THE EFFECTS OF SHAPED PIEZOCERAMIC ACTUATORS ON THE EXCITATION OF BEAMS

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Abstract

The effect of the shape of piezoceramic actuators on the vibration response of a simply supported beam is investigated. An equation is derived to convert between the shape of the piezoceramic actuator and the resulting moment distribution caused on the structure. A beam simulation program is then created to model the vibrations caused by various shaped moment distributions exciting a simply supported beam. The length of the moment distribution is iterated from the length of the beam to zero length, within the program, to show the trends in modal amplitudes. The amplitude of each mode is then plotted for each length of the moment distribution.

An equation is then derived to explain the resulting minimums and maximums of the modal amplitudes. The equation is shown to be a useful tool in designing shapes to meet specific control criteria. An example is given showing how the shape of the actuator can be designed to give superior performance for specific control criteria than a traditional rectangular shape. Two possible actuator shapes are shown for the situation. One shape is optimized for the given control criteria by causing the maximum response for the critical mode. The results from the beam simulation for both shapes are shown. The
shape of the actuator may now be used as a variable in the cost function for control optimization.
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Finally, I would like to dedicate this work to the memory of my brother, Jeff. I will always cherish the time we spent together. I deeply wish we could have had more.
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Nomenclature

Roman Letters:

\( b_a \) \hspace{1cm} \text{actuator width}
\( c \) \hspace{1cm} \text{distance from center of actuator to the edge in beam length direction}
\( d_{31} \) \hspace{1cm} \text{voltage to strain piezoelectric constant}
\( E_a \) \hspace{1cm} \text{Young's modulus of the actuator}
\( E_b \) \hspace{1cm} \text{Young's modulus of the beam}
\( f(x,t) \) \hspace{1cm} \text{external force on beam}
\( L \) \hspace{1cm} \text{length of beam}
\( l \) \hspace{1cm} \text{length of moment distribution}
\( M \) \hspace{1cm} \text{bending moment}
\( M_{eq} \) \hspace{1cm} \text{equivalent bending moment}
\( M_i \) \hspace{1cm} \text{generalized mass}
\( m(x) \) \hspace{1cm} \text{mass per unit length}
\( \eta \) \hspace{1cm} \text{moment distribution}
\( q_i \) \hspace{1cm} \text{generalized coordinate}
\( t \) \hspace{1cm} \text{time}
\( t_a \) \hspace{1cm} \text{thickness of piezoceramic actuator}
\( t_b \) \hspace{1cm} \text{thickness of beam}
\( t_s \) \hspace{1cm} \text{thickness of shear bonding layer}
\( V(t) \) \hspace{1cm} \text{voltage applied to actuator}
\( x,y,z \) \hspace{1cm} \text{Cartesian coordinates}
**Greek Letters:**

- $\phi$: mode shape
- $\Lambda$: actuation strain
- $\lambda$: spatial wavelength
- $\omega_i$: natural frequency
- $\Psi$: non-dimensionalized property relating the thickness and stiffness of the beam and actuator

**Subscripts:**

- $a$: actuator
- $b$: beam
- $eq$: equivalent
- $i$: mode number
- $s$: shear bonding layer
Chapter 1

Introduction

1.1 Motivation

Piezoceramic materials have been widely investigated for use in both vibration and noise control because they strain when excited by an electric field. When piezoceramic actuators are bonded to or embedded in a structure, the strain is induced in the structure which induces forces onto the structure. This fact has led to research in applications involving piezoelectric actuators, including active vibration control of large space structures (Bailey and Hubbard, 1985, Crawley and de Luis, 1987), and active acoustic control of plates and beams (Clark, Fuller, and Burdisso, 1991). Other likely applications involve controlling acoustical noise on a cylinder in an underwater environment (Sumali, Cudney, and Vipperman, 1992) and noise reduction in jet engines, pumps, turbines, and automobiles (Wang, 1991).

Traditionally, when controlling structures by bonding piezoceramic actuators to the surface or embedding them within the structural members, only rectangular shaped actuators have been used, while little work has been done exploring other shapes. Shaped actuators potentially have several advantages over rectangular actuators. They may lower stress concentrations on the surface to which they are bonded. This can be very important in controlling composite structures where a high stress concentration may cause delaminations. Shaped piezoceramic actuators may also be able to control certain modes of vibration more effectively, especially when actuator location and size may be
physically limited. Finally, there may be less control spillover when using shaped piezoceramic actuators, allowing certain modes to be excited while other modes are not.

1.2 Objective

The objective of this work is to investigate the effects of the shape, length, and location of piezoceramic actuators on the resulting vibration of simply-supported beams.

1.3 Approach

The approach taken is as follows. Computer simulations will be used to model the response of a simply-supported beam excited by piezoceramic actuators of various shapes, lengths, and locations. Modal amplitudes will be acquired from the simulation. Then, a derived beam vibration equation which calculates the beam response from the moments caused by a voltage applied to the piezoceramic actuator will be simplified and used to explain the maximums and minimums of modal amplitudes from the simulation. Using these observations we will establish the capabilities of using shaped actuators to control modal vibrations.

1.4 Previous Work

Crawley and de Luis developed static and dynamic uniform strain models for one-dimensional piezoceramic patches bonded to the surface or embedded in the body of beams (Crawley and de Luis, 1987). They showed that the stress in the beam caused by perfectly bonded actuators attached symmetrically to both sides of the beam can be
modelled by two concentrated moments acting at the centerline of the beam at the edges of an actuator. They also showed that piezoceramics can be used extensively on a structure while having a minimal effect on its structural properties. They also experimentally verified their model. The uniform strain model developed by these researchers is the model used in this research.

Several researchers have worked with shaped sensors and actuators. Burke and Hubbard applied a spatially shaped distributed actuator for vibration control of a simply-supported beam and analyzed it experimentally and analytically. They implemented a non-symmetric linearly varying spatial distribution that facilitates control of both even and odd vibrational modes. This configuration ensured that all modes will be excited. They did not explore other shapes (Burke and Hubbard, 1987).

Lee and Moon developed equations for modal sensors which are applicable when the sensor covers the entire length of the beam (Lee and Moon, 1990). They also discussed the reciprocal relationship between sensors and actuators. A sensor shape that senses only certain modes will excite only those same modes when used as an actuator.

Clark, et al. (1992) developed shaped PVDF sensors for acoustic control. They minimized the mean squared error for each mode as their control criterion. They give two separate ways of doing this which yield the same result. Their finalized acoustic sensor is very close to a first mode sensor. The biggest problem with their results is that the sensor magnifies the higher modes too much for a small error in the shape. Their work only deals with sensors covering the entire length of the beam. Using the
reciprocal relationship first noted by Lee and Moon (1990), this work on sensors could be applied to actuators.

Collins, et al. (1992) investigated piezoelectric modal sensors and showed that they perform as well as typical estimators for the flexible states and require less computational overhead. They shape the sensors so that the output when bonded to a structure is selectively proportional to particular deformation patterns. All the work is done with sensors spanning the entire length of the beam.

Burke, et al. (1993) extend the concept of colocation to distributed piezoelectric sensors and actuators. They show that sensors and actuators are colocated only when they have physically coincident spatial apertures and their spatial derivative orders are equal. They prove this mathematically and present several examples showing how this theory is applied.

Kim, et al. (1993) analyzed various shaped piezo-actuators (rectangular, rhombus, triangular, circular, and elliptic) in terms of equivalent forces and wavenumber spectra to develop a basic understanding of the influence of actuator shape on modal characteristics of a flat plate. They assumed that piezo-actuators cause discrete moments along the edges of the actuators for all shapes as is shown in Fig. 1.1.

![Moment Distribution](image)

Figure 1.1 Moments Shown at Edges of Actuator
They also discuss what dimensions a rectangular actuator must be to suppress specific wavenumbers. Specifically, the length of the actuator must equal a multiple of the spatial wavelength of the mode. The modal responses from the other shapes are discussed although explanations for the responses are not proposed. There was no investigation of general actuator shapes.

In the past, research has focused on rectangular actuators, and more generally shaped actuators that span the entire length of the beam. Kim, et al. (1993) did experimental studies of various shaped actuators applied to plates, but did not go into detail exactly how the shape of the actuator affects the modal response and hence the controllability of the structure. The objective of this thesis is to determine the influence of the shape of the moment distribution caused by the shaped actuators. First, we describe how the shape of the actuator affects the moment distribution. Then, we establish relationships between the shape, length, and placement of the moment distribution and the amplitudes of the modes excited. By knowing these relationships, engineers can design actuator shapes as well as choose actuator length and placement for specific applications which will optimize the control of critical modes.
Chapter 2
Theory

This chapter describes the theory used in this work. It begins with a general introduction to how piezoceramic actuators are used to control beam vibrations. The equation for the moments produced at the endpoints of a rectangular actuator perfectly bonded to a beam is given and the variables are defined. Next, the reason the moment distribution is modeled along the centerline of the beam, even though the actual distribution spans the width of the beam, is given. Then, the conversion between actuator shapes and moment distributions is discussed in detail. Finally, the derivation of the equation used to predict the modal response from a given moment distribution is presented.

2.1 General Piezoelectric Theory

Piezoceramic actuators can be used to control vibrations in structures due to their ability to strain when excited by an electric field. Figure 2.1 shows the deformation of a piezoceramic under an electric field. The change in size in the x-axis is shown and is exaggerated for illustrative purposes.

![Figure 2.1 A Piezoceramic Patch Straining Under an Electric Field](image)
The following explains how piezoceramic elements are used to excite beams. Piezoceramic actuators are attached to a structure, usually symmetrically on both sides. They are either bonded to the surface or embedded within the structure. A voltage is then applied to the actuators causing them to strain. The structure constrains the actuator and resists the strain. This causes stress in the structure. Finally, the stress in the structure may be resolved into moments and forces at the neutral axis. The forces cancel when the actuators are symmetrically bonded and excited out of phase, and only the moment remains. Figure 2.2 shows how these moments may be modelled from a pair of symmetrically bonded rectangular actuators attached to a beam.

![Figure 2.2 Moments Caused by Piezoceramics Attached to a Beam](image)

By varying the input voltage to control the moments produced, the vibrations of the beam may be controlled. The extent of the control of certain modes varies greatly with the location, shape, and size of the actuator.

2.2 Moment Equations

Crawley and de Luis (1987) developed equations for rectangular actuators showing that the shear stress from the actuators bonded to a beam is concentrated at the end lines of the actuator and therefore the moments act along the end lines of the actuator. This is important because it makes the moment distribution from the actuators easy to find.
assuming a perfect bond. Figure 2.3 shows the actuator attached to the beam through a bonding layer and defines the dimensions that will appear in the following equations.

\[ M_{eq} = \frac{t^2 E_b}{6 + \Psi} b_a \Lambda, \quad (2.1) \]

where the subscripts \( a \) and \( b \) refer to the actuator and beam respectively, \( t \) is thickness, \( b_a \) is the actuator width, \( E_b \) is Young's modulus of the beam, and \( \Psi \) is a non-
dimensionalized property relating the thickness and stiffness of the beam and actuator, where

$$\Psi = \frac{t_b E_b}{t_a E_a}.$$  \hspace{1cm} (2.2)

The actuation strain, $\Lambda$, is

$$\Lambda = \frac{V(t)d_{31}}{t_a},$$ \hspace{1cm} (2.3)

where $V(t)$ is the voltage applied to each actuator, and $d_{31}$ is the piezoelectric constant relating strain to voltage. For a case where there is just one actuator on one side of the beam, the resulting moment equation is simply one-half the original moment equation. This will also cause a resultant axial force on the beam since there is no canceling force from a symmetrically bonded actuator.

While the shape of the actuator spreads the moment distribution across the width of the beam, the moment distribution in this research is modelled as acting along the centerline of the beam as shown in Fig. 2.4.
This can be done since the moments can be moved anywhere across the width of the beam without affecting the transverse vibrations of the beam. The result is a simplified one-dimensional moment distribution which is easy to implement within the beam vibration program and is also used in the beam vibration equation that will be derived.

The theory in this section gave the main assumptions used in this work. The assumption of a perfectly bonded actuator was presented which allows the moments from the actuator attached to the beam to be modelled as concentrated moments at the endlines of the actuator. The fact that the moments act at the endlines of the actuator is important when finding the moment distribution from different shaped actuators. It was also important to establish that a two-dimensional moment distribution can be converted to one-dimension, which will be used extensively in the other chapters.

2.3 Relationship Between Actuator Shape and Moment Distribution

The theory used in this work is based on moment distributions applied to beams. These moment distributions simulate the moments caused by different actuator shapes. It is important to be able to convert between the moment distribution and the actuator shape and to understand the limitations involved.

In any moment distribution formed by a single actuator, the total positive moment produced equals the total negative moment produced. The reason for this is that the moment produced is proportional to the width of the actuator, as is shown in Eq. (2.1). Also, the width of the actuator causing a positive moment has to be equal to the width causing the negative moment. Therefore, when the moment values are summed, they
equal zero. This is easily seen if the actuator is approximated by a series of rectangles as shown in Fig. 2.5.

![Piezoceramic and Equivalent Rectangular Shape](image)

**Figure 2.5** Piezoceramic Shape Approximated as a Sum of Rectangles

By looking at the moment equations it can be seen that each rectangular strip has equal moments on each side and thus the combined moments have an amplitude of zero. Therefore, any moment distribution with a summed moment of zero has a corresponding shape. Using this observation, moment distributions are directly studied instead of the shape. This is convenient and desirable since the moment distribution more directly relates to how the various modes are affected.

Conversion between the actuator shape and the resulting moment distribution relies on the fact that the shape can be modelled as a series of rectangles. Equation (2.1) shows that the magnitude of the moment at the endlines of a rectangular actuator is proportional to its width, $b_x$. Because the total moment produced is directly proportional to the width of the actuator, the change in width of the shape at a point is proportional to the moment...
at that point. Figure 2.6 shows an example of this if the shape is assumed to be modelled by a series of rectangles when the width of one rectangle is assumed to yield a moment of $M$.

![Figure 2.6 Example of Actuator Shape being Converted to Moment Distribution](image)

The moment at a point is proportional to the width of each differential rectangle. By transforming Eq. (2.1) to the continuous domain the equation for the moment distribution then becomes

$$m(x, t) = CV(t) \frac{db(x)}{dx}, \quad (2.4)$$

where $V(t)$ is the voltage input, $b(x)$ is the function describing the actuator shape, and $x$ is the position along the beam. The constant term, $C$, is derived from combining Equations (2.1) and (2.3) and is given here as

$$C = \frac{t^2 E_b d_{11}}{(6 + \Psi)\nu}, \quad (2.5)$$
which has units of force per length.

The total moment produced must equal zero and is the integration of the moment distribution over the length of the beam, given by

$$M(t) = \int_0^L m(x, t) \, dx = 0,$$

which has the units of force multiplied by length. This expression may also be used to find the total moment acting on any portion of the beam by changing the integration limits.

Examples of how these equations are used to find the moment distribution will be applied to two different shapes. First a simple case will be considered and then a more general case. The first shape is half a sine wave. This has been simplified by making it non-symmetric about the x axis and is shown in Fig. 2.7.

![Diagram of a beam with a sine shaped actuator](image)

*Figure 2.7 Sine Shaped Actuator*
The spatial equation describing this shape is

\[ b(x) = \sin \left( \frac{x}{L} \pi \right). \]  \hspace{1cm} (2.7)

Here \( b(x) \) is in units of length. Substituting Eq. (2.7) into Eq. (2.4) yields,

\[ m(x, t) = CV(t) \frac{d \sin \left( \frac{x}{L} \pi \right)}{dx}, \]  \hspace{1cm} (2.8)

which can be further simplified to

\[ m(x, t) = CV(t) \frac{\pi}{L} \cos \left( \frac{x}{L} \pi \right), \]  \hspace{1cm} (2.9)

which yields the moment distribution shown in Fig. 2.8.

![Figure 2.8 Cosine Moment Distribution from Sine Shaped Actuator](image)

The total moment over the beam goes to zero, which can be seen both by observation and by applying Eq. (2.6).
The second example is more complicated since it deals with both finite and infinite slopes of the shape. The shape analyzed is shown in Fig. 2.9.

![Beam and Actuator Shape Diagram](image)

**Figure 2.9 Actuator Shape with Finite and Infinite Slopes**

The function that describes the shape is given by

$$b(x) = \frac{b_{\text{max}}}{(x_2 - x_1)} [(x - x_1) \cdot H(x-x_1) - (x - x_2) \cdot H(x-x_2)$$

$$- (x_2 - x_1) \cdot H(x-x_2)],$$

(2.10)

where $b_{\text{max}}$ is the maximum width of the shape, $x_1$ and $x_2$ are the positions of the edges of the shape, and $H$ is the Heaviside function defined by

$$H(a) = \begin{cases} 0 & \forall a < 0 \\ 1 & \forall a \geq 0, \end{cases}$$

(2.11)

where $H$ is unitless. Taking the derivative with respect to $x$ of Eq. (2.10) gives
\[
\frac{db(x)}{dx} = \frac{b \max}{(x_2 - x_1)} \left[ H(x - x_1) - H(x - x_2) - (x_2 - x_1)\delta(x - x_1) \\
-(x - x_1)\delta(x - x_1) - (x - x_2)\delta(x - x_2) \right].
\] (2.12)

Here \(\delta\) represents the Dirac delta function defined by

\[
\delta(a) = 0 \quad \forall \quad a \neq 0 \\
= 1 \quad \forall \quad a = 0,
\] (2.13)

and has the units of \(1/\text{length}\). Here it is important to note that the last two Dirac delta functions in Eq. (2.12) will not affect the results since the terms they are multiplied with will always be zero when the Dirac delta function is one, and for all other cases the Dirac delta function will be zero, causing the expressions to go to zero for all cases of \(x\). Substituting Eq. (2.12) into Eq. (2.4) and simplifying yields

\[
m(x,t) = \frac{CV(t)b \max}{(x_2 - x_1)} \left[ H(x - x_1) - H(x - x_2) - (x_2 - x_1)\delta(x - x_1) \\
-(x - x_1)\delta(x - x_1) - (x - x_2)\delta(x - x_2) \right].
\] (2.14)

The resulting moment distribution from Eq. (2.14) is shown in Fig. 2.10.
Figure 2.10 Moment Distribution for Finite and Infinite Slopes

Equation (2.6) yields the following result for the total moment, giving

$$M(t) = \frac{CV(t)b_{\text{max}}}{x_2 - x_1} \int_{x_1}^{x_2} [H(x - x_1) - H(x - x_1) - (x_2 - x_1)\delta(x - x_2)] dx. \quad (2.15)$$

When the integral of the Heaviside function is taken between these two bounds, the result is simply $x$ since the first Heaviside function will equal 1 and the second Heaviside function in the expression will be zero. The integration for the Dirac delta function is 1. Equation (2.15) can then be simplified to

$$M(t) = \frac{CV(t)b_{\text{max}}}{x_2 - x_1} \left[ x_1^2 - (x_2 - x_1) \right] = 0. \quad (2.16)$$

The result is zero, as we expected.

Actuator shapes are not unique. A particular moment distribution can have an infinite number of actuator shapes associated with it. For example, Fig. 2.11 shows a moment distribution and a few of its equivalent shapes.
Figure 2.11 Several Shaped Actuators which Produce Equivalent Moment Distributions

These shapes are simply derived by breaking up the actuator into rectangles as was shown in Fig. 2.5. Because the resulting moments are assumed to act on the centerline of the beam, the rectangular strips can be arranged in any order without affecting the moment distribution. Therefore, when the number of strips becomes infinite, the number of possible actuator shapes for a given moment distribution also becomes infinite.

Figure 2.12 shows examples of various shapes and the resulting moment distributions arrived at using Eq. (2.4). The equivalent non-symmetric shapes are also given.

Figure 2.12 How Actuator Shapes Affect Moment Distribution
As can be seen, the amplitude of the moment distribution is simply proportional to the slope of the shape.

2.4 Relationship Between Moment Distribution and Modal Amplitudes

An equation is derived in the following paragraphs which is used extensively in the next chapter to explain the results from the simulations. The resulting equation is especially useful in predicting the modal amplitudes produced by specific moment distributions.

The governing equation for modal vibrations in beams is derived in a text by Thompson (1988), and is given here as

\[
\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i} \left[ \int f(x,t)\phi_i(x)dx + \int m(x,t)\phi'_i(x)dx \right],
\]

(2.17)

where the subscript \(i\) is the mode number, \(q_i\) is the generalized coordinate, \(\mathfrak{M}(x,t)\) is the moment distribution over the length of the beam as a function of time, \(\omega_i\) is the natural frequency of the mode, and the terms \(\phi_i\) and \(\phi'_i\) are the mode shape and its first spatial derivative respectively. \(M_i\) is the generalized mass and satisfies the equation

\[
M_i = \int_0^L \phi_i^2(x)m(x)dx,
\]

(2.18)
where \( m(x) \) is the mass of the beam per unit length as a function of location, and \( l \) is the length of the beam. \( M_i \) is a constant for each particular mode. Since external forces are not being considered here, the term \( f(x,t) \) goes to zero, which results in,

\[
\ddot{q}_i(t) + \omega_i^2 q_i(t) = \frac{1}{M_i} \int m(x,t) \phi_i'(x) dx. \tag{2.19}
\]

Here, the amplitude of \( q_i \) is

\[
q_{i_{\text{max}}} = \frac{1}{\omega_i^2 M_i} \int m(x,t) \phi_i'(x) dx. \tag{2.20}
\]

The equation that governs the how each point on the beam vibrates with respect to time, \( y(x,t) \), is given by

\[
y(x,t) = \sum_i q_i(t) \phi_i(x) dx. \tag{2.21}
\]

The modal amplitude will be zero when \( q_i \) is zero and the amplitude will be maximum when \( q_i \) is a maximum. Thus, looking at Eq. (2.19), it can be seen that the following applies: for zero actuation of beam mode \( i \), the relationship

\[
\int m(x,t) \phi_i'(x) dx = 0 \tag{2.22}
\]

must hold, and for maximum modal actuation the equation
\[ \int m(x,t) \phi'(x) dx = \text{Maximum} \] 

(2.23)

must be true. In general, the modal amplitude at a mode peak is

\[ \text{Modal Amplitude} = A \int m(x,t) \phi'(x) dx , \] 

(2.24)

where \( A \) is a constant given by

\[ A = q_{\text{max}} \phi_i . \] 

(2.25)

Equation (2.24) is used extensively to explain the maximum and minimum modal amplitudes from the beam program. It will also be used to establish the capabilities of shaped actuators.
Chapter 3

Analytical Study

In the previous chapter, details of how the shape of an actuator affects the moment distribution along a beam were presented. Equation (2.24) was developed to predict the modal responses of a beam from any applied moment distribution. The primary thesis objective is to show how the shape, length, and location of an attached piezoceramic actuator affects the modal vibrations of a beam. This chapter describes the theoretical model, beam simulation program, and the actuator shapes modelled which are used to achieve this objective.

The form of this chapter is as follows. First, the theoretical model will be discussed to explain the assumptions used in this work. Second, a program used to simulate the response of a beam excited by an actuator with an arbitrary shape will be discussed. Finally, the specific moment distributions will be discussed.

3.1 Theoretical Model

A thin simply-supported beam is considered in this work. The mode shapes of this Euler-Bernoulli beam are sinusoidal, which simplifies calculations and gives a better qualitative understanding of the relationship between the moment distribution and modal amplitudes. The piezoceramic actuator is assumed to be thin compared to the beam and does not affect the mode shapes. It should be noted that the theory used here can be adapted to a beam with any boundary conditions provided the mode shapes are known.
The beam program simulates a shaped actuator, perfectly bonded to the beam, excited by an ideal impulse voltage. The impulse voltage ensures that a large bandwidth of frequencies is excited. The beam is nondimensional. The width and thickness of the beam are not important in this work since they will not affect the relative amplitude differences between the modes. The sensor location is chosen near the end of the beam so that it will not be on a node of any of the modes examined. Modal amplitudes are then scaled to their maximum values along the beam.

3.2 Simulation Program

The program was written using the MATLAB language created by the Mathworks, Inc. Different moment distributions are used as inputs to the beam and the resulting modal amplitudes are then calculated. This program, shown in the appendix, works as follows. Assuming the beam properties, the selected moment distribution calculated from the actuator shape, and the displacement location, a state space model is formed. Then, the "lsim" function in MATLAB calculates the response of the beam measured at the sensor location. The program then iterates for separate moment pairs, forming a moment distribution, and sums the responses. This is done discretely in 100 steps to form a smooth moment distribution. From this the FFT of the response is obtained giving the modal amplitudes and the phase. The first eight modes are simulated to give a set for comparison.
3.3 Simulated Actuator Shapes

The first moment distribution simulated is that from a rectangular shaped actuator. The rectangular shape has been the primary shape used to date. Any advantages other shapes may have over the rectangular shape will be considered important. The moment distribution from a rectangular shaped actuator is shown in Fig. 3.1.

![Figure 3.1 Moment Distribution from Rectangular Actuator](image)

Note that the moments are concentrated entirely at the ends of the actuator.

Next, a cosine shaped moment distribution will be implemented within the program. Figure 3.2 shows the actuator shape and moment distribution for a cosine distribution.

![Figure 3.2 Cosine Moment Distribution and Associated Actuator Shape](image)

A cosine moment distribution is used since previous research has shown that when applied to the entire beam, only the first mode will be actuated. This aspect made the
cosine distribution important to simulate. Also, the moment is distributed over the beam unlike the rectangular patch which produces moments only at its ends.

A sinusoidal moment distribution is also implemented due to its close relation to the cosine moment distribution and the sinusoidal mode shapes. The distribution as well as the shape which causes it are shown in Fig. 3.3.

Figure 3.3 Sinusoidal Moment Distribution and Associated Actuator Shape

Symmetric actuators are limited due to the fact that they are symmetric. Non-symmetric actuators may be more flexible and are simulated in this work. The first non-symmetric actuator simulated is one that gives a non-symmetric cosine shaped moment distribution. This was chosen because symmetric cosine distributions are also simulated in this work. The actuator shape and moment distribution are shown in Fig. 3.4.

Figure 3.4 Actuator Shape and Corresponding Non-Symmetric Cosine Distribution
An actuator which gives a non-symmetric sine moment distribution is also simulated. The reasons for choosing this are the same as for the non-symmetric cosine moment distribution. The actuator shape and moment distribution are shown in Fig. 3.5.

![Figure 3.5 Actuator Shape and Corresponding Non-Symmetric Sine Distribution](image)

The simulation results from these moment distributions will be used in conjunction with Eq. (2.24) to explain the modal response and to give ways to predict how a specific actuator shape will affect the modal response. Ultimately, the utility of custom shaping and placing piezoceramic actuators will be understood.
Chapter 4
Results and Discussion

The previous chapter explained the approach used in this work. This chapter explains how the results from the beam simulation are plotted and interpreted. The results from the beam simulations are presented and then explained using Eq. (2.24). Finally, an example is given showing how Eq. (2.24) can be used to shape actuators for specific control criterion.

4.1 Graphical Presentation of Results

The modal information from the beam simulations is presented as follows. The maximum amplitude of each mode when excited by the moment distribution is obtained. The length of the moment distribution is iterated from zero to the length of the beam. The length is then plotted versus each modal amplitude to show trends in the modal response and to show the length of the distribution that causes minimum and maximum excitation of each mode. When the distribution is not centered on the beam, the length of the distribution will vary as much as is physically possible. Figure 4.1 graphically shows how the length of the distribution is varied during the simulation. Notice that the amplitude of the distribution increases as the length of the distribution decreases. This is because the amplitude is in units of moment per unit length. The total moment produced has to be the same regardless of the actuator length since the actuator width is assumed to be constant as the actuator decreases in length. Equal width actuators must produce equal total moments and thus the area under the moment distribution remains constant as the length changes.
4.2 Rectangular Actuator Results

Rectangular Actuator Centered on Beam

Initially, the beam simulation was run simulating a rectangular actuator centered at the middle of the beam. Figure 4.2 shows the results from the simulation as the actuator length is varied from zero to the length of the beam. The relative amplitudes of the modes are not important since they can all be increased the same percentage by increasing the amplitude of the moment distribution. The beam is non-dimensionalized, making the length unity. Notice that the even numbered modes are not excited. The reason for this can be found by looking again at Eq. (2.24) which is

\[ \text{Modal Amplitude} = A \int m(x,t) \phi'(x) dx \]  \hspace{1cm} (2.24)

When the moments multiplied by the derivative of the mode shape, \( \phi' \), evaluated at the moment locations, add up to zero, the modal amplitude becomes zero. Figure 4.3 shows what is happening in detail. For any even numbered mode when the rectangular actuator
Figure 4.2 Rectangular Actuator Excitation
is centered on the beam, the sum of Eq. (2.24) will equal zero since \( y_1 = y_2 \) and the two moments are of equal magnitude and opposite sign.

![Figure 4.3 Rectangular Actuator at Center of Beam for an Even Mode](image)

In the aforementioned case, Eq. (2.24) simplifies to

\[
y_1 \cdot (-M) + y_2 \cdot (+M) = 0. \tag{4.1}
\]

For any case in this situation, Eq. (2.24) will equal zero and the even numbered modes will not be actuated. A more general observation is that a particular mode will not be actuated when the center of the rectangular actuator is at a peak of \( \phi' \), or more simply at a node of the mode shape, \( \phi \).

For odd modes, the specific result where the rectangular actuator is centered on the beam looks like Fig. 4.4.
Figure 4.4 Rectangular Actuator at Center of Beam for an Odd Mode

In this case Eq. (2.24) will not be zero because $y_1 = -y_2$ and the moments are opposite in sign. Equation (2.24) simplifies to

$$y_1 (-M) + y_2 (+M) = -2 y_1 M.$$  

(4.2)

Therefore, all of the odd modes will always be actuated when the rectangular actuator is centered on the beam as long as the ends of the actuator do not fall on the zero points of $\phi'$, causing $y_1$ and $y_2$ to go to zero and forcing Eq. (4.2) to zero.

Looking at each odd mode in Fig. 4.2, maximums and minimums can be seen. Figure 4.5 shows the relationship between actuator size and $\phi'$ for when odd modes will not be actuated. When the moments are on the nodes of $\phi'$, $\phi'$ is zero, forcing Eq. (2.24) to go to zero. Thus, the particular mode will not be affected. Looking at Fig. 4.2 for mode 3, the insensitivities (points of zero actuation) are at actuator lengths of 0.67 and zero.
Figure 4.5 Lengths of Rectangular Actuators that Cause Zero Actuation for Odd Modes

Figure 4.6 shows $\phi$ and $\phi'$ for the 3rd mode and shows the actuator length for the insensitivity.

Figure 4.6 The 3rd Mode with Actuator Length for the Insensitivity

The other odd modes follow the same rules as the 3rd mode for their insensitivities.

The resulting equation for the length where insensitivities occur for odd modes is

$$L = 1 - \frac{1 + n}{\text{mode #}}, \quad (n=0, 2, \ldots, \text{mode # - 1}).$$

(4.3)
Equation (4.3) is specific and applies for a rectangular actuator placed in the center of the beam. The beam is non-dimensional, making its length unity.

Figure 4.2 also shows actuator sizes that cause maximum modal amplitudes. These occur when the actuator is placed so the moments occur at the peaks of $\phi'$. Figure 4.7 shows the actuator size for the maximum excitation of mode 3.

![Figure 4.7 Mode 3 Shown Actuated at Maximum Amplitude](image)

When placed at the peaks of $\phi'$, Eq. (2.24) is maximized because the moment, M, is constant and the amplitudes of $\phi'$ at the moment locations are at their peaks. The length of the actuator that causes maximum actuation for a specific odd mode when the actuator is at the center of the beam is

\[
L = \frac{1 + n}{\text{mode} \#}, \quad (n=0, 2, \ldots, \text{mode} \# -1). \tag{4.4}
\]

The case for when the actuator is centered at the middle of the beam is very specific. The center of the patch is at a peak of $\phi'$ for even modes and at zero for odd modes.
General Rectangular Actuator Placement

While it is easy to see what happens for the case where the rectangular actuator is centered on the beam, it is more complicated for the general case. Figure 4.8 shows a general case where the left edge of the actuator is placed anywhere on the beam and the possible locations for the right edge that will not cause actuation of the mode are shown.

![Diagram of rectangular actuator placement](image)

Figure 4.8 Rectangular Actuator Moment Placement to Minimize Modal Amplitude

This case deals with both the length of the actuator and the location. To cause an insensitivity it is necessary to make the length of the actuator

\[ L = n\lambda, \quad (n=1, 2, \ldots) \]  

(4.5)

for any case, regardless of actuator placement, where \( \lambda \) is the spatial wavelength of the mode shape. An actuator of this length will never excite the given mode regardless of its placement. If the location of the left edge of the actuator is known (i.e. the distance \( a \) is known) there are other sizes that cause insensitivities. Alternate lengths are
\[ L = n\lambda + 2a, \quad (n=1, 2, \ldots) \] (4.6)

where \( \lambda \) is the same as in Eq. (4.5) and \( a \) is the distance from the left edge of the actuator to the peak of \( \phi' \). These placements cause zero actuation because Eq. (2.24) goes to zero. The amplitudes of the \( \phi' \)'s are equal and the moments are of equal magnitude and opposite in sign, effectively canceling the mode.

Maximizing a specific mode with a rectangular actuator is simple. It is maximized when the moments fall on the maximum of \( \phi' \). Figure 4.9 shows graphically where the moments must fall to maximize the mode, assuming that the location of the left edge of the actuator is known.

![Figure 4.9 Maximizing Modes with Rectangular Actuator](image)

The equation for the length of the patch needed to maximize the particular mode is

\[ L = (n - 1/2)\lambda + a, \quad (n=1, 2, 3, \ldots), \] (4.7)
where a is the distance to the nearest peak of $\phi'$. For this case patch placement and size are important.

The beam vibration program was run to show a more general case of actuator placements. Figure 4.10 shows results from a rectangular patch with the right edge held constant at 0.9 and the length of the actuator was allowed to vary from zero to 0.9. The output from the program agrees with the previous analysis. Looking at mode 3 it is seen that there are insensitivities for lengths of 0.47, and 0.67. There are maximums at 0.23 and 0.9. Figure 4.11 shows mode 3 graphically and the actuator lengths that cause the insensitivities.

![Figure 4.11 General Case Showing Insensitivities on Beam for Mode 3](image)

Equations (4.6) and (4.7) are shown above in Fig. 4.11 and these are verified by the beam program.

Figure 4.12 shows the actuator lengths that cause maximum excitation of mode 3. As can be seen, the maximums occur when the left edge is on the peaks of $\phi'$ opposite in sign from the right edge.
Figure 4.10 Rectangular Actuator Excitation, Right Edge Held at 0.9
4.3 Cosine Moment Distribution

The beam simulation was modified to using a cosine moment distribution where the area under the moment was held constant as the length of the distribution varied from zero to the length of the beam. Figure 4.13 shows the modal amplitude results from the program. As can be seen, the even modes are not actuated. Figure 4.14 shows graphically why the even modes are not excited for this case.

Figure 4.12 Actuator Lengths for Maximizing Mode 3 for Right Edge Held at 0.9

Figure 4.14 Cosine Moment at Center of Beam cannot Excite Even Modes
Figure 4.13 Cosine Moment Distribution Actuation
From Eq. (2.24), the integration between $\phi'$ for an even mode and the cosine moment distribution centered on the beam will always be zero, regardless of the length of the distribution. The reason can be found by looking at Fig. 4.14. The values of the integration under the curves with the "+" sign will always cancel the areas with the "-" sign since the cosine distribution is symmetrical and is centered at the peaks of $\phi'$ for all even modes.

Figure 4.13 shows an insensitivity for mode 3 at a length of 1. Figure 4.15 shows graphically why this is the case.

![Diagram showing integration cancels]

**Figure 4.15 Moment Distribution Causing Insensitivity for Mode 3**

Here the positive and the negative results of the integral from Eq. (2.24) cancel. A numeric integration was run to show this and it was also solved mathematically. This is a very special case in which the distribution covers the entire beam, making analytically solving the integration straightforward. Because this is a special case where only mode 1 is actuated to zero, the analytical results will be shown here. The equation for the integral of the moment multiplied by the derivative of the mode shape, Eq. (2.24), is
\[ A \int M(x) \phi'(x) dx = \text{modal amplitude}. \quad (2.24) \]

Due to symmetry, only one-half of the distribution will be analyzed, simplifying the equations. The constant, \( A \), can be disregarded for this case. The specific equation for the amplitude of an odd mode for the left half of the moment distribution on the beam becomes

\[ \int_0^\pi \cos(x) \cos(nx) dx, \quad (4.8) \]

where \( n \) is the mode number, in this case an odd mode. The result is easily integrated, which yields

\[ \frac{\sin((n-1)x)}{2(n-1)} + \frac{\sin((n+1)x)}{2(n+1)} \bigg|_0^\pi. \quad (4.9) \]

Further simplification results in

\[ \frac{2\sin((n-1)\pi) + \sin((n+1)\pi)}{(n-1)(n+1)} - \frac{2\sin 0 + \sin 0}{(n-1)(n+1)} = 0. \quad (4.10) \]

Therefore none of the odd modes, except mode 1, are actuated when the length of the cosine distribution covers the entire beam. When mode 1 is examined, Eq. (4.8) simply becomes.
\[ \int_0^\pi \cos^2(x)dx, \]

which does not go to zero.

Looking again at Fig. 4.13, it can be seen that the maximum actuation of the 3rd mode occurs when the length of the actuator is 0.45. Figure 4.16 shows graphically the length of the distribution for the maximum modal response in relation to the moment distribution and \( \phi' \).

![Figure 4.16 Moment Placement to Maximize 3rd Mode Actuation](image)

By running a numerical integration of \( \int M(x)\phi'(x)dx \), this was shown to be the length of the moment distribution that maximized the integration, matching the results from the simulation.

The other odd modes follow the same rules for their maximums and insensitivities. Figure 4.17 shows the derivative of the mode shape for the 5th mode, along with the actuator
placement for the maximums and insensitivities. Again, these results were verified by performing a numerical integration.

![Diagram showing modal insensitivities and derivative of mode shape](image)

Figure 4.17 Moment Distributions Causing Maximums and Insensitivities for Mode 5

4.4 Sine Moment Distribution

The properties of sine moment distributions were also examined. Figure 4.18 shows the results when the moment distribution was centered at the middle of the beam and the length was allowed to vary from zero to the length of the beam.

The first thing to note is that the even modes are not actuated for the same reasons as the case involving the cosine moment distribution. Figure 4.19 shows why even modes are not actuated. The particular even numbered mode is not important since the center of the moment distribution will always be at a peak of $\phi'$. Even modes will not be actuated for
Figure 4.18 Sine Moment Distribution Actuation
any distribution length, since the integration of the left and right sides will cancel regardless of the length, due to symmetry.

![Diagram of Moment Distribution](image)

**Figure 4.19** Even Mode Shown with Sine Distribution

Next, the reasons for the maximum excitation of each mode in Fig. 4.18 are important. The peaks will occur when Eq. (2.24) is maximized. Figure 4.20 shows the situation that maximizes the excitation of mode 3.

![Diagram of Sine Moment Distribution](image)

**Figure 4.20** Sine Moment Distribution Causing Maximum Actuation of Mode 3

Using a numerical integration yielded the same results as the simulation. At the distribution shown, Eq. (2.24) is at a maximum. The other odd modes behave in the same manner.
Next, the points of zero actuation will be explained. Again looking at Fig. 4.18, it is seen that the insensitivity for mode 5 lies at 0.8, and the insensitivities of mode 7 lie at 0.57 and 0.85. Figure 4.21 explains the insensitivities for mode 5 and the first insensitivity for mode 7.

![Figure 4.21 Sine Distribution Resulting in Zero Actuation](image)

Here the minimum occurs because Eq. (2.24) goes to zero. The positive and negative integrations cancel due to the symmetry of $\phi'$ and the moment distribution. The zero actuation point for mode 7, where the distribution length is 0.85, has a slightly different explanation. Figure 4.22 shows this situation.

![Figure 4.22 The Second Minimum for Mode 7](image)
The integration cancels on each side of the beam causing zero actuation at the distribution length of 0.85. Another important trend is that after the first insensitivity for mode 7, and as the length of the distribution expands, the amplitude remains oppressed. This can be seen by looking at the left half of the previous figure which is shown in Fig. 4.23.

![Diagram showing the derivative of mode 7 and sine moment distribution as length expands.]

Figure 4.23 Left Half of Mode 7 and Sine Moment Distribution as Length Expands

As the length varies, there will be little effect on the resulting value of Eq. (2.24). Quadrant 1 will cause little excitation since the amplitude of the moment distribution is low and $\phi'$ is small. The other quadrants will change minimally, causing little change in the modal amplitude when the length of the actuator varies.

4.5 Non-Symmetric Actuators

Symmetrically shaped actuators exhibit limited performance because their ability to selectively excite (or sense) modes is limited. There are more possible shapes of non-symmetric actuators, giving them more flexibility in selective modal excitation. Non-
symmetric actuators were implemented within the beam simulation program to compare their responses to that of symmetric shaped actuators.

Figure 4.24 shows how the non-symmetric moment distributions are modelled by the beam simulation.

![Cosine moment distribution](image)

**Figure 4.24 Non-Symmetric Distribution Simulation Model**

In this example, the right edge is held at 1, and the left edge is varied from zero to the midpoint of the beam, giving a range of modal amplitude results that can be plotted.

**Non-Symmetric Cosine Moment Distribution**

A cosine moment distribution was implemented in the beam simulation for the same reasons as the symmetric case. Figure 4.25 shows the simulation results from a non-symmetric cosine distribution with the right half held constant and the left half varied. The maximums and minimums on the graph can be explained once again by Eq. (2.24). Nevertheless, a few of the characteristics of the modal response will be shown in detail and discussed. Looking at mode 4, there is an insensitivity for an actuator length of 0.7. Figure 4.26 shows this case in detail.
Figure 4.25 Cosine Moment Distribution Actuation

a. Modes 1-4
b. Modes 5-8
Figure 4.26 Moment Distribution for Zero Actuation of Mode 4

Again, the integration from Eq. (2.24) is zero. The different partitions of the areas of integration are marked by whether the integration result for the section is positive or negative. They all cancel out for this distribution length, causing zero actuation for mode 4.

Looking again at Fig. 4.25, it is seen that the maximum of mode 4 occurs for an actuator length of 0.5. Here Eq. (2.24) is at a maximum for this mode. Figure 4.27 shows the situation in detail.

Figure 4.27 Actuator Length for Maximum Actuation of Mode 4
Here, the left side of the actuator has all of its moment concentrated on the peak of $\phi'$ at the center of the beam, causing Eq. (2.24) for the concentrated moment to be at a maximum. The right half never changes, and thus the modal response is a maximum at this length.

**Non-Symmetric Sine Moment Distribution**

A non-symmetric sine distribution is also implemented in the beam simulation. The length is iterated in the same way as the non-symmetric cosine distribution. The modal results are shown in Fig. 4.28. Looking at mode 4, there is a minimum where the length is 0.83. Figure 4.29 shows this case in more detail.

![Figure 4.29 Actuator Length that Causes Zero Actuation of Mode 4](image)

Again, for the length shown, Eq. (2.24) goes to zero.

The maximum excitation of mode 4 occurs when the actuator length is 0.5. Figure 4.30 shows this situation in detail.
Figure 4.28 Sine Moment Distribution Actuation
a. Modes 1-4
b. Modes 5-8
Figure 4.30 Actuator Length that Causes Maximum Actuation of Mode 4

Again, the results can be explained using Eq. (2.24). The maximum of Eq. (2.24) for mode 4 occurs at a length of 0.5. The left half of the moment distribution is at a peak of the mode shape, causing a negative maximum of the integration. The right half is constant, giving a negative result in the integration. This causes an overall maximum in the modal excitation.

4.6 Example for Choosing Actuator Shapes and Locations

An example will now be given to show how shaped actuators may be useful. This also goes through the process of using Eq. (2.24) to choose a suitable shape that will meet the given criteria.

Suppose the 4th mode of a beam must be excited while not affecting the 2nd mode. This presents problems if a single rectangular actuator is being used. A rectangular actuator cannot excite the 4th mode without also exciting the 2nd mode. Figure 4.31 shows the situation.
The rectangular actuators producing the moment pairs shown that cause zero actuation of the 2nd mode must also cause zero actuation of the 4th mode. Shaped actuators allow the 4th mode to be excited, while the 2nd mode is not affected. Figure 4.32 shows the moment distribution from an actuator that will excite the 4th mode while not exciting the 2nd mode.
Looking first at the 2nd mode, it is seen that there will be no actuation. The moment on the left is at a zero of $\phi'$, causing no actuation of the mode. The moment distribution on the right is symmetric about a zero point of $\phi'$, with negative values to the right and positive values to the left. Here Eq. (2.24) goes to zero causing no actuation of the mode. Looking at mode 4, the left moment will cause the maximum actuation possible since it is a concentrated moment at a peak of $\phi'$. The moment distribution on the right will tend to lessen the actuation, since the result of Eq. (2.24) will be opposite in sign of that for the left half. However, since the moments are not concentrated at a peak, the right half will not fully cancel the left half. The result will be actuation of mode 4 without actuating mode 2, although the amplitude of mode 4 will be small compared to the following solution.

A solution which will give a better actuation of mode 4 is shown in Fig. 4.33.

Figure 4.33 Mode 4 Actuated without Actuating Mode 2
Mode 2 will not be actuated since the left moment is on a zero of $\phi'$ and the response from the other moments cancel due to $\phi'$ being equal and opposite for each moment. Using Eq. (2.24) shows that there will be zero actuation of mode 2. The 4th mode will be excited more than in the previous example. All of the moments are placed on peaks of $\phi'$, causing the maximum actuation of mode 4. Both these situations were implemented in the beam program and the results are shown in Fig. 4.34 and Fig. 4.35. As predicted, the amplitude of the 4th mode is much greater for the second case. The 2nd mode is not actuated in either case. Also notice that the other modes shown vary widely in both examples. Depending on the needed criteria, the output from shaped actuators is predictable and adjustable.

The beam simulation takes a lot of computational time. Equation (2.24) can be used in place of the simulation to achieve the same output very quickly. A simulation was run using the equation in the same iterative scheme as in the beam simulation. The integration of Eq. (2.24) was taken for varying actuator lengths for a cosine moment to compare with the output of the beam simulation. The results are shown in Fig. 4.36. Compared with Fig. 4.13, it is seen that both results are nearly identical. Therefore, application of Eq. (2.24) may be used instead of the beam simulation to predict the results of the actuator shapes on the beam vibrations, greatly reducing the computational time.

When optimization of control is needed, Eq. (2.24) can be used to create a cost function. In this case, the cost function would be the modal amplitudes desired. The optimization loop would include the shape, location, and length of the piezoceramic actuator as variables and would minimize the cost function. This allows more flexibility and lower cost functions than previously possible.
Figure 4.34 Peaks of FFT for Example 1
Figure 4.35 Peaks of FFT for Example 2
Figure 4.36 Cosine Moment Distribution Plot Using Equation
4.7 Summary

This chapter showed the results of the beam simulation for rectangular actuators, symmetric sine and cosine moment distributions, and non-symmetric sine and cosine moment distributions. Equation (2.24) was used to explain the modal results for each simulation, proving that the equation is accurate. It may also be implemented in place of the beam simulation to decrease computational time. A beam control problem with specific criteria that could not be solved with a traditional rectangular actuator was presented, testing the shaped actuator rules presented in this work. Two possible solutions for shaped actuators were offered using Eq. (2.24). The better solution was predicted and the reasons discussed. The beam simulation was run and the results were as expected. It is now possible to use shape of the actuator as a variable for optimization.
Chapter 5

Conclusions and Recommendations

5.1 Conclusions

The purpose of this research was to study the effects of shape, size, and location of piezoceramic actuators on the modal response of beams, and to develop tools used to design actuator shapes to meet specific control criteria.

To accomplish these goals, an equation was derived to convert between the actuator shape and the corresponding moment distribution. Then, a beam simulation was created which models a simply supported beam excited by moment distributions caused by shaped actuators, and predicts the resulting modal amplitudes. Different moment distributions were simulated, including the moment distributions from traditional rectangular shaped actuators, cosine moment distributions, and sine moment distributions. Both symmetric and non-symmetric shaped actuators were simulated. The modal responses were obtained and another equation was derived to explain the maximums and minimums for each mode. This equation allows specific modal amplitudes to be extracted for a specific moment distribution without resorting to a full scale simulation which saves computational time. It is extremely useful in designing the shape and size, as well as the placement of the actuator to meet control criteria.

Shaped actuators can produce modal responses in beams not possible using rectangular actuators. An example was given that shows how the shapes of actuators were chosen to actuate the 4th mode while not actuating the 2nd mode, which cannot be done using a
rectangular actuator. The moment distribution from a shaped actuator can be distributed over a portion of a structure. This gives it more flexibility in meeting control criteria than a rectangular actuator with moments concentrated only at the edges. Using this theory, the shape of the actuator may now be made part of the design space for optimizing actuator configurations.

5.2 Recommendations

For better understanding of how shaped piezoceramic actuators affect the modal amplitudes of beams, the following work should be undertaken. Experimental testing must be done to test the analytical results presented in this work. This theory should be applied to plates to increase the number of applicable structures. Finally, shaped piezoceramic actuators should be applied to specific control systems and the optimization problem should be formulated.
References


Appendix A

The beam simulation program developed for this work is presented in this appendix. It was written using a standard text editor and is designed for use on Matlab. The program shown is for a cosine moment distribution. The programs for sine, rectangular, and non-symmetric distributions are similar with only slight changes in the function for the shape of the moment distribution. The program gives results for the matrix [peak3] which contains the peak amplitudes for the first 8 modes for all the different length of moment distributions.
Bcosmax.m

% bcosmax1  cosine moment distribution
% This program calculates the natural frequencies for a simply supported beam using
% Benoulli-Euler beam theory, uses these to form the A, B, C, and D matrices, and then
% computes the response to excitation by piezoelectric moment pairs for a cosine shaped
% moment distribution.
%
% ybe is the total response, xbe is the response of each mode
%
tottime=.2048;  % total time for simulation
step=.0002;  % time step for simulation
modes=8;  % number of modes simulated
npts=tottime/step;  % number of points in simulation
format short e;
%
% Beam properties
% Aluminum
rho=2710;  % density (kg/m^3)
bmlength=.452;  % beam length (m)
thick=.003/bmlength;  % thickness
width=.04/bmlength;  % width
ernod=69000000000;  % modulus of elasticity
actwidth=.04/bmlength;  % width of actuator
mid=.25;  % actuator midline
nummom=100;  % number of divisions of actuator
l=.7;  % non dimensional length of actuator
senpos=[.25]';  % position of displacement sensor
deltab=actwidth/nummom;  % calculate width of each rectangle
tb=.003/bmlength;  % thickness of beam
d31=190e-12;  % piezoelectric constant
ta=.001905;  % thickness of actuator(m)
eact=6.3e10;  % modulus of actuator
ps/-tb/ta*ernod/eact;  % effective stiffness ratio
sumy=0;  % set summation to 0
sumx=0;  % set summation to 0
dmom=.0001;  % distance force apart to simulate moments
lstep=step;  % length of time for pulse
mdist=0;  % set moment distribution saving to zero
dist=0;  % set distance for each moment to zero
%
% Calculated properties
area=thick*width*bmlength^2;  % cross-sectional area
inertia=1/12*width*thick^3*bmlength^4;  % moment of inertia
radgyr=inertia/area;  % radius of gyration
mass=rho*area; % mass per unit length (kg/m)
evcoeff=sqrt(2/(mass*bmlength)); % mass normalized eigenvector coefficient
zeta=.001; % damping coefficient

% Loop for Bernoulli-Euler frequencies
for n=1:1:modes:
    wbe(n)=n^2*pi^2*sqrt(emod*inertia/(mass*bmlength^4));
end;
%
% Form matrices for Bernoulli-Euler
%
% Form A matrix
abe11=zeros(modes,modes); % set submatrix (1,1) to zero
abe12=eye(modes,modes); % set submatrix (1,2) to identity
abe21=-diag(wbe.^2); % set submatrix (2,1) to natural frequencies
abe22=-diag(2*zeta*wbe); % set submatrix (2,2) to damping
abe=[abe11 abe12;abe21 abe22]; % put it all together
%
% Form B matrix for distribution
nact=4; % number of actuators
bbe1=zeros(modes,nact); %
%
% Form C matrix for point sensor
nsen=length(senpos); % number of sensors
for ns=1:1:nsen;
    for m=1:1:modes;
        cbel(ns,m)=evcoeff*(sin(m*pi/bmlength*(senpos(ns)*bmlength)));
    end;
end;
cbel=[cbel zeros(nsen,modes)]; % put C matrix together
%
% Form D matrix
dbe=zeros(nsen,nact);
u=zeros(nact,(tottime/step))'; % set size of force history matrix
for ii=1:step+1:npts;
    u(ii,nact)=0;
    u(ii,nact-1)=0;
    u(ii,nact-2)=0;
    u(ii,nact-3)=0;
end

t=0:step:(tottime-step); % create time history
count=0;
peak3=0;
for l=.5:.005:.01; % loop for length change
    suny=0;
    %
sumx=0;
count=count+1;
for xmom=1:nommom; % loop for moment distribution
    % Place forces to simulate moments
    left1=-xmom/nommom*l/2+mid;
    left2=xmom/nommom*l/2+mid+dmom;
    dist(nommom-xmom+1)=left1; % record moment locations
    right2=xmom/nommom*l/2+mid;
    right1=xmom/nommom*l/2+mid-dmom;
    dist(nommom+xmom)=right2; % record moment locations
    actpos=[left1 left2 right1 right2]; % place forces to represent moments
    for m=1:1:nomodes;
        mpiol=m*pi/bmlength;
        for na=1:1:nact;
            bbe2(m,na)=cevcoeff*sin(mpiol*(actpos(na)*bmlength));
        end;
    end;
    bbe=[bbe1;bbe2]; % put B matrix together
% Describe force
% implement step input
    for ii=1:1:step/step;
        meq=100*sin(xmom/nommom*3.141593/2); % calculate moment
        mdist(nommom-xmom+1)=meq;
        mdist(nommom+xmom)=-meq;
        u(ii,nact)=meq/dmom;
        u(ii,nact-1)=-meq/dmom;
        u(ii,nact-2)=-meq/dmom;
        u(ii,nact-3)=meq/dmom;
    end
% get time response
    ybe=xbe=lsim(abe,bbe,cbe,dbbe,u,t);
    sumy=sumy+ybe;
    sumx=sumx+xbe;
% check moment number
end

ybe=sumy;
xbbe=sumx;
frf1=fft(ybe); % do fft of response
mag1=abs(frf1)/(npts/2); % scale magnitude

% find peak of each mode
peak3(count,1)=1;
peak3(count,2)=max(mag1(5:10));
peak3(count,3)=max(mag1(27:31));
peak3(count,4) = max(mag1(62:65));
peak3(count,5) = max(mag1(171:175));
peak3(count,6) = max(mag1(247:251));
peak3(count,7) = max(mag1(336:341));
end
end
VITA

Gregory William Diehl was born on July 22, 1969, in Wilmington, Delaware. After graduating from North Springs High School in 1987, he studied mechanical engineering at Virginia Tech in Blacksburg, Virginia and graduated with a Bachelor of Science in Mechanical Engineering in 1991. He continued his education at Virginia Tech and enrolled in the Master of Science program in the Department of Mechanical Engineering. He completed his degree in November, 1993. After graduation, he will spend some time in Europe and then seek employment.

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