the furnace, with the test bar being removed from the mold after it had cooled for two minutes. The mold was further cooled in cold water and cleaned if necessary. Several test bars were made without cleaning the mold or coating it with silicone grease. Starting with the cold mold and the preheated wagon for each specimen, the procedure was repeated for other specimens.

C. Testing the Flexural Test Bars

Using an abrasive wheel mounted on a drill press, the test bars were ground to an approximate thickness of 0.25 inch. In grinding a rough grind was used to remove the bulk of the excess, and a finish grind was used to obtain the final thickness.

Each specimen was weighed to the nearest 0.1 gram, and the thickness of each specimen was obtained to the nearest 0.001 inch by averaging five measurements on each. After these measurements, the test bar was placed in the flexural testing device with the roller on the center of the bar and loaded until it failed. The weight of the lead shot required to cause failure was determined to the nearest 0.1 gram. These weights were corrected by adding the tare weight of roller, chain, and bucket assembly of the flexural testing device. After making the flexural test of the center of the bar, the two pieces that remained were tested in the same manner as the former. It should be noted here that the variables, D_1, D_2, and D_3 were thus accounted for by a test at the center and tests near the two ends.

IX. Results

A. Calculation of Flexural Strength

The measurement made for flexural strength on the flexural testing device was weight in pounds. As previously described, a thickness measurement was made on each specimen. Using these measurements, the flexural strength was calculated from the bending stress formula[2]:

$$S_{max} = \frac{MC}{I}$$

where S_{max} is the maximum bending stress, M is the bending moment, C is one-half the height (thickness) of the specimen, and I is the moment of inertia. I is given by the equation:

$$I = \frac{bh^3}{12}$$

where b is the base of the specimen and h is the height. M is given by the equation:

$$M = 1/2\ lW$$

where l is the distance between the two fixed ends of the specimen and W is the weight of the load required to break the specimen when loaded in the center between the two fixed ends. Since l and b, 3.5 inches and 1.0 inch respectively, were the same in all specimens, a more simplified formula may be used to calculate the flexural strength and is given by the equation:

$$S_{max} = \frac{5.25W}{h^2}$$

Appendix A shows a sample calculation of flexural strength using this formula.

B. Calculation of Bulk Density

The measurement made for bulk density was weight in grams, which was converted to pounds. Using the weight, w, thickness measurement, h, the length, L, and base measurement, b, the bulk density was calculated from the formula:

$$\text{Bulk Density} = \frac{w}{bhL}$$

w and h were variable, while b had a constant value of 1 inch and L a constant value of 13 inches. A sample calculation of bulk density is given in Appendix B.

C. Effects of Several Variables on Flexural Strength

Results of tests of flexural strength made to determine the effect of dwelling time, curing temperature, amount of resin, and sand composition on flexural strength are given in Tables 4, 5 and 6. These data were analyzed using the statistical method shown in Appendix C; the resulting analysis of variance is shown in Table 7 of Appendix E. From this analysis of variance it is evident at the 99.5 per cent level that replication, curing temperature, amount of resin used, sand composition, and the interaction of temperature and sand composition is significant in considering the strength of shell molds. At the 97.5 per cent level, in addition, two other effects are significant; the interaction of amount of resin used and composition of sand and the three-way interaction of dwelling time, sand composition, and amount of resin used. All other variables or combination of variables are negligible since some are significant only at the 75 per cent level and others are significant at even lower levels.

Data, as follows, to show the effect of curing temperature, sand composition, and the combined effect of curing temperature and sand composition on the flexural strength of shell molds was obtained by averaging across replicates, dwelling time, and distance in the furnace:

Sand Composition	C_1	C_2	C_3	C_4
Average Size, Microns	(298)	(316)	(180)	(243)
		Flexural Strength		
T_1 (500°F)	720 psi	721 psi	669 psi	688 psi
T_2 (600°F)	592 psi	676 psi	600 psi	691 psi
Average	656 psi	699 psi	635 psi	690 psi

The curves shown in Figure 4 give a graphical illustration of these results.

Data to show the single effects, sand composition (C_1, C_2, etc.) and amount of resin (R_1, R_2) and their combined effect were obtained in the same manner as for the case above and are as follows:

Sand Composition	C_1	C_2	C_3	C_4
Average Size, Microns	(298)	(316)	(180)	(243)
		Flexural Strength		
R_1 (6% resin)	775 psi	856 psi	763 psi	801 psi
R_2 (4% resin)	537 psi	542 psi	507 psi	579 psi

These data are shown graphically in Figure 5.

D. Effect of Grain Size Distribution on Flexural Strength

The grain size distributions used have been previously described.

Table 8 of Appendix E shows the values obtained for flexural strength

and bulk density for this part of the investigation.

Graphically illustrated in Figure 6 is the effect of grain size distribution on the flexural strength of shell molds, while Figure 7 shows the bulk density vs. grain size distribution curve. In making these specimens, a curing treatment of 500°F for one minute, dwelling time of 20 seconds, and 5 per cent resin were used.

E. Correlation of Bulk Density and Flexural Strength

Bulk density measurements were made on all specimens; there were three flexural strength measurements made on each. Tables 8 and 9 of Appendix E show the average flexural strength and bulk density for each specimen. The data shown in Table 8 were used for a regression analysis of the dependency of flexural strength on bulk density, which is shown by the curve in Figure 8. The regression analysis, which is described in Appendix D, yielded an equation which can be used to predict flexural strength of shell molds when the density is known and the treatments in making the molds are identical to those used in the part of the investigation concerning grain size distribution. This equation is as follows:

$$Y = 670.2 + 36410.9\ (X - .05213)$$

where Y is the flexural strength of the shell mold in psi and X is the bulk density in pounds per cubic inch.

X. Discussion of Results

A. The Effect of Several Variables on Flexural Strength

From the results of this experimental work, it is evident that the curing temperature does have an appreciable effect upon the flexural strength of shell molds. In comparing the mean values of flexural strength for the two curing temperatures used, it was obvious that greater strength resulted from shell mold test bars which were cured at 500°F than those which were cured at 600°F. This result indicates that a curing temperature near 500°F is sufficient in getting the reactions in the resin to proceed. It is doubtful that the maximum strength occurs at a curing temperature appreciably below 500°F. It is clear that a curing treatment of 600°F results in burn-up of the resin. Besides decreasing the flexural strength of shell molds, resin burn-up causes poorer surface finishes on castings, since most of the burn-up occurs on that part of the shell which will be in contact with the casting.

In practice, the hot strength of shell molds may be of primary importance. However, it should be pointed out that when a metal is cast in a mold, which is not preheated, solidification occurs rapidly near the mold wall and thenceforth proceeds from that surface with the metal first solidified containing the liquid metal. This means that the shell mold must resist the metallostatic pressure for only a short period of time. In some practices, it is customary to build up a thick shell by using several cycles of dumping and curing. This, of course, sacrifices casting surface for increased shell strength; from

this experimental work it appears that the increase in strength would be small because of resin burn-up.

There is considerable interaction between curing temperature and sand composition as shown by Figure 4. The major source of this interaction is sand compositions, C_4 and C_2, and temperature. (No explanation can be given for this phenomenon.)

The amount of resin used is highly significant in its influence on the strength of shell molds. This is to be expected since greater amounts of resin provide more points of bonding between sand and resin or more interfacial area between the two. The principal reason for designing the experimental work with amount of resin as a variable was to determine its interaction effects. In this investigation there are only two in which the amount of resin is involved. One is the interaction between amount of resin and sand composition, the major source of which is the interaction of sand composition, C_2, with amount of resin used. The other is a three-way interaction between dwelling time, sand composition, and amount of resin. No explanation has been found for either of these interactions.

The means of flexural strength calculated for sand composition do not give much information about the nature of the influence of average grain size on shell mold strength. Contained in these means are strong interaction effects of sand composition with other variables. It does appear, however, that the strength of shell molds, as inferred from the flexural bar tests, increases with increase in average grain size of the sand, as has been shown by other investigators. The effect of grain

size distribution cannot be resolved from this part of the investigation since it was not anticipated in the original experimental design. All measurements of flexural strength, therefore, are confused with grain size distribution which effect is most likely contained in the effect of sand composition. In complete analysis, since grain size distribution and average grain size both affect the flexural strengths obtained from the various sand compositions, it is evident that no quantitative measurement can be made of the effect of average grain size on the strength of shell molds. However, the variation in grain size distributions among the sands was small, so that some information about the effect of average grain size was confidently obtained.

B. Effect of Grain Size Distribution on Flexural Strength

Figure 6 shows generally that the wider the distribution of grain size the greater will be the flexural strength of the shell mold. It is evident that the lowest strengths were obtained with distributions No. 3 and No. 5. It is believed that the strength for No. 3 is slightly lower than normal, and for No. 4 it is slightly higher. The dashed curve in Figure 6 shows what is believed to be the normal variation of flexural strength with grain size distribution. It should be observed also that the average grain size of distribution No. 5 is significantly different from that of the other distributions. Hence, if a correction were applied, distribution No. 5 would have a slightly higher flexural strength.

For an explanation of variation in strength with grain size distribution, it was found that the bulk density of the specimens increased

with wider distributions, as shown in Figure 7. A greater bulk density indicates that there is less void space and better packing in the shell mold specimen. With this better packing there is a smaller average distance between sand grains so that less resin is needed to fill the void between any two typical grains. This makes available more resin to actually adhere to the surface of the sand particles, so that there is more interfacial area between the sand and resin. This greater efficiency of the resin in bonding then increases strength.

C. Correlation of Bulk Density and Flexural Strength

An equation was used to describe how flexural strength of shell molds varies with bulk density. This equation is suitable only where the treatments are identical to those used for this part of the investigation. The flexural strength was found to be a linear function of density, this being substantiated by a statistical test for linearity of regression. The value of this correlation is that it shows that if a series of such correlations can be found for various treatments, shell molds may be tested for flexural strength non-destructively.

D. Experimental Precision

In the investigation of the several variables affecting the strength of shell molds for which the factorial design was used, it was found that the effect of replications was significant. This indicates that the experimental precision was low. However, the effect of this lack of precision was removed from the other variables in the analysis of variance so that they were essentially unaffected.

The large difference in the means of the replicates is believed to be caused by partial break-down of the resin while in storage. The experimental work for replicate No. 1 and replicate No. 2 was done during the same month, and there is only a small difference between the means of their flexural strengths. But the experimental work for replicate No. 3 was done approximately four months after the other two. The mean of the flexural strengths of replicate No. 3 is quite different from the means of the other two.

The error term of the analysis of variance was larger than expected, indicating that some additional variable was not anticipated in the design. Probably this large error was caused by contributing interactions of the period of time separating the completion of each replicate and the variables which were known. This has been partially substantiated by analyzing the variance of just two of the replicates. Replicate No. 1 and Replicate No. 3 were used for this analysis which yielded comparatively large interactions of time between replicates and the other variables. This means that true replication did not exist insofar as replicates No. 1 and No. 3 are concerned. Since replicates No. 1 and No. 2 were done in the same month, they can be considered true replicates of the experimental design.

The error term evidently could have been made smaller by doing all three replicates at the same time, which probably would have given more information in the analysis of variance. The large error term in this investigation then means that estimation of all results from the analysis of variance are conservative.

XI. Conclusions

Within the limits of this investigation, the work of this thesis leads to the following conclusions:

1. There is a significant difference between the effects of different curing temperatures on the strength of shell molds. A curing temperature of 500°F is more desirable than one of 600°F.
2. The strength of shell molds increases with increasing resin consumption.
3. The strength of shell molds increases with increasing average sand grain size.
4. A wider distribution of sand grain sizes present in a single mixture yields a shell mold with a high flexural strength than a mold with the same average grain size but narrower distribution.
5. The following interaction effects have a significant influence on the strength of shell molds:
   a) Sand composition and curing temperature.
   b) Sand composition and amount of resin.
   c) Sand composition, dwelling time, and amount of resin.
6. The correlation between bulk density and flexural strength is linear. Flexural strength decreases with decreasing density.

XII. Acknowledgements

Sincere appreciation is expressed to the many people in the Engineering School of V.P.I. whose advice and direction made this thesis possible; in particular, to Professor John F. Eckel for his supervision of the thesis, Professor J. K. Sprinkle for his help in planning the investigation, and Professors A. J. Metzger and P. S. Dear for their helpful counsel.

Appreciation is also expressed to the Monsanto Chemical Company for furnishing the resin used in this investigation.

XIII. Suggestions for Future Work

From work to date, it appears that the following research should be conducted:

1. Determine the influence of several other levels of the variables used in this investigation on the strength of shell molds.
2. Determine the influence on the strength of shell molds of various sand distributions having other average grain sizes.
3. Investigate the interaction effects found in this work to determine why they exist.
4. Investigate other means of non-destructive testing shell molds.

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Appendix A

The formula used for calculation of flexural strength was as follows:

$$S_{max} = \frac{5.25\ W}{h^2}$$

Taking specimen No. 1 in replication No. 1 at distance D, as an example, the weight, W, of the load required to produce failure was 14.864 pounds; and the average thickness, h, of the test bar was 0.270 inch. Substituting these values in the above equation:

$$S_{max} = \frac{5.25(14.864)}{(.270)^2} = 1057 \text{ psi}$$

All other calculations of flexural strength in this investigation were made in the same manner and with the same accuracy.

Appendix B

The formula used for calculation of bulk density was as follows:

$$\text{Bulk Density} = \frac{W}{bhl}$$

Taking specimen No. 1 in replication No. 1 as an example, the weight of the specimen, W, was 0.196 pound; L had the constant value of 13 inches, and b the constant value of 1 inch. Substituting these values in the above equation:

$$\text{Bulk Density} = \frac{.196}{(1)\ (.270)\ (13)} = 0.0558\ \frac{\text{lb.}}{\text{in.}^3}$$

All other bulk densities were found in a manner identical to this example.

Appendix C

The method used for analysis of the statistical factorial design in this investigation can be best shown by illustration of the various calculations and operations done to obtain the analysis of variance, which is shown in Table 7.

The first step was to obtain the various one-way, two-way, three-way, four-way, and five-way classifications which were needed in finding the sums of squares. The first classification obtained was the five-way by adding the treatments across the replicates and maintaining all other effects intact. The result of this operation is shown in Table 10.

The four-way classification was obtained similarly. For example, taking the five-way classification in Table 10 and adding across T, the four-way classification shown in Table 11 results.

The three-way, two-way, and one-way classifications were obtained in the same manner. The three-way classification is obtained by adding across a variable in the four-way; for example, across t, which results in the three-way classification of Table 12. The two-way and one-way classifications are both shown in Table 13.

After obtaining all the various classifications, the next step was the computation of the sums of squares. To find the sum of squares for a single effect and using for an example, R, refer to the one-way classification of Table 13:

$$S.S_R = \frac{(114987)^2 + (77845)^2}{144} - \frac{(192832)^2}{288} = 4,790,028$$

There are 144 measurements in R_1 and 144 in R_2; there are 288 in the grand total. Continuing with the sample classifications and referring to Table 13, the sum of squares of the two-way interaction of D and R can be found as follows:

$$S.S._{DxR} = \frac{(38921)^2 + (38540)^2 + \text{----} + (25775)^2}{48} - S.S._R - S.S._D - \frac{(192832)^2}{288} = 6,265$$

Similarly, referring to Table 12, the sum of squares for the interaction, DRC, is obtained:

$$S.S._{DxRxC} = \frac{(9673)^2 + (9091)^2 + \text{----} + (6898)^2}{12} - S.S._D - S.S._R - S.S._C - S.S._{DR} - S.S._{DC}$$

$$- S.S._{RC} - \frac{(192832)^2}{288} = 15,403$$

Higher order interactions are obtained in a manner similar to the above procedure. The total sum of squares is obtained by adding the squares of all measurements and subtracting the correction factor as follows:

$$S.S._{Total} = (1057)^2 + (1057)^2 + \text{----} + (476)^2 - \frac{(192832)^2}{288} = 7,957,817$$

The error sum of squares is obtained by subtraction of all interactions which do not contain the replication factor and the single effects from the total sum of squares.

To obtain the number of degrees of freedom for single effects, it is simply one less than the number of levels of that variable. There are four levels of C; so there are three degrees of freedom. For an interaction effect the number of degrees of freedom is the product of the number of degrees of freedom possessed by the variables involved in the interaction. For example, C has three degrees of freedom and D has two; therefore, the interaction C x D, has six degrees of freedom. The error degrees of freedom is obtained by subtracting the degrees of freedom for all interactions not containing the replication factor and for the single effects from the total degrees of freedom.

The mean square is obtained by dividing the sum of squares of an effect by the number of degrees of freedom possessed by that effect. The F-ratio is the result of dividing the particular mean square by the mean

square of the error. The finding of the F-ratio concludes the computation of the analysis of variance for this design. The F-ratio was used for tests of significance in this investigation, all critical values for the F-ratio were obtained from tables of Bennett and Franklin.

Table 10. Five-way Classification of T, t, C, D, and R.

		T_1 (500° F)						T_2 (600° F)					
Rep1 Rep2 Rep3		t_1(20 sec.)			t_2(40 sec.)			t_1(20 sec.)			t_2(40 sec.)		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
		Flexural strength in psi											
C_1	R_1	2450	2580	2603	2839	2305	2647	2148	2128	1940	2236	2078	1953
	R_2	1996	1780	1938	1692	1508	1577	1600	1561	1601	1251	1386	1414
C_2	R_1	2342	2793	2710	2989	2760	2306	2921	2401	2278	2524	2353	2440
	R_2	1611	1571	1527	1794	1750	1828	1810	1468	1694	1642	1509	1312
C_3	R_1	2619	2468	2347	2008	2343	2393	2327	2316	2117	2091	2422	1994
	R_2	1520	1652	1546	1625	1874	1667	1212	1351	1422	1510	1490	1351
C_4	R_1	2257	2477	2517	2194	2429	2431	2418	2344	2424	2558	2343	2426
	R_2	1836	1884	1723	1613	1652	1752	1833	1742	1740	1623	1724	1683

Table 11. Four-way Classification of t, D, C, and R.

Rep. T		t_1			t_2		
		D_1	D_2	D_3	D_1	D_2	D_3
C_1	R_1	4598	4708	4543	5075	4383	4600
	R_2	3596	3341	3539	2943	2894	2991
C_2	R_1	5263	5194	4988	5513	5113	4746
	R_2	3421	3039	3221	3436	3259	3140
C_3	R_1	4946	4784	4464	4099	4765	4387
	R_2	2732	3003	2968	3135	3364	3018
C_4	R_1	4657	4821	4941	4752	4772	4857
	R_2	3669	3626	3463	3236	3376	3435

Table 12. Three-way Classification of D, C, and R.

Rep. T, t		D_1	D_2	D_3
C_1	R_1	9673	9091	9143
	R_2	6539	6235	6530
C_2	R_1	10776	10307	9734
	R_2	6857	6298	6361
C_3	R_1	9045	9549	8851
	R_2	5867	6367	5986
C_4	R_1	9427	9593	9798
	R_2	6905	7002	6898

Table 13. Two-way Classification of D and R.

Rep. T,t,C	D_1	D_2	D_3	
R_1	38921	38540	37526	114987
R_2	26168	25902	25775	77845
			Grand Total	192832

Appendix D

The regression analysis of the effect of bulk density on the flexural strength of shell molds was done in the following manner. Starting with the data in Table 8; and letting X be bulk density and Y be flexural strength, the following table was obtained:

X_i lb/in^3	Y_i lb/in^2	X_iY_i
.0553	799	44.1847
.0530	699	37.0470
.0514	593	30.4802
.0509	644	32.7796
.0504	607	30.5928
.0549	768	42.1632
.0526	704	37.0304
.0517	616	31.8472
.0506	672	34.0032
.0505	600	30.3000
ΣX_i = .5213	ΣY_i = 6702	ΣX_iY_i = 350.4283

$\bar{X}$ = .05213 $\bar{Y}$ = 670.2

ΣX_i^2 = .02720429 ΣY_i^2 = 4,538,316

$\Sigma X_i \Sigma Y_i$ = 3493.7526

The regression equation is:

$$Y = \bar{Y} + b(X - \bar{X})$$

where Y is the flexural strength, dependent on X,

$\bar{Y}$ is the average flexural strength,

X is a particular bulk density,

$\bar{X}$ is the average bulk density,

and b is the slope.

The slope is obtained from the equation:

$$b = \frac{\Sigma X_iY_i - \frac{\Sigma X_i \Sigma Y_i}{N}}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{N}}$$

where N is the number of samples.

Solving for the slope using this equation:

$$b = \frac{350.4283 - 349.37526}{.02720429 - .027175369} = 36410.9$$

The regression equation for these data is then

$$Y - 670.2 + 36410.9(X - .05213)$$

The 90 per cent confidence limits were obtained for this equation, the procedure for which is given in many references[10] on statistics.

Appendix E

The following tables show the experimental data.

Table 4. Flexural Strengths from Replicate No. 1 of the Factorial Design.

		T_1 (500° F)						T_2 (600° F)					
		t_1 (20 sec.)			t_2 (40 sec.)			t_1 (20 sec.)			t_2 (40 sec.)		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
		Flexural strength in psi											
C_1	R_1	1057	1057	1007	945	846	967	625	660	606	670	638	629
	R_2	729	599	678	501	494	475	664	427	587	372	457	383
C_2	R_1	704	934	1022	1010	1031	772	880	711	694	742	829	663
	R_2	659	530	543	522	535	580	653	515	572	417	413	483
C_3	R_1	827	710	706	674	731	800	898	858	765	632	788	741
	R_2	521	568	544	453	595	550	360	538	521	522	463	441
C_4	R_1	753	848	842	766	756	776	855	1000	871	801	704	786
	R_2	541	620	634	492	460	590	640	647	556	635	655	579

Table 5. Flexural Strengths from Replicate No. 2 of the Factorial Design.

		T_1 (500° F)						T_2 (600° F)					
		t_1 (20 sec.)			t_2 (40 sec.)			t_1 (20 sec.)			t_2 (40 sec.)		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
		Flexural Strength in psi											
C_1	R_1	793	945	869	114	774	815	875	687	781	747	708	733
	R_2	855	667	695	632	501	567	554	527	622	399	515	487
C_2	R_1	880	940	910	984	836	879	1136	905	961	759	710	1020
	R_2	481	555	578	754	675	645	637	457	536	614	583	488
C_3	R_1	835	967	793	645	900	774	566	711	562	759	807	657
	R_2	400	476	501	655	747	556	424	445	465	499	496	448
C_4	R_1	748	820	835	728	806	808	843	719	838	795	780	835
	R_2	618	623	569	535	552	576	613	630	553	492	576	628

Table 6. Flexural Strength from Replicate No. 3 of the Factorial Design.

		T_1 (500° F)						T_2 (600° F)					
		t_1 (20 sec.)			t_2 (40 sec.)			t_1 (20 sec.)			t_2 (40 sec.)		
		D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3	D_1	D_2	D_3
		Flexural strength in psi											
C_1	R_1	600	578	727	780	685	805	648	781	553	819	732	591
	R_2	412	505	565	559	513	535	382	607	392	480	414	544
C_2	R_1	758	919	778	995	893	655	905	785	623	1023	814	757
	R_2	471	486	406	518	540	603	520	496	586	611	513	341
C_3	R_1	957	791	848	689	712	819	863	747	770	700	827	596
	R_2	599	608	501	517	532	561	428	368	436	489	531	462
C_4	R_1	756	809	840	700	867	847	720	625	715	962	859	805
	R_2	677	641	520	586	640	586	580	465	631	496	493	476

Table 7. Analysis of Variance

Source	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Rep.	112,522	2	56,261	6.512
T	257,642	1	257,642	29.823
t	17,641	1	17,641	2.042
C	194,149	3	64,716	7.491
D	17,074	2	8,537	.988
R	4,790,028	1	4,790,028	554.466
T x t	1,760	1	1,760	.204
T x C	159,637	3	53,212	6.160
T x D	23,510	2	11,755	1.361
T x R	1,423	1	1,423	.165
t x C	19,612	3	6,537	.757
t x D	712	2	356	.041
t x R	967	1	967	.112
C x D	83,870	6	13,978	1.618
C x R	85,430	3	28,477	3.296
D x R	6,265	2	3,133	.362
T x t x C	41,877	3	13,978	1.618
T x t x D	12,415	2	6,208	.719
T x t x R	3,917	1	3,917	.453
t x C x R	96,478	3	32.159	3.723
t x C x D	28,928	6	4,821	.558
C x D x R	15,403	6	2,567	.297
T x D x R	16,566	2	8,283	.959
T x D x C	50,052	6	8,342	.966
C x T x R	29,395	3	9,798	1.134
t x D x R	5,146	2	2,573	.298
T x t x C x D	61,306	6	10,218	1.183
T x t x C x R	8,306	3	2,769	.321
T x C x D x R	28,289	6	4,715	.546
T x t x D x R	17,243	2	8,622	.998
t x C x D x R	58,085	6	9,681	1.121
T x t x C x D x R	70,777	6	11,796	1.365
Error	1,641,392	190	8,639	
Total	7,957,817	287		

Table 8. Flexural Strengths and Bulk Density from the Investigation of Grain Size Distribution.

Replication No. 1

Distribution Number	Distance in Oven D_1	D_2	D_3	Y Average Strength	X Density lb/in^3
	Flexural strength in psi				
1	871	166	161	799	.0553
2	574	751	773	699	.0530
3	558	572	650	593	.0514
4	610	647	675	644	.0509
5	593	582	647	607	.0504

Replication No. 2

Distribution Number	Distance in Oven D_1	D_2	D_3	Y Average Strength	X Density lb/in^3
	Flexural strength in psi				
1	753	750	800	768	.0549
2	795	721	596	704	.0526
3	629	664	556	616	.0517
4	637	710	669	672	.0506
5	656	570	575	600	.0505

Table 9. Bulk Density and Average Flexural Strength in the Factorial Design.

Specimen Number	Replicate 1		Replicate 2		Replicate 3	
	Density lb/in^3	Average lb/in^2	Density lb/in^3	Average lb/in2	Density lb/in^3	Average lb/in^2
1	.0558	1040.3	.0536	869.0	.0541	635.0
2	.0566	919.3	.0538	921.0	.0546	756.7
3	.0556	668.7	.0536	742.0	.0536	494.0
4	.0533	490.0	.0530	566.7	.0543	532.3
5	.0538	630.3	.0541	781.0	.0547	660.7
6	.0543	645.7	.0536	729.3	.0536	714.0
7	.0546	559.3	.0526	567.7	.0538	460.3
8	.0535	404.0	.0515	467.0	.0542	579.3
9	.0563	886.7	.0555	910.0	.0552	818.3
10	.0563	937.7	.0551	899.7	.0546	847.7
11	.0541	577.3	.0525	538.0	.0547	454.3
12	.0541	545.7	.0541	691.3	.0547	553.7
13	.0546	761.7	.0547	950.7	.0549	771.0
14	.0558	744.7	.0541	829.7	.0552	864.7
15	.0559	580.0	.0524	543.3	.0536	534.0
16	.0544	437.7	.0517	561.7	.0549	488.3
17	.0487	747.7	.0505	865.0	.0502	865.3
18	.0498	735.0	.0506	773.0	.0506	740.0
19	.0495	544.3	.0506	459.0	.0498	569.3
20	.0498	532.7	.0517	652.7	.0501	536.7
21	.0509	840.3	.0503	619.7	.0501	793.3
22	.0493	720.3	.0495	741.0	.0494	707.7
23	.0493	473.0	.0497	444.7	.0498	410.7
24	.0488	475.3	.0493	481.0	.0495	494.0
25	.0525	814.3	.0527	801.0	.0531	801.7
26	.0544	766.0	.0532	780.7	.0523	804.7
27	.0532	598.3	.0529	603.3	.0528	612.7
28	.0538	514.0	.0541	554.3	.0542	604.0
29	.0533	908.7	.0527	800.0	.0525	686.7
30	.0520	763.7	.0531	803.3	.0535	875.3
31	.0535	614.3	.0529	598.7	.0529	558.7
32	.0532	623.0	.0534	565.3	.0538	488.3

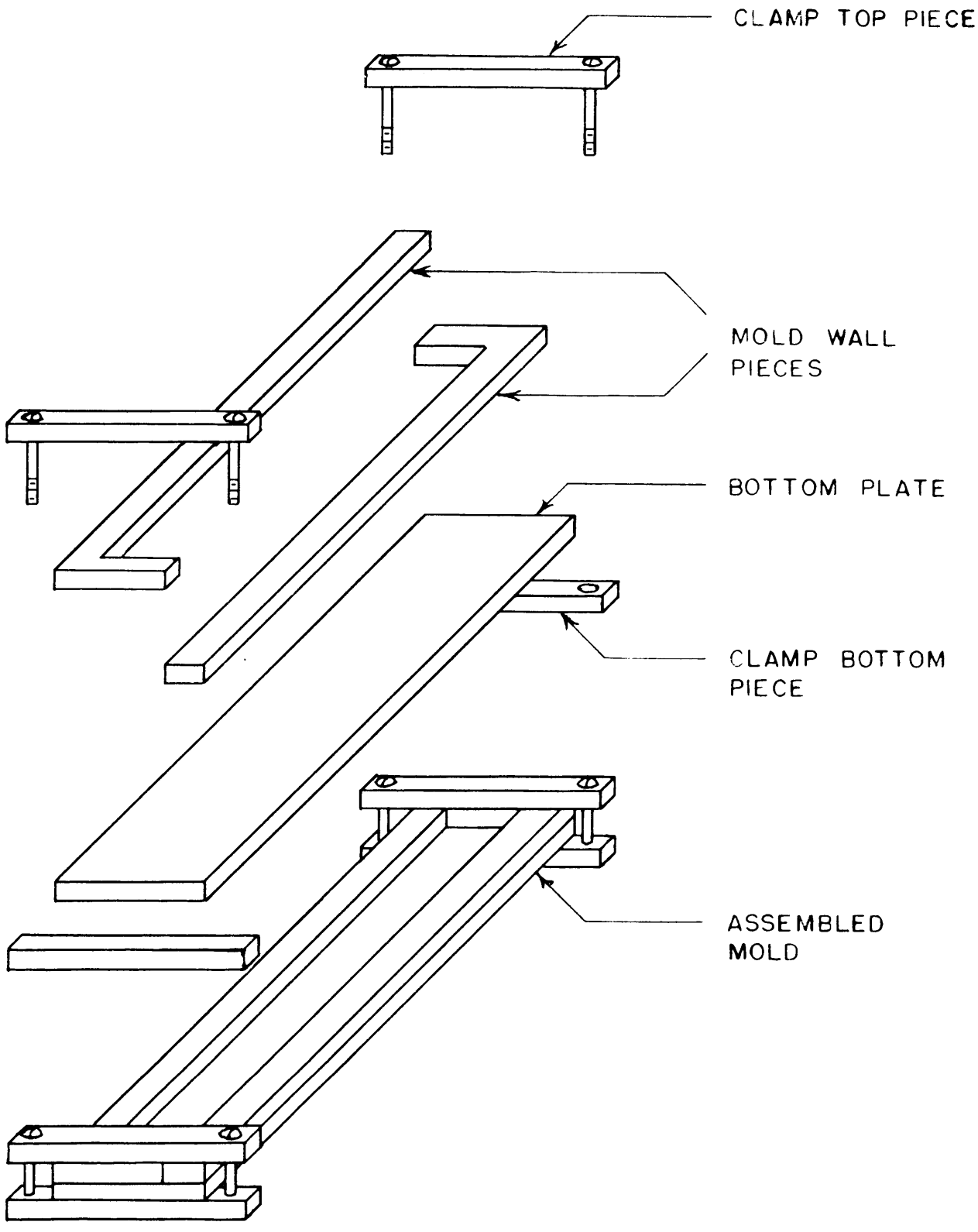

FIGURE I. FLEXURAL TEST BAR MOLD ASSEMBLY

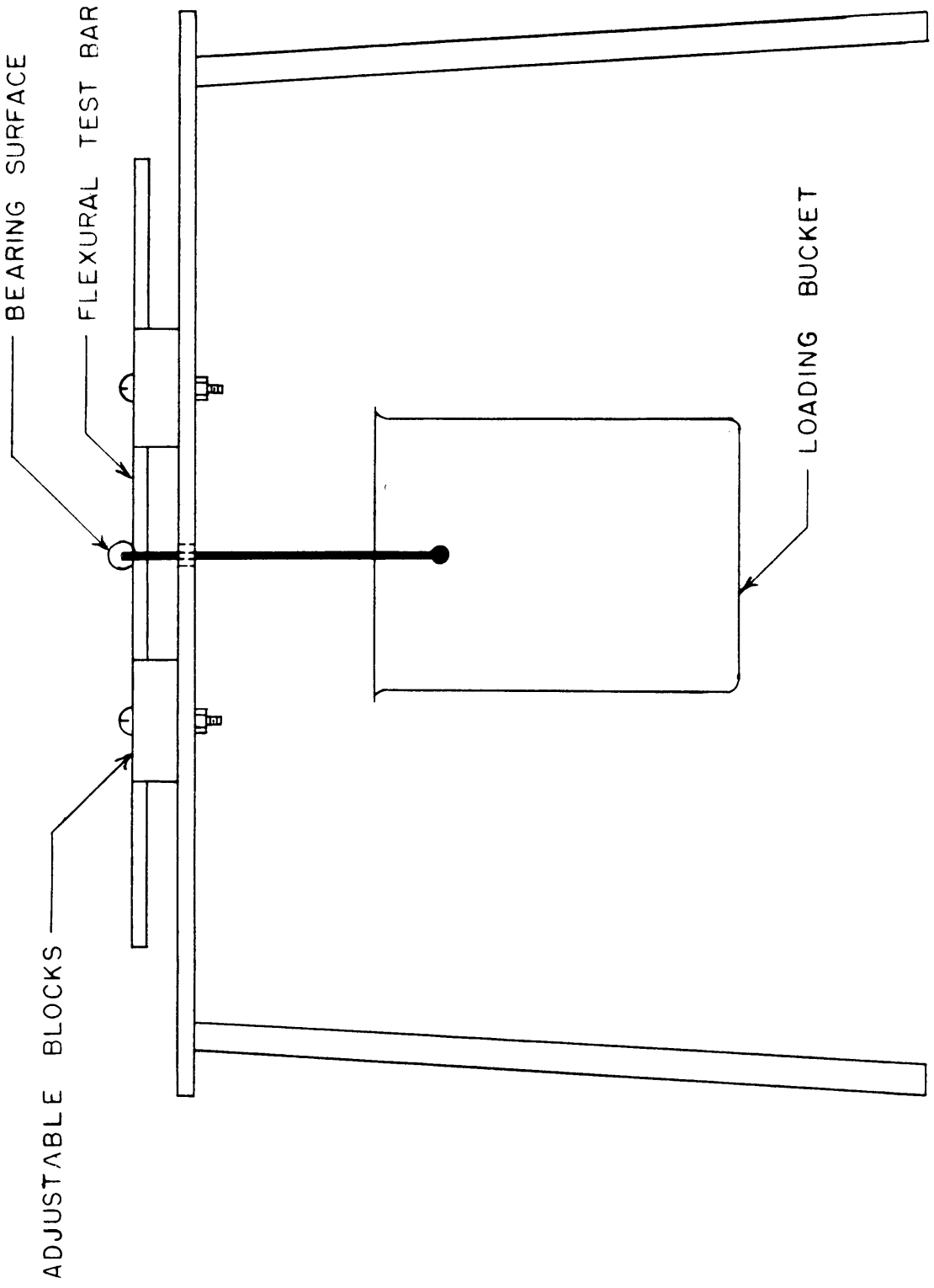

FIGURE 2. FLEXURAL TESTING DEVICE

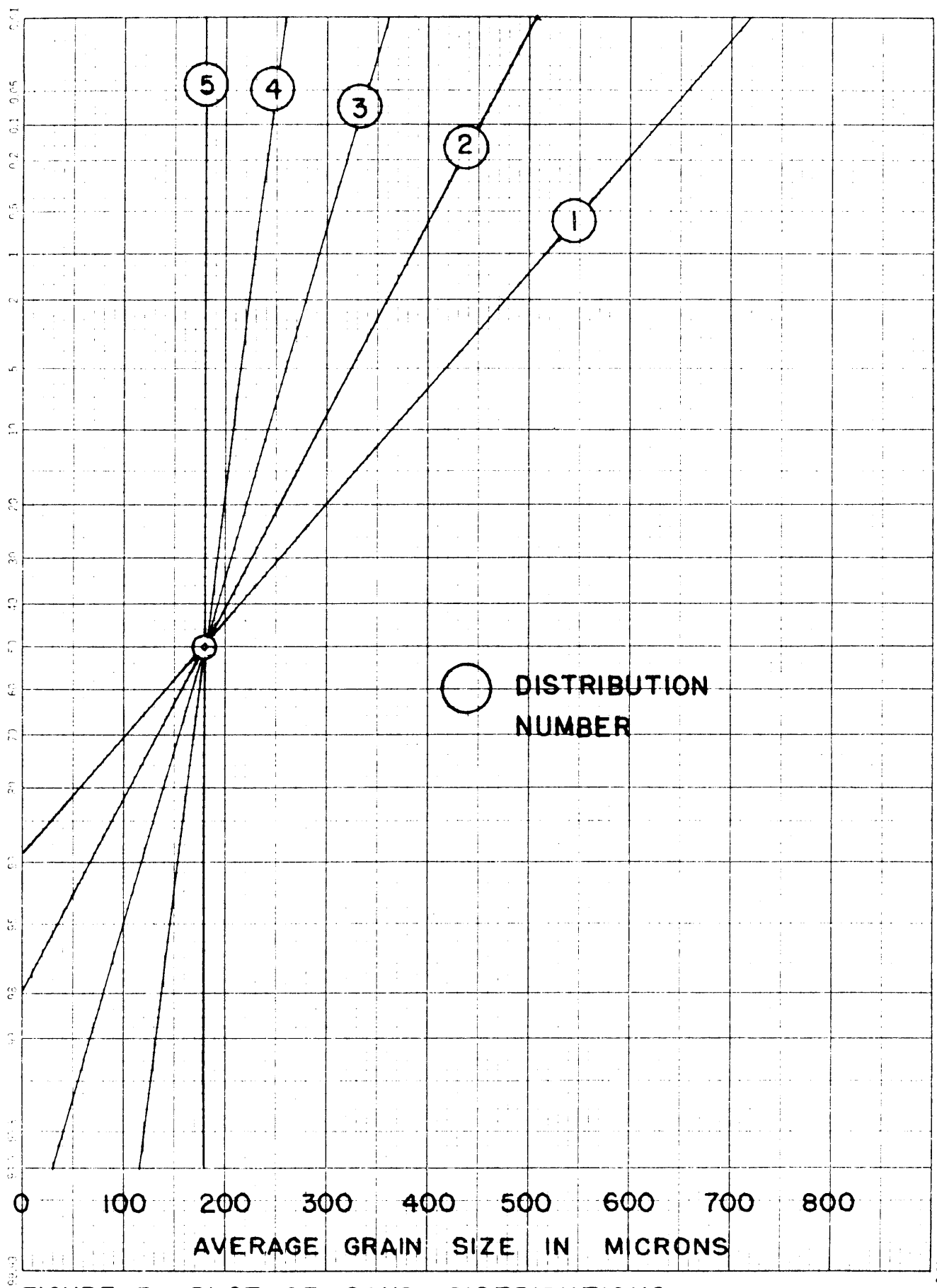

FIGURE 3. PLOT OF SAND DISTRIBUTIONS

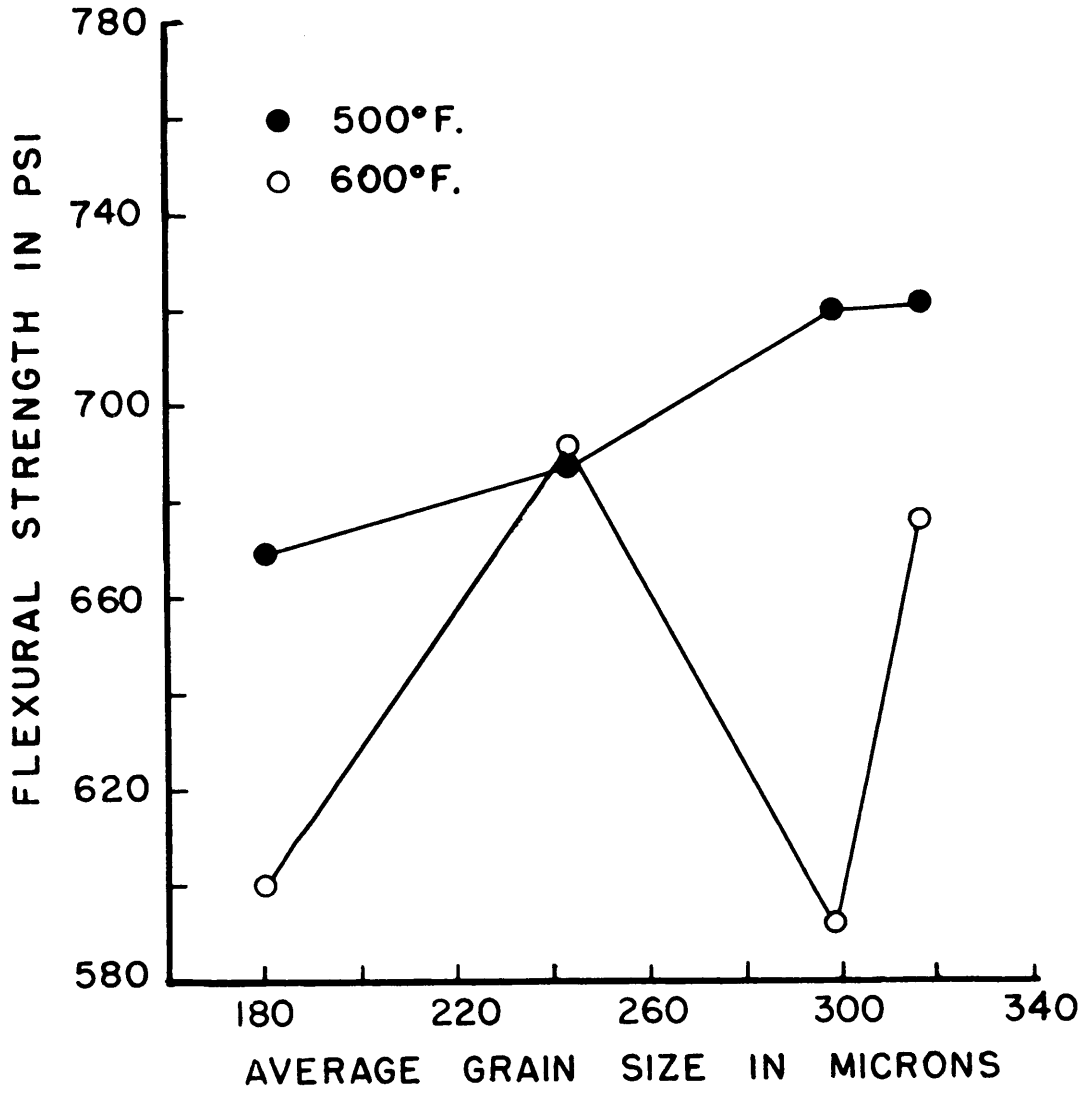

FIGURE 4. SAND COMPOSITION VS. FLEXURAL STRENGTH CURVES

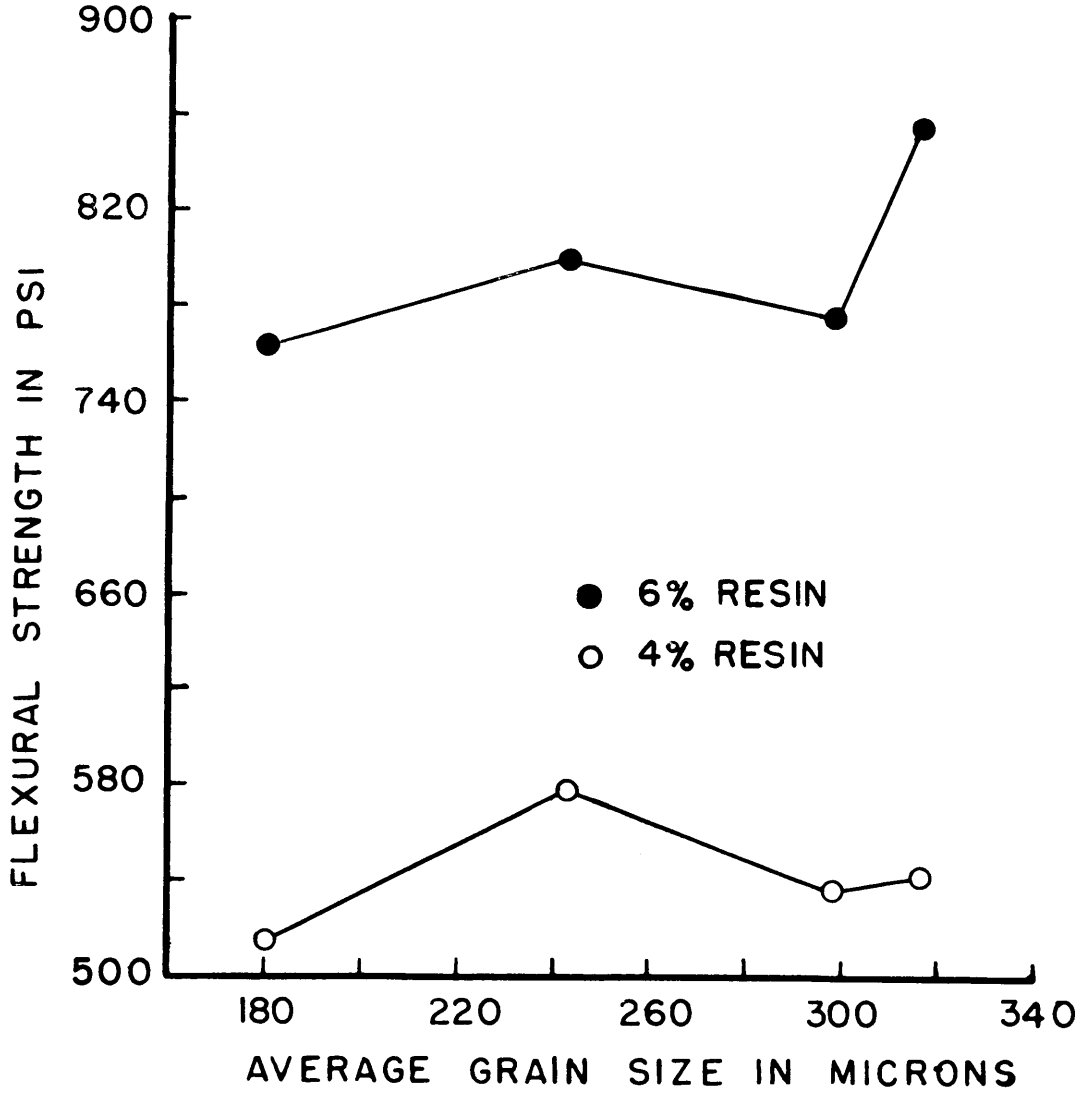

FIGURE 5. SAND COMPOSITION VS. FLEXURAL STRENGTH CURVES

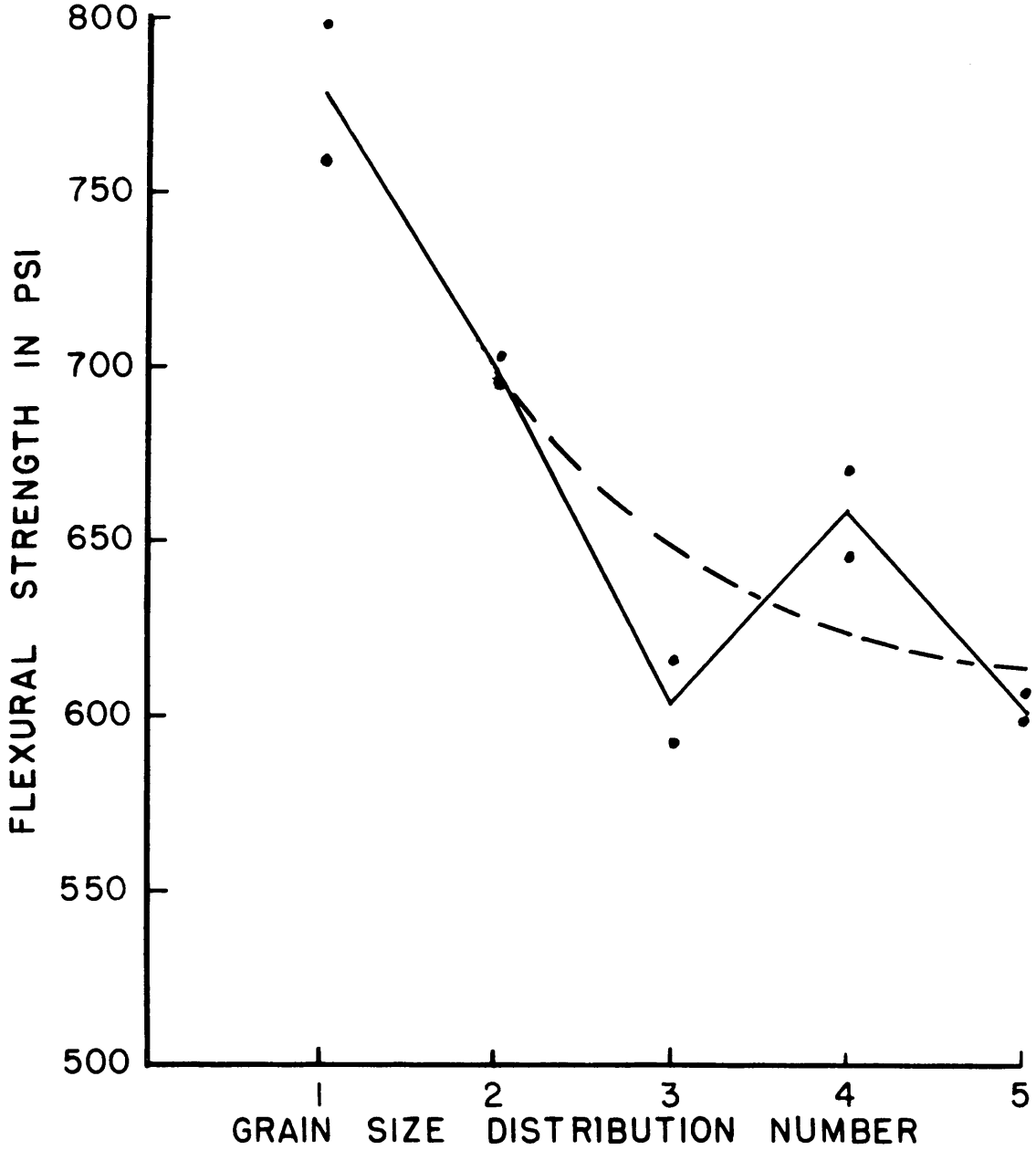

FIGURE 6. FLEXURAL STRENGTH VS. GRAIN SIZE DISTRIBUTION

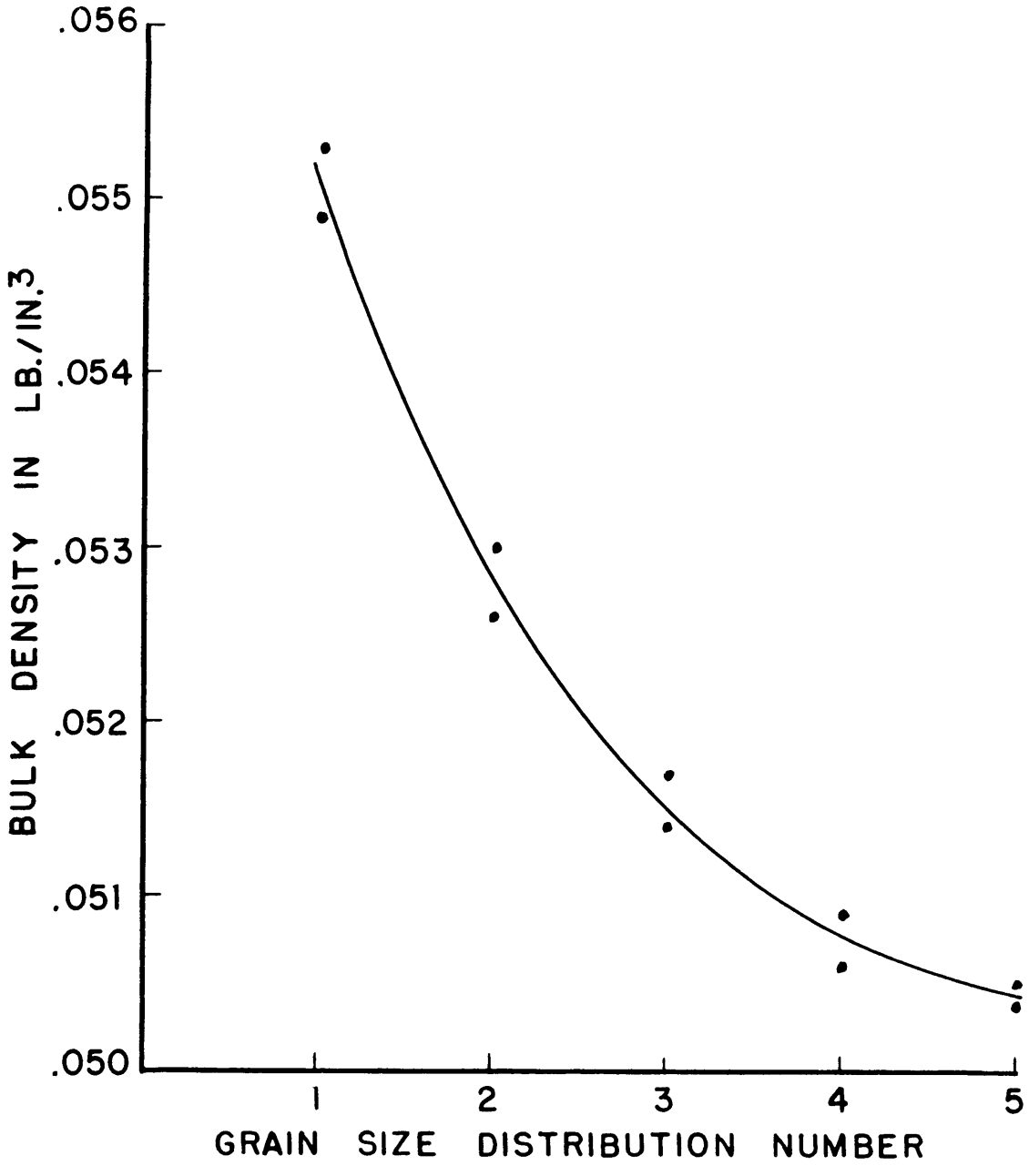

FIGURE 7. BULK DENSITY VS. GRAIN SIZE DISTRIBUTION

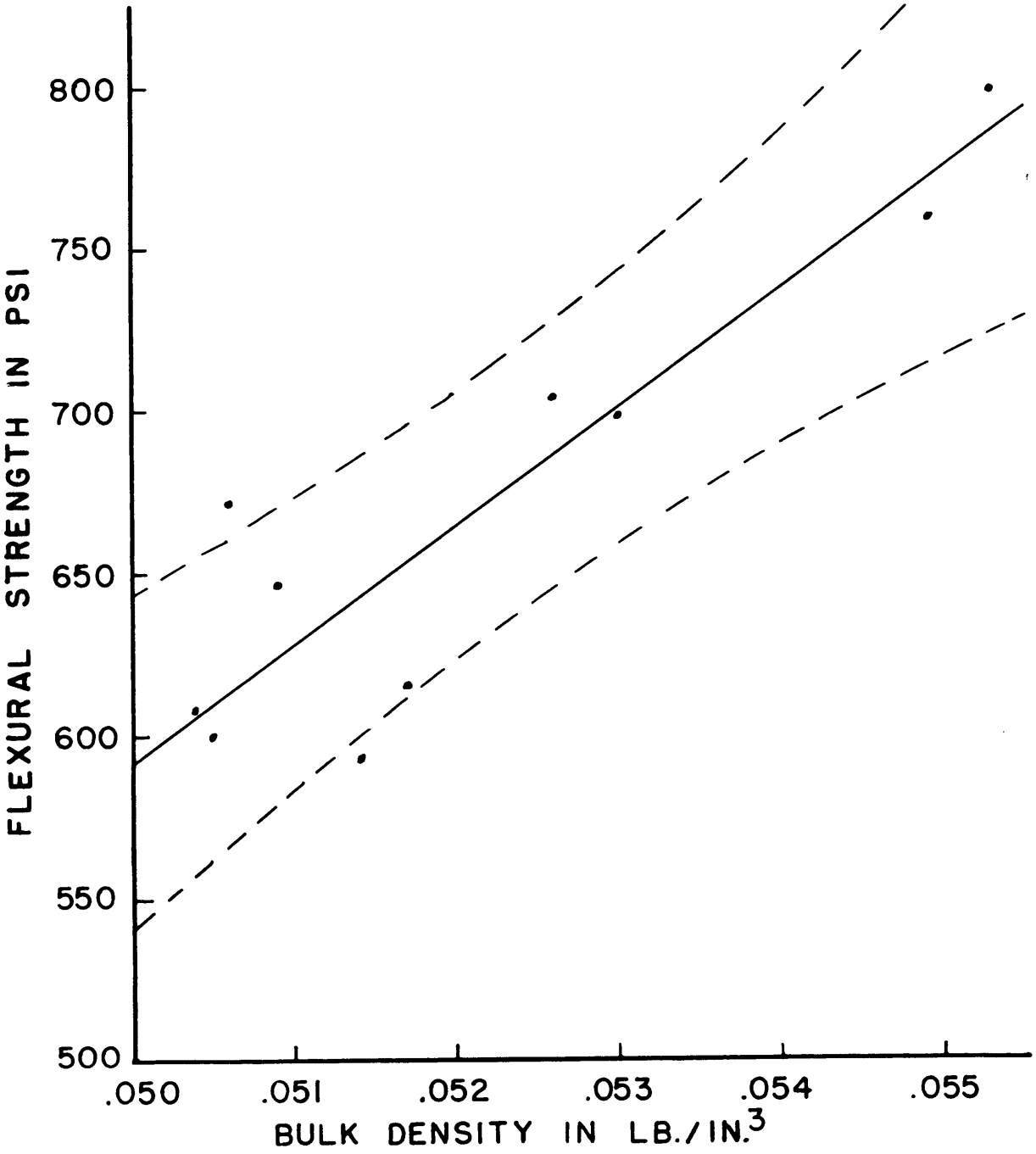

FIGURE 8. FLEXURAL STRENGTH VS. BULK DENSITY