Low Frequency Cutoff Effects in Fiber Optic

Communication Systems

by

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(ABSTRACT)

The presence of low frequency cutoffs in the forward path of the information signal leads to inter-symbol interference (ISI) and degradation of the signal to noise ratio at the sampling instant in digital on-off keying (OOK) systems. The low frequency cutoffs occur as a result of the presence of power separation filters in a line wire system and gain instability of APD's to D.C. in fiber optic systems. Also, it is easier to design amplifiers that do not extend to D.C. The ISI which manifests itself in the form of baseline wander can cause appreciable degradation in the signal to noise ratio. This thesis investigates two ways of combating the baseline wander problem. They are quantized feedback and line coding schemes. A detailed performance evaluation of quantized feedback scheme is performed . An overview of line coding schemes is given and some specific codes are evaluated in terms of S/N degradation at the receiver.

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chapter I

Overview

Introduction

The problem of maximizing the signal to noise ratio in a communication system has been studied extensively and is one the most researched topics in the field of telecommunications. The problem assumes different forms for different modulation schemes, both analog and digital, and is crucial because it gives a measure of the fidelity of the transmitter-receiver link.

In a baseband digital communication system that uses a twisted wire pair or a coaxial cable as a medium for transmission of the information signal, there are two principal mechanisms which can cause errors in the decision at the receiver. They are thermal noise, which is modelled as additive white Gaussian noise (AWGN), and inter-symbol interference (ISI), which is the corruption of the received signal by all previous signals. ISI, a manifestation of the finite bandwidth of a channel, can under ideal circumstances be completely removed from the signal by pulse shaping and filtering.

However, the deviations in the system parameters from the ideal values can cause ISI in practice.

In an optical fiber communication system, the noise is signal dependent and non-stationary. The arrival of photons at the photodetector is a Poisson process. The random nature of the arrival process places a fundamental limit on the number of photons that are required to detect the presence of a bit. For example, an error probability of 10^{-9} requires a minimum of 10 photons per bit. This calculation assumes that an ideal photon counter is being used and there are no other noise mechanisms prevelant in the detection process. In the case of an APD (Avalanche Photo Diode) there is a random gain phenomenon associated with gain amplification process, thus complicating the detection statistics. As a result, the analysis for characterising the system performance is significantly different from the thermal noise case.

This thesis is primarily concerned with a class of problems arising from low frequency cutoff effects in the channel, which tend to cause ISI in the form of baseline wander in both copper wire as well as optical fiber communication systems. Two specific methods that are used to counter the problem have been analysed; they are quantized feedback and line coding schemes. Quantized feedback is a form of decision feedback equalization where a part of the regenerated signal is fed back to the input of the decision device, in order to cancel the effect of the forward path low frequency cutoff. Line coding schemes involve restricting the possible sequences of a binary train to cause it to have have some desirable properties such as a zero or constant D.C., sufficient transitions to facilitate timing recovery, error monitoring capability and spectral shaping. A third popularly used scheme for the purpose of timing recovery and D.C. wander is scrambling, which is regularly used in the U.S., as opposed to Japan and Western Europe.

Overview of an Optical Fiber Communication System

A typical baseband digital optical fiber communication system is shown in fig 1. The source generates at a rate 1/T, statistically independent symbols a(k) taking values from the binary alphabet (0, 1) at discrete times kT. In an optical fiber communication system the baseband electrical signal modulates the intensity of an optical source (laser diode or an LED). The optical signal is transmitted over a fiber. At the receiving end it is converted back to an electrical signal by an avalanche photodiode or a PIN detector.

The input signal modulates the intensity of the light and the output power is transmitted through the fiber. As a result of the imperfect modulation of the physical devices, the optical line signal cannot be completely extinguished. The signal maps on to the sequence $(\alpha,1)$ where $\alpha << 1$, is the extinction ratio of the light emitting device. The received pulses are smeared out in time as a result of fiber dispersion, which causes a pulse to spread. Pulse spreading results in the spilling of a pulse into a neighbouring time slot thus causing inter-symbol interference.

A typical receiver consists of a photo detector, an amplifier, a receiver filter or an equalizer to compensate for the channel distortion and help maximize the signal to noise ratio and a decision device. In general the received pulses do not add linearly in power which makes the equalization problem difficult to solve. Personick(8) has shown that under the assumption of an incoherent intensity modulated source the

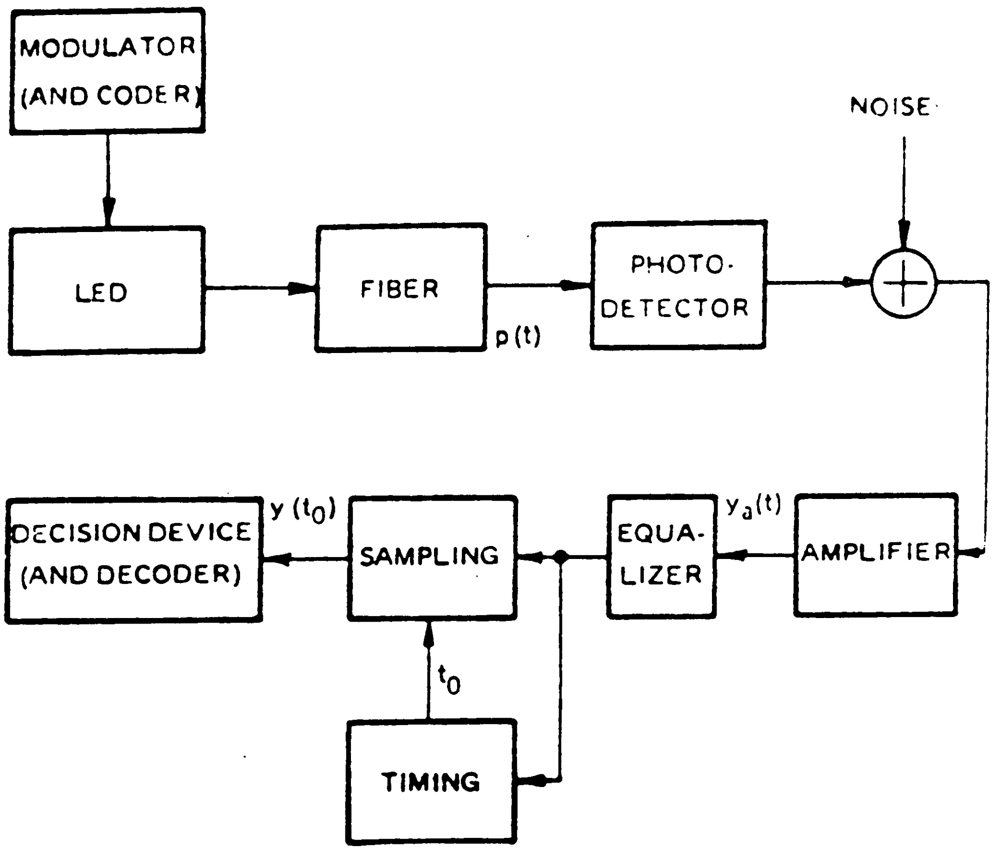


Figure 1. A Digital Optical Fiber Communication System

linearity approximation is a reasonable one and the power can be treated as linear in form.The received signal can be considered to be a superposition of a desired component related to the transmission bit to be detected and three types of impairments: ISI, shot noise and thermal noise. The threshold of the decision device is determined by taking into account all the above impairments and is chosen such that the error probability is a minimum.

As stated earlier, the arrival of photons at the receiver follows a Poisson process. The uncertainity in the number of photons per bit leads to the fundamental sensitivity of the receiver. This is called the quantum limit, and is equal to about 10 photons per bit for an error probabilty of 10^{-9}. In practice, the number of photons per bit is generally in the range of several hundred to several thousand as a result of several, noise processes adding to the received signal. Many of the impairments are a result of variations in the frequency response from the ideal case., The one of particular interest in this thesis is the problem associated with the presence of low frequency cutoffs in the channel.

Baseline Wander

Digital transmission of information over long distances on a wire pair or a coaxial cable requires the use of repeaters along the transmission route. In order to fully realise the advantages of digital transmission the repeaters must regenerate the pulses rather than merely amplify or equalise them.

The repeaters in metallic systems are generally powered by a DC current that is carried on the same line as the signal. The signal rides on the DC component. As a result the digital signal is AC coupled to the transmission line either through power separation capacitors or transformers.A second advantage of the AC coupling is that it isolates the repeaters from interference and potential damage due to low frequency transients arising from lightning, power surges and maintainance activities. It is also easier to build amplifier stages that are AC coupled. As a result of the AC coupling the regenerator has non zero cutoff frequency at the input and thus suffers from a poor low frequency response. The effect of low frequency cutoff on an isolated pulse is that a positive pulse must be followed by a long negative tail equal in area to the area of the positive pulse. In a pulse stream that contains a long sequence of bits of the same sign the intersymbol interference from these long tails may add up to a significant fraction of the pulse height and degrade the signal to noise ratio at the decision instant. As an example a long sequence of NRZ 1's can cause the eye to close as a result of this phenomena. The low frequency fluctuation of the average of the ISI, which in turn causes the fluctuation in the average of the detected signal as the pulse pattern varies, is called baseline wander.

In an optical fiber communication system baseline wander arises as a result of the use of AC coupling as a solution to the problem of gain instability in APD's. The current gain of an avalanche photodiode has a relatively large sensitivity to the DC component of the information signal, so AC coupling has to be used in order to avoid gain instability.

This thesis attempts to make a detailed analysis of the quantized feedback approach. An alternative approach, the use of redundant block codes is also considered.

Chapter Two gives a review of the literature that was studied and is pertinent to the topic under study.

Chapter Three gives a detailed analysis of the quantized feedback approach.

Chapter Four deals with the performance of line coding schemes for fiber optic Systems.

Chapter Five gives the results and the conclusions of the research.

chapter II

Literature Review

Additive White Gaussian Noise Channels

The problem of detecting a signal in the presence of additive white Gaussian noise has been studied extensively in the literature. Nyquist(1) was the first to address the problem for band limited channels and derived a class of channel spectral shapes that avoid any additive ISI at the the sampling instants.

Aaron and Tufts (2,3) used the criterion of minimum average error probability treating ISI as a special type of correlated non Gaussian noise to specify an optimum, linear, time invariant filter at the receiver.

A number of papers exist in which the exact characterestics of the overall system are discussed in order to compensate for ISI. However with this approach it is difficult to design exact transmitting and receiving filters thereby obtaining optimal system response. It is more appropriate to design equalizers that keep ISI within tolerable bounds with respect to an achievable error probability. In the literature two equalization criteria are used; zero forcing criteria (4) and minimum mean square error(MMSE) (5). The zero forcing criteria is known to minimize the equalized peak distortion if the distortion of the unequalized pulse is < 1. The second criteria minimizes the mean square between the received sequence and the transmitted sequence.

Fiber Optic System Receiver Design

The nature of the noise processes in a fiber channel do not allow it to be treated as an AWGN channel. Specfically the statistics of the photon arrival process and the avalanche detection process are non Gaussian and as a result the problem takes on a significantly greater complexity.

Receiver design considerations for optical fiber sytems were first enunciated by Personick (6,7). Personick investigated the design of the linear channel of the repeater for a fiber optic OOK system, focusing on the the choice of the front end amplifier and biasing circuitry of the APD; and how the receiver sensitivity for a desired BER varied with bit rate, pulse shape and desired baseband equalization for the output pulse shape.

The avalanche detection statistics for an APD have been studied in great detail. McIntyre(8) was the first to develop an exact expression for the density function of the shot noise treating it as a fitered doubly stochastic process. The density function is rather complicated and not amenable to mathematical manipulations.

Personick (9) in his classic paper treated the detection process as a combination of photon counting and random gain processes in order to determine the moments and used the moments to determine the optimal gain for the APD. Different schemes to calculate the bit error rates by the use of bounds or exact evaluations have been formulated (10,11). The detection of symbols in the presence ISI, shot noise and thermal noise has been treated as a problem in optimal receiver design (12,13); the main drawback with the schemes lies in their implementation. A procedure for evaluating the error probability has been given by Doggliotti etal (14). This procedure does not entail the commonly used hypothesis that the Poisson process may be approximated as a Gaussian; the process itself is non Gaussian and has an amplitude probability function with larger tails than the Gaussian density especially at low Poisson intensities.

Low Frequency Cut-off Effects

The quantized feedback approach was first suggested by W.R.Bennet(15) whose treatment of the problem is from a circuit design point of view and not from a communication theory standpoint.

P.L.Zador(16) was the first to consider the problem in terms of communication theory. He treated the ISI as a random walk whose transition probabilities depend on the Gaussian statistics of the additive White noise and the ISI due to all previous pulses. Aaron and Simon(17) evaluated an approximate value for the error probability in a digital regenerator using quantized feedback with the help of Zador's algorithm. To date no exact analysis of the quantized feedback problem appears to have been published.

Lucky, Salz and Weldon (18) suggested the use of quantized feedback approach in vestigial sideband transmission where D.C. and low frequency are removed at the receiver to facilitate carrier recovery. Satisfactory carrier recovery can then be obtained with relatively low carrier signal power. The demodulated signal is processed by a feedback circuit which essentially restores the low frequency components removed at the transimitter.

Waldhauer(19) designed and constructed a 280 Mbps digital repeater for coaxial transmission. The linear channel forward path included a 10 MHz low frequency cutoff and the missing low frequencies were supplied after filtering. This approach effectively removed the D.C. Wander. The eye margins also improved significantly and the use of a two level transmission implied a greater margin for power budgeting than a three level transmission.

In a recent paper Ennig and Vodhanel(20) suggested the use of quantized feedback in a fiber optic FSK heterodyne system in order to combat the problems resulting from signal distortions in the low frequency response of distributed feedback lasers. The non uniform frequency modulation response of semiconductor lasers causes these distortions in the FSK signal. Quantized feedback equalization was implemented at data rates of 150 Mbps and 1Gbps.

Line Coding Schemes

Line coding schemes have been extensively studied in literature. For transmission over the line, the binary information is coded into a sequence of symbols. The coding scheme is referred to as a pulse transmission plan. The use of a coding schemes addresses the following issues.

1.Minimization of performance dependence on source statistics.

2.D.C.wander of the pulse sequence.

3.Repeater timing jitter performance.

4.On line performance monitoring.

5.Spectral shaping for high frequency energy minimizatiion.

The simplest coding scheme is the bipolar plan (21) where alternate pulses are inverted after transmission and spaces are transmitted without any alteration. The bipolar plan has no D.C. component and error monitoring is implicit. This is also called AMI or alternate mark inversion.

Gabor(22) suggested a polar plan called "frequency doubling saturation" which used twice the Bandwidth of AMI or straight binary transmission. Neu and Kundig(23) first described a telephone network that used a ternary code based system with D.C. balanced binary transmission.

Johannes (24) described a Modal plan where alternate pulses are inverted before transmission and blocks of n successive spaces are replaced by distinctive patterns. This avoids the all zero's problem of AMI.

J.M.Sipress(25) suggested the use of a new type of selected ternary pulse transmission plan called paired selected ternary (PST) in which the the binary sequence to be transmitted is framed into pairs and translated to appropriately selected pairs at three levels.

J.Buchner (26) wrote an overview of ternary line codes with a comparative analysis of BnZS, HDBn, CHDBn type codes. Block codes of the 4B/3T type have been compared for performance. BnZS codes are bipolar codes with n-zero substitution to avoid the problem of timing in an all zero pattern. The substituted sequence is dependent on the previous pulse and contains at least one code violation so it's DC value is a constant. High Density Bipolar (HDB) and Compatible High Denstiy Bipolar(CHDB) codes are coding schemes where an all zero sequence is replaced by a sequence that depends both on the previous pulse and the previous substitution sequence.

B.S.Bosik (27) gave a mathematically elegant method of systematically determining the average power spectral density of different line codes. This method is particularly attractive because it gives an algorithm for computing the power spectral density.

P.A.Fransazek (28) used a sequence state approach to treat the coding problem in a general form.

Partial response or Duo binary signalling is well documented in all standard text-books (29,30,31). It is a method for decreasing the high frequency content of the signal and thereby decreasing the required channel bandwidth. Partial response signalling results in an increase in the number of signal transmission levels on the line. Lender (32) was the first to suggest the use of precoding to avoid the propagation of an error in the duobinary signalling channel.

Optical line codes tend to be two level rather three level since minimizing average power per bit is of a greater importance than minimizing bandwidth.

Tran Muoi (33) studied the receiver sensitivity of an M-ary pulse transmission. The analysis showed that M-ary pulse transmission plans were not suited for optical fiber systems, the use of a multilevel transmission implies the reduction of the used bandwidth but an increase in the signal power level. This scheme takes us in a direction opposite to the desired one, because in a fiber system the available bandwidth is high but the amount of light that can be injected into the fiber is limited.

Game and Jessop (34) suggested possible coding schemes that included PPM, Manchester Codes and scrambled binary with parity check. Rosseau (35) evaluated error probability for Manchester codes for a PIN detector.

Takasaki (36) studied optical pulse formats that included binary block coding schemes(mBnB), and verified the theoritical predictions of the performance with experimental 6.3 Mbps and 100 Mbps rates.

R.Dioglotti (37) etal gave an excellent overview of signal processing for fiber communication systems. Bosotti and Pirani (38) suggested a combined PAM-PPM technique as a feasible solution to the problem of line encoding in fiber communications. They used PAM and PPM jointly to give rise to three different waveforms in a signalling time period without resorting to more than two levels of signalling.

N.Yoshikai etal (39) dicuss the applicabilty of bit insertion codes to a high speed fiber system and compare the bit insertion codes (mB1C, DmB1M) with binary block codes (mBnB). A discussion is given with regards to the advantageous properties of bit insertion codes in terms of ease of implementation and expansibility of channels.

chapter III

Quantized Feedback

Introduction

A simplified block diagram of a configuration that uses quantized feedback to counter the baseline wander problem is given in figure 2.

In metallic systems the transmitted signal passes through two high pass power separation filters in traversing the path from the output of one repeater to the input of the linear channel of the succeeding repeater. Thus, in providing the remote D.C. powering of the repeater chain, the D.C component of the signal is removed and the low frequency components of the signal are selectively attenuated. This introduces a pattern dependent baseline wander and in the absence of quantized feedback impairs the ability of the regenerator to detect the signal correctly, when the signal is in a random binary form and is not encoded. The receiver power separation filter is used to decouple the information signal from the constant D.C. power being supplied to the repeater and contains a long negative tail in addition to the positive pulse. (Fig 3) The cable equalizer equalizes the dispersion effects caused by the medium and does the

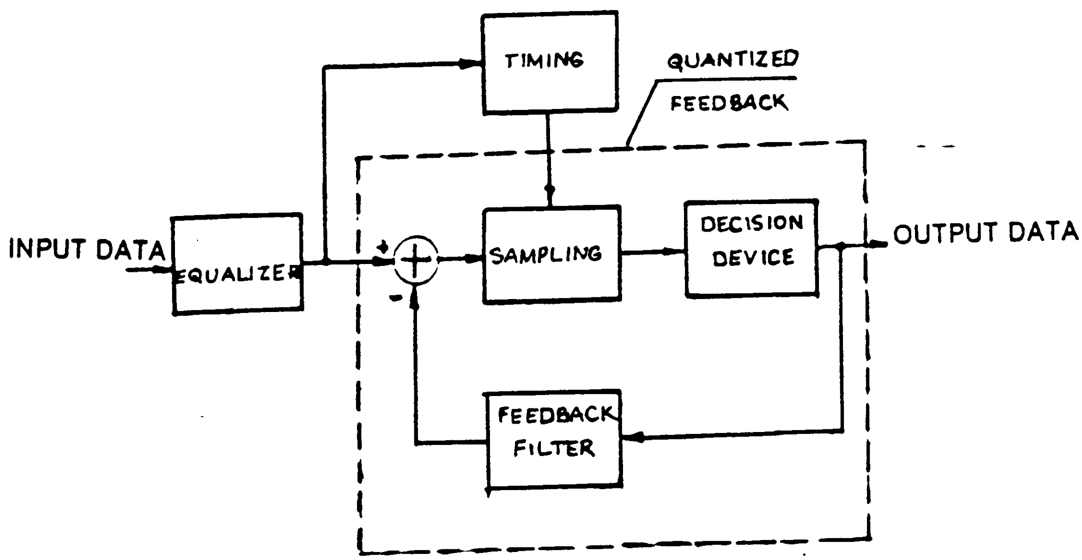


Figure 2. A Quantized Feedback Configuration.

required pulse shaping. At the input to the regenerator the negative tail of the pulse is cancelled by the positive pulse fed back by the quantized feedback filter. The input to the decision circuit is a reconstructed pulse that has no low pass impairment. The combined effect of the forward path input and the quantized feedback signal is to simulate a path from the transmitter to the regenerator input that extends to D.C. or to as low a frequency as desired. The feedback signal is opposite in sign to the tail only if the decision device has made a correct decision on the signal. This leads to the possibility of an error multiplication effect caused by the propagation of error as a result of the decision feedback. The decision feedback process involves a delay;hence the forward path equalization must introduce an additional delay to avoid any mismatch in tail cancellation.The filters must be designed in such a way that the cross-over frequency is high enough to take care of the unavoidable low frequency cutoffs in the forward paths but low enough to make the system not overly susceptible to a timing mismatch.

It has been stated earlier that a long sequence of 1's from an NRZ source can impair the decision of the repeater to the extent that the eye is effectively closed and the transmitted and received data become independent of each other. Fig(3) shows the low frequency cutoff effects on an isolated pulse and a series of pulses of the same sign, and reconstruction as a result of quantized feedback. The occurence of an all 1's sequence in a NRZ scheme will also pose a timing recovery problem, this problem has not been discussed in the thesis but is an issue of importance in the design considerations of a communication system.

The primary cause for concern in the use of quantized feedback for D.C. restoration is the error propagation that is expected in the case of an error in the decision at the

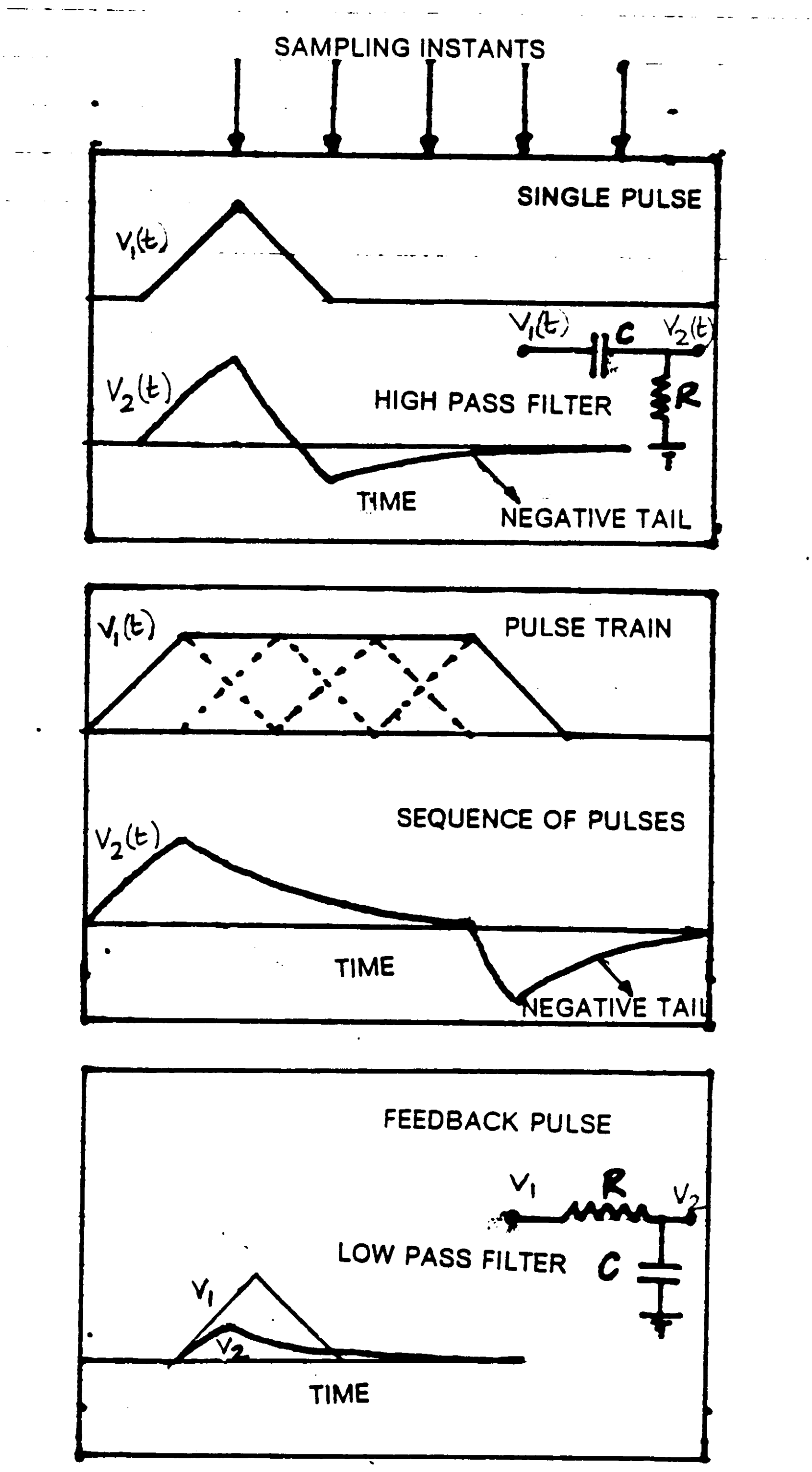


Figure 3. Low frequency cutoff effects.

regenerator. The question arises whether a random error could cause degradation in the signal to noise ratio resulting in an unacceptable error rate. The following analysis seeks to investigate that possibility. The analysis for a general case of a binary detection problem with ISI is considered and an expression for the error probability is derived in terms of the moments of the ISI using a Gram-Charlier expansion. The results are applied to the case of ISI introduced as a result of low frequency cutoff, and the residual ISI in the case of quantized feedback. The degradation in quantized feedback performance as a result of timing mismatch in the forward path and the feed back signal is analysed and the dependence of the degradation on the filter parameters is studied.

Calculation of Error Probability due to Baseline Wander

The input data a_i is chosen from the binary alphabet (+1,-1). The sampled input to the decision device is given by

$$y(t) = \sum_{i=-\infty}^{\infty} a_i \; f(t - iT) + n(t) \tag{3.1}$$

where $a_i f(t - iT)$ is equal to the output of the forward path high pass filter at the sampling instant iT.

$$y = a_0 f(t) + \sum_{\substack{i=-\infty \\ i\neq 0}}^{\infty} a_i f(t - iT) + n(t) \tag{3.2}$$

The second term gives the intersymbol interference on the '0'th pulse and the third term is the AWGN. Defining X as the ISI on the pulse

$$X = \sum_{i=-\infty}^{\infty} a_i f(t - iT)_{i \neq 0} \tag{3.3}$$

The sampled signal y at the sampling instant t_0 is given by

$$y_{t_0} = a_0 f(t)_{t=t_0} + X + n(t) \tag{3.4}$$

Thus the Gausssian noise n(t) may be expressed as

$$n(t) = y(t) - g_0 - X \tag{3.5}$$

g_0 is the output of the forward path high pass filter at the sampling time t= t_0. The thermal noise is Gaussian distributed with a zero mean and a variance σ^2 . For a specific sequence $\widetilde{X}$ the error probability is given by

$$P_e(\widetilde{x}) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{0} \exp(-\frac{(y - g_0 - \widetilde{x})^2}{2\ \sigma^2})dy \tag{3.6}$$

Averaging the error probability over all possible values of $\widetilde{X}$,the average probabality of error is given by

$$P_e(avg) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{\infty}\int_{-\infty}^{0} \exp(-\frac{(y - g_0 - \tilde{x})^2}{2\,\sigma^2})dy\, dF_x \tag{3.7}$$

F_x is cumulative distribution function of the additive ISI. Setting $z = y - g_0$

$$P_e(avg) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{\infty}\int_{-\infty}^{-g_0} \exp(-\frac{(z - \tilde{x})^2}{2\,\sigma^2})\, dz\, dF_x \tag{3.8}$$

The equation (3.8) can be rewritten as

$$P_e(avg) = \frac{1}{\sqrt{2\pi}\ \sigma} \int_{-\infty}^{\infty}\int_{-\infty}^{-g_0} \exp(-\frac{z^2}{2\,\sigma^2})\ \ \exp(-\frac{(2zx - x^2)}{2\,\sigma^2})dz\, dF_x \tag{3.9}$$

The second exponential in the equation (3.9) may be expressed as a sum of gaussian derivatives weighted by corresponding Hermite polynomials. This amounts to taking a Gram-Charlier expansion of the density function of $y(t_0)$. The density function for $y(t_0)$ may then be expressed as

$$g(y) = \exp(-\frac{z^2}{2\sigma^2}) \left[1 + \sum_{n=0}^{\infty} \frac{1}{n!} H_n(\frac{z}{\sqrt{2}\,\sigma})\ \frac{x^n}{(\sqrt{2}\,\sigma)^n} \right] \tag{3.10}$$

Where $H_n(x)$ is a Hermite polynomial and

$$H_0(x) = 0$$

$$H_1(x) = x$$

$$H_n(x) = x\, H_{n-1}(x) - (n-1)\, H_{n-2}(x)$$

Rewriting equation (3.8) and substituting for the n^{th} moment of the ISI M_n

$$M_n = \int X^n \, dF_x \tag{3.11}$$

The error probability can now written as a weighted sum of the moments of the ISI M_n. Using the identity $\frac{d^n}{dz^n} e^{(-\frac{z^2}{2\sigma^2})} = H_n(\frac{z}{\sqrt{2}\,\sigma})\, e^{(-\frac{z^2}{2\sigma^2})}$ equation (3.10) may be rewritten as

$$P_e = P_e(\text{thermal}) + \sum_{1}^{\infty} \frac{1}{n!} \frac{M_n}{(\sqrt{2}\,\sigma)^n} H_n(-\frac{g_0}{\sigma}) \frac{e^{-(\frac{g_0}{\sigma})^2}}{\sqrt{\pi}} \tag{3.12}$$

The first term in the expression gives the error probability in a purely thermal noise environment without the presence of any other noise mechanism to impair the system performance. The second term in the expression is the additive term that gives the increase in the probability of a bit error in the presence of any form of impairment that reduces the signal to noise ratio. The general form of the above expression helps in determining the degradation in the system performance for a wide range of problems. The advantage in this approach is that the moments of a distributon are in general easier to determine than the distribution itself for a large class of problems occuring in communication theory. The above method may be applied to the class of problems in optical communications that deal with the presence of signal dependent shot noise in addition to thermal noise, but the averaging over all possible sequences would give optimistic results in the case of signal dependent shot noise. In the above expression the assumption has been made that the additive term is bounded and does not blow up as n approaches large values. Although a mathematical proof for the same is not given here, the numerical computations clearly show that the sequence is indeed bounded and convergent.

ISI due to Baseline Wander.

X is the cumulative ISI as defined in eqn.(3.3) due the previous (n-1) pulses

$$X = x_1 + x_2 + x_3 + \cdots + x_{n-1} \tag{3.13}$$

where x_i is the contribution to the eye degradation due to the $(n-i)^{th}$pulse. The x_i are i.i.d. random variables, therefore the probability density function of X is given by

$$G(X) = g(x_1)^* g(x_2)^* g(x_3)^* \ldots g(x_n) \tag{3.14}$$

$g(x_i)$ is the probability density function of of the random variable x_i. A Fourier transform of equation (3.14) gives the moment generating function for the ISI.

$$G(w) = \prod_{i=1}^{\infty} \tilde{g}_i(w) \tag{3.15}$$

$\tilde{g}_i$ is the moment generating function of x_i

$$\tilde{g}_i(w) = \int p(x_i) e^{jwx_i} dx_i \tag{3.16}$$

If the binary digits a_i are assumed to be equiprobable then

$$p(x_i) = 1/2 \qquad x_i = + f(t - iT)$$

$$p(x_i) = 1/2 \qquad x_1 = - f(t - iT) \tag{3.17}$$

It can be easily shown that

$$\tilde{g}_i(w) = \cos w\,(\,f(t - iT)\,) \qquad G(w) = \prod_{i=1}^{n} \cos w\,(\,f(t - iT)\,) \tag{3.18}$$

The n^{th} derivative of the G(w) gives the n^{th} moment of the ISI. Differentiating ln G(w) gives

$$G'(w) = \left[\sum_{i=1}^{n} \tan w\,(\,f(t - iT)\,)\, f(t - iT) \right] G(w) \tag{3.19}$$

Denoting the first term in the expression (3.19) as h(w) and differentiating it (2k-1) times

$$G^{2k}(w) = -\left[\sum_{i=1}^{k} \binom{(2k-1)}{(2i-1)} h^{(2i-1)}(0)\, G^{2(k-i)}(0) \right] \tag{3.20}$$

$$h^{2i-1}\,(0) = \frac{d^{2i-1}}{dw^{2i-1}}\, h(w)_{w=0} \tag{3.21}$$

Expanding tan $w\,(\,f(t-iT)\,)$ as a Taylor's series around t = iT and differentiating k times with respect to w to obtain the first k moments of the ISI

$$M_{2k} = (\,-1)^{k} h^{2k}(0) \tag{3.22}$$

Combining 3.20 and 3.22

$$M_{2k} = -\left[\sum_{i=1}^{k} \binom{(2k-1)}{(2i-1)} (\,-1)^{i}\, M_{2(k-i)}\, h^{2i-1}(0) \right] \tag{3.23}$$

Equation 3.23 gives a recursive relation for the k^{th} moment of the ISI. It must be noted that only the odd derivatives of $h(w)_{w=0}$ are non zero, therefore only the even moments of the ISI are non zero. Thus, noting that $M_0 = 1$ and using equation 3.23 all the moments of X can be obtained. Substitution of equation 3.23 in equation 3.11 gives the error probability for a generalised pulse shape f(t). The analysis can now be applied to the case of a random binary sequence that suffers from ISI impairment due to low frequency cutoffs in the forward path of the channel. The forward path high pass filter is modelled as a simple RC filter and the effect of the filter time constant on the error probability studied. For the simple RC filter configuration, treating the ISI as a Gaussian with a zero mean and a variance determined from equation (3.23) gives results for the error probability close to those arrived at, by substituting for the higher moments of the ISI in the equation (3.12).

Analysis of Quantized Feedback

This section deals with the application of the previous analysis for the case of a generalised ISI to the quantized feedback problem. The analysis is carried out for the feedback equalization process assuming that the forward path and the feedback filters are si.:gle pole RC filters. This assumption makes the exact analysis possible. The calculation of error probability with quantized feedback is a more difficult problem, because the individual components that add to the cumilative ISI are no longer independent, but depend on the ISI on the previous pulse as well as the error probability on the previous pulse.

A simple discrete time quantized feedback circuit is shown in fig(4).

The incoming bit sequence is denoted by a_k. The output of the high pass filter is given by

$$s(t) = \sum_{k} a_k \int_0^{\infty} g(\tau)p(t - jT - \tau)\, d\tau \qquad (3.24)$$

Where p(t) is the input pulse shape and the data rate is given by 1/T.

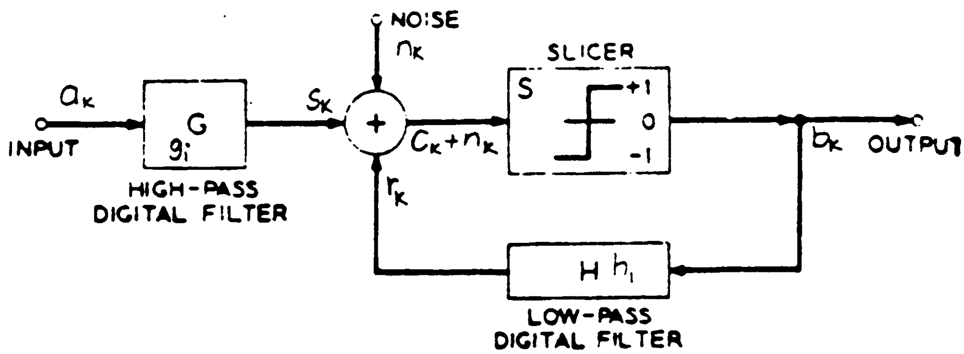


Figure 4. Model of a Quantized Feedback scheme

$$r(t) = \sum_k b_k \int_0^\infty h(\tau)P(t - jT - \tau)\, d\tau \qquad (3.25)$$

Where b_k is the restored data sequence. The signal input to the regenerator is given by

$$c_k = r_k + s_k \qquad (3.26)$$

$$c_k = \sum_{j=0}^{j=i-1} a_j g_{i-j} + b_j h_{i-j} + a_k g_0 + a_k h_0 \qquad (3.27)$$

In order to obtain perfect cancellation of the tails $g_i + h_i$ must be zero and h_0 must be zero. For an RC filter having the time constant T_c = RC The coefficients of h_i and g_i can be expressed as

$$h_i = 0 \qquad g_i = g_0 \qquad i = 0 \qquad (3.28)$$

$$g_i = g_0 r^i \qquad h_i = -\, g_0 r^i \qquad i \neq 0 \qquad (3.29)$$

The constant r given by $e^{-\frac{T}{T_c}}$ determines the cutoffs of the RC filters. A higher value of r would imply a lower cutoff frequency.

The regenerator makes a decision on the symbol by comparing the input to a threshold. Ideally the tail cancellation is perfect if there are no errors in determining the input sequence but in the presence of thermal noise there is a finite probability of error and the regenerated output bit sequence has errors which are fed back into the decision process of the bits that follow leading to an error multiplication effect.The output of the regenerator is given by

$$b_k = \mathrm{sgn}(c_k + n_k) \tag{3.30}$$

From equations (3.27), (3.28) & (3.29)

$$c_k = g_0 a_k + \sum_{\substack{i=0 \\ a \neq b}}^{k-1} (h_{k-i} b_i + g_{k-i} a_i) \tag{3.31}$$

For polar transmission $a_i \neq b_i$ implies $a_i = -b_i$ therefore (3.33) may be rewritten as

$$c_k = g_0 a_k + \sum_{i=0}^{k-1} 2 g_{k-i} a_i \tag{3.32}$$

Denoting the second term in the above equation as x_k, the cumulative effect of all previous errors on the input to the decision device, it can be seen that the output b_k is in error if

$$n_k + x_k < -g_0 \qquad a_k = 1 \tag{3.33}$$

$$n_k + x_k > g_0 \qquad a_k = -1 \tag{3.34}$$

The effective noise in the system is $n_k + x_k$. If the input sequence bits a_i are i.i.d then the random variables n_k and x_k are independent. If $F_k(x)$ is the distribution function of x_k and p(k) is the probability of a k^{th} bit error, then for an equiprobable distribution of $+1$'s and -1's in the incoming bit stream

$$p(k) = \frac{1}{2}\int N(-g_0 - x)\, dF_x + \frac{1}{2}\int (1 - N(g_0 - x))\, dF_x \qquad (3.35)$$

Where

$$N(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{x} e^{-}\left(\frac{x^2}{\sigma^2}\right)dx \qquad (3.36)$$

In other words p(k) is the error probability in the thermal noise environment averaged over all possible 2^k sequences of the input bit stream. Each sequence $\tilde{x}$ has a distribution function $F_k(\tilde{x})$. We will now obtain a recursion relation for x_k. From the definition of x_k as the second term in (3.32) it follows that

$$x_{k+1} = 2\sum_{i=0}^{i=k} g_{k+1-i}a_i \qquad (3.37)$$

From (3.37) and (3.35)

$$x_{k+1} = r\,x_k - d \qquad a_k = 1 \neq b_k \qquad p_1(x_k) \qquad (3.38)$$

$$x_{k+1} = r\,x_k + d \qquad a_k = -1 \neq b_k \qquad p_3(x_k) \qquad (3.39)$$

$$x_{k+1} = r\,x_k \qquad a_k = b_k \qquad p_2(x_k) \qquad (3.40)$$

Where $p_1(x)$, $p_2(x)$, $p_3(x)$ are the transition probabilities for the the random variable x_k, d is equal to $2g_1$r. Zador(16) treated the problem as a random walk process. The sequence X_k has a distribution $F_k(x)$ then x_k has a limiting distribution L(x) as k approaches infinity. If the sequence of r.v.'s X_0 is assumed to be independent of all input variables then sequence X_1 , X_2, ... where x_{k+1} is related to x_k by equations (3.38), (3.39), (3.40) forms a Markov chain.

$$p_1(x) = \frac{1}{2} N(-g_0 - x) \tag{3.41}$$

$$p_3(x) = \frac{1}{2} (1 - N(g_0 - x)) \tag{3.42}$$

$$p_2(x) = 1 - p_1(x) - p_3(x) \tag{3.43}$$

Thus $x_{k+1}$is re!ated to x_k by the above transitions and thus forms a Markov chain. Given the value of x_k the random variable x_{k+1} may be formed in three possible ways each characterised by a linear function of x_k, whose choice is dependent on the transition probabilities. The linear relationship is a result of the assumption that the filters are simple RC filters.

Zador observed that for $0 \le r \le 1$ the random variable x_k has a limiting distribution function L(x) and the average of any continuous function of x, f(x) can be determined without having to compute L(x). Specifically applying the idea to the quantized feedback case

$$U\,f(x) = p_1(x)f(xr - d) + p_2(x)f(xr) + p_3(x)f(xr + d) \tag{3.44}$$

If the k^{th} iterate of f(x) is denoted by $U^k f(x)$ then

$$\lim_{k \to \infty} U^k f(x) = \int f(x)\, dL(x) \tag{3.45}$$

Zador's algorithm can be used to determine the exact expression for the error probability in the quantised case. The error probabilities $p_1(x)$ and $p_3(x)$ are to be averaged over the distribution for all the sequences; thus

$$\int [p_1(x) + p_3(x)]\, dL(x) = \lim_{k \to \infty} U^k f(x) \tag{3.46}$$

Note that the k^{th} iterate of U(f(x)) is to be evaluated at the average value of x, which in this case is zero.

From the previous generalised expression for the error probability in the presence of ISI (equation 3.12) it can be seen that the error probability may be calculated from the $2n^{th}$ moments of x. Substituting f(x) $= x^n$ in the equation (3.45)

$$U(f(x)) = (xr - d)^n p_1(x) + (xr)^n p_2(x) + (xr + d)^n p_3(x) \tag{3.47}$$

Since $\left| \frac{xr}{d} \right| << 1$, it can be easily shown that

$$U\, f(x)_{x=0} = d^n(\, p_1(0) + p_3(0)\,)_{x=0} \tag{3.48}$$

$$U^2 f(x)_{x=0} = d^n(\, p_1(0) + p_3(0)\,)(1 + r^n) \tag{3.49}$$

$$U^3 f(x)_{x=0} = d^n(\, p_1(0) + p_3(0)\,)(1 + r^n + r^{2n}) \tag{3.50}$$

$$\lim_{k \to \infty} U^k f(x)_{x=0} = \frac{d^n(\, p_1(0) + p_3(0)\,)}{(1 - r^n)} \tag{3.51}$$

Thus the n^{th} moment of of x $M_n(x)$ is given by

$$M_n = \frac{d^n(\, p_1(0) + p_3(0)\,)}{(1 - r^n)} \tag{3.52}$$

Substituting in equation (3.13) the probability of error of a bit in an infinitely long random binary bit sequence is given by

$$p_e = p_e(\text{thermal}) + \sum_{1}^{\infty} \frac{1}{n!(\sqrt{2}\,\sigma)^n} \; H_n\left(-\frac{g_0}{\sigma}\right) \frac{e^{-\left(\frac{g_0}{\sigma}\right)^2}}{\sqrt{\pi}} \frac{d^n(p_1(0) + p_3(0))}{(1 - r^n)} \tag{3.53}$$

The additive term now contains the error probability in the absence of ISI as a multiplicative factor. The additive term which gives the degradation in system performance depends directly on the signal to noise ratio. If the system is operating at a signal to thermal noise amplitude ratio of about 6, the error probability it corresponds to is about 10^{-9}, which makes the additive term negligible. Thus the suggestion that a random burst of errors may cause the closure of the eye seems to be nearly impossible in practice because most systems operate at error rates close to 10^{-9}. This is quantified more precisely by the results presented at the end of this chapter.

Analysis of quantized feedback with a timing mismatch

The design of the crossover frequency for the quantized feedback system requires the investigation of the dependence of the error probability on the value of the timing mismatch in the tail cancellation. The analysis for the timing mismatch problem tells us the degradation in the system performance when the tails are not cancelled exactly. One would expect two mechanisms of degradation, the first would arise as an additive baseline wander term due to residual ISI as a result of imperfect cancellation and the second would be the term due to error multiplication. The timing mismatch is evaluated in terms of the filter parameter r, a mismatch of one bit period would correspond to multiplication of the feedback filter response coefficients by r. In general a timing mismatch of mT $0 \le m \le 1$ would be modelled by

$$h_i = 0 \qquad g_i = g_0 \qquad i = 0 \tag{3.54}$$

$$g_i = g_0 r^i \quad h_i = -g_0 r^{i+m} \quad i \neq 0 \tag{3.55}$$

Substituting for the above in eqns.(3.38),(3.39),(3.40) we get

$$x_{k+1} = r\, x_k - (1 + r^m) g_1 \qquad a_k = 1 \neq b_k \qquad p_1(x_k) \tag{3.56}$$

$$x_{k+1} = r\, x_k + (1 + r^m) g_1 \qquad a_k = -1 \neq b_k \qquad p_3(x_k) \tag{3.57}$$

$$x_{k+1} = r\, x_k + (1 - r^m) g_1 \qquad a_k = \quad b_k \qquad p_2(x_k) \tag{3.58}$$

Using Zador's algorithm we know that

$$\lim_{k \to \infty} U^k f(x)_{x=0} = p(k) \qquad f(x) = p_1(x) + p_3(x) \tag{3.59}$$

$$Uf(x) = p_1(x) f(xr - (1 + r^m) g_1) + p_2(x) f(xr + (1 - r^m) g_1) + p_3(x) f(xr + (1 + r^m) g_1) \tag{3.60}$$

Expanding f(x) as a Taylor's series and neglecting derivatives above the second derivative and noting that the odd derivatives of f(x) are zero

$$Uf(x) = f(xr) + f''(xr) \left[\frac{p^2}{2!} \left(p_1(0) + p_3(0) \right) + \frac{q^2}{2!} \left(p_2(0) \right) \right] \tag{3.61}$$

Where $q = (1 + r^m)g_1$ and $p = (1 - r^m)g_1$. The error probability for an infinitely long sequence of input data bits is given from equations (3.61) ,(3.46) by

$$p(k) = f(0) + f''(0)\left[\frac{\frac{p^2}{2!}\left(p_1(0) + p_3(0) \right) + \frac{q^2}{2!}\left(p_2(0) \right)}{(1 - r^2)} \right] \tag{3.62}$$

f(0) is the error probability due to thermal noise alone, and the second term is the additive error probability.

$$f''(0) = \frac{2g_0^3 r^2}{\sigma^3 \sqrt{2\pi}\, 1 - r^2} \; e^{-\left(\frac{g_0^2}{\sigma^2}\right)} \tag{3.63}$$

For $m = 0$ the expression (3.62) reduces as expected to eqn.(3.53) which gives the error probability for quantized feedback without mismatch.

Results and Conclusions

The results of the analysis for the degradation of receiver performance due to low frequency cutoffs in the forward path of the input signal are given in figures 5 and 6. In fig(5) the plot for $r = 1$ is the ideal case error probability curve for the case of a zero frequency cutoff, in other words it corresponds to a case where only thermal noise impairment exists. The plots for different values of r correspond to to impairments due to baseline wander for different values of low frequency cutoffs in the forward path of the input signal. The other curves correspond to frequency cutoffs that are 10%, 20%, 30% and 50% of the bit rate respectively. Note that the impairments

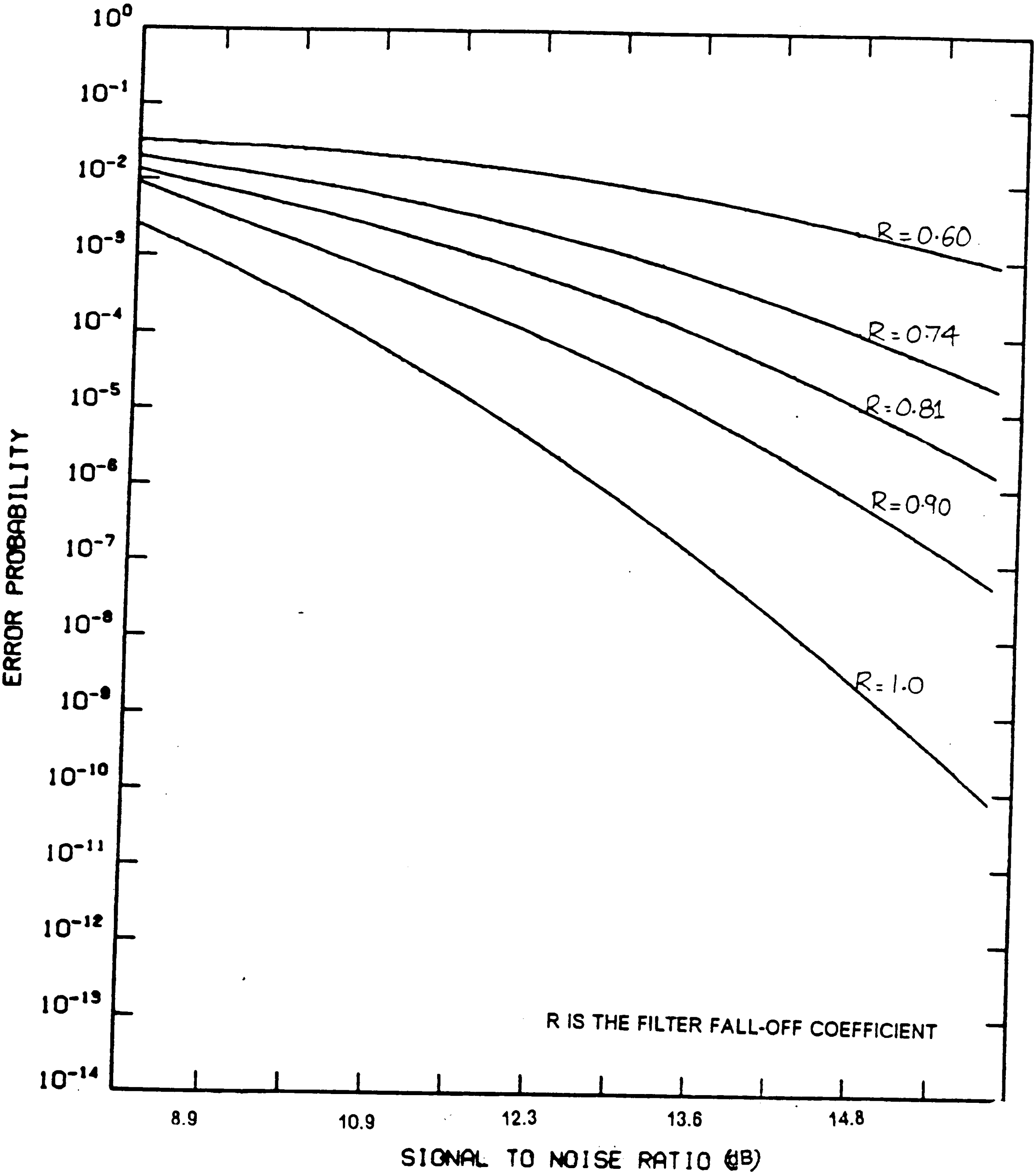
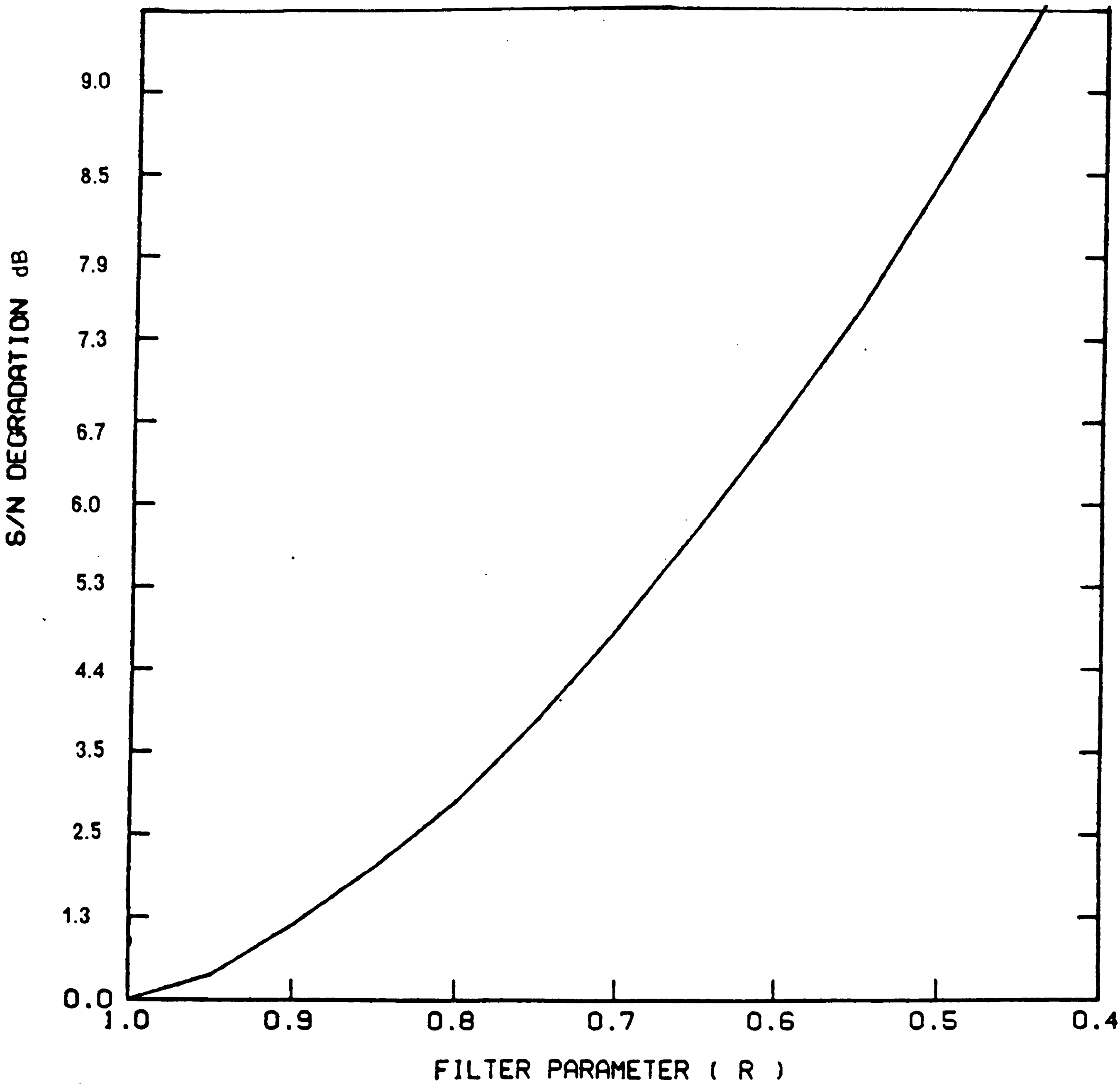


Figure 5. Error probability vs. S/N ratio for low frequency cutoffs



R IS THE FILTER FALL-OFF COEFFICIENT

Figure 6. S/N degradation vs. filter response. $P_e = 10^{-9}$

worsen as the value of the filter response r drops from 1 to 0.60 and cause unacceptable system performance as the frequency cutoff increases beyond 10% of the bit period. Figure 6 gives the degradation in the signal to noise ratio as a function of the filter response r for an error probability of about 10^{-9}. The filter response r varies from 0.4 to 1.0. This implies that the corresponding low frequency cutoffs vary from 50% of the data rate to D.C. The signal undergoes a considerable degradation as the forward path cutoff increases and reaches unacceptable levels of performance for values of r beyond 0.20.

The plot of error rate vs. SNR for the case of a repeater with quantized feedback equalization is given in fig 7. As the theory suggests the error multiplication effect is negligible and is noticed only at low values of SNR (< 3). At the values for signal to noise ratio that correspond to the operation of typical digital communication systems, the curve for the quantized feedback case coincides perfectly with the ideal case curve of a zero frequency cutoff. When the tail cancellation is perfect the error rate curves for different values of the filter response are almost the same, the error rate then is independent of the filter response for high values of the SNR.

In the event of a timing mismatch, the error rate and the degradation in the receiver signal to noise ratio depend critically on the filter parameter r. For values of r up to 0.9 (low frequency cutoff is 10% of the bit rate) a mismatch in the timing and tail cancellation does not cause a large degradation in performance. However, for values of the low frequency cutoffs greater than 10-15% of the bit rate even a timing mismatch of 0.05T can cause unacceptable error rates. Figures 8 and 9 give the variation of error rates and S/N degradation as a function of the timing mismatch.

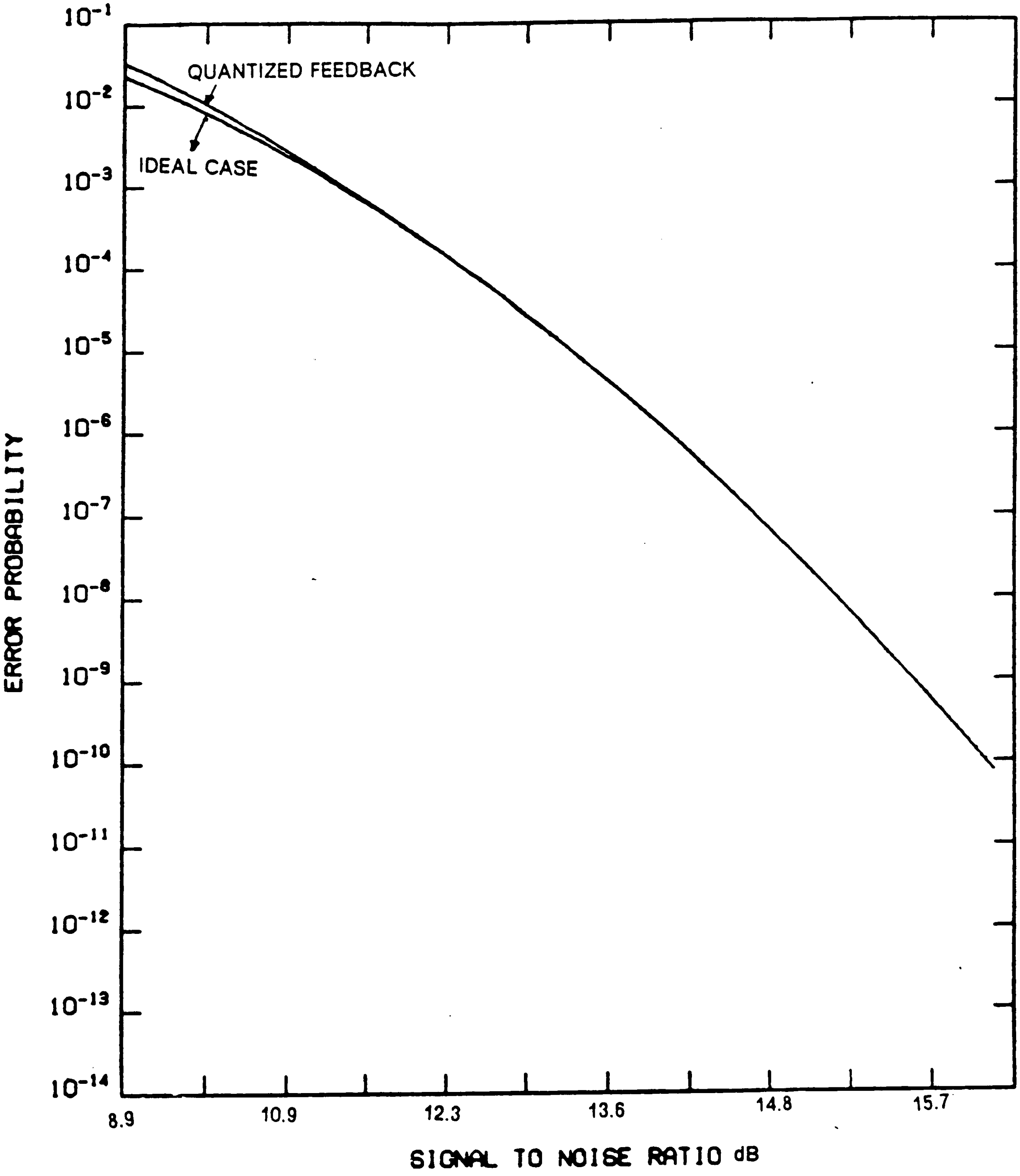


Figure 7. Error rate vs. SNR with quantized feedback improvement

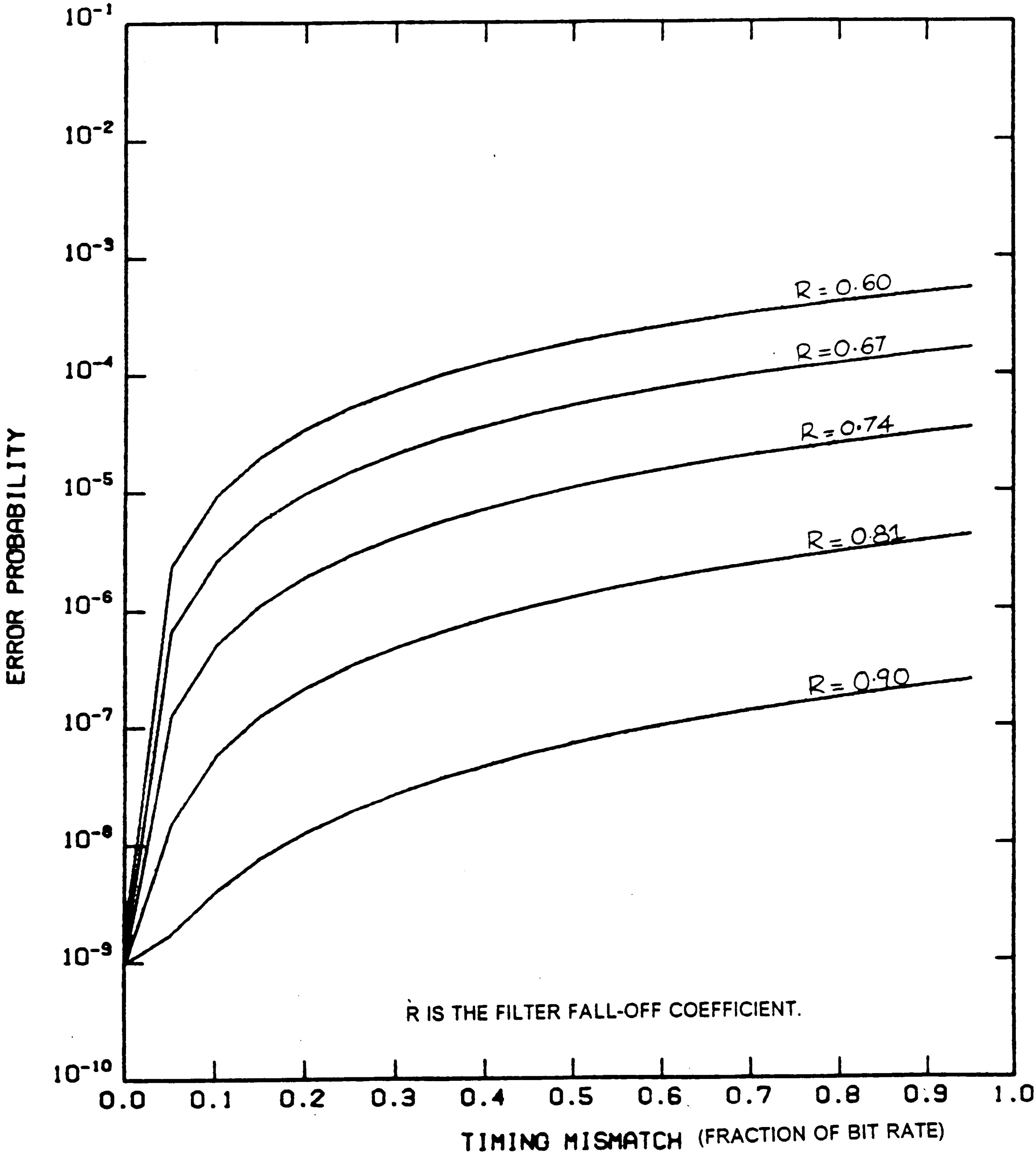


Figure 8. Error rate vs. timing mismatch. $P_e = 10^{-9}$

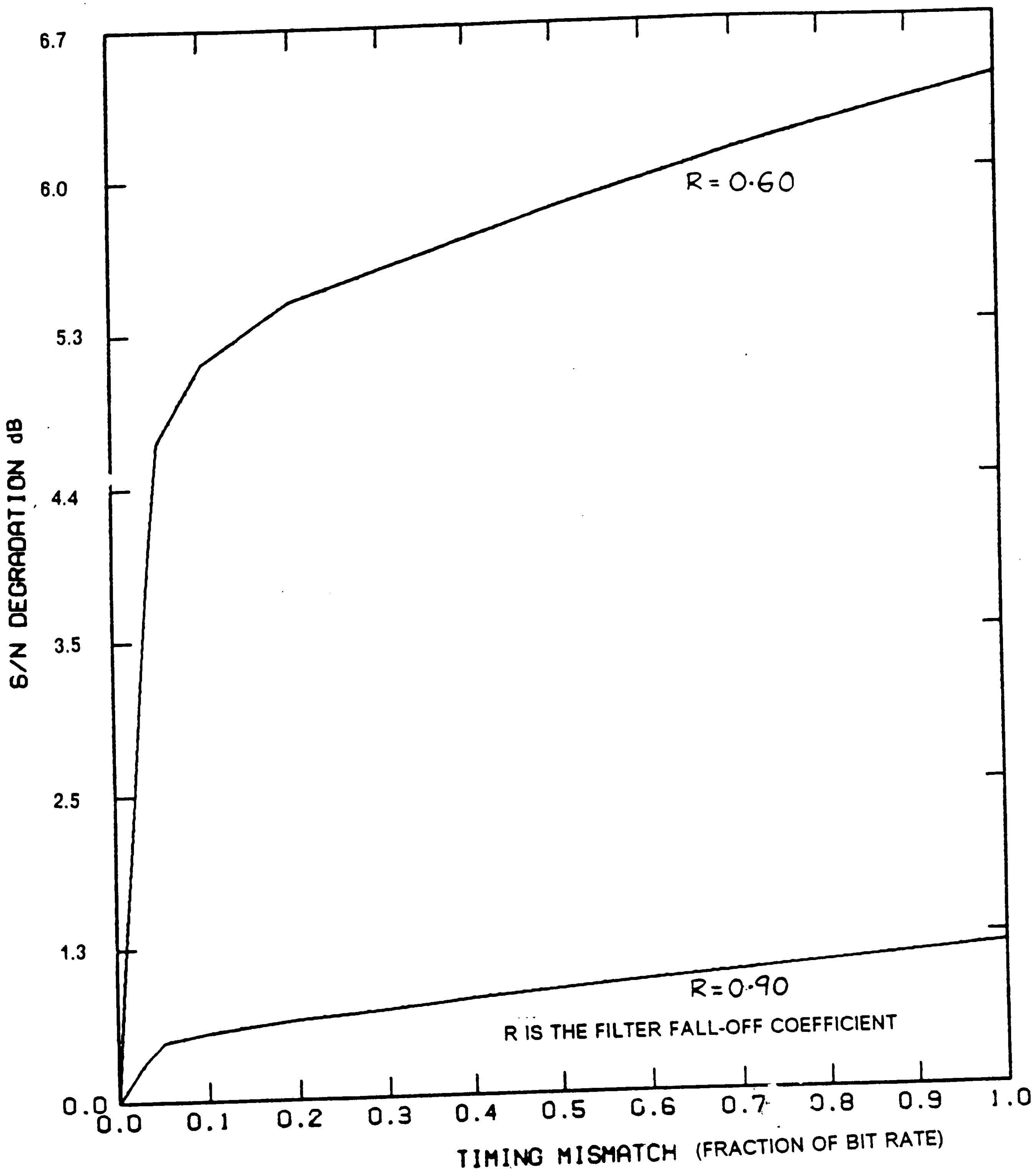


Figure 9. S/N degradation vs. timing mismatch. $P_e = 10^{-9}$

The results of the analysis carried out on the baseline wander problem and the application of the quantized feedback suggest the following.

1. The presence of baseline wander in a an OOK system can degrade it's performance to a considerable extent. The degradation worsens dramatically with an increase in the cutoff frequency and cutoff frequencies greater than 15% of the bit rate can lead to an unacceptable system performance.

2. Quantized feedback is a viable alternative to counter the problem. The so called error multiplication effect is negligible for most practical purposes and almost a zero degradation in performance may be observed if tail cancellation is perfect.

3.The choice of the low frequency cutoff must be made such that the filter crossover frequency is high enough to allow for the unavoidable cutoffs in the forward path but low enough to not cause a serious impairment in the system performance as a result of a timing mismatch. Typically values of the forward path low frequency cutoffs of about 15% of the data rate give an acceptable performance even with a timing mismatch.

The results of the analysis suggest that the low frequency cutoffs cause significant impairment in the signal to noise ratio for values of cutoffs greater than 15% of the bit rate, but in order to avoid a timing mismatch performance degradation the cutoffs have to be lesser than 15% of the bit rate. This indicates that quantized feedback schemes may not have a significant practical use, and is perhaps the reason that quantized feedback schemes are not as popular as one would expect them to be.

Quantized feedback in fiber optic systems

Although the analysis for determining the threshold position and corresponding error probabilities in the case of a digital fiber optic system is significantly different than in a line wire or a coaxial system ,the Gaussian approximation for the error probability is generally a conservative one(9). As long as the threshold and the optimal APD gain is determined by the shot noise analysis, the error rates still follow the " signal to noise" approximation quite closely. It is therefore safe to assume that the probability of error is given by the area under the Gaussian tail. Thus, the analysis carried out for quantized feedback performance in an optical fiber system should give the same kind of results pertaining to the error multiplication effect and timing mismatch. Indeed, the decision feedback nature of quantized feedback should give excellent performance as long the initial error rates are reasonably low.

chapter IV

Line Coding

Introduction

Line coding techniques are a clever means to realise a set of multiple objectives in digital communication systems. A line code is a modification of the data sequence to be transmitted over a communication link which contains redundant bits along with the information signal that provide information with regards to the sequence such as timing etc. The bit redundancy also causes the signal to have certain desirable properties like a zero or constant D.C., which improve system performance. Line codes for a copper wire system have been studied extensively and there is a multiplicity of such codes in existence and although their usage in the U.S. is not very extensive they are popular in Europe and Japan. This is also true for systems that use fiber optic links. In the U.S. scrambling with a parity check is often used to perform the same k.nds of function, but this approach lacks an exactness and occurences of pathological signal sequences could adversely affect the performance of the system. A line code may be defined as a mapping of the binary input information to an ensemble of symbol sequences which satisfy certain design criteria. The allowable sequences can in turn be be specified by a set of permissible states for each point. In addition it is convenient to define a set of allowable words which take the sequence through a succession of allowable states, one for each symbol in the word. The state which the code word occupies at the end is called a terminal state. The simplest codes of this type have fixed lengths and are called block codes, they are mappings of input binary words of length N to a set of allowable words of length M, where $M > N$ if the coded sequence is binary.

Performance criteria for line codes

A line code is optimally designed to best suit a set of criteria that allow upgrading of the system performance. The following is a list of some performance criteria for line codes.

1. A zero or a constant D.C. value to avoid the baseline wander problem and subsequent degradation in the system performance as a result of imbalance in the D.C. characteristics of the data sequence.
2. Adequate pulse transitions for timing recovery, to help the timing extraction circuit make correct decisions on the sampling instants
3. Built in error monitoring in the coding scheme. Violations of the scheme provide an error detecting capability.
4. Minimization of the dependence of system performance on the statistics of the information bit patterns. It is best to make the line signal completely independent of the statistics of the data sequence to allow for the occurence of some pathological signals. Codes which achieve this are called BSI codes (Bit Sequence Independent).

5. Reducing certain high frequency components in the spectra of the information signals to avoid crosstalk phenomena in metallic systems and to control bandwidth in radio sytems.

6. The information capacity of the code should be made as high as possible.

7. Word alignment information must also be obtained from the code. In other words the received symbol sequence must be partitioned correctly before decoding.

Performance measures for line codes

In order to characterise the performance of a code design some measures can be defined to quantify the code performance.They are

1. Running Digital Sum (RDS) is defined as the sum of all previous bits of the code. In other words

$$RDS(k) = \sum_{1}^{k} c_n + RDS(0) \tag{4.1}$$

The running digital sum gives a measure of the amount of D.C. content in the code. A zero or fixed RDS is desirable to minimize the baseline wander problem. Most of the constraints on the code can be translated to constraints on the running digital sum.

2. Digital Sum Variation (DSV) is defined as the difference between the the maximum and the minimum values of the running digital sum, and denotes an upper bound on the length of strings of like pulses and gives a measure of the low frequency content of the code.

3. The Power Spectral Density of the code which gives the energy content at different frequencies in the code. It is desirable for the PSD of the code to go to zero and for the derivatives of the power spectral density be as close to zero for D.C.

Optical Line Codes

Optical line codes tend to be two level rather than three level for two reasons. The need to go to an M-ary modulation scheme in a baseband digital communication system arises due to bandwidth restrictions. Transmission at more than two amplitude levels uses the same amount of bandwidth to transmit more information, the price for this gain is paid in terms of signal energy. In order to maintain the same error rate a greater amount of average signal power is transmitted. In fiber systems bandwidth optimisation is not generally a consideration because of the availability of very high bandwidths far exceeding the required bandwidth for transmission. However, the amount of signal power that can be put into a fiber is limited and a constraint on the system performance. Thus, going to a higher number of intensity levels in fiber systems seems to be a step in the direction opposite to the desired one. The second reason for not going to larger levels of transmission schemes is that lasers operating at higher power levels suffer from a non linearity problem causing a further degradation in the received signal power. Therefore it is apparent that optical line codes tend to be two level rather than three or more level codes. Pulse broadening or fiber dispersion is another design consideration to be taken into account for code optimisation, but the extensive use of single mode optical fibers tends to make that design consideration less important.

Binary block codes

These codes known as mB-nB codes, convert blocks of m binary digits to blocks of n binary digits where $n > m$. The n bit words correspond to 2^m information words to be chosen in such a way that clock extraction in the repeaters is facilitated and the low frequency component in the continuous code spectrum is reduced. Since the low frequency component of a code is proportional to the DSV, a limit is set on the DSV. For n even, zero DSV codes are possible if all the 2^m words can be expressed in terms of n bit words that have an equal number of 0's and 1's. A zero DSV can be obtained if $\binom{n}{n/2} > 2^m$. When the above condition is not met a bimodal operation has to be resorted to wherein some input words can generate two complementary bit sequences as output words. The jump from one mode to the other is determined by the value of the DSV at the end of the previous word. The ratio n/m is the increase in the symbol rate and thus is a bandwidth as well as a power penalty with respect to the uncoded signal. A lower value of n makes the decoding circuitry easier. It must be noted that the n bit zero disparity words that have equal first n/2 digits are excluded from the choice of codewords to reduce the bounds on disparity variation.

The error detection in these codes can be impemented by driving an up-down counter with the received code sequence and an error is registered when the DSV exceeds that of the adopted code. The efficiency of the error correction is directly dependent on the DSV, a smaller DSV implies a more efficient code correction capability.

1B-2B Codes

1B-2B codes are useful for two reasons, they have a very high redundancy and as a consequence show very good performance as line codes. Also, they require very simple circuits for encoding and decoding. The main drawback is that the symbol rate and thus the required bandwidth is double with respect to that of an uncoded signal. This causes a power penalty of about 3dB, so these codes may not be well suited when transmission rates are high and fiber bandwidth limitations arise. Two interesting examples of these codes are CMI (Coded Mark Inversion) and the Biphase code. In CMI the digit 0 is transmitted as 01 and the digit 1 is transmitted alternately as 11 or 00. The word 10 is forbidden and allows easy alignment procedures at the receiver. The DSV of the code is three which allows for an efficient error detection mechanism. In a Biphase code a zero is sent as 01 and a 1 as 10. The code thus has a DSV of two and two prohibited code words 00 and 11, and enjoys minor advantages over CMI. The Biphase code is also called Manchester code.

Partial response signalling or correlative coding is yet another means for line coding. In this scheme the transmission of the binary sequence a_n is accomplished at two levels by transmitting the sum $b_n = a_n + a_{n-1}$. The sequence b_n has three possible levels, if a_{n-1} has already been decoded a_n can be decoded by subtracting it from b_n. An error in decoding a_{n-1} can lead to a propagation of error. This problem is circumvented by using a precoding technique. Defining $c_n = \tilde{a}_n + c_{n-1}$, where $\tilde{a}$ is the complement of a and the addition is modulo two, b_n is transmitted as $c_n + c_{n-1} - 1$. The effect is that if $a_n = 0$, $b_n = 0$. When $a_n = 1$, the polarity of b_n depends on the number of intervening zeros since a_{k-1}, the last member of $\{a\}$ to be zero. If the number of intervening zeroes is even then $b_n = b_{n-k}$; otherwise $b_n = -b_{n-k}$ The errror properties are similar to a bipolar code, in that a single error will cause a violation in the coding rule. The drawbacks of this code are the higher power penalty (37) and complexity of a three level decision circuitry. It's principal utility is in reducing bandwidth requirement, not typically important in fiber systems.

Analysis of binary block codes

In this section an analysis of the performance of binary block coding schemes in terms of receiver sensitivity considerations is carried out.This analysis is purely from receiver design standpoint for maximising receiver sensitivity in terms of worst case noise situations that arise as a result of the signal dependent nature of shot noise in an optical communication system. The detailed expression for the worst case noise in an optical fiber system was derived by Personick(9) in his classic paper on receiver design considerations. He used the expression to determine the receiver sensitivity for a random binary train of pulses incorporating into his analysis impairments such as a non zero extinction ratio. Personick's derivation can however be modified to determine the noise impairment due to any sequence of data bits and it is this approach that has been applied to optical line codes.

Statistical model for optical receivers

The model for an avalanche photodiode is given in fig 10. The collision ionisation process is also shown. In fig 11, the current source can be considered to be a sequence of impulses corresponding to electrons generated within the photodiode due

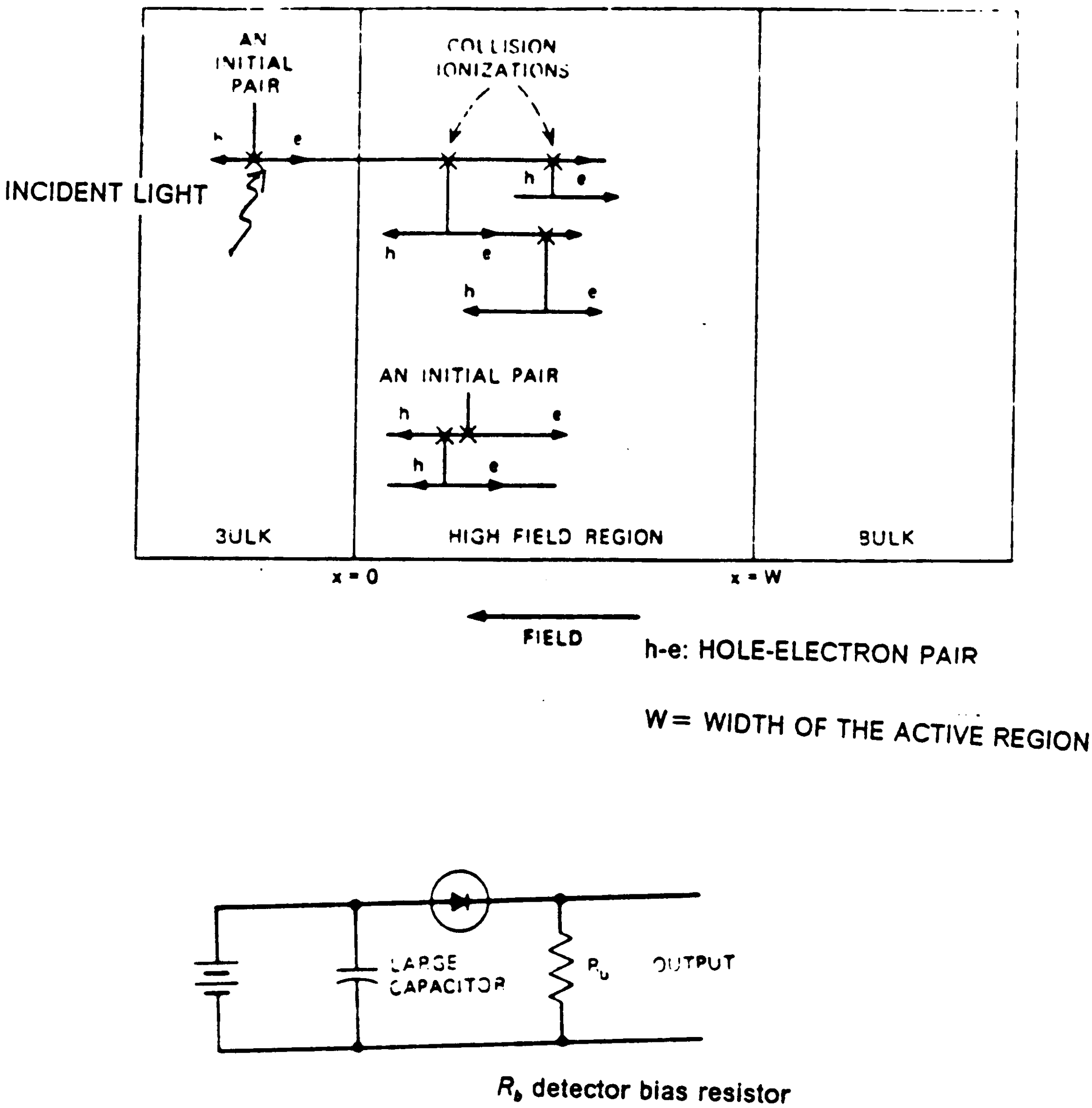


Figure 10. Modelling of an APD

to optical and thermal excitation or collision ionisation. In fig (10) electrons are produced at an average rate of $\lambda(t)$ per second where

$$\lambda(t) = \left(\frac{\eta}{h\Omega}\right)p(t) + \lambda_0 \tag{4.2}$$

where η = quantum efficiency of the counter

$h\Omega$ = energy of a photon

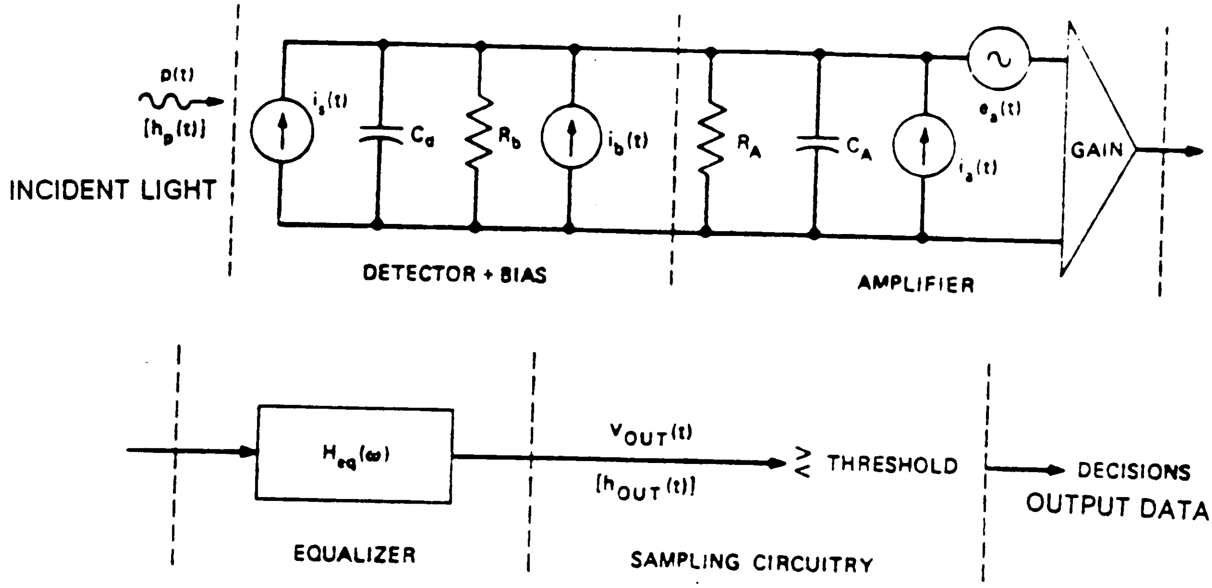
λ_0 = dark current counts per second

$p(t)$ = incident optical power

Note that $\lambda(t)$ is only the average rate at which the electrons are produced; the actual number is given in terms of a probability of occurence of exactly N counts which is given by a Poisson distribution with a mean λ. Each of these primary generated electrons enters a random multiplier which accounts for the APD avalanche gain, where it is replaced by a certain number of secondary electrons following the random gain statistics of the APD. This implies that the current leaving the photodioide consists of bunches of electrons, the number in each bunch follows avalanche statistics. Different APD's have different statistics governing the gain. For our purpose it will suffice to know that for a large class of photodiodes of interest

$$\overline{(g^2)} \simeq \bar{g}^{2+x} \tag{4.3}$$

where $\bar{g}$ is the avalanche gain of the APD and $\overline{(g^2)}$ is the mean square gain, x is defined as the excess gain factor of the APD. Fig 11 shows the configuration for a typical optical receiver the amplifier is modeled as an ideal high gain infinite impedance amplifier with an equivalent shunt capacitance and resistance at the input with two



R_b detector bias resistor
$i_{b(t)}$ the detector thermal noise current
C_d APD capacitance
R_a Amplifier input resistance
C_a Amplifier shunt capacitance
$i_a(t)$ Noise current source due to R_a
$e_a(t)$ Noise voltage source of the channel
$i_s(t)$ APD shot noise source

Figure 11. Reciever Model

noise sources referred to the input, these noise sources are white and Gaussian. The power falling on the detector is assumed to be of the form of a digital pulse stream.

$$p(t) = \sum_{-\infty}^{\infty} b_k h_p(t - kT) \tag{4.4}$$

where b_k takes one of two values for each integer value of k, T = pulse spacing and $h_p(t - kT)$ is the pulse shape where the normalisation $\int_{-\infty}^{\infty} h_p(t - kT) = 1$ is used. The average voltage at the equalizer output is given by

$$v_{out}(t) = \frac{A\eta g p(t) e}{h\Omega} * h_{fe}(t) * h_{eq}(t) \tag{4.5}$$

where A is an arbitrary constant, $h_{fe}(t)$ is the amplifier impulse response and $h_{eq}(t)$ is the equalizer impulse response. The output voltage is of the form

$$v(t) = \sum_{-\infty}^{\infty} b_k h_{out}(t - kT) + n(t) \tag{4.6}$$

The noise portion of the voltage is given by $N = \overline{(v^2)} - \bar{v}^2$. The noise is signal dependent and non stationary which is the reason that optical fiber systems have to be analysed differently in comparision to line wire systems. The noise n(t) is dependent on the sequence b_k; Personick carries out his analysis for the sequences that lead to worst case noise situations and maximise the mean square noise term. The analysis for optical line codes is carried out in a similar manner by evaluating the mean square noise terms for worst case sequences.

The output noise can be written as

$$v_{out}(t) - \overline{v_{out}(t)} = n_s(t) + n_r(t) + n_i(t) + n_e(t) \tag{4.7}$$

where

$n_s(t)$ is the output noise due to the Poisson nature of the the current produced by the detector.

$n_r(t)$ is the Johnson's noise of the biasing resistor R_b

$n_i(t)$ is the output noise due to the amplifier current noise source.

$n_e(t)$ is the output noise due to the amplifier input noise voltage source.

The last three terms can be easily determined by using the well known result for the power output of a filter driven by a white Gaussian noise source. The first term is the signal dependent shot noise term that is determined using Poisson arrival statistics and the second moment approximation for the APD gain. Personick has shown that for a generalised sequence b_k the worst case noise is given by

$$NW(b_0) = NW_{shot} + N_t \tag{4.8}$$

NW_{shot} is the worst case shot noise

$$NW_{shot} = A_1 I_1 + A_3 \Sigma_1 \tag{4.9}$$

N_t is the thermal noise due to the output noise of the current source, the output noise of the voltage source and the resistance R_b

$$N_t = B_1 I_1 + B_2 I_2 \tag{4.10}$$

A_1, A_2, B_1, B_2 are constants depending on the circuit parameters and

$$I_1 = \int_{-\infty}^{\infty} \left| \frac{H_{out}(w)}{H_p(w)} \right|^2 df \qquad (4.11)$$

$$I_2 = \int_{-\infty}^{\infty} \left| \frac{H_{out}(w)}{H_p(w)} \right|^2 f^2 df \qquad (4.12)$$

$$\Sigma_1 = \int_{-\infty}^{\infty} H_p(w)\{\sum_{-\infty}^{\infty} b_k e^{jwkT}\} \frac{H_{out}(w)}{H_p(w)} * \frac{H_{out}(w)}{H_p(w)} dw \qquad (4.13)$$

Where $H_{out}(w)$ and $H_p(w)$are the normalised fourier transforms of the input and output pulse shapes. The analysis carried out here assumes rectangular input pulse shapes of a unit area and output pulse shapes that are raised cosine with $\beta = 1$. Personick derives the worst case receiver sensitivity for a random binary bit sequence, the minimum reciever power for an error probability of 10^{-9} is given by

$$P(rec) = \frac{h\Omega}{\eta}(6)^{-1/(1+x)}(Z)^{x/(2+x)}(\gamma_1)^{x/(2+2x)}(\gamma_2)^{(2+x)/(1+x)} \qquad (4.14)$$

Where γ_1, γ_2 and Z depend on $I_1, I_2, \Sigma_1, A_1, A_2, B_1, B_2$ in a complicated way.

In the analysis considered the APD excess gain factor was 0.5, the bit rate was 250 Mbps, the quantum efficiency of the APD was 0.75. The circuit specifications are the

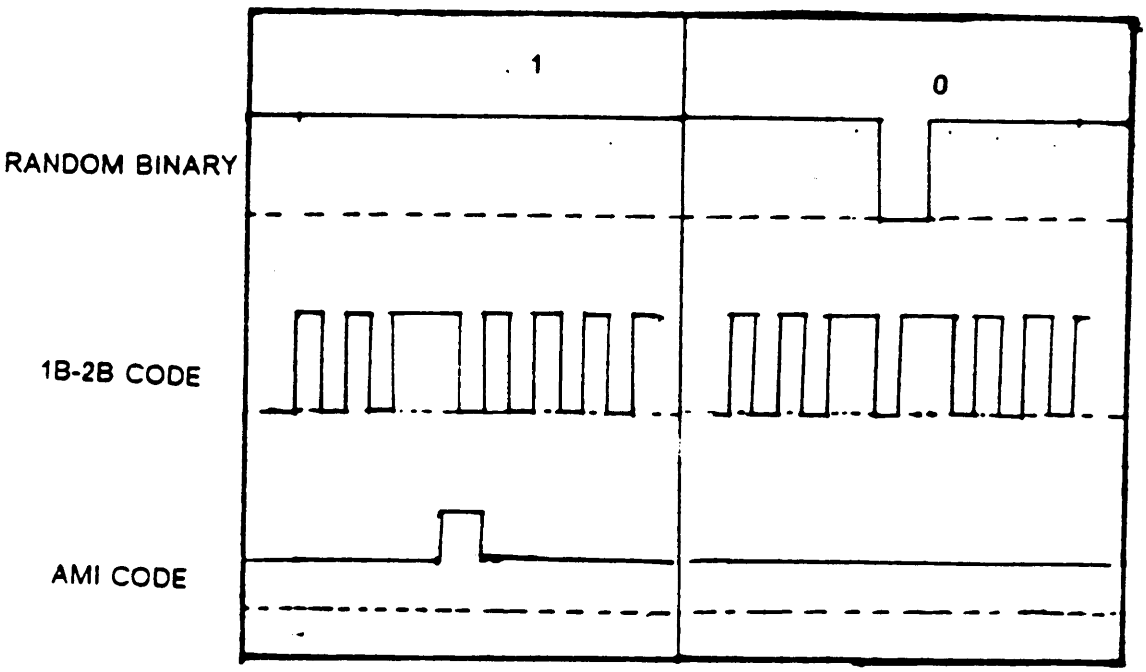


Figure 12. Worst case sequences for optical detection

same as those in fig(12). The results of the numerical computations of the receiver sensitivity for different binary block codes are given below.

Random binary	-57.13 dbm
1B-2B (2 level AMI)	-54.78 dbm
2B-3B code	-55.09 dbm
3B-4B code	-55.39 dbm
5B-6B code	-55.99 dbm

The tradeoffs in the use of these codes lies in deciding between the complexity of the receiving circuitry and the required bandwidth for transmission. For communication links where the data rates are high and close to fiber bandwidth, codes with a lesser redundancy are preferred because the S/N penalty now dictates the usage of a particular code. When the fiber bandwidth is sufficiently higher than the transmission rate 1B-2B codes are preferred.

The analysis carried out here does not specifically address the issue of low frequency cutoffs and the use of line codes to alleviate that problem. In that sense it does not seem to be in the purview of the specific problem. However, it is an approach for the evaluation of coding schemes for fiber systems that has a potential for further research. Another point to be noted is that coding schemes are used not only to address the D.C.wander problem, but also for other system considerations such as timing. Hence this analysis, although not specific to the low frequency cutoff issue, does address it in an indirect way and holds promise for further investigation.

One other interesting coding scheme is the complementary bit coding. In this scheme every tenth bit is a complement of the previous bit. This coding scheme has lesser redundancy but poorer low frequency properties and error correcting capability. The advantage that it enjoys is the simplicity of circuit design and implementation.

chapter V

Conclusions

The presence of low frequency cutoffs in both line wire and optical fiber communication systems is an example of the many impairments that exist in a practical data transmission link, that tend to degrade the performance of the system. It must be noted that low frequency cutoffs occur as a result of engineering solutions to certain practical problems or due to device limitations. Though the problem has been a long standing one there are only a few papers in the literature that specifically address this issue, although there is a host of them that deal with line codes that meet multiple design objectives.

The results of the analysis suggest that quantized feedback is an excellent solution to the baseline wander problem provided the cancellation of tails is attained perfectly. The error multiplication effect that was feared was shown to be negligible at signal to noise ratios of the operation of practical systems. In practical systems, however there is a definite delay in the decision process and thus a finite timing mismatch problem always exists. The problem becomes more critical as data rates go up, and this imposes an upper limit on the bit rate that can be used in a system employing quantized feedback. The timing mismatch problem may not be as severe as the analysis suggests for filters whose fall off is less sharp in comparision to that of the simple RC configuration. The design of appropriate filters for quantized feedback systems is a topic that needs to be investigated through the use of simulation techniques. It is a topic that the author would like to investigate after the arrival of the BOSS (41) simulation package at the Research Center. The timing mismatch effect would be the prime mover in the choice of the filtering scheme, specifically the shapes of the filters for times less than half the bit period are crucial.

An interesting aspect in terms of system performance evaluation for the study of low frequency cutoffs is the choice of the input bit stream. The question arises of the choice of a pseudo-random bit sequence as a test sequence; and how long these sequences should be to properly evaluate system performance.

Line codes are both interesting and intelligent engineering solutions. However, barring the use of certain block codes in Europe and the complimentary bit code in Japanese systems the interest in their use for fiber optic systems seems to be lacking at least in the telephone industry. Scrambling, which is a time tested, albeit inexact, solution seems to be a clear winner in that industry. It seems that the gains that can be had by the use of optical line codes are not sufficient to cause the replacement of scrambling techniques in most practical voice communication systems. The advent of high speed fiber data links, however is arousing interest in line coding. FDDI (Fiber Distributed Data Interface) which is the latest industry standard for a 100 Mbit token ring local area network with a fiber backbone, uses a 4B-3B code.

An interesting topic for investigation in the area of codes for fiber systems is the use of position and interval modulation schemes. The presence of large fiber bandwidths could make such systems viable alternatives to the regular OOK transmission schemes.

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