A CAPITAL ALLOCATION PROCESS FOR PUBLIC PROJECTS

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(ABSTRACT)

This research was initiated to address a shortfall in some of the previous work reviewed with respect to capital budgeting and allocation. This shortfall has to do with the very general manner in which the public sector is addressed. The public sector poses problems which are unique to its environment and the simple extension of private sector tools to this domain is considered ineffective.

The central aim of this research is to develop an initial methodology for capital allocation to projects in the public sector. In developing this methodology the fundamental differences between the public and private sectors are pointed out. The methodology devised utilizes knowledge from the fields of Engineering Economy, Capital Budgeting, and Multi-Attribute Decision Analysis.

Basic principles of Engineering Economy and Capital Budgeting are reviewed. The Multi - Attribute Decision Model selected for implementation in this research

methodology is the Analytic Hierarchy Process (AHP). The procedure followed by the AHP is presented and its benefits and limitations are discussed.

The research addresses the quantitative and qualitative aspects of assessing projects in the public sector. A hypothetical example is presented to demonstrate the manner in which the methodology works. The thesis concludes with recommendations and proposes areas for further research.

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Most of all I wish to express my appreciation to my family and to two very special women. It is because of the love and support of my wife Vivian, and my mother Joan, that I was able to pursue and complete this degree. I also wish to thank Vivian for her effort, above and beyond the call of duty, in proof-reading the thesis. Thank you both for your love and confidence. Finally, I wish to dedicate this work to my father, E. J. Fleming.

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I. INTRODUCTION

1.1 Thesis Organization

The thesis is organized into five chapters. Chapter 1 addresses the introductory elements of the problem. It provides a clear statement of the problem and objectives of this research. Chapter 2 provides a detailed review of the literature concerning capital allocation and discusses the relevant aspects of the literature with respect to the problem. In Chapter 3 the methodology to address the difficulties of the problem is outlined. The theoretical basis for applying this particular methodology and its validity is also discussed. Chapter 4 contains a simple hypothetical example problem which demonstrates the effectiveness of the methodology. Finally, in Chapter 5 conclusions are drawn, recommendations made, and areas for further research are outlined.

1.2 Problem Identification

Capital budgeting and the capital allocation problems are old and well known in the private sector. This is not as true for the public sector, where capital budgeting has only been around for approximately fifty years. In the United States most states and local governments apply capital budgeting techniques. Despite repeated arguments by economists and others, the Federal Government has yet to apply this accepted business practice.

The allocation of resources to public projects is an area of concern for public servants in this time of reducing assets and increasing demands. The capital allocation process for public projects has been twinned with the process employed in the private sector. That is to say, the private sector problem is addressed, and the public sector problem is reduced to general comments on how the public sector problem is the same. This is despite the fact that the ostensible goals of the private and public sector are not identical.

It is thought that the consideration of public projects, and their capital allocation process, would entail some basic differences in approach. The obvious difference is the goal. Public projects do not seek to maximize future wealth, or present worth, but to maximize public benefit. Therefore, the research question that has to be answered is; how is the public capital allocation process different from that of the private sector, and how do these differences affect the tools used in capital allocation?

1.3 Research Purposes

There are two purposes for this research:

(1) to demonstrate that the private sector and public sector capital allocation processes are significantly different and

(2) to present a methodology for the capital allocation process in the public sector.

This is accomplished by an examination of the basic differences between the capital allocation process in the private sector and the public sector. This examination is necessary to illustrate the aspects of the private sector processes that are applicable or not applicable to the public sector. The second purpose is accomplished through traditional Engineering Economy principles combined with multi-attribute evaluation techniques. The application of existing private sector techniques does not adequately address the non-quantified aspects of the public sector problem.

1.4 Research Premises/Delimitations

The first delimitation is one of scope. It is assumed that the highest level applicable would be a local, or state government at the most. This delimitation is necessary as this thesis will not be addressing issues such as national defense, which poses difficult, theoretical economic questions beyond the scope of this research. Higher levels of government typically allocate funds to sectors, unlike local/regional governments which utilize project allocation. It s thought that the use of project capital allocation is easier for local governments than the sector method employed at higher levels. At this level of government the budget is considered fixed and the discount factor is also fixed. This follows along the same lines as the argument put forward by Skipper [1] and Kladivko [2].

The first delimitation is of importance because it leads to the second. As the level of government being addressed is local, it is required that it meet a strict budget constraint. This constraint has an impact on the methodology and its effectiveness when considering the life-cycle cost implications of a project.

1.5 Problem Statement

This research reviews the dynamics of the public sector capital allocation process. The research problem is to show how the public sector allocation process is different from that of the private sector, and to provide a methodology which addresses those differences. The conceptual model for the problem is provided as Figure 1.1.

1.6 Research Hypotheses

It is hypothesized that consideration of the unique dynamics of the public sector capital allocation problem, in an engineering methodology, will yield better results than the

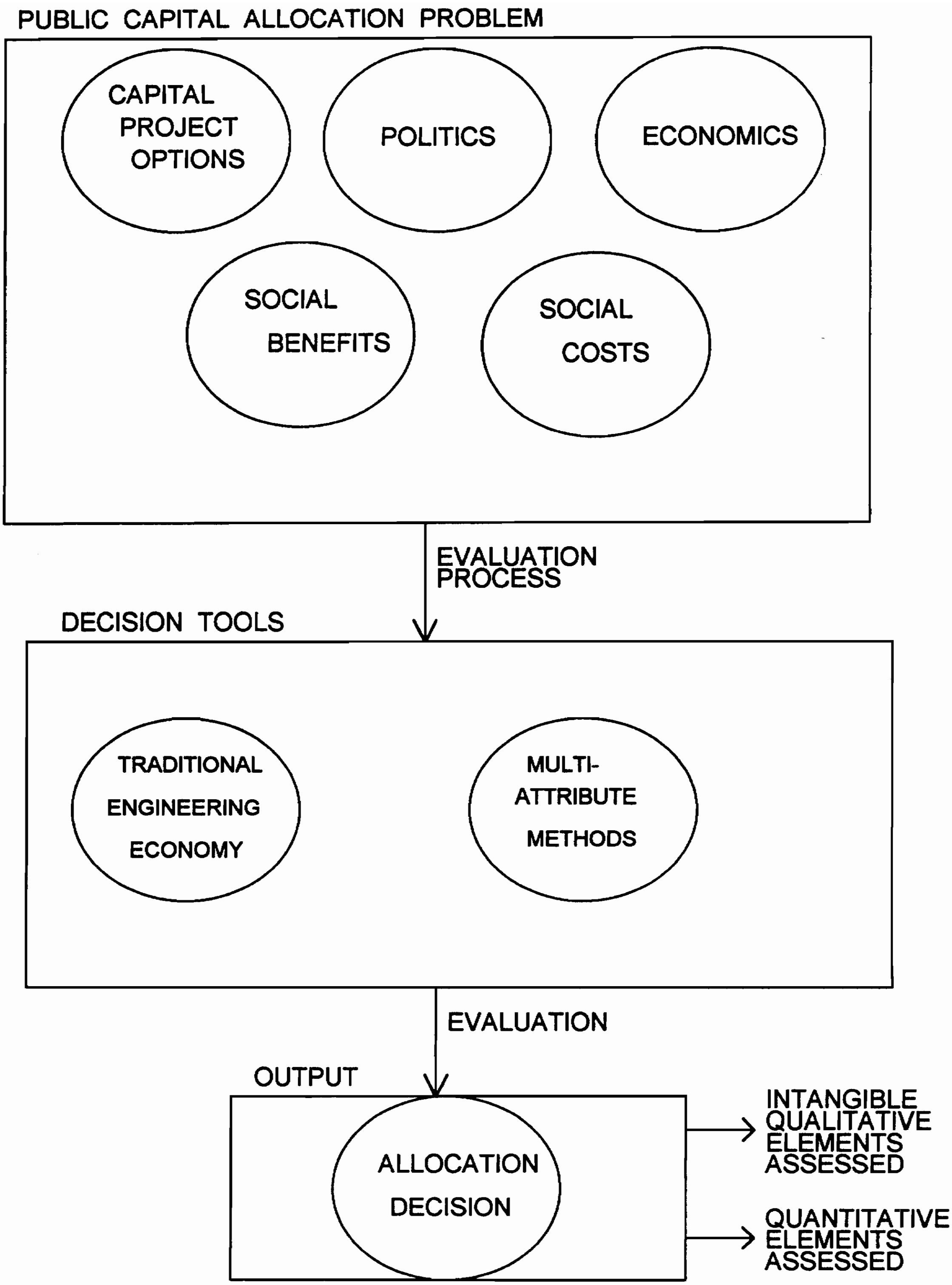


Figure 1.1 - Thesis Conceptual Model

present system which encourages sub-optimization and/or nepotism.

The above hypothesis is difficult, if not impossible, to support in the traditional manner. This thesis also sets forth three further hypotheses which may be easier to validate than the one above. They are as follows:

(1) the dynamics of the public sector are different from that of the private sector,

(2) the utilization of private sector capital allocation tools does not take into consideration these differences, and

(3) the public capital allocation process is best addressed through a combination of traditional Engineering Economy principles with the techniques of multi-attribute decision models.

1.7 Research Objectives

This research has five objectives as follows:

(1) it outlines the key differences between capital allocation in the public and private sectors,

(2) determines how the computer based budgeting algorithm employed by Skipper/Kladivko can be employed in the public budgeting process,

(3) determines how the Analytic Hierarchy Process (AHP) can be adapted to address the public sector capital allocation problem,

(4) presents a capital allocation process which combines the above into a coherent tool for the assessment of public projects, and

(5) presents a hypothetical example which will illustrate how the proposed process will work.

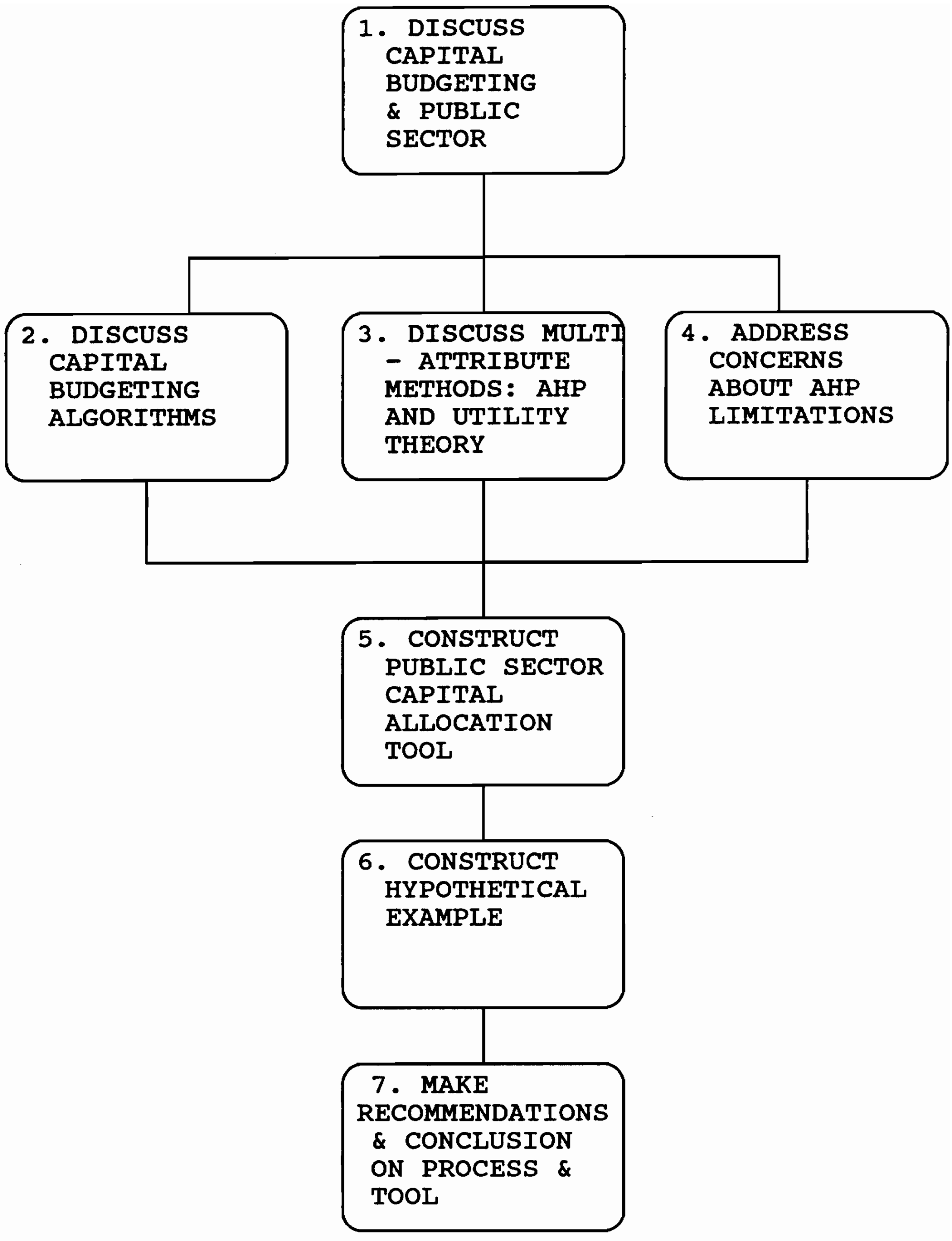


Figure 1.2 - Research Approach

1.8 Research Approach

This thesis builds upon previous work done by Skipper [1] and Kladivko [2] in the area of computer based capital budgeting. It also employs the AHP developed by T.L. Saaty [3]. An integrated approach combining the features of both will be developed. This approach will be developed to address the uniqueness of the public sector problem which is outlined in subsequent chapters. The process is demonstrated through the use of a hypothetical example.

The noted limitations of the AHP are addressed. Some of the problems are disputed in the literature, and there exists multiple techniques to solve the perceived shortfalls. All aspects of the AHP are considered, and its integration in the assessment of public sector capital allocation projects is both necessary and useful.

1.9 Research Significance

This research is significant for several reasons. The first is that it addresses the myth that private sector tools and methodologies can be easily applied in the public sector. Both Skipper [1] and Kladivko [2] maintained that their algorithms could be easily applied to public sector capital allocation.

The second reason that this research is significant is because it proposes a coherent, rational methodology for the allocation of public funds. Presently a methodology, which takes into account the intangibles of public sector capital allocation, and the dynamics of the public sector does not seem to exist.

This research also incorporates consideration for more than the user in the assessment of the public project. The use of the AHP enables the analyst to include consideration of the user, provider, and the general public or society in the analysis. This results in a more thorough analysis of the project benefits and costs.

Finally, this research combines two different processes/algorithms in an exhaustive and unique manner. It provides the decision maker with a complete list of viable alternatives, and presents the results in a method difficult to manipulate surreptitiously, yet easy to understand. Presently most decisions of this nature are made through the use of political power; be it personal by reason of position, or through alliances ("you scratch my back, I'll scratch yours"). The net result is decisions that are not always in the interest of the general public, but favor a small segment of the population. This present system is one that encourages sub-optimization or, more seriously, nepotism and corruption. The methodology proposed may help to reduce such activity. It encourages decision makers to make decisions improving the social welfare of all, rather than the few.

1.10 Definition of Terms

This section provides definitions of key terms. The thesis employs expressions, which may have different meanings to different people. The intent of this section is to ensure that the reader fully understands these expressions as used in this thesis.

Portfolio. In this thesis the term portfolio is used to define a group of projects. All possible portfolios are identified using the 0-1 matrix approach as employed by Skipper [1] and Kladivko [2]. The effect of using this matrix is, that for 'n' projects there are 2^n possible portfolios. This methodology is considered suitable for the evaluation of government project portfolio feasibility.

Project. A project typically involves some form of physical entity that is created, upgraded, or otherwise changed. A project also has a definite, finite existence.

II. LITERATURE REVIEW

2.1 Review Outline

The purpose of this Chapter is to review the critical bodies of knowledge relevant to this research. The first topic to be covered is the public sector, as this is the environment with which the research is concerned. The next Section outlines the differences between the public and private sectors. Capital budgeting, from both a private and public sector perspective, will be discussed in later Sections.

A review of the Skipper/Kladivko algorithm and their key components is also done. This is followed by a review of the Analytic Hierarchy Process (AHP); how it works, its benefits and limitations, and the difference between AHP and Utility Theory.

2.2 The Public Sector

This Section will defer discussing, in detail, the differences between the private and public sectors. These differences are discussed in Section 2.3. The public sector capital allocation problem has, typically, been treated in one of two ways in the literature. The most popular method is by Cost-Benefit Analysis (CBA) which condenses benefits by means of some conversion factor(s) to obtain an overall benefit. This is also known as the Cost-Benefit Ratio (CBR). The second is the cost effectiveness method which makes no attempt to combine the various benefits into a single measure.

As the form of this work is oriented towards the CBA form we will concentrate on this area. CBA has been used by the U.S. government since the 1930s, primarily in the area of water projects. Indeed, the concept of CBA is often illustrated by the use of a letter written by Benjamin Franklin in 1772. In this letter Franklin discusses what he calls

calls "moral or prudential algebra" [4].

In his book on public CBA, Gramlich [5] refers to two types of evaluations. They are as follows;

(1) summative evaluations, and

(2) formative evaluations.

A summative evaluation is normally done before a project has been started. Its purpose is to evaluate the projects suitability to be undertaken. A formative evaluation is done after a project has been initiated. Its purpose is to assess whether or not the project can be done better. The formative analysis begins with the conceptual design for a project, and should be an activity that is conducted throughout the project life. The summative type of evaluation is appropriate in considering the allocation process for public projects.

The decision to invest, or not invest, is greatly complicated by the decision environment for public projects. Unlike the private sector, where decisions can be strongly oriented around the profit motive, the public sector must address broad social and political questions. The environment in which public investments are made is incredibly complex and very fluid.

The public sector is also characterized by indirect benefits and costs to a much greater degree than the private sector. These indirect benefits are called 'externalities'. This refers to benefits or costs that spill over to the rest of society. The classical example of a positive externality is a child's education. The benefits of a good education for a child has a positive effect, a spillover, on his entire life and upon the communities in which he lives. Many public projects involve externalities which must be considered when they are being evaluated.

2.3 Public vs. Private Sector

It is believed, and generally supported by the literature, that public sector problems are different from those in the private sector. These differences are many, however only those having the most impact upon the capital allocation process for public projects are discussed herein.

First and foremost is the difference in objectives. Although some could argue, it can be stated that the objective of the private sector is to increase the future, or net worth, of the firm. This performance measure is easily translated into the medium of money. The prime objective of the public sector is to promote the general welfare of the populace or the citizenry. The performance measure for social welfare is not easily defined and it may be different from region to region.

The dynamics of the private sector are also different from the public sector. It is relatively easy for a firm to decide not to produce widgets any more. It is not so easy for a public agency to delay, or cancel, a project. Considered as a psychological problem it makes sense; whenever someone makes a public commitment to do something it becomes very difficult for them to retract that commitment. Therefore, it can be said that commitment, in the public sector, is more significant. This makes the public decision environment less dynamic and more rigid. As a result of this characteristic, it is crucial that a decision made in the public sector be 'correct'.

The phases of a product, or project life cycle are: conceptual and preliminary design, detail design and development, production and/or construction, and product use/support/phase out. The different activities typically performed in each phase are thoroughly explained in Blanchard [40]. Commitment, for the public sector problem, is when a project enters the production/construction phase. This makes this phase much more significant than the private sector. As mentioned, a firm can decide that a product is not economical to produce at any point in the life cycle. It is not so easy in the public sector. It is very difficult to stop constructing a bridge when it is only partially completed.

When dealing with public projects and their cost, the topic of discount rates is also raised. Unlike the private sector, where a firm can decide upon its own minimum attractive rate of return (MARR), a public agency must use a social discount rate. Section 2.4.3 addresses the topic of discount rates in greater detail.

Another area of concern, with respect to the public sector, is its failure to address life-cycle costs. Public agencies are notorious for budgeting/allocating funds on the basis of current requirements. Any capital allocation process must consider the project's life-cycle costs for it to be effective. The consideration of only acquisition costs in selecting a project results in other significant costs, such as operating costs, personnel costs, training costs, and disposal costs being hidden. This concept is aptly described as the "ice-berg effect" by Blanchard [41].

Given the often extended operating life of public projects, the full consideration of life-cycle costs is admittedly more difficult. The same reason also makes it vital that life-cycle costs are considered. As has been discussed, the dynamics of the public sector are, perhaps, even less forgiving than those of the private sector. It is very difficult to stop a public project once it enters the production/construction phase. The total effect of these differences is that the costs incurred for operation and support are often even greater in the public sector.

The net effect of these life-cycle differences is to exacerbate the relationships illustrated in Figure 2.1, namely the inverse relationship between life-cycle cost commitment and life-cycle cost incurred. Figure 2.1 illustrates how life-cycle costs are committed early in a project's life. Essentially by the time a project has finished its detail

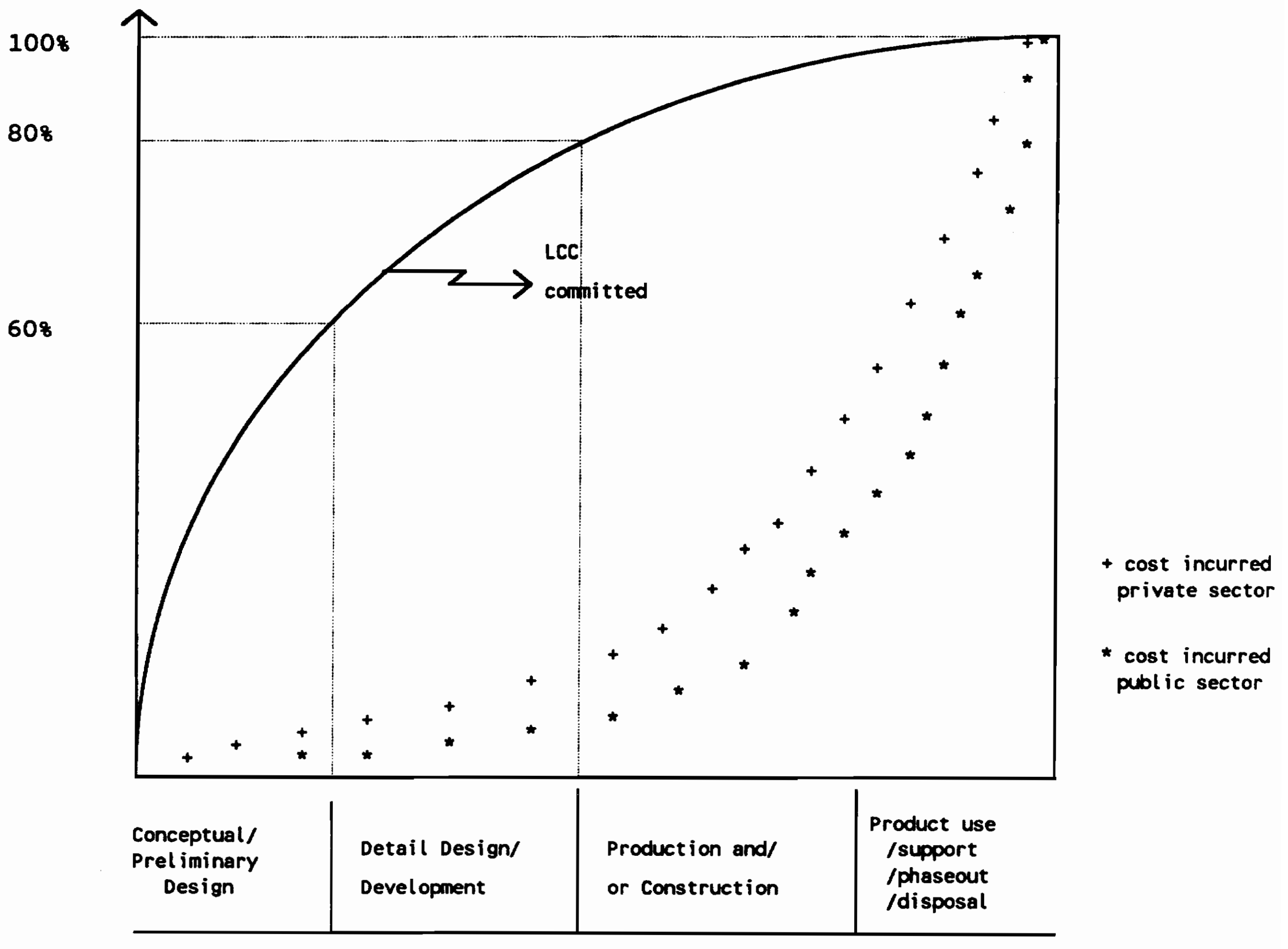


Figure 2.1 - LCC Commitment vs Cumulative LCC Curves [34]

design and development phase approximately 80% of its life-cycle cost has been committed. This is particularly relevant in the public sector because by the time projects are compared and evaluated against each other they have already undergone most, if not all, of their detail design activities. In this manner a public agency can determine how much the project will cost without incurring the significant expenditures of production/construction and project use phases.

Figure 2.1 also demonstrates how, in the public sector, costs are incurred differently from the private sector. This is because of the longer life cycle. A successful product may be produced for 10, or even 20 years. A public project, such as a school building may have a planned useful life of 50 or 60 years and in many cases that life is extended. The result of this generally longer life cycle is that life-cycle costs incurred do not increase as rapidly for public projects as for the private sector.

One final difference between the public and private sectors must be noted. This is the question of distribution. In the private sector the firm decides what is best for the firm. In the public sector a range of public goods exist, from pure public goods to public goods which can only be used by certain individuals. A pure public good provides benefits to more than those directly consuming it and for which it is impossible to exclude others from using (externalities). Therefore, in the public sector, the distribution of a benefit (a project for example) can be perceived as a cost by some and cause some dissension.

2.4 Capital Budgeting

What is capital budgeting? It is a concept that involves differentiating between current and capital expenditures. Although not interested in the specifics of a capital budget it is relevant to know what it entails. The purpose of the process being proposed is to assess its usefulness in capital budgeting allocation.

Capital budgeting is a practice followed by many states and local governments within the United States (31 states in 1963 and approximately 25% of cities with a 1950's population over 100,000 [6]). The interest in capital budgeting was stimulated by public works grants in the 1930s, originating with the Federal Government.

Therefore, the next aspect which must be considered is; what is a capital expenditure? In its most general terms, a capital expenditure provides benefits beyond the current accounting period [7]. This definition lacks precision. The lack of precision is desired because of the many different levels that are found in the public sector. Often the definition of capital expenditures for one agency is considered current expenditures for another. Provided that the agency involved maintains internal consistency this poses no problem. The converse of the capital expenditure definition, is a current expenditure, which provides benefits only for the current accounting period.

In his lengthy paper on the theory and practice of capital budgeting, Gurnani [8] provides a great deal of detail and insight into this topic. He indicates that the generic elements of capital budgeting include many diverse factors. The elements Gurnani discusses are reviewed to ensure that they have been addressed.

The first element is the determination of budget size and its financing. For the specified level of this problem the budget level is considered to be fixed. Also, as this thesis deals with the middle management concept espoused in Skipper [1]/Kladivko [2], it is not necessary to be concerned about the financing question. This is also realistic for this thesis environment. The analyst or town manager is not concerned with how he gets the funds, just how much. The question of financing is a difficult issue, most especially in the public sector where politics and emotion can be very strong.

The next element was identification of possible projects. This element is often over-looked by analysts as being obvious. Its apparent importance has not prevented it from being forgotten on many occasions. In the public sector the identification of possible projects, or projects being considered for funding, is aided by the administrative process. It is a rare public organization that does not have established procedures for proposal submissions.

The next element identified is "classification according to some scheme " [9]. This classification is used to assist in allocating resources. As the public projects are competing for the same resource they are not further classified. The next two elements are cash flow analysis and economic analysis. These steps have been incorporated in the analysis, but not as the main element in the capital budgeting process. The classification of projects is done through the establishment of a 2^n portfolio matrix, and further, by the identification of attributes. Although concerned with cash flow analysis and economic analysis, they are only components of the total analysis.

In his paper, Gurnani [8] pointed out the continuing attempts by academicians to derive more sophisticated processes and their focus on quantitative criteria. Although he indicates that the basics of these methods are well understood, they are not extensively used. Table 2-1 is a reproduction of Table 2 from Gurnani's paper demonstrating the use of increasingly sophisticated and more academically rigorous capital budgeting techniques. Note the drop in use of payback as the primary technique for Internal Rate of Return (IRR) over time.

The use of qualitative factors in capital budgeting is significant. From Gurnani, the following is noted:

> " Intangibles play a vital role in the long term capital allocation. Different divisions/subsidiaries of a corporation may have different investment performance evaluation criteria" [10]

This statement echoes the problem that is encountered in the public sector.

Table 2-1

Use of Quantitative Tools in Capital Budgeting - Overall [11]

	Klammer 1959	Klammer 1964	Klammer 1970	Kim/ Fremgen 1971	Gitmani Farragher 1971	Kim/ Forrester 1976	Farragher 1979
I. Evaluation							
A. Primary Technique							
Payback	34%	24%	12%	14%	15%	9%	12%
Account ROR	34%	30%	26%	22%	10%	25%	8%
IRR	19%	38%	57%	38%	37%	53%	9%
NPV				5%	26%	13%	19%
B. Secondary Methods							
Payback	-	-	-	53%	33%	44%	39%
ARR	-	-	-	27%	3%	14%	3%
IRR	-	-	-	33%	7%	14%	8%
NPV/PI	-	-	-	21%	7%	28%	8%
II. Risk Assessment							
Risk Analysis	7%	7%	13%	22%	5%	-	10%
Sensitivity Analysis	4%	7%	28%	-	10%	-	3%
III. Risk Adjustment							
Shorten Payback	9%	9%	10%	18%	11%	13%	4%
Risk Adjusted Discount Rate	12%	16%	21%	24%	12%	43%	19%
Utility Theory	-	1%	4%	-	2%	-	3%
Certainty Equiv.	-	-	-	-	1%	26%	3%
IV. Management Science							
Decision Theory	3%	4%	9%	-	10%	-	12%
Math Programming	1%	3%	4%	-	11%	-	13%
Game Theory	-	2%	3%	-	6%	-	7%
PERT	4%	13%	28%	-	19%	-	23%
Linear Programming	5%	8%	17%	-	12%	-	19%
Goal Programming	-	-	-	-	6%	-	7%

Different projects have different goals and may have different attributes which affect their assessments. This research process incorporates the techniques of capital budgeting, as discussed, with non-economic criteria in obtaining the overall best possible investment portfolio.

From Weingartner [12], it is seen that capital budgeting has two different implications for a firm. It can be employed as a plan, or as a control. This differentiation is of interest because of the approaches taken by Skipper and Kladivko. Skipper's work is strongly, and almost exclusively, oriented towards control. Kladivko expanded the work of Skipper, and in doing so, he included the planning aspect of a budget in his work. This difference is of note because capital budgeting as a control is easier to implement than a capital budget oriented toward planning.

2.4.1 Equivalence Criteria

Basic Engineering Economy principles require that cash flows for alternatives be compared on an equivalent basis. This is a topic that is well covered in the literature [12, 14, 15, 16, 19, 34, 40]. As the basis for Engineering Economy models, the equivalence measures of Present Worth (PW), Future Worth (FW), and Annual Worth (AW) are based upon the premise that a dollar today is worth more than in the future. This is called the time value of money.

AW is a procedure whereby, through application of interest rate formulas, costs and returns are expressed as an annual equivalent amount over a defined period. PW requires that costs, and returns, over time, be reduced to an equivalent present value based upon some interest rate called the discount rate. In the case of PW, funds are equated to an arbitrary point called the present. FW is the same as PW, however instead of discounting to the present, the funds are equated to a specified time in the future.

Mathematically the three are stated as follows [15]:

$$(1)\ PW(i) = \sum_{t=0}^{n} F_t\left[\frac{1}{(1+i)^n}\right] \qquad [2\text{-}1]$$

$$(2)\ FW(i) = \sum_{t=0}^{n} F_t(1+i)^n \qquad [2\text{-}2]$$

$$(3)\ AW(i) = \left[\sum_{t=0}^{n} F_t(1+i)^{-t}\right]\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] \qquad [2\text{-}3]$$

i = effective interest rate per period

n = number of compounding periods

F_t = amount of funds at time t, where t = 0,1,2, ..., n

Skipper [1] and Kladivko [2] both employ a FW criterion as the equivalence measure. The majority of the literature uses PW. In this thesis, as with Skipper/Kladivko, there are multiple projects with unequal lives. This is not conducive to using either AW or PW as explained below.

In cases where the literature does address unequal lives it uses the technique of multiple projects or early retirement as a means to make project cash flows equivalent. Multiple projects is a technique whereby projects are assumed to repeat until all projects have a coincidental termination point. Early retirement is a technique that selects an arbitrary point in the future; usually not longer than the shortest lived project. Projects which survive longer are considered to be 'retired' and their salvage value is adjusted and is included as a revenue in their life-cycle cost assessment.

As discussed by Fleischer [13], if AW is used, a planning horizon which is a common multiple of the lives of all the portfolios must be used. It must assume the cash flow of each portfolio will repeat, which will not, in all probability, happen. Therefore, this is not considered a good assumption for this analysis. The PW criterion does not take into account the unequal lives of projects and is not a valid measure of equivalence. The use of the FW criterion, as advocated and done by Skipper [1] and Kladivko [2], with the planning horizon (H) being the longest lived project, is the most accurate measure. Projects which end before H will have returns reinvested at the social discount rate, which will be explained in Section 2.4.3.

2.4.2 Incremental Analysis

The concept of incremental analysis seems to be a difficult one for the private sector to grasp. Therefore, a brief description of the process would appear to be in order. As stated in Fabrycky and Blanchard [14], it is the difference between mutually exclusive alternatives which is the basis for making a decision. As a list of feasible portfolios, which are mutually exclusive, is developed by the research methodology, an incremental analysis is relevant.

Four steps are involved in an incremental analysis. The first step is the ranking of projects, in ascending order based upon first cost. The second step is the selection of the best project with the lowest first cost. The third step compares the best project with the project having the second lowest first cost. The difference between the money flows is calculated. The rate of return of the incremental money flow is calculated and, if it exceeds the MARR, it is accepted; if not it is rejected. (A simpler method is to evaluate the incremental values of the money flow at the MARR. If the result is positive, the challenger is accepted. If the result is negative, the challenger is rejected.) The fourth and final step is to continue with the step three process, until all projects have been evaluated. The end result will be the project which yields the best overall return, with respect to the MARR.

2.4.3 Discount Rate and Budget Levels

This thesis is focused on a regional or local level of government. It is thought that this level of government will have a clearly set discount rate and budget level for capital projects. The provision of mandated budget levels and discount rates greatly reduces the complexity of the analysis. However, the topic of discount rates, especially in the public sector, requires further discussion.

The discount rate, or MARR for the private sector, is critically important to any economic analysis. When dealing with public projects and their cost, the topic of discount rates is also raised. Unlike the private sector, where a firm can decide upon its own MARR, a public agency must use a social discount rate. The interest rate for lending money is determined by four factors as discussed in Thuesen and Fabrycky [15]. They are as follows:

(1) risk of loss of funds,

(2) lender administration expenses,

(3) lender profit, and

(4) risk of not having funds available for better alternatives later.

These factors drive the market to determine an interest rate, which then affects money over time. This basic Engineering Economy principle is known as the time value of money. It is argued that risk for the public sector is negligible, therefore this factor should reduce the discount rate applied to public projects. Still others maintain that the social discount rate should be determined by government policy. A final view is that the rate should be the same as for the private sector, or else investment in the public sector will be too high. For a complete discussion of social discount rates see Au and Au [16]. This thesis uses a government mandated discount rate. This reflects the level of government that is being addressed in the research. The use of a mandated discount rate is an accepted practice in most public agencies today.

2.4.4 Skipper/Kladivko Algorithm

The research methodology builds upon the capital budgeting process as described in Skipper [1] and Kladivko [2]. In their work they identified four steps in the capital budgeting process, which were employed in the development of their algorithm. The four steps are as follows:

(1) project identification,

(2) parameter specification,

(3) constraint determination, and

(4) project portfolio selection.

In Project Identification, the life cycle of any and all projects must be examined. Skipper introduces his two tiered approach of having both discrete (end period) and continuous function models. Costs and returns must be estimated monetarily for both public and private projects in this algorithm. The concept of benefits/disbenefits for public projects is discussed, but only in a perfunctory manner. It is acknowledged that the estimation of benefits/disbenefits in monetary terms, for public projects, is very difficult. Skipper also emphasizes the proper estimation of project costs and returns as the key element in obtaining valid results. This step is not dissimilar to what would be done in the public sector. Of course the significant difference is the estimation of benefits/disbenefits which is being addressed.

In Parameter Specification the parameters of concern are budget limitation and the cost of capital. As Skipper focused on the middle manager he could, and did, reasonably state that these parameters were constants. Budget limits and the cost of capital are imposed by higher management. The cost of capital in this paper is considered to be the long run, average rate of return on investments. Skipper uses another term for the cost of capital, the marginal attractive rate of return or MARR; although they are not identical expressions, Skipper employs them interchangeably. As the focus of the research is on local/regional governments, it can be stated that the parameters for the public sector will also be given. Budget limitations are identified by the legislative body and the cost of capital will, typically, be set as a matter of policy.

Constraint Identification, apart from the obvious budget constraint, are as follows: mutual exclusivity, contingencies, and non-economic factors (must choose). Mutually exclusive projects are those in which the acceptance of one precludes the acceptance of another. Contingency constraints involve those projects which can not be selected unless the one upon which they are dependent is also selected. (For example, if A is contingent on B, then A can not be selected unless B is first selected. B can exist without A, but A can not exist without B.) Non-economic constraints seek to address those situations where the selection decision for a particular project is based upon factors other than financial ones. The same series of constraints are employed for public projects. The non-economic constraint is changed to the political constraint. It is used to identify projects that must be chosen for some political reason or another.

In the Project Portfolio Selection step the analyst presents the manager the preferred combination of projects, or portfolio. Although there are many different ways to do this Skipper used the FW equivalence criterion as outlined in Section 2.4.1. Given the intent and formulation of his algorithm this is an accepted method. The literature supports the use of FW for multiple projects with unequal lives [1,2,12,13]. Further discussion on the choice of which equivalence based measure to use is not done in this thesis.

The other aspect to this part of the problem is the computation of all the possible portfolios. This is accomplished through the use of a zero-one matrix which lists all the possible combination of projects. This results in a 2^n matrix for 'n' projects. This step in Skipper's algorithm is not used as the portfolio selection step for the public capital allocation problem. Instead it is used with the other steps to identify all feasible project portfolios. The portfolio selection step involves the use of AHP evaluated projects with the feasible project portfolios.

2.5 The Analytic Hierarchy Process

The Analytic Hierarchy Process (AHP) was developed by Thomas L. Saaty [3] over a period of years. AHP is a:

> " theory for dealing with complex technological, economic and socio-political problems" [17].

Individuals tend to group things in hierarchies. This natural tendency enables people to look at a complex problem logically, without being overwhelmed. This supposition was Saaty's premise when he developed AHP. The following from Saaty:

> "We show that the age old adage that one cannot compare apples and oranges is not true. ...It is our thesis that this sort of complicated comparison occurs in real life over and over again and some kind of mathematical approach is required." [18]

AHP enables an analyst to structure complicated problems with multi-attributes. The decision maker decides what his goal, or focus to use AHP terminology, is and what options exist. He then decides upon the criteria by which he wishes to evaluate those options with respect to his goal. AHP uses a sequence of pairwise comparisons moving down from the focus level to the alternative level. The decision-maker, or a group of decision makers, make ratio comparisons; from which they obtain their relative priorities, and alternative weights are ultimately calculated.

2.5.1 Justification for Using AHP

It was decided to use the AHP over many other different methodologies for a number of reasons. First and foremost is the fact that the AHP has a sound theoretical base with a relatively easy to understand procedure. It has been used and accepted in areas similar to the one being addressed. The AHP is a process which can be easily applied to a group activity. It produces unequivocal results which enable the group to make a decision.

The AHP is also a robust process allowing multiple types of problems and criteria to be addressed. From a Systems Engineering point of view it is a good design. It also possesses a unique capability to validate itself at each step with the Consistency Ratio (CR), and, if desired, the entire decision problem and solution can be validated with a similar process to the CR. Finally, it is felt that the AHP, of all the theories and algorithms that have been examined, has the best possibility of being actually used by the decision makers for which it was intended.

2.5.2 The AHP Methodology

The solution process, or methodology, employed in the AHP is summarized in Canada and Sullivan [19]. They indicate that there are four stages to the solution process. An additional stage of establishing the hierarchy and a goal, albeit obvious, can be added to their sequence giving the following:

(1) establish a goal and set up hierarchical structure with attributes and sub-attributes,

(2) determine relative importance of attributes and sub-attributes through pairwise comparisons,

(3) determine relative weight of alternatives with respect to sub-attributes using normalized eigen-vector method,

(4) verify consistency of pairwise comparisons using the consistency ratio (CR) method, and

(5) determine the overall relative weights of each alternative by aggregating their totals through the hierarchy.

An optional sixth step exists, but it is seldom mentioned. It is possible for the analyst to conduct a verification of the consistency of the overall hierarchy pairwise comparisons. This step is not included in this algorithm for reasons of simplicity. However, if the algorithm were to be further developed this feature may prove to be beneficial.

The first stage in the selection process is the generation of the hierarchy. A generic four level hierarchy is provided in Figure 2.2. The establishment of the goal or focus of the hierarchy is the first step. This is level one in the hierarchy. The second and third levels contain attributes that are considered important to the resolution of the problem. The second level in this process will always have the same three attributes. They are constant and will be discussed in Section 3.3. Level Three contains the sub-attributes. This level, and any other levels desired by the analyst, will vary according to the problem and the criteria by which it is evaluated. It is critical for the evaluation of the matrices that the attributes be independent. If they are not independent then the eigen-

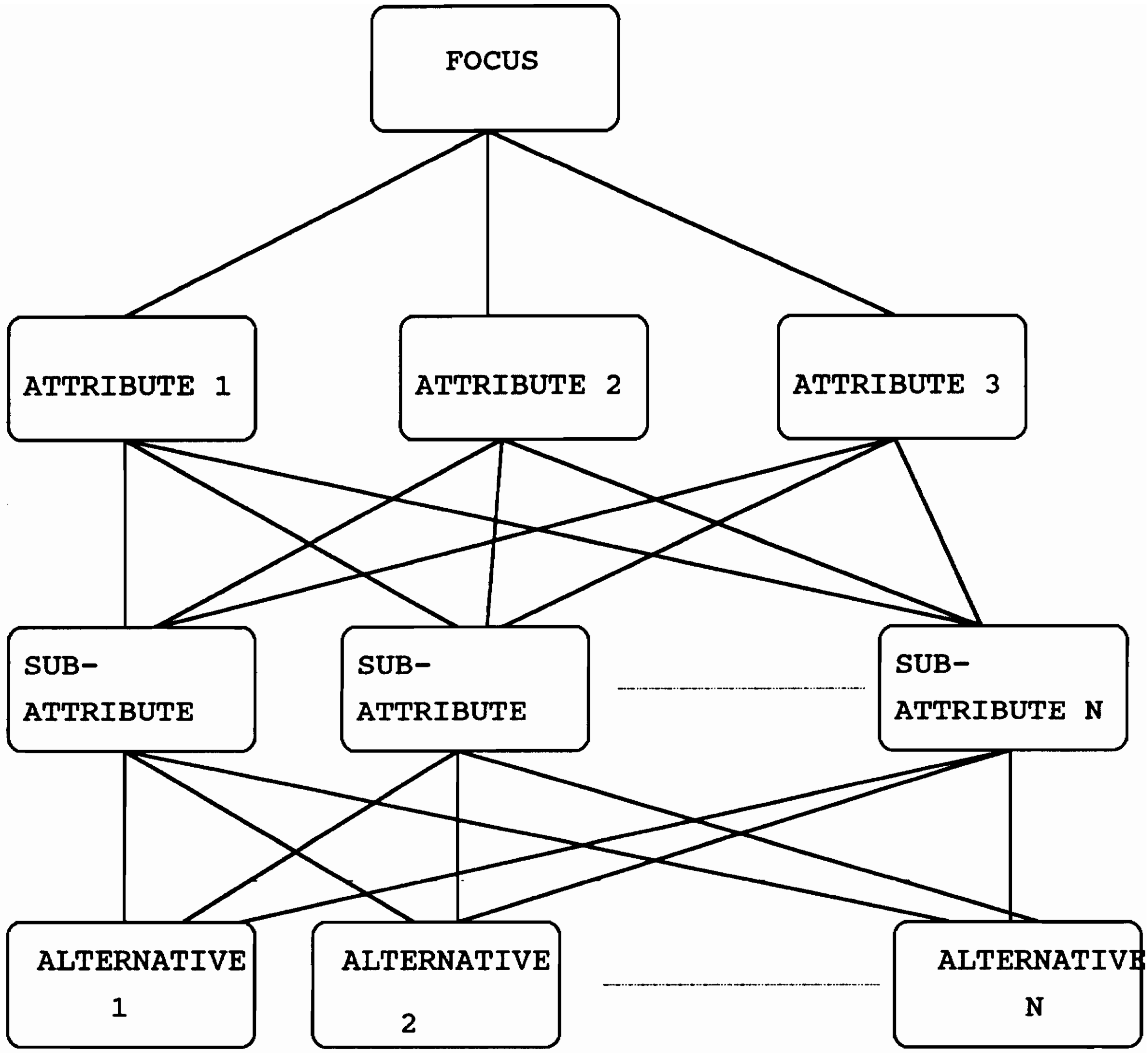


Figure 2.2 - Generic AHP Hierarchy

vector, or priority vector, will be incorrect and the assessment will be invalidated. The example has only four levels, consequently the bottom level is the mutually exclusive alternatives being assessed.

The second, and third stage, in the solution process is the determination of relative importance of attributes through pairwise comparisons. In the first stage the attributes are compared against each other. Then successive comparisons are made between the sub-attributes considering one attribute at a higher level. Respondents indicate how strongly they prefer an attribute over another with respect to the higher level attribute. The scale proposed by Saaty is from one to nine. The larger the number the greater the preference of one attribute over another. Table 2-2 illustrates the one to nine scale proposed by Saaty.

From this scale the matrix is obtained as follows:

$$A = \begin{bmatrix} w_1/w_1 & w_1/w_2 & w_1/w_3 & \cdots & w_1/w_n \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ \vdots & \vdots & \vdots & & \vdots \\ w_m/w_1 & w_m/w_2 & w_m/w_3 & \cdots & w_m/w_n \end{bmatrix}$$

Obtaining the vector of priorities is the next step. This is done by calculating the principal vector or eigen-vector. The eigen-vector is obtained by following the sequence outlined below.

Sum the columns of the matrix and obtain:

$$\sum_m w_m/w_1 \, , \quad \sum_m w_m/w_2 \, , \cdots\cdots , \sum_m w_m/w_n$$

Table 2-2

One-to-Nine Ratio Scale [20]

Intensity of Importance	Definition	Explanation
1	Equal Importance	Two activities contribute equally to the objective
3	Weak importance of one over another	Experience and judgment slightly favor one activity over another
5	Essential or strong importance	Experience and judgment strongly favor one activity over another
7	Very strong or demonstrated importance	An activity is favored very strongly over another; its dominance demonstrated in practice
9	Absolute importance	The evidence favoring one activity over another is of the highest possible order of affirmation
2,4,6,8	Intermediate values between adjacent scale values	When compromise is needed
Reciprocals of above non-zero	If activity *i* has one of the above non-zero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*.	A reasonable assumption
Rationals	Ratios arising from the scale	If consistency were to be forced by obtaining n numerical values to span the matrix.

Normalize matrix A by dividing each column by its sum, yielding:

$$A = \begin{bmatrix} w_1 / \sum_m w_m & \cdots & \cdots & \cdots & w_1 / \sum_m w_m \\ w_2 / \sum_m w_m & \cdots & \cdots & \cdots & w_2 / \sum_m w_m \\ \vdots & & & & \vdots \\ w_n / \sum_m w_m & \cdots & \cdots & \cdots & w_n / \sum_m w_m \end{bmatrix}$$

The priority vector, or vector of relative weights, is finally obtained by summing the rows and dividing by the number of elements in the row.

Row Sums	Vector of Relative Weights
$n \cdot w_1 / \sum_m w_m$	$w_1 / \sum_m w_m$
$n \cdot w_2 / \sum_m w_m$	$w_2 / \sum_m w_m$
$\vdots$	$\vdots$
$n \cdot w_n / \sum_m w_m$	$w_n / \sum_m w_m$

A numerical example of this procedure is provided to help clarify the procedure. The example has four attributes with three alternatives. The determination of the relative importance of the attributes is done through pairwise comparisons using the one-to-nine ratio scale proposed by Saaty. The preference matrix for the attributes that are used in the numerical example is as follows:

	Attribute			
	A	B	C	D
Attribute A	1	1/3	5	6
Attribute B	3	1	6	7
Attribute C	1/5	1/6	1	3
Attribute D	1/6	1/7	1/3	1

Numerical Example

(1) The preference matrix is converted to its decimal equivalent and its columns are summed.

	Attribute			
	A	B	C	D
Attribute A	1	0.333	5	6
Attribute B	3	1	6	7
Attribute C	0.200	0.170	1	3
Attribute D	0.170	0.140	0.330	1
	Σ 4.370	Σ 1.640	Σ 12.33	Σ 17

(2) Using the column sums from the above matrix the normalized matrix can be calculated by dividing the column elements by the column sum.

(3) The normalized rows are summed as illustrated below.

(4) The sum of the row vectors is divided by the number of row elements to obtain the priority vector; also illustrated below.

	A	B	C	D	Row Sums	Priority Vector
Attribute A	0.229	0.201	0.405	0.353	1.188	0.297
Attribute B	0.686	0.61	0.487	0.412	2.195	0.549
Attribute C	0.046	0.104	0.081	0.176	0.407	0.102
Attribute D	0.039	0.085	0.027	0.059	0.210	0.052
	Σ 1.0	Σ 1.0	Σ 1.0	Σ 1.0		Σ 1.0

A simple verification of the mathematics can be done by adding the columns of the normalized matrix and adding the elements of the priority vector. They should always add to one.

The fourth stage in the AHP process is the verification of the comparisons through the calculation of the CR. From the pairwise comparisons a_{ij} is denoted as the relative strength of preference of attribute *i* (c_i) to attribute *j* (c_j). The matrix of these comparisons is as follows:

$$A = \begin{bmatrix} c_1/c_1 & c_1/c_2 & \cdots & \cdots & c_1/c_n \\ c_2/c_1 & c_2/c_2 & \cdots & \cdots & c_2/c_n \\ \vdots & \vdots & \ddots & & \vdots \\ \vdots & \vdots & & \ddots & \vdots \\ c_n/c_1 & c_n/c_2 & \cdots & \cdots & c_n/c_n \end{bmatrix}$$

If $a_{ik} = a_{ij} \times a_{jk}$ for all i,j,k then the matrix A is perfectly consistent. Essentially, if the matrix is perfectly consistent it can be said that; if public safety is twice as important as the environment, and the environment is four times as important as recreation, then public safety is eight times as important as recreation.

The first step in the process followed by the AHP to calculate the CR is to multiply the matrix of comparisons by the priority vector. This yields a new vector. The next step is the division of this vector by the priority vector to obtain the final vector. (This vector should, generally, be of the same scale as the matrix size.) From this final vector, the average of the elements is computed. This yields the maximum eigen-value, λ_{max}. The formula below is used to calculate the consistency index (CI).

$$CI = \lambda_{max} - N / N - 1 \qquad [2\text{-}4]$$

Utilizing the random indexes (RI) calculated by Saaty the CR can be computed as:

$$CR = CI / RI \qquad [2\text{-}5]$$

A table of RIs is provided below. The RI values were calculated by Saaty [3] using random entries input into a matrix.

N	1	2	3	4	5	6	7	8	9
RI	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45

The numerical example is continued to help clarify the procedure. The calculation of the CR for the comparison matrix developed in the first part of the numerical example is now provided.

<u>Numerical Example</u>

(1) The decimal equivalent of the preference matrix is multiplied by the priority vector, yielding vector [C]. (The preference matrix is given on page 31 and the priority vector is given on page 32.)

$$
\overset{[A]}{\begin{bmatrix} 1 & 0.33 & 5 & 6 \\ 3 & 1 & 6 & 7 \\ 0.2 & 0.17 & 1 & 3 \\ 0.17 & 0.14 & 0.33 & 1 \end{bmatrix}} \times \overset{[B]}{\begin{bmatrix} 0.297 \\ 0.549 \\ 0.102 \\ 0.052 \end{bmatrix}} = \overset{[C]}{\begin{bmatrix} 1.300 \\ 2.416 \\ 0.411 \\ 0.213 \end{bmatrix}}
$$

(2) Divide [C] by [B] (the priority vector) yielding vector [D]:

[D] = [1.3/0.297 2.416/0.549 0.411/0.102 0.213/0.052]

= [4.377 4.401 4.029 4.096]

(3) Calculate λ_{max} (sum elements vector [D] and divide by number of vector elements):

$\lambda_{max} = (4.377 + 4.401 + 4.029 + 4.096) / 4 = 4.226$

(4) Calculate CI using Equation 2-4:

$CI = (\lambda_{max} - N) / N - 1 = 0.075$

(5) Calculate CR using Equation 2-5:

$CR = CI / RI = 0.075 / 0.90 = 0.083$

As the CR is less than 0.10 the matrix is considered to be consistent. Saaty [3] determined that 0.10 was an acceptable level for internal consistency. This was determined based upon a series of experiments that were conducted using random entries in a matrix.

The fifth and final stage in the AHP process is the determination of the overall relative weights for each alternative. The analyst now does the pairwise comparisons for each alternative considering each attribute. The results for these series of comparisons are provided in Table 2-3. For brevity's sake the CRs for these matrix comparisons have not been included. The alternative priority vectors in Table 2-3 are compiled into a matrix, giving the alternative weights considering each attribute. This is then multiplied by the attribute priority vector giving the alternative ratio weights.

Numerical Example

The matrix of the alternative/attribute weights is multiplied by the attribute priority vector (attribute weight) yielding the alternative weighting.

$$\begin{array}{l} \text{ALTERNATIVE A} \\ \text{ALTERNATIVE B} \\ \text{ALTERNATIVE C} \end{array} \quad \overset{[A]}{\begin{bmatrix} 0.624 & 0.571 & 0.316 & 0.070 \\ 0.239 & 0.286 & 0.102 & 0.166 \\ 0.137 & 0.143 & 0.582 & 0.764 \end{bmatrix}} \times \overset{[B]}{\begin{bmatrix} 0.297 \\ 0.549 \\ 0.102 \\ 0.052 \end{bmatrix}} = \overset{[C]}{\begin{bmatrix} 0.535 \\ 0.247 \\ 0.218 \end{bmatrix}}$$

Vector [C] is the alternative ranking. The reader is referred to Saaty's original work for a complete review of the theoretical base for the AHP. Another useful reference for those desiring a clear, concise explanation of the procedure is Canada and Sullivan [21].

2.5.3 AHP and Utility Theory

AHP and Utility Theory have been viewed as both conflicting and complementary theories in the literature. This conflict is best seen in a series of articles by Dyer [22], Saaty [23], and Harker and Vargas [24]. In his article Dyer attempts to tie AHP with Multi-Attribute Utility Theory. His attempt is strongly rebuffed by the proponents of

Table 2-3

Calculation of Example Alternative Weights

With respect to		Alt A	Alt B	Alt C	Vector *
Attribute A	Alt A	1	3	4	0.624
	Alt B	1/3	1	2	0.239
	Alt C	1/4	1/2	1	0.137
		Σ 1.58	Σ 4.50	Σ 7	
Attribute B	Alt A	1	2	4	0.571
	Alt B	1/2	1	2	0.286
	Alt C	1/4	1/2	1	0.143
		Σ 1.75	Σ 3.50	Σ 7	
Attribute C	Alt A	1	5	1/3	0.316
	Alt B	1/5	1	1/4	0.102
	Alt C	3	4	1	0.582
		Σ 4.20	Σ 10	Σ 1.58	
Attribute D	Alt A	1	1/3	1/9	0.070
	Alt B	3	1	1/6	0.166
	Alt C	9	6	1	0.764
		Σ 13	Σ 7.33	Σ 1.28	

* Priority vector is calculated using procedure outlined in numerical example; sum columns, normalize them, sum rows, divide by number of row elements.

AHP. Much of the controversy centers around the fundamental question of scales of measure. Saaty [25], states that the AHP, which is based on ratio scales would be worsened by the addition or incorporation of a theory based upon interval scales.

The different scales of measurement must be reviewed if it is desired to fully understand the disagreement. Generally four scales of measurement are considered to exist; nominal measurement, ordinal measurement, interval measurement, and ratio measurement [26]. Nominal measurement is when groups are formed and measured. Nominal measurement is often basic and serves to separate data into desirable groupings. Ordinal measurement is thought of in terms of greater than, or less than, a group to which it is being compared. The application of the ordinal scale generally increases the amount of statistical techniques that can be used. Interval measurement has two features, it employs equal units of measure, and the origin or zero point is established arbitrarily. The ratio scale of measurement compares one thing against another.

Utility theory is based upon the application of an interval scale. In the AHP benefits or costs of one attribute are compared across projects. These ratio measures are then taken and used to synthesize an ordinal scale.

This discussion is best summarized by the following quote from Leedy:

> "If you can say that:
> one object is *different* from another, you have a *nominal* scale,
> one object is *bigger* or *better* or *more* of anything than another, you have an *ordinal* scale,
> one object is *so many units* (degrees, inches) more than another, you have an *interval* scale,
> one object is *so many times* as bright or big or tall or heavy as another, you have a *ratio* scale." (italics inserted) [27]

The basic difference between these two scales is that the interval scale requires an origin as a point of reference. This is confirmed by this excerpt from Keeney and Raiffa:

" Then, because utility is relative and not absolute, to establish an origin and unit of measure, we can arbitrarily assign utilities to two of the consequences and then assess utilities for the other consequences relative to those two." [28]

Therefore, the basics of utility theory are the setting of an origin and unit of measure. Then the assessment of each successive consequence, say x_i, so that the decision maker is indifferent to the consequence x_i, at a certain probability p, to the 1-p probability of getting x_0. The calculation of a utility function is a time consuming, and often, frustrating practice. This difficulty becomes even worse when considering groups instead of individuals. The calculation of a group utility function is thought to be quite demanding and it is perceived as being a very volatile entity. Utility theory is also considered to have five axioms as follows; orderability, transitivity, continuity, monotonicity, and decomposability [42].

In his reply to Dyer's article Saaty [29] provides a list of reasons why utility theory and the AHP, can not be, or should not be combined. The first reason given by Saaty is that the AHP employs the ratio scale. If utility theory concepts are interposed into the AHP as proposed, it is as if an interval scale is multiplied by a ratio scale, obtaining an interval scale. Saaty does not consider this to be effective. His second argument is that attributes are abstract concepts more easily represented by ratio scales than by the difference of measurement not on the same scale. His final argument is that the establishment of ratio measurements is easily and effectively achieved unlike the interval measurement requirements for utility theory.

Obviously, there is a close relationship between the AHP and utility theory. This relationship continues to be competitive, as opposed to cooperative. Both theories have their strengths and weaknesses. It will take considerable work and ingenuity to make these two different approaches complement each other to the satisfaction of all participants.

2.5.4 AHP Benefits

AHP has numerous advantages. First, and foremost, it is simple to use and understand. This advantage is strengthened by the existence of software which applies the AHP such as Expert Choice [30] and Automan [31]. This advantage, often derided by academics as an excuse to avoid rigor, is supported as critical by Henry Mintzberg. In a landmark management article he discussed the relationship of the manager with the analyst and he notes the following;

> " For the analyst, adaptation means worrying less about the elegance of the method and more about its speed and flexibility." [32]

The selection of a multi-attribute model can be based upon many criteria. The criteria used to evaluate the AHP comes from a list provided for evaluating multi-attribute models and is provided in DeGarmo, Sullivan, and Bontadelli [38]. The ten criteria include the following; theoretical soundness, credibility, verifiable, comprehensive, reasonable data requirements, explicit assumptions, uncertainty explicitly treated, effects of time taken into account, suitability to a group process, and ease of communication.

When reviewing these criteria it is noted that the AHP satisfies most of them. It is theoretically sound, and is widely accepted as such in the literature. Problems with the AHP are generally centered around its effectiveness and not its theoretical base. Its credibility has been established in numerous applications and published articles. The confidence of those that use the AHP is also seen by the fact that the AHP is taught in engineering courses. There are also two different software packages which employ the AHP theory, one developed by the Federal Government [31]. The AHP provides a method of internal verification. It has a process that is transparent and explicit. One of the strongest features of the AHP is that it is comprehensive. It is a process which enables all quantifiable and non-quantifiable criteria to be simultaneously evaluated. The AHP has very reasonable data requirements. It does not require extensive data but, as with most models, the better the data the more informed the decision. The assumptions underlying the AHP have been made explicit in many articles and books on the subject. The AHP does not explicitly take into account uncertainty, but it can be evaluated fairly easily through sensitivity analysis. Effects of time are taken into account by the inclusion of equivalent money flows and by the fact that the sequential process can reflect changes over time. (In this case, projects which have been selected can be included in future analyses by making them political or must choose projects.) The AHP is eminently suited to a group process. There are many different methods by which the AHP can be used in a group process; one method would employ the geometric mean of the group decision. Finally, the last and one of the stronger points of the AHP, is its ease of communication. The model is easy to apply and its findings are explicit. The user receives an ordinal ranked list of his alternatives from which he can base his decision.

For the purpose of this research, the AHP is a recognized and accepted technique. It offers substantial advantages, with some limitations. Awareness of its limitations will reduce their impact and render the analysis that much more effective. The application of the AHP proposed in Chapter 3, offers a novel twist to the AHP method of assessing two hierarchies and their output vectors. It takes advantage of the best aspects of AHP.

2.5.5 AHP Limitations

The AHP does have its limitations. Problems that have been identified with AHP include:

(1) difficulties with the questioning procedure (does not take into account economies of scale),

(2) problems exist when an alternative does not possess the benefit/disbenefit given in the hierarchy, and

(3) rank reversal can happen when new alternatives are introduced into the problem.

These limitations are discussed in great detail in the literature [22, 33, 37]. However, for a good synopsis of the limitations of AHP, as viewed by its detractors, the work of E. MacStravic [33] is useful. In her thesis MacStravic talks about one other problem; the analysis of benefits and costs. This is not perceived as a problem with the AHP so much as a problem with a proposed application of the process. Accordingly, this issue will be looked at in Chapter 3, which discusses the methodology.

The first problem, difficulties with the questioning procedure, can be addressed through an improved questioning procedure as suggested by MacStravic [33]. She proposes that, unlike the sort of questions seen in texts on the AHP, the questions be tied to the alternatives. For example, do not ask if life-cycle cost is preferred to transportation capacity. It would be better to ask if X units of life cycle-cost are preferred to Y units of transportation capacity. This is based upon the fact that the traditional questioning procedure leaves the decision maker to mentally input his own quantities. Typically, one thinks in terms of average amounts which may not be applicable to the problem he faces.

Generally the procedure of asking questions based upon the alternatives is supported. However, it must be noted that in her example, MacStravic uses tangible or hard data criteria only. The public sector environment has criteria, such as public safety, which do not have units readily established for quantification. This is where the strength of the AHP is realized; in an analysis of a decision problem which includes tangible and intangible criteria assessed simultaneously.

The difficulties with a lack of a common hierarchy of attributes are a more difficult issue to address. The obvious solution would be to change the weighting scale, such that the weight assigned to the attribute would be insignificant. Another possibility is that of not including an individual alternative in the matrix calculations for that attribute. This would give the alternative an effective weight of zero for that attribute.

In the application of the software for Chapter 4, Hypothetical Example, the first method is utilized. The software provides the means by which data can be input directly into the program and the ratio then computed. It is possible to use this feature to give an attribute, for a selected alternative, an effective weight of zero. The use of this performance data option, to improve the ratio comparisons or to revise the hierarchy, is discussed in Chapter 4, The Hypothetical Example.

The final difficulty noted with the AHP is that of rank reversal when a new alternative is introduced. This fault is attributed to the fact that criteria weights are assessed independently of their alternatives. When using AHP it is doubtful that the situation is absolute. The introduction of a new alternative may change the picture sufficiently that rank reversal is experienced. The only effective method of dealing with such a problem is to ensure that the analyst has all the possible alternatives included prior to conducting his analysis.

It must also be noted that the introduction of a new project will also change the number and composition of feasible portfolios that are evaluated in the methodology presented in Chapter 3. It is felt that this deficiency can be addressed through rigorous application of the first step of the algorithm process, project identification.

III. METHODOLOGY

3.1 The AHP/Capital Budgeting Algorithm

The algorithm created herein utilizes the work of Skipper, Kladivko, and Saaty. Combining these techniques results in a synergistic effect which is discussed in this Chapter.

Figure 3.1 illustrates the conceptual flow of the new algorithm. This conceptual flow consists of three Branches. The Branches all flow from the initial step of identifying all the possible projects. These Branches are as follows; the Portfolio Feasibility Branch, the Project Assessment Branch, and the Portfolio Assessment Branch. An overview of the entire algorithm is presented in this Section. A detailed review of the activities for each different Branch or critical activity in the analysis is covered in later Sections. The intent of this Section is to provide an overview of the algorithm proposed for the treatment of capital allocation for public projects.

A note on the methodology should now be made. The methodology proposed in this thesis melds two different processes into one. Unfortunately the computer code for the methodology, as outlined, does not exist. Different existing computer codes have been used to accomplish the analysis in Chapter 4, Hypothetical Example. The life-cycle cost analysis, discussed in Section 3.3, is actually done using Engineering Economy software provided by Virginia Tech [39]. The AHP analysis is accomplished through the use of the Automan [31] software. The calculation of feasible portfolios is done with the code provided in Kladivko's thesis [2].

The first step involves the identification of all projects (note how this ties into the attempt to address the limitation of rank reversal discussed in Section 2.5.5). It must be

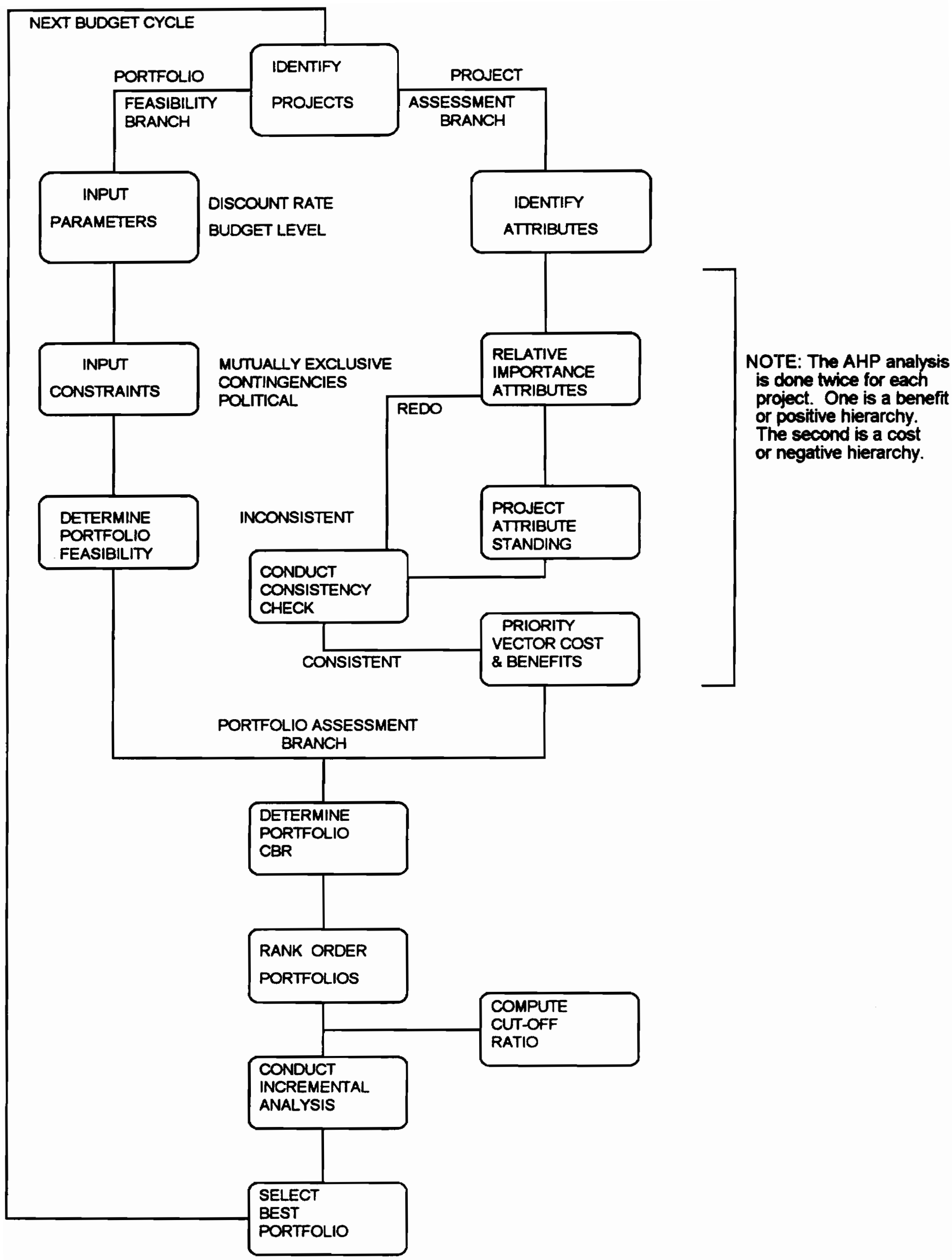


Figure 3.1 - AHP/Capital Budgeting Logic Flow

emphasized that the project be evaluated over its complete life cycle. Failure to do this simple, yet important, task has resulted in many poor analyses in the public sector.

The Portfolio Feasibility Branch of the conceptual flow diagram is reviewed first. This Branch essentially follows the same procedure as that advocated by Skipper [1] and Kladivko [2]. The input of parameters is primarily concerned with the discount rate to be used, and the amount of funds budgeted. As discussed, these will be constants determined by the appropriate elected bodies. The evaluation of constraints takes into consideration all mutual exclusivity, contingencies, and political constraints that must be weighed. The final step in the Portfolio Feasibility Branch is, as one would expect, the determination of all feasible portfolios.

The Project Assessment Branch employs AHP to establish priority weights for benefits and disbenefits for each project. The establishment of the attributes by which the projects are evaluated is a critical issue. It is possible that attributes with the same unit of measure would be in both the negative and positive hierarchy. An example of this is cost and revenue. Once the attributes are defined, the benefit and disbenefit hierarchies are analyzed. This produces the priority vectors for cost and benefits. The last step in the Project Assessment Branch is the conducting of consistency checks by use of the CR.

Finally, the Portfolio Assessment Branch is activated. First, the Portfolio Cost/Benefit Ratios are determined. This is done by summing the appropriate benefit and cost elements from the priority vectors. Next, the portfolios are rank ordered. From here the Cut-Off Ratio must be determined. Further details on the calculation of the Cut-Off Ratio are provided in Section 3.4.1. The Cut-Off Ratio is the bench-mark which is used in the incremental analysis. Now, the incremental benefit analysis is conducted to determine if the incremental benefit received from the other portfolios is greater than the Cut-Off Ratio. Based upon this analysis, the preferred Portfolio is selected.

3.2 The Portfolio Feasibility Branch

The Portfolio Feasibility Branch, as mentioned, incorporates the capital budgeting work of Skipper [1]/Kladivko [2]. The first step in the process is to input essential parameters. These parameters are the social discount rate and the budget limit. There is no difficulty in using a mandated discount rate on the fiscal aspects of a project. However, the intangible factors that are being measured, must also be considered. If public safety is being evaluated as a factor, how does the analyst quantify the fact that project A has high ratings for public safety, but its life is half of project B, which has lower safety ratings?

There are two possible ways to assess the unequal lives of projects with respect to their intangible benefits. One method would involve the social discount rate, or some variation thereof, to discount the benefits (or costs) to obtain equivalence. The other method would involve having decision makers subjectively assess the intangible, taking into account the different lives.

It is thought that the second method is better. Discount rates and equivalence assessments are based upon the time value of money. This basic principle states that a dollar today is worth more than a dollar a year from now. It has a sound theoretical base. Can it be stated that there is a time value to public safety, crime prevention or recreation? In terms of public safety, if a man loses his arm on the job today, or one year from today, the accident is no less severe. However, he has lost the ability to obtain income, therefore it could be said that there is a time value to these intangible criteria.

Skipper/Kladivko use the FW criterion because, it was argued, funds could be reinvested in a manner which should yield at least the MARR. This is considered to be acceptable, and is certainly better, than alternative approaches based on multiple lives or early retirement. Is it possible that a project will yield public safety or crime prevention at some social discount rate in the future? Given that each project is a unique entity that provides intangible benefits and costs dependent upon the conditions at that particular time; discounting, although an applicable concept, would be very difficult to apply. It is considered to be akin to the problem of developing group utility functions. As the benefits and costs are dependent upon the conditions existing at a particular time the discounting of intangibles would be a volatile issue. Another reason why it is felt that this method of assessment is better, is due to the typical life of public projects. The private sector does not often think in terms of 25, 50, or 100 years. However, these life spans are not uncommon for public projects. When starting to evaluate projects with such life spans it becomes very difficult to quantify values that far in the future. Indeed, the scale of benefits in the distant future would have to be tremendous, to have any significance.

The budget limit as presently defined is only for the present year. Kladivko has taken the Skipper algorithm a step further by permitting the analyst to input budget estimates for up to five years. This is a notable improvement in the usefulness of the algorithm. It becomes particularly important for the analysis as it must differentiate between capital costs and O&M type costs. For this reason, this Branch of the algorithm does not consider the life-cycle costs of a project. It is focused on merely determining whether or not the portfolio of projects meet the budget constraint or not. As identified in Section 1.4 Research Premises/Delimitations, this restriction has a negative impact on the ability of the methodology to reflect the best options from a life-cycle cost perspective.

It is perhaps pertinent to now discuss why the budget constraint has been separated from the life-cycle cost analysis. In the public sector, the analyst or bureaucrat has two different financial criteria which he has to consider. The first and foremost is his budget constraint. He can not exceed his budget limit for the present year. His second financial criteria is the life-cycle cost aspect of a project. It is not unheard of for a good project, from a life-cycle cost perspective, to be rejected, because it exceeds the present year budget. Then a second rate, or more expensive life-cycle cost project, gets accepted because it is within its first year budget limit. This methodology attempts to quantify both criteria in a discrete manner and reduce the short-sightedness associated with the budget constraint.

The next step in the Portfolio Feasibility Branch has been well outlined in previous Sections. Essentially, it considers which projects are mutually exclusive, have contingencies upon other projects and those which must be chosen. This last constraint has been renamed from "must choose" to the "political constraint". The reason for this renaming is because it was felt that the term "must choose" was misleading in the public sector environment. What must be chosen from an engineering analysis view point can, and is often considerably different, from what must be chosen from a political view point.

The last step in this Branch is the determination of portfolio feasibility. Utilizing the 2^n matrix, the feasible portfolios are determined. Feasible portfolios are those that do not violate any of the constraints that have been outlined. As discussed, the "do nothing" option or first portfolio, is not considered a viable alternative for the capital allocation process for public projects. Although it does not violate any of the explicit constraints, for the environment considered herein, the do nothing option does not exist. More succinctly, the political constraint says "do something".

3.3 The Project Assessment Branch

The Project Assessment Branch is where the different attributes are evaluated for each project on an individual basis. The life-cycle cost analysis, being an attribute for the analysis, forms part of this Branch's activities. The projects are assessed individually, using the AHP. However, even at this early stage the manner in which it is normally utilized is modified.

The individual steps of the Branch and their specific activities are now reviewed. The first step, which is critical for a successful analysis, is the identification of attributes. The number of attributes and sub-attributes is also of interest. It is proposed that the number of elements in any one level not exceed nine. This is because of human limitations with respect to perception. These limitations are well noted in a paper by George A. Miller [35], which discusses the limits on the capacity to process information.

The hierarchy for evaluating public projects involves at least four levels. A typical example is depicted in Figure 3-2. The second level of the hierarchy (referred to as attributes in AHP terminology) employs a technique used in transportation systems planning [36]. This level enables the analyst to capture the indirect benefits and costs or the externalities of a public project. It explicitly considers the full range of public projects; from the pure public project which provides unrestricted access, to the project that can be restricted. These attributes represent the group receiving the benefit (user), the group providing the benefit (provider)(sometimes they will be the same), and society at large (society). The inclusion of the provider, user, and society in the analysis is considered to be a significant improvement over methods presently in use. Although this complicates the analysis, it provides a much more effective, and accurate, assessment with respect to overall social welfare. Forcing the analyst to look beyond those merely receiving the benefit (users) may help the public agency make an informed decision based upon the desire to improve the social welfare of <u>all</u> the people in a community.

Typical sub-attributes that may be considered include the following; pollution, cost/revenue, public safety, public recreation, accessibility, job creation, tourism, education, and crime prevention. This list is not meant to be exhaustive, just typical. The

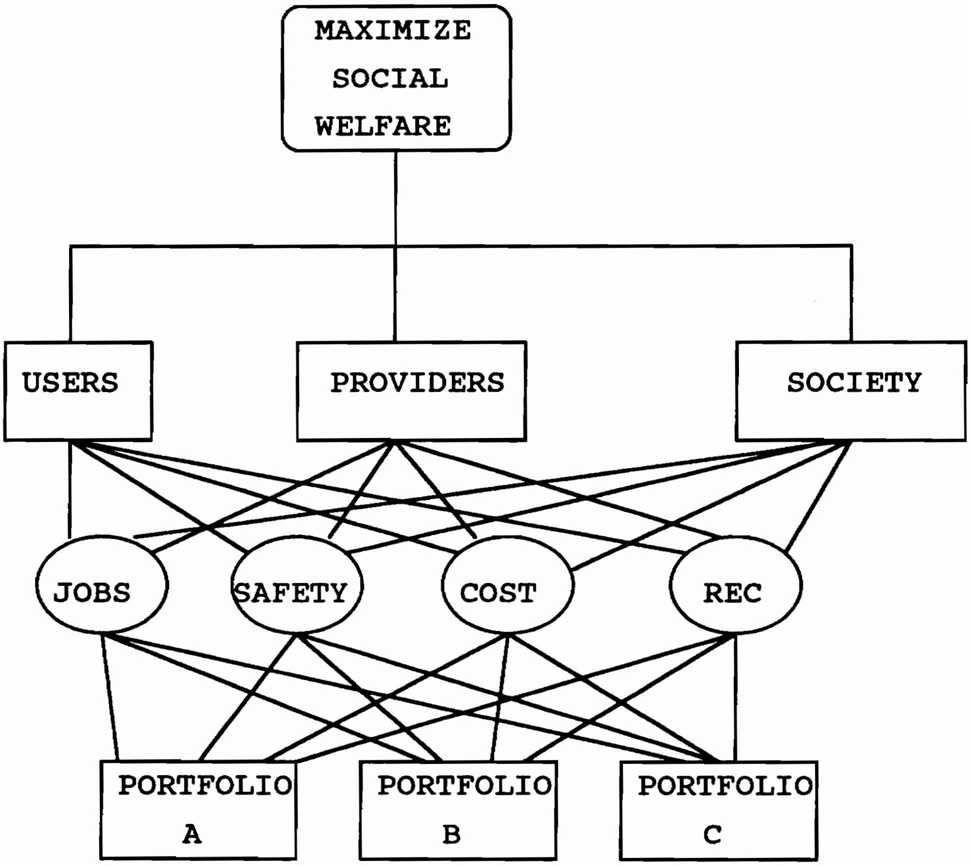


Figure 3.2 - Sample Hierarchy for Public Capital Allocation Process

attributes mentioned represent a sample based upon this author's reading and experience. The list is not, nor will there ever be, a definitive hierarchy. Each level of government can define their own particular attributes for a selected problem. The definition of the desired attributes will not affect the process; presuming of course that the attributes are defined equitably.

Once the attributes are defined, for both the positive and negative hierarchies, the AHP analysis begins. The first step is to establish the relative importance of the attributes. This means that the analyst first establishes the relative weights of the user, provider, and society with respect to the overall goal: maximizing social welfare. Then he establishes the relative merits of each attribute, for both the positive and negative hierarchies, with respect to the user, provider, and society. The imposed limitation of no more than nine attributes at any one level can now be appreciated. Figure 3.2 illustrates the hierarchy from the sub-attribute level (jobs, safety, rec, and cost), to the portfolio level. It must be remembered that further levels of sub-sub-attributes can be added to the analysis, if desired. The decision on whether or not to include further levels of attributes must weigh the cost of the more detailed analysis, with the expected benefit from the analysis.

The next step is to evaluate the project attribute standing. As has been done with the other levels, the standing of the projects, with respect to the specific attributes, must now be established. Consistency checks at each level of the analysis are conducted coincidentally. The CR for each series of comparisons is done to ensure the analysis is valid.

The final step of this Branch involves taking all the relative weights that have been computed. By moving back up through the hierarchy, the relative weight for each project with respect to the goal can be established. This relative weight is called the priority vector. There are two priority vectors for each project, one a positive or benefit vector and the other negative or a cost vector. The positive vector is easy to establish. More care must be exercised in the establishment of the cost vector. The temptation is to say that "less is better" and proceed with the analysis. However, it is desired to identify the project with the highest costs as having the largest priority vector for the cost hierarchy. Therefore, this analysis must proceed in the same manner as the benefit analysis.

One last point must be made about the Project Assessment Branch . One attribute will always be in the positive and negative hierarchies. Project life-cycle costs are identified and included in the negative hierarchy. Project life-cycle revenues are identified and included in the positive hierarchy. The degree to which life-cycle costs, or life-cycle revenues, affect the decision depends upon the weight given these particular attributes by the analyst. The inclusion of a proper life-cycle economical analysis is considered to be fundamental to the accurate assessment of the projects. The results of the life-cycle cost analysis are utilized in this Branch.

3.4 The Portfolio Assessment

The capital budgeting and the AHP methodologies have been utilized with only minor modifications. The Portfolio Assessment Branch combines the methodologies to achieve the synergistic effect and obtain an analysis which encompasses more than either methodology could by itself.

The first step in the Portfolio Assessment Branch process is the determination of the Portfolio Cost/Benefit Ratios (CBR). Feasible portfolios from the Portfolio Feasibility Branch activities are taken and combined with the positive and negative priority vectors from the Project Assessment Branch. Mathematically:

$$\text{Portfolio CBR} = \frac{\sum (b_j)(\text{project}_j)}{\sum (c_j)(\text{project}_j)} \qquad [3\text{-}1]$$

Where project j has a value of 0 or 1 for a particular project depending upon the portfolio feasibility depending upon the results of the Portfolio Feasibility Branch. The b_j identifies benefits for project j from the benefit vector and c_j identifies costs for project j.

Now that the Portfolio CBR has been determined, the portfolios are rank ordered. Portfolios are rank ordered on the basis of their negative priority vector. Ranking of the portfolios by ascending cost component is considered to be the same as ranking a project based upon initial investment. Any other method of ranking the portfolios introduces difficulties which can not be easily resolved.

The remainder of the Branch is as follows: a Cut-Off Ratio is determined, an incremental analysis is performed, and the preferred portfolio is selected. Further details on the Cut-Off Ratio and incremental analysis are given in the next two sub-sections.

3.4.1 Determination of Cut-Off Ratio

The concept of Cut-Off Ratio is explained in an article by Bernhard and Canada [23]. In the article they refute the process Saaty proposes to use for the allocation of resources between three mutually exclusive projects. In the article Bernhard and Canada outline how the process, described by Saaty, does not take into account the basic principle of incremental benefits. They utilize an example, with all benefits and disbenefits included in terms of dollars. They state that the best way to establish which project is the best, is to sum the individual project benefits and sum the individual project costs. The sum of the benefits is divided by the sum of the costs. This is what they called a "Cut-Off Ratio". Any project which had a CBR lower than the Cut-Off Ratio, was not economical to fund.

It is of interest to note that the Cut-Off Ratio identified in the article was 0.677 or lower than 1.0. This contradicts the traditional theory that the CBR must exceed 1.0 for a project to be beneficial. This is considered valid for the capital allocation problem in the public sector. As the do-nothing alternative is not viable for the public sector environment, a solution process which chooses the best option, from the alternative investment options considered, must be sought.

It is proposed that a variation of the Canada-Bernhard Cut-Off Ratio be used in this thesis. Multiple, mutually exclusive portfolios, are available to choose from, but the portfolios consist of individual projects which have normalized weights. Benefits and costs for all feasible portfolios shall be summed. This will give the Portfolio Cut-Off Ratio. Mathematically:

$$\text{Portfolio Cut-Off Ratio} = \frac{\sum_{\text{portfolio}} \left(\sum b_j\right)\left(\text{project}_j\right)}{\sum_{\text{portfolio}} \left(\sum c_j\right)\left(\text{project}_j\right)} \qquad [3\text{-}2]$$

This Cut-Off Ratio will now be utilized in the incremental analysis of the feasible portfolios.

3.4.2 Incremental Analysis

The concept of incremental analysis is a fundamental aspect of Engineering Economics. The basic principle involves the determination of the return that can be received on gradually higher levels of investment. It can be shown that the return on an increment, or delta (Δ), of investment is such that it is economically feasible to make the investment [14]. Since mutually exclusive alternatives are being considered, it is possible to make the decision based upon the *incremental* difference in the portfolios.

In the analysis, the portfolios are ranked based on their initial cost. Then the deltas of the cost and of the benefit vector from the next higher portfolio are calculated. If the portfolio benefit (positive) vector, is less than the next portfolio benefit vector, then a negative CBR exists. Obviously, this alternative is not considered. In the event that the incremental benefit vector is not negative; the incremental cost (negative) vector, and incremental benefit vectors, are used to calculate the incremental CBR (Δ CBR). This value is compared to the Cut-Off Ratio, which was previously determined. If the incremental CBR exceeds the Cut-Off Ratio, then the additional investment is recommended. If it does not, then the alternative is not selected. This procedure will continue until such time as all alternatives have been assessed. The end result is a list of feasible portfolios which have been analyzed for incremental benefit against the Cut-Off Ratio. From this analysis he can select the best portfolio or provide the decision makers with a listing of feasible portfolios and indicate the best portfolio.

IV. HYPOTHETICAL EXAMPLE

4.1 Example Overview

This Section provides the data for the hypothetical example. Sections two, three, and four provide the solution process for the Branches represented in Figure 3.1. Respectively they are: Portfolio Feasibility Analysis, Project Assessment Analysis, and Portfolio Assessment Analysis. Section 4.5, Alternative Treatments, considers the use of quantitative data when doing ratio comparisons and explores a method which can be used to assess attributes which may not be common to all alternatives.

For the example, suppose that a town is considering five possible capital projects. It is assumed that the projects have been processed through an administrative summative procedure and that, individually, each project is suitable for implementation. As previously discussed, it is not relevant to the capital allocation problem to consider methods to improve the individual projects. Improvements to the project is a formative process which must be done before the projects reach this stage of analysis.

The capital projects that are being considered are as follows:

(1) Project A: renovate police HQ

(2) Project B: construct new HQ

(3) Project C: build a new town park/recreation area

(4) Project D: upgrade road system serving park area

(5) Project E: purchase of snow removal fleet (3 pieces - 2 plows, 1 snowblower)

This concludes the initial and vitally important step of identifying <u>all</u> possible projects for consideration. If it is desired to avoid the problem of rank reversal as noted in Chapter 2, then it must be ensured that all projects are considered.

Now, the projects are reviewed individually. Each project has been through the conceptual design phase and is considered to now be in the detail design/development phase of their life cycle. In order to proceed they must pass through this analysis. In preparation for later analysis the life-cycle cost and life-cycle revenue cash flows are evaluated. Life-cycle cost and revenue data with FW are provided in Table 4.1.

Project A is the renovation of the local police headquarters. This project has a useful life of only 4 years. Increased operating and maintenance costs will result in an expenditure of $150,000 per year, with a gradient of $25,000 per year. In year 4 the building will be salvaged for some other use, for a cost of $300,000. This includes costs associated with moving police operations. No revenue will be generated.

Project B is the construction of a new police headquarters. It will cost $200,000 in the first year and $300,000 in the next year. Finishing touches will cost approximately $25,000 in year three and it will cost $75,000 to relocate. Operating and maintenance costs for this year are only expected to be $25,000 and then $50,000 per year until the end of the project life at ten years. This project will generate no significant revenue.

Project C is the building of a new town park/recreation area. The park will cost $140,000 to build over two years, with $100,000 being incurred in the first year. It will cost $10,000 per year to operate and maintain after construction. Forecasted growth in the area will result in greater use about 5 years from now, resulting in an additional cost of $5000 per year for 4 years. In the last two years a declining enrollment, as the facility ends its useful life, will result in operating expenses being reduced to $10,000. The facility will generate no revenues until year 3, when it will generate $10,000 for 2 years, $15,000 for the next 2 years, $10,000 for the next 2 years, and $7000 for its last 2 years.

Project D is the upgrading of the road system in the park area. The cost is $160,000. It will cost $5000 per year, to maintain the roads, with an additional $5000

Table 4.1

Life Cycle-Cost and Revenue Data

LCC (1000)						REVENUE (1000)					
Year	A	B	C	D	E	Year	A	B	C	D	E
0	150	200	100	160	125	0	0	0	0	0	0
1	175	300	50	10	50	1	-	-	0	0	-
2	200	125	10	5	30	2	-	-	0	5	-
3	225	50	10	5	30	3	-	-	10	5	-
4	300	50	10	10	30	4	-	-	10	5	-
5	-	50	15	5	30	5	-	-	15	5	-
6	-	50	15	5	30	6	-	-	15	5	-
7	-	50	15	10	30	7	-	-	10	5	-
8	-	50	15	5	30	8	-	-	10	5	50
9	-	50	10	5	-	9	-	-	7	5	-
10	0	50	10	20	0	10	0	0	7	5	0
FW	2200	2066	541	537	787	FW	0	0	123	68	61

being incurred in years 1, 4, and 7 for patch-work. In year 10 major maintenance costs will result in an expenditure of $20,000. At this time the town will consider a project to upgrade the roads, or to re-pave. The upgrading of the road system is expected to attract some new construction and result in higher property tax for existing properties. The net revenue from both these actions is $5000 per year starting in year 2.

The final project is Project E, purchase of snow removal equipment. The equipment will cost $125,000. Training costs for new operators are only expected to be $20,000 occurring in year 1 only. Operating and maintenance costs are expected to be $30,000 per year, for the eight year life of the project. No revenues will be generated by the project, but the salvage value of the equipment is expected to be $50,000 in year 8.

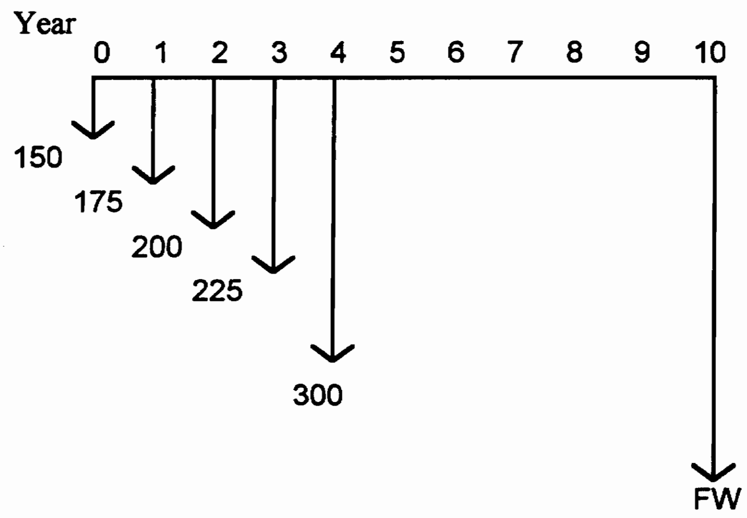
For illustrative purposes, the calculation of the FW for Project A's life-cycle costs is shown. The cash flow diagram, and calculations for the project are provided at Figure 4.1. Equation 2-3, from Chapter 2, has been used in the evaluation of the cash flows. The FW for all projects life-cycle cost and life-cycle revenue cash streams are provided in Table 4.1 The discount rate used in establishing the equivalence of the money flows is 10%. This is considered the mandated social discount rate. The town uses this mandated discount rate to aid in the procurement of state and federal funds.

4.2 Portfolio Feasibility Analysis

The first step in the analysis of portfolio feasibility is to input the parameters of budget and MARR. Suppose the town has established a maximum budget of $600,000 for capital projects this fiscal year. As discussed in Section 4.1 the town uses a social discount rate of 10%.

Next, the constraints which are part of the problem are considered. Project A and Project B have been put forward as alternative solutions and are mutually exclusive.

Project A - Cash Flow Life-Cycle Cost



$F_{10} = \$150{,}000\ (1+0.10)^{10} = \$389{,}061$

$F_9 = \$175{,}000\ (1+0.10)^9 = \$412{,}641$

$F_8 = \$200{,}000\ (1+0.10)^8 = \$428{,}718$

$F_7 = \$225{,}000\ (1+0.10)^7 = \$438{,}461$

$F_6 = \$300{,}000\ (1+0.10)^6 = \underline{\$531{,}468}$

$\Sigma\ \$2{,}200{,}349$

Figure 4.1 - Project A - Cash Flow Diagram and Life-Cycle Cost

Project C is contingent upon Project D, as the proposed park does not have adequate road facilities. This means that without the upgraded road work proposed in project D that Project C can not go ahead. Project E is a political constraint and it must be selected. (Next year is an election year and snow removal has become a hot issue due to difficulties last season.)

Each project has an initial cost associated with it. This cost is not the same as the life-cycle cost which was evaluated in Section 4.1. Rather, it is the cost which must come out of the available \$600,000 budget. The costs for the projects are as follows: Project A = \$150,000, Project B = \$200,000, Project C = \$100,000, Project D = \$160,000, and Project E = \$125,000.

From this information the number of feasible portfolios for the example can be calculated. The feasible portfolios are calculated from the 2^n or 32 different possibilities. Table 4.2 shows the feasible portfolios for this example. Note that, as has been demanded, Project E, the political constraint, appears in all of the feasible portfolios.

4.3 Project Assessment Analysis

The first step in this analysis is the identification of the attributes to be used in the analysis. The town manager, in consultation with the city council, has decided upon five attributes for the benefit analysis. The positive attributes are as follows; public safety (Pub Safe), economic impact (economic), crime reduction (crime(-)), revenues (revenue), and recreation (rec). The attributes for the negative hierarchy are as follows; life-cycle costs (LCC), environmental (environ), crime increase (crime(+)), and accessibility (access). (Expressions in parentheses show how the attributes are represented in the solution process.)

As discussed in previous chapters, the hierarchy also has the attributes of user,

Table 4.2

Feasible Portfolios

Portfolio	Project A	B	C	D	E
9	1	0	1	1	1
13	1	0	0	1	1
15	1	0	0	0	1
17	0	1	1	1	1
21	0	1	0	1	1
23	0	1	0	0	1
25	0	0	1	1	1
29	0	0	0	1	1
31	0	0	0	0	1

provider, and society. The benefit hierarchy is illustrated in Figure 4.2. The negative hierarchy appears the same, except it has the four negative attributes in level three. Now that the attributes have been identified, the next step is to establish their relative importance. This Section will present the results of the analysis. An example of the detailed calculations is provided in Appendix A.

The first attributes for which it is necessary to establish relative importance are; user, provider, and society. The user is established as having the largest weight, the provider the second largest, and society the smallest. While this assessment may not be very altruistic, it is considered accurate. The weight for these three attributes are provided in Table 4.3. These weights are obtained by making pairwise comparisons for the three levels.

The relative importance of the attributes in consideration of the above criteria is the next step. The results of these comparisons are also provided in Table 4.3. It must be remembered that all the weights are achieved using ratio comparisons. The author subjectively evaluated the ratios of the FW life-cycle costs and life-cycle revenues provided in Table 4.1. The direct use of quantitative data to calculate the ratio comparisons is done in Section 4.5.1.

The next step in the Project Assessment Branch is to evaluate each of the now weighted attributes with respect to the various alternatives. The conducting of consistency checks for each set of matrix comparisons in the hierarchy has been done. (Automan [31] automatically calculates the CR for each set of matrix comparisons and the user can not proceed unless the CR is less than 0.10.)

From the alternative attribute comparisons, the matrix is now collapsed to produce the priority vectors for benefits and costs. The priority vectors are provided in Table 4.3. Based upon these vectors, the benefit and cost priority vectors for the alternatives is now

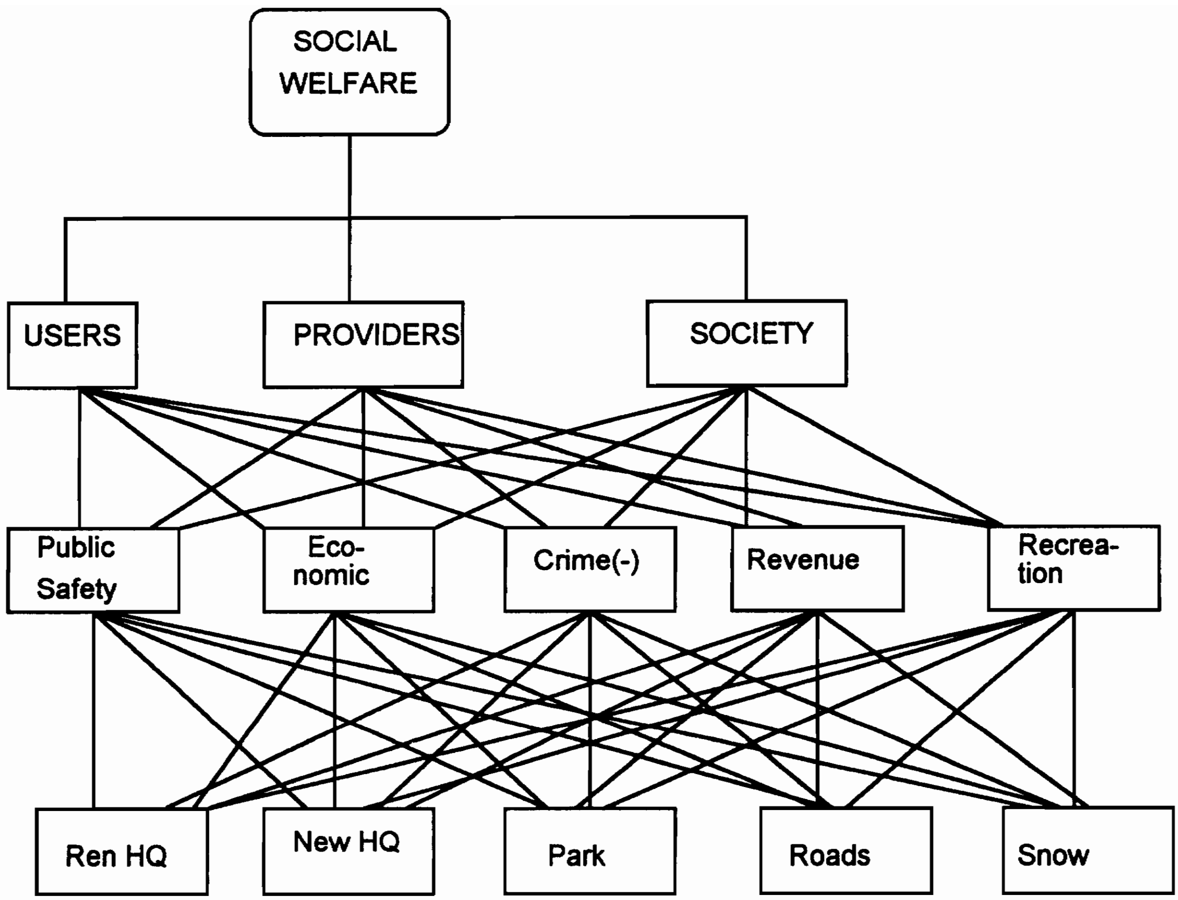


Figure 4.2 - Benefit Hierarchy

Table 4.3

Attribute Comparison Weights

Compared to	Positive	Negative
User (0.655)	Public Safety (0.333)	LCC (0.120)
	Economic (0.098)	Environment (0.316)
	Crime(-) (0.174)	Crime(+) (0.185)
	Revenues (0.029)	Accessibility (0.035)
	Recreation (0.020)	
Provider (0.250)	Public Safety (0.127)	LCC (0.123)
	Economic (0.038)	Environment (0.068)
	Crime(-) (0.067)	Crime(+) (0.049)
	Revenues (0.011)	Accessibility (0.010)
	Recreation (0.008)	
Society (0.095)	Public Safety (0.047)	LCC (0.012)
	Economic (0.016)	Environment (0.050)
	Crime(-) (0.026)	Crime(+) (0.027)
	Revenues (0.004)	Accessibility (0.006)
	Recreation (0.003)	

calculated. They are as follows:

	Benefit	Cost
Renovate HQ	0.252	0.450
New HQ	0.398	0.196
Park	0.167	0.113
Roads	0.098	0.118
Snow	0.085	0.123

This concludes the last step of the Project Assessment Branch. The projects are evaluated individually in the Project Assessment Branch, and as a group, using the concepts of capital budgeting, in the Portfolio Feasibility Branch. The next Branch, the Portfolio Assessment Analysis amalgamates the results from the two Branches. Complete and detailed calculations demonstrating how the above results were obtained are provided in Appendix A.

4.4 Portfolio Assessment Analysis

The first step in the Portfolio Assessment Branch begins the process of combining the capital budgeting algorithm with the AHP. The Portfolio CBR is now determined. This is done by taking the feasible portfolios from Table 4.2 and combining them with the priority vectors for the alternatives. This produces the Portfolio CBRs seen in Table 4.4. Equation 3-1 is used to calculate the Portfolio CBRs given in Table 4.4. The calculation of Portfolio # 9's CBR is provided as an example:

$$\text{Portfolio CBR} = \frac{\sum (b_j)(\text{project}_j)}{\sum (c_j)(\text{project}_j)}$$

$$= \frac{(1)(0.252) + (0)(0.398) + (1)(0.167) + (1)(0.098) + (1)(0.085)}{(1)(0.450) + (0)(0.196) + (1)(0.113) + (1)(0.118) + (1)(0.125)}$$

$$= 0.602/0.804 = 0.749$$

The next step in the process is to rank order the portfolios. Portfolios are ranked in ascending order based on the cost vector. This is the same concept as ranking them based upon the initial investment found in Engineering Economy texts. Before the incremental analysis continues, the Cut-Off Ratio is computed. The Cut-Off Ratio is explained in Section 3.4.1. It represents the CBR that would be achieved from all the projects simultaneously. The Cut-Off Ratio is (values taken from Table 4.4):

Cut-Off Ratio = 3.804/4.092 = 0.930

The next step is conducting the incremental analysis. This is illustrated in Table 4.5, where the portfolios are rank ordered on the basis of their cost vector. If the incremental CBR (Δ CBR) is not greater than the Cut-Off Ratio of 0.93, the incremental investment is not justified; if it is greater, than the investment is justified. The 0 in Table 4.5 represents the do-nothing alternative which, although not usually considered a viable alternative for the public sector situation, is the correct place to begin the analysis.

The result of the analysis illustrated in Table 4.5 is that the analyst first selects Portfolio #23. This is the first portfolio to have a CBR greater than 0.93. Then Portfolio #17 is selected. The incremental investment required for Portfolio #17 is much greater than 0.93. The incremental analysis continues until all the feasible portfolios are evaluated. Finally, Portfolio #17 is selected as the best alternative.

Table 4.4

Portfolio CBR

Portfolio #	Σ Benefits	Σ Costs	CBR
9	0.602	0.804	0.749
13	0.435	0.691	0.629
15	0.337	0.573	0.588
17	0.748	0.55	1.360
21	0.581	0.437	1.330
23	0.483	0.319	1.514
25	0.350	0.354	0.989
29	0.183	0.241	0.759
31	0.085	0.123	0.691
	Σ 3.804	Σ 4.092	

Table 4.5

Incremental Analysis

Increment	Δ Benefits	Δ Costs	Δ CBR	Justified ?
0 → 31	0.085	0.123	0.691	No
0 → 15	0.337	0.573	0.588	No
0 → 29	0.183	0.241	0.759	No
0 → 23	0.483	0.319	1.514	Yes
23 → 25	- 0.133	0.035	Neg	No
23 → 13	- 0.048	0.372	Neg	No
23 → 21	0.098	0.118	0.831	No
23 → 9	0.119	0.485	0.245	No
23 → 17	0.265	0.231	1.147	Yes

4.5 Alternative Treatments

The purpose of this Section is to assess any changes in the solution if two different things are done:

(1) use quantitative data for applicable attributes, and

(2) have the attributes which are not relevant to a particular alternative not be assessed for that alternative.

Only modifications to the Project Assessment Branch of the solution process are proposed. Both of these modifications are evaluated. The first modification involves using the quantitative data that was developed in Table 4.1. It is presented in Sub-Section 4.5.1. Sub-Section 4.5.2 outlines a means by which the hierarchy can be revised, such that not all attributes are common to all alternatives.

4.5.1 Use of Quantitative Data

The use of quantitative data directly to compute the ratios is an accepted practice in using the AHP. Automan [31] has the ability to let the user enter the data and then the program calculates the ratio comparison. Continuing with the hypothetical example, the only quantitative data available for the problem outlined are life-cycle cost and life-cycle revenues. This data is shown in Table 4.1. The FW values of LCC and Revenue is used to directly compute the ratios for these attributes. The reader is reminded that in the original example the analyst evaluated the ratio comparisons for these attributes subjectively. A better perspective may be provided with a direct comparison of the different results for each attribute when quantitative data is used directly. Therefore, the changes to the comparisons can be seen as follows:

	A	B	C	D	E
Ratio (LCC)	0.48	0.343	0.049	0.049	0.078
Data (LCC)	0.359	0.337	0.088	0.088	0.128
% difference	25%	- 2%	80%	80%	- 64%
Ratio (Revenue)	0.05	0.05	0.640	0.210	0.05
Data (Revenue)	0	0	0.488	0.270	0.242
% difference	100%	100%	- 24%	29%	384%

The percent difference is calculated using the ratio measurements as the baseline. It is obvious from the above that there are significant differences in the weights assigned. The net result of this discussion is that the analyst should, whenever possible, use quantitative data directly to make his comparisons. Of course the problem being addressed is one that does not utilize only quantitative data, therefore some direct comparisons will be required.

Rather than recreate the many tables that are required to carefully illustrate the original example, two tables are combined to show the new weighted rankings and one other table is used to show the new incremental comparisons. When the first part of Table 4.6 is compared with Table 4.3 there is very little difference. From Table 4.6 the new Cut-Off Ratio is calculated as follows:

Cut-Off Ratio = 3.87/4.2 = 0.920

Also, the new Cut-Off Ratio is only slightly different. Next, the incremental comparisons are redone. This is seen in Table 4.7. The analysis still results in Portfolio #17 being selected. This is not surprising given the fact that only one attribute in each

Table 4.6

Alternative Weights with Data and Portfolio CBR

Project	Benefit	Cost
A	0.250	0.420
B	0.395	0.195
C	0.161	0.123
D	0.101	0.127
E	0.094	0.136

Portfolio	Σ Benefit	Σ Cost	CBR
9	0.606	0.806	0.752
13	0.445	0.683	0.652
15	0.344	0.556	0.619
17	0.751	0.581	1.293
21	0.590	0.458	1.288
23	0.489	0.331	1.477
25	0.356	0.386	0.922
29	0.195	0.263	0.741
31	0.094	0.136	0.691
	Σ 3.87	Σ 4.2	

Table 4.7

Incremental Comparisons with Data

Increment	Δ Benefits	Δ Costs	Δ CBR	Justified?
0 → 31	0.094	0.136	0.691	No
0 → 15	0.344	0.556	0.619	No
0 → 29	0.195	0.263	0.741	No
0 → 23	0.489	0.331	1.477	Yes
23 → 25	- 0.133	0.055	Neg	No
23 → 13	- 0.044	0.352	Neg	No
23 → 21	0.101	0.127	0.795	No
23 → 9	0.117	0.475	0.246	No
23 → 17	0.262	0.250	1.048	Yes

hierarchy had quantitative data. One of these attributes, revenues, is not weighted very significantly.

4.5.2 Revising the Hierarchies

As discussed in Chapter 2 the data entry or performance data aspect of Automan [31] is used to give an attribute, for a selected alternative, an effective weight of zero. This performance data option is used to modify the hierarchy. This addresses the problem where the alternative does not possess the same attributes. The new matrices are seen in Figure 4.3, the Revised Benefit Hierarchy and Figure 4.4, the Revised Cost Hierarchy. These figures only show the lines for attributes that do <u>not</u> go to every alternative.

It has been decided that the purchase of snow removal equipment will have no effect on crime so this will be changed in both hierarchies. In addition, the renovating or building of the police HQs has no effect on recreation and these generate no revenues. The hierarchies are revised to reflect these changes in relations.

The revision to the hierarchy is accomplished by using the performance data option of Automan [31] to drive the value for the relevant attributes to zero. The analyst merely takes the values from his original analysis, multiplies them by some factor (1000 is used for this example) and reduces the desired attribute to one. For example, in the original analysis, the attribute crime increase has the ratio weights as follows for the alternatives: Renovate HQ = 0.502, New HQ = 0.165, Park = 0.255, Roads = 0.042, and Snow = 0.035. (The calculation of these weights can be seen in Appendix A.) Using the performance data option, as discussed, the analyst enters the following values: Renovate HQ - 502,000, New HQ - 165,000, Park - 255,000, Roads - 42,000, and Snow - 1. This generates new ratio weights as follows: Renovate HQ = 0.521, New HQ = 0.171, Park = 0.265, Roads = 0.044, and Snow = 0. The same technique is used to drive the attribute

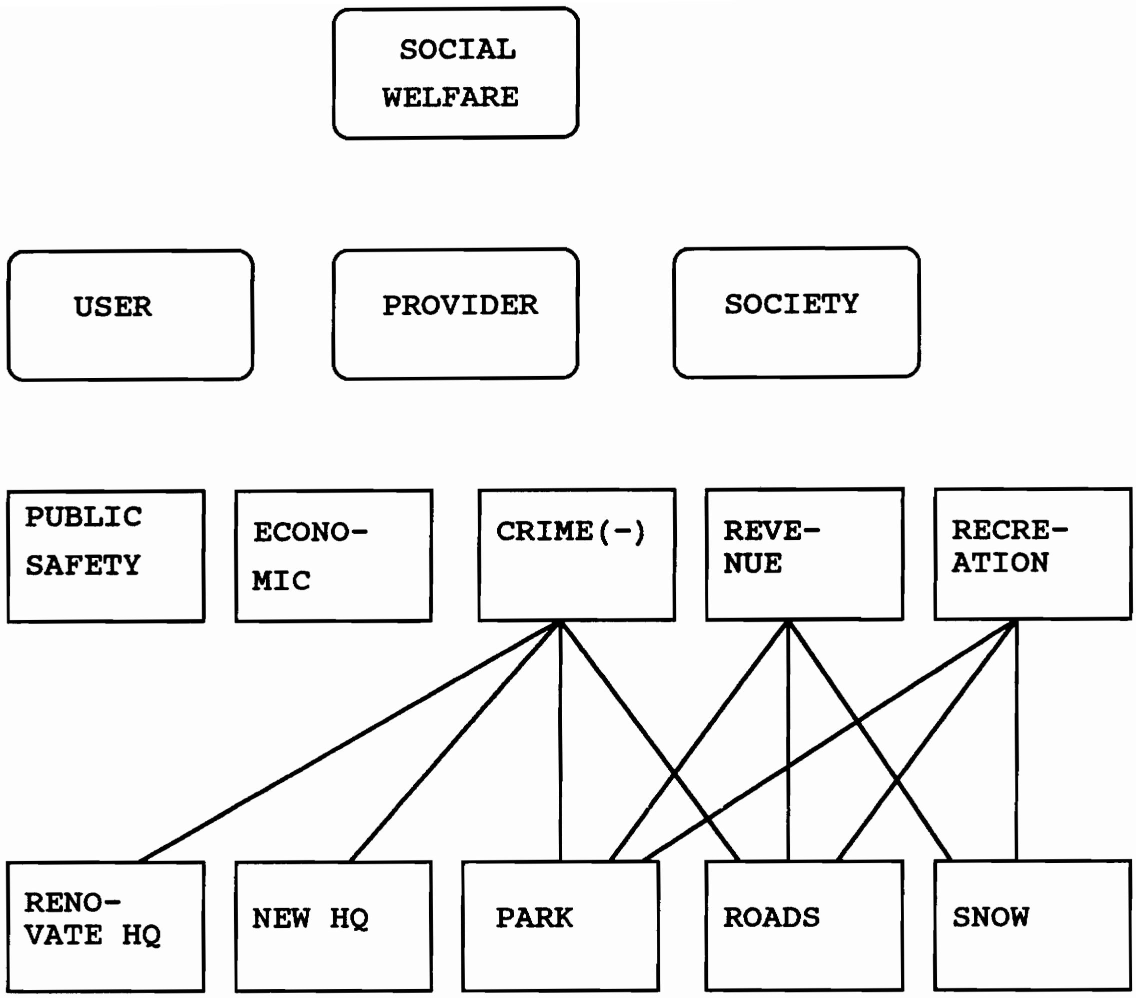


Figure 4.3 - Revised Benefit Hierarchy

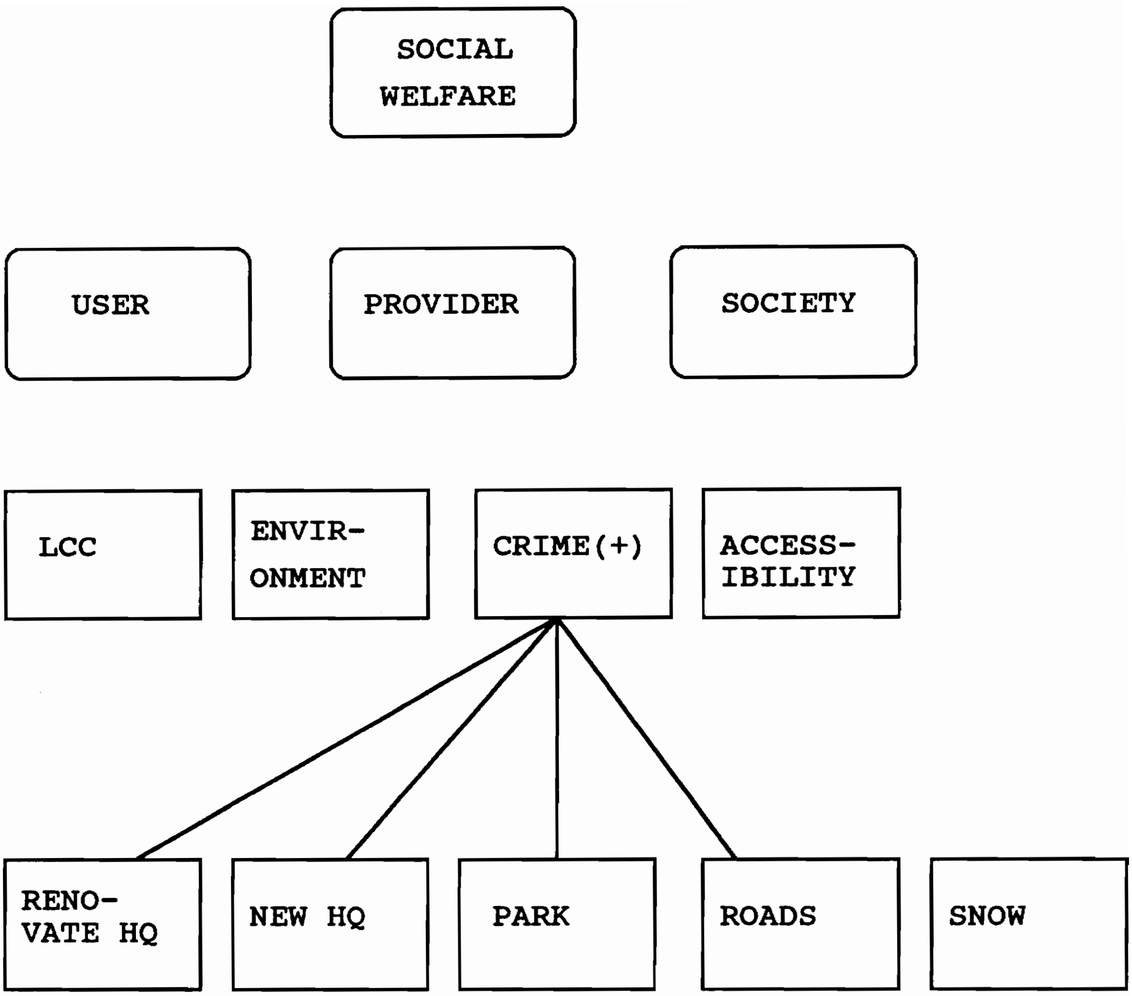


Figure 4.4 - Revised Cost Hierarchy

weight to zero for the alternatives and attributes represented in Figures 4.3 and 4.4.

The results from this final, revised analysis are contained in Table 4.8 - Alternative Weights with Revised Hierarchies and Portfolio CBR and Table 4.9 - Incremental Comparisons with Revised Hierarchies. From Table 4.8 the new Cut-Off Ratio is calculated as follows:

$$\text{Cut-Off Ratio} = 3.831 / 4.152 = 0.923$$

The final result of these different treatments is that the same portfolio is selected as the original analysis.

It has been illustrated that the effect of using quantitative data improves the comparison analysis and the ranking. The use of the performance data option to revise the hierarchies is effective, as it gives the selected alternatives a weight of zero, as desired. However, this did not change the decision output. The reason the attributes did not affect the decision output is because the selected attributes are not significantly weighted. The changing of the hierarchies to reflect differences in the attributes for the alternatives results in only the lower weighted projects changing their rank order. Project A and B possess the significant portion of the weighting for the benefit and negative hierarchies. Their combined weight washes out any effect on the portfolio CBR of the other three projects.

Table 4.8

Alternative Weights with Revised Hierarchies

and Portfolio CBR

Project	Benefit	Cost
A	0.251	0.196
B	0.398	0.425
C	0.163	0.126
D	0.102	0.128
E	0.087	0.127

Portfolio	Σ Benefit	Σ Cost	CBR
9	0.603	0.806	0.748
13	0.440	0.680	0.647
15	0.338	0.552	0.612
17	0.750	0.577	1.300
21	0.587	0.451	1.301
23	0.485	0.323	1.500
25	0.352	0.381	0.924
29	0.189	0.255	0.741
31	0.087	0.127	0.685
	Σ 3.831	Σ 4.152	

Table 4.9

Incremental Comparisons with Revised Hierarchies

Increment	Δ Benefits	Δ Cost	Δ CBR	Justified?
0 → 31	0.087	0.127	0.685	No
0 → 15	0.338	0.552	0.612	No
0 → 29	0.189	0.255	0.741	No
0 → 23	0.485	0.323	1.500	Yes
23 → 25	- 0.133	0.058	Neg	No
23 →13	- 0.045	0.357	Neg	No
23 → 21	0.102	0.128	0.797	No
23 → 9	0.118	0.483	0.244	No
23 → 17	0.265	0.254	1.043	Yes

V. SUMMARY AND CONCLUSION

5.1 Summary

A brief summary of the thesis is provided. The stated purpose of this research is:

(1) to demonstrate that the private sector and public sector capital allocation processes are significantly different, and

(2) to present a methodology for the capital allocation process for the public sector.

The differences in the public and private sector have been noted and a methodology for the capital allocation process has been developed.

Three basic hypotheses were also put forward. As this thesis does not involve a field study, the validation of these hypotheses can only be determined in a qualitative sense. The first hypothesis is that the dynamics of the public sector are different from that of the private sector. It is believed that it has been demonstrated fairly thoroughly that the public and private sectors have notable differences. The second hypothesis is that the utilization of purely private sector tools does not take into account these differences. The utilization of private sector models examined in this research, Skipper [1] and Kladivko [2], do not address these differences. The last hypothesis states that the process could best be addressed by a combination of capital budgeting, Engineering Economy principles and the techniques of multi-attribute decision models. The public sector differences were addressed, using elements of a capital budgeting process, Engineering Economy and a multi-attribute decision model (the AHP).

A review of the relevant literature was also conducted. It included a review of capital budgeting, a detailed review of the AHP including a discussion of its benefits and liabilities, and a brief discussion on the differences between Utility Theory and the AHP. The basic characteristics of the public sector were outlined and the differences between the public and private sectors were enumerated. The methodology developed is based upon the literature review and the stated objectives of the research.

A hypothetical example outlining the solution procedure was presented and some alternative treatments were considered. In the hypothetical example, the improvement to the process that can be achieved through the use of quantitative data was demonstrated. This example also demonstrates the consistency of the process. Despite making changes to four attributes, a decision reversal did not occur. This was explained by the weight assigned to the first two projects, which had over 60% of the weight for benefits and costs. Although the example did not dramatically show the effectiveness of using quantitative data, the changes to the attribute weights, which was outlined for life-cycle costs and life-cycle revenues, should be sufficient to motivate a user to use quantitative data, as much as possible. The example also demonstrates that it is not necessary to use hundreds of attributes as has been encountered in other public project evaluations. Although the effectiveness of the methodology can be improved by the use of data; it was designed to consider more than hard facts. There exist many other means whereby projects possessing only quantitative data can be analyzed.

5.2 Validity

Validity is concerned with the soundness or effectiveness of an instrument. Does the instrument or process measure what it is supposed to measure? Does it measure it well? This is a real concern in a study that gathers data and attempts to analyze the results.

This research is an amalgamation of two proven concepts yielding a process that is better than could be achieved by either alone. It is believed that this is true because one algorithm did not address qualitative elements and the other, if employed on its own, would not address all the constraints encountered in the first.

The methodology developed possesses strong face validity. It combines the work from capital budgeting and AHP using Engineering Economy principles. The algorithm possesses face validity because there is little doubt that it measures what it is supposed to measure. This is because the goal of the developed methodology contains the key elements put forward by each individual process. The question is one of effectiveness; in other words does it effectively measure what it is supposed to measure? This question can not be answered at this time.

The algorithm also possesses convergent validity. This is defined as the extent to which another, similar model provides equivalent results. Given that the methodology has combined the models of two theories which are already oriented to the same goal it is not difficult to see that it has convergent validity.

The algorithm put forward in this thesis is designed to capture the intangible qualities or non-quantified aspects of public projects and to better address the dynamics of the public sector. Simple devices, such as separately evaluating capital budgeting considerations from individual project considerations, and then combining the results ensures that the dynamics of the public sector are addressed.

It becomes difficult to fully assess the validity of the methodology without field trials. The methodology is the topic of a knowledge-based expert system project. A computer based tool will hopefully be developed, which will enable trials to be conducted in the future. At the present time the methodology, although coherent conceptually, does not have a single computer code which would enable a user to evaluate public projects as proposed. The hypothetical example and other calculations were done using three different programs [2] [31] [39]. This is, of course, unacceptable from a user viewpoint. Users generally want the entire package to be one program.

5.3 Areas for Further Research

5.3.1 Uncertainty and Risk

The consideration of risk and uncertainty is not well addressed in this methodology. Uncertainty and risk are connected, yet different entities. Uncertainty applies to situations where good probability information does not exist. Risk is applied to situations which, although not known with certainty, does have estimated probabilities.

For the analysis of money flows over time (life-cycle costs, revenues) there are accepted techniques, such as risk adjusted discount rates, which could be applied. However, there does not exist, at this time, a means that would enable the analyst to consider the risk associated with the pairwise comparisons made in the methodology. This is a serious gap in the methodology. Any development of a program for this methodology should have a method to explicitly consider risk and uncertainty. One possible method, which may be useful, is the use of three estimates. The user would be prompted to perform comparisons for the projects when they all performed as expected, least favorably, and most favorably. These estimates are useful in that they reveal consequences of having a better, or worse, situation result. The gap between the least favorable and the most likely is of considerable interest to the user.

The traditional method to overcome the deficiency of not explicitly considering risk and uncertainty, is to use sensitivity analysis. The sensitivity analysis would be accomplished through an examination of the weight given to the attributes. As was seen in the hypothetical example, the weight assigned to attributes higher in the hierarchy may be more important than the weight of that attribute with respect to the alternatives. Therefore, in the event that a program is designed to accomplish the proposed methodology, it should contain a means whereby sensitivity analysis can be done. The sensitivity analysis would have to be restricted to any one level in the hierarchy at a time. This would avoid having the user confound his sensitivity analysis.

5.3.2 Further Areas to Consider

The areas for further research are many and varied. These areas where further research may be applied are structured according to the different bodies of knowledge they represent.

In the area of capital budgeting this algorithm does not examine the use of continuous functions. It is possible that an analyst may find it more beneficial to use parametric functions as opposed discrete functions. Also, although the algorithm can capture the dynamics of the public sector, through its re-application at each successive cycle; it does not intrinsically capture the dynamics. A method to capture the three or five year planning cycle with inclusion of methods to assess the risk associated with these estimates would be most useful.

The inability of the methodology, in its present format, to adequately address the conflict between the short term budget constraints and life-cycle costs is another area for further research. Budget restrictions at this level of government are very real and can not be exceeded; therefore, budget becomes a constraint, just like mutual exclusivity, contingency, and political constraints. A step in the right direction would be the development of the methodology to consider more than one budget as discussed above. If the algorithm is modified to include more of the planning cycle than it may better enable a public agency to make decisions which more accurately reflect the life cycle of the projects with which they are dealing.

The AHP body of knowledge affords even more areas to examine. Does the questioning procedure, as has been proposed, provide any greater accuracy with respect to ratio comparisons? How can the difficulty of attributes not being common to all the alternatives be better resolved? This was addressed in the thesis, but it was resolved through a trick of computer programming. The fundamental question that must be answered is; can a hierarchy be established with attributes not common to each alternative? A preliminary examination of this problem would indicate that it is possible. The calculation of values when they are brought up through the hierarchy still results in a normalized priority vector. However, this work must be developed in a much more rigid mathematical style, considering the assumptions and axioms of the AHP.

Also, the concept of using a discount factor or other appropriate means to provide a more quantitative assessment of the time value of benefits and costs which are not monetary in nature is required. The discount factor may be based upon the social discount rate or a series of benefit and cost geometric flows could be developed based upon the type of project and its life. The addition and development of such discount factors would be novel and it would greatly increase the effectiveness of the methodology.

Another area which may be appropriate for further research is the use of AHP priority vectors to establish Cost/Benefit Ratios. The algorithm design of having a positive and negative hierarchy is useful but it may result in some attributes being "double counted". A more concise method of ensuring that the analyst only considers the characteristics of an attribute once should be devised.

Further research may also be done on the topic of the Cut-Off Ratio. This concept has been extrapolated in the thesis from other works and its result contravenes the typical result required by traditional Engineering Economy thinking. The traditional view is that a project with a cost-benefit ratio of less than one is not economical to undertake.

Another area for further research is that of the algorithm development. The algorithm as depicted in Figure 3.1 is not finalized. For example, in the Portfolio Feasibility Branch, when the parameters are input there is no need to input the discount rate. This Branch does not address the life-cycle costs. Even if the model is refined to include multiple budgets the discount rate would not be used. The Project Assessment Branch requires a box to be inserted after the "Identify Attributes" box. This box would be, "Attribute Characteristic Evaluation". Here the discount rate would be used to assess life-cycle costs. Any other characteristics which would assist in the evaluation of the attributes could be done at this point. For example, this may be where the analyst considers the rate at which intangible criteria are discounted. This is where the analyst could consider any performance data that he felt was relevant.

Finally, the entire concept should be coded into one computer program. The amalgamation of the concepts into one coherent program would greatly improve the effectiveness and usefulness of the algorithm. This is being pursued in the area of knowledge-based expert systems (KBES) by the author. Given the successful completion of the computer code in a KBES environment, or other language, it is recommended that this tool be applied in a small town or county that employs capital budgeting. If the town is presently using capital budgeting techniques it will be easier to apply the tool.

5.4 Conclusion

It is believed that this thesis has accomplished its stated purpose. The differences between the public sector and private sector have been enumerated. A methodology has been developed which addresses these differences through the combination of techniques from three different fields of knowledge. In addition, the methodology devised captures the consideration of more than the direct beneficiary in its analysis. Although the methodology can be better refined, and a computer code needs to be developed, the basic process and concepts will remain the same. The capital allocation process for public projects is a different approach which successfully addresses the needs of the public sector.

APPENDIX A

Project Assessment Calculations

The Project Assessment Branch of the algorithm involves the application of the AHP. The AHP solution methodology is presented in Section 2.5.2. The assessment procedure involves applying the AHP twice; once for the benefit hierarchy and once for the cost hierarchy. Only the cost hierarchy is evaluated in this Appendix. The process is as follows:

(1) establish a goal and set up hierarchical structure with attributes and sub-attributes,

(2) determine relative importance of attributes and sub-attributes through pairwise comparisons,

(3) determine relative weight of alternatives with respect to sub-attributes using normalized eigen-vector method,

[a] Sum the columns from the preference matrix

[b] Normalize the matrix

[c] Divide row sum by number of row elements to obtain priority vector.

(4) verify consistency of pairwise comparisons using the consistency ratio (CR) method, and

[a] multiply priority vector by comparison matrix

[b] divide resulting vector by priority vector

[c] Calculate λ_{max} by summing components of step 2 and dividing by number of elements

[d] Calculate CI using Equation 2-4

[e] Calculate CR using Equation 2-5

(5) determine the overall relative weights of each alternative by aggregating their totals through the hierarchy.

The first comparison matrix determines the relative importance between users, providers, and society. The comparison matrix is as follows:

	User	Provider	Society
User	1	3	6
Provider	1/3	1	3
Society	1/6	1/3	1
	Σ 1.500	Σ 4.333	Σ 10

The normalized matrix with row sums and priority vector calculated is as follows:

	User	Provider	Society	Row Sums	Priority vector
User	0.667	0.692	0.600	1.959	0.655
Provider	0.222	0.231	0.300	0.753	0.250
Society	0.111	0.077	0.100	0.288	0.095

Now the analyst performs a consistency check, as outlined in step #4, to verify his comparisons. First multiply the comparison matrix by the priority vector.

$$
\text{Comparison Matrix} \quad \text{Priority Vector} \quad [C]
$$

$$
\begin{bmatrix} 1.000 & 3.000 & 6.000 \\ 0.333 & 1.000 & 3.000 \\ 0.167 & 0.333 & 1.000 \end{bmatrix} \times \begin{bmatrix} 0.655 \\ 0.250 \\ 0.095 \end{bmatrix} = \begin{bmatrix} 1.975 \\ 0.753 \\ 0.288 \end{bmatrix}
$$

Divide vector [C] by priority vector:

[D] = [1.975/0.655 0.753/0.250 0.288/0.095] = [3.015 3.012 3.032]

λ_{max} = (3.015 + 3.012 + 3.032)/3 = 3.020

CI = λ_{max} - N/N-1 = 0.01

CR = CI/RI = 0.01/0.58 = 0.017 (see Section 2.5.2 for RI values)

As the CR is less than 0.1 the comparison matrix is accepted.

The next step is to evaluate the attributes. This is done for the attributes considering each user, provider, and society. The comparison matrix for the user is detailed below, as follows:

	LCC	Environment	Crime(+)	Access
LCC	1.000	0.333	0.500	5.000
Environment	3.000	1.000	2.000	7.000
Crime(+)	2.000	0.500	1.000	5.000
Access	0.200	0.143	0.200	1.000
	Σ 6.2	Σ 1.976	Σ 3.700	Σ 18.000

The normalized matrix with row sums and priority vector:

	LCC	Environment	Crime(+)	Access	Row Sums	Priority Vector
LCC	0.161	0.168	0.135	0.278	0.742	0.186
Environment	0.484	0.506	0.541	0.389	1.920	0.480
Crime(+)	0.323	0.254	0.270	0.278	1.125	0.281
Access	0.032	0.072	0.054	0.055	0.213	0.053

Calculate the CR (multiply comparison matrix by priority vector, yielding vector [C]):

	LCC	Environment	Crime(+)	Access	Priority Vector	[C]
LCC	1.000	0.333	0.500	5.000	0.186	0.751
Environment	3.000	1.000	2.000	7.000	0.480	1.971
Crime(+)	2.000	0.500	1.000	5.000	0.281	1.158
Access	0.200	0.143	0.200	1.000	0.053	0.215

Divide vector [C] by priority vector.

[D] = [0.751/0.186 1.971/0.480 1.158/0.281 0.215/0.053]

= [4.038 4.106 4.121 4.057]

λ_{max} = (4.038 + 4.106 + 4.121 + 4.057)/4 = 4.0805

CI = λ_{max} - N/N-1 = 0.027

CR = CI/RI = 0.027/0.9 = 0.03

The comparison matrix is acceptable.

The provider attribute and society attribute comparison matrices and their resultant priority vectors are provided below. Differences in the comparison matrices have resulted in different priority vectors. Each matrix has a CR less than 0.10.

Provider - Attribute Matrix.

	LCC	Environment	Crime(+)	Access	Priority Vector
LCC	1.000	2.000	3.000	9.000	0.492
Environment	0.500	1.000	1.500	7.000	0.272
Crime(+)	0.333	0.667	1.000	6.000	0.195
Access	0.111	0.143	0.167	1.000	0.041

Society - Attribute Matrix

	LCC	Environment	Crime(+)	Access	Priority Vector
LCC	1.000	0.200	0.333	3.000	0.124
Environment	5.000	1.000	2.500	5.000	0.525
Crime(+)	3.000	0.400	1.000	5.000	0.286
Access	0.333	0.200	0.200	1.000	0.064

The next step in the analysis is the computation of the priority vectors for the alternatives with respect to each attribute. The comparison matrices and their resulting priority vectors are provided below. The reader is invited to follow the procedure outlined in this Appendix to verify the results. (The reader is reminded that if X is preferred over Y by a factor of 4 then the position below the diagonal for Y preference over X is 1/4.)

Comparison matrix for alternatives considering LCC.

	Ren HQ	New HQ	Park	Roads	Snow	Priority Vector
Ren HQ	1.000	2.000	8.000	8.000	6.000	0.480
New HQ		1.000	7.500	7.500	5.250	0.343
Park			1.000	1.000	0.571	0.049
Roads				1.000	0.571	0.049
Snow					1.000	0.078

Comparison matrix for alternatives considering Environment.

	Ren HQ	New HQ	Park	Roads	Snow	Priority Vector
Ren HQ	1.000	3.000	4.000	2.000	2.000	0.378
New HQ		1.000	2.000	0.571	0.571	0.128
Park			1.000	0.333	0.333	0.074
Roads				1.000	1.000	0.210
Snow					1.000	0.210

Comparison matrix for alternatives considering Crime(+).

	Ren HQ	New HQ	Park	Roads	Snow	Priority Vector
Ren HQ	1.000	5.000	3.000	7.000	9.000	0.502
New HQ		1.000	0.333	6.000	8.000	0.165
Park			1.000	5.000	7.000	0.255
Roads				1.000	1.000	0.042
Snow					1.000	0.035

Comparison matrix for alternatives considering Access.

	Ren HQ	New HQ	Park	Roads	Snow	Priority Vector
Ren HQ	1.000	7.000	9.000	9.000	9.000	0.646
New HQ		1.000	5.000	5.000	5.000	0.200
Park			1.000	0.500	0.500	0.039
Roads				1.000	1.000	0.057
Snow					1.000	0.057

As the alternatives are re-evaluated for each attribute it is decided to use the comparison matrices shown above. Now the analyst begins the process of collapsing the matrices to obtain the alternative weights. The priority vectors from the alternatives are compiled in the matrix and multiplied by the user, provider, and society priority vectors. This generates the user, provider, and society alternative vectors.

1. Generation of user-alternative vector.

	LCC	Environ	Crime(+)	Access	User Attribute	User Alternative
Ren HQ	0.480	0.378	0.502	0.646	0.186	0.446
New HQ	0.343	0.128	0.165	0.200	0.480	0.182
Park	0.049	0.074	0.255	0.039	0.281	0.118
Roads	0.049	0.210	0.042	0.057	0.053	0.125
Snow	0.078	0.210	0.035	0.057		0.128

2. Generation of provider-alternative vector.

	LCC	Environ	Crime(+)	Access	Provider Attribute	Provider Alternative
Ren HQ	0.480	0.378	0.502	0.646	0.124	0.443
New HQ	0.343	0.128	0.165	0.200	0.525	0.170
Park	0.049	0.074	0.255	0.039	0.286	0.120
Roads	0.049	0.210	0.042	0.057	0.064	0.132
Snow	0.078	0.210	0.035	0.057		0.134

3. Generation of society-alternative vector

	LCC	Environ	Crime(+)	Access	Society Attribute	Society Alternative
Ren HQ	0.480	0.378	0.502	0.646	0.492	0.463
New HQ	0.343	0.128	0.165	0.200	0.272	0.244
Park	0.049	0.074	0.255	0.039	0.195	0.096
Roads	0.049	0.210	0.042	0.057	0.041	0.092
Snow	0.078	0.210	0.035	0.057		0.105

Now that these vectors have been obtained the last step in the process can be accomplished. The analyst can resolve the last level of the hierarchy and obtain the weighted evaluation for the alternatives.

	User	Provider	Society	Attribute Weight	Weighted Alternatives
Ren HQ	0.446	0.443	0.463	0.655	0.447
New HQ	0.182	0.170	0.244	0.250	0.186
Park	0.118	0.120	0.096	0.095	0.116
Roads	0.125	0.131	0.082		0.123
Snow	0.128	0.134	0.105		0.127

As this example was calculated using a hand calculator there is some difference in the final weighted evaluation obtained using the software, Automan [31]. The software generated solution of [0.450 0.196 0.113 0.118 0.123] is considered more accurate.

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He is happily married to Vivian, his high-school sweetheart. They have three sons, Ian, John, and Adam which is why Bill is an avid martial arts student. An active member in the Society of Logistics Engineers, Bill is the professional development officer for the Blue Ridge Chapter.