

Strategic and Tactical Models and Algorithms for the Coal

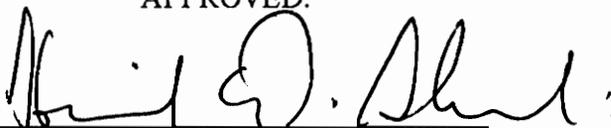
Industry Under the 1990 Clean Air Act

by

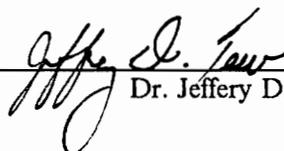
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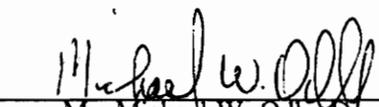
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(ABSTRACT)

This thesis is concerned with a study of the effect of The Acid Rain Provision of the 1990 Clean Air Act on the investment, production, and distribution operations in the coal industry, with a concentration on the development of new mines, shutting down of inefficient strips of existing mines, and on blending and distribution problems. The problem here is to determine which new mines to open and when, and what decisions and schedules to make for the shipment of coal from mines to silos, cleaning and blending operations at silos, and the subsequent shipment of coal to customers over a multi-period time horizon, so as to satisfy the demand at a minimum total operational cost. To meet this objective, a long-term strategic model is developed, and is coordinated with a modification of a short-term tactical model developed by Sherali and Puri (1991). The final product is a computer-based decision tool which will serve as a mechanism for implementing cost effective decisions in light of complex variations in the production levels of existing and potential mines, ore quality, and demand and quality requirements. The strategic model will play a useful role in planning future growth and in making capital investment decisions, and the tactical model will help in making better operational decisions. The models can also be used to study the effect of various policies, by testing the sensitivity, feasibility, and the cost of system operations under different perturbations of system configuration, data, and demand specifications. Real operational data and future projections from the Westmoreland Coal Company are used for testing purposes.

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Chapter I

INTRODUCTION

The research conducted in this study was motivated by the Acid Rain Provision of the 1990 Clean Air Act, passed by Congress on October 28, and signed into law by President Bush on November 15, 1990. This Provision in the Act places stringent restrictions on the sulfur dioxide and nitrous oxide emissions, particularly on electric utilities operating coal-fired generators, and it promotes the role of Southwest Virginia as becoming a central player in the coal industry due to the area's low sulfur coal resources, as compared with other coal mined east of Mississippi River. However, in order for the different companies in the industry to remain solvent and competitive in light of the anticipated sudden change in quality characteristics of future coal demand, they will need to judiciously plan for the usage of their reserves, and the operation of their cleaning and distribution facilities.

The 1990 Clean Air Act is a complex 700-page bill which not only presents new problems, but also new opportunities as alluded above. The states neighboring Virginia collectively include 302 coal-fired generators with a total of 87,693 MW generation capacity, resulting in a sulfur dioxide emission of about 5.68 million tons per year. As part of the new Clean Air Act, sulfur dioxide control will be implemented through a system of allowances in terms of annual sulfur dioxide emissions,

as issued by the Environmental Protection Agency (EPA) to existing pollution sources. One allowance unit equals one ton of sulfur dioxide emissions. In Phase I of this process, beginning January 1, 1995, all 111 U.S. plants emitting more than 2.5 pounds of sulfur dioxide per million BTU heat input (lbs. SO_2 /mil.) will be granted annual emission allowances equal to 2.5 pounds of sulfur dioxide per million BTU (lbs. SO_2 /mil.) times their baseline annual 1985-87 fuel consumption (in million BTU units).

During Phase II, which begins in January 1, 2000, all plants emitting more than 1.2 pounds of sulfur dioxide per million BTU (lbs. SO_2 /mil.) will be granted emission allowances equal to 1.2 pounds of sulfur dioxide per million BTU (lbs. SO_2 /mil.) times their baseline annual 1985-87 fuel consumption (in million BTU units). This Act will cap annual emissions at about 9.4 million tons of SO_2 in the early part of the next century, and at 8.9 million tons thereafter. For reference, in 1988, all utility plants emitted 16.6 million tons of SO_2 .

This research addresses the combined problems of choosing new mines to develop, determining mine production, ore purification, and the blending of different grades of coal, along with the problem of distributing the coal from mines to silos to customers, in order to satisfy customer demands placed over several time periods, each having specified quality requirements as driven by the Clean Air Act.

The main objective is to develop a long-term strategic and a short-term tactical model for the generic problem of the coal industry. The long-term model considers periods equal to six months in duration, with a horizon of three to five years and is designed to help coal companies in making strategic decisions such as the development of new mines or the development of new production units at mines. Also, this model is intended to assist coal companies by providing guidelines for the usage of their existing and potential reserves, and in the operation of their cleaning and distribution facilities, along with the development of new technologies.

In addition to this long-term model, we also develop a short-term model which extends a related model developed by Sherali and Puri (1991). Here, each time period is equal to one day, and this model can be run daily with a one week horizon in a rolling horizon fashion, in order to determine optimal schedules for shipping coal from mines to silos, for cleaning and blending coal at the silos, and for the distribution of coal to the customers, subject to storage, production capacity, material flow balance, and quality restrictions. Two new features have been added to this model. First, we include a penalty-reward structure function which either imposes a penalty or gives a reward in the objective function depending on the quality sent to each customer relative to the quality desired. Second, the model is augmented to accommodate more than one type of cleaning technology.

Data for these models have been provided by the Westmoreland Coal Company, one of the largest coal mining companies in Southwest Virginia. The company owns several coal mines, some of which are currently in use (existing mines), and some of which can be developed later as need arises. Each mine can produce coal at a specific rate and has a certain quality specification which varies over time at each mine. This coal needs to be appropriately shipped to silo facilities, where it is subjected to a beneficiation process in order to be partially cleaned to a desirable degree. Besides the conventional mine to silo to customer shipments, there might also be a coal transfer between the silo facilities at two different geographical sites. Such a transfer occurs due to the "stoker" customers. In the example of the Westmoreland coal company, "stoker" customers are provided with cleaned, sifted coal from the Wentz facility. Simultaneously, an almost equivalent amount of the sifted by-product is shipped to the Bullitt facility. The normal blending processes resume at this stage. Finally, the different grades of coal at individual silo facilities need to be blended and shipped to customers in order to satisfy demands for various quantities having stipulated quality specifications.

The problem is to find out which production units of potential mines should be opened and at what time periods, and to determine optimal schedules for shipping coal from mines to silos, for cleaning and blending coal at the silos, and for the distribution of coal to the customers. This has to be accomplished subject to restrictions involving storage at silos, production capacity limits, and

material flow balance constraints, so that customer demands having required quality specifications are satisfied at minimal total cost.

A general description of both the long-term and short-term models is given next.

Long-Term Strategic Model :

This model has the structure of a fixed-charge, mixed-integer, zero-one programming problem. Zero-one integer variables are used to model the decision of opening a production unit at each particular mine. In addition to the cleaning and shipping costs, the objective function incorporates a fixed-charge component to reflect the cost of opening a new production unit at a potential mine. Also, a storage cost component is added to the objective function in order to penalize unused quantities. The constraints include production capacity, storage, material flow balance, and quality requirement restrictions, along with restrictions on the sequence in which units can be opened, and on the maximum number of units that can be opened in any time period.

The decisions involved in this model correspond to :

- Whether or not to open a production unit at a potential mine in a particular time period.
- Shipments from new or existing mines to silos to customers in each time period.
- Stoker customer by-product shipments from new or existing mines to a Wentz silo and then to a Bullitt silo, and finally to customers in each time period.

Short-Term Tactical Model :

This model has the structure of a linear programming problem, and is an extension of "Model 1" developed by Sherali and Puri (1991). In this model, coal is shipped out of mines to silos at the beginning of time periods, and is shipped out of silo units to customers at the end of time periods. The model is flexible in terms of the allowable *shipment lag* between flows. It permits a maximum of a three period lag between coal production at mines and the final shipment to customers. The flow of coal corresponds to the mine to silo, the silo to customer, and the Wentz to Bullitt silo shipments. The proposed extension of the model includes a penalty-reward component in the objective function, which either penalizes or rewards the quality sent to the customers relative to what is desired. The model has also been augmented to accommodate more than one type of cleaning technology.

The decisions involved in this model correspond to :

- Shipment from each mine to each silo at the beginning of a particular time period t , and the continuing shipment from the silo to a customer at the end of a period τ , such that the *shipment lag* between t and τ does not exceed three time periods.
- Shipment representing a stoker customer by-product flow from each mine to a Wentz silo in period t_1 , followed by a Wentz to Bullitt silo transfer during period t_2 , and then from the Bullitt silo to a customer during period τ , such that the *shipment lag* between t_1 and τ does not exceed three periods. (This is tied in with the "stoker" customer shipment.)
- Shipments from silos to customers of coal in initial storage at the silos, in each time period.

To better understand the problem, we begin by giving a detailed specification of the related data requirements. The basic entities involved are the various mines, silos, and the customers. Below, we first specify the data requirements that are common to both the long-term and the short-term models, and next, we specify the special long-term and the short-term model requirements.

Common data requirements for the long-term and the short-term models:

- Number of time periods
- Number of silo units (storage and cleaning facilities)
- Number of customers
- Number of stoker customers
- Attenuation factors (output per ton input) corresponding to total weight and ash and sulfur contents for coal at clean silo units
- Cleaning costs at silos
- Shipping costs for mine to silo, silo to customer, and Wentz to Bullitt silo shipments
- Customer demand requirements during each time period

Special long-term model requirements :

- Number of existing mines (active mines)
- Number of potential mines (undeveloped mines)
- Number of production units at each potential mine, and the order in which they can be developed
- Amount produced at each existing mine in each period

- Amount that can be produced at each production unit of a potential mine in each time period, following its initiation.
- Storage capacity at silos
- Percentage of ash and sulfur content in coal produced at each existing mine
- Percentage of ash and sulfur content in coal that can be produced at each production unit of a potential mine
- Fixed-charge to open a production unit at a potential mine
- Penalty for left overs at the mines
- Upper limit on the number of production units that can be opened in any time period without any surcharge penalty

Special short-term model requirements :

- Number of production sites
- Amount produced at each production site in each time period
- Storage capacities at production sites and silos
- Initial storage amount at each silo
- Percentage of ash and sulfur content in coal produced at production sites, and that for coal in initial storage at the silos
- Storage costs at mines and silos

- Slope of reward functions for ash and sulfur content in the customer shipments, in dollars per ton of coal of better quality relative to what is desired by the customer
- Slope of penalty function for ash content in the customer shipments, in dollars per ton of coal of poorer quality relative to what is desired by the customer

The remainder of this thesis is organized as follows. Chapter II presents a brief review of the literature that is related to the present research. Chapters III and IV respectively describe model formulations for the short-term tactical problem, and the long-term strategic problem. Suitable solution procedures for these problems are developed in Chapter V, and related computational results are presented in Chapter VI.

Chapter II

Literature Review

This chapter presents a brief review of the literature related to the present research. The material reviewed is comprised of a synopsis of articles pertaining to work done in related areas. These articles are categorized according to the techniques used to approach the different problems, which vary over linear, and mixed integer 0-1 programming methods. The discussion begins with articles pertaining to the use of linear programming models in the coal industry, followed by others relating to the use of mixed-integer programming models, and then articles that pertain to problems in other related fields that are similar in structure to our problem.

Articles Pertaining to the Use of Linear Programming in the Coal Industry:

Many papers have been written on the use of Linear Programming in the coal industry. Young et al.(1963), Faulkner (1967) and Johnson (1969) are few of the pioneering papers in the area. Sherali and Puri (1991) is one of the latest papers in this area.

Tomlinson [1977]

This paper discusses a number of practical case studies of Operations Research in the United Kingdom coal mining industry. The paper is divided into three parts. The first part describes the way in which the Operations Research team provides service to the coal field management. The second part is concerned with a number of projects, directed toward problems of coal mining and how Operations Research can be used in solving these problems. In the third part, the author talks about interactive models and how they can help Operations Research teams in being closely involved with the management in making decisions

Young, Ferguson and Corbishley [1963]

Different aspects related to coal mining are discussed. Recommendations related to planning at different levels in the coal industry are made. Several factors which make the coal production uncertain, are described briefly. To illustrate one of the approaches which is used in the planning process, a linear programming model for a hypothetical colliery is formulated, to determine a scheme for working on the faces of the colliery so as to maximize profit. Due to insufficiency of available computer facilities, an alternative method was sought, which was based on the concept that the constraints must be satisfied in the most profitable way.

Faulkner [1967]

A linear programming formulation for the problem of scheduling shipments of ore from sources to reduction plants is developed. Five different formulations of the objective function, which include transportation costs, mine cycling and mine sequencing, a combination of the previous two, recovery of a particular substance, and the total value of production, are discussed.

Johnson [1969]

Mine planning concepts are discussed and a linear programming model is proposed to improve the same. The model developed in this paper allows optimal planning of the complete mining-concentrating-refining system over the entire planning horizon, and permits the system to dictate

how and when to process a block of material. The solution methodology adopted uses the decomposition principle to decompose the large-scale linear programming problem into a master problem and a sub-problem, solvable through available techniques.

Knight and Manula [1976]

Models are developed to simulate potential coal production and utilization systems in Pennsylvania. The models predict potential coal production and transportation for alternative future demand scenarios, while accommodating costs for various coal supply systems under specific economic, transportation, and environmental constraints. Furthermore, it provides an insight into such impacts of coal production as employment, land use, and environmental quality.

Lietaer [1987]

The planning of an underground mine using linear programming is described. The objective is to prepare a combination of mining works fulfilling certain requirements which take into account operational restrictions on the mines and the concentrators, while minimizing the total operational cost. The model developed allows the possibility of exploring the feasibility of a wide range of concentration options. This model has been successfully implemented in the Casapalca mine at Centromin, Peru, the largest integrated mining, smelting and refining company in Peru.

Williams and Haley [1989]

A linear programming problem related to the allocation of coking coals from collieries to washeries and blending plants is discussed in this paper. A total of 740 variables and 171 inequalities are involved in the problem. The problem is then reduced to a transportation problem, and is subsequently solved by the methods of transshipment. This was the first application of linear programming to one of the projects of the National Coal Board of Britain.

Sherali and Puri [1991]

This paper is concerned with the study of typical coal flow operations in the coal industry, with a concentration on the blending and distribution problems. The problem is to determine decisions and schedules for coal shipments to silos, cleaning and blending operations at silos, and the subsequent shipment of coal to customers over a multi period time horizon, so as to satisfy the demand at minimum total cost. Three short-term, tactical linear programming models of varying complexity and capability are developed and tested on real operational data from the Westmoreland coal company. All the three linear programming models were coded using GAMS 2.21 (General Algebraic Modeling System) and implemented on an IBM 3090 computer using MINOS 5.2, which is the default linear programming solver within GAMS.

Articles Pertaining to the Use of Mixed-Integer Programming in the Coal Industry :

Osleeb et al.(1983), Gershan (1983) and Osleeb et al.(1986) are few of the papers describing the application of mixed-integer programming models in the coal industry. Candler (1991) is one of the latest papers in this area.

Steinmann and Schwinn [1969]

Computational experience with different algorithms for zero-one programming problems that arise in mining contexts is reported. The objective formulated is to minimize the total resources necessary for balancing the capacity structure of the coal mine, subject to capacity constraints for a particular mine. The algorithms considered for the solution include Gomory's algorithm, the additive algorithm of Balas, Salkin and Speilberg's algorithm, the simplex method, and a branch and bound procedure. Optimal solutions for the two problems presented in the paper are found by using the branch and bound methodology.

Osleeb and Ratick [1983]

A mixed-integer programming model called Port Expansion System (PES) is formulated to determine the optimal capacity, placement, and railroad and marine interface of coal handling facilities within and between the New England ports and converting power plants. The mathematical structure of the model is discussed in detail. PES has two linked models, a transportation/transshipment model defined for the network, and a location/allocation model defined for the potential location of coal handling facilities within receiving and shipping nodes of the network. An example has been presented to illustrate the model. The program SESAME, developed at M.I.T., is used to solve the problem.

Gershan [1983]

A mixed-integer formulation of the mine scheduling problem is discussed and applied for the purpose of simultaneously optimizing both the mine production sequencing and the mill blending and processing problems. A mine scheduling optimization (MSO) approach has been adapted and presented. The system optimizes the net present value of the profits over the life of the mines, while considering multiple pits, polymetallic ores, ore handling and processing facilities, environmental limitations, and product sales. An interface of a long-term and a short-term model is also presented.

Osleeb, Ratick, Buckley, Lee and Kuby [1986]

A mixed-integer location-allocation problem is formulated to evaluate the potential for reducing water-borne coal transportation costs, and concomitantly, the costs of delivering coal to European markets. The cost reduction is accomplished through the use of supercolliers which would require the dredging of channels at selected ports or the use of offshore loading sites at East and Gulf coast ports, or both. The model presented in the paper, Coal Logistics Systems (COLS), is a comprehensive multi-quality commodity transportation/transshipment programming model that is defined for a network consisting of supply, transshipment, and final demand nodes connected by multi-modal transportation links. COLS is solved using the IBM MPSX/MIP software. Five scenario runs are made to demonstrate that some cost savings may be realized with the options tested.

Candler [1991]

This paper presents a blending problem with (a) integer constraints on minimum usage of certain amount of coal in a mix and (b) acceptance sampling with penalty function and rejection options. The problem is to trade off ingredient costs against the probability of rejection on the basis of a small sample. A deterministic model is developed, with individual ingredient costs as known quantities and then stochastic elements are introduced to account for sampling errors. This allows an orderly search for the preferred cost-risk combination.

Articles Pertaining to the Use of Mixed-Integer Programming in Related Fields:

Aboudi et al.(1989) develops a long-term planning model for the petroleum production and distribution problem using mixed-integer programming. Khan (1987) presents a mixed-integer model to optimize the total disposal costs in an urban solid-waste disposal problem. The objective function and constraints of both these models have the similar structure as our model. Lee et al.(1981) presents a multiple criteria modelling approach using a mixed-integer programming model for a facility location problem.

Aboudi, Hallefjord, Helgesen, Helming, Jornsten, Pettersen, Raum and Spence [1989]

This paper describes the results of an operations research project performed at the Chr Michelsen Institute during the period 1985-1987. The project's aim was to develop a long-term planning model for the sequencing of petroleum production activities on the Norwegian shelf. The emphasis was on the development of transport systems for petroleum products from the producing fields to gas customers and onshore oil terminal. The paper discusses all the phases of the operations research project. The model developed is a mixed-integer programming problem. The objective function is to maximize revenue minus operating and transportation costs. Constraints are grouped into three categories (a) conservation of flow constraints (b) budget constraints (c) logical constraints which ensure that a field/link is started at most once, and ensure that two or more fields

are started in a proper sequence. In order to solve the problem, a set of heuristics are used in combination with the ZOOM/XMP package to provide a near optimal solution. The system is made interactive and dialogue-based. A special purpose report generator has also been built.

Khan [1987]

This paper examines the problem of urban solid-waste disposal by landfill. A model has been presented which optimizes the disposal costs by trading off transportation costs against the capital and operating costs of introducing the transfer station. The goal is to minimize solid-waste transportation and disposal costs by the possible use of intermediate transfer stations that reduce the mass transported, albeit at the additional cost of operating the stations (fixed cost). Constraints ensuring the disposal of all the waste generated at each source, equality of input and output at each transfer station, fulfilling minimum and maximum capacity requirement at each transfer station, and landfill capacity constraints, besides nonnegativity constraints, are included. A branch and bound procedure is used to obtain the optimum and an imbedded simplex code serves to provide the solution at each branch.

Lee, Green and Kim [1981]

This paper presents a multiple criteria modelling approach to the facility location problem, which involves multiple conflicting objectives. The model describes a situation in which a firm evaluates six potential production plant location sites in four states that would serve four distribution centers, and considers multiple objectives including several qualitative factors of location analysis and cost related objectives. An integer goal programming package (IBRANCH) based on a branch and bound method is used to solve the model, and the results are compared with the linear programming relaxed solutions. Results of sensitivity analysis involving changes in the hierarchy of certain priorities assigned to different objectives are also presented.

Chapter III

Formulation of a Short-Term Tactical Model

The first section of this chapter gives a description of the overall problem and is followed by a section that outlines the modifications that have been made on 'Model 1' developed by Sherali and Puri (1991), as prompted by the problem at hand. The third section presents a verbal statement of the problem, the required data specifications, details about the mathematical formulation of the short-term tactical model, and its usage.

3.1 Problem Statement

The structure of the Westmoreland Coal Company is used here to illustrate the prevalent work environment or framework for a typical coal mining company. The company has its headquarter in Philadelphia and its branches in Virginia, West Virginia, Kentucky, and Montana. A branch of the Westmoreland Coal Company, located at Big Stone Gap, South-West Virginia, has been visited to study the problem.

The company owns several coal mines in this area. Thirteen of them are in use presently, producing coal at various rates, and having known quality specifications which vary over mines, as well as over time periods at each mine. The company also owns two large silo facilities, known as the Bullitt facility and the Wentz facility. The coal is shipped to these facilities via conveyors or trains, where it is subjected to a beneficiation process, if need be, in order to partially clean it to a desirable degree.

The Bullitt facility has six Silos, each with a capacity of 14000 tons. Silo 1 and Silo 2 store coal that does not require to be cleaned. Silo 3 contains high sulfur coal and Silo 4 contains low sulfur coal. After beneficiation, coal from Silo 3 is transferred to Silo 5 and coal from Silo 4 is transferred to Silo 6 for storage. Coal is beneficiated at this facility at the rate of 1200 tons/hour.

The Wentz facility has five silos, three with a capacity of 6000 tons and two with a capacity of 9000 tons. One of them is used to store coal which does not need to be cleaned, two of them are used to store the remaining coal prior to cleaning, and the remaining two are used for storing coal after it has been cleaned.

In the model, silos within each facility are partitioned into two categories based on the two different kinds of coal stored. These categories are denoted as the J_1 and the J_2 types of silos. The J_1 silos are "run-of-mine" (ROM) blend silos, where a shipment is received, stored, and is directly blended with other coal in its run-of-mine form itself. Each J_2 type of silo, on the other hand, constitutes a pair of silos, one of which receives the run-of-mine coal, while the other stores the cleaned coal following the beneficiation process, holding it ready for the blending operation. These silo pairs are indexed by the sets J_{2W} and J_{2B} for the Wentz and Bullitt facilities, respectively.

The different grades of coal at individual silo facilities then need to be blended and shipped to customers in order to satisfy demands for various quantities having stipulated quality specifications. The train or conveyor passes under the silos, where the required amount of coal is dropped from each silo to obtain an acceptable blend for satisfying the customers demands and specifications.

Besides the conventional mine to silo to customer shipments, there is also a coal transfer between the two silo facility sites. Such a transfer occurs due to what are known as “stoker” customers. In the example of the Westmoreland coal company, “stoker” customers are provided with cleaned, sifted coal with a size specification of .25 square inch to 1.25 square inch from a Wentz facility. Simultaneously, an equivalent amount of the sifted by-product is shipped to a Bullitt facility. The normal blending processes resume at this stage.

The aim of the problem is to determine optimal schedules for shipping coal from mines to silos, for cleaning and blending coal at the silos, and for the distribution of coal to the customers, subject to storage, production capacity, material flow balance, and quality requirement restrictions, so as to satisfy the demand at a minimum total cost. The objective function considers the distribution, storage, and cleaning (beneficiation) costs, as well as revenues or costs for possibly shipping coal to customers which is, respectively of a better or inferior quality than the minimum acceptable specified level. The problem constraints relate to the flow balance and storage restrictions at the mines and at the silos, the Wentz to Bullitt facility transfer, customer demand satisfaction, initial silo storage dissipation, and customer product quality requirements.

3.2 Modifications made to the Sherali-Puri model

In ‘Model 1’ constructed by Sherali and Puri (1991), coal is shipped out of mines to the silos at the beginning of time periods, and is shipped out of silo units to the customers at the end of time periods. A maximum of a three period *shipment lag* between coal production at mines and the final shipment to customers is permitted, based on an estimate of the clearance time at the silos. If t_1 is the time period for a certain mine to silo shipment, and t_2 is that for a continuing silo to customer shipment, the *shipment lag* is given by $t_2 - t_1$. The *transfer lag*, another time lag factor, that indicates the difference between the time of dispatch of a coal shipment from a mine to a silo and its actual arrival time at the silo, is negligible for the problem under study, and is therefore assumed

to be zero. We also adopt this same structure of the model. However, the following modifications have been made to enhance the problem representation.

1. A penalty-reward component has been introduced into the objective function, which either penalizes or rewards the quality sent to the customer relative to what is desired. The need of this function arose as the customers are now more liberal about the content of ash in the coal, but are more stringent about its sulfur content. In "Model 1" of Sherali and Puri (1991), there was only a reward structure for shipping better ash quality coal, and so, we have incorporated a reward or penalty depending on the ash quality of coal relative to the maximum specified limit. Also, due to 1990 Clean Air Act, to reflect the incentive for shipping coal having a better sulfur quality, a reward function is introduced in the objective function.
2. Due to the Clean Air Act 1990, coal companies might have to look for different types of cleaning technologies instead of just using a single type. "Model 1" can be given an interpretation so that it can accommodate more than one type of cleaning technology. For example, if there are two types of cleaning technologies used by a coal company, then the silos in J_2 can be divided further into subsets J_{21} and J_{22} , where each J_{21} type of silo constitutes a pair of silos, one of which receives the run-of-mine coal, while the other stores the coal following the cleaning operation, and where each J_{22} type of silo constitutes a similar pair of silos, but representing an alternative cleaning technology. Note that this is principally a data processing, rather than a model oriented, modification.
3. The equality constraint forcing strictly as much amount of coal to be transferred from the Wentz to the Bullitt silos as is shipped to stoker customers has been relaxed to an interval constraint, to better reflect the actual practice in this transfer.

3.3 Model Formulation

3.3.1 Verbal Statement

Given below is a verbal statement of the model, followed by its data specification and mathematical formulation. This short-term tactical model has the structure of a linear programming problem.

Objective Function :

Minimize total cost = (shipping and cleaning cost at mines) + (storage cost at mines) +
(storage cost at silos) + (penalty-reward function for ash quality shipped)
-(reward for better sulfur quality shipped)

Subject to the Constraints:

- Flow balance and storage at mines :

(Total production by period δ) - (Total output by period δ) = (Storage remaining in period δ)

- Storage constraints for silo facilities :

(Total input at a silo in period t) - (Total output at the silo in period t) = (Storage at the silo in period t)

- Dissipation of initial amount in storage at the silo facilities :

(Shipment to a customer from the amount in initial storage at a silo over a length of three periods) = (Total initial amount in storage at the silo)

- Wentz to Bullitt transfer balance with stoker customer shipments :

(Total amount transferred from the Wentz to the Bullitt silo facility in period t) \leq Shipment to stoker customer in the same period ≤ 1.1 (Total amount transferred from the Wentz to the Bullitt silo facility in period t)

- Demand satisfaction constraints for customers :

(Total amount to be shipped to customer in period t) = (Customer demand in period t)

- Customer product quality constraints, controlling the ash and sulfur contents in customer shipments, for each period t .
- Nonnegativity constraints on the decision variables.

3.3.2 Data Specification

We now describe below the data requirements along with our notation.

$t = 1, \dots, T \equiv \#$ of time periods (≤ 6).

$i = 1, \dots, m \equiv \#$ of mines (≤ 13).

$j = 1, \dots, J \equiv \#$ of silo units (≤ 18).

$k = 1, \dots, K \equiv \#$ of customers (≤ 10).

K_{st} = "stoker customers" served by Wentz silos.

p_{it} = production (tons) at mine i , in period t .

a_{it}, s_{it} = ash and sulfur content (%), respectively, in coal produced at mine i , in period t .

SM_i = storage capacity at mine i .

c_i^m = storage cost (per ton) at mine i .

$J_1 \equiv$ ROM silo storage units.

$J_{2W} \equiv$ cleaned silo units at the Wentz facility.

J_{2B} \equiv cleaned silo units at the Bullitt facility.

J_2 \equiv cleaned silo units ($J_{2W} \cup J_{2B}$).

SS_j = storage capacity of silo unit j . (For $j \in J_2$, this is taken as the sum of the two associated silo storage capacities).

c_j^s = storage cost per ton of coal at silo j .

q_j^0 = initial amount in storage at silo j .

a_j^0, s_j^0 = ash and sulfur content (%), respectively, in initial storage amount at silo j .

c_{ij}^{s1} = shipping cost (per ton), from mine i to silo j .

c_{ij}^c = cleaning cost (per ton) at silo j for coal from mine i .

$\alpha_{ij} \in (0,1]$ = total weight attenuation factor (output per ton input) at silo $j \in J_2$ for coal from mine i .

$\beta_{ij} \in (0,1]$ = ash content attenuation factor (output per ton input) at silo $j \in J_2$ for coal from mine i .

$\gamma_{ij} \in (0,1]$ = sulfur content attenuation factor (output per ton input) at silo $j \in J_2$ for coal from mine i .

Note: $\alpha_{ij}, \beta_{ij}, \gamma_{ij}$ are assumed to be 1 for $j \in J_1$.

A_1 = flow arcs (i,j) from mine i to silo j ; $F_1^i = \{j : (i,j) \in A_1\}$, $R_1^j = \{i : (i,j) \in A_1\}$.

A_2 = flow arcs (j,k) from silo j to customer k ; $F_2^j = \{k : (j,k) \in A_2\}$, $R_2^k = \{j : (j,k) \in A_2\}$.

A_3 = arcs (j,j') representing by-product flow from Wentz silo $j \in J_{2W}$ to Bullitt silo $j' \in J_{2B}$, corresponding to stoker customer shipments; $F_3^j = \{j' : (j,j') \in A_3\}$, $R_3^{j'} = \{j : (j,j') \in A_3\}$.

c_{jk}^{s2} = shipping cost (per ton), from silo j to customer k .

$c_{jj'}^{s3}$ = shipping cost (per ton), from Wentz silo $j \in J_{2W}$ to Bullitt silo $j' \in J_{2B}$.

d_{kt} = demand placed by customer k during period t .

ASH_{ukt} = upper limit on ash percentage in coal to be delivered to customer k in period t .

SUL_{ukt} = upper limit on sulfur percentage in coal to be delivered to customer k in period t .

cl_k = slope (revenue/ton) of reward function for each % point below the maximum specified limit of ash in coal shipped to customer k .

c_k^1 = slope (cost/ton) of penalty function for each % point above the maximum specified limit of ash in coal shipped to customer k .

c_k^2 = slope (revenue/ton) of reward function for each % point below the maximum specified limit of sulfur in coal shipped to customer k .

3.3.3 Mathematical Formulation

Decision Variables

- $y_{ij\tau}^{k\tau}$ = amount (tons) shipped from mine i to silo j in period t , with continued shipment to customer k in period τ , for $j = 1, \dots, J, i \in R_j^1, k \in F_j^2, t = 1, \dots, T$.
- y_{jk}^0 = amount (tons) in initial storage at silo j , shipped to customer k in period t , for $j = 1, \dots, J, k \in F_j^2, t = 1, 2, 3$.
- $Y_{ij\tau_1\tau_2}^{k\tau}$ = amount (tons) shipped from mine i to silo $j \in J_{2W}$ in time period t_1 , which is then shipped to $j' \in J_{2B}$ in period t_2 , and finally shipped to customer k in period τ , for $(j, j') \in A_3, i \in R_j^1, k \in F_j^2, t_1 = 1, \dots, T, (t_2, \tau) \in t_L(t_1) \equiv \{(t_1 + 1, t_1 + 1), (t_1 + 1, t_1 + 2), (t_1 + 2, t_1 + 2)\}$

Auxiliary Decision Variables

- $x_{i\delta}$ = slack variable equal to the amount (tons) of coal remaining in storage at mine i during period δ , for $i = 1, \dots, m, \delta = 1, \dots, T$.
- $u_{j\delta}$ = accumulated storage amount (tons) in silo unit j during period δ , for $j = 1, \dots, J, \delta = 1, \dots, T$.
- $ASH_{k\tau}$ = % ash content in blended coal delivered to customer k in period τ , for $k = 1, \dots, K, \tau = 1, \dots, T$.

- $(ASH_{k\tau})_1 =$ % ash content in blended coal delivered to customer k in period τ , that is below the maximum specified limit, for $k = 1, \dots, K, \tau = 1, \dots, T$.
- $(ASH_{k\tau})_2 =$ % ash content in blended coal, delivered to customer k in period τ , that exceeds the maximum specified limit, for $k = 1, \dots, K, \tau = 1, \dots, T$.
- $SUL_{k\tau} =$ % sulfur content in blended coal delivered to customer k in period τ , for $k = 1, \dots, K, \tau = 1, \dots, T$.

Objective Function

Minimize

$$\begin{aligned}
& \sum_{j=1}^J \sum_{i \in R_j^1} \sum_{k \in F_j^2} \sum_{t=1}^T \sum_{\tau=t}^{\min(t+2, T)} (c_{ij}^{sh1} + c_{ij}^{cl} + c_{jk}^{sh2}) y_{ijt}^{k\tau} + \\
& \sum_{j \in J_{2W}} \sum_{i \in R_j^1} \sum_{j' \in F_j^3} \sum_{k \in F_j^2} \sum_{t_1=1}^T \sum_{(t_2, \tau) \in t_L(t_1)} (c_{ij}^{sh1} + c_{j'k}^{sh2} + c_{jj'}^{sh3} + c_{ij}^{cl}) Y_{ijt_1j't_2}^{k\tau} + \\
& \sum_{j=1}^J \sum_{k \in F_j^2} \sum_{t=1}^3 c_{jk}^{sh2} y_{jkt}^0 + \sum_{i=1}^m \sum_{t=1}^T c_i^{sm} x_{it}^s + \sum_{j=1}^J \sum_{t=1}^T c_j^{ss} u_{jt}^s + \\
& \sum_{k=1}^K \sum_{\tau=1}^T [-c_{1k}^1 (ASH_{uk\tau} - (ASH_{k\tau})_1) d_{k\tau} + c_{2k}^1 (ASH_{k\tau})_2 d_{k\tau}] + \\
& \sum_{k=1}^K \sum_{\tau=1}^T [-c_{1k}^2 (SUL_{uk\tau} - SUL_{k\tau}) d_{k\tau}]
\end{aligned}$$

Constraint Equations

1. *Flow balance and storage at mines :*

$$\sum_{t \leq \delta} P_{it} - \sum_{j \in F_i^1} \sum_{t \leq \delta} \sum_{k \in F_j^2} \sum_{\tau=t}^{\min(t+2, T)} y_{ijt}^{k\tau} - \sum_{j \in F_i^1 \cap J_{2W}} \sum_{t_1 \leq \delta} \sum_{j' \in F_j^3} \sum_{k \in F_{j'}^2} \sum_{(t_2, \tau) \in t_L(t_1)} Y_{ijt_1 j' t_2}^{k\tau} = x_{i\delta}^s$$

for $i = 1, \dots, m$, and $\delta = 1, \dots, T$

$$0 \leq x_{i\delta}^s \leq SM_i \text{ for } i = 1, \dots, m, \text{ and } \delta = 1, \dots, T$$

2. *Storage constraints for silo units :*

$$\sum_{i \in R_j^1} \sum_{k \in F_j^2} \sum_{\delta=t-2}^t \sum_{\tau=t}^{\min(\delta+2, T)} \frac{(1 + \alpha_{ij})}{2} y_{ij\delta}^{k\tau} + \left(q_j^0 - \sum_{\tau < t} \sum_{k \in F_j^2} y_{jk\tau}^0 \right) + \theta_{jt} = u_{jt}^s$$

$$0 \leq u_{jt}^s \leq SS_j, \text{ for } j = 1, \dots, J, \text{ and } t = 1, \dots, T$$

where,

$$\theta_{jt} = \begin{cases} \sum_{i \in R_j^1} \sum_{j' \in F_j^3} \sum_{k \in F_{j'}^2} \sum_{\delta=t-2}^t \sum_{(t_2, \tau) \in t_L(\delta), t_2 \geq t} \frac{(1 + \alpha_{ij})}{2} Y_{ij\delta j' t_2}^{k\tau} & , j \in J_{2W} \\ \sum_{j' \in R_j^3} \sum_{i \in R_j^1} \sum_{k \in F_j^2} \sum_{(t_2, \tau) \in t_{L_1}(t)} \sum_{t_1=(\tau-2)}^{t_2-1} \frac{(1 + \alpha_{ij'})}{2} Y_{ij' t_1 j t_2}^{k\tau} & , j \in J_{2B} \\ 0 & , \text{ otherwise} \end{cases}$$

and where,

$$t_{L_1}(t) = \{ (t-1, t), (t, t), (t, (t+1)) \}$$

for defined combinations.

3. *Dissipation of initial storage at silos :*

$$\sum_{k \in F_j^2} \sum_{t=1}^3 y_{jkt}^0 = q_j^0 \quad , j = 1, \dots, J$$

4. *Wentz to Bullitt transfer :*

$$\sum_{j' \in F_j^3} \sum_{k \in F_{j'}^2} \sum_{(t_2, \tau) \in t_L(t)} Y_{ij' t_2}^{k\tau} \leq \sum_{k \in F_j^2 \cap K_{st}} \sum_{\tau=t}^{\min(T, t+2)} y_{ijt}^{k\tau} \leq 1.1 \sum_{j' \in F_j^3} \sum_{k \in F_{j'}^2} \sum_{(t_2, \tau) \in t_L(t)} Y_{ij' t_2}^{k\tau}$$

for $j \in J_{2W}$, $i \in R_j^1$, and $t = 1, \dots, T$

5. *Demand constraints for customers :*

$$\sum_{j \in R_k^2} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau} \alpha_{ij} y_{ijt}^{k\tau} + \sum_{j \in R_k^2} y_{jk\tau}^0 +$$

$$\sum_{j' \in R_k^2} \sum_{j \in R_j^3} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau-1} \sum_{t_2=t+1}^{\min(t+2, \tau)} \alpha_{ij} Y_{ijj't_2}^{k\tau} = d_{k\tau}$$

for $k = 1, \dots, K$, and $\tau = 1, \dots, T$

6. *Customer product quality constraints :*

$$ASH_{k\tau} = \frac{\left[\sum_{j \in R_k^2} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau} y_{ijt}^{k\tau} a_{it} \beta_{ij} + \sum_{j \in R_k^2} y_{jk\tau}^0 a_j^0 + \right.}{d_{k\tau}}$$

$$\left. \sum_{j' \in R_k^2} \sum_{j \in R_j^3} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau-1} \sum_{t_2=t+1}^{\min(t+2, \tau)} Y_{ijj't_2}^{k\tau} a_{it} \beta_{ij} \right]$$

for $k = 1, \dots, K$, $\tau = 1, \dots, T$

$$SUL_{k\tau} = \frac{\left[\sum_{j \in R_k^2} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau} y_{ijt}^{k\tau} s_{it} \gamma_{ij} + \sum_{j \in R_k^2} y_{jk\tau}^0 s_j^0 + \right.}{d_{k\tau}}$$

$$\left. \sum_{j' \in R_k^2} \sum_{j \in R_j^3} \sum_{i \in R_j^1} \sum_{t=\tau-2}^{\tau-1} \sum_{t_2=t+1}^{\min(t+2, \tau)} Y_{ijj't_2}^{k\tau} s_{it} \gamma_{ij} \right]$$

for $k = 1, \dots, K$, $\tau = 1, \dots, T$

$$ASH_{kt} = (ASH_{kt})_1 + (ASH_{kt})_2$$

$$0 \leq (ASH_{kt})_1 \leq ASH_{ukt}$$

$$0 \leq (ASH_{kt})_2 \leq \infty$$

for $k = 1, \dots, K$, and $t = 1, \dots, T$

$$0 \leq SUL_{kt} \leq SUL_{ukt}$$

for $k = 1, \dots, K$, and $t = 1, \dots, T$

7. *Nonnegativity constraints :*

$$y_{jt}^{ki} \geq 0, y_{kt}^{ij} \geq 0, \text{ for } i = 1, \dots, m, j = 1, \dots, J, k = 1, \dots, K, t = 1, \dots, T,$$

$$\tau = t, \dots, \min \{T, t + 2\}$$

$$Y_{j_1 j_2}^{k_1 k_2} \geq 0, \text{ for } i = 1, \dots, m, j \in J_{2W}, j' \in J_{2B}, k = 1, \dots, K, t_1 = 1, \dots, T,$$

$$(t_2, \tau) \in t_L(t_1).$$

3.3.4 Usage of the Short-Term Tactical Model

This model will help in making better tactical decisions, such as determining schedules for shipping coal from mines to silos, for cleaning and blending coal at the silos, and for the distribution of coal to the customers over a period of one week in a rolling time horizon fashion. As already described, this model is a modification of 'Model 1', the most accurate of the three models formulated by Sherali and Puri (1991). The introduction of a penalty-reward component into the objective function and the enhancement of the model to accommodate more than one type of cleaning technology are done to represent the new market situation emerging due to the 1990 Clean Air Act. Hence, this model should help the decision maker in making better operational decisions under the 1990 Clean Air Act.

As described in the chapter on Computational Experience, we have generated five test problems in order to examine the performance of the model in providing decisions under different perturbations of system configuration, data, and demand specifications. Note that various sensitivity analysis scenarios such as reducing the number of time periods in the given time horizon, widening the time horizon, varying shipment flow connections, reducing the production rate at mines, and temporarily shutting down a few mines can easily be explored by making appropriate changes in the available specifications.

Chapter IV

Formulation of a Long-Term Strategic Model

The first section of this chapter gives a description of the overall long-term problem and its relation with the 1990 Clean Air Act. The second section then develops a long-term strategic model to address this problem. It presents a verbal statement of the problem, the required data specifications, details about the mathematical formulation of the long-term strategic model, and its usage.

4.1 Problem Statement

The 1990 Clean Air Act mandates a stringent time table for utilities to achieve an accumulated decline in sulfur dioxide and nitrous oxide emissions over time. This burden has been transferred to the coal industry in terms of enhanced quality requirements on the sulfur and ash content of the coal produced. To meet this sudden change in quality characteristics of future coal demand, coal companies need to re-examine their resources and decide on which production units to develop at each potential mine, and at what point in time, and also to decide which existing mines need to be shut down.

The development of a new mine involves a significant fixed cost in terms of constructing access roads and preparing a site for production. A potential mine can have more than one production unit, and there is a pre-specified sequence for opening production units at any potential mine. An upper limit is also fixed on the total number of units that can be opened in a given time period over all the potential mines. The cost of developing a production unit at a potential mine is some percentage of the overall cost of developing all the production units at that particular potential mine. This percentage is dependent on the size of the unit and its place in the sequence of units to be developed. For example, if there are three units at a potential mine, then normally, the percentage of the overall cost for developing the first unit in the sequence would be more than the second, and the percentage of the overall cost for developing the second unit in the sequence would be more than the third. This also accounts for present value computations. Usually most of the common excess roads and other facilities required at the start of opening a potential mine are constructed while developing the first few units in the sequence, and therefore, units at the end of the sequence can be developed by incurring a very small percentage of the overall cost. This acts as an incentive to develop all the units in a mine, if some of them are already developed.

The coal produced at different mines has quality specifications which change from mine to mine as well as over time at each mine. Also, to achieve a long-term uniform usage of available reserves, a penalty function for underutilization at a mine is placed in the objective function. Hence, whenever the unused slack at any existing mine at a particular time period approaches the production at that mine in the given period, this indicates that it is time to cease production at that mine. Note that most of the existing mines have just one production unit, and so, to simplify the model, we treat every existing mine as a single unit.

The problem is to find out which production units of potential mines should be opened and at what time periods, and to determine optimal schedules for shipping coal from mines to silos, for cleaning and blending coal at the silos, and for the distribution of coal to the customers, subject to various constraints. These constraints include production capacity, silo capacity, material flow balance, demand satisfaction for customers, and quality requirement restrictions, along with restrictions on

the sequence in which units can be opened, and on the maximum number of units that can be opened in any time period, without any substantial surcharge penalties.

4.2 Model Formulation

4.2.1 Verbal Statement

Given below is a verbal statement of the model, followed by its data specification and mathematical formulation. This long-term strategic model has the structure of a fixed-charge, mixed-integer zero-one programming problem.

Objective Function :

Minimize total cost = (shipping and cleaning costs for coal from existing mines) + (shipping and cleaning costs for coal from potential mines) + (annualized fixed-charge to open a production unit at a potential mine) + (penalty for storage/underutilization at mines) + (penalty for opening additional units of mines beyond the specified limit) - (reward for better ash and sulfur quality shipped)

Subject to the Constraints :

1. Flow balance at existing mines and at individual units of potential mines:

(Prescribed production at the mine if active in period t) - (Total output in period t) = (Unused slack in period t)

2. Capacity constraints for silo facilities :

(Total output from the silo in period t) \leq (Capacity of the silo)

3. Wentz to Bullitt transfer balance with stoker customer shipments :

(Total amount transferred from the Wentz to the Bullitt silo facility in period t) \leq Shipment to stoker customer in the same period ≤ 1.1 (Total amount transferred from the Wentz to the Bullitt silo facility in period t)

4. Demand satisfaction constraints for customers :

(Total amount to be shipped to customer in period t) = (Total demand placed in period t)

5. Customer product quality constraints, controlling the ash and sulfur contents in customer shipments, for each period t .
6. Constraints restricting any unit at each potential mine to be opened only once.
7. Constraints that place an upper limit on the total number of units of potential mines that can be opened in period t without any surcharge penalty, and account for units that are opened beyond this limit at an added surcharge penalty incurred in the objective function.
8. Constraints sequencing units at a potential mine.
9. Constraints added to obtain a tighter linear programming relaxation of the problem.
10. Nonnegativity constraints on the decision variables.

4.2.2 Data Specification

The model requires specific data pertaining to the existing and potential mines, silos, and customers. This model considers periods equal to six months in duration, with a horizon of three to five years.

As this is a long-term model, all costs such as cleaning and shipping costs, revenues for shipping better quality coal, and penalties for storage and underutilization at mines, are present values using a certain rate of return. The subscript t attached to these cost factors reflects this representation.

The fixed-charge cost to open a production unit at a potential mine is the present value of the semi-annualized payments to be made over the horizon. For example, if a unit has a life equal to the duration of the horizon used in the model, then the fixed-charge is simply the first cost incurred initially. On the other hand, if the life of a unit exceeds the duration of the horizon in the model, then the equal-payment-series capital-recovery formula (Thuesen, Fabrycky and Thuesen, 1977), is used with the effective semi-annual interest rate and the number of interest periods in the life of that unit, to find semi-annualized payments over its life. The present value of these semi-annualized payments corresponding to the periods in the horizon of the model yield the net fixed-charge cost.

Real data from the Westmoreland Coal Company and nine other similar, hypothetical data sets have been used for making the model runs. Given below are the data requirements, along with the notation used.

$t = 1, \dots, T \equiv \#$ of time periods (≤ 8).

$i = 1, \dots, I \equiv \#$ of existing and potential mines (≤ 23).

$q = 1, \dots, h_i \equiv \#$ of units at a potential mine i (≤ 30).

$j = 1, \dots, J \equiv \#$ of silo units (≤ 10).

$k = 1, \dots, K \equiv \#$ of customers (≤ 12).

K_{st} = "stoker customers" served by Wentz silos.

p_{it} = production (tons) at existing mine i , in period t .

$p_{iq}(t)$ = a function which represents production (tons) at unit q of potential mine i , in period t of its life. (Note that t denotes the period of its life after this unit has been opened, and not the period of the model.)

a_{it} = ash content (%), in coal produced at existing mine i , in period t .

s_{it} = sulfur content (%), in coal produced at existing mine i , in period t .

a_{iq_t} = ash content (%), in coal produced at unit q of potential mine i , in period t . (Note that t denotes the period of horizon, and not the period of unit's life after it has been opened.)

s_{iq_t} = sulfur content (%), in coal produced at unit q of potential mine i , in period t . (The same comment applies here as for a_{iq_t})

$I_1 \equiv$ existing mines.

$I_2 \equiv$ potential mines.

$J_1 \equiv$ ROM silo storage units.

$J_{2W} \equiv$ cleaned silo units at the Wentz facility.

$J_{2B} \equiv$ cleaned silo units at the Bullitt facility.

$J_2 \equiv$ cleaned silo units ($J_{2W} \cup J_{2B}$).

SC_{jt} = storage capacity of silo unit j in period t . (For $j \in J_2$, this is taken as the sum of the two associated silo storage capacities).

c_{ij}^{sh1} = present value of shipping cost (per ton), from existing mine i to silo j in period t .

c_{iqj}^{sh1} = present value of shipping cost (per ton), from a unit of q of potential mine i to silo j in period t .

c_{jt}^{cl} = present value of cleaning cost (per ton) at silo j for coal from existing mine i in period t .

c_{iqjt}^{cl} = present value of cleaning cost (per ton) at silo j for coal from a unit q of a potential mine i in period t .

$\alpha_{ij} \in (0,1]$ = total weight attenuation factor (output per ton input) at silo $j \in J_2$ for coal from existing mine i .

$\beta_{ij} \in (0,1]$ = ash content attenuation factor at silo $j \in J_2$ for coal from existing mine i .

$\gamma_{ij} \in (0,1]$ = sulfur content attenuation factor at silo $j \in J_2$ for coal from existing mine i .

Note: α_{ij} , α_{iqj} , β_{ij} , β_{iqj} , γ_{ij} , γ_{iqj} are assumed to be 1 for $j \in J_1$.

A_1 = flow arcs (i,j) from mine i to silo j ; $F_1^i = \{j : (i,j) \in A_1\}$, $R_1^j = \{i : (i,j) \in A_1\}$.

A_2 = flow arcs (j,k) from silo j to customer k ; $F_2^j = \{k : (j,k) \in A_2\}$, $R_2^k = \{j : (j,k) \in A_2\}$.

A_3 = arcs (j,j') representing by-product flow from Wentz silo $j \in J_{2W}$ to Bullitt silo $j' \in J_{2B}$, corresponding to stoker customer shipments; $F_3^j = \{j' : (j,j') \in A_3\}$, $R_3^{j'} = \{j : (j',j) \in A_3\}$.

c_{jk}^{sh2} = present value of shipping cost (per ton), from silo j to customer k in period t .

$c_{ij't}^{3j}$ = present value of shipping cost (per ton), from Wentz silo $j \in J_{2W}$ to Bullitt silo $j' \in J_{2B}$ in period t .

d_{kt} = demand placed by customer k during period t .

ASH_{ukt} = upper limit on ash percentage in coal to be delivered to customer k in period t .

SUL_{ukt} = upper limit on sulfur percentage in coal to be delivered to customer k in period t .

R_{kt} = present value of additional revenue earned per percentage point below the maximum specification of ash content, in coal delivered to customer k in period t .

$f_{iq't}$ = fixed charge (equivalent present value as described above) to open a unit q at a potential mine i in period t .

f_t^{\max} = maximum $\{f_{iq't} : q = 1, \dots, h_i, i \in I_2\}$ for all $t = 1, \dots, T$.

PE_{it} = present value of penalty per ton of coal unused at existing mine i in period t .

$PE_{iq't}$ = present value of penalty per ton of coal unused at unit q of potential mine i in period t .

U_i = upper limit on the # of units that can be opened at a potential mine i in period t .

4.2.3 Mathematical Formulation

A detailed mathematical formulation of the long-term strategic model is given next, using the above notations.

Decision Variables

- $y_{ijk't}$ = amount (tons) shipped from existing mine i through silo j to customer k in time period t , for $j = 1, \dots, J$, $i \in R_j^1 \cap I_1$, $k \in F_j^2$, $t = 1, \dots, T$.
- $Y_{ij'k't}$ = stoker customer by-product amount (tons) shipped from existing mine i to silo $j \in J_{2W}$, which is then shipped to silo $j' \in J_{2B}$, and then to customer k in period t , for $(j, j') \in A_3$, $i \in R_j^1 \cap I_1$, $t = 1, \dots, T$, $k \in F_{j'}^2$.

- w_{ijk_t} = amount (tons) shipped from unit q of potential mine i through silo j to customer k in time period t , for $j = 1, \dots, J$, $i \in R^j \cap I_2$, $q = 1, \dots, h_i$, $k \in F_j^2$, $t = 1, \dots, T$.
- $W_{iaj'kt}$ = stoker customer by-product amount (tons) shipped from unit q of potential mine i to silo $j \in J_{2W}$, which is then shipped to silo $j' \in J_{2B}$, and then to customer k in period t , for $(j, j') \in A_3$, $i \in R^j \cap I_2$, $t = 1, \dots, T$, $k \in F_{j'}^2$
- x_{iq_t} = a binary variable that takes on a value of one if unit q of a potential mine $i \in I_2$ is opened or initiated in period t , and is zero otherwise.

Auxiliary Decision Variables

- z_{it} = slack variable equal to the amount (tons) of coal produced at existing mine i during period t that remains unused in that period, for $i \in I_1$, $t = 1, \dots, T$.
- z_{iq_t} = slack variable equal to the amount (tons) of coal produced at a unit q of potential mine i during period t that remains unused in that period, for $i \in I_2$, $q \in h_i$, $t = 1, \dots, T$.
- ASH_{kt} = % ash content in blended coal delivered to customer k in period t , for $k = 1, \dots, K$, $t = 1, \dots, T$.
- SUL_{kt} = % sulfur content in blended coal delivered to customer k in period t , for $k = 1, \dots, K$, $t = 1, \dots, T$.
- θ_{1t} = number of mines opened within the set limit U_t in period t , for $t = 1, \dots, T$.
- θ_{2t} = number of mines opened beyond the set limit U_t in period t , for $t = 1, \dots, T$.

Objective Function

Minimize

$$\begin{aligned}
& \sum_{j=1}^J \sum_{i \in R_j^1 \cap I_1} \sum_{k \in F_j^2} \sum_{t=1}^T (c_{ijt}^{sh1} + c_{ijt}^{cl} + c_{jkt}^{sh2}) y_{ijkt} + \\
& \sum_{j \in J_{2W}} \sum_{i \in R_j^1 \cap I_1} \sum_{j' \in F_j^3} \sum_{k \in F_j^2} \sum_{t=1}^T (c_{ijt}^{sh1} + c_{j'kt}^{sh2} + c_{jj't}^{sh3} + c_{ijt}^{cl}) Y_{ij'kt} + \\
& \sum_{j=1}^J \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{k \in F_j^2} \sum_{t=1}^T (c_{iajt}^{sh1} + c_{iajt}^{cl} + c_{jkt}^{sh2}) w_{iajkt} + \\
& \sum_{j \in J_{2W}} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{j' \in F_j^3} \sum_{k \in F_j^2} \sum_{t=1}^T (c_{iajt}^{sh1} + c_{j'kt}^{sh2} + c_{jj't}^{sh3} + c_{iajt}^{cl}) W_{iajj'kt} + \\
& \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{t=1}^T f_{iqt} x_{iqt} + \left(\sum_{i \in R_j^1 \cap I_1} \sum_{t=1}^T PE_{it} z_{it} + \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{t=1}^T PE_{iqt} z_{iqt} \right) + \\
& \sum_{t=1}^T 0.2 f_t^{\max} \theta_{2t} - \sum_{k=1}^K \sum_{t=1}^T (ASH_{ukt} - ASH_{kt}) d_{kt} R_{kt}
\end{aligned}$$

Constraint Equations

1. Flow balance at mines :

$$P_{it} - \left[\sum_{j \in F_i^1} \sum_{k \in F_j^2} y_{ijkt} + \sum_{j \in F_i^1} \sum_{j' \in F_j^3} \sum_{k \in F_j^2} Y_{ij'kt} \right] = z_{it}$$

for $i \in I_1$, and $t = 1, \dots, T$

$$\sum_{t \leq \delta} p_{iq}(\delta - t + 1) x_{iqt} - \left[\sum_{j \in F_i^1} \sum_{k \in F_j^2} w_{iajkt} + \sum_{j \in F_i^1} \sum_{j' \in F_j^3} \sum_{k \in F_j^2} W_{iaj'kt} \right] = z_{iq\delta}$$

for $i \in I_2$, $q = 1, \dots, h_i$ and $\delta = 1, \dots, T$

2. *Capacity constraints on silos :*

$$\sum_{i \in R_j^1 \cap I_1} \sum_{k \in F_j^2} \frac{(1 + \alpha_{ij})}{2} y_{ijkt} + \sum_{i \in R_j^1 \cap I_1} \sum_{j' \in J_{2W}} \sum_{k \in F_j^2} \frac{(1 + \alpha_{ij})}{2} Y_{ij'jkt} +$$

$$\sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{k \in F_j^2} \frac{(1 + \alpha_{iaq})}{2} w_{iajkt} + \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{j' \in J_{2W}} \sum_{k \in F_j^2} \frac{(1 + \alpha_{iaq})}{2} W_{iaj'jkt} \leq SC_{jt}$$

for $j \in (J_{2B} \cup J_{2W}) \cap J_2$, and $t = 1, \dots, T$

$$\sum_{i \in R_j^1 \cap I_1} \sum_{k \in F_j^2} y_{ijkt} + \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{k \in F_j^2} w_{iajkt} \leq SC_{jt}$$

for $j \in J_1$, and $t = 1, \dots, T$

3. *Wentz to Bullitt transfer :*

$$\sum_{j' \in F_j^3} \sum_{k \in F_j^2} Y_{ijj'kt} \leq \sum_{k \in F_j^2 \cap K_{st}} y_{ijkt} \leq 1.1 \sum_{j' \in F_j^3} \sum_{k \in F_j^2} Y_{ijj'kt}$$

for $j \in J_{2W} \cap F_1^1$, $i \in R_j^1 \cap I_1$, and $t = 1, \dots, T$

$$\sum_{j' \in F_j^3} \sum_{k \in F_j^2} W_{iaj'jkt} \leq \sum_{k \in F_j^2 \cap K_{st}} w_{iajkt} \leq 1.1 \sum_{j' \in F_j^3} \sum_{k \in F_j^2} W_{iaj'jkt}$$

for $j \in J_{2W} \cap F_1^1$, $i \in R_j^1 \cap I_2$, $q = 1, \dots, h_i$ and $t = 1, \dots, T$

4. *Demand constraints for customers :*

$$\sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_1} \alpha_{ij} y_{ijkt} + \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_1} \sum_{j' \in F_j^3} \alpha_{ij} Y_{ijj'kt} + \sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \alpha_{iaq} w_{iajkt}$$

$$+ \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{j' \in F_j^3} \alpha_{iaq} W_{iaj'jkt} = d_{kt}$$

for $k = 1, \dots, K$, and $t = 1, \dots, T$

5. *Customer product quality constraints :*

$$ASH_{kt} = \frac{\left[\sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_1} y_{ijkt} a_{it} \beta_{ij} + \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_1} \sum_{j' \in F_j^3} Y_{ij'kt} a_{it} \beta_{ij} + \sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} w_{iajkt} a_{iqt} \beta_{iaj} + \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{j' \in F_j^3} W_{iaj'kt} a_{iqt} \beta_{iaj} \right]}{d_{kt}}$$

for $k = 1, \dots, K, t = 1, \dots, T$

$$SUL_{kt} = \frac{\left[\sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_1} y_{ijkt} s_{it} \gamma_{ij} + \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_1} \sum_{j' \in F_j^3} Y_{ij'kt} s_{it} \gamma_{ij} + \sum_{j \in R_k^2} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} w_{iajkt} s_{iqt} \gamma_{iaj} + \sum_{j \in J_{2W} \cap R_j^3} \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{j' \in F_j^3} W_{iaj'kt} s_{iqt} \gamma_{iaj} \right]}{d_{kt}}$$

for $k = 1, \dots, K, t = 1, \dots, T$

$$0 \leq ASH_{kt} \leq ASH_{ukt} \text{ for } k = 1, \dots, K, \text{ and } t = 1, \dots, T$$

$$0 \leq SUL_{kt} \leq SUL_{ukt} \text{ for } k = 1, \dots, K, \text{ and } t = 1, \dots, T$$

6. *Restricting a unit of mine to be opened only once :*

$$\sum_{t=1}^T x_{iqt} \leq 1$$

for $i = 1, \dots, I_2, q = 1, \dots, h_i$

7. *Upper limit on the number of units that can be opened in a time period :*

$$\sum_{i \in I_2} \sum_{q=1}^{h_i} x_{iqt} \leq \theta_{1t} + \theta_{2t}$$

for $t = 1, \dots, T$

$$0 \leq \theta_{1t} \leq U, \theta_{2t} \geq 0, \text{ for } t = 1, \dots, T$$

8. *Sequencing of units at a potential mine (with some implied constraints added to derive a tighter relaxation)*

$$x_{iqt} \leq \sum_{\delta \leq t} x_{iq'\delta}$$

for $i \in I_2, q' = 1, \dots, q-1, q = 2, \dots, h_i, t = 1, \dots, T$

9. *Disaggregated constraints (further restrictions added to derive a tighter relaxation)*

$$\sum_{j \in F_i^1} w_{ijjk\delta} + \sum_{j \in F_i^1 \cap J_{2W}} \sum_{j' \in F_j^3} W_{ijj'k\delta} \leq d_{k\delta} \sum_{t \leq \delta} x_{iqt}$$

for $i \in I_2, q = 1, \dots, h_i, k = 1, \dots, K, \delta = 1, \dots, T$

10. *Nonnegativity and binary constraints :*

$$y_{ijkt} \geq 0, \text{ for } i \in I_1, j = 1, \dots, J, k = 1, \dots, K, t = 1, \dots, T$$

$$y_{iqjkt} \geq 0, \text{ for } i \in I_2, q = 1, \dots, h_i, j = 1, \dots, J, k = 1, \dots, K, t = 1, \dots, T$$

$$Y_{ijj'kt} \geq 0, \text{ for } i \in I_1, j \in J_{2W}, j' \in J_{2B}, k = 1, \dots, K, t = 1, \dots, T$$

$$Y_{iqij'kt} \geq 0, \text{ for } i \in I_2, q = 1, \dots, h_i, j \in J_{2W}, j' \in J_{2B}, k = 1, \dots, K, t = 1, \dots, T$$

$$z_{it} \geq 0, \text{ for } i \in I_1, t = 1, \dots, T$$

$$z_{iqt} \geq 0, \text{ for } i \in I_2, q = 1, \dots, h_i, t = 1, \dots, T$$

$$x_{iqt} = 0 \text{ or } 1, \text{ for } i \in I_2, q = 1, \dots, h_i, t = 1, \dots, T$$

4.2.4 Comments on the formulation

Given below is an explanation of some of the finer points related to the objective function and the constraints.

As this is a long-term model, all the cost coefficients in the objective function are present values. A fixed charge cost is incurred whenever a unit at a potential mine is opened. Shipment of better ash quality coal is rewarded, and storage and underutilization at a mine is discouraged by accommodating a penalty term in the objective function. Also, in any time period t , if the number of units opened exceeds the specified upper limit on the number of units that can be opened given the available budgetary resources, then a surcharge equal to 20 % of the maximum fixed-cost over the

different units of the potential mines is applied to the number of units that exceed the stated limit. This surcharge reflects the burden for acquiring additional capital for developing units beyond what the available budget permits.

In the flow balance constraints (1) for the mines, in contrast with the same type of constraints for the short-term model, we do not consider any storage restrictions. Rather, we require that all quantity produced within this six month period should account for flow within this duration, without any carryover of inventory. If the flow falls far shorter than the production, then this mine will be a candidate for a shutdown. (Note that in the tactical model, an operating mine is assumed to maintain a production schedule, and so, we need to actively account for shipment and inventory of coal from day to day.) Also, in the flow balance constraint for a potential new unit of a mine, the production rate function at that unit is multiplied by a binary variable that takes on a value of one when this unit is opened and is zero otherwise, such that the constraint reflects the appropriate rate of production at a unit according to its age in the given period.

As each J_2 type of silo constitutes a pair of silos, one of which receives the run-of-mine coal, while the other stores the cleaned coal following the beneficiation process, we have used the coefficient $\frac{(1 + \alpha_{ij})}{2}$ with the flow variables in the capacity constraints (2) for the J_2 type of silos. Furthermore, the normal storage or handling capacity of the silo is multiplied by the number of working days in a time period to derive the parameter SC_{jr} .

The Wentz to Bullitt transfer constraints (3) are interval constraints, and they reflect a transfer of coal from the Wentz silos to the Bullitt silos as actually practiced by the Westmoreland Coal Company. These constraints can easily be generalized for any coal company, and can be removed from the formulation if no such type of transfer is practiced in a particular company.

In the demand constraints (4) for customers, the flow variables are multiplied by coefficients α_{ij} to take into account the total weight attenuation during the beneficiation process at silos.

In the customer product quality constraints (5), we have simply used coefficients a_{iq_t} and s_{iq_t} instead of using $\sum_{\delta \leq t} a_{iq}(t - \delta + 1) x_{iq\delta}$ and $\sum_{\delta \leq t} s_{iq}(t - \delta + 1) x_{iq\delta}$ respectively, as it would lead to nonlinear constraints that would have to be linearized. (These latter functions are of the same nature as the production rate function used in the flow balance constraints for a unit of a given potential mine.) However, since variations in the ash/sulfur content over the horizon is not too significant, we can approximate these nonlinear terms by a_{iq_t} and s_{iq_t} , which in effect assumes that the ash/sulfur content fraction varies as if the unit was opened at time period 1.

In the constraints (6), every unit of a given potential mine is restricted to be opened only once. Each constraint (7) has a right-hand side equal to the sum of θ_{1t} and θ_{2t} , where θ_{1t} is bounded from above by the maximum number of units of all the potential mines that can be opened in the time period t . This upper limit depends on the availability of resources such as capital, equipment, and manpower. The variable θ_{2t} is simply nonnegatively restricted, but is penalized as described in the comment on the objective function.

The constraints (8) enforce the sequencing of units at a potential mine. Here, it is sufficient to use only the constraints having $q' = q - 1$ in order to obtain a valid representation. However, additional constraints have been added for $1 \leq q' \leq q - 1$ as well, in order to obtain a tighter relaxation. Given below is an example to illustrate this.

Taking $i = 1$, $q = 1,2,3$ and $t = 2$, we would obtain the following constraints when $q' = q - 1$:

$$x_{122} \leq x_{111} + x_{112} \text{ and } x_{132} \leq x_{121} + x_{122}$$

However, if we let $1 \leq q' \leq q - 1$, we would include an additional constraint :

$$x_{132} \leq x_{111} + x_{112}$$

Although this is implied in the integer sense, it is not implied in the continuous sense, and hence it assists in obtaining a tighter relaxation.

The disaggregated constraints (9) are included in the formulation to further tighten the continuous relaxation. In the spirit of the standard disaggregated fixed-charge problem formulation (see Nemhauser and Wolsey, 1988), these constraints could be written as

$$\sum_{j \in F_i^1} w_{iqjk\delta} + \sum_{j \in F_i^1 \cap J_{2W}} \sum_{j' \in F_j^3} W_{iqj'k\delta} \leq \min \left[\sum_{t \leq \delta} p_{iq}(\delta - t + 1) x_{iqt}, d_{k\delta} \sum_{t \leq \delta} x_{iqt} \right]$$

for $i \in I_2, q = 1, \dots, h_i, k = 1, \dots, K, \delta = 1, \dots, T$.

However, as we already have flow balance constraints of the form

$$\sum_{j \in F_i^1} w_{iqjk\delta} + \sum_{j \in F_i^1 \cap J_{2W}} \sum_{j' \in F_j^3} W_{iqj'k\delta} \leq \sum_{t \leq \delta} p_{iq}(\delta - t + 1) x_{iqt}$$

for $i \in I_2, q = 1, \dots, h_i, k = 1, \dots, K, \delta = 1, \dots, T$, we have simply added the constraints

$$\sum_{j \in F_i^1} w_{iqjk\delta} + \sum_{j \in F_i^1 \cap J_{2W}} \sum_{j' \in F_j^3} W_{iqj'k\delta} \leq d_{k\delta} \sum_{t \leq \delta} x_{iqt}$$

for $i \in I_2, q = 1, \dots, h_i, k = 1, \dots, K, \delta = 1, \dots, T$ in our formulation.

4.2.5 Usage of the Long-Term Strategic Model

As the name of the model suggests, this model can be useful in making strategic decisions concerning the planning of future growth and in making capital investment decisions over a three to five year horizon. To satisfy the demand of high quality coal for customers starting January 1, 1995, when phase I of the 1990 Clean Air Act will commence, the coal companies in Virginia would like to utilize the existing and potential resources in the best possible way. The companies would like to continue using existing mines, but also, they need to start anticipating the opening of suitable production units of potential mines that have low sulfur and ash content. This model will help in deciding the closure of existing mines by identifying those mines for which the amount of coal un-

used is close to its production capacity. As the model has a horizon of three to five years, it can also help in *a priori* adjusting the production rate, if possible, of existing mines and that of the different units of potential mines, in order to reduce the amount of unused coal.

As described in the chapter on Computational Experience, we have generated ten test problems for the purpose of testing the model and the solution procedures. Various what-if analyses can be performed in order to get more insight into the problem. The following is a list of various scenarios for performing what-if analyses.

- What is the effect of increasing or decreasing the length of the time period on the solution?
- How does increasing the number of time periods, i.e., widening the horizon of the model affect the solution?
- What is the effect on the number of units opened by changing the quality requirements of coal drastically?
- How does the slope of the various reward or penalty functions for shipping coal of better or worse quality relative to customer specifications, respectively, affect the opening of new units?
- What is the effect of variations in demand according to different scenarios on the solution?
- How do different mine-silo-customer connections affect the solution?
- What is the effect of recession on the solution?
- How does the inclusion of a new technology for cleaning coal affect the opening of new units?

These scenarios may be easily explored by making suitable changes in the data and would help the decision maker in making better and more foresighted decisions under various eventualities.

Chapter V

Solution Procedures

The first section of this chapter presents a solution procedure for the short-term tactical model. It is followed by a section on methodologies for solving the long-term strategic model, using both exact and heuristic techniques aimed at deriving near optimal solutions.

5.1 Solution Procedure for the Short-Term Tactical Model

The short-term tactical model is a linear programming problem. We coded this model in GAMS (General Algebraic Modeling System; see Brook, Kendrick and Meeraus, 1988), a software available on the VM mainframe in a CMS environment. Besides its convenience in usage, GAMS has the following additional merits.

- It is comprehensible to users of the model from other disciplines.
- It provides a system structure and programming language that maintains the conciseness of expression, generality, and portability.

- Links to different state of the art linear solvers such as MINOS, OSL, CPLEX, are available.
- Links to different state of the art mixed-integer solvers such as ZOOM, MPSX, OSL and CPLEX are available.

The five generated test problems are solved using MINOS 5.2, which is a default solver in GAMS for linear programming problems.

5.2 Solution Procedures for the Long-Term Tactical Model

5.2.1 Solving the Model Using Commercial Softwares

Unlike the short-term model, the long-term model poses added difficulties, being a mixed-integer 0-1 programming problem. We first tried solving the various test problems generated using ZOOM, which is the default solver in GAMS. However ZOOM could only find an optimal solution for two out of ten test problems. Hence, we had to resort to more sophisticated, state-of-the-art mixed-integer solvers, namely, OSL and CPLEX (MIP option). Different links to connect these softwares with GAMS have been developed by the GAMS Development Corporation. After coding the problem in GAMS, these solvers can be called to execute the solution process, provided of course, they have been installed within the system. The optimality tolerance while solving these test problems was kept at 10 %.

5.2.2 Heuristic Solution Procedures

When the number of binary variables that represent the initialization of the production units of different potential mines at different time periods is very large in number, it is advisable to derive a near optimal solution using a suitable heuristic instead of insisting on an optimal solution. We

tried to derive approximate solutions for our test problems using the Pivot and Complement Heuristic of Balas and Martin (1980), but it failed to solve all but the smallest of our test problems. Hence, we designed our own heuristic procedures, exploiting the nature of our problem. (Some of the ideas behind these procedures are portable to other 0-1 mixed-integer programming contexts as well.)

Heuristic I

In this heuristic, we replace the objective function by

$$\text{Maximize} \quad \sum_{i \in R_j^1 \cap I_2} \sum_{q=1}^{h_i} \sum_{t=1}^T (x_{iqt} - 1/2)^2 \quad (11)$$

and incorporate the following additional constraint in the problem

$$z \leq v_{LP}(1 + \lambda) \quad (12)$$

where z represents the objective function of the strategic model, v_{LP} is the value of its linear programming relaxation, and 100λ is a specified % deviation permitted from v_{LP} .

This problem is then solved as a linearly constrained nonlinear programming problem using MINOS 5.2. As the form of the objective function indicates, a solution to this problem encourages all the binary variables to take a value of either 0 or 1, in order to maximize the objective function, while satisfying the problem constraints, including the additional constraint (12). In fact, a solution is integer feasible to the original strategic model and has an objective value within the specified tolerance $v_{LP}(1 + \lambda)$ if and only if it solves the above problem. However, this problem is non-convex, and so MINOS might get stuck at a nonoptimal local maximum, which is not integer valued. To extract the heuristic from this situation, we used the strategy embodied in the following statement of this heuristic procedure.

Step 1 : Solve the problem with (11) as the objective function and with the additional constraint (12) using $\lambda = 0.0$ as described above. If the solution obtained has all the x -variables at binary values, then stop; an alternative optimal solution to the LP relaxation has been obtained which solves the original problem. Otherwise, put INDICATOR = 0 and go to Step 2.

Step 2 : Using the basis of the previous problem solved as an advanced basis, increase the value of λ by 0.01 and re-solve the problem. If the solution obtained has all the x -variables at binary values, then stop. Otherwise, continue this process by incrementing λ according to the sequence $\{0.04, 0.05, 0.08, 0.09, 0.12, 0.13\}$ until we either obtain all x -variables at binary values for some λ , or λ reaches a value of 0.13 with some x -variables still fractional. In the first case stop, and in the latter case, proceed to Step 3. If the number of x -variables that are fractional represent just one unit of a potential mine at different time periods then go to Step 5.

Step 3 : Select a solution that has the highest number of x -variables at binary values from the solutions obtained for the various λ values in Step 2. If there is a tie, then select the solution obtained using the smallest λ value from such ties. Fix all the x -variables that are equal to 1 at optimality at this value. If for mine i , x_{iq_t} is fixed at 1 here, then by (8), for each $q' < q$, we must have $x_{iq'\delta} = 1$ for some $\delta \leq t$. If this is not the case, then it must be that the fractional values of $x_{iq'\delta}$ over $\delta \leq t$ sum to 1. Pick the smallest $\delta' \leq t$ for which $x_{iq'\delta'}$ is fractional and put $x_{iq'\delta'} = 1$. (The smallest time period is chosen to ensure feasibility of supply.) If INDICATOR = 1, go to Step 5. Otherwise, proceed to Step 4.

Step 4 : Solve the problem defined at Step 1 with some of the x -variables fixed according to Step 3, using $\lambda = 0.0$ and using the new value of the linear programming relaxation obtained after fixing the variables at Step 3. If the solution obtained has all the x -variables at binary values then stop; otherwise, put INDICATOR = 1 and return to Step 2.

Step 5 : Enter Heuristic II at its Step 3.

Heuristic II

This heuristic procedure employs a sequential rounding scheme based on a series of continuous linear programming relaxations, exploiting the structure of the problem in determining which variables to round up to 1 at each step of the process. The following are the steps involved in this heuristic procedure :

Step 1 : Solve the linear programming relaxation of the problem. If the solution obtained has all the x -variables at binary values, then stop; the optimal solution obtained for the LP relaxation solves the mixed-integer programming problem. Otherwise, go to Step 2.

Step 2 : Solve the problem with (11) as the objective function, along with the additional constraint (12) using $\lambda = 0.0$ as described in Heuristic I. If the solution obtained has all the x -variables at binary values, then stop; an alternative optimal solution to the LP relaxation has been obtained which solves the original problem. Otherwise, go to Step 3.

Step 3 : Define $F = \{ \text{a set of mines } i \text{ for which some unit } q \text{ has a fractional variable } x_{iq_t} \text{ for some period } t \}$. For each $i \in F$, let $q(i)$ be the *imminent unit* defined as the smallest index q for which x_{iq_t} is fractional for some t . Note that by the constraints of the problem, and the definition of $q(i)$, we must have $x_{iq_\delta} = 1$ for each $q < q(i)$, for some $\delta \leq t$, where t is the smallest time period for which the x -variable for $q(i)$ is fractional. Now, for each $i \in F$, let $t_{i1} < t_{i2} < \dots < t_{in_i}$ be the time periods t for which the variables $x_{iq(i)t}$ are fractional, and let v_{i1}, \dots, v_{in_i} be the fractional values of $x_{iq(i)t_{ik}}$, $k = 1, \dots, n_i$. Accordingly, compute

$$T_i = \sum_{k=1}^{n_i} v_{ik} t_{ik} + T \left(1 - \sum_{k=1}^{n_i} v_{ik} \right) \quad \text{for each } i \in F$$

where T is number of time periods in the horizon. Note that the smaller the value of T_i , the relatively earlier is the tendency for the imminent unit of mine i to be opened. Hence, find

$$T^* = \underset{i \in F}{\text{minimum}}(T_i) \quad (13)$$

Instead of selecting a mine that is determined simply by (13) as the one for which the imminent unit should be opened, we examine a band of T_i values by finding

$$\bar{F} = \{i \in F: T_i \leq T^* + 1\}$$

For mines within this band, identify a mine r according to

$$r \in \underset{i \in \bar{F}}{\text{argmax}} \{v_{ik(t)}\}, \quad \text{where } v_{ik(t)} = \underset{k=1, \dots, n_i}{\text{maximum}} \{v_{ik}\} \text{ for } i \in \bar{F}$$

If the fractional x -variables represent only one unit of a potential mine at different time periods, then select $k(r) = 1$, for fixing $x_{rq(r)tk(r)} = 1$.

Step 4 : Fix $x_{rq(r)tk(r)} = 1$, along with all the other binary variables which turned out be 1 in the most recent linear programming relaxation solved. Re-solve the linear programming relaxation after fixing the above binary variables. If the problem is infeasible, go to Step 5. If the solution obtained has all the x -variables at binary values, then stop; the optimal solution obtained is prescribed as a heuristic solution. If the solution obtained still has fractional values for some of the x -variables then return to Step 3.

Step 5 : Replace $k(r)$ by $k(r) - 1$. (Note that by the structure of the problem and the nature of the solution to the last feasible linear programming relaxation, it must be that the revised $k(r) \geq 1$.) Return to Step 4.

Chapter VI

Computational Experience

The first section of this chapter presents our computational experience in solving the short-term tactical problem. It is followed by a section presenting our computational experience in solving the long-term strategic problem. Experience with different commercial softwares and heuristics, used to derive near optimal solutions is described.

6.1 Computational Results for the Short-Term Model

Real data from the Westmoreland Coal Company and four other similar, hypothetical data sets are generated for the purpose of testing the model and the solution procedure. The real data provided by Westmoreland Coal Company has 11 existing mines, 8 silos and 9 customers. The horizon considered in running the model with this data is 6 periods (6 days) in duration. Four hypothetical test problems are created using a different number of existing mines, silo units, and customers. Also, various combinations and connections between existing mines, silo units, and customers are incorporated in these test problems. These problems vary in the range of 121 constraints and 259

variables, to 631 constraints 3300 variables. Table 1 provides a list of specifications and results for all the test problems generated.

6.2 Computational Results for the Long-Term Model

6.2.1 Test Problem Specification

Real data from the Westmoreland Coal Company and nine other similar, hypothetical data sets are generated to test different heuristics and the commercial softwares used to solve the problem. The real data provided by Westmoreland Coal Company has 11 existing mines and 8 potential mines. Of the latter, one has four units, two of them have two units each, and the remaining five have one unit each. There are 8 silos and 6 customers. The horizon considered in running the model with this data is 6 periods (3 years) in duration.

Nine hypothetical test problems are created using a different number of existing mines, potential mines, units in each potential mine, silo units, and customers. Also, various combinations and connections between existing mines, potential mines, units in a potential mine, silo units, and customers are incorporated in these test problems. These problems vary in the range of 388 constraints, 27 binary variables, and 481 total number of variables, to 4155 constraints, 240 binary variables and 6565 total number of variables. Table 2 provides a list of specifications for all the test problems generated.

In all these data sets, we have included a special hypothetical potential mine with a single production unit that has a very high rate of production, and has a zero ash and sulfur percentage content in the coal produced by it. An inordinately large fixed-charge is required to open this mine. Also, the coal produced by it is given very high shipping and cleaning costs. In effect, it is ensured that this special hypothetical unit is used only when the model would otherwise have been infeasible.

Table 1. Specifications and results of test problems for the short-term tactical model

| Test problems | Number of mines | Number of customers | Number of time periods | Number of silos | Number of constraints | Number of variables | Objective value | CPU time (secs) | Iters |
|---------------|-----------------|---------------------|------------------------|-----------------|-----------------------|---------------------|-----------------|-----------------|-------|
| 1 | 4 | 4 | 4 | 4 | 121 | 259 | 1910483 | 0.7 | 197 |
| 2 | 7 | 10 | 6 | 4 | 315 | 2531 | 20981568 | 14.4 | 1306 |
| 3 | 11 | 6 | 6 | 8 | 325 | 998 | 736416 | 5.8 | 803 |
| 4 | 6 | 10 | 8 | 8 | 561 | 3060 | 154220192 | 17.2 | 1090 |
| 5 | 7 | 12 | 8 | 10 | 631 | 3300 | 30300222 | 23.6 | 1424 |

Legend:

CPU time = Resource usage in CPU seconds for an IBM 3090

Iters = Number of iterations required to solve the problem

Table 2. Specifications of test problems for the long-term strategic model

| Test problem | Number of existing mines | Number of potential mines | Number of units in potential mines | Number of customers | Number of time periods | Number of silos | Number of constraints | Number of variables | Number of binary variables | CPU time (secs) | Iters |
|--------------|--------------------------|---------------------------|------------------------------------|---------------------|------------------------|-----------------|-----------------------|---------------------|----------------------------|-----------------|-------|
| 1 | 8 | 5 | 9 | 7 | 3 | 8 | 388 | 481 | 27 | 3.7 | 560 |
| 2 | 11 | 8 | 13 | 6 | 6 | 8 | 950 | 1145 | 78 | 31.2 | 2047 |
| 3 | 5 | 8 | 14 | 7 | 6 | 8 | 1195 | 2309 | 84 | 57.6 | 2599 |
| 4 | 4 | 8 | 15 | 5 | 6 | 8 | 776 | 1337 | 90 | 25.5 | 1885 |
| 5 | 7 | 8 | 15 | 10 | 6 | 4 | 1310 | 2987 | 90 | 93.3 | 3367 |
| 6 | 5 | 8 | 16 | 6 | 6 | 4 | 1053 | 863 | 96 | 36.6 | 2238 |
| 7 | 7 | 8 | 13 | 6 | 8 | 8 | 1290 | 1957 | 104 | 56.7 | 2634 |
| 8 | 4 | 8 | 14 | 6 | 8 | 6 | 1251 | 1752 | 112 | 105.8 | 5016 |
| 9 | 6 | 13 | 24 | 10 | 8 | 8 | 2981 | 4149 | 192 | 497.8 | 9850 |
| 10 | 7 | 16 | 30 | 12 | 8 | 10 | 4155 | 6565 | 240 | 584.6 | 6947 |

Legend:

CPU time = Resource usage in CPU seconds for an IBM 3090

Iters = Number of iterations required to solve the linear programming relaxation of the problem

ble. In other words, whenever the model uses this particular mine, it implies that the model is infeasible and the infeasibility lies where the flow is satisfied using the coal from this special mine.

6.2.2 Derivation of a Tighter Relaxation

After coding all the test problems in GAMS, we also performed an investigation of the effect of using constraints that are implied in the integer sense, though not in the continuous sense, for the purpose of deriving a tighter relaxation of the model. Tighter relaxations of a discrete programming problem help in improving the performance of any exact or heuristic solution procedure. To ascertain the effect of implied constraints on the model, linear programming relaxations of all the test problems are run using MINOS 5.2 as a solver, both with and without these implied constraints. The results in Table 3 show the effect of the implied constraints on the model. The results exhibit that the implied constraints used in the formulation produce a tighter relaxation for 6 out of the 10 test problems. Test problem number 6 derived the highest percentage increase (6.69 %) in its relaxed linear programming value.

6.2.3 Comparison of Different Commercial Softwares

The test problems were first solved using ZOOM (Zero/One Optimization Method), which is a default solver in GAMS for mixed-integer programming problems. As it turned out, ZOOM faced significant difficulties even in getting a feasible solution to the linear programming relaxation of the original mixed-integer programming problem. After trying different combinations of options, like CHEAT, DIVE, EXPAND, FACTOR, GAP, PARTIAL, QUIT (see Brooke, Kendrick and Meeraus 1988) in the GAMS/ZOOM options file, we were able to find an integer solution for only two out of the ten test problems using ZOOM, with an optimality tolerance of 10 %. It should be noted that if OSL, CPLEX, or ZOOM does not find an optimal solution within the specified range then it gives the best solution found up to the point of termination, mentioning that there is no optimal solution within the specified range.

Table 3. Comparison of solutions obtained with and without implied constraints

| Test problem number | Solved without implied constraints (objective value) | Solved with implied constraints (objective value) | Percentage increase in LP value |
|---------------------|--|---|---------------------------------|
| 1 | 258368136 | 258368136 | 0.00 |
| 2 | 296801038 | 316045551 | 6.48 |
| 3 | 76531010 | 76531010 | 0.00 |
| 4 | 31379965 | 31379965 | 0.00 |
| 5 | 77480004 | 77740986 | 0.33 |
| 6 | 328998463 | 351037130 | 6.69 |
| 7 | 391664242 | 392528791 | 0.22 |
| 8 | 56816347 | 56816347 | 0.00 |
| 9 | 655812636 | 673834448 | 2.74 |
| 10 | 409673011 | 412477641 | 0.68 |

For the purpose of obtaining an optimal integer solution for the test problems and comparing ZOOM with two of the best softwares available in the market to solve mixed-integer programming problems, these test problems were also solved using OSL (Optimization Subroutine Library) and CPLEX. Separate GAMSLINKS, developed by the GAMS Development Corporation, are used to link GAMS with OSL and GAMS with CPLEX. The OSL runs were made on an IBM RS/6000 workstation model 320H, running AIX 3.1. The runs with CPLEX as a solver were made on a SUN Sparc 1, running SUNOS 4.1.1. The runs using ZOOM as the solver were made on an IBM 3090. (The OSL and CPLEX runs were made at the GAMS Development Corporation.) The results obtained after solving the problems with ZOOM/GAMS, OSL/GAMS and CPLEX/GAMS are tabulated in Table 4.

OSL was able to solve all the test problems as shown in Table 4, while CLPEX was unable to solve three of these problems due to memory limitations on the SUN Sparc 1 computers. On the other hand, ZOOM could only solve three test problems, out of which two of them were solved within 10 % of optimality, while for the third (Problem number 4) problem only a feasible integer solution within about 43 % of the linear programming lower bound could be found. No feasible solution was found for the remaining problems. Comparing CPLEX and OSL in terms of the number of iterations and the relative gap from the linear programming relaxation value, the results in the table show that for 4 out of the 7 problems, it solved, CPLEX require more total iterations than did OSL, but for 5 out of these 7 problems it obtained a better solution than did OSL. Blank spaces in the table specify those problems which could not be solved either by CPLEX due to shortage of memory, or by ZOOM due to different kinds of numerical difficulties faced by it.

It should be noted from the table that for all the three solvers, we have listed the relative gap between the initial linear programming relaxation value and the best integer solution value found by the procedure, and not the relative gap with respect to the improved lower bound at termination. This is intentionally done so that the results obtained through these commercial softwares can be compared with the results of heuristics I and II on a common basis.

Table 4. Comparison of solutions obtained and effort required for solving the test Problems with OSL/GAMS, CPLEX/GAMS and ZOOM/GAMS

| Test problem | OSL | | | CPLEX | | | ZOOM | | | | | |
|--------------|--------------|--------|------------------|-------|--------------|--------|------------------|-------|--------------|--------|------------------|-------|
| | MIP solution | RG | CPU1 time (secs) | Iters | MIP solution | RG | CPU2 time (secs) | Iters | MIP solution | RG | CPU3 time (secs) | Iters |
| 1 | 259637230 | 0.0049 | 1.8 | 178 | 259637230 | 0.0049 | 6.0 | 426 | 259637230 | 0.0049 | 5.4 | 1121 |
| 2 | 368434780 | 0.165 | 1335.9 | 47348 | 373343025 | 0.1812 | 414.2 | 8765 | - | - | - | - |
| 3 | 86359033 | 0.1284 | 27.9 | 1419 | 83701728 | 0.0936 | 230.1 | 4436 | - | - | - | - |
| 4 | 33697853 | 0.0738 | 20.6 | 1455 | 33642804 | 0.0721 | 243.0 | 6825 | 44930032 | 0.4318 | 355.7 | 49077 |
| 5 | 78278455 | 0.0103 | 32.6 | 2126 | 78278455 | 0.0103 | 83.9 | 2045 | 79326334 | 0.0238 | 139.2 | 7554 |
| 6 | 446829414 | 0.2728 | 196.0 | 13046 | - | - | - | - | - | - | - | - |
| 7 | 448381433 | 0.1422 | 121.9 | 5912 | 440208987 | 0.1214 | 179.9 | 4271 | - | - | - | - |
| 8 | 60458788 | 0.0641 | 41.1 | 2238 | 60584061 | 0.0663 | 127.7 | 2634 | - | - | - | - |
| 9 | 965484334 | 0.4328 | 166.9 | 3994 | - | - | - | - | - | - | - | - |
| 10 | 449239319 | 0.0891 | 221.5 | 4004 | - | - | - | - | - | - | - | - |

Legend:
 CPU1 time = Resource usage in CPU seconds for an IBM RS/6000 workstation model 320H
 CPU2 time = Resource usage in CPU seconds for a SUN Sparc 1
 CPU3 time = Resource usage in CPU seconds for an IBM 3090
 Iters = Total number iterations required to solve the problem
 RG = Relative gap from linear programming relaxation value

6.2.4 Results for Heuristics I and II

The heuristics described in the chapter on Solution Procedures were run on the ten test problems. The solutions obtained and the effort required to solve the test problems using these heuristics are given in Table 5.

As the results in the table exhibit, Heuristic II performs better than Heuristic I in all respects. For 9 out of 10 test problems Heuristic II derives better solutions and uses less CPU seconds than does Heuristic I. Also, for 6 problems, Heuristic II requires fewer total number of iterations than does Heuristic I. In test problem 10, both heuristics obtain the same solution, which happens to be an alternative optimal solution to the linear programming relaxation itself. However, as Heuristic II recognizes this optimal solution at its Step 2, while Heuristic I recognizes this in its Step 1, Heuristic II therefore consumes more time to solve this particular problem.

For 5 out of the 10 test problems, Heuristic I enters Heuristic II after or within the second loop at Step 3. For 3 test problems it enters Heuristic II at the first loop, and for 2 test problems it does not need to execute Heuristic II. The linear programming relaxations of the original problem and the linearly constrained nonlinear programming problems solved within Heuristics I and II use MINOS 5.2 as the solver.

6.2.4 Comparison of Results Obtained from the Commercial Softwares and the Heuristics

In all the test problems except for problem number 2, the proposed Heuristics consume more iterations to solve the problems than do the commercial softwares OSL and CPLEX, given the 10 % optimality tolerance used within the latter methods. It should be noted, however, that while solving the problems through our heuristics, we did not use any advanced bases except for the second loop of Heuristic I. The effort for the proposed heuristics can be substantially reduced by using the optimal basis obtained for one run in subsequent problem solved, since only a minor sensitivity analysis type of perturbation is made from one run to the next in the sequence of prob-

Table 5. Comparison of solutions obtained and effort required for solving the test problems using Heuristic I and Heuristic II

| Test problems | Heuristic I | | | | | Heuristic II | | | | |
|---------------|--------------|--------|-----------------|-------|--------------|--------------|-----------------|-------|-----|-----|
| | MIP solution | RG | CPU time (secs) | Iters | MIP solution | RG | CPU time (secs) | Iters | LPR | FRA |
| 1 | 268702860 | 0.0399 | 10.2 | 1119 | 259637230 | 0.0049 | 4.1 | 1108 | 3 | 1 |
| 2 | 380658989 | 0.2044 | 455.3 | 16430 | 370792903 | 0.1732 | 88.2 | 5305 | 3 | 7 |
| 3 | 86359033 | 0.1284 | 836.4 | 25941 | 84327533 | 0.1018 | 227.5 | 12340 | 6 | 3 |
| 4 | 34978483 | 0.1146 | 271.6 | 12667 | 34704832 | 0.1059 | 102.5 | 9435 | 7 | 6 |
| 5 | 78609767 | 0.0145 | 368.1 | 11627 | 78278455 | 0.0103 | 540.7 | 22516 | 8 | 6 |
| 6 | 579852096 | 0.6518 | 579.7 | 26670 | 469914352 | 0.3386 | 293.1 | 19006 | 12 | 9 |
| 7 | 465382130 | 0.1855 | 933.9 | 23152 | 448381433 | 0.1422 | 279.4 | 12883 | 6 | 4 |
| 8 | 61816163 | 0.0879 | 130.1 | 15400 | 60272957 | 0.0608 | 770.8 | 42031 | 14 | 10 |
| 9 | 1053001248 | 0.5627 | 6393.3 | 74000 | 1023861971 | 0.5194 | 4003.4 | 77540 | 12 | 11 |
| 10 | 412477641 | 0.0000 | 584.8 | 6980 | 412477641 | 0.0000 | 1009.4 | 13447 | 2 | 8 |

Legend:

- CPU time = Resource usage in CPU seconds for an IBM 3090
- Iters = Total number of iterations required to solve the problem
- RG = Relative gap from linear programming relaxation value
- LPR = Number of linear programming runs required before getting the mixed-integer solution
- FRA = Number of units of potential mines having fractional values at the first Step

lems solved within each heuristic. As far as the solution quality is concerned, Heuristic II obtains a better solution than does OSL for 3 of the test problems and the same solution for 2 of them. It also obtains a better solution than does CPLEX for a single case. Table 6 compares the solutions obtained using three commercial softwares and heuristics. As can be seen from this table, for the most part, the solutions obtained via the different procedures are comparable in quality. Finally note that while OSL was able to solve all the test problems, CPLEX was unable to solve 3 of them due to memory limitations on the SUN Sparc 1 station. Because OSL and CPLEX procedures employ a branch-and-bound technique, it can be expected that as the problem size increases, the optimality tolerance will need to be further relaxed, while the proposed Heuristic II can be expected to remain robust since it relies on the solution of a limited sequence of linear programming relaxations.

6.3 Conclusions and Recommendations for Further Research

In this thesis we have developed a long-term strategic model, which has a structure of a fixed-charge mixed-integer zero-one programming problem, and we have modified 'Model 1' of Sherali and Puri (1991) to develop a short-term tactical model that has the structure of a linear programming problem. The short-term model has been solved using a standard linear programming package, and for the long-term problem, we have developed specialized heuristics as well as tested the use of commercial packages. Depending on the availability of the commercial packages, we recommend any of these strategies for implementation. They are comparable in terms of both effort and solution quality. If a good linear programming package is available, we recommend the use of Heuristic II.

The overall objective of this research has been to model the problem in a way so that the developed tool can help in making capital investment decisions and in planning the future growth of the coal industry. Using the models developed, the effect of various policies can be studied by performing appropriate sensitivity analyses. Various scenarios can be explored by making suitable changes in the data and using different configurations of the system.

Table 6. Comparison of solutions obtained for the test problems using Commercial softwares and Heuristics I & II

| Test problems | Percentage relative gap from the LP relaxation value | | | | |
|---------------|--|-------|-------|-------------|--------------|
| | OSL | CPLEX | ZOOM | Heuristic I | Heuristic II |
| 1 | 0.49 | 0.49 | 0.49 | 3.99 | 0.49 |
| 2 | 16.5 | 18.12 | - | 20.44 | 17.32 |
| 3 | 12.84 | 9.36 | - | 12.84 | 10.18 |
| 4 | 7.38 | 7.21 | 43.18 | 11.46 | 10.59 |
| 5 | 1.03 | 1.03 | 2.38 | 1.45 | 1.03 |
| 6 | 27.28 | - | - | 65.18 | 33.86 |
| 7 | 14.22 | 12.14 | - | 18.55 | 14.22 |
| 8 | 6.41 | 6.63 | - | 8.79 | 6.08 |
| 9 | 43.28 | - | - | 56.27 | 51.94 |
| 10 | 8.91 | - | - | 0.00 | 0.00 |

In using the proposed heuristics, we point out that the effort for obtaining a near optimal solution can be reduced to a large extent by automating the algorithmic steps. This automation can be done by incorporating the MPS file generated through GAMS 2.25 within a FORTRAN or C program, and updating this file from one call to the next of the solver MINOS 5.2, OSL or CPLEX. Hence, in Heuristic II for example, every time a linear relaxation of the problem is solved, an advanced basis will be used, which will assist in reducing the overall effort substantially.

Large models can be run on a 386 or a 486 PC using the proposed heuristics or the commercial softwares such as OSL and CPLEX. As a branch-and-bound tree needs a great deal of memory, one should have enough memory and disc space to accommodate this type of a procedure.

We also remark here that GAMS 2.25 (latest version) can interact with different spreadsheets such as Lotus, Quatro, etc. This can help managers who don't have a working knowledge of GAMS in making runs with different data and in performing what-if analyses without ever going into the GAMS program itself.

For further research, we recommend additional testing of the commercial MIP packages as well as the proposed heuristics on a larger sized problems. We also suggest an investigation of implementing these procedures and testing them on a personal computer. (Appendix A contains instructions for such an implementaion.)

Appendix A

GAMS Program File for the Strategic Model

In this appendix, a sample GAMS program file containing data for a medium sized long-term strategic problem is presented. The data can be easily changed for running any other instance of this problem. Given below is a set of instructions for implementing the run of a continuous relaxation on a PC.

1. Install the GAMS software on the PC, say in subdirectory c:\GAMS.
2. **Input file :** Test problem 6 = c:\GAMS\TESTN6.GMS
3. **Running the program :** c:>GAMS>GAMS TESTN6.GMS
4. **Output file:** c:\GAMS\TESTN6.LST

A sample program with data for test problem number 6 is presented next. This program solves the linear programming relaxation of the problem. To make runs after fixing designated binary variables at 1, one only needs to insert constraints restricting the appropriate variables to unit values.

*THIS IS GAMS FILE FOR TEST PROBLEM 6
\$OFFSYMXREF OFFSYMLIST OFFFUELLIST OFFFUELXREF
\$OFFUPPER
\$OFFDIGIT
\$TITLE WESTMORELAND PRODUCTION PLAN

SETS I1 EXISTING MINES / MINE1
 MINE2
 MINE3
 MINE4
 MINE5 /

 I2 POTENTIAL MINES / I21
 I22
 I23
 I24
 I25
 I26
 I27
 GOLDMIN /

 Q PRODUCTION UNITS / I211
 I221,I222
 I231
 I241,I242
 I251,I252

I261,I262,I263

I271,I272

GOLDMINE /

Q1(Q) /I211 /

Q2(Q) /I221, I222 /

Q3(Q) /I231 /

Q4(Q) /I241, I242 /

Q5(Q) / I251,I252 /

Q6(Q) / I261,I262,I263 /

Q7(Q) / I271,I272 /

J INDEX OF SILOS /

ROMB1 RAW BLEND SILO 1 AT BULLITT

ROMB2 RAW BLEND SILO 2 AT BULLITT

CLEANB1 SILO PAIR 3-5 AT BULLIT

CLEANB2 SILO PAIR 4-6 AT BULLIT

CLEANB3

CLEANW1 C1 AT WENTZ

CLEANW2 C2 AT WENTZ
CLEANW3 C3 AT WENTZ /

K INDEX OF CUSTOMERS / CUSTOMER1

CUSTOMER2
CUSTOMER3
CUSTOMER4
CUSTOMER5
CUSTOMER6
CUSTOMER7 /

KST(K) INDEX OF STOKER CUSTOMER

/ CUSTOMER6 /

T INDEX OF PERIODS / PERIOD1

PERIOD2
PERIOD3
PERIOD4
PERIOD5
PERIOD6 /

J1(J) INDEX OF ROM BLEND SILOS

/ ROMB1
ROMB2 /

J2W(J) INDEX OF CLEAN SILOS AT WENTZ FACILITY

/ CLEANW1
CLEANW2
CLEANW3 /

J2B(J) INDEX OF CLEAN SILOS AT BULLIT FACILITY

/ CLEANB1
CLEANB2
CLEANB3 /

J2WK(J2W) INDEX OF SILO SUPPLYING CUSTOMER

/ CLEANW1
CLEANW2 /

AI2Q(I2,Q) CONNECTION MAP BETWEEN MINE I2 AND SILO UNITS Q

/ I21.I211
I22.(I221,I222)
I23.I231
I24.(I241,I242)
I25.(I251,I252)
I26.(I261,I262,I263)
I27.(I271,I272)
GOLDMIN.GOLDMINE /

AI1J(I1,J) CONNECTION MAP BETWEEN MINE I1 AND SILO UNIT J

/ MINE1.(ROMB1,ROMB2)
MINE2.(CLEANB1,CLEANB2,CLEANB3,CLEANW1)

MINE3.(CLEANB1,CLEANB2,CLEANB3,CLEANW2)

MINE4.(CLEANW1,CLEANW2,CLEANW3)

MINE5.(CLEANW1,CLEANW2,CLEANW3) /

AI1J1(I1,J1) CONNECTION MAP BETWEEN MINE I1 AND ROM BLEND SILO J1

/ MINE1.(ROMB1,ROMB2) /

AI1J2W(I1,J2W) CONNECTION MAP (MINE I1 -> CLEAN SILO AT W ENTZ)

/ MINE2.CLEANW1

MINE3.CLEANW2

MINE4.(CLEANW1,CLEANW2,CLEANW3)

MINE5.(CLEANW1,CLEANW2,CLEANW3) /

AI1J2WK(I1,J2W) MAP BETWEEN BETWEEN I1 AND SILO SUPPLYING CUST

/ MINE4.(CLEANW1,CLEANW2)

MINE5.(CLEANW1,CLEANW2) /

AI1J2B(I1,J2B) CONNECTION MAP (MINE I1 -> CLEAN SILO AT BULLIT)

/ MINE2.(CLEANB1,CLEANB2,CLEANB3)

MINE3.(CLEANB1,CLEANB2,CLEANB3) /

AQJ(Q,J) MAP BETWEEN Q AND SILOS

/ I211.(ROMB1,ROMB2)

(I221,I222).(ROMB1,ROMB2)

I231.(CLEANB1,CLEANB2,CLEANB3,CLEANW1)

(I241,I242).(CLEANB1,CLEANB2,CLEANB3,CLEANW1)

(I251,I252).(CLEANB1,CLEANB2,CLEANB3,CLEANW2)

(I261,I262,I263).(CLEANW1,CLEANW2,CLEANW3)

(I271,I272).(CLEANW1,CLEANW2,CLEANW3)

GOLDMINE.(CLEANW1,CLEANW2,CLEANW3,CLEANB1,CLEANB2,CLEANB3)

/

AQJ1(Q,J1) CONNECTION MAP BETWEEN Q AND SILO J1

/ I211.(ROMB1,ROMB2)

(I221,I222).(ROMB1,ROMB2) /

AQJ2W(Q,J2W) CONNECTION MAP BETWEEN Q AND SILO J2W

/ I231.CLEANW1

(I241,I242).CLEANW1

(I251,I252).CLEANW2

(I261,I262,I263).(CLEANW1,CLEANW2,CLEANW3)

(I271,I272).(CLEANW1,CLEANW2,CLEANW3)

GOLDMINE.(CLEANW1,CLEANW2,CLEANW3) /

AQJ2WK(Q,J2W) CONNECTION MAP BETWEEN Q AND SILOS SUPPLYING
STOK

/ I231.CLEANW1
(I241,I242).CLEANW1
(I251,I252).CLEANW2
(I261,I262,I263).(CLEANW1,CLEANW2)
(I271,I272).(CLEANW1,CLEANW2)
GOLDMINE.(CLEANW1,CLEANW2) /

AQJ2B(Q,J2B) CONNECTION MAP (I2 AND SILO J2B)

/ I231.(CLEANB1,CLEANB2,CLEANB3)
(I241,I242).(CLEANB1,CLEANB2,CLEANB3)
(I251,I252).(CLEANB1,CLEANB2,CLEANB3)
GOLDMINE.(CLEANB1,CLEANB2,CLEANB3) /

AJK(J,K) CONNECTION MAP (SILO J -> CUSTOMER K)

/ ROMB1.(CUSTOMER1,CUSTOMER2)
ROMB2.(CUSTOMER1,CUSTOMER2)
CLEANB1.(CUSTOMER1,CUSTOMER3,CUSTOMER4)
CLEANB2.(CUSTOMER2,CUSTOMER3,CUSTOMER4)
CLEANB3.(CUSTOMER3,CUSTOMER4)
CLEANW1.(CUSTOMER5,CUSTOMER6)
CLEANW2.(CUSTOMER5,CUSTOMER6,CUSTOMER7)
CLEANW3.(CUSTOMER5,CUSTOMER7) /

AJ1K(J1,K) CONNECTION MAP (ROM SILO J1 -> CUSTOMER K)

/ ROMB1.(CUSTOMER1,CUSTOMER2)
ROMB2.(CUSTOMER1,CUSTOMER2) /

AJ2WK(J2W,K) CONNECTION MAP (CLEAN SILO J2W-> CUSTOMER K)

/ CLEANW1.(CUSTOMER5,CUSTOMER6)
CLEANW2.(CUSTOMER5,CUSTOMER6,CUSTOMER7)
CLEANW3.(CUSTOMER5,CUSTOMER7) /

AJ2BK(J2B,K) CONNECTION MAP (CLEAN SILO J2B-> CUSTOMER K)

/ CLEANB1.(CUSTOMER1,CUSTOMER3,CUSTOMER4)
CLEANB2.(CUSTOMER2,CUSTOMER3,CUSTOMER4)
CLEANB3.(CUSTOMER3,CUSTOMER4) /

AJ2WKST(J2W,KST) CONNECTION MAP (CLEAN SILO J2W-> ST. CUST KST)

/ CLEANW1.(CUSTOMER6)
CLEANW2.(CUSTOMER6) /

A2W2B(J2W,J2B) CONNECTION MAP (CLEAN SILO J2W -> CLEAN SILO J2B)

/ (CLEANW1,CLEANW2).(CLEANB1,CLEANB2,CLEANB3) /

BI2QT(T,I2,Q) CONNECTION MAP BETWEEN MINE I2 AND SILO UNITS Q

/ PERIOD1.(GOLDMIN.GOLDMINE)
 PERIOD2.(GOLDMIN.GOLDMINE)
 PERIOD3.(GOLDMIN.GOLDMINE)
 PERIOD4.(GOLDMIN.GOLDMINE)
 PERIOD5.(GOLDMIN.GOLDMINE)
 PERIOD6.(GOLDMIN.GOLDMINE) /

ALIAS (T,TP) ;

TABLE P(I1,T) PRODUCTION FROM MINE I IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-------|---------|---------|---------|---------|
| MINE1 | 180000 | 180000 | 180000 | 180000 |
| MINE2 | 260000 | 260000 | 260000 | 235000 |
| MINE3 | 570000 | 570000 | 570000 | 550000 |
| MINE4 | 570000 | 570000 | 570000 | 530000 |
| MINE5 | 320000 | 320000 | 320000 | 200000 |
| + | | | | |
| | PERIOD5 | PERIOD6 | | |
| MINE1 | 160000 | 160000 | | |
| MINE2 | 235000 | 235000 | | |
| MINE3 | 550000 | 450000 | | |
| MINE4 | 480000 | 480000 | | |
| MINE5 | 200000 | 200000 | ; | |

TABLE PQ (I2,Q,T) PRODUCTION FROM UNIT Q OF MINE I2 IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|----------------------|-----------|-----------|-----------|-----------|
| I21.I211 | 55000 | 55000 | 55000 | 55000 |
| I22.(I221,I222) | 85000 | 85000 | 85000 | 85000 |
| I23.I231 | 50000 | 50000 | 50000 | 50000 |
| I24.(I241,I242) | 96000 | 96000 | 96000 | 80000 |
| I25.(I251,I252) | 103000 | 103000 | 100000 | 90000 |
| I26.(I261,I262,I263) | 100000 | 100000 | 100000 | 100000 |
| I27.(I271,I272) | 75000 | 75000 | 75000 | 75000 |
| GOLDMIN.GOLDMINE | 100000000 | 100000000 | 100000000 | 100000000 |
| + | | | | |
| | PERIOD5 | PERIOD6 | | |
| I21.I211 | 55000 | 55000 | | |
| I22.(I221,I222) | 85000 | 85000 | | |
| I23.I231 | 50000 | 50000 | | |
| I24.(I241,I242) | 65000 | 65000 | | |
| I25.(I251,I252) | 90000 | 90000 | | |
| I26.(I261,I262,I263) | 100000 | 100000 | | |
| I27.(I271,I272) | 75000 | 75000 | | |
| GOLDMIN.GOLDMINE | 100000000 | 100000000 | ; | |

TABLE D(K,T) DEMAND OF CUSTOMER K IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-----------|---------|---------|---------|---------|
| CUSTOMER1 | 130000 | 130000 | 130000 | 160000 |
| CUSTOMER2 | 155000 | 155000 | 155000 | 155000 |

| | | | | |
|-----------|--------|--------|--------|--------|
| CUSTOMER3 | 320000 | 320000 | 320000 | 330000 |
| CUSTOMER4 | 190000 | 190000 | 190000 | 195000 |
| CUSTOMER5 | 80000 | 95000 | 100000 | 105000 |
| CUSTOMER6 | 75000 | 75000 | 75000 | 75000 |
| CUSTOMER7 | 50000 | 50000 | 50000 | 50000 |

+

| | PERIOD5 | PERIOD6 |
|-----------|---------|---------|
| CUSTOMER1 | 160000 | 160000 |
| CUSTOMER2 | 165000 | 165000 |
| CUSTOMER3 | 330000 | 330000 |
| CUSTOMER4 | 200000 | 200000 |
| CUSTOMER5 | 110000 | 110000 |
| CUSTOMER6 | 75000 | 75000 |
| CUSTOMER7 | 70000 | 70000 ; |

TABLE A(I1,T) ASH PERCENTAGE IN COAL FROM MINE I1 IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-------|---------|---------|---------|---------|
| MINE1 | 16 | 16 | 16 | 16 |
| MINE2 | 45 | 45 | 45 | 45 |
| MINE3 | 44 | 44 | 44 | 44 |
| MINE4 | 35 | 35 | 35 | 35 |
| MINE5 | 48 | 48 | 48 | 48 |

+

| | PERIOD5 | PERIOD6 |
|-------|---------|---------|
| MINE1 | 16 | 16 |
| MINE2 | 45 | 45 |

| | | |
|-------|----|------|
| MINE3 | 44 | 44 |
| MINE4 | 35 | 35 |
| MINE5 | 48 | 48 ; |

TABLE AQ (I2,Q,T) ASH PERCENT IN COAL FROM UNITQ OF MINE I2 IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|----------------------|---------|---------|---------|---------|
| I21.I211 | 15 | 15 | 15 | 15 |
| I22.(I221,I222) | 14 | 14 | 14 | 14 |
| I23.I231 | 47 | 47 | 47 | 47 |
| I24.(I241,I242) | 41 | 41 | 41 | 41 |
| I25.(I251,I252) | 42 | 42 | 42 | 42 |
| I26.(I261,I262,I263) | 45 | 45 | 45 | 45 |
| I27.(I271,I272) | 38 | 38 | 38 | 38 |
| GOLDMIN.GOLDMINE | | 42 | 42 | 42 42 |

+

| | PERIOD5 | PERIOD6 |
|----------------------|---------|---------|
| I21.I211 | 15 | 15 |
| I22.(I221,I222) | 14 | 14 |
| I23.I231 | 47 | 47 |
| I24.(I241,I242) | 41 | 41 |
| I25.(I251,I252) | 42 | 42 |
| I26.(I261,I262,I263) | 45 | 45 |
| I27.(I271,I272) | 38 | 38 |
| GOLDMIN.GOLDMINE | 42 | 42 ; |

TABLE S(I1,T) SULPHUR PERCENTAGE IN COAL FROM MINE I IN PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-------|---------|---------|---------|---------|
| MINE1 | 1.17 | 1.17 | 1.17 | 1.17 |
| MINE2 | 1.15 | 1.15 | 1.15 | 1.15 |
| MINE3 | 1.2 | 1.2 | 1.2 | 1.2 |
| MINE4 | 1.11 | 1.11 | 1.11 | 1.11 |
| MINE5 | 1.2 | 1.2 | 1.2 | 1.2 |
| + | | | | |
| | PERIOD5 | PERIOD6 | | |
| MINE1 | 1.17 | 1.17 | | |
| MINE2 | 1.15 | 1.15 | | |
| MINE3 | 1.2 | 1.2 | | |
| MINE4 | 1.11 | 1.11 | | |
| MINE5 | 1.2 | 1.2 | ; | |

TABLE SQ(I2,Q,T) SULFUR PERCENT IN COAL FROM UNITQ OF MINE I2 IN PERIODT

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|----------------------|---------|---------|---------|---------|
| I21.I211 | 1.0 | 1.0 | 1.0 | 1.0 |
| I22.(I221,I222) | 1.05 | 1.05 | 1.05 | 1.05 |
| I23.I231 | 0.7 | 0.7 | 0.7 | 0.7 |
| I24.(I241,I242) | 0.65 | 0.65 | 0.65 | 0.65 |
| I25.(I251,I252) | 0.8 | 0.8 | 0.8 | 0.8 |
| I26.(I261,I262,I263) | 0.9 | 0.9 | 0.9 | 0.9 |
| I27.(I271,I272) | 1 | 1 | 1 | 1 |

| | | | | |
|----------------------|---------|---------|----|----|
| GOLDMIN.GOLDMINE | .9 | .9 | .9 | .9 |
| + | | | | |
| | PERIOD5 | PERIOD6 | | |
| I21.I211 | 1.0 | 1.0 | | |
| I22.(I221,I222) | 1.05 | 1.05 | | |
| I23.I231 | 0.7 | 0.7 | | |
| I24.(I241,I242) | 0.65 | 0.65 | | |
| I25.(I251,I252) | 0.8 | 0.8 | | |
| I26.(I261,I262,I263) | 0.9 | 0.9 | | |
| I27.(I271,I272) | 1 | 1 | | |
| GOLDMIN.GOLDMINE | .9 | .9 | ; | |

TABLE CSH1(I1,J) SHIPPING COST PER TON OF COAL (MINE I -> SILO J)

| | | | | | |
|-------|---------|---------|---------|---------|---------|
| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
| MINE1 | 1.0 | 1.0 | | | |
| MINE2 | | .5 | .5 | .5 | |
| MINE3 | | .7 | .7 | .7 | |
| MINE4 | | | | | |
| MINE5 | | | | | |
| + | | | | | |
| | CLEANW1 | CLEANW2 | CLEANW3 | | |
| MINE1 | | | | | |
| MINE2 | 1 | | | | |
| MINE3 | | 1.2 | | | |
| MINE4 | .6 | .6 | .6 | | |

MINE5 .8 .8 .8 ;

TABLE CSH1Q (I2,Q,J) SHIPPING COST PER TON OF COAL FROM Q OF MINE I2 TO J

| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
|----------------------|-------|-------|---------|---------|---------|
| I21.I211 | 1 | 1 | | | |
| I22.(I221,I222) | .9 | .9 | | | |
| I23.I231 | | .9 | .9 | .9 | |
| I24.(I241,I242) | | 1.2 | 1.2 | 1.2 | |
| I25.(I251,I252) | | 1.3 | 1.3 | 1.3 | |
| I26.(I261,I262,I263) | | | | | |
| I27.(I271,I272) | | | | | |
| GOLDMIN.GOLDMINE | | | 10 | 10 | 10 |

+

| | CLEANW1 | CLEANW2 | CLEANW3 |
|----------------------|---------|---------|---------|
| I21.I211 | | | |
| I22.(I221,I222) | | | |
| I23.I231 | 1.2 | | |
| I24.(I241,I242) | 1.4 | | |
| I25.(I251,I252) | 1.5 | | |
| I26.(I261,I262,I263) | 1.0 | 1.0 | 1.0 |
| I27.(I271,I272) | 1.1 | 1.1 | 1.1 |
| GOLDMIN.GOLDMINE | | 10 | 10 |

10 ;

TABLE CSH2(J,K) SHIPPING COST PER TON OF COAL (SILO J -> CUSTOMER K)

CUSTOMER1 CUSTOMER2 CUSTOMER3 CUSTOMER4

ROMB1
 ROMB2
 CLEANB1
 CLEANB2
 CLEANB3
 CLEANW1
 CLEANW2
 CLEANW3

+

CUSTOMER5 CUSTOMER6 CUSTOMER7

ROMB1
 ROMB2
 CLEANB1
 CLEANB2
 CLEANB3
 CLEANW1
 CLEANW2
 CLEANW3

;

TABLE CSH3(J2W,J2B) SHIPPING COST PER TON OF COAL (SILO J2W -> SILO J2B)

| | CLEANB1 | CLEANB2 | CLEANB3 |
|---------|---------|---------|---------|
| CLEANW1 | 1.00 | 1.00 | 1.00 |
| CLEANW2 | 1.00 | 1.00 | 1.00 ; |

TABLE CCL(I1,J) CLEANING COST PER TON OF COAL (MINE I1-> SILO J)

| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
|-------|-------|-------|---------|---------|---------|
| MINE1 | | | | | |
| MINE2 | | 2.0 | 2.0 | | |
| MINE3 | | 2.0 | 2.0 | | |
| MINE4 | | | | | |
| MINE5 | | | | | |

+

| | CLEANW1 | CLEANW2 | CLEANW3 | |
|-------|---------|---------|---------|---|
| MINE1 | | | | |
| MINE2 | 2.25 | | | |
| MINE3 | | 2.25 | | |
| MINE4 | 2.25 | 2.25 | 2.25 | |
| MINE5 | 2.25 | 2.25 | 2.25 | ; |

TABLE CCLQ (I2,Q,J) CLEANING COST AT SILO J FOR COAL FROM Q OF I2

| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
|----------------------|-------|-------|---------|---------|---------|
| I21.I211 | | | | | |
| I22.(I221,I222) | | | | | |
| I23.I231 | | 1.6 | 1.6 | 1.6 | |
| I24.(I241,I242) | | 1.5 | 1.5 | 1.5 | |
| I25.(I251,I252) | | 1.5 | 1.5 | 1.5 | |
| I26.(I261,I262,I263) | | | | | |
| I27.(I271,I272) | | | | | |
| GOLDMIN.GOLDMINE | | | 10 | 10 | 10 |
| + | | | | | |

| | CLEANW1 | CLEANW2 | CLEANW3 | |
|----------------------|---------|---------|---------|------|
| I21.I211 | | | | |
| I22.(I221,I222) | | | | |
| I23.I231 | 1.8 | | | |
| I24.(I241,I242) | 1.8 | | | |
| I25.(I251,I252) | | 1.9 | | |
| I26.(I261,I262,I263) | 2.0 | 2.0 | 2.0 | |
| I27.(I271,I272) | 2.0 | 2.0 | 2.0 | |
| GOLDMIN.GOLDMINE | | 10 | 10 | 10 ; |

TABLE FQ (I2,Q,T) FIXED CHARGE TO OPEN A UNIT AT TIME PERIOD T

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|------------------|---------|-----------|-----------|---------------------|
| I21.I211 | 100000 | 100000 | 100000 | 100000 |
| I22.I221 | 100000 | 100000 | 100000 | 100000 |
| I22.I222 | 70000 | 70000 | 70000 | 70000 |
| I23.I231 | 100000 | 100000 | 100000 | 100000 |
| I24.I241 | 100000 | 100000 | 100000 | 100000 |
| I24.I242 | 70000 | 70000 | 70000 | 70000 |
| I25.I251 | 250000 | 250000 | 250000 | 250000 |
| I25.I252 | 150000 | 150000 | 150000 | 150000 |
| I26.I261 | 350000 | 350000 | 350000 | 350000 |
| I26.I262 | 250000 | 250000 | 250000 | 250000 |
| I26.I263 | 100000 | 100000 | 100000 | 100000 |
| I27.I271 | 250000 | 250000 | 250000 | 250000 |
| I27.I272 | 125000 | 125000 | 125000 | 125000 |
| GOLDMIN.GOLDMINE | | 100000000 | 100000000 | 100000000 100000000 |
| + | | | | |

| | PERIOD5 | PERIOD6 | |
|------------------|-----------|-----------|---|
| I21.I211 | 100000 | 100000 | |
| I22.I221 | 100000 | 100000 | |
| I22.I222 | 70000 | 70000 | |
| I23.I231 | 100000 | 100000 | |
| I24.I241 | 100000 | 100000 | |
| I24.I242 | 70000 | 70000 | |
| I25.I251 | 250000 | 250000 | |
| I25.I252 | 150000 | 150000 | |
| I26.I261 | 350000 | 350000 | |
| I26.I262 | 250000 | 250000 | |
| I26.I263 | 100000 | 100000 | |
| I27.I271 | 250000 | 250000 | |
| I27.I272 | 125000 | 125000 | |
| GOLDMIN.GOLDMINE | 100000000 | 100000000 | ; |

TABLE ALPHA(I1,J) WEIGHT ATTENUATION FACTOR OF COAL (MINE I1 -> SILO J)

| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
|-------|---------|---------|---------|---------|---------|
| MINE1 | 1.0 | 1.0 | | | |
| MINE2 | | .55 | .55 | .55 | |
| MINE3 | | .55 | .55 | .55 | |
| MINE4 | | | | | |
| MINE5 | | | | | |
| + | | | | | |
| | CLEANW1 | CLEANW2 | CLEANW3 | | |

```

MINE1
MINE2      .55
MINE3      .55
MINE4      .6   .6   .6
MINE5      .55  .55  .55  ;

```

PARAMETER ASHROM(I1) ASH % IN ROM COAL

```

/ MINE1  16
   MINE2  45
   MINE3  44
   MINE4  35
   MINE5  48 / ;

```

PARAMETER ASHCLEAN(I1) ASH % IN CLEAN COAL

```

/ MINE1  16
   MINE2  14
   MINE3  12
   MINE4  11.5
   MINE5  13 / ;

```

PARAMETER BETA(I1,J) ASH ATTENUATION FACTOR OF COAL (MINE I1 -> SILO J);

BETA(I1,J) = ALPHA(I1,J) * ASHCLEAN(I1) /ASHROM(I1);

TABLE ALPHAQ(I2,Q,J) WEIGHT ATTENUATION FACTOR FOR COAL FROM Q OF I2 AT J

| | ROMB1 | ROMB2 | CLEANB1 | CLEANB2 | CLEANB3 |
|----------------------------------|---------|---------|---------|------------------|---------|
| I21.I211 | 1.0 | 1.0 | | | |
| I22.(I221,I222) | 1.0 | 1.0 | | | |
| I23.I231 | | 0.6 | 0.6 | 0.6 | |
| I24.(I241,I242) | | 0.6 | 0.6 | 0.6 | |
| I25.(I251,I252) | | 0.55 | 0.55 | 0.55 | |
| I26.(I261,I262,I263) | | | | | |
| I27.(I271,I272) GOLDMIN.GOLDMINE | | | | .5 | .5 .5 |
| + | | | | | |
| | CLEANW1 | CLEANW2 | CLEANW3 | | |
| I21.I211 | | | | | |
| I22.(I221,I222) | | | | | |
| I23.I231 | 0.6 | | | | |
| I24.(I241,I242) | 0.6 | | | | |
| I25.(I251,I252) | | 0.6 | | | |
| I26.(I261,I262,I263) | 0.6 | 0.6 | 0.6 | | |
| I27.(I271,I272) | 0.55 | 0.55 | 0.55 | GOLDMIN.GOLDMINE | .5 |
| .5 | .5 | ; | | | |

PARAMETER ASHROMQ (I2,Q) ASH PERCENT INROM COAL FROM UNIT Q OF I2

| | |
|-----------------|----|
| / I21.I211 | 15 |
| I22.(I221,I222) | 14 |
| I23.I231 | 47 |

I24.(I241,I242) 41
 I25.(I251,I252) 42
 I26.(I261,I262,I263) 45
 I27.(I271,I272) 38
 GOLDMIN.GOLDMINE 42 /;

PARAMETER ASHCLEANQ(I2,Q) ASH PERCENT IN CLEAN COAL FROM Q OF I2

/ I21.I211 15
 I22.(I221,I222) 14
 I23.I231 12
 I24.(I241,I242) 11
 I25.(I251,I252) 13
 I26.(I261,I262,I263) 11.5
 I27.(I271,I272) 9
 GOLDMIN.GOLDMINE 11 / ;

PARAMETER BETAQ (I2,Q,J) ASH ATTEN FACTOR OF COAL FROMQ OF I2 IN SILO J;

$BETAQ(I2,Q,J) = (ALPHAQ(I2,Q,J) * ASHCLEANQ(I2,Q) / ASHROMQ(I2,Q)) / (ASHROMQ(I2,Q) \neq 0);$

PARAMETER SULROM(I1) SULPHUR % IN ROM COAL

/ MINE1 1.17
 MINE2 1.15

MINE3 1.2
MINE4 1.11
MINE5 1.2 / ;

PARAMETER SULCLEAN(I1) SULPHUR % IN CLEAN COAL

/ MINE1 1.17
MINE2 1.15
MINE3 1.2
MINE4 1.11
MINE5 1.2 / ;

PARAMETER GAMMA(I1,J) SULPHUR ATTENUATION OF COAL (MINE I1 -> SILO
J) ;

$GAMMA(I1,J) = ALPHA(I1,J) * SULCLEAN(I1) /SULROM(I1);$

PARAMETER SULROMQ (I2,Q) SULFUR PERCENT IN ROM COAL FROM QOF I2

/ I21.I211 1.0
I22.(I221,I222) 1.05
I23.I231 0.7
I24.(I241,I242) 0.65
I25.(I251,I252) 0.8
I26.(I261,I262,I263) 0.9
I27.(I271,I272) 1.00
GOLDMIN.GOLDMINE .9 / ;

PARAMETER SULCLEANQ(I2,Q) SULFUR PERCENT IN CLEAN COAL FROM Q
OF I2

/ I21.I211 1.0
 I22.(I221,I222) 1.05
 I23.I231 0.7
 I24.(I241,I242) 0.65
 I25.(I251,I252) 0.8
 I26.(I261,I262,I263) 0.9
 I27.(I271,I272) 1.00
 GOLDMIN.GOLDMINE .9 /;

PARAMETER GAMMAQ(I2,Q,J) SULFUR ATTENUATION OF COAL (MINE I1 -> SILO J) ;

$GAMMAQ(I2,Q,J) = (ALPHAQ(I2,Q,J) * SULCLEANQ(I2,Q) /SULROMQ(I2,Q)) \$$
 (ASHROMQ (I2,Q) NE 0);

PARAMETER SILOQ(J) STORAGE CAPACITY IF SILO UNIT J

/ ROMB1 2750000
 ROMB2 2750000
 CLEANB1 3500000
 CLEANB2 3500000
 CLEANB3 1000000
 CLEANW1 2125000
 CLEANW2 2125000
 CLEANW3 2125000 / ;

PARAMETER U(T) UPPER LIMIT ON THE # OF UNITS WHICH CAN BE OPENED IN T

```
/ PERIOD1 4
PERIOD2 3
PERIOD3 3
PERIOD4 3
PERIOD5 3
PERIOD6 3 / ;
```

PARAMETER FIX(T) PENALTY FOR OPENING MORE THAN U(T) IN A TIME PERIOD

```
/ PERIOD1 350000
PERIOD2 350000
PERIOD3 350000
PERIOD4 350000
PERIOD5 350000
PERIOD6 350000 / ;
```

PARAMETER ASHUP(K) UPPER LIMIT ON ASH PERCENTAGE IN COAL

```
/ CUSTOMER1 17
CUSTOMER2 15
CUSTOMER3 14
CUSTOMER4 13.5
CUSTOMER5 12
CUSTOMER6 11
CUSTOMER7 14 / ;
```

PARAMETER ASHLO(K) LOWER LIMIT ON ASH PERCENTAGE IN COAL

```
/ CUSTOMER1  0
CUSTOMER2  0
CUSTOMER3  0
CUSTOMER4  0
CUSTOMER5  0
CUSTOMER6  0
CUSTOMER7  0  / ;
```

PARAMETER SULUP(K) UPPER LIMIT ON SULPHUR PERCENTAGE IN COAL

```
/ CUSTOMER1  1.15
CUSTOMER2  1.20
CUSTOMER3  1.1
CUSTOMER4  0.9
CUSTOMER5  1.05
CUSTOMER6  0.95
CUSTOMER7  1.15  / ;
```

PARAMETER SULLO(K) LOWER LIMIT ON SULPHUR PERCENTAGE IN COAL

```
/ CUSTOMER1  0
CUSTOMER2  0
CUSTOMER3  0
CUSTOMER4  0
CUSTOMER5  0
CUSTOMER6  0
CUSTOMER7  0  / ;
```

PARAMETER R(K) RELATIVE REWARD FOR BETTER ASH QUALITY

/ CUSTOMER1 .5
CUSTOMER2 .5
CUSTOMER3 .6
CUSTOMER4 .6
CUSTOMER5 .7
CUSTOMER6 .5
CUSTOMER7 .6 / ;

TABLE XQL(I2,Q,T) FIXING GOLDMINE TO ZERO

PERIOD1

GOLDMIN.GOLDMINE

+

PERIOD2

GOLDMIN.GOLDMINE

+

PERIOD3

GOLDMIN.GOLDMINE

+

PERIOD4

GOLDMIN.GOLDMINE

+

PERIOD5

GOLDMIN.GOLDMINE

+

PERIOD6

GOLDMIN.GOLDMINE

;

SCALAR F HALF /1.5/ ;

SCALAR G BOUND /1.1/ ;

SCALAR PE PENALTY /25/ ;

SCALAR PE1 PENALTY1 /0.2/ ;

\$STITLE VARIABLE SPECIFICATION

VARIABLES

Y(I1,J,K,T) AMOUNT OF COAL SHIPPED (I1->J->K) IN PERIOD T

YTR(I1,J2W,J2B,K,T) AMOUNT OF COAL (I1->J2W->J2B->K) IN PERIOD T

YQ(I2,Q,J,K,T) AMOUNT OF COAL FROM UNIT Q OF I2->J->K IN PERIOD T

YTRQ(I2,Q,J2W,J2B,K,T) AMOUNT OF COALFROM UNIT Q OF I2->J2W->J2B->K
IN T

XQ (I2,Q,T) WHETHER TO OPEN UNIT Q OF I2 OR NOT

THE1(T) NUMBER OF MINES OPENED WITHIN THE LIMIT

THE2(T) NUMBER OF MINES OPENED OVER THE LIMIT WITHPENALT

COST1 SHIPPING & CLEANING COST

FIXEDCHARG FIXED CHARGE TO OPEN A MINE

PENALTY1 PENALTY FOR OPENING MORE THAN U(T)

PENALTY PENALTY FOR LEFT OVERS

| | |
|------------|--|
| TOTALCOST | TOTAL COST |
| QUALITY | REWARD FOR BETTER QUALITY |
| Z(I1,T) | LEFT OVER FROM MINES I1 IN TIME PERIOD T |
| ZQ(I2,Q,T) | LEFT OVER FROM UNIT Q OF MINES I2 IN TIME PERIOD T |
| ASH(K,T) | ASH PERCENTAGE IN BLENDED COAL FOR CUSTOMER K IN |
| SUL(K,T) | SULPHUR PERCENTAGE IN BLENDED COAL FOR CUSTOMER; |

POSITIVE VARIABLES Y,YQ,YTRQ,YTR,xq,Z,ZQ,THE1,THE2 ; * BINARY VARIABLES XQ;

FREE VARIABLES

| | |
|------------|------------------------------------|
| COST1 | SHIPPING & CLEANING COST |
| FIXEDCHARG | FIXED COST TO OPEN Q'S OF MINE I2 |
| PENALTY | PENALTY FOR LEFT OVERS |
| PENALTY1 | PENALTY FOR OPENING MORE THAN U(T) |
| TOTALCOST | TOTAL COST |
| QUALITY | REWARD FOR BETTER QUQLITY; |

* LOWER LIMIT ON % ASH CONTENT IN COAL TO CUSTOMER K IN PERIOD T

$$\text{ASH.LO}(K,T) \cdot D(K,T) = \text{ASHLO}(K);$$

* UPPER LIMIT ON % ASH CONTENT IN COAL TO CUSTOMER K IN PERIOD T

$$\text{ASH.UP}(K,T) \cdot D(K,T) = \text{ASHUP}(K);$$

* LOWER LIMIT ON % SULPHUR CONTENT IN COAL TO CUSTOMER K IN PERIOD
T

$$\text{SUL.LO}(K,T) \leq D(K,T) = \text{SULLO}(K);$$

* UPPER LIMIT ON % SULPHUR CONTENT IN COAL TO CUSTOMER K IN PERIOD
T

$$\text{SUL.UP}(K,T) \leq D(K,T) = \text{SULUP}(K); \text{ \$\$TITLE EQUATION SPECIFICATION}$$

EQUATIONS

ZS(I1,T) AMOUNT OF COAL REMAINING FROM MINE I1 IN PERIOD T
ZSQ(I2,Q,T) AMOUNT OF COAL REMAINING FROM UNIT Q OF MINE I2 IN T
SILOJ2BQ(J2B,T) CAPACITY CONSTRAINT ON BULLITT SILOS
SILOJ2WQ(J2W,T) CAPACITY CONSTRAINT ON WENTZ SILOS
SILOJ1Q (J1,T) CAPACITY CONSTRAINT ON ROM SILOS
WBC1(I1,J2W,T) WENTZ BULLITT EQUALITY CONSTRAINT
WBC2(I1,J2W,T) WENTZ BULLITT EQUALITY CONSTRAINT
WBCQ1(I2,J2W,Q,T) WENTZ BULLITT EQUALITY CONSTRAINT
WBCQ2(I2,J2W,Q,T) WENTZ BULLITT EQUALITY CONSTRAINT
DMD(K,T) DEMAND CONSTRAINT FOR CUSTOMER K AT TIME T
ASHC(K,T) ASH QUALITY CONSTRAINT
SULC(K,T) SULPHUR QUALITY CONSTRAINT
XQLC1(I2,Q,T) FIXING THE VALUES OF THE BINARY VARIABLE
XQLC2(I2,Q,T) FIXING THE VALUES OF THE BINARY VARIABLE
XTT1 (I2,Q) A MINE CAN ONLY BE OPENED ONCE

XTT2 (T) UPPER LIMIT ON THE NUMBER OF MINES CAN BE OPENE

XTT3 (I2,Q,T) ORDERING OPENING OF MINES

XTT4 (I2,Q,T) ORDERING OPENING OF MINES

THETA (T)

ADDITION1(I2,Q,K,TP) FROM DISAGGREGATED NTWRK FIXED CHARGED
PROBLEM

COST1C SHIPPING & CLEANING COST

FIXEDCHARC FIXED COST TO OPEN Q'S OF MINE I2

PENALTYC TOTAL PENALTY FOR THE LEFT OVERS

PENALTYIC

TOTCC TOTAL COST

QUALITYC REWARD FOR BETTER QUALITY ;

\$\$TITLE EQUATION DEFINITION

XQLC1(I2,Q,T) \$((XQL(I2,Q,T)EQ 0)\$ B12QT(T,I2,Q))..

XQ(I2,Q,T)=E=XQL(I2,Q,T) ;

XQLC2(I2,Q,T) \$((XQL(I2,Q,T)EQ 1)\$ B12QT(T,I2,Q))..

XQ(I2,Q,T)=E=XQL(I2,Q,T) ;

ZS(I1,T).. Z(I1,T) =E= P(I1,T)-SUM (J \$A11J (I1,J), SUM (K \$AJK (J,K),Y(I1,J,K,T)))
- SUM (J2W \$ A11J2W(I1,J2W), SUM(J2B \$ A2W2B(J2W,J2B),
SUM(K \$ AJ2BK(J2B,K), YTR(I1,J2W,J2B,K,T))));

ZSQ(I2,Q,T) \$ AI2Q(I2,Q)..

ZQ(I2,Q,T)=E= SUM (TP\$(ORD(TP) LE ORD(T)), PQ(I2,Q,TP)*XQ(I2,Q,TP))
- SUM (J\$AQJ (Q,J),SUM (K\$AJK(J,K),YQ(I2,Q,J,K,T)))

- SUM(J2W\$AQJ2WK(Q,J2W),SUM (J2B\$A2W2B(J2W,J2B),
SUM(K\$ AJ2BK(J2B,K), YTRQ(I2,Q,J2W,J2B,K,T)));

SILOJ2BQ (J2B,T)..

SILOQ (J2B) = G= SUM(I1 \$ AI1J2B (I1,J2B), SUM (K \$ AJ2BK (J2B,K),
Y (I1,J2B,K,T) * F * (1 + ALPHA (I1,J2B)))) +
SUM (J2W \$ A2W2B (J2W,J2B), SUM (I1 \$ AI1J2WK (I1,J2W),
SUM (K \$ AJ2BK (J2B,K), YTR (I1,J2W,J2B,K,T) * F *
(1 + ALPHA (I1,J2W))))) +
SUM (Q \$ AQJ2B (Q,J2B), SUM (I2 \$ AI2Q (I2,Q), SUM (K \$ AJ2BK(J2B,K),
YQ (I2,Q,J2B,K,T) * F * (1 + ALPHAQ(I2,Q,J2B))))) +
SUM (J2W \$ A2W2B (J2W,J2B), SUM (Q \$ AQJ2W (Q,J2W),
SUM (K \$ AJ2BK (J2B,K), SUM (I2 \$ AI2Q (I2,Q),
YTRQ (I2,Q,J2W,J2B,K,T) * F * (1 + ALPHAQ (I2,Q,J2W))))));

SILOJ2WQ (J2W,T)..

SILOQ (J2W) = G= SUM (I1 \$ AI1J2W (I1,J2W),
SUM (K \$ AJ2WK (J2W,K),Y(I1,J2W,K,T) * F *(1 + ALPHA(I1,J2W)))) +
SUM (I1 \$ AI1J2WK (I1,J2W),SUM (J2B \$ A2W2B (J2W,J2B),
SUM (K \$ AJ2BK (J2B,K),YTR (I1,J2W,J2B,K,T) * F *
(1 + ALPHA (I1,J2W))))) +
SUM (Q \$ AQJ2W (Q,J2W), SUM (I2 \$ AI2Q (I2,Q), SUM (K \$ AJ2WK (J2W, K),
YQ (I2,Q,J2W,K,T) * F *(1 + ALPHAQ (I2,Q,J2W))))) +
SUM (Q \$ AQJ2WK (Q,J2W),SUM (J2B \$ A2W2B (J2W,J2B),
SUM (K \$ AJ2BK (J2B,K), SUM (I2 \$ AI2Q (I2,Q),
YTRQ (I2,Q,J2W,J2B,K,T) * F * (1 + ALPHAQ (I2,Q,J2W))))));

SILOJ1Q (J1,T)..

SILOQ (J1)= G= SUM (I1 \$ AI1J1 (I1,J1), SUM (K \$ AJ1K (J1,K),
 Y (I1,J1,K,T))) +
 SUM (Q \$ AQJ1(Q,J1), SUM (K \$ AJ1K (J1,K), SUM (I2 \$ AI2Q (I2,Q),
 YQ (I2,Q,J1,K,T)));

WBC1(I1,J2W,T) \$ AI1J2WK(I1,J2W).. SUM(J2B \$ A2W2B(J2W,J2B), SUM(K \$
 AJ2BK(J2B,K), YTR(I1,J2W,J2B,K,T)))
 = L= SUM(KST \$ AJ2WKST (J2W,KST), Y(I1,J2W,KST,T));

WBC2 (I1,J2W,T) \$ AI1J2WK (I1,J2W)..
 G * SUM (J2B \$ A2W2B (J2W,J2B), SUM (K \$ AJ2BK (J2B,K),
 YTR (I1,J2W,J2B,K,T))) = G =
 SUM (KST \$ AJ2WKST (J2W,KST), Y (I1,J2W,KST,T));

WBCQ1 (I2,J2W,Q,T) \$(AQJ2WK (Q,J2W) \$ AI2Q(I2,Q))..
 SUM (J2B \$ A2W2B (J2W,J2B),SUM (K \$ AJ2BK (J2B,K),
 YTRQ (I2,Q,J2W,J2B,K,T))) = L =
 SUM (KST \$ AJ2WKST (J2W,KST), YQ (I2,Q,J2W,KST,T));

WBCQ2 (I2,J2W,Q,T) \$(AQJ2WK (Q,J2W) \$ AI2Q(I2,Q))..
 G * SUM (J2B \$ A2W2B (J2W,J2B),SUM (K \$ AJ2BK (J2B,K),
 YTRQ (I2,Q,J2W,J2B,K,T))) = G =
 SUM (KST \$ AJ2WKST (J2W,KST), YQ (I2,Q,J2W,KST,T));

DMD(K,T)..
 D(K,T) = E = SUM(J \$ AJK(J,K), SUM(I1 \$ AI1J(I1,J), ALPHA(I1,J)*
 Y (I1,J,K,T)))
 + SUM (J2B \$ AJ2BK(J2B,K), SUM(J2W \$ A2W2B(J2W,J2B),

SUM (I1 \$ AI1J2W(I1,J2W),
 ALPHA(I1,J2W)*YTR(I1,J2W,J2B,K,T)))
 + SUM (J\$AJK(J,K), SUM (Q \$ AQJ(Q,J),SUM (I2 \$ AI2Q(I2,Q),
 ALPHAQ(I2,Q,J) *YQ (I2,Q,J,K,T))))
 + SUM (J2B\$AJ2BK(J2B,K),SUM (J2W\$A2W2B(J2W,J2B),
 SUM (Q\$AQJ2W(Q,J2W),SUM (I2 \$ AI2Q (I2,Q),
 ALPHAQ (I2,Q,J2W)*YTRQ(I2,Q,J2W,J2B,K,T)))));

ASHC(K,T) \$ D(K,T)..

ASH(K,T)*D(K,T) = E=

SUM (J \$ AJK(J,K), SUM (I1 \$ AI1J(I1,J), Y(I1,J,K,T) *A(I1,T)*
 BETA (I1,J))) +
 SUM (J2B \$ AJ2BK(J2B,K), SUM(J2W \$ A2W2B(J2W,J2B),
 SUM (I1 \$ AI1J2W(I1,J2W), YTR(I1,J2W,J2B,K,T)*A(I1,T)*BETA(I1,J2W)))) +
 SUM (J \$ AJK(J,K),SUM(Q \$ AQJ(Q,J),SUM(I2 \$ AI2Q(I2,Q),
 YQ(I2,Q,J,K,T)*AQ(I2,Q,T)*BETAQ(I2,Q,J)))) +
 SUM(J2B \$ AJ2BK(J2B,K),SUM(J2W\$A2W2B (J2W,J2B),
 SUM (Q \$ AQJ2W (Q,J2W), SUM (I2 \$ AI2Q (I2,Q),
 YTRQ (I2,Q,J2W,J2B,K,T)*AQ(I2,Q,T)*BETAQ(I2,Q,J2W)))));

SULC(K,T) \$ D(K,T)..

SUL(K,T)*D(K,T) = E=

SUM (J \$ AJK(J,K), SUM (I1 \$ AI1J(I1,J), Y(I1,J,K,T) *S(I1,T)*
 GAMMA(I1,J))) +
 SUM (J2B \$ AJ2BK(J2B,K), SUM(J2W \$ A2W2B(J2W,J2B),
 SUM (I1 \$ AI1J2W(I1,J2W), YTR(I1,J2W,J2B,K,T)*S(I1,T)*GAMMA(I1,J2W)))) +
 SUM (J\$ AJK (J,K),SUM (Q \$ AQJ (Q,J),SUM (I2 \$ AI2Q (I2,Q),
 YQ (I2,Q,J,K,T) * SQ(I2,Q,T)* GAMMAQ(I2,Q,J)))) +

SUM(J2B \$ AJ2BK (J2B,K), SUM (J2W \$ A2W2B (J2W,J2B),
SUM (Q \$ AQJ2W (Q,J2W) ,SUM (I2 \$ AI2Q (I2,Q),
YTRQ(I2,Q,J2W,J2B,K,T) * SQ(I2,Q,T) * GAMMAQ(I2,Q,J2W)))));

XTT1(I2,Q)..

SUM (T, XQ(I2,Q,T) \$ AI2Q(I2,Q)) = L= 1;

THETA (T)..

THE1(T) = L= U(T) ;

XTT2 (T)..

SUM (I2, SUM (Q \$ AI2Q (I2,Q), XQ (I2,Q,T))) = L= THE1(T)+THE2(T) ;

XTT3 (I2,Q+1,T) \$ (AI2Q(I2,Q) \$ AI2Q(I2,Q+1))..

XQ (I2,Q+1,T) \$ AI2Q (I2,Q)-SUM (TP \$ (ORD(TP) LE ORD(T)),

XQ (I2,Q,TP) \$ AI2Q(I2,Q))= L= 0 ;

XTT4 (I2,Q+2,T) \$ (AI2Q(I2,Q) \$ AI2Q(I2,Q+2))..

XQ (I2,Q+2,T) \$ AI2Q (I2,Q)-SUM (TP \$ (ORD(TP) LE ORD(T)),

XQ (I2,Q,TP) \$ AI2Q(I2,Q))= L= 0 ;

ADDITION1 (I2,Q,K,TP) \$ AI2Q(I2,Q)..

SUM(J \$ AQJ (Q,J),YQ (I2,Q,J,K,TP) \$ AJK(J,K)) +

SUM (J2W \$ AQJ2WK(Q,J2W), SUM(J2B \$ A2W2B (J2W,J2B),

YTRQ (I2,Q,J2W,J2B,K,TP) \$ AJ2BK (J2B,K))) = L=

D(K,TP) * SUM (T \$ (ORD(T) LE ORD(TP)), XQ (I2,Q,T));

COST1C.. COST1 =E= SUM(J,SUM(I1 \$ AI1J(I1,J),SUM(K \$ AJK(J,K),SUM(T,
Y(I1,J,K,T)*
(CSH1(I1,J) + CCL(I1,J) + CSH2(J,K)))))) +
SUM(J2W \$ J2WK(J2W),SUM(I1 \$ AI1J2WK(I1,J2W),
SUM(J2B \$ A2W2B(J2W,J2B),
SUM(K \$ AJ2BK(J2B,K), SUM(T, YTR(I1,J2W,J2B,K,T)*
(CSH1(I1,J2W) + CSH2(J2B,K) + CSH3(J2W,J2B) + CCL(I1,J2W))))))+
SUM (J, SUM(Q \$AQJ(Q,J), SUM(I2 \$ AI2Q (I2,Q), SUM(K\$AJK (J,K),
SUM(T, YQ (I2,Q,J,K,T)*(CSH1Q(I2,Q,J) + CCLQ(I2,Q,J) +
CSH2(J,K)))))) +
SUM (J2W \$ J2WK(J2W), SUM (Q \$ AQJ2WK (Q,J2W),
SUM (J2B \$ A2W2B(J2W,J2B),
SUM(K \$ AJ2BK (J2B,K), SUM (I2 \$ AI2Q (I2,Q),
SUM(T, YTRQ (I2,Q,J2W,J2B,K,T) * (CSH1Q(I2,Q,J2W) + CSH2 (J2B,K) +
CSH3(J2W,J2B) + CCLQ (I2,Q,J2W)))))));

FIXEDCHARC..

FIXEDCHARG =E= SUM (I2,SUM (Q \$ AI2Q (I2,Q), SUM (T, FQ (I2,Q,T) *
XQ (I2,Q,T))));

PENALTYC..

PENALTY =E= PE * (SUM((I1,T), Z(I1,T)) +SUM (I2, SUM (Q \$ AI2Q(I2,Q),
SUM (T, ZQ (I2, Q,T)))));

PENALTY1C..

PENALTY1 =E= PE1 * SUM (T, FIX(T) * THE2(T)) ;

QUALITYC..

QUALITY =E= SUM (K,SUM (T,(ASHUP(K)- ASH(K,T))*D(K,T)*R(K)));

TOTCC.. TOTALCOST=E=COST1 + FIXEDCHARG +PENALTY+PENALTY1 -
QUALITY ;

\$STITLE MODEL SPECIFICATION

MODEL WESTMORE PRODUCTION PLANNING /ALL/;

OPTIONS ITERLIM=20000 ;

OPTIONS SYSOUT=ON,LIMCOL=0,LIMROW=0;

SOLVE WESTMORE USING LP MINIMIZING TOTALCOST ;

DISPLAY Y.L;

DISPLAY YQ.L;

DISPLAY YTR.L;

DISPLAY YTRQ.L;

DISPLAY XQ.L;

DISPLAY Z.L;

DISPLAY ZQ.L;

DISPLAY ASH.L;

DISPLAY SUL.L;

DISPLAY COST1.L;

DISPLAY PENALTY1.L;

```
DISPLAY FIXEDCHARG.L;
```

```
DISPLAY TOTALCOST.L;
```

Appendix B

GAMS Program File for the Tactical Model

In this appendix, a sample GAMS program file containing data for a medium sized short-term tactical problem is presented. The data can be easily changed for running any other instance of this problem. Given below is a set of instructions for implementing the run on a PC.

1. Install the GAMS software on the PC, say in subdirectory `c:\GAMS`.
2. **Input file :** Test problem 2 = `c:\GAMS\TESTN2.GMS`
3. **Running the program :** `c:> GAMS > GAMS TESTN2.GMS`
4. **Output file:** `c:\GAMS\TESTN2.LST`

A sample program with data for test problem number 2 is presented next.

* SHORT TERM MODEL TEST PROBLEM 2.....

\$OFFSYMXREF OFFSYMLIST OFFUELLIST OFFUELXREF

\$OFFUPPER

\$OFFDIGIT

\$TITLE WESTMORELAND PRODUCTION PLAN (SHORT TERM MODEL)

SETS I list of mines /

MINEONE

MINETWO

MINETHREE

MINEFOUR /

J INDEX OF SILOS / ROMW1

ROMB1

CLEANW1

CLEANB1 /

K INDEX OF CUSTOMERS / CUSTOMER1

CUSTOMER2

CUSTOMER3

CUSTOMER4 /

KST(K) INDEX OF THE STOKER CUSTOMER

/ CUSTOMER3 /

T index of periods / PERIOD1

PERIOD2

PERIOD3

PERIOD4 /

TI(T) index of periods initial coal consumption

/ PERIOD1

PERIOD2

PERIOD3 /

J1(J) index of Rom Blend silos

/ ROMW1

ROMB1 /

J2W(J) index of clean silos at Wentz facility

/ CLEANW1 /

J2WK(J) INDEX OF SILO SUPPLYING STOKER CUSTOMER

/ CLEANW1 /

J2B(J) index of clean silos at Bullit facility

/ CLEANB1 /

AIJ(I,J) connection map between mine i and silo unit j

/ MINEONE.ROMW1

MINETWO.(ROMW1, CLEANW1)
MINETHREE.ROMB1
MINEFOUR.(ROMB1, CLEANB1) /

AIJ1(I,J1) connection map between mine i and ROM BLEND silo j1

/ MINEONE.ROMW1
MINETWO.ROMW1
MINETHREE.ROMB1
MINEFOUR.ROMB1 /

AIJ2W(I,J2W) connection map (mine i -> clean silo at Wentz)

/ MINETWO.CLEANW1 /

AIJ2WK(I,J2W) CONNECTION MAP (MINE I -> CLEAN SILO AT WENTZ)

/ MINETWO.CLEANW1 /

AIJ2B(I,J2B) connection map (mine i -> clean silo at Bullit)

/ MINEFOUR.CLEANB1 /

AJK(J,K) connection map (silo J -> customer k)

/ ROMW1.(CUSTOMER1, CUSTOMER2)

```
ROMB1.(CUSTOMER1, CUSTOMER2)
CLEANW1.(CUSTOMER2, CUSTOMER3)
CLEANB1.(CUSTOMER2,CUSTOMER4) /
```

AJ1K(J1,K) connection map (ROM silo J1 -> customer k)

```
/ ROMW1.(CUSTOMER1, CUSTOMER2)
ROMB1.(CUSTOMER1, CUSTOMER2) /
```

AJ2WK(J2W,K) connection map (clean silo J2W-> customer k)

```
/ CLEANW1.(CUSTOMER2, CUSTOMER3) /
```

AJ2BK(J2B,K) connection map (clean silo J2B-> customer k)

```
/ CLEANB1.(CUSTOMER2,CUSTOMER4) /
```

AJ2WKST(J2W,KST) CONNECTION MAP(J2W -> STOKER CUST)

```
/ CLEANW1.CUSTOMER3 /
```

A2W2B(J2W,J2B) connection map (clean silo J2W -> clean silo J2B)

```
/ CLEANW1.CLEANB1 / ;
```

ALIAS (T,TP) ;

ALIAS (TI,TIP);

ALIAS (T,TAU) ;

ALIAS (T,T1);

ALIAS (T,T2);

TABLE P(I,T) production from mine i in period t

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-----------|---------|---------|---------|---------|
| MINEONE | 10000 | 10000 | 10000 | 9000 |
| MINETWO | 20000 | 20000 | 11000 | 11000 |
| MINETHREE | 10000 | 10000 | 9000 | 9000 |
| MINEFOUR | 10000 | 10000 | 11000 | 11000 ; |

TABLE D(K,T) demand of customer k in period t

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-----------|---------|---------|---------|---------|
| CUSTOMER1 | 6000 | 6000 | 6000 | 6000 |
| CUSTOMER2 | 6000 | 6000 | 4000 | 4000 |
| CUSTOMER3 | 6000 | 6000 | 8000 | 8000 |
| CUSTOMER4 | 9000 | 10000 | 9000 | 9000 ; |

TABLE A(I,T) ash percentage in coal from mine i in period t

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-----------|---------|---------|---------|---------|
| MINEONE | 15 | 15 | 15 | 15 |
| MINETWO | 13 | 13 | 13 | 13 |
| MINETHREE | 17 | 17 | 17 | 17 |
| MINEFOUR | 16 | 16 | 16 | 16 ; |

TABLE S(I,T) sulphur percentage in coal from mine i in period t

| | PERIOD1 | PERIOD2 | PERIOD3 | PERIOD4 |
|-----------|---------|---------|---------|---------|
| MINEONE | 2 | 2 | 2 | 2 |
| MINETWO | 2 | 2 | 2 | 2 |
| MINETHREE | 3 | 3 | 3 | 3 |
| MINEFOUR | 3 | 3 | 3 | 3 ; |

TABLE CSH1(I,J) shipping cost per ton of coal (mine i -> silo j)

| | ROMW1 | ROMB1 | CLEANW1 | CLEANB1 |
|-----------|-------|-------|---------|---------|
| MINEONE | .2 | .2 | .1 | .1 |
| MINETWO | 1.0 | 1.0 | 0.7 | 2.0 |
| MINETHREE | 1.1 | 1.4 | 2.1 | 2.0 |
| MINEFOUR | 1.2 | 1.3 | 1.7 | 1.8 ; |

TABLE CSH2(J,K) shipping cost per ton of coal (silo j -> customer k)

| | CUSTOMER1 | CUSTOMER2 | CUSTOMER3 | CUSTOMER4 |
|---------|-----------|-----------|-----------|-----------|
| ROMW1 | | | | |
| ROMB1 | | | | |
| CLEANW1 | | | | |
| CLEANB1 | | | | |

TABLE CSH3(J2W,J2B) shipping cost per ton of coal (silo j2w -> silo j2b)

| | CLEANB1 |
|---------|---------|
| CLEANW1 | 1.0 |

TABLE CCL(I,J) cleaning cost per ton of coal (mine i -> silo j)

| | ROMW1 | ROMB1 | CLEANW1 | CLEANB1 |
|-----------|-------|-------|---------|---------|
| MINEONE | 0 | 0 | 1.0 | 1.2 |
| MINETWO | 0 | 0 | 1.0 | 1.8 |
| MINETHREE | 0 | 0 | 1.1 | 1.0 |
| MINEFOUR | 0 | 0 | 1.7 | 1.8 |

TABLE ALPHA(I,J) weight attenuation factor of coal (mine i -> silo j)

| | ROMW1 | ROMB1 | CLEANW1 | CLEANB1 |
|---------|-------|-------|---------|---------|
| MINEONE | 1 | 1 | .98 | .98 |
| MINETWO | 1 | 1 | .98 | .98 |

```
MINETHREE    1    1    .98    .98
MINEFOUR     1    1    .98    .98  ;
```

PARAMETER ASHROM(I) ash % in ROM coal

```
/ MINEONE    16
  MINETWO    25
  MINETHREE  17.5
  MINEFOUR   28      /
```

PARAMETER ASHCLEAN(I) ash % in CLEAN coal

```
/ MINEONE    16
  MINETWO    12
  MINETHREE  17.5
  MINEFOUR   11      /
```

PARAMETER BETA(I,J) ash attenuation factor of coal (mine i -> silo j);

$BETA(I,J) = ALPHA(I,J) * ASHCLEAN(I) / ASHROM(I);$

PARAMETER SULROM(I) sulphur % in ROM coal

```
/ MINEONE    3.7
  MINETWO    5.4
```

MINETHREE 3.65
MINEFOUR 5.5 /

PARAMETER SULCLEAN(I) sulphur % in CLEAN coal

/ MINEONE 3.7
MINETWO 5.1
MINETHREE 3.65
MINEFOUR 5.0 /

PARAMETER GAMMA(I,J) sulphur attenuation of coal (mine i -> silo j) ;

$GAMMA(I,J) = ALPHA(I,J) * SULCLEAN(I) /SULROM(I);$

PARAMETER SM(I) storage capacity at mine i

/ MINEONE 200000
MINETWO 200000
MINETHREE 200000
MINEFOUR 200000 /

PARAMETER SS(J) storage capacity of silo unit j

/ ROMW1 200000
ROMB1 200000
CLEANW1 200000

CLEANB1 200000 /

PARAMETER Q0(J) initial amount in storage at silo j

/ ROMW1 3000
ROMB1 3000
CLEANW1 3000
CLEANB1 3000 /

PARAMETER A0(J) initial ash percentage of coal at silo j

/ ROMW1 15
ROMB1 15
CLEANW1 15
CLEANB1 15 /

PARAMETER S0(J) initial sulphur percentage of coal at silo j

/ ROMW1 2
ROMB1 2
CLEANW1 2
CLEANB1 2 /

PARAMETER CSM(I) storage cost per ton of coal from mine i

/ MINEONE 4
MINETWO 5
MINETHREE 5

MINEFOUR 4 /

PARAMETER CSS(J) storage cost per ton of coal at silo j

/ ROMW1 4
ROMB1 3
CLEANW1 9
CLEANB1 3 /

PARAMETER ASHUP(K) upper limit on ash percentage in coal

/ CUSTOMER1 20
CUSTOMER2 20
CUSTOMER3 20
CUSTOMER4 20 / ;

PARAMETER ASHLO(K) lower limit on ash percentage in coal

/ CUSTOMER1 1
CUSTOMER2 1
CUSTOMER3 1
CUSTOMER4 1 / ;

PARAMETER SULUP(K) upper limit on sulphur percentage in coal

/ CUSTOMER1 5
CUSTOMER2 5
CUSTOMER3 5
CUSTOMER4 5 / ;

PARAMETER SULLO(K) lower limit on sulphur percentage in coal

```
/ CUSTOMER1 .8
CUSTOMER2 .8
CUSTOMER3 .8
CUSTOMER4 .8 / ;
```

PARAMETER RA(K) SLOPE OF THE REWARD FOR THE GOOD QUALITY COAL
(ASH)

```
/ CUSTOMER1 .1
CUSTOMER2 .1
CUSTOMER3 .1
CUSTOMER4 .1 / ;
```

PARAMETER PA(K) SLOPE OF THE PENALTY FOR THE BAD QUALITY COAL
(ASH)

```
/ CUSTOMER1 .15
CUSTOMER2 .15
CUSTOMER3 .15
CUSTOMER4 .15 / ;
```

PARAMETER RS(K) SLOPE OF THE REWARD FOR THE GOOD QUALITY COAL
(SUL)

/ CUSTOMER1 .2
 CUSTOMER2 .2
 CUSTOMER3 .2
 CUSTOMER4 .2 / ;

SCALAR NIT TOTAL NUMBER OF INITIAL PERIODS / 3 / ;

SCALAR F HALF / .5 / ;

SCALAR G TOLERANCE FACTOR / 1.1 / ;

\$STITLE VARIABLE SPECIFICATION

VARIABLES

Y(I,J,T,K,TAU) AMT OF COAL SHPD I->J PRD T J->K PRD TAU

Y0(J,K,T) AMOUNT OF INITIAL COAL SHIPPED (J->K)

IN PERIOD T YRT(I,J2W,T1,J2B,T2,K,TAU) AMOUNT OF
 COAL I->J2W PRD. T1 & J2W->J2B

IN T2 & J2B -> K IN TAU

XS(I,T) AMOUNT OF COAL REMAINING IN MINE I
 DURING PERIOD T

THT2W(J2W,T) AMOUNT OF COAL (J2W->J2B) AT J2B

THT2B(J2B,T) AMOUNT OF COAL (J2W->J2B) AT J2W

US(J,T) ACCUMULATED STORAGE OF SILO J
 DURING PERIOD T

| | |
|----------------|--|
| COST1 | SHIPPING & CLEANING COST |
| COST2 | STORAGE COST AT MINES |
| COST3 | STORAGE COST AT SILOS |
| QUALITY BAD | REWARD FOR BETTER QUALITY AND PENALTY FOR BAD |
| TOTALCOST | TOTAL COST |
| ASH(K,TAU) | ASH PERCENTAGE IN BLENDED COAL FOR CUSTOMER K IN PERIOD TAU |
| ASH1(K,TAU) | ASH PERCENTAGE BELOW THE UPPER LEVEL |
| ASH2(K,TAU) | ASH PERCENTAGE ABOVE THE UPPER LEVEL |
| SUL(K,TAU) | SULPHUR PERCENTAGE IN BLENDED COAL FOR CUSTOMER K IN PERIOD TAU ; |

POSITIVE VARIABLES Y,Y0,YRT,XS,US,ASH2;

FREE VARIABLES

| | |
|-----------|--|
| COST1 | shipping & cleaning cost |
| COST2 | storage cost at mines |
| COST3 | storage cost at silos |
| QUALITY | EXTRA REVENUE FOR BETTER ASH QUALITY SHIPPED |
| TOTALCOST | total cost ; |

* upper bound of storage capacity of mine i

$XS.UP(I,T) = SM(I);$

* upper bound of storage capacity of silo j

$$US.UP(J,T) = SS(J);$$

* QUALITY LIMIT ON % ASH CONTENT IN COAL TO CUSTOMER K IN PERIOD
T

$$ASH1.UP(K,TAU) \$ D(K,TAU) = ASHUP(K);$$

* QUALITY LIMIT ON % ASH CONTENT IN COAL TO CUSTOMER K IN PERIOD
T

$$ASH1.LO(K,TAU) \$ D(K,TAU) = ASHLO(K);$$

* lower limit on % sulphur content in coal to customer k in period t

$$SUL.LO(K,TAU) \$ D(K,TAU) = SULLO(K);$$

* upper limit on % sulphur content in coal to customer k in period t

$$SUL.UP(K,TAU) \$ D(K,TAU) = SULUP(K);$$

\$\$TITLE EQUATION SPECIFICATION

EQUATIONS

XSC(I,T) amount of coal from mine i in period t

THT2WC(J2W,T) amount of coal (j2w->j2b) at J2B

THT2BC(J2B,T) AMOUNT OF COAL (J2W- > J2B) AT J2W
 USJ1C(J1,T) accumulated storage amount in silo j1 during period t
 USJ2WC(J2W,T) accumulated storage amount in silo j2W during period t
 USJ2BC(J2B,T) accumulate storage amount in silo j2B during period t
 DISP(J) dissipation of initial storage at silo j
 DMD(K,T) demand constraint for customer k at time t
 ASHC(K,T) ASH QUALITY CONSTRAINT
 SULC(K,T) SULPHUR QUALITY CONSTRAINT
 WBC1(I,J2W,T) WENTZ BULLIT BALANCING CONSTRAINT
 WBC2(I,J2W,T) WENTZ BULLIT BALANCING CONSTRAINT
 ASHT(K,TAU)
 COST1C shipping & cleaning cost
 COST2C storage cost at mines
 COST3C storage cost at silos
 QUALITYC EXTRA REVENUE FOR BETTER QUALITY SHIPPED
 TOTCC total cost ;

\$STITLE EQUATION DEFINITIONS

XSC(I,T).. XS(I,T) = E =

SUM (TP \$ (ORD(TP) LE ORD(T)), P(I,TP)) -
 SUM (J \$ AIJ(I,J), SUM(TP \$ (ORD(TP) LE ORD(T)),
 SUM (K \$ AJK(J,K), SUM (TAU \$ ((ORD(TAU) GE ORD(TP))
 AND (ORD(TAU) LE MIN (ORD(TP) + 2,6))),
 Y(I,J,TP,K,TAU))))) -
 SUM (J2W \$ AIJ2WK(I,J2W), SUM (T1 \$ (ORD(T1) LE ORD(T)),
 SUM (J2B \$ A2W2B(J2W,J2B),
 SUM (K \$ AJ2BK(J2B,K),

SUM ((T2,TAU) \$ ((ORD(TAU) GE ORD(T1)) AND (ORD(TAU) LE
 MIN(ORD(T1)+ 2,6)) AND (ORD(TAU) GE ORD(T2)) AND
 (ORD(T2) GT ORD(T1))),
 YRT(I,J2W,T1,J2B,T2,K,TAU))))) ;

THT2WC(J2W,T)\$ J2WK(J2W).. THT2W(J2W,T) = E =

SUM (I \$ AIJ2WK(I,J2W),
 SUM (J2B \$ A2W2B(J2W,J2B),
 SUM (K \$ AJ2BK(J2B,K), SUM(TP \$ (
 (ORD(TP) GE (ORD(T) - 2)) AND (ORD(TP) LE ORD(T))),
 SUM((T2,TAU) \$ ((ORD(T2) GE ORD(TP)+ 1)
 AND (ORD(TAU) LE
 MIN(ORD(TP)+ 2,6)) AND (ORD(T2) GE ORD(T))
 AND (ORD(T2) LE ORD(TAU))),
 YRT(I,J2W,TP,J2B,T2,K,TAU)*F*(1 + ALPHA(I,J2W))))) ;

THT2BC(J2B,T).. THT2B(J2B,T) = E =

SUM (J2W \$ A2W2B(J2W,J2B), SUM (I \$ AIJ2WK(I,J2W),
 SUM(K \$ AJ2BK(J2B,K), SUM((T2,TAU) \$ ((ORD(TAU) GE ORD(T))
 AND (ORD(TAU) LE MIN(ORD(T)+ 1,6)) AND (ORD(T2) GE ORD(T)-1)
 AND (ORD(T2) LE ORD(T))),
 SUM (T1 \$ ((ORD(T1) GE MIN(ORD(T2),ORD(TAU)-2)) AND
 (ORD(T1) LE ORD(T2)-1)) ,
 YRT(I,J2W,T1,J2B,T2,K,TAU)*F*(1 + ALPHA(I,J2W))))) ;

USJ1C(J1,T).. US(J1,T) = E =

SUM (I \$ AIJ1(I,J1), SUM (K \$ AJ1K(J1,K), SUM(TP \$ ((ORD(TP)
GE (ORD(T) - 2)) AND (ORD(TP) LE ORD(T)))
, SUM (TAU \$ ((ORD(TAU) GE ORD(T)) AND
(ORD(TAU) LE MIN(ORD(TP)+ 2 , 6))),
Y(I,J1,TP,K,TAU)*F*(1 + ALPHA(I,J1))))))
+ Q0(J1)
- SUM(TI \$ (ORD(TI) LT ORD(T)),SUM (K \$ AJ1K(J1,K),
Y0(J1,K,TI))) ;

USJ2WC(J2W,T).. US(J2W,T) = E =

SUM (I \$ AIJ2W(I,J2W), SUM (K \$ AJ2WK(J2W,K), SUM (TP \$ ((ORD(TP) GE (ORD(T) - 2)) AND (ORD(TP) LE ORD(T)))
, SUM (TAU \$ ((ORD(TAU) GE ORD(T))
AND (ORD(TAU) LE MIN(ORD(TP) + 2,6))),
Y(I,J2W,TP,K,TAU) *F*(1 + ALPHA(I,J2W))))))
+ Q0(J2W) - SUM(TI \$ (ORD(TI) LT ORD(T)),
SUM(K \$ AJ2WK(J2W,K), Y0(J2W,K,TI))) + THT2W(J2W,T) ;

USJ2BC(J2B,T).. US(J2B,T) = E =

SUM (I \$ AIJ2B(I,J2B), SUM (K \$ AJ2BK(J2B,K), SUM (TP \$ ((ORD(TP) GE (ORD(T) - 2)) AND (ORD(TP) LE ORD(T)))
, SUM (TAU \$ ((ORD(TAU) GE ORD(T))
AND (ORD(TAU) LE MIN(ORD(TP) + 2,6))),
Y(I,J2B,TP,K,TAU)*F*(1 + ALPHA(I,J2B))))))
+ Q0(J2B) - SUM (TI \$ (ORD(TI) LT ORD(T)), SUM (K \$
AJ2BK(J2B,K), Y0(J2B,K,TI))) + THT2B(J2B,T) ;

DISP(J).. Q0(J) = E = SUM (K \$ AJK(J,K), SUM(TI, Y0(J,K,TI)));

WBC1(I,J2W,T) \$ AIJ2WK(I,J2W)..

SUM (J2B \$ A2W2B(J2W,J2B),
SUM (K \$ AJ2BK(J2B,K),SUM ((T2,TAU)
\$ ((ORD(TAU) LE ORD(T) + 2)
AND (ORD(T2) GE ORD(T) + 1) AND
(ORD(T2) LE ORD(TAU))) ,
YRT(I,J2W,T,J2B,T2,K,TAU)))) = L =
SUM (KST \$ AJ2WKST(J2W,KST),
SUM (TAU \$ ((ORD(TAU) GE ORD(T))
AND (ORD(TAU) LE MIN(6,ORD(T) + 2))) ,
Y(I,J2W,T,KST,TAU)));

WBC2(I,J2W,T) \$ AIJ2WK(I,J2W)..

G*SUM (J2B \$ A2W2B(J2W,J2B),
SUM (K \$ AJ2BK(J2B,K),SUM ((T2,TAU)
\$ ((ORD(TAU) LE ORD(T) + 2)
AND (ORD(T2) GE ORD(T) + 1) AND
(ORD(T2) LE ORD(TAU))) ,
YRT(I,J2W,T,J2B,T2,K,TAU)))) = G =
SUM (KST \$ AJ2WKST(J2W,KST),
SUM (TAU \$ ((ORD(TAU) GE ORD(T))
AND (ORD(TAU) LE MIN(6,ORD(T) + 2))) ,
Y(I,J2W,T,KST,TAU)));

DMD(K,TAU)..

D(K,TAU) = E= SUM (J \$ AJK(J,K), SUM (I \$ AIJ(I,J),
SUM (T \$ ((ORD(T) GE (ORD(TAU) - 2)) AND (ORD(T) LE ORD(TAU)))
, ALPHA(I,J)*Y(I,J,T,K,TAU))))
+ SUM (J \$ AJK(J,K), Y0(J,K,TAU) \$ (ORD(TAU) LE NIT)) +
SUM (J2B \$ AJ2BK(J2B,K), SUM (J2W \$ A2W2B(J2W,J2B),
SUM (I \$ AIJ2WK(I,J2W), SUM (T \$ ((ORD(T) GE (ORD(TAU)-2)) AND
(ORD(T) LE (ORD(TAU)-1))), SUM (T2 \$ ((ORD(T2) GE ORD(T) + 1)
AND (ORD(T2) LE MIN(ORD(T) + 2,ORD(TAU)))) ,
ALPHA(I,J2W)*YRT(I,J2W,T,J2B,T2,K,TAU)))))) ;

ASHC(K,TAU) \$ D(K,TAU).. ASH(K,TAU)*D(K,TAU) = E=

SUM (J \$ AJK(J,K),SUM (I \$ AIJ(I,J), SUM (T \$ ((ORD(T)
GE (ORD(TAU) - 2)) AND (ORD(T) LE ORD(TAU)))
, Y(I,J,T,K,TAU)*A(I,T)*BETA(I,J)))) +
SUM (J \$ AJK(J,K),A0(J)*Y0(J,K,TAU)
\$ (ORD(TAU) LE NIT))
+ SUM (J2B \$ AJ2BK(J2B,K), SUM (J2W \$ A2W2B(J2W,J2B),
SUM (I \$ AIJ2WK(I,J2W),SUM (T1 \$
((ORD(T1) GE (ORD(TAU)-2))
AND (ORD(T1) LE (ORD(TAU)-1))), SUM (T2 \$ ((ORD(T2) GE
(ORD(T1) + 1)) AND (ORD(T2) LE MIN(ORD(T1) + 2,ORD(TAU)))) ,
YRT(I,J2W,T1,J2B,T2,K,TAU)*A(I,T1)*BETA(I,J2W)))))) ;

SULC(K,TAU) \$ D(K,TAU).. SUL(K,TAU)*D(K,TAU) = E=

SUM (J \$ AJK(J,K),SUM (I \$ AIJ(I,J), SUM (T \$ ((ORD(T)

GE (ORD(TAU) - 2)) AND (ORD(T) LE ORD(TAU)))
 , Y(I,J,T,K,TAU)*S(I,T)*GAMMA(I,J)))) +
 SUM (J \$ AJK(J,K), S0(J)*Y0(J,K,TAU)
 \$ (ORD(TAU) LE NIT)) +
 SUM (J2B \$ AJ2BK(J2B,K), SUM (J2W \$ A2W2B(J2W,J2B),
 SUM (I \$ AIJ2WK(I,J2W), SUM (T1 \$ (
 (ORD(T1) GE (ORD(TAU)-2)) AND (ORD(T1) LE (ORD(TAU)-1)))
 , SUM (T2 \$ ((ORD(T2) GE (ORD(T1) + 1)) AND (ORD(T2) LE
 MIN(ORD(T1) + 2,ORD(TAU)))) ,
 YRT(I,J2W,T1,J2B,T2,K,TAU)*S(I,T1)*
 GAMMA(I,J2W)))))) ;

ASHT(K,TAU)\$D(K,TAU).. ASH(K,TAU)=E= ASH1(K,TAU) + ASH2(K,TAU) ;

COST1C.. COST1 =E= SUM(J,SUM(I\$AIJ(I,J),SUM(K \$ AJK(J,K), SUM (T, SUM
 (TAU \$ (
 (ORD(TAU) GE ORD(T)) AND (ORD(TAU) LE MIN(ORD(T) + 2,6))),
 Y(I,J,T,K,TAU) * (CSH1(I,J) + CSH2(J,K) + CCL(I,J)))))) +
 + SUM (J2WK(J2W), SUM (I \$ AIJ2WK(I,J2W),
 SUM (J2B \$ A2W2B(J2W,J2B),
 SUM (K \$ AJ2BK(J2B,K), SUM (T1, SUM ((T2,TAU) \$ ((ORD(TAU)
 GE (ORD(T1) + 1)) AND (ORD(TAU) LE MIN((ORD(T1) + 2),6))
 AND (ORD(T2) LE (ORD(T1) + 2)) AND (ORD(T2) GT ORD(T1))) ,
 YRT(I,J2W,T1,J2B,T2,K,TAU) * (CSH1(I,J2W) + CSH2(J2B,K) +
 CSH3(J2W,J2B) + CCL(I,J2W))))))) +
 SUM (J, SUM (K \$ AJK(J,K), SUM (T \$ (ORD(T) LE NIT),
 Y0(J,K,T) * CSH2(J,K)))) ;

COST2C.. COST2 =E= SUM((I,T),XS(I,T)*CSM(I));

COST3C..

COST3 =E= SUM((J,T),US(J,T)*CSS(J)) ;

QUALITYC.. QUALITY =E= SUM(K,SUM(TAU, (ASHUP(K) - ASH1(K,TAU)) *
D(K,TAU)*(-1)*RA(K) + ASH2(K,TAU) * D(K,TAU)*PA(K))) + SUM(K, SUM(TAU,
(SULUP(K) -SUL(K,TAU)) * D(K,TAU) * (-1)* RS(K))) ;

TOTCC.. TOTALCOST =E= COST1+ COST2+ COST3 + QUALITY ;

\$STITLE MODEL SPECIFICATION

MODEL WESTMORE3 PRODUCTION PLANNING /ALL/;

OPTION LP=MINOS5;

OPTIONS SYSOUT=ON, LIMCOL=0,LIMROW=0 ;

SOLVE WESTMORE3 USING LP MINIMIZING TOTALCOST;

DISPLAY Y.L;

DISPLAY YRT.L ;

DISPLAY Y0.L;
DISPLAY XS.L ;
DISPLAY US.L;
DISPLAY ASH.L;
DISPLAY SUL.L;
DISPLAY COST1.L;
DISPLAY COST2.L;
DISPLAY COST3.L;
DISPLAY QUALITY.L;
DISPLAY TOTALCOST.L;

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Vita

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