Mobility Analysis

of

Variable Geometry Trusses

by

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(ABSTRACT)

Parallel manipulators are being thought of as a solution to many problems involving control and manipulation. They offer significant advantages over serial manipulators in terms of increased strength and rigidity. Variable geometry trusses (VGTs) are a special class of parallel trusses.

Literature on VGTs has covered many interesting problems, yet there has been no conscious effort to formulate a definition for a VGT. The major emphasis of this thesis is developing a set of precise kinematic rules for defining and analyzing VGTs.

Traditional mobility criteria, when applied to parallel geometries, predict results which are often confusing and sometimes inaccurate. Based on the set of rules developed, mobility equations are derived for planar and spatial VGTs. The equations are tested on a sufficiently large number of VGT configurations to convince the author about the validity of the set of rules and the equations derived. Using the mobility equations, different candidate geometries can be analyzed and compared. In addition, the equations can be used in the type and number synthesis of VGTs.

Acknowledgements

I would take this oppurtunity to express my gratitude towards Dr. Reinholtz who assisted and motivated me throughout this research effort. I wish to acknowledge the contribution of Dr. Myklebust and Dr. Robertshaw who lent me their sincere cooperation and help.

Finally I would like to thank my parents for their moral support and motivation throughout my academic career.

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Chapter 1

Introduction

As far back as 1965, researchers were intuitively thinking of adaptive structures as a solution to many problems involving control and manipulation. The concept of an adaptive structure is very simple; it is a structure that can vary its configuration. For example, when Stewart first presented the idea of the Stewart's platform [27] (Fig.1) he had envisioned it as an aircraft flight simulator. This device has six degrees of freedom and the platform can, in general, be manipulated and oriented arbitrarily in space within the limitations of its workspace. This could simulate the pitch, roll and yaw angles of rotation of an aircraft in flight. One finds closed-loop linkages used in numerous applications and in a variety of configurations. In the late 70's, researchers began to seriously consider the use of closed-loop linkages as robotic manipulators.

Initially, it is necessary to define clearly certain frequently used terms.

Link

A link is a rigid body having two or more pairing elements by means of which it may be connected to other bodies for purposes of transmitting force or motion.

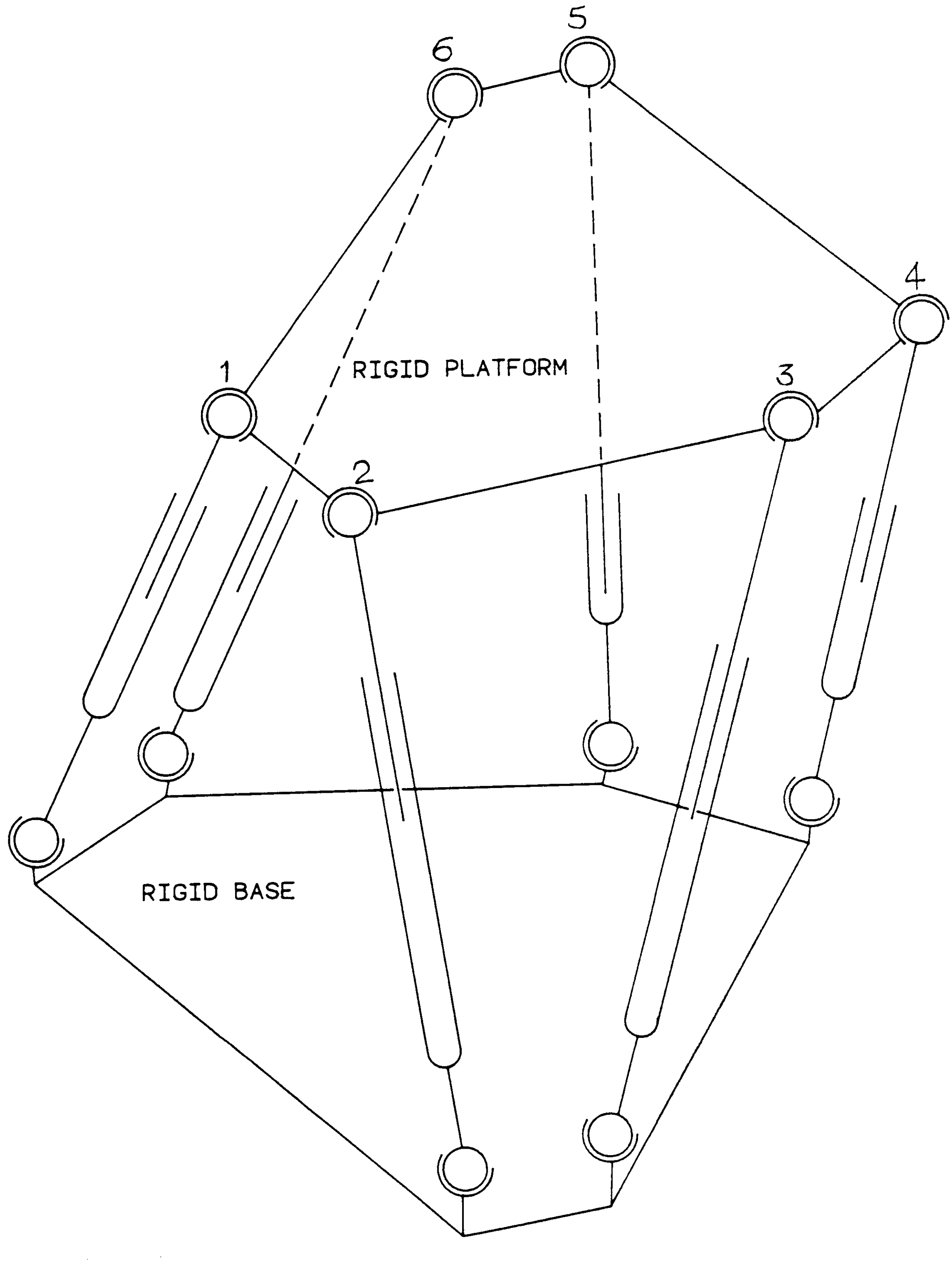


Figure 1. Stewart's Platform

Pairing Element

The geometrical forms by which two members of a mechanism are joined together so that relative motion between these two members is consistent are known as pairing elements.

When a number of links are connected by means of pairs, the resulting system is a kinematic chain. If these links are connected in such a way that no motion is possible, a locked chain (structure) results. A constrained chain is obtained when the links are so connected that, no matter how many motion cycles are passed through, the relative motion will always be the same between the links. A kinematic chain is over-constrained when it has less than one degree of freedom. If one of the links of a constrained chain is made a fixed link, the result is a mechanism. A mechanism has exactly one degree of freedom.

Manipulators differ from mechanisms in that they have more than one degree of freedom. Manipulators can be broadly classified into serial (open-loop) and parallel (closed-loop) devices. Variable geometry trusses (VGTs) are a subset of parallel connection devices. What exactly constitutes a VGT is discussed in greater detail in Chapter 3. A deployable structure is a structure which is capable of being deployed, e.g. an umbrella. An adaptive structure is a controllable structure capable of modifying itself to meet certain needs, e.g. the Hoop-Column antenna described by Miura and Furuya [16]. Note that adaptive and deployable do not necessarily mean the same thing. An adaptive structure must have actuated links and some means of control. A deployable structure must simply open and close. A more subtle distinction exists between an adaptable truss and a variable geometry truss robot. A VGT that performs its functions through an end-effector can be classified as a VGT robot. Examples of such are the pyramid-pyramid(PP) truss (refer to Fig 3) equipped with some kind of an end-effector. Structures like the hoop-column antenna using a VGT as a central deployable column have no obvious end-effectors as such and could be classified as deployable trusses.

Exactly what constitutes a robotic manipulator or industrial robot is sometimes debated. The distinction between kinematic linkages and robotic manipulators lies in the sophistication of the programmability of the device. Craig [7] defines a robotic manipulator as " a mechanical device that can be programmed to perform a wide variety of applications ". A kinematic linkage, on the contrary, cannot be programmed in such a manner.

Serial and Parallel Manipulators

Many researchers believe that parallel manipulators offer significant advantages over serial manipulators, and that serial manipulators may be eventually replaced by parallel manipulators in certain applications. Solving the inverse kinematics problem for parallel manipulators in closed form may cause the most difficulties. Researchers like Hunt [13] and Fichter [10] talk about serial manipulators and weaknesses that restrict their potential applications. It is important to realize that comparisons between serial and parallel configurations are meaningful only in the context of a particular property or application.

There are several reasons why the series connected manipulator arrangement appeared first. It was an obvious attempt to simulate the anthropomorphic design of the human arm. All of the primary joints are powered, giving a direct relationship between the number of joints and the number of degrees of freedom of the end effector. Tesar and Cox [24] have documented the pros and cons of serial and parallel manipulators in great detail. Some of the factors mentioned by them and other researchers are listed below. This is by no means a quantitative analysis. Rather it is a subjective treatment of the possible merits of parallel kinematic arrangements.

- Workspace

This is an application-dependent criterion that influences design in instances where a specific task needs to be performed. Workspace is a measure of the range of motion of a device. For example, it could be the farthest reach of a serial robotic manipulator. In most cases it is desirable to maximize this property. Due to the additional kinematic constraints placed on multi-loop devices, they generally have restricted ranges of motion compared to the serial devices.

- Rigidity of the Structure

The potential for increased strength and rigidity is perhaps the strongest argument in favor of parallel devices. Since parallel devices are able to distribute loads among several links, they can retain their rigidity even when the configuration is being transformed. When the serial manipulator is subjected to loads, each link behaves like a cantilever beam, which is the most inefficient form of a structure. As a result, there are severe limitations on lifting capacity. In general, serial linkages have higher weight-to-stiffness ratios compared to parallel linkages.

- Kinematic Indeterminancy

Another disadvantage of open kinematic chains is that the positions of links are usually indeterminate. This means that with the gripping mechanism in one position the other links can be in any one of several positions. This property causes a problem in the control of the manipulator. Fig.(2) shows how for one position of the gripper C, there are two possible positions of joint A. The control system is normally given the position where the gripper must be, and it must decide how to position each of the joints. This is more difficult when many joint positions give the same gripper position. The robot must have explicit instructions on where is should go and where it should not. The parallel manipulator suffers from similar multiple position problems; that is, given a set of values for the joint variables, there may be multiple positions of the end effector. This is discussed by Fichter [10] along with a sketch of a parallel manipulator which demonstrates this problem.

- Compactness

When a robotic manipulator is being used as an adaptive space structure, an important consideration is how compactly the robot can be stored. The paper by Miura [15] discusses the sequential mode deployment of the octahedral or pyramid-pyramid truss. A geometric transformation in a sequential mode is one in which each module is deployed successively and and other modules remain fixed. For clarity of explanation, the whole truss is divided into three zones - the deployed zone, the transient zone and the folded zone. When the *i*th module contracts, the *(i-1)*th module deploys partially and the rest of the modules remain unaffected. Many forms of the VGT can be made to collapse and hence can be stored very efficiently.

- Stability

Another problem common to both serial and parallel manipulators is stability.In a serial manipulator there are singular positions where the number of degrees of freedom get reduced from the general case and the manipulator suffers from an instantaneous loss in mobility. The parallel manipulator also suffers from the problem of singularities. For some given sets of joint coordinates, it is possible for a parallel manipulator to loose or

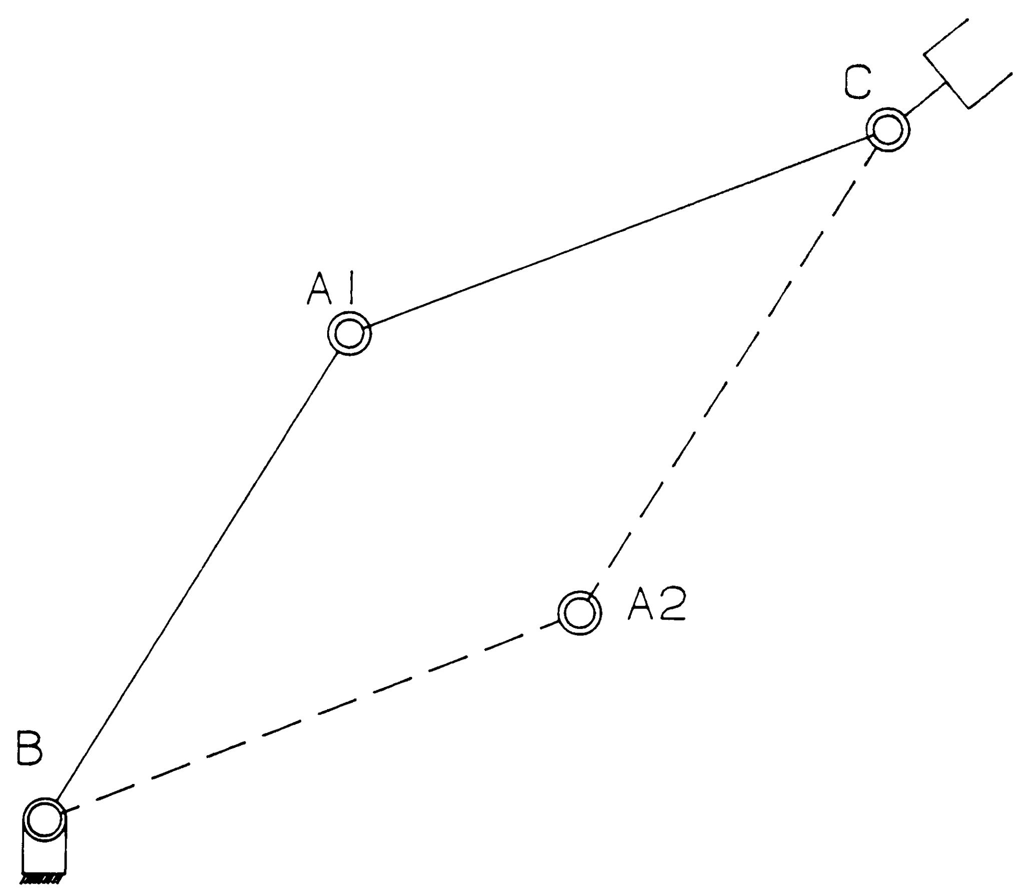


Figure 2. Kinematic Indeterminancy of Serial Robots

gain one or more degrees of freedom. These singular sets of joint coordinates must be avoided or used creatively.

Many researchers like Miura and Furuya [16] and Sincarsin and Hughes[21] have provided convincing arguments in favor of parallel deployable trusses. Literature on VGTs has covered many interesting problems like forward kinematics, dynamics, control, mechanical design, etc. Yet there has been no conscious effort to formulate a definition or conceptual model for the VGT. This is the major emphasis of this thesis. How does one recognize a VGT? How does one differentiate a VGT configuration from the prevalent configurations for kinematic linkages? This is not an easy question to answer. The effort in this thesis is to lay down a set of rules to define a VGT, derive equations for degrees of freedom based on the model, and test the model for a sufficiently large number of configurations to convince oneself about the validity of the rules and the mobility equations.

Chapter 2

Literature Review

The concept of VGTs can be traced back to a paper by Stewart [22] presented in the Proceeding of the I. Mech. E.(London). Stewart proposed a mechanism, based on a regular octahedron, for a flight simulator. This mechanism came to be popularly known as the Stewart's Platform(Fig.1). It is discussed in greater detail in the next chapter.

Although many researchers had worked on the concept of adaptive structures, the most concrete assertion of the idea came from a paper by K.Miura and H.Furuya [16] The paper defines an adaptive structure as " a structure that can purposefully vary its configuration(and physical properties) arbitrarily in space while retaining its inherent stiffness during the transformation".

Another important point can be discovered from a review of literature on VGTs. Almost all of the researchers appear to be convinced that a particular configuration or set of configurations define the scope of deployable structures. Referring to the paper by D.C.H. Yang and T.W.Lee [27], we find the following quote, "It is found that the 6-SPS, or the Stewart's platform appears to be the only mechanism of its type that can possible be adopted as as general maneuverable device." These statements reflect the general implication of the researchers that, as far as robotic manipulators are concerned, a few geometric configurations stand alone. This thesis will demonstrate that many arrangements of VGTs are possible.

It would be interesting to review some of the literature on mobility. Published in 1978, a book by K.H.Hunt [13] presents an explanation of the concepts of mobility, relative mobility and connectivity based on principles of geometry and kinematics. Hunt prefers the concept of screw coordinates to define general kinematic motion(translatory and rotary). Equations of mobility are derived by imposing independent constraints over the bodies in a kinematic chain. The book by Mabie and Reinholtz [14] provides a documentation of the different types of joints and the relative degrees of freedom they permit. Researchers like Fichter and McDowell [11] and Yang and Lee [27] have presented mobility equations for analyzing deployable trusses. The equations by Yang and Lee [27] are in terms of connectivity between links in a mechanism. In another reference, Fichter [10] presents an idea that the Stewart's platform is constructed of triangle planes. All along researchers have intuitively thought about a definition of what could be described as a variable geometry truss. Unfortunately none of them have attempted to lay down a precise definition from the kinematic/geometric point of view. The significance of prescribing a set of restrictions to define a VGT may not be immediately apparent. This may perhaps be the reason for their lack of interest in this problem.

One finds a great deal of research being done in the area of constraints and mobility. Researchers are attempting to set up analytical methods to accurately compute the mobility and relative degrees of freedom of all classes of mechanisms and structures. The author wishes to elucidate some of the more important contributions in this area. In a paper [26] which can be regarded as a modern classic, Waldron discusses the concepts of constraint, mobility and relative degrees of freedom in a kinematic chain. He highlights the shortcomings of earlier mobility criteria and also points out the fact that the mobility of a practical mechanism is less important than the number of relative degrees of freedom between two specific links, such as the input and output members. In another paper, [4], Baker attempts to provide a set of analytical procedures to determine the relative degrees of freedom of a kinematic linkage. The work is inspired by Waldron's paper, [26]. A direct algorithm is presented to establish the relative degrees of freedom between two links. Another paper [3] by Baker attempts to place the notion of mobility in perspective with the algorithm presented in [4] and demonstrates the applicability of this algorithm to structures.

Researchers like Rhodes and Mikulas [18] and Sincarsin and Hughes [21] have expressed serious reservations about traditional deployable beam concepts. A major drawback to most of these concepts is that they require a relatively complicated deployment mechanism to unfold the beam structure. Following deployment, the deployer mechanism serves no purpose. The papers recommend the controllable/variable geometry truss (VGT). This does not require a special deployer mechanism and does not have to be deployed in a straight line along its longitudinal axis. The controllable geometry feature of VGTs is that by changing the length of a control member the longitudinal axis of the beam can be deformed in a predictable manner. Variable geometry allows the beam to correct for alignment errors.

The study conducted by Sincarsin and Hughes [21] examines four candidate geometries for VGTs in some detail. The engineering considerations taken into account include configurational constraints, joint simplicity, kinematics and singularities. This paper uses a mobility equation similar to the one used by Miura and Furuya [16]. On the basis of all the considerations the authors report that the pyramid-pyramid (PP) truss clearly excels and is recommended as the most suitable truss geometry. A more detailed review of this study is presented in the next chapter. The paper by Nayfeh [17] does a detailed kinematic analysis of the PP truss. A computer program is developed which maps the geometry of the VGT once controlling parameters are specified.

In the period 1982-83, there were four major studies conducted for NASA by Vought Corporation [5,6,25] and Rockwell International [19]. The Executive Summary Report [25] summarizes an 18 month study of deployable structures for large space platform systems conducted by Vought Corporation for the NASA George C. Marshall Space Flight Center. The work was performed in two parts. Part I [5] involved the review and generation of candidate deployable linear platform system concepts suitable for development to technology readiness by 1986. The deployable volume studies involved generation of concepts for deployable volumes which could be used as unpressurized or pressurized hangars, habitats and interconnecting tunnels. Concept generation emphasized using flexible materials and deployable truss structure technology. Part 2 [6] involved layout design of a ground test article based on the results of the concept selection from Part 1. The design was to meet the specification for a prior NASA-MSFC ground test article simulating a Science and Application Space Platform (SASP) arm. An aluminium structure design was derived from the Part 1 graphite/ epoxy flight article conceptual design. Deployable volume effort during Part 2 focussed on evolving the selected Part 1 truss/bladder concept for the habitat and hangar modules.

The studies conducted for NASA by Rockwell International [19] and Cox and Nelson [5,6] evaluate eight candidate structures and large deployable volumes such as Habitat modules, Tunnels and OTV hangers. A paper by Mikulas and Rhodes [18] expresses reservations about the proposed candidates on the grounds that they require a relatively complicated deployment mechanism to unfold the beam structure. These mechanisms must deploy the beam in a straight line along its longitudinal axis and provide structural support to the partially deployed beam. Following deployment, the deployer mechanism serves no purpose. The paper presents a case for the controllable geometry truss, discusses its geometric configuration in detail and develops an analysis of deployment of the truss.

Chapter 3

A Conceptual Model for the VGT

Referring to the previous chapter, it can be seen that, although researchers have studied the geometry, kinematics and dynamics of VGTs, none of them have attempted to precisely define what constitutes a variable geometry truss. Most of the researchers restrict themselves to studying the Stewart's platform or the more common PP truss(Fig.2). As mentioned before, Sincarsin and Hughes [21] have presented a study on the following four different geometric configurations for a VGT :

- P-P (pyramid-pyramid) truss;
- T-T (tetrahedron-tetrahedron) truss;
- P-T (pyramid-tetrahedron) truss; and
- C-C (cube-cube) truss.

Figures (3), (4), (5) and (6) represent the geometric configurations of the PP, TT, TP and CC trusses respectively.

The TT truss is formed of regular tetrahedrons placed on top of one another. The TP truss differs from the TT truss in that the tetrahedrons are irregular. When we consider these configurations as VGTs, where it is possible to make any of the link lengths variable, we find that the TT and TP configurations are kinematically equivalent. It is possible to convert one to the other by adjusting link lengths. The Stewart's Platform is essentially a rigid platform connected to ground through six SS links. The PP and TT trusses can be derived from this structure by coinciding different joints. These observations indicate that these different VGT structures are essentially kinematically equivalent.

Based on these candidates, the authors have selected the PP, or the octahedral truss as the most suitable design. But they do not consider the use of such a truss as a manipulator, where closed form solution of the forward and inverse kinematic equations is of primary importance.

In addition, studies by Miura [15] and Sincarsin and Hughes [21] do not make clear whether other possible geometries exist and, more importantly, they do not make clear which links of the candidate static trusses may be actuated and what effect this will have on the final design. A great deal of study remains to be done in this area. It would be unwise to assume that a few selected configurations can cater to the multitude of applications foreseen for VGTs. The process of designing and analyzing a particular VGT configuration would be completely intuitive unless there is either a model or a precise set of rules to be guided by. The present research effort attempts to formulate a method for designing viable geometries for VGTs.

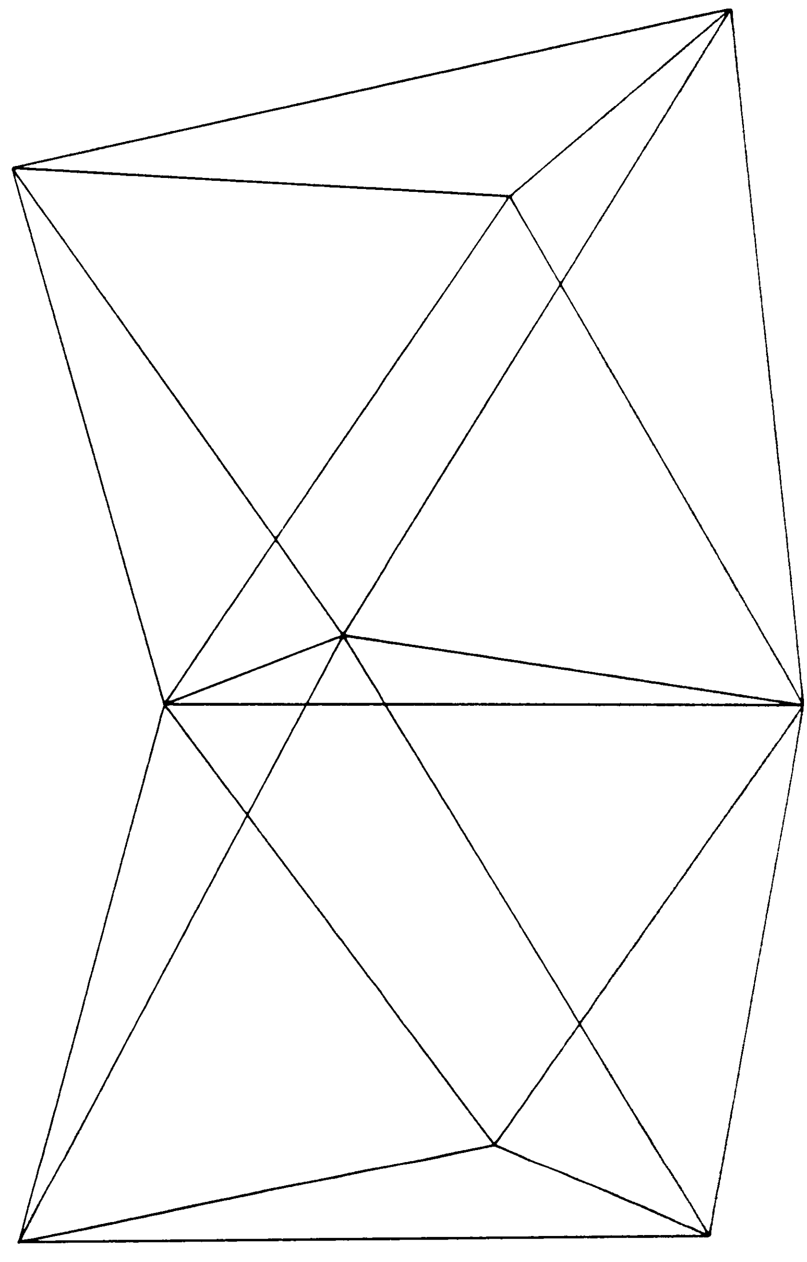


Figure 3. Pyramid-Pyramid (PP) Truss

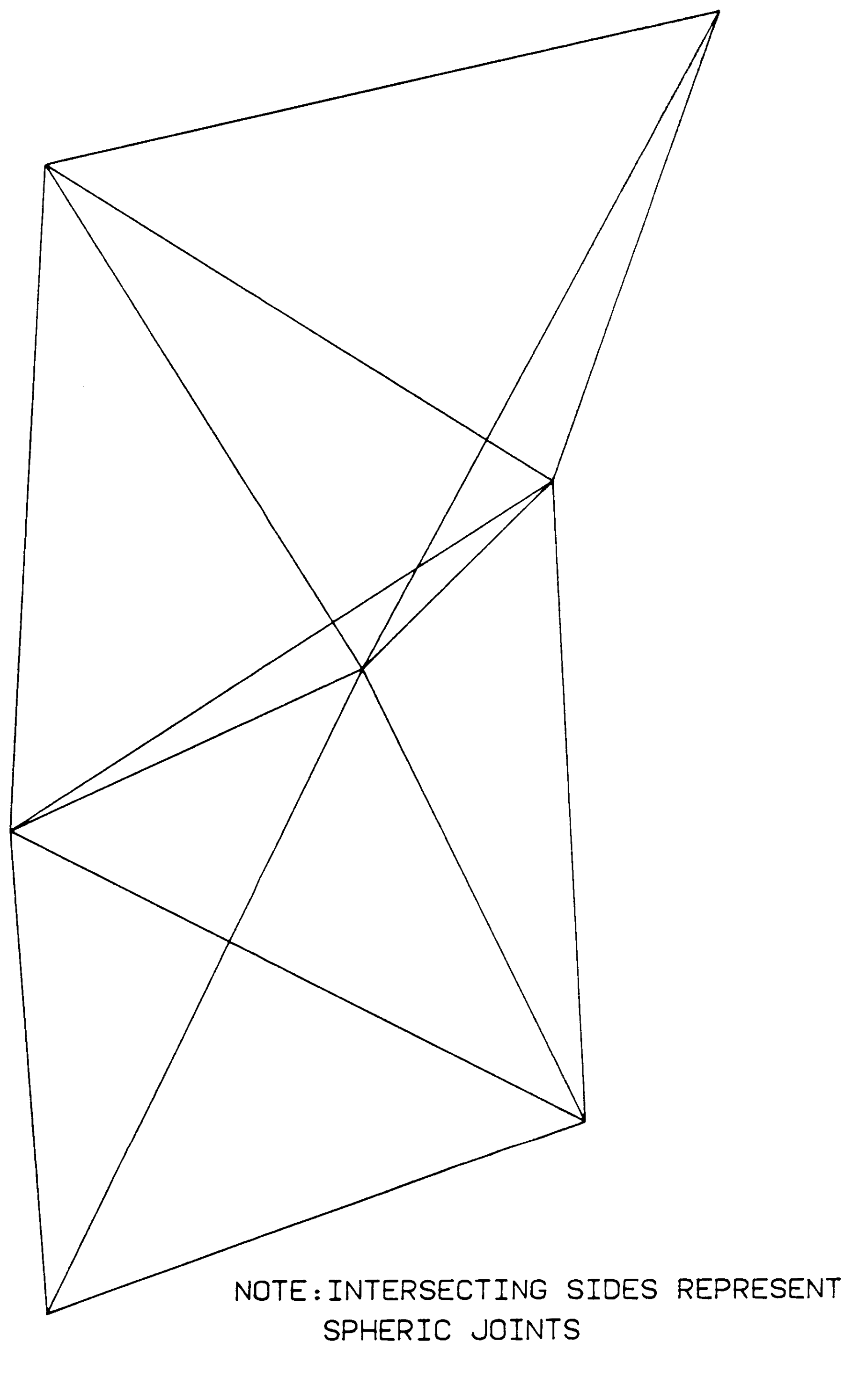


Figure 4. Tetrahedron-Tetrahedron (TT) Truss

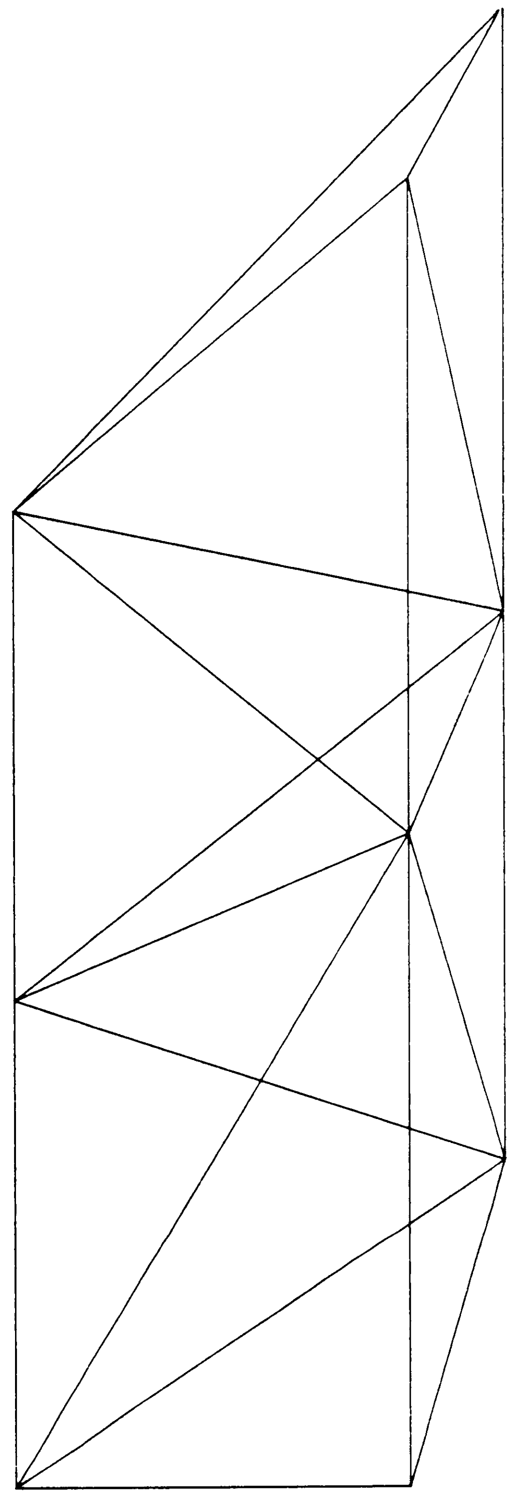


Figure 5. Tetrahedron-Pyramid (TP) Truss

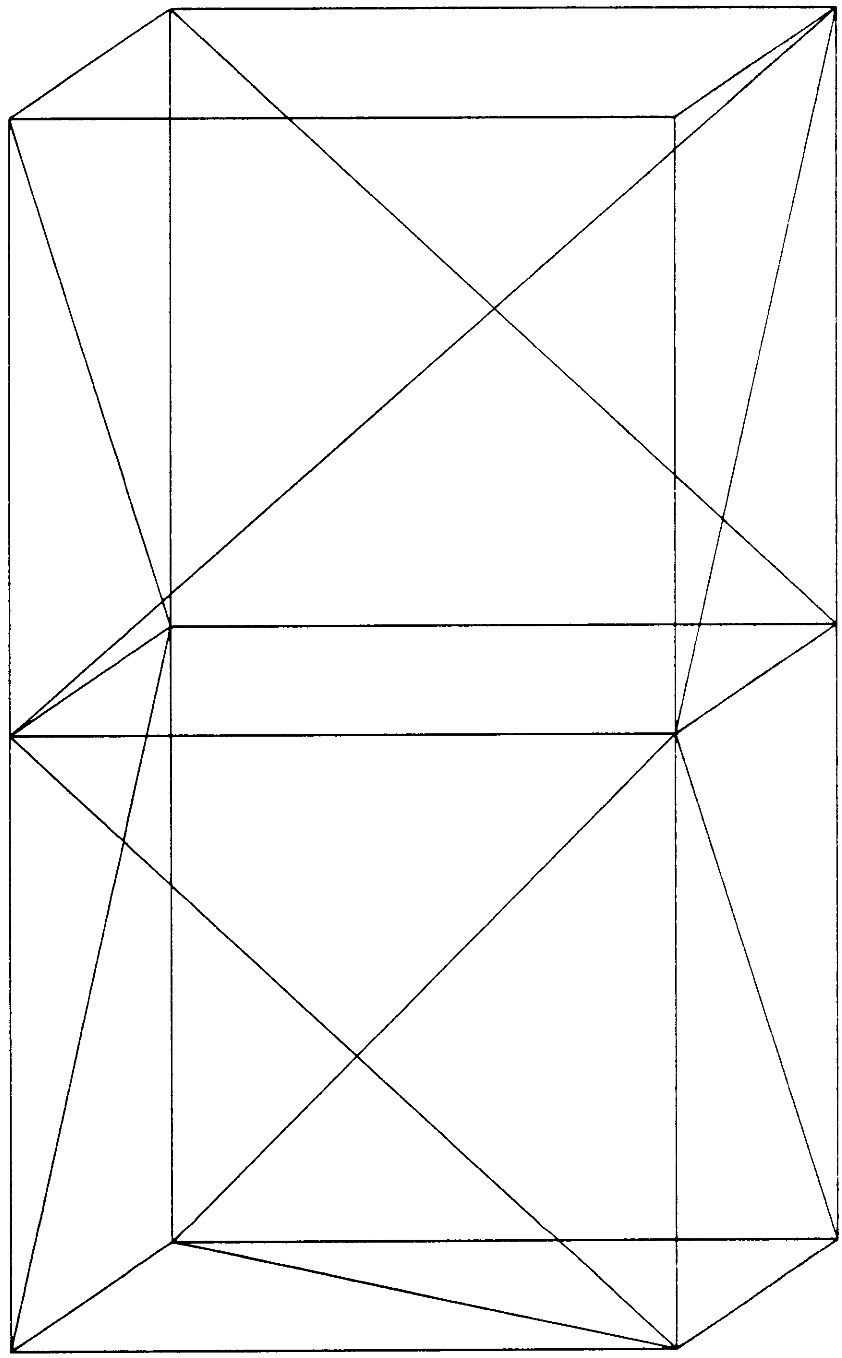


Figure 6. Cube-Cube (CC) Truss

Theory of triangulation

Schodek defines trusses as an assemblage of individual linear elements arranged in a triangle or combination of triangles to form a rigid framework that cannot be deformed by the application of external forces without deformation of one or more of its members. The individual members are rigid and are joined at their intersections by revolute joints in the plane and spheric joints in space. The members are so arranged that the loads and reactions occur only at the intersections.

Arranging truss members into triangular configurations results in a stable shape. By stable we mean that any deformations that occur in the structure are associated with changes in member length (typically, very small) caused by forces generated in the members by the external load. Bending is neither present nor can be developed if the loading is at the joints alone.

In an attempt to define VGTs in the most basic terms we propose the following rules:

1. VGTs are composed exclusively from the four types of "triangle planes" (shown in Fig.7).

2. Every triangle plane has at least one side common with another triangle plane or with the ground or must be attached at all three apexes to other triangle plane apexes or to the ground.

3. Common sides between triangle planes allow a single degree of freedom i.e. rotation about the common side.

4. Triangle planes joined at their apexes allow arbitrary rotation in 3-D space i.e. the spheric joint permits three degrees of freedom in space.

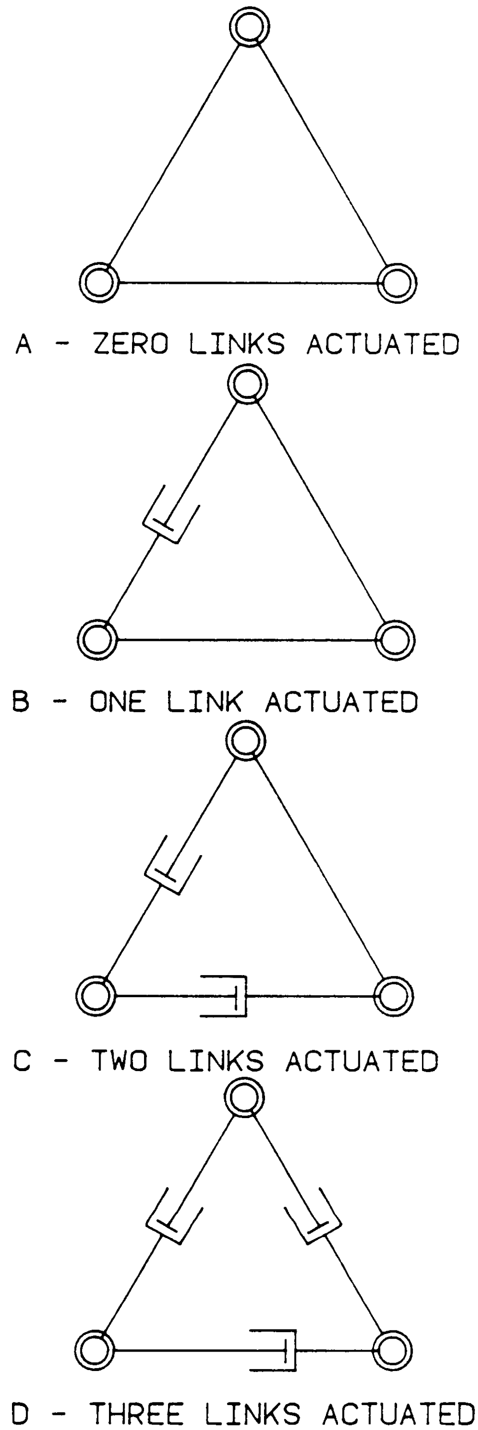


Figure 7. Triangle Links

Different joints (planar and spatial) and the relative motions they permit are illustrated in Table 1 and 2.

Based on the rules presented, we proceed to derive mobility equations for VGTs. Why do we need to derive new equations and not use the Grubler's or Kutzbach's criterion? These are a few arguments in favor of developing new mobility equations:

- A mobility equation specifically for VGTs will be a useful tool in type and number synthesis of VGTs.
- As an indicator of the validity of the rules prescribed for VGTs.
- Grubler's and Kutzbach's criteria include the idle degrees of freedom in the overall mobility of the VGT. These freedoms do not represent useful mobility and are often confusing.
- Traditional criteria for mobility fail to predict mobility accurately for certain special geometries.

Before proceeding to develop mobility equations based on these rules, a few assumptions and restrictions have to be made clear. For the case of simplicity, we assume that all joints between triangles links in space are spheric. Note that two spheric joints connecting the same side is equivalent to a revolute joint about that side. Analogous to this assumption, we also restrict triangle links in the plane to be connected by revolute pairs. The mobility equation developed in the next chapter can be easily modified to accommodate other types of connections. Connecting triangle pairs by means of revolute joints would constrain the structure more than spheric joints.

An actuated link is thought of as a component of a link of variable length instead of two links connected by a prismatic joint.

Table 1. Joints in Planar Mechanisms

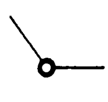
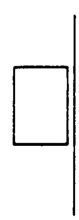
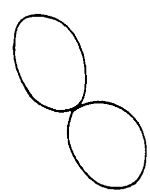
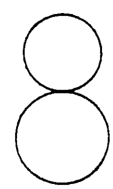
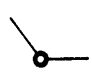
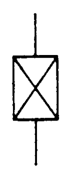
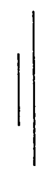
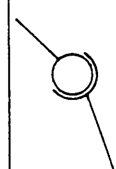
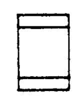
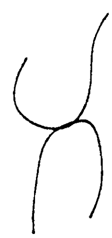
JOINT TYPE (SYMBOL)	SCHEMATIC REPRESENTATION	DEGREES OF FREEDOM
REVOLUTE (R)		1 (PURE ROTATION)
PRISMATIC (P)		1 (PURE TRANSLATION)
CAM OR GEAR		2 (ROLLING AND SLIDING)
ROLLING CONTACT		1 (ROLLING WITHOUT SLIDING)

Table 2. Joints in Spatial Mechanisms

JOINT TYPE (SYMBOL)	SCHEMATIC REPRESENTATION	DEGREES OF FREEDOM
REVOLUTE (R)		1 (PURE ROTATION)
PRISMATIC (P)		1 (PURE TRANSLATION)
CYLINDRIC (C)		2 (ROTATION AND TRANSLATION)
SPHERIC (S)		3 (X,Y AND Z ROTATIONS)
SCREW (H)		1 (HELICAL MOTION)
SPATIAL		5 (PREVENTS TRANSLATION ALONG NORMAL)

The triangle link is thought of as the basic building block for any particular configuration. This results in two-force members only in static loading. The VGT members would be subject to static forces in tension and compression. This is the main reason for the enhanced rigidity of VGTs. Any VGT can be constructed from triangle links.

Another important point to illustrate here is the way the triangular links are counted/accounted for. When analyzing a VGT, one must count the least number of triangular links necessary to account for the entire structure. There would be redundancies in the calculation of mobility if more than the absolutely necessary number of triangles are counted. In some of the configurations these considerations do not come into play but in cases like the TT or TP trusses it is necessary to be careful in the manner in which the triangle planes are accounted for. The main reason for this restriction is that some of the triangle planes are not the links with which the VGT was constructed but are formed as a result of the particular geometry. The important question is whether there can be an intelligent algorithm which can recognize the triangle links in a VGT. One method would be to store the nodes of the truss in some kind of a tree structure. The algorithm would traverse the tree and list all possible combinations of triangle planes to account for the structure. Each combination could be stored in a table and the algorithm could decide on some optimum combination. Such an algorithm would be best implemented by an artificial intelligence language like PROLOG.

One of the earlier proposed configurations for VGTs is the Stewart's platform. Fig. 1 shows a model of the Stewart's platform in one of its simpler forms. It is a regular octahedron with two opposite faces as a rigid base and rigid platform. The other six edges that outline the faces are linear actuators. The Stewart's platform has six degrees of freedom not counting the idle rotation of the grounded SS links about their axes. The movable rigid platform can be manipulated into any position and orientation within the workspace of the structure. As shown, the Stewart's Platform is not a VGT since a VGT is made up entirely of triangle links which account for all of the links. From a kinematic point of view, joints 1 and 2 can be made

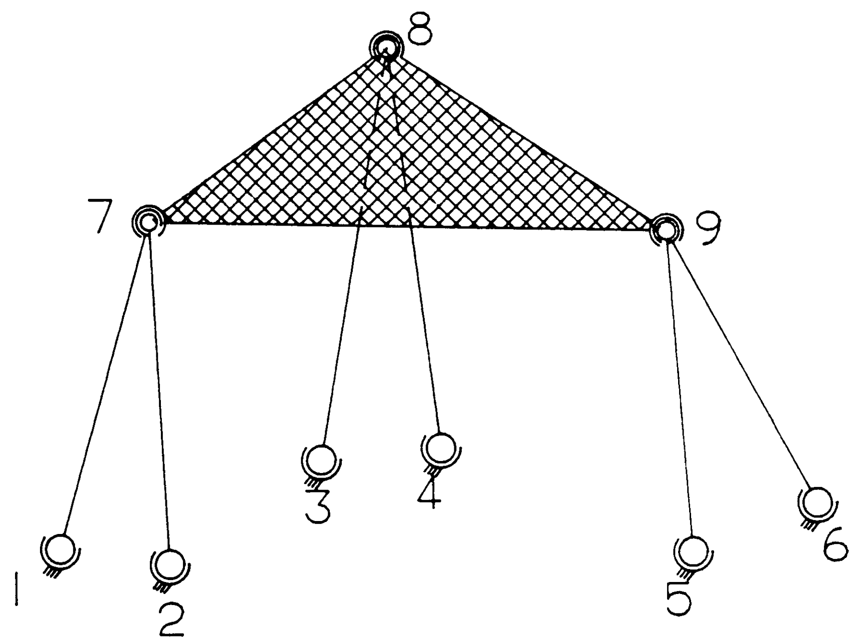


Figure 8. Kinematically Equivalent Stewart's Platform

to coincide without affecting the mobility or the motion characteristics of of the structure. The same argument can be applied to joints 3 & 4 and 5 & 6. Now we can consider the structure to be built up from triangles (refer to Fig.(8)). The point to be made here is that even though a geometry does not physically resemble the model, it can be converted to a kinematically equivalent model which confirms to the specifications.

Chapter 4

The Theories of Mobility

Presentation of the Different Approaches to Mobility

This chapter attempts to document and analyze the work on mobility to date. Interestingly, although the definition of mobility is agreed upon universally, researchers disagree on the analytical tools to measure it. Present-day researchers tend to believe that connectivity, or relative degrees of freedom, is a more important indicator than the total mobility of the linkage itself. Before attempting to derive equations for VGTs based on the model presented in the previous chapter, it is relevant to look at some of the ways different researchers have tackled the problem of mobility analysis. The equations derived in the present thesis are restricted to VGTs. Some of the researchers like Fichter and McDowell [11] and Yang and Lee [21] have presented equations that apply to VGTs. The results of those efforts and this thesis do not conflict but are supplementary to each other.

By definition, the mobility of a mechanism is the number of degrees of freedom it possesses. The most commonly offered definition of mobility (also the one presented by Mabie and Reinholtz [14]) is " mobility is the minimum number of independent parameters required to specify the location of every link within a mechanism ". The derivation of Grubler's and Kutzbach's mobility equations for planar and spatial mechanisms closely matches the definition. It is interesting to follow the derivations of these equations and to compare them with the other formulations presented. Both the Grubler's and Kutzbach's equations fail for certain special geometries ,some of which are documented by Mabie and Reinholtz[14].

Planar Mobility

A single link constrained to move in a plane has three degrees of freedom. For example, the x and y coordinates of the point P along with the angle θ form an independent set of parameters describing its location. If we consider two links in a plane, the system as a whole has six degrees of freedom (three for each link). If the two links are connected by means of a revolute joint, this has the effect of removing two degrees of freedom. We also need to consider joints that permit two degrees of freedom, like the cam or gear. Recognizing that one link of every mechanism will always be considered to be fixed to ground removes three degrees of freedom. This leaves the system with a total of 3(n-1) degrees of freedom. Every one-degree-of-freedom joint removes two degrees of freedom from the system. Similarly, each two-degree-of-freedom joint removes one degree of freedom from the system. The total mobility of the system is given by Grubler's equation

$$M = 3n - 3 - 2f_1 - f_2 \qquad [1]$$

where

M = mobility, or number of degrees of freedom

n = the total number of links, including the ground

f_1 = the number of one-degree-of-freedom joints

f_2 = the number of two-degree-of-freedom joints

Table 1 documents the common type of joints found in planar mechanisms and the relative freedoms they permit.

Spatial Mobility

Many of the concepts used in the mobility analysis of planar manipulators also apply to the spatial case. In space, each link will have six degrees of freedom (three translations and three rotations). Therefore, connecting two spatial links with a joint having one degree of freedom has the effect of removing five degrees of freedom. Similarly, connecting two links with a two-degree-of-freedom joint has the effect of removing four degrees of freedom, and so forth. One link of a spatial mechanism will have all six degrees of freedom removed because it is fixed to ground. The total mobility of a system of n interconnected spatial links is therefore given by the following equation :

$$M = 6(n-1) - 5f_1 - 4f_2 - 3f_3 - 2f_4 - f_5 \qquad [2]$$

where

M = mobility, or number of degrees of freedom

n = total number of links, including the ground

f_1 = number of one-degree-of-freedom joints

f_2 = number of two-degree-of-freedom joints

f_3 = number of three-degree-of-freedom joints

f_4 = number of four-degree-of-freedom joints

f_5 = number of five-degree-of-freedom joints

In planar mechanisms, only four types of joints, or pairs, are commonly used :

1. Revolute joints

2. Prismatic joints

3. Rolling Contact joints

4. Cam or Gear joints

In the case of revolute, prismatic, and rolling contact joints, each has one degree of freedom, while the cam, or gear, joint has two degrees of freedom. Many other joint types are possible in spatial mechanisms. The most common of these include the screw, or helical, joint (one degree of freedom), the cylinderic joint (two degrees of freedom), the spheric, or ball-and-socket, joint (three degrees of freedom), and the spatial cam joint (five degrees of freedom).

Another Interpretation of the Concepts of Mobility and Connectivity

The book by Hunt[12] presents a different interpretation and analytical yardstick for studying mobility. At least six independent parameters are needed to locate a rigid body in space. These are usually three translations and three rotations in space. Hunt stresses the fact that these freedoms do not necessarily need to be three rotations and three translations. These could instead be replaced by five lengths and one angle or two direction-angles for a line, three lengths and an angle. Any combination of six geometric quantities which uniquely describe the location of the rigid body in space are acceptable. But whatever system is chosen, no fewer than six coordinates are required to locate the body. If there are more than six, say $6+m$, then it is always true that there are m identities which interrelate the six coordinates leaving only six independent coordinates which, if they are all removed, leave the body with six independent freedoms. The commonly prescribed theory is a narrow and even misleading description. One of the reasons for this is that the six coordinates are not always homogenous, besides it is not always easy to make them lengths or angles.

True kinematic motion is rarely translatory or rotary, instead it is a combination of both. For kinematic generality we need a motion that is neither purely translational or rotary. This generality lies in the screw motion. When a rigid body moves so that each point of the body has motion of rotation about a fixed axis and at the same time has translation parallel to the axis, the body has helical motion. An example of helical motion is the motion of a nut as the nut is screwed onto a bolt. We need to provide six screw coordinates. Let (x1,x2,x3,x4,x5,x6) be the set of six screw coordinates describing a rigid body in space. A body is fixed when values are attached to all six of them, but the physical constraining system to which the body is subjected is unlikely to have for every constraint exactly one corresponding independent screw coordinate. There are six simultaneous equations in six unknowns in terms of x1,x2,.....,x6. Hence we have the a system of six independent constraint equations:

$$f_i(x1,x2,\ldots\ldots,x6) = 0,\ i = 1,6 \qquad [3]$$

As the constraints are progressively relaxed one by one, so the corresponding constraint equations can be eliminated one after the other, the body acquiring one,two,.... degrees of freedom, until eventually with no constraints at all (no equations) the body is completely free with its six freedoms. If the number of constraints, or unfreedoms, is denoted by u, and the number of freedoms by f, then

$$u + f = 6 \qquad [4]$$

As mentioned before in the literature review, papers by Waldron [26] and Baker [4] present novel concepts and analytical procedures for the study of mobility. The algorithm presented by Baker uses the concept of motor screw algebra. Initially, it is necessary to define and explain connectivity, which according to Waldron [26] and Baker [4], is a more important concept than mobility of a mechanism as a whole.

Every kinematic pair has its characteristic number of degrees of freedom. In mechanisms we are often interested in relative movements of members which do not touch one another directly. To determine the number of degrees of relative freedom between two such members we have to consider the freedoms contributed by several kinematic pairs that connect them. For the 4-bar linkage, the connectivity between the input and output link is one. However if the revolute pairs are replaced by spheric pairs, an additional spin-freedom about the SS axis is introduced, and the connectivity between them is now two.

Waldron and Baker present a strong case for using connectivity rather than mobility. Baker [4] develops an algorithm for finding the relative degrees of freedom of a kinematic linkage based on Waldron's theory and principles of motor screw algebra. It is interesting to look at the concepts of mobility, relative freedoms and connectivity as presented by Waldron [26].

Consider two rigid bodies, each of which has a coordinate frame fixed to it. If the position of every point of each body is known with respect to the coordinate frame fixed to that body, then it may be determined with respect to the coordinate frame fixed to the other body, if the transformation from one coordinate system to the other is known. In three dimensional space, six independent parameters are required to define such a transformation.

Some or all of the transformation parameters may be specified by the imposition of physical constraints on the relative motion of the two bodies. Waldron [26] defines the number of relative degrees of freedom of two bodies as the " number of transformation parameters which remain to be satisfied after the imposition of physical constraints".

A joint between two bodies may be defined to be a set of such constraints. Connectivity of a joint can be defined as the number of relative degrees of freedom of the bodies connected by that joint. The mobility of a mechanism may be defined to be the number of transformation parameters of joints of the mechanism which are required to determine the position of every point of every member with respect to a coordinate frame fixed to one of the members.

Chapter 5

Derivation of Mobility Equation for the VGT

Mobility is simple in concept but often difficult to apply in practice, especially as an aid in design. The basic definition of mobility(quoted in the previous chapter) is a perfectly satisfactory one. It is the determination of mobility that has been the stumbling block. The history of the constraint analysis of rigid body mechanisms and the related subject of number synthesis dates back to last century when Chebyshev in 1870 and later Grubler proposed formulae for the mobility of planar mechanisms. A formula for the mobility of general spatial mechanisms appeared about 1930 and is usually attributed to Kutzbach. The equation appears in different forms but these forms can be easily derived from one another. One form of the equation uses the concept of linkage mobility(number of degrees of freedom of the whole mechanism) whereas another form uses the concept of connectivity(relative degrees of freedom of one link with respect to another link). It has been recognized that this equation does not hold in either of its forms for a number of mechanisms. The criterion works for gross mobility of single-loop planar, spherical and true spatial linkages free of geometrical specialties. The equation is known to fail for certain multi-loop chains, overconstrained linkages or special configurations. Some examples of the final category are illustrated by Mabie and Reinholtz [14].

Another interesting point about the Kutzbach equation applied to a spatial VGT configuration can be illustrated by the following example. Consider the PP truss in Fig.(3). Applying Kutzbach's equation to this truss, we have

$n = 12$

$f_1 = f_2 = 0$

$f_3 = 6(3) = 18$

$f_4 = f_5 = 0$

Since we are considering the geodesic truss to be free-floating in space, we modify Kutzbach's equation and apply it to the truss. We do not include the term which accounts for one of the links fixed rigidly to ground. We have

$m = 6n - 5f_1 - 4f_2 - 3f_3 - 2f_4 - f_5$

$m = 6(12) - 3(18) = 18$

At first sight, this result seems astonishingly different from the predicted mobility. Any free-floating rigid structure in space should have a mobility of six. The reason is that the Kutzbach equation considers the idle degrees of freedom of the links. For example, an S-S link can rotate about itself. This rotation is counted as idle because it does not contribute to the useful mobility of the mechanism. Thus for twelve rigid links we have 12 additional rotations.

The equation derived in this chapter for the mobility of VGTs eliminates these idle freedoms of the VGT. The reason is that three rigid links are considered to be a single "triangle link", the relative freedoms of links between themselves in a triangle link are not considered. A triangle configuration is the simplest no-constraint no-freedom geometry. If the "triangle link" is made of three S-S links, then our mobility equation neglects the three idle rotations of the links about themselves. Hence, this equation would predict the same mobility for a triangle link made up of three S-S links as it would for a triangle link made of three R-R links. This property of the equation turns out to be an advantage, since the idle degrees of freedom serve no purpose and are often confusing as the previous example demonstrates.

Derivation of the Mobility Equation in the Plane

A rigid body in a plane has three degrees of freedom : translations in the x and y directions and rotation about the z axis. If one of the sides of a planar triangle link is fixed, it looses all of its freedoms. If the triangle link is connected to ground through one of its joints, it looses the two translations. For the reason of simplicity, we assume all connecting joints are revolute. Revolute joints permit one rotation between the links they connect.

If a triangle link shares a side with another, it looses all of its freedoms (3). If it is connected to another triangle link through just an apex joint, it looses two translational freedoms. Every actuated link adds a freedom to the mobility of the linkage. As in the case of the spatial truss, we consider actuated links to be a single link and not two links connected by a sliding pair.

Taking into consideration all of the conditions we come up with a mobility equation for the planar VGT:

$$M = 3n + p - 3q - 2r \quad [5]$$

n is the number of triangle links

p is the number of actuated links

q is the number of shared sides

r is the number of apex joints

Derivation of the Mobility Equation in Space

A detailed explanation of the derivation of the mobility equation for VGTs is presented in this section. This equation will be used to predict the mobility of any spatial VGT configuration that can be developed using the rules enumerated in Chapter 3.

A triangular link floating freely in space has six degrees of freedom. The triangle link is made up of three links connected to one another through spherical joints. If this triangular link is connected to another triangular link through a side, then the connection permits a single rotation about that side. Hence, we have to deduct five degrees of freedom for two triangular links sharing a side. The same constraint would apply to a triangular link sharing a side with the ground link. It should be noted that a triangle link can share zero, one, two or three sides with other triangular links.

Another type of connection is an apex joint between two or more triangle links. This is illustrated in Fig.(9). An apex joint is formed when a triangle plane connects with another triangle plane or the ground through a single joint. This has the effect of eliminating the relative

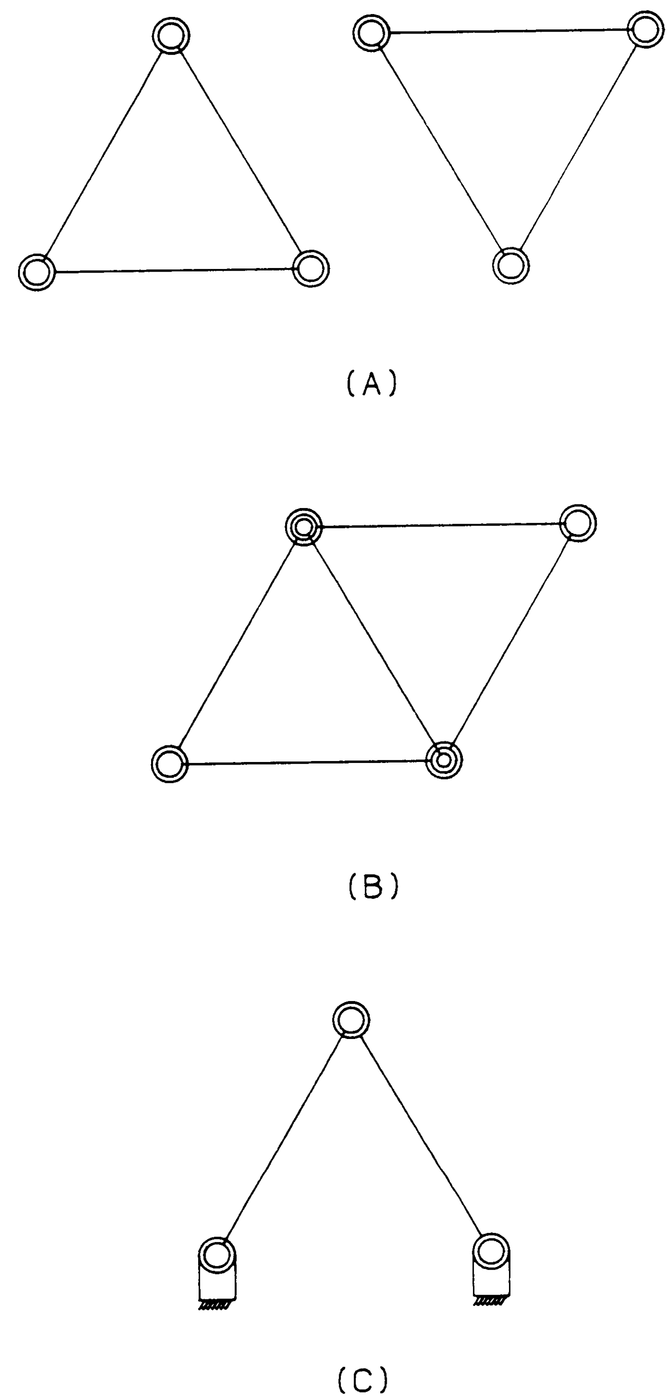


Figure 9. Shared Sides of Triangle Links

x,y and z translations of two triangle links even though it permits the three rotations. To account for these, we deduct three degrees of freedom for every apex joint.

An interesting point can be illustrated about apex joints. Referring to Fig.10, we see that the system in Fig.10(a) has nine degrees of freedom (six rigid body freedoms and three relative rotations). The system of Fig.11(a) has ten degrees of freedom (six rigid body freedoms + three rotations of apex joint + one rotation about side 1). It can be noted that the joint A will still be a single apex joint. Since the triangles 1 and 2 are connected to each other through a side and are attached to triangle 3 through an apex joint. This apex joint permits three rotations between triangle 3 and the system of 1 and 2. On the other hand, in the system in 11(b), three triangle planes are connected at the apex joint, which allows six rotations between the triangle planes. This is counted as two apex joints.

Each actuated side is part of a triangular link. Referring to the previous classification of triangle links (Fig.7) every type A,B,C or D triangle link adds 0,1,2, or 3 freedoms respectively. A term for the freedoms added by actuated sides is included in the mobility equation.

The VGT could be connected to the ground through one or more triangle links.

The three joints through which the connection is made can be thought of as a triangle link. Every such connection to ground would remove six freedoms of the VGT. A term to account for this is included in the mobility equation.

Accounting for all the terms discussed, we have the mobility equation for spatial VGTs :

$$m = 6n + p - 5q - 3r + 3v \qquad [6]$$

n is the number of triangle links

p is the number of actuated sides

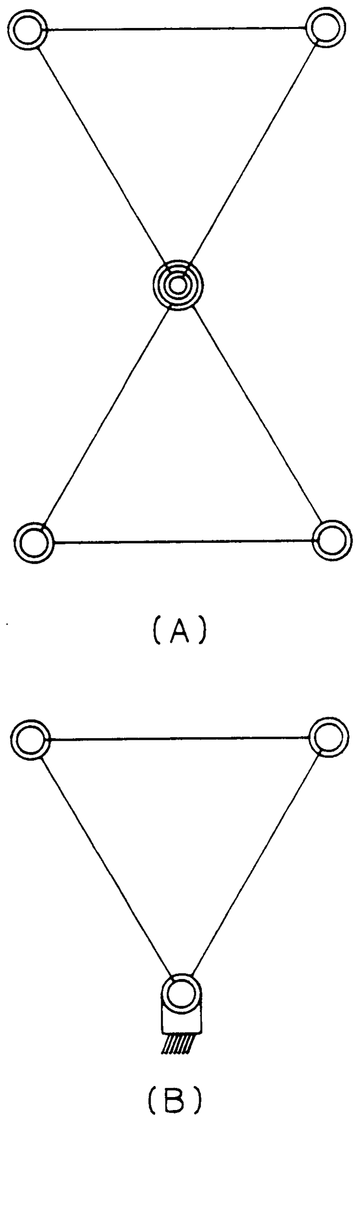


Figure 10. Examples of Apex Joints

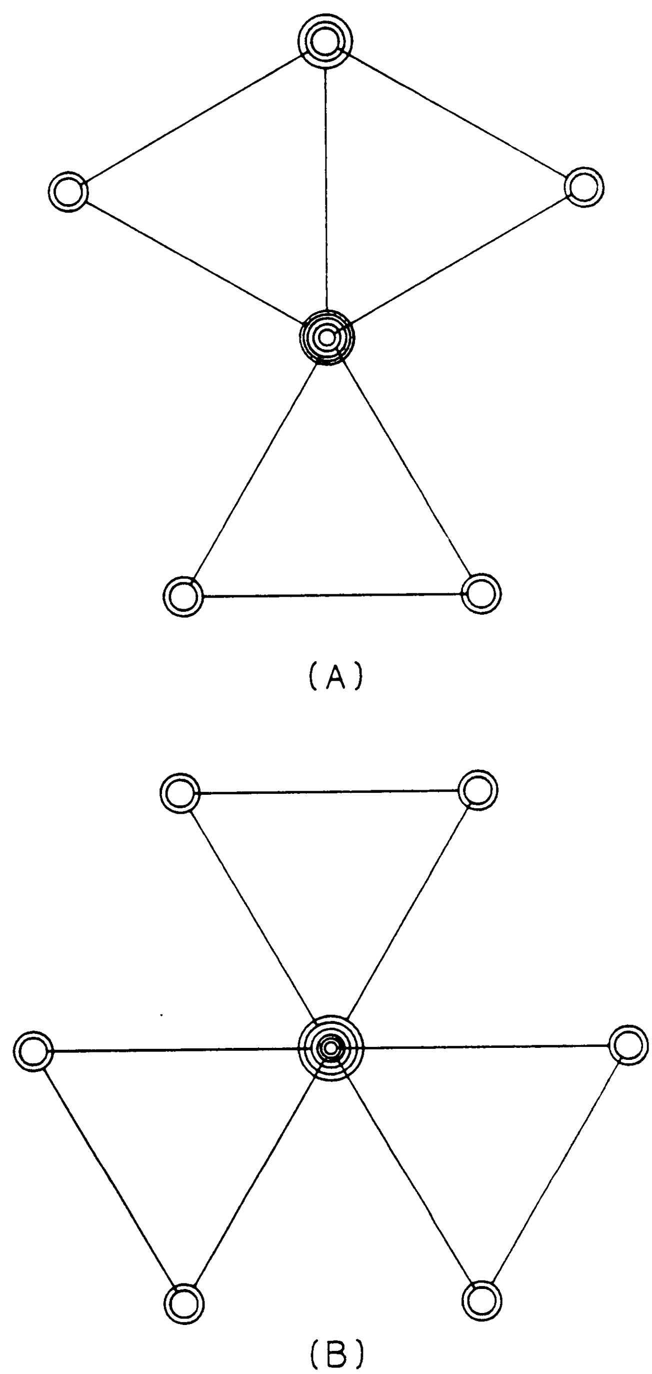


Figure 11. More Examples of Apex Joints

q is the number of shared sides

r is the number of apex joints

v is the number of times n triangles share more than (n-1) sides

The explanation for the v term in the mobility equation can be illustrated with the following example. Consider the tetrahedron in Fig. (11). The structure would be considered to be made of three links to account for all the sides. In addition, there are three sides shared and no apex joints. The number of links, shared sides and joints is given by:

Triangle links : 134, 123 and 234

Shared sides : 34, 13 and 23

Apex joints : none

Counting links, joints, shared sides and connections to ground, we have

$$M = 6(3) - 5(3) - 3(0) = 3$$

This result conflicts with expected mobility value of six. The anomaly can be explained by considering Fig.(11(a)). This system can be considered to be derived from the tetrahedron by decoupling sides 1-2 and 4-2. Nodes 2 and 2′ are identical in the tetrahedron geometry. The count of links, shared sides and joints is now:

Triangle Links : 123, 134 and 42′3

Shared sides : 13 and 34

Apex joints : none

Counting links, joints, shared sides and connections to ground, we have

$M = 6(3) - 5(2) = 8$

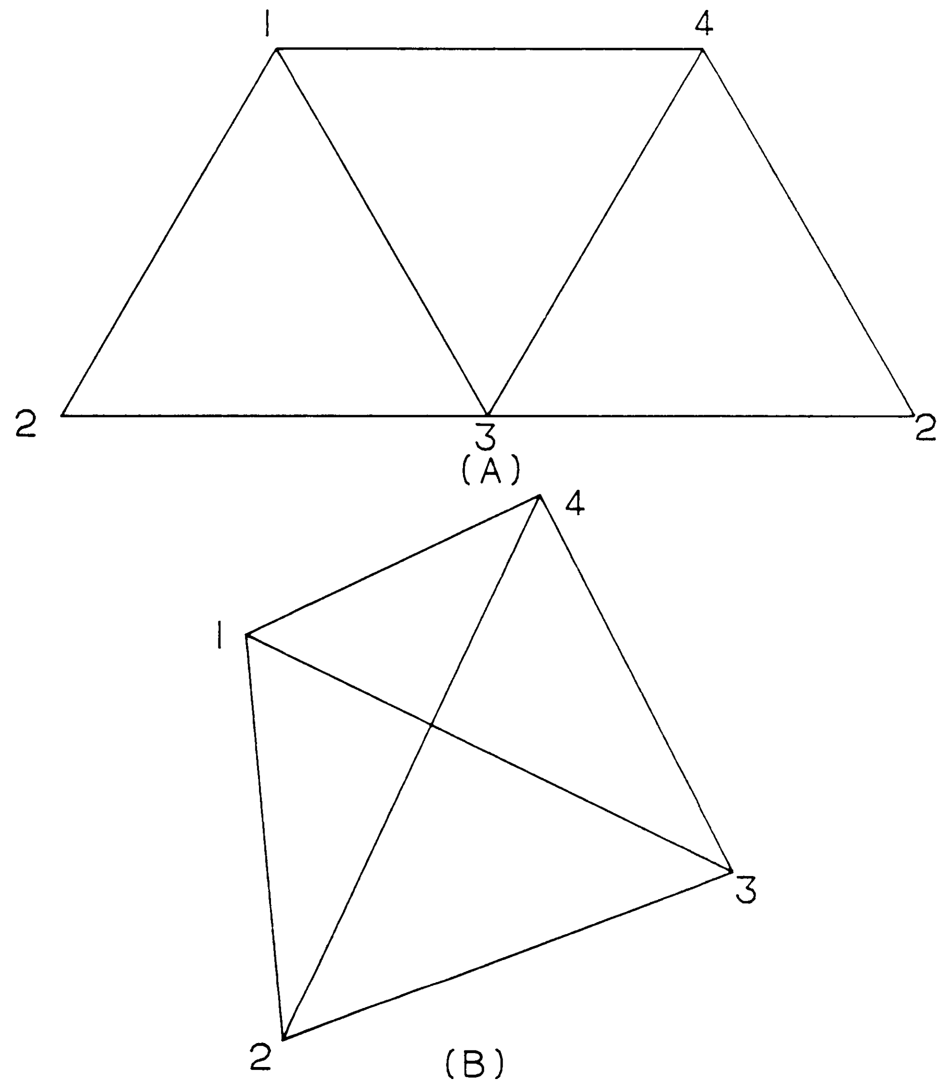
(6 rigid body freedoms and two rotations about 13 and 34)

If we join nodes 2 and 2′ to get back the tetrahedron, what we are effectively achieving is the removal of the rotations about 34 and 13. This reduces the mobility of the system to six. But, as seen previously, the result predicted by the mobility equation does not match with this result. The reason for this anomaly is the redundant removal of freedoms from the system.

The tetrahedron is considered to be made up of three triangle links sharing three sides. If we join nodes 2 and 2′ in Fig (12), we add a shared side 32(32′). From the mobility equation, every shared side removes five degrees of freedom. In this case, the calculation is incorrect since we have already removed the three freedoms corresponding to the translation of triangle links 123 and 342′. Therefore making 32 and 32′ coincident removes only two rotations. In a VGT configuration, whenever there are n triangles sharing more than (n-1) sides, the mobility is incremented by three to account for this effect. This anomaly occurs only when we are considering single cells of a configuration and, as such, would be considered a special case.

Another special configuration that involves n triangles sharing n sides is the VGT in Fig.(20). In the example, we choose all triangles on the top plane(canopy) to account for the VGT. In this specific example, we have 8 triangles sharing 8 sides. In general, any configuration similar to this with n triangle links would share n sides. To account for this, we make use of the v term to add in the three additional freedoms. Another solution to this problem would be to consider a different combination of triangle links.

This example illustrates an important property of the mobility equations i.e. it is possible to have a finite number of combinations of triangle links which satisfy the prescribed rules and



NOTE: INTERSECTING SIDES REPRESENT
SPHERIC JOINTS

Figure 12. Shared Sides of TT Cells

also predict the mobility accurately. This makes implementing an algorithm for calculating the triangle links an interesting problem since multiple solutions are possible. If there are more than one possibilities for the solution, which solution should the algorithm choose?

The following examples would illustrate how the mobility equations are applied to various configurations. They also serve the purpose of validating the results. As mentioned previously, the mobility equations have been applied to a sufficiently large number of mechanisms to convince the author about their validity. A few interesting cases of planar and spatial trusses have been included here.

Example 1 : Planar VGT

Referring to Fig.(13), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	123, 134, 345 and 356
Shared Sides	:	12, 13 43 and 35
Apex Joints	:	None

Referring to the planar mobility equation, we have

$n = 4$

$q = 4$

$r = 0$

p = number of actuated sides

Substituting the above values into the planar mobility equation, we have

$M = 3n + p - 3q - 2r$

$M = 3(4) + p - 3(4) - 2(0)$

$M = p = 0$

The above VGT is a structure.

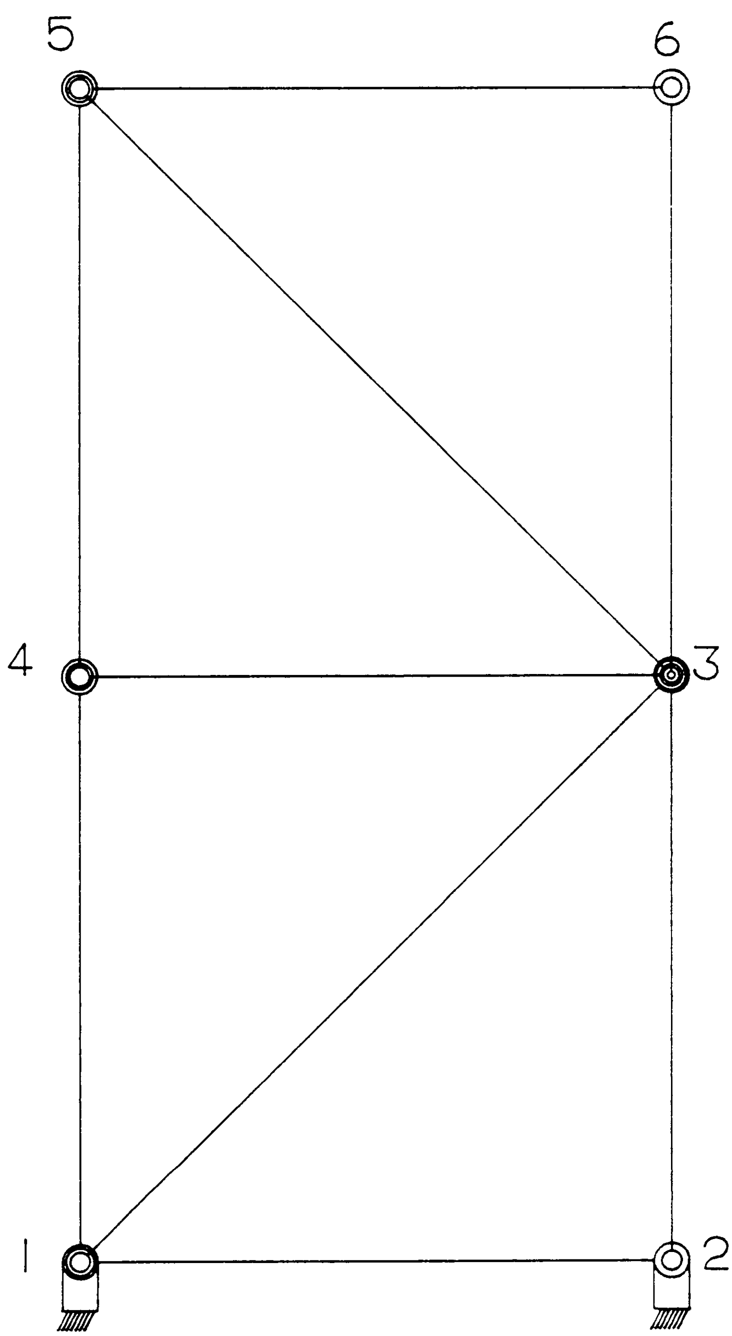


Figure 13. Example 1 : Planar VGT

Example 2 : Planar VGT

Referring to Fig.(14), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	123, 345, 567 and 489
Shared Sides	:	12, 67, and 89
Apex Joints	:	3, 4, and 5

Referring to the planar mobility equation, we have

$n = 4$

$q = 3$

$r = 3$

p = number of actuated sides

Substituting the above values into the planar mobility equation, we have

$$M = 3n + p - 3q - 2r$$

$$M = 3(4) + p - 3(3) - 2(3)$$

$$M = p-3 = -3$$

The above VGT is a statically indeterminate structure.

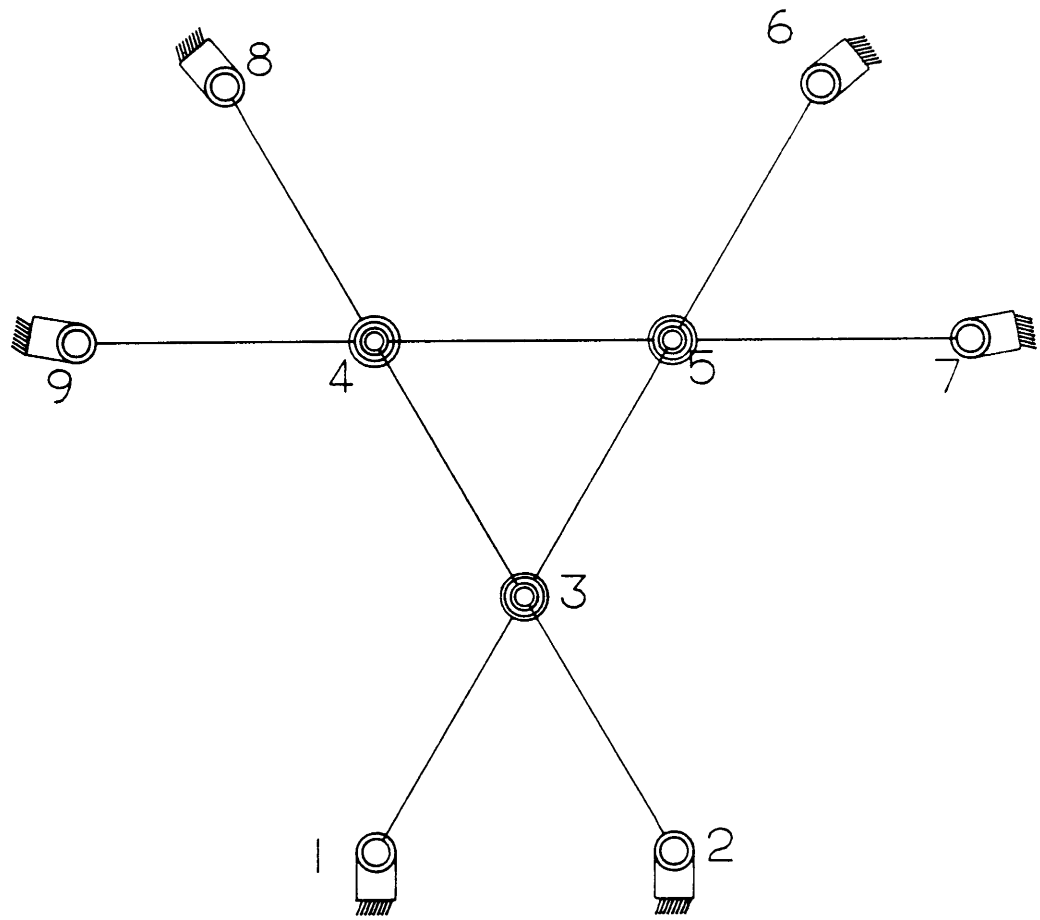


Figure 14. Example 2 : Planar VGT

Example 3 : Spatial VGT

This VGT is a spatial form of the previous VGT and is kinematically equivalent to the Stewart's Platform. Referring to Fig.(15), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	123, 345, 567 and 489
Shared Sides	:	12, 67, and 89
Apex Joints	:	3, 4, and 5

Referring to the spatial mobility equation, we have

$n = 4$

$q = 3$

$r = 3$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(4) + p - 5(3) - 3(3) + 3(0)$$

$$M = p = 0$$

The above VGT is a structure. There is one interesting point to note in this example. As mentioned before, this VGT is the spatial equivalent of the previous VGT. This VGT has 3 more freedoms than the previous VGT. The reason for this is that the joints 3, 4 and 7 permit rotation about them in space and not in the plane.

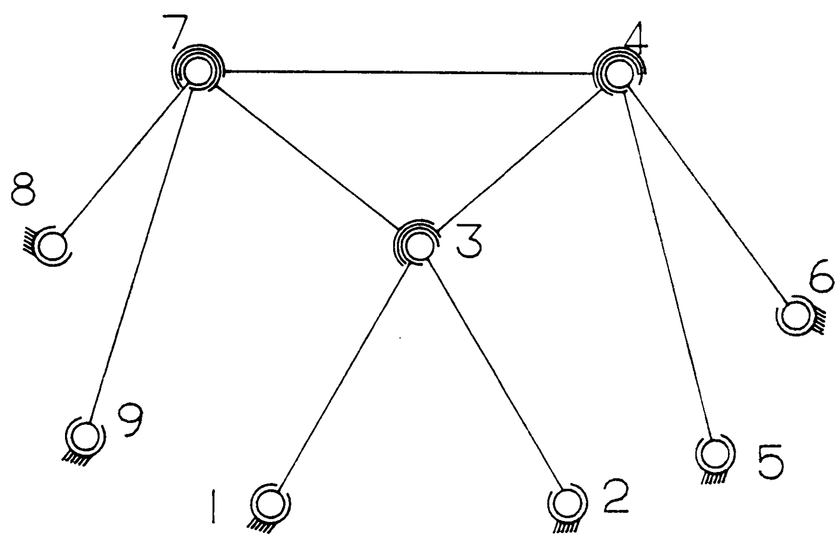


Figure 15. Example 3 : Spatial VGT

Example 4 : P-P Truss

This VGT is an example of a single bay of a PP truss. Referring to Fig.(16), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	236, 134, 125 and 456
Shared Sides	:	None
Apex Joints	:	1, 2, 3, 4, 5 and 6

Referring to the spatial mobility equation, we have

$n = 4$

$q = 0$

$r = 6$

$v = 0$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(4) + p - 5(0) - 3(6) + 3(0)$$

$$M = p + 6 = 6$$

The above VGT has six freedoms if it is floating freely in space and is a structure when attached to ground.

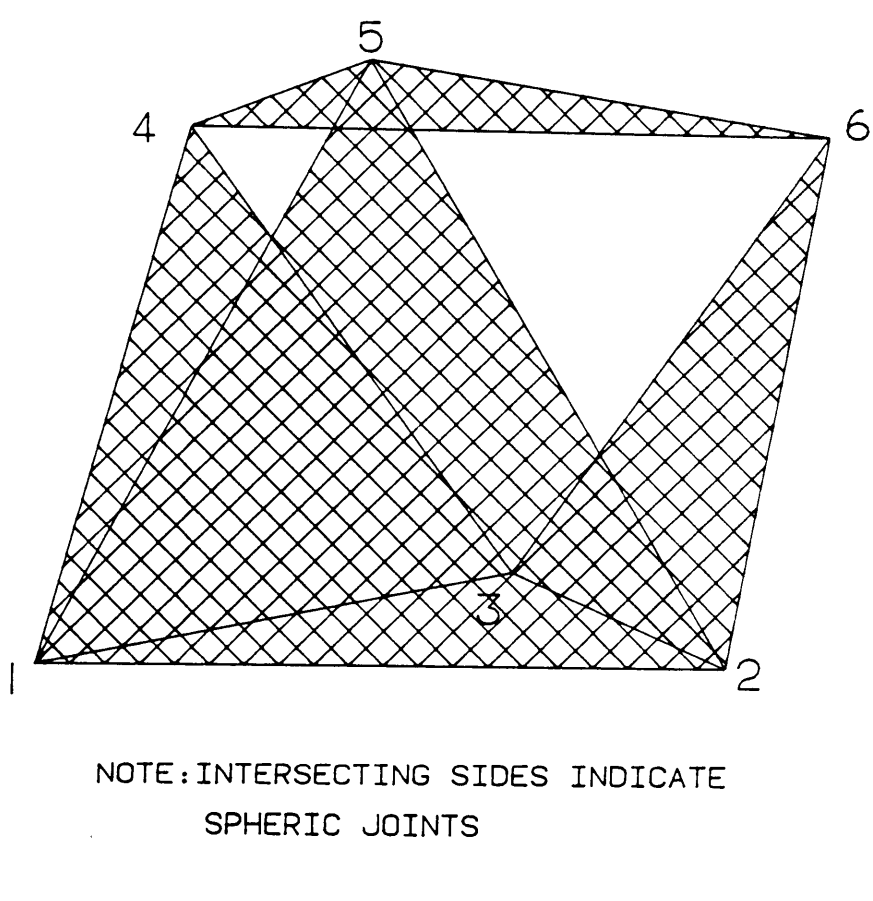


Figure 16. Example 4 : PP Truss

Example 5 : T-P Truss

This VGT is an example of a single bay of a TP truss. Referring to Fig.(17), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	123, 134, 125, 456 and 356
Shared Sides	:	13, 56 and 12
Apex Joints	:	3, 4, and 5

Referring to the spatial mobility equation, we have

$n = 5$

$q = 3$

$r = 3$

$v = 0$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(5) + p - 5(3) - 3(3) + 3(0)$$

$$M = p + 6 = 6$$

The above VGT has six freedoms if it is floating freely in space and is a structure when attached to ground.

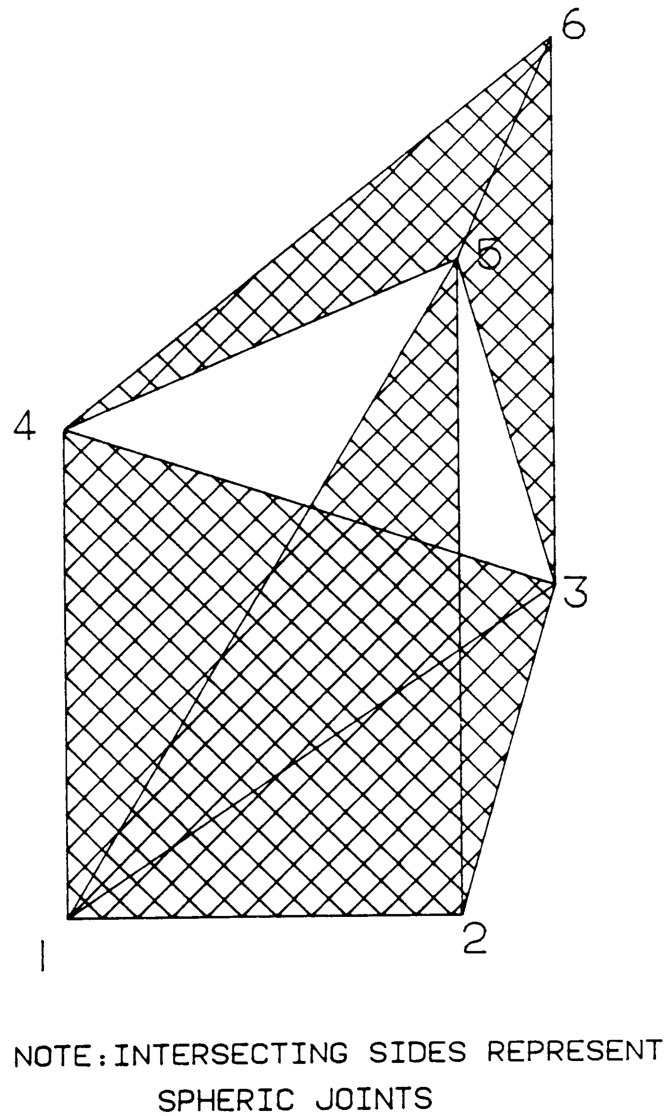


Figure 17. Example 5 : TP Truss

Example 6 : T-T Truss

This VGT is an example of a single bay of a TT truss. Referring to Fig.(18), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	123, 125, 234, 357, 456 and 467
Shared Sides	:	12, 46 and 23
Apex Joints	:	7, 4, 5(2) and 3

Referring to the spatial mobility equation, we have

$n = 6$

$q = 3$

$r = 5$

$v = 0$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(6) + p - 5(3) - 3(5) + 3(0)$$

$$M = p + 6 = 6$$

The above VGT has six freedoms if it is floating freely in space and is a structure when attached to ground.

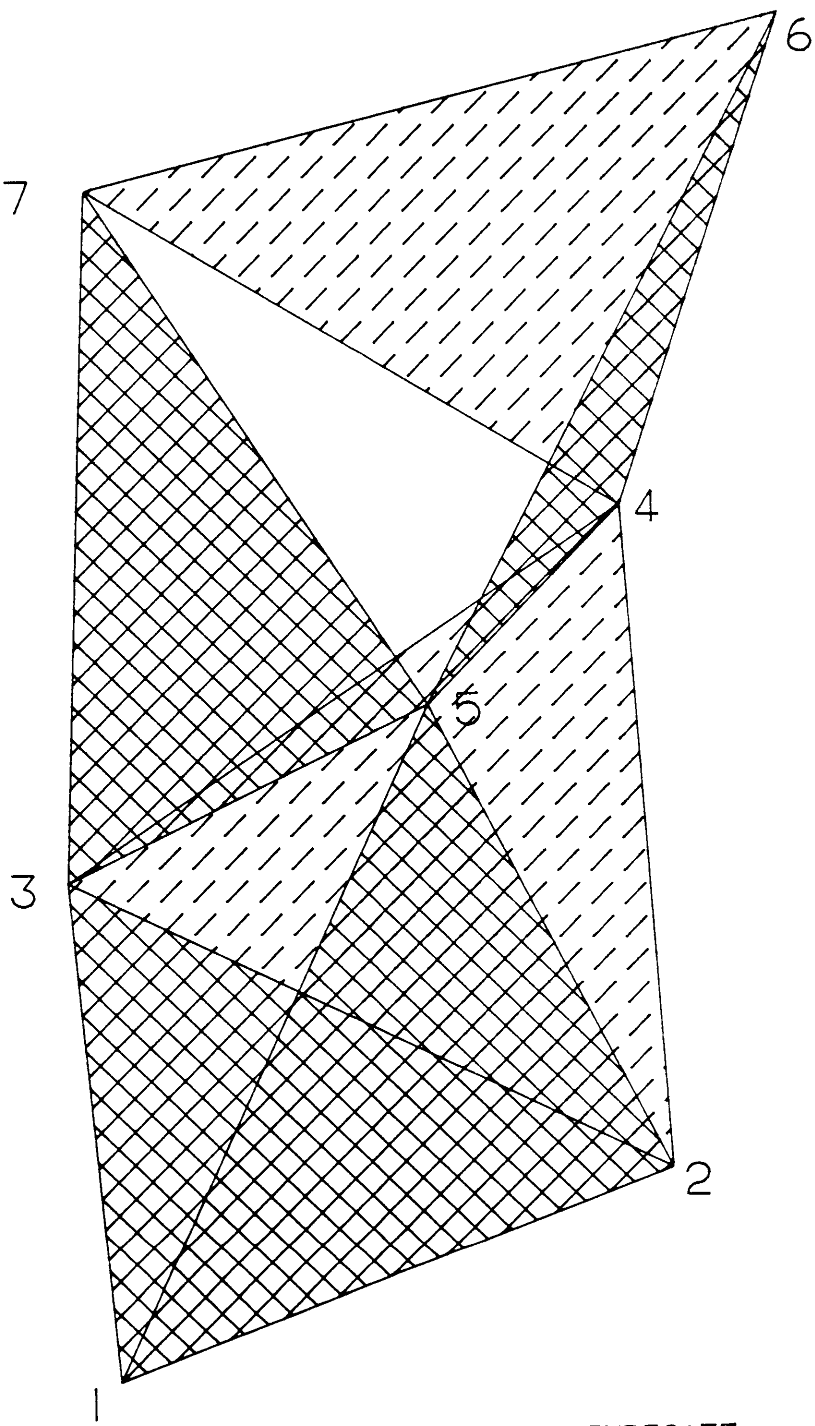


Figure 18. Example 6 : TT Truss

Example 7 : C-C Truss

This VGT is an example of a single bay of a CC truss. Referring to Fig.(19), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	128, 258, 234, 356, 678, 147 and 245
Shared Sides	:	24, 28 and 25
Apex Joints	:	8, 5, 6, 1, 4, 7 and 3

Referring to the spatial mobility equation, we have

$n = 7$

$q = 3$

$r = 7$

$v = 0$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(7) + p - 5(3) - 3(7) + 3(0)$$

$$M = p + 6 = 6$$

The above VGT has six freedoms if it is floating freely in space and is a structure when attached to ground.

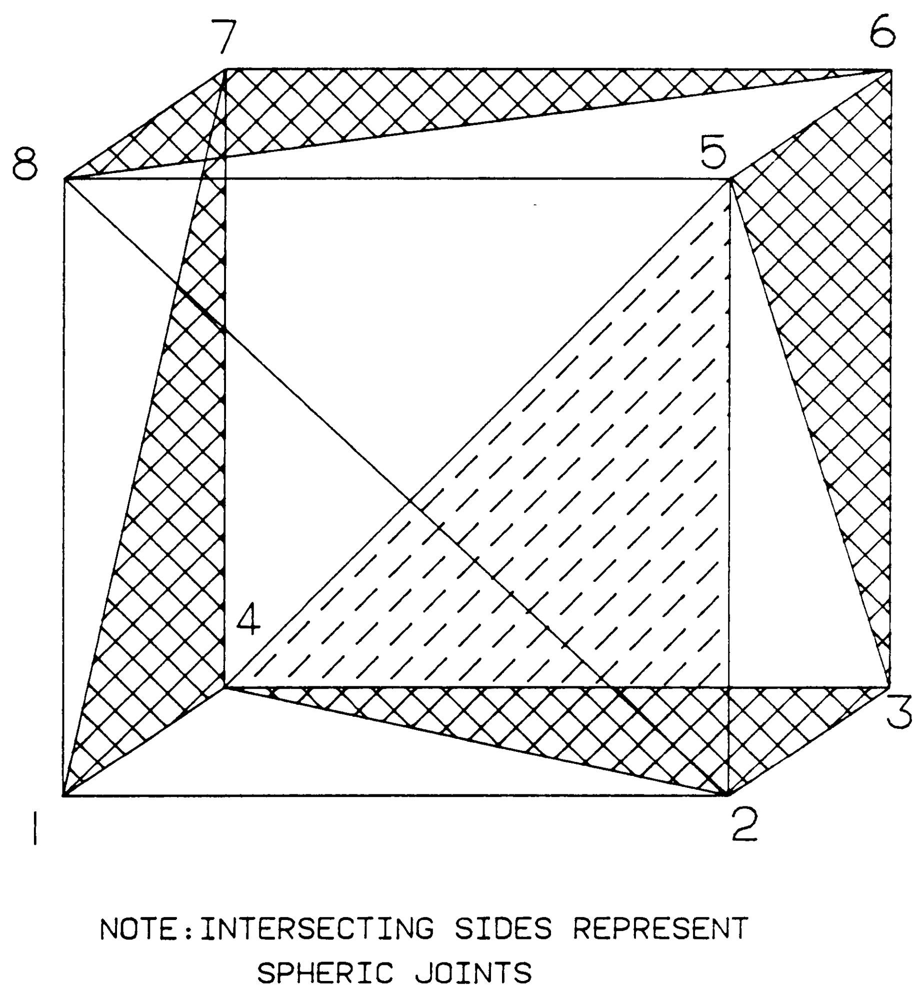


Figure 19. Example 7 : CC Truss

Example 8 : Canopy Truss

This VGT is another example where we have n triangle links sharing n sides. Referring to Fig.(20), we count the triangle links, shared sides and apex joints as follows:

Triangle Links	:	1-2-10, 2-3-11, 3-4-12, 4-5-13, 5-6-14, 6-7-15,
	:	7-8-16, 1-8-9, 7-3-1, 7-5-3, 17-9-10, 17-10-11,
	:	17-11-12, 17-12-13, 17-13-14, 17-14-15, 17-15-16,
	:	and 17-16-9
Shared Sides	:	9-17, 10-17, 11-17, 12-17, 13-17, 14-17, 15-17,
	:	7-3 and 16-17
Apex Joints	:	1(2), 2, 3(2), 4, 5(2), 6, 7(2), 8, 9, 10, 11, 12,
	:	13, 14, 15, 16

Referring to the spatial mobility equation, we have

$n = 18$

$q = 9$

$r = 20$

$v = 1$

p = number of actuated sides

Substituting the above values into the spatial mobility equation, we have

$$M = 6n + p - 5q - 3r + 3v$$

$$M = 6(18) + p - 5(9) - 3(20) + 3(1)$$

$$M = p + 6 = 6$$

The above VGT has six freedoms if it is floating freely in space and is a structure when attached to ground.

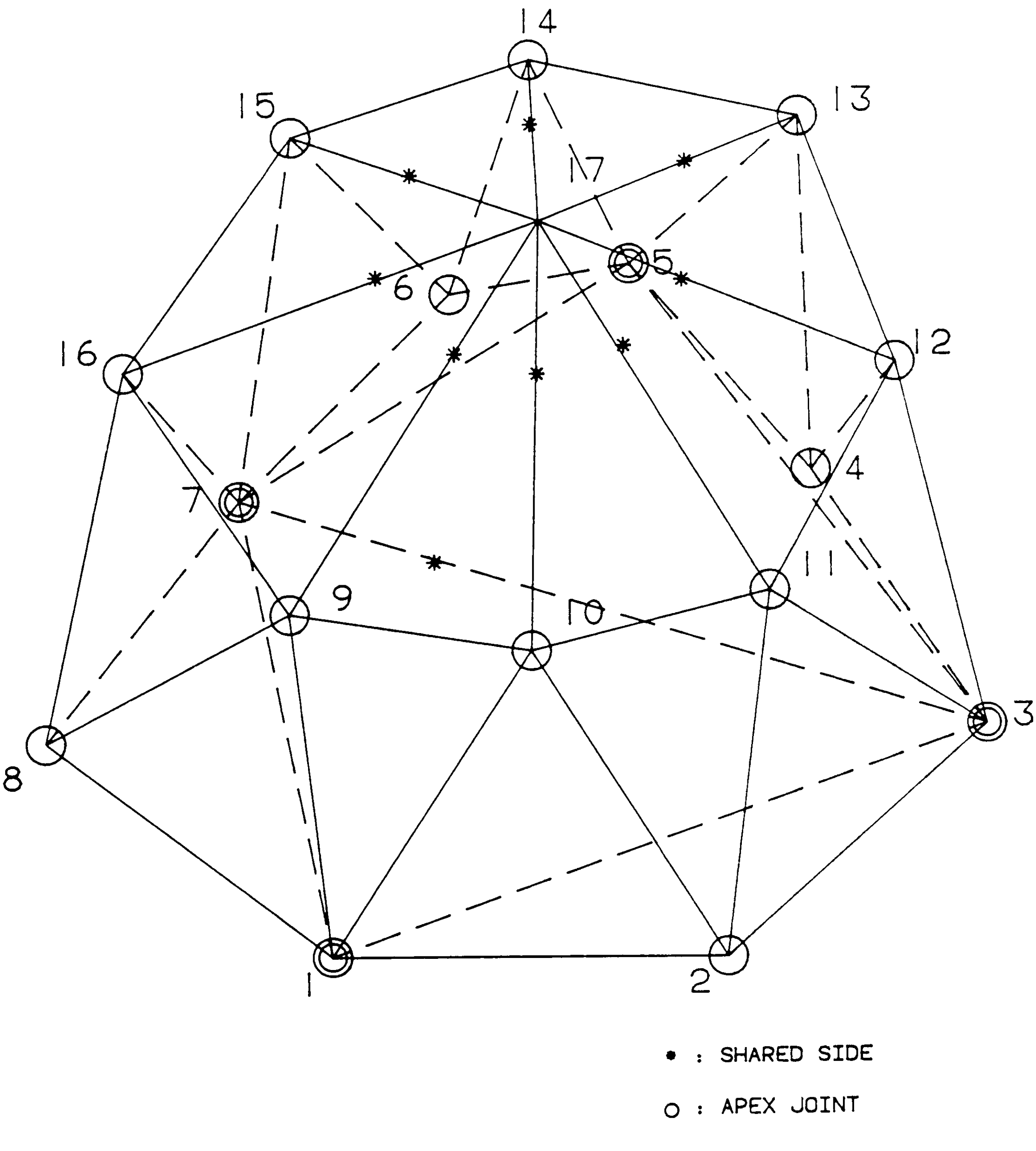


Figure 20. Example 8 : Canopy Truss

Chapter 6

Conclusions and Recommendations

As mentioned before, this thesis does not attempt to present a case for using VGTs. The emphasis here is to stress the importance of having a unifying model to integrate the research efforts on VGTs. It is not clear if this set of rules could be applied to developing constraint-equations for any particular configuration. The forward kinematics problem for the pyramid-pyramid truss has been studied in great detail, and there is on-going research on other geometries like the tetrahedral, irregular tetrahedral, etc. The equations developed have accurately predicted the mobility for a large subset of VGT geometries. This supplies strong evidence that the mobility equations can be applied with confidence. The purpose of the present chapter is to present ideas for future similar research efforts.

A topic mentioned in Chapter 3 on the presentation of the model was the development of an algorithm to calculate the number and position of the triangle links to account for all members of the truss. The development of an algorithm which accepts geometries as inputs and does a constraint analysis of these poses a very interesting problem. This would include an algorithm for calculating the triangle links in the geometry. Artificial intelligence languages like LISP and PROLOG implement such algorithms very efficiently.

One of the major handicaps in designing VGTs seems to be the fact that researchers are unaware of different possible geometric configurations. Also, it needs to be understood that for a researcher to design a VGT that is not derived from or does not resemble a previously applied VGT, becomes a very intuitive process. With the help of the rules developed here, a researcher may extend his imagination to create a VGT and then analyze it based on the constraint equations developed. If he finds that the VGT is over-constrained or has more freedoms he can refine his design following the rules imposed by the model. Thus, a designer can successively refine the VGT design until he reaches a satisfactory configuration.

An important problem in kinematic synthesis is the type and number synthesis of mechanisms. Given the required performance, what type of mechanism would be suitable? Also: How many links should the mechanism have? How many degrees of freedom are required? What configuration is desirable? Deliberations involving the number of links and degrees of freedom are often referred to as the province of a subcategory of type synthesis called number synthesis. The mobility equations developed in this thesis could be used to perform an inverse type and number synthesis. Given the number of triangle links, shared sides, apex joints and connections to ground, how many different VGT configurations are possible? The advantage of the "triangle link" mobility theory in this respect is that the number of choices are limited. This is because the solutions of the inverse analysis are in terms of triangle links, which limits the number of possible solutions.

Just as the mobility equations can be used to predict the degrees of freedom of a mechanism, so also they can be used to work backwards from a known mobility to the number of links, shared sides, etc. For the inverse problem, there may not be unique solutions. Usually, the designer would be given a certain number of possible choices for the different parameters.

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