MODELING THE DIAMETER AND LOCATIONAL DISTRIBUTIONS OF BRANCHES WITHIN THE CROWNS OF LOBLOLLY PINE TREES

by

Paul F. Doruska

Thesis submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

Master of Science

in

Forestry

APPROVED:

Harold E. Burkhart, Chairman

Paul V. Bolstad

John R. Seiler

February, 1993

Blacksburg, Virginia

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(ABSTRACT)

Crown structure for 9- to 30-year-old loblolly pine was quantified via analysis of branch diameters and location, both along and around the bole, using observational data from 68 trees. The trees analyzed ranged in size from 11.1 to 31.6 cm in DBH and from 8.30 to 25.67 m in height, and were growing in Piedmont and Atlantic Coastal Plain stands ranging from 70 to 200 sq. ft. BA/acre. A series of equations was used to describe the diameter distribution of branches. Circular statistics were used to examine branching patterns around the bole.

A recursive system of 2 equations was developed in order to predict the total number of branches within a crown. A series of 3 equations was used to describe the average of and range in diameter within a whorl. Attempts at modeling the height above ground to branches (whorls) were unsuccessful; therefore, equidistant spacing was assumed. Similarly, predicting the number of branches within a whorl of a certain height was difficult, and overall percentages were employed.

Analysis of branch azimuths on a whole tree basis indicated a uniform distribution was appropriate (and not a "circular normal" distribution). Finally circular correlation was used to analyze rotational patterns within and between whorls, and a strong positive correlation was found for consecutive whorls of the same number of branches.

From this study it was concluded that modeling crown structure will be difficult, with much variation occurring among trees. More data are necessary to better refine the baseline work herein presented.

Acknowledgements

I wish to sincerely thank my advisor, Dr. Harold Burkhart, for his guidance, support, and patience throughout the course of this study. I also wish to thank Dr. Paul Bolstad and Dr. John Seiler for serving on my committee, and Ralph Amateis for his statistical computing expertise.

Additionally I wish to thank the U.S.D.A. Forest Service for providing the financial support for the particular Southern Global Change Project under which this thesis falls.

Finally, I wish to thank the biometrics section faculty, staff, and students for tolerating me.

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Chapter I

Introduction

Most diameter distribution models used in forestry relate to stands of trees. Foresters can use these distributions to estimate the number of trees in respective size classes for a given stand. These diameter distribution values can then be used to calculate the standing volume of a stand. Furthermore, upon mapping a stand, the exact location of each tree can be recorded (if such detail is required.)

The same procedures used to model the diameter distribution of trees in a stand may be applicable to modeling the diameter distribution of branches in a tree crown. The heights to branches and circular relation of branches can also be evaluated, drawing the parallel to a mapped stand. The branch number, diameter, and locational information, when combined, can be very useful to foresters.

For example, pruning is a common stand management practice (Smith 1986). A model predicting the sizes of and heights to branches can be used to estimate pruning time and cost. Wood quality is a major concern, especially in regard to differing management practices (MacPeak *et al.* 1987, 1990, and Biblis 1990). Maguire *et al.* (1991) utilized such information to assess wood quality in Douglas-fir (*Pseudotsuga menziesii* (Mirb.) Franco.) Furthermore, with such information, the sizes of and heights to potential knots can lead to valuable information concerning log quality.

The use of detailed crown information can be used in areas other than wood quality. Hepp and Brister (1982) showed how this type of data is used for estimating crown volumes and weights. The carbon allocation model MAESTRO (Wang and Jarvis 1990) relies heavily on crown characteristics. Work currently underway (Burkhart *et al.* 1990) linking MAESTRO to the stand level growth and yield model PTAEDA2 relies upon the tree crown for the linkage.

Clearly, with much tree information provided by the crown and its characteristics, better definitions of crown structure and crown attributes would be extremely useful. Yet such information is only available for a handful of species, and that information is by no means complete. This thesis, observational and exploratory in nature, attempted to model the number, diameter, and location of branches within the crowns of loblolly pine (*Pinus taeda* L.) trees (9 to 30 years old) grown in unthinned plantations.

Objectives

The main objective of this thesis was to model the diameter and locational distribution of branches in loblolly pine crowns. Specific objectives were to:

1. Model the diameter distribution of branches within the crown of 9- to 30-year-old loblolly pine,

2. Model the heights to branches of specific diameters,

3. Develop an equation to predict the total number of live branches,

4. Determine if branch diameter distribution, heights to branches of specific diameters, and total number of branches in the crown vary with stand density (or some relative spacing parameter),

5. Determine if any branch azimuth distributional pattern exists on a whole-tree- and whorl-by-whorl-basis.

Chapter II

Literature Review

While much recent research has concentrated on the tree crown, very little research has gone within the tree crown to study branches. However, a tree crown can be considered a collection of branches much in the same way a stand can be considered a collection of stems. Thus, some of the methods used to describe tree stands can also be applied to tree crowns. Other stand descriptor methods, such as spatial relationships, cannot be transferred to crown branches. First, however, a brief review of crown shape and branching behavior will be given.

Crown Shape and Branching Behavior

There are 2 main components to the structural design of branches in woody plants (Bertram 1989).

1. Spatial organization of the branches, and

2. Structural proportions of individual stems and branches.

Photosynthetic capacity and competitive ability are influenced by branch angles and organizational patterns (Horn 1971), while the mechanical loads a tree is subjected to must be structurally supported by individual stems and branches, which can be viewed as support columns. Through allometric analysis, Bertram (1989) proposed that differential scaling of branches exists and is caused by the branch bending mechanics, branch sizes, and the branches' water conductance requirements.

Many have tried to describe tree crowns in relation to the geometric shapes they mimic. Mawson *et al.* (1976) developed a 15-shape system to classify tree crowns. This system of shapes combines 2 independent shape classifications, the profile of the crown and the ground projection of the tree crown, into a final shape classification. While developed for all types of trees, hardwoods and conifers, only 6 are actually applicable to conifers. Wang and Jarvis (1990) allocate 3 shapes, paraboloid, cone, and half-ellipsoid, in conjunction with more quantitative measures, to describe crown shape in their MAESTRO carbon allocation model. Stiell (1962) divided the vertical profile of red pine (*Pinus resinosa* Ait.) crowns into two portions, a triangle above point of maximum width, and a rectangle below the same point.

Others have chosen to describe crown shapes through more quantitative methods. Honer (1971) chose to use crown radius as a measure of crown shape for balsam fir (*Abies balsamea* (L.) Mill.) and black spruce (*Picea mariana* (Mill.) B.S.P.) with the following model form:

$$CR = a_1L - a_2LH - a_3\frac{L^2}{H} + a_4L^2$$

where: CR = crown radius at a certain branch,

L = perpendicular distance from branch tip to tree tip, and

H = total height of the tree.

Moeur (1981) also developed crown width equations as a measure of crown shape for a variety of northern Rocky mountain conifers. Berlyn (1962) employed a crown length to crown radius ratio:

$$k = \frac{z}{R^2}$$

where: k = crown shape factor,

z = crown length, and

R = geometric mean radius of the base.

to describe crown shapes. Finally, Biging and Wensel (1990) used crown radius and a species-specific profile shape parameter to estimate crown volumes via:

$$V(h) = \hat{CV}\left(\frac{H-h}{H-HCB}\right)^k$$

where: $HCB \leq h \leq H$,

V(h) = cumulative crown volume from crown base to height h,

$\hat{CV} = a(D^b)H^cCR^d$,

D = diameter at breast height,

H = total height,

CR= crown ratio,

HCB= height to base live crown, and

k= species specific profile shape parameter.

To better comprehend the tree crown and understand its dynamics, branches, which compose the crowns, need to be studied. Much of this work has been done outside of North America. Hashimoto (1990, 1991) reported changes in crown morphology in young sugi (*Cryptomeria japonica*) stands, determining,

1. tall trees had deeper crowns than short trees, yet crown diameters were similar between the two,

2. understory trees tended toward umbrella shapes, while overstory trees exhibited less apical roundness,

3. middle-story trees exhibited the largest branch inclinations, while understory trees possessed the smallest, and

4. crown length to diameter ratios and average branch inclinations increased with age.

Hashimoto concluded that the light environment was the primary determinant of crown morphology and structure. Kurttio and Kellomäki (1990) employed branching ratios to describe the crown structure of young Scots pine (*Pinus sylvestris*), and Pulkkinen and Pöykkö (1990) studied the narrow crown form of Norway spruce (*Picea abies.*) Ovington and Madgwick (1959) described young Scots pine crowns via branch length, branch weight, and leaf weight by whorls. Stiell (1962) performed a rather detailed analysis of the crown structure in 14 red pine trees, ranging from 8 to 20 years in age. He reported:

1. 8 to 14 live whorls per tree,

2. branches tended, slightly, to favor a southern exposure, on a per tree basis,

3. branch diameters tended to increase and then decrease as one progressed from to the tip of the crown downward, and

4. secondary branches tended to occur in pairs.

Hepp and Brister (1982) analyzed the crowns of 364 loblolly pine trees growing in site-prepared plantations across the North and South Carolina coastal regions. Equations predicting branchwood, needle and total dry weight in the crowns were developed across three scales of magnitude: branch-, tree- and, stand-level. Maguire *et al.* (1991), while studying the effects of management of wood quality of Douglas-fir, developed the following equation to predict branch size:

$$\ln(BD) = a + b\ln(DINC) + c\ln(DBH) + d\ln(RD) + eSI$$

where: BD = mean maximum branch diameter for annual whorl near crown base,

DINC = depth into crown (tree height minus height of branch),

RD = relative density, and

SI = site index.

Because this paper was primarily concerned with wood quality, the presented equation only applies to the lower third of the crown (that which may be part of the merchantable portion of the bole).

A crown's structure is also influenced by the trees surrounding it. Jack and Long (1991) examined the role of stand densities on crown size. Studies performed using lodgepole pine (*Pinus contorta* Dougl. ex. Loud.) and subalpine fir (*Abies lasiocarpa* (Hook.) Nutt.) indicate that lodgepole pine crowns were highly influenced by increasing densities, while subalpine fir crown sizes were not affected as much (attributed to the different relative shade tolerances of the two species). Furthermore, they found a strong relationship between stand height and crown size measurements (crown width, crown height).

No discussion of branching and crown form would be complete without mention of the pipe model theory (Shinozaki *et al.* 1964a, 1964b; Oohata and Shinozaki 1979). The pipe model theory has been paramount to understanding plant form. The amount of leaves present on a tree or in a plant community above a given level was found to be proportional to the sum of branch and stem cross-sectional areas at that level. Any plant could then be described as a series of pipes, each supporting foliage. A specific length of unit pipe supports a certain amount foliage. An application of this generic model was then made for trees.

Shinozaki *et al.* (1964b) proposed that since the amount of foliage above a certain point was proportional to the cross-sectional area of the stem and branches at that point, then the cross-sectional area of branches above that point should also be proportional to the cross sectional area of stem at that point. Thus, the amount of foliage and/or sum of the cross-sectional area of branches in the crown was shown to be directly proportional to the stem cross-sectional area at crown base.

The pipe model theory has been applied countless times to a multitude of species for a variety of purposes. Of interest to forestry are its applications to tree growth (Valentine 1985, 1988, 1990), canopy leaf area (Waring *et al.* 1982), and height growth (Mäkelä 1986).

Diameter Distributions

Although not used in this thesis (for reasons to be discussed later), the idea of diameter distributions should be mentioned as it might be applied to branches in tree crowns in the future if the data allow. Diameter distributions are commonly used to examine size class distributions of trees in a stand. This distributional information can then be used to estimate standing volumes.

Clutter and Bennett (1965) examined the diameter distribution of slash pine (*Pinus elliottii* Englem.), using the beta probability density function. Equations predicting the maximum and minimum diameters in a given stand from its age, number of trees per acre, and site index were developed with data from 187 old-field slash pine plantation 64-tree plots located in the middle coastal plain of Georgia. They then used these values to develop a distribution function for calculating the relative frequency of stems of a given diameter:

$$f(D_i) = 31.1504\left(\frac{D_i - D_{min}}{D_{max} - D_{min}}\right)^{2.472}\left(1 - \frac{D_i - D_{min}}{D_{max} - D_{min}}\right)^{1.6548}$$

The area under this curve between two diameters is the proportion of stems whose diameters are between those two limits. Multiplication of this proportion by the total number of trees per acre and the volume per tree at the midpoint of the diameter class yields volume by class per acre. Multiplication of the volume per acre by the total number of acres in the stand results in the volume of a given size class for the stand. Clutter and Bennett suggested the use of this curve and its related maximum and minimum diameter equations to evaluate size class distributional information for a stand of a given age, density, and site index.

Clutter and Bennett (1965) used a diameter distribution model for even-aged plantations; however, the same basic procedure can be used for uneven-aged stands as well. Leak (1965) recommended the use of the inverse J-shaped distribution for modeling diameter distributions of uneven-aged stands.

The framework put into place by Clutter and Bennett (1965) for even-aged stands and Leak (1965) for uneven-aged stands allowed for further experimentation into stand-level diameter distribution models. McGee and Della-Bianca (1967) fit the beta distribution to even-aged stands of yellow poplar (*Liriodendron tulipifera* L.) The maximum and minimum diameters as well as the pdf parameters were again estimated through stand characteristics.

The beta distribution is quite useful for for modeling stand-level diameter distributions. However, there are inherent problems with using the beta distribution. Frazier (1981) pointed out the relative inflexibility of the beta distribution. The shape of the distribution is assumed constant across all stand conditions when mean values of α and β are used. Furthermore, numerical techniques are needed to integrate the beta distribution (if the cumulative distribution function is desired) as a closed form is not available (Burkhart and Strub 1974). The Weibull distribution became more popular as use of the beta distribution waned.

The Weibull distribution was first used by Bailey (1972) to model the diameter distribution of unthinned *Pinus radiata* (D. Don) stands in New Zealand. A complete description of use of the Weibull was reported by Bailey and Dell (1973). Any distributional model chosen should possess certain qualities:

1. Cover all shapes (reverse J, mound shapes with positive skewness or negative skewness) assumed by diameter distributions,

2. Constants present in the model should vary with stand characteristics,

3. Be relatively easy to integrate (without requiring numerical techniques),

4. Be easily fitted and have parameter estimators exhibiting good statistical properties.

The beta distribution was discounted because it is difficult to integrate. Gram-Charlier curves were discounted because they are difficult to fit. The Weibull distribution, however, satisfies all qualities.

Fisher and Tippett (1928) and Weibull (1939) independently derived the Weibull distribution. The Weibull is commonly seen in two forms, the two parameter model:

$$f(x) = \left(\frac{c}{b}\right)\left(\frac{x}{b}\right)^{c-1} e^{-\left(\frac{x}{b}\right)^{c}}$$

or the three parameter form:

$$g(y) = \left(\frac{c}{b}\right)\left(\frac{(y-a)}{b}\right)^{c-1} e^{-\left(\frac{(y-a)}{b}\right)^{c}}$$

The two models are linked via the transformation $x = y - a$. Parameter a is the location parameter and can be set to the smallest diameter found in the stand. Parameter b is called the scale parameter. Parameter c is known as the shape parameter. Thus by changing the values of c, the shape of the distribution changes. Similarly, changes in the values of b and a change the scale and location of the distribution respectively. Clearly, the Weibull distribution can change shapes as required by diameter distributions of varying stand conditions. The Weibull distribution is easily integrated to obtain a two-parameter cumulative distribution:

$$F(x) = 1 - e^{-\left(\frac{x}{b}\right)^{c}}$$

Parameter estimation can be performed using various estimation techniques, including maximum likelihood estimation, transformations, and percentiles. Maximum likelihood estimation was deemed best, and use of this technique for the Weibull distribution requires iterative computations (see Cohen 1965).

Schreuder and Swank (1974) looked specifically at applying the Weibull distribution to coniferous stands. They evaluated the modeling of tree diameter, basal area, surface area, and biomass as well as the vertical distribution of crown biomass and surface area for eastern white pine (*Pinus strobus* L.) and loblolly pine via the Weibull and compared the results with models using the normal, lognormal, and gamma distributions.

The Weibull was favored over the others for modeling tree diameter, basal area, surface area, and biomass because the one distributional form could summarize all facets successfully. Schreuder and Swank (1974) also reported the Weibull distribution to outperform the lognormal and gamma distributions for modeling vertical distributions of crown biomass and surface area. The same conclusion was reached for both eastern white pine and loblolly pine.

Thus, the Weibull distribution has supplanted the beta distribution for stand-level diameter distribution modeling. The Weibull was used to model diameter distribution of yellow poplar (Knoebel *et al.* 1986) and loblolly pine (Smalley and Bailey 1974, Feduccia *et al.* 1979). Dell *et al.* (1979) used the Weibull for slash pine and Lohrey and Bailey (1976) modeled longleaf pine (*Pinus palustris* Mill.) diameter distributions with the Weibull.

A class-free method (proposed by Strub and Burkhart 1975) was advanced by Hyink (1980) with the "parameter recovery" model. This method assures compatibility between stand average models (for predicting yields on a whole stand basis) and diameter distribution models (predicting diameter distribution of stand and converting to yield). Whether constructed independently or with the same dataset, the two model types seldom agree (Daniels *et al.* 1979). Hyink (1980) and Frazier (1981) explain in detail the "parameter recovery" procedure.

The Strub and Burkhart (1975) model form was designed to estimate volume. However, other attributes can be predicted by using appropriate functions. Average stand attributes could then be calculated (*i.e.* basal area). From the average stand attribute equations, it is possible to "recover" the pdf parameters of the stand's diameter distribution as long as there are at least as many stand attribute equations as pdf parameters (Frazier 1981).

Two methods were presented by Frazier (1981): moment-based and volume-based parameter recovery. The former method estimates the first and second non-central moments of the random variable DBH from average stand diameter and basal area per acre. The latter method employs both non-central moments of DBH (same derivation as before) as well as at least one stand volume equation. From these equations, the pdf parameters of the diameter distribution are calculated.

A diameter distribution, then, might very well be applied to branches in a tree crown if the occurring distributions conform to certain shapes and proper conditioning variables are found (if "parameter recovery" is used.)

Circular Statistics

When describing a stand of trees spatially, an XY plane is employed (as in Reed *et al.* 1989). That is, linear relationships can be used to describe the distance between one tree and another. This, however, can not be transferred completely to the locational distribution of branches. While the spacing between whorls can be described in this manner, the spacing of branches within whorls cannot. XYZ systems would be necessary to map individual branch locations; however, since branch locations in whorls are circular in nature, circular statistics might be more appropriate. Batschelet (1981) was the primary source of the circular statistics herein reported. Little else was found.

Two topics of interest apply to this thesis. The first of these is circular distributions. Circular distributions are best represented as circular plots with a vector pointing to the part of the circle containing the most mass (mean direction). The length of the vector indicates the concentration of mass in that area (*i.e.* a relatively long vector implies a strong concentration, while a vector of little or no length tends to imply a near uniform distribution around the circular plot.)

One circular distribution already mentioned is the uniform distribution.

$$f(\phi) = \frac{1}{2\pi}$$

$$(0 \leq \phi \leq 2\pi)$$

In this case, all angles are reported in radians. No mean direction exists for the uniform distribution. Another circular distribution is the Von Mises distribution:

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\phi - \theta_1)}, \kappa \geq 0.$$

where: $I_0(\kappa)$ is the Bessel function.

When $\kappa = 0$ the Von Mises collapses to the uniform distribution. The mean angle is represented by θ_1, and κ is called parameter of concentration. The larger κ is, the more concentrated the angles are around the mean angle. The Von Mises distribution is the circular equivalent of the normal distribution, and is often called the circular normal distribution.

The Von Mises distribution described is for a unimodal circular distribution. However, multimodal circular distributions also exist. One such multimodal distribution is the multimodal Von Mises distribution, which adds another parameter ν, representing the number of modes:

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\nu(\phi - \theta))}$$

where: θ representing one of the ν modes.

The second topic of interest regarding circular statistics is circular correlation. Both positive and negative circular correlation can be studied. Positive correlation (r_+ evolves from calculating a difference between successive pairs of angles, while negative correlation (r_-) employs a sum.) Each circular correlation coefficient is then calculated via:

$$r = \frac{1}{n}\left[\left(\sum_{i=1}^{n}\cos(\delta_i)\right)^2 + \left(\sum_{i=1}^{n}\sin(\delta_i)\right)^2\right]^{\frac{1}{2}}$$

where: δ_i =the difference or sum of paired angles.

The proper circular correlation coefficient is the larger of the two calculated. That is:

$$r = \max(r_+, r_-)$$

Chapter III

DATA

The data used for this study are part of a dataset collected for a project entitled "A linked model for simulating growth processes and stands development of loblolly pine" (Burkhart *et al.* 1990) funded by the U.S.D.A. Forest Service as part of their Southern Global Change Program. The linkage of the PTAEDA2 and MAESTRO models, as proposed, will be made through the tree crowns. Crown width and competition, biomass distribution, branching angle and length, and internodal growth increments are some of the measures needed to develop the link. MAESTRO, a carbon allocation model, will be used to "grow" individual loblolly pine trees under varying atmospheric conditions, with stand-level effects found via PTAEDA2, a stand level growth and yield model. An iterative process could then be set-up allowing for analysis of the effects of atmospheric changes over an entire rotation age.

Clearly, a multitude of crown measurements were needed. Only those pertinent to this thesis are reported. Thirty-four subject trees were felled for the Linked-Model project. One neighboring tree per subject tree was also felled to increase the data available for this thesis. Thus, data from 68 trees were used. Twenty-eight trees were located in the Piedmont physiographic region (Appomattox, VA) and the remaining 40 trees were in the Atlantic Coastal Plain physiographic region (West Point, VA and Tillery, NC), The trees were found in stands ranging from 9 to 30 years in age and from 70 to 200 sq. ft. of BA/acre.

For each of the 68 trees, the field crew measured and tallied:

1. The height above ground to every visible branch or branch stub,

2. The diameter (outside bark, taken just beyond stem swell) of every visible branch or branch stub,

3. The azimuth of every visible branch or branch stub,

4. Diameter (outside and inside bark) at breast height and base live crown,

5. Total height and height to live crown,

6. Stump age, age at crown base, and crown class.

In addition to the tree measurements listed, A BAF 10 and 20 prism plot was taken around each of the 34 Linked-Model subject trees. None were taken for the 34 neighbor trees. However, the results should be applicable to the neighbor trees because of the proximity between the 34 pairs of trees. Because the dead branches are a very minor component of all crown branches, only branches tallied as alive were used in this thesis.

Table 1 gives summary statistics for the 68 trees. The genetic variation among the trees is unknown and was not controlled.

Table 1. Descriptive statistics for the 68 sampled loblolly pine trees collected from Appomattox and West Point, VA, and Tillery, NC.

	Mean	Std. Dev.	Minimum	Maximum
Age(yr)	16.26	4.89	9	30
DBH(cm)	17.77	4.00	11.10	31.60
Height(m)	13.79	2.87	8.30	25.67
Ht. to Live Crown(m)	7.08	2.55	3.00	14.76
Num. of Branches	47.69	11.25	22	75
Stand BA (sq.ft./acre)	117.06	30.47	70	200

Chapter IV

Methods and Procedures

The analysis herein performed is best broken into two distinct sections (1) branch numbers, diameter distributions, and vertical location, and (2) branch circular location. Because the data is observational in nature, no rigorous analysis was pre-planned; rather, most analysis is exploratory and investigative in nature.

Total Number of Branches

A one-equation system was sought predicting the total number of live branches within the crown of loblolly pine. This model would serve as the base model for this thesis. Although Hepp and Brister (1982) collected the data necessary to develop such a model for loblolly pine, none was presented. In fact, no such model was found in the literature, making this analysis truly investigative. The following independent variables were available: DBH, height, height to live crown, crown length, age, crown class, and BA/acre. Also available were any and all interactions/transformations possible from this list.

A recursive system of equations was another approach investigated. A recursive system of equations utilizes a dependent variable predicted by one equation as an independent variable in another equation. Such systems are useful when some variable not easily measured but fairly well predicted is strongly correlated with the dependent variable of interest. In this case, two variables not commonly measured were considered candidates for a recursive system (1) diameter at base live crown, and (2) number of whorls.

In all cases when regression fits were compared, standard measures of fit such as MSE, R^2 and residual analysis were used to compare candidate models. The SAS computer package (1988) was used for all model and data evaluation.

Diameter Distribution of Branches

The framework for this analysis was originally intended to be very similar to developing a stand-level diameter distribution of trees. Specifically, after predicting a stand's basal area and assuming some pre-specified distributional form, the actual diameter distribution can be found (Hyink 1980). This method is quite transferable to branches in a tree crown. The cross sectional area at crown base (serving as the equivalent to basal area and predictable via the pipe model) would condition the diameter distribution with the tree crown. However, as will be discussed later, the actual diameter distributions found in the sample trees rendered this approach fruitless.

It was then necessary to try new approaches which would yield the same information as a diameter distribution, namely the average and range of branch diameters. Three approaches were tried. First, models were sought in the literature. Only Maguire *et al.* (1991) contained an equation for predicting branch diameters, but as previously reported, this equation predicted the mean maximum branch diameter for an annual whorl near crown base in

Douglas-fir. Thus, the equation was developed for only the portion of the crown near crown base. Furthermore, the equational form itself cannot be applied to these data because of variable incompatibility. No other equation predicting branch diameters was found. Thus, models needed to be developed from this data. Second, equations predicting average diameter in a whorl (hereafter referred to as whorl diameter) and the range of branch diameters within a whorl (hereafter referred to as whorl diameter range) were sought such that:

Whorl diameter = f(tree variables, measure of whorl height), and

Whorl diameter range = f(tree variables, measure of whorl height).

Several possible ways of expressing whorl height include:

- whorl height above ground,
- whorl height above base live crown,
- relative whorl height (whorl height divided by tree height), and
- relative crown position (whorl height above base live crown divided by crown length).

Each of these measures of whorl height were candidate independent variables for the equations.

Second, if predicting the whorl diameter range was unsuccessful, predicting the minimum and maximum diameters within a whorl (hereafter referred to as minimum and maximum whorl diameter, respectively) would serve the same purpose.

Vertical Location of Branches

After knowing the total number of branches in a crown and their diameters (relatively speaking as a function of height), the next logical step was to predict or assign their actual heights. Whorl heights, as opposed to individual branch heights, were sought due to the complexity involved with the latter. Again, the literature provided very little help, providing only patterns but no true predictive equations.

Several approaches exist for predicting/analyzing the height to whorls.

- Assume equidistant spacing;
- Model length between whorls;
- Model whorl height/whorl number relation;
- Utilize a height increment model to set yearly intervals and model whorl heights within years (confounded via the indeterminate growth pattern of loblolly pine).

All approaches were examined.

One quantity not yet mentioned but clearly needed to complete the size, diameter, and height assignment of branches is the number of branches within a whorl (because of the heavy reliance on whorl characteristics). This quantity will also play a key role in the branch circular relation section to follow. Several avenues were examined for quantifying the number of branches in a whorl.

- Predicting the number as a function of tree, stand, and whorl characteristics (such as total number, number of whorls from crown base, and measures of whorl height);

- Utilizing overall percentages calculated from the data;

- Periodicity trends on an individual and/or dataset basis.

Again, all approaches were examined.

Circular Patterns of Branches

Once the above descriptors of branch number, size, and height have been found, the only factor left to consider is the position of the branches around the tree bole. As previously mentioned, the field of circular statistics is needed for such analysis. It should be noted that two unique topics must be examined (1) branch azimuth distribution on a whole tree basis, and (2) branch azimuth relationships within and between whorls. All methodology reported in this section is from Batschelet (1981).

Each of the 68 trees was examined to determine the best circular distribution to use to describe the branch azimuth distribution on a whole tree basis. The first quantity of interest was the mean vector of the branch azimuths for each tree. The coordinates of the mean vector (assuming the unit circle) determine both the dominant (modal) azimuth for the tree as well as the concentration about that azimuth (a measure of strength about the "mean".) After converting the azimuths into radians, the x and y coordinates of each branch (again assuming the branches are plotted on the unit circle) are found via:

$$x_i = \cos \theta_i$$

$$y_i = \sin \theta_i$$

The rectangular coordinates of the mean vector are then:

$$\bar{x} = \frac{1}{n}(x_1 + x_2 + \ldots + x_n)$$

$$\bar{y} = \frac{1}{n}(y_1 + y_2 + \ldots + y_n)$$

From these quantities, both the vector length

$$r = \sqrt{\bar{x}^2 + \bar{y}^2}$$

and the mean angle

$$\phi = arctan\frac{\bar{y}}{\bar{x}}, \ if\,\bar{x} > 0$$

$$\phi = 180° + arctan\frac{\bar{y}}{\bar{x}}, \ if\,\bar{x} < 0$$

can be found.

From these quantities, each tree will be examined to determine if the Von Mises ("circular normal") or uniform distribution is appropriate. As previously seen the Von Mises distribution is defined via two parameters:

$$f(\phi) = \frac{1}{2\pi I_0(\kappa)} e^{\kappa \cos(\phi - \theta_1)}, \kappa \geq 0.$$

where: $I_0(\kappa)$ is the Bessel function.

When $\kappa = 0$ the distribution collapses to the uniform distribution.

$$f(\phi) = \frac{1}{2\pi}$$

$$(0 \leq \phi \leq 2\pi)$$

Thus, it is first necessary to evaluate κ. The length of the mean vector, r, is used to calculate κ and the conversions are tabled in Batschelet (1981). As r increases, so does κ. Thus, small values for r translate into small values of κ implying a uniform distribution is appropriate. The other parameter defining the Von Mises distribution is ϕ, the mean angle. Once analysis on a whole tree basis was completed, the topic of concentration shifted to branch azimuth relations within and between whorls, as measured with circular correlation.

The methodology herein presented assumes paired, uniformly distributed data. As will be seen and discussed later, the branch azimuths were uniformly distributed on a whole tree basis, making the following methods appropriate. Analysis was limited to consecutive whorls with the same number of branches to meet the paired data criterion. No methodology was found in the literature for measuring circular correlation in an unpaired environment (*i.e.* consecutive whorls with differing number of branches.) As previously mentioned both positive and negative correlation must be examined simultaneously in order to determine the true circular correlation coefficient.

The branches of each pair of whorls (again only consecutive whorls with the same number of branches were used) were ordered by azimuth within whorl. Two difference angles (modulo 360°) were calculated from these paired branches:

$$\delta_i = \psi_i - \phi_i, \text{ for positive correlation}$$

$$\delta_i = \psi_i + \phi_i, \text{ for negative correlation}$$

where: ψ_i is the azimuth of branch i in one whorl, and

ϕ_i is the azimuth of branch i in the second whorl.

All whorl pairs were analyzed with the lower whorl being the first whorl, and the higher whorl being the second whorl. For both the positive and negative differences angles, the positive and negative circular correlation coefficients (on a paired whorl basis) are found via:

$$r_{\pm} = \frac{1}{n}\left[\left(\sum_{i=1}^{n} \cos \delta_i\right)^2 + \left(\sum_{i=1}^{n} \sin \delta_i\right)^2\right]^{\frac{1}{2}}$$

The proper circular coefficient correlation coefficient is the larger of the two calculated, namely:

$$r = \max(r_+, r_-)$$

Chapter V

Results and Discussion

Total Number of Branches

The following model was deemed "best" for predicting (from tree and stand variables) the total number of branches within a loblolly pine crown.

$$N_b = 20.244 + 4.091(C_l)$$

$R^2 = 0.315$

where: N_b = number of branches, and

C_l = crown length (in meters).

Although very simple in nature, this model performed as well as any other developed. After attempting many models and improvements to the presented model, a couple of factors became apparent. First, R^2 never exceeded 0.32. Regardless of the model form, it appears that only 32% of the variation in the dataset can be explained by any model. Variables such as AGE, DBH, and BA/acre were expected to be of use, but apparently all the information these variables could provide is included in crown length. The addition of another but not commonly measured tree attribute, namely number of whorls, was found to greatly improve the prediction of total number of branches. Thus, a recursive system of equations was built predicting number of whorls first, and then total number of branches from the number of whorls and/or other variables. The "best" models were as follows:

$$N_w = 3.933 + 0.429(DBH) + 0.943(C_l)$$

$R^2 = 0.53$

where: N_w = number of whorls,

DBH = diameter at breast height (in centimeters), and

C_l = as before.

The companion equation was

$$N_b = 7.288 + 2.259(N_w)$$

$R^2 = 0.650$

where: N_b, N_w = as before.

Again, these equations seem rather simplistic in nature, but in each case, no improvement in model fit was found via addition of other independent variables. While not exceptional, the recursive model fits are better than the one equational system presented above. However, the error of prediction from the first equation does carry-over into the prediction of the second equation, as is the case in any recursive system, and may be the cause of the wide spreading residual plot shown in Figure 1. The residual plot from the recursive system was slightly "tighter" than that generated by the single equation (Figure 2).

Another model comparison tool employed was the PRESS statistic.

$$\sum_{i=1}^{n}\left(\frac{e_i}{(1-h_{ii})}\right)^2$$

where: e_i = residual for observation i, and

h_{ii} = hat matrix $(\boldsymbol{X}(\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}')$ diagonal for observation i.

The PRESS statistic was easily calculated for the single-equation system; however, 3 variations exist for the recursive system. The first employed the original x's, used to develop the second equation, to calculate the hat diagonals, while the second variation utilized the predicted x's, from the first equation, to calculate the hat diagonals. The third variation required 68 sets of regressions, each fit exclusive of one of the sample trees. The excluded tree was then used to calculate the residual from each of the 68 systems. These residuals were then summed and squared to calculate a true PRESS (deleted) residual. All 3 variations of the PRESS statistic were lower than that of the single-equation system (indicating a better fit) and are shown in Table 2.

The recursive system is thus recommended over the single equation system.

Branch Diameter

As intimated earlier, the use of a diameter distribution model within a crown was futile because the underlying distributions simply cannot be described by any commonly used distribution (*i.e.* the Weibull.) The diameter distribution of all collected data (relative to maximum branch diameter on a tree by tree basis) is shown in Figure 3. While such a pattern can be modeled with the distribution, the objective was to fit diameter distributions on a tree-by-tree basis. Sample tree branch diameter distributions can be found in Figures 4-7. These plots

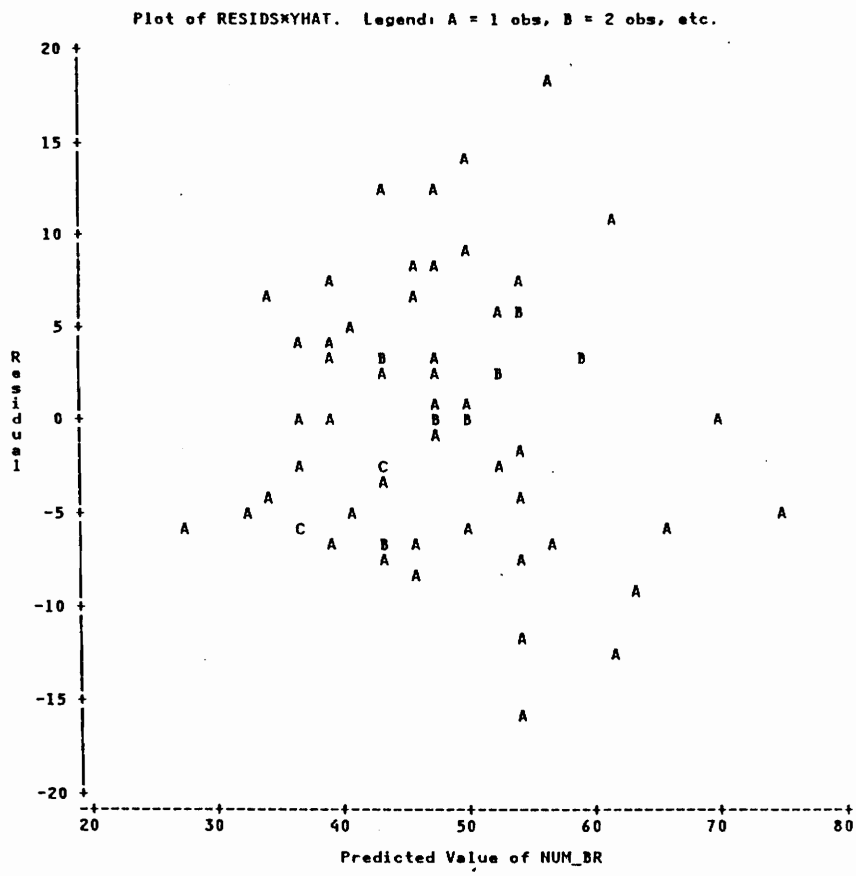

Figure 1. Residual plot from recursive system predicting total number of branches in loblolly pine crowns: The wide range in magnitude of the 68 residuals shows the variability present in the data. Most residuals are within ± 15 branches of the actual number present.

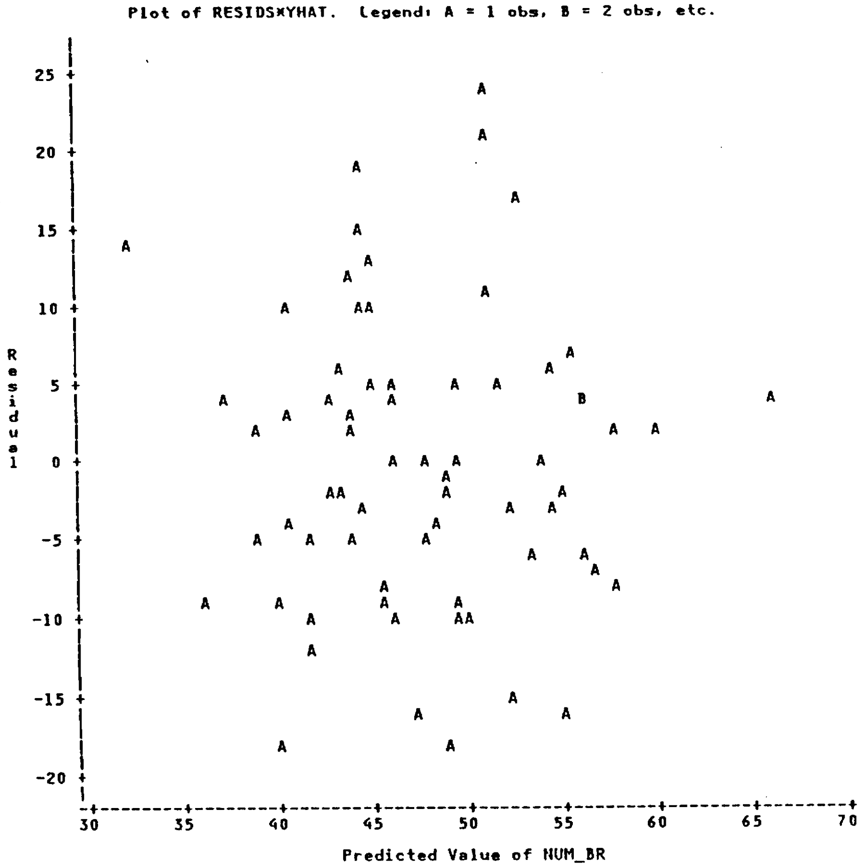

Figure 2. **Residual plot from single-equation system predicting total number of branches in loblolly pine crowns:** The wider range in magnitude of the 68 residuals (more outside of ± 15 branches) shows a poorer fit than the recursive system.

Table 2. Comparison of PRESS statistics between the single-equation and recursive system predicting total number of branches in loblolly pine crowns.

Single-equation	Recursive system
6142	6108[1]
	5945[2]
	5980[3]

[1] PRESS using original x's
[2] PRESS using predicted x's
[3] PRESS using 68 deleted residuals

clearly do not conform to shapes covered by the Weibull, or any other commonly used, distribution. Thus, predictive equations for branch diameters were developed.

The "best" model developed predicting whorl diameter was

$$\overline{WD} = 2.134 + 0.073(DBH) - 2.867(RWH^2)$$

$R^2 = 0.497$

where: $\overline{WD}$ = whorl diameter (cm),

RWH = relative whorl height, and

DBH = as before.

It was expected that relative whorl height would be outperformed by some other measure of whorl height more specific to the tree crown and not the whole tree, but this was not the case. Furthermore, an observed trend (also reported by Stiell (1962)) of whorl diameter increasing and then decreasing as one progresses from base live crown to tree tip is usually modeled with both a linear and quadratic term expressing whorl height. However, this trend must not be well-defined as inclusion of both terms did not improve fit.

As with the previous models, the proportion of variation explained by this or any other model form did not exceed the 0.5 presented. Also, this model is again relatively simple in form, but this is in agreement with Maguire *et al.* (1991) who reported a linear regression of similar objective and stated "log-log models, weighted and un-weighted linear models, and weighted and un-weighted non-linear models with various combinations of pertinent variables" were all attempted but the linear regression's performance could not be surpassed.

An equation predicting the range in branch diameters within a whorl was then sought. However, all attempts toward this end were futile, with no R^2 exceeding 0.10. Thus, the other aforementioned approach was utilized, namely predicting minimum and maximum whorl diameter.

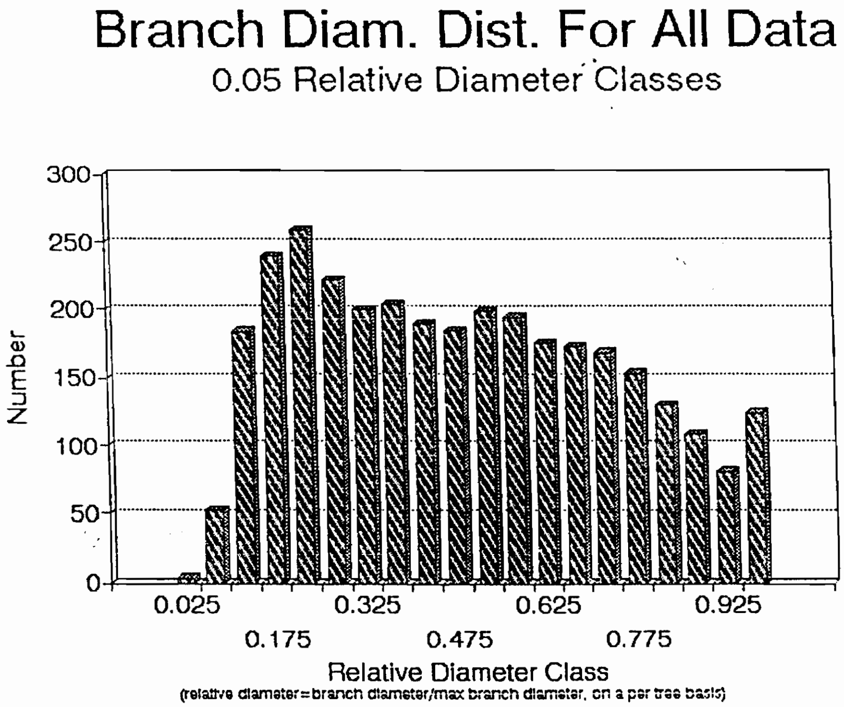

Figure 3. Observed branch diameter distribution of entire dataset of loblolly pine branches, relative to maximum branch diameter on a tree-by-tree basis: Such a pattern is can be fit with the uniform distribution, but the objective was to fit diameter distributions on a tree-by-tree basis.

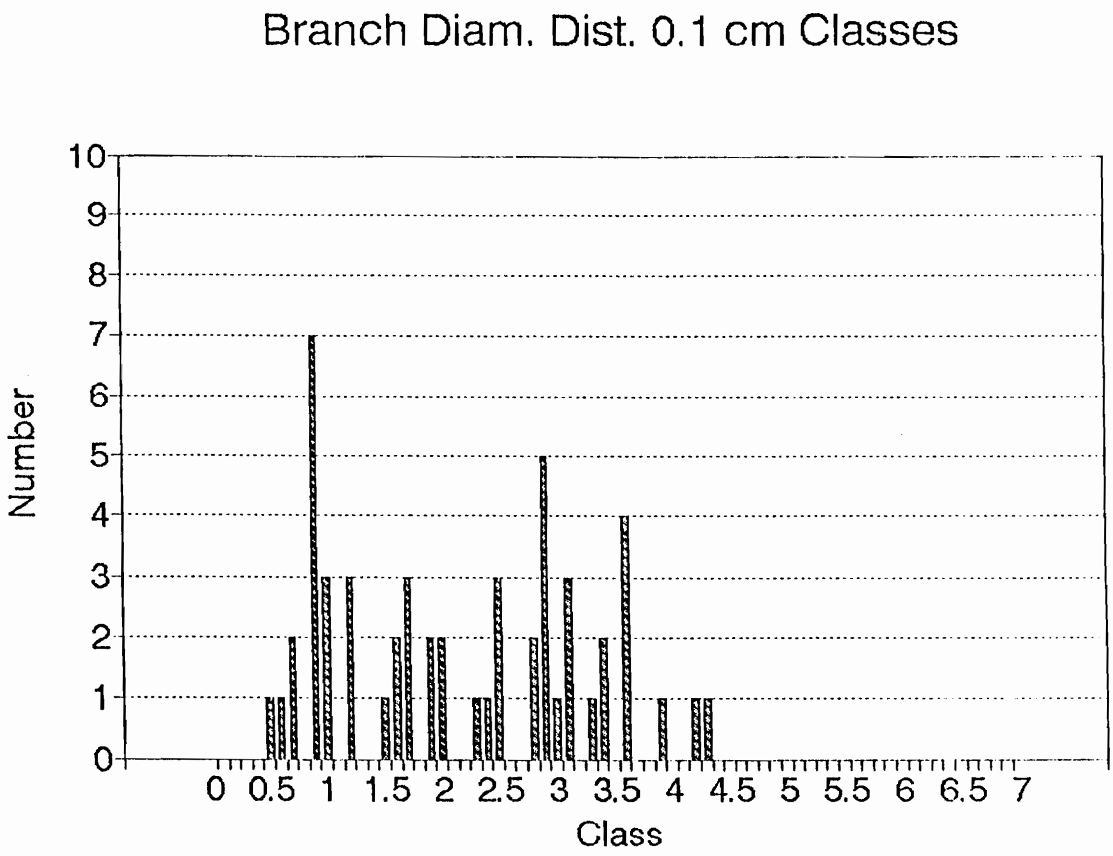

Figure 4. **Observed branch diameter distribution for a 14-year-old Atlantic Coastal Plain loblolly pine with a DBH of 20.9 cm and a height of 14.84 m, found in a 120 sq. ft. BA/acre stand:** Such a pattern is not easily fit by statistical distribution functions due to the number of peaks and gaps.

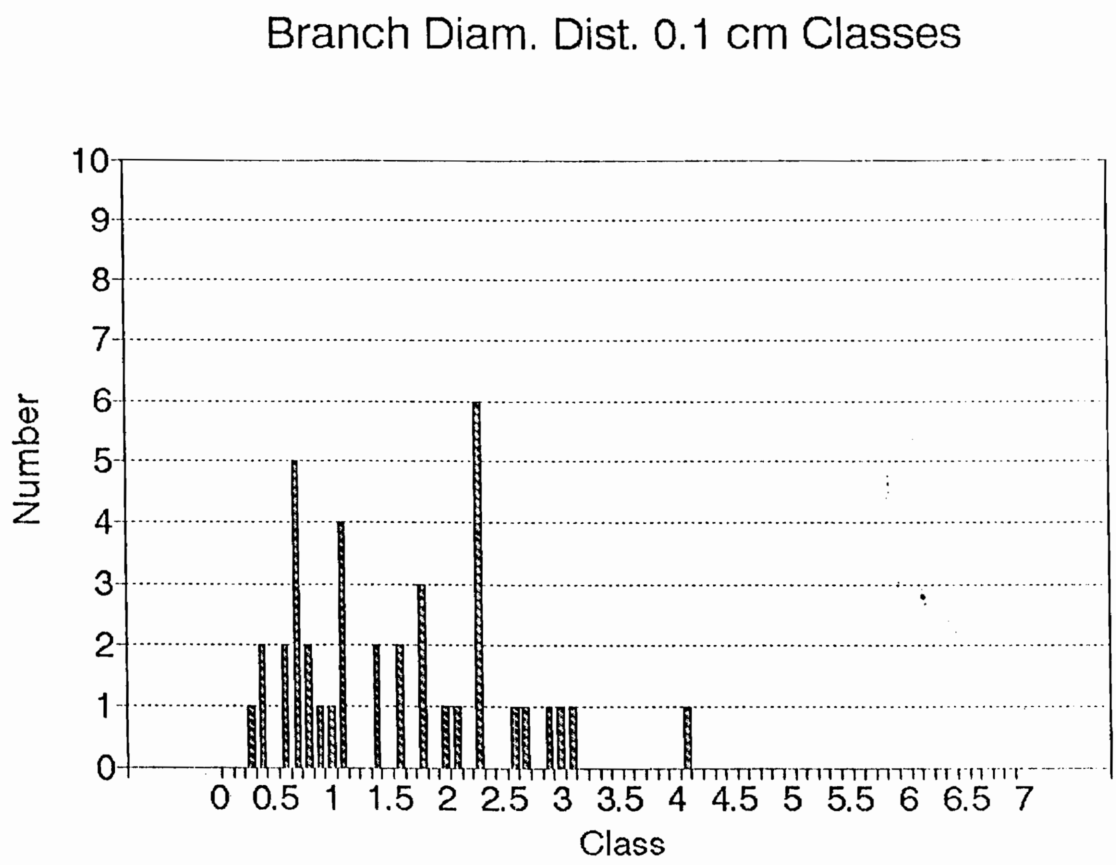

Figure 5. **Observed branch diameter distribution for a 19-year-old Atlantic Coastal Plain loblolly pine with a DBH of 18.2 cm and a height of 12.98 m, found in a 140 sq. ft. BA/acre stand:** Such a pattern is not easily fit by statistical distribution functions due to the number of peaks and gaps.

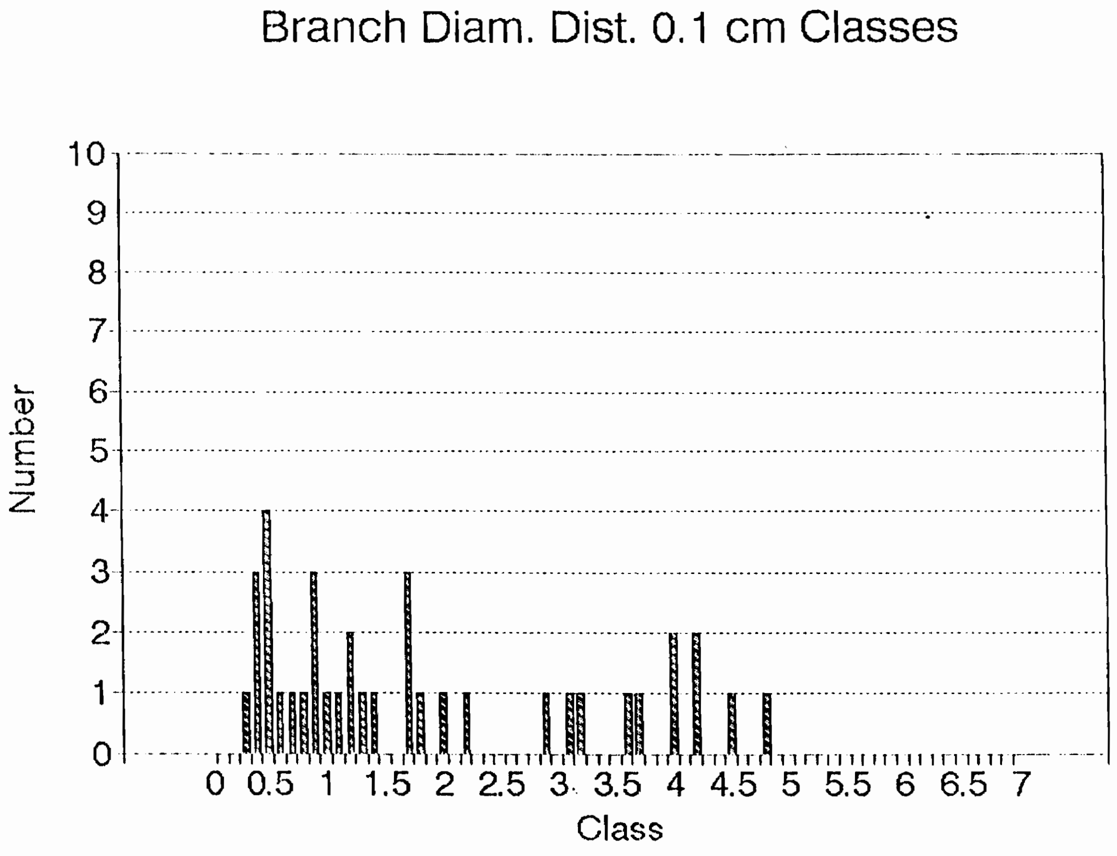

Figure 6. **Observed branch diameter distribution for an 18-year-old Atlantic Coastal Plain loblolly pine with a DBH of 18.5 cm and a height of 13.75 m, found in a 170 sq. ft. BA/acre stand:** Such a pattern is not easily fit by the statistical distribution functions due to the number and size of the gaps.

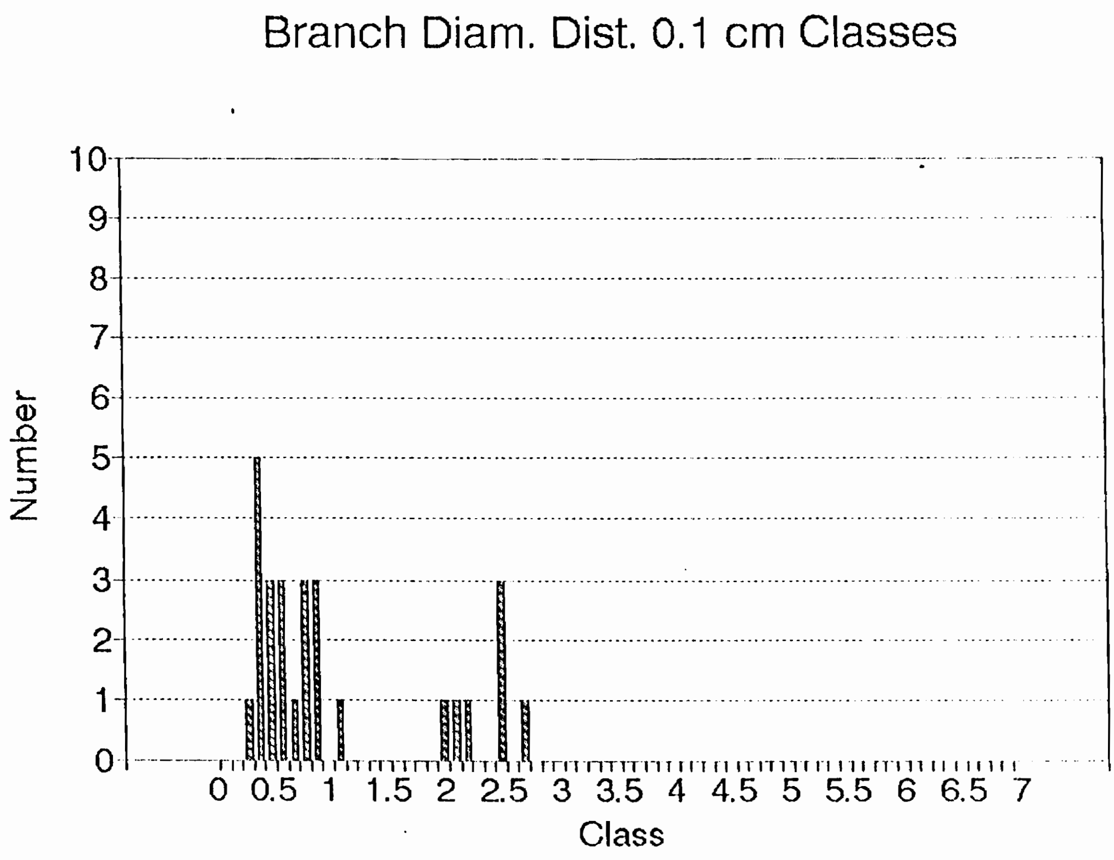

Figure 7. **Observed branch diameter distribution for an 18-year-old Atlantic Coastal Plain loblolly pine with a DBH of 11.1 cm and a height of 11.4 m, found in a 180 sq. ft. BA/acre stand:** Such a pattern is not easily fit by statistical distribution functions due to gaps.

The "best" model predicting minimum branch diameter within a whorl was

$$W_{dmin} = 3.121 + 0.055(DBH) - 3.598(RWH)$$

$R^2 = 0.386$

where: W_{dmin} = minimum whorl diameter (cm), and

DBH, RWH = as before.

The companion "best" model predicting maximum branch diameter within a whorl was

$$W_{dmax} = 3.821 + 0.091(DBH) - 4.452(RWH)$$

$R^2 = 0.464$

where: W_{dmax} = maximum whorl diameter (cm), and

DBH, RWH = as before.

Although these models do not explain much of the variation in the dataset, they do represent the "best" models found. When used in conjunction with one another (as required to estimate range in whorl diameters) the system does not perform much better than the single predictive equation previously dismissed. The variation in the dataset makes prediction of diameter range (or minimum and maximum diameters) within a whorl extremely difficult. No equations/attempts were found in the literature for model comparison.

Branch Heights

As mentioned in the methods section, 4 avenues were explored in an attempt to model the height above ground to whorls (and thereby branches). Regressions relating whorl height to tree/stand/whorl characteristics proved futile (R^2 never exceeded 0.05.) Likewise modeling the distance between whorls was also futile, probably a result of the indeterminate growth pattern exhibited by loblolly pine.

A height increment model could be used to set the first whorl of each year, but not any subsequent whorl of that same season. Patterns were sought but not found trying to relate position in crown to distance between whorls; the same factors probably blocking this avenue as well.

The only recourse available is to assume equidistant spacing of whorls within the crown, thereby employing total height and height to live crown to drive whorl height assignment. The total number of whorls was already predicted using the equation previously reported.

Number of Branches Within a Whorl

Estimating the number of branches within a whorl as a function of tree/stand/whorl attributes proved just as difficult as trying to predict whorl heights above ground. Direct regression analysis was futile, as were attempts at identifying periodicity. It was thought (through field observation) that some prediction was possible with the number stabilizing as one neared tree tip, but such was not the case.

Figure 8 shows the pattern of number of branches per whorl versus height for the entire dataset (made relative through relative whorl height), while Figures 9 and 10 show the pattern for two sample trees. While patterns might appear in Figures 9 and 10, the prediction of branches per whorl was foiled by unusual numbers (*i.e.* the 7-branch whorl in Figure 9) and the varying number of whorls in any recognized pattern.

In this case, the only recourse available is to utilize overall percentages as found in the dataset and employ these percentages with respect to the number of whorls and the total num-

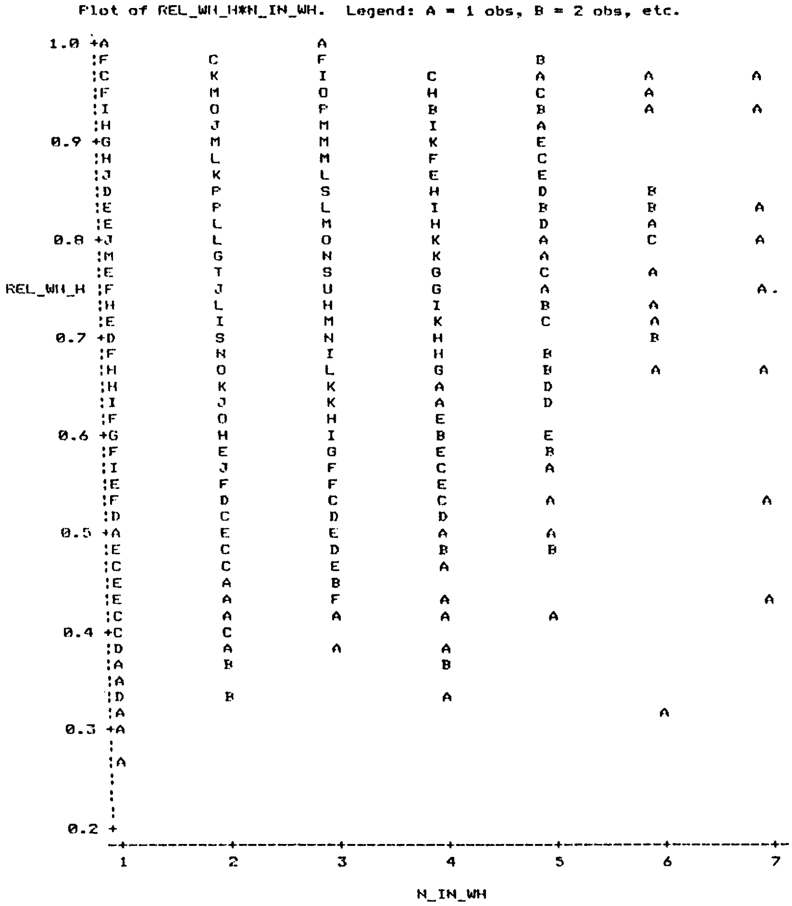

Figure 8. Observed relationship between number of branches in a whorl and relative whorl height in the entire 9- to 30-year-old loblolly pine dataset: Such a pattern is extremely difficult to model (modeling was to be done on a tree-by-tree basis) but is presented to show the randomness of the data.

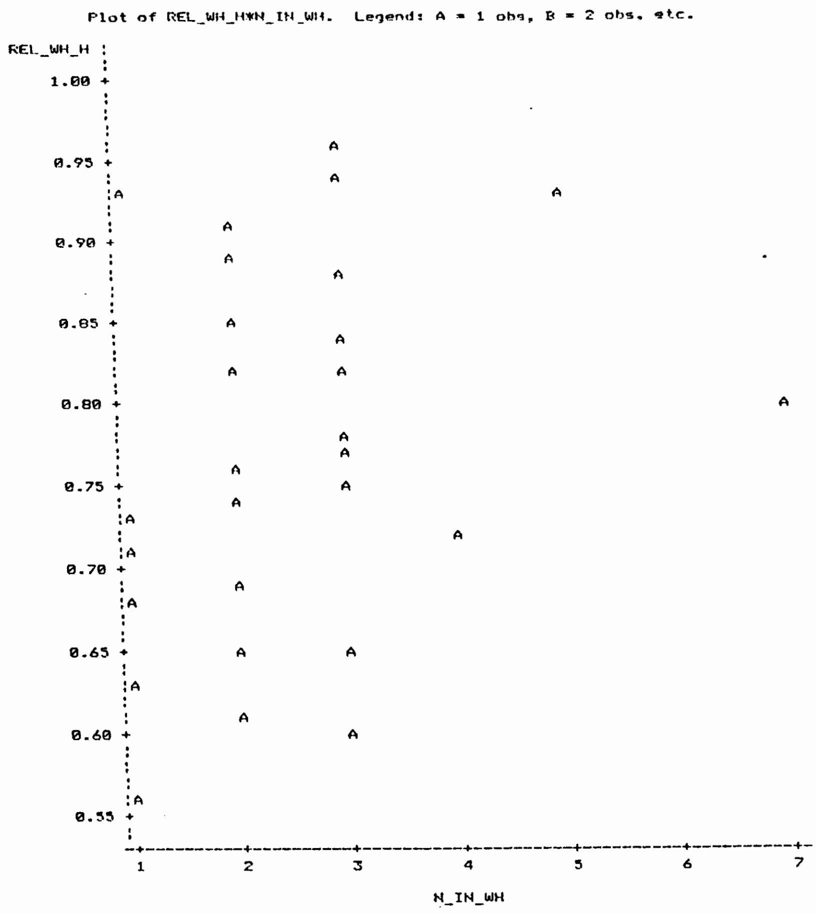

Figure 9. Observed relationship between number of branches in a whorl and relative whorl height in a 25-year-old Atlantic Coastal Plain loblolly pine with a 31.6 cm DBH and a 23 m height.: While a pattern is apparent, its prediction is made difficult by the 7-branch whorl and the differing number of whorls comprising the pattern.

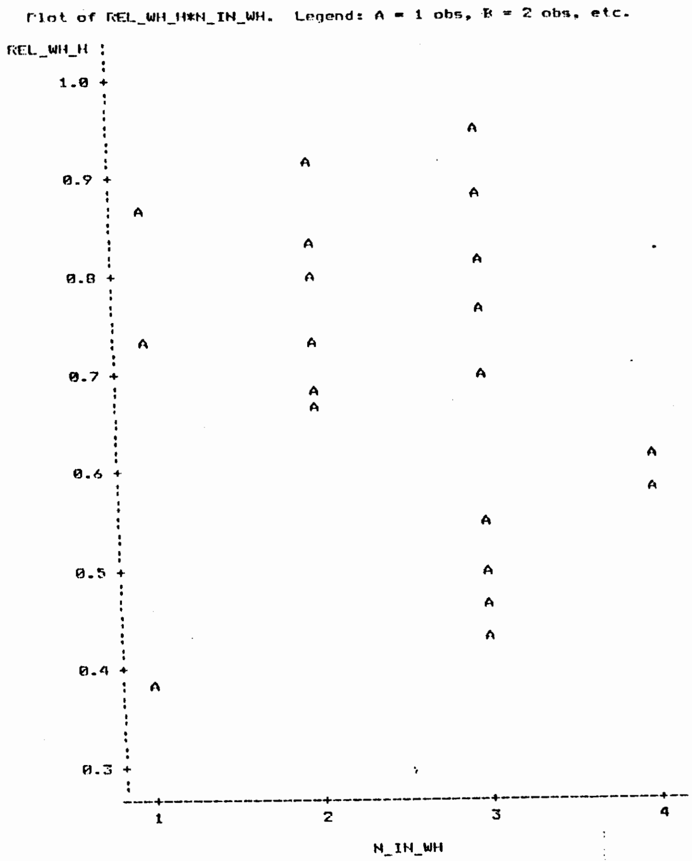

Figure 10. Observed relationship between number of branches in a whorl and relative whorl height in a 15-year-old Atlantic Coastal Plain loblolly pine with a 17.1 cm DBH and a 14.7 m height.: While a pattern is apparent near the top of the tree, the same cannot be said near the base of the crown. Such a pattern makes prediction difficult.

ber of branches. The percentage of whorls containing a given number of branches (across the entire dataset) can be found in Table 3.

Circular Distributions

The mean vector length and mean angle of each of the 68 trees can be found in Table 4. It is apparent from this table that, although not reported, the κ's corresponding to these vector lengths will be very near 0. This implies that a uniform distribution is appropriate. One can visually confirm that a uniform distribution is proper via Figures 11-13, presenting a few of the sample trees as they need to be viewed for this part of the analysis. Figure 14 presents the azimuth distribution of the entire dataset.

While it is not true that a uniform distribution is appropriate for all trees, it is certainly appropriate for most. Recall that a uniform distribution implies that even though a mean (dominant) azimuth can be calculated, the branch azimuths are not heavily concentrated around said angle on a whole tree basis.

Circular Correlation of Branches

The circular correlation methodology previously presented is indeed appropriate once one concludes the branches are uniformly distributed (supported by Wilson and Archer (1979), Bertram (1989), and Baldwin, Dougherty, and Seiler, personal communication.) Recall that the data must also be paired, thereby eliminating consecutive whorls with differing number of branches. Table 5 presents the available number of whorl pairs by branch number.

Table 3. Percentage, in entire dataset, of whorls containing the specified number of branches in the crowns of 9- to 30-year-old Atlantic Coastal Plain and Piedmont loblolly pine

# of branches in whorl	Percentage of whorls
1	19.3%
2	28.3%
3	29.3%
4	15.4%
5	5.6%
6	1.5%
7	0.7%

Table 4. Mean angle and mean vector length of the collection of branch azimuths from each of the 68 crowns present in the 9- to 30-year-old loblolly pine dataset

Tree	Mean angle(°)	Mean vector length
1	133	0.080
2	234	0.084
3	94	0.084
4	197	0.078
5	191	0.067
6	200	0.057
7	199	0.125
8	208	0.031
9	149	0.144
10	261	0.152
11	174	0.044
12	152	0.081
13	211	0.117
14	210	0.075
15	235	0.102
16	117	0.023
17	221	0.059
18	263	0.147
19	215	0.154
20	160	0.037
21	117	0.032
22	96	0.052
23	184	0.080
24	145	0.072

Tree	Mean angle(°)	Mean vector length
25	260	0.092
26	232	0.067
27	259	0.111
28	200	0.055
29	178	0.011
30	94	0.118
31	212	0.008
32	270	0.121
33	126	0.037
34	153	0.057
35	121	0.139
36	253	0.099
37	141	0.044
38	131	0.174
39	224	0.063
40	267	0.188
41	108	0.185
42	138	0.291
43	256	0.238
44	161	0.214
45	209	0.368
46	265	0.183

Tree	Mean angle(°)	Mean vector length
47	117	0.190
48	248	0.181
49	109	0.032
50	152	0.190
51	104	0.188
52	93	0.071
53	203	0.011
54	140	0.130
55	264	0.370
56	265	0.014
57	259	0.134
58	232	0.131
59	172	0.062
60	181	0.094
61	268	0.225
62	237	0.160
63	208	0.144
64	98	0.131
65	123	0.094
66	182	0.136
67	233	0.117
68	174	0.082

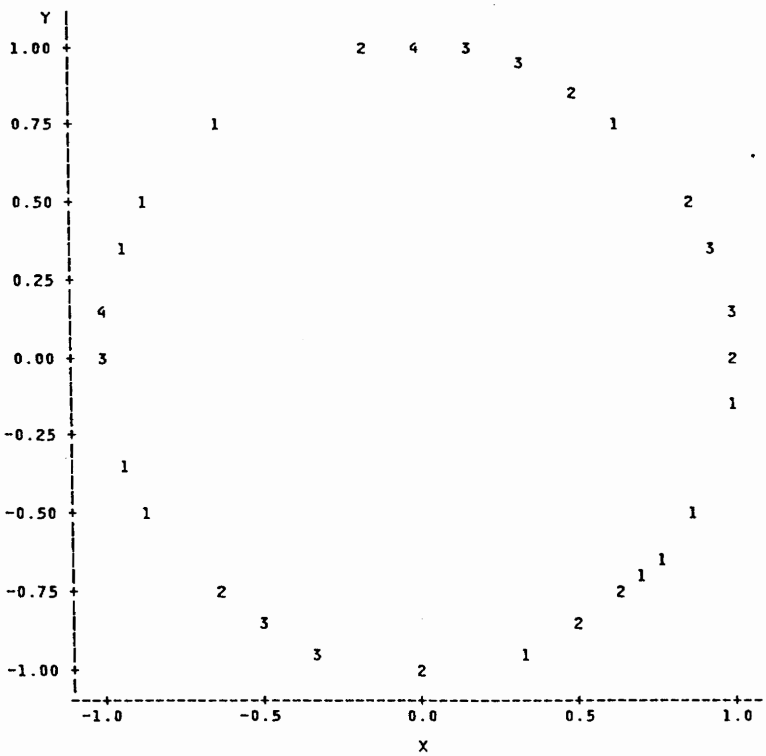

Figure 11. Observed branch azimuth distribution for a 10-year-old Piedmont loblolly pine with a 14.1 cm DBH and a 10.3 m height, found in a 70 sq.ft. BA/acre stand: A representation of the branch azimuths as if one was looking straight down the bole.

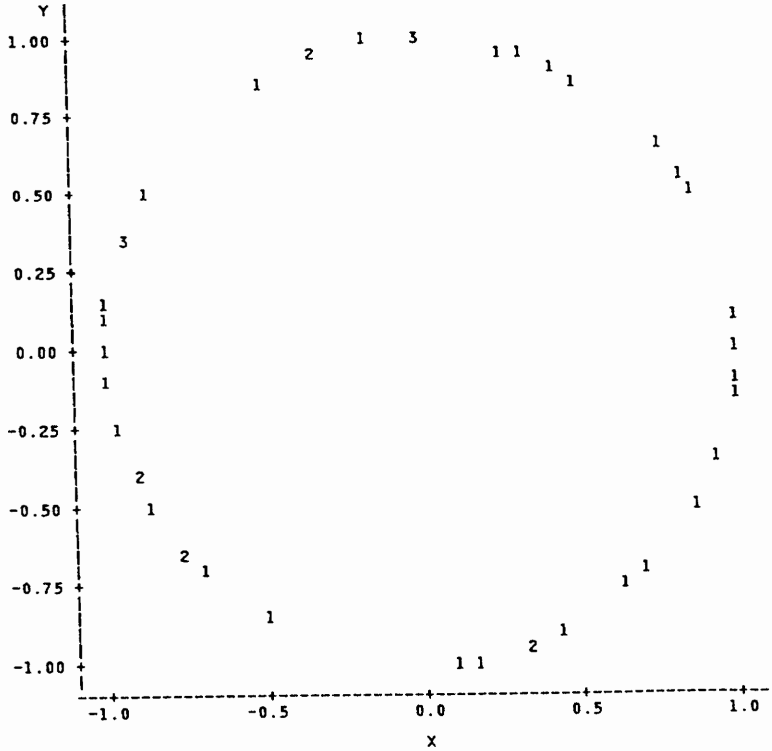

Figure 12. Observed branch azimuth distribution for a 9-year-old Piedmont loblolly pine with an 11.9 cm DBH and an 8.94 m height, found in a 70 sq. ft. BA/acre stand: A representation of the branch azimuths as if one was looking straight down the bole.

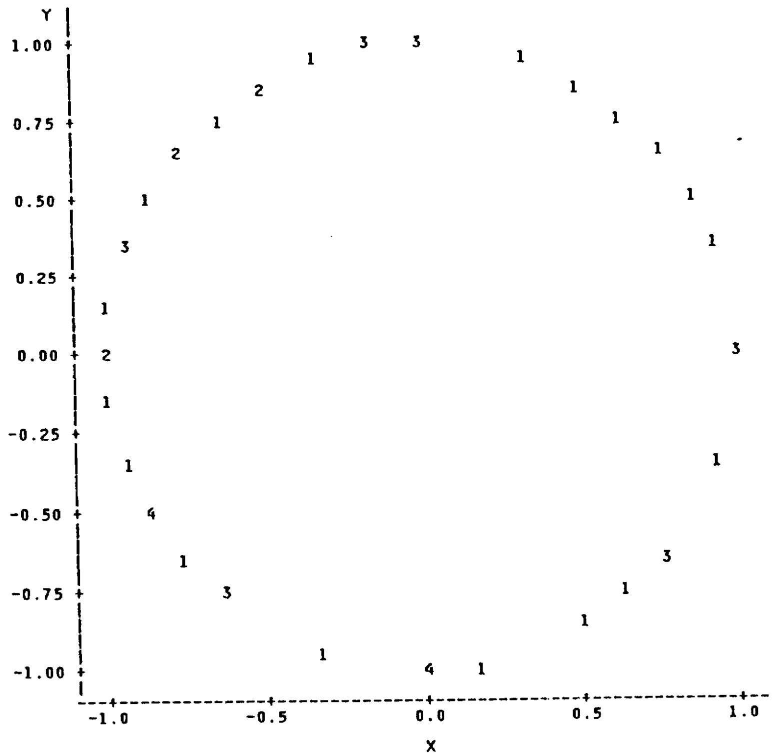

Figure 13. Observed branch azimuth distribution for a 19-year-old Piedmont loblolly pine with a 15.3 cm DBH and a 15 m height, found in a 110 sq. ft. BA/acre stand: A representation of the branch azimuths as if one was looking straight down the bole.

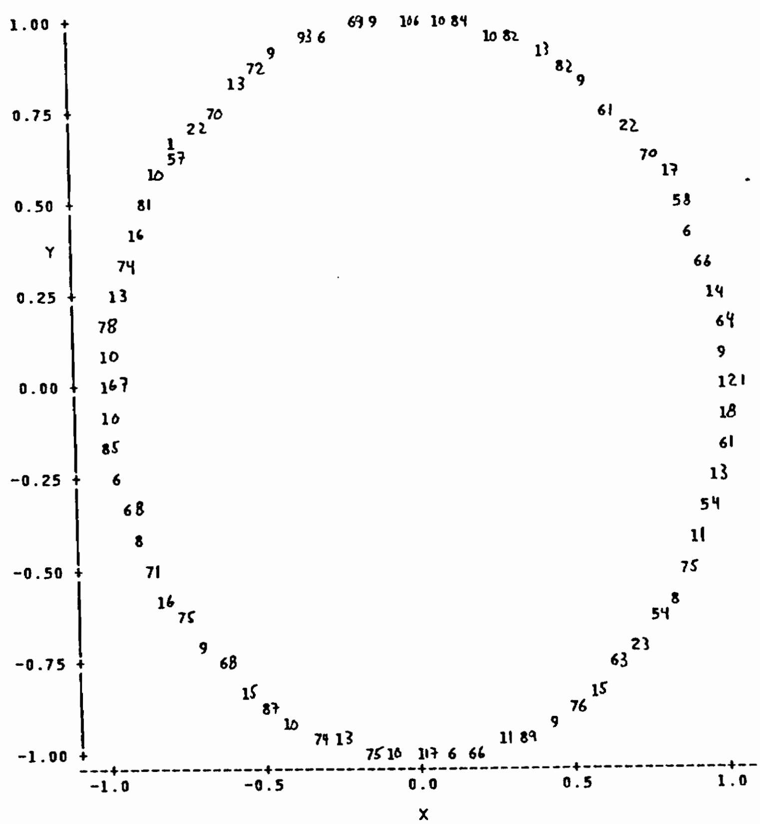

Figure 14. **Observed branch azimuth distribution for the entire dataset of 9- to 30-year-old loblolly pine, found in 70 to 200 sq. ft. BA/acre stands in the Piedmont and Atlantic Coastal Plain:** A representation of the branch azimuths as if one was looking straight down the bole of a tree containing all measured branches.

Table 5. Available pairs of whorls (consecutive whorls with the same number of branches) for circular correlation analysis, in the entire 9- to 30-year-old loblolly pine dataset

# of branches in whorl	# of available whorl pairs
2	87
3	99
4	29
5	5
6	1
7	0

Table 6 contains the means, ranges, and standard deviations of the data summarized in Table 5, while Figures 15-17 provide the graphic representation needed for to evaluate the correlation analysis. The same trees shown in Figures 11-13 are shown in Figures 15-17 for direct comparative purposes. In all cases, the positive correlation coefficient was larger than the negative correlation coefficient, implying that positive rotations dominated.

Most correlation coefficients exceeded 0.65 for consecutive whorls with 2 branches. However, the assumption of uniformly distributed branches is quite unlikely under this scenario (branches would have to be 180° apart). The assumption becomes more valid as the number of branches within a whorl increases.

The range tightened and the mean increased for correlation coefficients for consecutive whorls with 3 branches, While the strong positive correlation trend continues with consecutive whorls of 4 branches. The results of consecutive whorls with 5 and 6 branches should both be taken lightly because of the small sample sizes (5 pairs and 1 pair, respectively.)

It is apparent, then, that under the uniformity of distribution assumption, a positive rotation of corresponding branch azimuths exist for consecutive whorls with the same number of branches. No analysis is provided for consecutive whorls with differing number of branches, but both field observation and subsequent graphic representation suggest no or very little rotation is equally as likely.

Table 6. Mean, range, and std. dev. of correlation coefficients by whorl size from circular correlation analysis of consecutive whorls with the same number of branches in entire dataset

# of branches in each whorl	Correlation coefficient Mean	Std. Dev.	Min.	Max.
2	0.610	0.320	0.000	1.000
3	0.874	0.133	0.333	0.997
4	0.895	0.099	0.559	0.995
5	0.830	0.073	0.727	0.903
6	0.942	-----	0.942	0.942

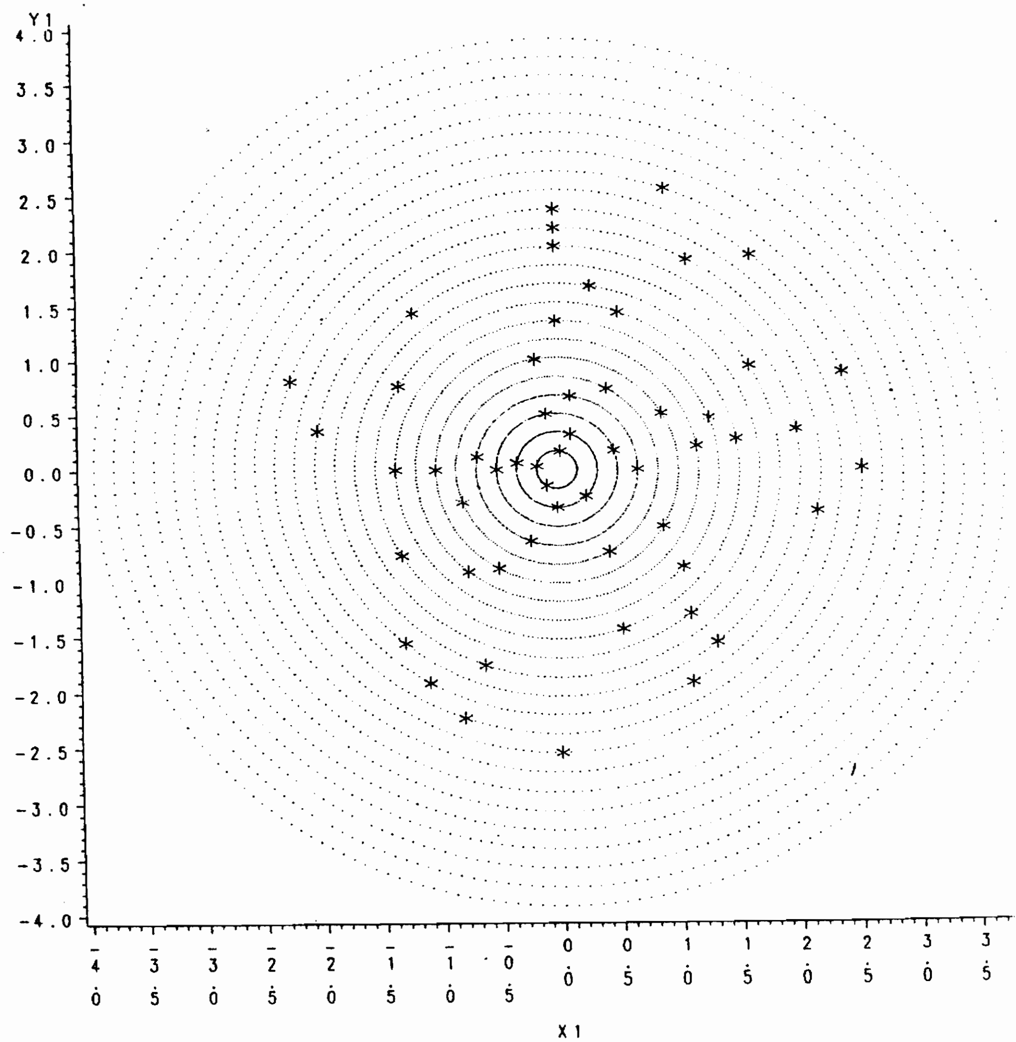

Figure 15. Observed branch azimuth distribution for a 10-year-old Piedmont loblolly pine with a 14.1 cm DBH and a 10.3 m height, found in a 70 sq. ft. BA/acre stand: Each ring represents one whorl of branches. Base of live crown is the innermost ring, and one moves up the tree with each successive ring.

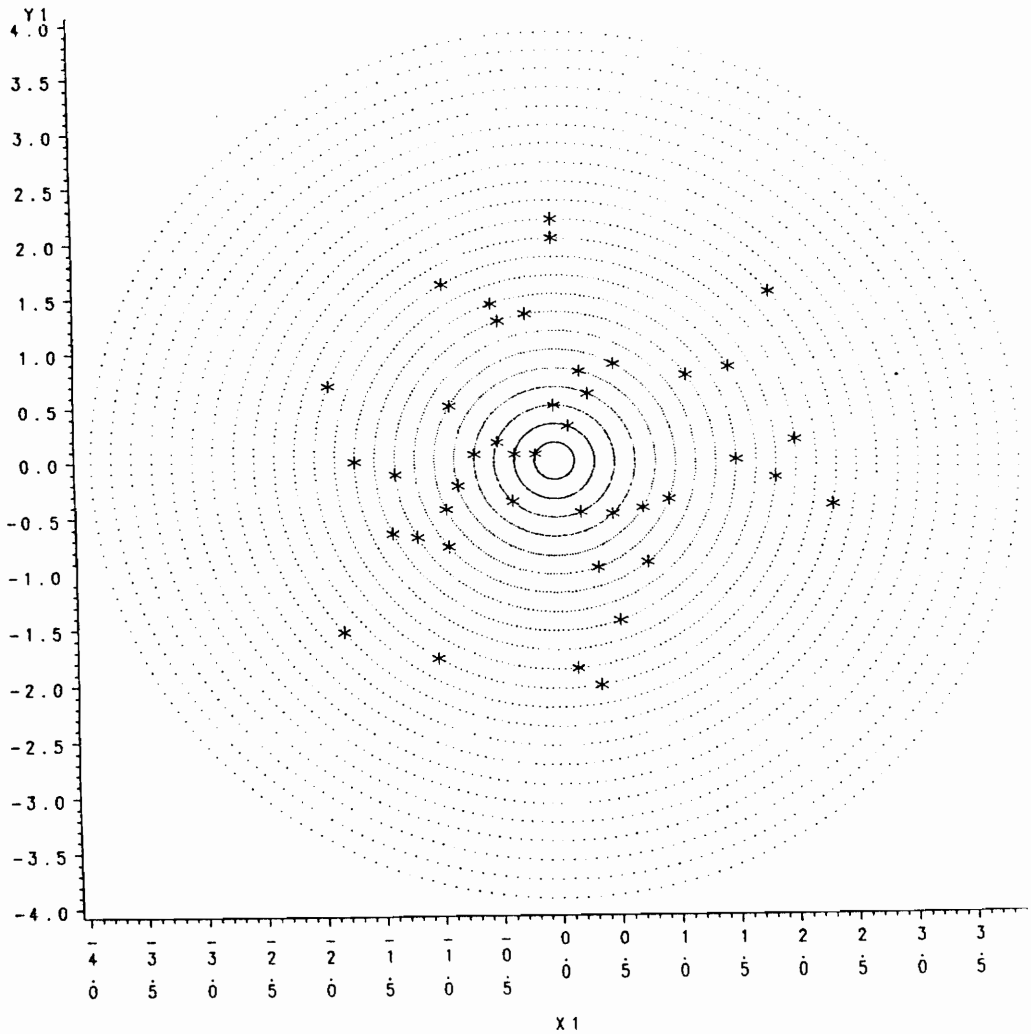

Figure 16. Observed branch azimuth distribution for a 9-year-old Piedmont loblolly pine with an 11.9 cm DBH and an 8.94 m height, found in a 70 sq. ft. BA/acre stand: Each ring represents one whorl of branches. Base of live crown is the innermost ring, and one moves up the tree with each successive ring.

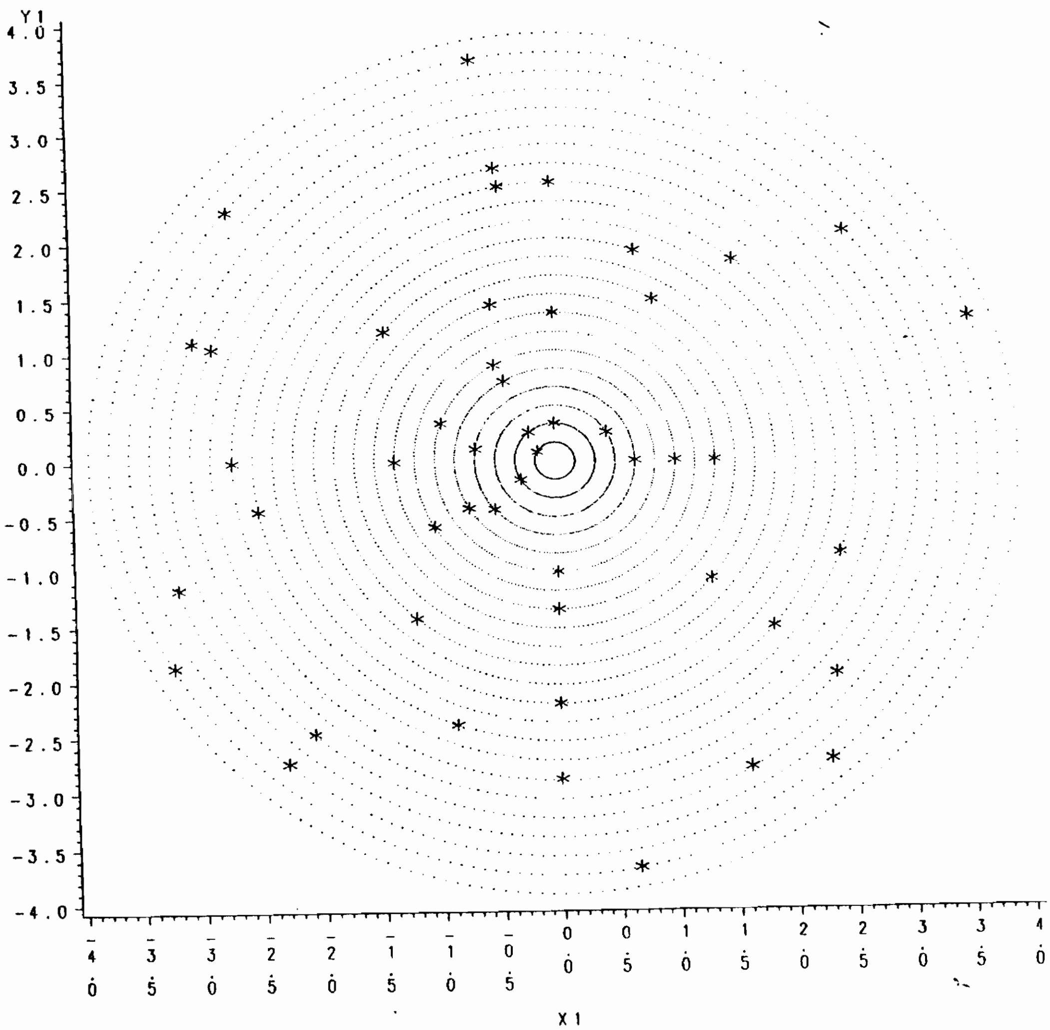

Figure 17. Observed branch azimuth distribution for a 19-year-old Piedmont loblolly pine with a 15.3 cm DBH and a 15 m height, found in a 110 sq.ft. BA/acre stand: Each ring represents one whorl of branches. Base of live crown is the innermost ring, and one moves up the tree with each successive ring.

Chapter VI

Summary and Conclusions

Data from 68 trees were used to develop a quantitative description of crown structure in loblolly pine. The observed branch diameter distributions prevented the application of diameter distribution methods to branches within a tree crown. A series of models were developed estimating the quantities normally derived from a fitted diameter distribution.

Total number of branches in a crown was best modeled using a 2-equation recursive system. The first equation utilized DBH and crown length to predict number of whorls, while the second equation used number of whorls to predict number of branches. Average diameter in a whorl was modeled using DBH and the square of relative whorl height as independent variables. A series of 2 equations, predicting minimum and maximum diameter within a whorl using DBH and relative whorl height as independent variables, was used to quantify the range in diameter within a whorl. This combination of two equations was deemed superior to using one equation to predict diameter range directly.

Heights above ground to whorls and the number of branches within a whorl were modeled with little or no success. Wide variation in the data and the indeterminate growth exhibited by loblolly pine were considered to be the primary confounding factors. Equidistant whorls were assumed to account for the former, while overall percentages from the dataset were used to estimate the latter attribute.

Each of the 68 trees was analyzed to see if a Von Mises or a uniform distribution best described the branch azimuth pattern on a whole tree basis. In most cases, a uniform distribution was appropriate. Circular correlation was used to analyze rotational patterns within and between whorls, assuming uniformly distributed paired data. A strong positive circular correlation was found for consecutive whorls of the same number of branches.

These are the equations herein developed. The following recursive system predicts total number of branches.

$$N_w = 3.933 + 0.429(DBH) + 0.943(C_l)$$

where: N_w = number of whorls,

DBH = diameter at breast height (in centimeters), and

C_l = crown length (in meters).

$$N_b = 7.288 + 2.259(N_w)$$

where: N_b = number of branches, and

N_w = number of whorls.

The next equations are used to estimate the sizes of those branches, as a function of branch (whorl) height.

$$\overline{WD} = 2.134 + 0.073(DBH) - 2.867(RWH^2)$$

where: $\overline{WD}$ = whorl diameter (cm),

RWH = relative whorl height,

DBH = diameter breast height (cm).

$$W_{dmin} = 3.121 + 0.055(DBH) - 3.598(RWH)$$

where: W_{dmin} = minimum whorl diameter (cm),

DBH = diameter breast height (cm), and

RWH = relative whorl height.

$$W_{dmax} = 3.821 + 0.091(DBH) - 4.452(RWH)$$

where: W_{dmax} = maximum whorl diameter (cm),

DBH = diameter breast height (cm), and

RWH = relative whorl height.

After assuming equidistant whorl spacing and using percentages to assign number of branches in a whorl, the crown, with respect to branch size and location, can be described.

This study was largely exploratory in nature, and thus must be evaluated as such. Much of the work presented represents a first attempt at such analyses and was performed on a relatively small dataset. There is much additional work that can be done along these lines, especially in regard to circular relations of branches, but the data and techniques are not readily available. It is hoped that this thesis would provide a good framework for future work in this area of research.

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Vita

Paul F. Doruska was born on February 3, 1968 in Chicago, Illinois. He enrolled at Michigan Technological University in August, 1986, and completed his Bachelor of Science degree in Forestry in May 1990. Upon graduation, he entered the graduate school at Virginia Polytechnic Institute and State University where he earned his Master of Science degree in Forest Biometrics in May 1993.

Paul F. Doruska